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1.5

Calculate the densities of Si and GaAs from the lattice constants (Appendix III), atomic weights, and Avogadro's number. Compare the results with densities given in Appendix III. The atomic weights of Si, Ga, and As are 28.1, 69.7, and 74.9, respectively.

✓ Answer ✓

$$\text{Si: } \frac{28.1 \text{ g}}{\text{mol}} \frac{1 \text{ mol}}{6.02214076 \times 10^{23}} \frac{8}{0.543^3 \text{ nm}^3} \frac{10^{21} \text{ nm}^3}{\text{cm}^3} = 2.33 \frac{\text{g}}{\text{cm}^3}$$
$$\text{GaAs: } \frac{(4)69.7 + (4)74.9 \text{ g}}{\text{mol}} \frac{1 \text{ mol}}{6.02214076 \times 10^{23}} \frac{1}{0.565^3 \text{ nm}^3} \frac{10^{21} \text{ nm}^3}{\text{cm}^3} = 5.33 \frac{\text{g}}{\text{cm}^3}$$

1.13

How many atoms are found inside a unit cell of an sc, a bcc, and an fcc crystal? How far apart in terms of lattice constant a are the nearest neighbor atoms in each case, measured from center to center?

✓ Answer

SC: 1, a

BCC: 2, $\frac{\sqrt{3}a}{2}$

FCC: 4, $\frac{\sqrt{2}a}{2}$

2.10

An electron is described by a plane-wave wavefunction $\psi(x, t) = Ae^{j(10x+3y-4t)}$. Calculate the expression value of a function defined as $\{4p_x^2 + 2p_z^2 + 7mE\}$. Where m is the mass of the electron, p_x and p_z are the x and z components, and E is energy. (Give values in terms of the Planck constant.)

✓ Answer

$$N = \int_{-\infty}^{\infty} \psi^*(\vec{x}, t) \psi(\vec{x}, t) d\vec{x}$$

$$N = \int_{-\infty}^{\infty} A^2 dx$$

$$p_x = \int_{-\infty}^{\infty} \psi^*(\vec{x}, t) \frac{\hbar}{j} \frac{\partial}{\partial x} \psi(\vec{x}, t) d\vec{x} / N$$

$$p_x = \int_{-\infty}^{\infty} Ae^{-j(10x+3y-4t)} \frac{\hbar}{j} \frac{\partial}{\partial x} Ae^{j(10x+3y-4t)} d\vec{x} / N$$

$$p_x = \int_{-\infty}^{\infty} 10\hbar A^2 dx / N$$

$$p_x = 10\hbar$$

$$p_z = \int_{-\infty}^{\infty} \psi^*(\vec{x}, t) \frac{\hbar}{j} \frac{\partial}{\partial z} \psi(\vec{x}, t) d\vec{x} / N$$

$$p_z = 0$$

$$E = \int_{-\infty}^{\infty} \psi^*(\vec{x}, t) \frac{\hbar}{j} \frac{\partial}{\partial t} \psi(\vec{x}, t) d\vec{x} / N$$

$$E = 4\hbar$$

$$f = 400\hbar^2 + 28\hbar m$$

2.11

A particle is trapped in the ground state (lowest energy level) of a potential well of width L . To understand how the particle is localized, a common measure is the standard deviation $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ where $\langle x^2 \rangle$ and $\langle x \rangle$ are the expectation values for x^2 and x , respectively. Find the uncertainty Δx in the position of the particle in terms of length L and estimate the minimum uncertainty in the momentum of the particle, using the Heisenberg uncertainty principle in terms of L and the Planck's constant \hbar .

✓ **Answer**

$$\psi(x, t) = A \sin\left(\frac{\pi x}{L}\right) \{0 \leq x \leq L\}$$

$$A = \sqrt{\frac{2}{L}}$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\Delta x = \sqrt{0.28L^2 - 0.25L^2}$$

$$\Delta x = \sqrt{0.03}L$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{\hbar}{2\sqrt{0.03}L}$$