assignment_1_Tri_Ninh

February 15, 2018

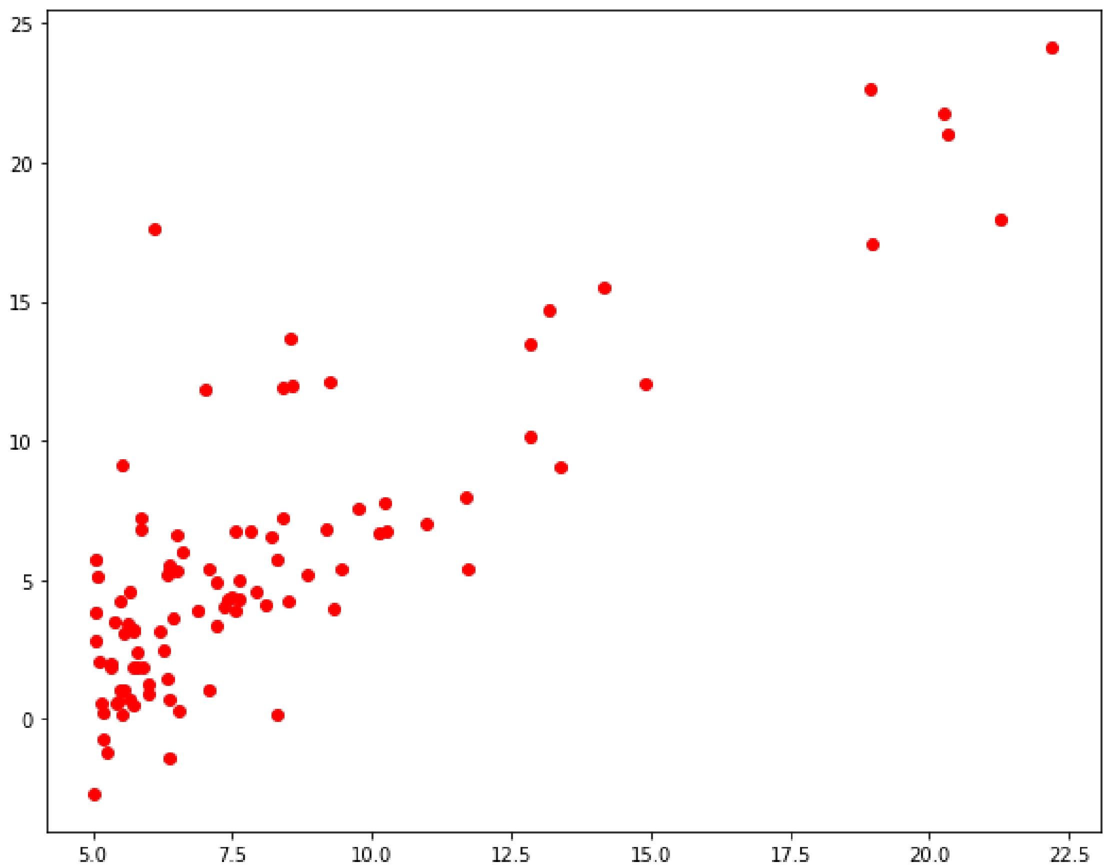
```
In [1]: import pandas as pd
        import matplotlib as mp
        from matplotlib import pyplot as plt
        import numpy as np
        %matplotlib inline
```

0.0.1 1. Linear Regression with one variable

```
In [2]: df0 = pd.read_csv('ex1data1.csv', header=None)
```

```
In [3]: df0.columns=['Population', 'Profit']
        #df0.plot(kind='scatter', x='', y='1', figsize=(12,8))
        df0.insert(0, 'w0', 1)
```

```
In [4]: x=df0['Population'].values
        y=df0['Profit'].values
        plt.figure(figsize=(10,8))
        plt.scatter(x, y,  c='red')
        plt.show()
```




```
In [5]: x = df0['Population'].values
        x

Out[5]: array([ 6.1101,  5.5277,  8.5186,  7.0032,  5.8598,  8.3829,
                7.4764,  8.5781,  6.4862,  5.0546,  5.7107, 14.164 ,
                5.734 ,  8.4084,  5.6407,  5.3794,  6.3654,  5.1301,
                6.4296,  7.0708,  6.1891, 20.27  ,  5.4901,  6.3261,
                5.5649, 18.945 , 12.828 , 10.957 , 13.176 , 22.203 ,
                5.2524,  6.5894,  9.2482,  5.8918,  8.2111,  7.9334,
                8.0959,  5.6063, 12.836 ,  6.3534,  5.4069,  6.8825,
               11.708 ,  5.7737,  7.8247,  7.0931,  5.0702,  5.8014,
               11.7   ,  5.5416,  7.5402,  5.3077,  7.4239,  7.6031,
                6.3328,  6.3589,  6.2742,  5.6397,  9.3102,  9.4536,
                8.8254,  5.1793, 21.279 , 14.908 , 18.959 ,  7.2182,
                8.2951, 10.236 ,  5.4994, 20.341 , 10.136 ,  7.3345,
                6.0062,  7.2259,  5.0269,  6.5479,  7.5386,  5.0365,
               10.274 ,  5.1077,  5.7292,  5.1884,  6.3557,  9.7687,
                6.5159,  8.5172,  9.1802,  6.002 ,  5.5204,  5.0594,
                5.7077,  7.6366,  5.8707,  5.3054,  8.2934, 13.394 ,  5.4369])
```

```
In [6]: def cost_function(x, y, w1, w0):
            total_cost = 0
            for i in range(len(x)):
                total_cost += np.power(((w1 * x[i] + w0) - y[i]),2)
            return total_cost / len(x)
```

```
In [7]: oldCost = cost_function(x, y, 1, 1)
        oldCost
```

Out[7]: 20.533040982767009

```
In [8]: def gradient_descent(x, y, w1, w0, alpha, iterations):
            inner_w1_deriv = 0
            inner_w0_deriv = 0
            #temp = 1
            cost = 0
            for _ in range(iterations):
                #cost
                inner_w1_deriv = 0
                inner_w0_deriv = 0
                for i in range(len(x)):
                    # Calculate partial derivatives
                    # -2x(y - (mx + b))
                    inner_w1_deriv += (x[i] * ((w1*x[i]) - y[i]))
                    inner_w0_deriv += ((w1*x[i]) - y[i])
                # We subtract because the derivatives point in direction of steepest ascent
                temp1 = w1 - (2 * (inner_w1_deriv / len(x)) * alpha)
                temp0 = w0 - (2 * (inner_w0_deriv / len(x)) * alpha)
                w1 = temp1
                w0 = temp0
                #temp = cost
                cost= cost_function(x, y, w1, w0)
        #         if j % 10 == 0:
        #             print "iter: "+str(j) + " cost: "+str(cost)
            return w1, w0, cost
```

```
In [9]: alpha = 0.01
        iterations = 100
        w1, w0, cost = gradient_descent(x, y, 1, 1, alpha, iterations)
```

```
In [10]: cost
```

Out[10]: 11.287549032521689

```
In [11]: print 'RMSE %f' % np.sqrt(cost)
```

RMSE 3.359695

```
In [12]: w1
```

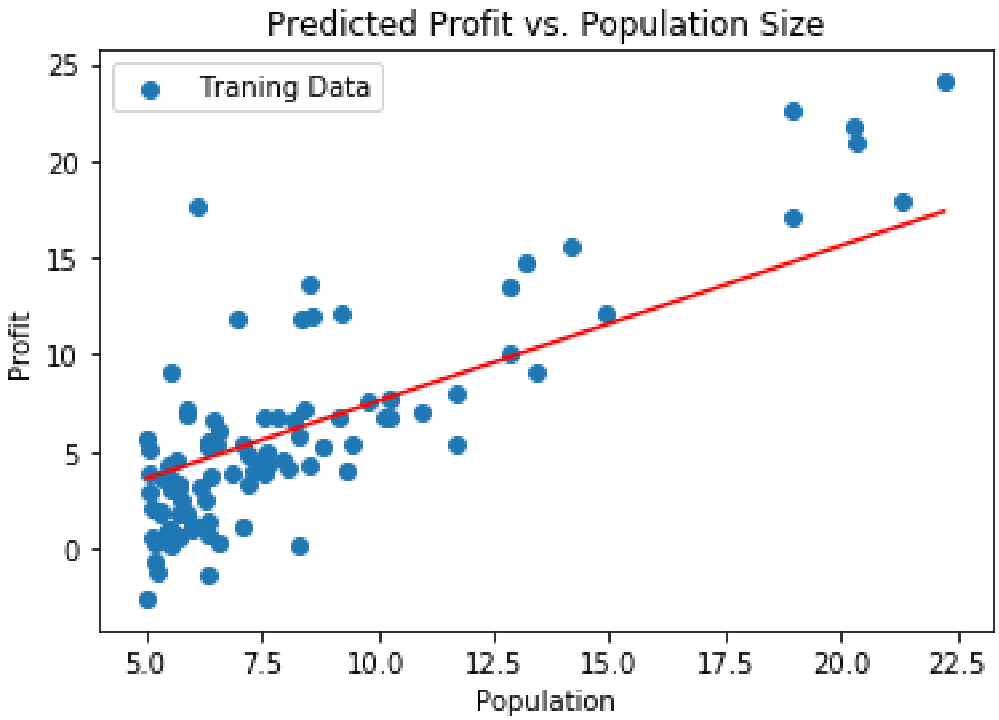
Out[12]: 0.80252684920135675

In [13]: w0

Out[13]: -0.4384414554300905

```
In [14]: x1 = np.linspace(df0.Population.min(), df0.Population.max(), 100)
         f = w1 * x1 + w0
         fig, ax = plt.subplots()
         ax.plot(x1, f, 'r')
         ax.scatter(df0.Population, df0.Profit, label='Traning Data')
         ax.legend(loc=2)
         ax.set_xlabel('Population')
         ax.set_ylabel('Profit')
         ax.set_title('Predicted Profit vs. Population Size')
         fig.show()
```

```
/Users/trininh/anaconda/lib/python2.7/site-packages/matplotlib/figure.py:403: UserWarning: mat
  "matplotlib is currently using a non-GUI backend, "
```



0.0.2 2-1

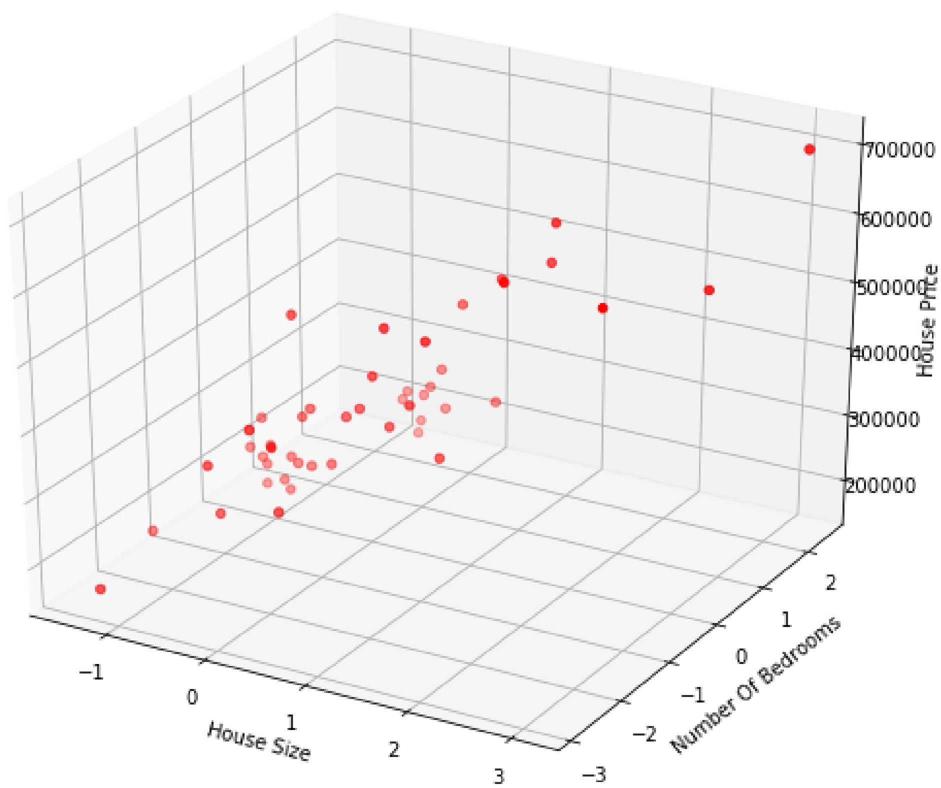
```
In [15]: df1 = pd.read_csv('ex1data2.csv', header=None)
```

```
In [16]: df1.columns=['HouseSize', 'NumberOfBedrooms', 'HousePrice']

In [17]: df1['HouseSize'] = (df1['HouseSize'] - df1['HouseSize'].mean()) / df1['HouseSize'].std
         df1['NumberOfBedrooms'] = (df1['NumberOfBedrooms'] - df1['NumberOfBedrooms'].mean()) /

In [18]: df1.insert(0, 'Ones', 1)

In [19]: from mpl_toolkits.mplot3d import Axes3D
         fig = plt.figure(figsize=(10,8))
         ax = fig.add_subplot(111, projection='3d')
         X = df1['HouseSize']
         Y = df1['NumberOfBedrooms']
         Z = df1['HousePrice']
         ax.scatter(X, Y, Z, c = 'r')
         ax.set_xlabel('House Size')
         ax.set_ylabel('Number Of Bedrooms')
         ax.set_zlabel('House Price')
         plt.show()
```




```
In [20]: x1 = np.matrix(df1.iloc[:, 0:3].values)
         y1 = np.matrix(df1.iloc[:, 3:4].values)
```

```
In [31]: # def cost_function_matrix(x, y, w):
         #     total_cost = ((np.matmul(x,w) - y).T)*(np.matmul(x,w) - y)
         # #     total_cost = (np.matmul(x, w) - y)**2
         #     m = len(x)
         #     cost = total_cost / m
         #     return cost
             #return np.sum(np.square(np.matmul(x, w) - y)) / (2 * len(y))
         m = y1.size
         def h(w,x): #Linear hypothesis function
             return np.dot(x,w)

         def cost_function_matrix(w,x,y): #Cost function
             """
             theta_start is an n- dimensional vector of initial theta guess
             X is matrix with n- columns and m- rows
             y is a matrix with m- rows and 1 column
             """
             #note to self: *.shape is (rows, columns)
             return float((1./(2*m)) * np.dot((h(w,x)-y).T,(h(w,x)-y)))

In [33]: w = np.zeros((3,1))
         #w.reshape((2,1))
         cost_function_matrix(w, x1, y1)

Out[33]: 65591585744.68085

In [23]: x1.shape, y1.shape, w.shape

Out[23]: ((47, 3), (47, 1), (3, 1))

In [67]: m

Out[67]: 47

In [99]: # w.reshape((2,1))
         # parameters = int(w.ravel().shape[1])
         # parameters
         # def gradient_descent_matrix1(x, y, w, alpha, iterations):
         #     temp = np.ones((3, 1))
         #     w = np.ones((3,1))
         #     m = len(x)
         #     costs = np.zeros(iterations)
         #     for i in range(iterations):
         #         deriv = (2.0/m) * np.matmul((np.matmul(x, temp) - y).T, x)
         #         #deriv = (2.0/m) * np.matmul(x.T, np.matmul(x, w) - y)
         #         #print deriv.shape
         #         temp = temp - (alpha * deriv).T
         #         costs[i] = cost_function_matrix(x, y, temp)
         #     w = temp
```

```python
#     return w, costs
def descendGradient(X, y, init_w):
    """
    theta_start is an n- dimensional vector of initial theta guess
    X is matrix with n- columns and m- rows
    """
    w = init_w
    costs = [] #Used to plot cost as function of iteration
    w_history = [] #Used to visualize the minimization path later on
    for i in xrange(iterations):
        temp = w
        costs.append(cost_function_matrix(w,X,y))
        # Buggy line
        #thetahistory.append(list(tmptheta))
        # Fixed line
        w_history.append(list(w[:,0]))
        #error = (h(w, X) - y)
        #Simultaneously updating theta values
        for j in xrange(len(temp)):
            temp[j] = w[j] - (alpha/m)*np.sum((h(w, X) - y)*np.array(X[:,j]).T)
        w = temp
    return w, w_history, costs
```

In [73]:
```python
iterations = 1000
alpha = 0.001
init_w = np.zeros((x1.shape[1],1))
w1, w1_history, costs1 = descendGradient(x1, y1, init_w)
```

In [74]:
```python
w1
```

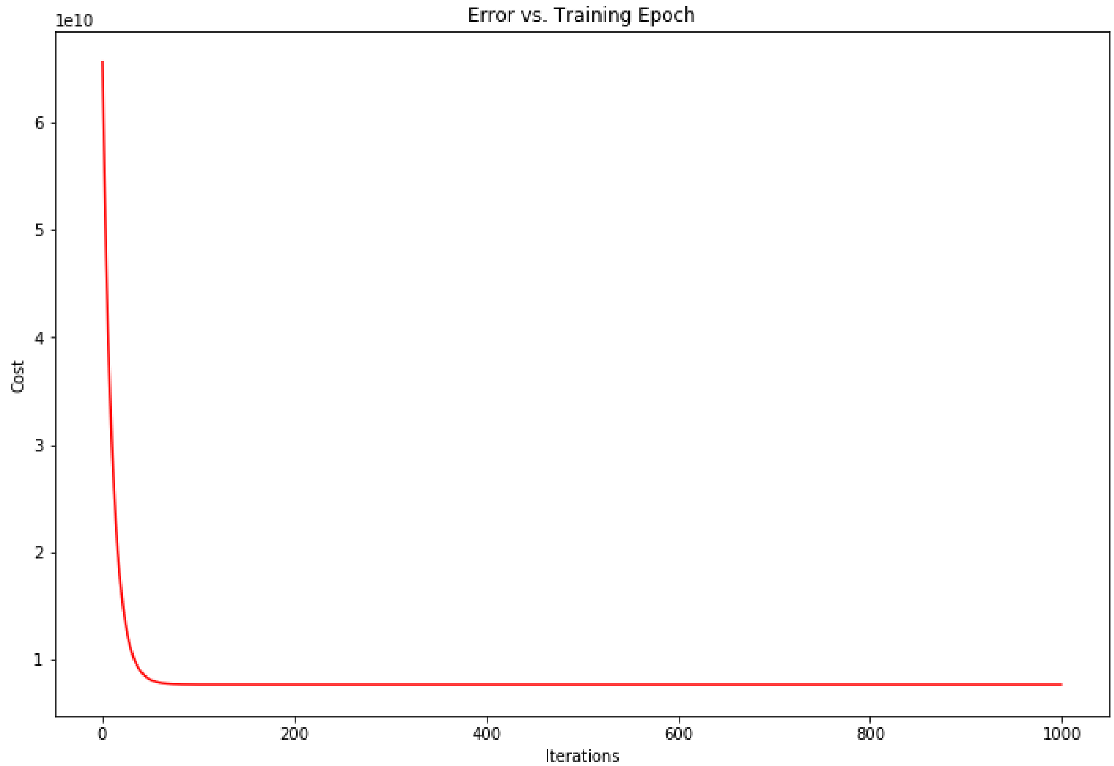
Out[74]:
```
array([[  3.40412766e+05],
       [  9.27978730e-12],
       [  8.47965644e-11]])
```

In [75]:
```python
print 'RMSE %f' % np.sqrt(cost_function_matrix(w1, x1, y1))
```

```
RMSE 87470.910200
```

In [76]:
```python
fig, ax = plt.subplots(figsize=(12,8))
ax.plot(np.arange(iterations), costs1, 'r')
ax.set_xlabel('Iterations')
ax.set_ylabel('Cost')
ax.set_title('Error vs. Training Epoch')
```

Out[76]:
```
<matplotlib.text.Text at 0x11d674290>
```



0.0.3 2-2

```
In [184]: from numpy.linalg import inv
          def normal_equation(x, y):
              return inv(x.T.dot(x)).dot(x.T).dot(y)
          w = normal_equation(x1, y1)
          cost=cost_function_matrix(w, x1, y1)
          print w
          print cost
          #print cost
```

```
[[ 340412.76595745]
 [ 110631.01899357]
 [  -6649.39536298]]
2043280477.39
```

```
In [185]: print 'RMSE %f' % np.sqrt(cost)
```

```
RMSE 45202.660070
```

0.0.4 3-1

In [100]:
```
df2 = pd.read_csv('ex1data3.csv', index_col=0)
```

In [101]:
```
X = df2.iloc[:,0:8].values
Y = df2.iloc[:, 8:9].values
```

In [102]:
```
X2 = np.matrix(X)
Y2 = np.matrix(Y)
w3 = np.ones(X2.shape[1])
```

In [103]:
```
X2
```

Out[103]:
```
matrix([[   8.3252    ,   41.        ,    6.98412698, ...,    2.55555556,
          37.88      , -122.23      ],
        [   8.3014    ,   21.        ,    6.23813708, ...,    2.10984183,
          37.86      , -122.22      ],
        [   7.2574    ,   52.        ,    8.28813559, ...,    2.80225989,
          37.85      , -122.24      ],
        ...,
        [   1.7       ,   17.        ,    5.20554273, ...,    2.3256351 ,
          39.43      , -121.22      ],
        [   1.8672    ,   18.        ,    5.32951289, ...,    2.12320917,
          39.43      , -121.32      ],
        [   2.3886    ,   16.        ,    5.25471698, ...,    2.61698113,
          39.37      , -121.24      ]])
```

In [104]:
```
def feature_normalize(x):
    numberOfFeatures = x.shape[1]
    means = np.array([np.mean(x[:,i]) for i in range(numberOfFeatures)])
    stds = np.array([np.std(x[:,i]) for i in range(numberOfFeatures)])
    normalized = (x - means) / stds

    return normalized
```

In [113]:
```
#X_norm = data.as_matrix(X)
X_norm = feature_normalize(df2.iloc[:,0:8].values)
```

In [114]:
```
#X_norm1 = np.concatenate(np.ones(shape=(20640,1)), X_norm, axis=1)
X_norm1 = np.insert(X_norm, 0, 1, axis=1)
```

In [115]:
```
X_norm1.shape
```

Out[115]:
```
(20640, 9)
```

In [118]:
```
matrix_x = np.matrix(X_norm1)
```

In [175]:
```
# def gradient_descent_matrix2(x, y, w, alpha, iterations):
#     w = np.zeros((x.shape[1], 1))
#     #w = w.reshape((x.shape[1],1))
```

```
#     m = len(X)
#     costs = np.zeros(iterations)
#     for i in range(iterations):
#         deriv = (2.0/m) * np.matmul(x.T, np.matmul(x, w)- y)
#         w = w - alpha * deriv
#         costs[i] = cost_function_matrix(x, y, w)
#     return w, costs
def gradientDescent2(X, y, theta, alpha, iters):
    temp = np.matrix(np.zeros((theta.shape[0],1)))
    parameters = int(theta.ravel().shape[0])
    cost = np.zeros(iters)

    for i in range(iters):
        error = (X * theta) - y

        for j in range(parameters):
            term = np.multiply(error, X[:,j])
            temp[j] = theta[j] - ((alpha / len(X)) * np.sum(term))

        theta = temp
        cost[i] = cost_function_matrix(theta, X, y)

    return theta, cost
```

In [176]:
```
alpha = 0.01
iterations = 1000
init_w = np.zeros((X_norm1.shape[1], 1))
w5, costs5 = gradientDescent2(matrix_x, Y2, init_w, alpha, iterations)
```

In [177]:
```
temp = np.matrix(np.zeros((init_w.shape[0],1)))
temp[2]
```

Out[177]:
```
matrix([[ 0.]])
```

In [178]:
```
w5
```

Out[178]:
```
matrix([[ 2.06846887],
        [ 0.81659877],
        [ 0.17689017],
        [-0.12729893],
        [ 0.14127008],
        [ 0.0166395 ],
        [-0.04392099],
        [-0.48604502],
        [-0.44967077]])
```

In [172]:
```
costs5
```

Out[172]:
```
array([ 1210.11655077,  1188.75282301,  1167.81874126,  1147.30555168,
        1127.20468216,  1107.50773849,  1088.20650045,  1069.29291818,
```

```
1050.7591085 ,  1032.59735139,  1014.80008644,   997.35990956,
 980.26956954,   963.52196491,   947.11014069,   931.0272853 ,
 915.26672757,   899.82193369,   884.68650436,   869.85417195,
 855.3187977 ,   841.07436902,   827.11499681,   813.4349129 ,
 800.02846746,   786.89012657,   774.01446974,   761.39618754,
 749.03007929,   736.9110508 ,   725.03411207,   713.39437519,
 701.98705217,   690.80745287,   679.85098294,   669.11314184,
 658.58952088,   648.2758013 ,   638.16775243,   628.26122983,
 618.5521735 ,   609.03660615,   599.71063146,   590.57043241,
 581.61226967,   572.83247992,   564.22747435,   555.79373707,
 547.52782364,   539.42635957,   531.48603888,   523.70362269,
 516.07593785,   508.59987557,   501.27239009,   494.09049739,
 487.05127394,   480.15185541,   473.38943552,   466.76126477,
 460.26464934,   453.8969499 ,   447.65558055,   441.53800765,
 435.54174881,   429.66437181,   423.9034936 ,   418.25677926,
 412.72194102,   407.29673734,   401.97897192,   396.76649277,
 391.65719136,   386.64900167,   381.73989937,   376.92790094,
 372.21106285,   367.58748075,   363.05528867,   358.61265821,
 354.25779783,   349.98895206,   345.80440078,   341.70245848,
 337.6814736 ,   333.73982781,   329.87593532,   326.08824226,
 322.375226  ,   318.73539455,   315.16728588,   311.66946737,
 308.24053519,   304.87911373,   301.58385499,   298.35343808,
 295.1865686 ,   292.08197818,   289.0384239 ,   286.05468779,
 283.12957633,   280.26191994,   277.45057253,   274.69441098,
 271.99233472,   269.34326524,   266.74614565,   264.19994028,
 261.70363419,   259.2562328 ,   256.85676148,   254.5042651 ,
 252.19780768,   249.936472  ,   247.71935921,   245.54558846,
 243.41429653,   241.32463752,   239.27578242,   237.26691885,
 235.29725068,   233.36599771,   231.47239537,   229.61569436,
 227.79516041,   226.0100739 ,   224.25972964,   222.54343651,
 220.86051724,   219.2103081 ,   217.59215861,   216.00543131,
 214.44950149,   212.92375691,   211.42759758,   209.96043549,
 208.52169439,   207.11080953,   205.72722747,   204.37040578,
 203.03981292,   201.73492792,   200.45524023,   199.20024951,
 197.9694654 ,   196.76240731,   195.57860428,   194.41759472,
 193.27892628,   192.16215562,   191.06684825,   189.99257835,
 188.9389286 ,   187.90549002,   186.89186175,   185.89765097,
 184.92247267,   183.96594952,   183.02771172,   182.10739685,
 181.20464972,   180.31912219,   179.4504731 ,   178.59836806,
 177.76247938,   176.94248587,   176.13807276,   175.34893154,
 174.57475985,   173.81526137,   173.07014566,   172.3391281 ,
 171.6219297 ,   170.91827705,   170.22790219,   169.55054248,
 168.88594053,   168.23384407,   167.59400583,   166.9661835 ,
 166.35013957,   165.74564127,   165.15246045,   164.57037353,
 163.99916134,   163.4386091 ,   162.8885063 ,   162.34864661,
 161.81882782,   161.29885174,   160.7885241 ,   160.28765453,
 159.79605642,   159.31354689,   158.83994669,   158.37508012,
 157.91877501,   157.47086258,   157.03117742,   156.59955741,
```

```
156.17584365,   155.7598804 ,   155.35151501,   154.95059788,
154.55698237,   154.17052475,   153.79108417,   153.41852255,
153.05270457,   152.6934976 ,   152.34077164,   151.99439927,
151.6542556 ,   151.32021823,   150.99216719,   150.66998489,
150.35355607,   150.04276776,   149.73750923,   149.43767198,
149.14314962,   148.85383789,   148.5696346 ,   148.2904396 ,
148.01615471,   147.74668371,   147.48193228,   147.22180797,
146.96622019,   146.7150801 ,   146.46830068,   146.22579659,
145.9874842 ,   145.75328154,   145.52310826,   145.29688562,
145.07453642,   144.85598501,   144.64115721,   144.42998034,
144.22238316,   144.01829582,   143.81764987,   143.62037822,
143.4264151 ,   143.23569605,   143.04815788,   142.86373865,
142.68237767,   142.50401542,   142.32859358,   142.15605499,
141.98634359,   141.81940447,   141.65518377,   141.49362874,
141.33468763,   141.17830974,   141.02444537,   140.87304579,
140.72406326,   140.57745097,   140.43316302,   140.29115445,
140.15138116,   140.01379995,   139.87836846,   139.74504516,
139.61378935,   139.48456114,   139.35732142,   139.23203186,
139.10865488,   138.98715365,   138.86749205,   138.74963471,
138.63354693,   138.5191947 ,   138.40654469,   138.29556421,
138.18622124,   138.07848438,   137.97232285,   137.86770648,
137.76460569,   137.66299149,   137.56283547,   137.46410976,
137.36678705,   137.27084058,   137.1762441 ,   137.08297189,
136.99099873,   136.90029991,   136.81085119,   136.72262881,
136.63560949,   136.54977041,   136.46508918,   136.38154387,
136.29911298,   136.21777542,   136.13751053,   136.05829806,
135.98011814,   135.9029513 ,   135.82677847,   135.75158092,
135.67734033,   135.60403871,   135.53165843,   135.46018222,
135.38959313,   135.31987456,   135.25101024,   135.18298419,
135.11578077,   135.04938465,   134.98378079,   134.91895445,
134.85489117,   134.79157678,   134.7289974 ,   134.66713942,
134.60598947,   134.54553447,   134.4857616 ,   134.42665826,
134.36821214,   134.31041113,   134.25324339,   134.1966973 ,
134.14076145,   134.08542469,   134.03067606,   133.97650482,
133.92290045,   133.86985263,   133.81735124,   133.76538636,
133.71394826,   133.66302741,   133.61261446,   133.56270025,
133.51327578,   133.46433225,   133.41586102,   133.36785361,
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```
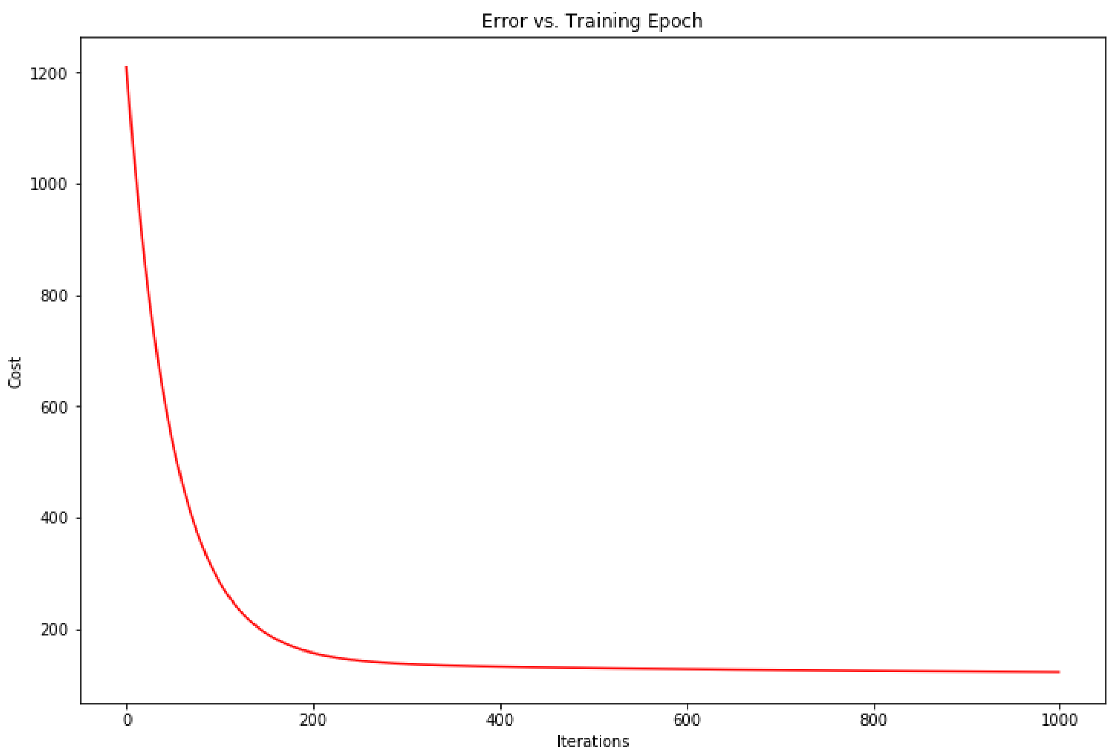

In [179]:
```
print 'RMSE %f' % float(np.sqrt(cost_function_matrix(w5, X_norm1, Y2)))
```

```
RMSE 11.019735
```

In [180]:
```
fig, ax = plt.subplots(figsize=(12,8))
ax.plot(np.arange(iterations), costs5, 'r')
ax.set_xlabel('Iterations')
ax.set_ylabel('Cost')
ax.set_title('Error vs. Training Epoch')
```

Out[180]: <matplotlib.text.Text at 0x11e2fcbd0>



0.0.5 3-2

```
In [187]: w6 = normal_equation(matrix_x, Y2)
          cost6=cost_function_matrix(w6, matrix_x, Y2)

In [188]: w6

Out[188]: matrix([[ 2.06855817],
                  [ 0.8296193 ],
                  [ 0.11875165],
                  [-0.26552688],
                  [ 0.30569623],
                  [-0.004503  ],
                  [-0.03932627],
                  [-0.89988565],
                  [-0.870541  ]])

In [190]: print 'RMSE %f' % np.sqrt(cost6)

RMSE 10.729748
```

0.0.6 3-3

```
In [191]: from sklearn import linear_model
          reg = linear_model.LinearRegression()
          reg.fit(X_norm, Y2)

Out[191]: LinearRegression(copy_X=True, fit_intercept=True, n_jobs=1, normalize=False)

In [195]: print 'Weights:'
          print reg.coef_

Weights:
[[ 0.8296193   0.11875165 -0.26552688  0.30569623 -0.004503   -0.03932627
  -0.89988565 -0.870541  ]]

In [194]: from sklearn.metrics import mean_squared_error
          print 'RMSE: %f' % np.sqrt(mean_squared_error(reg.predict(X_norm), Y2))

RMSE: 0.724100
```

0.0.7 3-4

```
In [2176]: import tensorflow as tf
```

In [2177]:
```
m = X_norm.shape[0]
n = X_norm.shape[1]
X_train = tf.placeholder(tf.float32, [m,n])
Y_train = tf.placeholder(tf.float32, [m,1])
weights = tf.Variable(tf.zeros([n,1], dtype=np.float32), name='weight')
bias = tf.Variable(tf.zeros([1], dtype=np.float32), name='bias')
```

In [2178]:
```
learning_rate = 0.1
iterations = 5000
cost_history = np.empty(shape=[1],dtype=float)
init = tf.initialize_all_variables()
```

```
WARNING:tensorflow:From <ipython-input-2178-b9a007e7fbf2>:4: initialize_all_variables (from ter
Instructions for updating:
Use `tf.global_variables_initializer` instead.
```

In [2179]:
```
y_hat = tf.add(tf.matmul(X_train, weights), bias)
cost = tf.reduce_mean(tf.square(y_hat - Y_train))
optimizer = tf.train.GradientDescentOptimizer(learning_rate).minimize(cost)
```

In [2180]:
```
#costs = np.empty(shape=[1], dtype=float)
#costs = np.zeros(iterations)

sess = tf.Session()
sess.run(init)

for i in range(1, iterations):
    sess.run(optimizer, feed_dict={
        X_train: X_norm,
        Y_train: Y2
    })
    cost_history = np.append(cost_history,sess.run(cost,feed_dict={X_train: X_norm,Y

print('bias:', sess.run(bias))
print('weights:', sess.run(weights))
```

```
('bias:', array([ 2.06855774], dtype=float32))
('weights:', array([[ 0.82962048],
       [ 0.11875186],
       [-0.26552892],
       [ 0.30569792],
       [-0.00450296],
       [-0.03932633],
       [-0.89988297],
       [-0.87053841]], dtype=float32))
```

In [2182]:
```
pred_y = sess.run(y_hat, feed_dict={X_train: X_norm})
mse = tf.reduce_mean(tf.square(pred_y - Y2))
print "RMSE: %.4f" % np.sqrt(sess.run(mse))
```

```
RMSE: 0.7241
```

```
In [2183]: sess.close()
```