1

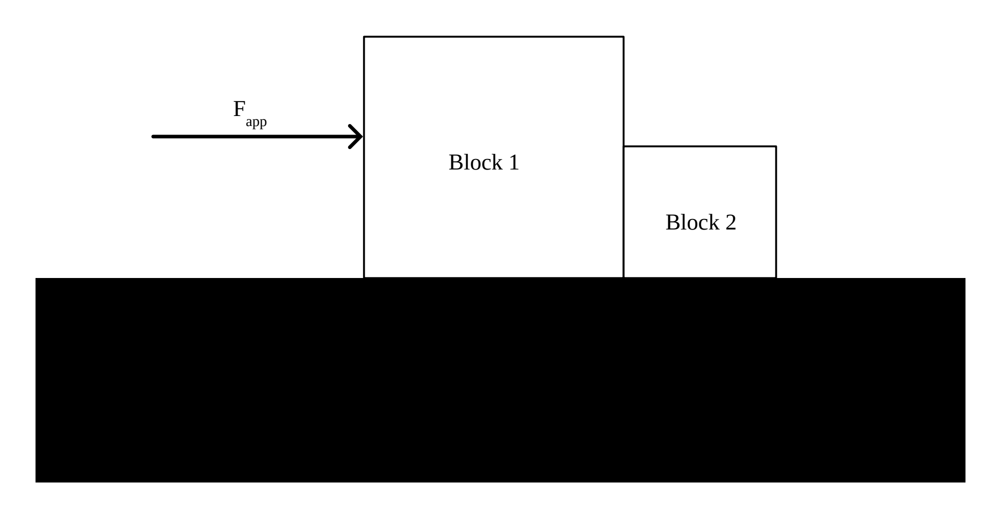
Two blocks, with given masses $m_1 = 14.29kg$ and $m_2 = 3.67kg$ sit on a frictionless surface. The blocks sit side-by-side in contact with each other. A given applied external horizontal force $F_{app} = 16.84N$ is applied to the side of the first block of mass m_1 which in turn pushes against the second block of mass m_2 . As a result, both blocks are accelerated.

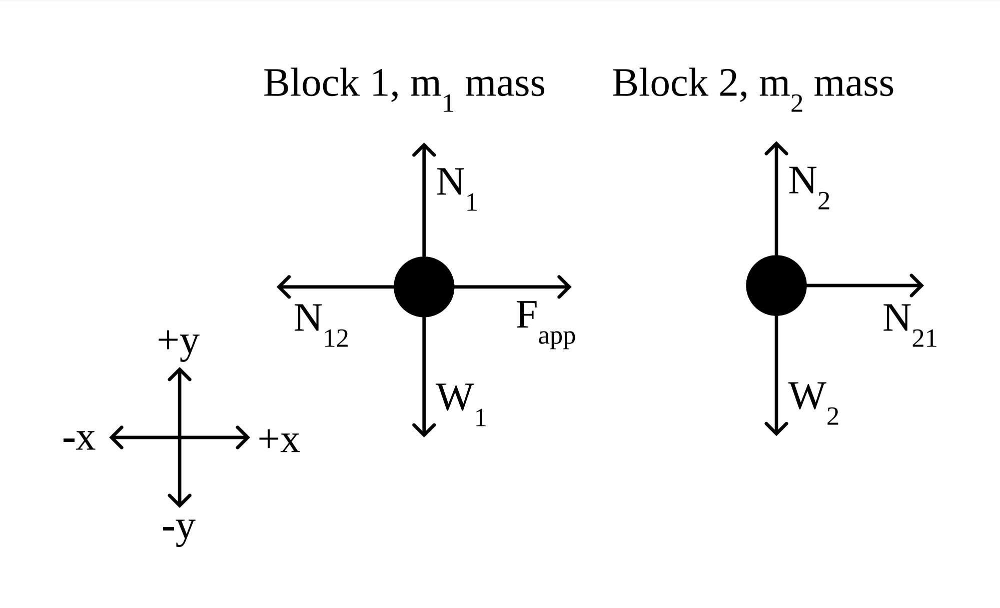
a

Draw a little sketch that indicates what is going on here. We need to see the two blocks in your sketch. We need to see how they are positioned relative to each other. We also need to see any coordinate system you plan to use. Also write down a list of *given parameters* for this problem (there are three of these). Also draw **Free Body Diagrams** for each of the two blocks indicating *all* of the relevant forces on each block. Be sure your forces are properly labeled: Use the "On-due" system. For example, one of your forces should be labeled like this: $\vec{N}_{21}$ which means "the Normal force *on* Block 2 *due* to Block 1.

✓ **Answer** ⌄

We are given:
$m_1 = 14.29kg$
$m_2 = 3.67kg$
$F_{app} = 16.84N$





b

What is the acceleration for the two blocks that results? You can safely assume that the two blocks move together (Yes, yes you can. This is called a *Kinematic Constraint*.) Explain your work.

✓ **Answer**

Given:
Two blocks move together $\implies \vec{a}_1 = \vec{a}_2 = \vec{a}, \vec{v}_1 = \vec{v}_2$
$m_1 = 14.29\ kg$
$m_2 = 3.67\ kg$
$F_{app} = 16.84\ N$

We know that both blocks don't move or accelerate up or down.
$\therefore F_{1\,net\,y} = F_{2\,net\,y} = 0$

$\vec{W}_1 = \langle 0, -m_1 g \rangle$
$\vec{N}_1 = \langle 0, m_1 g \rangle$
$\vec{W}_2 = \langle 0, -m_2 g \rangle$

$\vec{N}_2 = \langle 0, m_2 g \rangle$
$\vec{F}_{app} = \langle F_{app}, 0 \rangle$
$-\vec{N}_{12} = \vec{N}_{21} = \langle x, 0 \rangle$ Newton's third law

$\vec{F}_{1\,net} = \langle F_{app} - x, 0 \rangle$
$\vec{F}_{2\,net} = \langle x, 0 \rangle$

$\vec{F}_{1\,net} = m_1 \vec{a}$ Newton's second law
$\vec{F}_{2\,net} = m_2 \vec{a}$ Newton's second law
$\vec{F}_{1\,net} + \vec{F}_{2\,net} = (m_1 + m_2)\vec{a}$
$\vec{a} = \frac{\langle F_{app}, 0 \rangle}{m_1 + m_2}$

$\vec{a} = \frac{\langle 16.84, 0 \rangle}{14.29 + 3.67} \ \frac{m}{s^2}$
$\vec{a} = \langle 0.9376, 0 \rangle \ \frac{m}{s^2}$

$\vec{a} = \frac{\langle F_{app}, 0 \rangle}{m_1 + m_2}$
$\vec{a} = \langle 0.9376, 0 \rangle \ \frac{m}{s^2}$
$\square$

c

What is the magnitude of the Normal force exerted on Block 1 due to Block 2? Explain your work.

✓ **Answer**

From part (b):
$-\vec{N}_{12} = \vec{N}_{21} = \langle x, 0 \rangle$ Newton's third law
$\vec{F}_{2\,net} = \langle x, 0 \rangle$
$\vec{F}_{2\,net} = m_2 \vec{a}$ Newton's second law

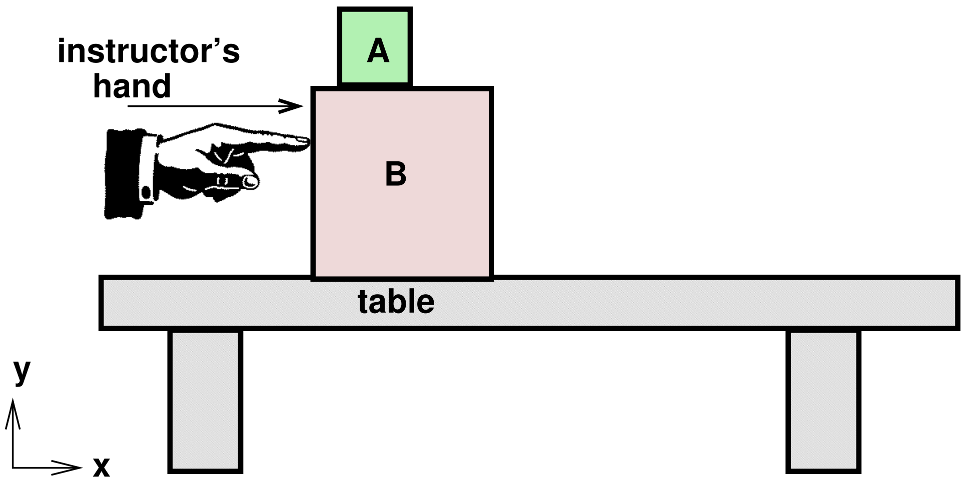
$-\vec{N}_{12} = m_2 \vec{a}$
$\vec{N}_{12} = -m_2 \vec{a}$

$\vec{N}_{12} = -(3.67\ kg)\langle 0.9376, 0 \rangle\ N$
$\vec{N}_{12} = \langle -3.44, 0 \rangle\ N$
$N_{12} = 3.44\ N$

$\vec{N}_{12} = -m_2\vec{a}$

$N_{12} = 3.44\ N$

2

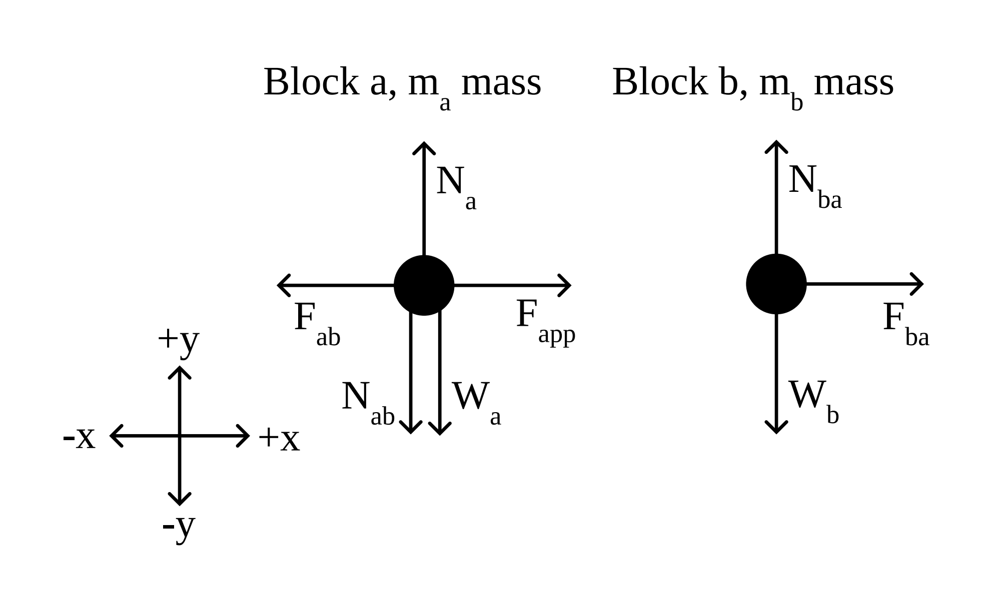


Two blocks are placed together on a table as shown. Block A has a given mass m_A and Block B has a given mass m_B. There is **friction** acting on the surfaces between the two blocks. However, the surface of the table is **frictionless**. An instructor pushes horizontally on Block B with his finger with a constant given (known) applied force F_{app} . Assume also that **known** values of both the coefficient of static friction μ_s and the coefficient of kinetic friction μ_k are given parameters. Finally, here we observe that the given applied force F_{app} is small enough so that Block A **neither slips nor slides** relative to Block B. (In other words, Block A stays stuck to Block B).

a

Draw two Free Body Diagrams (FBDs), one for each block, showing *all* vertical and horizontal forces on each. Make sure that your diagrams clearly indicate the nature (type) and direction of each force. Make sure that your forces are labeled with appropriate subscripts, as needed. (For example, $\vec{W}_A$ means the Weight Force on Block A. $\vec{N}_{AB}$ means the Normal Force on Block A due to Block B.)

✓ Answer



b

Calculate **all** of the forces on each of the two blocks. Also calculate the accelerations, a_A and a_B for each of the two blocks. Express your answers in terms of the given parameters. Show your work.

✓ **Answer**

Assuming

- (1) no slip no slide $\implies \vec{a}_a = \vec{a}_b, \vec{v}_a = \vec{v}_b$
- (2) Not going into the ground $\implies a_{ay} = a_{ab} = 0 = v_{ay} = v_{by}$

$W_a = m_a g$
$W_b = m_b g$
$N_{ba} = W_b$ Assumption 2
$N_{ba} = m_b g$
$N_{ba} = N_{ab}$ Third Law
$N_{ab} = m_b g$
$N_a = N_{ab} + W_a$ Assumption 2, Second Law
$N_a = (m_a + m_b)g$

Let: $x = F_{ab} = F_{ba}$ Third Law

$\vec{F}_{a\,net} = \langle F_{app} - x, 0\rangle$
$\vec{F}_{b\,net} = \langle x, 0\rangle$

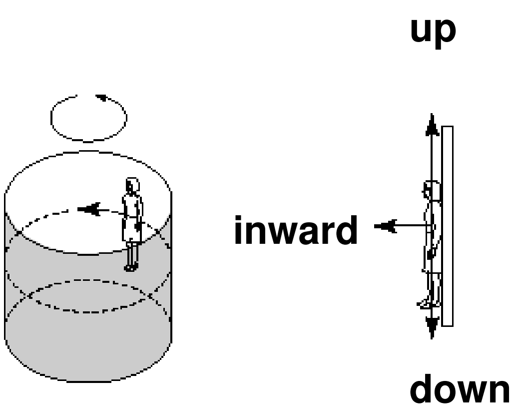
$\vec{a}_a = \vec{a}_b = \vec{a}$ Assumption 1
$\vec{F}_{a\,net} = m_a \vec{a}$
$\vec{F}_{b\,net} = m_b \vec{a}$
$\vec{F}_{a\,net} + \vec{F}_{b\,net} = (m_a + m_b)\vec{a}$

$\vec{a} = \frac{\langle F_{app}, 0 \rangle}{m_a + m_b}$
$a = \frac{F_{app}}{m_a + m_b}$

$F_{b\,net} = F_{ba} = m_b a$
$F_{ba} = \frac{m_b F_{app}}{m_a + m_b}$

$W_a = m_a g$
$W_b = m_b g$
$N_{ba} = m_b g$
$N_{ab} = m_b g$
$N_a = (m_a + m_b) g$
$F_{ba} = \frac{m_b F_{app}}{m_a + m_b}$
$F_{ab} = \frac{m_b F_{app}}{m_a + m_b}$
$a_a = a_b = a = \frac{F_{app}}{m_a + m_b}$
F_{app} is given
□

3

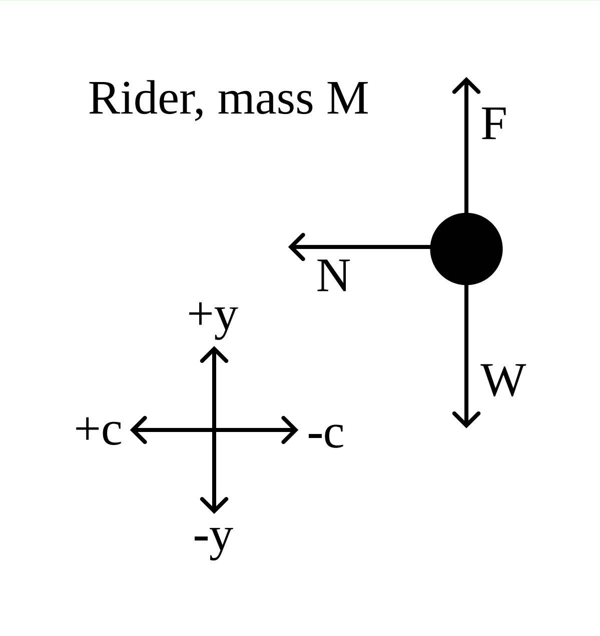


“**The Rotor Ride**: Perhaps the most feared ride at the amusement park is this small, seemingly non-intimidating attraction. It consists of a small barrel that spins in a circle about a vertical axis. Riders enter the ride and then stand with their backs against the inside wall of the barrel. As the barrel spins faster and faster, the centripetally inward force of the wall against the riders’ backs becomes greater and greater. All of a sudden, the floor is dropped out from under the riders. It is the friction between the barrel wall and the riders that keep the riders from falling. The rotor ride is a great place to learn the thrill of centripetal acceleration.”

a

Consider the forces on a single rider as shown on the right-most figure above. Draw a careful and complete Free-Body-Diagram to indicate all of the forces on the rider. Be sure to indicate a proper coordinate system.

✓ **Answer**



b

Assume that after the floor falls out, the rider is moving around inside the ride at a given constant speed V. Assume that the rider has a given mass M , and that the barrel of the rotor has a given radius R. Also assume that the coefficient of static friction between the rider and the wall is given as μ. Calculate the magnitude and direction of every force on the rider who is stuck to the wall after the floor falls away. Give your answer in terms of the given parameters. Explain your work.

✓ **Answer**

All vectors are in $\langle c, y\rangle$
$\vec{a}_c = \langle \frac{V^2}{R}, 0\rangle$

$\vec{W} = \langle 0, -Mg\rangle$
$\vec{F}_c = N = M\vec{a}_c = \langle \frac{MV^2}{R}, 0\rangle$

Assuming the rider does not fall downwards $\implies F_{net\ y} = 0$

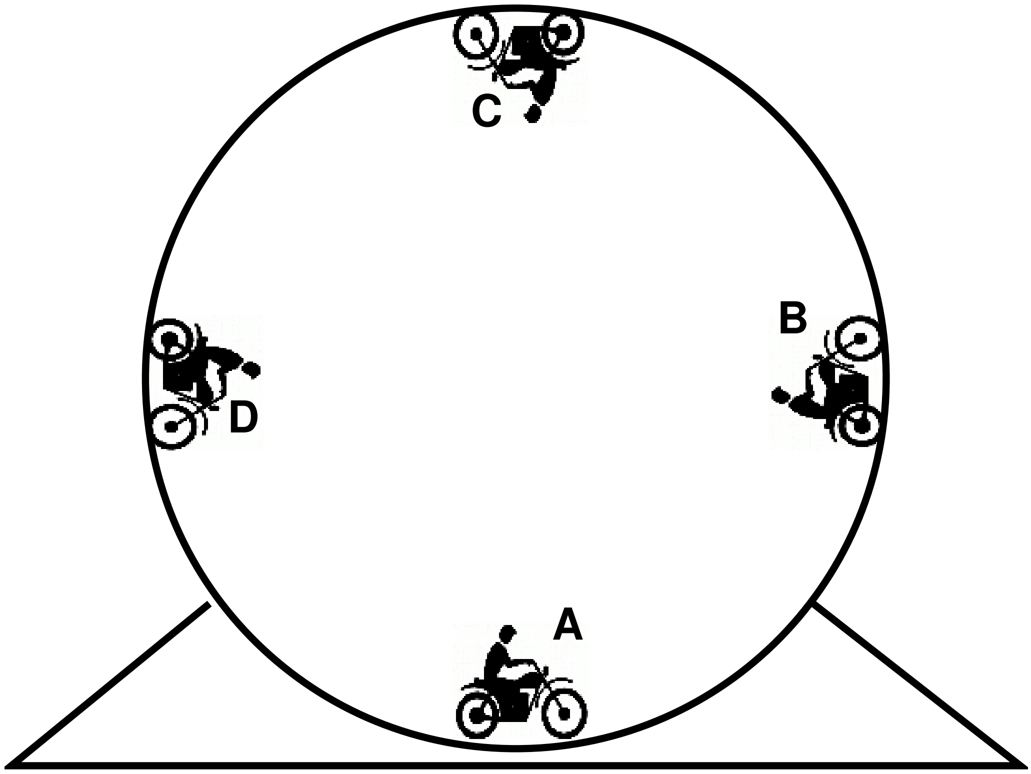
$\vec{W} = -\vec{F}$ Second law
$\vec{F} = \langle 0, Mg\rangle$

$\vec{W} = \langle 0, -Mg\rangle$
$\vec{F} = \langle 0, Mg\rangle$

$\vec{N} = \langle \frac{MV^2}{R}, 0 \rangle$

□

4

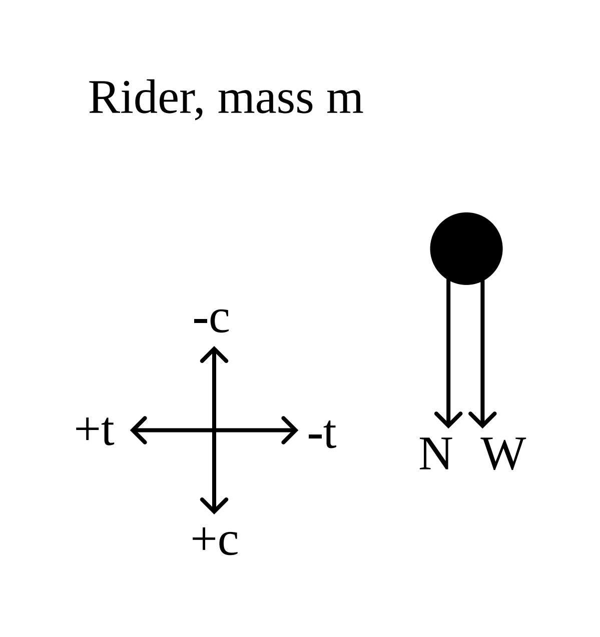


A motorcyclist performs a somewhat nauseating trick by driving on a vertical circular track (so-called “loop-the-loop”). The position of the motorcyclist is show at four positions (A, B, C, D) corresponding to bottom, right, top and left points on the track respectively. The mass of the motorcyclist and his motorcycle together is given as m. The radius of the circular path is given as R.

a

Assume that the motorcyclist somehow maintains *constant speed* V . Draw a careful Free Body Diagram showing all of the forces on system that is the motorcyclist and his motorcycle together when he is in **Position C**. Calculate the magnitude and direction of each force.

✓ **Answer**



$W = mg$

Assuming constant speed, there is no friction as we are not accelerating in the tangential direction.

$a_c = \frac{V^2}{R}$

Since there is no force in the tangential direction, a_c will be the a_{net}

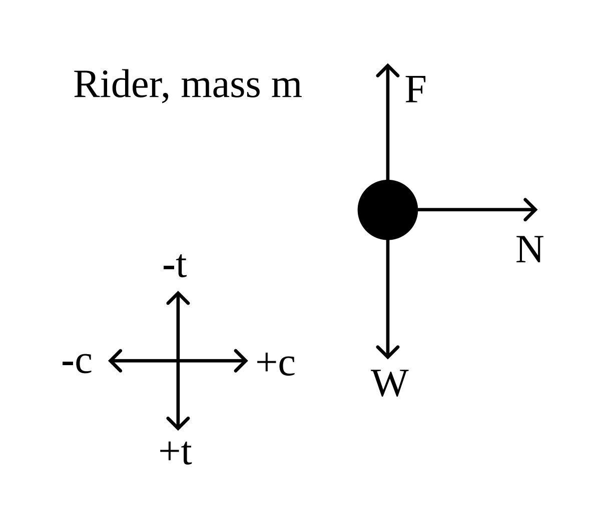
$$F_{net} = ma_c$$
$$N + W = \frac{mV^2}{R}$$
$$N + mg = \frac{mV^2}{R}$$
$$N = m(\frac{V^2}{R} - g)$$

$$W = mg$$
$$N = m(\frac{V^2}{R} - g)$$
$\square$

b

Assume that the motorcyclist maintains *constant speed* V . Draw a careful Free Body Diagram showing all of the forces on system that is the motorcyclist and his motorcycle together when he is in **Position D**. Calculate the magnitude and direction of each force on your FBD.

✓ **Answer**



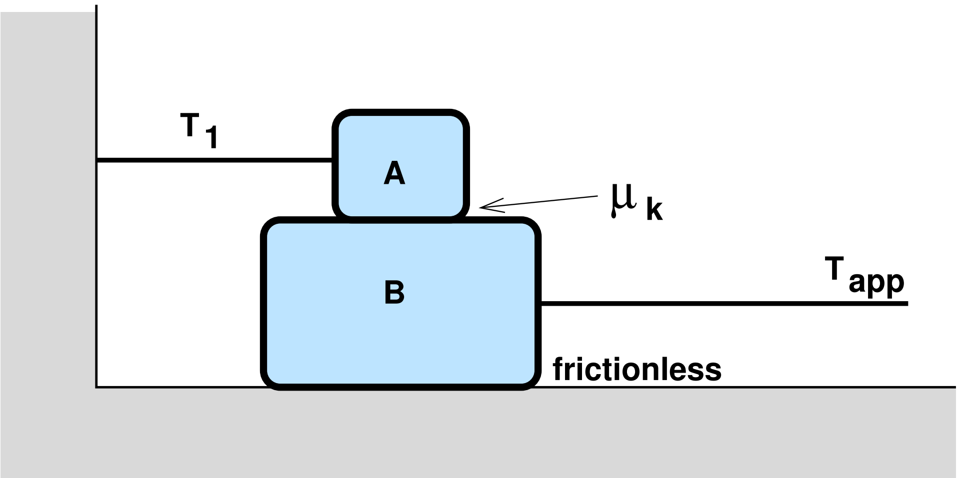
Since we know we have a constant velocity, we will not be accelerating in the tangential direction, which means the weight force will be fully cancelled out.

$W = mg$
$W = F$ Second law, constant velocity
$F = mg$

$a_c = \frac{V^2}{R}$
$N = F_{net\ c} = \frac{mV^2}{R}$

$W = mg$
$F = mg$
$N = \frac{mV^2}{R}$
$\square$

5

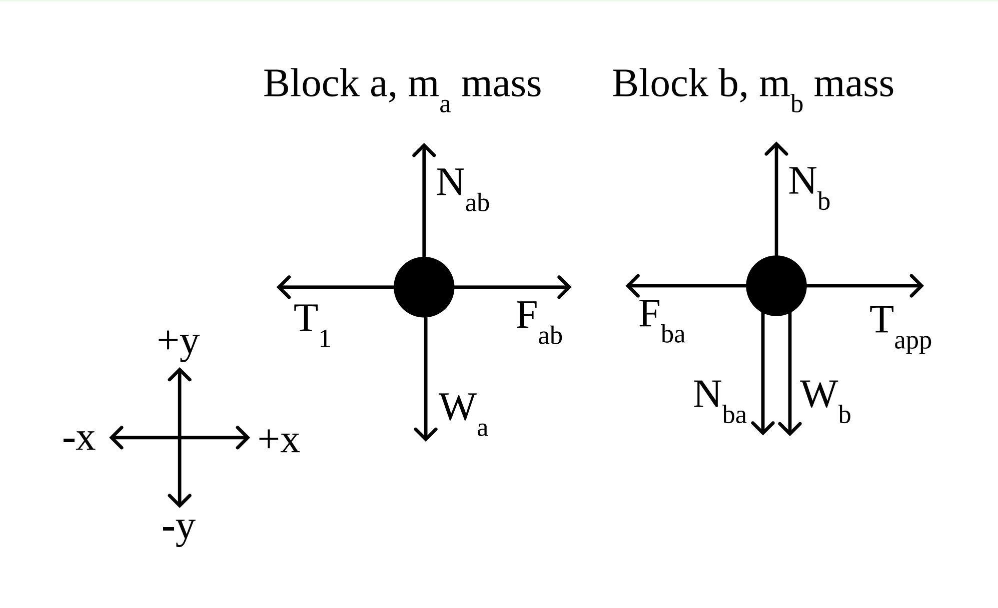


Block A with a given mass m_a is positioned on top of Block B with a given mass m_b. The coefficient of kinetic (sliding) friction between Block A and Block B is given as μ_k . Block B sits on a frictionless surface. Block A is tied to the wall with an ideal string which prevents Block A from moving to the right. A given constant horizontal tension force T_{app} is applied to Block B as shown, with the result that Block B slides to the right.

a

Carefully draw a neat “Free Body Diagram” (FBD) for each of the two blocks. Be sure to indicate and properly label all forces that are applied to each block.

✓ **Answer**



b

If you have drawn your two FBDs correctly, there will be two “Third Law Forces Pairs.” Which four forces correspond to these two pairs? Explain how you know this.

✓ **Answer**

$N_{ab} \iff N_{ba}$

This is a push force between A and B, they will have to push each other an equal force in order to be balanced within the system.

$F_{ab} \iff F_{ba}$

This is a friction force between A and B, they must equal and cancel out in order for the system to remain balanced.

c

Determine T_1 , the tension in the rope that is attached to Block A in terms of the given parameters. Explain your work. Hint: the rope does more that simply provide a force of tension on Block A. The rope also imposes a *kinematic constraint* on the acceleration of Block A.

Answer

$W_a = m_a g$

Since we know block A does not accelerate in any direction due to the string and block B.

$W_a = N_{ab}$ Second law
$N_{ab} = m_a g$

$T_1 = F_{ab} = \mu_k N_{ab}$
$T_1 = F_{ab} = \mu_k m_a g$ Kinetic friction formula

$N_{ab} = N_{ba}$ Force pairs elaborated in (b)
$F_{ab} = F_{ba}$ Force pairs elaborated in (b)
$N_{ba} = m_a g$
$F_{ba} = \mu_k m_a g$
$N_b = N_{ba} + W_b = (m_a + m_b)g$

$F_{b\,net} = T_{app} - F_{ba}$
$F_{b\,net} = T_{app} - \mu_k m_a g$

$T_1 = \mu_k m_a g$
□

d

Determine a_x , the acceleration of Block B in terms of the given parameters. Explain your work.

Answer

As we established in part (c),
$F_{b\,net} = T_{app} - \mu_k m_a g$
$F_{b\,net} = m_b a_x$
$a_x = \frac{T_{app} - \mu_k m_a g}{m_b}$

$$a_x = \frac{T_{app} - \mu_k m_a g}{m_b}$$

□