1

Consider a new force which is defined as a function of two-dimensional coordinates this way:

$\vec{F} \equiv Ay\hat{i} + Bx^4\hat{j}$

Here x and y are standard Cartesian coordinates and A and B are given real positive constants with appropriate units.

Consider the loop that goes around a square path as show in the figure. Each side of the square has a given length C.

a

What is the **Work** done by this specific force $\vec{F}$ for the closed path that goes around the loop in the counter-clockwise direction, starting from the origin $(0, 0)$?
Important: You must explicitly calculate the path integral here. Explain your work. Give your answer in terms of the given parameters.

✓ **Answer**

First Segment:

$$= \int_P^Q \vec{F}\, d\vec{r}$$

$$= \int_{\langle 0,0\rangle}^{\langle C,0\rangle} \vec{F}(\vec{r}) \cdot\, d\vec{r}$$

$$= \vec{r}(t) = \langle t, 0\rangle$$

$$= \int_0^C \langle 0, Bt^4\rangle \cdot \langle 1, 0\rangle\, dt$$

$$= \int_0^C 0\, dt$$

$$= 0$$

Second Segment:

$$= \int_P^Q \vec{F}\, d\vec{r}$$

$$= \int_{\langle C,0\rangle}^{\langle C,C\rangle} \vec{F}(\vec{r}) \cdot\, d\vec{r}$$

$$= \vec{r}(t) = \langle C, t\rangle$$

$$= \int_0^C \langle At, BC^4\rangle \cdot \langle 0, 1\rangle\, dt$$

$= \int_0^C BC^4\, dt$

$= BC^5$

Third Segment:

$= \int_P^Q \vec{F}\, d\vec{r}$

$= \int_{\langle C,C\rangle}^{\langle 0,C\rangle} \vec{F}(\vec{r}) \cdot\, d\vec{r}$

$= \vec{r}(t) = \langle C-t, C\rangle$

$= \int_0^C \langle AC, B(C-t)^4\rangle \cdot \langle -1, 0\rangle\, dt$

$= \int_0^C -AC\, dt$

$= -AC^2$

Fourth Segment:

$= \int_P^Q \vec{F}\, d\vec{r}$

$= \int_{\langle 0,C\rangle}^{\langle 0,0\rangle} \vec{F}(\vec{r}) \cdot\, d\vec{r}$

$= \vec{r}(t) = \langle 0, C-t\rangle$

$= \int_0^C \langle A(C-t), 0\rangle \cdot \langle 0, -1\rangle\, dt$

$= \int_0^C 0\, dt$

$= 0$

Total = Sum of segments

$BC^5 - AC^2$

$BC^5 - AC^2$

$\square$

b

Based on your calculation from Part (a), answer this question: Is this force $\vec{F}$ Conservative or Not Conservative? Cite specific evidence and/or examples to support your claim either way. Explain your work completely here to get full credit.

✓ **Answer**

No, it is non-conservative as a closed loop interval is non-zero.

2

Two astronauts, Lucy, and Ringo, are floating free in deep space, as shown. Each astronaut is defined to have zero velocity. Lucy has a mass m_L of $40\ kg$. She holds a wrench of mass m_w of $10\ kg$. Ringo has a mass m_R of $90\ kg$. Lucy throws a the wrench at Ringo. She throws the wrench so that it moves towards Ringo at a translational speed v_w of $5\ \frac{m}{s}$. She also puts a rather good spin on the wrench, so that it rotates with an angular velocity ω_w of 16 radians per second. Ringo then catches the wrench. Assume that the wrench has a rotational inertia given as $I_{wrench} = 0.2\ kgm^2$ and assume that Lucy has a rotational inertia that is 80 times that of the wrench. Assume Ringo has a rotational inertia that is 125 times that of the wrench.

a

What is Lucy's translational velocity after she throws the wrench?

✓ **Answer**

$p_i = 0$ Given
$p_{wf} = v_w m_w$
$p_f = p_i$
$p_{Lf} = -p_{wf}$
$p_{Lf} = -v_w m_w$
$v_L = -\frac{v_w m_w}{m_L}$

$v_L = -\frac{v_w m_w}{m_L}$
$= -\frac{5(10)}{40} = -1.25\ \frac{m}{s}$
$\square$

b

What is Ringo's translational velocity after he catches the wrench?

✓ **Answer**

$p_i = p_{wi} = v_w m_w$
$p_f = p_i$

$v_w m_w = v_R(m_w + m_R)$
$v_R = \frac{v_w m_w}{m_w + m_R}$

$v_R = \frac{v_w m_w}{m_w + m_R}$
$= \frac{5(10)}{10+90} = 0.5\ \frac{m}{s}$
□

c

What is Lucy's rotational velocity after she throws the wrench?

✓ **Answer**

$L = I\omega$
$L_i = 0 = L_f$
$L_{wf} = I_w \omega_w$
$L_{Lf} = -I_w \omega_w$
$\omega_L = -\frac{I_w \omega_w}{80 I_w}$
$\omega_L = -\frac{\omega_w}{80}$

$\omega_L = -\frac{\omega_w}{80}$
$= -\frac{16}{80} = -0.2\ s^{-1}$
□

d

What is Ringo's rotational velocity after he catches the wrench?

✓ **Answer**

$L_i = L_{wi} = I_w \omega_w = L_f$
$I_w \omega_w = \omega_R(I_w + I_R)$
$\omega_R = \frac{I_w \omega_w}{126 I_w}$
$\omega_R = \frac{\omega_w}{126}$

$\omega_R = \frac{I_w \omega_w}{126 I_w + I_R}$
$= \frac{16}{126} = 0.1270\ s^{-1}$

3

A common recreational fixture in children's outdoor playgrounds is the "merry-go-round" which is generally a large circular platform mounted on a central bearing so that the platform can spin freely.

A "bird's-eye-view" of a merry-go-round is shown above. Assume that the bearing is ideal and that the total rotational inertia of the merry-go-round alone is given as I_m. Assume a small child with given mass m leaps from the ground with horizontal velocity given as v_c and then lands and sticks on the merry-go-round at a given distance R from the center. Assume that the child impacts the merry-go-round at a given angle ϕ relative to the radial direction as shown. Treat the child as a point-like-object. Ignore all vertical motion of the child. Assume the merry-go-round is initially moving with an angular speed of ω_0 in the clock-wise direction as shown.

a

What is the magnitude of the total angular momentum L_{tot} for the combined child-plus-merry-go-round system in the instant just before the child comes in contact with the merry-go-round as calculated for the pivot point corresponding to the central axle of the merry-go-round? Express your answer in terms of the given parameters. Explain your work.

✓ **Answer**

$I = mR^2$
$L_c = mR^2 v_c \sin\phi$
$L_{tot} = I_m \omega_0 + mR^2 v_c \sin\phi$

$L_{tot} = I_m \omega_0 + mR^2 v_c \sin\phi$
$\square$

b

Assume the merry-go-round is initially moving with a given angular speed of ω_0 but then comes up to angular speed ω_1 just after the child leaps on. Calculate the value of ω_1. Express your answer in terms of the given parameters. Explain your work.

✓ **Answer**

$I_m\omega_0 + mR^2 v_c \sin\phi = (I_m + mR^2)\omega_1$ Conservation of angular momentum
$\omega_1 = \frac{I_m\omega_0 + mR^2 v_c \sin\phi}{I_m + mR^2}$

$\omega_1 = \frac{I_m\omega_0 + mR^2 v_c \sin\phi}{I_m + mR^2}$
☐

c

Now assume that the child subsequently pushes straight off the merry-go-round so as to land just off the outside edge on the ground with precisely zero horizontal velocity. What is the angular speed ω_2 of the merry-go-round after the child performs this maneuver? Express your answer in terms of the given parameters. Explain your work.

✓ **Answer**

$I_m\omega_0 + mR^2 v_c \sin\phi = I_m\omega_3$
$\omega_3 = \frac{I_m\omega_0 + mR^2 v_c \sin\phi}{I_m}$

$\omega_3 = \frac{I_m\omega_0 + mR^2 v_c \sin\phi}{I_m}$
☐