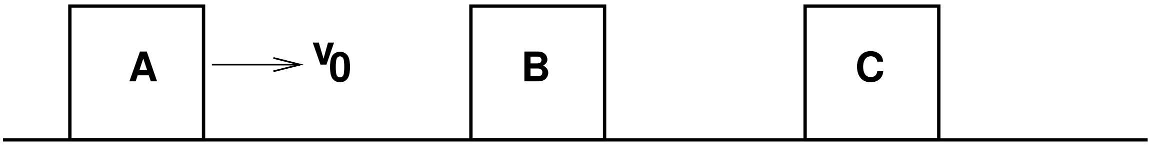
1 - "6"



Three blocks, are placed on a frictionless surface as shown above. Block A has a given mass of m, Block B has a given mass of $2m$ and Block C has a given mass of $5m$. Block A has initial velocity given as v_0 while the other two blocks are initially at rest. Then the following happens:

- First Collision: Block A collides with Block B and this collision is totally inelastic.
- Second Collision: Block B collides with Block C and this collision is totally elastic.

a

Determine the final velocity of every block immediately after the First Collision.

Answer

$p_{ai} = mv_0$ Momentum equation
$p_{bi} = 0$ Momentum equation

$v_{af} = v_{bf}$ Condition of totally inelastic collisions
$p_f = 3mv_f$ Final momentum equation
$mv_0 = 3mv_f$ Conservation of momentum within a system
$v_f = \frac{v_0}{3}$

$v_{af} = \frac{v_0}{3}$ A and B have the same end velocity
$v_{bf} = \frac{v_0}{3}$
$v_{cf} = 0$ C doesn't move
$\square$

b

Determine the final velocity of every block immediately after the Second Collision.

Answer

$p_i = mv_0$ Same initial momentum
$K_i = \frac{1}{6}mv_0^2$ Kinetic energy equation

$K_f = K_i$ Conditions of a totally elastic collision
$p_i = p_f$ Conditions of any interaction within a system
$v_{af} = v_{bf}$ Blocks A and B remain joined

$p_{abf} = 3mv_{abf}$
$p_{cf} = 5mv_{cf}$
$K_{abf} = \frac{3}{2}mv_{abf}^2$
$K_{cf} = \frac{5}{2}mv_{cf}^2$

$$\begin{cases} mv_0 = 3mv_{abf} + 5mv_{cf} \\ \frac{1}{6}mv_0^2 = \frac{3}{2}mv_{abf}^2 + \frac{5}{2}mv_{cf}^2 \end{cases}$$
$$\begin{cases} v_0 = 3v_{abf} + 5v_{cf} \\ v_0^2 = 9v_{abf}^2 + 15v_{cf}^2 \end{cases}$$
$$\begin{cases} \frac{v_0 - 3v_{abf}}{5} = v_{cf} \\ v_0^2 = 9v_{abf}^2 + 15(\frac{v_0 - 3v_{abf}}{5})^2 \end{cases}$$
$$\begin{cases} \frac{v_0 - 3v_{abf}}{5} = v_{cf} \\ v_0^2 = 9v_{abf}^2 + 3(\frac{v_0^2 - 6v_0v_{abf} + 9v_{abf}^2}{5}) \end{cases}$$
$$\begin{cases} \frac{v_0 - 3v_{abf}}{5} = v_{cf} \\ 0 = \frac{72}{5}v_{abf}^2 - \frac{18}{5}v_0v_{abf} - \frac{2}{5}v_0^2 \end{cases}$$
$$\begin{cases} \frac{v_0 - 3v_{abf}}{5} = v_{cf} \\ 0 = 36v_{abf}^2 - 9v_0v_{abf} - v_0^2 \end{cases}$$
$$\begin{cases} \frac{v_0 - 3v_{abf}}{5} = v_{cf} \\ 0 = (12v_{abf} + v_0)(3v_{abf} - v_0) \end{cases}$$
$(v_{abf}, v_{cf}) \in \{(-\frac{v_0}{12}, \frac{v_0}{4}), (\frac{v_0}{3}, 0)\}$
We know that blocks A and B cannot travel through block C, so
$(v_{abf}, v_{cf}) = (-\frac{v_0}{12}, \frac{v_0}{4})$

$v_{af} = -\frac{v_0}{12}$
$v_{bf} = -\frac{v_0}{12}$
$v_{cf} = \frac{v_0}{4}$
$\square$

c

Suppose we define V_{CM} as the velocity of the Center-of-Mass of the *entire system* of three blocks, A, B, and C. What is V_{CM} just before the First collision. What is V_{CM} just after this First Collision? What is V_{CM} just after this Second Collision?

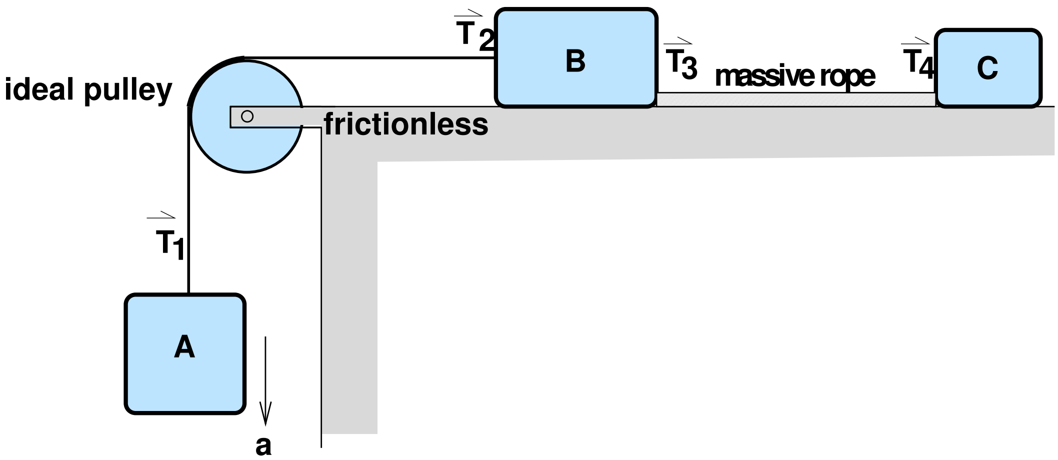
✓ **Answer**

Since the system never has any work done upon it from outside the system, the velocity of the system cannot change.

$p = mv_0$ Sum of momentums of all pieces
$V_{CM} = \frac{v_0}{8}$ Velocity from momentum equation

$V_{CM} = \frac{v_0}{8}$
□

2



Three blocks A, B, and C with a given mass m_A, m_B , and m_C respectively, are positioned as shown above. Blocks B and C are accelerated to the left along the frictionless surface. The pulley is ideal as is the string between Blocks A and B. A massive rope of mass m_r connects Blocks B and C. Assume that the rope sits flush on the frictionless surface so that there is no component of the tension in the vertical direction.

a

Rank the four tension forces from largest to smallest. Explain how you determined this. Hint: this is a *conceptual* question. No algebra or calculations required. Important hint: any solution that includes either of the words “system” or “motion” is probably not correct. You want to invoke a Law of Physics here.

✓ **Answer**

As the whole system is accelerating together constantly, the tension pulling the most mass will have the most tension $(F^{\uparrow} = m^{\uparrow}a)$

$T_1 = T_2 > T_3 > T_4$
□

b

Determine the magnitude of the acceleration a of Block A. Explain your work. Give your answer in terms of the given parameters. Hint: you will simplify your calculation if you consider these three items: Blocks B, Block C and the massive rope, altogether, as one “system”.

✓ **Answer**

If we treat the entire system as a system, the only external force being applied is on block A.

$F_{net} = gm_A$
$a_t = \frac{gm_A}{m_A+m_B+m_r+m_C}$ acceleration of the entire system in the direction of tension.
$a_A = \frac{gm_A}{m_A+m_B+m_r+m_C}$ acceleration of the system in the direction of tension will be the same as the acceleration of block A.

$a_A = \frac{gm_A}{m_A+m_B+m_r+m_C}$
$\square$

c

Assume that the massive rope has length L. Determine the tension *in the massive rope* as a function of horizontal position x along the rope measured relative to the right edge of Block B. Give your answer in terms of the given parameters. Explain your work.

✓ **Answer**

Assuming the rope's mass is equally distributed, the force will also be equally distributed as the acceleration is constant.
$F \propto m$
$T(x=0) = T_3 = \frac{gm_A(m_r+m_C)}{m_A+m_B+m_r+m_C}$
$T(x=L) = T_4 = \frac{gm_Am_C}{m_A+m_B+m_r+m_C}$

We know $T(x)$ is linear w.r.t. x due to the prior relation

$T(x) = \left((1-\frac{x}{L})m_r + m_C\right)\frac{gm_A}{m_A+m_B+m_r+m_C}$ By linear interpolation

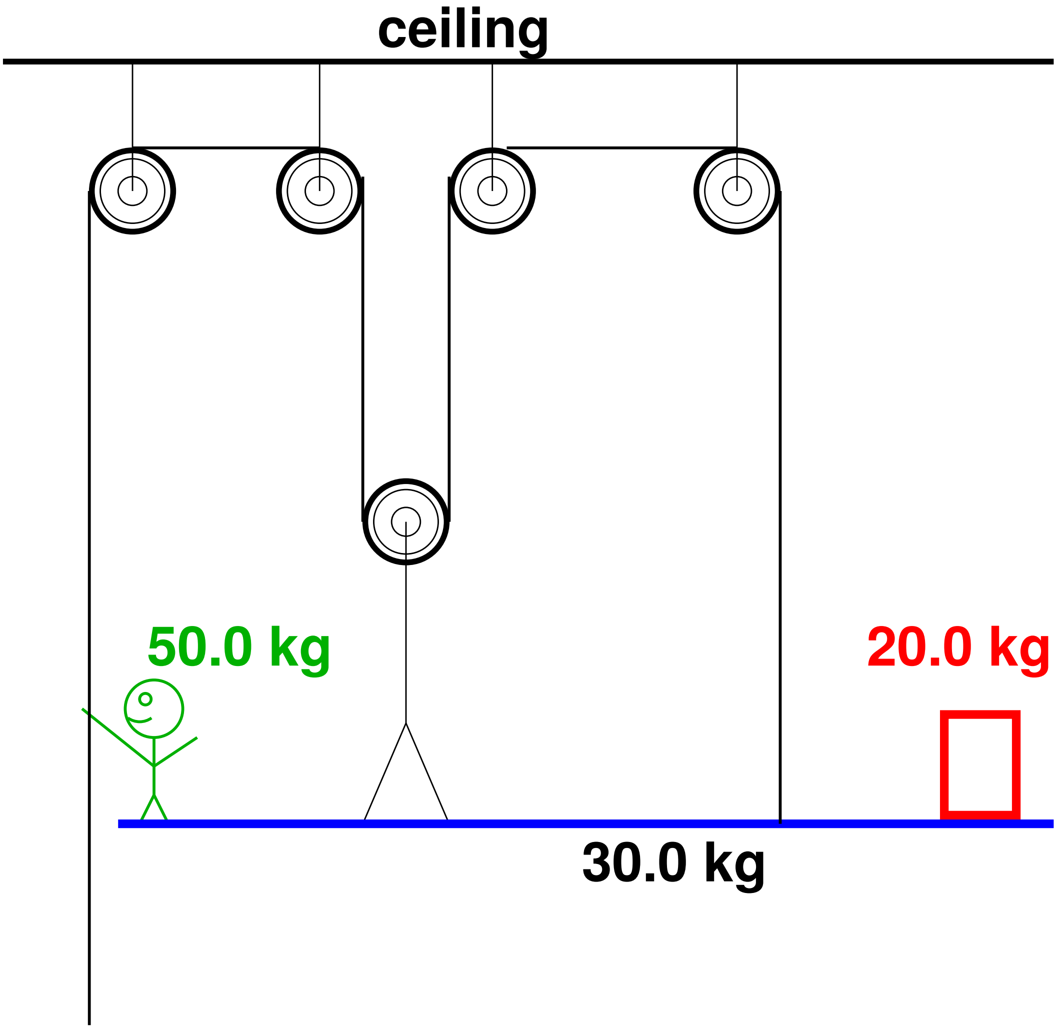
---

$T(x) = \left(\left(1 - \frac{x}{L}\right) m_r + m_C\right) \frac{g m_A}{m_A + m_B + m_r + m_C}$

$\square$

3

Leonard Nimoy stands on a platform and pulls on an ideal rope, which is attached to a system of ideal pulleys as shown:



The mass of Leonard is given as $m_l \equiv 50.0\ kg$. The mass of the platform is given as $m_p \equiv 30.0\ kg$. The mass of a box that sits on the platform is given as $m_b \equiv 20.0\ kg$. Assume that Leonard pulls the rope so that the platform moves upward with given constant acceleration $A_0 \equiv 0.200\ \frac{m}{s^2}$.

a

What are the magnitudes and directions of all forces on the box? Give your answer both in terms of given symbolic parameters and in terms of numeric values. Explain your work.

✓ Answer

The box only has its weight force straight down, and its normal force straight up.
$W_b = -gm_b$ N2L
$F_b = A_0 m_b$ N2L
$F_b = W_b + N_b$ N2L
$N_b = m_b(A_0 + g)$

$W_b = -gm_b$
$N_b = m_b(A_0 + g)$

$W_b = -20.0g\ N$
$N_b = 20.0(0.200 + g)\ N$
$N_b = 200.0\ N$
□

b

What is the tension in the rope? Give your answers in terms of given symbolic parameters and in terms of numeric values. Explain your work.

✓ **Answer**

If we treat everything except the rope as a system, the only external forces are due to gravity and the two tension forces, which must be equal.

$F_{net} = A_0(m_p + m_l + m_b)$ N2L
$W = -g(m_p + m_l + m_b)$ N2L
$T_{net} = T_1 + T_2$
$T_{net} = (A_0 + g)(m_p + m_l + m_b)$ Subtraction
$T_1 = T_2 = \frac{1}{2}(A_0 + g)(m_p + m_l + m_b)$ Algebra

$T_1 = T_2 = \frac{1}{2}(A_0 + g)(m_p + m_l + m_b)$

$T_1 = T_2 = 50.0(0.200 + g)\ N$
$T_1 = T_2 = 500.0\ N$
$\square$

c

What are the magnitudes and directions of all forces on Leonard? Give your answers in terms of given symbolic parameters and in terms of numeric values. Explain your work.

✓ **Answer**

The only forces acting upon Leo is the tension force, weight force, and normal force.

We already know the tension force is
$T_1 = \frac{1}{2}(A_0 + g)(m_p + m_l + m_b)$

We know the weight force is
$W_l = -gm_l$ N2L

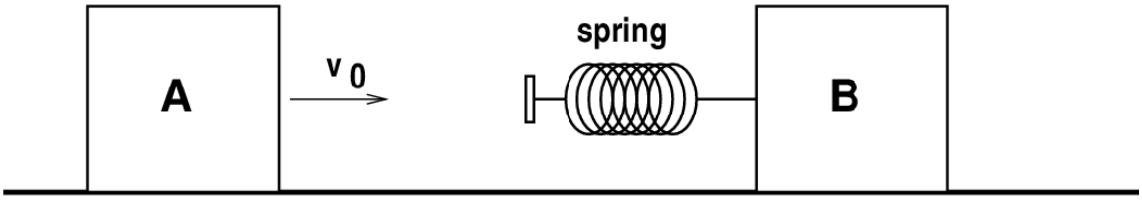
We know the net force must be
$F_l = A_0 m_l$

By N2L we can calculate what the normal force must be.
$N_l = \frac{1}{2}(A_0 + g)(m_p - m_l + m_b)$

All in the upwards direction,
$T_1 = \frac{1}{2}(A_0 + g)(m_p + m_l + m_b)$
$W_l = -gm_l$
$N_l = \frac{1}{2}(A_0 + g)(m_p - m_l + m_b)$

All in the upwards direction,
$T_1 = 500\ N$
$W_l = -494.9\ N$
$N_l = 0\ N$
$\square$

4



Two blocks A and B each with mass $m = 2.0\ kg$ sit on a frictionless surface. Block A has a velocity $v_0 = 5.0\ \frac{m}{s}$ towards block B which is at rest. There is a massless spring with spring constant $k = 1.00\ \frac{N}{m}$ attached to Block B as shown. Block A collides with the spring that is attached Block B. The spring is compressed, and then after a while it is uncompressed and the two blocks separate from each other. Note that because the spring force is Conservative, the collision is ***totally elastic***.

a

What is the final velocity of block A? What is the final velocity of block B? Be sure to show your work.

✓ **Answer**

Since the collision is totally elastic and the blocks have equal mass and there is no external work on the system, all velocity will be transferred from block A to block B. The fact that there is a spring has no effect on the collision as it is totally elastic, as given.

We can check the well known statement that all velocity will be transferred however.

$$p_i = mv_0 + 0$$
$$p_f = 0 + mv_0$$
$$p_i = p_f$$

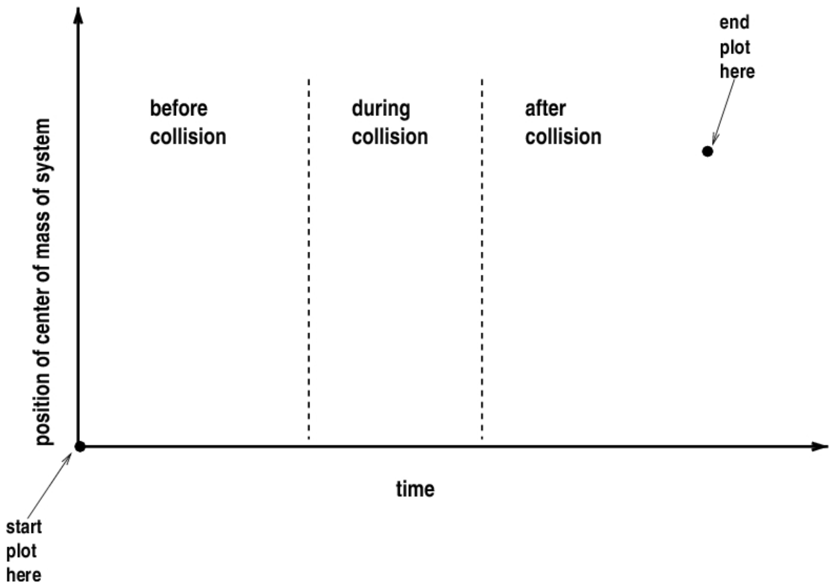
$$K_i = mv_0^2 + 0$$
$$K_f = 0 + mv_0^2$$
$$K_i = K_f$$

$$v_{af} = 0$$
$$v_{bf} = v_0$$

$$v_{af} = 0$$
$$v_{bf} = 5.0\ \frac{m}{s}$$

b

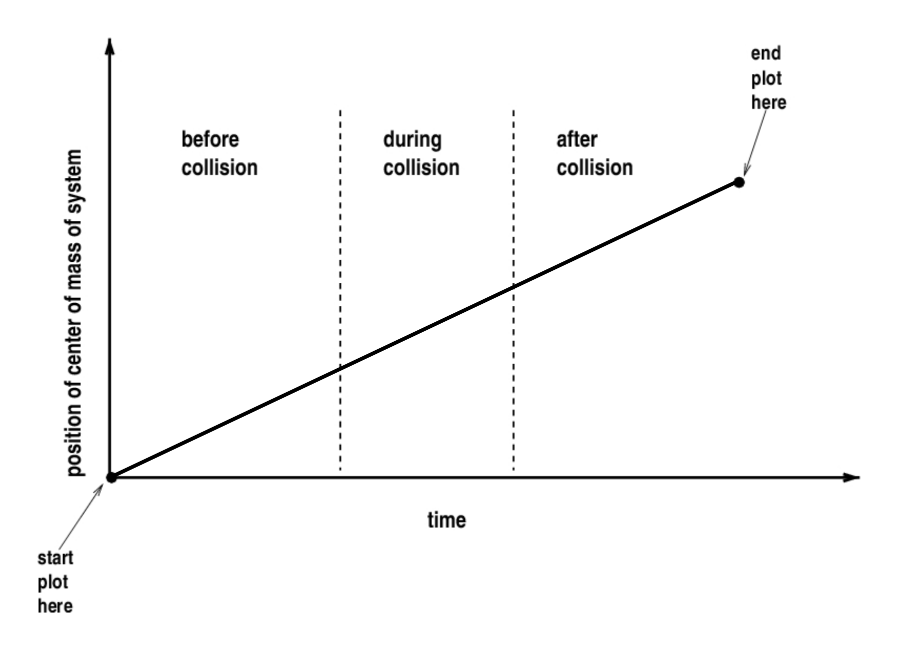
Consider the system which is made of block A, block B, and the spring. On the following figure make a *qualitative plot* of the position of the center of mass x_{cm} for this entire system as a function of time. Start and end your plot at the points indicated. What is the slope of your plot during the intervals of time corresponding to each of these three periods: (1) before, (2) during, and (3) after the collision? Be sure to explain your work. Use physics!



✓ **Answer**

If we define our system as the blocks and the spring, no external work is being done on the system, so the system cannot be accelerating. We do not care about internal forces as they will have internal force pairs by N3L.

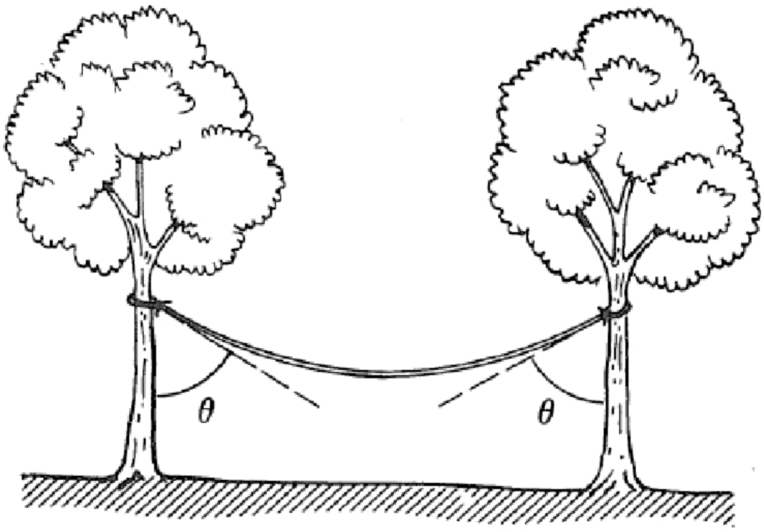
Since the system is not accelerating, it must have a constant velocity and a linear position.



The slope of a position vs. time graph is just the velocity, which we can obtain from the momentum of the system.

$p = mv_0$
$v = \frac{mv_0}{2m}$
$v = \frac{v_0}{2}$
$v_{bf} = 2.5\ \frac{m}{s}$

5



A massive uniform rope of total mass m and length L is attached to two trees as shown in the figure above. The connection point for the rope is the same height on each tree, each rope makes an given angle θ with each tree as shown.

a

Find the tension at either end of the rope.

✓ **Answer**

We know the sum of the tensions must be counteracting the weight force as there is no net force on the rope as a whole. By N2L.

$$0 = T_{net} + W$$
$$W = -mg$$
$$T_{net} = mg$$
$$\vec{T}_{net} = \langle 0, mg \rangle$$
$$\vec{T}_1 = T_1 \langle -\sin\theta, \cos\theta \rangle$$
$$\vec{T}_2 = T_2 \langle \sin\theta, \cos\theta \rangle$$
$$T_{net} = T_1 + T_2$$
$$\langle (T_2 - T_1)\sin\theta, (T_2 + T_1)\cos\theta \rangle = \langle 0, mg \rangle$$
$$\begin{cases} T_2 = T_1 \\ 2T_1 \cos\theta = mg \end{cases}$$
$$\begin{cases} T_2 = T_1 \\ T_1 = \frac{mg}{2\cos\theta} \end{cases}$$
$$T_2 = T_1 = \frac{mg}{2\cos\theta}$$

$$T_2 = T_1 = \frac{mg}{2\cos\theta}$$
$$\square$$

b

Find the tension at the center of the rope.

✓ **Answer**

If we treat each half of the rope as its own "system", the weight components of each half will be counteracted by the vertical component of the tensile force on that side.

$F_{1net} = 0$ N2L
$T_{1y} = \frac{mg}{2}$ N2L
$W_1 = -\frac{mg}{2}$

The only remaining force is the center tension and the horizontal component of the first tension.

$T_{1x} = -\frac{mg\tan\theta}{2}$

Since our net force is 0 as it is not accelerating

$T_{1x} + T_c = 0$

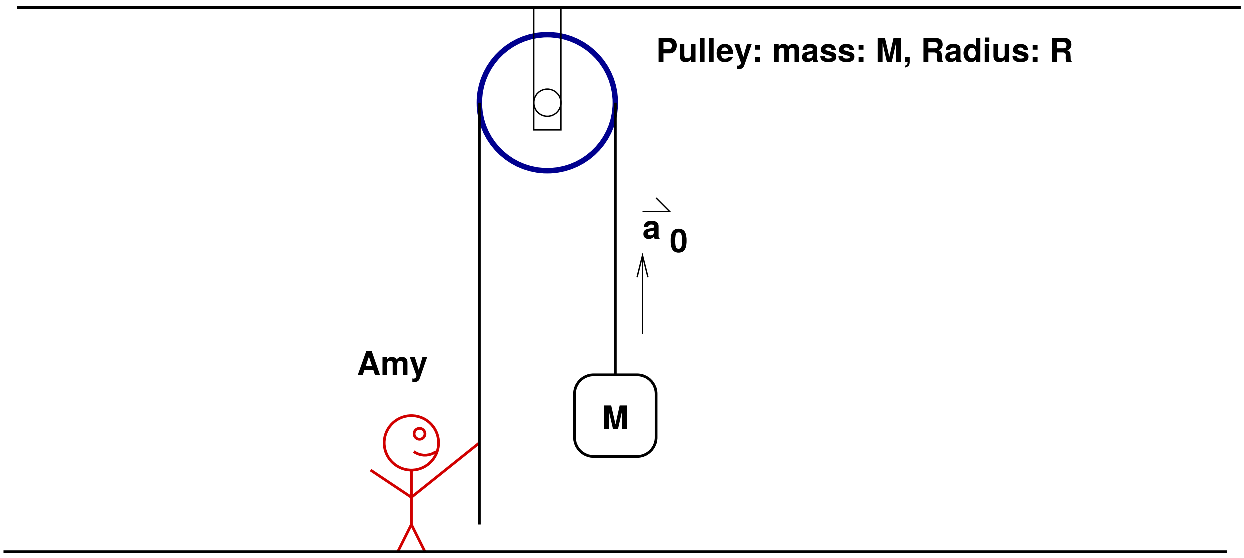
$T_c = \frac{mg\tan\theta}{2}$

The same logic could be used for side 2 of the rope, with the exact same logic.

$T_c = \frac{mg\tan\theta}{2}$

□

6



Amy stands on the floor and pulls hard on a rope that is attached to a massive pulley and a block as shown above. The mass of the block is given as M . The mass of the pulley has the same given mass M . Amy's mass is given as $3M$. Amy pulls so that the block accelerates upward with a given constant acceleration $\vec{a}_0$. Assume that the pulley wheel can be represented as a solid disk with rotational inertia given by $I = \frac{1}{2}mR^2$.

What are all of the forces on Amy? Determine the type, magnitude, and direction of each force. Explain your work. If you are using Newton's Laws here, be sure to include clearly labeled Free-Body-Diagrams and/or Extended-Free-Body-Diagrams as needed. Your work for this problem needs to be complete, neat, coherent, and completely explained to earn full credit.

✓ Answer

We know Amy is not accelerating, so her net force is 0 (by N2L)

$F = 0$

The only forces acting upon her are her weight force, the normal force on the floor, and the the friction force between her hand and the rope (could also be classified as tensile).

$W = -3Mg$ N2L

Added tension due to the pulley

$\tau = I\alpha$

$T_pR = \frac{MR^2a_0}{2R}$ As the acceleration tesile-wise must be the same as the block's acceleration

$T_p = \frac{Ma_0}{2}$

Added tension due to the block

$T_b + W_M = Ma_0$

$W_M = -Mg$

$T_b = M(a_0 + g)$

The tension near her hand will be the sum of these two tensions by N2L at the pulley.

$T = M\left(\frac{3a_0}{2} + g\right)$

This tension will be the same force of friction on her hand as they balance the forces at the end of the rope.

$f_f = M\left(\frac{3a_0}{2} + g\right)$

As we know Amy is not accelerating as we established earlier

$-3Mg + M\left(\frac{3a_0}{2} + g\right) + N = 0$

$N = M\left(2g - \frac{3a_0}{2}\right)$

$W = -3Mg$

$f_f = M\left(\frac{3a_0}{2} + g\right)$

$N = M\left(2g - \frac{3a_0}{2}\right)$

$\square$