1

A strange science-fiction battle takes on a tiny moon in deep space; here, the moon is so small that we can completely ignore gravity. A deranged invading space monster of given mass $28.2\ kg$ kicks off a rocky structure to achieve a horizontal speed of $3.45\ m/s$ directly toward a startled space cadet. The cadet is braced firmly on the surface of the moon and is armed with a rapid-fire automatic marshmallow blaster gun which fires multiple sticky marshmallows toward the monster, each of mass $11.5\ g$ and each with a speed of $53.4\ m/s$. The cadet attempts to stop the monster with the marshmallow gun, slowing the monster down by hitting the monster with marshmallow after marshmallow until the monster is finally brought to a stop. How many marshmallows will the cadet have to fire at the monster to stop the monster's horizontal motion? (Assume each fired marshmallow harmlessly sticks to the monster. Assume that the monster is flying horizontally, with no vertical motion and no friction or other forces on the monster except for those due to the collision with the marshmallows. Explain your work. Indicate what Physics Concept (law of physics) you used to solve this problem.

Answer

We can use the concept of momentum to solve this problem.

$m_M = 28.2\ kg$
$v_M = 3.45\ \frac{m}{s}$
$v_{Mf} = 0$
$m_m = 0.0115\ kg$
$v_m = -53.4\ \frac{m}{s}$
$v_{mf} = 0$

The initial momentum of the monster M can be calculated.
$p_M = m_m v_M = 97.29\ \frac{kgm}{s}$
The momentum of one marshmallow m can also be calculated.
$p_m = m_m v_m = -0.6141\ \frac{kgm}{s}$

The final momentum of the monster and marshmallow can also be calculated.
$p_{Mf} = m_M v_{Mf} = 0 = m_m v_{mf} = p_{mf}$

The impulse can also be calculated.
$\Delta p_M = -97.29\ \frac{kgm}{s}$
$\Delta p_m = 0.6141\ \frac{kgm}{s}$

Since the collision only contains the monster and the marshmallows, the net momentum of the entire system should remain constant.

$n = -\frac{m_M v_M}{m_m v_m}$
$n = \frac{97.29}{0.6141} = 158.4269662921348$

$n = -\frac{m_M v_M}{m_m v_m}$
$n = 158.4269662921348$
It takes at least 159 marshmallows to completely stop the monster.
Since this is a totally inelastic collision, the system loses kinetic energy.

2

A medicine ball with given mass $M = 8.11\ kg$ drops vertically, eventually hitting the floor. Just before impact, the ball has a given downward speed of $5.11\ m/s$. It then rebounds straight back with a given upward speed of $4.42\ m/s$.

a

What is conserved with respect to the system that is defined as the medicine ball during the entire collision? Is Mechanical Energy Conserved? Is Linear Momentum Conserved? Explain how you know this?

✓ **Answer**

Nothing is conserved as the floor is not in the system, so it will have an external force on the system. The medicine ball also loses KE and momentum as it slows down after the collision.

b

What is the total impulse imparted to the ball as a result of contact with the floor?

✓ **Answer**

$\Delta p = p_f - p_i = mv_f - mv_i$
$\Delta p = 77.2883\ \frac{kgm}{s} \approx 77.29\ \frac{kgm}{s}$
Where up is positive

c

If the ball is in contact for a time interval $\Delta t = 0.056s$, what is the average force exerted on the floor during this collision?

✓ **Answer**

$F = \frac{\Delta p}{\Delta t}$
$F = \frac{77.2883}{0.056}\ N = 1.38 \times 10^3\ N$

3

You are driving a car that is moves with constant given speed V . Your mass is given as m and the mass of the car (not including yourself) is given as M . The road is completely flat and level. The car enters a curve in the road that can be described as a circular arc with given radius R as shown above in a “bird’s-eye” view of the path of the car. Assume you are properly secured into your seat as you drive.

a

What is the magnitude of the net force on you, the driver, as you go around the curve? Explain how you know this.

✓ **Answer**

Since I am moving in circular motion we can use the circular motion formula
$a_c = \frac{V^2}{R}$
$F_{net} = ma_c = \frac{mV^2}{R}$

b

A student says:

> “As the car comes around the curve, there must be a net force on the driver to the driver’s right. In other words, the net force points away from the center of the circle.”

Is the student correct or incorrect? Explain how you know this.

✓ **Answer**

Wrong. If we had a force to the right then we would go to the right??? Since we accelerate towards the left (from the perspective of the driver) then we should have a corresponding net force towards the left? Gurl needs to logic.

c

What is the magnitude of the force of friction on the car due to the road as you go around the curve? Hint: it is not zero. Explain how you know this?

✓ **Answer**

The force of friction, being non-vertical, will be equal to the net force as the normal and the weight force are equal and opposite.
Since the driver and the car are attached to each other (accelerating equally and have equal velocity) we can treat them as a system.

Since the system moves in circular motion, we can deduce their net force, which is equal to the friction force.
$F_{net} = \frac{(M+m)V^2}{R} = F_f$

d

What is the Total Work done by all forces on the car as it moves on the curve from point P (corresponding to where the car enters the curve) to the point Q (corresponding to where the car leaves the curve)? Explain how you know this? Important: you must use one or more Physics Concept(s) here

✓ **Answer**

Since there is no movement in the vertical direction, both weight force and normal force have 0 work, as all movement is perpendicular to the force (completely non-vertical).

$\vec{v} \cdot \vec{W} = \cos\theta v W = 0$
$\vec{v} \cdot \vec{N} = \cos\theta v N = 0$

Since the friction force will always point towards the center of the circle, while velocity will always point tangent to the circle, the vectors will always be perfectly perpendicular to each other, meaning the work is 0.

$\vec{v} \cdot \vec{F}_f = \cos\theta v F_f = 0$

Therefore, the total work is 0

4

A constant given torque τ of 0.073 Newton-meters is applied to a free-spinning bicycle wheel of mass $m = 4.3\ kg$, radius R of $0.27m$ as shown. The rotational inertia of the wheel is given by the expression: $I = mR^2$.

a

What is the angular acceleration of the wheel? (Hint: Use the rotational analog of Newton's 2nd Law).

✓ **Answer**

$\tau = I\alpha$ N's 2nd Law
$\alpha = \frac{\tau}{I} = \frac{\tau}{mR^2}$
$\alpha = \frac{0.073}{(4.3)(0.27)^2}\ s^{-2} = 0.23\ s^{-2}$

b

Assuming the wheel starts at rest, how much time will it take for the wheel to be spun to a final rotational velocity of $\omega = 22.2$ radians/second? (Hint: This is analogous to translational motion with constant acceleration).

✓ **Answer**

$\omega = \omega_0 + \alpha t$
$22.2 = 0 + 0.23t$
$t = 95\ s$

c

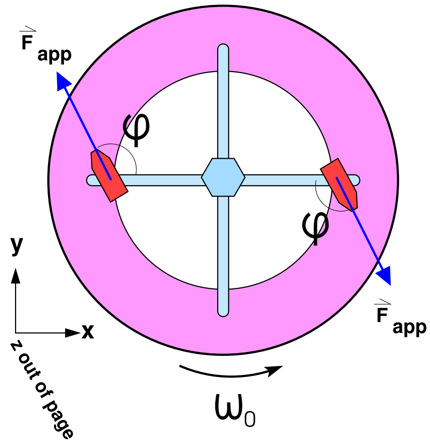
A small piece of mud is stuck to the outside edge of of the wheel. What is the centripetal acceleration on the piece of mud after time $T = 5.0s$ torque have been applied? (Hint: find the angular velocity, and then consider the piece of mud as instantaneously moving with Uniform Circular Motion).

✓ **Answer**

$\omega = \omega_0 + \alpha t$
$a_c = R\omega^2 = R\alpha^2 t^2$
$a_c = 0.27(0.23)^2(5)^2 \ \frac{m}{s^2} = 0.37 \ \frac{m}{s^2}$

5

Consider a hypothetical future space station that consists of a torus attached by spokes to a central hub as show above. Suppose the space station has an given initial angular speed of ω_0 and a rotational inertia $I = CMR^2$ where M is the given mass of the space station, R is the given outer radius of the torus, and C is a given numerical constant.
In this situation, the space engineers need to repair the station and in order to do this, they need to stop the rotation and bring the station to rest. Therefore, the engineers affix two large thrusters to the space station as shown. Each thruster is attached to a fixed position on the space station corresponding to a given distance r from the center and at a given angle ϕ relative to the radial direction. When activated, each thruster will result in a force of given constant magnitude F_{app} applied to the station as shown. Note that a coordinate system is given here, with x-direction corresponding to the right, y-direction corresponding to the top of the page, and z-direction corresponding to out of the page.



a

What is the magnitude of the total torque τ applied to the space station that results from the action of both of the thrusters? Give your answer in terms of given parameters only. Explain your work.

✓ **Answer**

$\tau_{net} = -2\vec{F}_{app} \times \vec{r}$ Two times the torque equations
$\tau_{net} = -2F_{app} r \sin\phi$ Definition of cross product

b

What is the magnitude of the angular acceleration α of the space station that results from the action of both of the thrusters? Give your answer in terms of given parameters only. Explain your work.

✓ **Answer**

$\tau_{net} = I\alpha$ Torque acceleration relation
$\alpha = -\frac{2F_{app} r \sin\phi}{CMR^2}$ Substitution

c

What is the linear acceleration $\vec{a}$ of the entire space station as a single body that results from the action of both of the thrusters? Indicate both the magnitude and the direction. Give your answer in terms of given parameters only. Explain your work. Hint: The Rolling Constraint does not apply here.

✓ **Answer**

The forces are equal and (assumed) opposite to each other, and thus will cancel out each other. Since there are no other forces, the acceleration will be 0.
$F = ma$
$F = F_{app} - F_{app} = 0$
$a = \frac{0}{m} = 0$

d

Assume that at time $t = 0$ both thrusters are activated. How many total rotations of the space station will occur before the space station is brought completely to rest? Give your answer in terms of given parameters only. Explain your work. Hint: one rotation equals 2π radians of angle.

✓ **Answer**

$\omega(t) = \omega_0 + \alpha t$

Since α is constant, and $\omega_f = 0$, the integral of $\omega(t)$ from start until stop will take the form of $\frac{1}{2}\omega_0 \frac{\omega_o}{-\alpha}$ (area of a triangle).

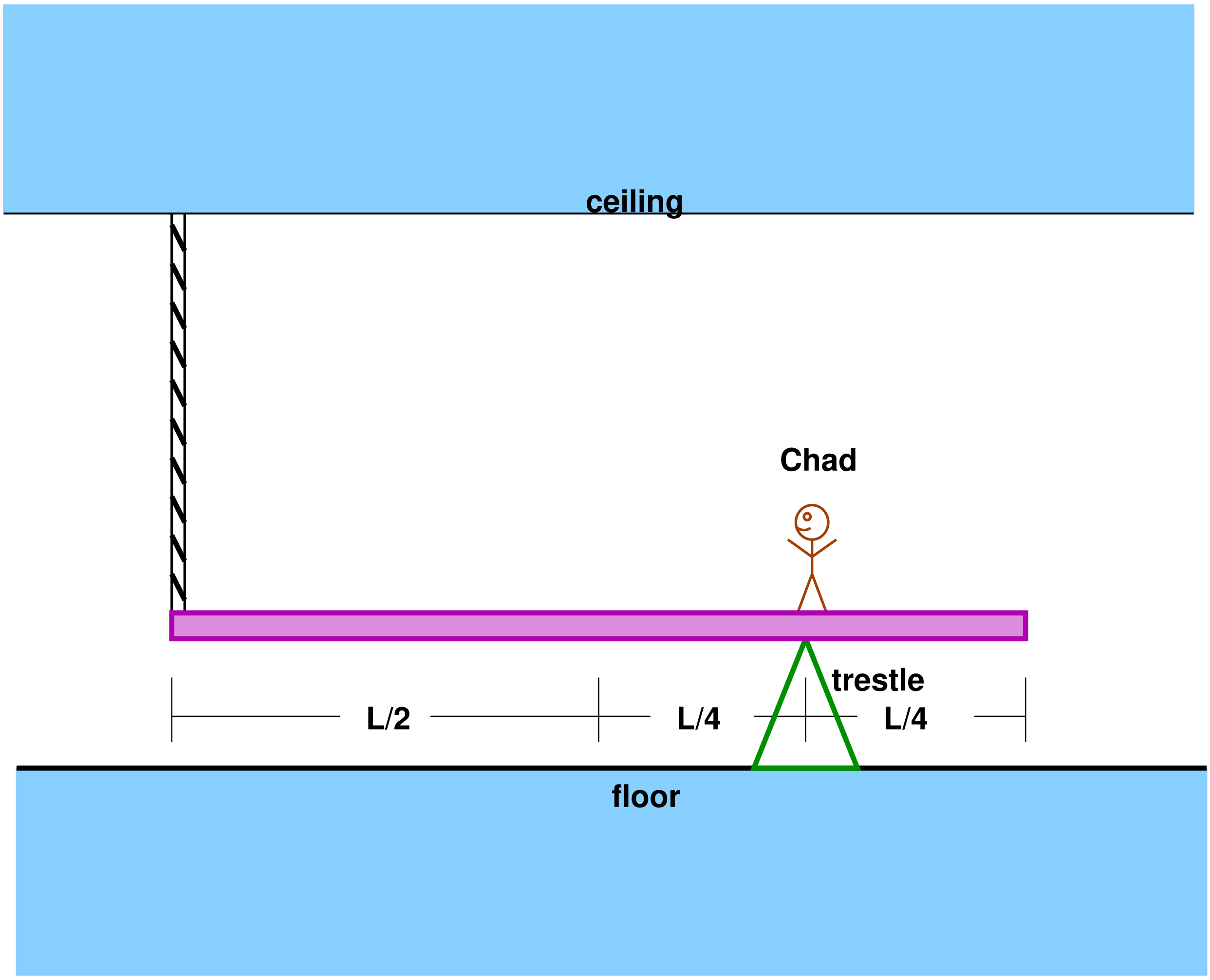
$\theta = \frac{\omega_0^2 CMR^2}{4F_{app} r \sin\phi}$
$\mathrm{Rot} = \frac{\omega_0^2 CMR^2}{8\pi F_{app} r \sin\phi}$

6

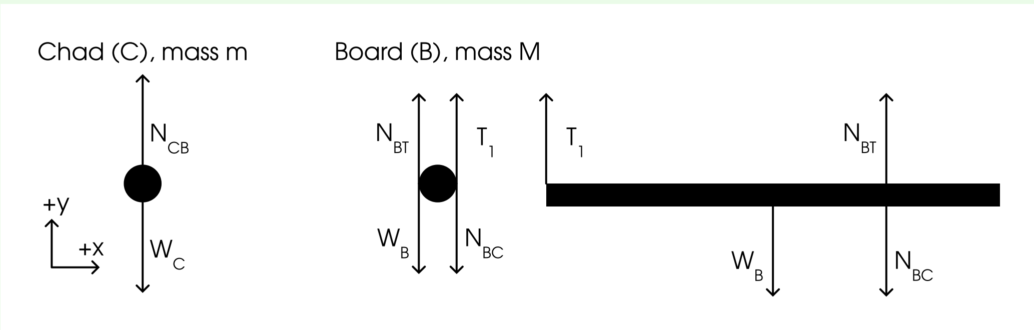
Chad stands on a a large heavy uniform board. Chad has a mass m. The board has a mass M and a total length L. The board is suspended by a rope on the left side and a trestle which pushes the board upward at a position that is precisely three-fourths the length of the board from the left side as shown in the figure above. Chad is positioned directly above the trestle.

Draw complete and carefully labeled Free Body Diagrams (FBDs) for both Chad and the board. Also draw a complete and carefully labeled Extended Free Body Diagram (XFBD) for the board.

Determine T, the magnitude of the Tension on the board due to the rope. Also determine N_{BT}, the magnitude of the Normal force on the board due to the trestle. Express your answer in terms of the given parameters. Be sure to explain your work.



✓ **Answer**



Since nothing is accelerating, we know that all net forces and net torques are 0.

$N_{CB} = W_C = mg$ 2nd law
$N_{BC} = N_{CB} = mg$ 2nd law
$W_B = Mg$
$(M+m)g = N_{BT} + T_1$ 2nd law

Choosing the pivot as the trestle,

$\tau_T = Fr = 0F = 0$
$\tau_{T_1} = \frac{3L}{4} T_1 \sin\frac{-\pi}{2} = -\frac{3LT_1}{4}$

$\tau_W = \frac{L}{4} W_B \sin\frac{\pi}{2} = \frac{LMg}{4}$
$\tau_N = Fr = 0F = 0$

$0 = \tau_{net} = \frac{LMg}{4} - \frac{3LT_1}{4}$
$LMg = 3LT_1$
$T_1 = \frac{Mg}{3}$

$(M+m)g = N_{BT} + \frac{LMg}{3L}$
$N_{BT} = (M+m)g - \frac{Mg}{3}$

$T_1 = \frac{Mg}{3}$
$N_{BT} = (M+m)g - \frac{Mg}{3}$
$\square$