1

$f(x,y,z) = (\sin xy)e^{-z^2}$
$P = (1, \pi, 0)$

$f'_x(x,y,z) = y(\cos xy)e^{-z^2}$
$f'_y(x,y,z) = x(\cos xy)e^{-z^2}$
$f'_z(x,y,z) = -2z(\sin xy)e^{-z^2}$

$\nabla f = \langle y(\cos xy)e^{-z^2}, x(\cos xy)e^{-z^2}, -2z(\sin xy)e^{-z^2} \rangle$
$\nabla f(P) = \langle -\pi, -1, 0 \rangle$
$\square$

You would need to go in the $\langle -\pi, -1, 0 \rangle$ direction as ∇f represents the direction of steepest ascent.

2

$x^2 + 2y^2 + 3z^2 = 10$
$P = (0, \sqrt{2}, \sqrt{2})$

$\begin{cases} 2x + 6zf'_x = 0 \\ 4y + 6zf'_y = 0 \end{cases}$
$\begin{cases} f'_x = -x/3z \\ f'_y = -2y/3z \end{cases}$

$L_{(a,b,c)}(x,y) = c - \frac{a}{3c}(x-a) - \frac{2b}{3c}(y-b)$
$L_{(0,\sqrt{2},\sqrt{2})}(x,y) = \sqrt{2} - \frac{2}{3}(y - \sqrt{2})$
$= -\frac{2}{3}y + \frac{5\sqrt{2}}{3}$
$\square$