#### **Discrete Distributions**

### Bernoulli (p)

Given  $x = 0, 1; \quad 0 \le p \le 1$ 

- p is the probability of getting selected trait
- Has p probability of being 1 and 1-p probability of being 0

$$egin{aligned} P(X = x) &= p^x (1-p)^{1-x} \ \mu &= p \ \sigma^2 &= p (1-p) \ M(t) &= (1-p) + p e^t \end{aligned}$$

#### Binomial (n, p)

Given  $x = 0, 1, 2, ..., n; 0 \le p \le 1$ 

- p is probability of selecting a particular trait
- n is number of samples in a round of sampling
- Predicts probability of getting certain number of chosen trait in sample set

$$egin{aligned} P(X=x) &= inom{n}{x} p^x (1-p)^{n-x} \ \mu &= np \ \sigma^2 &= np (1-p) \ M(t) &= (pe^t + (1-p))^n \end{aligned}$$

# Discrete Uniform (N)

Given x = 1, 2, ..., N; N = 1, 2, ...

- N is the largest possible sample
- All numbers from 1 to N are equally likely

$$egin{aligned} P(X=x) &= 1/N \ \mu &= rac{N+1}{2} \ \sigma^2 &= rac{(N+1)(N-1)}{12} \ M(t) &= rac{1}{N} \sum_{i=1}^N e^{it} \end{aligned}$$

### Geometric (p)

Given  $x=1,2,\ldots;\quad 0\leq p\leq 1$ 

p is probability of getting certain trait

Predicts number of samples needed to get a sample of particular trait

$$egin{aligned} P(X=x) &= p(1-p)^{x-1} \ \mu &= 1/p \ \sigma^2 &= rac{1-p}{p^2} \ M(t) &= rac{pe^t}{1-(1-p)e^t} \end{aligned}$$

#### Hypergeometric (N, K, M)

Given 
$$x = 0, 1, 2, ..., K$$
;  $M - (N - K) \le x \le M$ ;  $N, M, K = 0, 1, 2, ...$ 

- N is the population size
- M is the number of samples in the population with a certain trait
- K number of samples taken in a round of sampling
- Predicts the likelihood of selecting X samples of type M after selecting K samples from population N

$$P(X-x) = rac{inom{M}{x}inom{N-M}{K-x}}{inom{N}{K}}$$
  $\mu = KM/N$   $\sigma^2 = rac{KM(N-M)(N-K)}{N^2(N-1)}$ 

#### **Negative Binomial** (r, p)

Given 
$$x = 0, 1, 2, ...; 0 \le p \le 1$$

- p is the probability of getting a particular trait in one sample
- r is the desired number of samples with a particular trait
- Predicts number of likelihood of getting r samples of trait after X samples

$$egin{aligned} P(X=x) &= inom{r+x-1}{x} p^r (1-p)^x \ \mu &= rac{r(1-p)}{p} \ \sigma^2 &= rac{r(1-p)}{p^2} \ M(t) &= (rac{p}{1-(1-p)e^t})^r \end{aligned}$$

#### **Poisson Distribution** ( $\lambda$ )

Given 
$$x = 0, 1, 2, \ldots; \quad 0 \le \lambda$$

- $\lambda$  is the number of times on average an event will happen within an interval
- Predicts number of times an event will happen within an interval
- · Approximates the Binomial Distribution

$$P(X=x) = rac{e^{-\lambda}\lambda^x}{x!}$$
 $\mu = \lambda$ 

$$\sigma^2 = \lambda \ M(t) = e^{\lambda(e^t-1)}$$

#### **Continuous Distributions**

### Beta $(\alpha, \beta)$

Given  $0 \le x \le 1$ ;  $\alpha > 0$ ;  $\beta > 0$ 

$$egin{aligned} f(x) &= rac{x^{lpha-1}(1-x)^{eta-1}}{B(lpha,eta)} \ \mu &= rac{lpha}{lpha+eta} \ \sigma^2 &= rac{lphaeta}{(lpha+eta)^2(lpha+eta+1)} \ M(t) &= 1 + \sum_{k=1}^{\infty} rac{t^k}{k!} \prod_{r=0}^{k-1} rac{a+r}{lpha+eta+r} \end{aligned}$$

# Cauchy $(\theta, \sigma)$

Given  $-\infty < x < \infty$ ;  $-\infty < \theta < \infty$ ;  $\sigma > 0$ 

$$f(x)=rac{1}{\pi\sigma(1+(rac{x- heta}{\sigma})^2)}$$

### Chi squared (p)

Given  $0 \le x < \infty$ ;  $p = 1, 2, 3, \dots$ 

$$f(x) = rac{x^{p/2-1}e^{-x/2}}{\Gamma(p/2)2^{p/2}} \ \mu = p \ \sigma^2 = 2p \ M(t) = (rac{1}{1-2t})^{p/2}$$

### **Double Exponential** $(\mu, \sigma)$

Given  $-\infty < x < \infty$ ;  $-\infty < \mu < \infty$ ;  $\sigma > 0$ 

$$egin{aligned} f(x) &= rac{e^{-|x-\mu|/\sigma}}{2\sigma} \ \mu &= \mu \ \sigma^2 &= 2\sigma^2 \ M(t) &= rac{e^{\mu t}}{1-(\sigma t)^2} \end{aligned}$$

### **Exponential** $\beta$

Given  $0 \le x < \infty$ ;  $\beta > 0$ 

$$egin{aligned} f(x) &= rac{e^{-x/eta}}{eta} \ \mu &= eta \ \sigma^2 &= eta^2 \ M(t) &= rac{1}{1-eta t} \end{aligned}$$

#### $\mathsf{F}\left(v_{1},v_{2} ight)$

Given  $0 \le \infty$ ;  $v_1, v_2 = 1, 2, 3, ...$ 

$$f(x) = rac{\Gamma(rac{v_1+v_2}{2})}{\Gamma(rac{v_1}{2})\Gamma(rac{v_2}{2})} (rac{v_1}{v_2})^{v_1/2} rac{x^(v_1-2)/2}{(1+rac{v_1x}{v_2})^{(v_1+v_2)/2}} \ \mu = rac{v_2}{v_2-2} \ \sigma^2 = 2(rac{v_2}{v_2-2})^2 rac{v_1+v_2-2}{v_1(v_2-4)} \ EX^n = rac{\Gamma(rac{v_1+v_2}{2})\Gamma(rac{v_2-2n}{2})}{\Gamma(v_1/2)\Gamma(v_2/2)} (rac{v_2}{v_1})^n \quad ; n < rac{v_2}{2}$$

# Gamma Distribution $(\alpha, \beta)$

Given  $0 \le x < \infty$ ;  $\alpha, \beta > 0$ 

$$egin{aligned} f(x) &= rac{x^{lpha - 1}e^{-x/eta}}{\Gamma(lpha)eta^lpha} \ \mu &= lphaeta \ \sigma^2 &= lphaeta^2 \ M(t) &= (rac{1}{1-eta t})^lpha \end{aligned}$$

#### Logistic $(\mu, \beta)$

Given  $-\infty < x < \infty$ ;  $-\infty < \mu < \infty$ ;  $\beta > 0$ 

$$egin{align} f(x) &= rac{e^{-(x-\mu)/eta}}{eta(1+e^{-(x-\mu)/eta)})^2} \ \mu &= \mu \ \sigma^2 &= rac{\pi^2eta^2}{3} \ M(t) &= e^{\mu t}\Gamma(1+eta t) \ \end{array}$$

### Lognormal $(\mu, \sigma^2)$

Given  $0 \le x < \infty$ ;  $-\infty < \mu < \infty$ ;  $\sigma > 0$ 

$$f(x) = rac{1}{\sqrt{2\pi}\sigma} rac{e^{-(\log x - \mu)^2/(2\sigma^2)}}{x} \ \mu = e^{\mu + (\sigma^2/2)} \ \sigma^2 = e^{2(\mu + \sigma^2)} - e^{2\mu + \sigma^2} \ EX^n = e^{n\mu + n^2\sigma^2/2}$$

# Normal $(\mu, \sigma^2)$

Given  $-\infty < x < \infty$ ;  $-\infty < \mu < \infty$ ;  $\sigma > 0$ 

$$egin{aligned} f(x) &= rac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sqrt{2\pi}\sigma} \ \mu &= \mu \ \sigma^2 &= \sigma^2 \end{aligned}$$

# Paretto $(\alpha, \beta)$

Given  $a < x < \infty$ ;  $\alpha, \beta > 0$ 

$$egin{aligned} f(x) &= rac{eta lpha^eta}{x^{eta+1}} \ \mu &= rac{eta lpha}{eta-1} \quad ; eta > 1 \ \sigma^2 &= rac{eta lpha^2}{(eta-1)^2(eta-2)} \quad ; eta > 2 \end{aligned}$$

### t(v)

Given  $-\infty < x < \infty$ ;  $v = 1, 2, 3, \dots$ 

$$egin{align} f(x) &= rac{\Gamma(rac{v+1}{2})}{\Gamma(rac{v}{2})} rac{1}{\sqrt{v\pi}} rac{1}{(1+(rac{x^2}{v}))^{(v+1)/2}} \ \mu &= 0 \quad ; v > 1 \ \sigma^2 &= rac{v}{v-2} \quad ; v > 2 \ MX^n &= egin{cases} rac{\Gamma(rac{n+1}{2})\Gamma(rac{v-n}{2})}{\sqrt{\pi}\Gamma(v/2)} v^{n/2} & n < v; n ext{ is even} \ 0 & n < v; n ext{ is odd} \end{cases}$$

### Uniform (a, b)

Given  $a \le x \le b$ 

- a is the lower bound of the distribution
- b is the upper bound
- All values between a and b are equally distributed

$$f(x) = rac{1}{b-a} \ \mu = rac{b+a}{2} \ \sigma^2 = rac{(b-a)^2}{12} \ M(t) = rac{e^{bt}-e^{at}}{t(b-a)}$$

#### Weibull $(\gamma, \beta)$

Given  $0 \le x < \infty$ ;  $\gamma, \beta > 0$ 

$$egin{aligned} f(x) &= rac{\gamma}{eta} x^{\gamma-1} e^{-x^{\gamma}/eta} \ \mu &= eta^{1/\gamma} \Gamma(1+rac{1}{\gamma}) \ \sigma^2 &= eta^{2/\gamma} (\Gamma(1+rac{2}{\gamma}) - \Gamma^2(1+rac{1}{\gamma})) \ EX^n &= eta^{n/\gamma} \Gamma(1+rac{n}{\gamma}) \end{aligned}$$