

Quantitative Skills 4: The Chi-Square Test



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“Goodness of Fit”

The **Chi-Square (χ^2) Test** is used to examine the difference between an actual sample and a hypothetical sample that would be expected due to chance.



Probably due to
chance

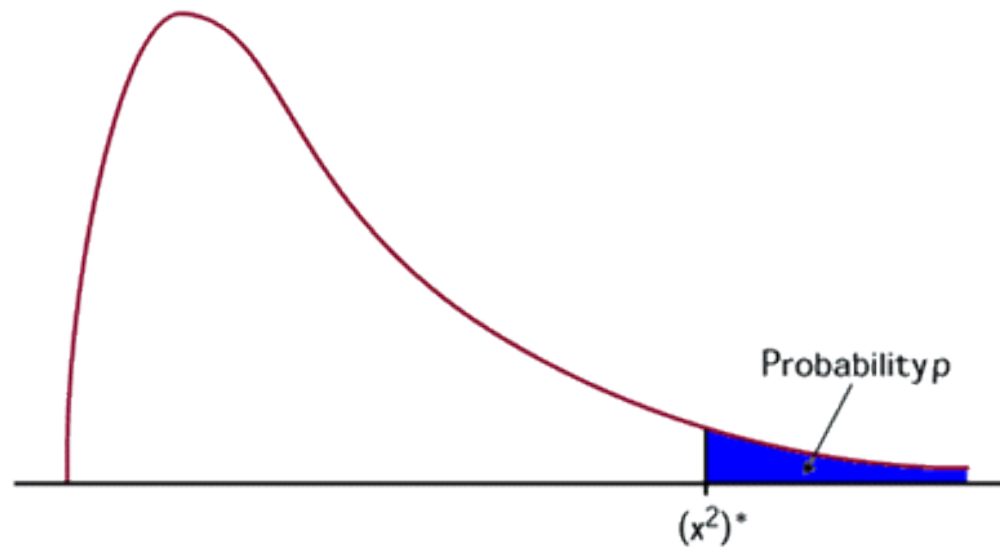


Possibly due to
chance

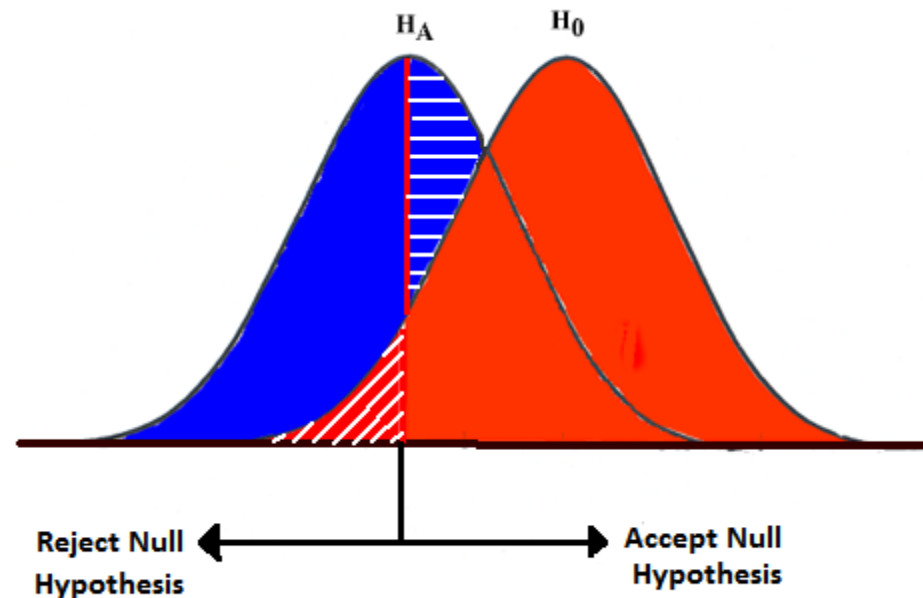


Probably not due to
chance

Using Chi-Square, it is possible to discern whether experimental results are valid, or whether they are probably due to chance alone.



The Chi-Square test compares two rival hypotheses (the **null hypothesis** and an **alternative hypothesis**) to see which hypothesis is best supported by the data.



Establishing a **null hypothesis** (H_0) and an **alternative hypothesis** (H_A)

- A null hypothesis states that there is **no relationship** between two variables.
 - The finding probably occurred by chance.
- An alternative hypothesis states that there **is** a relationship between two variables.
 - The finding probably did not occur by chance.

Example : “I think my cheese will mold if I leave it out on the counter too long.”

Example null hypothesis (H_0): If cheese is kept at room temperature for a week, then it will have the same amount of mold on it as the same amount of cheese kept in a refrigerator for a week.

Example alternative hypothesis (H_A): If cheese is kept at room temperature for a week, then it will have more mold on it than the same amount of cheese kept in a refrigerator for a week.



The goal of the Chi-Square Test is to either ***fail to reject*** or ***reject*** the null hypothesis.

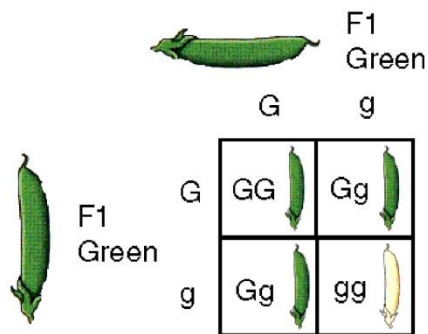
- If we **fail to reject** the null hypothesis, then there probably is **no relationship** between the two variables and the experimental results were probably due to chance alone.
- If we **reject** the null hypothesis, then there probably **is** a relationship between the two variables, and the experimental results are probably not due to chance.

Observed and Expected Results

- **Observed results** are what you actually observed in your experiment. These are the data you collect.
- **Expected results** are a theoretical prediction of what the data would look like if the experimental results are due only to chance.

How do you get expected results?

- If you are working with a genetics problem, then use the Punnett square ratio as your expected result.
- If you are working with a another type of problem, use probability.



$$P(\text{green}) = .75$$



$$P(\text{heads}) = .5$$

Obtaining the χ^2 value:

Example: We flip a coin 200 times to determine if the coin is fair.

H_0 : There is **no** statistically significant difference between our coin flips and what we would expect by chance. (The coin is fair.)

H_A : There **is** a statistically significant difference between our coin flips and what we would expect by chance. (The coin is not fair.)



The Chi-Square equation:

$$\chi^2 = \sum \frac{(o - e)^2}{e}$$

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$$\chi^2 = \text{(sum of all)} \quad \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

Example: We flip a coin 200 times to determine if a coin is fair.

Setting up this kind of table is a VERY good idea!

classes	Observed	Expected	$(o - e)$	$(o - e)^2$	$\frac{(o - e)^2}{e}$
Heads	108	100	8	64	.64
Tails	92	100	-8	64	.64
χ^2					1.28

Critical Value Table

Now you need to look up your χ^2 value in a critical value table to see if it is over a certain **critical value**.

Degrees of Freedom								
p	1	2	3	4	5	6	7	8
0.05	3.84	5.99	7.82	9.49	11.07	12.59	14.07	15.51
0.01	6.64	9.32	11.34	13.28	15.09	16.81	18.48	20.09

Typically, in biology we use the $p = 0.05$ confidence interval.

- **The p -value** is a predetermined choice of how certain we are. The smaller the p -value, the more confidence we can claim. $p = 0.05$ means that we can claim 95% confidence.

Degrees of Freedom								
p	1	2	3	4	5	6	7	8
0.05	3.84	5.99	7.82	9.49	11.07	12.59	14.07	15.51
0.01	6.64	9.32	11.34	13.28	15.09	16.81	18.48	20.09

Calculating **Degrees of Freedom**

Degrees of Freedom = # classes - 1

- In our example experiment, the classes were heads and tails (2 classes).
- Degrees of Freedom in our experiment would be:

$$\mathbf{DF = 2 - 1 = 1}$$

Degrees of Freedom								
p	1	2	3	4	5	6	7	8
0.05	3.84	5.99	7.82	9.49	11.07	12.59	14.07	15.51
0.01	6.64	9.32	11.34	13.28	15.09	16.81	18.48	20.09

Fail to reject or Reject the Null Hypothesis

- If the χ^2 value is **less than** the critical value, **fail to reject** the null hypothesis. (The difference is not statistically significant.)
- If the χ^2 value is **greater than or equal to** the critical value, **reject** the null hypothesis. (The difference is statistically significant.)

Degrees of Freedom								
p	1	2	3	4	5	6	7	8
0.05	3.84	5.99	7.82	9.49	11.07	12.59	14.07	15.51
0.01	6.64	9.32	11.34	13.28	15.09	16.81	18.48	20.09

In our example, the χ^2 value we calculated was 1.28, which is less than the critical value of 3.84. Therefore:

- We *fail to reject* our null hypothesis.
- We *reject* our alternative hypothesis .
- We determine that our coin is fair.

Degrees of Freedom								
p	1	2	3	4	5	6	7	8
0.05	3.84	5.99	7.82	9.49	11.07	12.59	14.07	15.51
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