HW 1

1.1

3

a

Find the energies of the pair of signals $x(t)$ and $y(t)$ depicted in Figs. P1.1-3a and P1.1-3b. Sketch and find the energies of signals $x(t) + y(t)$ and $x(t) - y(t)$. Can you make any observation from these results?

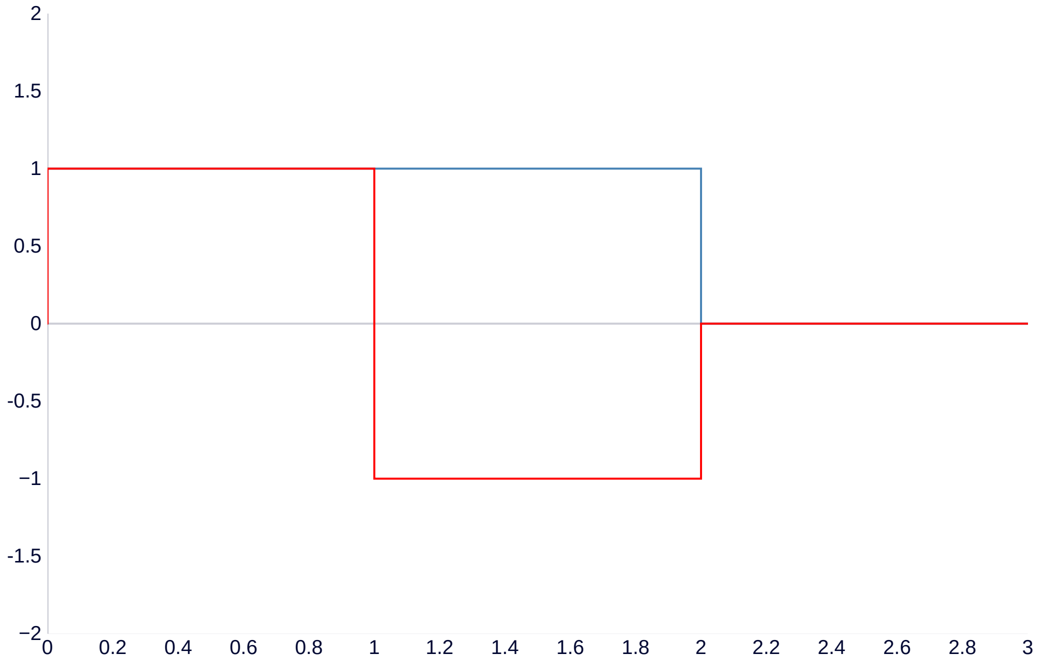


Fig. P1.1-3a: Where $x(t)$ is blue and $y(t)$ is red

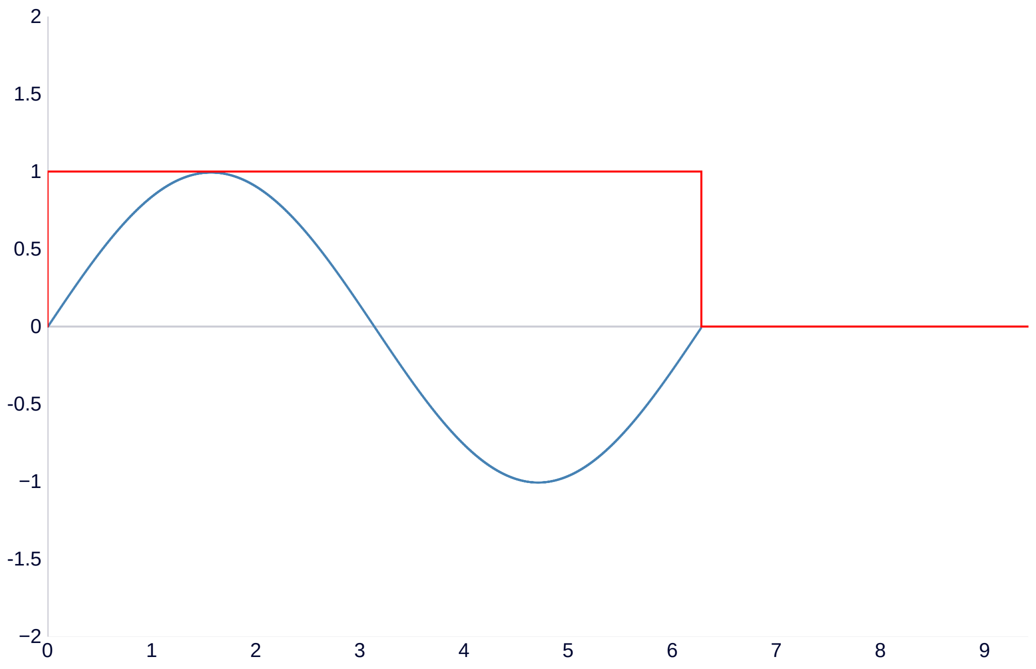
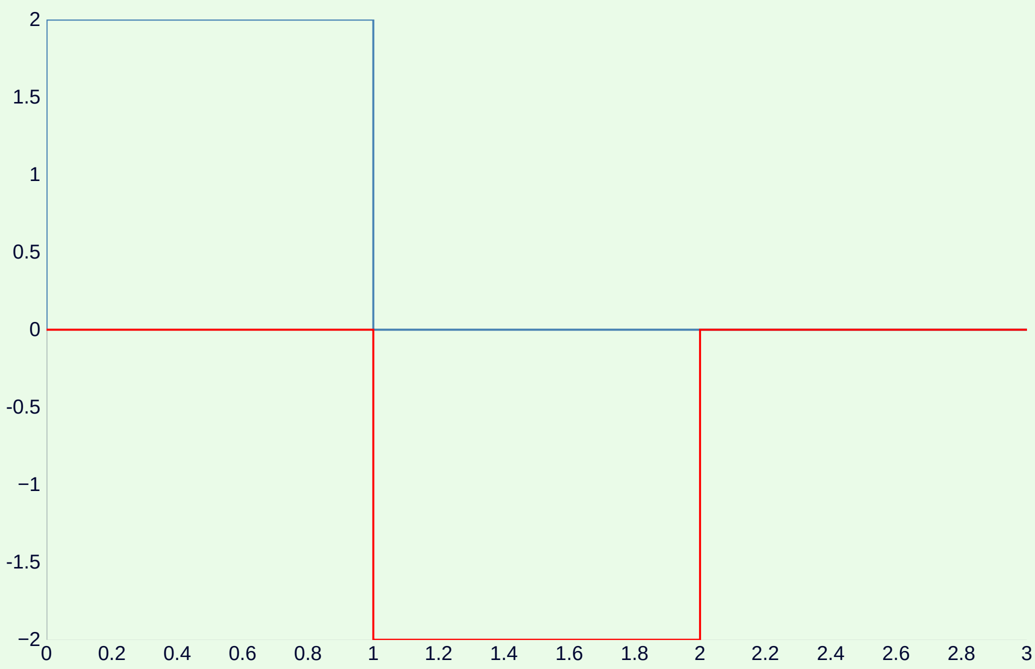


Fig. P1.1-3b: Where $x(t)$ is blue and $y(t)$ is red

✓ **Answer**

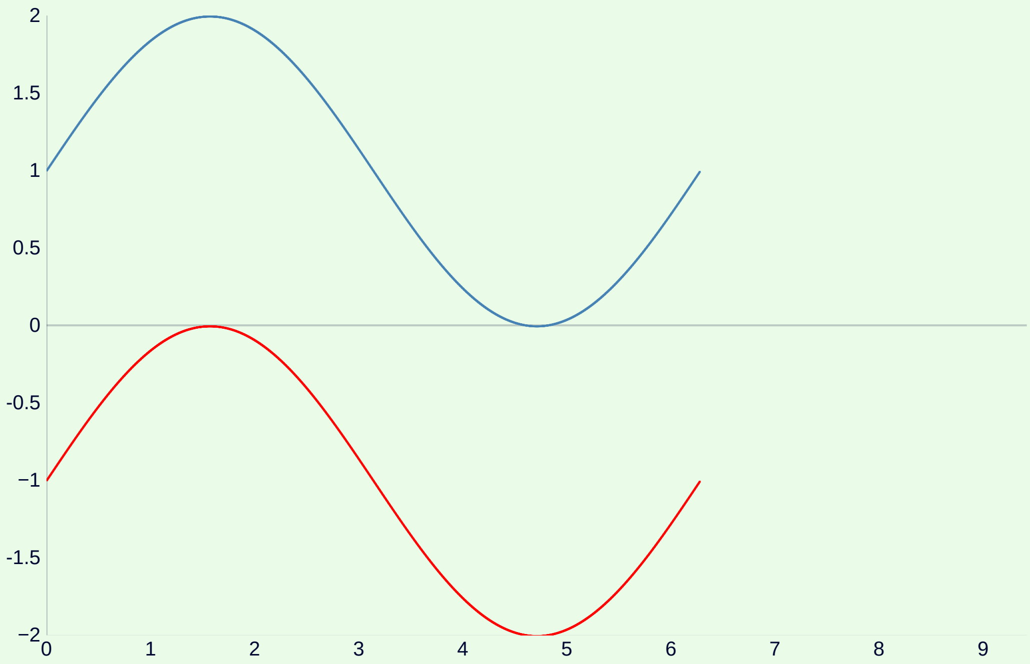
$$Ex = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$



Where $x(t) + y(t)$ is blue and $x(t) - y(t)$ is red

$$Ex = 2 \cdot 1^2 = 2$$
$$Ey = 2 \cdot 1^2 = 2$$

$E(x+y) = 1 \cdot 2^2 = 4$
$E(x-y) = 1 \cdot 2^2 = 4$



Where $x(t) + y(t)$ is blue and $x(t) - y(t)$ is red

$Ex = \pi$
$Ey = 2\pi \cdot 1^2 = 2\pi$
$E(x+y) = 3\pi$
$E(x-y) = 3\pi$

It appears that adding or subtracting functions would result in the addition of their powers, however based on the process of obtaining power, it feels like this result is very specifically cherry picked and should not generalize.

b

Repeat part (a) for the signal pair illustrated in Fig. P1.1-3c. Is your observation in part (a) still valid?

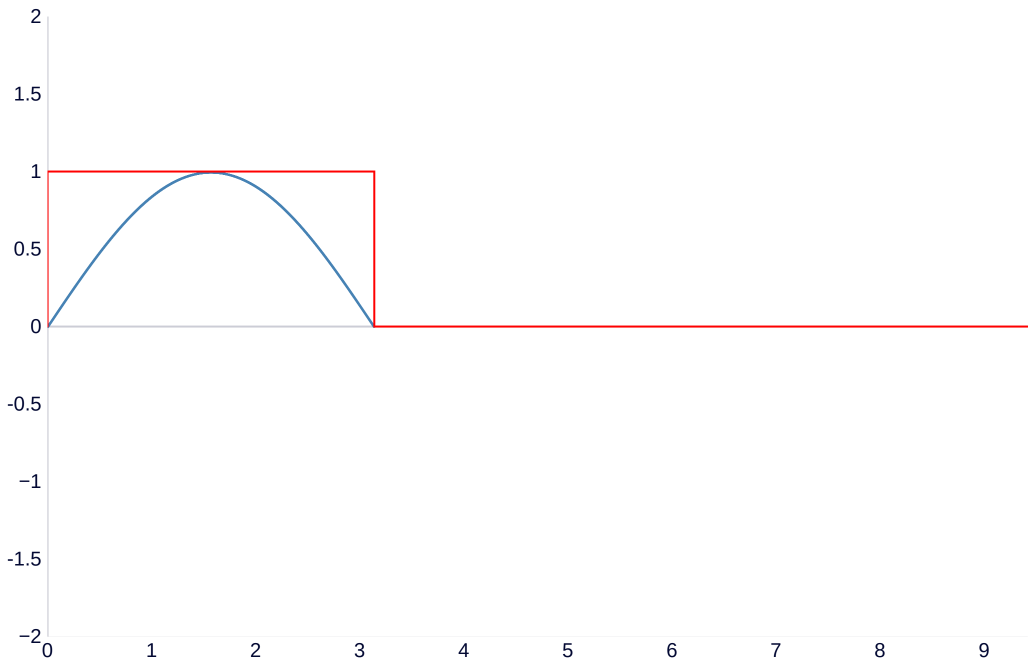
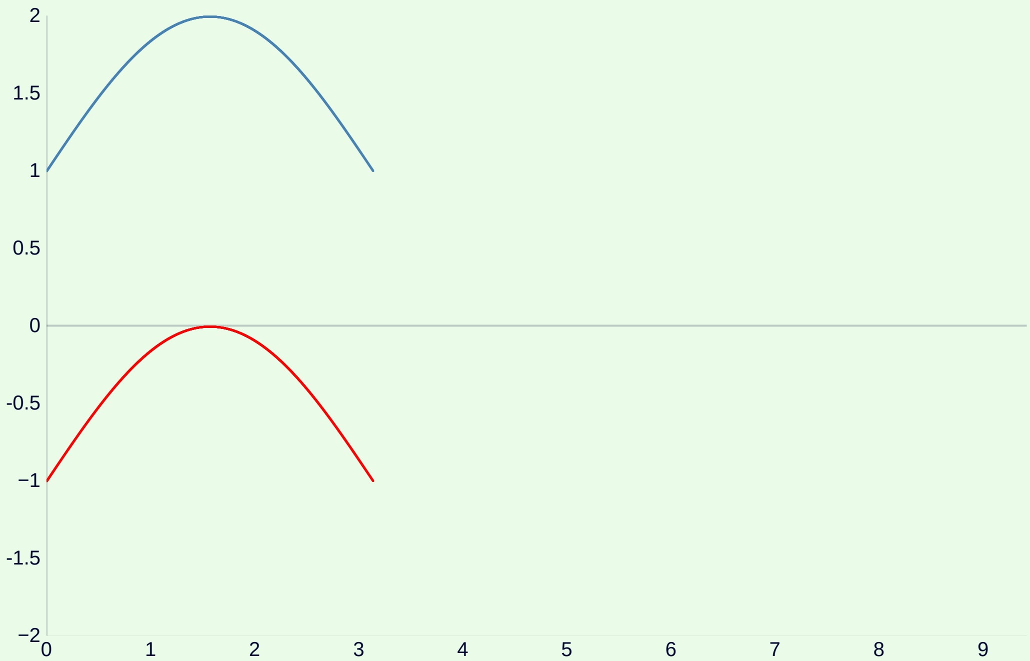


Fig. P1.1-3c: Where $x(t)$ is blue and $y(t)$ is red

✓ **Answer**



Where $x(t)+y(t)$ is blue and $x(t)-y(t)$ is red

$Ex = \frac{\pi}{2}$
$Ey = \pi \cdot 1^2 = \pi$
$E(x+y) = 4 + \frac{3\pi}{2}$
$E(x-y) = \frac{3\pi}{2} - 4$

Our observation no longer holds.

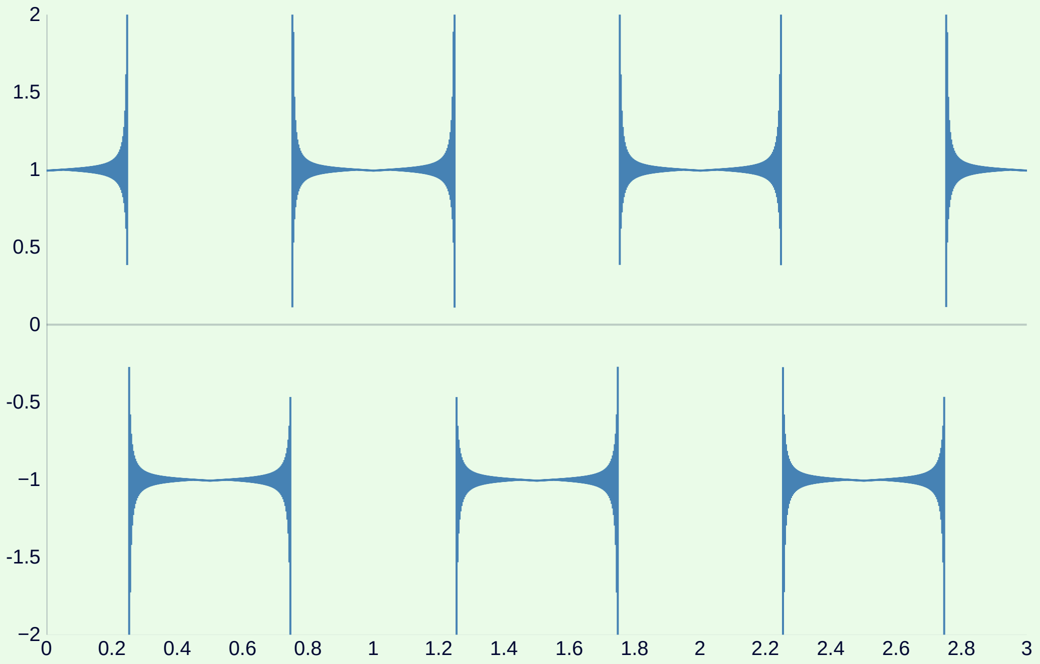
8

Two periodic signals that differ only by a 90° phase shift are considered to be quadrature signals. For example, $\cos(2\pi t)$ and $\sin(2\pi t)$ are quadrature signals. Another pair of quadrature signals is $x(t) = \text{sgn}(\cos(2\pi t))$ and $y(t) = \text{sgn}(\sin(2\pi t))$, where sgn is the sign function.

a

Plot $x(t)$ and determine its power P_x and energy E_x.

✓ **Answer**



Since domain of $x(t)$ is not bounded, its total energy will be infinite.

$Ex = \infty$

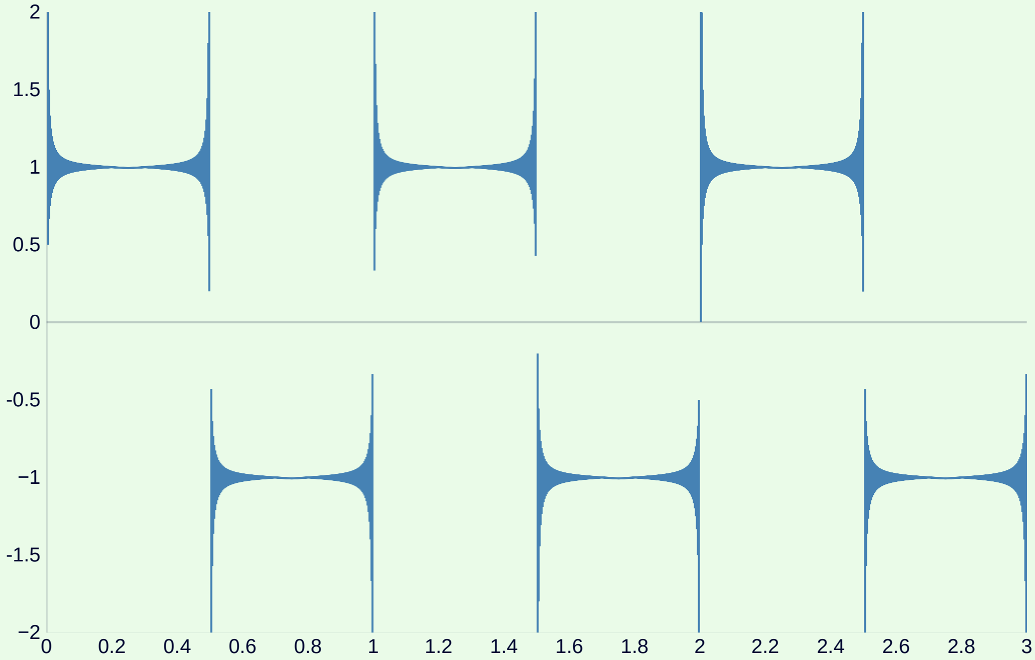
$$Px = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$$Px = \frac{1}{0.5} \int_{0}^{0.5} 1^2 = 1$$

b

Plot $y(t)$ and determine its power P_y and energy E_y.

✓ **Answer**



Since domain of $y(t)$ is not bounded, its total energy will be infinite.

$Ey = \infty$

$$Py = \frac{1}{0.5} \int_0^{0.5} 1^2 \, dt = 1$$

c

Consider the complex function $f(t) = x(t) + jy(t)$. Determine the power and energy of $f(t)$.

✓ **Answer**

$f(t) = \mathrm{sgn}(\cos(2\pi t)) + j\mathrm{sgn}(\sin(2\pi t))$

$|x(t)|^2 = 1$

$|y(t)|^2 = 1$

$|f(t)|^2 = \sqrt{|x(t)|^2 + |y(t)|^2}^2 = |x(t)|^2 + |y(t)|^2$

$|f(t)|^2 = 2$

As with x and y, the energy will be infinite.

$$Pf = \frac{1}{0.5} \int_0^{0.5} 2 \, dt = 2$$

d

When real functions $x(t)$ and $y(t)$ and combined as $f(t) = x(t) + jy(t)$, it is generally true that $E_f = E_x + E_y$ and $P_f = P_x + P_y$? Prove your answer.

✓ **Answer**

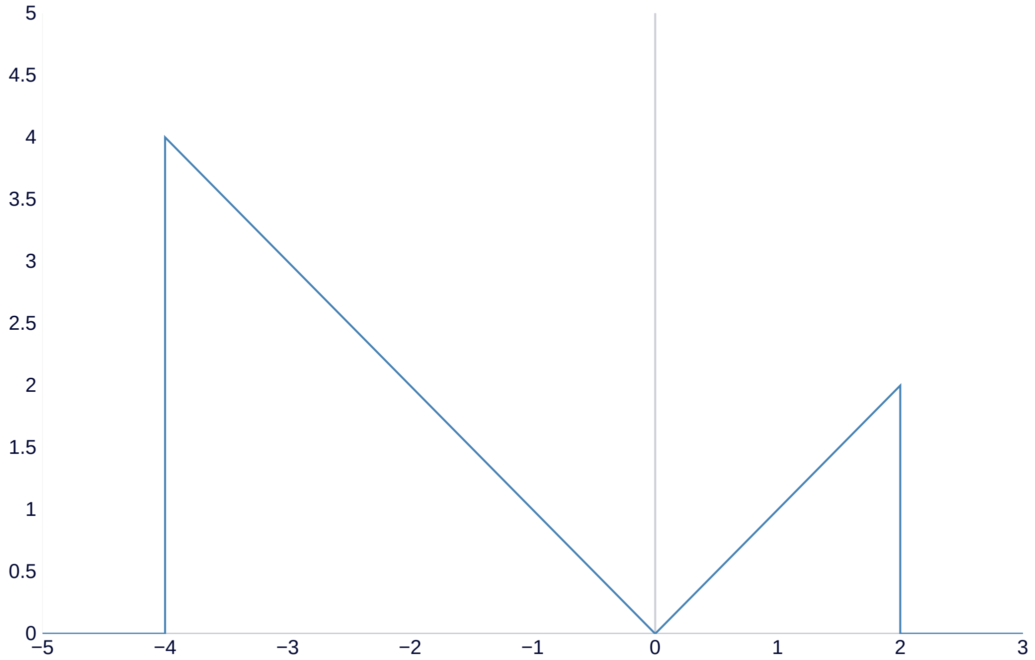
Yes

Since: $f(t) = x(t) + jy(t) \implies |f(t)|^2 = |x(t)|^2 + |y(t)|^2$

1. $$Ef = \int_{\infty}^{\infty} |f(t)|^2 \, dt$$
   $$= \int_{\infty}^{\infty} |x(t)|^2 + |y(t)|^2 \, dt$$
   $$= \int_{\infty}^{\infty} |x(t)|^2 \, dt + \int_{\infty}^{\infty} |y(t)|^2 \, dt$$
   $$= Ex + Ey$$
2. $$Pf = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |f(t)|^2 \, dt$$
   $$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 + |y(t)|^2 \, dt$$
   $$= \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt + \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |y(t)|^2 \, dt$$
   $$= Px + Py$$

1.2

2

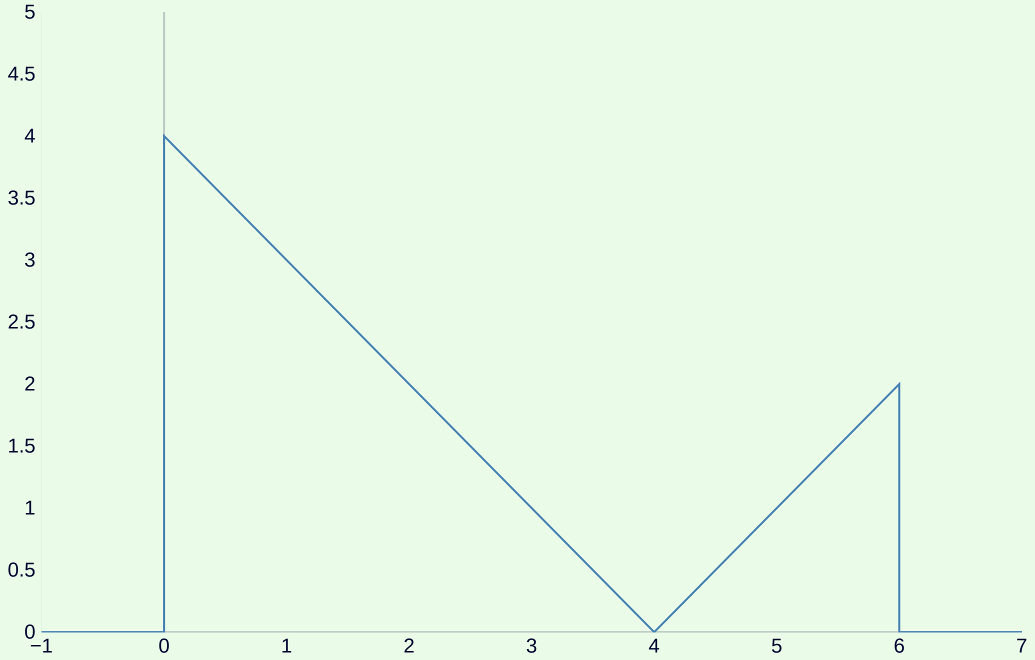


For signal $x(t)$, sketch the following

a

$x(t-4)$

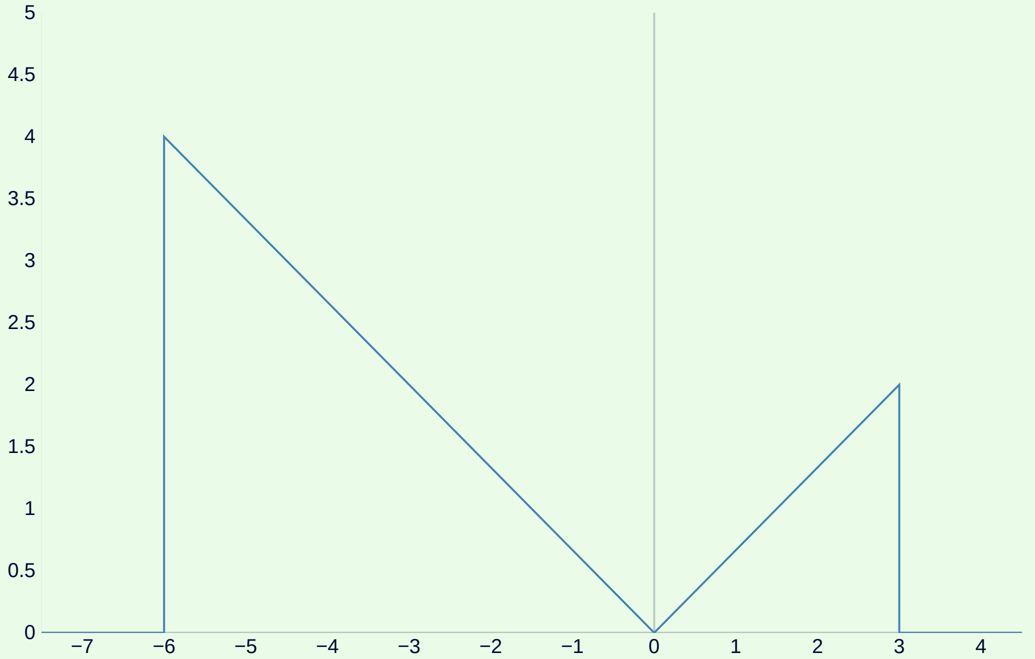
✓ **Answer**



b

$x\left(\frac{t}{1.5}\right)$

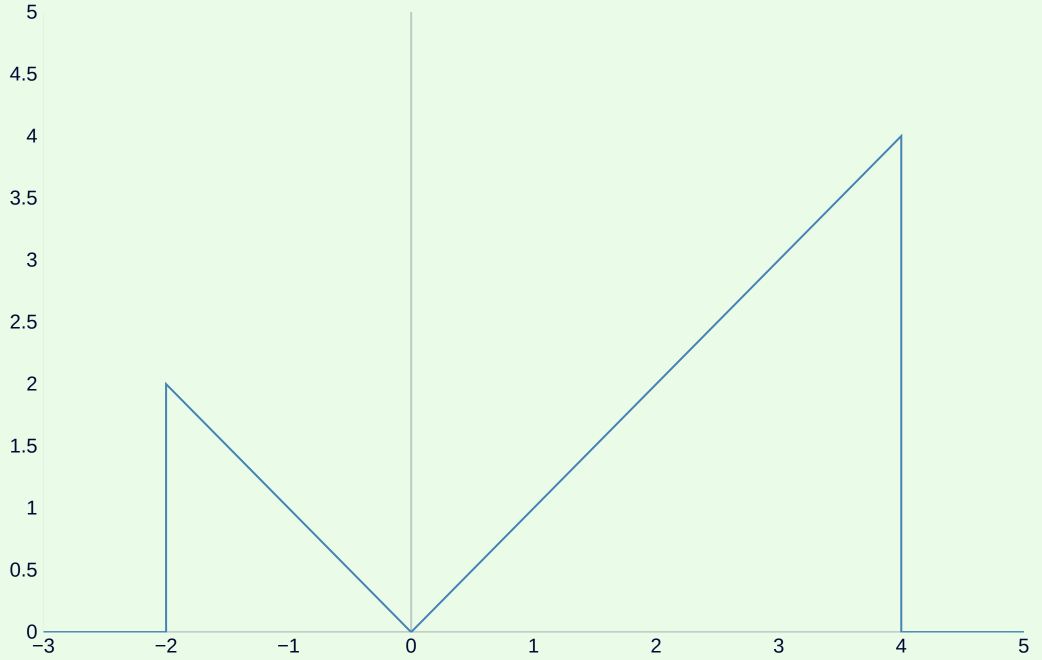
✓ **Answer**



c

$x(-t)$

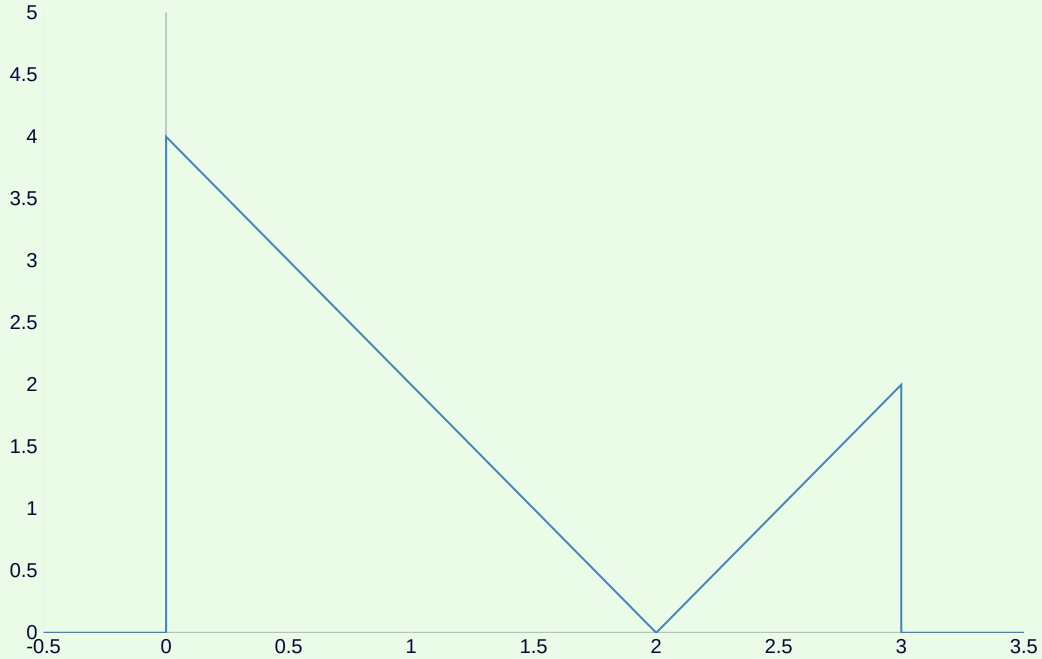
✓ **Answer**



d

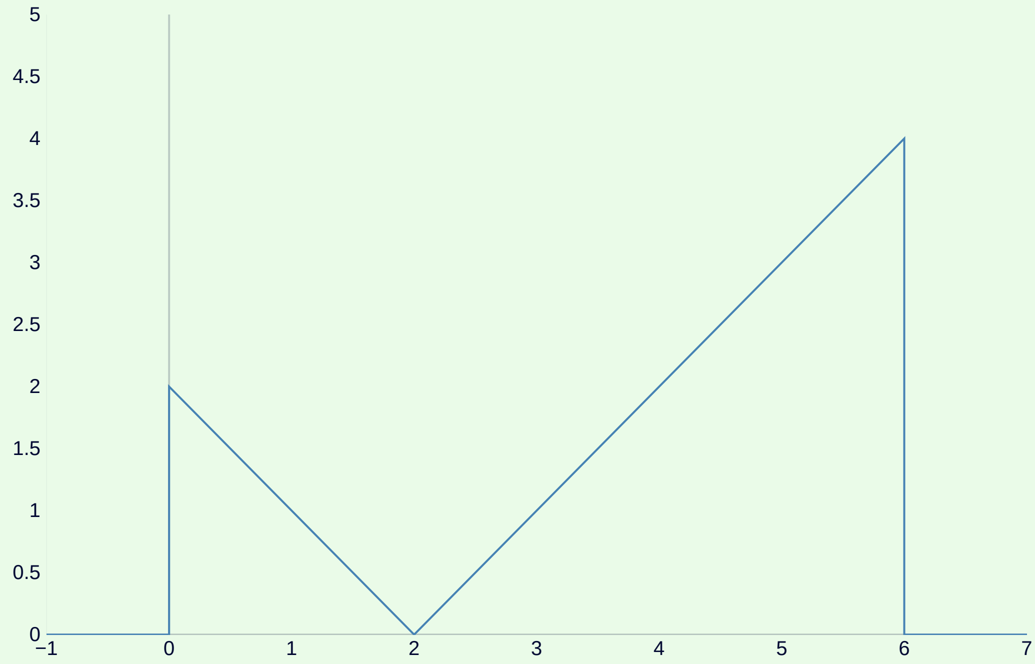
$x(2t-4)$

✓ **Answer**



e

✓ Answer



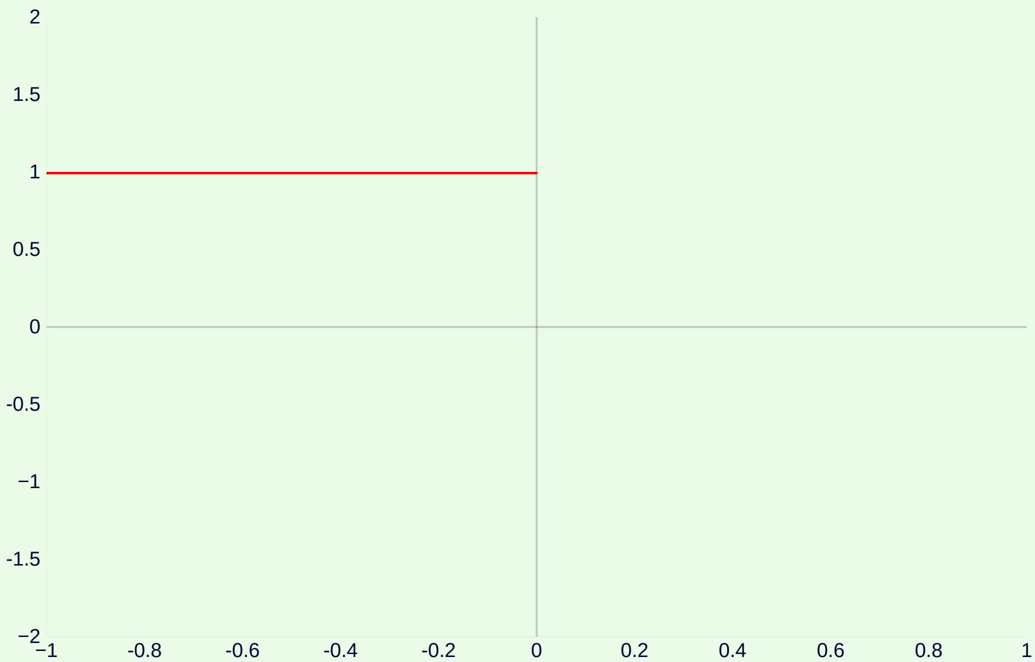
6

Consider the signal $x(t) = 2^{-tu(t)}$, where $u(t)$ is the unit step function.

a

Accurately sketch $x(t)$ over $(-1 \leq t \leq 1)$.

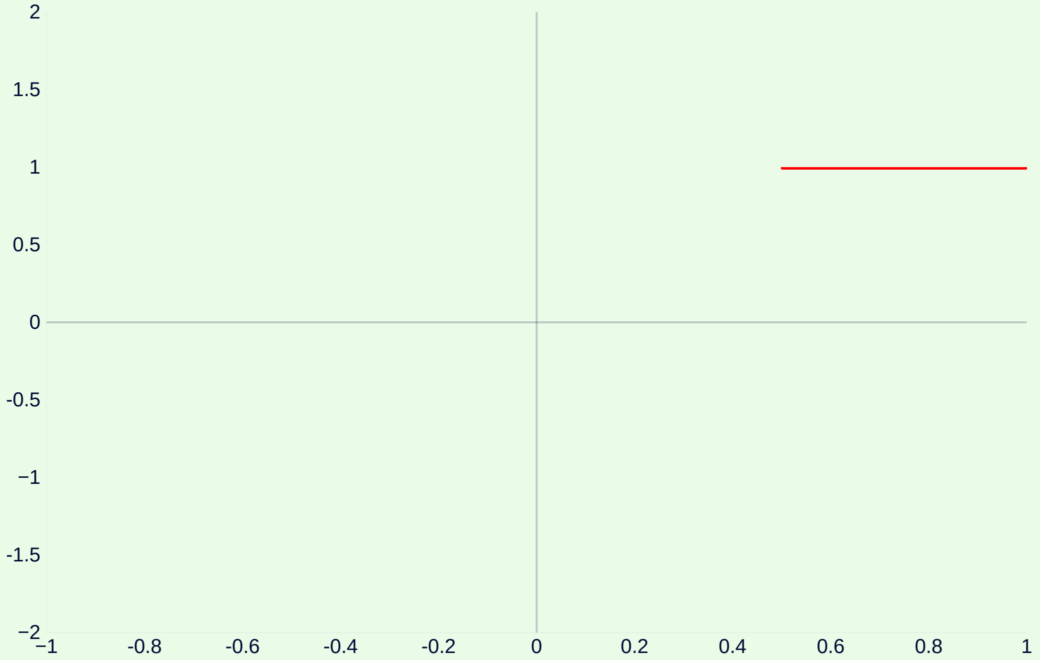
✓ Answer



b

Accurately sketch $y(t) = 0.5x(1 - 2t)$ over $(-1 \leq t \leq 1)$.

✓ Answer



1.3

4

Determine whether each of the following statements is true or false. If the statement is false, demonstrate by proof or example why the statement is false.

a

Every bounded periodic signal is a power signal.

✓ Answer

Assuming the signal is non-zero, then yes, a bounded periodic signal must necessarily be a power signal.

Since the signal is non-zero, $Px > 0$.

Since the signal is bounded by M across a period of n, the maximum power this signal can have must be $Px \leq \frac{M^2 n}{n} = M^2$.

Since $0 < Px \leq M^2$, x must be a power signal.

b

Every bounded power signal is a periodic signal.

✓ **Answer**
False, $\sin(x^2)$ is a power signal between 0 and 1 but is not periodic.

c

If an energy signal $x(t)$ has energy E, then the energy of $x(at)$ is $\frac{E}{a}$. Assume a is real and positive.

✓ **Answer**
True,

$$Ex = \int_{\infty}^{\infty} |x(t)|^2 \, dt$$

$$Ex(at) = \int_{\infty}^{\infty} |x(at)|^2 \, dt$$

Let $u = at, u' = a$

$$\implies Ex(at) = \frac{1}{a} \int_{\infty}^{\infty} |x(u)|^2 \, du$$

$$\implies Ex(at) = \frac{1}{a} Ex = \frac{E}{a}$$

d

If a power signal $x(t)$ has power P, then the power of $x(at)$ is $\frac{P}{a}$. Assume a is real and positive.

✓ **Answer**
False,

$x(t) = 1$

$Px = 1$

$Px(at) = 1 \neq \frac{1}{a}$

5

Given $x_1(t) = \cos(t)$, $x_2(t) = \sin(\pi t)$, and $x_3(t) = x_1(t) + x_2(t)$.

a

Determine the fundamental periods T_1 and T_2 of signals $x_1(t)$ and $x_2(t)$.

✓ **Answer**

$T_1 = 2\pi$, this is a known fact.

$T_2 = \frac{2\pi}{\pi} = 2$

b

Show that $x_3(t)$ is not periodic, which requires $T_3 = k_1 T_1 = k_2 T_2$ for some integers k_1 and k_2.

✓ **Answer**

Let T_3 be the period of $x_3(t)$ assuming it exists.

$T_3 = k_1 T_1 = k_2 T_2$ must be true where $k_n \in \mathbb{Z}$

$T_1 = T_2 \frac{k_2}{k_1}$

$\pi = \frac{k_2}{k_1}$

π is irrational, thus k_1 and k_2 cannot exist, thus T_3 cannot exist.

$x_3(t)$ is not periodic.

c

Determine the powers P_{x_1}, P_{x_2}, and P_{x_3} of signals $x_1(t)$, $x_2(t)$, and $x_3(t)$.

✓ **Answer**

$$Px = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

$P_{x_1} = \frac{\pi}{2\pi} = 0.5$

$P_{x_2} = \frac{1}{2} = 0.5$

$$P_{x_3} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} (\cos(t) + \sin(\pi t))^2 \, dt$$

$$= P_{x_1} + P_{x_2} + 2 \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \cos(t)\sin(\pi t) \, dt$$

Since $\cos(t)\sin(\pi t)$ is on average 0 due to being centered around 0,

$$\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \cos(t)\sin(\pi t) \, dt = 0$$

$P_{x_3} = P_{x_1} + P_{x_2} = 1$

1.4

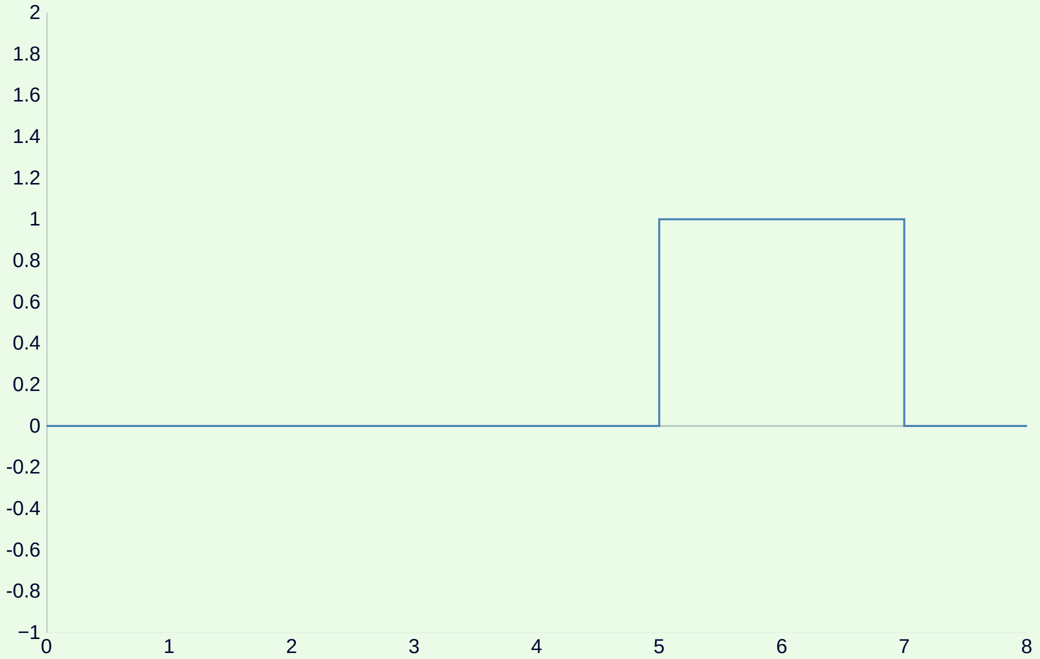
1

Sketch the following signals:

a

$u(t-5) - u(t-7)$

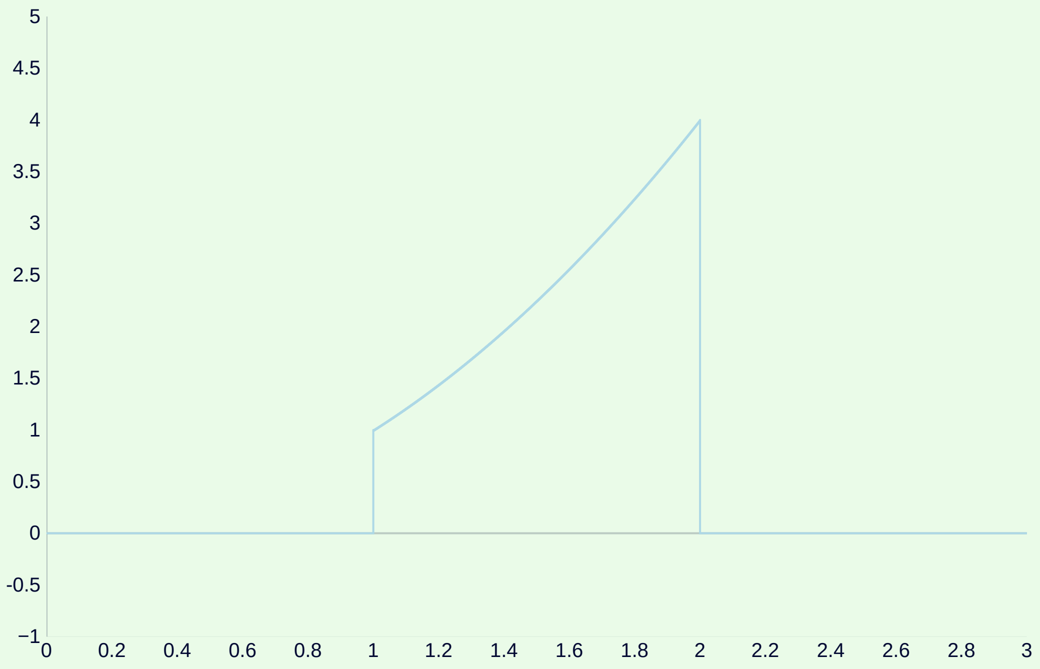
✓ **Answer**



c

$t^2(u(t-1) - u(t-2))$

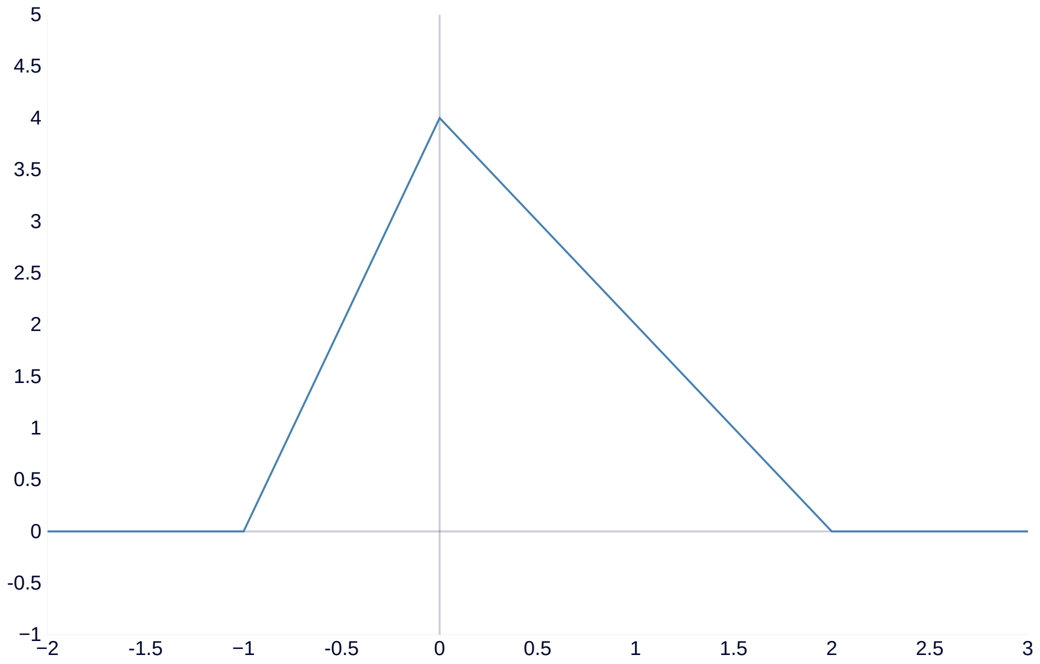
✓ **Answer**



2

Express each of the signals in Fig. P1.4-2 by a single expression valid for all t.

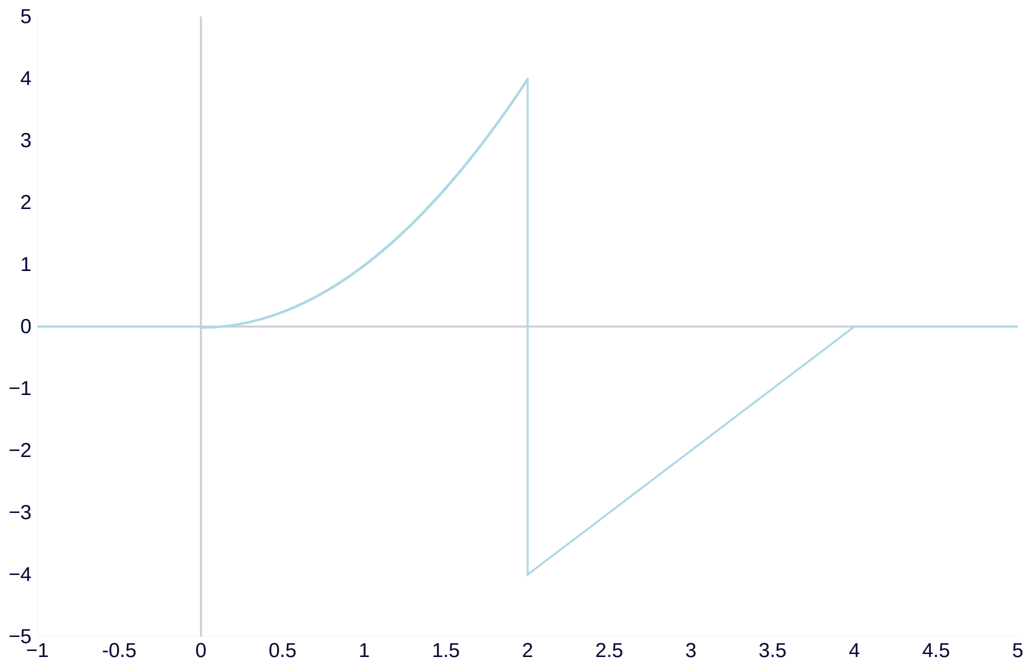
a



Answer

$x(t) = (4t+4)(u(t+1)-u(t)) + (-2t+4)(u(t)-u(t-2))$

b



✓ **Answer**

$t^2(u(t)-u(t-2))+(2t-8)(u(t-2)-u(t-4))$

13

A sinusoid $e^{\sigma t}\cos(\omega t)$ can be expressed as a sum of exponentials e^{st} and e^{-st} with complex frequencies $s=\sigma+j\omega$ and $s=\sigma-j\omega$. Locate in the complex plane the frequencies of the following sinusoids:

a

$\cos(3t)$

✓ **Answer**

$s=\pm j3$

b

$e^{-3t}\cos(3t)$

✓ **Answer**

$s=-3\pm j3$

c

$e^{2t}\cos(3t)$

Answer

$s = 2 \pm j3$

d

e^{-2t}

Answer

$s = -2$

e

e^{2t}

Answer

$s = 2$

f

5

Answer

$s = 0$

1.5

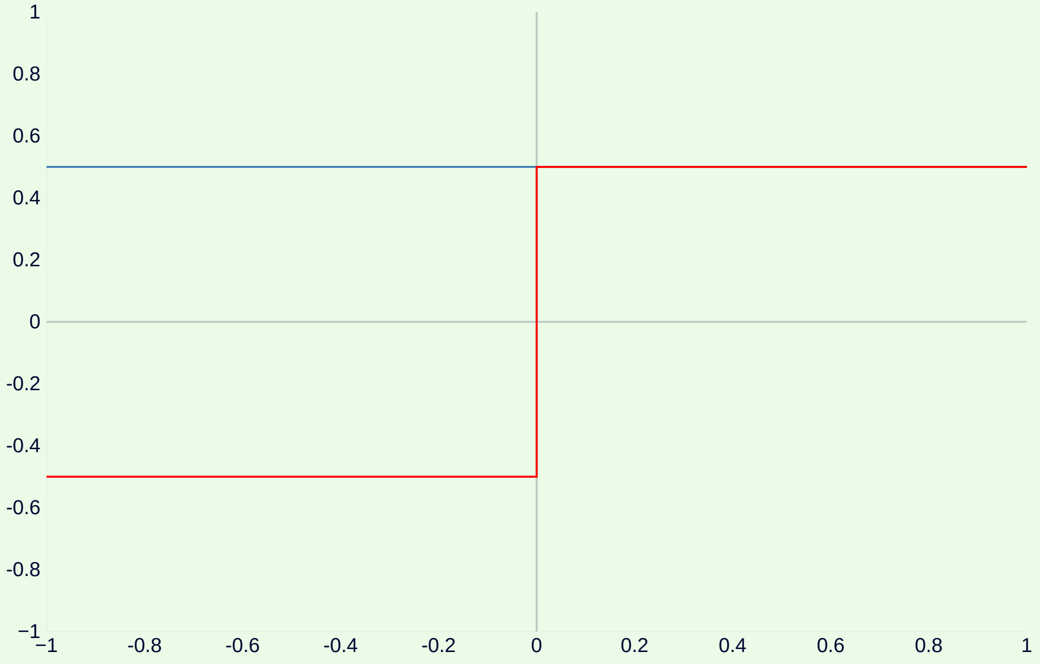
1

Find and sketch the odd and even components of the following:

a

$u(t)$

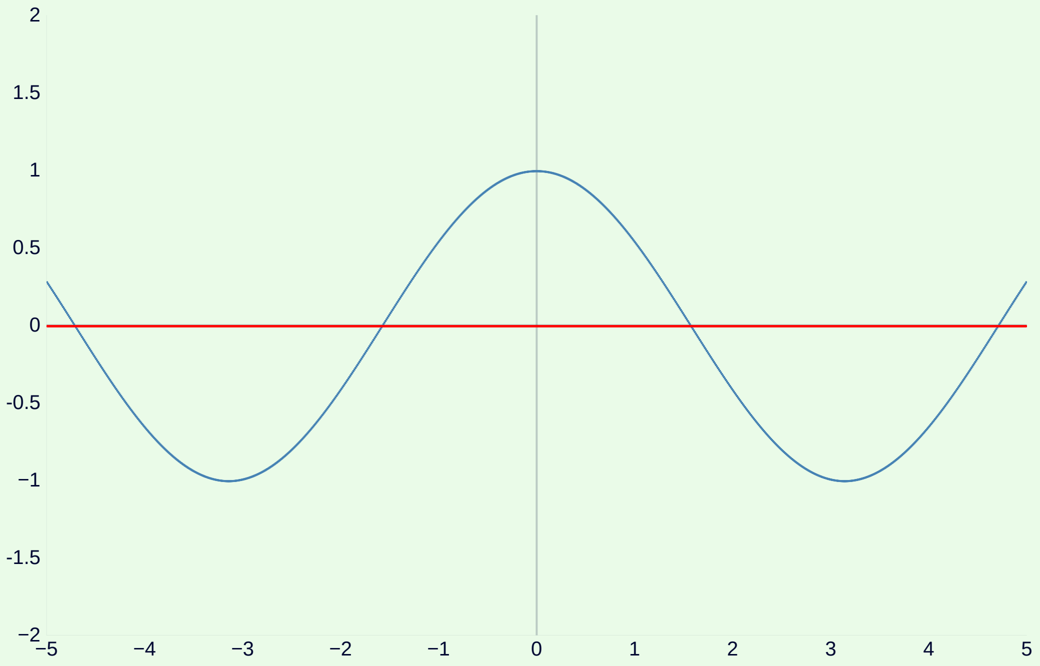
Answer



d

$\cos(\omega_0 t)$

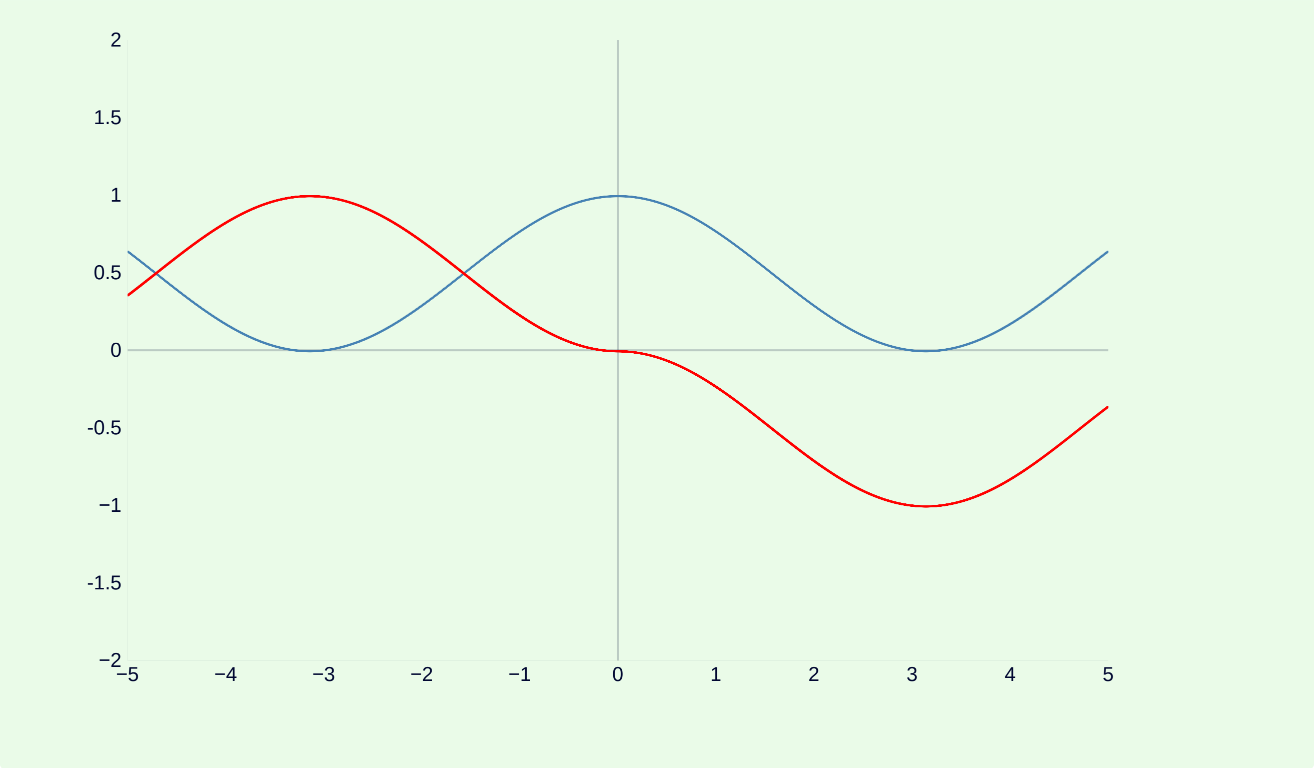
✓ **Answer**



g

$\cos(\omega_0 t u(t))$

✓ **Answer**



3

a

Determine even and odd components of the signal $x(t) = e^{-2t}u(t)$.

✓ Answer

Even: $0.5(e^{-2|t|})$

Odd: $0.5 \operatorname{sgn}(t)(e^{-2|t|})$

b

Show that the energy of $x(t)$ is the sum of energies for its odd and even components found in part (a).

✓ Answer

$$Ex = \int_0^\infty e^{-4t}\, dt = 0.25$$

$E\text{even} = \frac{1}{2}Ex = 0.125$ due to half height, double width

$E\text{odd} = \frac{1}{2}Ex = 0.125$ due to half height, double width

$E\text{even} + E\text{odd} = Ex$

c

Generalize the result in part (b) for any finite energy signal.

Answer

$$Eeven + Eodd = 0.25\left(\int_{-\infty}^{\infty} (x(t) + x(-t))^2 \, dt + \int_{-\infty}^{\infty} (x(t) - x(-t))^2 \, dt\right)$$

$$= 0.25\left(\int_{-\infty}^{\infty} x^2(t) + 2x(t)x(-t) + x^2(-t) + x^2(t) - 2x(t)x(-t) + x^2(-t) \, dt\right)$$

$$= 0.5\left(\int_{-\infty}^{\infty} x^2(t) + x^2(-t) \, dt\right)$$

$$= 0.5\left(\int_{-\infty}^{\infty} x^2(t) \, dt + \int_{-\infty}^{\infty} x^2(-t) \, dt\right)$$

$$= 0.5\left(\int_{-\infty}^{\infty} x^2(t) \, dt + \int_{-\infty}^{\infty} x^2(t) \, dt\right)$$

$$= \int_{-\infty}^{\infty} x^2(t) \, dt$$

$$= \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= Ex$$