Discrete Distributions

Bernoulli (p)

Given $x = 0, 1; \quad 0 \le p \le 1$

- p is the probability of getting selected trait
- Has p probability of being 1 and $1 - p$ probability of being 0

$P(X = x) = p^x(1-p)^{1-x}$
$\mu = p$
$\sigma^2 = p(1-p)$
$M(t) = (1-p) + pe^t$

Binomial (n, p)

Given $x = 0, 1, 2, \dots, n; \quad 0 \le p \le 1$

- p is probability of selecting a particular trait
- n is number of samples in a round of sampling
- Predicts probability of getting certain number of chosen trait in sample set

$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$
$\mu = np$
$\sigma^2 = np(1-p)$
$M(t) = (pe^t + (1-p))^n$

Discrete Uniform (N)

Given $x = 1, 2, \dots, N; \quad N = 1, 2, \dots$

- N is the largest possible sample
- All numbers from 1 to N are equally likely

$P(X = x) = 1/N$
$\mu = \frac{N+1}{2}$
$\sigma^2 = \frac{(N+1)(N-1)}{12}$
$M(t) = \frac{1}{N}\sum_{i=1}^{N} e^{it}$

Geometric (p)

Given $x = 1, 2, \dots; \quad 0 \le p \le 1$

- p is probability of getting certain trait
- Predicts number of samples needed to get a sample of particular trait

$P(X = x) = p(1-p)^{x-1}$
$\mu = 1/p$
$\sigma^2 = \frac{1-p}{p^2}$
$M(t) = \frac{pe^t}{1-(1-p)e^t}$

Hypergeometric (N, K, M)

Given $x = 0, 1, 2, \dots, K; \quad M - (N - K) \le x \le M; \quad N, M, K = 0, 1, 2, \dots$

- N is the population size
- M is the number of samples in the population with a certain trait
- K number of samples taken in a round of sampling
- Predicts the likelihood of selecting X samples of type M after selecting K samples from population N

$P(X - x) = \frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}$
$\mu = KM/N$
$\sigma^2 = \frac{KM(N-M)(N-K)}{N^2(N-1)}$

Negative Binomial (r, p)

Given $x = 0, 1, 2, \dots; \quad 0 \le p \le 1$

- p is the probability of getting a particular trait in one sample
- r is the desired number of samples with a particular trait
- Predicts number of likelihood of getting r samples of trait after X samples

$P(X = x) = \binom{r+x-1}{x} p^r (1-p)^x$
$\mu = \frac{r(1-p)}{p}$
$\sigma^2 = \frac{r(1-p)}{p^2}$
$M(t) = (\frac{p}{1-(1-p)e^t})^r$

Poisson Distribution (λ)

Given $x = 0, 1, 2, \dots; \quad 0 \le \lambda$

- λ is the number of times on average an event will happen within an interval
- Predicts number of times an event will happen within an interval
- Approximates the Binomial Distribution

$P(X = x) = \frac{e^{-\lambda}\lambda^x}{x!}$
$\mu = \lambda$
$\sigma^2 = \lambda$
$M(t) = e^{\lambda(e^t - 1)}$

Continuous Distributions

Beta (α, β)

Given $0 \le x \le 1; \quad \alpha > 0; \quad \beta > 0$

$f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$
$\mu = \frac{\alpha}{\alpha+\beta}$
$\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
$M(t) = 1 + \sum_{k=1}^{\infty} \frac{t^k}{k!} \prod_{r=0}^{k-1} \frac{a+r}{\alpha+\beta+r}$

Cauchy (θ, σ)

Given $-\infty < x < \infty; \quad -\infty < \theta < \infty; \quad \sigma > 0$

$f(x) = \frac{1}{\pi\sigma(1+(\frac{x-\theta}{\sigma})^2)}$

Chi squared (p)

Given $0 \le x < \infty; \quad p = 1, 2, 3, \dots$

$f(x) = \frac{x^{p/2-1}e^{-x/2}}{\Gamma(p/2)2^{p/2}}$
$\mu = p$
$\sigma^2 = 2p$
$M(t) = (\frac{1}{1-2t})^{p/2}$

Double Exponential (μ, σ)

Given $-\infty < x < \infty; \quad -\infty < \mu < \infty; \quad \sigma > 0$

$f(x) = \frac{e^{-|x-\mu|/\sigma}}{2\sigma}$
$\mu = \mu$
$\sigma^2 = 2\sigma^2$
$M(t) = \frac{e^{\mu t}}{1-(\sigma t)^2}$

Exponential β

Given $0 \le x < \infty; \quad \beta > 0$

$f(x) = \frac{e^{-x/\beta}}{\beta}$
$\mu = \beta$
$\sigma^2 = \beta^2$
$M(t) = \frac{1}{1-\beta t}$

F (v_1, v_2)

Given $0 \le \infty; \quad v_1, v_2 = 1, 2, 3, \dots$

$f(x) = \frac{\Gamma(\frac{v_1+v_2}{2})}{\Gamma(\frac{v_1}{2})\Gamma(\frac{v_2}{2})} (\frac{v_1}{v_2})^{v_1/2} \frac{x^{(v_1-2)/2}}{(1+\frac{v_1 x}{v_2})^{(v_1+v_2)/2}}$
$\mu = \frac{v_2}{v_2-2}$
$\sigma^2 = 2(\frac{v_2}{v_2-2})^2 \frac{v_1+v_2-2}{v_1(v_2-4)}$
$EX^n = \frac{\Gamma(\frac{v_1+2n}{2})\Gamma(\frac{v_2-2n}{2})}{\Gamma(v_1/2)\Gamma(v_2/2)} (\frac{v_2}{v_1})^n \quad ; n < \frac{v_2}{2}$

Gamma Distribution (α, β)

Given $0 \le x < \infty; \quad \alpha, \beta > 0$

$f(x) = \frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}$
$\mu = \alpha\beta$
$\sigma^2 = \alpha\beta^2$
$M(t) = (\frac{1}{1-\beta t})^\alpha$

Logistic (μ, β)

Given $-\infty < x < \infty; \quad -\infty < \mu < \infty; \quad \beta > 0$

$f(x) = \frac{e^{-(x-\mu)/\beta}}{\beta(1+e^{-(x-\mu)/\beta)})^2}$
$\mu = \mu$
$\sigma^2 = \frac{\pi^2\beta^2}{3}$
$M(t) = e^{\mu t}\Gamma(1 + \beta t)$

Lognormal (μ, σ^2)

Given $0 \le x < \infty; \quad -\infty < \mu < \infty; \quad \sigma > 0$

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \frac{e^{-(\log x-\mu)^2/(2\sigma^2)}}{x}$
$\mu = e^{\mu+(\sigma^2/2)}$
$\sigma^2 = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$
$EX^n = e^{n\mu+n^2\sigma^2/2}$

Normal (μ, σ^2)

Given $-\infty < x < \infty; \quad -\infty < \mu < \infty; \quad \sigma > 0$

$f(x) = \frac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sqrt{2\pi}\sigma}$
$\mu = \mu$
$\sigma^2 = \sigma^2$

Paretto (α, β)

Given $a < x < \infty; \quad \alpha, \beta > 0$

$f(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}$
$\mu = \frac{\beta\alpha}{\beta-1} \quad ; \beta > 1$
$\sigma^2 = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)} \quad ; \beta > 2$

t (v)

Given $-\infty < x < \infty; \quad v = 1, 2, 3, \dots$

$f(x) = \frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})} \frac{1}{\sqrt{v\pi}} \frac{1}{(1+(\frac{x^2}{v}))^{(v+1)/2}}$
$\mu = 0 \quad ; v > 1$
$\sigma^2 = \frac{v}{v-2} \quad ; v > 2$
$MX^n = \begin{cases} \frac{\Gamma(\frac{n+1}{2})\Gamma(\frac{v-n}{2})}{\sqrt{\pi}\Gamma(v/2)} v^{n/2} & n < v; n \text{ is even} \\ 0 & n < v; n \text{ is odd} \end{cases}$

Uniform (a, b)

Given $a \le x \le b$

- a is the lower bound of the distribution
- b is the upper bound
- All values between a and b are equally distributed

$f(x) = \frac{1}{b-a}$
$\mu = \frac{b+a}{2}$
$\sigma^2 = \frac{(b-a)^2}{12}$
$M(t) = \frac{e^{bt}-e^{at}}{t(b-a)}$

Weibull (γ, β)

Given $0 \le x < \infty; \quad \gamma, \beta > 0$

$f(x) = \frac{\gamma}{\beta} x^{\gamma-1} e^{-x^\gamma/\beta}$
$\mu = \beta^{1/\gamma}\Gamma(1 + \frac{1}{\gamma})$
$\sigma^2 = \beta^{2/\gamma}(\Gamma(1 + \frac{2}{\gamma}) - \Gamma^2(1 + \frac{1}{\gamma}))$
$EX^n = \beta^{n/\gamma}\Gamma(1 + \frac{n}{\gamma})$