PHYS 122 HW 2

1 Equilibrium of a Square

Four charges are placed at the vertices of a diagonal square with side of length a and a fifth charge is placed at the center. The corner charges each have the same charge q; the central charge is Q.

a

What is the total electrostatic force on the central charge?

Answer

$\vec{0}$ as it cancels out.

b

What is the total electrostatic force on the charge at the vertex A (top)? Your answer may depend on q, Q, $4\pi\epsilon_0$, a and the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

Answer

$F_{Elr} = \sqrt{2}\frac{1}{4\pi\epsilon_0}\frac{q}{a^2}\hat{\jmath}$ via symmetry, trig, and additivity
$F_{Eb} = \frac{1}{4\pi\epsilon_0}\frac{q}{2a^2}\hat{\jmath}$
$F_{Ec} = \frac{1}{4\pi\epsilon_0}\frac{2Q}{a^2}\hat{\jmath}$

$F_e = \frac{1}{4\pi\epsilon_0}\frac{(\sqrt{2}+0.5)q+2Q}{a^2}\hat{\jmath}$
$\square$

c

Write down the total force on each of the other three corner charges also. [Hint: Use your result of part (b) and symmetry to write down the answer. No calculation is needed].

Answer

Let $F_e = \frac{1}{4\pi\epsilon_0}\frac{(\sqrt{2}+0.5)q+2Q}{a^2}$
$F_{et} = F_e\hat{\jmath}$

$F_{eb} = -F_e\hat{j}$
$F_{el} = -F_e\hat{i}$
$F_{er} = F_e\hat{i}$
$F_{ec} = 0$
□

d

What value should the central charge Q have in order that the forces on all five charges are exactly zero?

✓ **Answer**

$F_e = \frac{1}{4\pi\epsilon_0}\frac{(\sqrt{2}+0.5)q+2Q}{a^2}$
$F_e = 0 \iff (\sqrt{2}+0.5)q + 2Q = 0$
$Q = -\frac{2\sqrt{2}+1}{4}q$
□

2 Field of a semi-circular arc

A semi-circular arc in the $+y$ half of radius R has a charge per unit length λ. We wish to compute the electric field $\vec{E}$ at P, the center of the arc.

a

What is the total charge of the arc?

✓ **Answer**

$l = \pi R$
$c = \lambda l$
$c = \pi R\lambda$

b

In what direction does $\vec{E}$ point? Explain briefly.

✓ **Answer**

It will either point $\hat{j}$ or $-\hat{j}$ depending on the sign of λ as it is horizontally symmetric, meaning there will be no horizontal force at point P

c

Consider the infinitesimal segment of arc located at an angle θ from the x-axis. $d\theta$ is the angle subtended by this segment at P as shown in figure 3.

i

What is the charge of this segment?

✓ **Answer**

The charge will be $R\lambda d\theta$

ii

What is the distance of this segment from P?

✓ **Answer**

R

iii

Write down the displacement vector $\vec{r}$ from the segment to P. Give your answer in terms of R, θ, $\hat{i}$ and $\hat{j}$.

✓ **Answer**

$-R(\hat{i}\cos\theta + \hat{j}\sin\theta)$

iv

Using Coulomb's law write down the electric field $d\vec{E}$ at P that is produced by the infinitesimal arc segment.

✓ **Answer**

$-R(\hat{\imath}\cos\theta+\hat{\jmath}\sin\theta)\frac{1}{4\pi\epsilon_0}\frac{R\lambda d\theta}{R^2}$
$-(\hat{\imath}\cos\theta+\hat{\jmath}\sin\theta)\frac{1}{4\pi\epsilon_0}\lambda d\theta$
$\square$

d

By integration of the result of part (c) determine $\vec{E}_x$, the x-component of the total electric field at P.

✓ **Answer**

$\int_0^\pi -(\hat{\imath}\cos\theta)\frac{1}{4\pi\epsilon_0}\lambda d\theta$
$= 0$
$\square$

e

By integration of the result of part (c) determine $\vec{E}_y$, the y-component of the total electric field at P.

✓ **Answer**

$\int_0^\pi -(\hat{\jmath}\sin\theta)\frac{1}{4\pi\epsilon_0}\lambda d\theta$
$= \frac{-\lambda}{2\pi\epsilon_0}$
$\square$

3 Field of a charged strip

An infinite strip of width $2a$ lies in the yz plane symmetrically about the z-axis. The strip has a uniform surface charge density σ and we are interested in the electric field at the point P, x units on the x_+-axis. To calculate the field we will mentally subdivide the strip into narrow strips of infinitesimal width dy.

a

In what direction do you expect the field at P to point? Briefly explain your reasoning.

✓ **Answer**

Either in the $+\hat{\imath}$ direction or the $-\hat{\imath}$ direction depending on the sign of σ

b

What is the charge per unit length of the infinitely long narrow strip?

✓ **Answer**

σdy

c

What is the electric field at P due to the narrow strip? [Hint: The narrow strip may be regarded as a line of charge. Use the result for the field of a line of charge derived in class].

✓ **Answer**

$$\frac{x\sigma}{4\pi\epsilon_0}\int\limits_{y=-a}^{a}\int\limits_{z=-\infty}^{\infty}\frac{dz\,dy}{(x^2+y^2+z^2)^{3/2}}$$
$$\frac{x\sigma}{2\pi\epsilon_0}\int\limits_{y=-a}^{a}\frac{dy}{x^2+y^2}$$
$$\frac{\sigma}{\pi\epsilon_0}\arctan\left(\frac{a}{x}\right)$$
$\square$

d

Integrate your result of part (c) to obtain the total electric field at point P.

✓ **Answer**

$$\frac{\sigma}{\pi\epsilon_0}\arctan\left(\frac{a}{x}\right)$$
$\square$

e

What electric field do you expect at P in the limit that $a >> x$? Does your answer in part (d) reduce to the expected result? [Hint: tan−1 ξ ≈ π/2 for ξ ≫ 1].

✓ **Answer**

$$\lim_{a \to \infty} \frac{\sigma}{\pi\epsilon_0} \arctan\left(\frac{a}{x}\right)$$
$$= \frac{\sigma}{2\epsilon_0}$$

This makes sense as it would approach an infinite sheet.

f

What electric field do you expect at P in the limit that $a << x$? Does your answer in part (d) reduce to the expected result? [Hint: tan−1 ξ ≈ ξ for ξ ≪ 1].

✓ Answer

$$\lim_{x \to \infty} \frac{\sigma}{\pi\epsilon_0} \arctan\left(\frac{a}{x}\right)$$
$$= 0$$

This makes sense as the sheet would approach relatively 0 width w.r.t. the point.

5 Electric Field Lines

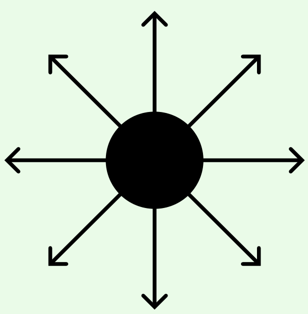
a

Draw the electric field lines assuming positive/negative charge of a charged infinite cylinder. Remember to indicate the direction of the field lines.

i

The cylinder is positively charged

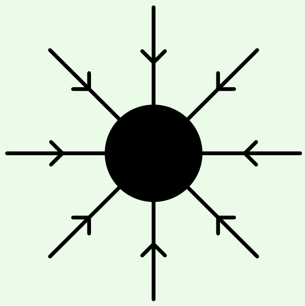
✓ Answer



ii

The cylinder is negatively charged

✓ Answer



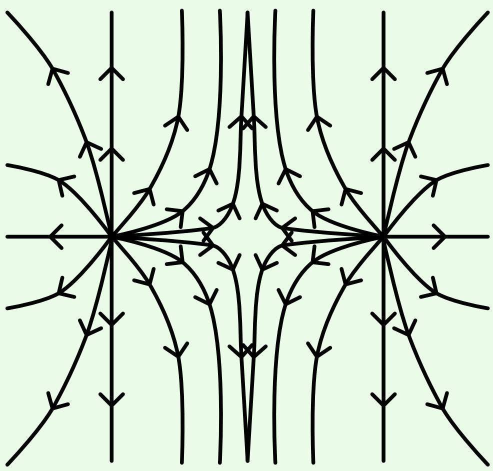
b

Draw field lines for the following particle configurations to visualise it's electric field:

i

A pair of positively charged particles

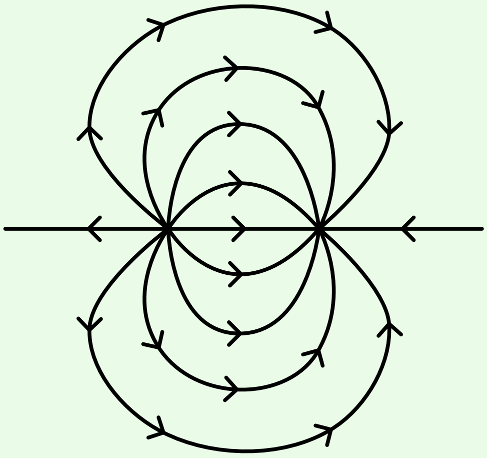
✓ Answer



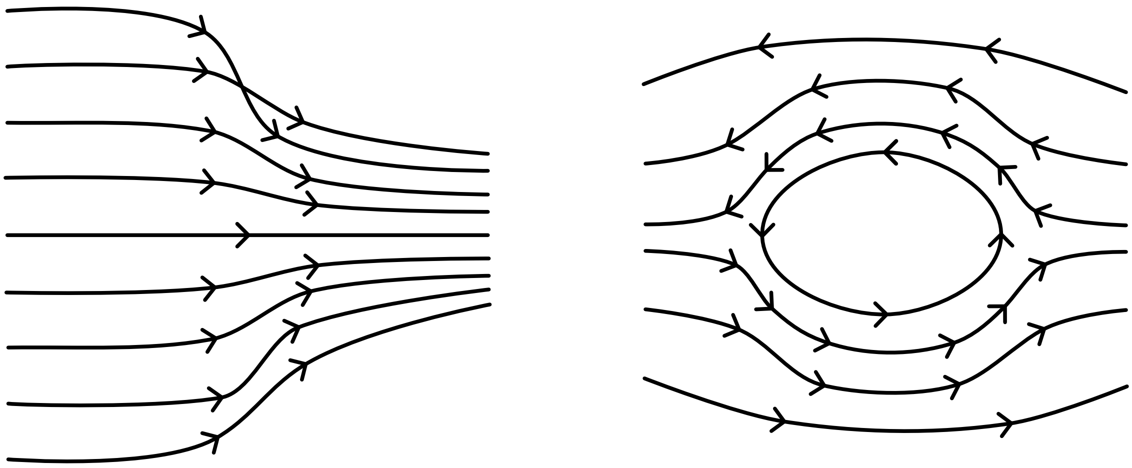
ii

An oppositely charged pair of particles

✓ Answer



c



The nutty professor: A professor draws the pictures shown that he claims are the field lines for an electrostatic field. Explain what is wrong with each drawing.

Answer

The first image has field lines crossing, which is not possible.

The second image has a loop, which is also not possible for electric fields.

6 Spherical shell

A spherical thick shell of inner radius a and outer radius b is uniformly charged with a volume charge density ρ.

a

What is the total charge of the shell, Q_{tot}? Give your answer in terms of a, b and ρ.

✓ **Answer**

$\frac{4\pi\rho}{3}(b^3 - a^3)$
□

b

Based on symmetry in what direction do you expect the electric field to point outside the shell?

✓ **Answer**

Directly away from the center of the shell

c

Use Gauss's law to determine the magnitude of the electric field outside the shell at a distance r from the center of the shell (i.e. for $r > b$). Give your answer in terms of Q_{tot}, r and ϵ_0.

✓ **Answer**

$\vec{E} = \frac{Q_{tot}}{4\pi\epsilon_0 r^2}\hat{r}$
□

d

Use Gauss's law to determine the magnitude of the electric field inside the shell at a distance r from the center of the shell (i.e. for $r < a$). Give your answer in terms of ρ, a, b, r and ϵ_0.

✓ **Answer**

$\vec{E} = \vec{0}$
□

e

Use Gauss's law to determine the magnitude of the electric field within the shell at a distance r from the center of the shell (i.e. for $a < r < b$). Give your answer in terms of ρ, a, b, r and ϵ_0 .

Answer

$Q_{tot} = \frac{4\pi\rho}{3}(r^3 - a^3)$

$\vec{E} = \frac{\rho(r^3 - a^3)}{3\epsilon_0 r^2}\hat{r}$

$\square$