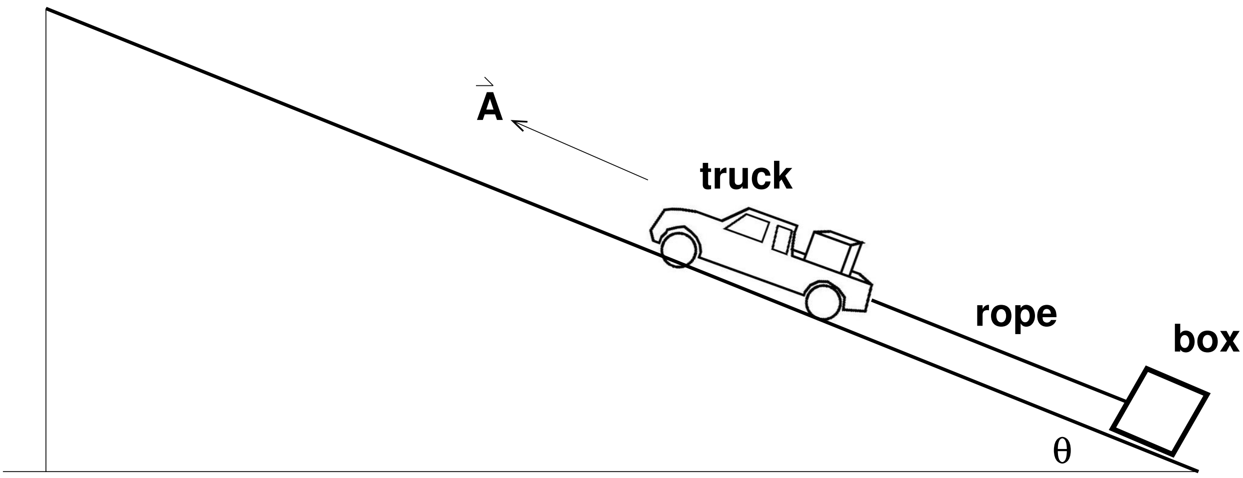
1

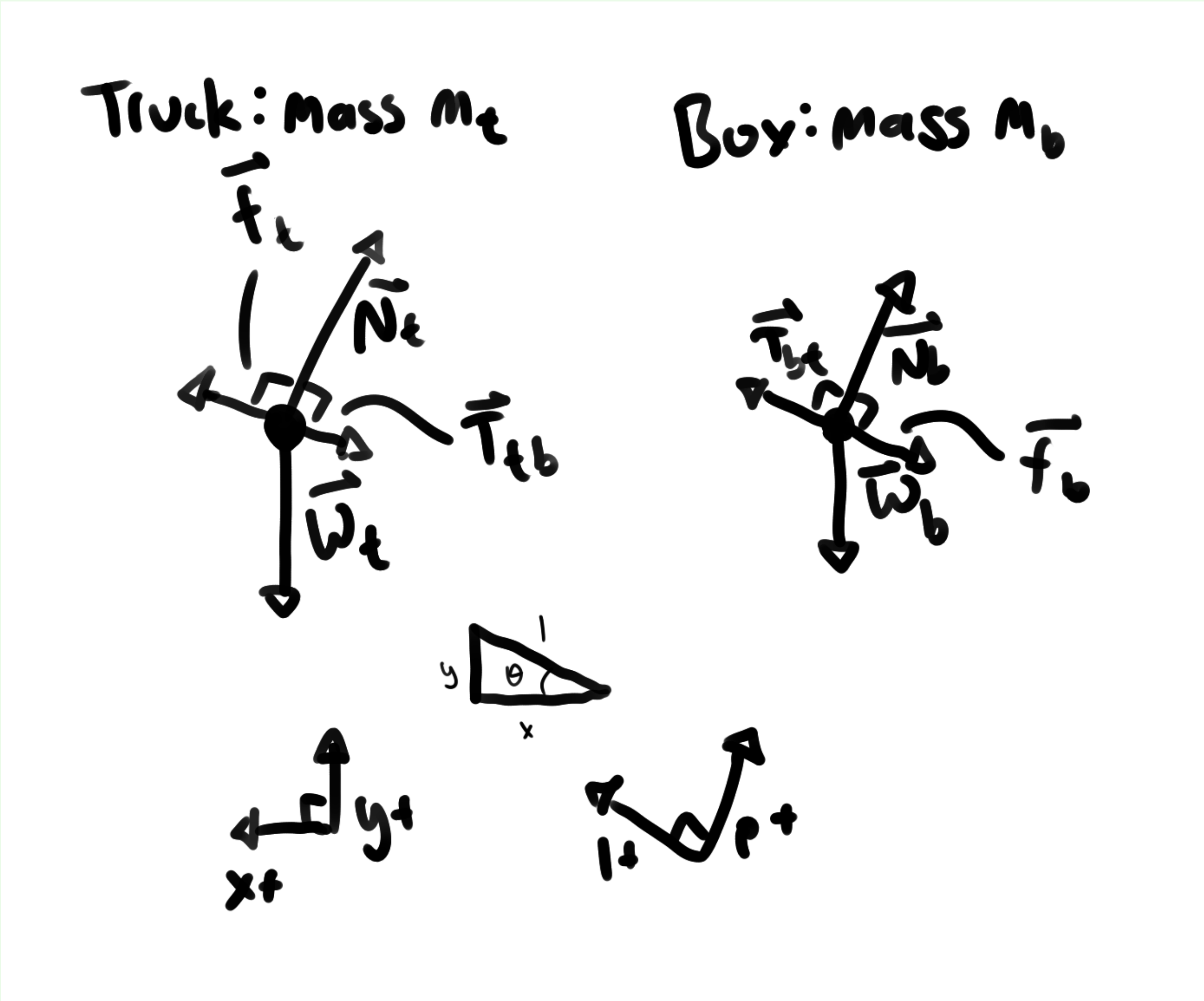
A truck pulls a box up a ramp at a constant given constant acceleration A as shown above with an ideal rope. The angle of the ramp is given as θ. The wheels of the truck pull the truck up with no slipping and no sliding. The box is greased on the bottom surface so that there is a given coefficient of sliding friction between the box and the ramp of μ. The mass of the truck is given as m_t and the mass of the box is given as m_b.



a

Draw a careful Free Body Diagram for both the truck and the box. Include a proper coordinate system and be sure to indicate every force on each body. Make very sure you have each force clearly and properly labeled. At this time we expect all students use the format and notation given in this class for every FBD.

✓ Answer



b

In terms of given parameters A, mt , mb , μ and θ, determine the magnitude of each and every force on both the truck and the box. Note: thee are four forces on each. Be sure you find all eight forces and express your answers in terms of the given parameters.

✓ **Answer**

$W_t = m_t g$ N2L
$W_b = m_b g$ N2L

Since there is only acceleration in the $l+$ direction, there is no acceleration in the $p+$ direction because the directions are perpendicular
$N_t = W_t \cos\theta$
$N_b = W_b \cos\theta$

$N_t = m_t g \cos\theta$
$N_b = m_b g \cos\theta$

$f_b = \mu N_b$
$f_b = \mu m_b g \cos\theta$

$F_{bl} = T_{bt} - f_b - W_b \sin\theta$ Sum all forces in the $l+$ direction for b
$F_{bl} = m_b A$ N2L
$T_{bt} - f_b - W_b \sin\theta = m_b A$
$T_{bt} - \mu m_b g \cos\theta - m_b g \sin\theta = m_b A$
$T_{bt} = m_b(\mu g \cos\theta + g \sin\theta + A)$

$T_{tb} = T_{bt}$ N3L
$T_{tb} = m_b(\mu g \cos\theta + g \sin\theta + A)$

$F_{tl} = f_t - T_{tb} - W_t \sin\theta$ Sum all forces in the $l+$ direction for t
$F_{tl} = m_t A$ N2L
$f_t - m_b(\mu g \cos\theta + g \sin\theta + A) - m_t g \sin\theta = m_t A$
$f_t = m_b(\mu g \cos\theta + g \sin\theta + A) + m_t(g \sin\theta + A)$

$W_t = m_t g$
$W_b = m_b g$
$N_t = m_t g \cos\theta$
$N_b = m_b g \cos\theta$
$f_b = \mu m_b g \cos\theta$
$T_{bt} = m_b(\mu g \cos\theta + g \sin\theta + A)$
$T_{tb} = m_b(\mu g \cos\theta + g \sin\theta + A)$
$f_t = m_b(\mu g \cos\theta + g \sin\theta + A) + m_t(g \sin\theta + A)$
$\square$

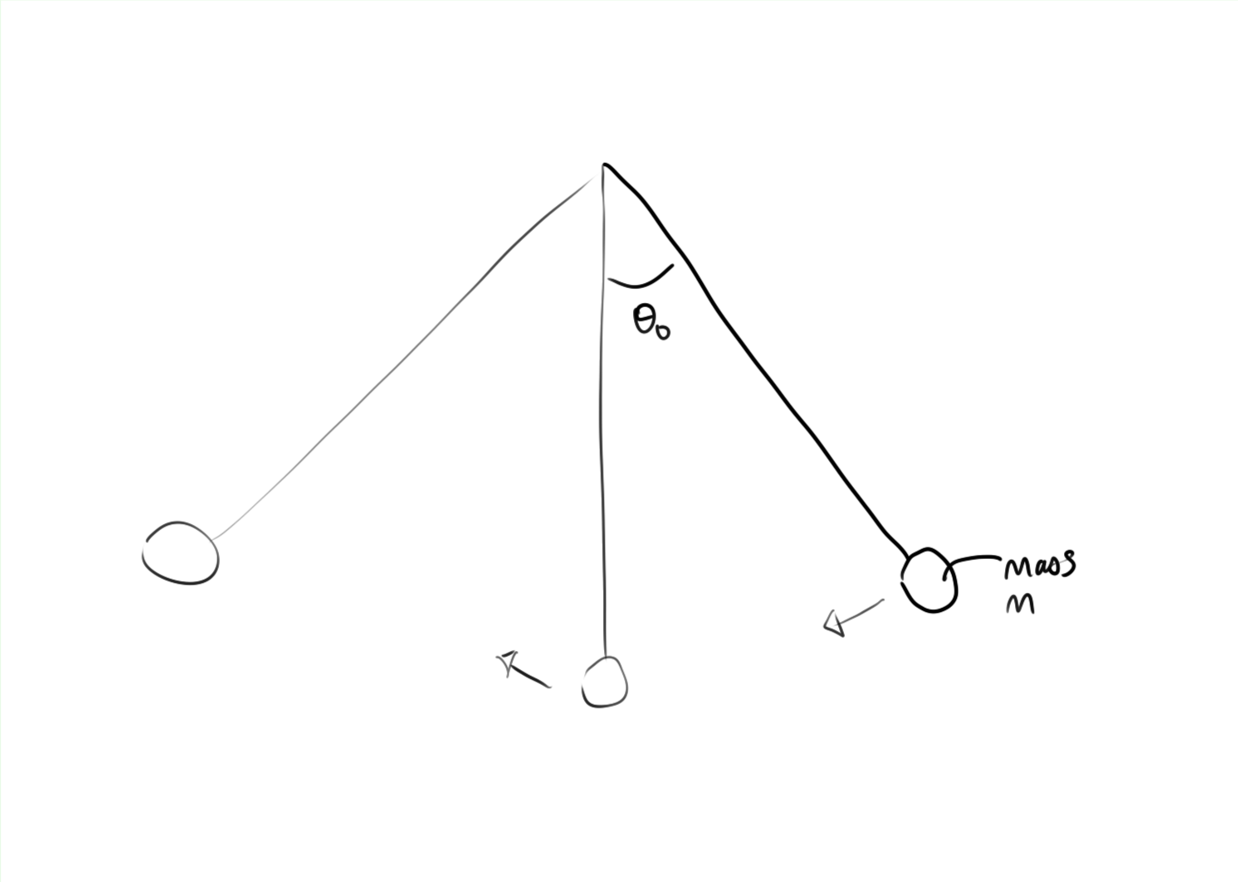
2

The Simple Pendulum represents a rich system for checking our understanding of forces, work, and energy. This from a previous final exam:
A pendulum is made from a bob of mass m and an ideal string of length l. Suppose the bob is pulled back and released from rest at a position with the string at an angle of θ_0 relative to the vertical where θ_0 has some particular but arbitrary value less than 90°. For all of these questions give your answer in terms of the given parameters m, and θ_0.

a

Draw a careful sketch of what is going on here.

✓ **Answer**



b

What is the magnitude of the acceleration of the bob immediately after the bob is released? Hint: no it is not zero. Another hint. If you choose the right coordinate system this problem is relatively simple.

✓ **Answer**

$W_p = mg$

There is only acceleration in the tangential direction t as the string restrains it from moving in the centrifugal direction.
There is also no centripetal acceleration as the initial velocity is 0

$a_p = \frac{F_p}{m}$ N2L
$a_p = \frac{mg\sin\theta_0}{m}$
$a_p = g\sin\theta_0$

$a_p = g\sin\theta_0$
$\square$

c

What is the tension on the string immediately after the bob is released? No, it is not zero. No, it is not mg.
Hint: What kind of acceleration applies in the direction of the string?

✓ **Answer**

We can use force separation to separate the weight force into tangential and centrifugal. The centrifugal forces cancel as there is no acceleration in the centrifugal direction.

$$a_{pc} = 0$$
$$0 = F_{pc}$$
$$0 = T_p - mg\cos\theta$$
$$T_p = mg\cos\theta$$

$$T_p = mg\cos\theta$$

□

d

What is the Work done by the force of Tension as the bob falls from its initial position to the position corresponding to the bottom of the arc?

✓ **Answer**

Zero, as there is never any movement in the centrifugal direction, and the tension will always be in the centrifugal direction.

$$W = F \cdot d$$
$$F \perp d$$
$$W = 0$$

$$W = 0$$

□

e

What is the Work done by the Weight force as the bob falls from its initial position to the position corresponding to the bottom of the arc?

✓ **Answer**

$W = W_m \cdot d$
$W = W_m d_l$ Where d_l is the distance traveled parallel to force F
$d_l = l - l\cos\theta$ by geometric trig

$W = mg(l - l\cos\theta)$

$W = mgl(1 - \cos\theta)$
□

f

What is the maximum speed V of the bob during the swing of the pendulum?

✓ **Answer**

The total energy in the system stays the same with an exception to work by an external force. In this case the only work is done by gravity.

We start at rest, so $W_i = 0$
$K_f = K_i + W$
$K_f = W$
$K_f = mgl(1 - \cos\theta)$
$K = mv^2$
$v^2 = gl(1 - \cos\theta)$
$v = \sqrt{gl(1 - \cos\theta)}$

$V = \sqrt{gl(1 - \cos\theta)}$
□

g

What is the magnitude of the force of Tension on the string when the bob is at the position corresponding to the bottom of the arc? Hint: No, $T = mg$ is not the correct answer.

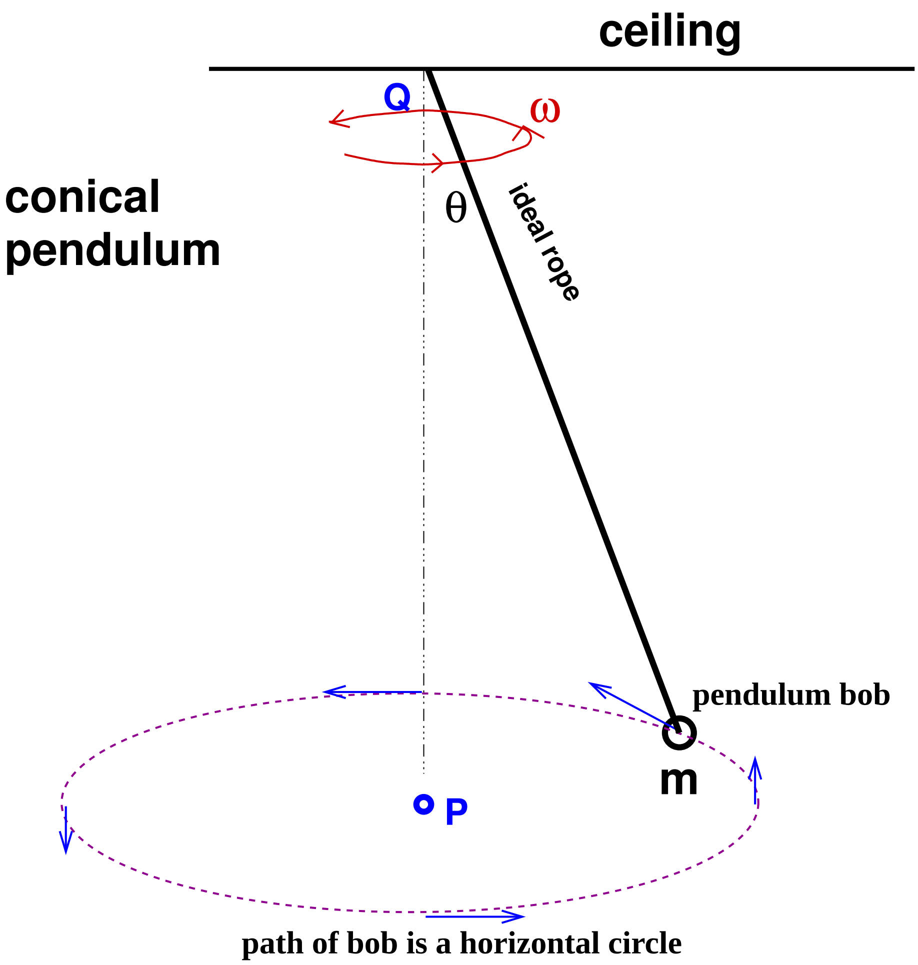
✓ **Answer**

$a_c = \frac{v^2}{l}$
$a_c = g(1 - \cos\theta)$
$F_c = \frac{a_c}{m}$
$F_c = T_m - W_m$
$T_m = \frac{g(1-\cos\theta)}{m} + mg$

$T_m = \frac{g(1-\cos\theta)}{m} + mg$
$\square$

3

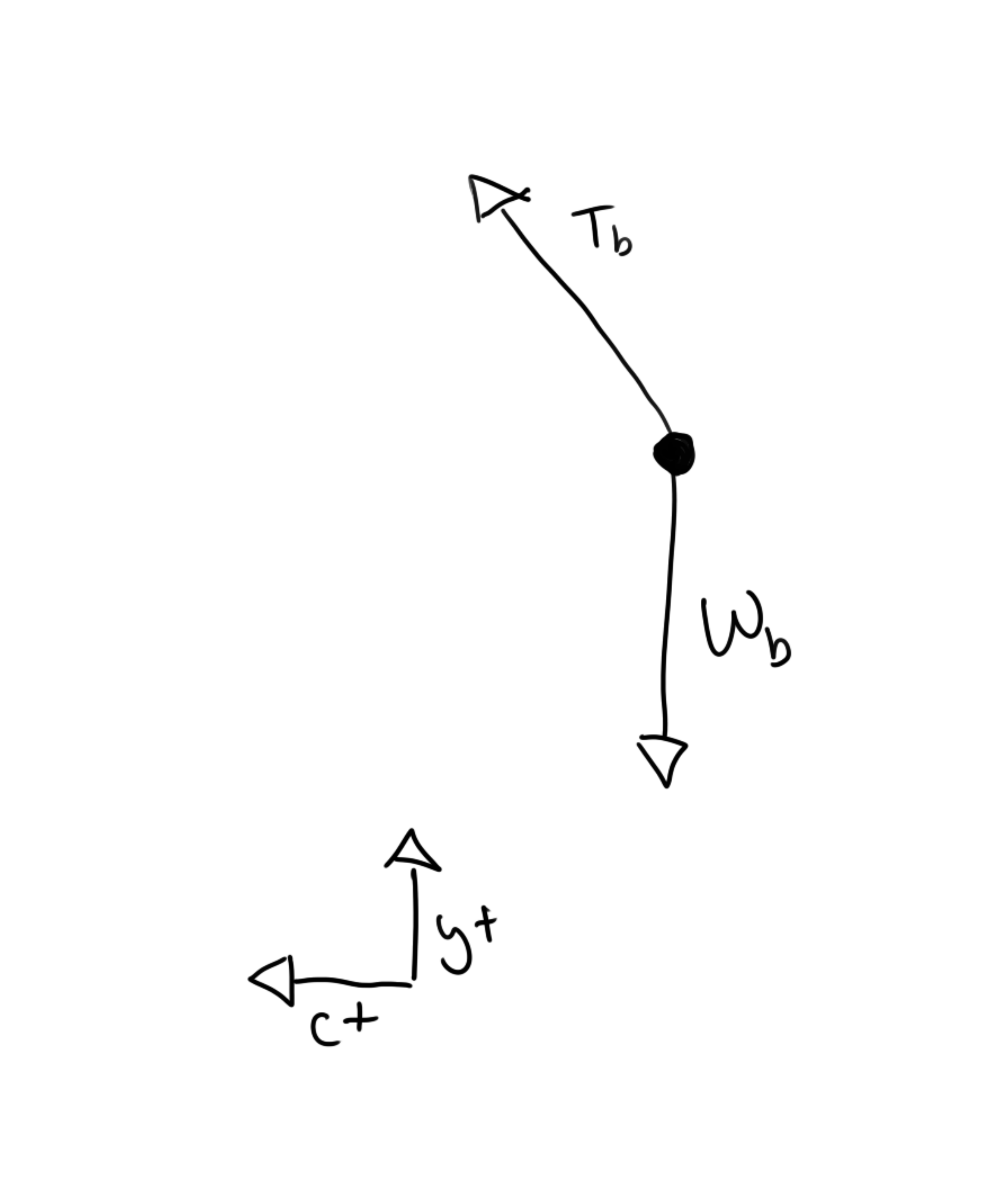
A “Conical Pendulum” is shown in the figure above. The bob of given mass m travels in a horizontal circular path. The point P indicate the point at the center of the circular path of the bob as shown. The angle of the ideal string with respect to the vertical is constant and given as θ. The length of the string l is also given.



a

Draw a careful and complete Free-Body-Diagram showing all of the forces on the bob. Be sure to include an appropriately labeled coordinate system.

✓ **Answer**



b

Calculate the magnitude of the force of Tension on the string. Explain your work. Give your answer in terms of the given parameters.

Hint: Apply Newton's Second Law in the vertical Component.

✓ **Answer**

There is no acceleration in the vertical direction

$a_y = 0$

$F_{by} = 0$ N2L

$0 = W_b - T_b \cos\theta$

$T_b = \frac{W_b}{\cos\theta}$

$W_b = mg$ Gravitation

$T_b = \frac{mg}{\cos\theta}$

$T_b = \frac{mg}{\cos\theta}$

□

c

Calculate the angular speed ω of the conical pendulum. Explain your work. Give your answer in terms of the given parameters.

Hint: Apply Newton's Second Law in the centripetal component. Use the Rolling Constraint: $v = \omega r$ that relates the (linear) speed of an object with Uniform Circular Motion to the angular speed of rotation about the axis.

✓ **Answer**

$a_c = r\omega^2$

$F_c = T_b \sin\theta$ Sum of all forces in $c+$ direction

$F_c = mg\tan\theta$

$a_c = g\tan\theta$

$r = l\sin\theta$

$l\omega^2 \sin\theta = g\tan\theta$

$\omega^2 = \frac{g}{l\cos\theta}$

$\omega = \sqrt{\frac{g}{l\cos\theta}}$

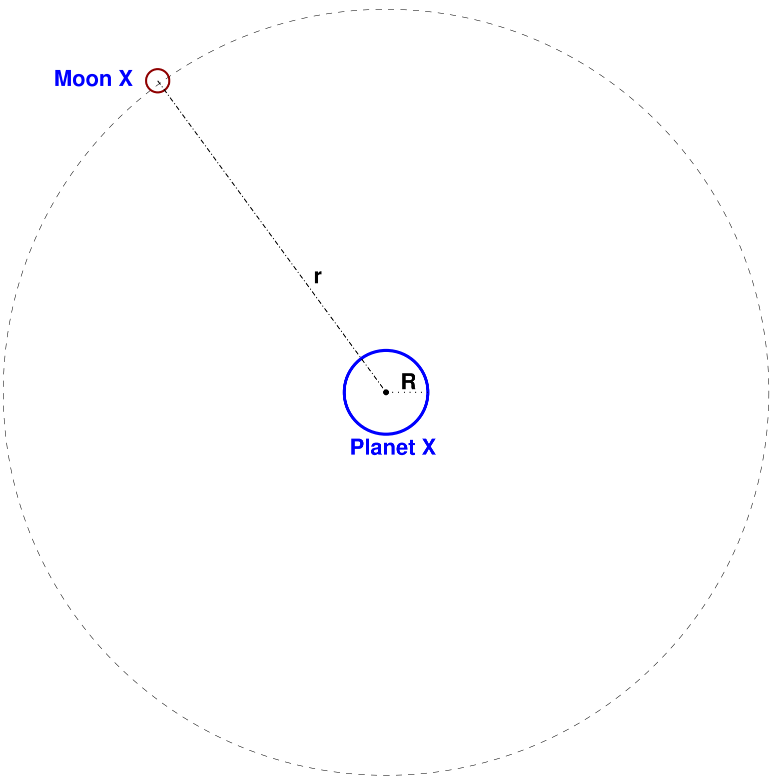
$\omega = \sqrt{\frac{g}{l\cos\theta}}$

□

4

Suppose a new planet is discovered at a great distance from the sun in deep space. The new planet is called Planet X. Suppose further that Planet X has moon, called Moon X, that orbits the planet in a circular orbit. Assume that the mass of the planet is quite large compared to the mass of the moon.

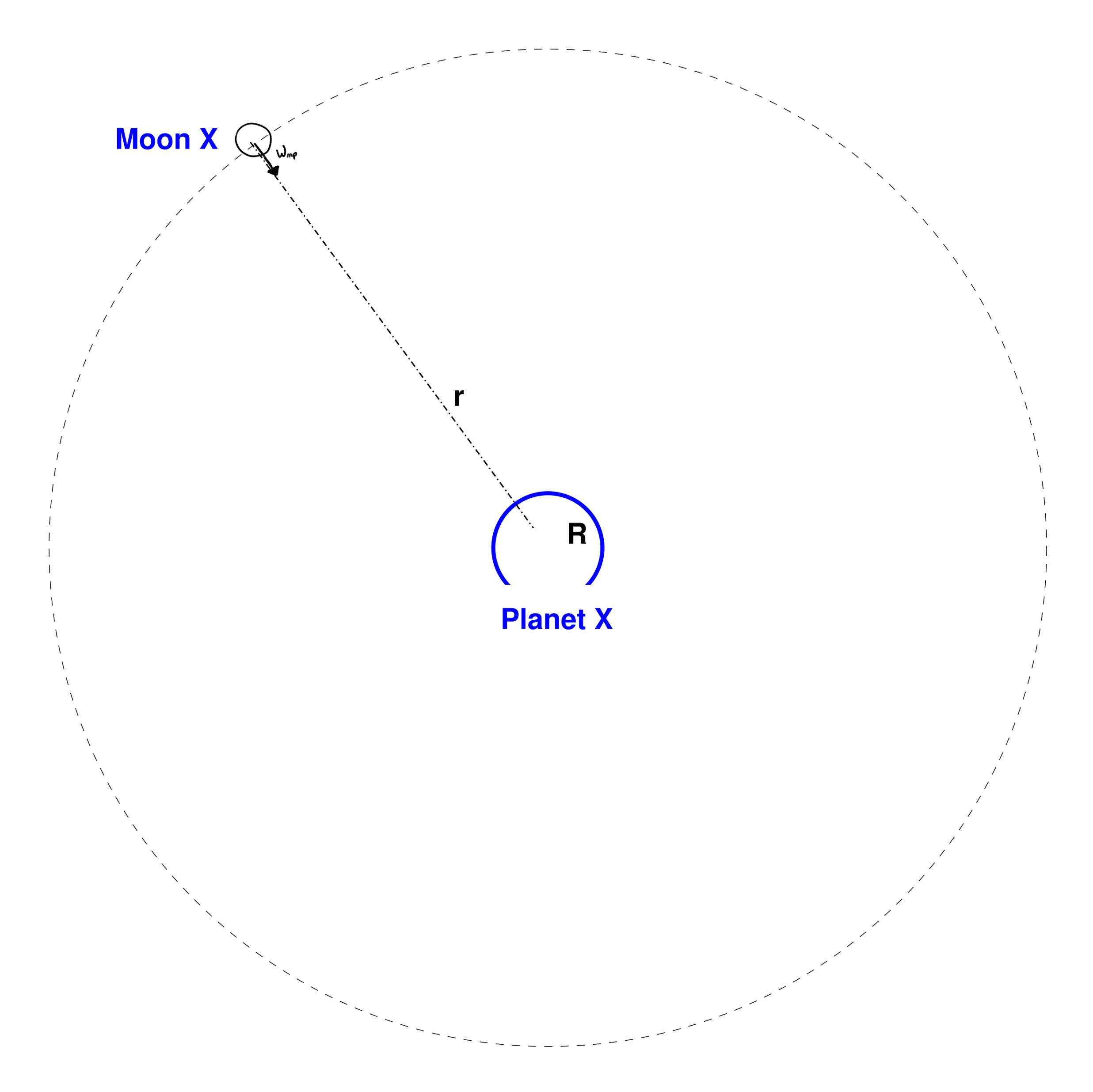
Suppose astronomers measure the radius of Planet X to be $R = 5.20 \times 105\ m$, and the orbital radius of Moon X to be $r = 4.61 \times 106\ m$, as shown. Suppose also that the orbital period of Moon X P is measured to be $14.32\ h$. Assume that the mass of Planet X is known to be much larger than the mass of Moon X. In other words $M >> m$.



a

On the figure, draw a vector to indicate the direction of the gravitational force on Moon X. Explain how you know this.

✓ **Answer**



By the law of universal gravitation, and planet X is the only other large mass

b

In terms of the given parameters, can you calculate the mass M of Planet X? Here, start from First Principles assuming Uniform Circular Motion and Newton's Law of Universal Gravity. Give your answers both in terms of symbolic constants and numerically. Explain how you calculated this. Show your work.

✓ Answer

$F_c = G\frac{m_1 m_2}{r^2}$ Law of universal gravitation
$a_c = \frac{v^2}{r}$ Centripetal acceleration
$v = \frac{2\pi r}{P}$
$a_c = \frac{4\pi^2 r}{P^2}$

$a_c = \frac{F_c}{m_2}$ N2L
$a_c = G\frac{m_1}{r^2}$

$\frac{4\pi^2 r}{P^2} = G\frac{m_1}{r^2}$
$m_1 = \frac{4\pi^2 r^3}{GP^2}$

$m_1 = \frac{4\pi^2 r^3}{GP^2}$
$m_1 = \frac{4\pi^2(4.61\times10^6)^3}{6.67430\times10^{-11}(14.32(3600))^2}$
$m_1 = 2.18 \times 10^{22}\ kg$

$m_1 = \frac{4\pi^2 r^3}{GP^2}$
$m_1 = 2.18 \times 10^{22}\ kg$
□

C

Suppose you are standing on Planet X and you drop your mobile phone. What is the observed acceleration due to the gravity of Planet X on that phone? Explain your work.

✓ **Answer**

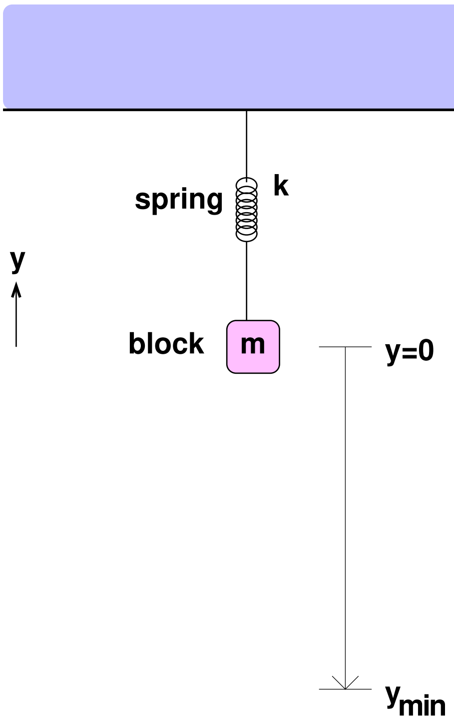
$F_c = G\frac{m_1 m_2}{R^2}$ Law of universal gravitation
$F_c = m_2 a_c$
$a_c = G\frac{\frac{4\pi^2 r^3}{GP^2}}{R^2}$
$a_c = \frac{4\pi^2 r^3}{R^2 P^2}$

$a_c = \frac{4\pi^2 r^3}{R^2 P^2}$
$a_c = \frac{4\pi^2(4.61\times10^6)^3}{(5.20\times10^5)^2(14.32(3600))^2}$
$a_c = 5.38\ \frac{m}{s^2}$

$a_c = \frac{4\pi^2 r^3}{R^2 P^2}$
$a_c = 5.38\ \frac{m}{s^2}$
□

5

A block of given mass m is released at rest at a position $y = 0$ where it is connected to an ideal spring with a given spring constant k. At the release, the spring is at its equilibrium position (neither compressed nor extended). Once released, the block falls vertically and the spring stretches.



a

What is y_{min} corresponding to the minimum position of the block after release?
Express your answer in terms of the given parameters. Explain your work. Hint: Note that y_{min} is negative.

✓ Answer

$U_s = 0.5kx^2$ Potential energy of a spring
$U_g = -mgx$ Gravitational Potential

$U_{si} = 0.5k(0)^2 = 0$
$U_{gi} = mg(0) = 0$

$U_{sf} = 0.5ky_{min}^2$
$U_{gf} = mg(y_{min})$

By conservation of energy, assuming no work,
$-mgy_{min} = 0.5ky_{min}^2$
$y_{min} = -\frac{2mg}{k}$

$y_{min} = -\frac{2mg}{k}$

☐

b

What is the magnitude and direction of the acceleration of the block when the block is at the position y_{min} as calculated in Part (a)? Express your answer in terms of the given parameters. Explain your work. Hint: no, the acceleration here is not zero.

✓ **Answer**

$F = F_{bs} - W_b$
$W_b = mg$
$F_{bs} = -kx$
$F_{bs} = -k\left(-\frac{2mg}{k}\right)$
$F_{bs} = 2mg$

$F = mg$
$a = \frac{F}{m}$
$a = g$ Upwards

$a = g$ Upwards

☐

c

What is y_{max} , corresponding to the position of the block when it has maximum speed? Express your answer in terms of the given parameters. Explain your work. Hint: You do not need to find the actual value of the maximum speed. Also Conservation of Energy is not what you want. Instead, think about the net force on the block as a function of position.

✓ **Answer**

Since the motion of the block is sinusoidal as it is a reciprocating spring, velocity is also represented by a sinusoidal function, offset earlier by a quarter of the period.

The maximum velocity thus corresponds to time of the average location of the position sinusoidal function.

The average location of a sinusoidal can be found by the average of the max and min locations.

$$y_{avg} = \frac{y_{min}+y_{max}}{2}$$
$$y_{avg} = \frac{y_{min}}{2}$$
$$y_{avg} = -\frac{mg}{k}$$

Thus, the position y_{max} is given by the value y_{avg} calculated

---

$$y_{avg} = -\frac{mg}{k}$$

□