Mathematics for Engineering

Assignment 1

1. Let $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{pmatrix}$. What is the $(1,2)-$entry of the matrix $AB - BA$?

   (a) -4 (b) 2 (c) -2 (d) 1

2. The (2,3)-entry of the product $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 5 & 1 \\ 4 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 5 & 1 & 0 \\ 0 & 4 & 3 \end{bmatrix}$ is

   (a) 8 (b) 10 (c) 11 (d) 7

3. If ABC can be formed and A is 4×4, C is 7×7. What is the size of B?

   (a) 4×7 (b) 4×4 (c) 7×4 (d) 7×7

4. If A is a 2×2 invertible matrix and $(3A)^{-1} = \begin{pmatrix} -1 & 3 \\ 4 & 5 \end{pmatrix}$, what is the $(1,1)-$entry of A?

   (a) -5/51 (b) -25/3 (c) 5/21 (d) 5/3

5. If an $n \times n$ matrix A satisfies $A^2 - 6A + 5I_n = 0$, then A^{-1}

   (a) does not exist (b) is $(6I_n - A)/5$ (c) $(A - 6I_n)/5$ (d) exists only if $n < 6$

6. Let A be an arbitrary square matrix. Which of the following matrices are symmetric:

   (i) $A + A^T$
   (ii) $A + 2A^T$

   (a) (i) (b) (ii)
   (c) (i) and (ii) (d) None of the other choices is correct

7. Given that $3\begin{pmatrix} x & 2 & 1 \\ 0 & z & y \end{pmatrix} = \begin{pmatrix} 9 & 2z & -y \\ 0 & t & s \end{pmatrix}$. Find $t + s$.

   (a) 0 (b) 5 (c) 9 (d) 12

8. Find all a, b, c such that the following matrix is in reduced row-echelon form: $\begin{bmatrix} a & 1 & b & b & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & c \end{bmatrix}$.

   (a) $(1,0,0)$ (b) $(0,0,0)$ and $(1,0,0)$ (c) $(0,0,1)$ (d) $(1,0,0)$ and $(0,0,1)$

9. Let $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 \\ 2 & -3 \end{pmatrix}$. Solve $AXB = BA$, where X is a matrix

   (a) $X = I$ (b) $X = \begin{pmatrix} 59 & 32 \\ -24 & -13 \end{pmatrix}$ (c) $X = \begin{pmatrix} 27 & -16 \\ -32 & 19 \end{pmatrix}$ (d) None of the others

10. Find rank of $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 0 \end{pmatrix}$

(a) 0 (b) 1 (c) 2 (d) 3

11. Which of the following statements are true for invertible $n \times n$ matrices A, B, and C?

(i) $(A+B)^{-1} = A^{-1} + B^{-1}$
(ii) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
(iii) $A^2B^2 = (AB)^2$
(iv) $(A+B)^2 = A^2 + 2AB + B^2$
(v) $(A+C)(A-C) = A^2 - C^2$

(a) (ii) and (v) (b) (ii) and (iii) only (c) (i) and (iv) only (d) (ii) only

12. Given that rank of the matrix $\begin{pmatrix} -1 & 4 & 5 \\ 2 & 3 & -2 \\ 3 & 10 & a \end{pmatrix}$ is 2, what is a?

(a) -1 (b) 1/2 (c) 0 (d) 1

13. Let A be a 3×5 matrix. Choose correct statements

(i) A can have rank 3
(ii) A can have rank 5
(iii) A can have linearly independent rows
(iv) A can have linearly independent columns

(a) (i) (b) (i) and (iii) (c) (ii) and (iv) (d) (iv)

14. Let $T : \mathbb{R}^4 \longrightarrow \mathbb{R}^3$ be a linear transformation with $T(1, 1, 0, -2) = (2, 3, -1)$ and $T(0, -1, 1, 1) = (5, 0, 1)$. Find $T(1, 3, -2, -4)$.

(a) $(7, 3, 0)$ (b) $(1, -6, 3)$ (c) $(-8, 3, -3)$ (d) None of the others

15. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that the matrix of T is $\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$. Find $T(3, 2)$.

(a) (7,3) (b) (8,4) (c) (3,2) (d) (4,3)

16. Find all values of m for which the following system of equations has nontrivial solutions:

$$\begin{cases} x - 2y + z &= 0 \\ x + my - 3z &= 0 \\ -x + 6y - 5z &= 0 \end{cases}$$

(a) $m = 2$ (b) $m = -2$ (c) $m \neq -2$ (d) $m \neq 2$

17. Consider a homogeneous system of 5 linear equations in 6 unknowns. Which of the following is true?

(a) The system can have no solution
(b) The system has between 0 and 5 solutions
(c) The system always has infinitely many solutions
(d) The system has only the trivial solution

18. Let A be the augmented matrix of a homogeneous of 3 equations in 6 variables. If $\text{rank}(A) = 1$, how many solutions and how many parameters does this system have?

    (a) Infinitely many solutions and 3 parameters(b) Infinitely many solutions and 2 parameters

    (c) Infinitely many solutions and 5 parameters(d) Unique solution

19. Consider the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -4 & 2 & 2 \\ 4 & 2 & 3 \end{pmatrix}$. If A is the augmented matrix of a system of linear equations, determine the number of equations and the number of variables.

    (a) 3 equations, 3 unknowns (b) 2 equations, 3 unknowns

    (c) 2 equations, 2 unknowns (d) 3 equations, 2 unknowns

20. Find all values m such that the system of equations $\begin{cases} x + y - z &= 1 \\ x + 2y + mz &= 0 \\ 2x + 3y - 2z &= m \end{cases}$ has exactly one solution

    (a) $m \neq 1$ (b) $m \neq 2$ (c) $m \neq -1$ (d) $m = -1$

21. Find all values of t such that the system $\begin{cases} x + ty &= 0 \\ tx + y &= 2 \\ x + y &= 1 \end{cases}$ is consistent.

    (a) $t = 1$ (b) $t = -1$ (c) $t \neq 1$ (d) Does not exist

22. Find all values of m such that the following system has no solution

$$\begin{cases} x - 2y + z &= 0 \\ x + y + 3z &= 1 \\ 2x - y + 4z &= m \end{cases}$$

    (a) $m = 1$ (b) Any number (c) $m \neq 1$ (d) $m = 0$

23. Solve the system of linear equations: $\begin{cases} 3x + y &= 9 \\ x - y &= 3 \end{cases}$

    (a) $x = 6, y = 3$ (b) $x = 0, y = -3$ (c) $x = 3, y = 0$ (d) $x = 1, y = 1$

24. The $(3,1)-$cofactor of $\begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 5 \\ 3 & 0 & 6 \end{pmatrix}$ is:

    (a) 2 (b) 5 (c) 3 (d) 9

25. Find the second row of the adjugate of the matrix $\begin{pmatrix} -6 & -9 & -8 \\ 2 & 9 & 6 \\ 0 & 1 & -1 \end{pmatrix}$.

    (a) $\begin{pmatrix} -17 & 6 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & 6 & -20 \end{pmatrix}$ (c) $\begin{pmatrix} 2 & 6 & 20 \end{pmatrix}$ (d) $\begin{pmatrix} 17 & 6 & 6 \end{pmatrix}$

26. Let $\begin{vmatrix} a & m & d \\ b & n & e \\ c & p & f \end{vmatrix} = 10$. Find $\begin{vmatrix} 2a+3d & d & -m \\ 2b+3e & e & -n \\ 2c+3f & f & -p \end{vmatrix}$.

    (a) -20 (b) -60 (c) 20 (d) 60

27. If $\det \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix} = 2$, compute $\det \begin{bmatrix} -p & -q & -r \\ 2p+a & 2q+b & 2r+c \\ p+3x & q+3y & r+3z \end{bmatrix}$

    (a) -6 (b) 3 (c) 6 (d) -3

28. Let $A = \begin{pmatrix} 1 & * & * & * \\ 0 & 3 & * & * \\ 0 & 0 & 5 & * \\ 0 & 0 & 0 & 7 \end{pmatrix}$, where $(*)$ denotes any real number. Compute $\det(2A^{-1})$

(a) 2/105 (b) 210 (c) 16/105 (d) None of the others

29. Suppose A and B are 3×3 matrix with $\det A = 2, \det B = 5$. What is $\det(2AB)$?

(a) 20 (b) 14 (c) 80 (d) 60

30. A is a 4×4 matrix with $\det A = 4$. If $\operatorname{adj}(A)$ denotes the transpose of the matrix of cofactors of A, find $\det(\operatorname{adj}(A))$.

(a) 16 (b) 1/16 (c) 1/64 (d) 64

31. Find m such that the matrix $\begin{pmatrix} 0 & m & -4 \\ 2 & 3 & -1 \\ 1 & 4 & 1 \end{pmatrix}$ is not invertible.

(a) All number but $-20/3$ (b) All numbers but 20/3

(c) 20/3 (d) -20/3

32. Let $A = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 0 \\ 2 & -1 & m \end{bmatrix}$. For which values of m is A invertible?

(a) $m = 2$ (b) $m \neq 2$ (c) $m = 1$ (d) $m \neq 1$

33. The characteristic polynomial of $A = \begin{pmatrix} 3 & -2 \\ 1 & 0 \end{pmatrix}$ is

(a) $(x-2)(x+1)$ (b) $x^2 - 3x + 2$ (c) $(x+2)(x+1)$ (d) $3x^2$

34. Find the eigenvalues of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ -12 & 11 & 4 \end{pmatrix}$.

(a) 3; 3; -1 (b) 3; 3; 1 (c) 3; -1; -1 (d) 3; -1; 1

35. Given that $\lambda = 1$ is an eigenvalues for the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$. Find a set of basic eigenvectors corresponding to this eigenvalue $\lambda = 1$

(a) $\{(0,0,1)\}$ (b) $\{(1,0,0),(0,0,1)\}$ (c) $\{(1,0,0)\}$ (d) $\{(0,-1,1)\}$

36. Find all values of a such that $\begin{pmatrix} a & 1 \end{pmatrix}^T$ is an eigenvector of matrix $A = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$.

(a) 1 or 0 (b) -1 or 0 (c) 1 or -1 (d) 2 or -2

37. Which of the following are subspaces of $\mathbb{R}^3$.

(i) $U = \{(x,y,z) \in \mathbb{R}^3 : x + y^2 - z = 0\}$

(ii) $V = \{(x,y,z) \in \mathbb{R}^3 : x + 2y - 3z = 0 \text{ and } 2x - z = 0\}$

(a) (ii) (b) (i) and (ii)

(c) (i) (d) None of the other choices is correct

38. Find the value of t for which $(4,6,t)$ is a linear combination of $(1,3,1); (2,8,-1)$ and $(-1,-5,2)$.

(a) 0 (b) 4 (c) 7 (d) 13

39. Find all values of m such that the set $\{(1,-1,2);(3,0,1);(-2,m,1)\}$ is linearly independent
(a) $m=1$ (b) $m\neq -1$ (c) $m=3$ (d) $m=-1$

40. Let $\{u,v,w\}$ be independent. Which of the following sets are independent?
(i) $\{u, v-w, w\}$
(ii) $\{u, u-v, u+v, w\}$
(a) (i) (b) (ii)
(c) (i) and (ii) (d) None of the other choices is correct

41. Find m such that the set $\{(2,m,1);(m,0,0);(1,1,m)\}$ is a basis of $\mathbb{R}^3$.
(a) $m\neq 0$ (b) $m\neq \pm 1$ (c) $m\neq 1$ (d) $m\in\mathbb{R}\setminus\{0,1,-1\}$

42. Find the dimension of $U=\operatorname{span}\{(1,2,-1);(3,1,1);(-1,2,0);(0,1,1)\}$
(a) 0 (b) 3 (c) 2 (d) 4

43. Let $V=\operatorname{span}\{(1,2,3,4);(3,2,5,1);(2,1,0,1)\}$. Find all t such that $(1,2,t,3)\in V$.
(a) 9 (b) 1 (c) -3 (d) 27/5

44. Which condition on the numbers a,b,c is the vector $(a,b,c)\in\operatorname{span}\{(1,0,2),(1,2,8)\}$
(a) $c=2a+3b$ (b) $c=-2a-3b$ (c) $c=2a-3b$ (d) $c=-2a+3b$

45. Find a basis for the subspace $U=\{(x,y,z): x+2y+3z=0\}$
(a) $(-2,-1,0)$ and $(3,0,1)$ (b) $(-2,1,0)$ and $(-3,0,1)$
(c) $(2,1,0)$ and $(-3,0,1)$ (d) $(-2,-1,0)$ and $(-3,0,-1)$

46. Let $U=\{(a,b,c,d)|3a-5b=0, b+c+d=0\}$ be a subspace of $\mathbb{R}^4$. Find the dimension of U
(a) 1 (b) 2 (c) 3 (d) 4

47. Let $U=\{(x,y,z)|2x-y+z=0\}$ be a subspace of $\mathbb{R}^3$. Which of the following statements are true?
(i) $U=\operatorname{span}\{(1,0,-2),(0,1,1)\}$
(ii) $U=\operatorname{span}\{(1,2,0)\}$
(a) (i) (b) (i) and (ii) (c) (ii) (d) None

48. Let $u=(1,1,1), v=(1,2,3), w=(1,3,7)$ and $x=(0,-3,-10)$. Which of the following statements is true?
(i) $\{u,v,w,x\}$ is linearly dependent.
(ii) $\dim(\operatorname{span}\{u,v,w,x\})=3$.
(a) (i) (b) (ii) (c) Both is true (d) None is true

49. Which one of the following is a basis for the subspace of $\mathbb{R}^3$ defined by $G=\{(x,y,z): 2x-y+3z=0\}$?
(a) $\{(1,0,0);(1,2,0)\}$ (b) $\{(1,2,0)\}$
(c) $\{(1,0,0);(0,1,0);(0,0,1)\}$ (d) $\{(1,2,0);(0,3,1)\}$

50. Find all $\begin{pmatrix}a & b & c & d\end{pmatrix}^T$ in $\mathbb{R}^4$ such that the given set is orthogonal.
$$\left\{\begin{pmatrix}1 & 2 & 1 & 0\end{pmatrix}^T, \begin{pmatrix}1 & -1 & 1 & 3\end{pmatrix}^T, \begin{pmatrix}2 & -1 & 0 & -1\end{pmatrix}^T, \begin{pmatrix}a & b & c & d\end{pmatrix}^T\right\}$$
(a) $\begin{pmatrix}s\\ s\\ -3s\\ s\end{pmatrix}$ (b) $\begin{pmatrix}2s+t\\ s-3t\\ s\\ t\end{pmatrix}$ (c) $\begin{pmatrix}s+2t\\ -s-t\\ s\\ t\end{pmatrix}$ (d) $\begin{pmatrix}4s\\ 4s-t\\ s\\ t\end{pmatrix}$