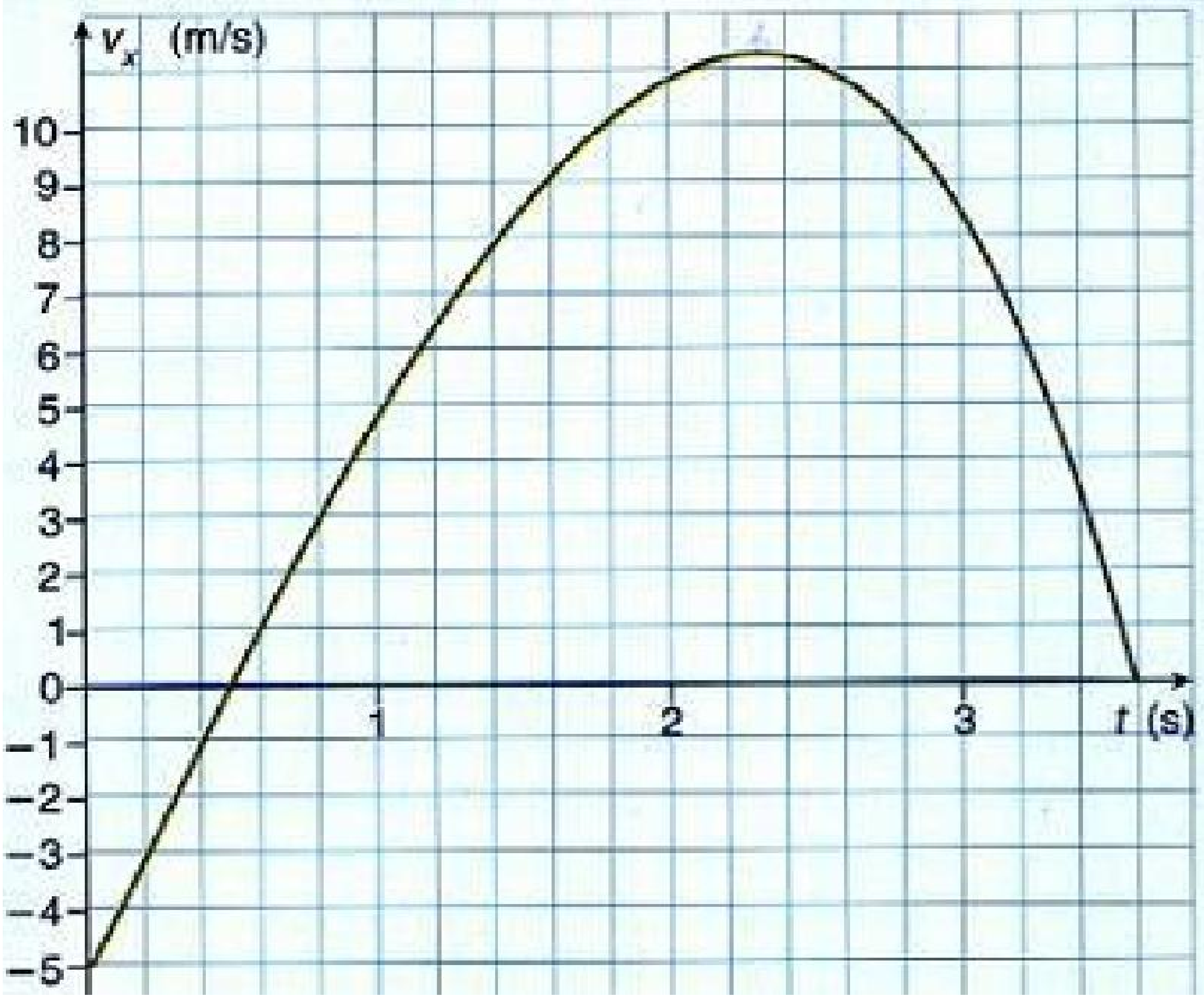
1

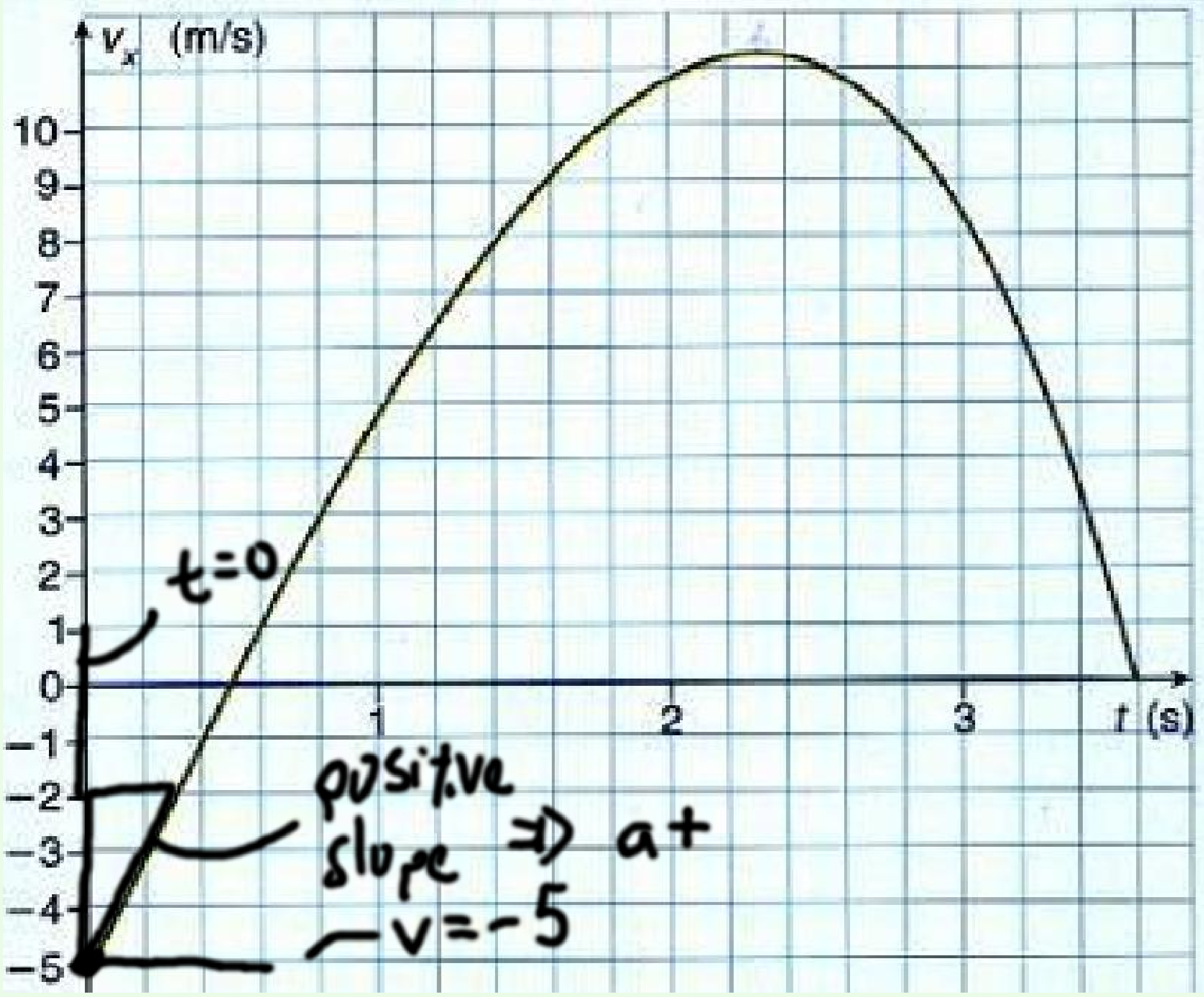


a

Three students make statements. Only one of these three students is correct. Which student is correct and why? Explain your answer.

1. At time $t = 0$ we know the particle has zero velocity because particles must start at rest.
2. No, you are wrong. In fact at time $t = 0$ we know the particle is already moving in the negative direction. In fact the particle is slowing down.
3. No, you are both wrong. At time $t = 0$ the particle is speeding up. We know this because the slope is positive.

✓ Answer ⌄



1. This statement is incorrect as we can see that at $t = 0$, $v_x(0) = -5$, which means the object is moving at $-5\frac{m}{s}$ — not at rest — at the start.
2. This is true. As we established in (1), it begins with moving at $-5\frac{m}{s}$. We can also see that the slope is upwards, which means that the acceleration is positive. When velocity is negative and acceleration is positive, it means that the object is slowing down, as they oppose each other.
3. This is false as we established the slope is positive while the velocity is negative, causing the velocity to *increase* but the speed to *decrease* as the velocity moves towards 0 at $t = 0$

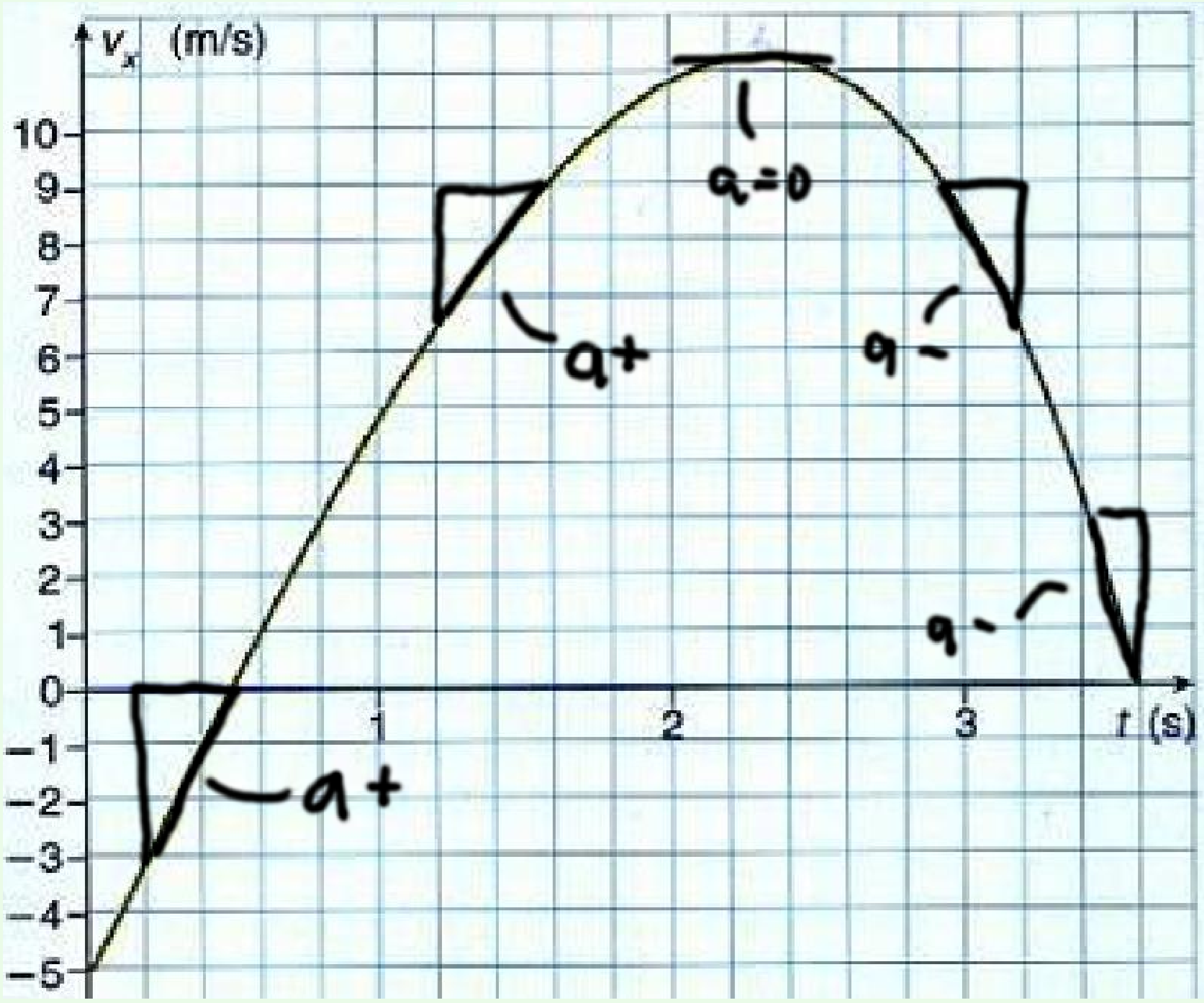
The correct statement is (2)

□

b

Is the acceleration of the particle constant during this the entire interval shown here? How do you know this? Explain your answer.

✓ **Answer**



No, the acceleration is not constant during the entire interval. As you can see in the image, it begins positive at $t = 0$ and stays positive until about $t = 2.3s$ where the the slope of velocity is flat, which means $a = 0$. After this, the acceleration turns negative. We can also see that the velocity curve is always concave down, which means the acceleration vs time graph is always decreasing, further proving the point it isn't constant.

No

☐

c

Consider the following equation:

$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$

In principle, can we apply this equation to describe the motion of the particle during this entire interval? Yes or No? How do you know this? Explain your answer.

✓ **Answer**

No, this equation assumes we have constant acceleration across the entire time interval (hence a not being a function of t). As we have established in part (b), a is not constant across the entire time interval, which means we cannot use this equation.

No

☐

d

Are there one or more instants in time that corresponds to **zero acceleration** of the particle? If so, indicate the time(s) (in seconds) and explain:
How do you know that the indicates time(s) corresponds to zero acceleration?
Very important: Explain your answer.

✓ **Answer**

Yes, as we established in part (b), the acceleration is 0 at around $t = 2.3s$. We know the acceleration is 0 here because acceleration is the derivative of velocity with respect to time, which means it is equivalent to the slope on a velocity vs. time graph. At around $t = 2.3s$ in the graph of part (b), we see the slope is flat — or 0.

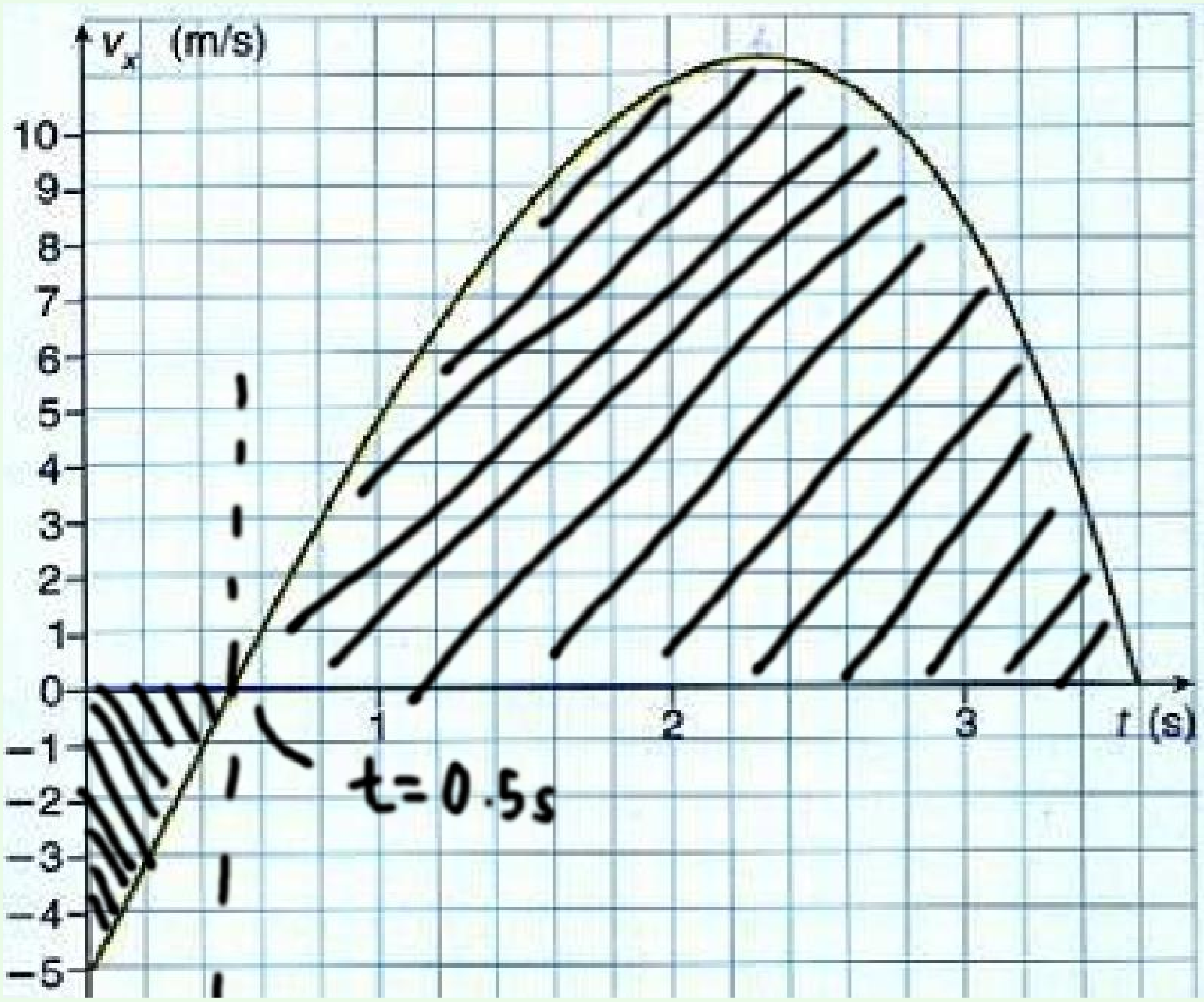
Yes, $a(2.3s) = 0$

☐

e

Is there any single instant in time that corresponds to the **minimum position** of the particle? If so, what time is this and how do you know that this is the minimum position? Explain your answer.

✓ Answer



Yes, at $t = 0.5s$, which is when the velocity switches from negative to positive. Since position is the integral of velocity with respect to time, when the velocity turns positive after $t = 0.5s$. The position of the particle will begin to increase. We can also know the minimum by the other characteristics of the velocity graph: when $v = 0$ we know the particle does not move, and thus its position vs. time graph has a slope of 0. Since velocity has a positive slope at this time, we know that the position is concave up at that point, making the zero slope at that point a local minima. Since there are no other local maxima or minima and we know that the position is less at $t = 0.5s$ than $t = 0$ or $t = 3.6s$ as the integral from $\Big|_{t=0}^{0.5}$ is negative, making it further right and $\Big|_{t=0.5}^{3.6}$ is positive, making it further right, we can conclude the local minima at $t = 0.5s$ is the global minimum.

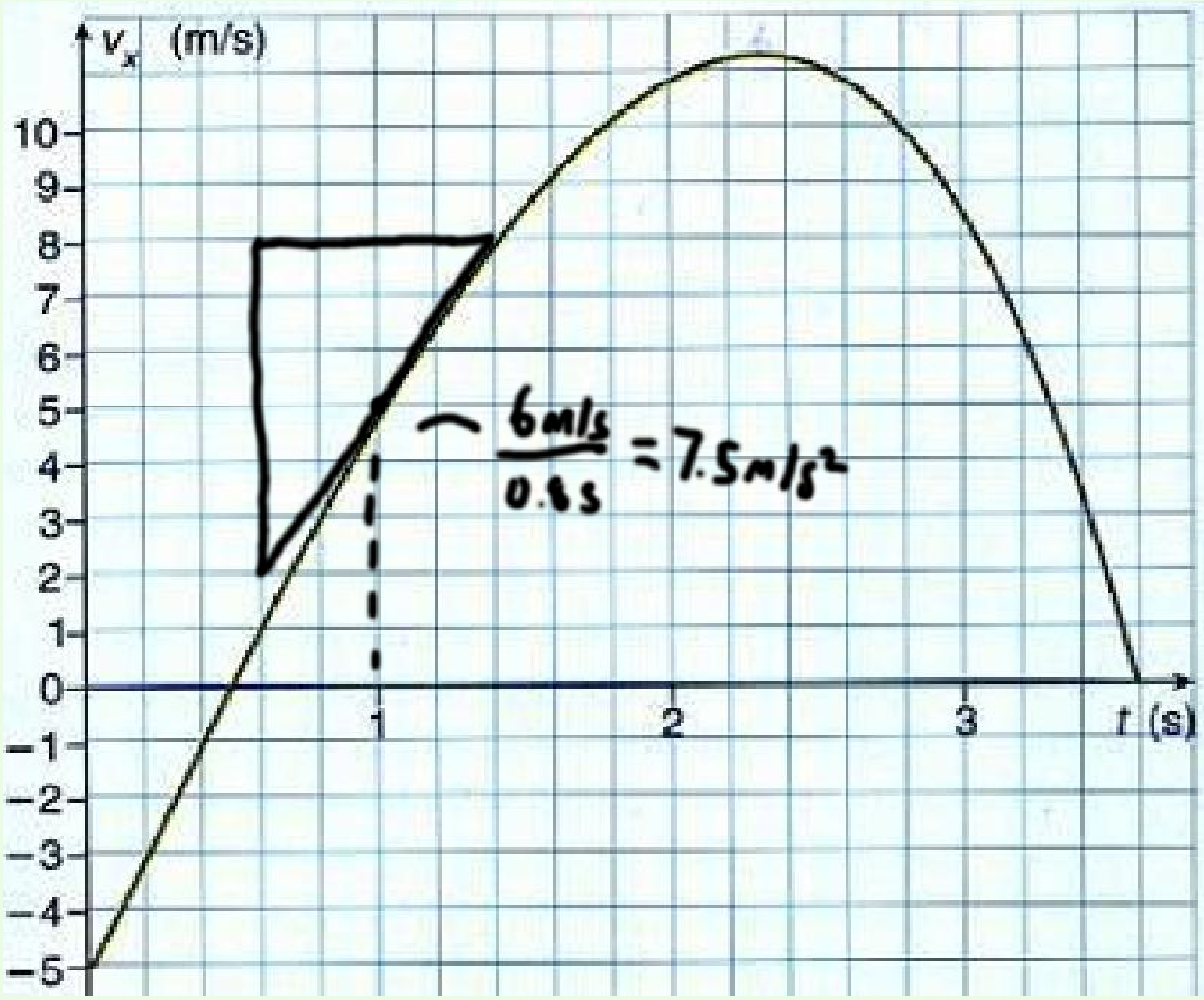
$t_{xmin} = 0.5s$

$\square$

f

Estimate the acceleration of the particle at the instant corresponding to $t = 1.0s$. Important: You **must** explicitly show your numerical calculations in a complete way and you must explicitly reference the graph to explain **how in terms of some graphical** method you used the graph to calculate this. Be careful with the units!

✓ **Answer**



We can estimate the acceleration by taking the slope of the tangent line to the velocity vs. time graph at $t = 1.0s$. I drew a tangent line at $t = 1.0s$ on the graph and saw that its slope was roughly equal to $+6$ blocks up and $+4$ blocks right, which corresponds to $\frac{6\frac{m}{s}}{0.8s} = 7.5\frac{m}{s^2}$.

$a(1.0s) = 7.5\frac{m}{s^2}$

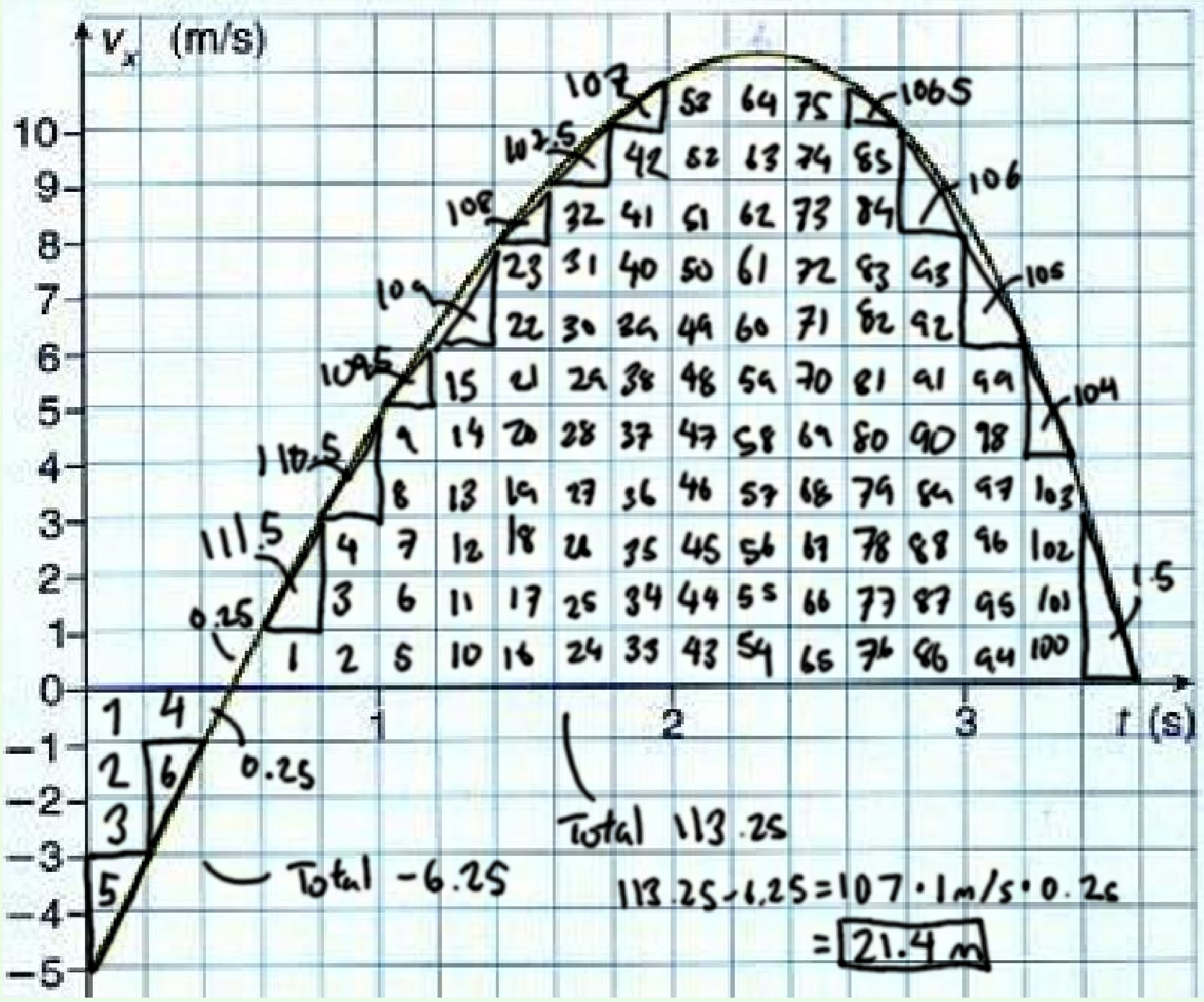
□

g

Harder and more time consuming: Estimate the *final position* of the particle $x(t)$ at time $t = 3.6s$ relative to the initial position $x = 0$ at $t = 0$ seconds. There are several approximately correct graphical ways to do this. As long as you are consistent, any method that gives reasonable accurate results is acceptable. Again, you must use words and **use the graph** to explain **how in**

terms of some graphical method you you calculate this. Clearly demonstrate to the grader how you obtained your answer. Be careful with the units!

✓ Answer



The position at time t is equal to the original position $x_0 = 0$ at $t = 0$ plus the integral over the time interval of velocity. Taking the integral of the velocity over the entire time frame could be estimated by taking the area under the curve of velocity over the time, which is approximately 107 blocks, at $1\frac{m}{s} \times 0.2s = 0.2m$ per block, this gives us the estimated integral at $21.4m$. Adding the initial position of 0 gives us the final position $21.4m$ at $t = 3.6s$

$x(3.6s) = 21.4m$

☐

2

A heavy package of given mass M is released at rest to fall straight down from a hovering helicopter located at a given height H meters from the ground.

a

How much time t will pass from when the package is released to when it hits the ground? Important: use the equations of Free-Fall Kinematics to solve this problem symbolically in terms of the given parameters. Here the given parameters are M and H. You can also always assume that any physical constants such as g are also given). Assume you an ignore air resistance. Explain what you are doing. Show your work.

✓ Answer

Since there is no air-resistance and we can assume that gravity is the only force acting upon the object, we can use the free-fall kinematic equation.

$\vec{F}_g = \langle 0, -Mg \rangle$ Listing all active forces
$\vec{F}_{net} = \langle 0, -Mg \rangle$ Calculating net force from all forces
$\vec{a}_{net} = \frac{\vec{F}_{net}}{M} = \langle 0, -g \rangle$ Calculating net acceleration from net force
$y(t) = x_0 + \int\limits_0^t v_{y0} + \int\limits_0^t a_y(t)\ dt\ dt$ Integrating the acceleration and velocity to calculate position — derives the free fall equations.
$y(t) = H + \frac{1}{2} a_y t^2$ We know we start at rest at height H
$y(T) = H + \frac{1}{2} a_y T^2$ Plug in final time
$-\frac{2H}{a_y} = T^2$ We know we end at $y(T) = 0$
$\frac{2H}{g} = T^2$ Substitute a_y
$T = \sqrt{\frac{2H}{g}}$

$T = \sqrt{\frac{2H}{g}}$
☐

b

How fast will it be moving at the instant just before it hits? Again, use the equations of Free-Fall Kinematics to solve this problem symbolically in terms of the given parameters. Explain your work.

✓ **Answer**

As we established in the previous section (a), we know we hit the ground at $T = \sqrt{\frac{2H}{g}}$, so we can integrate $a_y(t)$ from $t = 0$ to $t = T$ to get the velocity at time $t = T$

$v_y(T) = v_{y0} + \int_0^T a_y(t)\, dt$ Integrate acceleration to get velocity

$v_y(T) = Ta_y$ We start with no velocity

$v_y(T) = \sqrt{\frac{2H}{g}}g$ Substitution

$v_y(T) = \sqrt{2Hg}$

$v_y(T) = \sqrt{2Hg}$

□

c

Now suppose you are told that the mass of the heavy box is specified as $M = 47.84\ kg$ and the release height of the helicopter is given as $H = 52.8\ m$. Plug these numbers into the symbolic answer you just obtained above to calculate the speed of the box numerically. Be sure to take care with significant digits and physical units.

✓ **Answer**

$v_y(T) = \sqrt{2Hg}$

$v_y(T) = \sqrt{2(52.8\ m)\left(9.8\ \frac{m}{s^2}\right)}$

$v_y(T) = \sqrt{1034.88\ \frac{m^2}{s^2}}$

$v_y(T) = 32.2\frac{m}{s}$

$v_{yf} = 32.2\frac{m}{s}$
$\square$

3

José Ramírez swings the bat, but hits a pop-up. The baseball has a vertical upward velocity of $v_0 = 23.7\frac{m}{s}$ when the ball leaves his bat. Relative to his bat, how high in the air will the baseball travel? How much total time will elapse before the ball is caught by the opposing catcher? Assume the ball is caught at the same vertical location that it was hit. Ignore air resistance.

✓ **Answer**

$v_0 = 23.7\frac{m}{s}$
$h_0 = 0m$
$h_f = 0m$
$h(t) = h_0 + \int_0^t v_0 + \int_0^t a_y(c)\,dc\,dc$ writing h in terms of velocity and accel.
$F_g = -Mg$ writing all forces on baseball
$F_{net} = -Mg$ summing all forces on baseball
$a_y = -g$ calculating acceleration from net force
$h(t) = h_0 + \int_0^t v_0 + \int_0^t -g\,dt\,dt$ sub.
$h(t) = h_0 + v_0 t - \frac{1}{2}gt^2$ sub.
$h(t_f) = h_0 = h_0 + v_0 t_f - \frac{1}{2}gt_f^2$
$2v_0 = gt_f$
$t_f = \frac{2v_0}{g}$

$t_f = \frac{2v_0}{g}$
$t_f = \frac{2(23.7\frac{m}{s})}{9.8\frac{m}{s^2}}$
$t_f = 4.84s$

$t_f = \frac{2v_0}{g}$

$t_f = 4.84s$

□

4

The dwarf planet Sedna is at a distance $D = 85AU$ (Astronomical Units) from the Earth. You want to fly a space-ship to Sedna. Suppose you have a space-ship that is able to linearly accelerate in either a positive or a negative direction at exactly $A = 0.238\frac{m}{s^2}$. You plan a trip where you move with constant positive acceleration in a straight line toward Sedna, speeding up the whole time, and then, at precisely the half-way point, you turn the ship around and accelerate in the opposite direction e.g., with negative acceleration of the same magnitude so that you are now slowing down. **By symmetry**, if you reverse acceleration at the half-way point, you should arrive at your destination with zero velocity (right?).

Note: in this problem we are completely ignoring the practical reality of how one might build a spaceship that can sustain constant acceleration for an arbitrary period of time. We are also ignoring the motion of the Earth and of Sedna during the trip.

a

How long will the trip take? Important: You must given a **symbolic** answer in terms of the given parameters (here A and D corresponding to the acceleration of the spaceship and the distance traveled to Sedna. Box your symbolic answer. Also give a numerical answer in terms of seconds. Be sure that the answer has an appropriate number of significant digits. Box your numerical answer. Also convert your answer in seconds to an answer in units that are most appropriate to actual length of the trip (days, years, etc.) Again box your answer. Show your work. Hint: you will need to convert from “AU” to meters.

✓ **Answer**

$a(t) = \begin{cases} A & 0 \leq t < \frac{t_f}{2} \\ -A & \frac{t_f}{2} \leq t \leq t_f \end{cases}$ Given acceleration

$x(t) = x_0 + \int\limits_0^t v_0 + \int\limits_0^t a(t)\ dt\ dt$ Definition of acceleration

$x(t) = x_0 + \int\limits_0^t v_0 + \begin{cases} At & 0 \leq t < \frac{t_f}{2} \\ \frac{t_f}{2}A - A\left(t - \frac{t_f}{2}\right) & \frac{t_f}{2} \leq t \leq t_f \end{cases} dt$ Sub.

$x(t) = x_0 + \int\limits_0^t \begin{cases} v_0 + At & 0 \leq t < \frac{t_f}{2} \\ v_0 + A\left(t_f - t\right) & \frac{t_f}{2} \leq t \leq t_f \end{cases} dt$

$x(t) = x_0 + \begin{cases} v_0 t + \frac{1}{2}At^2 \\ \frac{v_0 t_f}{2} + \frac{1}{8}At_f^2 + v_0\left(t - \frac{t_f}{2}\right) + A\left(t_f\left(t - \frac{t_f}{2}\right) - \frac{1}{2}t^2 + \frac{1}{8}t_f^2\right) \end{cases}$

$x(t) = \begin{cases} x_0 + v_0 t + \frac{1}{2}At^2 & 0 \leq t < \frac{t_f}{2} \\ x_0 + v_0 t + A\left(t_f t - \frac{t_f^2}{2} - \frac{t^2}{4}\right) & \frac{t_f}{2} \leq t \leq t_f \end{cases}$

$x(t_f) = D = x_0 + v_0 t_f + A\frac{t_f^2}{4}$ Sub. t_f as we know D is position at t_f

$\sqrt{\frac{4D}{A}} = t_f$ we know we start at rest and at $x = 0$

$t_f = \sqrt{\frac{4D}{A}}$

$t_f = \sqrt{\frac{4(85\ AU)\left(1.495979 \times 10^{11}\ \frac{m}{AU}\right)}{\left(0.238\ \frac{m}{s^2}\right)}}$

$t_f = \sqrt{2.13711 \times 10^{14}}\ s$

$t_f = 13618867.46\ s$

$t_f = 169.200\ days$

$t_f = 1.7 \times 10^2\ days$

$t_f = \sqrt{\frac{4D}{A}}$

$t_f = 1.7 \times 10^2\ days$

□

b

What is the maximum velocity of the spaceship during the trip? Again, present your answer symbolically in terms of given parameters, box this, and then plug in to get a numerical answer in terms of appropriate SI units, and then box this. Be sure to explain your work.

✓ **Answer**

We can find all maxima of velocity by examining all discontinuities, the beginning, the end, and when acceleration is 0. Since acceleration is never 0, we only need to check $t = \left\{0, \frac{t_f}{2}, t_f\right\}$

As we derived in part (a) velocity can be defined as

$$v(t) = \begin{cases} v_0 + At & 0 \leq t < \frac{t_f}{2} \\ v_0 + A\left(t_f - t\right) & \frac{t_f}{2} \leq t \leq t_f \end{cases}$$

$$\begin{cases} v(0) = v(t_f) = v_0 \\ v\left(\frac{t_f}{2}\right) = v_0 + \frac{At_f}{2} \end{cases}$$

If A is positive, we can conclude that the greatest velocity is at $t = \frac{t_f}{2}, v = v_0 + \frac{At_f}{2}$, else we can conclude the greatest velocity is at the beginning and the end at $v = v_0$.

Since we know acceleration is positive in this case, the maximum will be at $t = \frac{t_f}{2}$

$$v\left(\frac{t_f}{2}\right) = v_0 + \frac{At_f}{2}$$

$$v\left(\frac{t_f}{2}\right) = \frac{A\sqrt{\frac{4D}{A}}}{2}$$

$$v\left(\frac{t_f}{2}\right) = \sqrt{DA}$$

$$v\left(\frac{t_f}{2}\right) = \sqrt{DA}$$

$$v\left(\frac{t_f}{2}\right) = \sqrt{(85\ AU)\left(1.495979 \times 10^{11}\,\frac{m}{AU}\right)\left(0.238\ \frac{m}{s^2}\right)}$$

$$v\left(\frac{t_f}{2}\right) = \sqrt{3.02637 \times 10^{12}\,\frac{m}{s}}$$

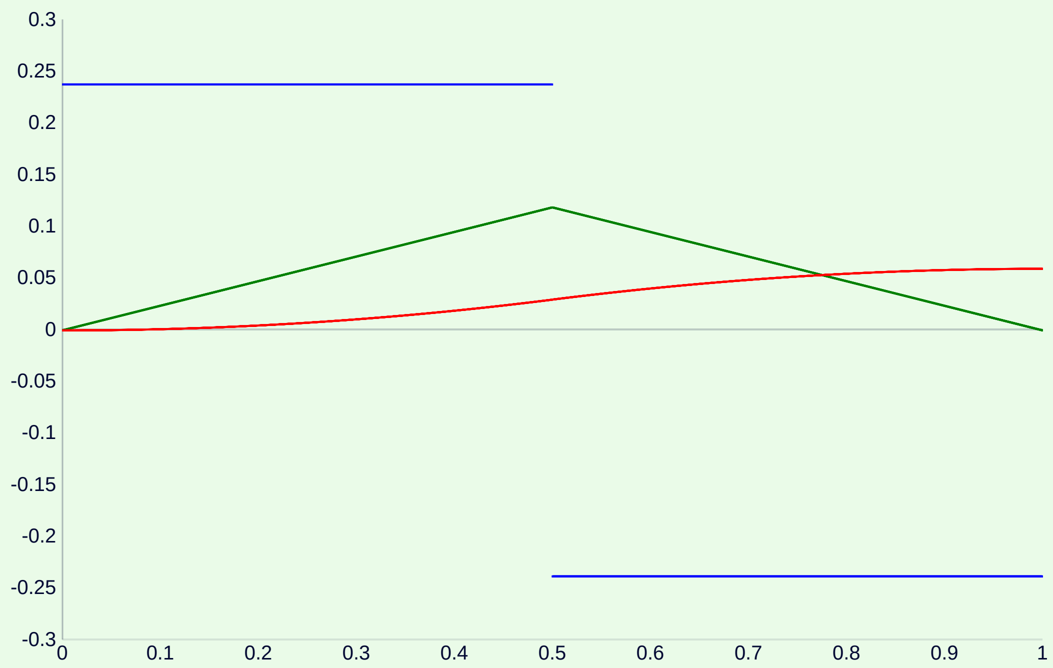
$$v\left(\frac{t_f}{2}\right) = 1739645.227\frac{m}{s}$$
$$v\left(\frac{t_f}{2}\right) = 1.7 \times 10^6\,\frac{m}{s}$$

$$\begin{cases} v(0) = v(t_f) = v_0 \\ v\left(\frac{t_f}{2}\right) = v_0 + \frac{At_f}{2} \end{cases}$$
$$v\left(\frac{t_f}{2}\right) = \sqrt{DA}$$
$$v\left(\frac{t_f}{2}\right) = 1.7 \times 10^6\,\frac{m}{s}$$
$\square$

c

Draw a qualitative but clear sketch showing plots of the acceleration, the velocity and the position of the spacecraft as a function of time during the entire trip from Earth To Sedna. For each plot, explain in a sentence or two how you determined the shape of the plot.

✓ Answer



Where:

- Blue is a: a piecewise function for A and -A

- Green is v: two lines with the slope of a
- Red is x: a concave up parabola and a concave down parabola with derivative of v

5

An apple sits at rest on top of a book which in turn sits on a table that sits on the floor. The mass of the apple is given as $m_A = 76\ g$ and the mass of the book is $m_B = 1.07\ kg$ and the mass of the table is $m_T = 13.4\ kg$.

a

Determine the **net force** on the book. Hint. This is a conceptual question. Literally: no calculations are required, whatsoever. Just apply **Newton's Second Law**.

✓ **Answer**

Since everything is in static equilibrium, there is no net force on the book.

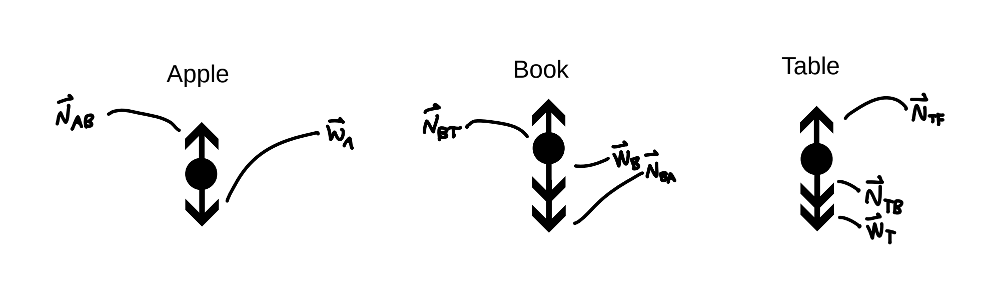
$\vec{F}_{Bnet} = \vec{0}$

□

b

Draw complete and proper **Free Body Diagrams** for both the apple and the book. Include only forces that are listed on the "Force Menu" (see Course Document #04, page 7) Each and every force on each object must be consistently and properly labeled. For example, if you want to indicate the force of **Weight** on the apple then you should label such a force as $\vec{W}_A$ which means "the Weight force on Body A." Likewise if you want to label the Normal force on the book due to the apple you would label this $\vec{N}_{BA}$.

✓ **Answer**



C

Apply **Newton's Second Law** together with your two **Free-Body Diagrams** to calculate the magnitude of each and every individual force on each of the two object (apple and book). Ignore air resistance and friction. Explain your work. Note that you must invoke and then apply Newton's Second Law to receive proper credit. Note: you **must** first set up and solve this problem *symbolically*, not numerically. In other words, give your answers in terms of the given parameters m_A , m_B and perhaps m_T, and the physical constant g. Only after you have done this, should you plug in the numbers to your answer in terms numbers with appropriate units at the very end. *Answers that are solved numerically and not symbolically will not receive full credit*

✓ **Answer**

Setting up the net forces for all objects, which are 0 since we are in equilibrium.

$\vec{F}_{Anet} = \vec{0} = \vec{W}_A + \vec{N}_{AB}$
$\vec{F}_{Bnet} = \vec{0} = \vec{W}_B + \vec{N}_{BA} + \vec{N}_{BT}$
$\vec{F}_{Tnet} = \vec{0} = \vec{W}_T + \vec{N}_{TB} + \vec{N}_{TF}$

$\vec{W}_A = \langle 0, -gm_A \rangle$
$\vec{N}_{AB} = \langle 0, gm_A \rangle$ Newton's second

$\vec{N}_{AB} = -\vec{N}_{BA}$ Newton's third
$\vec{N}_{BA} = \langle 0, -gm_A \rangle$
$\vec{W}_B = \langle 0, -gm_B \rangle$
$\vec{N}_{BT} = \langle 0, g(m_A + m_B) \rangle$ Newton's second

$\vec{N}_{TB} = -\vec{N}_{BT}$ Newton's third
$\vec{N}_{TB} = \langle 0, -g(m_A + m_B) \rangle$
$\vec{W}_T = \langle 0, -g m_T \rangle$
$\vec{N}_{TF} = \langle 0, g(m_A + m_B + m_T) \rangle$ Newton's second

---

$\vec{W}_A = \langle 0, -0.74\ N \rangle$
$\vec{N}_{AB} = \langle 0, 0.74\ N \rangle$
$\vec{N}_{BA} = \langle 0, -0.74\ N \rangle$
$\vec{W}_B = \langle 0, -10.5\ N \rangle$
$\vec{N}_{BT} = \langle 0, 11.2\ N \rangle$
$\vec{N}_{TB} = \langle 0, -11.2\ N \rangle$
$\vec{W}_T = \langle 0, -131.\ N \rangle$
$\vec{N}_{TF} = \langle 0, 143.\ N \rangle$

$W_A = 0.74\ N$
$N_{AB} = 0.74\ N$
$N_{BA} = 0.74\ N$
$W_B = 10.5\ N$
$N_{BT} = 11.2\ N$
$N_{TB} = 11.2\ N$
$W_T = 131.\ N$
$N_{TF} = 143.\ N$
$\square$

d-f

Repeat parts (a), (b) and (c) for Problem 5 above except now assume that table is sitting in an elevator that is accelerating upward with a given acceleration of precisely $A_0 = 6.96 \frac{m}{s^2}$. Be sure to explain your work. Note that you must explicitly invoke and then apply Newton’s Second Law to receive proper credit. Note that students are specifically disallowed from invoking “fictitious” forces of any kind. Students are disallowed from “changing the value of g” or anything like this. Students **must** solve the problem using **Newton’s Second Law** and they must solve the problem using kinematic variables as defined in an inertial reference frame only. Note: again you **must** set up and solve this problem symbolically, first, not numerically. **Plug in numbers only at the very end**.

d

✓ **Answer**

We know that the book, along with everything else in this system is accelerating upwards at $A_0 = 6.96 \frac{m}{s^2}$. From this we can calculate the net Force.

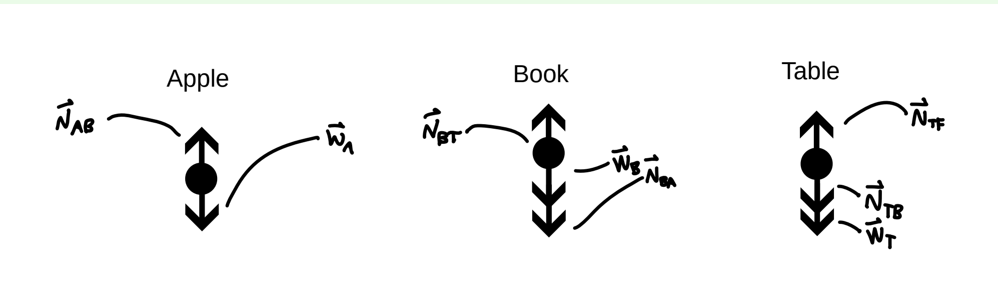
$$F_{Bnet} = m_B a_B$$

$$F_{Bnet} = m_B a_B$$
$$F_{Bnet} = (1.07\ kg)\left(6.96 \frac{m}{s^2}\right)$$
$$F_{Bnet} = 7.45\ N$$

$$F_{Bnet} = m_B a_B$$
$$F_{Bnet} = 7.45\ N$$
$$\square$$

e

✓ **Answer**



f

✓ **Answer**

Setting up the net forces for all objects, which are $\langle 0, ma\rangle$ since we are in constant acceleration.

$\vec{F}_{Anet} = \langle 0, m_A A_0\rangle = \vec{W}_A + \vec{N}_{AB}$
$\vec{F}_{Bnet} = \langle 0, m_B A_0\rangle = \vec{W}_B + \vec{N}_{BA} + \vec{N}_{BT}$
$\vec{F}_{Tnet} = \langle 0, m_T A_0\rangle = \vec{W}_T + \vec{N}_{TB} + \vec{N}_{TF}$

$\vec{W}_A = \langle 0, -gm_A\rangle$
$\vec{N}_{AB} = \langle 0, (A_0 + g)m_A\rangle$ Newton's second

$\vec{N}_{AB} = -\vec{N}_{BA}$ Newton's third
$\vec{N}_{BA} = \langle 0, -(A_0 + g)m_A\rangle$
$\vec{W}_B = \langle 0, -gm_B\rangle$
$\vec{N}_{BT} = \langle 0, (A_0 + g)(m_A + m_B)\rangle$ Newton's second

$\vec{N}_{TB} = -\vec{N}_{BT}$ Newton's third
$\vec{N}_{TB} = \langle 0, -(A_0 + g)(m_A + m_B)\rangle$
$\vec{W}_T = \langle 0, -gm_T\rangle$
$\vec{N}_{TF} = \langle 0, (A_0 + g)(m_A + m_B + m_T)\rangle$ Newton's second

---

$A_0 = 6.96\frac{m}{s^2}$

$\vec{W}_A = \langle 0, -0.74\ N\rangle$
$\vec{N}_{AB} = \langle 0, 1.3\ N\rangle$
$\vec{N}_{BA} = \langle 0, -1.3\ N\rangle$
$\vec{W}_B = \langle 0, -10.5\ N\rangle$
$\vec{N}_{BT} = \langle 0, 19.2\ N\rangle$
$\vec{N}_{TB} = \langle 0, -19.2\ N\rangle$
$\vec{W}_T = \langle 0, -131.\ N\rangle$
$\vec{N}_{TF} = \langle 0, 244.\ N\rangle$

$W_A = 0.74\ N$
$N_{AB} = 1.3\ N$
$N_{BA} = 1.3\ N$
$W_B = 10.5\ N$
$N_{BT} = 19.2\ N$
$N_{TB} = 19.2\ N$
$W_T = 131.\ N$
$N_{TF} = 244.\ N$
□

6

An force of given magnitude F_{app} is applied to a block of a given mass M . The force is applied to the right and upward at a given angle θ relative to the horizontal as shown. There is a given force of friction with magnitude f that acts on the block to the left. As a result of these forces, the block is observed to move to the right a given distance D.

a

What is the **Work** done by the *Normal force* on the block? Explain your answer.

✓ **Answer**

The Normal force is a conservative force: it doesn't ever cause movement, but only resists movement. As such, it pushes for a distance of 0, making its work also 0

$W_N = 0$
□

b

What is the **Work** done by the *Weight force* on the block? Explain your answer.

✓ **Answer**

The weight force of the block is always pointed straight "down", and since in this case the block doesn't move down — or parallel to the force. The dot product of the movement and force is always 0, which makes the work done by weight also 0

$W_W = 0$
□

c

What is the **Work** done by the *Applied Force* force on the block? Explain your answer.

✓ **Answer**

$$W_{app} = \vec{F}_{app} \cdot \vec{D}$$
$$W_{app} = F_{app} D \cdot \cos(\theta)$$

Since the block moves horizontally across the floor, and we are given θ, which is the deviation from this horizontal movement of which the applied force is applied, θ also represents the angle between movement and force, which is needed to calculate work through the dot product. As such, work is simply the product of distance traveled D, magnitude of force F_{app}, and the cosine of the angle $\cos(\theta)$

$W_{app} = F_{app} D \cdot \cos(\theta)$
□

d

What is the **Work** done by the *Friction Force* force on the block?

✓ **Answer**

$$W_f = \vec{F}_f \cdot \vec{D}$$
$$W_f = F_f D \cdot \cos(\pi)$$
$$W_f = -F_f D$$

Since the block moves horizontally across the floor, π represents the angle between movement and force as friction force is pointing π radians the other way, which is needed to calculate work through the dot product. As such, work is simply the product of distance traveled D, magnitude of force F_f, and the cosine of the angle $\cos(\pi)$

$$W_f = -F_f D$$
$\square$

e

What is the Total Work done on the block? Explain your answer.

✓ **Answer**

The total work is simply the sum of the work due to all forces — or the work of the net force. Since we already know all the individual works, we might as well sum those.

$$W_{net} = W_N + W_W + W_{app} + W_f$$
$$W_{net} = F_{app} D \cdot \cos(\theta) + -F_f D$$
$$W_{net} = (F_{app}\cos(\theta) - F_f)D$$

$$W_{net} = (F_{app}\cos(\theta) - F_f)D$$
$\square$

f

Assuming that the block starts at rest, what is the **final speed** of the block when it has traveled a distance D?

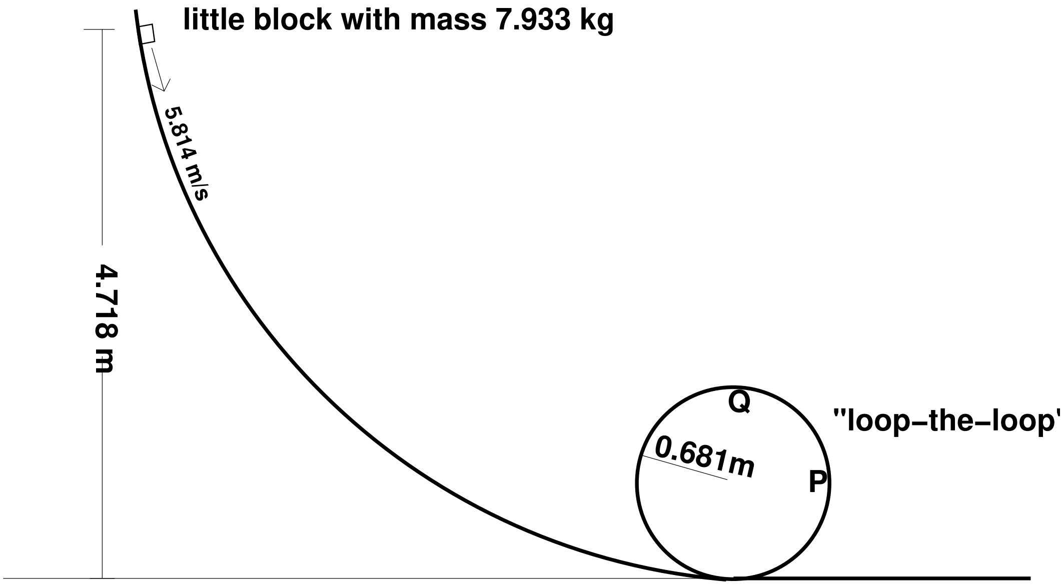
✓ **Answer**

Since we know the floor is horizontal and the block does not gain any potential energy, we can assume in this case that all added energy goes towards being kinetic.

$\Delta K = W_{net}$
$K_f = W_{net}$ Since we know we start at rest, $K_i = 0$
$K_f = \frac{1}{2}Mv_f^2$
$(F_{app}\cos(\theta) - F_f)D = \frac{1}{2}Mv_f^2$
$v_f = \sqrt{\frac{2(F_{app}\cos(\theta) - F_f)D}{M}}$

$v_f = \sqrt{\frac{2(F_{app}\cos(\theta) - F_f)D}{M}}$
$\square$

7



A small block of metal with a given mass of $m_b = 7.933\ kg$ is provided given initial speed $V_i = 5.814\ \frac{m}{s}$ at the top of a curved **frictionless** surface as shown above. The block moves down the track and then does a “loop-the-loop”. The height of the track h and the radius of the loop R are given as shown in the figure.

a

What is the **Work** done by the *Normal* force on the block due to the track as it moves from the top of the track to point P (precisely halfway up the “loop-the-loop”)? Explain your answer.

✓ **Answer**

Since the normal force is conservative — not moving, but only resisting motion, it has no movement, thus its work is 0

$W_N = 0$
$\square$

b

What is the **Work** done by the force of *Weight* on the block as it moves from the top of the track to point P? Is this positive work or negative work? Explain your answer.

✓ **Answer**

Since we know work is path-independent, we can calculate work based on the difference of the projections of the beginning and the end point.

Projecting the path to be parallel to the direction of weight force would give us:

$$W_W = (h_f - h_i)F_W$$

In the case of the weight force, "down" is the positive direction for the force, so if we are measuring "up" as positive, we much negate our height measurements to get them to be from the same perspective as weight.

$$W_W = (h_i - h_f) m_b g$$

$$W_W = (h_i - h_f) m_b g$$
$$W_W = (4.718\ m - 0.681\ m)(7.933\ kg)\left(9.8\ \frac{m}{s^2}\right)$$
$$W_W = 313.9\ Nm$$

$$W_W = (h_i - h_f) m_b g$$
$$W_W = 313.9\ Nm$$
$$\square$$

c

What is the *speed* of the block at point P? Show how you determined this. Explain what physics concept(s) and/or Laws you used to determine this. Explain your work.

✓ **Answer**

According to the work energy theorem, we can calculate the final KE using its initial state and work.

$\Delta K = W_{net} = K_f - K_i$ Work-energy theorem

We also know that $W_{net} = W_W$ as all other works are equal to 0

$K_f = W_{net} + K_i$
$\frac{1}{2} m_b v_f^2 = (h_i - h_f) m_b g + \frac{1}{2} m_b v_i^2$ Def of kinetic energy
$v_f^2 = 2(h_i - h_f) g + v_i^2$
$v_f = \sqrt{2(h_i - h_f) g + v_i^2}$

$$v_f = \sqrt{2(h_i - h_f) g + v_i^2}$$
$$v_f = \sqrt{2((4.617\ m) - (0.681\ m))\left(9.8\ \frac{m}{s^2}\right) + \left(5.814\ \frac{m}{s}\right)^2}$$
$$v_f = 10.53\ \frac{m}{s}$$

$$v_f = \sqrt{2(h_i - h_f)g + v_i^2}$$

$$v_f = 10.53\ \frac{m}{s}$$

$\square$