<u>UNC LAB:</u> <u>Error Analysis and Propagation Exercise</u>

Revised April 06, 2006

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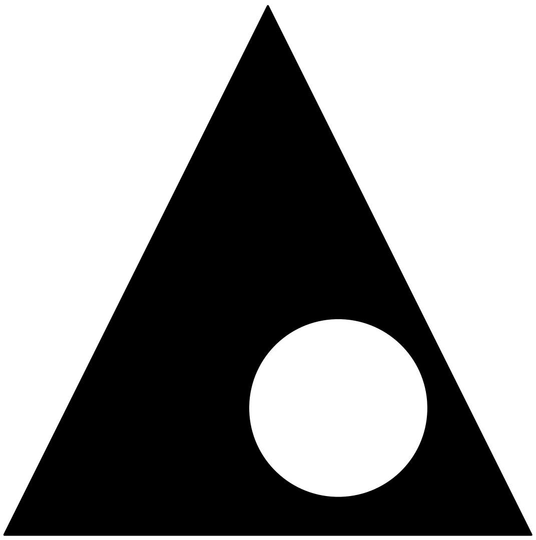

This exercise is designed to help you understand error analysis and error propagation. You need to determine the area of the shaded region in the figure above; that is, the area of a triangle minus the area of a circle. If the triangle has a height, *h*, and width, *w*, and the circle has diameter *d*, then the shaded area is given by the formula $A = hw/2 - \pi d^2/4$.

Every measurement has an associated uncertainty. The uncertainties can be labeled with the symbol, δ, which indicates a small change in the associated quantity. The uncertainties of *h*, *w*, and *d* are given by δ_h, δ_w, and δ_d respectively.

Use a metric ruler to measure *h*, *w*, and *d*, estimate the uncertainties δ_h, δ_w, and δ_d in your measurements of each quantity and enter these values below, in cm. For your convenience, copy these values onto the other side of this page.

69.3	±	0.1	cm	69.2	±	0.1	cm	23.3	±	0.1	cm
h	±	δ_h		w	±	δ_w		d	±	δ_d	

(measured w/ calipers)

Now calculate $A = hw/2 - \pi d^2/4 =$ 1.97×10³ cm² 1971.395191

To estimate the *uncertainty* in *A*, δ_A, we need to *propagate* each individual contribution to the uncertainty (δ_h, δ_w, and δ_d) through the equation for *A* to find out how much each contributes to the uncertainty in *A* (*these terms are labeled as* δ_{Ah}, δ_{Aw}, *and* δ_{Ad}) and then add these contributions in quadrature $\delta_A = (\delta_{Ah}^2 + \delta_{Aw}^2 + \delta_{Ad}^2)^{1/2}$.

The first step is to determine δ_{Ah}, δ_{Aw}, and δ_{Ad}. This may be done by one of two methods. In the computational method, you calculate the change in *A* caused by substituting for each term, such as *h*, the value plus its estimated uncertainty, such as $h + \delta_h$ (or $h - \delta_h$). The derivative method has you calculate terms such as δ_{Ah} using the idea that any small change in *A* due to a small change in *h* is given by the derivative of *A* with respect to *h*, treating all the other terms such as *w* and *d* as constants. This is properly called a *partial derivative* and uses the symbol ∂ as in $\frac{\partial A}{\partial h}$ rather than $\frac{dA}{dh}$. Once you know how *A* changes as a function of *h*, you can simply multiply this by the estimated uncertainty in *h*, δ_h, to find $\delta_{Ah} = |\partial A/\partial h|\delta_h$.

Now, for some practice in error propagation, fill in each of the blanks on the other side of this page.

h w d

69.3 ± 0.1 69.2 ± 0.1 23.3 ± 0.1

COMPUTATIONAL METHOD

$\delta_{Ah} = |(hw/2 - \pi d^2/4) - ((h+\delta_h)w/2 - \pi d^2/4)| = \{$ *this simplifies to* $\delta_h w/2\} =$ 3.46 cm^2
(units)

$\delta_{Aw} = |(hw/2 - \pi d^2/4) -$ $(h(w+\delta_w)/2 - \pi d^2/4)$ $| =$ 3.465 cm^2

$\delta_{Ad} = |(hw/2 - \pi d^2/4) -$ $(hw/2 - \pi(d+\delta_d)^2/4)$ $| =$ 3.668 cm^2

$\delta_A = (\delta_{Ah}^2 + \delta_{Aw}^2 + \delta_{Ad}^2)^{1/2} =$ 6.216 cm^2

You should quote your value for A in the form $A \pm \delta_A$ (units): 1971 ± 6 cm^2
(δ_A *is normally given with one or at most two significant figures while the most significant figure in the value of* A *should be determined from* δ_A*, as in* A = *3.65 ± 0.03* cm^2. *See Appendix V, Section D.*)

DERIVATIVE METHOD

(*Optional for P115 students*)

$$\delta_{Ah} = \left|\frac{\partial A}{\partial h}\right| \delta_h = \left|\frac{\partial}{\partial h}\left(\frac{hw}{2} - \frac{\pi d^2}{4}\right)\right| \delta_h = \frac{\delta_h w}{2} = \text{3.46 } cm^2$$

(units)

$$\delta_{Aw} = \left|\frac{\partial A}{\partial w}\right| \delta_w = \left|\frac{\partial}{\partial w}\left(\frac{hw}{2} - \frac{\pi d^2}{4}\right)\right| \delta_w = \frac{h\delta_w}{2} = \text{3.465 } cm^2$$

$$\delta_{Ad} = \left|\frac{\partial A}{\partial d}\right| \delta_d = \left|\frac{\partial}{\partial d}\left(\frac{wh}{2} - \frac{\pi d^2}{4}\right)\right| \delta_d = \frac{\pi d \delta_d}{2} = \text{3.660 } cm^2$$

$$\delta_A = \sqrt{\delta_{Ah}^2 + \delta_{Aw}^2 + \delta_{Ad}^2} = \text{6.113 } cm^2$$

$A =$ 1971 ± 6 cm^2

You should find that the computational and derivative methods give similar results.

GRADE:_______ **GRADED BY ________**
(out of 10 points) (TA's initials)