Discrete Mathematics 1

Chapter 2: Basic Structures: Sets, Functions, Sequences and Sums

Department of Mathematics
The FPT university

Definition

2.1 Sets

Definition

- A set is an unordered collection of objects.

Definition

- A set is an unordered collection of objects.
- An object of a set is called an element, or a member, of that set.

2.1 Sets

Definition

- A set is an unordered collection of objects.
- An object of a set is called an element, or a member, of that set.
- The cardinality of the set A is the number of distinct elements of A, denoted by $|A|$.

2.1 Sets

Definition

- A set is an unordered collection of objects.
- An object of a set is called an element, or a member, of that set.
- The cardinality of the set A is the number of distinct elements of A, denoted by $|A|$.
- The empty set, denoted by $\emptyset$, is the set whose cardinality is 0.

2.1 Sets

Definition

- A set is an unordered collection of objects.
- An object of a set is called an element, or a member, of that set.
- The cardinality of the set A is the number of distinct elements of A, denoted by $|A|$.
- The empty set, denoted by $\emptyset$, is the set whose cardinality is 0.

Example.

2.1 Sets

Definition

- A set is an unordered collection of objects.
- An object of a set is called an element, or a member, of that set.
- The cardinality of the set A is the number of distinct elements of A, denoted by $|A|$.
- The empty set, denoted by $\emptyset$, is the set whose cardinality is 0.

Example.

- The set $\{a, cat, catches, a, mouse\}$ has

Definition

- A set is an unordered collection of objects.
- An object of a set is called an element, or a member, of that set.
- The cardinality of the set A is the number of distinct elements of A, denoted by $|A|$.
- The empty set, denoted by $\emptyset$, is the set whose cardinality is 0.

Example.

- The set $\{a, cat, catches, a, mouse\}$ has 4 elements.

2.1 Sets

Definition

- A set is an unordered collection of objects.
- An object of a set is called an element, or a member, of that set.
- The cardinality of the set A is the number of distinct elements of A, denoted by $|A|$.
- The empty set, denoted by $\emptyset$, is the set whose cardinality is 0.

Example.

- The set $\{a, cat, catches, a, mouse\}$ has 4 elements.
- The set $\{a, b, \{a, b\}, c, \{a, b, c, \}, \emptyset\}$ has

Definition

- A set is an unordered collection of objects.
- An object of a set is called an element, or a member, of that set.
- The cardinality of the set A is the number of distinct elements of A, denoted by $|A|$.
- The empty set, denoted by $\emptyset$, is the set whose cardinality is 0.

Example.

- The set $\{a, cat, catches, a, mouse\}$ has 4 elements.
- The set $\{a, b, \{a, b\}, c, \{a, b, c, \}, \emptyset\}$ has 6 elements

- If x is an element of A we write $x \in A$.

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$ (T)

$\{a, b\} \in \{a, b, \{a, b\}, c\}$

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$ (T)

$\{a, b\} \in \{a, b, \{a, b\}, c\}$ (T)

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$ (T)

$\{a, b\} \in \{a, b, \{a, b\}, c\}$ (T)

$\emptyset \subseteq \{\emptyset\}$

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$ (T)

$\{a, b\} \in \{a, b, \{a, b\}, c\}$ (T)

$\emptyset \subseteq \{\emptyset\}$ (T)

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$ (T)

$\{a, b\} \in \{a, b, \{a, b\}, c\}$ (T)

$\emptyset \subseteq \{\emptyset\}$ (T)

$\emptyset \in \{\emptyset\}$

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$ (T)

$\{a, b\} \in \{a, b, \{a, b\}, c\}$ (T)

$\emptyset \subseteq \{\emptyset\}$ (T)

$\emptyset \in \{\emptyset\}$ (T)

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$ (T)

$\{a, b\} \in \{a, b, \{a, b\}, c\}$ (T)

$\emptyset \subseteq \{\emptyset\}$ (T)

$\emptyset \in \{\emptyset\}$ (T)

$\{a, b, c\} \subseteq \{a, b, c\}$

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$ (T)

$\{a, b\} \in \{a, b, \{a, b\}, c\}$ (T)

$\emptyset \subseteq \{\emptyset\}$ (T)

$\emptyset \in \{\emptyset\}$ (T)

$\{a, b, c\} \subseteq \{a, b, c\}$ (T)

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$ (T)

$\{a, b\} \in \{a, b, \{a, b\}, c\}$ (T)

$\emptyset \subseteq \{\emptyset\}$ (T)

$\emptyset \in \{\emptyset\}$ (T)

$\{a, b, c\} \subseteq \{a, b, c\}$ (T)

$\{a, b, c\} \in \{a, b, c\}$

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)	$\emptyset \subseteq \{\emptyset\}$ (T)
$x \subseteq \{x\}$ (F)	$\emptyset \in \{\emptyset\}$ (T)
$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$ (T)	$\{a, b, c\} \subseteq \{a, b, c\}$ (T)
$\{a, b\} \in \{a, b, \{a, b\}, c\}$ (T)	$\{a, b, c\} \in \{a, b, c\}$ (F)

- If x is an element of A we write $x \in A$. If x is not an element of A we write $x \notin A$.
- If all elements of A are also elements of B we write $A \subseteq B$, and A is called a subset of B.
- If A is a proper subset of B, meaning $A \subseteq B$ and $A \neq B$, we write $A \subset B$.
- The empty set $\emptyset$ is a subset of any set.

Example. Which statements are true?

$x \in \{x\}$ (T)

$x \subseteq \{x\}$ (F)

$\{a, b\} \subseteq \{a, b, \{a, b\}, c\}$ (T)

$\{a, b\} \in \{a, b, \{a, b\}, c\}$ (T)

$\emptyset \subseteq \{\emptyset\}$ (T)

$\emptyset \in \{\emptyset\}$ (T)

$\{a, b, c\} \subseteq \{a, b, c\}$ (T)

$\{a, b, c\} \in \{a, b, c\}$ (F)

Definitions

Definitions

- The Cartesian product of two sets A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.

Definitions

- The Cartesian product of two sets A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.
- The power set of the set A, denoted by $P(A)$, is the set of all subsets of A.

Definitions

- The Cartesian product of two sets A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.
- The power set of the set A, denoted by $P(A)$, is the set of all subsets of A.

Note.

Definitions

- The Cartesian product of two sets A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.
- The power set of the set A, denoted by $P(A)$, is the set of all subsets of A.

Note.

If $|A| = m$ and $|B| = n$ then $|A \times B| = mn$.

Definitions

- The Cartesian product of two sets A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$.
- The power set of the set A, denoted by $P(A)$, is the set of all subsets of A.

Note.

If $|A| = m$ and $|B| = n$ then $|A \times B| = mn$.

If $|A| = n$ then $|P(A)| = 2^n$.

2.2 Set Operations

2.2 Set Operations

Let A and B be two sets.

2.2 Set Operations

Let A and B be two sets.

- Union of A and B: $A \cup B = \{x | (x \in A) \vee (x \in B)\}$

Let A and B be two sets.

- Union of A and B: $A \cup B = \{x \mid (x \in A) \vee (x \in B)\}$
- Intersection of A and B: $A \cap B = \{x \mid (x \in A) \wedge (x \in B)\}$

2.2 Set Operations

Let A and B be two sets.

- Union of A and B: $A \cup B = \{x | (x \in A) \vee (x \in B)\}$
- Intersection of A and B: $A \cap B = \{x | (x \in A) \wedge (x \in B)\}$
- Difference of A and B: $A - B = \{x | (x \in A) \wedge (x \notin B)\}$

Let A and B be two sets.

- Union of A and B: $A \cup B = \{x | (x \in A) \vee (x \in B)\}$
- Intersection of A and B: $A \cap B = \{x | (x \in A) \wedge (x \in B)\}$
- Difference of A and B: $A - B = \{x | (x \in A) \wedge (x \notin B)\}$
- Symmetric difference of A and B: $A \oplus B = \{x | (x \in A) \oplus (x \in B)\}$

2.2 Set Operations

Let A and B be two sets.

- Union of A and B: $A \cup B = \{x | (x \in A) \vee (x \in B)\}$
- Intersection of A and B: $A \cap B = \{x | (x \in A) \wedge (x \in B)\}$
- Difference of A and B: $A - B = \{x | (x \in A) \wedge (x \notin B)\}$
- Symmetric difference of A and B: $A \oplus B = \{x | (x \in A) \oplus (x \in B)\}$
- Complement of A with respect to the universal set U: $\overline{A} = U - A$

Set Identities

Set Identities

Name	Identity

Set Identities

Name	Identity
Complementation law	$\overline{(\overline{A})} = A$

Set Identities

Name	Identity
Complementation law	$\overline{(\overline{A})} = A$
Identity laws	$A \cup \emptyset = A, \;\; A \cap U = A$

Set Identities

Name	Identity
Complementation law	$\overline{(\overline{A})} = A$
Identity laws	$A \cup \emptyset = A, \;\; A \cap U = A$
Domination laws	$A \cup U = U, \;\; A \cap \emptyset = \emptyset$

Set Identities

Name	Identity
Complementation law	$\overline{(\overline{A})} = A$
Identity laws	$A \cup \emptyset = A, \;\; A \cap U = A$
Domination laws	$A \cup U = U, \;\; A \cap \emptyset = \emptyset$
Complement laws	$A \cup \overline{A} = U, \;\; A \cap \overline{A} = \emptyset$

Set Identities

Name	Identity
Complementation law	$\overline{(\overline{A})} = A$
Identity laws	$A \cup \emptyset = A, \;\; A \cap U = A$
Domination laws	$A \cup U = U, \;\; A \cap \emptyset = \emptyset$
Complement laws	$A \cup \overline{A} = U, \;\; A \cap \overline{A} = \emptyset$
Idempotent laws	$A \cup A = A, \;\; A \cap A = A$

Set Identities

Name	Identity
Complementation law	$\overline{(\overline{A})} = A$
Identity laws	$A \cup \emptyset = A, \;\; A \cap U = A$
Domination laws	$A \cup U = U, \;\; A \cap \emptyset = \emptyset$
Complement laws	$A \cup \overline{A} = U, \;\; A \cap \overline{A} = \emptyset$
Idempotent laws	$A \cup A = A, \;\; A \cap A = A$
Commutative laws	$A \cup B = B \cup A, \;\; A \cap B = B \cap A$

Set Identities

Name	Identity
Complementation law	$\overline{(\overline{A})} = A$
Identity laws	$A \cup \emptyset = A, \;\; A \cap U = A$
Domination laws	$A \cup U = U, \;\; A \cap \emptyset = \emptyset$
Complement laws	$A \cup \overline{A} = U, \;\; A \cap \overline{A} = \emptyset$
Idempotent laws	$A \cup A = A, \;\; A \cap A = A$
Commutative laws	$A \cup B = B \cup A, \;\; A \cap B = B \cap A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$

Set Identities

Name	Identity
Complementation law	$\overline{(\overline{A})} = A$
Identity laws	$A \cup \emptyset = A, \ \ A \cap U = A$
Domination laws	$A \cup U = U, \ \ A \cap \emptyset = \emptyset$
Complement laws	$A \cup \overline{A} = U, \ \ A \cap \overline{A} = \emptyset$
Idempotent laws	$A \cup A = A, \ \ A \cap A = A$
Commutative laws	$A \cup B = B \cup A, \ \ A \cap B = B \cap A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Set Identities

Name	Identity
Complementation law	$\overline{(\overline{A})} = A$
Identity laws	$A \cup \emptyset = A, \;\; A \cap U = A$
Domination laws	$A \cup U = U, \;\; A \cap \emptyset = \emptyset$
Complement laws	$A \cup \overline{A} = U, \;\; A \cap \overline{A} = \emptyset$
Idempotent laws	$A \cup A = A, \;\; A \cap A = A$
Commutative laws	$A \cup B = B \cup A, \;\; A \cap B = B \cap A$
Associative laws	$(A \cup B) \cup C = A \cup (B \cup C)$ $(A \cap B) \cap C = A \cap (B \cap C)$
Distributive laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
De Morgan's laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$ $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Two methods to show the equality of two sets.

Two methods to show the equality of two sets.

- Show that each set is a subset of the other.

Two methods to show the equality of two sets.

- Show that each set is a subset of the other.
- Use Membership table, similar to the method of using truth table to establish propositional equivalences.

Computer Representation of Sets

Let U be a universal set. Fix an ordering of elements of U as $a_1, a_2, \ldots, a_n$.

Computer Representation of Sets

Let U be a universal set. Fix an ordering of elements of U as $a_1, a_2, \ldots, a_n$.

If A is a subset of U, represent A with a bit string of length n, where the ith bit is 1 if a_i is in A, and is 0 if a_i is not in A.

Computer Representation of Sets

Let U be a universal set. Fix an ordering of elements of U as $a_1, a_2, \ldots, a_n$.

If A is a subset of U, represent A with a bit string of length n, where the ith bit is 1 if a_i is in A, and is 0 if a_i is not in A.

Example.

Computer Representation of Sets

Let U be a universal set. Fix an ordering of elements of U as $a_1, a_2, \ldots, a_n$.

If A is a subset of U, represent A with a bit string of length n, where the ith bit is 1 if a_i is in A, and is 0 if a_i is not in A.

Example. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$.

Computer Representation of Sets

Let U be a universal set. Fix an ordering of elements of U as $a_1, a_2, \ldots, a_n$.

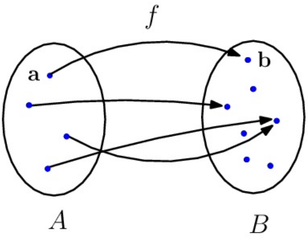
If A is a subset of U, represent A with a bit string of length n, where the ith bit is 1 if a_i is in A, and is 0 if a_i is not in A.

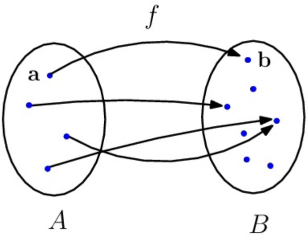
Example. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Then the subset $A = \{1, 3, 4, 6\}$ is represented as the bit string 10110100.

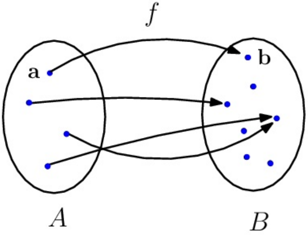
2.3 Functions

2.3 Functions

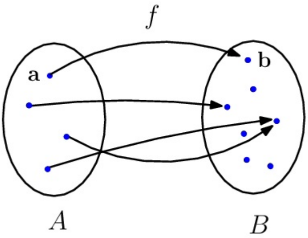
2.3 Functions



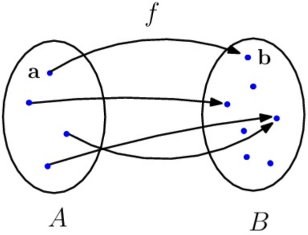

f
a
b
A
B



The set A is called domain and B is called codomain of f.



The set A is called domain
and B is called codomain of f.
If $f(a) = b$ we say b is the image of a
and a is a preimage of b.

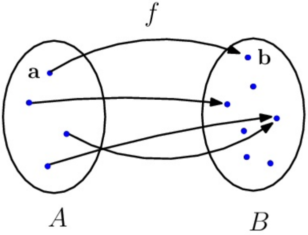


The set A is called domain and B is called codomain of f.
If $f(a) = b$ we say b is the image of a and a is a preimage of b.

Let S be a subset of A. The set

$$f(S) = \{b \in B | \exists a \in A(f(a) = b)\}$$

is called the image of S,



The set A is called domain and B is called codomain of f.
If $f(a) = b$ we say b is the image of a and a is a preimage of b.

Let S be a subset of A. The set

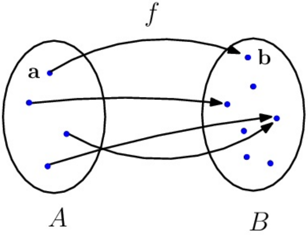
$$f(S) = \{b \in B | \exists a \in A(f(a) = b)\}$$

is called the image of S, and the set

$$f^{-1}(S) = \{a \in A | f(a) \in S\}$$

is called the preimage of S.

2.3 Functions



The set A is called domain
and B is called codomain of f.
If $f(a) = b$ we say b is the image of a
and a is a preimage of b.

Let S be a subset of A. The set

$$f(S) = \{b \in B | \exists a \in A(f(a) = b)\}$$

is called the image of S, and the set

$$f^{-1}(S) = \{a \in A | f(a) \in S\}$$

is called the preimage of S. The set $f(A)$ is called the range of f.

Some Important Functions

Some Important Functions

- Floor function: $\lfloor x \rfloor$

Some Important Functions

- Floor function: $\lfloor x \rfloor$ = the greatest integer that is not greater than x.

Some Important Functions

- Floor function: $\lfloor x \rfloor$ = the greatest integer that is not greater than x.
- Ceiling function: $\lceil x \rceil$

Some Important Functions

- Floor function: $\lfloor x \rfloor$ = the greatest integer that is not greater than x.
- Ceiling function: $\lceil x \rceil$ = the smallest integer that is not smaller than x.

Some Important Functions

- Floor function: $\lfloor x \rfloor =$ the greatest integer that is not greater than x.
- Ceiling function: $\lceil x \rceil =$ the smallest integer that is not smaller than x.

Note. For all real numbers x we have

$$x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$$

Some Important Functions

- Floor function: $\lfloor x \rfloor$ = the greatest integer that is not greater than x.
- Ceiling function: $\lceil x \rceil$ = the smallest integer that is not smaller than x.

Note. For all real numbers x we have

$$x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$$

Example. Which statements are true for all real numbers x, y and all integers n?

Some Important Functions

- Floor function: $\lfloor x \rfloor$ = the greatest integer that is not greater than x.
- Ceiling function: $\lceil x \rceil$ = the smallest integer that is not smaller than x.

Note. For all real numbers x we have

$$x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$$

Example. Which statements are true for all real numbers x, y and all integers n?

$\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$

Some Important Functions

- Floor function: $\lfloor x \rfloor$ = the greatest integer that is not greater than x.
- Ceiling function: $\lceil x \rceil$ = the smallest integer that is not smaller than x.

Note. For all real numbers x we have

$$x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$$

Example. Which statements are true for all real numbers x, y and all integers n?

$\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$
$\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$

Some Important Functions

- Floor function: $\lfloor x \rfloor$ = the greatest integer that is not greater than x.
- Ceiling function: $\lceil x \rceil$ = the smallest integer that is not smaller than x.

Note. For all real numbers x we have

$$x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$$

Example. Which statements are true for all real numbers x, y and all integers n?

$\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil$

$\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$

$\lceil x + n \rceil = \lceil x \rceil + n$

Some Important Functions

- Floor function: $\lfloor x \rfloor$ = the greatest integer that is not greater than x.
- Ceiling function: $\lceil x \rceil$ = the smallest integer that is not smaller than x.

Note. For all real numbers x we have

$$x - 1 < \lfloor x \rfloor \le x \le \lceil x \rceil < x + 1$$

Example. Which statements are true for all real numbers x, y and all integers n?

$$\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil \qquad \lceil x + n \rceil = \lceil x \rceil + n$$

$$\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor \qquad \lfloor x + n \rfloor = \lfloor x \rfloor + n$$

Some Important Functions

- Floor function: $\lfloor x \rfloor$ = the greatest integer that is not greater than x.
- Ceiling function: $\lceil x \rceil$ = the smallest integer that is not smaller than x.

Note. For all real numbers x we have

$$x - 1 < \lfloor x \rfloor \leq x \leq \lceil x \rceil < x + 1$$

Example. Which statements are true for all real numbers x, y and all integers n?

$$\lceil x + y \rceil = \lceil x \rceil + \lceil y \rceil \qquad \lceil x + n \rceil = \lceil x \rceil + n$$

$$\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor \qquad \lfloor x + n \rfloor = \lfloor x \rfloor + n$$

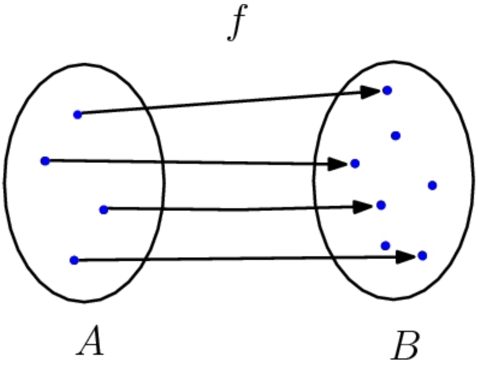
One-to-One, Onto, and Bijection

One-to-One, Onto, and Bijection

The function $f : A \to B$ is one-to-one if $f(a_1) \neq f(a_2)$ for all $a_1 \neq a_2$ in A.

One-to-One, Onto, and Bijection

The function $f : A \to B$ is one-to-one if $f(a_1) \neq f(a_2)$ for all $a_1 \neq a_2$ in A.



Example. Which functions are one-to-one?

Example. Which functions are one-to-one?

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

Example. Which functions are one-to-one?

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

(b) $f : \mathbb{R}^+ \to \mathbb{R}; \quad f(x) = x^2$

Example. Which functions are one-to-one?

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

(b) $f : \mathbb{R}^+ \to \mathbb{R}; \quad f(x) = x^2$

(c) $f : \mathbb{Z} \to \mathbb{Z}; \quad f(n) = \lfloor \frac{n+1}{2} \rfloor$

Example. Which functions are one-to-one?

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

(b) $f : \mathbb{R}^+ \to \mathbb{R}; \quad f(x) = x^2$

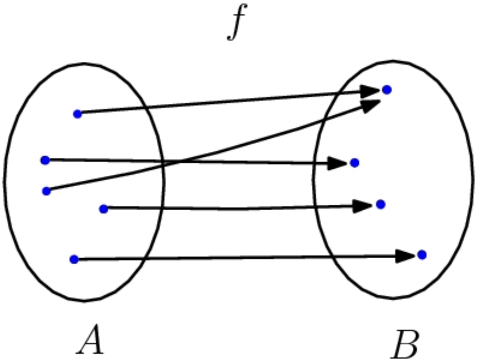
(c) $f : \mathbb{Z} \to \mathbb{Z}; \quad f(n) = \lfloor \frac{n+1}{2} \rfloor$

(d) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}; \quad f(m, n) = m + n$

The function $f : A \rightarrow B$ is onto if for each b in B there is a in A such that $f(a) = b$.

The function $f : A \to B$ is onto if for each b in B there is a in A such that $f(a) = b$. In other words, the function $f : A \to B$ is onto if $f(A) = B$.

The function $f : A \to B$ is onto if for each b in B there is a in A such that $f(a) = b$. In other words, the function $f : A \to B$ is onto if $f(A) = B$.



Example. Which functions are onto:

Example. Which functions are onto:

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

Example. Which functions are onto:

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

(b) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^3$

Example. Which functions are onto:

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

(b) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^3$

(c) $f : \mathbb{R} \to \mathbb{Z}; \quad f(x) = 2\lfloor x \rfloor$

Example. Which functions are onto:

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

(b) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^3$

(c) $f : \mathbb{R} \to \mathbb{Z}; \quad f(x) = 2\lfloor x \rfloor$

(d) $f : \mathbb{R} \to \mathbb{Z}; \quad f(x) = \lfloor 2x \rfloor$

Example. Which functions are onto:

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

(b) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^3$

(c) $f : \mathbb{R} \to \mathbb{Z}; \quad f(x) = 2\lfloor x \rfloor$

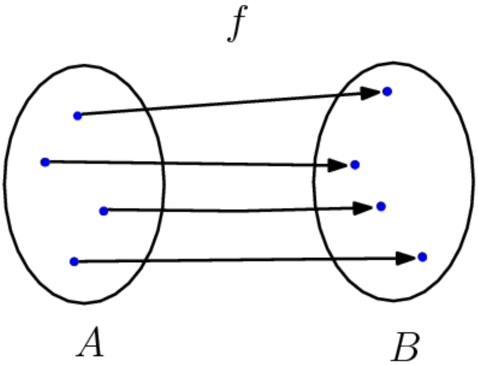
(d) $f : \mathbb{R} \to \mathbb{Z}; \quad f(x) = \lfloor 2x \rfloor$

(e) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}; \quad f(m, n) = m + n$

The function $f : A \to B$ is a bijection (or one-to-one correspondence) if it is both one-to-one and onto.

The function $f : A \to B$ is a bijection (or one-to-one correspondence) if it is both one-to-one and onto.

The function $f : A \to B$ is a bijection (or one-to-one correspondence) if it is both one-to-one and onto.



Example 1. Which functions are bijection:

Example 1. Which functions are bijection:

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

Example 1. Which functions are bijection:

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

(b) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^3$

Example 1. Which functions are bijection:

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

(b) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^3$

(c) $f : \mathbb{R} \to \mathbb{Z}; \quad f(x) = \lfloor 2x \rfloor$

Example 1. Which functions are bijection:

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

(b) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^3$

(c) $f : \mathbb{R} \to \mathbb{Z}; \quad f(x) = \lfloor 2x \rfloor$

(d) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}; \quad f(m, n) = m + n$

Example 1. Which functions are bijection:

(a) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^2$

(b) $f : \mathbb{R} \to \mathbb{R}; \quad f(x) = x^3$

(c) $f : \mathbb{R} \to \mathbb{Z}; \quad f(x) = \lfloor 2x \rfloor$

(d) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}; \quad f(m, n) = m + n$

(e) $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}; \quad f(m, n) = (m, m + n)$

Example 2. Does there exist:

Example 2. Does there exist:

(a) A bijection /one-to-one/onto function from a set of 7 elements to a set of 5 elements?

Example 2. Does there exist:

(a) A bijection /one-to-one/onto function from a set of 7 elements to a set of 5 elements? From a set of 5 elements to a set of 7 elements?

Example 2. Does there exist:

(a) A bijection /one-to-one/onto function from a set of 7 elements to a set of 5 elements? From a set of 5 elements to a set of 7 elements?

(b) A bijection from the set of even integers to the set of odd integers?

Example 2. Does there exist:

(a) A bijection /one-to-one/onto function from a set of 7 elements to a set of 5 elements? From a set of 5 elements to a set of 7 elements?

(b) A bijection from the set of even integers to the set of odd integers?

(c) A bijection from the set of odd integers to the set of all integers?

Example 2. Does there exist:

(a) A bijection /one-to-one/onto function from a set of 7 elements to a set of 5 elements? From a set of 5 elements to a set of 7 elements?

(b) A bijection from the set of even integers to the set of odd integers?

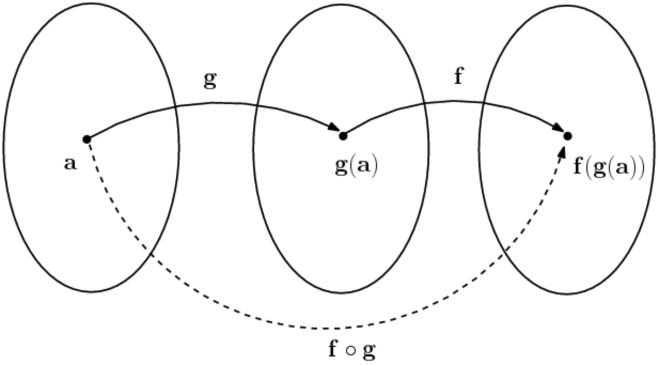
(c) A bijection from the set of odd integers to the set of all integers?

(d) A bijection from the set of all real numbers to the set of positive real numbers?

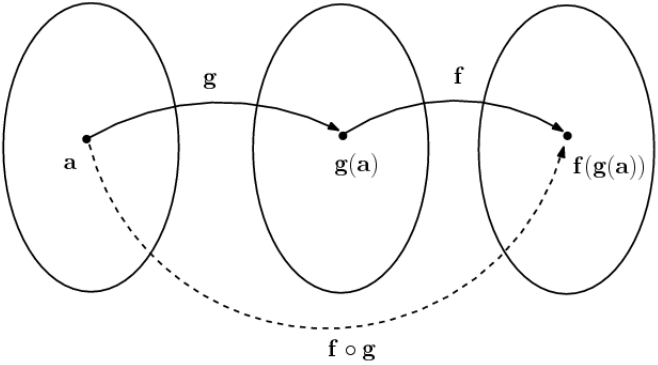
Compositions and Inverses

Composition

Composition

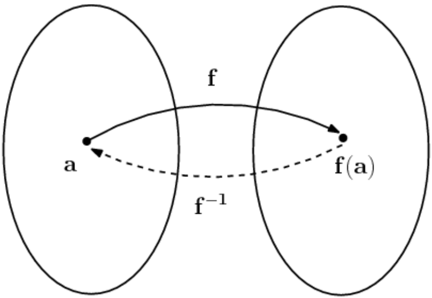


Composition

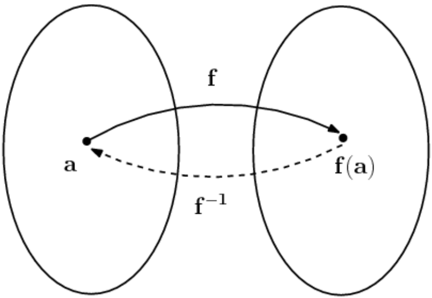


Inverse

Inverse

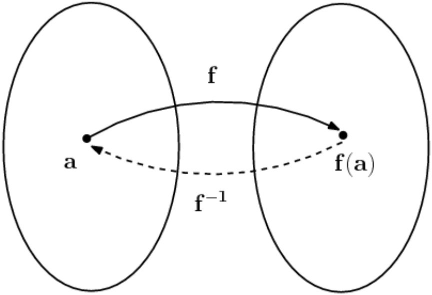


Inverse



Note.

Inverse



Note. The function $f : A \to B$ has an inverse if and only if f is a bijection.

Sequences

2.4 Sequences and Summations

Sequences

Sequence is a discrete structure used to represent an order list.

Sequences

Sequence is a discrete structure used to represent an order list. It is usually denoted as $\{a_1, a_2, \ldots\} = \{a_n, n = 1, 2, \ldots\}$.

Sequences

Sequence is a discrete structure used to represent an order list. It is usually denoted as $\{a_1, a_2, \ldots\} = \{a_n, n = 1, 2, \ldots\}$.

Example.

Sequences

Sequence is a discrete structure used to represent an order list. It is usually denoted as $\{a_1, a_2, \ldots\} = \{a_n, n = 1, 2, \ldots\}$.

Example. Find a general formula for a_n of each sequence:

Sequences

Sequence is a discrete structure used to represent an order list. It is usually denoted as $\{a_1, a_2, \ldots\} = \{a_n, n = 1, 2, \ldots\}$.

Example. Find a general formula for a_n of each sequence:

(a) $\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots$

Sequences

Sequence is a discrete structure used to represent an order list. It is usually denoted as $\{a_1, a_2, \ldots\} = \{a_n, n = 1, 2, \ldots\}$.

Example. Find a general formula for a_n of each sequence:

(a) $\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots$

(b) $-2, 1, 4, 7, 10, \ldots$

Sequences

Sequence is a discrete structure used to represent an order list. It is usually denoted as $\{a_1, a_2, \ldots\} = \{a_n, n = 1, 2, \ldots\}$.

Example. Find a general formula for a_n of each sequence:

(a) $\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots$

(b) $-2, 1, 4, 7, 10, \ldots$ (an arithmetic progression)

(c) $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots$

Sequences

Sequence is a discrete structure used to represent an order list. It is usually denoted as $\{a_1, a_2, \ldots\} = \{a_n, n = 1, 2, \ldots\}$.

Example. Find a general formula for a_n of each sequence:

(a) $\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots$

(b) $-2, 1, 4, 7, 10, \ldots$ (an arithmetic progression)

(c) $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots$

(d) $1, 1, 2, 3, 5, 8, 13, 21, \ldots$

Sequences

Sequence is a discrete structure used to represent an order list. It is usually denoted as $\{a_1, a_2, \ldots\} = \{a_n, n = 1, 2, \ldots\}$.

Example. Find a general formula for a_n of each sequence:

(a) $\frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \ldots$

(b) $-2, 1, 4, 7, 10, \ldots$ (an arithmetic progression)

(c) $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots$

(d) $1, 1, 2, 3, 5, 8, 13, 21, \ldots$ (Fibonacci sequence)

Summations

Summations

Given a sequence $\{a_n, n = 1, 2, \ldots\}$.

Summations

Given a sequence $\{a_n, n = 1, 2, \ldots\}$. The summation notation, $\sum$, is used as follows:

Summations

Given a sequence $\{a_n, n = 1, 2, \ldots\}$. The summation notation, $\sum$, is used as follows:

$$\sum_{i=1}^{100} a_i = a_1 + a_2 + \cdots + a_{100}$$

Summations

Given a sequence $\{a_n, n = 1, 2, \ldots\}$. The summation notation, $\sum$, is used as follows:

$$\sum_{i=1}^{100} a_i = a_1 + a_2 + \cdots + a_{100}$$

$$\sum_{i=1}^{100} a_{2i+1} = a_3 + a_5 + \cdots + a_{201}$$

Summations

Given a sequence $\{a_n, n = 1, 2, \dots\}$. The summation notation, $\sum$, is used as follows:

$$\sum_{i=1}^{100} a_i = a_1 + a_2 + \cdots + a_{100}$$

$$\sum_{i=1}^{100} a_{2i+1} = a_3 + a_5 + \cdots + a_{201}$$

$$\sum_{i=1}^{100} 1 = 1 + 1 + \cdots + 1 \text{ (100 terms)}$$

Summations

Given a sequence $\{a_n, n = 1, 2, \ldots\}$. The summation notation, $\sum$, is used as follows:

$$\sum_{i=1}^{100} a_i = a_1 + a_2 + \cdots + a_{100}$$

$$\sum_{i=1}^{100} a_{2i+1} = a_3 + a_5 + \cdots + a_{201}$$

$$\sum_{i=1}^{100} 1 = 1 + 1 + \cdots + 1 \text{ (100 terms)}$$

Properties

Summations

Given a sequence $\{a_n, n = 1, 2, \ldots\}$. The summation notation, $\sum$, is used as follows:

$$\sum_{i=1}^{100} a_i = a_1 + a_2 + \cdots + a_{100}$$

$$\sum_{i=1}^{100} a_{2i+1} = a_3 + a_5 + \cdots + a_{201}$$

$$\sum_{i=1}^{100} 1 = 1 + 1 + \cdots + 1 \text{ (100 terms)}$$

Properties

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$$

Summations

Given a sequence $\{a_n, n = 1, 2, \ldots\}$. The summation notation, $\sum$, is used as follows:

$$\sum_{i=1}^{100} a_i = a_1 + a_2 + \cdots + a_{100}$$

$$\sum_{i=1}^{100} a_{2i+1} = a_3 + a_5 + \cdots + a_{201}$$

$$\sum_{i=1}^{100} 1 = 1 + 1 + \cdots + 1 \text{ (100 terms)}$$

Properties

$$\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \qquad \sum_{i=1}^{n} ka_i = k\sum_{i=1}^{n} a_i$$

Summations

Given a sequence $\{a_n, n = 1, 2, \ldots\}$. The summation notation, $\sum$, is used as follows:

$$\sum_{i=1}^{100} a_i = a_1 + a_2 + \cdots + a_{100}$$

$$\sum_{i=1}^{100} a_{2i+1} = a_3 + a_5 + \cdots + a_{201}$$

$$\sum_{i=1}^{100} 1 = 1 + 1 + \cdots + 1 \text{ (100 terms)}$$

Properties

$$\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \qquad \sum_{i=1}^{n} k a_i = k \sum_{i=1}^{n} a_i$$

Some Important Sums

Some Important Sums

$$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

Some Important Sums

$$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Some Important Sums

$$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Some Important Sums

$$\sum_{i=1}^{n} i = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$\sum_{i=0}^{n} r^i = 1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

Example 1.

Example 1. Find:

Example 1. Find:

(a) $$\sum_{i=1}^{100} \frac{3^i}{4^{i+1}}$$

Example 1. Find:

(a) $\displaystyle\sum_{i=1}^{100} \frac{3^i}{4^{i+1}}$

(b) $\displaystyle\sum_{i=1}^{100} \frac{1}{i(i+1)}$ (Telescoping sum)

Example 2.

Example 1. Find:

(a) $$\sum_{i=1}^{100} \frac{3^i}{4^{i+1}}$$

(b) $$\sum_{i=1}^{100} \frac{1}{i(i+1)}$$ (Telescoping sum)

Example 2. Find double summations:

Example 1. Find:

(a) $$\sum_{i=1}^{100} \frac{3^i}{4^{i+1}}$$

(b) $$\sum_{i=1}^{100} \frac{1}{i(i+1)}$$ (Telescoping sum)

Example 2. Find double summations:

(a) $$\sum_{i=1}^{2} \sum_{j=0}^{2} (i+2j)$$

Example 1. Find:

(a) $\displaystyle\sum_{i=1}^{100} \frac{3^i}{4^{i+1}}$

(b) $\displaystyle\sum_{i=1}^{100} \frac{1}{i(i+1)}$ (Telescoping sum)

Example 2. Find double summations:

(a) $\displaystyle\sum_{i=1}^{2} \sum_{j=0}^{2} (i+2j)$

(b) $\displaystyle\sum_{i=1}^{10} \sum_{j=1}^{100} ij$