PHYS 122 HW 3

2

Dielectric breakdown of a gas.

a

An electron starts from rest and moves a distance l under the influence of a uniform electric field of magnitude E. Assume that the displacement is in the direction of the electric field. What is the final kinetic energy of the electron?

Give your answer in terms of q = charge of the electron, m = mass of the electron, E and l.

✓ Answer

$K_0 = 0$
$W = qEl$
$K_f = qEl$ By Work-Energy Thm.
$\square$

b

Suppose the final kinetic energy of the electron is $10\ eV$ and $l = 1.0 \times 10^{-6}\ m$. Determine E in V/m. Possibly useful data: $q = -1.6 \times 10^{-19}\ C$, $m = 9.1 \times 10^{-31}\ kg$.

✓ Answer

$10\ eV = -E1.6 \times 10^{-25}\ Cm$
$E = -\frac{10\ eV}{1.6\times10^{-25}\ Cm}$
$E = -\frac{1.6\times10^{-18}\ V}{1.6\times10^{-25}\ m}$
$E = -10^7\ \frac{V}{m}$
$\square$

c*

Air is primarily composed of neutral N_2 and O_2 molecules and is an insulator. There are always some free electrons in air (created if nothing else by radioactivity and cosmic rays)but their concentration is too low to make air significantly conducting. Free electrons move a typical distance l between collisions with air molecules; l is called the mean free path of the electrons. If an intense electric field is present (e.g. due to a thundercloud; see problem 8 of homework 2) the electron will be accelerated between collisions. If electrons are accelerated sufficiently, their collision will knock additional electrons out of the molecules as well as excite the molecules, causing them to subsequently relax by giving off light. If each collision yields several electrons, a chain reaction is established whereby each newly liberated electron undergoes further collisions liberating additional electrons. Under these conditions the air undergoes "dielectric breakdown": it gives off light and becomes conducting. Lightning bolts are conducting channels created by dielectric breakdown of air by the intense electric fields of thunderclouds. In fluorescent lights a low density gas (with a correspondingly long mean free path l) undergoes dielectric breakdown under comparatively weaker electric fields.

3

Electrostatic energy of complex ions. A complex ion consists of a central cation, M^{4+}, with a charge $4e$ and four "ligands", N^{-}, each with charge $-e$.

a

Planar complex: In a planar complex the four N anions sit at the corners of a square. And the M cation sits at the center. a denotes the distance between the central M cation and the four N anions. Calculate the electrostatic energy of the complex treating the ions as point charges. Give your answer in terms of e^2, $4\pi\epsilon_0$ and a.

Hint: The electrostatic energy is a sum of the energy of each pair of ions. What is the energy of the pair $M - N_A$? Here, N_A denotes the N anion marked A in the figure. What is the energy of the pair $N_A - N_B$? Of $N_A - N_C$? Notice that all other pairs are equivalent to one of these three.

Answer

$$E_a = M - N_A \rightarrow ke^2\left(-\frac{4}{a}\right)$$
$$E_b = N_A - N_B \rightarrow ke^2\left(\frac{1}{a\sqrt{2}}\right)$$
$$E_c = N_A - N_C \rightarrow ke^2\left(\frac{1}{2a}\right)$$

$$E = 4E_a + 4E_b + 2E_c$$
$$E = ke^2\left(\frac{2\sqrt{2}-15}{a}\right)$$
$$E = e^2\frac{2\sqrt{2}-15}{4a\pi\epsilon_0}$$
$\square$

b

Tetrahedral complex: In a tetrahedral complex the four N anions sit at the vertices of a tetrahedron and the M cation sits at the centre of the tetrahedron. Again a denotes the distance between the M cation and each of the N anions. We place the origin of our co-ordinates at M and align the z-axis with N_A where N_A denotes the N anion marked A in the figure. The position vectors of the four N anions are then

$A \rightarrow a\hat{k}$
$B \rightarrow -\frac{1}{3}a\hat{k} + \frac{\sqrt{8}}{3}a\hat{i}$
$C \rightarrow -\frac{1}{3}a\hat{k} - \frac{\sqrt{2}}{3}a\hat{i} + \sqrt{\frac{2}{3}}a\hat{j}$
$D \rightarrow -\frac{1}{3}a\hat{k} - \frac{\sqrt{2}}{3}a\hat{i} - \sqrt{\frac{2}{3}}a\hat{j}$

What is the electrostatic energy of the tetrahedral complex? You may treat the ions as point charges. Give your answer in terms of e^2, $4\pi\epsilon_0$ and a.

Answer

$E_{MN} = M - N \rightarrow ke^2 \left(-\frac{4}{a}\right)$
$E_{NN} = N - N \rightarrow ke^2 \left(\frac{3\sqrt{\frac{2}{3}}}{4a}\right)$

$E = 4E_{MN} + 6E_{NN}$
$E = ke^2 \left(\frac{9\sqrt{\frac{2}{3}}-32}{2a}\right)$
$E = e^2 \frac{9\sqrt{\frac{2}{3}}-32}{8a\pi\epsilon_0}$
$\square$

c

Which configuration has lower energy, the planar geometry or the tetrahedral? If electrostatics was the only consideration, the complex ion would take on the configuration that has lowest electrostatic energy.

Answer

Tetrahedral, as $9\sqrt{\frac{2}{3}} - 32 < 2\sqrt{2} - 15$

4

Electrostatic energy of a salt crystal. The figure shows an arrangement of $Na+$ ions (charge e) and $Cl-$ ions (charge $-e$) along a straight line. The separation between consecutive ions is a. This “one-dimensional crystal” is a cartoon of common salt which is a three dimensional alternating pattern of the same ions.

The total electrostatic energy of the one-dimensional crystal is ∞. However, the energy per ion is finite and the purpose of this problem is to calculate it. The total electrostatic energy of the crystal is the sum of the energy of every pair. To calculate the energy per Na ion focus on the pairs that involve the Na ion located at the origin.

a

Write an expression for the electrostatic energy of all pairs that involve the Na ion at the origin.

Hint: This expression is an infinite series.

✓ **Answer**

$\frac{2ke^2}{a}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$

□

b

Sum the infinite series making use of the identity

$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \dots$

You should find that the total electrostatic energy per Na ion is

$-\frac{e^2 \ln 2}{2a\pi\epsilon_0}$

✓ **Answer**

$-\frac{2ke^2\ln 2}{a}$

$= -\frac{e^2\ln 2}{2a\pi\epsilon_0}$

□