HW 3

2.1

We roll two dice. Find the conditional probability that at least one of the numbers is even, given that the sum is 8.

Answer

There are 5 different ways to roll a sum of 8.
$2, 3, 4, 5, 6$ for the first die and the corresponding amount to sum up to 8.

Out of those 5, 3 of them contain an even number.

$P(\text{One number even}|\text{Sum of two rolls is } 8) = \frac{3}{5}$

2.6

When Alice spends the day with the babysitter, there is a 0.6 probability that she turns on the TV and watches a show. Her little sister Betty cannot turn the TV on by herself. But once the TV is on, Betty watches with probability 0.8. Tomorrow the girls spend the day with the babysitter.

Hint. Define events precisely and use the product rule and the law of total probability.

a

What is the probability that both Alice and Betty watch TV tomorrow?

Answer

$P(A) = 0.6$
$P(B|A) = 0.8$

$P(A \cap B) = P(B|A)P(A) = 0.6 \cdot 0.8 = 0.48$

b

What is the probability that Betty watches TV tomorrow?

Answer

By the partition theorem,
$\{A, A^C\}$ is a partition

$P(B) = P(A \cap B) + P(A^C \cap B) = 0.48 + 0$
Since Betty will not be watching without Alice.

$P(B) = 0.48$

c

What is the probability that only Alice watches TV tomorrow?

✓ **Answer**
$P(B^C|A) = 1 - P(B|A)$
$P(B^C \cap A) = P(B^C|A)P(A) = 0.2 \cdot 0.6 = 0.12$

2.11

Suppose that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{3}$, and $P(A \cap B^C) = \frac{2}{9}$.
Decide if A and B are independent or not.

✓ **Answer**
$P(B^C) = 1 - P(B) = \frac{2}{3}$
$P(B^C)P(A) = \frac{2}{9} = P(A \cap B^C)$

Therefore A and B are independent.

2.14

Let A and B be two disjoint events. Under what condition are they independent?

✓ **Answer**
If $P(A) = 0$ or $P(B) = 0$

Since if $P(A \cap B) = 0$ and $P(A)P(B) = P(A \cap B)$ then $P(A)$ or $P(B)$ must be 0.

2.48

A crime has been committed in a town of 100,000 inhabitants. The police are looking for a single perpetrator, believed to live in town. DNA evidence is found on the crime scene. Kevin's DNA matches the DNA recovered from the crime scene. According to DNA experts, the probability that a random persons DNA matches the crime scene is 1 in 10,000. Before the DNA evidence, Kevin was no more likely to be the guilty person than any other person in town. What is the probability that Kevin is guilty after the DNA evidence appeared?

✓ **Answer**

Where T represents a positive test result and G represents guilty or not

$P(T|G^C) = \frac{1}{10000}$
$P(G) = \frac{1}{100000}$
$P(G^C) = \frac{99999}{100000}$
$P(T \cap G^C) = \frac{99999}{10^9}$

Assuming that the test works 100% of the time when testing against the true guilty person:

$P(T|G) = 1$

$P(T \cap G) = P(T|G)P(G) = \frac{1}{100000}$

By the partition theorem,
Since $\{G, G^C\}$ forms a partition,
$P(T) = P(T \cap G) + P(T \cap G^C)$

$P(T) = \frac{109999}{10^9}$

$P(G|T) = \frac{P(G \cap T)}{P(T)} = \frac{10000}{109999}$
$= 9.09\%$

2.54

Let A and B be events with these propeties: $0 < P(B) < 1$ and $P(A|B) = P(A|B^C) = \frac{1}{3}$.

a

Is it possible to calculate $P(A)$ from this information? Either declare that it is not possible, or find the value of $P(A)$.

✓ **Answer**

$P(A|B) = \frac{P(A \cap B)}{P(B)} = P(A|B^C) = \frac{P(A \cap B^C)}{P(B^C)}$
$\frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B^C)}{P(B^C)} = \frac{1}{3}$
$P(A \cap B^C) = \frac{1 - P(B)}{3}$
$P(A \cap B) = \frac{P(B)}{3}$

By the partition theorem over $\{B, B^C\}$
$P(A) = P(A \cap B) + P(A \cap B^C) = \frac{1 - P(B)}{3} + \frac{P(B)}{3} = \frac{1}{3}$

b

Are A and B independent, not independent, or is it impossible to determine?

✓ **Answer**

Since $P(A|B) = P(A|B^C) = P(A)$, A and B are independent.