

1

No, solving each game after the random dice rolls using alpha-beta pruning will take significant time, and doing it 50 times per possible move is a lot of processing. What would be recommended instead is to use a playout policy after the random moves to generate feasible endgames after the 50 random moves (much like Monte Carlo Tree Search) or use a heuristic to judge the gamestate.

2

Constraints

- $\text{Alldiff}(F, T, U, W, R, O)$
- $F, T, U, W, R, O \in [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]$
- $C_1, C_2, C_3 \in [0, 1]$
- $2O \bmod 10 = R$
- $C_1 = 2O // 10$
- $(2W + C_1) \bmod 10 = U$
- $C_2 = (2W + C_1) // 10$
- $(2T + C_2) \bmod 10 = O$
- $C_3 = (2T + C_2) // 10$
- $F = C_3$

Breaking Down Constraints into Binary Constraints

- $F, T, U, W, R, O \in [0, 1, 2, 3, 4, 5, 6, 7, 8, 9]$
- $C_1, C_2, C_3 \in [0, 1]$
- None of F, T, U, W, R, O equal each other
- $2O \bmod 10 = R$
- $C_1 = 2O // 10$
- $2W \bmod 10 \in [U, U - 1]$
- $U \bmod 2 = C_1$
- $C_2 = 2W // 10$
- $2T \bmod 10 \in [O, O - 1]$
- $O \bmod 2 = C_2$
- $C_3 = 2T // 10$
- $F = C_3$

Solving

We pick the first variable as O as it has 4 constraints related to it following the MRV heuristic.

1 O

Possible Values

Variable	Values	Remaining Restraints
F	[1]	1
T	[0,1,2,3,4,5,6,7,8,9]	2
U	[0,1,2,3,4,5,6,7,8,9]	2
W	[0,1,2,3,4,5,6,7,8,9]	2
R	[0,1,2,3,4,5,6,7,8,9]	1
O	[0,1,2,3,4,5,6,7,8,9]	4
C_1	[0,1]	2
C_2	[0,1]	2
C_3	[0,1]	2

Value Selection

0 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	[1]	1
T	[5]	1
U	[1,2,3,4,5,6,7,8,9]	2
W	[1,2,3,4,5,6,7,8,9]	2
R	[]	0
O	0	0
C_1	[0]	1
C_2	[0]	1
C_3	[0,1]	2

Variable Selection

Fails, backtracks to previous conflicting assignment.

2 O

Possible Values

Variable	Values	Remaining Restraints
F	[1]	1
T	[0,1,2,3,4,5,6,7,8,9]	2
U	[0,1,2,3,4,5,6,7,8,9]	2

Variable	Values	Remaining Restraints
W	[0,1,2,3,4,5,6,7,8,9]	2
R	[0,1,2,3,4,5,6,7,8,9]	1
O	[0,1,2,3,4,5,6,7,8,9]	4
C_1	[0,1]	2
C_2	[0,1]	2
C_3	[0,1]	2

Value Selection

1 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	[]	1
T	[0,5]	1
U	[0,2,3,4,5,6,7,8,9]	2
W	[0,2,3,4,5,6,7,8,9]	2
R	[2]	0
O	1	0
C_1	[0]	1
C_2	[1]	1
C_3	[0,1]	2

Variable Selection

Fails, backtracks to previous conflicting assignment.

3 O

Possible Values

Variable	Values	Remaining Restraints
F	[1]	1
T	[0,1,2,3,4,5,6,7,8,9]	2
U	[0,1,2,3,4,5,6,7,8,9]	2
W	[0,1,2,3,4,5,6,7,8,9]	2
R	[0,1,2,3,4,5,6,7,8,9]	1
O	[0,1,2,3,4,5,6,7,8,9]	4
C_1	[0,1]	2

Variable	Values	Remaining Restraints
C_2	[0,1]	2
C_3	[0,1]	2

Value Selection

2 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	[1]	1
T	[1,6]	1
U	[0,1,3,4,5,6,7,8,9]	2
W	[0,1,3,4,5,6,7,8,9]	2
R	[4]	0
O	2	0
C_1	[0]	1
C_2	[0]	1
C_3	[0,1]	2

Variable Selection

R is chosen as it has the least remaining restraints

3-1 R

Possible Values

Variable	Values	Remaining Restraints
F	[1]	1
T	[1,6]	1
U	[0,1,3,4,5,6,7,8,9]	2
W	[0,1,3,4,5,6,7,8,9]	2
R	[4]	0
O	2	0
C_1	[0]	1
C_2	[0]	1
C_3	[0,1]	2

Value Selection

4 is selected.

Forward Checking

Variable	Values	Remaining Constraints
F	[1]	1
T	[1,6]	1
U	[0,1,3,5,6,7,8,9]	2
W	[0,1,3,5,6,7,8,9]	2
R	4	0
O	2	0
C_1	[0]	1
C_2	[0]	1
C_3	[0,1]	2

Variable Selection

F is chosen as it has the least remaining constraints

3-1-1 F

Possible Values

Variable	Values	Remaining Constraints
F	[1]	1
T	[1,6]	1
U	[0,1,3,5,6,7,8,9]	2
W	[0,1,3,5,6,7,8,9]	2
R	4	0
O	2	0
C_1	[0]	1
C_2	[0]	1
C_3	[0,1]	2

Value Selection

1 is selected.

Forward Checking

Variable	Values	Remaining Constraints
F	1	0

Variable	Values	Remaining Restraints
T	[1,6]	1
U	[0,3,5,6,7,8,9]	2
W	[0,3,5,6,7,8,9]	2
R	4	0
O	2	0
C_1	[0]	1
C_2	[0]	1
C_3	[1]	1

Variable Selection

C_1 is chosen as it has the least remaining restraints

3-1-1-1 C_1

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	[1,6]	1
U	[0,3,5,6,7,8,9]	2
W	[0,3,5,6,7,8,9]	2
R	4	0
O	2	0
C_1	[0]	1
C_2	[0]	1
C_3	[1]	1

Value Selection

0 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	[1,6]	1
U	[6,8]	1
W	[0,3,5,6,7,8,9]	2
R	4	0

Variable	Values	Remaining Restraints
O	2	0
C_1	0	0
C_2	[0]	1
C_3	[1]	1

Variable Selection

C_2 is chosen as it has the least remaining restraints

3-1-1-1-1 C_2

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	[1,6]	1
U	[6,8]	1
W	[0,3,5,6,7,8,9]	2
R	4	0
O	2	0
C_1	0	0
C_2	[0]	1
C_3	[1]	1

Value Selection

0 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	[1,6]	1
U	[6,8]	1
W	[0,3]	1
R	4	0
O	2	0
C_1	0	0
C_2	0	0
C_3	[1]	1

Variable Selection

C_3 is chosen as it has the least remaining restraints

3-1-1-1-1-1 C_3

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	[1,6]	1
U	[6,8]	1
W	[0,3]	1
R	4	0
O	2	0
C_1	0	0
C_2	0	0
C_3	[1]	1

Value Selection

1 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	[6]	0
U	[6,8]	1
W	[0,3]	1
R	4	0
O	2	0
C_1	0	0
C_2	0	0
C_3	1	0

Variable Selection

T is chosen as it has the least remaining restraints

3-1-1-1-1-1 T

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	[6]	0
U	[6,8]	1
W	[0,3]	1
R	4	0
O	2	0
C_1	0	0
C_2	0	0
C_3	1	0

Value Selection

1 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	6	0
U	[8]	1
W	[0,3]	1
R	4	0
O	2	0
C_1	0	0
C_2	0	0
C_3	1	0

Variable Selection

U is chosen as it has the least remaining restraints

3-1-1-1-1-1-1 U

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	6	0
U	[8]	1
W	[0,3]	1

Variable	Values	Remaining Restraints
R	4	0
O	2	0
C_1	0	0
C_2	0	0
C_3	1	0

Value Selection

8 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	6	0
U	8	0
W	[]	0
R	4	0
O	2	0
C_1	0	0
C_2	0	0
C_3	1	0

Variable Selection

Fails, backtracks to previous conflicting assignment.

4 O

Possible Values

Variable	Values	Remaining Restraints
F	[1]	1
T	[0,1,2,3,4,5,6,7,8,9]	2
U	[0,1,2,3,4,5,6,7,8,9]	2
W	[0,1,2,3,4,5,6,7,8,9]	2
R	[0,1,2,3,4,5,6,7,8,9]	1
O	[0,1,2,3,4,5,6,7,8,9]	4
C_1	[0,1]	2
C_2	[0,1]	2

Variable	Values	Remaining Restraints
C_3	[0,1]	2

Value Selection

3 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	[1]	1
T	[1,6]	1
U	[0,1,2,4,5,6,7,8,9]	2
W	[0,1,2,4,5,6,7,8,9]	2
R	[6]	0
O	3	0
C_1	[0]	1
C_2	[1]	1
C_3	[0,1]	2

Variable Selection

R is chosen as it has the least remaining restraints

4-1 R

Possible Values

Variable	Values	Remaining Restraints
F	[1]	1
T	[1,6]	1
U	[0,1,2,4,5,6,7,8,9]	2
W	[0,1,2,4,5,6,7,8,9]	2
R	[6]	0
O	3	0
C_1	[0]	1
C_2	[1]	1
C_3	[0,1]	2

Value Selection

6 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	[1]	1
T	[1,6]	1
U	[0,1,2,4,5,7,8,9]	2
W	[0,1,2,4,5,7,8,9]	2
R	6	0
O	3	0
C_1	[0]	1
C_2	[1]	1
C_3	[0,1]	2

Variable Selection

F is chosen as it has the least remaining restraints

4-1-1 F

Possible Values

Variable	Values	Remaining Restraints
F	[1]	1
T	[1]	1
U	[0,1,2,4,5,7,8,9]	2
W	[0,1,2,4,5,7,8,9]	2
R	6	0
O	3	0
C_1	[0]	1
C_2	[1]	1
C_3	[0,1]	2

Value Selection

1 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	[]	1
U	[0,2,4,5,7,8,9]	2

Variable	Values	Remaining Restraints
W	[0,2,4,5,7,8,9]	2
R	6	0
O	3	0
C_1	[0]	1
C_2	[1]	1
C_3	[1]	1

Variable Selection

Fails, backtracks to previous conflicting assignment.

5 O

Possible Values

Variable	Values	Remaining Restraints
F	[1]	1
T	[0,1,2,3,4,5,6,7,8,9]	2
U	[0,1,2,3,4,5,6,7,8,9]	2
W	[0,1,2,3,4,5,6,7,8,9]	2
R	[0,1,2,3,4,5,6,7,8,9]	1
O	[0,1,2,3,4,5,6,7,8,9]	4
C_1	[0,1]	2
C_2	[0,1]	2
C_3	[0,1]	2

Value Selection

4 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	[1]	1
T	[2,7]	1
U	[0,1,2,3,5,6,7,8,9]	2
W	[0,1,2,3,5,6,7,8,9]	2
R	[8]	0
O	4	0
C_1	[0]	1

Variable	Values	Remaining Restraints
C_2	[0]	1
C_3	[0,1]	2

Variable Selection

R is chosen as it has the least remaining restraints

5-1 R

Possible Values

Variable	Values	Remaining Restraints
F	[1]	1
T	[2,7]	1
U	[0,1,2,3,5,6,7,8,9]	2
W	[0,1,2,3,5,6,7,8,9]	2
R	[8]	0
O	4	0
C_1	[0]	1
C_2	[0]	1
C_3	[0,1]	2

Value Selection

8 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	[1]	1
T	[2,7]	1
U	[0,1,2,3,5,6,7,9]	2
W	[0,1,2,3,5,6,7,9]	2
R	8	0
O	4	0
C_1	[0]	1
C_2	[0]	1
C_3	[0,1]	2

Variable Selection

F is chosen as it has the least remaining restraints

5-1-1 F

Possible Values

Variable	Values	Remaining Restraints
F	[1]	1
T	[2,7]	1
U	[0,1,2,3,5,6,7,9]	2
W	[0,1,2,3,5,6,7,9]	2
R	8	0
O	4	0
C_1	[0]	1
C_2	[0]	1
C_3	[0,1]	2

Value Selection

1 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	[2,7]	1
U	[0,2,3,5,6,7,9]	2
W	[0,2,3,5,6,7,9]	2
R	8	0
O	4	0
C_1	[0]	1
C_2	[0]	1
C_3	[1]	1

Variable Selection

C_1 is chosen as it has the least remaining restraints

5-1-1-1 C_1

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	[2,7]	1
U	[0,2,3,5,6,7,9]	2
W	[0,2,3,5,6,7,9]	2
R	8	0
O	4	0
C_1	[0]	1
C_2	[0]	1
C_3	[1]	1

Value Selection

0 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	[2,7]	1
U	[0,2,6]	1
W	[0,2,3,5,6,7,9]	2
R	8	0
O	4	0
C_1	0	0
C_2	[0]	1
C_3	[1]	1

Variable Selection

C_2 is chosen as it has the least remaining restraints

5-1-1-1-1 C_2

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	[2,7]	1
U	[0,2,6]	1
W	[0,2,3,5,6,7,9]	2

Variable	Values	Remaining Restraints
R	8	0
O	4	0
C_1	0	0
C_2	[0]	1
C_3	[1]	1

Value Selection

0 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	[2,7]	1
U	[0,2,6]	1
W	[0,2,3]	1
R	8	0
O	4	0
C_1	0	0
C_2	0	0
C_3	[1]	1

Variable Selection

C_3 is chosen as it has the least remaining restraints

5-1-1-1-1-1 C_3

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	[2,7]	1
U	[0,2,6]	1
W	[0,2,3]	1
R	8	0
O	4	0
C_1	0	0
C_2	0	0

Variable	Values	Remaining Restraints
C_3	[1]	1

Value Selection

0 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	[7]	0
U	[0,2,6]	1
W	[0,2,3]	1
R	8	0
O	4	0
C_1	0	0
C_2	0	0
C_3	1	0

Variable Selection

T is chosen as it has the least remaining restraints

5-1-1-1-1-1 T

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	[7]	0
U	[0,2,6]	1
W	[0,2,3]	1
R	8	0
O	4	0
C_1	0	0
C_2	0	0
C_3	1	0

Value Selection

0 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	7	0
U	[0,2,6]	1
W	[0,2,3]	1
R	8	0
O	4	0
C_1	0	0
C_2	0	0
C_3	1	0

Variable Selection

U is chosen as it has the least remaining restraints

5-1-1-1-1-1 U

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	7	0
U	[0,2,6]	1
W	[0,2,3]	1
R	8	0
O	4	0
C_1	0	0
C_2	0	0
C_3	1	0

Value Selection

0 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	7	0
U	0	0

Variable	Values	Remaining Restraints
W	\square	0
R	8	0
O	4	0
C_1	0	0
C_2	0	0
C_3	1	0

Variable Selection

Fails, backtracks to previous conflicting assignment.

5-1-1-1-1-2 U

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	7	0
U	[2,6]	1
W	[0,2,3]	1
R	8	0
O	4	0
C_1	0	0
C_2	0	0
C_3	1	0

Value Selection

2 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	7	0
U	2	0
W	\square	0
R	8	0
O	4	0
C_1	0	0

Variable	Values	Remaining Restraints
C_2	0	0
C_3	1	0

Variable Selection

Fails, backtracks to previous conflicting assignment.

5-1-1-1-1-1-3 U

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	7	0
U	[6]	1
W	[0,2,3]	1
R	8	0
O	4	0
C_1	0	0
C_2	0	0
C_3	1	0

Value Selection

2 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	7	0
U	6	0
W	[3]	0
R	8	0
O	4	0
C_1	0	0
C_2	0	0
C_3	1	0

Variable Selection

T is chosen as it has the least remaining restraints

5-1-1-1-1-1-3-1 T

Possible Values

Variable	Values	Remaining Restraints
F	1	0
T	7	0
U	6	0
W	[3]	0
R	8	0
O	4	0
C_1	0	0
C_2	0	0
C_3	1	0

Value Selection

2 is selected.

Forward Checking

Variable	Values	Remaining Restraints
F	1	0
T	7	0
U	6	0
W	3	0
R	8	0
O	4	0
C_1	0	0
C_2	0	0
C_3	1	0

Solution

Variable	Values
F	1
T	7
U	6
W	3

Variable	Values
R	8
O	4

$$734+734=1468$$

3

If $WA = \text{green}$ and $V = \text{red}$, then according to AC-3,

- $SA = \text{blue}$
- $NT = \text{red}$
- $Q = \text{green}$
- $NSW = \text{red}$

NSW and V touch, which invalidates the partial assignment.