13.1

9

$\vec{r}(t) = \langle t, t^3, t^2 + 1 \rangle$
If $\vec{r}$ intersects then xy plane, its z value must be 0 somewhere.

$t^2 + 1 = 0$
$t = i$

$\vec{r}$ does not intersect the xy plane unless t is a complex variable and can have the value of i
$\square$

11

$\vec{r}(t) = \langle 1 - \cos(2t), t + \sin t, t^2 \rangle$

$\vec{r}$ will intersect the yz plane if its x value equals 0 at some point.

$1 - \cos(2t) = 0$
$1 = \cos(2t)$
$t = R\pi, \quad \text{where } R \in \mathbb{I}$

This proves that $\vec{r}$ intersects the yz plane in infinitely many places
$\square$

In order for $\vec{r}$ to never cross the yz plane, it must have all values on one side of it, or disconnected values on both sides of it.

However, we can see that:

$-1 \leq \cos(x) \leq 1$
$0 \leq 1 - \cos(2t) \leq 2$

Where the domain of the x value of $\vec{r}$ is $[0, 2]$, and therefore never crosses the yz plane.
$\square$

19

$\vec{r}(t) = (9 \cos t)\hat{\imath} + (9 \sin t)\hat{\jmath}$

Since $\vec{r}$ has no $\hat{k}$ component, its z value is always 0, and it must lie on the xy plane.
$\square$

We can convert $\hat{\imath}\hat{\jmath}\hat{k}$ notation back into xyz notation

$\vec{r}(t) = \begin{cases} x = 9\cos t \\ y = 9\sin t \end{cases}$
$\vec{r}(t) = \begin{cases} x^2 = 9^2\cos^2 t \\ y^2 = 9^2\sin^2 t \end{cases}$
$\vec{r}(t) = \{x^2 + y^2 = 9^2(\sin^2 t + \cos^2 t)$
$\vec{r}(t) = \{x^2 + y^2 = 9^2$

Where $(x - x_0)^2 + (y - y_0)^2 = r^2$ is the formula for a circle
Meaning
$\begin{cases} x_0 = 0 \\ y_0 = 0 \\ r = 9 \end{cases}$

Therefore the center of the circle is $(0, 0)$ and has a radius of 9
$\square$

31

a

$C = \begin{cases} x^2 + y^2 = z^2 \\ y = z^2 \end{cases}$

$C(t) = \begin{cases} x^2 + y^2 = t^2 \\ y = t^2 \end{cases}$
$C(t) = \begin{cases} x^2 + y^2 = y \\ z = t \\ y = t^2 \end{cases}$
$C(t) = \begin{cases} x = \pm\sqrt{y - y^2} \\ z = t \\ y = t^2 \end{cases}$
$C(t) = \begin{cases} x = \pm\sqrt{t^2 - t^4} \\ z = t \\ y = t^2 \end{cases}$
$C(t) = \langle \pm\sqrt{t^2 - t^4}, t^2, t \rangle$

Or

$C(t) = \begin{cases} \langle \sqrt{t^2 - t^4}, t^2, t \rangle & x \geq 0 \\ \langle -\sqrt{t^2 - t^4}, t^2, t \rangle & x \leq 0 \end{cases}$
$\square$

b

The projection of $C(t)$ onto the xy plane would look like the curve with all z values set to 0

$C(t) = \langle \pm\sqrt{t^2 - t^4}, t^2, 0 \rangle$
Let $\tilde{t} = t^2$
$C(t) = \langle \pm\sqrt{\tilde{t} - \tilde{t}^2}, t, 0 \rangle$
$C(t) = \begin{cases} x = \pm\sqrt{\tilde{t} - \tilde{t}^2} \\ y = \tilde{t} \end{cases}$

$$C(t) = \begin{cases} x^2 = \tilde{t} - \tilde{t}^2 \\ y^2 = \tilde{t}^2 \end{cases}$$
$$C(t) = \{x^2 + y^2 = \tilde{t}$$
$$C(t) = \{x^2 + y^2 = t^2$$
$$C(t) = \{x^2 + y^2 = y$$
$$C(t) = \{x^2 + y^2 - y = 0$$
$$C(t) = \{x^2 + (y - 0.5)^2 - 0.25 = 0$$
$$C(t) = \{x^2 + (y - 0.5)^2 = 0.5^2$$

Where $(x - x_0)^2 + (y - y_0)^2 = r^2$ is the formula for a circle

Meaning:

$$\begin{cases} x_0 = 0 \\ y_0 = 0.5 \\ r = 0.5 \end{cases}$$

The projection of $C(t)$ onto the xy plane looks like a circle centered at $(0, 0.5)$ with a radius of 0.5
□

c

$$C(t) = \begin{cases} x = \pm\sqrt{t^2 - t^4} \\ z = t \\ y = t^2 \end{cases}$$
$$C(t) = \begin{cases} x^2 = t^2 - t^4 \\ y^2 = t^4 \\ z^2 = t^2 \end{cases}$$
$$C(t) = \begin{cases} x^2 = t^2 - t^4 \\ y^2 = t^4 \\ z^2 = t^2 \\ (y-1)^2 = y^2 - 2y + 1 \end{cases}$$
$$C(t) = \begin{cases} x^2 = t^2 - t^4 \\ y^2 = t^4 \\ z^2 = t^2 \\ (y-1)^2 = t^4 - 2t^2 + 1 \end{cases}$$
$$C(t) = \{x^2 + (y-1)^2 + z^2 = t^2 - t^4 + t^4 - 2t^2 + 1 + t^2$$
$$C(t) = \{x^2 + (y-1)^2 + z^2 = 1$$

Where $(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2$ is the formula for a sphere

Meaning:

$$\begin{cases} x_0 = 0 \\ y_0 = 1 \\ z_0 = 0 \\ r = 1 \end{cases}$$

Meaning $C(t)$ lies on the sphere centered on $(0, 1, 0)$ with a radius of 1
□

13.2

1

$\lim_{x \to 3} \langle t^2, 4t, 1/t \rangle$

$t^2, 4t, 1/t$ are all continuous at 3, so we can just evaluate at 3

$\langle 9, 12, 1/3 \rangle$
$\square$

5

$\lim_{h \to 0} (\vec{r}(t+h) - \vec{r}(t))/h$
$\vec{r}(t) = \langle t^{-1}, \sin t, 4 \rangle$

$\lim_{h \to 0} (\langle (t+h)^{-1}, \sin(t+h), 4 \rangle - \langle t^{-1}, \sin t, 4 \rangle)/h$
$\lim_{h \to 0} \langle (t+h)^{-1} - t^{-1}, \sin(t+h) - \sin t, 4 - 4 \rangle / h$
$\lim_{h \to 0} \langle (-1)/(t(t+h)), (\sin(t+h) - \sin t)/h, 0 \rangle$
$\lim_{h \to 0} \langle -1/(t^2 + th), (\sin(t+h) - \sin t)/h, 0 \rangle$
$\lim_{h \to 0} \langle -1/(t^2 + th), (\sin t \cos h + \cos t \sin h - \sin t)/h, 0 \rangle$
$\lim_{h \to 0} \langle -1/(t^2 + th), (\sin t(\cos h - 1) + \cos t \sin h)/h, 0 \rangle$
$\lim_{h \to 0} \langle -1/(t^2 + th), \sin t(0) + \cos t(1), 0 \rangle$
$\lim_{h \to 0} \langle -1/t^2, \cos t, 0 \rangle$
$\langle -1/t^2, \cos t, 0 \rangle$
$\square$

7

$\vec{r}(t) = \langle t, t^2, t^3 \rangle$
$\vec{r}'(t) = \langle (t)', (t^2)', (t^3)' \rangle$
$\vec{r}'(t) = \langle 1, 2t, 3t^2 \rangle$
$\square$

17

$\vec{r}(t) = \langle t - \sin t, 1 - \cos t \rangle$
Parallel to $\langle \sqrt{3}, 1 \rangle$
$\vec{r}'(t) = C\langle \sqrt{3}, 1 \rangle$

$\vec{r}'(t) = \langle 1 - \cos t, \sin t \rangle$
$\langle 1 - \cos t, \sin t \rangle = \langle C\sqrt{3}, C \rangle$
$(1 - \cos t)/\sin t = \sqrt{3}$
$t = 2\pi/3$
$\square$

25

$\vec{r}(t) = \langle t^2, 1-t \rangle \quad g(t) = e^t$
$\frac{d}{dx}\vec{r}(g(t)) = \vec{r}'(g(t))g'(t)$

$\vec{r}'(t) = \langle 2t, -1 \rangle$
$g'(t) = e^t$

$\frac{d}{dx}\vec{r}g(t)) = \langle 2e^t, -1 \rangle e^t$
$\frac{d}{dx}\vec{r}g(t)) = \langle 2e^{2t}, -e^t \rangle$
$\square$

39

$\vec{r}(t) = \langle t, 1, 1 \rangle$
$\|\vec{r}'(t)\| = \|\langle 1, 0, 0 \rangle\| = 1$
$\|\vec{r}(t)\|' = \sqrt{t^2+2}' = t/\sqrt{t^2+2}$

1 does not always equal $t/\sqrt{t^2+2}$
$\square$

13.3

1

$\vec{r}(t) = \langle 3t, 4t-3, 6t+1 \rangle, \quad 0 \leq t \leq 3$
$\vec{r}'(t) = \langle 3, 4, 6 \rangle$

$\int\limits_0^3 \sqrt{3^2+4^2+6^2}dt$
$\int\limits_0^3 \sqrt{61}dt$
$3\sqrt{61}$
$\square$

5

$\vec{r}(t) = \langle t, 4t^{3/2}, 2t^{3/2} \rangle, \quad 0 \leq t \leq 3$
$\vec{r}'(t) = \langle 1, 6t^{1/2}, 3t^{1/2} \rangle$

$\int\limits_0^3 \sqrt{1+36t+9t}dt$
$\int\limits_0^3 \sqrt{1+45t}dt$
$2(1+45t)^{3/2}/135\Big|_0^3$

$2(136)^{3/2}/135 - 2/135$

23.482

□

9

$\vec{r}(t) = \langle \cos(7t), \sin(7t), 2\cos(t) \rangle, \quad 0 \le t \le 2\pi$

$\vec{r}'(t) = \langle -7\sin(7t), 7\cos(7t), -2\sin(t) \rangle$

$\int\limits_0^{2\pi} \sqrt{49\sin^2(7t) + 49\cos^2(7t) + 4\sin^2(t)}$

$\int\limits_0^{2\pi} \sqrt{49 + 4\sin^2(t)}$

$\int\limits_0^{2\pi} \sqrt{49 + 4(1 - \cos(2t))/2}$

$\int\limits_0^{2\pi} \sqrt{51 - 2\cos(2t)}$

$\approx \int\limits_0^{2\pi} \frac{\sqrt{49}+\sqrt{53}}{2} - 2\cos(2t)/(\sqrt{49} + \sqrt{53})$

$\frac{\sqrt{49}+\sqrt{53}}{2} t - \sin(2t)/(\sqrt{49} + \sqrt{53}) \Big|_0^{2\pi}$

$= \pi(\sqrt{49} + \sqrt{53}) - 0 - 0 + 0$

44.862

□

17

$\vec{r}(t) = \langle \sin 3t, \cos 4t, \cos 5t \rangle, \quad t = \pi/2$

$\vec{r}'(t) = \langle 3\cos 3t, -4\sin 4t, -5\sin 5t \rangle$

$s(t) = \sqrt{9\cos^2 3t + 16\sin^2 4t + 25\sin^2 5t}$

$s(\pi/2) = \sqrt{0 + 0 + 25}$

$= 5$

□

19

$y = x^2$

$y' = 2x$

$y'|_1 = 2$

$v = \langle 1, 2 \rangle$

$\|\langle 1, 2 \rangle\| = 500 \text{ km/h}$

$v = \langle 100\sqrt{5} \text{ km/h}, 200\sqrt{5} \text{ km/h} \rangle$

□