15.1

15

$R = [-4, 4] \times [0, 5]$

$\iint\limits_{R} x^3 dA$

$$= \iint\limits_{[-4,4]\times[0,5]} x^3 dA$$
$$= \iint\limits_{[-4,0]\times[0,5]} x^3 dA + \iint\limits_{[0,4]\times[0,5]} x^3 dA$$
$$= \iint\limits_{[-4,0]\times[0,5]} x^3 dA - \iint\limits_{[-4,0]\times[0,5]} x^3 dA$$
$$= 0$$

19

$\int\limits_1^3 \int\limits_0^2 x^3 y \, dydx$

$$= \int\limits_1^3 4x^3/2 \, dx$$
$$= (81 - 1)/2$$
$$= 40$$

27

$\int\limits_0^1 \int\limits_0^2 x + 4y^3 \, dxdy$

$$= \int\limits_0^1 \Big|_0^2 x^2/2 + 4xy^3 \, dxdy$$
$$= \int\limits_0^1 2 + 8y^3 \, dy$$
$$= \Big|_0^1 2y + 2y^4 \, dy$$
$$= 4$$

33

$$\int_0^4 \int_0^5 (x+y)^{-1/2}\, dydx$$

$$= \int_0^4 \Big|_0^5 2(x+y)^{1/2}\, dydx$$

$$= \int_0^4 2((x+5)^{1/2} - x^{1/2})\, dx$$

$$= \Big|_0^4 4((x+5)^{3/2} - x^{3/2})/3\, dx$$

$$= 4((9)^{3/2} - (4)^{3/2} - (5)^{3/2})/3$$

$$= 4(27 - 8 - 5\sqrt{5})/3$$

$$= 4(19 - 5\sqrt{5})/3$$

$$\approx 10.426$$

37

$$R = [-2,4] \times [1,3]$$

$$\iint_R \frac{x}{y} dA$$

$$= \iint_{[-2,4]\times[1,3]} \frac{x}{y} dA$$

$$= \iint_{[2,4]\times[1,3]} \frac{x}{y} dA$$

$$= \int_1^3 \int_2^4 \frac{x}{y}\, dxdy$$

$$= \int_1^3 \Big|_2^4 \frac{x^2/2}{y}\, dxdy$$

$$= \int_1^3 \frac{8-2}{y}\, dy$$

$$= \int_1^3 \frac{6}{y}\, dy$$

$$= \Big|_1^3 6 \ln y\, dy$$

$$= 6(\ln 3 - \ln 1)$$

$$= 6 \ln 3$$

$$\approx 6.592$$

45

$$f(x,y) = mxy^2$$

$$R = [0,1] \times [0,2]$$

$$\iint\limits_R f(x,y)dA = 1$$

$$= \iint\limits_{[0,1]\times[0,2]} mxy^2 dA = 1$$

$$= \Big|_{[0,1]\times[0,2]} mx^2y^3/6\ dA$$

$$= 4m/3$$

$$m = 3/4$$

47

a

x because there is no product.

b

$$R = [0,1] \times [0,1]$$

$$\iint\limits_R y\sqrt{1+xy}\,dA$$

$$= \int_0^1 \int_0^1 y\sqrt{1+xy}\,dxdy$$

$$= \int_0^1 \Big|_0^1 2(1+xy)^{3/2}/3\ dxdy$$

$$= \int_0^1 2((1+y)^{3/2}-1)/3\ dy$$

$$= \Big|_0^1 2(2(1+y)^{5/2}/5-1)/3\ dy$$

$$= 2(2((2)^{5/2}-1)/5-1)/3$$

$$= \frac{4}{15}2^{5/2} - \frac{14}{15}$$

$$\approx 0.575$$

15.2

9

$$R = \begin{cases} y > x^2 \\ y < x+2 \end{cases}$$

$$f(x,y) = x$$

$$\iint\limits_R f(x,y)\,dA$$

$$= \iint\limits_R x\,dA$$
$$= \int\limits_{-1}^{2} \int\limits_{x^2}^{x+2} x\,dydx$$
$$= -\int\limits_{-1}^{2} x(x^2 - x - 2)\,dx$$
$$= -\int\limits_{-1}^{2} x^3 - x^2 - 2x\,dx$$
$$= -\Big|_{-1}^{2} x^4/4 - x^3/3 - x^2\,dx$$
$$= -(4 - 8/3 - 4 - 1/4 - 1/3 + 1)$$
$$= 9/4$$

11

$$\iint\limits_D \frac{y}{x} dA$$

$$= \int\limits_1^2 \int\limits_0^{\sqrt{4-x^2}} \frac{y}{x}\,dydx$$
$$= \int\limits_1^2 \Big|_0^{\sqrt{4-x^2}} \frac{y^2}{2x}\,dydx$$
$$= \int\limits_1^2 \frac{4-x^2}{2x}\,dx$$
$$= \int\limits_1^2 \frac{2}{x} - \frac{x}{2}\,dx$$
$$= \Big|_1^2 2\ln x - \frac{x^2}{4}\,dx$$
$$= 2\ln 2 - \frac{4}{4} - 2\ln 1 + \frac{1}{4}$$
$$= \ln 4 - \frac{3}{4}$$
$$= \ln 4 - \frac{3}{4}$$
$$\approx 0.636$$

17

$$\int\limits_0^5 \int\limits_x^{2x+3} x^3 y\,dydx$$

$$= \int_0^5 \Big|_x^{2x+3} x^3y^2/2 \, dydx$$

$$= \int_0^5 x^3(3x^2+12x+9)/2 \, dx$$

$$= \int_0^5 3(x^5+4x^4+3x^3)/2 \, dx$$

$$= \Big|_0^5 3(x^6/6+4x^5/5+3x^4/4)/2 \, dx$$

$$= 3(5^6/6+4(5)^5/5+3(5)^4/4)/2$$

$$= 8359.375$$

19

$$\int_0^1 \int_1^{e^{x^2}} x \, dydx$$

$$= \int_0^1 xe^{x^2} - x \, dx$$

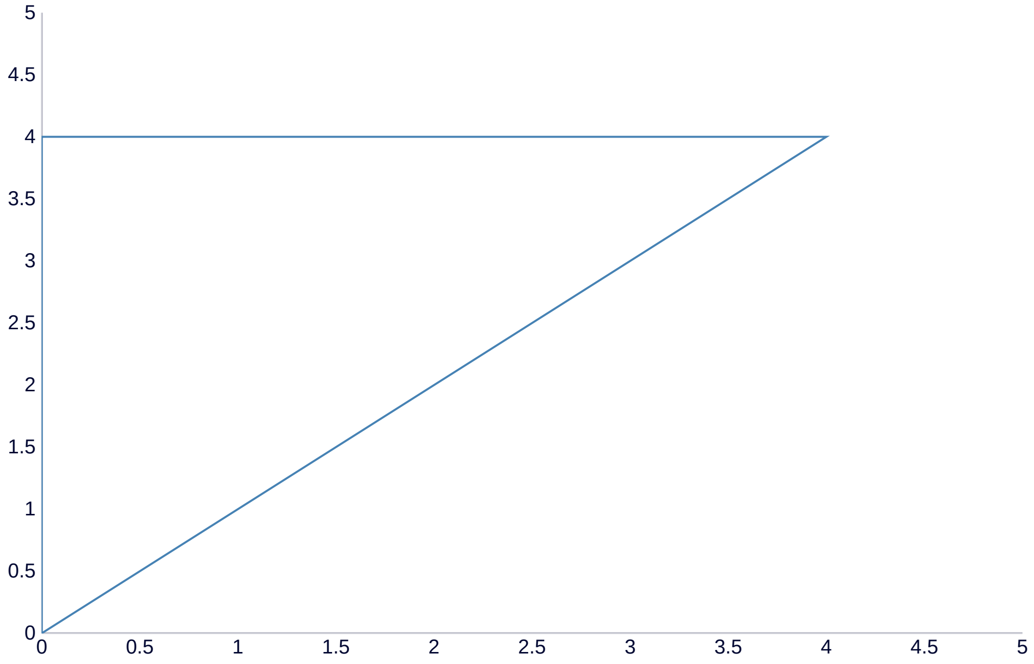
$$= \Big|_0^1 e^{x^2}/2 - x^2/2 \, dx$$

$$= \frac{e-2}{2}$$

$$\approx 0.359$$

25

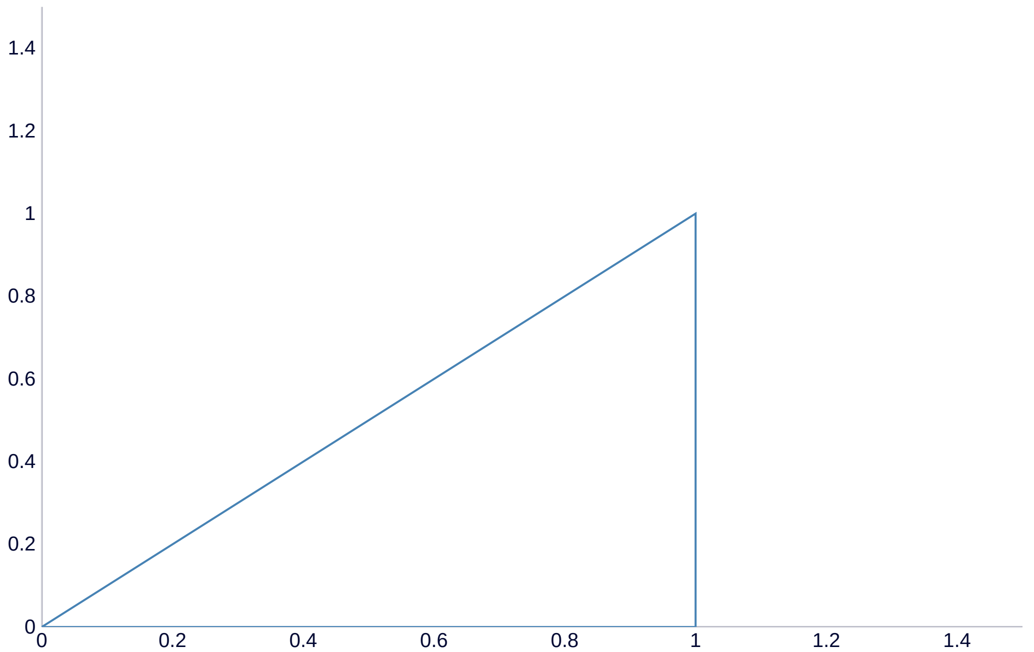
$$\int_0^4 \int_x^4 f(x,y) \, dydx$$



$$= \int_0^4 \int_0^y f(x, y)\, dxdy$$

33

$$\int_0^1 \int_y^1 \frac{\sin x}{x}\, dxdy$$



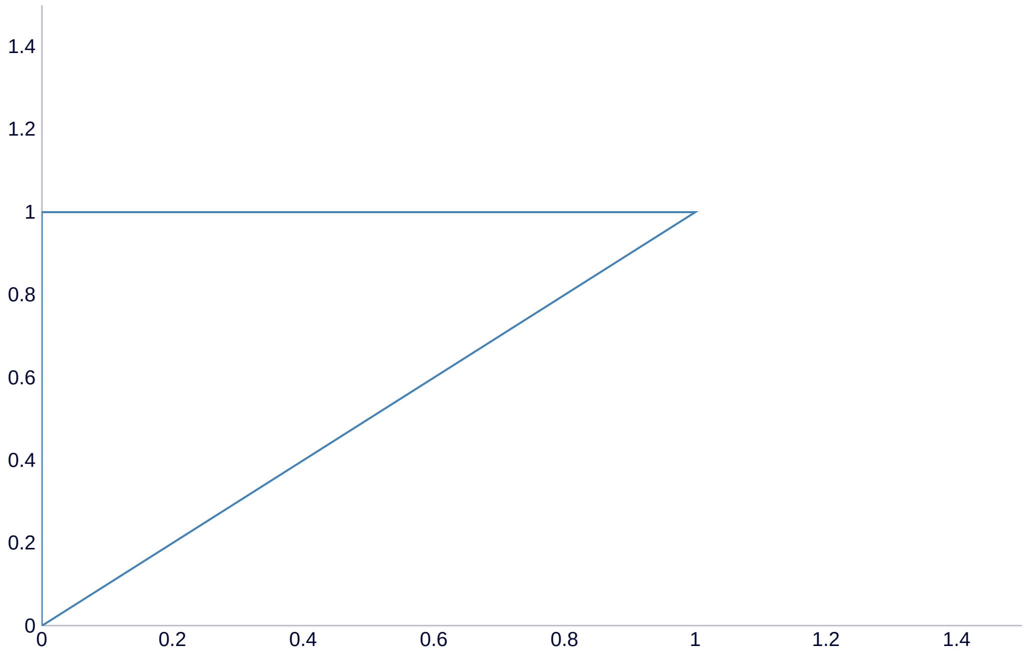
$$= \int_0^1 \int_0^x \frac{\sin x}{x}\, dydx$$

Since there is no y in the equation, we can integrate the whole thing as a constant.

$$= \int_0^1 \sin x \, dx$$
$$= -\cos 1 + 1 \, dx$$
$$\approx 0.460$$

35

$$\int_0^1 \int_x^1 x e^{y^3} \, dy dx$$



$$= \int_0^1 \int_0^y x e^{y^3} \, dx dy$$
$$= \int_0^1 y^2 e^{y^3}/2 \, dy$$
$$= \frac{e-1}{6}$$
$$\approx 0.286$$

43

$$\int_1^2 \int_y^{2y} \frac{\sin y}{y} \, dx dy$$

$$= \int_1^2 \sin y \, dy$$

$$= \cos 1 - \cos 2$$

$$\approx 0.956$$

49

$$\theta = \arctan(y/x)$$

$$r^2 = x^2 + y^2$$

$$\int_0^{2\pi} \int_0^2 8r - 2r^3 \, drdh$$

63

$$D = [0,1] \times [0,4]$$

$$\bar{f} = \frac{\iint_D xy^2 \, dA}{\iint_D 1 \, dA}$$

$$\bar{f} = \frac{1}{4} \iint_D xy^2 \, dA$$

$$\bar{f} = \frac{64/6}{4}$$

$$\bar{f} = \frac{8}{3}$$

A possible solution is $(\frac{2}{\sqrt[3]{3}}, \frac{2}{\sqrt[3]{3}})$

15.3

3

$$\int_0^1 \int_0^1 \int_0^2 xe^{y-2z} \, dxdydz$$

$$= \int_0^1 \int_0^1 2e^{y-2z} \, dydz$$

$$= \int_0^1 2(e^{1-2z} - e^{-2z}) \, dz$$

$$= -(e^{-1} - e^{-2} - e^1 + 1)$$

$$= (1 - e^{-2})(e - 1)$$

$$\approx 1.486$$

5

$$\int_0^1 \int_0^3 \int_0^3 xy - xz - y^2 + yz \, dzdydx$$

$$= \int_0^1 \int_0^3 3xy - 9x/2 - 3y^2 + 9y/2 \, dydx$$

$$= \int_0^1 27x/2 - 27x/2 - 27 + 81/4 \, dx$$

$$= 27/4 - 27/4 - 27 + 81/4$$

$$= 81/4 - 27$$

$$= -27/4$$

$$= -6.75$$

13

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} e^z \, dzdydx$$

$$= \int_0^1 \int_0^{1-x} e^{1-x-y} - 1 \, dydx$$

$$= \int_0^1 -e^{1-x-(1-x)} - (1-x) + e^{1-x} \, dx$$

$$= \int_0^1 -2 + x + e^{1-x} \, dx$$

$$= \int_0^1 -2 + 1/2 - 1 + e \, dx$$

$$= e - 5/2$$

$$\approx 5.218$$

17

$$D = \begin{cases} x > 0 \\ y > 0 \\ z > 0 \\ z < 8 - 2x^2 - y^2 \\ z > y^2 \end{cases}$$

$$D = \begin{cases} 0 < x \\ 0 < y \\ 0 < z < 8 \\ 0 < y^2 < z < 8 - 2x^2 - y^2 \end{cases}$$

$$D = \begin{cases} 0 < x < 2 \\ 0 < y < 2 \\ 0 < z < 8 \\ x^2 + y^2 < 4 \end{cases}$$

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{y^2}^{8-2x^2-y^2} x \, dzdydx$$

$$= \int_0^2 \int_0^{\sqrt{4-x^2}} \int_{y^2}^{8-2x^2-y^2} x \, dzdydx$$

$$= \int_0^2 \int_0^{\sqrt{4-x^2}} (8 - 2x^2 - 2y^2)x \, dydx$$

$$r^2 = x^2 + y^2$$

$$\theta = \arctan(y/x)$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$= \int_0^{\pi/2} \int_0^2 (8 - 2r^2)(r\cos\theta)r \, drd\theta$$

$$= \int_0^{\pi/2} \int_0^2 8r^2\cos\theta - 2r^4\cos\theta \, drd\theta$$

$$= \int_0^{\pi/2} 64\cos\theta/3 - 64\cos\theta/5 \, d\theta$$

$$= \int_0^{\pi/2} \frac{128}{15}\cos\theta \, d\theta$$

$$= \frac{128}{15}$$

23

$$T(x, y) = (x, 3y/2)$$

$$J(T) = \begin{vmatrix} 1 & 0 \\ 0 & 3/2 \end{vmatrix} = 3/2$$

$$r^2 = x^2 + y^2$$

$$\theta = \arctan(y/x)$$

$x = r\cos\theta$

$y = r\sin\theta$

$$\iint_D \int_0^{\sqrt{16-x^2-y^2}} xz\; dzdA$$
$$= \iint_D x(16-x^2-y^2)/2\; dA$$
$$= \int_0^{\pi/2} \int_D x(16-x^2-y^2)/2\; dA$$

25

The positive x side of a sphere with radius of $\sqrt{5}$ above $x = 1$

29

$dzdxdy$

$$\int_0^2 \int_0^{y/2} \int_0^{4-y^2} xyz\; dzdxdy$$

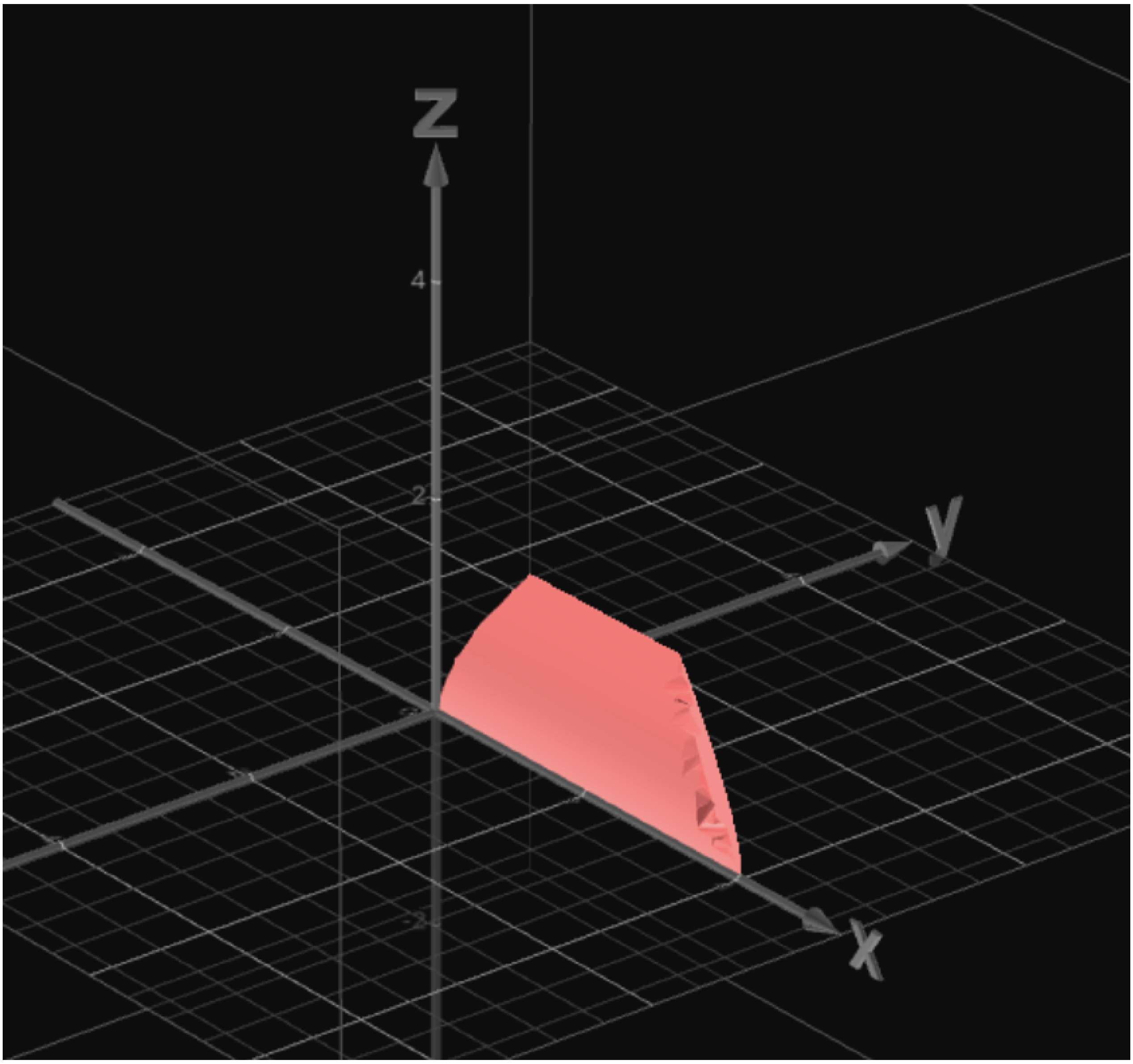
$dxdydz$

$$\int_0^4 \int_0^{\sqrt{4-z}} \int_0^{y/2} xyz\; dxdydz$$

$dydxdz$

$$\int_0^4 \int_0^{4-4x^2} \int_{2x}^{\sqrt{4-z}} xyz\; dydxdz$$

37



z
y
x
4
2