Given $0 \leq x < \infty; \quad \alpha, \beta > 0$

$f(x) = \frac{x^{\alpha-1}e^{-x/\beta}}{\Gamma(\alpha)\beta^\alpha}$
$\mu = \alpha\beta$
$\sigma^2 = \alpha\beta^2$
$M(t) = (\frac{1}{1-\beta t})^\alpha$

Logistic (μ, β)

Given $-\infty < x < \infty; \quad -\infty < \mu < \infty; \quad \beta > 0$

$f(x) = \frac{e^{-(x-\mu)/\beta}}{\beta(1+e^{-(x-\mu)/\beta)})^2}$
$\mu = \mu$
$\sigma^2 = \frac{\pi^2\beta^2}{3}$
$M(t) = e^{\mu t}\Gamma(1+\beta t)$

Lognormal (μ, σ^2)

Distribution where the log of a variable is normally distributed. Similar in appearance to the gamma distribution.

Given $0 \leq x < \infty; \quad -\infty < \mu < \infty; \quad \sigma > 0$

$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\frac{e^{-(\log x-\mu)^2/(2\sigma^2)}}{x}$
$\mu = e^{\mu+(\sigma^2/2)}$
$\sigma^2 = e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}$
$EX^n = e^{n\mu+n^2\sigma^2/2}$

Normal (μ, σ^2)

Can be used to approximate many different kinds of distributions as their population sizes increase.

Given $-\infty < x < \infty; \quad -\infty < \mu < \infty; \quad \sigma > 0$

$f(x) = \frac{e^{-(x-\mu)^2/(2\sigma^2)}}{\sqrt{2\pi}\sigma}$
$\mu = \mu$
$\sigma^2 = \sigma^2$
$M(t) = e^{\mu t+\sigma^2 t^2/2}$

Paretto (α, β)

Given $a < x < \infty; \quad \alpha, \beta > 0$

$f(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}$
$\mu = \frac{\beta\alpha}{\beta-1} \quad ; \beta > 1$
$\sigma^2 = \frac{\beta\alpha^2}{(\beta-1)^2(\beta-2)} \quad ; \beta > 2$

T (v)

Given $-\infty < x < \infty; \quad v = 1, 2, 3, \ldots$

Given $\vec{\mu} \in \Re^k; \quad \mathbf{\Sigma} \in \Re^{k^2}; \quad k \in \mathbb{N}$

$f(\vec{x}) = (2\pi)^{-k/2}\det(\mathbf{\Sigma})^{-1/2}\exp\left(-\frac{1}{2}(\vec{x}-\vec{\mu})^T\mathbf{\Sigma}^{-1}(\vec{x}-\vec{\mu})\right)$
$\vec{\mu} = \vec{\mu}$
$\boldsymbol{\sigma^2} = \mathbf{\Sigma}$
$M(\vec{t}) = \exp\left(\vec{\mu}^T\vec{t} + \frac{1}{2}\vec{t}^T\mathbf{\Sigma}\vec{t}\right)$

Bivariate case $(\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \rho)$

$f(x,y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{1-\rho^2}}\exp\left(-\frac{1}{2(1-\rho^2)}\left(\left(\frac{\vec{x}-\vec{\mu}_X}{\sigma_X}\right)^2 - 2\rho\left(\frac{\vec{x}-\vec{\mu}_x}{\sigma_X}\right)\left(\frac{\vec{y}-\vec{\mu}_Y}{\sigma_Y}\right) + \left(\frac{\vec{y}-\vec{\mu}_Y}{\sigma_Y}\right)^2\right)\right)$

$\vec{\mu} = \langle\mu_X, \mu_Y\rangle$
$\Sigma = \begin{bmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{bmatrix}$

Families of Distributions

Exponential Family

Exponential Family
Any statistical distribution or family of distributions that can fit into the form:
$f(x|\theta) = h(x)c(\theta)\exp\left(\sum_{i=1}^{k} w_i(\theta)t_i(x)\right)$

Binomial

$\binom{n}{x}(1-p)^n\exp\left(\log(\frac{p}{1-p})x\right)$

Normal

$\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\mu^2}{2\sigma^2}\right)\exp\left(-\frac{x^2}{2\sigma^2}+\frac{\mu x}{\sigma^2}\right)$

Location Scale Family

Location Scale Family
Any statistical distribution or family of distributions that can fit into the form:
$g(x|\mu,\sigma) = \frac{1}{\sigma}f(\frac{x-\mu}{\sigma})$

Probability Inequalities

Chebychev's Inequality

Let X be a random variable and let $g(x)$ be a non-negative function. Then, for any $r > 0$

$P(g(X) \geq r) \leq \frac{Eg(X)}{r}$

Normal Probability Inequality

With Z as a normal distribution,

$f(x) = \frac{\Gamma(\frac{v+1}{2})}{\Gamma(\frac{v}{2})}\frac{1}{\sqrt{v\pi}}\frac{1}{(1+(\frac{x^2}{v}))^{(v+1)/2}}$
$\mu = 0 \quad ; v > 1$
$\sigma^2 = \frac{v}{v-2} \quad ; v > 2$
$MX^n = \begin{cases} \frac{\Gamma(\frac{n+1}{2})\Gamma(\frac{v-n}{2})}{\sqrt{\pi}\Gamma(v/2)}v^{n/2} & n < v; n \text{ is even} \\ 0 & n < v; n \text{ is odd} \end{cases}$

$\mathrm{T}(v) = \frac{N(0,1)}{\sqrt{\frac{\chi_v^2}{v}}}$

Uniform (a, b)

All values between a and b are evenly distributed and x has equal chance of landing anywhere on that range.

Given $a \leq x \leq b$

- a is the lower bound of the distribution
- b is the upper bound
- All values between a and b are equally distributed

$f(x) = \frac{1}{b-a}$
$\mu = \frac{b+a}{2}$
$\sigma^2 = \frac{(b-a)^2}{12}$
$M(t) = \frac{e^{bt}-e^{at}}{t(b-a)}$

Weibull (γ, β)

Given $0 \leq x < \infty; \quad \gamma, \beta > 0$

$f(x) = \frac{\gamma}{\beta}x^{\gamma-1}e^{-x^\gamma/\beta}$
$\mu = \beta^{1/\gamma}\Gamma(1+\frac{1}{\gamma})$
$\sigma^2 = \beta^{2/\gamma}(\Gamma(1+\frac{2}{\gamma}) - \Gamma^2(1+\frac{1}{\gamma}))$
$EX^n = \beta^{n/\gamma}\Gamma(1+\frac{n}{\gamma})$

Multivariable Distributions

Covariance and Correlation

$\mathrm{Cov}(X,Y) = E((X-\mu_X)(Y-\mu_Y))$
$\rho_{XY} = \frac{\mathrm{Cov}(X,Y)}{\sigma_X\sigma_Y}$

Multinomial Distribution

Very similar to a binomial, except there is more than one possible outcome per trial — as compared to success or failure in the binomial.

- Taking the marginal of any of the possible outcomes results in a regular binomial

Multivariable Normal $(\vec{\mu}, \mathbf{\Sigma})$

$P(|Z| \geq t) \leq \sqrt{\frac{e}{\pi}}\frac{e^{-t^2/2}}{t}$ for all $t > 0$

Random Samples

Properties of the sample

Mean: $\bar{X} = \frac{X_1+\ldots+X_n}{n} = \frac{1}{n}\sum_{i=1}^{n} X_i$

Variance: $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

$E(\sum_{i=1}^{n} g(X_i)) = nEg(X_1)$
$\mathrm{Var}(\sum_{i=1}^{n} g(X_i)) = n\mathrm{Var}g(X_1)$

Properties of properties of the sample of random variables

$E(\bar{X}) = \mu$
$\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$
$E(S^2) = \sigma^2$

Sample distributions of common distributions

Normal (μ, σ^2)

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$
S^2 is independent from $\bar{X}$

Gamma (α, β)

$\bar{X} \sim \mathrm{Gamma}\left(n\alpha, \frac{\beta}{n}\right)$

Cauchy

$\mathrm{Cauchy}(0,\sigma_1) + \ldots + \mathrm{Cauchy}(0,\sigma_n) = \mathrm{Cauchy}(0, \sum_{i=1}^{n}\sigma_i)$

Chi-squared

$\mathrm{Chi}(a_1) + \ldots + \mathrm{Chi}(a_n) = \mathrm{Chi}(\sum_{i=1}^{n} a_i)$