5

2.37

Prove that $x^3 + x + 1$ is irreducible in $\mathbb{F}_2[x]$.

Answer

In order for $x^3 + x + 1$ to be irreducible, it must not be divisible by x or $x + 1$.
Since $x^3 + x + 1 = (x^2 + 1)(x) + 1 = (x^2 + x)(x + 1) + 1$,
and 1 is not divisible by x or $x + 1$, our polynomial is also not divisible.

Thus, our polynomial is irreducible.

2.38

The multiplication table for the field $\mathbb{F}_2[x]/(x^3 + x + 1)$ is given in Table 2.5, but we have omitted fourteen entries. Fill in the missing entries.

Answer

$\times$	0	1	x	x^2	$1+x$	$1+x^2$	$x+$
0	0	0	0	0	0	0	0
1	0	1	x	x^2	$1+x$	$1+x^2$	$x+$
x	0	x	x^2	$1+x$	$x+x^2$	1	$1+$
x^2	0	x^2	$1+x$	$x+x^2$	$1+x+x^2$	x	$1+$
$1+x$	0	$1+x$	$x+x^2$	$1+x+x^2$	$1+x^2$	x^2	1
$1+x^2$	0	$1+x^2$	1	x	x^2	$1+x+x^2$	$1+$
$x+x^2$	0	$x+x^2$	$1+x+x^2$	$1+x^2$	1	$1+x$	x
$1+x+x^2$	0	$1+x+x^2$	$1+x^2$	1	x	$x+x^2$	x^2

(last column may be cut off in the pdf export, the table is diagonally symmetric however)