3.1

4

Let $Q(n)$ be the predicate "$n^2 \leq 30$."

a

Write $Q(2)$, $Q(-2)$, $Q(7)$, and $Q(-7)$, and indicate which of these statements are true and which are false.

Answer

$Q(2): 4 \leq 30$ True
$Q(-2): 4 \leq 30$ True
$Q(7): 49 \leq 30$ False
$Q(-7): 49 \leq 30$ False

b

Find the truth set of $Q(n)$ if the domain of n is $\mathbf{Z}$, the set of all integers.

Answer

$\|n\| \leq \sqrt{30}$
$\|n\| \leq 5$
$n \in [-5, 5]$

7

Find the truth set of each predicate.

c

Predicate: $1 \leq x^2 \leq 4$, domain: $\mathbf{R}$

Answer

$(1 \leq x \leq 2) \vee (1 \leq -x \leq 2)$
$x \in [-2, -1] \cup [1, 2]$

d

Predicate: $1 \leq x^2 \leq 4$, domain: $\mathbf{Z}$

✓ Answer

$(1 \leq x \leq 2) \vee (1 \leq -x \leq 2)$
$x \in [-2, -1] \cup [1, 2]$

12

Find counterexamples to show that the statement is false

$\forall$ real numbers x and y, $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$.

✓ Answer

Let $x = 1$ and $y = 1$
$\sqrt{1+1} \neq \sqrt{1} + \sqrt{1}$
$\sqrt{2} \neq 2$

17

Rewrite the following in the form "$\exists$ ____ x such that ____."

b

Some real numbers are rational.

✓ Answer

$\exists$ some $x \in \mathbf{R}$ such that x is rational

20

Rewrite the following statement informally in at least two different ways without using variables or the symbol $\forall$ or the words "for all."

$\forall$ real numbers x, if x is positive then the square root of x is positive.

✓ Answer

- If x is real and positive, then its square root is positive
- If x is positive and its square root is not positive, then x is not a real number

22

Rewrite the following in the form "$\forall$ ____ x, if ____ then ____."

a

All Java programs have at least 5 lines.

Answer

$\forall$ Java programs x, x has at least 5 lines.

28

Rewrite each statement without using quantifiers or variables. Indicate which are true and which are false, and justify your answers as best as you can.

Let the domain of x be the set D of objects discussed in mathematics courses, and let $\text{Real}(x)$ be "x is a real number," $\text{Pos}(x)$ be "x is a positive real number," $\text{Neg}(x)$ be "x is a negative real number," and $\text{Int}(x)$ be "x is an integer."

a

$\text{Pos}(0)$

Answer

0 is a positive real number.

b

$\forall x, \text{Real}(x) \wedge \text{Neg}(x) \rightarrow \text{Pos}(-x)$

Answer

If the inverse of an object discussed in mathematics courses is not positive, then the object is either not a real number, or the object is not negative.

c

$\forall x, \text{Int}(x) \rightarrow \text{Real}(x)$

Answer

All integers are real.

d

$\exists x \text{ s.t. } \mathrm{Real}(x) \wedge \neg \mathrm{Int}(x)$

✓ **Answer**

Some real numbers are not integers

3.2

1

Which of the following is a negation for “All discrete mathematics students are athletic”? More than one answer may be correct.

1. There is a discrete mathematics student who is nonathletic.
2. All discrete mathematics students are nonathletic.
3. There is an athletic person who is not a discrete mathematics student.
4. No discrete mathematics students are athletic.
5. Some discrete mathematics students are nonathletic.
6. No athletic people are discrete mathematics students.

✓ **Answer**

1, 5

5

Write a negation for each of the following statements.

a

Every valid argument has a true conclusion.

✓ **Answer**

There is an argument with a false conclusion

b

All real numbers are positive, negative, or zero.

Answer

There is a real number that is neither positive, negative, or zero

12

Determine whether the proposed negation is correct. If it is not, write a correct negation.

Statement: The product of any irrational number and any rational number is irrational.

Proposed negation: The product of any irrational number and any rational number is rational.

Answer

False, There exists a product of some irrational number and some rational number that is not irrational.

17

Write a negation for each statement

$\forall$ integers d, if $\frac{6}{d}$ is an integer, then $d = 3$

Answer

There exists an an integer d where if $\frac{6}{d}$, $d \neq 3$

29

Write the contrapositive, converse, and inverse. Indicate as best as you can which of these statements are true and which are false. Give a counterexample for each that is false.

$\forall n \in Z$, if n is prime then n is odd or $n = 2$.

Answer

Contrapositive (True): $\forall n \in \mathbf{Z}$ if n is even and $n \neq 2$ then n is not prime
Converse (False $n = 9$): $\forall n \in \mathbf{Z}$ if n is odd or $n = 2$ then n is prime
Inverse (False $n = 9$): $\forall n \in \mathbf{Z}$ if n is not prime then n is even and $n \neq 2$

48

Use the facts that the negation of a $\forall$ statement is a $\exists$ statement and that the negation of an if-then statement is an and statement to rewrite the statement without using the word necessary or sufficient.

Being a polynomial is not a sufficient condition for a function to have a real root.

✓ **Answer**

$x \in$ set of all functions
$P : x$ is a polynomial
$Q : x$ has a real root
$P \wedge \neg Q$

3.3

2

Let $G(x, y)$ be "$x^2 > y$" Indicate which of the following statements are true and which are false.

a

$G(2, 3)$

✓ **Answer**

$4 > 3$ True

b

$G(1, 1)$

✓ **Answer**

$1 > 1$ False

c

$G\left(\frac{1}{2}, \frac{1}{2}\right)$

✓ **Answer**

$\frac{1}{4} > \frac{1}{2}$ False

d

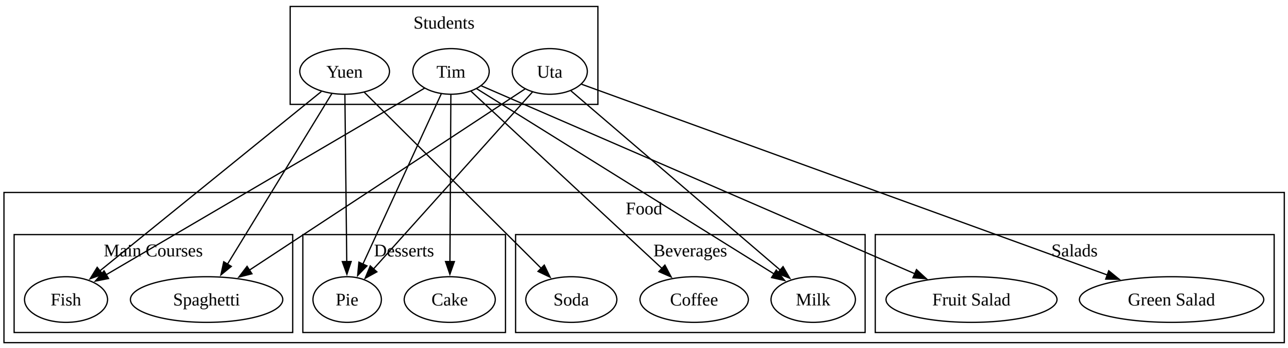
$G(-2, 2)$

✓ **Answer**

$4 > 2$ True

10

Determine whether each of the following statements is true or false.



a

$\forall$ student S, $\exists$ a dessert D such that S chose D.

✓ **Answer**

True, Yuen chose a Pie, Tim chose a Pie and a Cake, and Uta chose a Pie

b

$\forall$ student S, $\exists$ a salad T such that S chose T.

✓ **Answer**

False, Yuen did not choose a Salad

19

Rewrite the statement in English without using the symbol $\forall$ or $\exists$ or variables and expressing your answer as simply as possible. Also, write a negation for the statement.

$\exists x \in \mathbf{R}$ such that for every real number y,
$x + y = 0$

✓ **Answer**

For every real number, there is another real number such that their sum is 0.
There is a real number that does not have another a real number where their sum is 0.

23

Rewrite the statement in English without using the symbol $\forall$ or $\exists$ or variables. Also, indicate whether the statement is true or false.

a

$\forall$ nonzero real number r, $\exists$ a real number s such that $rs = 1$.

✓ **Answer**

Every nonzero real number has another real number where their product is 1.
True.

b

$\exists$ a real number r such that $\forall$ nonzero real number s, $rs = 1$.

✓ **Answer**

There is a real number where multiplying it with any other real number results in 1.
False, literally any number except 1.

3.4

2

Use universal instantiation or universal modus ponens to
fill in valid conclusions for the argument

If an integer n equals $2k$ and k is an integer, then n is even.
0 equals 2×0 and 0 is an integer.

✓ **Answer**

0 is even.

14

The argument may be valid by universal modus ponens or universal modus tollens; others are invalid and exhibit the converse or the inverse error. State whether the argument is valid or invalid. Justify your answer.

If compilation of a computer program produces error messages, then the program is not correct.
Compilation of this program does not produce error messages.
∴ This program is correct.

✓ **Answer**

This has an inverse error, this assumes the inverse.

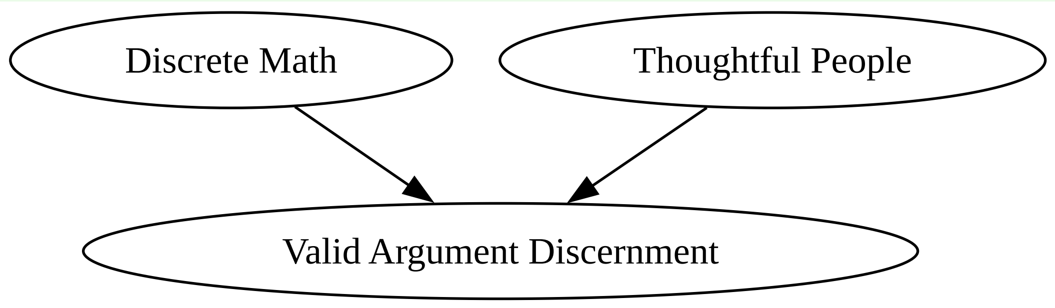
22-24

Indicate whether the arguments in 22–24 are valid or invalid. Support your answers by drawing diagrams.

22

All discrete mathematics students can tell a valid argument from an invalid one.
All thoughtful people can tell a valid argument from an invalid one.
∴ All discrete mathematics students are thoughtful.

✓ **Answer**

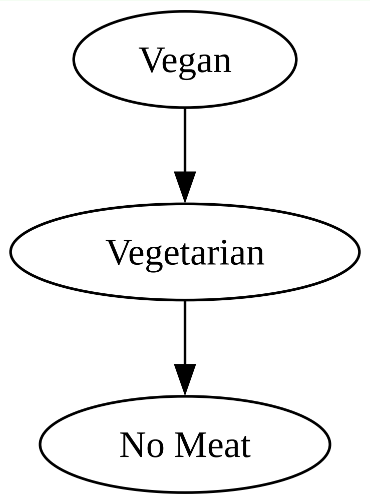


False, assumes the converse of the second statement.

24

No vegetarians eat meat.
All vegans are vegetarian.
∴ No vegans eat meat.

✓ **Answer**



True, by chain rule