Discrete Mathematics 1

Chapter 1: The Foundations: Logic and Proofs

Department of Mathematics
The FPT university

0. Course Introduciton

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Course name: Discrete Mathematics 1 (MAD111)

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- Chapter 2: Sets, Functions, Sequences, and Sums

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- Chapter 6: Discrete Probability
- Chapter 7: Advanced Counting Techniques

Topics covered:

Chapter 1: Introduction

Topics covered:

1.1 Propositional Logic

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1.2 Propositional Equivalences

1.3 Predicates and Quantifiers

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1.6 Introduction to Proofs

1.7 Proof Methods and Strategy

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Logic and Bit Operations

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Computers represent information using bits. A bit is a symbol of two possible values, 0 and 1. A bit can represent a truth value, that is, 1 represents T (true) and 0 represents F (false). Information is often represented using bit strings, and operations on bit strings can be used to manipulate this information.

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Note. Other notation for $\wedge, \vee, \oplus$ are AND, OR, XOR.

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Two methods for proving logical equivalences:

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Double negation law	$\neg(\neg p) \equiv p$
Identity laws	$p \wedge T \equiv p$ $p \vee F \equiv p$
Domination laws	$p \vee T \equiv T$ $p \wedge F \equiv F$
Negation laws	$p \vee \neg p \equiv T$ $p \wedge \neg p \equiv F$
Idempotent laws	$p \vee p \equiv p$ $p \wedge p \equiv p$
Commutative laws	$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$
Associative laws	$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributive laws	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
De Morgan's laws	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Note:

$$p \rightarrow q \equiv \neg p \vee q.$$

Note:

$$p \to q \equiv \neg p \vee q.$$

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$$

Note:

$$p \to q \equiv \neg p \lor q.$$
$$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$$
$$p \oplus q \equiv \neg(p \leftrightarrow q)$$

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Example 1. Prove that $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

Example 2. Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

1.3 Predicates and Quantifiers

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Example. $R(x, y, z) =$ "$x + y < z$" is a propositional function with variables x, y, z and R is the predicate.

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(b) $\forall x((x > 0) \wedge (x^2 \geq x))$

(c) $\forall x((x > 0) \vee (x^2 \geq x))$

(d) $\exists x((x > 0) \to (x^2 \geq x))$

Quantifiers $\forall$, $\exists$.

Let $P(x)$ be a propositional function where x gets values in a particular domain.

The universal quantification $\forall x P(x)=$ For all values of x in the domain, $P(x)$ is true

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(c) $\forall x((x > 0) \lor (x^2 \geq x))$
(d) $\exists x((x > 0) \to (x^2 \geq x))$
(e) $\exists x((x > 0) \land (x^2 \geq x))$
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Negating Quantified Expressions

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Example. Rewrite the expression

$$\neg\forall x(P(x) \to Q(x))$$

so that the negation precedes the predicates.

Translating Sentences into Logical Expressions

Example 1. "Every students of class SE0000 passed Calculus"

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(a) If domain consists of all students of SE0000

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Example 3. "Some student of SE0000 has visited Canada or Mexico"

1.4 Nested Quantifiers

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$\forall x \forall y P(x, y)$

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Example. Determine the truth values of the following propositions on the set of real numbers.

$\forall x \forall y (x + y = 1)$

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Translate Logical Expressions into Sentences

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where x, y are real numbers.

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$C(x) =$ " x has a laptop"

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Translate the logical expression $\forall x[C(x) \vee \exists y(C(y) \wedge F(x, y))]$

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Example 3. Let x, y represent students in a university, and

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Translate the logical expression

$$\exists x \forall y \forall z[(F(x, y) \wedge F(x, z) \wedge (y \neq z)) \rightarrow \neg F(y, z)]$$

Translate Sentences into Logical Expression using Nested Quantifiers

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Example 1. "Each student has sent emails to each other, but not to him/herself."

Translate Sentences into Logical Expression using Nested Quantifiers

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Use:

Translate Sentences into Logical Expression using Nested Quantifiers

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Use: $E(x, y)$="x has sent emails to y"

Translate Sentences into Logical Expression using Nested Quantifiers

Example 1. "Each student has sent emails to each other, but not to him/herself."

Use: $E(x, y)$="x has sent emails to y"

Example 2. "Each student either has a car or has a room-mate in the same class who has a car"

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Use: $C(x)$ = "x has a car"
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Example 3. (a) There is exactly one student in the class that was born in Hanoi.

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Use: $C(x)$ = "x has a car"
$R(x, y)$="x and y are room-mates "

Example 3. (a) There is exactly one student in the class that was born in Hanoi.

(b) There are exactly two students in the class that was born in Hanoi.

Negating Nested Quantifiers

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$$\neg(\forall x \forall y P(x, y)) = \exists x \exists y \neg P(x, y)$$

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Example.

Negating Nested Quantifiers

$$\neg(\forall x \forall y P(x,y)) \equiv \exists x \exists y \neg P(x,y) \quad \neg(\forall x \exists y P(x,y)) \equiv \exists x \forall y \neg P(x,y)$$
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Example. Translate the following statements into logical expressions, then find the negation statement.

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Example. Translate the following statements into logical expressions, then find the negation statement.

(a) " For all real numbers x there is a real number y such that $x = y^3$"

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Example. Translate the following statements into logical expressions, then find the negation statement.

(a) " For all real numbers x there is a real number y such that $x = y^3$"
(b) " For all $\epsilon > 0$, for all real numbers x there exists a rational number p such that $|p - x| < \epsilon$"

- An argument is a sequence of statements that end with a conclusion.

1.5 Rules of Inference

- An argument is a sequence of statements that end with a conclusion. An argument is valid if the conclusion follows from the truth of the preceding statements (premises or hypotheses).

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- In propositional logic, an argument is valid if it is based on a tautology.
- Arguments that are not based on tautology are called fallacies.

Name	Rule of Inference	Tautology
Addition	p $\therefore \overline{p \vee q}$	$p \to (p \vee q)$
Simplification	$\underline{p \wedge q}$ $\therefore p$	$(p \wedge q) \to p$
Modus ponens	p $\underline{p \to q}$ $\therefore q$	$p \wedge (p \to q) \to q$
Modus tollens	$\neg q$ $\underline{p \to q}$ $\therefore \neg p$	$(\neg q) \wedge (p \to q) \to \neg p$
Hypothetical syllogism	$p \to q$ $\underline{q \to r}$ $\therefore p \to r$	$(p \to q) \wedge (q \to r) \to (p \to r)$
Disjunctive syllogism	$\neg p$ $\underline{p \vee q}$ $\therefore q$	$(p \vee q) \wedge (\neg p) \to q$

Example 1.

Example 1. Given the hypotheses:

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- "It is not sunny and is cold"

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- "It is not sunny and is cold"
- "We go swimming only if it is sunny"

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- "If we do not go swimming then we will play soccer"

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- "It is not sunny and is cold"
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- "If we do not go swimming then we will play soccer"
- "If we play soccer then we will go home by sunset"

Example 1. Given the hypotheses:

- "It is not sunny and is cold"
- "We go swimming only if it is sunny"
- "If we do not go swimming then we will play soccer"
- "If we play soccer then we will go home by sunset"

Show that these hypotheses lead to the conclusion: "We will go home by sunset".

Example 2.

Example 2. Given the hypotheses:

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- "If you send me an email, I will finish writing the program"

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- "If you send me an email, I will finish writing the program"
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Show that these hypotheses lead to the conclusion: "If I do not finish writing the program then I will go jogging tomorrow morning".

Some fallacies

Some fallacies

- Fallacy of affirming the conclusion: $[(p \to q) \land q] \to p$

Some fallacies

- Fallacy of affirming the conclusion: $[(p \to q) \wedge q] \to p$
- Fallacy of denying the hypothesis: $[(p \to q) \wedge \neg p] \to \neg q$

Some fallacies

- Fallacy of affirming the conclusion: $[(p \to q) \land q] \to p$
- Fallacy of denying the hypothesis: $[(p \to q) \land \neg p] \to \neg q$

Rules of Inference for Quantified Statements

Name	Rule of Inference
Universal instantiation	$\dfrac{\forall x P(x)}{\therefore P(c),\ c \text{ is arbitrary}}$
Universal generalization	$\dfrac{P(c),\ \text{c is arbitrary}}{\therefore \forall x P(x)}$
Existential instantiation	$\dfrac{\exists x P(x)}{\therefore P(c),\ \text{for some } c}$
Existential generalization	$\dfrac{P(c),\ \text{for some } c}{\therefore \exists x P(x)}$

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Example 1. Given the hypotheses:

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- "Each student of SE0000 must take Discrete Math",

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- "Each student of SE0000 must take Discrete Math",
- "Jenifer is a student of SE0000".

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Show that these hypotheses lead to the conclusion "Jenifer must take Discrete Math".

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- "Jenifer is a student of SE0000".

Show that these hypotheses lead to the conclusion "Jenifer must take Discrete Math".

Example 2. Given the hypotheses:

Example 1. Given the hypotheses:

- "Each student of SE0000 must take Discrete Math",
- "Jenifer is a student of SE0000".

Show that these hypotheses lead to the conclusion "Jenifer must take Discrete Math".

Example 2. Given the hypotheses:

- "Some student of SE0000 has not read this book",

Example 1. Given the hypotheses:

- "Each student of SE0000 must take Discrete Math",
- "Jenifer is a student of SE0000".

Show that these hypotheses lead to the conclusion "Jenifer must take Discrete Math".

Example 2. Given the hypotheses:

- "Some student of SE0000 has not read this book",
- "Every student of SE0000 passed the exam".

Example 1. Given the hypotheses:

- "Each student of SE0000 must take Discrete Math",
- "Jenifer is a student of SE0000".

Show that these hypotheses lead to the conclusion "Jenifer must take Discrete Math".

Example 2. Given the hypotheses:

- "Some student of SE0000 has not read this book",
- "Every student of SE0000 passed the exam".

Show that these hypotheses lead to the conclusion "Some student of SE0000 who passed the exam has not read this book".

1.6 Introduction to Proofs

Direct method

Direct method

Problem: Prove that the statement $p \rightarrow q$ is correct.

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Example. Prove that if n is an odd number then n^2 is also an odd number.

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Indirect method - Proof by contraposition

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Indirect method - Proof by contraposition

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Problem: Prove that the statement $p \to q$ is correct.

Proof: Assume that p is true. We will show that q is true.

Example. Prove that if n is an odd number then n^2 is also an odd number.

Indirect method - Proof by contraposition

Problem: Prove that the statement $p \to q$ is correct.

Proof: Assume that q is false. We will show that p is also false.

1.6 Introduction to Proofs

Direct method

Problem: Prove that the statement $p \to q$ is correct.

Proof: Assume that p is true. We will show that q is true.

Example. Prove that if n is an odd number then n^2 is also an odd number.

Indirect method - Proof by contraposition

Problem: Prove that the statement $p \to q$ is correct.

Proof: Assume that q is false. We will show that p is also false.

Example. Show that if x is an irrational number then $1/x$ is also irrational.

Proof by contradiction

Proof by contradiction

Problem: Prove that the statement p is correct.

Proof by contradiction

Problem: Prove that the statement p is correct.

Proof: Suppose that p is not correct. We will find a contradiction.

Proof by contradiction

Problem: Prove that the statement p is correct.

Proof: Suppose that p is not correct. We will find a contradiction.

Example. Show that $\sqrt{2}$ is irrational.

Proof by contradiction

Problem: Prove that the statement p is correct.

Proof: Suppose that p is not correct. We will find a contradiction.

Example. Show that $\sqrt{2}$ is irrational.

1.7 Proof Methods and Strategies

Read textbook!