☐PHYS115 ✓PHYS121 ☐PHYS123
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Lab Cover Letter

Author (*You*) Trevor N. Signature: Trev N.

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Lab Partner(s) Katherine

Date Performed 10/04/2024 Date Submitted 16/04/2024

Lab (*such as #1: UNC*) #6 SHM

TA: Phillip

GRADE (*to be filled in by your TA*) See your TA for detailed feedback.
An '*x*' next to a subcategory means you need to improve this aspect of your work.

***Paper Subtotals* (points)**

( ) ***General* (6)**
_____ Sig. figs.
_____ Units
_____ Clarity of Presentation
_____ Format

( ) ***Abstract* (4)**
_____ Quantity or principle
_____ How measurement was made
_____ Numerical Results
_____ Conclusion

( ) ***Intro & Theory* (9)**
_____ Basic principle
_____ Main equations to be used
_____ Apparatus
_____ What will be plotted
_____ Fitting parameters related

( ) ***Exp. Procedures* (15)**
_____ Description
_____ Stating and justifying uncertainties
_____ Data Record
_____ Quality of Lab Work

( ) ***Analysis & Error Analysis* (20)**
_____ Discussion
_____ Equations & Calculations
_____ Presentation inc. Graphs, Tables
_____ Results Reported & Reasonable
_____ Underlined items addressed

( ) ***Discussion & Conclusions* (6)**
_____ Numerical comparison of results
_____ Logical conclusions
_____ Discussion of pos. errors
_____ Suggestions to reduce errors

( ) **Paper Total (60 points)**
(30 points for CME or EPF)

( ) **Notebook (10 points)**
_____ Format (*proper style, following directions*)
_____ Apparatus (*brief description of equipment, including sketches*)
_____ Data (*including computer file names and manually recorded data*)
_____ Experimental Technique (*describing your procedures; stating & justifying uncerts.*)
_____ Analysis (*results and errors*)

( ) **Worksheet(s)/Fill-in-the-Blank-Report (30 points)** *if applicable*

( ) **Adjustments** – late submissions, improper procedures, etc. – or bonus points for exceptional work.

( ) Total Grade

Graded by ___________ *(TA's initial)*

1 Abstract

The purpose of this lab is to determine if the current theory for harmonic motion within linear springs and oscillating torsion forces is consistent with experimental data.

n	Copper
$(4.10 \pm 0.09) \times 10^{10}\ Nm^2$	4.24×10^{10}

k_s	ω_2	ω_{2e}
$0.3068 \pm 0.0003\ \frac{kg}{s^2}$	$5.285 \pm 0.001\ s^{-1}$	$5.282 \pm 0.004\ s^{-1}$

Our estimated ω_2 value was accurate to less than 1 SD of the experimental value and our n value was accurate to around 2.5 SD from what we would expect from theory.

We believe our variance would be due to the human factor in the releasing of both weights, leading to inconsistencies across trials. Other reasons for variance would include air resistance of the masses, and the dampening force both within the spring and the rod.

Thus we fail to reject the current theory on simple harmonic motion within linear springs and oscillating restoring torsion forces

2 Theory

2.1 Variables

2.1.1 Constants

$$\begin{array}{ll}
L = 0.9850 \pm 0.0005\ m & \text{Length of the rod} \\
d_p = 0.2685 \pm 0.0005\ m & \text{Diameter of the plate} \\
d_r = 0.0045 \pm 0.0002\ m & \text{Diameter of the rod} \\
m = 4.7 \pm 0.1\ kg & \text{Mass of the plate} \\
m_1 = 0.100 \pm 0.0001\ kg & \text{Mass of first oscilating weight} \\
m_2 = 0.110 \pm 0.0001\ kg & \text{Mass of second oscilating weight}
\end{array}$$

2.1.2 Measurable Values

$$\begin{array}{ll}
T & \text{Period of Oscillation} \\
m, \theta_m & \text{Amplitude of Oscillation}
\end{array}$$

2.2 Formulae

2.2.1 Finding the position vs. time relation for a spring

$$F = -k(\Delta x) \qquad \text{Force of a spring} \tag{2.2.1a}$$

$$a = -\frac{k\Delta x}{m_1} \qquad \text{Definition of acceleration} \tag{2.2.1b}$$

$$-k\Delta x = m_1 \ddot{x} \tag{2.2.1c}$$

$$\begin{cases} x = x_m \sin(\omega t + \phi) + x_0 \\ \omega = \sqrt{\frac{k_s}{m_1}} \end{cases} \qquad \text{The solution is well known} \tag{2.2.1}$$

2.2.2 Finding the position vs. time relation for torqued bar

$$\tau = -k_t\theta \qquad \text{Force of restoring Torque} \tag{2.2.2a}$$

$$\tau = I\alpha \qquad \text{Definition of Torque} \tag{2.2.2b}$$

$$-k_t\theta = I\alpha \tag{2.2.2c}$$

$$\omega = \frac{2\pi}{T} \tag{2.2.2d}$$

$$k_t = \frac{nA^2}{2\pi L} \qquad \text{Definition of torsion constant} \tag{2.2.2e}$$

$$\begin{cases} \theta = \theta_m \sin(\omega t + \phi) \\ \omega = \sqrt{\frac{k_t}{I}} \end{cases} \qquad \text{The solution is well known} \tag{2.2.2}$$

2.2.3 Finding the torsion modulus

$$I = \frac{1}{2}m\left(\frac{d_p}{2}\right)^2 \qquad \text{Inertia of a disk} \tag{2.2.3a}$$

$$\frac{2\pi}{T} = \sqrt{\frac{k_t}{I}} \qquad (2.2.2d, 2.2.2) \tag{2.2.3b}$$

$$I4\pi^2 = T^2 k_t \tag{2.2.3c}$$

$$\frac{md_p^2\pi^2}{2T^2} = k_t \qquad (2.2.3a) \tag{2.2.3d}$$

$$\frac{md_p^2\pi^2}{2T^2} = \frac{n\left(\frac{d_r}{2}\right)^4\pi^2}{2\pi L} \qquad (2.2.2e) \tag{2.2.3e}$$

$$n = \frac{16md_p^2\pi L}{d_r^4 T^2} \tag{2.2.3}$$

2.2.4 Error in torsion modulus

$$\delta_n = \sqrt{\delta_{nm}^2 + \delta_{nd_p}^2 + \delta_{nL}^2 + \delta_{nd_r}^2 + \delta_{dT}^2} \tag{2.2.4a}$$

$$\delta_n = n\sqrt{\left(\frac{\delta_m}{m}\right)^2 + \left(\frac{2\delta_{d_p}}{d_p}\right)^2 + \left(\frac{\delta_L}{L}\right)^2 + \left(\frac{4\delta_{d_r}}{d_r}\right)^2 + \left(\frac{2\delta_T}{T}\right)^2} \tag{2.2.4}$$

2.2.5 Finding the spring constant

$$k_s = \omega^2 m_1 \qquad (3.2.2) \tag{2.2.5}$$

2.2.6 Error in spring constant

$$\delta_{k_s} = \sqrt{\delta^2_{k_s\omega} + \delta^2_{k_s m_1}} \qquad \text{Combination of errors} \tag{2.2.6a}$$

$$\delta_{k_s} = k_s\sqrt{\left(\frac{2\delta_\omega}{\omega}\right)^2 + \left(\frac{\delta_{m_1}}{m_1}\right)^2} \tag{2.2.6}$$

2.2.7 Prediction of ω_{2e} for m_2

$$\omega_{2e} = \sqrt{\frac{k_s}{m_2}} \qquad (2.2.1) \tag{2.2.7}$$

2.2.8 Error in ω_{2e}

$$\delta_{\omega_{2e}} = \sqrt{\delta^2_{\omega_{2e}k_s} + \delta^2_{\omega_{2e}m_2}} \qquad \text{Combination of errors} \tag{2.2.8a}$$

$$\delta_{\omega_{2e}} = \omega_{2e}\sqrt{\left(\frac{\delta_{k_s}}{2k_s}\right)^2 + \left(\frac{\delta_{m_2}}{2m_2}\right)^2} \tag{2.2.8}$$

3 Procedure

3.1 Spring Constant

1. Set up a linear spring secured and hung by one end hanging downwards
2. Place a distance sensor directly below the spring
3. Hang a mass with m_1 from the bottom of the spring such that the distance to the sensor is measurable
4. Ensure that if the mass bobs on the spring, the sensor is able to capture its distance accurately
5. Lift the mass enough that the mass can bob consistently for over 25 cycles
6. Record the distance over time as the mass bobs
7. Regress the data match that of Eq. (2.2.1)
8. Calculate k_s from the regressed value of ω with Eq. (2.2.5) and its error with Eq. (2.2.6)
9. Now experiment with a higher drop distance
10. Now swap out the mass with one that is heavier (mass m_2) and repeat the experiment
11. Estimate ω_2 for m_2 and compare it with the experimental data with Eq. (2.2.7) and its error with Eq. (2.2.8)

3.2 Torsion Modulus

1. Secure a thin metal rod hanging from end
2. Record its length as L and its diameter as d_r
3. Attach a cylindrical mass at the bottom of the rod, with their centers lining up
4. Record its mass and diameter as m and d_p
5. Set up a photogate and negligible mass light blocking prong vertically on the plate such that it would block the photogate when the system is at rest

6. Twist the plate vertically such that the rod stays in place
7. Record the times between each cycle of movement with the photogate
8. Calculate the Torsion modulus of the rod with Eq. (2.2.3) and its Error with Eq. (2.2.4)

4 Analysis

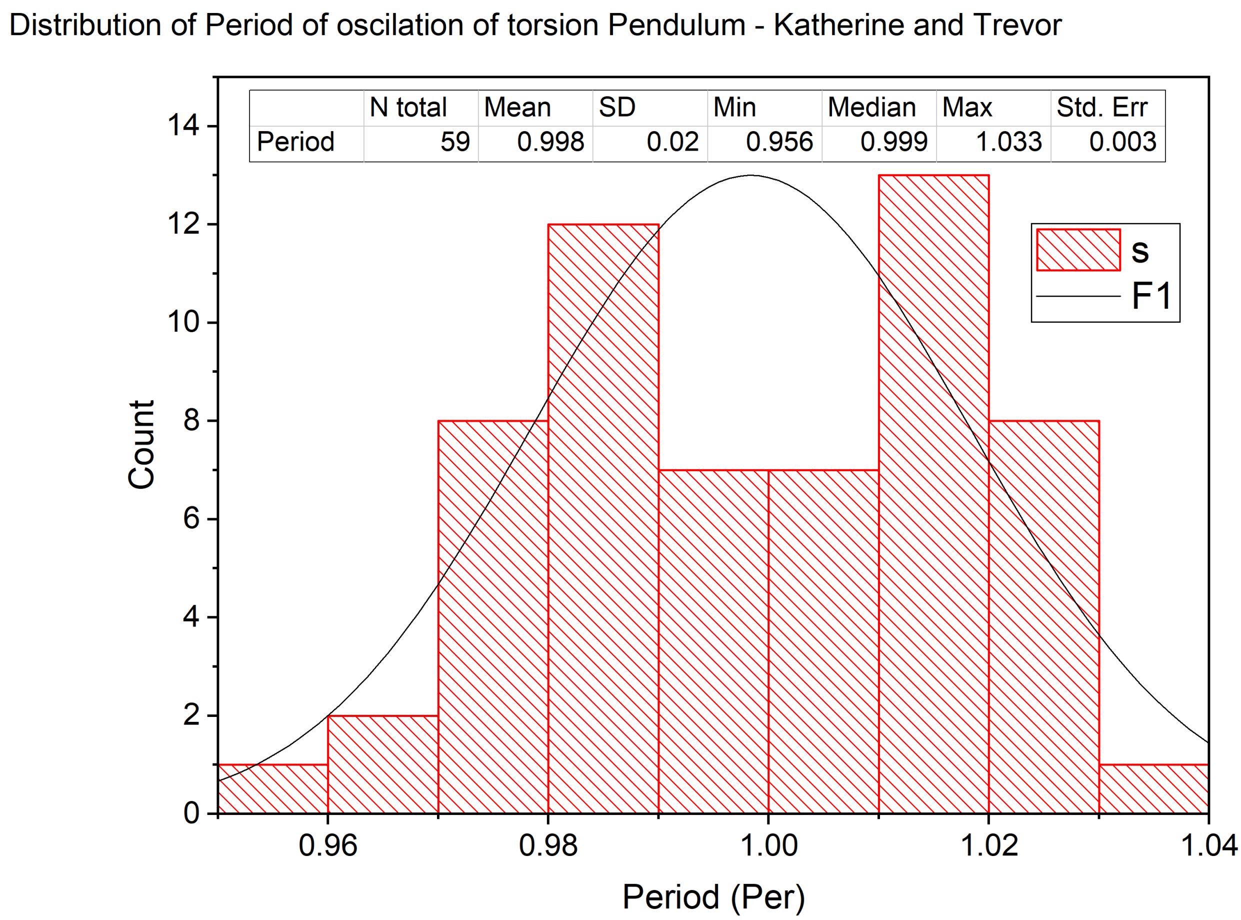
4.1 Measured and Calculated values Values

4.1.1 Torsion

4.1.1.1 Raw Data

Trial	T (s)
1	1.0013676
2	0.9993538
3	0.9991016
4	0.9974852
5	0.9991
6	1.0002148
7	1.0027448
8	1.0042864
9	1.0019008
10	1.0000696
11	0.9964816
12	0.9945008
13	0.9956008
14	0.9975616
15	1.0007008
16	1.00344
17	1.0012672
18	0.9980848
19	0.9942208
20	0.9912816
21	0.9916992
22	0.9951184
23	0.9993328
24	1.0030688

Trial	T (s)
25	1.002736
26	0.9977488
27	0.9926528
28	0.9892992
29	0.9879008
μ	0.998 ± 0.003



4.1.1.2 Calculated Data

n
$(4.10 \pm 0.09) \times 10^{10}\ Nm^2$

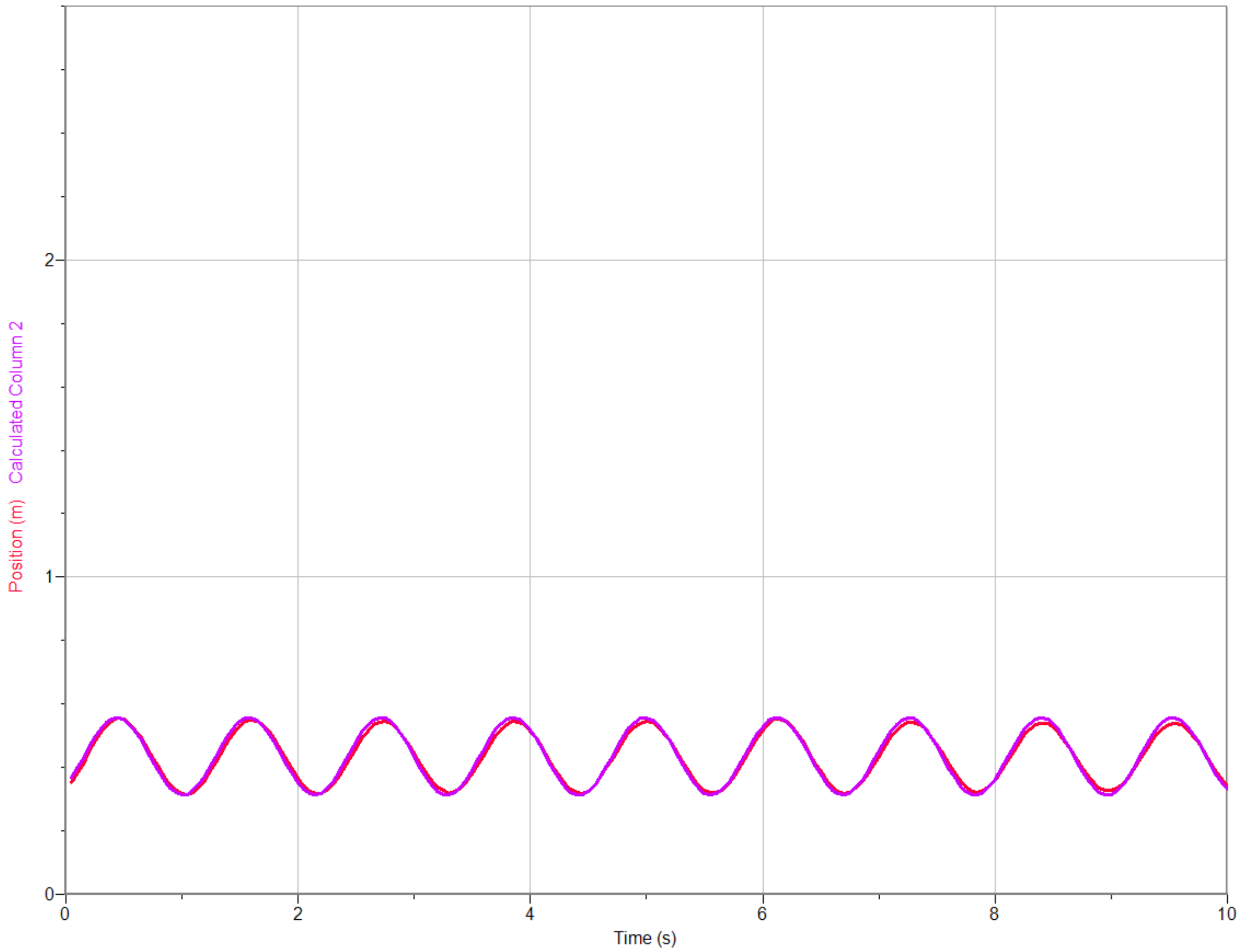
4.1.2 Spring

4.1.2.1 Raw Data after fitting

Trial	x_m (m)	ω (s^-1)	ϕ	x_0 (m)	RMSE (m)	r Cor
1	0.1133 ± 0.0004	5.536 ± 0.001	5.277 ± 0.006	0.4321 ± 0.0003	0.003622	0.9990

Trial	x_m (m)	ω (s^-1)	ϕ	x_0 (m)	RMSE (m)	r Cor
2	0.0977 ± 0.0002	5.541 ± 0.0008	5.623 ± 0.004	0.4316 ± 0.0002	0.002326	0.9994
3[1]	0.1118 ± 0.0002	5.539 ± 0.0006	5.291 ± 0.003	0.4301 ± 0.0001	0.001916	0.9997
4	0.1354 ± 0.0003	5.538 ± 0.0006	4.778 ± 0.004	0.4303 ± 0.0002	0.002528	0.9997
5	0.1580 ± 0.0003	5.538 ± 0.0007	4.124 ± 0.004	0.4303 ± 0.0002	0.003212	0.9996
10cm	0.2240 ± 0.0006	5.536 ± 0.0009	3.156 ± 0.005	0.4303 ± 0.0004	0.005793	0.9994
m_2	0.2144 ± 0.0008	5.285 ± 0.001	2.431 ± 0.008	0.4032 ± 0.0006	0.008438	0.9985
μ		5.539 ± 0.0006				
SE		0.0009				
σ		0.0019				

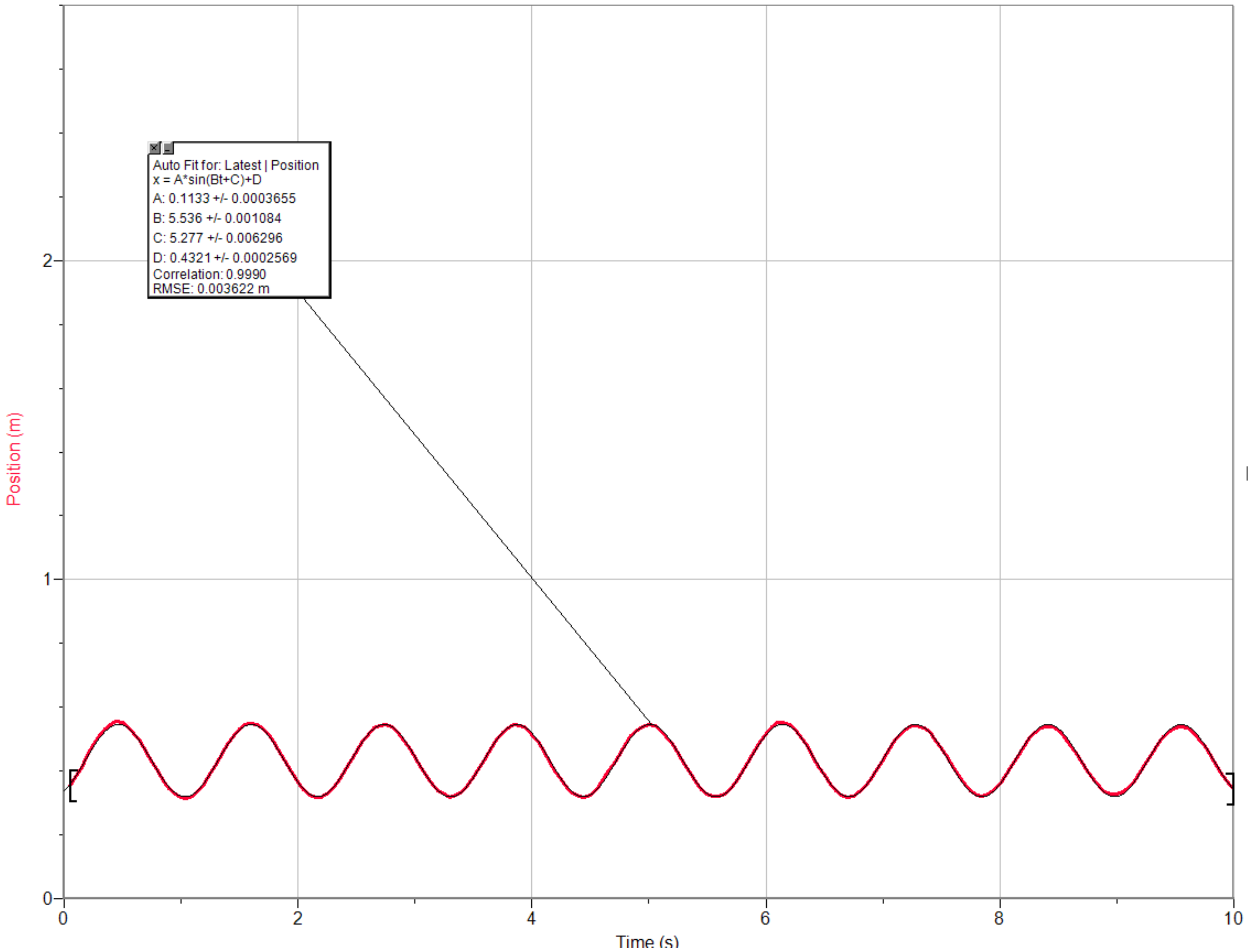
[1]Best sample



Manual Fit Curve

Estimated values

x_{max}	x_m	T	ϕ
0.557	0.1225	1.3625	0.1599375



Auto Fit curve

4.1.2.2 Calculated Data

k_s	ω_2	ω_{2e}
$0.3068 \pm 0.0003\ \frac{kg}{s^2}$	$5.285 \pm 0.001\ s^{-1}$	$5.282 \pm 0.004\ s^{-1}$

4.2 Process

4.2.1 Torsion

Once we took an average value for one period of oscillation, we used Eq. (2.2.3, 2.2.4) to calculate the torsion modulus.

Using a table of commonly accepted torsion moduli,

Material	Torsion Modulus (Nm^2)
Lead	0.54×10^{10}
Magnesium	1.67×10^{10}
Aluminium	2.37×10^{10}
Brass	3.53×10^{10}
Copper	4.24×10^{10}

Material	**Torsion Modulus (Nm^2)**
Iron	7.0×10^{10}
Nickel	$7.0 - 7.6 \times 10^{10}$
Steel	7.8×10^{10}
Molybdenum	14.7×10^{10}
Tungsten	14.8×10^{10}

We found that our n would strongly match Copper's within 2 SD, which is the correct material

n	**Copper**
$(4.10 \pm 0.09) \times 10^{10}\ Nm^2$	4.24×10^{10}

4.2.2 Spring

Once we fit the data either automatically or manually to Eq. (2.2.1), we calculated our final values using Eq. (2.2.5, 2.2.6)

k_s
$0.3068 \pm 0.0003\ \frac{kg}{s^2}$

We recognize that our estimated value for ω_2 was within 1 SD of the actual value measured using Eq. (2.2.7, 2.2.8)

ω_2	ω_{2e}
$5.285 \pm 0.001\ s^{-1}$	$5.282 \pm 0.004\ s^{-1}$

5 Conclusion

We concluded that our experimental data is consistent with the current theory for the torsion restoring force and the linear spring restoring force. Our estimated parameters based on the theory was consistent with experimental data in both variance in spring weight and amplitude. For the torsion force, we found that our experimental value of the torsion modulus was consistent with that of the commonly accepted value for Copper, which was accurate.

n	**Copper**
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k_s	ω_2	ω_{2e}
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Our estimated ω_2 value was accurate to less than 1 SD and our n value was accurate to around 2.5 SD.

We believe our variance would be due to the human factor in the releasing of both weights, leading to inconsistencies across trials. Other reasons for variance would include air resistance of the masses, and the dampening force both within the spring and the rod.

Thus we fail to reject the current theory on simple harmonic motion within linear springs and oscillating restoring torsion forces

6 Acknowledgements and Info

- Lab #6 SHM
- 10/04/2024
- Station 15+16 Rockefeller 404
- PHYS 121-118

Lab Partner: Katherine Chen
Lab Manual: Lab 6 SHM PHYS 121