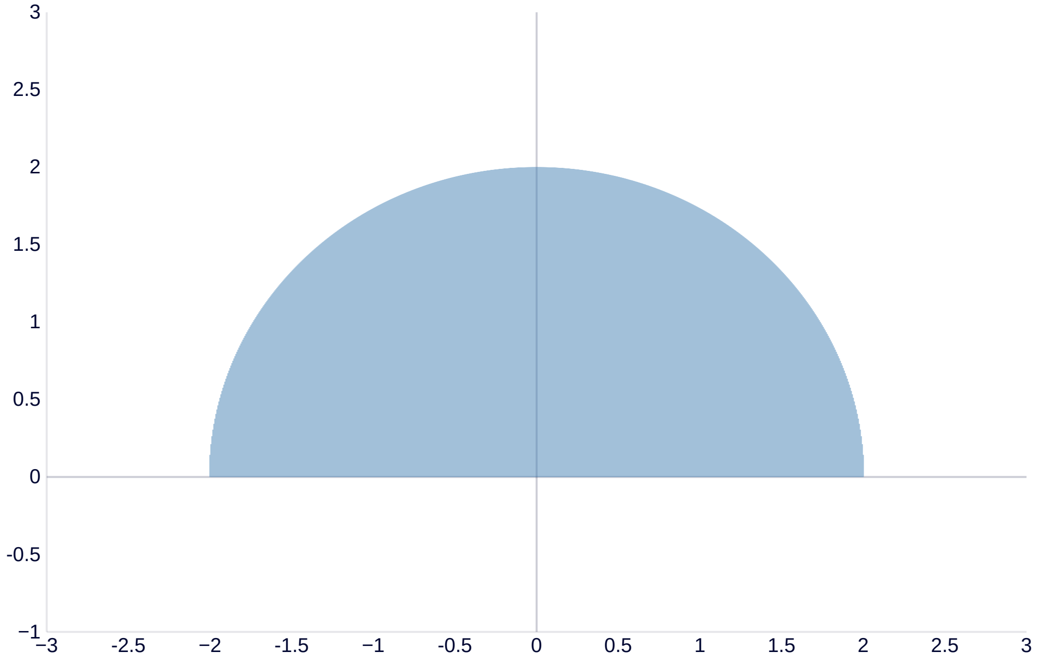
15.4

7

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} x^2+y^2\ dydx$$



$0 < r < 2$

$0 < \theta < \pi$

$$\int_{0}^{\pi}\int_{0}^{2} r^2 r\ drd\theta$$

$$= \int_{0}^{\pi} \Big|_{0}^{2} r^4/4\ drd\theta$$

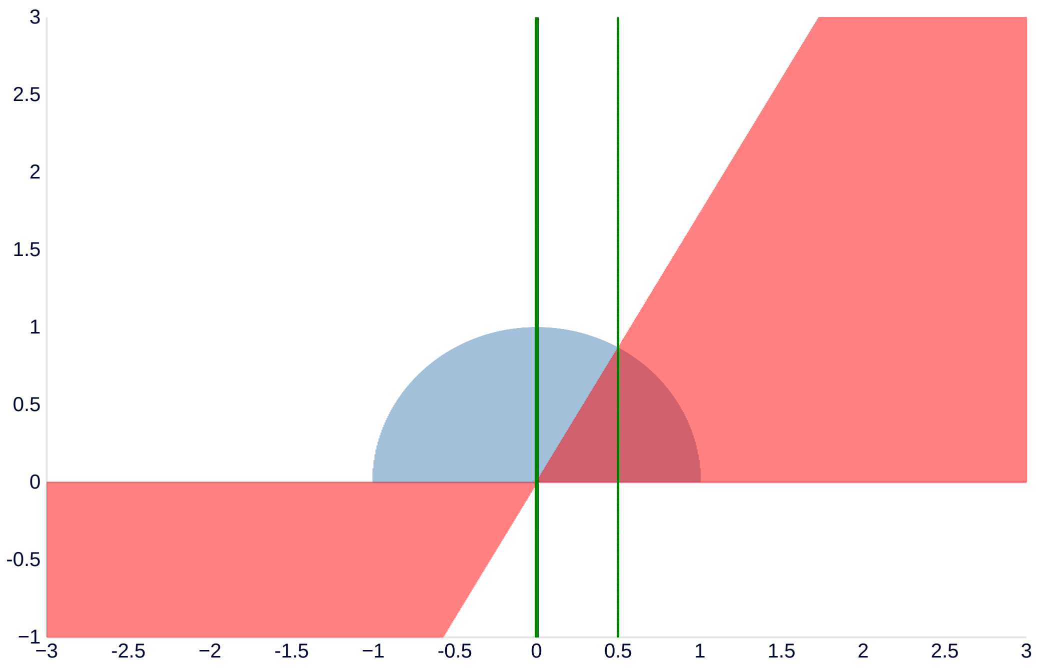
$$= \int_{0}^{\pi} 4\ d\theta$$

$$= 4\pi$$

$$\approx 12.566$$

9

$$\int_{0}^{1/2}\int_{\sqrt{3}x}^{\sqrt{1-x^2}} x\ dydx$$



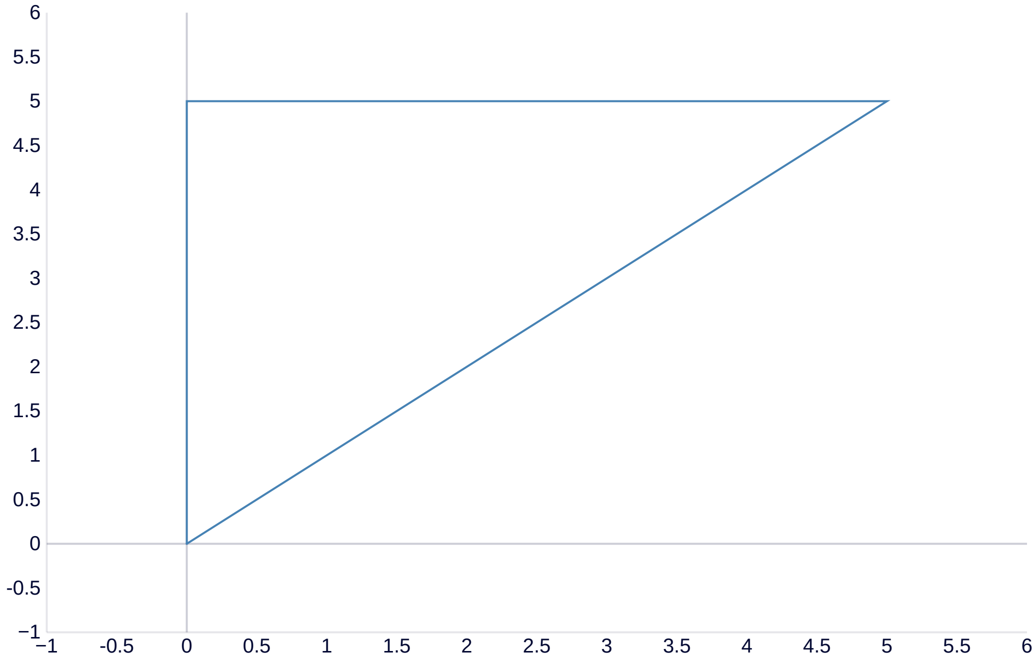
Where the area is the blue area between the green lines that is not in the red area.

$0 < r < 1$
$\pi/3 < \theta < \pi/2$

$$\int_{\pi/3}^{\pi/2} \int_0^1 r^2 \cos\theta \, dr d\theta$$
$$= \int_{\pi/3}^{\pi/2} \frac{1}{3} \cos\theta \, d\theta$$
$$= \Big|_{\pi/3}^{\pi/2} \frac{1}{3} \sin\theta \, d\theta$$
$$= \Big|_{\pi/3}^{\pi/2} \frac{1}{3} \sin\theta \, d\theta$$
$$= \frac{1}{3} - \frac{\sqrt{3}}{6}$$
$$\approx 0.045$$

11

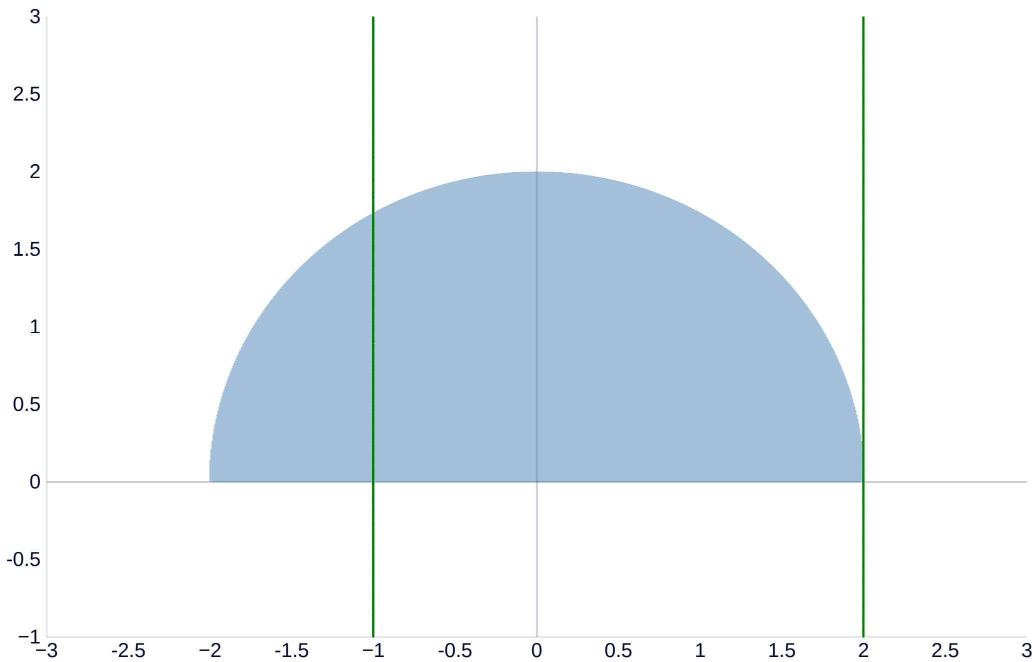
$$\int_0^5 \int_0^y x \, dx dy$$



$$= \int_{0}^{5} y^2/2 \, dy$$
$$= (5)^3/6$$
$$= (5)^3/6$$
$$= 125/6$$

13

$$\int_{-1}^{2} \int_{0}^{\sqrt{4-x^2}} x^2 + y^2 \, dydx$$



Where the area is the blue semicircle between the green lines

$$\begin{cases} 0 < r < 2 \\ 0 < \theta < 2\pi/3 \end{cases}$$

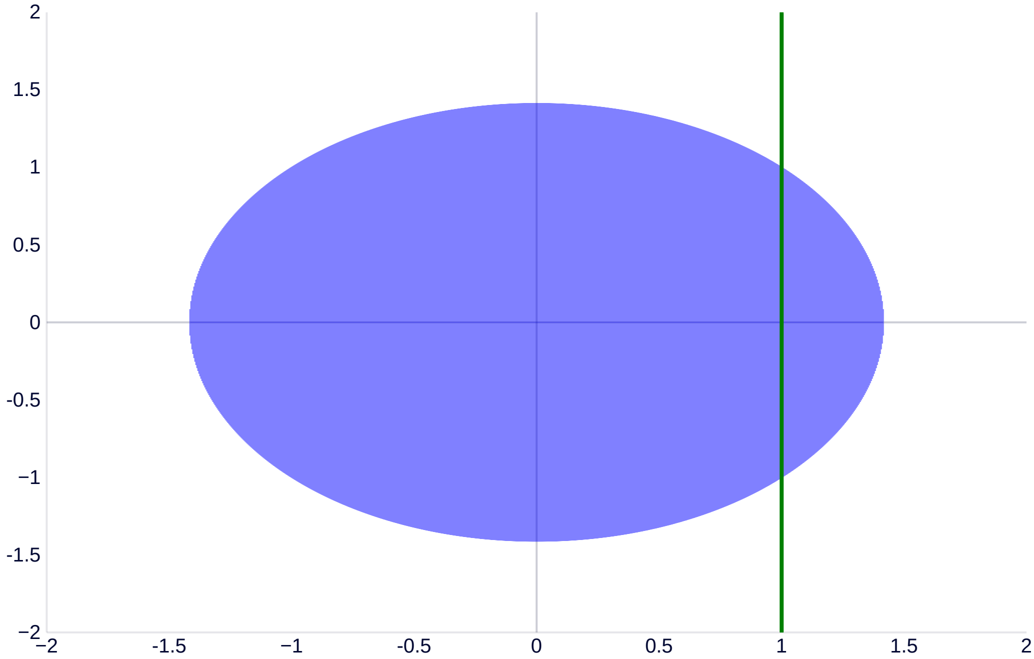
$$= \int_0^{2\pi/3} \int_0^2 r^3 \; dr d\theta + \int_0^1 \int_0^{x \tan \pi/3} x^2 + y^2 \; dy dx$$

$$= \int_0^{2\pi/3} 4 \; d\theta + \int_0^1 2x^3 \sqrt{3} \; dx$$

$$= 8\pi/3 + \sqrt{3}/2$$

$$\approx 9.244$$

15



Where the area is the circle right of the green line

$$2 \int_0^{\pi/4} \int_{\cos\theta}^{\sqrt{2}} r^{-3} \; dr d\theta$$

$$= \int_0^{\pi/4} \Big|_{\cos\theta}^{\sqrt{2}} - 1/r^2 \; dr d\theta$$

$$= \int_0^{\pi/4} -1/2 + \sec^2\theta \; d\theta$$

$$= -\pi/8 + \Big|_0^{\pi/4} \tan\theta \; d\theta$$

$$= -\pi/8 + 1$$

$$\approx 0.607$$

17

$$4\int_0^{\pi/2}\int_0^1 r^3\cos\theta\sin\theta\,drd\theta$$
$$=\int_0^{\pi/2}\cos\theta\sin\theta\,d\theta$$
$$=\Big|_0^{\pi/2}\tfrac{1}{2}(\sin\theta)^2\,d\theta$$
$$=1/2$$
$$=0.5$$

19

$$u=x/\sqrt{2}+y/\sqrt{2}$$
$$v=x/\sqrt{2}-y/\sqrt{2}$$

$$J=\begin{vmatrix}1/\sqrt{2} & 1/\sqrt{2}\\ 1/\sqrt{2} & -1/\sqrt{2}\end{vmatrix}$$
$$=-1$$

$$\int_{-\pi/4}^{\pi/4}\int_{r\cos\theta}^{1} r^2\sin\theta\,drd\theta$$
$$=\int_{r\cos\theta}^{1}\int_{-\pi/4}^{\pi/4} r^2\sin\theta\,d\theta dr$$

Since $\sin$ is an even function the first integral evaluates to 0

$$=0$$

27

$$\int_0^5\int_0^{2\pi}\int_0^3 r^3\,drd\theta dz$$
$$=5\times 2\pi\times(3)^4/4$$
$$=405\pi/2$$
$$\approx 636.173$$

29

$$\int_{-3}^{3}\int_{0}^{\pi/2}\int_{0}^{4} r^2\cos\theta\, drd\theta dh$$
$$= \int_{-3}^{3}\int_{0}^{\pi/2} \frac{64}{3}\cos\theta\, d\theta dh$$
$$= 128$$

31

$$\int_{0}^{9}\int_{0}^{2\pi}\int_{0}^{\sqrt{h}} hr\, drd\theta dh$$
$$= \int_{0}^{9} \pi h^2\, dh$$
$$= 243\pi$$
$$\approx 763.407$$

33

$$\int_{0}^{4}\int_{0}^{2\pi}\int_{0}^{1} rf(r\cos\theta, r\sin\theta, h)\, drd\theta dh$$

41

Where h is the height of the band and R is the radius of the sphere. In cylindrical coordinates of (r,θ,z)

This integral represents the volume of the band.

$$\int_{-h/2}^{h/2}\int_{0}^{2\pi}\int_{\sqrt{R^2-(h/2)^2}}^{\sqrt{R^2-z^2}} r\, drd\theta dz$$
$$= \int_{-h/2}^{h/2}\int_{0}^{2\pi} \Big|_{\sqrt{R^2-(h/2)^2}}^{\sqrt{R^2-z^2}} r^2/2\, drd\theta dz$$
$$= \int_{-h/2}^{h/2}\int_{0}^{2\pi} ((h/2)^2 - z^2)/2\, d\theta dz$$
$$= \pi \int_{-h/2}^{h/2} (h/2)^2 - z^2\, dz$$
$$= \pi \Big|_{-h/2}^{h/2} z(h/2)^2 - z^3/3\, dz$$
$$= \pi(h(h/2)^2 - h^3/12)$$

Which only depends on h (the height of the band) and not the radius of the sphere R.

43

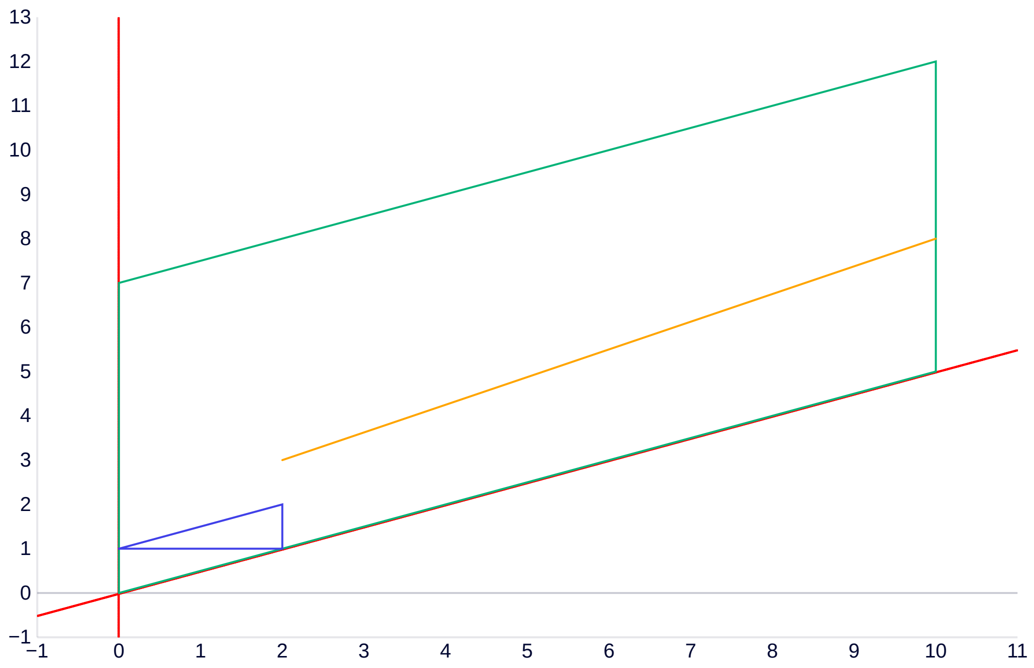
$$\int_0^{2\pi}\int_0^{\pi/3}\int_{\sec\phi}^{2}\rho^2\sin\phi\,d\rho d\phi d\theta$$
$$=\int_0^{2\pi}\int_0^{\pi/3}\frac{8-\sec^3\phi}{3}\sin\phi\,d\phi d\theta$$
$$=\int_0^{2\pi}\Big|_0^{\pi/3}\frac{-8\cos\phi}{3}-\frac{1}{6(1-\sin^2\phi)}\,d\phi d\theta$$
$$=\int_0^{2\pi}-\frac{4}{3}+\frac{8}{3}-\frac{2}{3}+\frac{1}{6}\,d\theta$$
$$=\int_0^{2\pi}\frac{5}{6}\,d\theta$$
$$=\frac{5\pi}{3}$$
$$\approx 5.236$$

45

$$\int_0^{\pi/2}\int_0^{\pi/2}\int_0^{1}r^3\sin^2\phi\cos\theta\,drd\theta d\phi$$
$$=\int_0^{\pi/2}\int_0^{\pi/2}\frac{1}{4}\sin^2\phi\cos\theta\,d\theta d\phi$$
$$=\int_0^{\pi/2}\frac{1}{4}\sin^2\phi\,d\phi$$
$$=\int_0^{\pi/2}\frac{1-\cos 2\phi}{8}\,d\phi$$
$$=\frac{\pi}{16}$$
$$\approx 0.196$$

15.6

1



13

$$J = \begin{vmatrix} 3 & 4 \\ 1 & -2 \end{vmatrix} = -10$$

17

$$J = \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta = r$$

$$\Big|_{(4,\frac{\pi}{6})} = 4$$

21

$$G(u, v) = (5u + 3v, u + 4v)$$

$$J(G) = \begin{vmatrix} 5 & 3 \\ 1 & 4 \end{vmatrix} = 17$$

$$\iint_{D_0} (5u + 3v)(u + 4v)17\, dA$$

$$= \int_0^1 \int_0^1 (5u + 3v)(u + 4v)17\, dudv$$

$$= \int_0^1 (5/3 + 23v/2 + 12v^2)17\, dv$$

$$= (5/3 + 23/4 + 4)17$$

$= 2329/12$

≈ 194.083

25

$$J(G) = \begin{vmatrix} \frac{1}{v} & \frac{-u}{v^2} \\ v & u \end{vmatrix} = \frac{2u}{v}$$

$$\int_1^4 \int_1^4 \frac{2u}{v} \, dudv$$
$$= \int_1^4 \frac{15}{v} \, dv$$
$$= 15 \ln 4$$

31

$$J(G) = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1$$

$$D_0 = [6, 10] \times [1, 3]$$

$$\iint_{D_0} (u + v) \, dA$$
$$= \int_6^{10} \int_1^3 (u + v) \, dvdu$$
$$= \int_6^{10} (4 + 2v) \, du$$
$$= (16 + 64)$$
$$= 80$$