2.1

2

Represent the common form of each argument using letters to stand for component sentences, and fill in the blanks so that the argument in part (b) has the same logical form as the argument in part (a).

a

If all computer programs contain errors, then this program contains an error. This program does not contain an error. Therefore, it is not the case that all computer programs contain errors.

Answer

Let A be the set of all computer programs,
$E(x)$ be the argument "x contains an error":

$\forall a \in A : E(a) \implies a : E(a)$
$\exists a \in A : \neg E(a)$
$\therefore \neg(\forall a \in A : E(a))$

b

If ____ then ____. 2 is not odd. Therefore, it is not the case that all prime numbers are odd.

Answer

If all prime numbers are odd then 2 is odd. 2 is not odd. Therefore, it is not the case that all prime numbers are odd.

5

Indicate which of the following sentences are statements.

b

She is a mathematics major.

Answer

True

c

$128 = 2^6$

✓ **Answer**

True

8

Write the statements in symbolic form using the symbols $\sim, \wedge, \vee$ and the indicated letters to represent component statements.

Let h =“John is healthy,” w =“John is wealthy,” and s=“John is wise.”

b

John is not wealthy but he is healthy and wise.

✓ **Answer**

$\neg w \wedge h \wedge s$

e

John is wealthy, but he is not both healthy and wise.

✓ **Answer**

$w \wedge \neg(h \wedge s)$

10

Let p be the statement “DATAENDFLAG is off,” q the statement “ERROR equals 0,” and r the statement “SUM is less than 1,000.” Express the following sentences in symbolic notation.

b

DATAENDFLAG is off but ERROR is not equal to 0.

✓ **Answer**

$p \wedge \neg q$

e

Either DATAENDFLAG is on or it is the case that both ERROR equals 0 and SUM is less than 1,000.

✓ **Answer**

$(\neg p \vee (q \wedge r)) \wedge \neg(\neg p \wedge (q \wedge r))$
Since the statement is in the form of either ... or. Assuming not both

15

Write truth tables for the statement $p \wedge (\sim q \vee r)$

✓ **Answer**

p	q	r	$p \wedge (\neg q \vee r)$
F	F	F	F
F	F	T	F
F	T	F	F
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

19-24

Determine whether the statements are logically equivalent. Construct a truth table and include a sentence justifying your answer. Your sentence should show that you understand the meaning of logical equivalence.

19

$p \wedge \mathbf{t}$ and p

✓ **Answer**

Yes, they are logically equivalent by definition of tautology

p	p	$p \wedge \mathbf{t}$
F	F	F

p	p	$p \wedge \mathbf{t}$
T	T	T

Yes, by the definition of a tautology, these are equivalent, which is supported by the table

20

$p \wedge \mathbf{c}$ and $p \vee \mathbf{c}$

✓ **Answer**

No, they are not logically equivalent

p	$p \wedge \mathbf{c}$	$p \vee \mathbf{c}$
T	F	T
F	F	F

The definition of a contradiction states that the second statement will be equivalent to just p while the definition of and will make the first statement also a contraction. Which are not equivalent to each other

22

$p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$

✓ **Answer**

Yes, this is equivalent according to the distributive law.

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
F	T	T	F	F
T	F	F	F	F
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

Yes, these are the same by the distributive law, and can be seen in the table as the two output columns are the same.

24

$(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$

✓ **Answer**

No, these are not equivalent.

p	q	r	$(p \vee q) \vee (p \wedge r)$	$(p \vee q) \wedge r$
F	F	F	F	F
F	F	T	F	F
F	T	F	T	F
F	T	T	T	T
T	F	F	T	F
T	F	T	T	T
T	T	F	T	F
T	T	T	T	T

The first statement can be reorganized to

$q \vee (p \vee (p \wedge r))$
$q \vee p$ (Absorbtion)
Let $s = q \vee p$
$s \not\equiv s \wedge r$

29

Use De Morgan's laws to write negations for the statement
"This computer program has a logical error in the first ten lines or it is being run with an incomplete data set."

✓ **Answer**

This computer program doesn't have a logical error in the first ten lines and it isn't being run with an incomplete data set.

35

Assume x is a particular real number and use De Morgan's laws to write negations for the statement
$x \leq -1$ or $x > 1$

✓ Answer

$x > -1$ and $x \leq 1$

43

Use a truth table to establish if the statement form is a tautology or if it is a contradiction.

$(\sim p \vee q) \vee (p \wedge \sim q)$

✓ Answer

p	q	$(\sim p \vee q) \vee (p \wedge \sim q)$
F	F	T
F	T	T
T	F	T
T	T	T

This is a tautology as it is always true.

49

A logical equivalence is derived from Theorem 2.1.1. Supply a reason for each step.

$(p \vee \sim q) \wedge (\sim p \vee \sim q)$

$\equiv (\sim q \vee p) \wedge (\sim q \vee \sim p)$

$\equiv \sim q \vee (p \wedge \sim p)$

$\equiv \sim q \vee \mathbf{c}$

$\equiv \sim q$

Therefore, $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q$

✓ Answer

$(p \vee \sim q) \wedge (\sim p \vee \sim q)$

$\equiv (\sim q \vee p) \wedge (\sim q \vee \sim p)$ Communicative

$\equiv \sim q \vee (p \wedge \sim p)$ DIstributive

$\equiv \sim q \vee \mathbf{c}$ Negation

$\equiv \sim q$ Identity

Therefore, $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q$

51

Use Theorem 2.1.1 to verify the logical equivalence. Supply a reason for each step.

$p \wedge (\sim q \vee p) \equiv p$

✓ **Answer**

$p \wedge (\sim q \vee p)$
$p \wedge (p \vee \neg q)$ Associative
p Absorbtion

$\therefore p \wedge (\sim q \vee p) \equiv p$

2.2

2

Rewrite the statement in if-then form

I am on time for work if I catch the 8:05 bus

✓ **Answer**

If I catch the 8:05 bus, I will be on time for work.

8

Construct truth tables for the statement form

$\sim p \vee q \rightarrow r$

✓ **Answer**

p	q	r	$\sim p \vee q \rightarrow r$
F	F	F	F
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

14

b

$p \to q \vee r \equiv p \wedge \sim q \to r \equiv p \wedge \sim r \to q$

Use the logical equivalences above to rewrite the following sentence in two different ways. (Assume that n represents a fixed integer.)

If n is prime, then n is odd or n is 2.

✓ **Answer**

Let:
$p =$"n is prime"
$q =$"n is odd"
$r =$"n is 2"

The original statement is in the form of $p \to q \vee r$

$p \wedge \neg q \to r$
If n is prime and n isn't odd, then n is 2

$p \wedge \neg r \to q$
If n is prime and n isn't 2, then n is odd

16

Write the statement in symbolic form and determine whether it is logically equivalent. Include a truth table and a few words of explanation to show that you understand what it means for statements to be logically equivalent.

If you paid full price, you didn’t buy it at Crown Books. You didn’t buy it at Crown Books or you paid full price.

✓ **Answer**

Let:
$p =$"Paid full price"
$q =$"Buy it at Crown Books"
$p \to \neg q$
$\therefore \neg q \vee p$

False, as $p \to \neg q \equiv \neg p \vee \neg q \not\equiv p \to \neg q$

20

Write negations for each of the following statements. (Assume that all variables represent fixed quantities or entities, as appropriate.)

b

If today is New Year’s Eve, then tomorrow is January.

✓ Answer

Let:
$p =$"today is New Year's Eve"
$q =$"tomorrow is January"
The statement is of the form $p \to q$

The negation will be of the form $p \land \neg q$
Today is New Year's Eve and tomorrow isn't January.

e

If x is non-negative, then x is positive or x is 0.

✓ Answer

Let:
$p =$"x is non-negative"
$q =$"x is positive"
$r =$"x is 0"
The statement is of the form $p \to q \lor r$

The negation will be of the form $p \land \neg q \land \neg r$
x is non-negative, x isn't positive, and x isn't 0.

22

Write contrapositives for the statements

b

If today is New Year’s Eve, then tomorrow is January.

✓ Answer

Let:
$p =$"today is New Year's Eve"
$q =$"tomorrow is January"
The statement is of the form $p \to q$

The contrapositive will be of the form $\neg q \to \neg p$
If tomorrow isn't January, then today isn't New Year's Eve

e

If x is non-negative, then x is positive or x is 0.

✓ **Answer**

Let:
$p =$"x is non-negative"
$q =$"x is positive"
$r =$"x is 0"
The statement is of the form $p \to q \vee r$

The contrapositive will be of the form $\neg q \wedge \neg r \to \neg p$
If x isn't positive and x isn't 0, then x isn't non-negative

23

Write the converse and inverse for each statement

b

If today is New Year's Eve, then tomorrow is January.

✓ **Answer**

Let:
$p =$"today is New Year's Eve"
$q =$"tomorrow is January"
The statement is of the form $p \to q$

The converse will be of the form $q \to p$
If tomorrow is January, then today is New Year's Eve

The inverse will be of the form $\neg p \to \neg q$
If tomorrow isn't January, then today isn't New Year's Eve

e

If x is non-negative, then x is positive or x is 0.

✓ **Answer**

Let:
$p =$"x is non-negative"
$q =$"x is positive"
$r =$"x is 0"
The statement is of the form $p \to q \vee r$

The converse will be of the form $q \vee r \to p$
If x is positive or x is 0, then x is non-negative

The inverse will be of the form $\neg p \to \neg q \wedge \neg r$
If x isn't non-negative, then x isn't positive and x isn't 0

27

Use truth tables to establish the truth of this statement

The converse and inverse of a conditional statement are logically equivalent to each other.

✓ **Answer**

p	q	$q \to p$	$\neg p \to \neg q$
F	F	T	T
F	T	F	F
T	F	T	T
T	T	T	T

The columns for the inverse and the converse in the table above are exactly the same, meaning they are logically equivalent.

30

If statement forms P and Q are logically equivalent, then $P \leftrightarrow Q$ is a tautology. Conversely, if $P \leftrightarrow Q$ is a tautology, then P and Q are logically equivalent. Use $\leftrightarrow$ to convert the logical equivalence to a tautology. Then use a truth table to verify each tautology.

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

✓ **Answer**

$p \wedge (q \vee r) \leftrightarrow (p \wedge q) \vee (p \wedge r)$
This is true by the distributive property

p	q	r	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
F	T	T	F	F
T	F	F	F	F
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

The truth tables also show that they are logically equivalent

41

Rewrite the statement in if-then form

Having two 45° angles is a sufficient condition for this triangle to be a right triangle.

✓ **Answer**

If a triangle has two 45° angles, then it is a right triangle

46

“If compound X is boiling, then its temperature must be at least 150°C.” Assuming that this statement is true, which of the following must also be true?

c

Compound X will boil only if its temperature is at least 150°C.

✓ **Answer**

True, its the same statement

d

If compound X is not boiling, then its temperature is less than 150°C.

✓ **Answer**

False, the inversion isn't always true

2.3

4

Use modus ponens or modus tollens to fill in the blanks in the argument so as to produce valid inferences.

If this graph can be colored with three colors, then it can colored with four colors. This graph cannot be colored with four colors.

✓ **Answer**

If this graph can be colored with three colors, then it can colored with four colors.
This graph cannot be colored with four colors.
Therefore, this graph cannot be colored with three colors.

9

Use truth tables to determine whether the argument form is valid. Indicate which columns represent the premises and which represent the conclusion, and include a sentence explaining how the truth table supports your answer. Your explanation should show that you understand what it means for a form of argument to be valid or invalid.

$p \wedge q \to \sim r$
$p \vee \sim q$
$\sim q \to p$
$\therefore \sim r$

✓ **Answer**

p **input**	q **Input**	r **Input**	$p \wedge q \to \neg r$ **Premise**	$p \vee \neg q$ **Premise**	$\neg q \to p$ **Premise**	$\neg r$ **Conclusion**
F	F	F	T	T	F	T
F	F	T	T	T	F	F
F	T	F	T	F	T	T
F	T	T	T	F	T	F
T	**F**	**F**	**T**	**T**	**T**	**T**
T	*F*	*T*	*T*	*T*	*T*	*F*

p input	q Input	r Input	$p \wedge q \to \neg r$ Premise	$p \vee \neg q$ Premise	$\neg q \to p$ Premise	$\neg r$ Conclusion
T	**T**	**F**	**T**	**T**	**T**	**T**
T	T	T	F	T	T	F

As you can see in the table, I have italicized or bolded the rows where all the premises are true. The first and third line both have true conditions, but the middle one is false, which means that given the necessary premises, we cannot guarantee the conclusion. Therefore this argument is false.

23

Use symbols to write the logical form of each argument, and then use a truth table to test the argument for validity. Indicate which columns represent the premises and which represent the conclusion, and include a few words of explanation showing that you understand the meaning of validity.

Oleg is a math major or Oleg is an economics major.
If Oleg is a math major, then Oleg is required to take Math 362.
∴ Oleg is an economics major or Oleg is not required to take Math 362.

✓ **Answer**

Let:
$p =$"Oleg is a math major"
$q =$"Oleg is an economics major"
$r =$"Oleg is required to take Math 362"
$p \vee q$
$p \to r$
$\therefore q \vee \neg r$

p **Input**	q **Input**	r **Input**	$p \vee q$ **Premise**	$p \to r$ **Premise**	$q \vee \neg r$ **Conclusion**
F	F	F	F	T	T
F	F	T	F	T	F
F	**T**	**F**	**T**	**T**	**T**
F	**T**	**T**	**T**	**T**	**T**
T	F	F	T	F	T
T	**F**	**T**	**T**	**T**	**F**
T	T	F	T	F	T
T	**T**	**T**	**T**	**T**	**T**

As you can see in the table, when all the premises are true (bolded in yellow), the conclusion is not always true in the second last row which is highlighted, the conclusion is false.

29-32

Some of the arguments in 29–32 are valid, whereas others exhibit the converse or the inverse error. Use symbols to write the logical form of each argument. If the argument is valid, identify the rule of inference that guarantees its validity. Otherwise, state whether the converse or the inverse error is made.

29

If at least one of these two numbers is divisible by 6, then the product of these two numbers is divisible by 6.
Neither of these two numbers is divisible by 6.
∴ The product of these two numbers is not divisible by 6.

✓ **Answer**

Let:
$p =$"At least one of these two numbers is divisible by 6"
$q =$"The product of these two numbers is divisible by 6"
$p \to q$
$\neg p$
$\therefore \neg q$

False, as this is an inverse error.
$p \to q$
$\equiv \neg p \vee q$
Given $\neg p$
$\equiv \mathbf{t} \vee q$
$\equiv \mathbf{t} \not\equiv \neg q$

We cannot make a conclusion from the given Premises, much less one that concludes $\neg q$. By making the conclusion $\neg q$ we are assuming the inversion $\neg p \to \neg q$

32

If I get a Christmas bonus, I'll buy a stereo.
If I sell my motorcycle, I'll buy a stereo.
∴ If I get a Christmas bonus or I sell my motorcycle, then I'll buy a stereo.

✓ **Answer**

Let
$p =$"I get a Christmas bonus"
$q =$"I sell my motorcycle"
$r =$"I'll buy a stereo"
$p \to r$
$q \to r$
$\therefore (p \lor q) \to r$

True.
$p \to r$
$q \to r$
Using division of cases,
$p \lor q \to r$

40

Sharky, a leader of the underworld, was killed by one of his own band of four henchmen. Detective Sharp interviewed the men and determined that all were lying except for one. He deduced who killed Sharky on the basis of the following statements:

1. Socko: Lefty killed Sharky.
2. Fats: Muscles didn’t kill Sharky.
3. Lefty: Muscles was shooting craps with Socko when Sharky was knocked off.
4. Muscles: Lefty didn’t kill Sharky.

Who did kill Sharky?

✓ **Answer**

Muscles must've killed Sharky as all the statements would be false except Muscle's own statement that Lefty didn't kill Sharky

44

A set of premises and a conclusion are given. Use the valid argument forms listed in Table 2.3.1 to deduce the conclusion from the premises, giving a reason for each step as in Example 2.3.8. Assume all variables are statement variables.

1. $p \to q$
2. $r \lor s$
3. $\sim s \to \sim t$
4. $\sim q \lor s$

5. $\sim s$
6. $\sim q \wedge r \rightarrow u$
7. $w \vee t$
8. $\therefore u \wedge w$

Answer

1. $p \rightarrow q$
2. $r \vee s$
3. $\sim s \rightarrow \sim t$
4. $\sim q \vee s$
5. $\sim s$
6. $\sim p \wedge r \rightarrow u$
7. $w \vee t$

$q \rightarrow s$ Conditional equivalence (4)
$\neg s \rightarrow \neg q$ Contrapositive
$\neg q$ (5)

$\neg q \rightarrow \neg p$ Contrapositive (1)
$\neg p$

$\neg s \rightarrow r$ Conditional equivalence (2)
r (5)
u (6)

$\neg t \rightarrow w$ Conditional equivalence (7)
$\neg t$ (5)(3)
w

$\therefore u \wedge w$ Conjunction