$$P = (-1, 2)$$

 $f(x, y) = xy^2 + x^3y^2$
 $f'_x(x, y) = y^2 + 3x^2y^2$
 $f'_y(x, y) = 2xy + 2x^3y$
 $f'_x(P) = 16$
 $f'_y(P) = -8$
 $f(P) = -8$
 $A = -8 + 16(x + 1) - 8(y - 2)$
 $A = 16x - 8y + 24$

$$P = (4,1)$$

 $f(x,y) = x^2 + y^{-2}$
 $f'_x(x,y) = 2x$
 $f'_y(x,y) = -2y^{-3}$
 $f'_x(P) = 8$
 $f'_y(P) = -2$
 $f(P) = 17$
 $A = 17 + 8(x - 4) - 2(y - 1)$
 $A = 8x - 2y - 13$

$$egin{aligned} P &= (2,1) \ F(r,s) &= r^2 s^{-1/2} + s^{-3} \ F'_r(r,s) &= 2r s^{-1/2} \ F'_s(r,s) &= -0.5 r^2 s^{-3/2} - 3 s^{-4} \ F'_r(P) &= 4 \ F'_s(P) &= -5 \ F(P) &= 5 \ A &= 5 + 4(r-2) - 5(s-1) \ A &= 4r - 5s + 2 \end{aligned}$$

$$f(x, y) = 3x^2 - xy - y^2$$

 $f'_x(x, y) = 6x - y$
 $f'_y(x, y) = -x - 2y$

$$\begin{cases} 6x - y = 0 \\ -x - 2y = 0 \end{cases}$$

$$\begin{cases} 6x - y = 0 \end{cases}$$

$$\begin{cases} -x - 12x = 0 \\ y = 0 \\ x = 0 \end{cases}$$

$$P=(0,0)$$

$$P = (2,1) \ f(x,y) = x^2 y^3 \ f'_x(x,y) = 2xy^3$$

$$f_y'(x,y)=3x^2y^2$$

$$L_{(a,b)}(x,y) = a^2b^3 + 2ab^3(x-a) + 3a^2b^2(y-b)$$

$$L_P(x,y) = 4 + 4(x-2) + 12(y-1)$$

$$L_P(x,y) = 4x + 12y - 16$$

$$L_P(2.01, 1.02) = 4.28$$

$$L_P(1.97, 1.01) = 4$$

$$f(2,4) = 5$$

$$f_x^\prime(2,4)=0.3$$

$$f_y^{\prime}(2,4) = -0.2$$

$$L_{(2,4)}(x,y) = 5 + 0.3(x-2) - 0.2(y-4)$$

$$L_{(2,4)}(x,y) = 0.3x - 0.2y + 5.2$$

$$L_{(2,4)}(2.1,3.8) = 5.07$$

a

$$f(x,y) = 5x + 4y^2$$
 $f'_x(x,y) = 5$
 $f'_y(x,y) = 8y$
 $L_{(a,b)}(x,y) = 5a + 4b^2 + 5(x-a) + 8b(y-b)$
 $L_{(2,1)}(x,y) = 14 + 5(x-2) + 8(y-1)$
 $L_{(2,1)}(x,y) = 5x + 8y - 4$
 \Box
 $L_{(2,1)}(x,y) + 4(y-1)^2 = 5x + 8y - 4 + 4y^2 - 8y + 4y^2$
 $= 5x + 4y^2$
 $= f(x,y)$

b

$$\frac{4(y-1)^2}{\sqrt{(x-2)^2+(y-1)^2}}$$

Since $(x-2)^2$ and $(y-1)^2$ are positive, the denominator is also positive. Since $(y-1)^2$ is positive, so is the numerator and thus the whole fraction ≥ 0

$$egin{align} rac{4(y-1)^2}{\sqrt{(x-2)^2+(y-1)^2}} \ &\sqrt{(x-2)^2+(y-1)^2} \geq \sqrt{(y-1)^2} = |y-1|^2 \ &rac{4(y-1)^2}{\sqrt{(x-2)^2+(y-1)^2}} \leq rac{4(y-1)^2}{|y-1|} = 4|y-1| \ &rac{1}{|y-1|} = 4|y-1| \ & rac{1}{|y-1|} = 4|y-1| \ & rac{1}{|y-1|} = 4|y-1| \ & rac{1}{|y-1|} = 4|y-1| \ & rac{1}{|y-1|} = 4|y-1| \ & rac{1}{|y-1|} = 4|y-1| \ & rac{1}{|y-1|} = 4|y-1| \ & rac{1}{|y-1|} = 4|y-1| \ & rac{1}{|y-1|} = 4|y-1| \ & rac{1}{|y-1|} = 4|y-1| \ & rac{1}{|y-1|} = 4|y-1| \ & |y-1| = 4|y-1| \ & |y-1$$

C

$$\begin{array}{ll} \lim\limits_{(x,y)\to(a,b)} \frac{f(x,y)-L_{(a,b)}(x,y)}{\sqrt{(x-a)^2+(y-b)^2}} \\ \lim\limits_{(x,y)\to(a,b)} \frac{(5x+4y^2)-(5a+4b^2+5(x-a)+8b(y-b))}{\sqrt{(x-a)^2+(y-b)^2}} \\ \lim\limits_{(x,y)\to(a,b)} \frac{4y^2-8by+4b^2}{\sqrt{(x-a)^2+(y-b)^2}} \\ \lim\limits_{(x,y)\to(a,b)} \frac{4(y-b)^2}{\sqrt{(x-a)^2+(y-b)^2}} \end{array}$$

The limit must ≥ 0 since the fraction is positive

$$\lim_{(x,y) o (a,b)}rac{4(y-b)^2}{\sqrt{(x-a)^2+(y-b)^2}}\leq \lim_{(x,y) o (a,b)}4|y-b|=0$$

Thus, the limit must lie on the interval [0,0], meaning it equals 0 everywhere.

Since the limit equals 0 for any (a,b), the function f is differentiable on any (a,b) \Box

$$f(x,y) = 3x - 7y$$
 $ec{r}(t) = \langle \cos t, \sin t
angle$

$$t = 0$$

$$rac{d}{dt}f(ec{r}(t)) =
abla f(ec{r}(t)) \cdot ec{r}'(t)$$

$$abla f(ec{r}(t)) = \langle 3, -7
angle$$

$$ec{r}'(t) = \langle -\sin t, \cos t
angle$$

$$\nabla f(\vec{r}(t)) \cdot \vec{r}'(t) = -3\sin t - 7\cos t$$

$$abla f(\vec{r}(0)) \cdot \vec{r}'(0) = -7$$

$$f(x,y) = x^2 - 3xy$$

$$ec{r}(t) = \langle \cos t, \sin t
angle$$

$$t = 0$$

$$rac{d}{dt}f(ec{r}(t)) =
abla f(ec{r}(t)) \cdot ec{r}'(t)$$

$$\nabla f(\vec{r}(t)) = \langle 2\cos t - 3\sin t, -3\cos t \rangle$$

$$ec{r}'(t) = \langle -\sin t, \cos t
angle$$

$$abla f(ec{r}(0)) \cdot ec{r}'(0) = 0 + -3$$

$$=-3$$

$$f(x,y) = x - xy$$

$$ec{r}(t) = \langle t^2, t^2 - 4t
angle$$

$$t=4$$

$$rac{d}{dt}f(ec{r}(t)) =
abla f(ec{r}(t)) \cdot ec{r}'(t)$$

$$abla f(ec{r}(t)) = \langle 1 - (t^2 - 4t), -(t^2)
angle$$

$$ec{r}'(t) = \langle 2t, 2t-4
angle$$

$$abla f(ec{r}(4)) \cdot ec{r}'(4) = 1*8 + -16*4$$

$$= -56$$

$$f(x,y) = \ln x + \ln y$$
 $\vec{r}(t) = \langle \cos t, t^2 \rangle$
 $t = \frac{\pi}{4}$

$$\nabla f(\vec{r}(0)) \cdot \vec{r}'(0) = 0 + -3$$

$$\nabla f(\vec{r}(t)) = \langle \frac{1}{\cos t}, \frac{1}{t^2} \rangle$$
 $\vec{r}'(t) = \langle -\sin t, 2t \rangle$

$$\nabla f(\vec{r}(\frac{\pi}{4})) \cdot \vec{r}'(\frac{\pi}{4}) = -1 + \frac{8}{\pi}$$

$$= \frac{8}{\pi} - 1$$

$$f(x,y)=x^2+y^3$$
 $ec{v}=\langle 4,3
angle$ $P=(1,2)$ $\nabla f(P)=\langle 2,12
angle$

$$egin{aligned}
abla f(P) &= \langle 2, 12
angle \\
abla f(P) \cdot ec{v} / \| ec{v} \| &= 8/5 + 36/5 \\
&= 44/5 = 8.8 \end{aligned}$$

$$egin{aligned} f(x,y) &= \ln(x^2+y^2) \ ec{v} &= 3i-2j = \langle 3,-2
angle \ P &= (1,0) \end{aligned}$$
 $\left. egin{aligned}
abla f(P) &= \langle rac{2x}{x^2+y^2}, rac{2y}{x^2+y^2}
angle
ight|_P \ &= \langle 2,0
angle \end{aligned}$

$$egin{aligned}
abla f(P) \cdot ec{v}/\|ec{v}\| &= 6/\sqrt{13} \ &= rac{6}{\sqrt{13}} &= rac{6\sqrt{13}}{13} \ &\Box \end{aligned}$$

$$egin{aligned} g(x,y,z) &= xe^{-yz} \ ec{v} &= \langle 1,1,1
angle \ P &= (1,2,0) \end{aligned}$$

$$egin{aligned}
abla g(P) &= \langle e^{-yz}, -xze^{-yz}, -xye^{-yz}
angle \ &= \langle 1, 0, -2
angle \end{aligned}$$

$$egin{aligned}
abla g(P) \cdot ec{v} / \|ec{v}\| &= -1/\sqrt{3} \ = rac{-1}{\sqrt{3}} &= rac{-\sqrt{3}}{3} \end{aligned}$$

$$egin{array}{l} rac{x^2}{4} + rac{y^2}{9} + z^2 = 1 \ f(x,y) = z = \pm \sqrt{1 - rac{x^2}{4} - rac{y^2}{9}} \end{array}$$

$$abla f((x,y,\pm\sqrt{1-rac{x^2}{4}-rac{y^2}{9}})) = \langle \mprac{x}{4\sqrt{1-rac{x^2}{4}-rac{y^2}{9}}}, \mprac{y}{9\sqrt{1-rac{x^2}{4}-rac{y^2}{9}}}
angle$$

Meaning the z value is the opposite sign of the normal whilst x and y are the same

Which means the tangent plane at (x, y) is

$$L_{(x,y)}(a,b) = f(x,y) +
abla f_x(a-x) +
abla f_y(b-y)$$

Making the normal to the plane

$$ec{N}=\langle\pmrac{x}{4\sqrt{1-rac{x^2}{4}-rac{y^2}{lpha}}},\pmrac{y}{9\sqrt{1-rac{x^2}{4}-rac{y^2}{lpha}}},1
angle$$

$$ec{N}=\langle 1,1,-2
angle$$

$$\begin{cases} \pm \frac{x}{4\sqrt{1 - \frac{x^2}{4} - \frac{y^2}{9}}} = \frac{-1}{2} \\ \pm \frac{y}{9\sqrt{1 - \frac{x^2}{4} - \frac{y^2}{9}}} = \frac{-1}{2} \end{cases}$$
$$= \begin{cases} x = \mp 2\sqrt{1 - \frac{x^2}{4} - \frac{y^2}{9}} \\ y = \mp 4.5\sqrt{1 - \frac{x^2}{4} - \frac{y^2}{9}} \end{cases}$$

Which means that x and y are the same sign

$$=\begin{cases} x^2 = 4(1 - \frac{x^2}{4} - \frac{y^2}{9}) \\ y^2 = \frac{81}{4}(1 - \frac{x^2}{4} - \frac{y^2}{9}) \end{cases}$$

$$=\begin{cases} x^2 = 2 - \frac{2y^2}{9} \\ y^2 = \frac{81}{4} - \frac{81x^2}{16} - \frac{9y^2}{4} \end{cases}$$

$$=\begin{cases} x^2 = 2 - \frac{2y^2}{9} \\ y^2 = \frac{81}{4} - \frac{81(2 - \frac{2y^2}{9})}{16} - \frac{9y^2}{4} \end{cases}$$

$$=\begin{cases} x^2 = 2 - \frac{2y^2}{9} \\ y^2 = \frac{81}{4} - \frac{81}{8} + \frac{9y^2}{8} - \frac{9y^2}{4} \end{cases}$$

$$=\begin{cases} x^2 = 2 - \frac{2y^2}{9} \\ y^2 = \frac{81}{8} - \frac{9y^2}{8} \end{cases}$$

$$=\begin{cases} x^2 = 2 - \frac{2y^2}{9} \\ \frac{17}{8}y^2 = \frac{81}{8} \end{cases}$$

$$=\begin{cases} x^2 = 2 - \frac{2x^2}{17} \\ y^2 = \frac{81}{17} \end{cases}$$

$$=\begin{cases} x^2 = \frac{16}{17} \\ y^2 = \frac{81}{17} \end{cases}$$

$$(\pm \frac{4}{\sqrt{177}}, \pm \frac{9}{\sqrt{177}})$$

Where x and y are the same sign and z is the opposite sign

$$(\pm \frac{4}{\sqrt{17}}, \pm \frac{9}{\sqrt{17}}, \mp \sqrt{1 - \frac{x^2}{4} - \frac{y^2}{9}})$$

$$(\pm \frac{4}{\sqrt{17}}, \pm \frac{9}{\sqrt{17}}, \mp \sqrt{1 - \frac{4}{17} - \frac{9}{17}})$$

$$(\pm \frac{4}{\sqrt{17}}, \pm \frac{9}{\sqrt{17}}, \mp \sqrt{\frac{4}{17}})$$

$$(\pm \frac{4}{\sqrt{17}}, \pm \frac{9}{\sqrt{17}}, \mp \frac{2}{\sqrt{17}})$$

Therefore, the Normals are $\langle 1,1,-2\rangle$ at

$$(\tfrac{4}{\sqrt{17}},\tfrac{9}{\sqrt{17}},-\tfrac{2}{\sqrt{17}})$$

and

$$(-\frac{4}{\sqrt{17}}, -\frac{9}{\sqrt{17}}, \frac{2}{\sqrt{17}})$$

47

$$xz + 2x^2y + y^2z^3 = 11$$

 $P = (2, 1, 1)$

$$egin{cases} f_x x + z + 4xy + 3y^2 z^2 f_x = 0 \ f_y x + 2x^2 + 2z^3 y + 3y^2 z^2 f_y = 0 \end{cases}$$

$$\begin{cases} 2f_{xP} + 1 + 8 + 3f_{xP} = 0 \\ 2f_{yP} + 8 + 2 + 3f_{yP} = 0 \end{cases}$$
$$\begin{cases} f_{xP} = \frac{-9}{5} \\ f_{yP} = \frac{-10}{5} \end{cases}$$
$$\nabla f(P) = \langle -\frac{9}{5}, -2 \rangle$$

$$L_P(x,y) = 1 - rac{9}{5}(x-2) - 2(y-1) \ L_P(x,y) = -rac{9}{5}x - 2y + 33$$

_

14.6

3

$$f(x,y,z)=xy+z^2$$

$$x = s^2$$

$$y = 2rs$$

$$z=r^{2}$$

$$f(s,r)=2rs^3+r^4$$

$$f_s^\prime(s,r)=6rs^2$$

$$f_r^\prime(s,r)=2s^3+4r^3$$

$$g(heta,\phi)= an(heta+\phi)$$

$$\theta = xy$$

$$\phi = x + y$$
 $g(x,y) = an(xy + x + y)$ $g'_x(x,y) = \sec^2(xy + x + y)(y + 1)$ $g'_y(x,y) = \sec^2(xy + x + y)(x + 1)$

$$egin{align*} f(x,y,z) &= x^3 + yz^2 \ x &= u^2 + v \ y &= u + v^2 \ z &= uv \ (u,v) &= (-1,-1) \ f(u,v) &= (u^2 + v)^3 + (u + v^2)(uv)^2 \ f'_u(u,v) &= 6u(u^2 + v)^2 + 2(u + v^2)(uv) + (uv)^2 \ igg|_{(-1,-1)} &= 1 \ f'_v(u,v) &= 3(u^2 + v)^2 + 2(u + v^2)(uv) + 2v(uv)^2 \ igg|_{(-1,-1)} &= -2 \ \hline \end{array}$$

$$egin{aligned} g(x,y) &= x^2 - y^2 \ x &= e^u \cos v \ y &= e^u \sin v \ (u,v) &= (0,1) \ g(u,v) &= e^{2u} \cos^2 v - e^{2u} \sin^2 v \ g_u'(u,v) &= 2e^{2u} \cos^2 v - 2e^{2u} \sin^2 v \ g_u'(u,v) &= 2e^{2u} (\cos(2v)) \ igg|_{(0,1)} &= 2\cos 2 \ igcup \end{array}$$

$$egin{aligned} x &= s + t \ y &= s - t \ f(x,y) \end{aligned}$$
 $egin{aligned} rac{\partial f}{\partial s} &= rac{\partial f}{\partial x} rac{\partial x}{\partial s} + rac{\partial f}{\partial y} rac{\partial y}{\partial s} \ rac{\partial f}{\partial s} &= rac{\partial f}{\partial x} + rac{\partial f}{\partial y} \end{aligned}$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \frac{\partial y}{\partial t}
\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} - \frac{\partial f}{\partial y}
= \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}
\frac{\partial f}{\partial t} \frac{\partial f}{\partial s} = \left(\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y}\right) \left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\right)
\frac{\partial f}{\partial t} \frac{\partial f}{\partial s} = \left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2$$

a

$$F(x,y,z) = xz^2 + y^2z + xy - 1$$

$$egin{aligned} F_x'(x,y,z) &= z^2 + y \ F_y'(x,y,z) &= 2yz + x \ F_z'(x,y,z) &= 2xz + y^2 \ \hline \end{aligned}$$

b

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{z^2 + y}{2xz + y^2}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{2yz + x}{2xz + y^2}$$

35

$$egin{aligned} \|ec{r}\| &= r = \sqrt{x^2 + y^2 + z^2} \
abla f &= \langle rac{\partial f}{\partial x}, rac{\partial f}{\partial y}, rac{\partial f}{\partial z}
angle \end{aligned}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial F} \frac{\partial F}{\partial x}$$

$$f(x,y,z) = F(r) \ rac{\partial f}{\partial F} = 1$$

$$rac{\partial F}{\partial x} = rac{\partial F}{\partial r} rac{\partial r}{\partial x} \ rac{\partial f}{\partial x} = F'(r) rac{\partial r}{\partial x}$$

$$rac{\partial r}{\partial x}=rac{1}{2\sqrt{x^2+y^2+z^2}}2x=rac{x}{\sqrt{x^2+y^2+z^2}}$$

$$rac{\partial f}{\partial x} = F'(r) rac{x}{\sqrt{x^2 + y^2 + z^2}}$$

By that same logic we can conclude

$$rac{\partial f}{\partial y} = F'(r) rac{y}{\sqrt{x^2 + y^2 + z^2}} \ rac{\partial f}{\partial z} = F'(r) rac{z}{\sqrt{x^2 + y^2 + z^2}}$$

Which makes

$$egin{aligned}
abla f &= \langle F'(r) rac{x}{\sqrt{x^2+y^2+z^2}}, F'(r) rac{y}{\sqrt{x^2+y^2+z^2}}, F'(r) rac{z}{\sqrt{x^2+y^2+z^2}}
angle \ &= F'(r) \langle x,y,z
angle / \sqrt{x^2+y^2+z^2} \end{aligned}$$

Since

$$ec{r} = \langle x,y,z
angle \ \|ec{r}\| = \sqrt{x^2 + y^2 + z^2}$$

$$egin{aligned}
abla f &= F'(r)ec{r}/\|ec{r}\| \ &= F'(r)e_{ec{r}} \end{aligned}$$