Discrete Mathematics 1

Chapter 5: Counting

Department of Mathematics
The FPT university

Chapter 5: Introduction

Topics covered:

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5.1 The Basics of Counting

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5.2 The Pigeonhole Principle

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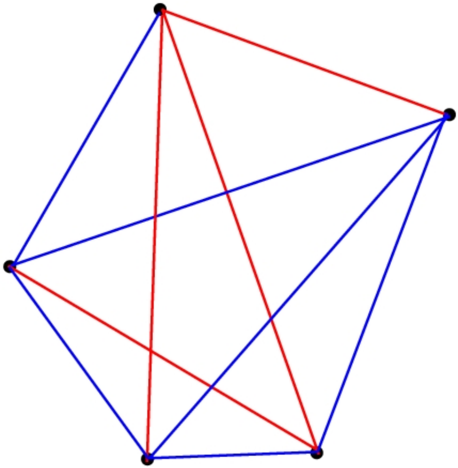

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Problem: Given a permutation or an r-combination, find the next permutation or r-combination in the lexicographic order.

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- Among the numbers $a_{i+1}, a_{i+2}, \ldots, a_n$ choose the smallest element a_k with the property that $a_k > a_i$.

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- Swap a_i and a_k
- Sort the list $a_{i+1}, a_{i+2}, \ldots, a_n$ in the increasing order.

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