System Effects

Stability

There are two kinds of system stability: internal (Asymptotic) and external (BIBO)

BIBO Stability

BIBO stands for bounded input, bounded output.

A system is considered BIBO stable if for any bounded input, the output would be bounded as well.

More specifically, it can be shown that if the following condition holds, a system is BIBO stable:

$$\int\limits_{-\infty}^{\infty} |h(t)| \, dt < \infty$$

Where $h(t)$ is the impulse response.

BIBO stability (exernal) typically only is useful to analyse when the internal and exterior behaviors of systems are equivalent (most well behaved systems).

Asymptotic Stability

Although there are multiple definitions for internal stability for systems, for LTI systems, having asymptotic stability is a useful metric.

The concept of asymptotic stability is that the system tends to 0 as $t \to \infty$

In practice this means that each node in the system must approach 0 as $t \to \infty$.

Where $\vec{\lambda}$ are the characteristic roots of the system

1. If $\forall \lambda \in \vec{\lambda} : \Re(\lambda) < 0$ then the system is asymptotically stable
2. If $\exists \lambda \in \vec{\lambda} : \Re(\lambda) > 0$ or $\exists \lambda \in \vec{\lambda} : \Re(\lambda) = 0$ and is duplicated, then the system is unstable
3. Else, $\forall \lambda \in \vec{\lambda} : \Re(\lambda) = 0$ and are not duplicated, it is marginally stable

Marginally stable systems are systems that neither tend towards ∞ not 0, and just oscillates.

System Effect Timing

Theoretically, all effects have infinite time as they are constantly approacing 0 if they decay. It becomes useful to have a measure for the effective inflence time of effects, however.

We define the following as an effective time constant:

$$T_h = \frac{\int\limits_{-\infty}^{\infty} h(t)\ dt}{\max[h(t)]}$$

We also define the cutoff frequency to be

$$f_c = \frac{1}{T_h}$$