

17.1

1

$$\begin{aligned}
 & \oint_C \langle xy, y \rangle d\vec{r} \\
 &= \int_0^{2\pi} \langle \cos t \sin t, \sin t \rangle \cdot \langle -\sin t, \cos t \rangle dt \\
 &= \int_0^{2\pi} -\cos t \sin^2 t + \sin t \cos t dt \\
 &= \left|_0^{2\pi} -\frac{\sin^3 t}{3} + \frac{\sin^2 t}{2} dt \right. \\
 &= 0
 \end{aligned}$$

$$= \iint_D 0 - x dA$$

Since A is symmetric over x and F is even, the integral is 0

$$= 0$$

3

$$\begin{aligned}
 & \oint_C \langle y^2, x^2 \rangle \\
 & \iint_D 2x - 2y dA \\
 & \left|_D yx^2 - xy^2 dA \right. \\
 &= 1 - 1 \\
 &= 0
 \end{aligned}$$

5

$$\begin{aligned}
 & \oint_C \langle 5y, 2x \rangle \\
 &= \iint_D 2 - 5 dA \\
 &= \iint_D -3 dA \\
 &= 2 * 1 * 0.5 * -3 \\
 &= -3
 \end{aligned}$$

7

$$\begin{aligned}
& \oint_C \langle x^2 y, 0 \rangle \\
&= \iint_D -x^2 \, dA \\
&= \int_0^{2\pi} \int_0^1 -r^3 \cos^2 \theta \, dr d\theta \\
&= \int_0^{2\pi} -\frac{1}{4} \cos^2 \theta \, d\theta \\
&= \int_0^{2\pi} -\frac{1}{4} \left(\frac{1}{2} + \frac{\cos(2\theta)}{2} \right) d\theta \\
&= \left|_0^{2\pi} -\frac{1}{4} \left(\frac{1}{2} \theta + \frac{\sin(2\theta)}{2} \right) d\theta \right. \\
&= \frac{\pi}{4}
\end{aligned}$$

11

$$\begin{aligned}
& \oint_C \langle x^{x+y}, e^{x-y} \rangle \\
&= - \iint_D e^{x-y} - e^{x+y} \, dA \\
&= - \int_0^2 \int_y^{y+2} e^{x-y} - e^{x+y} \, dx dy \\
&= \int_0^2 e^{2y+2} - e^{2y} + 1 - e^2 \, dy \\
&= \frac{1}{2} e^6 - \frac{1}{2} e^2 - \frac{1}{2} e^4 + \frac{1}{2} + 2 - 2e^2 \\
&= \frac{1}{2} e^6 - \frac{5}{2} e^2 - \frac{1}{2} e^4 + \frac{5}{2} \\
&= \frac{1}{2} (e^4 - 5)(e^2 - 1)
\end{aligned}$$

13

a

$$\begin{aligned}
\nabla f &= \langle 2xe^y, x^2 e^y \rangle \\
f &= x^2 e^y
\end{aligned}$$

b

$$\begin{aligned}
\vec{r} &= \langle t, 0 \rangle \\
& \oint_{OA} \langle 0, t \rangle \cdot \langle 1, 0 \rangle dt \\
&= 0
\end{aligned}$$

$$\vec{r} = \langle 0, t \rangle$$

$$\oint_{OB} \langle 0, 0 \rangle \cdot \langle 0, 1 \rangle dt$$

$$= 0$$

C

$$\oint_C \vec{F} \cdot d\vec{r}$$

$$= \oint_C (\nabla f + \vec{G}) \cdot d\vec{r}$$

$$= f(B) - f(A) + \oint_C \vec{G} \cdot d\vec{r}$$

$$= f(B) - f(A) + \iint_D 1 \, dA - \oint_{BO} \langle 0, 0 \rangle \cdot \langle 0, 1 \rangle dt - \oint_{OA} \langle 0, t \rangle \cdot \langle 1, 0 \rangle dt$$

$$= f(B) - f(A) + \frac{16\pi}{4}$$

$$= f(B) - f(A) + 4\pi$$

$$= 4\pi - 16$$