1.2

3

a

Is $4 = \{4\}$

✓ **Answer**

No, 4 is not a set, and 4 itself is not the same as a set containing 4.

b

How many elements are in the set $\{3, 4, 3, 5\}$

✓ **Answer**

There are 3 unique elements in the set.

c

How many elements are in the set $\{1, \{1\}, \{1, \{1\}\}\}$

✓ **Answer**

3 elements: 1, $\{1\}$, and $\{1, \{1\}\}$

6

For each integer n, let $T_n = \{n, n^2\}$ How many elements are in each of T_2, T_{-3}, T_1, T_0?

✓ **Answer**

$T_2 = \{2, 4\}$, 2 Elements
$T_{-3} = \{-3, 9\}$, 2 Elements

$T_1 = \{1\}$, 1 Element

$T_0 = \{0\}$, 1 Element

7

Use the set-roster notation to indicate the elements in each of the following sets

a

$S = \{n \in \mathbf{Z} | n = (-1)^k, \text{ for some integer } k\}$

✓ **Answer**

$S = \{-1, 1\}$

e

$W = \{t \in \mathbf{Z} | 1 < t < -3\}$

✓ **Answer**

$W = \varnothing$

f

$X = \{u \in \mathbf{Z} | u \leq 4 \cup u \geq 1\}$

✓ **Answer**

$X = \mathbf{Z}$

9

c

Is $\{2\} \in \{1, 2\}$?

✓ **Answer**

No, the set of 2 is not in the other set

g

Is $\{1\} \subseteq \{1, 2\}$

✓ **Answer**

Yes, the set of 1 is a subset of the set of 1 and 2

10

b

Is $(5, -5) = (-5, 5)$

✓ **Answer**

No, the order matters in ORDERED pairs

d

Is $\left(\frac{-2}{-4}, (-2)^3\right) = \left(\frac{3}{6}, -8\right)$

✓ **Answer**

Yes, both ordered pairs are equal to $(0.5, -8)$

12

Let $S = \{2, 4, 6\}$ and $T = \{1, 3, 5\}$. Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set.

a

$S \times T$

✓ **Answer**

$\{(2,1),(2,3),(2,5),(4,1),(4,3),(4,5),(6,1),(6,3),(6,5)\}$

$\|S \times T\| = 9$

c

$S \times S$

✓ **Answer**

$\{(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),(6,2),(6,4),(6,6)\}$

$\|S \times S\| = 9$

1.3

2

Let $C = D = \{-3,-2,-1,1,2,3\}$ and define a relation S from C to D as follows: $\forall (x,y) \in C \times D \cap \frac{1}{x} - \frac{1}{y} \in \mathbf{Z}, (x,y) \in S$

a

Is $2S2$? Is $-1S-1$? Is $(3,3) \in S$ Is $(3,-3) \in S$

✓ **Answer**

- Yes, as $\frac{1}{2} - \frac{1}{2} = 0 \in \mathbf{Z}$
- Yes, as $-\frac{1}{1} + \frac{1}{1} = 0 \in \mathbf{Z}$
- Yes, as $\frac{1}{3} - \frac{1}{3} = 0 \in \mathbf{Z}$
- No, as $\frac{1}{3} + \frac{1}{3} = \frac{2}{3} \notin \mathbf{Z}$

b

Write S as a set of ordered pairs.

✓ **Answer**

$S = \{(-3,-3),(-2,-2),(-2,2),(-1,-1),(-1,1),$
$(1,-1),(1,1),(2,-2),(2,2),(3,3)\}$

c

Write the domain and co-domain of S

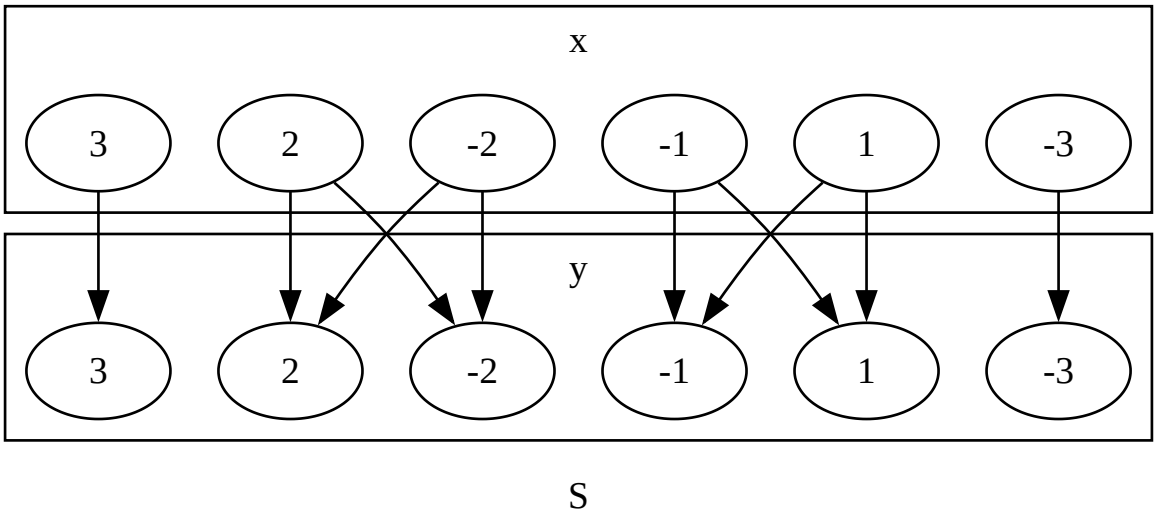
✓ **Answer**

The domain and co-domain of S are both $C = D$

d

Draw an arrow diagram for S

✓ **Answer** ⌄



8

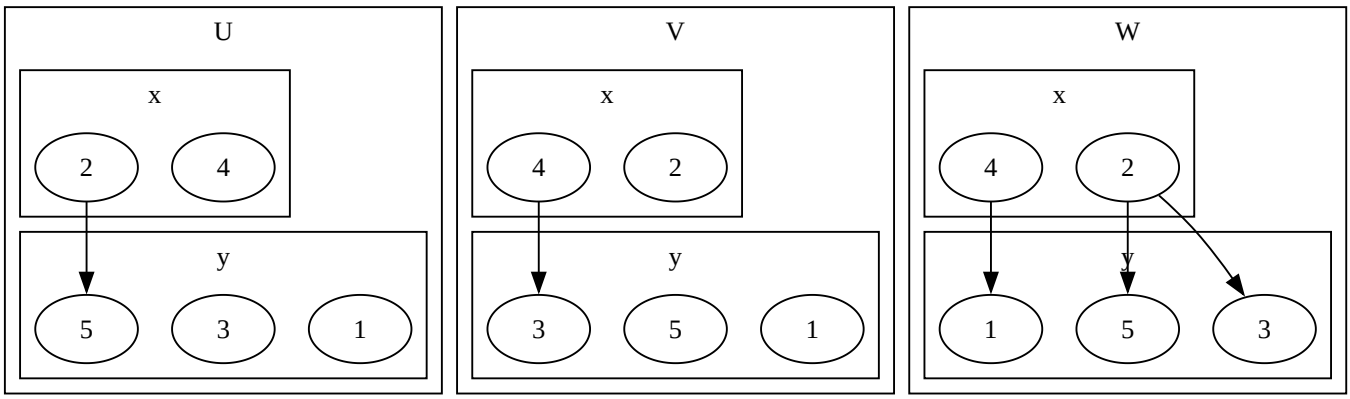
Let $A = \{2,4\}$ and $B = \{1,3,5\}$ and define relations U, V, and W from A to B as follows:

- $(x,y) \in U \iff y - x > 2, (x,y) \in A \times B$
- $(x,y) \in V \iff y - 1 = \frac{x}{2}, (x,y) \in A \times B$
- $W = \{(2,5),(4,1),(2,3)\}$

a

Draw arrow diagrams for U, V, W

✓ **Answer**



b

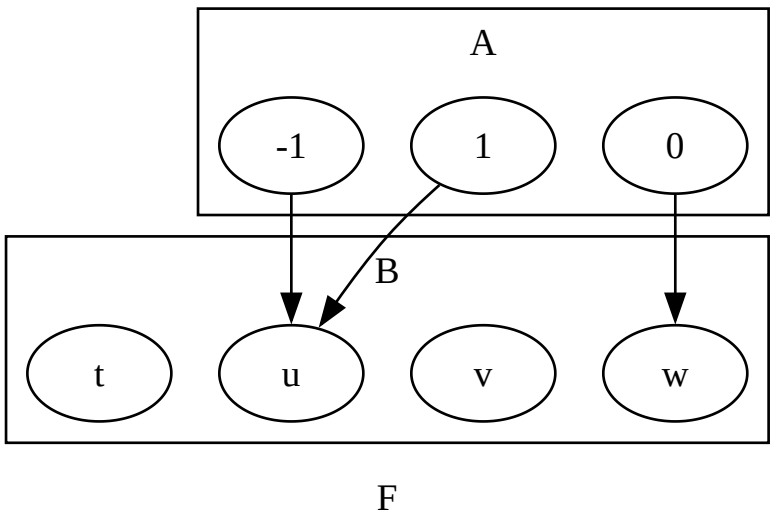
Indicate whether any of the relations U, V, W are functions

✓ **Answer**

U, V are not functions as they do not map some items in the domain and W is not a function as an item in the domain is mapped to multiple items in the co-domain

13

Let $A = \{-1, 0, 1\}$ and $B = \{t, u, v, w\}$ define a function $F : A \to B$ by the following arrow diagram:



F

a

Write the domain and co-domain of F

✓ **Answer**

A, B are the domain and co-domain, in that order

b

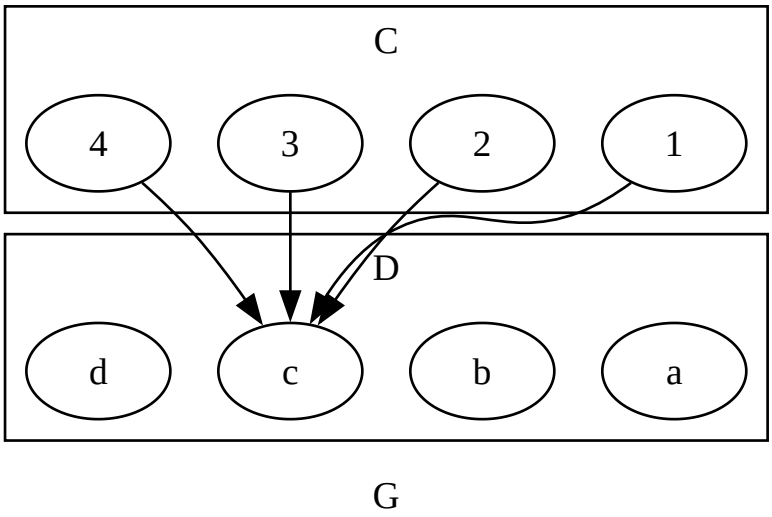
Find $F(-1), F(0), F(1)$

✓ **Answer**

$F(-1) = u$
$F(0) = w$
$F(1) = u$

14

Let $C = \{1, 2, 3, 4\}$ and $D = \{a, b, c, d\}$ define a function $G : C \to D$ by the following arrow diagram:



G

a

Write the domain and co-domain of G

✓ **Answer**

C, D are the domain and co-domain, in that order

b

Find $G(1), G(2), G(3), G(4)$

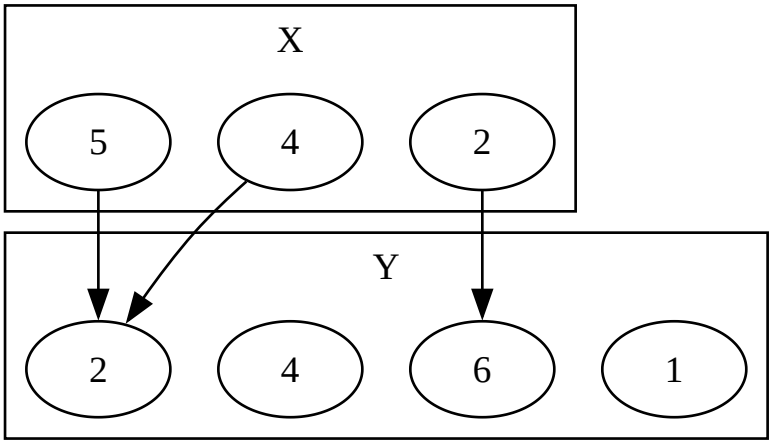
✓ **Answer**

$G(1) = c$
$G(2) = c$
$G(3) = c$
$G(4) = c$

15

Let $X = \{2, 4, 5\}$ and $Y = \{1, 2, 4, 6\}$. Which of the following diagrams determine functions from X to Y

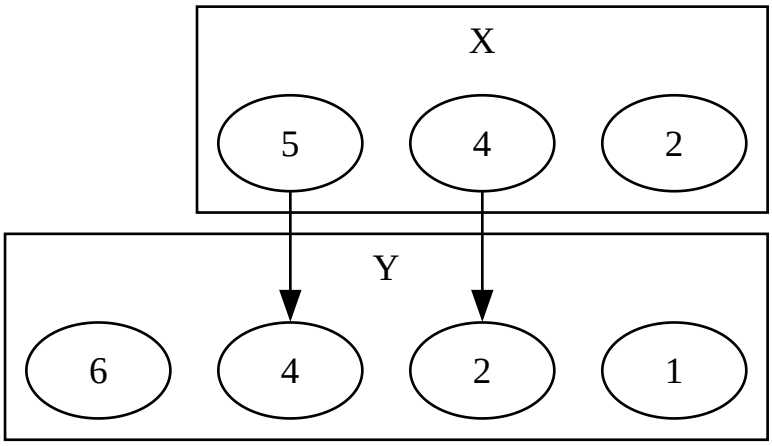
d



✓ **Answer**

Yes, this is a function as all inputs have one and only one output

e



✓ **Answer**

No, this is not a function as not all items in the domain are mapped

18

Let $h(r) = 2 : r \in \mathbf{Q}$
Find $h\left(-\frac{12}{5}\right), h\left(\frac{0}{1}\right), h\left(\frac{9}{17}\right)$

✓ **Answer**

$h\left(-\frac{12}{5}\right) = 2$
$h\left(\frac{0}{1}\right) = 2$
$h\left(\frac{9}{17}\right) = 2$

20

Define functions H and K from $\mathbf{R}$ to $\mathbf{R}$ by the following formulas:

- $H(x) = (x-2)^2 : x \in \mathbf{R}$
- $K(x) = (x-1)(x-3) + 1 : x \in \mathbf{R}$

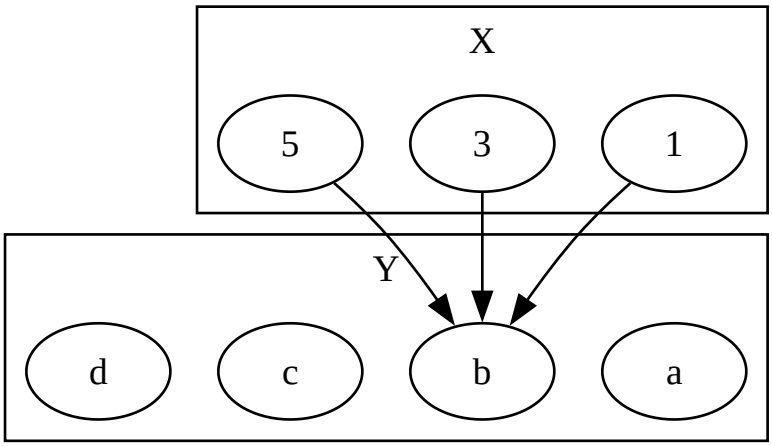
✓ **Answer**

$H(x) = x^2 - 4x + 4 : x \in \mathbf{R}$
$K(x) = x^2 - 4x + 4 : x \in \mathbf{R}$
Thus, yes, $H = K : \mathbf{R}$

7.1

2

Let $X = \{1, 3, 5\}$ and $Y = \{a, b, c, d\}$ defind $g : X \to Y$ by the following arrow diagram



g

a

Write the comain of g and the co-domain of g

✓ **Answer**

The domain and co-domain are X, Y in that order

b

Find $g(1), g(3), g(5)$

✓ **Answer**

$g(1) = b$
$g(3) = b$
$g(5) = b$

c

What is the range of g?

✓ **Answer**

The range of g is $\{b\} \subset Y$

d

Is 3 an inverse image of a? Is 1 and inverse image of b?

✓ **Answer**

No not both because the inverse image is a set, but 1 would be in the inverse image of b
$1 \in g^{-1}(b)$

e

What is the inverse image of b? of c?

✓ **Answer**

$g^{-1}(b) = \{1, 3, 5\}$
$g^{-1}(c) = \varnothing$

f

Represent g as a set of ordered pairs

✓ **Answer**

$g = \{(1, b), (3, b), (5, b)\}$

4

b

Find all functions from $X = \{a, b, c\} \to Y = \{u\}$

✓ **Answer**

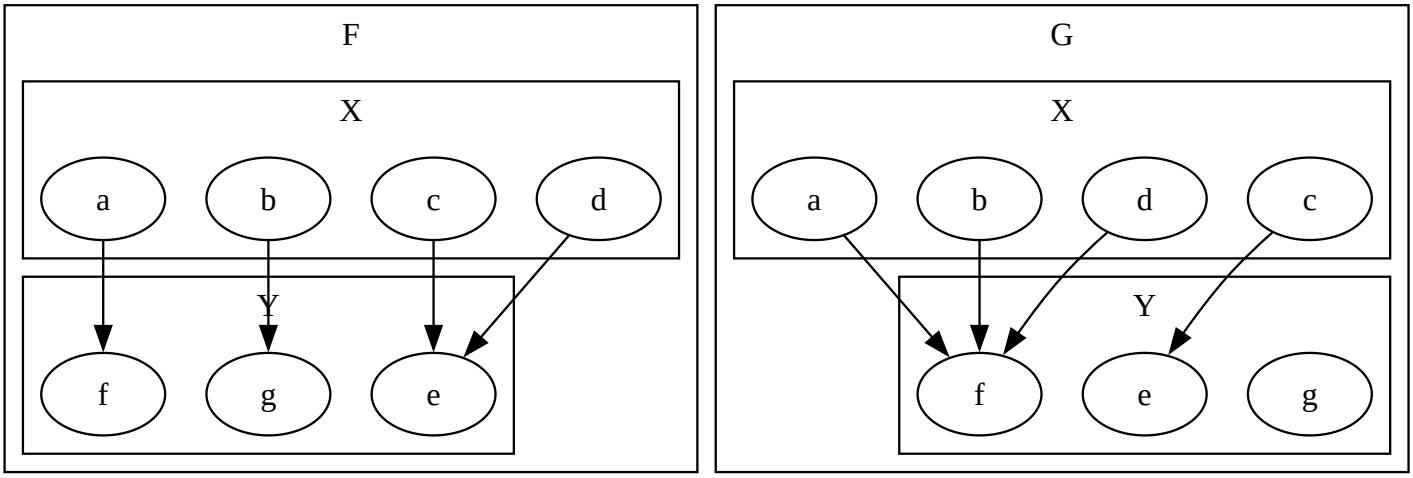
As all inputs must be mapped to the co-domain and there is only one possible choice in the co-domain, there is only one valid function:

$f(x) = u : x \in X$

7.2

7

Let $X = \{a, b, c, d\}$ and $Y = \{e, f, g\}$ define functions F and G on X to Y by the arrow diagrams below



b

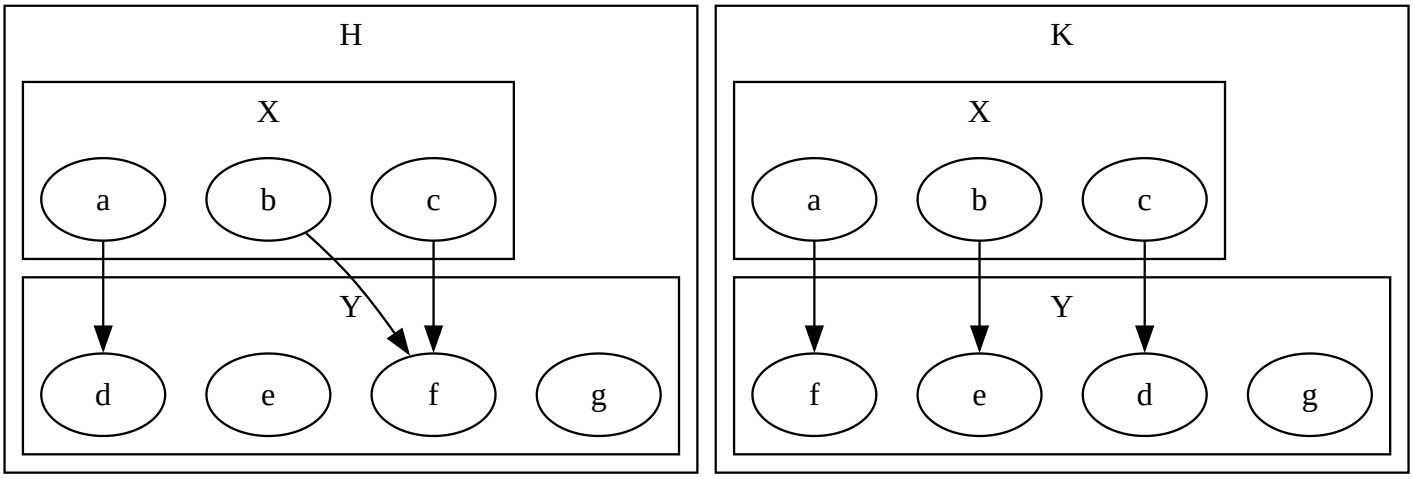
Is G one-to-one? Why or why not? Is it onto? Why or why not?

✓ **Answer**

No, G is not one-to-one as a, b, d all map to f through G.
No, G is not onto as there is no $x \in X$ mapped to $g \in Y$

8

Let $X = \{a, b, c\}$ and $Y = \{d, e, f, g\}$ define functions H and K on X to Y by the arrow diagrams below



a

Is H one-to-one? Why or why not? Is it onto? Why or why not?

✓ Answer

No, H is not one-to-one, $H(b) = H(c) = f$
No, H is not onto, $H^{-1}(e) = H^{-1}(g) = \varnothing$

b

Is K one-to-one? Why or why not? Is it onto? Why or why not?

✓ Answer

Yes, H is not one-to-one as $\forall x_1 \in X, x_2 \in X, x_1 \neq x_2 : K(x_1) \neq K(x_2)$
No, H is not onto, $H^{-1}(g) = \varnothing$