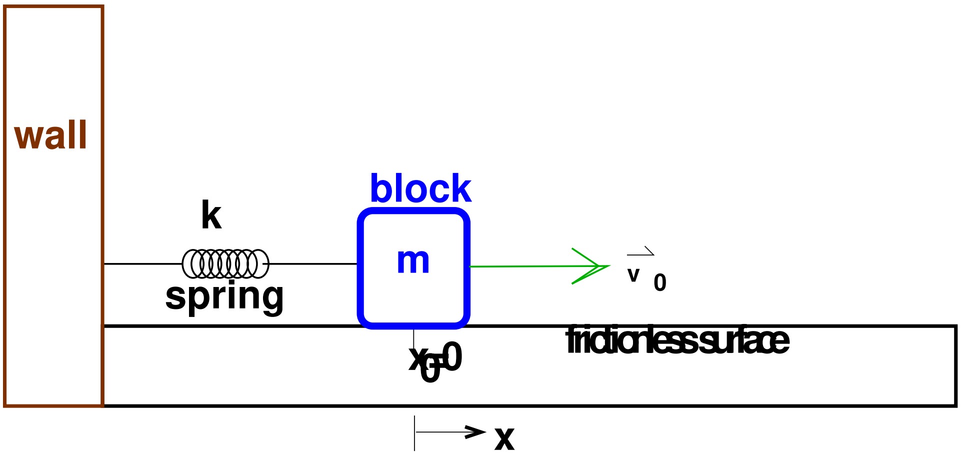
1

Suppose a block of given mass m is attached to a spring of given spring constant k and released on a frictionless surface with an initial horizontal velocity given as v_0 at time $t = 0$. At time $t = 0$ the block is located at the equilibrium position corresponding to $x = 0$ as shown:



We showed in class that the Equation of Motion for a Simple Harmonic Oscillator is this:

$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$

We also showed that the general solution for the Equation of Motion of a Simple Harmonic Oscillator is given by this equation:

$x(t) = B\cos(\omega t) + C\sin(\omega t)$

Determine the particular values of ω, B, and C that correspond to this particular system. Give your answer in terms of the given parameters: m, k, and v_0. Explain how you know this. Hint: Consider the value of x at time $t = 0$. Now consider the value of $v = \frac{dx}{dt}$ at time $t = 0$.

Answer

$x(t) = B\cos(\omega t) + C\sin(\omega t)$
$\dot{x}(t) = -B\omega\sin(\omega t) + C\omega\cos(\omega t)$
$\ddot{x}(t) = -B\omega^2\cos(\omega t) - C\omega^2\sin(\omega t)$
$\ddot{x}(t) + \frac{k}{m}x(t) = 0$

$x(0) = 0$
$0 = B$

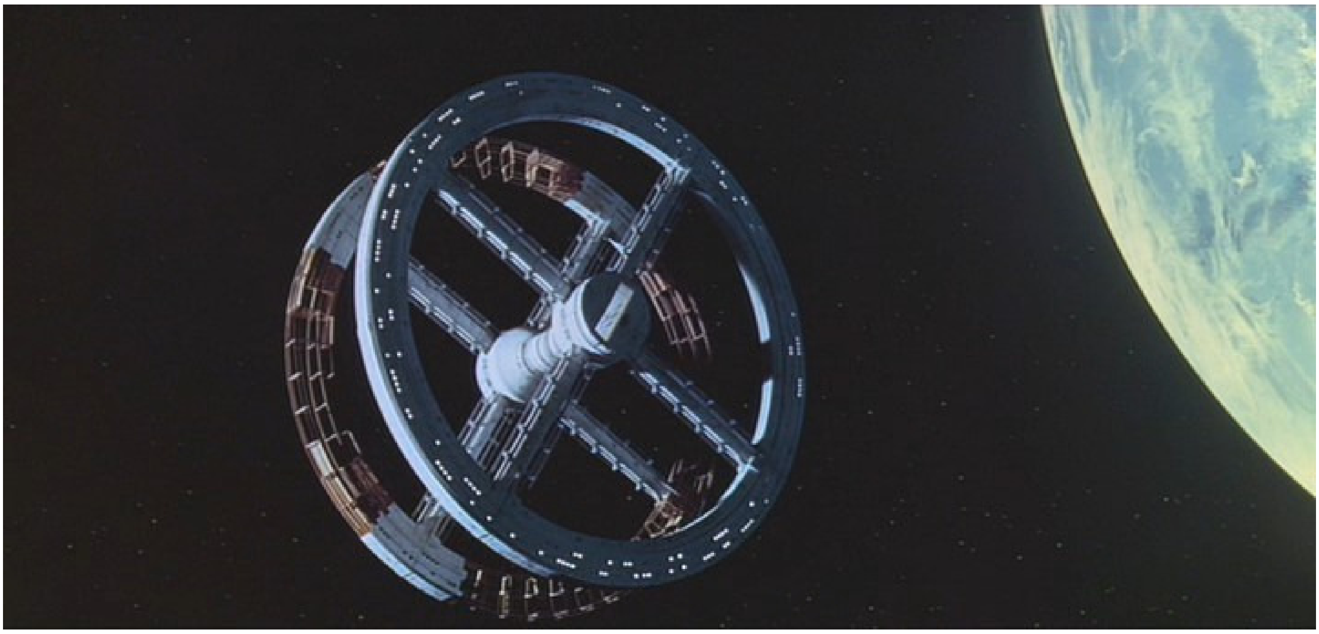
$\dot{x}(0) = v_0$
$v_0 = C\omega$

$$\frac{Ck\sin(\omega t)}{m} = C\omega^2\sin(\omega t)$$
$$\frac{k}{m} = \omega^2$$
$$\omega = \sqrt{\frac{k}{m}}$$
$$C = v_0\sqrt{\frac{m}{k}}$$

$$B = 0$$
$$C = v_0\sqrt{\frac{m}{k}}$$
$$\omega = \sqrt{\frac{k}{m}}$$
$\square$

2

Suppose a space-station is designed in a shape of a torus such as this one depicted in Stanley Kubrick's "2001: A space Odyssey" below:



a

Suppose the space station has a diameter of 175 meters. At what rotational rate should the station be spun so that occupants standing on the inside surface of the outer wall (maximum distance from the center) experience earth-normal artificial gravity? Give your answer in terms of rotations per minute.

✓ Answer

$$a_c = \frac{v^2}{r}$$
$$a_c = g$$

$v = \omega r$
$g = \omega^2 r$
$\omega = \sqrt{\frac{g}{r}}$

$\omega = \sqrt{\frac{g}{r}}$
$\omega = 0.2366\ s^{-1}$
$\omega = 2.260\ rpm$

b

Suppose that inside the space station there is a transport cart for transport around the rim that can reach a maximum speed of 25.0 meters per second. Using such a cart, how could you tell which way the space station from spinning from inside without looking out any windows? Be quite specific about what you experience when you drive around with the cart in one direction or the other.

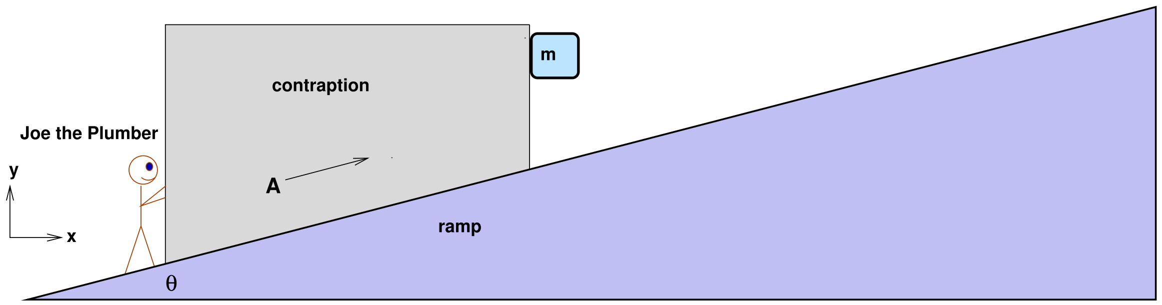
✓ **Answer**

When you are moving the same direction the station is spinning, with a non-moving reference frame from outside the station, your velocity will be higher than if you were not moving, which leads to a higher centripetal acceleration, making you feel more weight.

If you were moving the opposite direction, your total linear velocity will be less, and so will your centripetal acceleration, leading to a feeling of less weight.

3

Joe the plumber pushes a contraption up a ramp at a given small angle θ as shown at a given constant acceleration $\vec{A}$. A small box of mass m is in contact with the vertical surface of the contraption as shown.



a

Assume that there is static friction between the small box and the contraption. The coefficient of static friction between the box and the contraption is μ. Calculate the direction and the magnitude of all forces on the small box in terms of the given parameters. Explain your work.

✓ Answer

There is a weight force, normal force, and friction force.

$W = mg$ Downwards
$N = Am\sin\theta$ Rightwards
$f < \mu Am\sin\theta$ Upwards

If $f < N$ then the box will fall off, else the box will stay still in reference to the big box.

b

Suppose the small box contains a mini-aquarium with water and fish. Assuming that the system is in equilibrium, with what angle will the surface of the water inside the aquarium make with respect to the horizontal? Explain how you know this.

✓ Answer

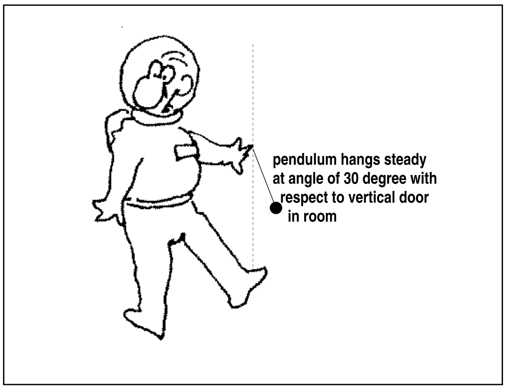
Using the sine and cosine rule of triangles, we can calculate the overall acceleration of the box.

$$a^2 + b^2 - ab\cos C = c^2$$
$$c = \sqrt{W^2 + A^2 + 2WA\sin\theta}$$
$$\theta' = \arcsin\left(\frac{\cos\theta}{A}\sqrt{W^2 + A^2 + 2WA\sin\theta}\right)$$
$$\theta' = \arcsin\left(\frac{A\cos\theta}{\sqrt{(mg)^2 + A^2 + 2mgA\sin\theta}}\right)$$

Since we calculated the deviance of the normal vector of the surface compared to the direction of the weight force, this angle should be the same as the angle between the water and the horizontal. We know this by the addition of vectors.

4

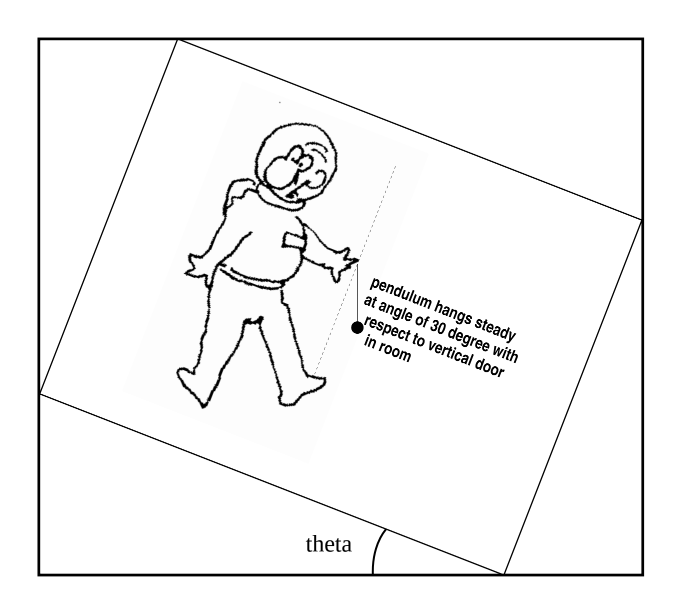
After a disorienting and ill-advised episode involving mind-altering substances, Ringo inexplicably finds himself in a locked room with no windows. The room appears normal in every respect except that the whole room appears to be tilted at an angle of 30 degrees relative to level. Ringo determines this tilt by holding up a small pendulum relative to the door-frame that is aligned with the room and measuring with his protractor that the pendulum hangs steady at 30 degrees relative to the door frame. There is a loud vibrating fan in the room that makes it impossible for Ringo to get any noise or vibration from outside the room. Ringo concludes that either he is in in a tilted room, or else he is in a room that is moving in a special way to give the illusion that the room is
tilting.



a

Ringo is trying to guess what is going on. First, he assumes that the room is not moving at all but is simply tilted. Draw a simple sketch indicating how the room looks from the outside (just draw a box and indicate the tilt angle).

✓ Answer



b

Now Ringo is wondering if his observations can be explained as a result of the motion of the room rather than a tilt. Suppose we assume that the room is not really tilted but is actually moving horizontally with constant velocity. Can Ringo's observations be explained under this assumption? Explain your answer.

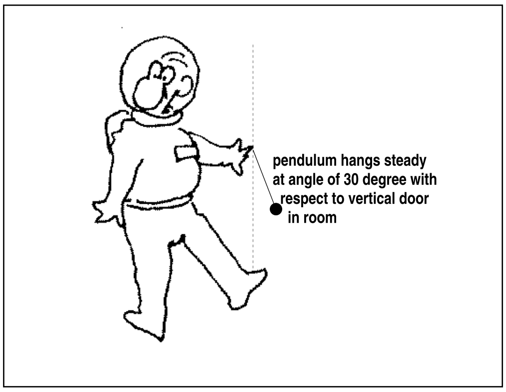
✓ **Answer**

Yes, he could be under the influence of gravity as well as accelerating on a non-vertical direction. The addition of both his acceleration and gravitational force would lead his pendulum to swing to a non-vertical direction

c

Now assume that the room is not really tilted but is actually accelerating with constant acceleration in the horizontal direction. Can Ringo's observations be explained under this assumption? If so, what is the magnitude of this acceleration?
Hint: work this problem in the inertial frame where you can use Newton's second law. Consider the forces and acceleration on Ringo's pendulum. Draw a sketch that shows the room and the pendulum as it would appear to an outside observer.

✓ **Answer**



We can use the law of sines to find the magnitude of the acceleration.

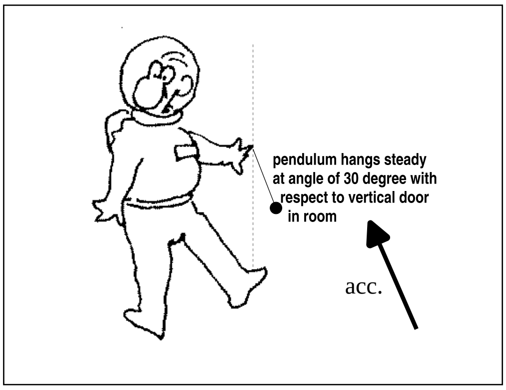
$$\frac{W}{\cos\theta} = \frac{A}{\sin\theta}$$
$$A = W\tan\theta$$

$$A = \frac{W}{\sqrt{3}}$$
□

d

Suppose in fact that Ringo is not on the surface of the Earth but has been kidnapped by space aliens so that the whole room is really on an accelerating space ship in deep space. Draw a sketch to indicate the direction that space ship is accelerating relative to the room. What the is magnitude of the acceleration?

✓ **Answer**



e

Is there any physics experiment or any other test that Ringo can do to verify that he is in an accelerating spaceship in deep space vs. sitting in a room on Earth? If not, why not?

✓ **Answer**

No, because he does not know what is outside the box, he cannot tell what the acceleration is from, it could be from a combination of things or it may just be gravity or acceleration.

5

A bear travels 100 miles due south (precisely). Then he travels 100 miles due east. Then he travels 100 miles due north, to arrive at the exact same location from where he started. What color is the bear?

(This problem is demonstrates the effects of “curved space” on geometric objects, such as triangles). Based on this example, describe how – in principle – a space traveler might be able to measure the curvature of space due to gravity by traveling and very carefully measuring distances and angles. Here, imagine that somehow the practical constraints on traveling and communicating over great cosmological distances could be easily overcome.

✓ **Answer**

By his travel itinerary, this bear must be from the North Pole, and according to my trusty map of bear color vs. location, he must be white.