a

$$egin{aligned} D &= \sum\limits_{n=1}^{N} \sum\limits_{k=1}^{K} r_{n,k} \|ec{x}_n - ec{\mu}_k\|^2 \ D &= \sum\limits_{n=1}^{N} \sum\limits_{k=1}^{K} \sum\limits_{i=1}^{I} r_{n,k} (x_{ni} - \mu_{ki})^2 \ rac{\partial D}{\partial \mu_{ki}} &= \sum\limits_{n=1}^{N} -2 r_{n,k} (x_{ni} - \mu_{ki}) = 0 \ rac{\sum\limits_{n=1}^{N} r_{n,k} x_{ni} - r_{n,k} \mu_{ki} = 0}{\sum\limits_{n=1}^{N} r_{n,k} x_{ni} - \sum\limits_{n=1}^{N} r_{n,k} \mu_{ki}} = 0 \ rac{\sum\limits_{n=1}^{N} r_{n,k} x_{ni} = \sum\limits_{n=1}^{N} r_{n,k} \mu_{ki}}{\sum\limits_{n=1}^{N} r_{n,k} x_{ni}} = \mu_{ki} \sum\limits_{n=1}^{N} r_{n,k} r_{n,k} \ rac{\sum\limits_{n=1}^{N} r_{n,k} x_{ni}}{\sum\limits_{n=1}^{N} r_{n,k} x_{ni}} = \mu_{ki} \ rac{\sum\limits_{n=1}^{N} r_{n,k} x_{ni}}{\sum\limits_{n=1}^{N} r_{n,k}} = \mu_{ki} \end{aligned}$$

b

$$egin{align} D &= \sum\limits_{n=1}^{N} \sum\limits_{k=1}^{K} r_{n,k} \| ec{x}_n - ec{\mu}_k \|^2 \ &rac{\partial D}{\partial ec{\mu}_k} = -2 \sum\limits_{n=1}^{N} r_{n,k} (ec{x}_n - ec{\mu}_k) = ec{0} \ &\sum\limits_{n=1}^{N} r_{n,k} (ec{x}_n - ec{\mu}_k) = ec{0} \ &\sum\limits_{n=1}^{N} r_{n,k} ec{x}_n = \sum\limits_{n=1}^{N} r_{n,k} ec{\mu}_k \end{aligned}$$

$$\sum\limits_{n=1}^{N} r_{n,k} ec{x}_n = ec{\mu}_k \sum\limits_{n=1}^{N} r_{n,k} \ rac{\sum\limits_{n=1}^{N} r_{n,k} ec{x}_n}{\sum\limits_{n=1}^{N} r_{n,k}} = ec{\mu}_k$$

2

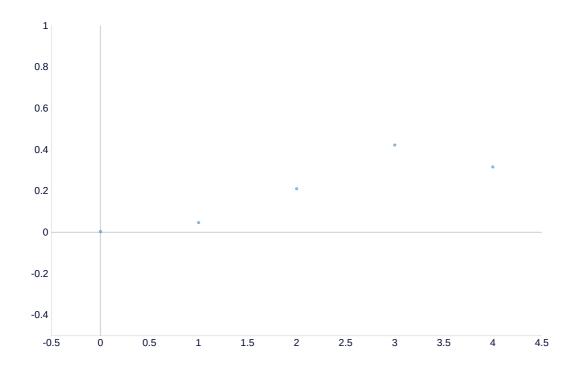
a

$$P(\theta|y,n) = P(y|\theta,n)P(\theta|n)/P(y|n)$$

$$egin{aligned} P(heta|y,n) &= inom{n}{y} heta^y (1- heta)^{n-y} rac{1}{1} / rac{1}{1+n} \ P(heta|y,n) &= (1+n) inom{n}{y} heta^y (1- heta)^{n-y} \end{aligned}$$

b

with
$$n=4$$
, $\theta=3/4$



C

1.
$$y = 1$$
, $n = 1$

2.
$$y = 2$$
, $n = 2$

3.
$$y = 2$$
, $n = 3$

3

$$egin{aligned} f(x|\mu,\sigma^2) &= rac{1}{(2\pi\sigma^2)^{1/2}} e^{-rac{1}{2\sigma^2}(x-\mu)^2} \ f(ec x|\mu,\sigma^2) &= \prod_{n=0}^N f(ec x_n|\mu,\sigma^2) \ f(ec x|\mu,\sigma^2) &= \prod_{n=0}^N rac{1}{(2\pi\sigma^2)^{1/2}} e^{-rac{1}{2\sigma^2}(ec x_n-\mu)^2} \ \ln f &= \ln(\prod_{n=0}^N rac{1}{(2\pi\sigma^2)^{1/2}} e^{-rac{1}{2\sigma^2}(ec x_n-\mu)^2}) \ \ln f &= \sum_{n=0}^N \ln(rac{1}{(2\pi\sigma^2)^{1/2}} e^{-rac{1}{2\sigma^2}(ec x_n-\mu)^2}) \ \ln f &= \sum_{n=0}^N -rac{1}{2} \ln(2\pi\sigma^2) - rac{1}{2\sigma^2}(ec x_n-\mu)^2 \end{aligned}$$

a

$$0 = \frac{\partial \ln f}{\partial \mu} = \frac{\partial}{\partial \mu} \sum_{n=0}^{N} -\frac{1}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} (\vec{x}_n - \mu)^2$$

$$= -\frac{1}{2\sigma^2} \frac{\partial}{\partial \mu} \sum_{n=0}^{N} (\vec{x}_n - \mu)^2$$

$$= -\frac{1}{2\sigma^2} \sum_{n=0}^{N} -2(\vec{x}_n - \mu)$$

$$= \frac{1}{\sigma^2} \sum_{n=0}^{N} (\vec{x}_n - \mu)$$

$$\implies 0 = \sum_{n=0}^{N} \vec{x}_n - \sum_{n=0}^{N} \mu$$

$$\implies 0 = \sum_{n=0}^{N} \vec{x}_n - N\mu$$

$$\implies \mu = \frac{1}{N} \sum_{n=0}^{N} \vec{x}_n$$

b

$$0 = \frac{\partial \ln f}{\partial \sigma} = \frac{\partial}{\partial \sigma} \sum_{n=0}^{N} -\frac{1}{2} \ln(2\pi\sigma^{2}) - \frac{1}{2\sigma^{2}} (\vec{x}_{n} - \mu)^{2}$$

$$= \sum_{n=0}^{N} \frac{\partial}{\partial \sigma} \frac{1}{2} \ln(2\pi\sigma^{2}) + \sum_{n=0}^{N} \frac{\partial}{\partial \sigma} \frac{1}{2\sigma^{2}} (\vec{x}_{n} - \mu)^{2}$$

$$= \sum_{n=0}^{N} \frac{1}{2} \frac{4\pi\sigma}{2\pi\sigma^{2}} + \sum_{n=0}^{N} \frac{-2}{2\sigma^{3}} (\vec{x}_{n} - \mu)^{2}$$

$$= \sum_{n=0}^{N} \frac{1}{\sigma} - \sum_{n=0}^{N} \frac{1}{\sigma^{3}} (\vec{x}_{n} - \mu)^{2}$$

$$= \frac{N}{\sigma} - \frac{1}{\sigma^{3}} \sum_{n=0}^{N} (\vec{x}_{n} - \mu)^{2}$$

$$\implies \frac{N}{\sigma} = \frac{1}{\sigma^{3}} \sum_{n=0}^{N} (\vec{x}_{n} - \mu)^{2}$$

$$\implies N\sigma^{2} = \sum_{n=0}^{N} (\vec{x}_{n} - \mu)^{2}$$

$$\implies \sigma^{2} = \frac{1}{N} \sum_{n=0}^{N} (\vec{x}_{n} - \mu)^{2}$$

$$\implies \sigma^{2} = \frac{1}{N} \sum_{n=0}^{N} (\vec{x}_{n} - \mu)^{2}$$

4

a

$$X \in \{C_1,C_2\}$$

$$P(X = C_1) = 2P(X = C_2)$$

$$egin{aligned} 1 &= \sum_{x \in \{C_1, C_2\}} P(X = x) \ &= P(X = C_1) + P(X = C_2) \ &= 3P(X = C_2) \ &\Longrightarrow egin{cases} P(X = C_1) &= rac{2}{3} \ P(X = C_2) &= rac{1}{3} \end{aligned}$$

| | C_1 | C_2 |
|---|---------------|---------------|
| X | $\frac{2}{3}$ | $\frac{1}{3}$ |

b

Assuming $\mu_1 < \mu_2$,

$$E=P(X_1> heta\cap X\in C_1)+P(X_2< heta\cap X\in C_2) \ E=P(X_1> heta)P(X\in C_1)+P(X_2< heta)P(X\in C_2)$$

$$egin{aligned} X_n &\sim N(\mu_n, \sigma_n) \ P(X_1 > heta) = 1 - F_{X_1}(heta) \ P(X_2 < heta) = F_{X_2}(heta) \end{aligned}$$

$$egin{aligned} F_N(x) &= \Phi(x) \ F_{X_n}(x) &= F_N(rac{x-\mu}{\sigma}) &= \Phi(rac{x-\mu_n}{\sigma_n}) \end{aligned}$$

$$E = rac{2}{3}(1 - \Phi(rac{\theta - \mu_1}{\sigma_1})) + rac{1}{3}(\Phi(rac{\theta - \mu_2}{\sigma_2})) \ = 2/3 - 2\Phi(rac{\theta - \mu_1}{\sigma_1})/3 + \Phi(rac{\theta - \mu_2}{\sigma_2})/3$$

C

Let
$$N(z)=rac{1}{\sqrt{2\pi}}e^{-rac{1}{2}x^2}$$
 $rac{d\Phi}{dz}=N(z)$

$$egin{aligned} 0 &= rac{\partial E}{\partial heta} = 2/3 - 2\Phi(rac{ heta-\mu_1}{\sigma_1})/3 + \Phi(rac{ heta-\mu_2}{\sigma_2})/3 \ &= N(rac{ heta-\mu_2}{\sigma_2})/3\sigma_2 - 2N(rac{ heta-\mu_1}{\sigma_1})/3\sigma_1 \end{aligned}$$

Where

$$egin{align} A &= rac{1}{2\sigma_2^2} - rac{1}{2\sigma_1^2} \ B &= rac{1}{2\sigma_1^2} \mu_1 - rac{1}{2\sigma_2^2} \mu_2 \ C &= rac{1}{2\sigma_2^2} \mu_2^2 - rac{1}{2\sigma_1^2} \mu_1^2 - \ln(rac{\sigma_1}{\sigma_2}) \ \end{array}$$

$$heta = rac{-B\pm\sqrt{B^2-4AC}}{2A}$$