1.33

Where M is defined as being a man, and C is defined as being color blind
$P(C|M) = 0.05$
$P(C|M^C) = 0.0025$
$P(M) = P(M^C) = 0.5$

$P(C|M)P(M) = P(C \cap M)$
$P(C|M^C)P(M^C) = P(C \cap M^C)$

$P(C) = P(C \cap M) + P(C \cap M^C)$
$= P(C|M)P(M) + P(C|M^C)P(M^C)$
$= 0.02625$

$P(M|C) = P(C|M)P(M)/P(C)$
$= 0.05 * 0.5/0.02625$
$= 0.9524$

1.35

The first axiom is true by definition that $P(\cdot) \geq 0$ is a valid probability function

In the sample space of B, $\mathbb{U} = B$,
$P(\mathbb{U}|B) = P(B|B) = P(B \cap B)/P(B) = P(B)/P(B) = 1$
Thus the second axiom is true

$P((\bigcup\limits_{i=1}^{\infty} A_i)|B)$
$= P((\bigcup\limits_{i=1}^{\infty} A_i) \cap B)/P(B)$
$= P(\bigcup\limits_{i=1}^{\infty} (A_i \cap B))/P(B)$

Since the collection A is disjoint, $A \cap B$ is also disjoint
$= (\sum\limits_{i=1}^{\infty} P(A_i \cap B))/P(B)$
$= \sum\limits_{i=1}^{\infty} (P(A_i \cap B)/P(B))$
$= \sum\limits_{i=1}^{\infty} P(A_i|B)$

Thus the third axiom holds true

1.39

a

If A and B are mutually exclusive, then $A \cap B = \varnothing$, but if A and B are independent, then
$P(A \cap B) = P(A)P(B)$
$P(\varnothing) = P(A)P(B)$

$P(A)P(B) = 0$
which is false unless
$P(A)$ or $P(B) = 0$

b

If A and B are independent, then $P(A \cap B) = P(A)P(B)$, but if A and B are mutually exclusive, then
$A \cap B = \varnothing$
$P(A \cap B) = 0$
$P(A \cap B) = P(A)P(B)$
$P(A)P(B) = 0$
which is false unless
$P(A)$ or $P(B) = 0$

1

A club has 25 members.

1. How many ways are there to choose four members of the club to serve on an executive committee?
2. How many ways are there to choose a president, vice president, secretary, and treasurer of the club?

a

Assuming the order of which the people get selected onto the committee does not matter, then we will be performing an unordered selection without replacement as a person cannot be selected twice.

There will be $\binom{25}{4} = 12650$ ways to pick the people on the committee

b

Now that there are positions, the order of which they are selected does matter. We will be performing an ordered without replacement search.

There will be $25!/(25-4)! = 303600$ ways to pick the committee

2

There are three cabinets A, B and C, each of which has two drawers. Each drawer contains one coin; A has two gold coins, B has two silver coins, and C has one gold and one silver. A cabinet is chosen at random, one drawer is opened, and a silver coin is found. What is the probability that the other drawer in that cabinet contains a silver coin?

$P(A) = P(B) = P(C) = 1/3$
$P(S|A) = 0$
$P(S|B) = 1$
$P(S|C) = 0.5$

$P(S \cap A) = 0$
$P(S \cap B) = 1/3$
$P(S \cap C) = 1/6$

Since $\{A, B, C\}$ form a slice
$P(S) = 1/3 + 1/6 = 1/2$

$P(B|S) = P(S|B)P(B)/P(S)$
$= (1/3)/(1/2)$
$= 2/3$