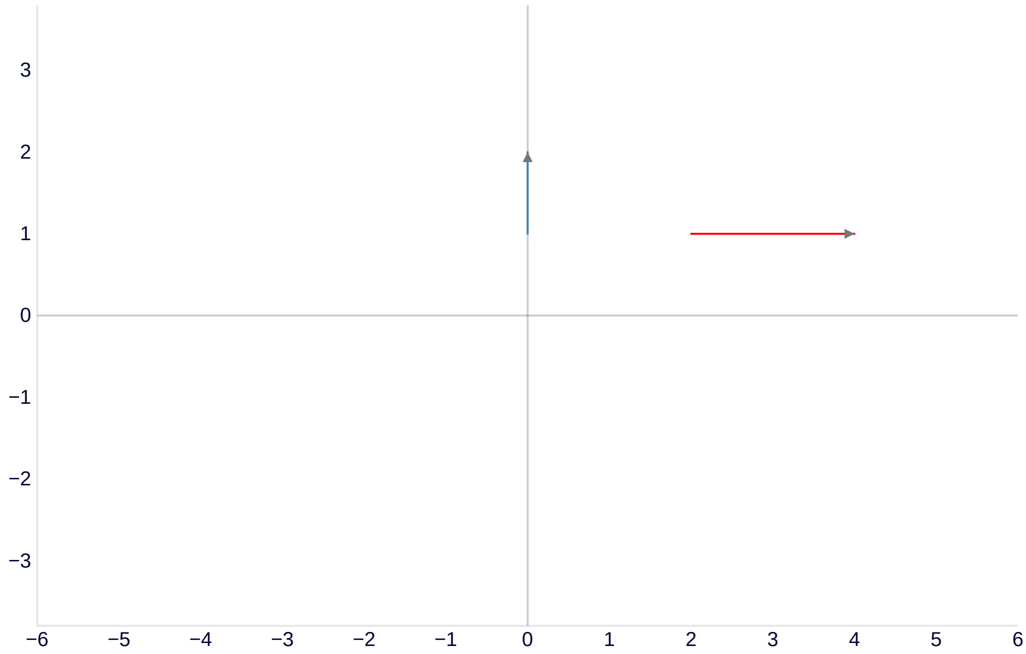
16.1

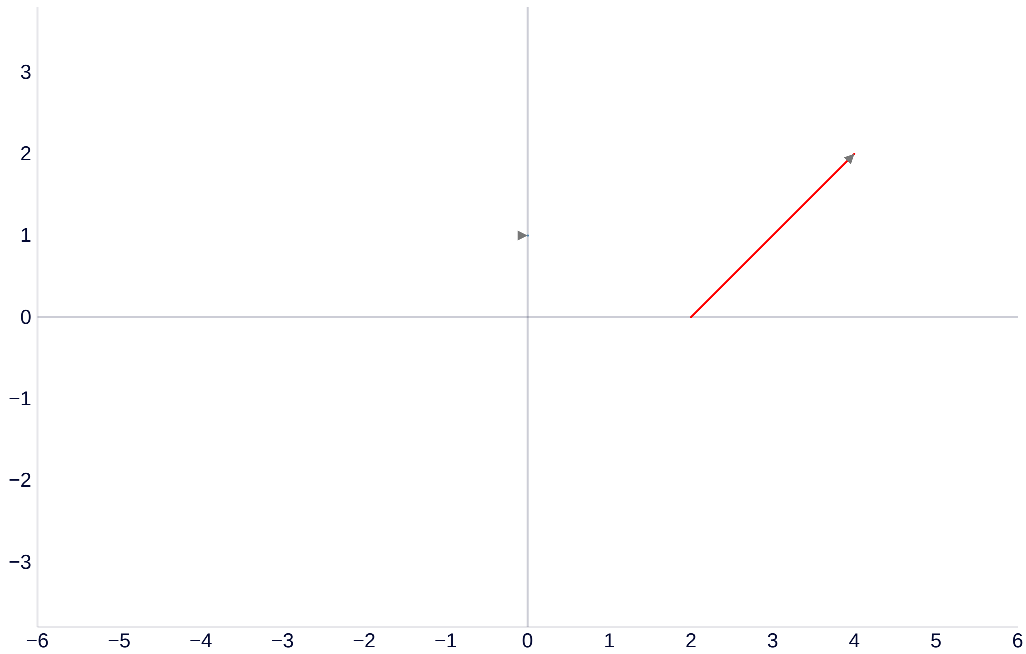
3

$\vec{F}(\vec{P}) = \langle 0, 1, 0 \rangle$

$\vec{F}(\vec{Q}) = \langle 2, 0, 2 \rangle$



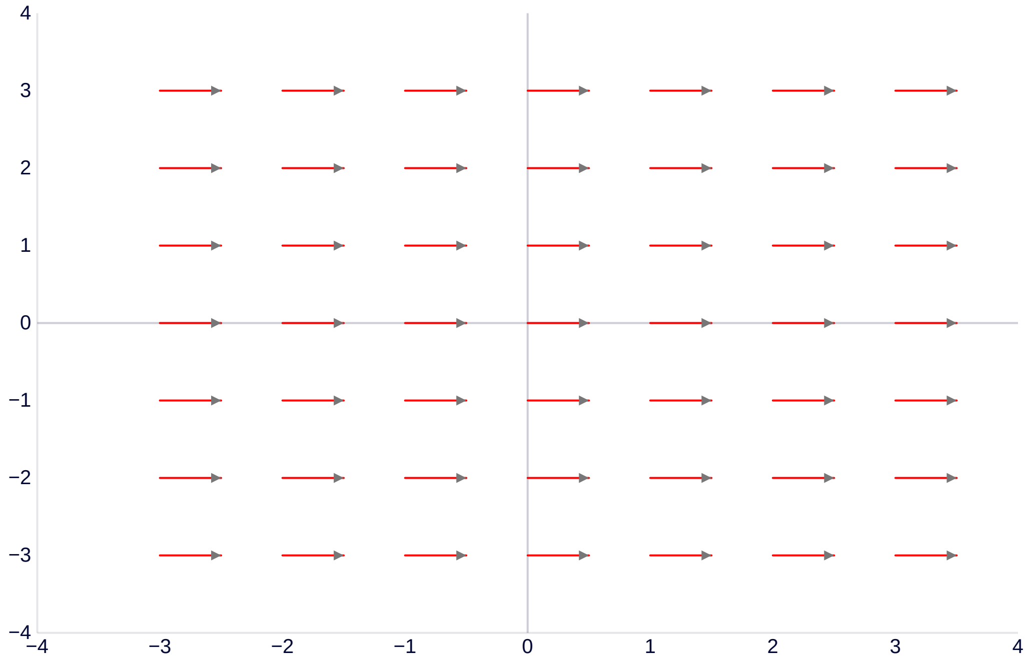
On the XY plane



On the XZ plane

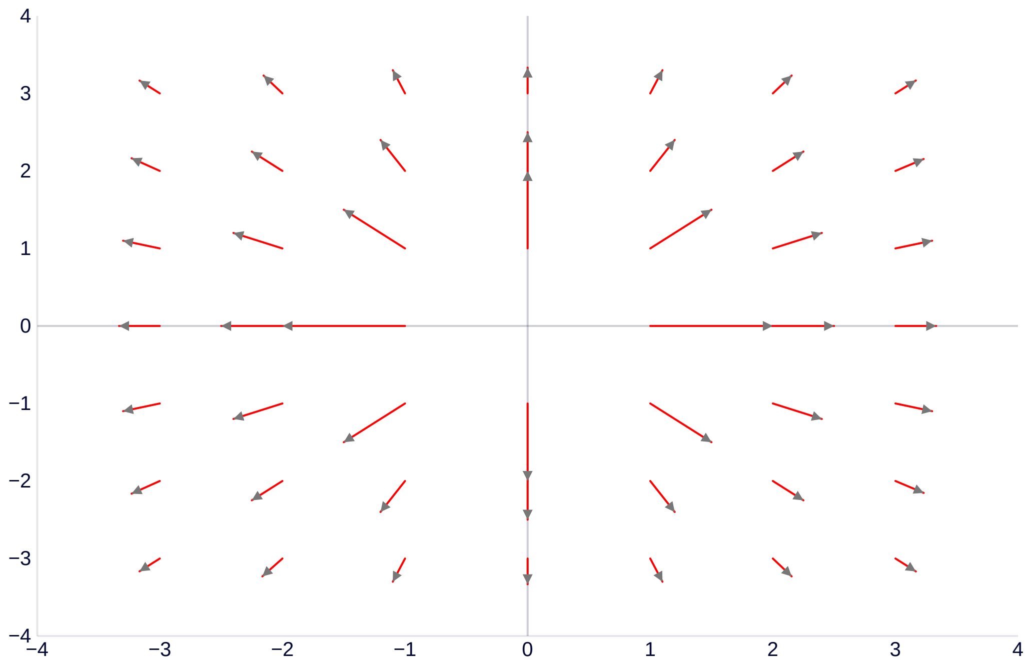
5

$\vec{F} = \langle 1, 0 \rangle$



11

$\vec{F} = \left\langle \frac{x}{x^2+y^2}, \frac{y}{x^2+y^2} \right\rangle$



17

C

23

$$\text{div}(\langle x, y, z\rangle) = 1 + 1 = 1 = 3$$
$$\text{curl}(\langle x, y, z\rangle) = \langle 0, 0, 0\rangle = 0$$

25

$$\text{div}(\langle x - 2zx^2, z - xy, z^2x^2\rangle) = 1 - 4zx - x + 2zx^2$$
$$\text{curl}(\langle x - 2zx^2, z - xy, z^2x^2\rangle) = \langle -1, 2x^2 - 2xz^2, -y\rangle$$

31

$$\text{div}(\vec{F} + \vec{G}) = \text{div}(\vec{F}) + \text{div}(\vec{G})$$

$$\text{div}(\vec{F} + \vec{G}) = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle \vec{F}_x + \vec{G}_x, \vec{F}_y + \vec{G}_y, \vec{F}_z + \vec{G}_z\rangle$$
$$= \langle \tfrac{\partial}{\partial x}(\vec{F}_x + \vec{G}_x), \tfrac{\partial}{\partial y}(\vec{F}_y + \vec{G}_y), \tfrac{\partial}{\partial z}(\vec{F}_z + \vec{G}_z)\rangle$$
$$= \langle \tfrac{\partial}{\partial x}\vec{F}_x + \tfrac{\partial}{\partial x}\vec{G}_x, \tfrac{\partial}{\partial y}\vec{F}_y + \tfrac{\partial}{\partial y}\vec{G}_y, \tfrac{\partial}{\partial z}\vec{F}_z + \tfrac{\partial}{\partial z}\vec{G}_z\rangle$$
$$= \langle \tfrac{\partial}{\partial x}\vec{F}_x, \tfrac{\partial}{\partial y}\vec{F}_y, \tfrac{\partial}{\partial z}\vec{F}_z\rangle + \langle \tfrac{\partial}{\partial x}\vec{G}_x, \tfrac{\partial}{\partial y}\vec{G}_y, \tfrac{\partial}{\partial z}\vec{G}_z\rangle$$
$$= \text{div}(\vec{F}) + \text{div}(\vec{G})$$

33

$$\text{div}(\text{curl}(\vec{F})) = 0$$

$$\text{div}(\text{curl}(\vec{F})) = \nabla \cdot (\nabla \times \vec{F})$$
$$= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \left(\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \times \vec{F} \right)$$
$$= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \left\langle \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}, \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}, \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right\rangle$$
$$= \frac{\partial F_z}{\partial yx} - \frac{\partial F_y}{\partial zx} + \frac{\partial F_x}{\partial zy} - \frac{\partial F_z}{\partial xy} + \frac{\partial F_y}{\partial xz} - \frac{\partial F_x}{\partial yz}$$
$$= 0$$

39

$$\tfrac{\partial}{\partial y}x = 0$$
$$\tfrac{\partial}{\partial x}y = 0$$

$\int x\, dx = \frac{x^2}{2} + C$
$\int y\, dy = \frac{y^2}{2} + C$

$f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$

16.2

9

$f(x, y) = \sqrt{1 + 9xy}$
$\vec{r} = \langle t, t^3 \rangle$
$C = [\vec{r}(0), \vec{r}(2)]$

$\int\limits_C f\, ds = \int\limits_0^2 f(\vec{r}(t)) \|\vec{r}'(t)\|\, dt$
$= \int_0^2 \sqrt{1 + 9y^4}\sqrt{1 + 9t^4}\, dt$
$= \int_0^2 1 + 9t^4\, dt$
$= \Big|_0^2 t + \frac{9t^5}{5}\, dt$
$= 2 + \frac{288}{5}$
$= \frac{298}{5}$

11

$f(x, y, z) = z^2$
$\vec{r} = \langle 2t, 3t, 4t \rangle$
$C = [\vec{r}(0), \vec{r}(2)]$

$\int\limits_C f\, ds = \int\limits_0^2 f(\vec{r}(t)) \|\vec{r}'(t)\|\, dt$
$= \int_0^2 16t^2\sqrt{19t^2}\, dt$
$= \int_0^2 16t^2\sqrt{29t^2}\, dt$
$= \int_0^2 16\sqrt{29}t^3\, dt$
$= \Big|_0^2 4\sqrt{29}t^4\, dt$
$= 64\sqrt{29}$

19

$\vec{F}(x,y) = \langle 1+x^2, xy^2 \rangle$
$\vec{r} = \langle t, 3t \rangle$
$C = [\vec{r}(0), \vec{r}(1)]$

$$\int_C F\ ds = \int_0^1 \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)\ dt$$
$$= \int_0^1 \langle 1+t^2, 9t^3 \rangle \cdot \langle 1, 3 \rangle\ dt$$
$$= \int_0^1 1 + t^2 + 27t^3\ dt$$
$$= \Big|_0^1 t + \tfrac{t^3}{3} + \tfrac{27t^4}{4}\ dt$$
$$= 1 + \tfrac{1}{3} + \tfrac{27}{4}$$
$$= \tfrac{97}{12}$$

21

$\vec{F}(x,y) = \langle x^2, xy \rangle$
$\vec{r} = \langle -\sin t, \cos t \rangle$
$C = \left[\vec{r}(0), \vec{r}\left(\frac{\pi}{2}\right)\right]$

$$\int_C F\ ds = \int_0^{\frac{\pi}{2}} \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)\ dt$$
$$= \int_0^{\frac{\pi}{2}} \langle \sin^2 t, -\sin t \cos t \rangle \cdot \langle -\cos t, -\sin t \rangle\ dt$$
$$= \int_0^{\frac{\pi}{2}} -\sin^2 t \cos t + \sin^2 t \cos t\ dt$$
$$= \int_0^{\frac{\pi}{2}} 0\ dt$$
$$= 0$$

23

$\vec{F}(x,y,z) = \left\langle \frac{3z}{y}, 4x, -y \right\rangle$
$\vec{r} = \langle e^t, e^t, t \rangle$
$C = \left[\vec{r}(-1), \vec{r}\,(1)\right]$

$$\int_C F\,ds = \int_{-1}^{1} \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)\,dt$$
$$= \int_{-1}^{1} \left\langle \frac{3t}{e^t}, 4e^t, -e^t \right\rangle \cdot \langle e^t, e^t, 1\rangle\,dt$$
$$= \int_{-1}^{1} 3t + 4e^{2t} - e^t\,dt$$
$$= \Big|_{-1}^{1} \frac{3t^2}{2} + 2e^{2t} - e^t\,dt$$
$$= \frac{3}{2} + 2e^2 - e - \frac{3}{2} - 2e^{-2} + e^{-1}$$
$$= 2e^2 - e - 2e^{-2} + e^{-1}$$
$$\approx 12.157$$

25

$$\vec{F}(x,y,z) = \left\langle \frac{1}{y^3+1}, \frac{1}{z+1}, 1 \right\rangle$$
$$\vec{r} = \langle t^3, 2, t^2 \rangle$$
$$C = [\vec{r}(0), \vec{r}\,(1)]$$

$$\int_C F\,ds = \int_0^1 \vec{F}(\vec{r}(t)) \cdot \vec{r}'(t)\,dt$$
$$= \int_0^1 \left\langle \frac{1}{9}, \frac{1}{t^2+1}, 1 \right\rangle \cdot \langle 3t^2, 0, 2t\rangle\,dt$$
$$= \int_0^1 \frac{t^2}{3} + 2t\,dt$$
$$= \Big|_0^1 \frac{t^3}{9} + t^2\,dt$$
$$= \frac{10}{9}$$

27

$$\vec{r} = \langle t, t^3 \rangle$$
$$C = [\vec{r}(0), \vec{r}\,(3)]$$

$$\int_C x\,dx = \int_0^3 t\,dt$$
$$= \Big|_0^3 \frac{t^2}{2}\,dt$$
$$= \frac{9}{2}$$

29

$\vec{r} = \langle t, t^2 \rangle$
$C = \left[\vec{r}(0), \vec{r}\,(2)\right]$

$\int\limits_C y\; dx - x\; dy = \int\limits_0^2 t^2 - 2t^2\; dt$
$= \int\limits_0^2 -t^2\; dt$
$= \Big|_0^2 - \frac{t^3}{3}\; dt$
$= -\frac{8}{3}$

31

$\vec{r} = \langle t, 4t, 4t \rangle$
$C = \left[\vec{r}(0), \vec{r}\,(1)\right]$

$\int\limits_0^1 (x - y)\; dx + (y - z)\; dy + z\; dz$
$= \int\limits_0^1 -3t + 16t\; dt$
$= \Big|_0^1 \frac{13t^2}{2}\; dt$
$= \frac{13}{2}$

16.3

7

$\vec{F} = \langle x, y, z \rangle$

$f = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2}$

11

$\vec{F} = \langle y^2, 2xy + e^z, ye^z \rangle$

$\int y^2\; dx = xy^2 + C$
$\int 2xy + e^z\; dy = xy^2 + e^z y + C$
$\int ye^z\; dz = e^z y + C$

$f = xy^2 + ye^z + C$

15

$\vec{F} = \langle z \sec^2 x, z, y + \tan x \rangle$

$\int z \sec^2 x \; dx = z \tan x + C$
$\int z \; dy = yz + C$
$\int y + \tan x \; dz = yz + z \tan x + C$

$f = yz + z \tan x + C$

17

$\vec{F} = \langle 2xy + 5, x^2 - 4z, -4y \rangle$

$\int 2xy + 5 \; dx = x^2 y + 5x + C$
$\int x^2 - 4z \; dy = x^2 y - 4yz + C$
$\int -4y \; dz = -4yz + C$

$f = x^2 y + 5x - 4yz + C$

21

$f = x^2 y - z$
$\vec{F} = \langle 2xy, x^2, -1 \rangle$

$\vec{r}_1 = \langle t, t, 0 \rangle$
$0 < r < 1$

$\int\limits_C \vec{F}(\vec{r}) \; ds = \int\limits_0^1 \vec{F}(\vec{r}) \cdot \vec{r}' \; dt$
$= \int\limits_0^1 \langle 2t^2, t^2, -1 \rangle \cdot \langle 1, 1, 0 \rangle \; dt$
$= \int\limits_0^1 3t^2 \; dt$
$= \Big|_0^1 t^3 \; dt$
$= 1$

> $\vec{r}_2 = \langle t, t^2, 0 \rangle$
> $0 < r < 1$
>
> $$\int_C \vec{F}(\vec{r})\ ds = \int_0^1 \vec{F}(\vec{r}) \cdot \vec{r}'\ dt$$
> $$= \int_0^1 \langle 2t^3, t^2, -1 \rangle \cdot \langle 1, 2t, 0 \rangle\ dt$$
> $$= \int_0^1 4t^3\ dt$$
> $$= \Big|_0^1 t^4\ dt$$
> $$= 1$$
>
> $f(1, 1, 0) - f(0, 0, 0) = 1$
>
> Thus all pathes with the same endpoints have the same integral on a conservative vector field, which is equivalent to the difference in the potential function.