

# Inclined Plane

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## Abstract:

I have tested the theory of Newton's Second Law of Motion with a system of a cart on an inclined plane connected to a counterweight by a string over a pulley. After releasing the system from rest, I measured the velocity as a function of time. According to Newton's theory, the velocity should vary *linearly* with time. The data that I have collected does not support a linear dependence between velocity and time to within the uncertainties on the data points. I have also measured the average acceleration of the system as  $a_{meas} = \underline{0.290} \pm \underline{0.008} \text{ m/s}^2$ . Newton's Second Law predicts that the acceleration of the system should be  $a_{pred} = \underline{0.382} \pm \underline{0.003} \text{ m/s}^2$ . I find that the measured acceleration is not consistent with the predicted acceleration. Because of a few factors, namely being friction of the string rubbing against the table and the track not being completely frictionless, the pulley and string not being massless.

*(If your measured velocity supports a linear model and/or your accelerations are consistent with the predicted acceleration, cross out the "not's" in the above abstract paragraph. Give a one-sentence conclusion about the lab.)*

## Theory and Background:

One can determine the acceleration of the system depicted in Figure 1 by using Newton's Second Law to analyze the motion. Assuming that the frictional force  $\vec{f}$  is negligible and that the pulley is massless and frictionless, the acceleration  $a$  of the system is<sup>1</sup>

$$g \frac{(m_1 - m_2 \sin \theta)}{m_1 + m_2} \quad (1)$$

where  $m_1$  is the mass of block 1 in the diagram  
and  $m_2$  is the mass of block 2 in the diagram  
and  $\theta$  is the angle of the slope

(Write down the appropriate equation; define all variables that haven't been defined yet.)

One can find the sine of the angle of the incline  
using Eq. 1. If we adjust  $m_1$  and  $m_2$  so that the

acceleration is zero and call this hanging mass the balancing mass  $m_b$ , then

$$0 = \frac{g(m_1 - m_2 \sin \theta)}{m_1 + m_2} \quad (2)$$

$$\Rightarrow 0 = m_b - m_2 \sin \theta \quad (3)$$

$$\Rightarrow m_b = m_2 \sin \theta \quad (4)$$

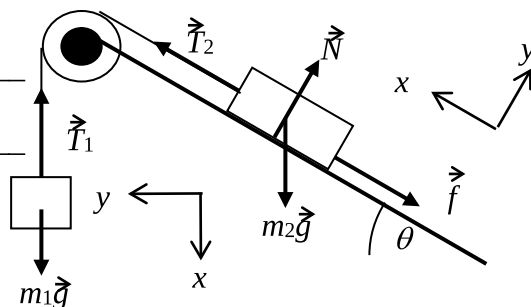
(Eq. 2 should be Eq. 1 with  $a = 0$ ; Eq. 3 should be an intermediate algebra step; Eq. 4 should be  $\sin \theta$  in terms of  $m_b$  and  $m_2$ .)

Equation 1 also implies that the acceleration of the system will be constant, so the velocity  
as a function of time will be

$$v(t) = v_0 + at \quad (5)$$

where  $v_0$  is the initial velocity

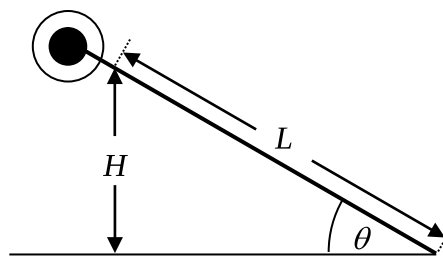
(For equation 5 write down the expression for velocity in terms of time and other variables.  
Explain any new variables you introduce.) From Eq. 5, we can see that if we fit a straight line to a  
plot of  $v$  vs.  $t$ , the slope of the line will be the acceleration and the intercept will be the velocity at  
time zero.



**Figure 1:** Schematic of Forces in Experiment. Courtesy Driscoll, (year).

## Procedure:

To get an estimate of the angle of the incline I estimated the length  $L$  and height  $H$  of the incline as in Figure 2. I used a meter stick to measure  $H$  = 30.45 cm and  $L$  = 100.00 cm. Getting accurate and precise



**Figure 2:** Estimating the angle.  
Courtesy Driscoll, (year).

measurements of  $H$  and  $L$  was difficult because The track was slanted and it was difficult to line it up  
I compensated for these difficulties by Asking my partner to step back and let me know if it was straight

Because of these issues, I estimate that my uncertainty in  $H$  is  $\delta_H =$  0.01 cm and my uncertainty in  $L$  is  $\delta_L =$  0.01 cm.

My first estimate of  $\sin\theta$  is then

$$\sin\theta = \frac{H}{L} \quad (6)$$

$$\Rightarrow \sin\theta = \frac{30.45 \text{ cm}}{100.00 \text{ cm}} = \underline{0.3045}$$

(Put the appropriate variables in the first line; put your actual measurements and final value for  $\sin\theta$  in the second line.)

The uncertainty in  $\sin\theta$  is

$$\delta_{\sin\theta} = \sqrt{\delta_{\sin\theta,H}^2 + \delta_{\sin\theta,L}^2} \quad (7)$$

where  $\delta_{\sin\theta,H}$  is the uncertainty in  $\sin\theta$  due to  $\delta_H$  and  $\delta_{\sin\theta,L}$  is the uncertainty in  $\sin\theta$  due to  $\delta_L$ .

Using the “computational method” to determine  $\delta_{\sin\theta,H}$  and  $\delta_{\sin\theta,L}$ , I obtain

$$\delta_{\sin\theta,H} = \delta_H \frac{1}{L} \quad (8)$$

and

$$\delta_{\sin \theta, L} = \delta_L \frac{H}{L^2} \quad (9)$$

yielding  $\delta_{\sin \theta} = \sqrt{\left(\delta_{H \frac{1}{L}}\right)^2 + \left(\delta_{L \frac{-H}{L^2}}\right)^2} = 0.0001$ , or  $\sin \theta = 0.3045 \pm 0.0001$ . Since  $\theta =$

$\sin^{-1}(\sin \theta)$ , the uncertainty in  $\theta$  is

$$\delta_{\theta} = \frac{\delta_{\sin \theta}}{\sqrt{1 - \sin^2 \theta}}$$

or  $\theta = 0.3094 \pm 0.0001$  rad.

I measured  $m_2$  with an electronic balance and determined that  $m_2 = 489. \pm 1. \text{ g}$ .

I estimated the uncertainty  $\delta_{m_2}$  as 1. g because That was the adjusted std. dev. & also the resolution of the scale.

I now used the first estimate of  $\theta$  to determine an estimate of the mass  $m_b$  required to balance the system by taking Eq. 4 and solving for  $m_b$ :

$$m_b = \sin \theta m_2 \quad (11)$$

$$\Rightarrow m_b = 149.9$$

(Solve Eq. 4 for  $m_b$ , substitute in the appropriate numbers, and solve.)

I then set the mass of  $m_1$  to 149 g by adding masses to the hanger. After releasing the cart, the system was not in balance; the ~~system~~ cart moved down.

(If the system was balanced, cross out "not." Describe the system's motion in the blank.)

I then found the minimum and maximum masses that lead to zero acceleration ( $m_{\min}$  and  $m_{\max}$ ) by adding and removing mass from  $m_1$  in order to obtain a better estimate of the angle of the incline and to account for the small amount of friction in the system. \_\_\_\_\_

Started with a stationary cart.

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*(State if you tested zero acceleration by a stationary cart or cart moving with constant velocity. If you used the constant velocity test, also state how you determined the cart was moving at constant speed. Write a sentence about your reasons for choosing your methods, i.e., the advantages and disadvantages of your methods over other choices.)*

Using the procedure above, I determined that  $m_{\min} = \underline{150.6 \pm 0.1 \text{ g}}$  and  $m_{\max} = \underline{158.1 \pm 0.1 \text{ g}}$ , each ~~with negligible uncertainty~~. I then set the average of  $m_{\min}$  and  $m_{\max}$  to be the “balancing mass”  $m_b$  and half the difference between  $m_{\min}$  and  $m_{\max}$  as the uncertainty  $\delta_{mb}$ , so  $m_b = \underline{154} \pm \underline{4} \text{ g}$ .

Substituting  $m_b$  into Eq. 4, we see that sine of the angle of the incline is

$$\sin \theta = \frac{m_b}{m_2} = \underline{0.315}.$$

The uncertainty in  $\sin \theta$  is

$$\delta_{\sin \theta} = \sqrt{\delta_{\sin \theta, m_b}^2 + \delta_{\sin \theta, m_2}^2} \quad (12)$$

where  $\delta_{\sin \theta, m_b}$  is the uncertainty in  $\sin \theta$  due to  $\delta_{mb}$  and  $\delta_{\sin \theta, m_2}$  is the uncertainty in  $\sin \theta$  due to  $\delta_{m_2}$ .

Using the “computational method” to determine  $\delta_{\sin \theta, m_b}$  and  $\delta_{\sin \theta, m_2}$ , I obtain

$$\delta_{\sin \theta, m_b} = \delta_{m_b} \frac{1}{m_2} \quad (13)$$

and

$$\delta_{\sin \theta, m_2} = \delta_{m_2} \frac{m_1}{m_1^2} \quad (14)$$

yielding  $\delta_{\sin \theta} = \sqrt{\left(\delta_{\frac{1}{m_2}}\right)^2 + \left(\delta_{\frac{m_1}{m_2^2}}\right)^2} = 0.008$ , or  $\sin \theta = 0.315 \pm 0.008$ .

which is moderately higher than the previous calculation.

(Compare this value of  $\sin \theta$  with the value from direct measurement of  $L$  and  $H$ .)

I will adopt this value for  $\sin \theta$ .

I then set the counterweight  $m_1$  to a value of 180 g to allow the cart to accelerate up the plane. I will refer to this value of  $m_1$  as the experiment value  $m_e$ . I recorded the motion of the cart using an encoded pulley and *Logger Pro* software.<sup>2</sup> I subsequently exported the data from *Logger Pro* to *Origin* for a more complete analysis. Specifically, I plotted velocity vs. time to determine if the velocity has a linear dependence and to measure the slope. Previous experimenters<sup>3</sup> using this equipment have determined that the uncertainty in  $v$  has a value of 0.008 m/s; we adopted this value in our analysis.

## Results:

The average acceleration recorded directly by *Logger Pro* statistics software was  $a_{\text{meas1}} = 0.382 \pm 0.003$  m/s<sup>2</sup>. Figure 3 shows a plot of velocity vs. time and a best linear fit using the *Origin* software. (Attach a copy of your  $v$  vs.  $t$  graph labeled “Figure 3” to the end of the report.) For this plot, vertical error bars are assigned based on an estimated uncertainty of the velocity measurements of  $\pm 0.003$  m/s<sup>2</sup> for each point, where this value was determined by previous

measurements done by the laboratory staff. As can be seen in the plot, the since not every data point lies within about one error bar of the best linear fit we conclude that these data are not consistent with Newton's model. *(If the data points fit the line to within about one error bar then delete the words "not" above. If the data are not consistent be sure to address this in your conclusion. Is Newton wrong? Or might there be systematic error in your data?)*

The slope of the graph as determined by Origin's fitting software is  $a_{\text{meas2}} = \underline{0.388} \pm \underline{0.003} \text{ m/s}^2$ . In comparing this value to the value obtained directly from Logger Pro I note that these two values agree (agree/do not agree to within their uncertainties/are exactly the same). We expect that  $a_{\text{meas2}}$  should be more accurate because calculated slope over time instead of an average inst. and I will adopt it as the measured value,  $a_{\text{meas}}$ .

By substituting in known values into Eq. 1, one can determine a theoretical value for the acceleration of the system,  $a_{\text{pred}}$ . Since I did not measure  $\theta$  directly, I will substitute Eq. 4 into Eq. 1 to obtain:

$$a_{\text{pred}} = g \frac{m_1 - m_2}{m_1 + m_2} \quad (15)$$

$$\Rightarrow a_{\text{pred}} = \underline{0.381} \text{ m/s}^2.$$

### Error Analysis:

To find the uncertainty in  $a_{\text{pred}}$ ,  $\delta_{a_{\text{pred}}}$ , I must find the contribution to  $\delta_{a_{\text{pred}}}$  for each of the quantities in Eq. 15 and add them in quadrature. The uncertainties in  $m_1$  &  $m_2$  are negligible compared to the other quantities because they were directly measured.

(Identify any quantities that you will treat as having negligible uncertainty and justify your treatment. Then show your work in estimating the uncertainty in  $a_{pred}$  on the next page.)

$m_1$  &  $m_2$  both have errors around 0.1 while  $m_b$  has an error of 4.

$$\delta a \approx \delta_{a, m_b} = \delta_{m_b} \frac{-g}{m_1 + m_2} = 0.06 \text{ m/s}^2$$

So  $a_{pred} = 0.38 \pm 0.06 \text{ m/s}^2$ .

### Conclusions:

The predicted value for the acceleration was  $a_{pred} = 0.38 \pm 0.06 \text{ m/s}^2$  and the measured value for the acceleration of the system was  $a_{meas} = 0.388 \pm 0.003 \text{ m/s}^2$ .

They agree quite closely.

(State whether or not your values agree within their uncertainties. If they do not agree, suggest at least one source of systematic error that were not adequately accounted for and suggest a way to reduce the effect of this error. If the two values do agree, suggest at least one source of random error and suggest a way to reduce the effect of this error. Make a quantitative statement about the effect friction should have had on this experiment. Make a conclusion about Newton's Second Law, especially regarding whether or not the data points support a linear model. If the model does not show a linear dependence give at least one reason why this might not be.)



## Acknowledgements:

I would like to thank Katherine, Case Department of Physics, for \_\_\_\_\_  
help in obtaining the experimental data and preparing the figures. \_\_\_\_\_

& Mr Phillip  
\_\_\_\_\_

*(Thank your lab partner(s). If they or anyone else gave you additional assistance, say who they were and specifically what their assistance was.)*

## References:

*(If you have any additional references, list them below. Make sure to indicate with an endnote where in the report you referred to the reference.)*

1. Driscoll, D., *General Physics I: Mechanics Lab Manual*, "Inclined Plane," CWRU Bookstore, 2014.
- 2.

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## End Notes:

<sup>1</sup> Driscoll, D., p. 2.

<sup>2</sup> Driscoll, D., p. 3, describes the encoded pulley.

<sup>3</sup> Driscoll, D., p. 5

Trevor and Katherine Feb 7, 2024

