Probabilistic Reasoning

Bayesian Networks

1. Each node corresponds to a random variable
2. Directed links connect pairs of nodes
    I. Is a DAG, so no loops
3. Each node has associated probability information (**CPT** - Conditional Probability Table) $\theta(\text{Node}|\text{parents})$

- Represents a connection to every node in the network, working as another representation of the joint distribution

Nodes

- Contains parents which are what are given to the node
- Each node contains probabilities for the domain of the distribution given the parents of the node

Relationships between Nodes

1. **A->B** Direct Cause
    - $P(B|A)$
2. **A->B->C** Indirect Cause
    - $P(B|A)$
    - $P(C|B)$
    - C is independent of A given B
3. **B<-A->C** Common Cause
    - $P(B|A)$
    - $P(C|A)$
4. **A->C<-B** Common Effect
    - $P(C|A,B)$

Probabilities of a state

$$P(x_1, \ldots, x_n) = \prod_{i=1}^{n} \theta(x_i|\text{parents}(X_i))$$

Where parents are the values in the set of $x_1, \ldots, x_n$ of the parents of the node.

- Can be proven that $\theta() = P()$

Constructing a network

Defintion

Chain rule

$$P(x_1, \dots, x_n)$$
$$= P(x_n|x_{n-1}, \dots, x_1)P(x_{n-1}|x_{n-2}, \dots, x_1)\dots P(x_2|x_1)P(x_1)$$
$$= \prod_{i=1}^{n} P(x_i|x_{i-1}, \dots, x_1)$$

1. Remove unrelated nodes
2. Attempt to order nodes in the order of causes to effects
3. For each node
    I. Find a minimal amount of related parent nodes
    II. Insert a link from parent to current node
    III. Create the conditional probability table