2.17

a

$$f(x) = 3x^2 \qquad , 0 < x < 1$$

$$P(X > m) = P(X < m)$$

$$F(m) = 1 - F(m)$$

$$F(m) = 0.5$$

$$F(x)=\int\limits_0^x f(t)dt$$

$$F(x)=x^3$$

$$0.5=m^3$$

$$m=0.7937$$

b

$$f(x) = rac{1}{\pi(1+x^2)} \qquad , -\infty < x < \infty$$

$$P(X > m) = P(X < m)$$

$$F(m) = 1 - F(m)$$

$$F(m) = 0.5$$

$$F(x) = \int\limits_{-\infty}^x f(t) dt$$

$$F(x) = \arctan(x)/\pi + 0.5$$

$$0.5 = \arctan(m)/\pi + 0.5$$

$$m = 0$$

2.24

a

$$f(x) = ax^{a-1}$$
 , $0 < x < 1, a > 0$

EX

$$EX=\int\limits_{0}^{1}xf(x)dx$$

$$EX=\int\limits_{0}^{1}ax^{a}dx$$

$$EX=\left.rac{ax^{a+1}}{a+1}
ight|_0^1$$

$$EX = \frac{a}{a+1}$$

EX^2

$$egin{aligned} EX^2 &= \int\limits_0^1 x^2 f(x) dx \ EX^2 &= \int\limits_0^1 a x^{a+1} dx \ EX^2 &= \left. rac{a x^{a+2}}{a+2}
ight|_0^1 \ EX^2 &= rac{a}{a+2} \end{aligned}$$

 σ^2

$$\sigma^2 = EX^2 - (EX)^2$$
 $\sigma^2 = \frac{a}{a+2} - (\frac{a}{a+1})^2$
 $\sigma^2 = \frac{a}{a+2} - \frac{a^2}{(a+1)^2}$
 $\sigma^2 = \frac{a^3 + 2a^2 + a}{(a+2)(a+1)^2} \frac{a^3 + 2a^2}{(a+2)(a+1)^2}$
 $\sigma^2 = \frac{a}{(a+2)(a+1)^2}$

b

$$f(x)=1/n \qquad , x=1,2,\ldots,n, n>0 ext{ an integer}$$

EX

$$EX = \sum\limits_{x=1}^{n} x f(x)$$
 $EX = \sum\limits_{x=1}^{n} x/n$ $EX = n(n+1)/2n$ $EX = (n+1)/2$

EX^2

$$egin{aligned} EX^2 &= \sum\limits_{x=1}^n x^2 f(x) \ EX^2 &= \sum\limits_{x=1}^n x^2/n \ EX^2 &= (n+1)(2n+1)/6 \end{aligned}$$

σ^2

$$egin{aligned} \sigma^2 &= EX^2 - (EX)^2 \ \sigma^2 &= (n+1)(2n+1)/6 - ((n+1)/2)^2 \ \sigma^2 &= rac{2(n+1)(2n+1)-3(n+1)^2}{12} \ \sigma^2 &= rac{n^2-1}{12} \end{aligned}$$

C

$$f(x) = 3(x-1)^2/2$$
 , $0 < x < 2$

EX

$$egin{aligned} EX &= \int\limits_0^2 x f(x) dx \ EX &= rac{3}{2} \int\limits_0^2 x^3 - 2x^2 + x dx \ EX &= rac{3}{2} (x^4/4 - 2x^3/3 + x^2/2)igg|_0^2 \ EX &= 1 \end{aligned}$$

EX^2

$$egin{aligned} EX^2 &= \int\limits_0^2 x^2 f(x) dx \ EX^2 &= rac{3}{2} \int\limits_0^2 x^4 - 2x^3 + x^2 dx \ EX^2 &= rac{3}{2} (x^5/5 - x^4/2 + x^3/3) igg|_0^2 \ EX^2 &= 8/5 \end{aligned}$$

σ^2

$$\sigma^2 = EX^2 - (EX)^2$$
 $\sigma^2 = 8/5 - 1$
 $\sigma^2 = 3/5$

2,28

b

$$f(x)=e^{-x} \qquad , x\geq 0$$

EX

$$EX=\int\limits_{0}^{\infty}xe^{-x}dx$$

u	dv
\boldsymbol{x}	e^{-x}
1	$-e^{-x}$
0	e^{-x}

$$EX = -xe^{-x} - e^{-x}igg|_0^\infty \ EX = 1$$

μ_2

$$egin{aligned} \mu_2 &= E((X-\mu)^2) \ \mu_2 &= \int\limits_0^\infty (x-1)^2 e^{-x} dx \end{aligned}$$

$$egin{align} \mu_2 &= \int\limits_0^\infty (x^2-2x+1)e^{-x} \ \mu_2 &= -(x^2+2x+2-2x-2+1)e^{-x}igg|_0^\infty \ \mu_2 &= -(x^2+1)e^{-x}igg|_0^\infty \ \mu_2 &= 1 \end{aligned}$$

μ_3

$$egin{align} \mu_3 &= E((X-\mu)^3) \ \mu_3 &= \int\limits_0^\infty (x-1)^3 e^{-x} dx \ \mu_3 &= \int\limits_0^\infty (x^3-3x^2+3x-1)e^{-x} \ \mu_3 &= -(x^3+3x^2+6x+6-3x^2-6x-6+3x+3-1)e^{-x}igg|_0^\infty \ \mu_3 &= -(x^3+3x+2)e^{-x}igg|_0^\infty \ \mu_3 &= 2 \end{aligned}$$

α_3

$$lpha_3=rac{\mu_3}{\mu_2^{3/2}} \ lpha_3=2$$

C

$$f(x)=rac{1}{\sqrt{2\pi}}e^{-x^2/2}$$

 μ

$$egin{aligned} EX &= \int \limits_{-\infty}^{\infty} x f(x) dx \ EX &= \int \limits_{-\infty}^{\infty} x rac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \ EX &= 0 \end{aligned}$$

μ_2

$$egin{aligned} \mu_2 &= E((X-\mu)^2) \ \mu_2 &= \int\limits_{-\infty}^{\infty} x^2 rac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \ \mu_2 &= 1 \end{aligned}$$

μ_4

$$egin{aligned} \mu_4 &= E((X-\mu)^4) \ \mu_4 &= \int\limits_{-\infty}^{\infty} x^4 rac{1}{\sqrt{2\pi}} e^{-x^2/2} dx \end{aligned}$$

$$\mu_4=3$$

 $lpha_4$

$$lpha_4=rac{\mu^4}{\mu_2^2}$$

$$\alpha_4=3$$

The graph does not look very peaked to me

ii

$$f(x) = 1/2$$

 μ

$$EX = \int\limits_{-1}^{1} x f(x) dx$$
 $EX = \int\limits_{-1}^{1} x/2 dx$ $EX = 0$

 μ_2

$$\mu_2 = E((X-\mu)^2)$$
 $\mu_2 = \int\limits_{-1}^1 x^2/2dx$
 $\mu_2 = 1/3$

 μ_4

$$egin{aligned} \mu_4 &= E((X-\mu)^4) \ \mu_4 &= \int\limits_{-1}^1 x^4/2 dx \ \mu_4 &= 1/5 \end{aligned}$$

 $lpha_4$

$$lpha_4=rac{\mu^4}{\mu_2^2} \ lpha_4=9/5$$

This graph does not look peaked at all

iii

$$f(x)=1/2e^{-|x|}$$

 μ

$$egin{aligned} EX &= \int \limits_{-\infty}^{\infty} x f(x) dx \ EX &= \int \limits_{-\infty}^{\infty} x/2e^{-|x|} dx \ EX &= 0 \end{aligned}$$

 μ_2

$$egin{aligned} \mu_2 &= E((X-\mu)^2) \ \mu_2 &= \int\limits_{-\infty}^{\infty} x^2/2e^{-|x|}dx \ \mu_2 &= 2 \end{aligned}$$

 μ_4

$$egin{aligned} \mu_4 &= E((X-\mu)^4) \ \mu_4 &= \int\limits_{-\infty}^{\infty} x^4/2e^{-|x|}dx \ \mu_4 &= 24 \end{aligned}$$

 $lpha_4$

$$\alpha_4 = \frac{\mu^4}{\mu_2^2}$$

$$\alpha_4 = 6$$

This graph looks the most peaked compared to everything else

2.33

C

i

$$egin{aligned} M(x) &= E(e^{tX}) \ M(x) &= \sum_{x=0}^{\infty} e^{tx} e^{-\lambda} \lambda^x/x! \ M(x) &= e^{-\lambda} \sum_{x=0}^{\infty} e^{tx} \lambda^x/x! \ M(x) &= e^{-\lambda} e^{\lambda e^t} \ M(x) &= e^{\lambda e^t - \lambda} \ M(x) &= e^{\lambda (e^t - 1)} \end{aligned}$$

EX

$$egin{aligned} EX &= rac{d}{dt}M(t)igg|_0 \ EX &= e^{\lambda(e^t-1)}\lambda e^tigg|_0 \ EX &= \lambda \end{aligned}$$

EX^2

$$egin{align} EX^2 &= rac{d^2}{d^2t} M(t)igg|_0 \ EX^2 &= rac{d}{dx} e^{\lambda(e^t-1)} \lambda e^tigg|_0 \ EX^2 &= rac{d}{dx} e^{\lambda(e^t-1)} \lambda^2 e^2 t + e^{\lambda(e^t-1)} \lambda e^tigg|_0 \ EX^2 &= \lambda^2 + \lambda \ \end{aligned}$$

 σ^2

$$\sigma^2 = EX^2 - (EX)^2 \ \sigma^2 = \lambda^2 + \lambda - (\lambda)^2 \ \sigma^2 = \lambda$$

ii

$$egin{aligned} M(x) &= E(e^{tX}) \ M(x) &= \sum_{x=0}^{\infty} e^{tx} p (1-p)^x \ M(x) &= p \sum_{x=0}^{\infty} (e^t (1-p))^x \ M(x) &= p/(1-e^t (1-p)) \end{aligned}$$

EX

$$egin{aligned} EX &= rac{d}{dt}M(t)igg|_0 \ EX &= rac{p(e^t(1-p))}{(1-e^t(1-p))^2}igg|_0 \ EX &= rac{1-p}{p} \end{aligned}$$

EX^2

$$egin{align*} EX^2 &= rac{d^2}{d^2t} M(t) igg|_0 \ EX^2 &= rac{d}{dx} rac{p(e^t(1-p))}{(1-e^t(1-p))^2} igg|_0 \ EX^2 &= rac{d}{dx} rac{pe^t-p^2e^t}{(1-e^t(1-p))^2} igg|_0 \ EX^2 &= rac{(pe^t-p^2e^t)(1-e^t(1-p))^2+2(pe^t-p^2e^t)(1-e^t(1-p))(e^t(1-p))}{(1-e^t(1-p))^4} igg|_0 \ EX^2 &= rac{(p-p^2)(p)^2+2(p-p^2)(p)(1-p)}{p^4} \ EX^2 &= rac{(p-p^2)+2(1-p)(1-p)}{p^2} \ EX^2 &= rac{(p-p^2)+2-4p+2p^2}{p^2} \ EX^2 &= rac{p-p^2+2-4p+2p^2}{p^2} \ EX^2 &= rac{p^2-3p+2}{p^2} \ EX^2 &= rac{(p-2)(p-1)}{p^2} \ EX^2 &= rac{(p-2)(p-1)}$$

$$\sigma^2 = EX^2 - (EX)^2 \ \sigma^2 = rac{(p-2)(p-1)}{p^2} - (rac{1-p}{p})^2 \ \sigma^2 = rac{(p-2)(p-1)-(1-p)^2}{p^2} \ \sigma^2 = rac{1-p}{p^2}$$

iii

$$\begin{split} &M(x)=E(e^{tX})\\ &M(x)=\int\limits_{-\infty}^{\infty}e^{tx}\frac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sqrt{2\pi}\sigma}dx\\ &\text{Let }u=x-\mu \text{ and }x=u+\mu\\ &M(x)=\int\limits_{-\infty}^{\infty}e^{t(x+\mu)}\frac{e^{-u^2/(2\sigma^2)}}{\sqrt{2\pi}\sigma}du\\ &M(x)=\frac{e^{t\mu}}{\sqrt{2\pi}\sigma}\int\limits_{-\infty}^{\infty}e^{tu}e^{-u^2/(2\sigma^2)}du\\ &M(x)=\frac{e^{t\mu}}{\sqrt{2\pi}\sigma}\int\limits_{-\infty}^{\infty}e^{(2tu\sigma^2-u^2)/(2\sigma^2)}du\\ &M(x)=\frac{e^{t\mu}}{\sqrt{2\pi}\sigma}\int\limits_{-\infty}^{\infty}e^{(-(u-t\sigma^2)^2+t^2\sigma^4)/(2\sigma^2)}du\\ &M(x)=\frac{e^{t\mu}e^{\sigma^2t^2/2}}{\sqrt{2\pi}\sigma}\int\limits_{-\infty}^{\infty}e^{-(u-t\sigma^2)^2/(2\sigma^2)}du\\ &M(x)=\frac{e^{t\mu}e^{\sigma^2t^2/2}}{\sqrt{2\pi}\sigma}\int\limits_{-\infty}^{\infty}e^{-(\frac{u-t\sigma^2}{\sqrt{2}\sigma})^2}du\\ &\text{Let }v=\frac{u-t\sigma^2}{\sqrt{2}\sigma} \text{ and }dv=\frac{du}{\sqrt{2}\sigma} \text{ and }du=\sqrt{2}\sigma dv\\ &M(x)=\frac{e^{t\mu}e^{\sigma^2t^2/2}}{\sqrt{\pi}\sigma}\int\limits_{-\infty}^{\infty}e^{-v^2}dv\\ &M(x)=e^{t\mu}e^{\sigma^2t^2/2}\\ &M(x)=e^{t\mu}e^{\sigma^2t^2/2}z \end{split}$$

EX

$$egin{aligned} EX &= \left. rac{d}{dt} M(t)
ight|_0 \ EX &= \left. rac{d}{dt} e^{\mu t + \sigma^2 t^2/2}
ight|_0 \ EX &= (\mu + \sigma^2 t) e^{\mu t + \sigma^2 t^2/2}
ight|_0 \ EX &= \mu \end{aligned}$$

EX^2

$$egin{aligned} EX^2 &= rac{d^2}{d^2t}M(t)igg|_0 \ EX &= rac{d}{dt}(\mu+\sigma^2t)e^{\mu t+\sigma^2t^2/2}igg|_0 \ EX &= (\sigma^2+(\mu+\sigma^2t)^2)e^{\mu t+\sigma^2t^2/2}igg|_0 \ EX &= \sigma^2+\mu^2 \end{aligned}$$

$$\sigma^2 = EX^2 - (EX)^2$$
 $\sigma^2 = \sigma^2 + \mu^2 - (\mu)^2$
 $\sigma^2 = \sigma^2$