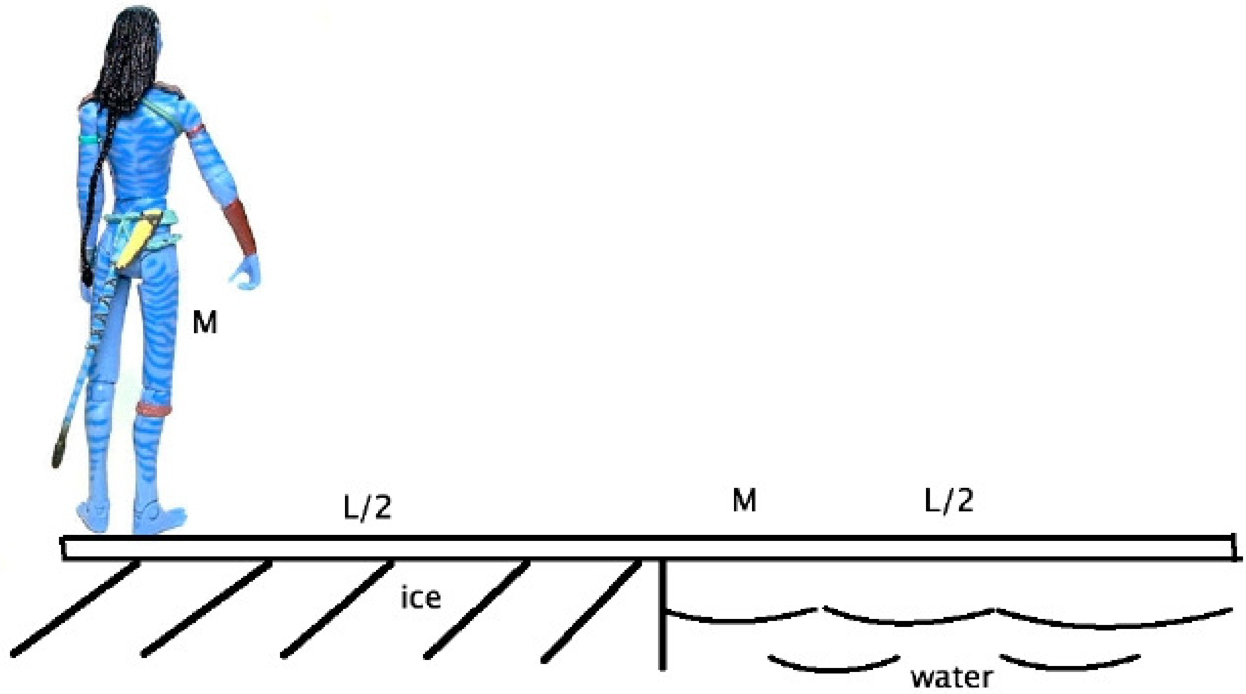
1

Neytiri is standing on the end of a long thing uniform board of length L. Half of the board including the end where Neytiri is standing rests on very slippery ice with the other half over open water as shown. Lucky for our calculations, the board and Neytiri have the same mass M.



a

Find the x coordinate, relative to the ice, of the overall CM of Neytiri and the board. Define $x = 0$ at her end, and $x = L$ at the other end.

✓ **Answer**

The weighted average will be:

$$\frac{0M + \frac{LM}{2}}{2M}$$

$$= \frac{L}{4}$$

$$= \frac{L}{4}$$

□

b

If Neytiri walks all the way to the other end, will she fall into the water? Give your reasoning.

✓ **Answer**

Neytiri's walking is only a frictional force that is constrained to the system, meaning the CM will not move as there are no external forces (no friction from ice)

When Neytiri is at the other end of the board, the board will be $\frac{L}{2}$ to her left, which means Neytiri will be $\frac{L}{4}$ to the right of the CM (which does not move).

$CM + \frac{L}{4} = \frac{L}{2}$

Neytiri will be at position $x = \frac{L}{2}$, which is at the edge of the water, so no, she will not fall in.

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☐

c

Suppose Neytiri moves at a speed v **relative to the ice**, while she is walking to the right as described in part (b) from her original end position. What is the CM velocity?

✓ **Answer**

It does not matter how fast neytiri is moving, because the force that makes Neytiri move is an internal force and thus the system as a whole will not move.

The CM's velocity is still 0.

d

How fast is the board moving relative to the ice and in what direction, if Neytiri is moving at speed v to the right?

✓ **Answer**

Since the CM does not move, and the board and Neytiri have the same mass, they must have opposite velocities by the law of momentum of systems.

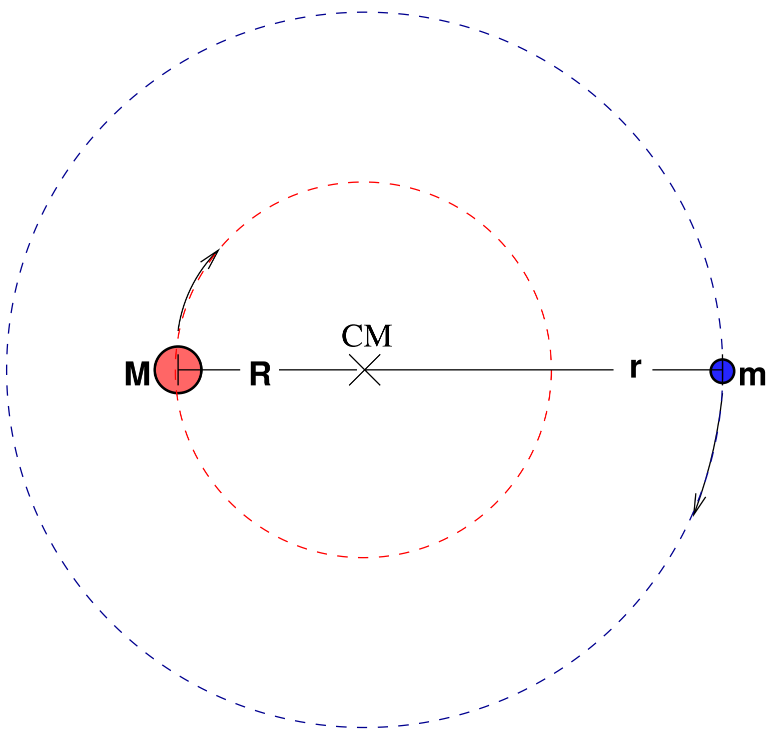
$p = 0$
$v_n = v$

$p_n = Mv$
$p_n + p_b = 0$
$p_b = -Mv$
$v_b = -v$

$v_b = -v$
$\square$

2

Consider two stars, each in its own *circular* orbit around a common CM point as shown below:



Supposed the masses of the two stars M and m are given. The radii of each of the two orbits is indicated by R and r. Assume that the radius r is given but the radius R is *not* given.

Calculate the magnitude of the linear velocity v_m of the star with mass m (on the right) in terms of the two given masses and the radius r. Explain your work.

✓ **Answer**

Let $\hat{R}$ be the direction of star S with length R and $\hat{r}$ be the direction of star s with length r
Let the CM be at position $\vec{0}$

$M\hat{R}+m\hat{r}=\vec{0}$ By calculation of center of mass by its composition

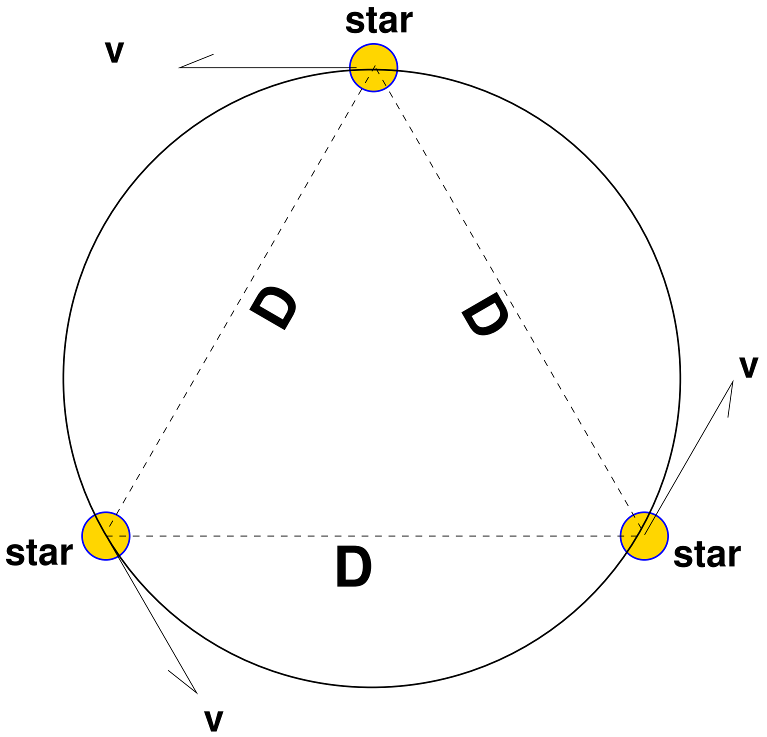
$\hat{R}=-\frac{m}{M}\hat{r}$

$R=-\frac{mr}{M}$

$R=-\frac{mr}{M}$

$\square$

3



Consider a very special arrangement of three stars place in orbit about each other as shown in the figure above. Each star has given mass m and each star is positioned on the three corners of an equilateral triangle so that the distance between any two stars is given as D

Consider a very special arrangement of three starts place in orbit about each other as shown in the figure above. Each star has given mass m and each star is positioned on the three corners of an equilateral triangle so that the distance between any two stars is given as D — the length of one side of the triangle. All three stars travel along a circular orbit of radius $=\frac{D}{\sqrt{3}}$ in accordance with geometry.

Calculate the *speed* v of any of the three stars in orbit. Give your answer in terms of the given parameters. Explain your work.

✓ **Answer**

$$F = G\frac{m_1 m_2}{r^2}$$

$$F_{12} = G\frac{m_1 m_2}{D^2}$$
$$F_{13} = G\frac{m_1 m_3}{D^2}$$
$$F = \frac{\sin\frac{2\pi}{3}}{\sin\frac{\pi}{6}} G\frac{m^2}{D^2}$$

$$a_c = \frac{\sqrt{3}v^2}{D}$$
$$F = \frac{\sqrt{3}v^2 m}{D}$$
$$\frac{\sin\frac{2\pi}{3}}{\sin\frac{\pi}{6}} G\frac{m}{\sqrt{3}D} = v^2$$
$$2\cos\frac{\pi}{6} G\frac{m}{\sqrt{3}D} = v^2$$
$$v = \sqrt{\frac{2mG\cos\frac{\pi}{6}}{D\sqrt{3}}}$$

$$v = \sqrt{\frac{2mG\cos\frac{\pi}{6}}{D\sqrt{3}}}$$

$\square$