Assignment 2-LA Chapter 2

Mathematics Engineering (Trường Đại học FPT)

ASSIGNMENT 02 Chapter 2 Linear Algebra

Subject: MAE101

Name of student:

1. Solve the exercises yourself and submit your solutions in LMS before deadlines (do not send via email).
2. Each student is required to represent solutions of at least 2 of the exercises in lecture classes.

Chapter 2:

	Content	**Score**
19	1.What is a linear transformation from R^n to R^m 2. Give an example of linear transformation from R^n to R^m.	
20	3. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 3 \\ -1 & 0 \\ 1 & 4 \end{bmatrix}$, and $E = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Compute the following (where possible). (a) $3A - 2B$ ◆(b) $5C$ (c) $3E^T$ ◆(d) $B + D$ (e) $4A^T - 3C$ ◆(f) $(A + C)^T$ (g) $2B - 3E$ ◆(h) $A - D$ (i) $(B - 2E)^T$	

21	14. In each case determine all s and t such that the given matrix is symmetric: (a) $\begin{bmatrix} 1 & s \\ -2 & t \end{bmatrix}$ ◆(b) $\begin{bmatrix} s & t \\ st & 1 \end{bmatrix}$ (c) $\begin{bmatrix} s & 2s & st \\ t & -1 & s \\ t & s^2 & s \end{bmatrix}$ ◆(d) $\begin{bmatrix} 2 & s & t \\ 2s & 0 & s+t \\ 3 & 3 & t \end{bmatrix}$	
22	1. Solve for the matrix X if: (a) $PXQ = R$; (b) $XP = S$; where $P = \begin{bmatrix} 1 & 0 \\ 2 & -1 \\ 0 & 3 \end{bmatrix}$, $Q = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix}$, $R = \begin{bmatrix} -1 & 1 & -4 \\ -4 & 0 & -6 \\ 6 & 6 & -6 \end{bmatrix}$, $S = \begin{bmatrix} 1 & 6 \\ 3 & 1 \end{bmatrix}$	
23	2. Consider $p(X) = X^3 - 5X^2 + 11X - 4I$. (a) If $p(U) = \begin{bmatrix} 1 & 3 \\ -1 & 0 \end{bmatrix}$, compute $p(U^T)$. ◆(b) If $p(U) = 0$ where U is $n \times n$, find U^{-1} in terms of U.	

24	4. Assume that a system $A\mathbf{x} = \mathbf{b}$ of linear equations has at least two distinct solutions $\mathbf{y}$ and $\mathbf{z}$. (a) Show that $\mathbf{x}_k = \mathbf{y} + k(\mathbf{y} - \mathbf{z})$ is a solution for every k. ◆(b) Show that $\mathbf{x}_k = \mathbf{x}_m$ implies $k = m$. [*Hint*: See Example 7 Section 2.1.] (c) Deduce that $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions.	
25	5. (a) Let A be a 3×3 matrix with all entries on and below the main diagonal zero. Show that $A^3 = 0$. (b) Generalize to the $n \times n$ case and prove your answer.	
26	9. If A is 2×2, show that $A^{-1} = A^T$ if and only if $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ for some θ or $A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$ for some θ. [*Hint*: If $a^2 + b^2 = 1$, then $a = \cos\theta$, $b = \sin\theta$ for some θ. Use $\cos(\theta - \varphi) = \cos\theta\cos\varphi + \sin\theta\sin\varphi$.]	
27	10. (a) If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, show that $A^2 = I$. (b) What is wrong with the following argument? If $A^2 = I$, then $A^2 - I = 0$, so $(A - I)(A + I) = 0$, whence $A = I$ or $A = -I$.	

28	1. In each case, show that the matrices are inverses of each other. (a) $\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 0 \\ 1 & -4 \end{bmatrix}, \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 1 & -3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 2 & 3 \\ 1 & 3 & 1 \end{bmatrix}, \begin{bmatrix} 7 & 2 & -6 \\ -3 & -1 & 3 \\ 2 & 1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}, \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$	
29	2. Find the inverse of each of the following matrices. (a) $\begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$ ◆(b) $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 0 \\ -1 & -1 & 0 \end{bmatrix}$ ◆(d) $\begin{bmatrix} 1 & -1 & 2 \\ -5 & 7 & -11 \\ -2 & 3 & -5 \end{bmatrix}$ (e) $\begin{bmatrix} 3 & 5 & 0 \\ 3 & 7 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ ◆(f) $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & 5 & -1 \end{bmatrix}$	
30	3. In each case, solve the systems of equations by finding the inverse of the coefficient matrix. (a) $\begin{aligned} 3x - y &= 5 \\ 2x + 2y &= 1 \end{aligned}$ ◆(b) $\begin{aligned} 2x - 3y &= 0 \\ x - 4y &= 1 \end{aligned}$	
31	In each case, solve the systems of equations by finding the inverse of the coefficient matrix. (c) $\begin{aligned} x + y + 2z &= 5 \\ x + y + z &= 0 \\ x + 2y + 4z &= -2 \end{aligned}$ ◆(d) $\begin{aligned} x + 4y + 2z &= 1 \\ 2x + 3y + 3z &= -1 \\ 4x + y + 4z &= 0 \end{aligned}$	

32	4. Given $A^{-1} = \begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 5 \\ -1 & 1 & 0 \end{bmatrix}$: (a) Solve the system of equations $A\mathbf{x} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$. ◆(b) Find a matrix B such that $AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. (c) Find a matrix C such that $CA = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 1 \end{bmatrix}$.	
33	9. In each case either prove the assertion or give an example showing that it is false. (a) If $A \neq 0$ is a square matrix, then A is invertible. ◆(b) If A and B are both invertible, then $A + B$ is invertible. (c) If A and B are both invertible, then $(A^{-1}B)^T$ is invertible. ◆(d) If $A^4 = 3I$, then A is invertible. (e) If $A^2 = A$ and $A \neq 0$, then A is invertible. ◆(f) If $AB = B$ for some $B \neq 0$, then A is invertible. (g) If A is invertible and skew symmetric ($A^T = -A$), the same is true of A^{-1}. ◆(h) If A^2 is invertible, then A is invertible. (i) If $AB = I$, then A and B commute.	

34	15. Consider $A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 5 & 0 & 0 \end{bmatrix}$. Find the inverses by computing (a) A^6; ♦(b) B^4; and (c) C^3.	
35	1. Let $T_\theta : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation. (a) Find $T\begin{bmatrix} 8 \\ 3 \\ 7 \end{bmatrix}$ if $T\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $T\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$. ♦(b) Find $T\begin{bmatrix} 5 \\ 6 \\ -13 \end{bmatrix}$ if $T\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$ and $T\begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.	
36	3. In each case assume that the transformation T is linear, and use Theorem 2 to obtain the matrix A of T. (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is reflection in the line $y = -x$. ♦(b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $T(\mathbf{x}) = -\mathbf{x}$ for each $\mathbf{x}$ in $\mathbb{R}^2$. (c) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is clockwise rotation through $\frac{\pi}{4}$. ♦(d) $T : \mathbb{R}^2 \to \mathbb{R}^2$ is counterclockwise rotation through $\frac{\pi}{4}$.	
37	4. In each case use Theorem 2 to obtain the matrix A of the transformation T. You may assume that T is linear in each case. (a) $T : \mathbb{R}^3 \to \mathbb{R}^3$ is reflection in the x-z plane. ♦(b) $T : \mathbb{R}^3 \to \mathbb{R}^3$ is reflection in the y-z plane.	

38	6. Use Theorem 2 to find the matrix of the **identity transformation** $1_{\mathbb{R}^n} : \mathbb{R}^n \to \mathbb{R}^n$ defined by $1_{\mathbb{R}^n} : (\mathbf{x}) = \mathbf{x}$ for each $\mathbf{x}$ in $\mathbb{R}^n$.	
39	7. In each case show that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is not a linear transformation. (a) $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} xy \\ 0 \end{bmatrix}$. ◆(b) $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ y^2 \end{bmatrix}$	