

A

$$\vec{F}(x, y) = \langle -y \sin x + 2x, \cos x \rangle$$

$$\frac{\partial F_1}{\partial y} = -\sin x$$

$$\frac{\partial F_2}{\partial x} = -\sin x$$

Since $\frac{\partial F_1}{\partial y} = \frac{\partial F_2}{\partial x}$, this means that the vector field is conservative

B

$$\begin{aligned} \int -y \sin x + 2x \, dx \\ = y \cos x + x^2 + C \end{aligned}$$

$$\begin{aligned} \int \cos x \, dy \\ = y \cos x + C \end{aligned}$$

$$f(x, y) = y \cos x + x^2 + C$$

C

Since \vec{F} is conservative, a line integral on R^2 of \vec{F} is equivalent to the difference in the potential function of the beginning and end of the curve.

$$a = (0, -1)$$

$$b = (3, 0)$$

$$\int_C \vec{F}(x, y) \, d\vec{r}$$

$$= f(3, 0) - f(0, -1) = 9 - -1 = 10$$