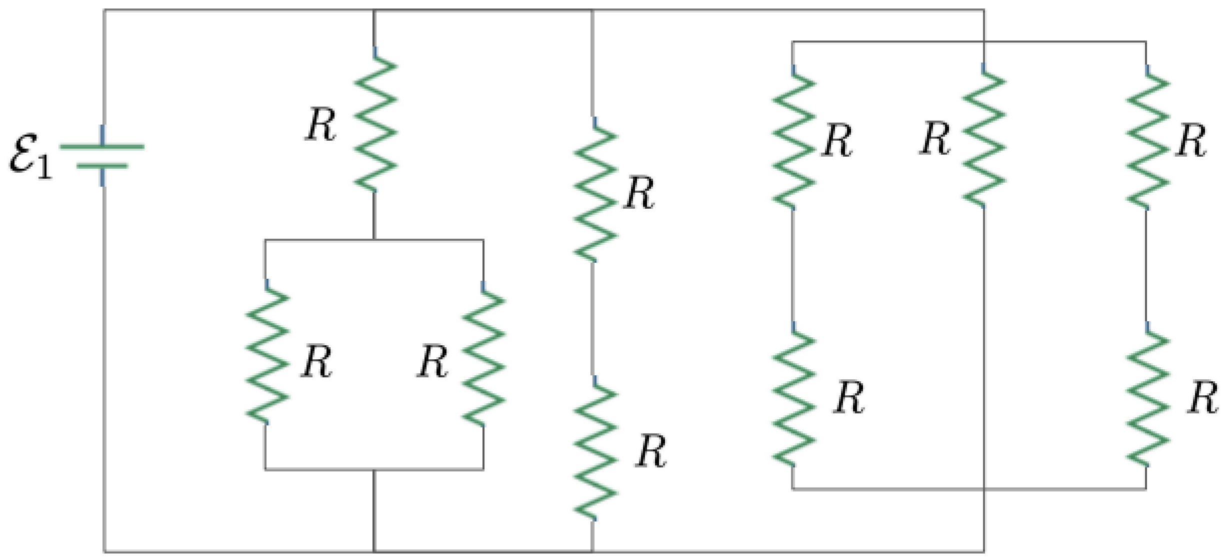
PHYS 122 HW 9

1



Consider the following circuit of ten resistors all with the same resistance R. Is the resistance of the circuit greater or less than R? By what factor?

Answer

$$R_T = \frac{1}{\frac{1}{R+\frac{1}{\frac{1}{R}+\frac{1}{R}}}+\frac{1}{R+R}+\frac{1}{R+R}+\frac{1}{R}+\frac{1}{R+R}}$$

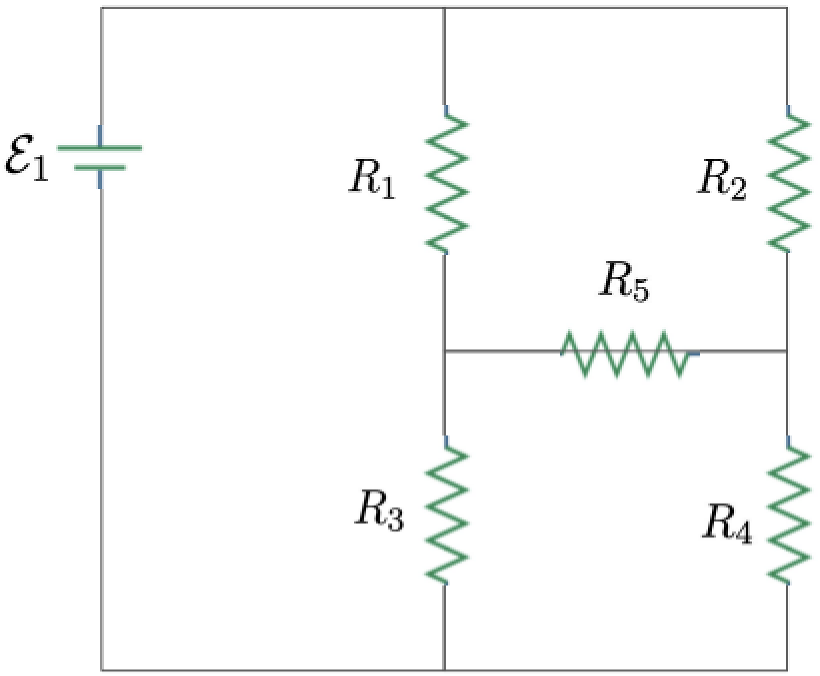
$$R_T = \frac{1}{\frac{1}{R+\frac{1}{\frac{2}{R}}}+\frac{1}{2R}+\frac{1}{2R}+\frac{1}{R}+\frac{1}{2R}}$$

$$R_T = \frac{1}{\frac{2}{3R}+\frac{1}{2R}+\frac{1}{2R}+\frac{1}{R}+\frac{1}{2R}}$$

$$R_T = \frac{1}{\frac{2}{3R}+\frac{3}{2R}+\frac{1}{R}}$$

$$R_T = \frac{6R}{19}$$

2



Consider the following circuit of five resistors $R_1 = 3\Omega$, $R_2 = 4\Omega$, $R_3 = 7\Omega$, $R_4 = 5\Omega$, $R_5 = 6\Omega$ and one battery $V = 12\ V$. What is the current flowing through each resistor?

✓ **Answer**

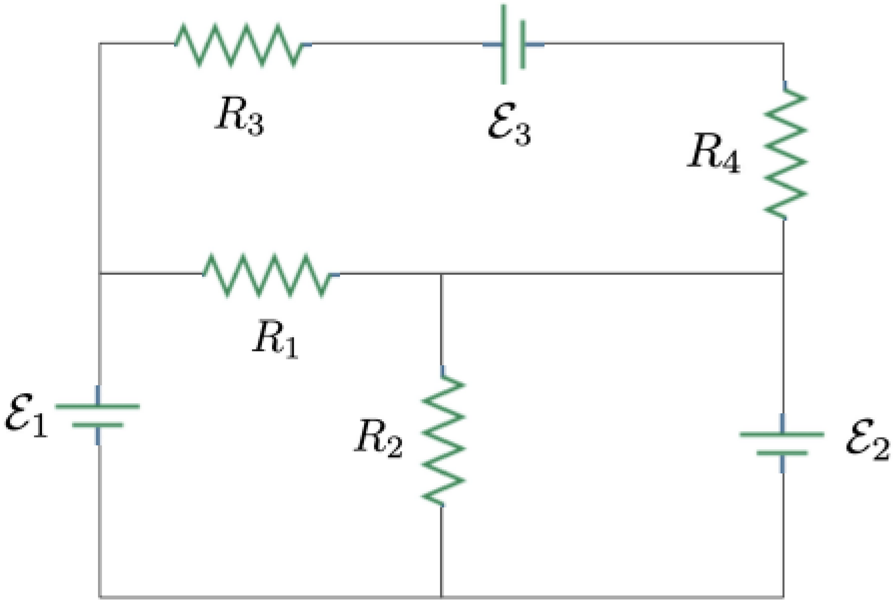
Where we define I_1 as the left clockwise loop, I_2 as the topright clockwise loop, and I_3 as the bottom right clockwise loop:

$$\begin{cases} \epsilon_1 - (I_1 - I_2)R_1 - (I_1 - I_3)R_3 = 0 \\ -I_2R_2 - (I_2 - I_3)R_5 - (I_2 - I_1)R_1 = 0 \\ -I_3R_4 - (I_3 - I_1)R_3 - (I_3 - I_2)R_5 = 0 \end{cases}$$
$$= \begin{cases} -\epsilon_1 = -I_1(R_1 + R_3) + I_2(R_1) + I_3(R_3) \\ 0 = I_1(R_1) - I_2(R_1 + R_2 + R_5) + I_3(R_5) \\ 0 = I_1(R_3) + I_2(R_5) - I_3(R_3 + R_4 + R_5) \end{cases}$$
$$\implies \begin{bmatrix} \epsilon_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -R_1 - R_3 & R_1 & R_3 \\ R_1 & -R_1 - R_2 - R_5 & R_5 \\ R_3 & R_5 & -R_3 - R_4 - R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \frac{\epsilon_0 \begin{bmatrix} R_1R_3+R_1R_4+R_1R_5+R_2R_3+R_2R_4+R_2R_5+R_3R_5+R_4R_5 \\ R_1R_3+R_1R_4+R_1R_5+R_3R_5 \\ R_1R_3+R_1R_5+R_2R_3+R_3R_5 \end{bmatrix}}{R_1R_2R_3+R_1R_2R_4+R_1R_2R_5+R_1R_3R_4+R_1R_4R_5+R_2R_3R_4+R_2R_3R_5+R_3R_4R_5}$$
$$= \begin{bmatrix} \frac{2376}{929} \\ \frac{1152}{929} \\ \frac{1308}{929} \end{bmatrix}$$

R_1 down: $\frac{1224}{929}\ A$
R_2 down: $\frac{1152}{929}\ A$
R_3 down: $\frac{1068}{929}\ A$

R_4 down: $\frac{1308}{929}\ A$
R_5 right: $\frac{156}{929}\ A$

3



Consider the following circuit of four resistors $R_1 = 1\Omega$, $R_2 = 5\Omega$, $R_3 = 3\Omega$, $R_4 = 6\Omega$ and three batteries $E_1 = 12\ V$, $E_2 = 6\ V$, $E_3 = 24\ V$. What is the current flowing through each resistor?

✓ **Answer**

With I_1 being the bottom left clockwise loop, I_2 being the bottom right clockwise loop, and I_3 being the top clockwise loop.

$$\begin{bmatrix} \epsilon_1 \\ -\epsilon_2 \\ -\epsilon_3 \end{bmatrix} = \begin{bmatrix} R_1 + R_2 & -R_2 & -R_1 \\ -R_2 & R_2 & 0 \\ -R_1 & 0 & R_1 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} \frac{\epsilon_1 R_1 + \epsilon_1 R_3 + \epsilon_1 R_4 - \epsilon_2 R_1 - \epsilon_2 R_3 - \epsilon_2 R_4 - \epsilon_3 R_1}{R_1 R_3 + R_1 R_4} \\ \frac{\epsilon_1 R_1 R_2 + \epsilon_1 R_2 R_3 + \epsilon_1 R_2 R_4 - \epsilon_2 R_1 R_2 - \epsilon_2 R_1 R_3 - \epsilon_2 R_1 R_4 - \epsilon_2 R_2 R_3 - \epsilon_2 R_2 R_4 - \epsilon_3 R_1 R_2}{R_1 R_2 R_3 + R_1 R_2 R_4} \\ \frac{\epsilon_1 - \epsilon_2 - \epsilon_3}{R_3 + R_4} \end{bmatrix}$$

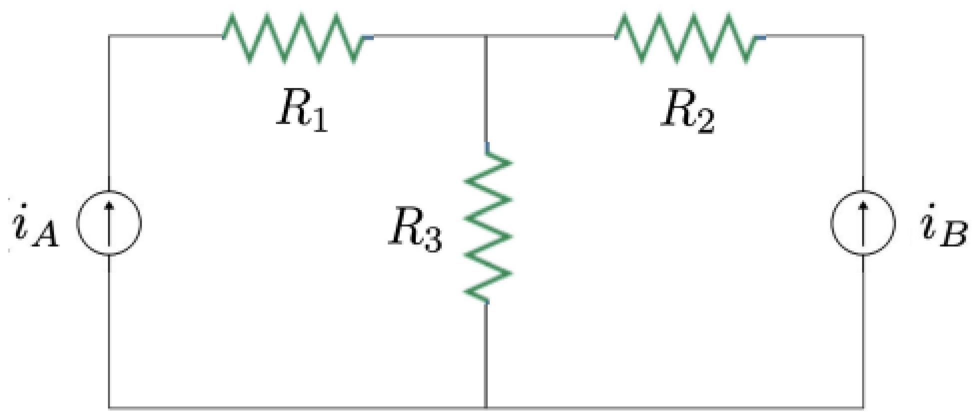
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ \frac{14}{5} \\ -2 \end{bmatrix}$$

R_1 right: $3\ A$
R_2 down: $\frac{6}{5}\ A$

R_3, R_4 counterclockwise: $2\ A$

4

An ideal current source is a circuit element that is complementary to a battery. An ideal battery maintains a constant voltage across its terminals. The current that flows in the battery is determined by the details of the circuit to which the battery is connected. An ideal current source supplies a definite current. The voltage across the terminals of the current source is determined by the details of the circuit to which it is connected. The symbol for a current source is a circle with an arrow inside. The direction of the arrow indicates the direction in which the current flows through the current source.



a

Determine the currents through the three resistors in terms of their resistances (R_1, R_2, R_3) and the currents of the two sources (i_A, i_B).

Answer

$R_1 : i_A$
$R_2 : i_B$
$R_3 : i_A + i_B$

b

Determine the voltage across each of the current sources.

Answer

$R_1 : i_A R_1$
$R_2 : i_B R_2$
$R_3 : (i_A + i_B) R_3$

$i_A : i_A R_1 + (i_A + i_B) R_3$
$i_B : i_B R_2 + (i_A + i_B) R_3$

5

A laboratory power supply delivers a constant voltage of $V = 12\ V$. You are provided with several identical resistors, each having a resistance of $R = 4\Omega$.

a

Connect a single resistor to the power supply. What is the power dissipated across this single resistor?

✓ **Answer**

$V = IR$
$I = 3\ A$

$P = I^2 R$
$P = 36\ W$

b

Connect 3 of these resistors in series with the power supply. What is the power dissipated across the entire series configuration? How much power is dissipated across each resistor?

✓ **Answer**

$V = 3IR$
$I = \frac{V}{3R}$
$I = 1\ A$

$P = I^2 R$
$P = 12\ W$

For one resistor:
$P = 3\ W$

c

Connect 4 of these resistors in parallel with the power supply. What is the power dissipated across the entire parallel configuration? How much power is dissipated across each resistor?

✓ **Answer**

$V = 4IR$
$I = 0.75\ A$

$P = I^2R$
$P = 6.75\ W$

For one resistor:
$P = 1.6875\ W$