3.1

If $X \sim \mathrm{DUniform}(N_0, N_1)$, then

EX

$$EX = \sum_{n=N_0}^{N_1} n \frac{1}{N_1-N_0+1}$$
$$= \frac{1}{N_1-N_0+1} \sum_{n=N_0}^{N_1} n$$
$$= \frac{1}{N_1-N_0+1} (\sum_{n=0}^{N_1} n - \sum_{n=0}^{N_0-1} n)$$
$$= \frac{1}{N_1-N_0+1} (\frac{N_1(N_1+1)}{2} - \frac{N_0(N_0-1)}{2})$$
$$= \frac{1}{N_1-N_0+1} \frac{N_1^2+N_1-N_0^2+N_0}{2}$$
$$= \frac{N_1^2+N_1-N_0^2+N_0}{2(N_1-N_0+1)}$$
$$= \frac{(N_0+N_1)(N_1-N_0+1)}{2(N_1-N_0+1)}$$
$$= \frac{N_0+N_1}{2}$$
□

EX^2

$$EX^2 = \sum_{n=N_0}^{N_1} n^2 \frac{1}{N_1-N_0+1}$$
$$= \frac{1}{N_1-N_0+1} \sum_{n=N_0}^{N_1} n^2$$
$$= \frac{1}{N_1-N_0+1} (\sum_{n=0}^{N_1} n^2 - \sum_{n=0}^{N_0-1} n^2)$$
$$= \frac{1}{N_1-N_0+1} (\frac{N_1(N_1+1)(2N_1+1)}{6} - \frac{N_0(N_0-1)(2N_0-1)}{6})$$
$$= \frac{1}{N_1-N_0+1} \frac{2N_1^3+3N_1^2+N_1-2N_0^3+3N_0^2-N_0}{6}$$
$$= \frac{2N_1^3+3N_1^2+N_1-2N_0^3+3N_0^2-N_0}{6(N_1-N_0+1)}$$
$$= \frac{(N_1-N_0+1)(2N_0^2+2N_0N_1-N_0+2N_1^2+N_1)}{6(N_1-N_0+1)}$$
$$= \frac{2N_0^2+2N_0N_1-N_0+2N_1^2+N_1}{6}$$

$\mathrm{Var} X$

$$= \frac{2N_0^2+2N_0N_1-N_0+2N_1^2+N_1}{6} - (\frac{N_1+N_0}{2})^2$$
$$= \frac{2N_0^2+2N_0N_1-N_0+2N_1^2+N_1}{6} - \frac{N_1^2+2N_1N_0+N_0^2}{4}$$
$$= \frac{4N_0^2+4N_0N_1-2N_0+4N_1^2+2N_1-3N_1^2-6N_1N_0-3N_0^2}{12}$$
$$= \frac{N_0^2-2N_0N_1-2N_0+N_1^2+2N_1}{12}$$
$$= \frac{(N_1-N_0)(N_1-N_0+2)}{12}$$
□

3.5

$p = 0.8$
$N = 100$
$x = 85$

This can be modeled with the binomial distribution with $p = 0.8$ and $n = 100$, with a possibility of this happening being $P(X \geq 85)$ where $X \sim \mathrm{Binom}(100, 0.8)$

$P(X \geq 85) = 1 - P(X < 85)$
$= \sum_{n=85}^{100} \binom{100}{n} 0.8^n 0.2^{100-n}$
$= 12.85\%$

12.85% is insignificant with a 5% significance level is insignificant. The 85 positives may have been down to chance and we cannot conclude that the new drug is better than the old one.

3.7

$X \sim \mathrm{Poisson}(\lambda)$
$P(X \geq 2) \geq 0.99$
$P(X \leq 1) = P(X = 1) + P(X = 0) \leq 0.01$
$= e^{-\lambda} + \lambda e^{-\lambda} \leq 0.01$

Graphically solved, this gives us a λ of 6.638
$\square$

3.12

$X \sim \mathrm{Binom}(n, p)$
$Y \sim \mathrm{NegBinom}(r, p)$

$F_X(r-1)$ would represent the likelihood of sampling $r-1$ or less successes after n trials with probability p
$= P(X \leq r-1)$
Which means taking less than or equal to $r-1$ successes in n trials with p probability

$1 - F_Y(n-r)$ would represent the likelihood of not sampling r or more successes in n trials with probability p
$= 1 - P(Y \leq n-r)$
$= P(Y > n-r)$
Which means the probability of taking more than n trials to get r successes with probability p, which also means getting less than or equal to $r-1$ successes in n trials with p probability

3.13

a

PDF

$X \sim \text{Poisson}(\lambda)$

$P(X = 0) = e^{-\lambda}$
$P(X > 0) = 1 - e^{-\lambda}$

$f_Y = \frac{e^{-\lambda}\lambda^x}{x!(1-e^{-\lambda})}$
$\square$

Mean

$\mu_X = \sum_{n=0}^{\infty} \frac{ne^{-\lambda}\lambda^n}{n!} = \lambda$

$\mu_Y = \sum_{n=1}^{\infty} \frac{ne^{-\lambda}\lambda^n}{n!(1-e^{-\lambda})}$
$\mu_Y = \frac{1}{1-e^{-\lambda}} \sum_{n=1}^{\infty} \frac{ne^{-\lambda}\lambda^n}{n!}$
$\mu_Y = \frac{1}{1-e^{-\lambda}} (\sum_{n=0}^{\infty} \frac{ne^{-\lambda}\lambda^n}{n!}) - 0$
$\mu_Y = \frac{1}{1-e^{-\lambda}} \lambda - 0$
$\mu_Y = \frac{\lambda}{1-e^{-\lambda}}$
$\square$

Variance

$\mu_{2X} = \sum_{n=0}^{\infty} \frac{n^2e^{-\lambda}\lambda^n}{n!} = \lambda^2 + \lambda$

μ_{2Y}
$\mu_{2Y} = \sum_{n=1}^{\infty} \frac{n^2e^{-\lambda}\lambda^n}{n!(1-e^{-\lambda})}$
$\mu_{2Y} = \frac{1}{1-e^{-\lambda}} \sum_{n=1}^{\infty} \frac{n^2e^{-\lambda}\lambda^n}{n!}$
$\mu_{2Y} = \frac{1}{1-e^{-\lambda}} \sum_{n=0}^{\infty} \frac{n^2e^{-\lambda}\lambda^n}{n!} - 0$
$\mu_{2Y} = \frac{\lambda^2+\lambda}{1-e^{-\lambda}}$

$\sigma^2 = \frac{\lambda^2+\lambda}{1-e^{-\lambda}} - (\frac{\lambda}{1-e^{-\lambda}})^2$
$\sigma^2 = \frac{(1-e^{-\lambda})(\lambda^2+\lambda)-\lambda^2}{(1-e^{-\lambda})^2}$
$\sigma^2 = \frac{\lambda-\lambda^2e^{-\lambda}-\lambda e^{-\lambda}}{(1-e^{-\lambda})^2}$
$\square$