a

$$egin{aligned} D &= \sum\limits_{n=1}^{N} \sum\limits_{k=1}^{K} r_{n,k} \|ec{x}_n - ec{\mu}_k\|^2 \ D &= \sum\limits_{n=1}^{N} \sum\limits_{k=1}^{K} \sum\limits_{i=1}^{I} r_{n,k} (x_{ni} - \mu_{ki})^2 \ rac{\partial D}{\partial \mu_{ki}} &= \sum\limits_{n=1}^{N} -2 r_{n,k} (x_{ni} - \mu_{ki}) = 0 \ rac{\sum\limits_{n=1}^{N} r_{n,k} x_{ni} - r_{n,k} \mu_{ki} = 0}{\sum\limits_{n=1}^{N} r_{n,k} x_{ni} - \sum\limits_{n=1}^{N} r_{n,k} \mu_{ki}} = 0 \ rac{\sum\limits_{n=1}^{N} r_{n,k} x_{ni} = \sum\limits_{n=1}^{N} r_{n,k} \mu_{ki}}{\sum\limits_{n=1}^{N} r_{n,k} x_{ni}} = \mu_{ki} \sum\limits_{n=1}^{N} r_{n,k} r_{n,k} \ rac{\sum\limits_{n=1}^{N} r_{n,k} x_{ni}}{\sum\limits_{n=1}^{N} r_{n,k} x_{ni}} = \mu_{ki} \ rac{\sum\limits_{n=1}^{N} r_{n,k} x_{ni}}{\sum\limits_{n=1}^{N} r_{n,k} x_{ni}} = \mu_{ki} \end{aligned}$$

b

$$egin{align} D &= \sum\limits_{n=1}^{N} \sum\limits_{k=1}^{K} r_{n,k} \| ec{x}_n - ec{\mu}_k \|^2 \ &rac{\partial D}{\partial ec{\mu}_k} = -2 \sum\limits_{n=1}^{N} r_{n,k} (ec{x}_n - ec{\mu}_k) = ec{0} \ &\sum\limits_{n=1}^{N} r_{n,k} (ec{x}_n - ec{\mu}_k) = ec{0} \ &\sum\limits_{n=1}^{N} r_{n,k} ec{x}_n = \sum\limits_{n=1}^{N} r_{n,k} ec{\mu}_k \end{aligned}$$

$$\sum\limits_{n=1}^{N} r_{n,k} ec{x}_n = ec{\mu}_k \sum\limits_{n=1}^{N} r_{n,k} \ rac{\sum\limits_{n=1}^{N} r_{n,k} ec{x}_n}{\sum\limits_{n=1}^{N} r_{n,k}} = ec{\mu}_k$$

2

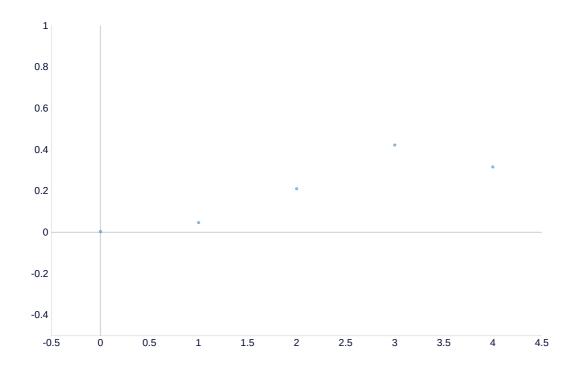
a

$$P(\theta|y,n) = P(y|\theta,n)P(\theta|n)/P(y|n)$$

$$egin{aligned} P(heta|y,n) &= inom{n}{y} heta^y (1- heta)^{n-y} rac{1}{1} / rac{1}{1+n} \ P(heta|y,n) &= (1+n) inom{n}{y} heta^y (1- heta)^{n-y} \end{aligned}$$

b

with
$$n=4$$
, $\theta=3/4$

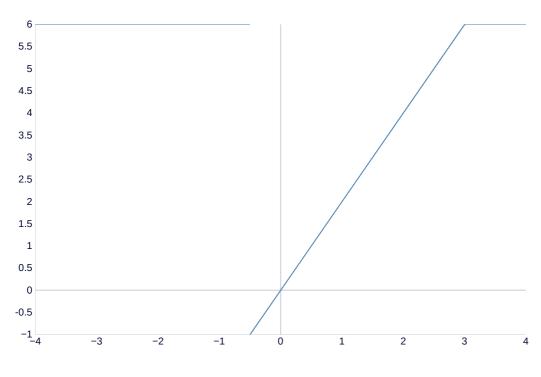


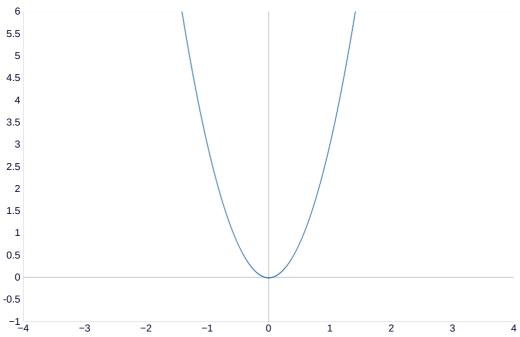
C

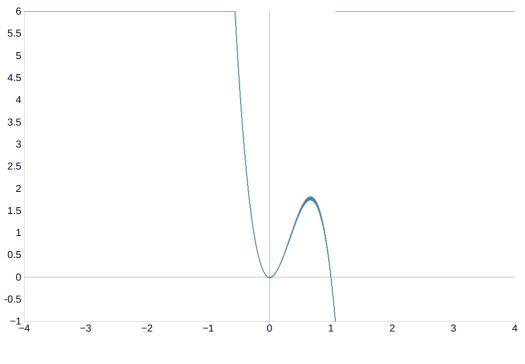
1.
$$y = 1$$
, $n = 1$

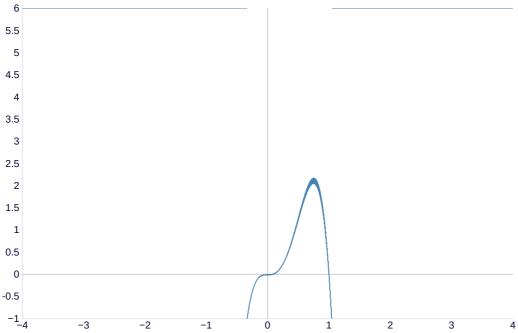
2.
$$y = 2$$
, $n = 2$

3.
$$y = 2$$
, $n = 3$









$$egin{align} f(x|\mu,\sigma^2) &= rac{1}{(2\pi\sigma^2)^{1/2}} e^{-rac{1}{2\sigma^2}(x-\mu)^2} \ f(ec{x}|\mu,\sigma^2) &= \prod_{n=0}^N f(ec{x}_n|\mu,\sigma^2) \ f(ec{x}|\mu,\sigma^2) &= \prod_{n=0}^N rac{1}{(2\pi\sigma^2)^{1/2}} e^{-rac{1}{2\sigma^2}(ec{x}_n-\mu)^2} \ \ln f &= \ln(\prod_{n=0}^N rac{1}{(2\pi\sigma^2)^{1/2}} e^{-rac{1}{2\sigma^2}(ec{x}_n-\mu)^2}) \ \end{array}$$

$$egin{aligned} \ln f &= \ln (\prod_{n=0}^N rac{1}{(2\pi\sigma^2)^{1/2}} e^{-rac{1}{2\sigma^2} (ec{x}_n - \mu)^2}) \ \ln f &= \sum_{n=0}^N \ln (rac{1}{(2\pi\sigma^2)^{1/2}} e^{-rac{1}{2\sigma^2} (ec{x}_n - \mu)^2}) \end{aligned}$$

$$\ln f = \sum_{n=0}^{N} -rac{1}{2} {
m ln}(2\pi\sigma^2) - rac{1}{2\sigma^2} (ec{x}_n - \mu)^2$$

a

$$0 = \frac{\partial \ln f}{\partial \mu} = \frac{\partial}{\partial \mu} \sum_{n=0}^{N} -\frac{1}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} (\vec{x}_n - \mu)^2$$

$$= -\frac{1}{2\sigma^2} \frac{\partial}{\partial \mu} \sum_{n=0}^{N} (\vec{x}_n - \mu)^2$$

$$= -\frac{1}{2\sigma^2} \sum_{n=0}^{N} -2(\vec{x}_n - \mu)$$

$$= \frac{1}{\sigma^2} \sum_{n=0}^{N} (\vec{x}_n - \mu)$$

$$\implies 0 = \sum_{n=0}^{N} \vec{x}_n - \sum_{n=0}^{N} \mu$$

$$\implies 0 = \sum_{n=0}^{N} \vec{x}_n - N\mu$$

$$\implies \mu = \frac{1}{N} \sum_{n=0}^{N} \vec{x}_n$$

b

$$0 = \frac{\partial \ln f}{\partial \sigma} = \frac{\partial}{\partial \sigma} \sum_{n=0}^{N} -\frac{1}{2} \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} (\vec{x}_n - \mu)^2$$

$$= \sum_{n=0}^{N} \frac{\partial}{\partial \sigma} \frac{1}{2} \ln(2\pi\sigma^2) + \sum_{n=0}^{N} \frac{\partial}{\partial \sigma} \frac{1}{2\sigma^2} (\vec{x}_n - \mu)^2$$

$$= \sum_{n=0}^{N} \frac{1}{2} \frac{4\pi\sigma}{2\pi\sigma^2} + \sum_{n=0}^{N} \frac{-2}{2\sigma^3} (\vec{x}_n - \mu)^2$$

$$= \sum_{n=0}^{N} \frac{1}{\sigma} - \sum_{n=0}^{N} \frac{1}{\sigma^3} (\vec{x}_n - \mu)^2$$

$$= \frac{N}{\sigma} - \frac{1}{\sigma^3} \sum_{n=0}^{N} (\vec{x}_n - \mu)^2$$

$$\implies \frac{N}{\sigma} = \frac{1}{\sigma^3} \sum_{n=0}^{N} (\vec{x}_n - \mu)^2$$

$$\implies N\sigma^2 = \sum_{n=0}^{N} (\vec{x}_n - \mu)^2$$

$$\implies \sigma^2 = \frac{1}{N} \sum_{n=0}^{N} (\vec{x}_n - \mu)^2$$

4

a

$$X \in \{C_1,C_2\}$$

$$P(X=C_1)=2P(X=C_2)$$

$$egin{aligned} 1 &= \sum_{x \in \{C_1, C_2\}} P(X = x) \ &= P(X = C_1) + P(X = C_2) \ &= 3P(X = C_2) \ &\Longrightarrow egin{cases} P(X = C_1) &= rac{2}{3} \ P(X = C_2) &= rac{1}{3} \end{aligned}$$

$$egin{array}{|c|c|c|c|} egin{array}{|c|c|c|c|} egin{array}{|c|c|c|c|} egin{array}{|c|c|c|c|} C_1 & C_2 \end{array}$$

 $egin{array}{|c|c|c|c|} C_1 & C_2 \ X & rac{2}{3} & rac{1}{3} \ \end{array}$

b

Assuming $\mu_1 < \mu_2$,

$$E=P(X_1> heta\cap X\in C_1)+P(X_2< heta\cap X\in C_2)$$

$$E=P(X_1>\theta)P(X\in C_1)+P(X_2<\theta)P(X\in C_2)$$

$$X_n \sim N(\mu_n, \sigma_n)$$

$$P(X_1 > \theta) = 1 - F_{X_1}(\theta)$$

$$P(X_2 < heta) = F_{X_2}(heta)$$

$$F_N(x) = \Phi(x)$$

$$F_{X_n}(x) = F_N(rac{x-\mu}{\sigma}) = \Phi(rac{x-\mu_n}{\sigma_n})$$

$$E=rac{2}{3}(1-\Phi(rac{ heta-\mu_1}{\sigma_1}))+rac{1}{3}(\Phi(rac{ heta-\mu_2}{\sigma_2}))$$

$$=2/3-2\Phi(rac{ heta-\mu_1}{\sigma_1})/3+\Phi(rac{ heta-\mu_2}{\sigma_2})/3$$

C

Let
$$N(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$\frac{d\Phi}{dz} = N(z)$$

$$0 = \frac{\partial E}{\partial \theta} = 2/3 - 2\Phi(\frac{\theta - \mu_1}{\sigma_1})/3 + \Phi(\frac{\theta - \mu_2}{\sigma_2})/3$$

$$= N(\frac{\theta - \mu_2}{\sigma_2})/3\sigma_2 - 2N(\frac{\theta - \mu_1}{\sigma_1})/3\sigma_1$$

$$\Rightarrow N(\frac{\theta - \mu_2}{\sigma_2})/\sigma_2 = 2N(\frac{\theta - \mu_1}{\sigma_1})/\sigma_1$$

$$\Rightarrow \frac{1}{\sigma_2} e^{-\frac{1}{2\sigma_2^2}(x - \mu_2)^2} = \frac{1}{\sigma_1} e^{-\frac{1}{2\sigma_1^2}(x - \mu_1)^2}$$

$$\Rightarrow \ln(\frac{\sigma_1}{\sigma_2}) = \frac{1}{2\sigma_2^2} (\theta - \mu_2)^2 - \frac{1}{2\sigma_1^2} (\theta - \mu_1)^2$$

$$\Rightarrow \ln(\frac{\sigma_1}{\sigma_2}) = \frac{1}{2\sigma_2^2} (\theta^2 - \mu_2\theta + \mu_2^2) - \frac{1}{2\sigma_1^2} (\theta^2 - \mu_1\theta + \mu_1^2)$$

$$\Rightarrow 0 = (\frac{1}{2\sigma_2^2} - \frac{1}{2\sigma_1^2})\theta^2 + (\frac{1}{2\sigma_1^2} \mu_1 - \frac{1}{2\sigma_2^2} \mu_2)\theta + (\frac{1}{2\sigma_2^2} \mu_2^2 - \frac{1}{2\sigma_1^2} \mu_1^2 - \ln(\frac{\sigma_1}{\sigma_2}))$$

Where

$$egin{aligned} A &= rac{1}{2\sigma_2^2} - rac{1}{2\sigma_1^2} \ B &= rac{1}{2\sigma_1^2} \mu_1 - rac{1}{2\sigma_2^2} \mu_2 \ C &= rac{1}{2\sigma_2^2} \mu_2^2 - rac{1}{2\sigma_1^2} \mu_1^2 - \ln(rac{\sigma_1}{\sigma_2}) \ heta &= rac{-B \pm \sqrt{B^2 - 4AC}}{2A} \end{aligned}$$