

☐ PHYS115 ☒ PHYS121 ☐ PHYS123
☐ PHYS116 ☐ PHYS122 ☐ PHYS124
Lab Cover Letter

Author (You) Tam N.

Signature: Tam N.

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Lab Partner(s) Katherine

Date Performed Feb 7, 2024

Date Submitted Feb 14, 2024

Lab (such as #1: UNC) #2 IP

TA: Phillip

GRADE (to be filled in by your TA) See your TA for detailed feedback.
 An 'x' next to a subcategory means you need to improve this aspect of your work.

Paper Subtotals (points)

() **General (6)**

- _____ Sig. figs.
- _____ Units
- _____ Clarity of Presentation
- _____ Format

() **Abstract (4)**

- _____ Quantity or principle
- _____ How measurement was made
- _____ Numerical Results
- _____ Conclusion

() **Intro & Theory (9)**

- _____ Basic principle
- _____ Main equations to be used
- _____ Apparatus
- _____ What will be plotted
- _____ Fitting parameters related

() **Exp. Procedures (15)**

- _____ Description
- _____ Stating and justifying uncertainties
- _____ Data Record
- _____ Quality of Lab Work

() **Analysis & Error Analysis (20)**

- _____ Discussion
- _____ Equations & Calculations
- _____ Presentation inc. Graphs, Tables
- _____ Results Reported & Reasonable
- _____ Underlined items addressed

() **Discussion & Conclusions (6)**

- _____ Numerical comparison of results
- _____ Logical conclusions
- _____ Discussion of pos. errors
- _____ Suggestions to reduce errors

() **Paper Total (60 points)**

(30 points for CME or EPF)

() **Notebook (10 points)**

- _____ Format (*proper style, following directions*)
- _____ Apparatus (*brief description of equipment, including sketches*)
- _____ Data (*including computer file names and manually recorded data*)
- _____ Experimental Technique (*describing your procedures; stating & justifying uncerts.*)
- _____ Analysis (*results and errors*)

() **Worksheet(s)/Fill-in-the-Blank-Report (30 points) if applicable**

- () **Adjustments** – late submissions, improper procedures, etc. – or bonus points for exceptional work.

() **Total Grade**

Graded by _____ (TA's initial)

Inclined Plane

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Abstract:

I have tested the theory of Newton's Second Law of Motion with a system of a cart on an inclined plane connected to a counterweight by a string over a pulley. After releasing the system from rest, I measured the velocity as a function of time. According to Newton's theory, the velocity should vary *linearly* with time. The data that I have collected does not support a linear dependence between velocity and time to within the uncertainties on the data points. I have also measured the average acceleration of the system as $a_{meas} = \underline{0.290} \pm \underline{0.008} \text{ m/s}^2$. Newton's Second Law predicts that the acceleration of the system should be $a_{pred} = \underline{0.382} \pm \underline{0.003} \text{ m/s}^2$. I find that the measured acceleration is not consistent with the predicted acceleration. Because of a few factors, namely being friction of the string rubbing against the table and the track not being completely frictionless, the pulley and string not being massless.

(If your measured velocity supports a linear model and/or your accelerations are consistent with the predicted acceleration, cross out the "not's" in the above abstract paragraph. Give a one-sentence conclusion about the lab.)

Theory and Background:

One can determine the acceleration of the system depicted in Figure 1 by using Newton's Second Law to analyze the motion. Assuming that the frictional force \vec{f} is negligible and that the pulley is massless and frictionless, the acceleration a of the system is¹

$$g \frac{(m_1 - m_2 \sin \theta)}{m_1 + m_2} \quad (1)$$

where m_1 is the mass of block 1 in the diagram
and m_2 is the mass of block 2 in the diagram
and θ is the angle of the slope

(Write down the appropriate equation; define all variables that haven't been defined yet.)

One can find the sine of the angle of the incline
using Eq. 1. If we adjust m_1 and m_2 so that the

acceleration is zero and call this hanging mass the balancing mass m_b , then

$$0 = \frac{g(m_1 - m_2 \sin \theta)}{m_1 + m_2} \quad (2)$$

$$\Rightarrow 0 = m_b - m_2 \sin \theta \quad (3)$$

$$\Rightarrow m_b = m_2 \sin \theta \quad (4)$$

(Eq. 2 should be Eq. 1 with $a = 0$; Eq. 3 should be an intermediate algebra step; Eq. 4 should be $\sin \theta$ in terms of m_b and m_2 .)

Equation 1 also implies that the acceleration of the system will be constant, so the velocity
as a function of time will be

$$v(t) = v_0 + at \quad (5)$$

where v_0 is the initial velocity

(For equation 5 write down the expression for velocity in terms of time and other variables.
Explain any new variables you introduce.) From Eq. 5, we can see that if we fit a straight line to a
plot of v vs. t , the slope of the line will be the acceleration and the intercept will be the velocity at
time zero.

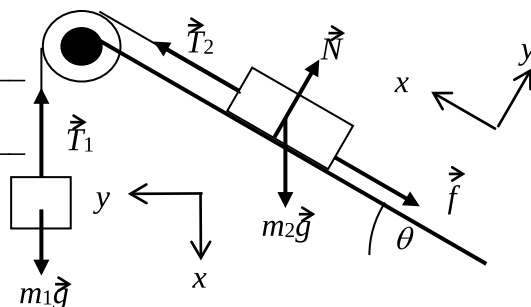


Figure 1: Schematic of Forces in Experiment. Courtesy Driscoll, (year).

Procedure:

To get an estimate of the angle of the incline I estimated the length L and height H of the incline as in Figure 2. I used a meter stick to measure H = 30.45 cm and L = 100.00 cm. Getting accurate and precise

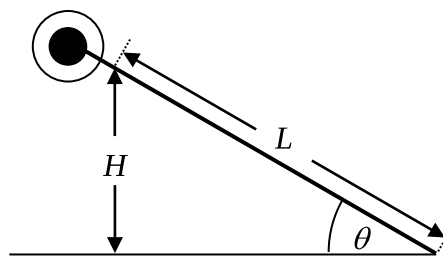


Figure 2: Estimating the angle.
Courtesy Driscoll, (year).

measurements of H and L was difficult because The track was slanted and it was difficult to line it up
I compensated for these difficulties by Asking my partner to step back and let me know if it was straight

Because of these issues, I estimate that my uncertainty in H is $\delta_H =$ 0.01 cm and my uncertainty in L is $\delta_L =$ 0.01 cm.

My first estimate of $\sin\theta$ is then

$$\sin\theta = \frac{H}{L} \quad (6)$$

$$\Rightarrow \sin\theta = \frac{30.45 \text{ cm}}{100.00 \text{ cm}} = \underline{0.3045}$$

(Put the appropriate variables in the first line; put your actual measurements and final value for $\sin\theta$ in the second line.)

The uncertainty in $\sin\theta$ is

$$\delta_{\sin\theta} = \sqrt{\delta_{\sin\theta,H}^2 + \delta_{\sin\theta,L}^2} \quad (7)$$

where $\delta_{\sin\theta,H}$ is the uncertainty in $\sin\theta$ due to δ_H and $\delta_{\sin\theta,L}$ is the uncertainty in $\sin\theta$ due to δ_L .

Using the “computational method” to determine $\delta_{\sin\theta,H}$ and $\delta_{\sin\theta,L}$, I obtain

$$\delta_{\sin\theta,H} = \delta_H \frac{1}{L} \quad (8)$$

and

$$\delta_{\sin \theta, L} = \delta_L \frac{-H}{L^2} \quad (9)$$

yielding $\delta_{\sin \theta} = \sqrt{\left(\delta_{H \frac{1}{L}}\right)^2 + \left(\delta_L \frac{-H}{L^2}\right)^2} = 0.0001$, or $\sin \theta = 0.3045 \pm 0.0001$. Since $\theta =$

$\sin^{-1}(\sin \theta)$, the uncertainty in θ is

$$\delta_{\theta} = \frac{\delta_{\sin \theta}}{\sqrt{1 - \sin^2 \theta}}$$

or $\theta = 0.3094 \pm 0.0001$ rad.

I measured m_2 with an electronic balance and determined that $m_2 = 489. \pm 1. \text{ g}$.

I estimated the uncertainty δ_{m_2} as 1. g because That was the adjusted std. dev. & also the resolution of the scale.

I now used the first estimate of θ to determine an estimate of the mass m_b required to balance the system by taking Eq. 4 and solving for m_b :

$$m_b = \sin \theta m_2 \quad (11)$$

$$\Rightarrow m_b = 149.9$$

(Solve Eq. 4 for m_b , substitute in the appropriate numbers, and solve.)

I then set the mass of m_1 to 149 g by adding masses to the hanger. After releasing the cart, the system was not in balance; the ~~system~~ cart moved down.

(If the system was balanced, cross out "not." Describe the system's motion in the blank.)

I then found the minimum and maximum masses that lead to zero acceleration (m_{\min} and m_{\max}) by adding and removing mass from m_1 in order to obtain a better estimate of the angle of the incline and to account for the small amount of friction in the system. _____

Started with a stationary cart.

(State if you tested zero acceleration by a stationary cart or cart moving with constant velocity. If you used the constant velocity test, also state how you determined the cart was moving at constant speed. Write a sentence about your reasons for choosing your methods, i.e., the advantages and disadvantages of your methods over other choices.)

Using the procedure above, I determined that $m_{\min} = \underline{150.6 \pm 0.1 \text{ g}}$ and $m_{\max} = \underline{158.1 \pm 0.1 \text{ g}}$, each ~~with negligible uncertainty~~. I then set the average of m_{\min} and m_{\max} to be the “balancing mass” m_b and half the difference between m_{\min} and m_{\max} as the uncertainty δ_{mb} , so $m_b = \underline{154} \pm \underline{4} \text{ g}$.

Substituting m_b into Eq. 4, we see that sine of the angle of the incline is

$$\sin \theta = \frac{m_b}{m_2} = \underline{0.315}.$$

The uncertainty in $\sin \theta$ is

$$\delta_{\sin \theta} = \sqrt{\delta_{\sin \theta, m_b}^2 + \delta_{\sin \theta, m_2}^2} \quad (12)$$

where $\delta_{\sin \theta, m_b}$ is the uncertainty in $\sin \theta$ due to δ_{mb} and $\delta_{\sin \theta, m_2}$ is the uncertainty in $\sin \theta$ due to δ_{m2} .

Using the “computational method” to determine $\delta_{\sin \theta, m_b}$ and $\delta_{\sin \theta, m_2}$, I obtain

$$\delta_{\sin \theta, m_b} = \delta_{m_b} \frac{1}{m_2} \quad (13)$$

and

$$\delta_{\sin \theta, m_2} = \delta_{m_2} \frac{m_1}{m_1^2} \quad (14)$$

yielding $\delta_{\sin \theta} = \sqrt{\left(\delta_{\frac{1}{m_2}}\right)^2 + \left(\delta_{\frac{m_1}{m_2^2}}\right)^2} = 0.008$, or $\sin \theta = 0.315 \pm 0.008$.

which is moderately higher than the previous calculation.

(Compare this value of $\sin \theta$ with the value from direct measurement of L and H .)

I will adopt this value for $\sin \theta$.

I then set the counterweight m_1 to a value of 180 g to allow the cart to accelerate up the plane. I will refer to this value of m_1 as the experiment value m_e . I recorded the motion of the cart using an encoded pulley and *Logger Pro* software.² I subsequently exported the data from *Logger Pro* to *Origin* for a more complete analysis. Specifically, I plotted velocity vs. time to determine if the velocity has a linear dependence and to measure the slope. Previous experimenters³ using this equipment have determined that the uncertainty in v has a value of 0.008 m/s; we adopted this value in our analysis.

Results:

The average acceleration recorded directly by *Logger Pro* statistics software was $a_{\text{meas1}} = 0.382 \pm 0.003$ m/s². Figure 3 shows a plot of velocity vs. time and a best linear fit using the *Origin* software. (Attach a copy of your v vs. t graph labeled “Figure 3” to the end of the report.) For this plot, vertical error bars are assigned based on an estimated uncertainty of the velocity measurements of ± 0.003 m/s² for each point, where this value was determined by previous

measurements done by the laboratory staff. As can be seen in the plot, the since not every data point lies within about one error bar of the best linear fit we conclude that these data are not consistent with Newton's model. *(If the data points fit the line to within about one error bar then delete the words "not" above. If the data are not consistent be sure to address this in your conclusion. Is Newton wrong? Or might there be systematic error in your data?)*

The slope of the graph as determined by Origin's fitting software is $a_{\text{meas2}} = \underline{0.388} \pm \underline{0.003} \text{ m/s}^2$. In comparing this value to the value obtained directly from Logger Pro I note that these two values agree (agree/do not agree to within their uncertainties/are exactly the same). We expect that a_{meas2} should be more accurate because calculated slope over time instead of an average inst. and I will adopt it as the measured value, a_{meas} .

By substituting in known values into Eq. 1, one can determine a theoretical value for the acceleration of the system, a_{pred} . Since I did not measure θ directly, I will substitute Eq. 4 into Eq. 1 to obtain:

$$a_{\text{pred}} = g \frac{m_1 - m_2}{m_1 + m_2} \quad (15)$$

$$\Rightarrow a_{\text{pred}} = \underline{0.381} \text{ m/s}^2.$$

Error Analysis:

To find the uncertainty in a_{pred} , $\delta_{a_{\text{pred}}}$, I must find the contribution to $\delta_{a_{\text{pred}}}$ for each of the quantities in Eq. 15 and add them in quadrature. The uncertainties in m_1 & m_2 are negligible compared to the other quantities because they were directly measured.

(Identify any quantities that you will treat as having negligible uncertainty and justify your treatment. Then show your work in estimating the uncertainty in a_{pred} on the next page.)

m_1 & m_2 both have errors around 0.1 while m_b has an error of 4.

$$\delta a \approx \delta_{a, m_b} = \delta_{m_b} \frac{-g}{m_1 + m_2} = 0.06 \text{ m/s}^2$$

So $a_{pred} = 0.38 \pm 0.06 \text{ m/s}^2$.

Conclusions:

The predicted value for the acceleration was $a_{pred} = 0.38 \pm 0.06 \text{ m/s}^2$ and the measured value for the acceleration of the system was $a_{meas} = 0.388 \pm 0.003 \text{ m/s}^2$.

They agree quite closely.

(State whether or not your values agree within their uncertainties. If they do not agree, suggest at least one source of systematic error that were not adequately accounted for and suggest a way to reduce the effect of this error. If the two values do agree, suggest at least one source of random error and suggest a way to reduce the effect of this error. Make a quantitative statement about the effect friction should have had on this experiment. Make a conclusion about Newton's Second Law, especially regarding whether or not the data points support a linear model. If the model does not show a linear dependence give at least one reason why this might not be.)

Acknowledgements:

I would like to thank Katherine, Case Department of Physics, for _____
help in obtaining the experimental data and preparing the figures. _____

& Mr Phillip

(Thank your lab partner(s). If they or anyone else gave you additional assistance, say who they were and specifically what their assistance was.)

References:

(If you have any additional references, list them below. Make sure to indicate with an endnote where in the report you referred to the reference.)

1. Driscoll, D., *General Physics I: Mechanics Lab Manual*, "Inclined Plane," CWRU Bookstore, 2014.
- 2.

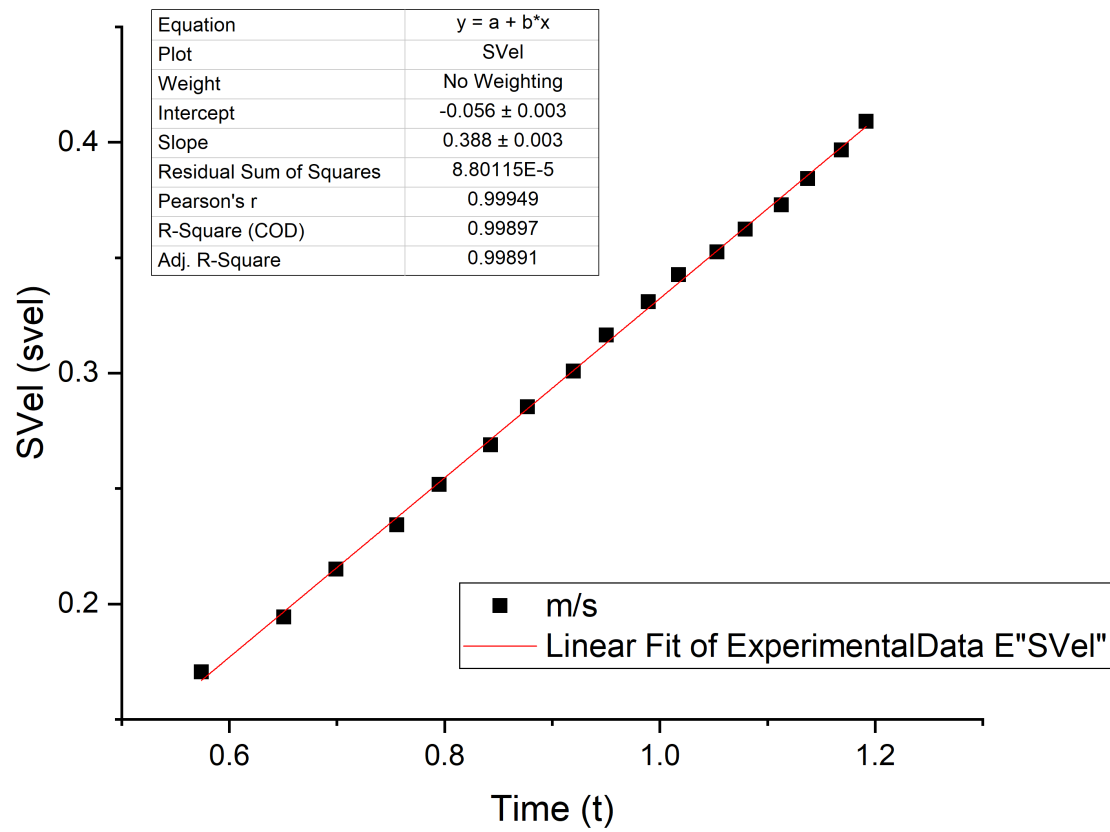
End Notes:

¹ Driscoll, D., p. 2.

² Driscoll, D., p. 3, describes the encoded pulley.

³ Driscoll, D., p. 5

Trevor and Katherine Feb 7, 2024



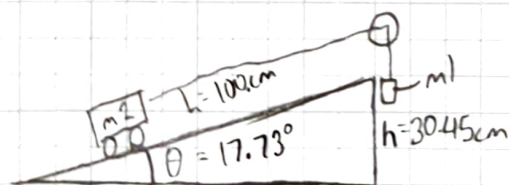
Trial	height (cm)	m_2 (g)	m_1 (g)	m_{max} (g)	m_{min} (g)
1	30.45	489.	180.	159.	150.
2		489.	180.	158.	150.
3		491.	180.	158.	151.
4		489.	180.	158.	150.
5		476.	180.	158.	151.
6		490.	180.	158.	151.
7		490.	180.	158.	151.
8		489.	180.	158.	151.
9		491.	180.	158.	150.
10		491.	180.	158.	151.
Mean	30.45	489.	180.	158.1	150.6
SD	-	4.48	0.		
SE	0.01	1	0.	0.1	0.2

Balanced: $\sin\theta m_2 = m_1$



$148.75 \approx 149g$

a m_1 of 149g should balance the cart at an incline



$\theta = \arcsin(\frac{h}{L})$



$\theta = 17.7281^\circ$

$h = 100.00 \pm 0.01 \text{ cm}$

$L = 30.45 \pm 0.01 \text{ cm}$

$\delta\theta = \sqrt{\left(\frac{1}{L(1-\frac{h^2}{L^2})}\right)^2 + \left(\frac{-0.01}{h^2(1-\frac{h^2}{L^2})}\right)^2} = 0.0001$

$m_b = \frac{m_{max} + m_{min}}{2} = 154.35 \approx 154$

$\delta m_b = \frac{m_{max} - m_{min}}{2} = 3.75 \approx 4$



$m_b = 154 \pm 4g$

$\delta \sin\theta = \frac{\partial \sin\theta}{\partial \theta} \cdot \delta\theta = \cos\theta \delta\theta$



0.0001

$\theta = 17.7281 \pm 0.0001^\circ$

$\sin\theta = \frac{0.3045 \pm 0.0001}{1}$

$$a = \frac{g(m_1 - m_2 \sin \theta)}{m_1 + m_2} = \boxed{0.382 \text{ m/s}^2}$$

$\frac{m_b}{m_2}$
↑
 $m = m_c = m_b + 26$

$$= \frac{g(m_1 - m_2(\frac{m_b}{m_2}))}{m_1 + m_2} = \frac{g(26)}{m_1 + m_2} = 0.3815 \text{ m/s}^2 = \frac{26g}{m_1 + m_2 + 26}$$

$$\delta a = \sqrt{\left(\frac{g(26\delta m_1)}{(m_b + m_2 + 26)^2}\right)^2 + \left(\frac{g(26\delta m_2)}{(m_b + m_2 + 26)^2}\right)^2} = 0.0032 \text{ m/s}^2$$

⇓

$$\boxed{a = 0.382 \pm 0.003 \text{ m/s}^2}$$

Experimental accel.

From a : 0.2895

$$\delta a = \frac{\sigma}{\sqrt{N}} = 0.0079$$

$$a = 0.290 \pm 0.008 \text{ m/s}^2$$

From v :

$$a = 0.287 \pm 0.002 \text{ m/s}^2$$

↗ Values agree.

~~From \dots :~~

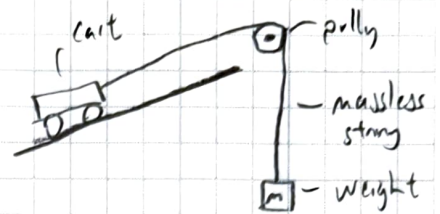
Origin:

$$\text{Slope: } 0.388 \pm 0.003 \text{ m/s}^2 = a_{\text{exp}}$$

$$a_{\text{pred}} = 0.382 \pm 0.003 \text{ m/s}^2$$

Our data and predicted values line up and their error ranges touch.

Procedure



- Measure mass of cart, calc mean & uncertainty
- predict value of weight given mass of cart & slope that will put the system in equilibrium
- ~~add~~ add weight to the weight until the cart does not move, then remove weight until right before it begins moving. Record this weight as m_{min}
- repeat the previous step, but with adding instead of removing weight. Record as m_{max}
- ~~calculate expected accelerations from~~
- average m_{min} & m_{max} to roughly find the equilibrium weight.
- approximate the acceleration with Newton's laws if you added ~~roughly~~ 25g of weight.
- add 25g of weight and measure the acceleration
- compare accelerations.