PHYS 122 HW 1

1 Triboelectricity

a

Fun with scotch tape.

1. You peel two strips of scotch tape off the same roll. The strips repel one another.
2. A long time goes by. Then you stick the sticky side of one strip to the smooth side of the other and rip them apart. The strips now attract one another.

Explain why in one or two sentences.

✓ **Answer**

The adhesive and the cellophane portion of the tape have different triboelectric properties, so when the two are ripped off the roll, they will be similarly charged, while the roll itself will have the opposite charge. Since the two pieces have the same charge, they will repel each other.

After time passes, their charge will disperse. When sticking one to the other and ripping it apart, since one side is adhesive and the other cellophane, they will be charged oppositely, and thus attract each other.

b

Physics of Laundry.

i

Your clothing tends to cling together after going through the dryer. Why?

✓ **Answer**

The frequent rubbing against each other in the dryer will charge up the clothes and make some positively and some negatively charged, therefore they will attract each other.

ii

In one load the clothing is all of the same material, cotton; in the other load there is clothing of various different kinds of materials. In which case do you expect the clothes to cling together more?

✓ Answer

One with different materials, as when all the materials are the same they are likely to have the same triboelectric effect, thus all either gaining or losing charge together instead of transferring between the clothes.

c

Hanging by a thread. Two metal spheres hang from nylon threads and attract each other when brought close together.

i

What can you say about the charges on the two spheres?

✓ Answer

They are likely oppositely charged as they attract each other

ii

The spheres are now brought into contact. Will they continue to attract each other?

✓ Answer

Since the spheres are metal, they will be conductive, thus neutralising the charge when brought together. No they will no longer continue to attract each other.

2 Electroscope

Two plastic spheres A and B, both of mass m, hang by threads of length l from a common point of support (see figure 2). An unknown charge $+q$ is imparted to each sphere. As a result the spheres hang with each thread making an angle θ to the vertical.

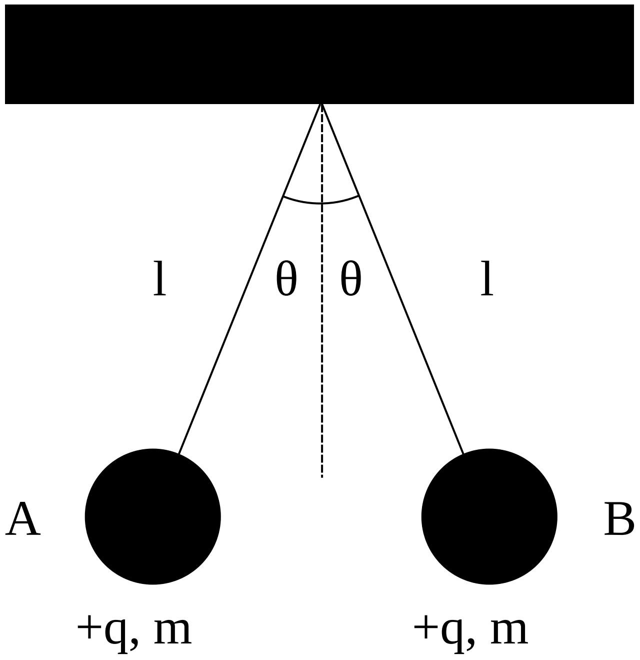

Fig. 2

Determine q. Give your answer in terms of m, θ, l and g = acceleration due to gravity.
Hint: First draw a free body diagram for sphere A.

✓ **Answer**

A and B are mirrors of each other, so I will solve for one.

$F_g = mg$
$F_T = \frac{mg}{\cos\theta}$
$F_{Tx} = mg\tan\theta$
$F_C = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = F_{Tx}$ N2L
$r = 2l\sin\theta$

$\frac{1}{4\pi\epsilon_0}\frac{q^2}{(2l\sin\theta)^2} = mg\tan\theta$
$q^2 = 16\pi\epsilon_0 mgl^2\tan\theta\sin^2\theta$
$q = \sqrt{16\pi\epsilon_0 mgl^2\tan\theta\sin^2\theta}$
$q = 4l\sin\theta\sqrt{\pi\epsilon_0 mg\tan\theta}$
$\square$

3 Millikan and the quantization of charge

a

Terminal velocity: A small oil drop of mass m falls under the influence of gravity and air drag at a uniform speed v called the terminal velocity. The magnitude of the air drag force is Kv; the force points in the direction opposite to the motion of the oil drop. K is a constant that depends on the size of the oil drop and the viscosity of air.

i

Calculate the speed v. Give your answer in terms of m, g and K.
Hint: Apply Newton's second law

Answer

$F_g = mg$
$F_d = F_g = Kv$ N2L
$Kv = mg$
$v = \frac{mg}{K}$
□

ii

Invert your answer to part (i) to give K in terms of m, g and v.

Answer

$K = \frac{mg}{v}$
□

b

Electrostatic forces: A strong uniform electric field E that points vertically upward is now applied and a charge q is imparted to the oil drop. The oil drop is now observed to float upward at a uniform speed u.

i

Determine u. Give your answer in terms of m, g, q, E and K.

Answer

$F_g = mg$
$F_d = Ku$
$F_e = qE$

$qE = mg + Ku$ N2L
$u = \frac{qE-mg}{K}$
□

ii

Suppose that the charge of the droplet is changed from q to $q + e$ where e is the fundamental unit of charge (i.e. the charge of the electron is $-e$ in our notation). What is Δu, the change in the speed of the oil drop? Give your answer in terms of e, E and K.

✓ **Answer**

Assuming the particle approaches a terminal velocity, and that Δu is the difference in terminal velocities:
$u_f = \frac{(q+e)E-mg}{K}$
$\Delta u = u_f - u = \frac{eE}{K}$
□

iii

Use the expression for K in part (a-ii) to determine Δu in terms of e, E, m, g and v.

✓ **Answer**

$K = \frac{mg}{v}$
$\Delta u = u_f - u = \frac{eE}{K}$

$\Delta u = \frac{eEv}{mg}$
□

c

In his famous oil drop experiment Millikan observed a drop that drifted at a terminal velocity $v = 1.1 \times 10^{-4}\ \frac{m}{s}$. He then applied an electric field of $E = 6 \times 10^5\ \frac{N}{C}$ and varied the charge of the drop by various means. He found that the drop drifted upward and that the speed of the drop changed in steps of size $\Delta u = 3.0 \times 10^{-4}\ \frac{m}{s}$ as charge was added or subtracted. Assuming, as Millikan did, that the jumps in the velocity correspond to the addition or subtraction of a fundamental unit of charge, determine e in Coulombs. Take the mass of the drop to be $m = 3.5 \times 10^{-15}\ kg$.

✓ **Answer**

$e = \frac{mg\Delta u}{Ev}$
$e = \frac{3.5(9.81)(3.0)\times10^{-19}}{6(1.1)\times10}\ C$
$e = 1.6 \times 10^{-19}\ C$
$\square$

4 Cathode ray tube (optional)

In the cathode ray tube shown in figure 4, a beam of electrons is injected horizontally into the exact center of the parallel-plate region.

Here $L = 3.0\ cm$ and $d = 0.20\ cm$. In the region between the plates there is a uniform electric field E that points vertically downward. The magnitude of this electric field is $E = 1.7 \times 10^4\ \frac{N}{C}$.

What minimum speed v_0 should the electrons have in order to ensure that they do not strike the upper plate? (Source: Adapted from Ohanian and Markert, Physics for Engineers and Scientists).

5 Hierarchy problem

Two protons in an atomic nucleus are separated by a distance of $1.0 \times 10^{-15}\ m$. Each proton has a charge $e = 1.6 \times 10^{-19}\ C$ and a mass $m = 1.67 \times 10^{-27}\ kg$.

Useful data: $\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9\ \frac{kg\ m^3}{C^2\ s^2}$ and $G = 6.67 \times 10^{-11}\ \frac{m^3}{kg\ s^2}$

a

What is the electric force between the protons, F_e? Is it attractive or repulsive?

✓ **Answer**

$F_e = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}$
$F_e = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}$
$= 2.3 \times 10^2\ N$ Repulsive

b

What is the gravitational force between the protons, F_g? Is it attractive or repulsive?

✓ **Answer**

$F_g = G\frac{m_1 m_2}{r^2}$
$F_g = G\frac{m^2}{r^2}$
$= 1.9 \times 10^{-34}\ N$ Attractive

c

What is the ratio of the magnitude of the two forces, $\frac{F_e}{F_g}$?

✓ **Answer**

$1 : 8.2 \times 10^{-37}$

d*

Inside a nucleus there is an additional attractive force between the protons called the strong nuclear force. This force is stronger than the electrostatic repulsion between the protons and is the force that holds together atomic nuclei. To a good approximation the strong force between a pair of protons, or a pair of neutrons, or a proton and a neutron are all the same. In contrast the neutron, being electrically neutral, does not experience the electrostatic force calculated in part (a). The relative weakness of gravity compared to other forces on the subatomic scale is a great mystery.

6 Electric quadrupole moment

A point charge $+2q$ is placed at the origin. Two additional point charges $-q$ are placed symmetrically about the origin on the x axis each at a distance a from the origin.

Point P is x units to the right of the origin.
Point Q is y units above the origin.

This configuration of charges is called an electric quadrupole.

a

i

In which direction does the total electric field point at the location P?

✓ **Answer**

$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$
$E_1 = \frac{1}{4\pi\epsilon_0}\frac{-q}{(x+a)^2}$
$E_2 = \frac{1}{4\pi\epsilon_0}\frac{2q}{x^2}$
$E_3 = \frac{1}{4\pi\epsilon_0}\frac{-q}{(x-a)^2}$

$E_T = \frac{q}{4\pi\epsilon_0}\left(-\frac{1}{(x+a)^2} + \frac{2}{x^2} - \frac{1}{(x-a)^2}\right)$

If $x > a$, then E_T will always be negative, so the field points left.

ii

Determine the electric field at P in terms of q, a, ϵ_0 and x = the x-coordinate of P.

✓ **Answer**

$E_T = \frac{q}{4\pi\epsilon_0}\left(-\frac{1}{(x+a)^2} + \frac{2}{x^2} - \frac{1}{(x-a)^2}\right)$
$\square$

iii

Simplify your expression in the limit that a is small compared to x.
Hint: You may use the small a/x expansion
$\frac{1}{(x+a)^2} = \frac{1}{x^2} - \frac{2a}{x^3} + \frac{3a^2}{x^4} + \ldots$

✓ **Answer**

$E_T = \frac{q}{4\pi\epsilon_0}\left(-\frac{1}{(x+a)^2} + \frac{2}{x^2} - \frac{1}{(x-a)^2}\right)$
$E_T = -\frac{6a^2q}{4\pi\epsilon_0 x^4}$
$\square$

b

i

At the location Q in which direction does the total electric field point?

✓ **Answer**

Upwards, as the two negative charges on the side would sum to be weaker than the one larger charge.

ii

Determine the electric field at Q in terms of q, a, ϵ_0 and y = the y-coordinate of Q.

✓ **Answer**

$$E_{C1} = \frac{1}{4\pi\epsilon_0}\frac{-qy}{(a^2+y^2)^{3/2}}$$
$$E_{C2} = \frac{1}{4\pi\epsilon_0}\frac{2q}{y^2}$$
$$E_{C3} = \frac{1}{4\pi\epsilon_0}\frac{-qy}{(a^2+y^2)^{3/2}}$$

$$E_T = \frac{q}{2\pi\epsilon_0}\left(\frac{1}{y^2} - \frac{y}{(a^2+y^2)^{3/2}}\right)$$

□

iii

Simplify your expression in the limit that a is small compared to y.
Hint: You may use the small a/y expansion
$\frac{1}{(y^2+a^2)^{3/2}} = \frac{1}{y^3} - \frac{3a^2}{2y^5} + \ldots$

✓ **Answer**

$$E_T = \frac{q}{2\pi\epsilon_0}\left(\frac{1}{y^2} - \frac{y}{(a^2+y^2)^{3/2}}\right)$$
$$E_T = \frac{3a^2q}{4y^4\pi\epsilon_0}$$

□