

# 1

$$f(x, y) = x^2 + y^2 - 2x$$

$$D = \triangle[(2, 0), (0, 2), (0, -2)]$$

$$\nabla f = \langle 2x - 2, 2y \rangle$$

$$\nabla f = 0 \iff (x, y) = (1, 0)$$

$$f(1, 0) = -1$$

$$f(2, 0) = 0$$

$$f(0, 2) = 4$$

$$f(0, -2) = 4$$

$$y = -x + 2$$

$$f(x, y) = x^2 + (-x + 2)^2 - 2x = 2x^2 - 6x + 4$$

$$f'_x(x, y) = 4x - 6$$

$$f'_x(x, y) = 0 \iff (x, y) = (1.5, 0.5)$$

$$f(1.5, 0.5) = -0.5$$

$$x = 0$$

$$f(x, y) = y^2$$

$$f'_y(x, y) = 2y$$

$$f'_y(x, y) = 0 \iff (x, y) = (0, 0)$$

$$f(0, 0) = 0$$

$$y = x - 2$$

$$f(x, y) = x^2 + (x - 2)^2 - 2x = 2x^2 - 6x + 4$$

$$f'_x(x, y) = 4x - 6$$

$$f'_x(x, y) = 0 \iff (x, y) = (1.5, -0.5)$$

$$f(1.5, -0.5) = -0.5$$

---

$$f(1, 0) = -1$$

$$f(2, 0) = 0$$

$$f(0, 2) = 4$$

$$f(0, -2) = 4$$

$$f(1.5, 0.5) = -0.5$$

$$f(0, 0) = 0$$

$$f(1.5, -0.5) = -0.5$$

---

Maximums:  $(0, 2), (0, -2)$  at 4

Minimums:  $(1, 0)$  at  $-1$

□

## 2

$$f(x, y) = y^2 + 2x^2$$

$$x^2 + y^2 = 1$$

$$f(x, y) = 1 + x^2 \quad x \in [-1, 1]$$

$$f'_x(x, y) = 2x$$

$$f'_x(x, y) = 0 \iff (x, y) = (0, 1) | (0, -1)$$

$$f(0, 1) = 1$$

$$f(0, -1) = 1$$

$$f(-1, 0) = 2$$

$$f(1, 0) = 2$$

---

Maximums:  $(-1, 0), (1, 0)$  at 2

Minimums:  $(0, 1), (0, -1)$  at 1

□