1

a

$$P(X,Y|e) = \frac{P(X,Y,e)}{P(e)}$$
$$= \frac{P(X,Y,e)}{P(e)} \frac{P(Y,e)}{P(Y,e)}$$
$$= \frac{P(X,Y,e)}{P(Y,e)} \frac{P(Y,e)}{P(e)}$$
$$= P(X|Y,e)P(Y|e)$$

b

$$P(Y|X,e)P(X|e) = P(Y,X|e)$$
$$P(X,Y|e) = P(X|Y,e)P(Y|e)$$

$$P(Y|X,e) = \frac{P(Y,X|e)}{P(X|e)}$$
$$= \frac{P(X|Y,e)P(Y|e)}{P(X|e)}$$

2

a

C

According to the probability law for Bayes's nets

$$P(x_1,\ldots,x_n) = \prod_{i=1}^{n} \theta(x_i|\text{parents}(X_i))$$

Therefore

$$P(G_{father},G_{mother},G_{child}) = P(G_{child})P(G_{mother})P(G_{father})$$

b

A, as it shows that H_{child} is mainly influenced by G_{child} and also that G_{child} is determined mainly by G_{father} and G_{mother}

c

A, for the same reasons

d

s is not involved as that is the probability that the G_{child} matches H_{child} and we are only calculating G_{child}

$P(G_{child} = r)$	$G_{father} = r$	$G_{father} = l$
$G_{mother} = r$	$1.0 - m$	0.5
$G_{mother} = l$	0.5	m

e

$$P(G_{child} = l) = \sum_{G_{mother}} \sum_{G_{father}} P(G_{child} = l, G_{father}, G_{mother})$$
$$= \sum_{G_{mother}} \sum_{G_{father}} P(G_{child} = l | G_{father}, G_{mother}) P(G_{father}) P(G_{mother})$$

$P(G_{child} = l)$	$G_{father} = r$	$G_{father} = l$
$G_{mother} = r$	m	0.5
$G_{mother} = l$	0.5	$1.0 - m$

$$= m(1-q)^2 + q(1-q) + q^2(1-m)$$
$$\square$$

$$q = m(1-q)^2 + q(1-q) + q^2(1-m)$$
$$= m - 2mq + mq^2 + q - q^2 + q^2 - mq^2$$
$$= m - 2mq + q$$

$$0 = m - 2mq$$
$$2mq = m$$
$$q = 0.5$$

This is not true because the real left-handedness in humans is significantly lower than 0.5 of the population, meaning it is likely not simply inherited through the means predicted.

3

a

ii, iii as adding more givens does nothing once that area of the net is already detached from the query

b

$$P(b, u, \neg m, g, j) = P(b)P(\neg m)P(i|b, \neg m)P(g|i, b, \neg m)P(j|g)$$
$$= 0.9 * 0.9 * 0.5 * 0.8 * 0.9 = 0.2916$$

c

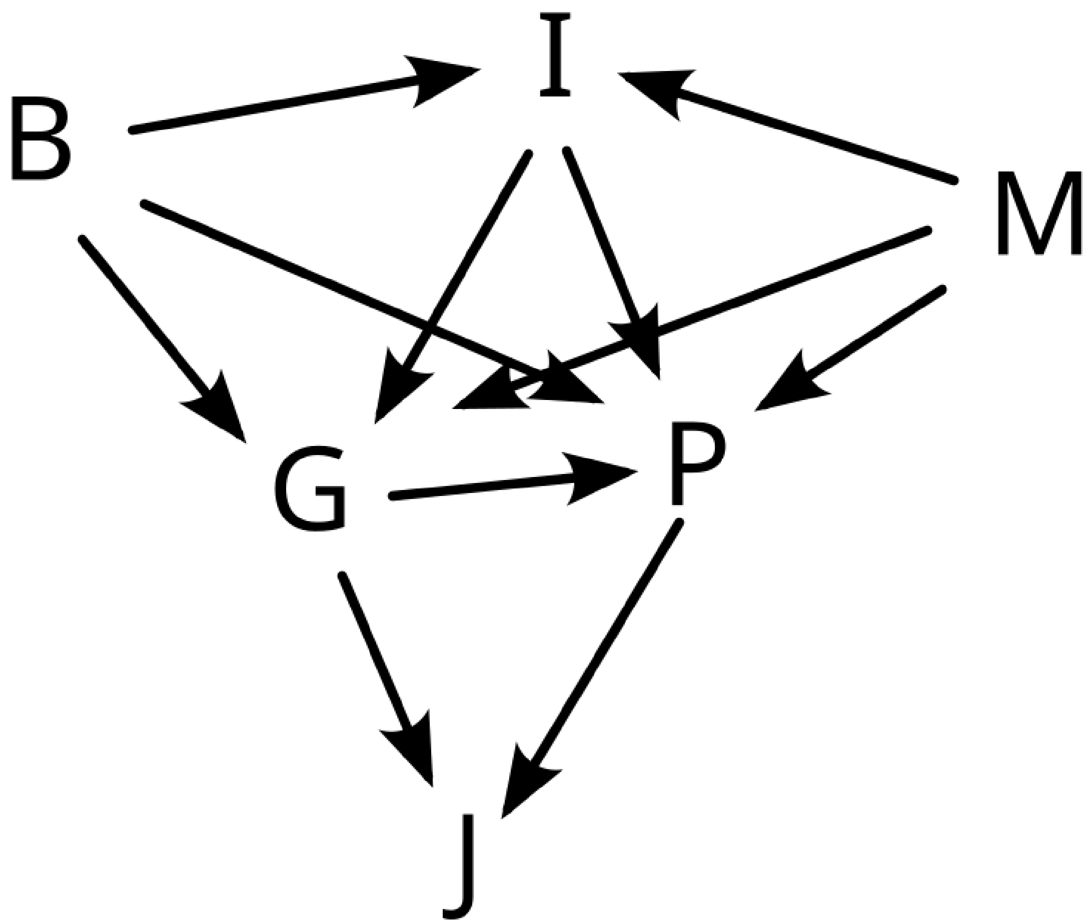
$$P(j|b, i, m) = \frac{\sum_{g \in \{t,f\}} P(j,b,i,m,g)}{\sum_{j,g \in \{t,f\}} P(j,b,i,m,g)}$$

$$= \frac{0.9*0.9*0.1*(0.9*0.9+0.1*0)}{0.9*0.9*0.1*(0.9*0.9+0.9*0.1+0.1)}$$
$$= \frac{0.06561}{0.081}$$
$$= 0.81$$

d

Whenever Indicted is not true.

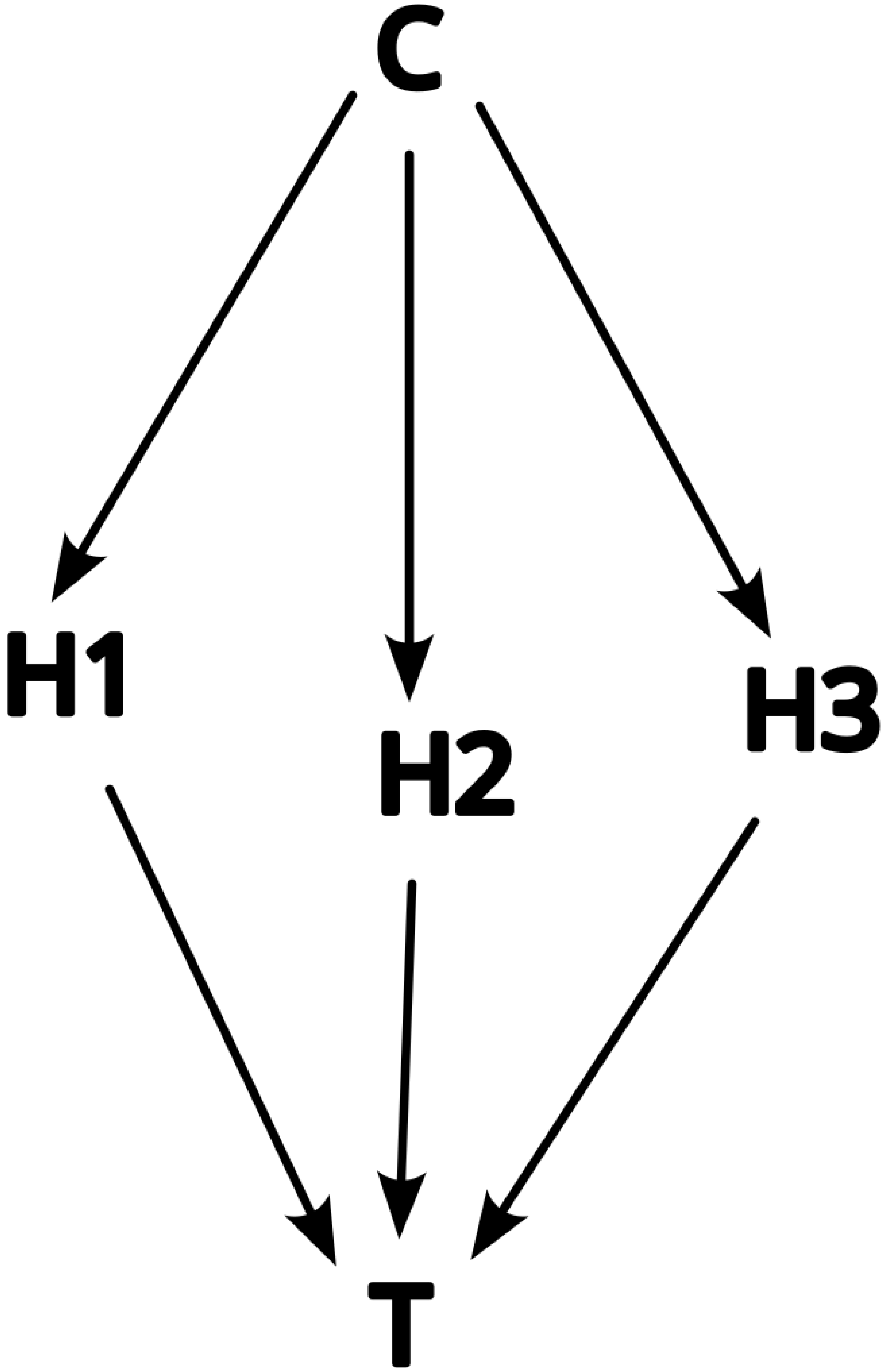
e



Because the president will likely take into account most of the other factors when giving a pardon and will only impact whether they go to jail or not

4

a



C for coin selected and Hx for result of a traial and T for total heads flipped.

C	P(C)
a	1/3

C	P(C)
b	1/3
c	1/3

Hx|C	P(Hx=h)
a	0.2
b	0.6
c	0.8

$$T = \sum_{H \in \{H_1, H_2, H_3\}} H$$
$$T \sim \text{binom}(3, P(H_x = h))$$

b

$$P(C|T = 2)$$

$$P(T = 2|C = a) = \binom{3}{2} 0.2^2 0.8 = 0.096$$
$$P(T = 2|C = b) = \binom{3}{2} 0.6^2 0.4 = 0.432$$
$$P(T = 2|C = c) = \binom{3}{2} 0.6^2 0.4 = 0.384$$

$$P(C = a, T = 2) = P(T = 2|C = a)P(C = a) = 0.032$$
$$P(C = b, T = 2) = P(T = 2|C = b)P(C = b) = 0.144$$
$$P(C = c, T = 2) = P(T = 2|C = c)P(C = c) = 0.128$$

$$P(C = a|T = 2) = \frac{P(C=a, T=2)}{\sum_{C \in \{a,b,c\}} P(C, T=2)} = 0.105$$
$$P(C = b|T = 2) = \frac{P(C=b, T=2)}{\sum_{C \in \{a,b,c\}} P(C, T=2)} = 0.474$$
$$P(C = c|T = 2) = \frac{P(C=c, T=2)}{\sum_{C \in \{a,b,c\}} P(C, T=2)} = 0.421$$

Thus, it is most likely that $C = b$, the coin selected is the b coin.

5

a

$M \in \{w, d, l\}$ where a win is defined as the first team winning

$T_x \in \{0, 1, 2, 3\}$ where T_x is defined as the quality of the team

$$P(M = w|T_1, T_2) = P(T_1 > T_2)$$
$$P(M = d|T_1, T_2) = P(T_1 = T_2)$$
$$P(M = l|T_1, T_2) = P(T_1 < T_2)$$

Defines a model where a team wins a match if their quality is higher, and tie if they are equal.

b

It would look exactly the same as M solely depends on T_1 and T_2, just like the probabilistic model. Also since we do not know the distribution of the quality points.

c

$$P(M = d|T_1 = T_a, T_2 = T_c) = 1 = P(T_a = T_c)$$
$$\implies T_a = T_c$$

$$P(M = w|T_1 = T_a, T_2 = T_b) = 1 = P(T_a > T_b)$$
$$\implies T_a > T_b$$

$$P(M = w|T_1 = T_b, T_2 = T_c) = P(T_b > T_c)$$
$$\implies T_c = T_a > T_b$$
$$\implies T_c > T_b$$
$$P(T_b > T_c) = 0$$
$$P(M = w|T_1 = T_b, T_2 = T_c) = 0$$