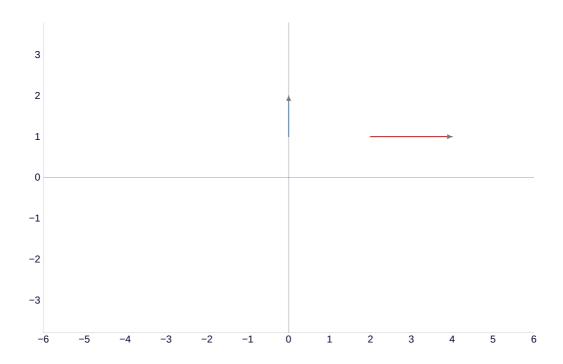
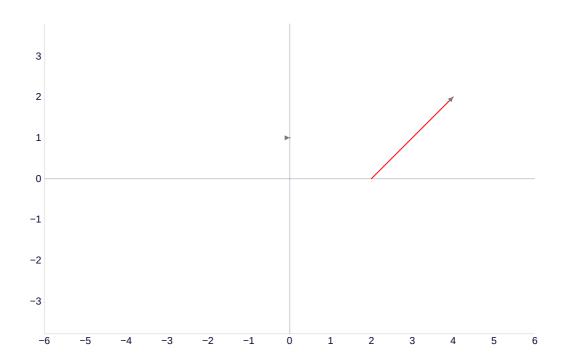
$$ec{F}(ec{P}) = \langle 0, 1, 0
angle$$

$$ec{F}(ec{Q}) = \langle 2, 0, 2
angle$$

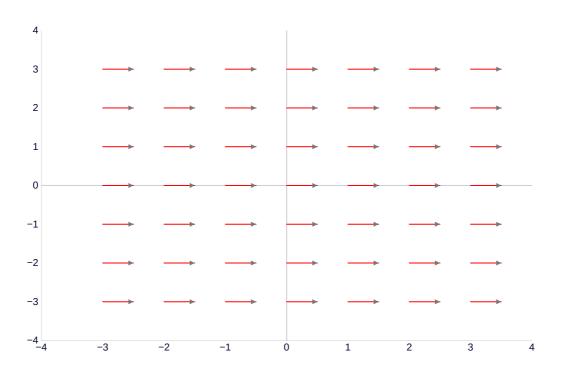


On the XY plane

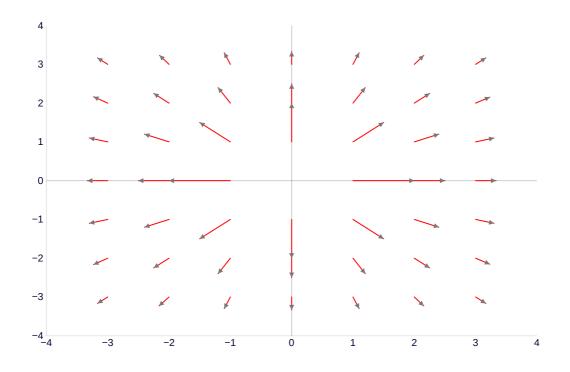


On the XZ plane

$$ec{F}=\langle 1,0
angle$$



$$ec{F} = \left\langle rac{x}{x^2 + y^2}, rac{y}{x^2 + y^2}
ight
angle$$



C

$$egin{aligned} \operatorname{div}(\langle x,y,z
angle) &= 1+1 = 1 = 3 \ \operatorname{curl}(\langle x,y,z
angle) &= \langle 0,0,0
angle &= 0 \end{aligned}$$

$$egin{aligned} \operatorname{div}(\langle x-2zx^2,z-xy,z^2x^2
angle) &= 1-4zx-x+2zx^2 \ \operatorname{curl}(\langle x-2zx^2,z-xy,z^2x^2
angle) &= \langle -1,2x^2-2xz^2,-y
angle \end{aligned}$$

$$\operatorname{div}(\vec{F} + \vec{G}) = \operatorname{div}(\vec{F}) + \operatorname{div}(\vec{G})$$

$$\begin{aligned} \operatorname{div}(\vec{F} + \vec{G}) &= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \left\langle \vec{F}_x + \vec{G}_x, \vec{F}_y + \vec{G}_y, \vec{F}_z + \vec{G}_z \right\rangle \\ &= \left\langle \frac{\partial}{\partial x} (\vec{F}_x + \vec{G}_x), \frac{\partial}{\partial y} (\vec{F}_y + \vec{G}_y), \frac{\partial}{\partial z} (\vec{F}_z + \vec{G}_z) \right\rangle \\ &= \left\langle \frac{\partial}{\partial x} \vec{F}_x + \frac{\partial}{\partial x} \vec{G}_x, \frac{\partial}{\partial y} \vec{F}_y + \frac{\partial}{\partial y} \vec{G}_y, \frac{\partial}{\partial z} \vec{F}_z + \frac{\partial}{\partial z} \vec{G}_z \right\rangle \\ &= \left\langle \frac{\partial}{\partial x} \vec{F}_x, \frac{\partial}{\partial y} \vec{F}_y, \frac{\partial}{\partial z} \vec{F}_z \right\rangle + \left\langle \frac{\partial}{\partial x} \vec{G}_x, \frac{\partial}{\partial y} \vec{G}_y, \frac{\partial}{\partial z} \vec{G}_z \right\rangle \\ &= \operatorname{div}(\vec{F}) + \operatorname{div}(\vec{G}) \end{aligned}$$

$$\operatorname{div}(\operatorname{curl}(\vec{F})) = 0$$

$$\begin{aligned} \operatorname{div}(\operatorname{curl}(\vec{F})) &= \nabla \cdot (\nabla \times \vec{F}) \\ &= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \left(\left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \times \vec{F} \right) \\ &= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \left\langle \frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}, \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}, \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right\rangle \\ &= \frac{\partial F_z}{\partial yx} - \frac{\partial F_y}{\partial zx} + \frac{\partial F_x}{\partial zy} - \frac{\partial F_z}{\partial xy} + \frac{\partial F_y}{\partial xz} - \frac{\partial F_x}{\partial yz} \\ &= 0 \end{aligned}$$

$$\frac{\partial}{\partial y}x = 0$$

$$\frac{\partial}{\partial x}y = 0$$

$$\int x\ dx = rac{x^2}{2} + C$$
 $\int y\ dy = rac{y^2}{2} + C$ $f(x,y) = rac{x^2}{2} + rac{y^2}{2}$

16.2

$$egin{aligned} f(x,y,z) &= z^2 \ ec{r} &= \langle 2t, 3t, 4t
angle \ C &= [ec{r}(0), ec{r}(2)] \ \ \int_C f \, ds &= \int_0^2 f(ec{r}(t)) \| ec{r}'(t) \| \, dt \ \ &= \int_0^2 16 t^2 \sqrt{19 t^2} \, dt \ \ &= \int_0^2 16 \sqrt{29} t^3 \, dt \ \ &= \int_0^2 4 \sqrt{29} t^4 \, dt \ \ &= 64 \sqrt{29} \end{aligned}$$

$$ec{F}(x,y) = \langle 1+x^2, xy^2
angle \ ec{r} = \langle t, 3t
angle \ C = [ec{r}(0), ec{r}(1)] \ \int_C F \, ds = \int_0^1 ec{F}(ec{r}(t)) \cdot ec{r}'(t) \, dt \ = \int_0^1 \langle 1+t^2, 9t^3
angle \cdot \langle 1, 3
angle \, dt \ = \int_0^1 1 + t^2 + 27t^3 \, dt \ = \int_0^1 t + \frac{t^3}{3} + \frac{27t^4}{4} \, dt \ = 1 + \frac{1}{3} + \frac{27}{4} \ = \frac{97}{12}$$

$$egin{aligned} ec{F}(x,y) &= \langle x^2, xy
angle \ ec{r} &= \langle -\sin t, \cos t
angle \ C &= \left[ec{r}(0), ec{r}\left(rac{\pi}{2}
ight)
ight] \ \int\limits_C F \, ds &= \int\limits_0^{rac{\pi}{2}} ec{F}(ec{r}(t)) \cdot ec{r}'(t) \, dt \ &= \int\limits_0^{rac{\pi}{2}} \langle \sin^2 t, -\sin t \cos t
angle \cdot \langle -\cos t, -\sin t
angle \, dt \ &= \int\limits_0^{rac{\pi}{2}} -\sin^2 t \cos t + \sin^2 t \cos t \, dt \ &= \int\limits_0^{rac{\pi}{2}} 0 \, dt \ &= 0 \end{aligned}$$

$$egin{aligned} ec{F}(x,y,z) &= \left\langle rac{3z}{y}, 4x, -y
ight
angle \ ec{r} &= \left\langle e^t, e^t, t
ight
angle \ C &= \left[ec{r}(-1), ec{r}\left(1
ight)
ight] \end{aligned}$$

$$egin{aligned} \int\limits_{C} F \, ds &= \int\limits_{-1}^{1} ec{F}(ec{r}(t)) \cdot ec{r}'(t) \, dt \ &= \int\limits_{-1}^{1} \left\langle rac{3t}{e^t}, 4e^t, -e^t
ight
angle \cdot \left\langle e^t, e^t, 1
ight
angle \, dt \ &= \int\limits_{-1}^{1} 3t + 4e^{2t} - e^t \, dt \ &= \left| rac{1}{2} rac{3t^2}{2} + 2e^{2t} - e^t \, dt
ight. \ &= rac{3}{2} + 2e^2 - e - rac{3}{2} - 2e^{-2} + e^{-1} \ &= 2e^2 - e - 2e^{-2} + e^{-1} \ &pprox 12.157 \end{aligned}$$

$$egin{aligned} ec{F}(x,y,z) &= \left\langle rac{1}{y^3+1}, rac{1}{z+1}, 1
ight
angle \ ec{r} &= \left\langle t^3, 2, t^2
ight
angle \ C &= \left[ec{r}(0), ec{r}(1)
ight] \ \int\limits_C F \, ds &= \int\limits_0^1 ec{F}(ec{r}(t)) \cdot ec{r}'(t) \, dt \ &= \int\limits_0^1 \left\langle rac{1}{9}, rac{1}{t^2+1}, 1
ight
angle \cdot \left\langle 3t^2, 0, 2t
ight
angle \, dt \ &= \int\limits_0^1 rac{t^2}{3} + 2t \, dt \ &= \left| rac{1}{9} rac{t^3}{9} + t^2 \, dt \ &= rac{10}{9} \end{aligned}$$

$$egin{aligned} r &= \langle t, t^3
angle \ C &= \left[ec{r}(0), ec{r}\left(3
ight)
ight. \ \int\limits_C x \ dx &= \int\limits_0^3 t \ dt \ &= \left|\int\limits_0^3 rac{t^2}{2} \ dt
ight. \ &= rac{9}{2} \end{aligned}$$

$$egin{aligned} ec{r} &= \langle t, t^2
angle \ C &= \left[ec{r}(0), ec{r}\left(2
ight)
ight] \ \int\limits_C y \, dx - x \, dy &= \int\limits_0^2 t^2 - 2t^2 \, dt \ &= \int\limits_0^2 - t^2 \, dt \ &= \left| \int\limits_0^2 - rac{t^3}{3} \, dt
ight. \ &= -rac{8}{3} \end{aligned}$$

16.3

$$ec{F}=\langle x,y,z
angle \ f=rac{x^2}{2}+rac{y^2}{2}+rac{z^2}{2}$$

$$ec{F}=\langle y^2,2xy+e^z,ye^z
angle \ \int y^2\ dx=xy^2+C \ \int 2xy+e^z\ dy=xy^2+e^zy+C \ \int ye^z\ dz=e^zy+C$$

$$f = xy^2 + ye^z + C$$

$$ec{F} = \langle z \sec^2 x, z, y + an x
angle$$
 $\int z \sec^2 x \ dx = z an x + C$
 $\int z \ dy = yz + C$
 $\int y + an x \ dz = yz + z an x + C$
 $f = yz + z an x + C$

$$ec{F} = \langle 2xy + 5, x^2 - 4z, -4y
angle \ \int 2xy + 5 \ dx = x^2y + 5x + C \ \int x^2 - 4z \ dy = x^2y - 4yz + C \ \int -4y \ dz = -4yz + C \ f = x^2y + 5x - 4yz + C$$

$$egin{aligned} f &= x^2y - z \ ec{F} &= \langle 2xy, x^2, -1
angle \ ec{r}_1 &= \langle t, t, 0
angle \ 0 &< r < 1 \end{aligned} \ egin{aligned} \int ec{F}(ec{r}) \; ds &= \int 0^1 ec{F}(ec{r}) \cdot ec{r}' \; dt \ &= \int 0^1 \langle 2t^2, t^2, -1
angle \cdot \langle 1, 1, 0
angle \; dt \ &= \int 0^1 0^1 dt \end{aligned} \ &= \int 0^1 0^1 dt \end{aligned}$$

$$egin{aligned} ec{r}_2 &= \langle t, t^2, 0
angle \ 0 < r < 1 \ &\int_C ec{F}(ec{r}) \; ds = \int_0^1 ec{F}(ec{r}) \cdot ec{r}' \; dt \ &= \int_0^1 \langle 2t^3, t^2, -1
angle \cdot \langle 1, 2t, 0
angle \; dt \ &= \int_0^1 4t^3 \; dt \ &= igg|_0^1 t^4 \; dt \ &= 1 \end{aligned}$$

f(1,1,0) - f(0,0,0) = 1

Thus all pathes with the same endpoints have the same integral on a conservative vector field, which is equivalent to the difference in the potential function.