HW 4

2.23

The probability that there is no accident at a certain busy intersection is 95% on any given day, independently of the other days.

a

Find the probability that there will be no accidents at this intersection during the next 7 days.

Answer

$P(7 \text{ days no accidents}) = 0.95^7$
$= 69.83\%$

b

Find the probability that next September there will be exactly 2 days with accidents.

Answer

30 days
$X \sim \text{Binomial}(30, 0.05)$
$P(X = 2) = 25.86\%$

c

Today was accident free. Find the probability that there is no accident during the next 4 days, but there is at least one by the end of the 10th day.

Answer

$P(4 \text{ days no accidents}) = 0.95^4 = 81.45\%$
$X \sim \text{Binomial}(6, 0.05)$
$P(X \geq 1) = 26.49\%$
$P(4 \text{ days no accidents} \cap X \geq 1) = 21.58\%$

2.28

We play a card game where we receive 13 cards at the beginning out of the deck of 52. We play 50 games one evening. For each of the following random variables identify the name and the parameters of the distribution.

a

The number of aces I get in the first game.

> ✓ **Answer**
> $A \sim \text{Hypergeometric}(52, 13, 4)$

b

The number of games in which I receive at least one ace during the evening.

> ✓ **Answer**
> $B \sim \text{Binomial}(50, P(A \geq 1))$

c

The number of games in which all my cards are from the same suit.

> ✓ **Answer**
> $C \sim \text{Binomial}(50, 4 \cdot P(\text{Hypergeometric}(52, 13, 13) = 13))$

d

The number of spades I receive in the 5th game.

> ✓ **Answer**
> $D \sim \text{Hypergeometric}(52, 13, 13)$

2.47

A medical trial of 80 patients is testing a new drug to treat a certain condition. This drug is expected to be effective for each patient with probability p, independently of the other patients. You personally have two friends in this trial. Given that the trial is a success for 55 patients, what is the probability that it was successful for both of your two friends?

> ✓ **Answer**
> $X_{\text{likelihood}} \sim \text{Binomial}(80, p)$
> $p_{\text{prior}} \sim \text{Beta}(1, 1)$
> $A \sim p_{\text{posterior}} X_{\text{likelihood}} = \text{Beta}(56, 26)$

$P(\text{Bimonial}(2, A) = 2)$
$= E(A^2) = V(A) - E(A)^2 = \frac{(56)(26)}{(56+26)^2(26+26+1)} + \left(\frac{56}{56+26}\right)$
$= 47.0474672533\%$

2.66

You are given a fair die. You must decide ahead of time how many times to roll. If you get exactly 2 sixes, you get a prize. How many rolls should you take to maximize your chances and what are the chances of winning? There are two equally good choices for the best number of rolls.

✓ **Answer**

$X = \text{NegBinomial}\left(2, \frac{1}{6}\right) + 2$
$\text{Max}(X) = 6, 7 : P(X) = 6.70\%$

2.71

As in Example 2.38, assume that 90% of the coins in circulation are fair, and the remaining 10% are biased coins that give tails with probability 3/5. I hold a randomly chosen coin and begin to flip it.

a

After one flip that results in tails, what is the probability that the coin I hold is a biased coin? After two flips that both give tails? After n flips that all come out tails?

✓ **Answer**

$P(T|\text{fair}) = 0.5$
$P(T|\text{biased}) = 0.6$
$P(\text{fair}) = 0.9$
$P(T) = 0.51$

$P(\text{biased}|\text{n flips}) = \frac{P(\text{n flips}|\text{biased})P(\text{biased})}{P(\text{n flips})}$
$= \frac{(0.6^n)(0.1)}{0.9(0.5)^n + 0.1(0.6)^n}$

$|_{n=1} = 11.76\%$
$|_{n=2} = 13.79\%$
$|_{n\to\infty} = 1$

b

After how many straight tails can we say that with 90% probability the coin I hold is biased?

✓ **Answer**

$P(\text{biased}|\text{n flips}) = 0.9$

Solved graphically with $n = 24.1027$

We can say with 90% surity once we get 25 flips.

c

After n straight tails, what is the probability that the next flip is also tails?

✓ **Answer**

$P(\text{biased}|\text{n flips}) = \frac{(0.6^n)(0.1)}{0.9(0.5)^n+0.1(0.6)^n}$

$P(T|\text{n flips}) = 0.5(1 - P(\text{biased}|\text{n flips})) + 0.6P(\text{biased}|\text{n flips})$

$= \frac{0.9(0.5)^{n+1}+0.1(0.6)^{n+1}}{0.9(0.5)^n+0.1(0.6)^n}$

d

Suppose we have flipped a very large number of times (think number of flips n tending to infinity), and each time gotten tails. What are the chances that the next flip again yields tails?

✓ **Answer**

$|_{n\to\infty} = 0.6$

2.81

A Martian year is 669 Martian days long. Solve the birthday problem for Mars. That is, for each n find the probability that among n randomly chosen Martians there are at least two with the same birthday. Estimate the value of n where this probability becomes larger than 90%.

✓ **Answer**

Out of the d^n ways to arrange n birthdays, only $\frac{d!}{(d-n)!}$ don't have repeated birthdays.

$\implies P(\text{Birthday Collision}) = \frac{d!}{(d-n)!d^n}$

$n = 56$ solved with brute force in python

```
def collision(d, n):
    return 1 - reduce(lambda a, b: a*b, range(d-n+1, d+1))/pow(d, n)
```

```python
[(i, collision(669, i)) for i in range(2, 100)]
```