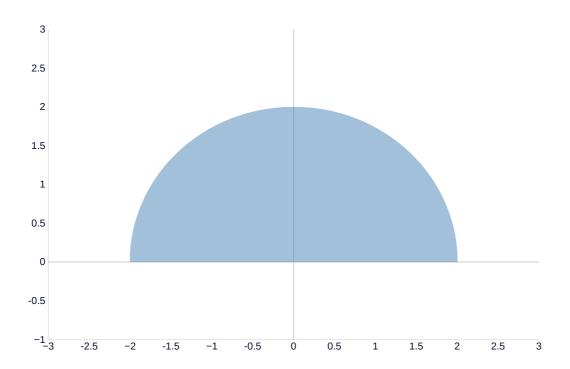
$$\int\limits_{-2}^{2}\int\limits_{0}^{\sqrt{4-x^2}}x^2+y^2\;dydx$$

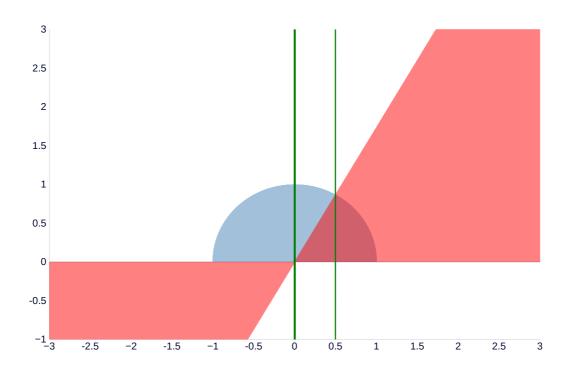


$$0 < r < 2$$

$$0 < \theta < \pi$$

$$\int \limits_{0}^{\pi} \int \limits_{0}^{2} r^{2} r \ dr d heta = \int \limits_{0}^{\pi} \Big| _{0}^{2} r^{4} / 4 \ dr d heta = \int \limits_{0}^{\pi} 4 \ d heta = 4\pi
onumber pprox 12.566$$

$$\int\limits_0^{1/2}\int\limits_{\sqrt{3}x}^{\sqrt{1}-x^2}x\;dydx$$



Where the area is the blue area between the green lines that is not in the red area.

$$0 < r < 1$$

$$\pi/3 < \theta < \pi/2$$

$$\int_{\pi/3}^{\pi/2} \int_{0}^{1} r^{2} \cos \theta \, dr d\theta$$

$$= \int_{\pi/3}^{\pi/2} \frac{1}{3} \cos \theta \, d\theta$$

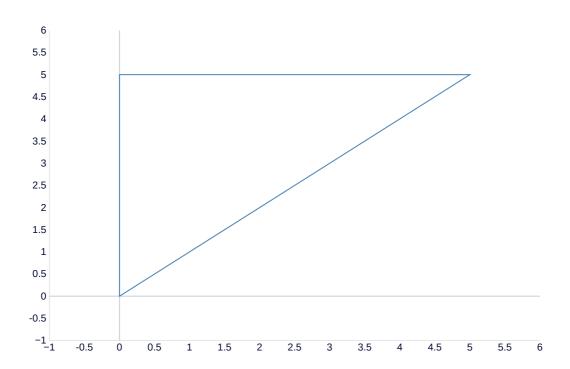
$$= \Big|_{\pi/3}^{\pi/2} \frac{1}{3} \sin \theta \, d\theta$$

$$= \Big|_{\pi/3}^{\pi/2} \frac{1}{3} \sin \theta \, d\theta$$

$$= \frac{1}{3} - \frac{\sqrt{3}}{6}$$

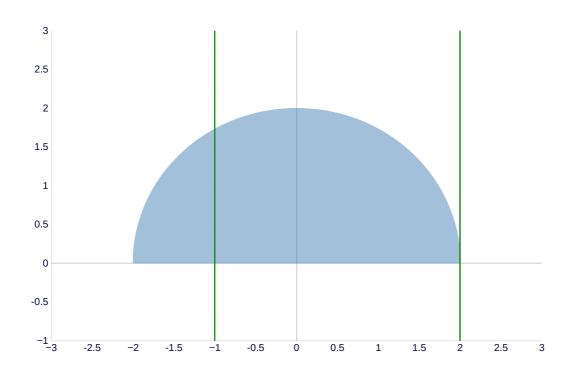
$$\approx 0.045$$

$$\int\limits_0^5\int\limits_0^y x\;dxdy$$



$$= \int_{0}^{5} y^{2}/2 \, dy$$
$$= (5)^{3}/6$$
$$= (5)^{3}/6$$
$$= 125/6$$

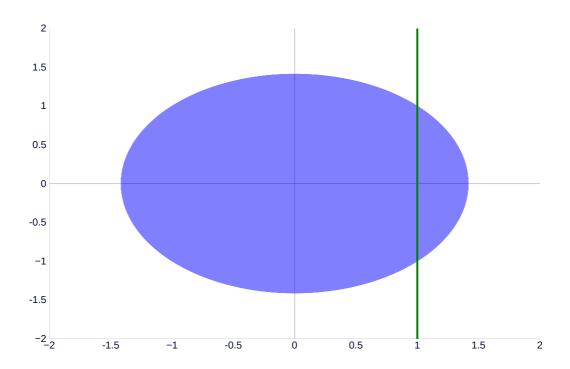
$$\int\limits_{-1}^{2}\int\limits_{0}^{\sqrt{4-x^2}}x^2+y^2\;dydx$$



Where the area is the blue semicircle between the green lines

$$egin{aligned} & \left\{ egin{aligned} 0 < r < 2 \ 0 < heta < 2\pi/3 \end{aligned}
ight. \ & = \int\limits_{0}^{2\pi/3} \int\limits_{0}^{2} r^3 \; dr d heta + \int\limits_{0}^{1} \int\limits_{0}^{x an\pi/3} x^2 + y^2 \; dy dx \ & = \int\limits_{0}^{2\pi/3} 4 \; d heta + \int\limits_{0}^{1} 2x^3 \sqrt{3} \; dx \ & = 8\pi/3 + \sqrt{3}/2 \ pprox 9.244 \end{aligned}$$

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Where the area is the circle right of the green line

$$egin{array}{l} rac{\pi/4}{2} & \sqrt{2} \ 2 \int \int \limits_{0}^{\pi/4} \int \limits_{\cos heta}^{r-3} dr d heta \ = \int \limits_{0}^{\pi/4} \left| \int \limits_{\cos heta}^{\sqrt{2}} - 1/r^2 \ dr d heta \ = \int \limits_{0}^{\pi/4} -1/2 + \sec^2 heta \ d heta \ = -\pi/8 + \left| \int \limits_{0}^{\pi/4} an heta \ d heta \ = -\pi/8 + 1 \ pprox 0.607 \end{array}
ight.$$

$$4 \int_{0}^{\pi/2} \int_{0}^{1} r^{3} \cos \theta \sin \theta \, dr d\theta$$

$$= \int_{0}^{\pi/2} \cos \theta \sin \theta \, d\theta$$

$$= \Big|_{0}^{\pi/2} \frac{1}{2} (\sin \theta)^{2} \, d\theta$$

$$= 1/2$$

$$= 0.5$$

$$egin{aligned} u &= x/\sqrt{2} + y/\sqrt{2} \ v &= x/\sqrt{2} - y/\sqrt{2} \end{aligned} \ J &= egin{aligned} 1/\sqrt{2} & 1/\sqrt{2} \ 1/\sqrt{2} & -1/\sqrt{2} \ \end{vmatrix} \ &= -1 \end{aligned} \ &= -1 \ \int\limits_{-\pi/4}^{\pi/4} \int\limits_{r\cos\theta}^{1} \int\limits_{-\pi/4}^{r2} \sin\theta \ dr d\theta \ &= \int\limits_{r\cos\theta-\pi/4}^{1} \int\limits_{r\sin\theta}^{\pi/4} d\theta dr d\theta dr \end{aligned}$$

Since \sin is an even function the first integral evaluates to 0

= 0

$$\int\limits_{0}^{5} \int\limits_{0}^{2\pi} \int\limits_{0}^{3} r^{3} \ dr d\theta dz \ = 5 imes 2\pi imes (3)^{4}/4 \ = 405\pi/2 \ pprox 636.173$$

$$\int\limits_{-3}^{3} \int\limits_{0}^{\pi/2} \int\limits_{0}^{4} r^{2} \cos heta \ dr d heta dh$$
 $= \int\limits_{-3}^{3} \int\limits_{0}^{\pi/2} \frac{64}{3} \cos heta \ d heta dh$
 $= 128$

$$\int\limits_{0}^{9} \int\limits_{0}^{2\pi} \int\limits_{0}^{\sqrt{h}} hr \, dr d heta dh \ = \int\limits_{0}^{9} \pi h^2 \, dh \ = 243\pi \ pprox 763.407$$

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$$\int\limits_{0}^{4}\int\limits_{0}^{2\pi}\int\limits_{0}^{1}rf(r\cos heta,r\sin heta,h)\;drd heta dh$$

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Where h is the height of the band and R is the radius of the sphere. In cylindrical coordinates of (r,θ,z)

This integral represents the volume of the band.

$$egin{aligned} & rac{h/2}{\int} \int\limits_{0}^{2\pi} \int\limits_{\sqrt{R^2-(h/2)^2}}^{\sqrt{R^2-z^2}} r \, dr d heta dz \ & = \int\limits_{-h/2}^{h/2} \int\limits_{0}^{2\pi} \left| \sqrt{rac{N^2-z^2}{\sqrt{R^2-z^2}}} r^2/2 \, dr d heta dz
ight. \ & = \int\limits_{-h/2}^{h/2} \int\limits_{0}^{2\pi} ((h/2)^2 - z^2)/2 \, d heta dz \ & = \pi \int\limits_{-h/2}^{h/2} (h/2)^2 - z^2 \, dz \ & = \pi igg|_{-h/2}^{h/2} z(h/2)^2 - z^3/3 \, dz \ & = \pi (h(h/2)^2 - h^3/12) \end{aligned}$$

Which only depends on h (the height of the band) and not the radius of the sphere R.

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$$\int_{0}^{2\pi} \int_{0}^{\pi/3} \int_{\sec \phi}^{2} \rho^{2} \sin \phi \, d\rho d\phi d\theta$$

$$= \int_{0}^{2\pi} \int_{0}^{\pi/3} \frac{8 - \sec^{3} \phi}{3} \sin \phi \, d\phi d\theta$$

$$= \int_{0}^{2\pi} \left| \int_{0}^{\pi/3} \frac{-8 \cos \phi}{3} - \frac{1}{6(1 - \sin^{2} \phi)} \, d\phi d\theta$$

$$= \int_{0}^{2\pi} -\frac{4}{3} + \frac{8}{3} - \frac{2}{3} + \frac{1}{6} \, d\theta$$

$$= \int_{0}^{2\pi} \frac{5}{6} \, d\theta$$

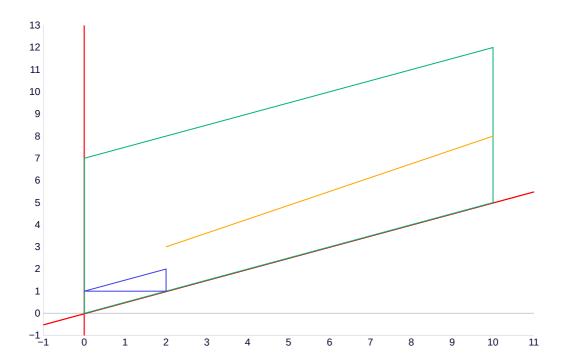
$$= \frac{5\pi}{3}$$

$$\approx 5.236$$

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$$egin{array}{l} \pi/2 & \pi/2 & 1 \ \int\limits_{0}^{\pi/2} \int\limits_{0}^{\pi/2} \int\limits_{0}^{\pi/2} r^3 \sin^2\phi \cos\theta \ dr d\theta d\phi \ &= \int\limits_{0}^{\pi/2} \int\limits_{0}^{\pi/2} rac{1}{4} \sin^2\phi \cos\theta \ d\theta d\phi \ &= \int\limits_{0}^{\pi/2} rac{1}{4} \sin^2\phi \ d\phi \ &= \int\limits_{0}^{\pi/2} rac{1-\cos2\phi}{8} \ d\phi \ &= rac{\pi}{16} \ pprox 0.196 \end{array}$$

15.6



$$J=egin{bmatrix} 3 & 4 \ 1 & -2 \end{bmatrix}=-10$$

$$J = egin{array}{ccc} \cos heta & -r \sin heta \ \sin heta & r \cos heta \ \end{vmatrix} = r \cos^2 heta + r \sin^2 heta = r \ \end{vmatrix}_{(4,rac{\pi}{6})} = 4$$

$$G(u,v) = (5u+3v,u+4v)$$

$$J(G)=egin{bmatrix} 5 & 3 \ 1 & 4 \end{bmatrix}=17$$

$$egin{aligned} &\iint_{D_0} (5u+3v)(u+4v)17\ dA \ &= \int\limits_0^1 \int\limits_0^1 (5u+3v)(u+4v)17\ dudv \ &= \int\limits_0^1 (5/3+23v/2+12v^2)17\ dv \ &= (5/3+23/4+4)17 \end{aligned}$$

$$=2329/12 \\ \approx 194.083$$

$$J(G) = egin{bmatrix} rac{1}{v} & rac{-u}{v^2} \ v & u \end{bmatrix} = rac{2u}{v}$$

$$egin{array}{l} \int\limits_{1}^{4} \int\limits_{1}^{4} rac{2u}{v} \; du dv \ = \int\limits_{1}^{4} rac{15}{v} \; dv \ = 15 \ln 4 \end{array}$$

$$J(G)=egin{bmatrix} 1 & -2 \ 0 & 1 \end{bmatrix}=1$$

$$D_0 = [6,10] imes [1,3]$$

$$egin{aligned} &\iint_{D_0}(u+v)\;dA\ &=\int\limits_6^{10}\int\limits_1^3(u+v)\;dvdu\ &=\int\limits_6^{10}(4+2v)\;du\ &=(16+64)\ &=80 \end{aligned}$$