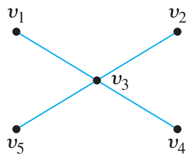
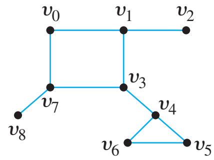
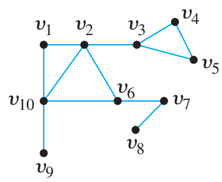
a.



b.



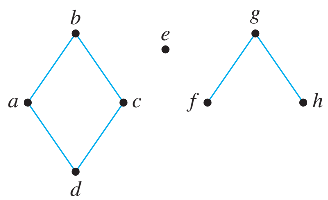
c.



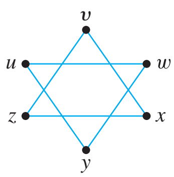
7. Given any positive integer n, (a) find a connected graph with n edges such that removal of just one edge disconnects the graph; (b) find a connected graph with n edges that cannot be disconnected by the removal of any single edge.

8. Find the number of connected components for each of the following graphs.

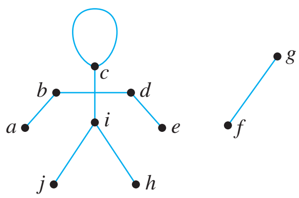
a.



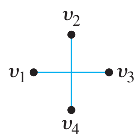
b.



c.



d.



9. Each of (a)–(c) describes a graph. In each case answer *yes*, *no*, or *not necessarily* to this question: Does the graph have an Euler circuit? Justify your answers.

a. G is a connected graph with five vertices of degrees 2, 2, 3, 3, and 4.

b. G is a connected graph with five vertices of degrees 2, 2, 4, 4, and 6.

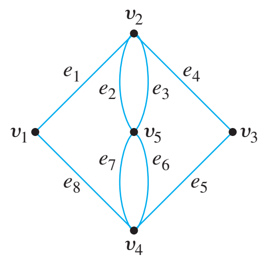
c. G is a graph with five vertices of degrees 2, 2, 4, 4, and 6.

10. The solution for Example 10.1.6 shows a graph for which every vertex has even degree but which does not have an Euler circuit. Give another example of a graph satisfying these conditions.

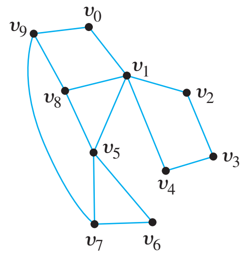
11. Is it possible for a citizen of Königsberg to make a tour of the city and cross each bridge exactly twice? (See Figure 10.1.1.) Explain.

Determine which of the graphs in 12–17 have Euler circuits. If the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one.

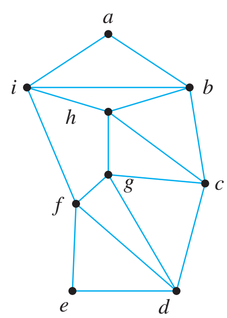
12.



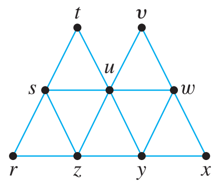
13.



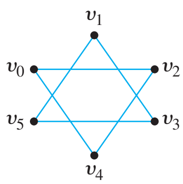
14.



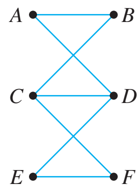
15.



16.



17.



18. Is it possible to take a walk around the city whose map is shown below, starting and ending at the same point and crossing each bridge exactly once? If so, how can this be done?