1.51

30 ovens, 5 defective
4 chosen without replacement

Since order doesn't matter, we use 30 choose 4 to get the number of possibilities.

$$\binom{30}{4}$$

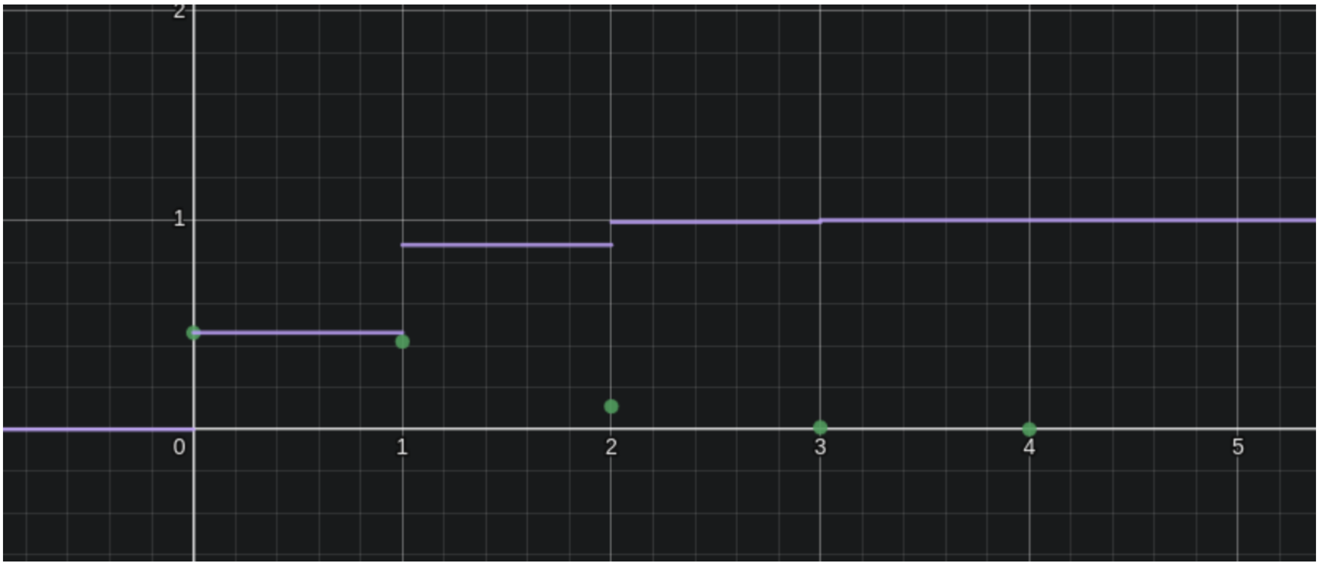
The number of valid solutions will be equal to $\binom{5}{X}\binom{26}{4-X}$

$$f(x) = \frac{\binom{5}{X}\binom{26}{4-X}}{\binom{30}{4}}$$

$$f(x) = \begin{cases} 0.462 & x = 0 \\ 0.420 & x = 1 \\ 0.109 & x = 2 \\ 0.009 & x = 3 \\ 0.000 & x = 4 \end{cases}$$

Where the CDF is the sum of all values below it.

$$F(x) = \begin{cases} 0.000 & x < 0 \\ 0.462 & 0 \leq x < 1 \\ 0.882 & 1 \leq x < 2 \\ 0.991 & 2 \leq x < 3 \\ 1.000 & 3 \leq x < 4 \\ 1.000 & 4 \leq x \end{cases}$$



1.52

$$g(x) = \begin{cases} f(x)/(1 - F(x_0)) & x \geq x_0 \\ 0 & x < x_0 \end{cases}$$

A in order for a function to be a pdf, it must be strictly positive, and have its integral from $-\infty$ to ∞ be equal to 1.

Since $f(X)$ is a PDF, it is already strictly positive, and since $F(X_0) < 1$, $g(x)$ is also positive.

The integral of $g(x)$, $G(x)$ can be defined as

$$G(x) = \begin{cases} F(x)/(1 - F(x_0)) & x \geq x_0 \\ 0 & x < x_0 \end{cases}$$

Since $F(X)$ is a CDF, it approaches 1 as x approaches ∞. The integral from $-\infty$ to ∞ will be equal to

$$1/(1 - F(x_0)) - F(x_O)/(1 - F(x_o)) = \frac{1-F(x_0)}{1-F(x_0)} = 1$$

Thus $\int\limits_{-\infty}^{\infty} g(x) = 1$

Therefore $g(x)$ is a PDF.

1.53

a

$$F_Y(y) = 1 - \frac{1}{y^2} \qquad 1 \leq y < \infty$$

In order for $F_Y(y)$ to be a CDF, it must approach 1 as y approaches ∞ and be strictly increasing, meaning $\frac{d}{dy}F_Y(y) > 0$

As y approaches ∞, the fraction on the right side approaches 0, meaning $F_Y(y)$ approaches 1

Let $f_Y(y) = \frac{d}{dy}F_Y(y) = \frac{2}{y^3}$

On the interval $1 \leq y < \infty$, $f_Y(y) > 0$, meaning $F_Y(y)$ is increasing.

Therefore $F_Y(y)$ is a CDF.

b

$$f_Y(y) = \frac{d}{dy}F_Y(y) = \frac{2}{y^3}$$

1

Given $f(x)$ and $g(x)$ are PDFs

Let $h(x) = \alpha f(x) + (1 - \alpha)g(x) \qquad 0 \leq \alpha \leq 1$

$h(x)$ is a PDF if and only if it is strictly positive, and its integral from $-\infty$ to ∞ is 1.

Since f and g are PDFs, they are strictly positive, and since $0 \leq \alpha \leq 1$, $\alpha f(x) \geq 0$ and $(1 - \alpha)g(x) \geq 0$, therefore $h(x) \geq 0$

$$\int\limits_{-\infty}^{\infty} h(x) = \alpha \int\limits_{-\infty}^{\infty} f(x) + 1 - \alpha \int\limits_{-\infty}^{\infty} g(x)$$

Since f and g are valid PDFs, their integrals from $-\infty$ to ∞ are 1.

$$= \alpha + 1 - \alpha = 1$$

Therefore the integral of $h(x)$ from $-\infty$ to ∞ is 1 and is a valid PDF.

2

a

Given $f(x)$ is a PDF.

$$f(x) = \begin{cases} cx^2 & 0 \leq x \leq 1 \\ 0 & else \end{cases}$$

Since $f(x)$ is a PDF, its integral from $-\infty$ to ∞ is 1

$$\int f(x) = \begin{cases} 0 & x < 0 \\ cx^3/3 & 0 \leq x \leq 1 \\ 1 & 1 < x \end{cases}$$

$$\int\limits_{-\infty}^{\infty} f(x) = c/3 = 1$$

$$\implies c = 3$$

b

$$F(x) = \begin{cases} 0 & x < 0 \\ x^3 & 0 \leq x \leq 1 \\ 1 & 1 < x \end{cases}$$

c

$$P(0.1 \leq X < 0.5) = F(0.5) - F(0.1) = 0.124$$