1.1

a

The sample space would be a sequence of four digits, where each digit can either be H or T

b

Any natural number

c

Any real number above 0

d

Any real number above 0

e

Any rational number between 0 and 1

1.4

a

$$
\begin{aligned}
&P(A \cup B) \\
&= P(A) + P(B) - P(A \cap B)
\end{aligned}
$$

b

$$
\begin{aligned}
&P((A \setminus B) \cup (B \setminus A)) \\
&= P(A \setminus B) + P(B \setminus A) \\
&= P(A \cap B^C) + P(B \cap A^C) \\
&= P(A) - P(A \cap B) + P(B) - P(A \cap B) \\
&= P(A) + P(B) - 2P(A \cap B)
\end{aligned}
$$

c

$$
\begin{aligned}
&P(A \cup B) \\
&= P(A) + P(B) - P(A \cap B)
\end{aligned}
$$

d

$$
\begin{aligned}
&P((A \setminus B) \cup (B \setminus A)) \\
&= P(A \setminus B) + P(B \setminus A) \\
&= P(A \cap B^C) + P(B \cap A^C)
\end{aligned}
$$

$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$
$= P(A) + P(B) - 2P(A \cap B)$

1

Let (S, B, P) be a probability model and let $A, B \in B$. Using only the Kolmogorov axioms and Theorem 1 on slide 9 in Lecture 2.1 show the following:

1. $P(A) = P(A \cap B) + P(A \cap B^C)$
2. $P(A \setminus B) = P(A) - P(A \cap B)$
3. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
4. If $A \subseteq B$ then $P(A) \leq P(B)$

You may prove the four parts in any order you like. Just make sure you don't go in a circle, e.g. use (b) to prove (a) and then use (a) to prove (b)

a

$P(A) = P(A \cap B) + P(A \cap B^C)$
Since B and B^C are disjoint, $A \cap B$ and $A \cap B^C$ are also disjoint
Define $B_1^* = B$ and $B_2^* = B^C$
$P(A \cap B) + P(A \cap B^C)$
$= P(A \cap B_1^*) + P(A \cap B_2^*)$
$= \sum_{i=1}^{2} P(A \cap B_i^*)$
$= P(\bigcup_{i=1}^{2} A \cap B_i^*)$
$= P((A \cap B_1^*) \cup ((A \cap B_2^*)))$
$= P((A \cap B) \cup (A \cap B^C))$
$= P(A \cap (B \cup B^C))$
$= P(A \cap \mathbb{U})$
$= P(A)$
$\square$

b

$P(A \setminus B) = P(A) - P(A \cap B)$

$A \setminus B = \{x : x \in A : x \notin B\}$
$= A \cap B^C$

$P(A) = P(A \cap B) + P(A \cap B^C)$
$= P(A \cap B) + P(A \setminus B)$

$P(A \setminus B) = P(A) - P(A \cap B)$
$\square$

c

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P((A \cup B) \cap B) + P((A \cup B) \cap B^C)$
$= P(B) + P((A \cap B^C) \cup (B \cap B^C))$
$= P(B) + P(A \cap B^C)$
$= P(B) + P(A \cap B^C) + P(A \cap B) - P(A \cap B)$
$= P(B) + P(A) - P(A \cap B)$
$\square$

d

If $A \subseteq B$ then $P(A) \leq P(B)$

If $A = B$ then $P(A) = P(B)$

If $A \subset B$ then $x \in A \implies x \in B$
$A = \{x : x \in A\}$
$A = \{x : x \in A : x \in B\}$
$A \cap B = \{x : x \in A : x \in B\}$
$A \cap B = A$
$P(A \cap B) = P(A)$

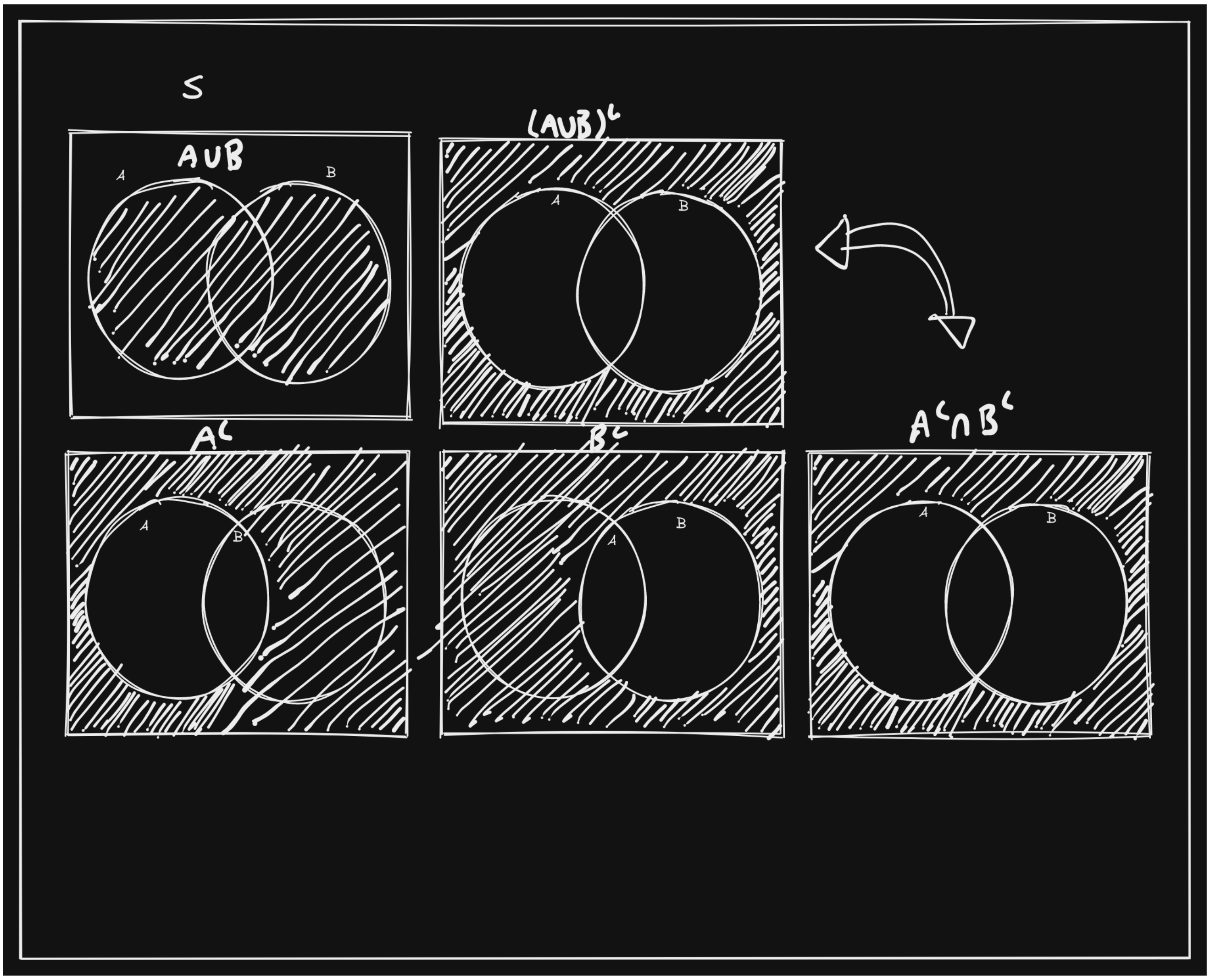
$P(B) = P(A \cap B) + P(A^C \cap B)$
$= P(A) + P(A^C \cap B)$

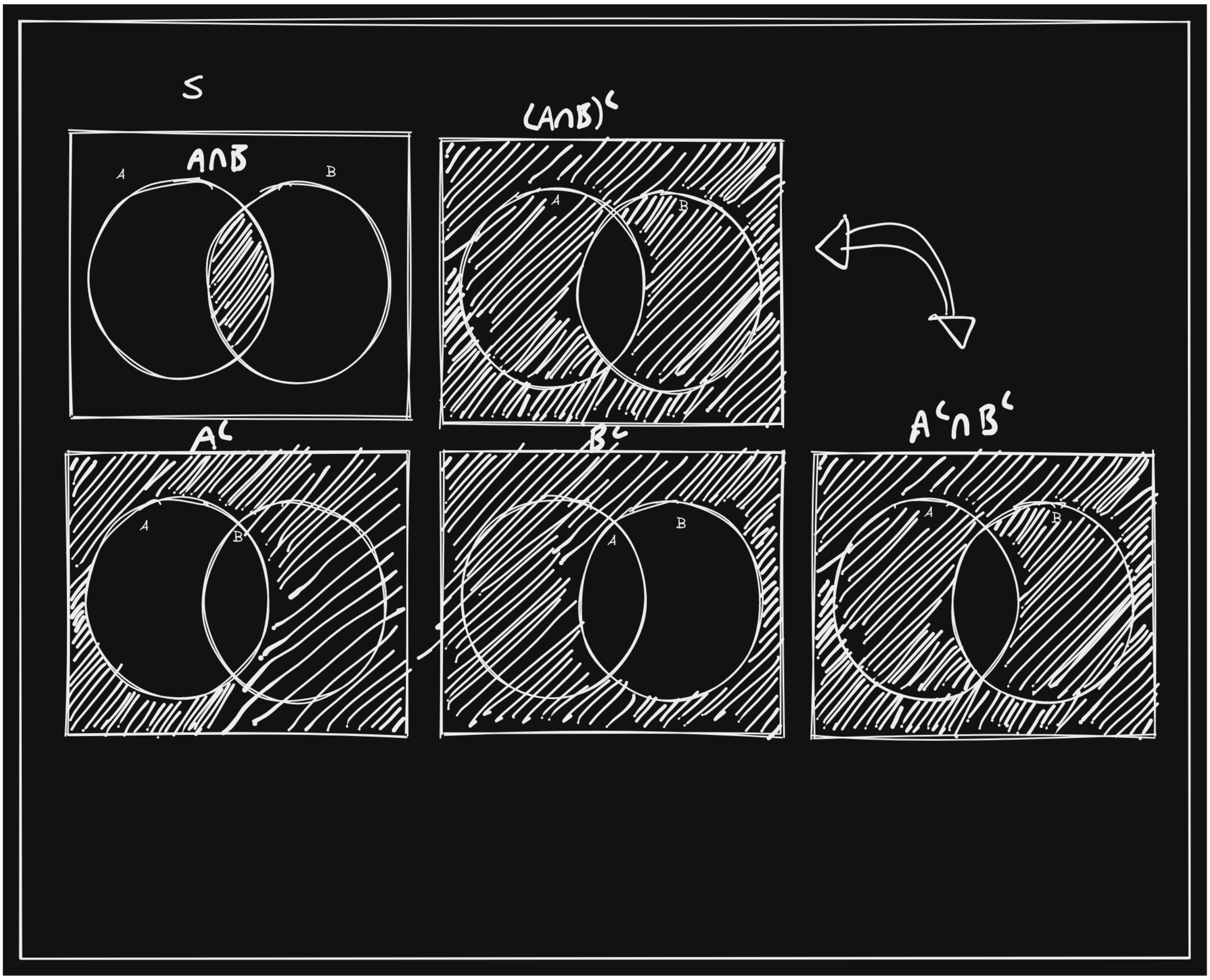
$P(A^C \cap B) > 0$

$P(B) > P(A)$

2

Draw Venn diagrams to illustrate DeMorgan's laws
$(A \cup B)^C = A^C \cap B^C$ and $(A \cap B)^C = A^C \cup B^C$


S
A∪B
A
B
(A∪B)^c
A^c
B^c
A^c∩B^c



S
A∩B
A
B
(A∩B)^c
A
B
A^c
A
B
B^c
A
B
A^c∩B^c
A
B