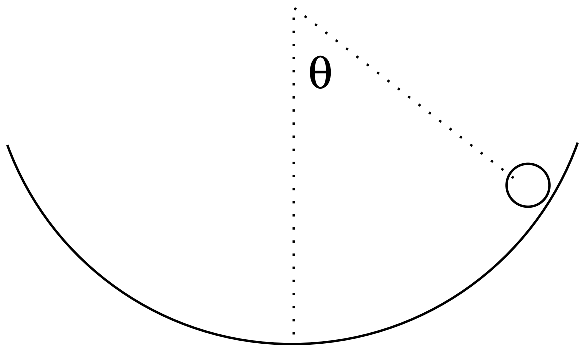
1

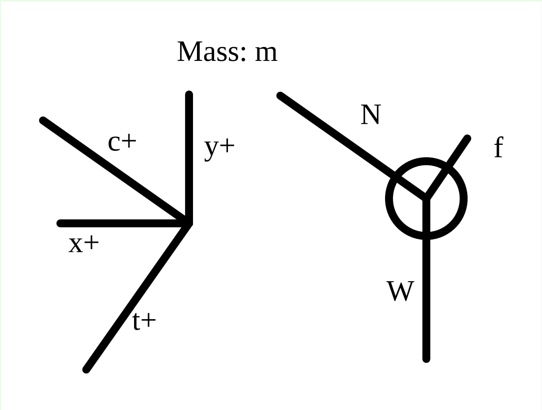
A round marble with a given mass m and given radius r is placed inside a smooth spherical bowl as shown. The radius of curvature for the inside of the bowl is given as R and the marble is released at rest at a position corresponding to a given angle θ_0 relative to the the vertical radius line as shown. Assume $r << R$. Assume that θ_0 is small enough so that the marble rolls without slipping.



a

Draw a Free Body Diagram (FBD) indicating the forces on the marble at the point it is released. Be sure to include a proper coordinate system that is aligned with the direction of acceleration.

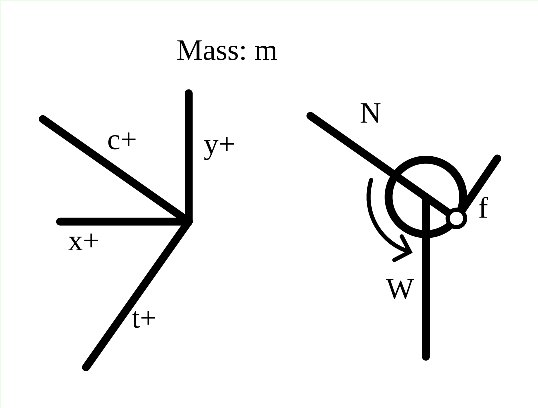
Answer



b

Draw an Extended Free Body Diagram (XFBD) indicating the forces on the marble at the point it is released. Be sure to indicate a “pivot point” and also indicate the direction of positive torque.

✓ **Answer**



C

Calculate the magnitude and direction of the torque about the center of the marble that is applied to the marble at the point instantly after it is released. Give you answer in terms of the given parameters. Explain your work.

✓ **Answer**

The marble is constrained to move in the t^+ direction, which is perpendicular to $\vec{N}$

$W = mg$

$N = W\cos\theta$
$N = mg\cos\theta$

Let $f_c = \frac{f}{m}$
$F = mg\sin\theta - f_c m$
$\tau = rf_c m$
$a = g\sin\theta - f_c$ N2L

$a = r\alpha$ Rolling Constraint
$\tau = I\alpha$ N2L (rot)
$I_s = \frac{2}{5}mr^2$
$\tau = \frac{2mar^2}{5r}$
$\tau = \frac{2mr(g\sin\theta - f_c)}{5}$

$rf_c m = \frac{2mr(g\sin\theta - f_c)}{5}$
$5f_c = 2g\sin\theta - 2f_c$
$f_c = \frac{2}{7}g\sin\theta$

$\tau = \frac{2}{7}grm\sin\theta$

$\tau = \frac{2}{7} grm \sin\theta$
$\square$

d

Calculate the magnitude of the translational acceleration of the marble at the point instantly after it is released? Give you answer in terms of the given parameters. Explain your work.

✓ **Answer**

As from part c,
$a = g\sin\theta - f_c$
$f_c = \frac{2}{7} g\sin\theta$
$a = \frac{5}{7} g\sin\theta$

$a = \frac{5}{7} g\sin\theta$
$\square$

e

Use Conservation of Energy to determine the linear speed of the marble when it is at the bottom of the bowl. Don't forget that you must include both linear and rotational kinetic energy. Give you answer in terms of the given parameters. Explain your work.

✓ **Answer**

Let $R' = R - r$
$U_{gi} = mgR'(1 - \cos\theta)$
$K_{ti} = 0$
$K_{ri} = 0$

$E_i = mgR'(1 - \cos\theta) = E_f$ Conservation of energy with no external work (gravity accounted for)
$U_{gf} = 0$
$v = r\omega$
$K_{tf} = \frac{1}{2} mv^2$ Kinetic energy formula
$K_{rf} = \frac{1}{2} I\omega^2$ Kinetic energy (rot) formula
$K_{rf} = \frac{1}{5} mv^2$

$E_f = \frac{7}{10} mv^2 = mgR'(1 - \cos\theta)$
$v = \sqrt{\frac{10}{7} g(R - r)(1 - \cos\theta)}$

$v=\sqrt{\frac{10}{7}g(R-r)(1-\cos\theta)}$

□

f

What is the magnitude of the Normal Force on the marble when it is at the bottom of the bowl? Give you answer in terms of the given parameters. Explain your work. Hint: $N=mg$ is the wrong answer.

✓ **Answer**

$a_c=\frac{v^2}{R-r}$ centripetal acceleration formula
$v=\sqrt{\frac{10}{7}g(R-r)(1-\cos\theta)}$ from part (e)
$a_c=\frac{10}{7}g(1-\cos\theta)$

$F=\frac{10}{7}mg(1-\cos\theta)$

The only vertical forces are W, N
$\therefore \vec{W}+\vec{N}=\vec{F}$

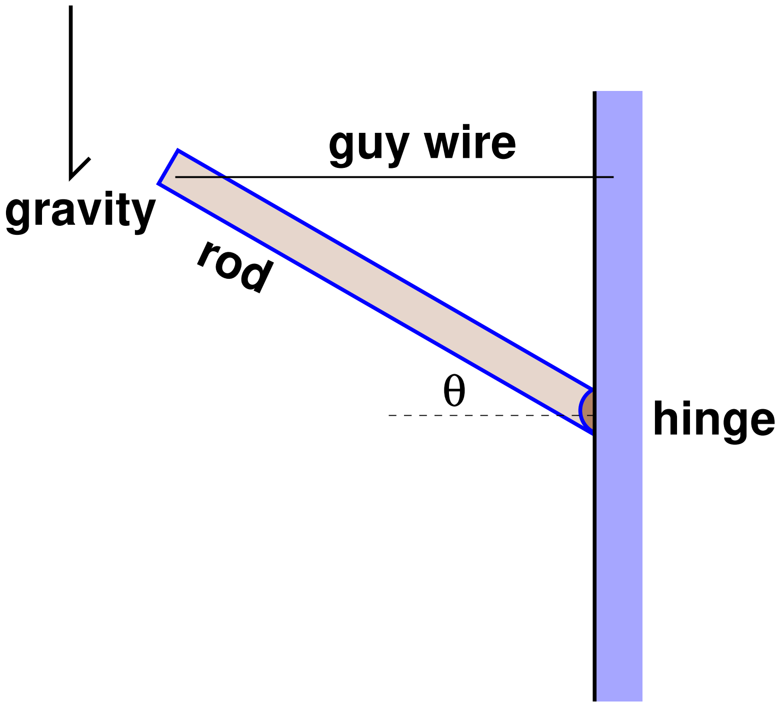
$N=\frac{10}{7}mg(1-\cos\theta)+mg$
$N=\frac{17-10\cos\theta}{7}mg$

$N=\frac{17-10\cos\theta}{7}mg$

□

2

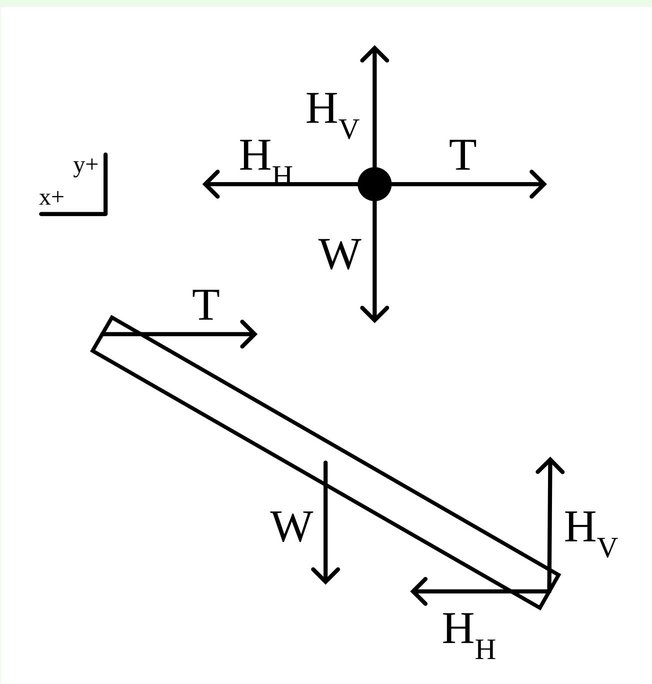
A thin rod of given mass m is attached to an hinge so that it can swing vertically. The length of the rod is given as l and the rotational inertial of the rod about one end is $I=\frac{1}{3}ml^2$. The rod is held in place at a given angle θ with respect to the horizontal by an guy wire which we represent as an ideal string which connects the far end of the bar to the wall as shown.



a

Draw a careful and properly labeled Free-Body Diagram (FBD) that shows all of the forces on the thin rod. Also, draw a careful and properly labeled Extended Free-Body Diagram (XFBD) for the thin rod. In your diagrams, the unknown hinge force should be represented by two components, H_H and H_V, corresponding to the horizontal and vertical components respectively.

✓ Answer



b

Calculate the magnitudes of both H_V and H_H. Express your answers in terms of the given parameters. Explain. Hint: for this part use a coordinate system that is lined up with the Hinge Force components (not tilted). Another hint: $\sin(90^\circ - \theta) = \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$.

✓ **Answer**

$\vec{F} = \vec{0}$
$\vec{W} + \vec{H}_V = 0$
$\vec{T} + \vec{H}_H = 0$

$W = mg = H_V$ N2L

$\tau = 0$
$\tau_W = mg\cos\theta$ Torque formula
$\tau_T = T\sin\theta$
$mg\cos\theta = T\sin\theta$ Sum of torques is zero, as no rotational acceleration
$T = \frac{mg}{\tan\theta} = H_H$ N2L

$H_V = mg$
$H_H = \frac{mg}{\tan\theta}$
$\square$

c

Suppose the guy wire is cut so that the rod is suddenly free to begin to rotate on the hinge. What is the magnitude of the acceleration of the center-of-mass point of the rod just at the instant after the wire is cut? Express your answer in term of the given parameters. Explain your work. Hint: for this part use a coordinate system that is tilted so that it is aligned with the instantaneous direction of acceleration.

✓ **Answer**

$\tau_W = mg\cos\theta$ Torque formula
$\tau = \tau_W$ weight is the only torque

$\tau = I\alpha$
$I = \frac{1}{3}ml^2$
$\alpha = \frac{3g}{l^2}\cos\theta$

$R = \frac{l}{2}$ center of mass is of radius of half the length
$a = R\alpha$ linear/rotational relation
$\alpha = \frac{3g}{2l}\cos\theta$

$\alpha = \frac{3g}{2l}\cos\theta$
$\square$

d

After the wire is cut, the rod swings down and collides flush with the wall. What is ω_{hit}, the angular speed of the rod at the instant just before it hits the wall? Express your answer in term of the given parameters. Explain your work.

✓ **Answer**

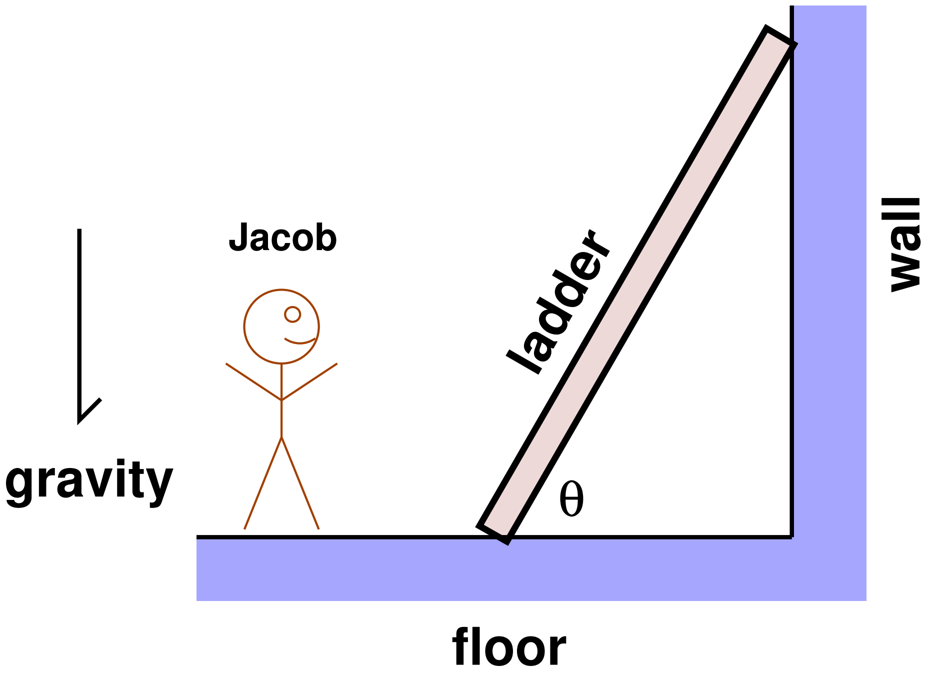
$U_{gi} = \frac{l}{2}(1+\sin\theta)mg$ initial height
$K_{ti} = 0$
$K_{ri} = 0$
$K_i = \frac{l}{2}(1+\sin\theta)mg = K_f$ Conservation of energy (gravity accounted)

$U_{gf} = 0$
$K_{tf} = \frac{1}{2}mv^2$ kinetic energy formulas
$K_{rf} = \frac{1}{2}I\omega^2$
$v = \frac{l\omega}{2}$ translational/rotational relation
$K_{rf} = \frac{1}{24}mv^2$
$K_f = \frac{13}{24}mv^2$
$\frac{l}{2}(1+\sin\theta)mg = \frac{13}{24}mv^2$
$v = \sqrt{\frac{12}{13}(1+\sin\theta)gl}$
$\omega = \sqrt{\frac{48}{13l}(1+\sin\theta)g}$

$\omega = \sqrt{\frac{48}{13l}(1+\sin\theta)g}$
$\square$

3

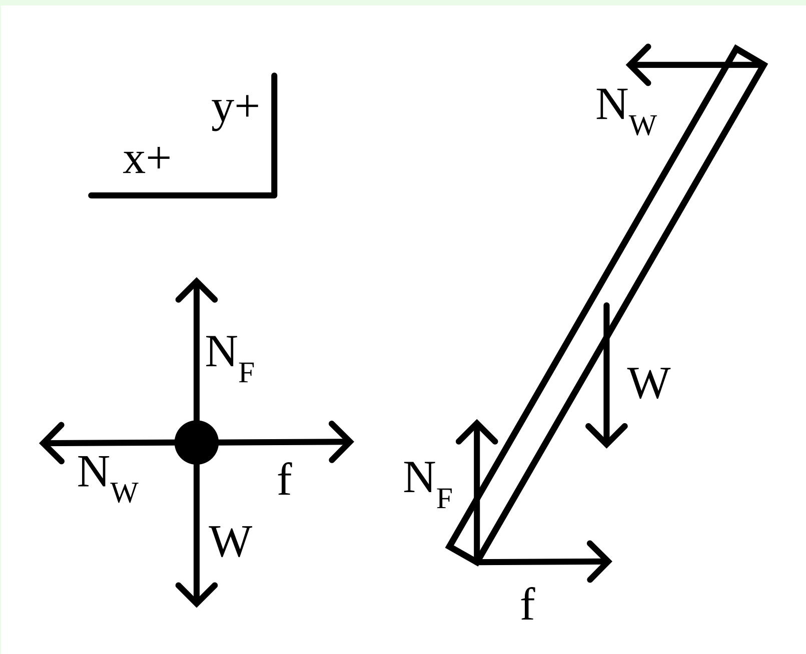
A ladder of given length L and mass M leans at rest against the wall at a given angle θ as shown. Assume that the surface of the wall is frictionless, but assume that the floor applies both a Normal force and an Friction force to the ladder.



a

Draw a Free Body Diagram and also an Extended Free Body Diagram for the ladder. Be sure to label each of your forces appropriately.

✓ Answer



b

Calculate the magnitude of each and every force on the ladder. Present your answers in terms of given parameters. Explain your work.

✓ **Answer**

$\vec{F} = \vec{0}$ Object is not accelerating
$W = mg$ N2L
$N_F = W = mg$ N2L (vertical)
$N_W = f$ N2L (horiz)

With the pivot as the floor contact point,
$\tau = 0$
$\tau_W = mg\cos\theta$
$\tau_{N_W} = N_W \sin\theta$
$N_W = \frac{mg}{\tan\theta} = f$

$W = mg$
$N_F = mg$
$N_W = \frac{mg}{\tan\theta}$
$f = \frac{mg}{\tan\theta}$
$\square$

c

Suppose the coefficients of kinetic friction and static friction are given as μ_k and μ_s respectively. What is the constraint that applies to the value of θ that ensures that the ladder stays at rest and does not slip and fall down? Explain your work.

✓ **Answer**

$f \leq N_F \mu_s$ Static friction formula
$\frac{1}{\tan\theta} \leq \mu_s$ Algebra

$\frac{1}{\tan\theta} \leq \mu_s$
$\square$