2.17

a

$f(x) = 3x^2 \qquad ,0 < x < 1$

$P(X > m) = P(X < m)$
$F(m) = 1 - F(m)$
$F(m) = 0.5$

$F(x) = \int\limits_0^x f(t)dt$
$F(x) = x^3$
$0.5 = m^3$
$m = 0.7937$

b

$f(x) = \frac{1}{\pi(1+x^2)} \qquad , -\infty < x < \infty$

$P(X > m) = P(X < m)$
$F(m) = 1 - F(m)$
$F(m) = 0.5$

$F(x) = \int\limits_{-\infty}^x f(t)dt$
$F(x) = \arctan(x)/\pi + 0.5$
$0.5 = \arctan(m)/\pi + 0.5$
$m = 0$

2.24

a

$f(x) = ax^{a-1} \qquad ,0 < x < 1, a > 0$

EX

$EX = \int\limits_0^1 xf(x)dx$
$EX = \int\limits_0^1 ax^a dx$
$EX = \frac{ax^{a+1}}{a+1}\Big|_0^1$
$EX = \frac{a}{a+1}$

EX^2

$$EX^2 = \int_0^1 x^2 f(x) dx$$
$$EX^2 = \int_0^1 a x^{a+1} dx$$
$$EX^2 = \frac{a x^{a+2}}{a+2}\Big|_0^1$$
$$EX^2 = \frac{a}{a+2}$$

σ^2

$$\sigma^2 = EX^2 - (EX)^2$$
$$\sigma^2 = \frac{a}{a+2} - (\frac{a}{a+1})^2$$
$$\sigma^2 = \frac{a}{a+2} - \frac{a^2}{(a+1)^2}$$
$$\sigma^2 = \frac{a^3+2a^2+a}{(a+2)(a+1)^2} \frac{a^3+2a^2}{(a+2)(a+1)^2}$$
$$\sigma^2 = \frac{a}{(a+2)(a+1)^2}$$

b

$f(x) = 1/n \qquad , x = 1, 2, \ldots, n, n > 0 \text{ an integer}$

EX

$$EX = \sum_{x=1}^{n} x f(x)$$
$$EX = \sum_{x=1}^{n} x/n$$
$$EX = n(n+1)/2n$$
$$EX = (n+1)/2$$

EX^2

$$EX^2 = \sum_{x=1}^{n} x^2 f(x)$$
$$EX^2 = \sum_{x=1}^{n} x^2/n$$
$$EX^2 = (n+1)(2n+1)/6$$

σ^2

$$\sigma^2 = EX^2 - (EX)^2$$
$$\sigma^2 = (n+1)(2n+1)/6 - ((n+1)/2)^2$$
$$\sigma^2 = \frac{2(n+1)(2n+1) - 3(n+1)^2}{12}$$
$$\sigma^2 = \frac{n^2-1}{12}$$

c

$f(x) = 3(x-1)^2/2 \qquad , 0 < x < 2$

EX

$$EX = \int_0^2 x f(x) dx$$

$$EX = \frac{3}{2} \int_0^2 x^3 - 2x^2 + x dx$$

$$EX = \frac{3}{2} (x^4/4 - 2x^3/3 + x^2/2) \Big|_0^2$$

$$EX = 1$$

EX^2

$$EX^2 = \int_0^2 x^2 f(x) dx$$

$$EX^2 = \frac{3}{2} \int_0^2 x^4 - 2x^3 + x^2 dx$$

$$EX^2 = \frac{3}{2} (x^5/5 - x^4/2 + x^3/3) \Big|_0^2$$

$$EX^2 = 8/5$$

σ^2

$$\sigma^2 = EX^2 - (EX)^2$$
$$\sigma^2 = 8/5 - 1$$
$$\sigma^2 = 3/5$$

2.28

b

$$f(x) = e^{-x} \qquad , x \geq 0$$

EX

$$EX = \int_0^\infty x e^{-x} dx$$

u	dv
x	e^{-x}
1	$-e^{-x}$
0	e^{-x}

$$EX = -xe^{-x} - e^{-x} \Big|_0^\infty$$

$$EX = 1$$

μ_2

$$\mu_2 = E((X - \mu)^2)$$

$$\mu_2 = \int_0^\infty (x - 1)^2 e^{-x} dx$$

$\mu_2 = \int\limits_0^\infty (x^2 - 2x + 1)e^{-x}$

$\mu_2 = -(x^2 + 2x + 2 - 2x - 2 + 1)e^{-x}\Big|_0^\infty$

$\mu_2 = -(x^2 + 1)e^{-x}\Big|_0^\infty$

$\mu_2 = 1$

μ_3

$\mu_3 = E((X - \mu)^3)$

$\mu_3 = \int\limits_0^\infty (x - 1)^3 e^{-x} dx$

$\mu_3 = \int\limits_0^\infty (x^3 - 3x^2 + 3x - 1)e^{-x}$

$\mu_3 = -(x^3 + 3x^2 + 6x + 6 - 3x^2 - 6x - 6 + 3x + 3 - 1)e^{-x}\Big|_0^\infty$

$\mu_3 = -(x^3 + 3x + 2)e^{-x}\Big|_0^\infty$

$\mu_3 = 2$

α_3

$\alpha_3 = \frac{\mu_3}{\mu_2^{3/2}}$

$\alpha_3 = 2$

c

i

$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

μ

$EX = \int\limits_{-\infty}^{\infty} x f(x) dx$

$EX = \int\limits_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$

$EX = 0$

μ_2

$\mu_2 = E((X - \mu)^2)$

$\mu_2 = \int\limits_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$

$\mu_2 = 1$

μ_4

$\mu_4 = E((X - \mu)^4)$

$\mu_4 = \int\limits_{-\infty}^{\infty} x^4 \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$

$\mu_4 = 3$

α_4

$\alpha_4 = \frac{\mu^4}{\mu_2^2}$
$\alpha_4 = 3$

The graph does not look very peaked to me

ii

$f(x) = 1/2$

μ

$EX = \int_{-1}^{1} xf(x)dx$
$EX = \int_{-1}^{1} x/2dx$
$EX = 0$

μ_2

$\mu_2 = E((X-\mu)^2)$
$\mu_2 = \int_{-1}^{1} x^2/2dx$
$\mu_2 = 1/3$

μ_4

$\mu_4 = E((X-\mu)^4)$
$\mu_4 = \int_{-1}^{1} x^4/2dx$
$\mu_4 = 1/5$

α_4

$\alpha_4 = \frac{\mu^4}{\mu_2^2}$
$\alpha_4 = 9/5$

This graph does not look peaked at all

iii

$f(x) = 1/2e^{-|x|}$

μ

$$EX = \int_{-\infty}^{\infty} xf(x)dx$$
$$EX = \int_{-\infty}^{\infty} x/2e^{-|x|}dx$$
$$EX = 0$$

μ_2

$$\mu_2 = E((X-\mu)^2)$$
$$\mu_2 = \int_{-\infty}^{\infty} x^2/2e^{-|x|}dx$$
$$\mu_2 = 2$$

μ_4

$$\mu_4 = E((X-\mu)^4)$$
$$\mu_4 = \int_{-\infty}^{\infty} x^4/2e^{-|x|}dx$$
$$\mu_4 = 24$$

α_4

$$\alpha_4 = \frac{\mu^4}{\mu_2^2}$$
$$\alpha_4 = 6$$

This graph looks the most peaked compared to everything else

2.33

c

i

$$M(x) = E(e^{tX})$$
$$M(x) = \sum_{x=0}^{\infty} e^{tx}e^{-\lambda}\lambda^x/x!$$
$$M(x) = e^{-\lambda}\sum_{x=0}^{\infty} e^{tx}\lambda^x/x!$$
$$M(x) = e^{-\lambda}e^{\lambda e^t}$$
$$M(x) = e^{\lambda e^t - \lambda}$$
$$M(x) = e^{\lambda(e^t-1)}$$

EX

$$EX = \frac{d}{dt}M(t)\bigg|_0$$

$$EX = e^{\lambda(e^t-1)}\lambda e^t\bigg|_0$$
$$EX = \lambda$$

EX^2

$$EX^2 = \frac{d^2}{d^2t} M(t)\Big|_0$$

$$EX^2 = \frac{d}{dx} e^{\lambda(e^t-1)} \lambda e^t\Big|_0$$
$$EX^2 = \frac{d}{dx} e^{\lambda(e^t-1)} \lambda^2 e^2 t + e^{\lambda(e^t-1)} \lambda e^t\Big|_0$$
$$EX^2 = \lambda^2 + \lambda$$

σ^2

$$\sigma^2 = EX^2 - (EX)^2$$
$$\sigma^2 = \lambda^2 + \lambda - (\lambda)^2$$
$$\sigma^2 = \lambda$$

ii

$$M(x) = E(e^{tX})$$
$$M(x) = \sum_{x=0}^{\infty} e^{tx} p(1-p)^x$$
$$M(x) = p \sum_{x=0}^{\infty} (e^t(1-p))^x$$
$$M(x) = p/(1-e^t(1-p))$$

EX

$$EX = \frac{d}{dt} M(t)\Big|_0$$
$$EX = \frac{p(e^t(1-p))}{(1-e^t(1-p))^2}\Big|_0$$
$$EX = \frac{1-p}{p}$$

EX^2

$$EX^2 = \frac{d^2}{d^2t} M(t)\Big|_0$$
$$EX^2 = \frac{d}{dx} \frac{p(e^t(1-p))}{(1-e^t(1-p))^2}\Big|_0$$
$$EX^2 = \frac{d}{dx} \frac{pe^t - p^2e^t}{(1-e^t(1-p))^2}\Big|_0$$
$$EX^2 = \frac{(pe^t-p^2e^t)(1-e^t(1-p))^2 + 2(pe^t-p^2e^t)(1-e^t(1-p))(e^t(1-p))}{(1-e^t(1-p))^4}\Big|_0$$
$$EX^2 = \frac{(p-p^2)(p)^2 + 2(p-p^2)(p)(1-p)}{p^4}$$
$$EX^2 = \frac{(p-p^2) + 2(1-p)(1-p)}{p^2}$$
$$EX^2 = \frac{p-p^2+2-4p+2p^2}{p^2}$$
$$EX^2 = \frac{p^2-3p+2}{p^2}$$
$$EX^2 = \frac{(p-2)(p-1)}{p^2}$$

σ^2

$\sigma^2 = EX^2 - (EX)^2$
$\sigma^2 = \frac{(p-2)(p-1)}{p^2} - (\frac{1-p}{p})^2$
$\sigma^2 = \frac{(p-2)(p-1)-(1-p)^2}{p^2}$
$\sigma^2 = \frac{1-p}{p^2}$

iii

$M(x) = E(e^{tX})$
$M(x) = \int\limits_{-\infty}^{\infty} e^{tx} \frac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sqrt{2\pi}\sigma} dx$
Let $u = x - \mu$ and $x = u + \mu$
$M(x) = \int\limits_{-\infty}^{\infty} e^{t(x+\mu)} \frac{e^{-u^2/(2\sigma^2)}}{\sqrt{2\pi}\sigma} du$
$M(x) = \frac{e^{t\mu}}{\sqrt{2\pi}\sigma} \int\limits_{-\infty}^{\infty} e^{tu} e^{-u^2/(2\sigma^2)} du$
$M(x) = \frac{e^{t\mu}}{\sqrt{2\pi}\sigma} \int\limits_{-\infty}^{\infty} e^{(2tu\sigma^2-u^2)/(2\sigma^2)} du$
$M(x) = \frac{e^{t\mu}}{\sqrt{2\pi}\sigma} \int\limits_{-\infty}^{\infty} e^{(-(u-t\sigma^2)^2+t^2\sigma^4)/(2\sigma^2)} du$
$M(x) = \frac{e^{t\mu}e^{\sigma^2t^2/2}}{\sqrt{2\pi}\sigma} \int\limits_{-\infty}^{\infty} e^{-(u-t\sigma^2)^2/(2\sigma^2)} du$
$M(x) = \frac{e^{t\mu}e^{\sigma^2t^2/2}}{\sqrt{2\pi}\sigma} \int\limits_{-\infty}^{\infty} e^{-(\frac{u-t\sigma^2}{\sqrt{2}\sigma})^2} du$
Let $v = \frac{u-t\sigma^2}{\sqrt{2}\sigma}$ and $dv = \frac{du}{\sqrt{2}\sigma}$ and $du = \sqrt{2}\sigma dv$
$M(x) = \frac{e^{t\mu}e^{\sigma^2t^2/2}}{\sqrt{\pi}} \int\limits_{-\infty}^{\infty} e^{-v^2} dv$
$M(x) = e^{t\mu}e^{\sigma^2t^2/2}$
$M(x) = e^{\mu t+\sigma^2t^2/2} z$

EX

$EX = \frac{d}{dt} M(t)\Big|_0$
$EX = \frac{d}{dt} e^{\mu t+\sigma^2t^2/2}\Big|_0$
$EX = (\mu + \sigma^2 t)e^{\mu t+\sigma^2t^2/2}\Big|_0$
$EX = \mu$

EX^2

$EX^2 = \frac{d^2}{d^2t} M(t)\Big|_0$
$EX = \frac{d}{dt} (\mu + \sigma^2 t)e^{\mu t+\sigma^2t^2/2}\Big|_0$
$EX = (\sigma^2 + (\mu + \sigma^2 t)^2)e^{\mu t+\sigma^2t^2/2}\Big|_0$
$EX = \sigma^2 + \mu^2$

σ^2

$$\sigma^2 = EX^2 - (EX)^2$$
$$\sigma^2 = \sigma^2 + \mu^2 - (\mu)^2$$
$$\sigma^2 = \sigma^2$$