

Firm-Level Uncertainty and the Transmission of Forward Guidance to Investment^{*}

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Abstract

I study the role of firms' uncertainty in the transmission of forward guidance to investment. To do so, I employ a quarterly firm-level panel of U.S. publicly traded firms. I measure forward guidance shocks based on unexpected changes in the slope of the yield curve in a 30-minute window around Federal Reserve announcements. I show that firms which are more uncertain adjust their investment as if they are more pessimistic. Put differently, more uncertain firms adjust their investment relatively more downward for expected monetary tightenings, and relatively less upward for expected loosening. To explain my empirical findings, I construct a medium-scale New Keynesian model with two sectors, a high-uncertainty and low-uncertainty one. Agents in the high-uncertainty sector are ambiguous (Knightian uncertain) about the informativeness of forward guidance, and choose to take a pessimistic stance due to their ambiguity aversion. The model implies that expansionary forward guidance is less powerful in recessions due to a larger share of uncertain agents.

JEL Codes: D20, D80, E30, E40, E50

Keywords: Monetary policy, forward guidance, uncertainty, investment, ambiguity aversion, New Keynesian

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1 Introduction

Since the late 1990s central bank communication about future intentions of monetary policy, i.e. forward guidance, has become an increasingly important part of central banks’ toolkit worldwide, especially after the Great Recession. By now, there is a large consensus among policymakers and academics that forward guidance should remain a central part of monetary policy going forward.¹ Despite its relevance, our understanding of how forward guidance affects economic agents, and hence transmits to the macroeconomy, is limited.

In this paper, I study the transmission of forward guidance to investment, traditionally the most responsive part of GDP to monetary policy. In particular, I examine the role of firm-level uncertainty in the transmission. This focus is motivated by a large body of work which emphasizes the importance of a firm’s uncertainty on its investment decision, as well as the implications of agents’ uncertainty for the overall effectiveness of macroeconomic policies (Bloom, 2014). However, the literature on forward guidance has, so far, neither theoretically nor empirically paid much attention to firms’ uncertainty. This paper fills this gap by studying the following two questions: How does a firm’s uncertainty matter for the investment response to forward guidance? What does this imply for the aggregate transmission?

To address the first question, I employ a quarterly firm-level panel of U.S. publicly traded firms from 1996–2019 combining various datasets. The resulting micro-level dataset is ideal for my purposes with its rich set of firm characteristics, as well as its long and relatively high-frequency time dimension. To measure firm-level uncertainty, I employ firms’ option-implied volatility, an ex-ante measure of the expected volatility of a firm’s stock, which captures a broad notion of uncertainty. The construction of the quarterly forward guidance shock follows key insights from recent empirical papers. Using 30-minute changes around FOMC meetings, I estimate a surprise in the slope of the yield curve which has no effect on the federal funds rate (Gürkaynak, Sack, and Swanson, 2005). Further, I purge the surprises from potentially problematic variation by macroeconomic news (Bauer and Swanson, 2020), and sum them up to a quarterly measure. On the aggregate, a positive shock, an expected tightening, leads to a rise in longer-term interest rates, a contraction in economic activity, and a fall in prices.

Using Jordà’s (2005) local projection, I estimate the heterogeneous effect of forward guidance on firms’ capital stock depending on their uncertainty level. The key result can

¹In a survey conducted by Blinder, Ehrmann, De Haan, and Jansen (2017), 72 percent of central bank governors and 87 percent of academics think that forward guidance should remain in the central banks’ toolkit.

be summarized as follows: For both positive and negative forward guidance shocks, firms with high uncertainty reduce their capital stock relatively to less uncertain firms. Put differently, more uncertain firms responds “as if” they are more pessimistic. Hence, higher firm uncertainty amplifies firms’ investment response to contractionary forward guidance and attenuates it to expansionary one. Importantly, in my estimation I control for a multitude of other firm characteristics and allow the effect of forward guidance to vary by each of them. Hence, my findings cannot be explained by firm characteristics emphasized in previous papers, which study the transmission of conventional monetary policy, such as firm size (Gertler and Gilchrist, 1994), age (Cloyne, Ferreira, Froemel, and Surico, 2018), liquidity (Jeenas, 2018), or leverage (Ottonello and Winberry, 2020). Further, I do not find evidence of the same firm-level heterogeneity in response to conventional monetary policy shocks.

To study the implication of my empirical findings for the aggregate transmission of forward guidance, I build a medium-scale New Keynesian model with two sectors, a high- and low-uncertainty one. Each sector is populated by a representative household who owns the firms and their capital stock. Both sectors are identical except for their expectation formation of the path of future policy rates. I model the forward guidance shock as a set of noisy signals about future policy deviations. Here, the noise reflects that agents might not find the communication perfectly credible or understandable. The key difference between both sectors is that while the household in the low-uncertainty sector knows how informative the forward guidance signals are, the household in the high-uncertainty sector is ambiguous (Knightian uncertain) about the signal informativeness and acts according to a worst-case belief due to her ambiguity aversion. Hence, the high-uncertainty sector is more responsive to contractionary forward guidance and less responsive to expansionary forward guidance.

I move on to calibrate the model in order to use it in a quantitative manner. The forward guidance shock is set to match the impact effect on the yield curve of its empirical counterpart. I calibrate the ambiguity of the high-uncertainty sector such that the model can match the heterogeneity in response to a forward guidance observed in the data. With the calibrated model at hand, I assess the aggregate transmission of forward guidance at different states of the economy. To do so, I change the relative size of each sector consistent with shifts in the empirical distribution of the firm-level uncertainty measure during expansions and recessions. The model implies a reduced effectiveness of expansionary forward guidance of 33 percent during recessions. In contrast, contractionary forward guidance is 22 percent more effective during expansions. Hence, my results suggest that forward guidance is less powerful when needed.

Related Literature My paper relates to several strands of the literature in monetary economics. First, my paper relates to the empirical literature on the transmission of forward guidance. So far, most papers focus on the financial market and macroeconomic effects of forward guidance (e.g., [Gürkaynak, Sack, and Swanson, 2005](#); [Campbell, Evans, Fisher, and Justiniano, 2012](#); [Nakamura and Steinsson, 2018a](#); [Bundick and Smith, 2020](#); [Lunsford, 2020](#)). While there is substantial work on the heterogeneous transmission of conventional monetary policy, very few papers have focused on the heterogeneous transmission of forward guidance so far. [Andrade, Gaballo, Mengus, and Mojon \(2019\)](#) study the differential response of professional forecasters to forward guidance, while [Coibion, Georgarakos, Gorodnichenko, and Weber \(2020\)](#) study the effect of forward guidance on households' expectations. My contribution is to show that there is also substantial heterogeneity in the transmission of forward guidance to investment. Further, my paper emphasizes the role of firms' uncertainty in the transmission of shocks, which the literature on forward guidance has not focused on so far but which is widely documented in the uncertainty literature ([Bloom, 2014](#)).

Second, this paper relates to a growing literature which incorporates agents' ambiguity aversion in monetary economics, and in macroeconomics more generally. This is done either using the multiple-priors preferences as axiomatized by [Gilboa and Schmeidler \(1989\)](#) and [Epstein and Schneider \(2003\)](#) as done in this paper or using the robust control theory by [Hansen and Sargent \(2001, 2010\)](#). Examples include [Adam and Woodford \(2012\)](#), [Ilut and Schneider \(2014\)](#), [Benigno and Paciello \(2014\)](#), [Bianchi, Ilut, and Schneider \(2018\)](#), [Bhandari, Borovička, and Ho \(2019\)](#), [Baqae \(2019\)](#), [Ilut, Valchev, and Vincent \(2020\)](#), [Masolo and Monti \(2020\)](#), and [Ilut and Saijo \(2021\)](#). Empirically, my contribution is to provide new evidence which is consistent with ambiguity aversion in firm's decision making. This is also in line with recent work by [Bachmann, Carstensen, Lautenbacher, and Schneider \(2020\)](#) who survey German firm executives and show that Knightian uncertainty is pervasive among them. In terms of the model, [Michelacci and Paciello \(2020\)](#) also introduce ambiguity about the credibility of forward guidance. However, they focus on the idea that creditors and debtors have different worst-case scenarios about the expected interest rate. While this might be the case, such a channel cannot explain my empirical findings as I control for the debt structure of the firms in my analysis.

Lastly, my paper relates to recent papers which study the role of firm heterogeneity in the transmission of monetary policy to investment (e.g., [Jeenas, 2018](#); [Cloyne et al., 2018](#); [Lakdawala and Moreland, 2019](#); [Crouzet and Mehrotra, 2020](#); [Ottonello and Winberry, 2020](#); [Howes, 2021](#)). This paper differs from previous work at least in two dimensions. First, I focus

on forward guidance in my analysis while previous papers focus on conventional monetary policy.² Second, I study the role of firm uncertainty, a firm characteristic which has not been studied in the monetary policy transmission so far. As mentioned earlier, my empirical findings cannot be explained by the mechanisms suggested by the aforementioned papers. That being said, the results do necessarily contradict previous work either.

Roadmap The rest of the paper is structured as follows. Section 2 and Section 3 detail the construction of the forward guidance shock and the firm-level data, respectively. Section 4 presents the empirical analysis and reports the main findings. Section 5 outlines the structural model. Section 6 describes the model calibration, as well as the quantitative results. Lastly, Section 7 concludes.

2 Identifying Forward Guidance Shocks

In this section, I describe the construction of the quarterly forward guidance shocks which I employ throughout my main analysis in Section 4. The construction follows in two steps. First, I use 30-minute changes in interest rate futures around FOMC meetings to construct an unexpected tilting of the yield curve which leaves the federal funds rate unaffected. Second, I aggregate these high-frequency shocks to a quarterly measure. Here, I purge out the correlation between the high-frequency shocks and macroeconomic news released prior in the quarter to account for the potential omitted variable problem pointed out by Bauer and Swanson (2020).

2.1 Construction of High-frequency Shocks

To construct my high-frequency shocks, I employ intraday data on interest rate futures from *Thomson Reuters Tick History*. The sample period ranges from January 1996 and to December 2019. Following Kuttner (2001) and Gürkaynak, Sack, and Swanson (2005), I use Federal funds rate and Eurodollar futures to cover surprises in the yield curve within one and half years. For longer horizons, I employ Treasury futures following Gürkaynak, Kısacıkoglu, and Wright (2018). Appendix A provides an overview of the employed data and details the construction of the interest rate surprises.

²Lakdawala and Moreland (2019) use a joint measure of conventional and unconventional monetary policy in their analysis.

I obtain dates and times of the FOMC press releases from Bloomberg. I also cross-check them with information from the Federal Reserve website,³ as well as data from [Gürkaynak, Sack, and Swanson \(2005\)](#), [Nakamura and Steinsson \(2018a\)](#), and [Jarociński and Karadi \(2020\)](#). My sample contains both scheduled and unscheduled FOMC announcements. Following [Gürkaynak, Sack, and Swanson \(2005\)](#) and subsequent papers, I exclude the unscheduled 9/11 FOMC meeting on September 17, 2001. I also drop the unscheduled meeting on April 18, 2001 and the QE1 announcement on March 18, 2009 as both announcements are stark outliers in my dataset of 30-minute changes.⁴ Dropping the latter announcement also mitigates concerns that my forward guidance shock is driven by QE announcements since the QE1 announcement is normally found to have the largest effects ([Krishnamurthy and Vissing-Jorgensen, 2011](#)). I come back to this point below. Overall, this leaves me with 201 FOMC announcements ranging from January 1996 until December 2019.

For each FOMC meeting, I construct changes in the interest rate futures in a 30-minute window around each announcement starting 10 minutes prior and ending 20 minutes after the announcement time. Following [Gürkaynak, Sack, and Swanson \(2005\)](#), I then extract two factors using principal components and rotate both factors such that one factor does not load on the surprise in the Federal funds rate. I refer to this factor as the high-frequency forward guidance shock. Appendix A.5 provides details on the factor estimation. Figure 1 shows how the resulting forward guidance shocks maps to the 30-minute changes in the yield curve.

In Appendix A.6, I compare my high-frequency measures with other measures from previous papers. For the overlapping sample period, my measure is highly correlated with the “path factor” by [Gürkaynak, Sack, and Swanson \(2005\)](#). Importantly, it is also highly correlated with the forward guidance shock by [Swanson \(2021\)](#) who separately identifies a forward guidance and QE factor. This evidence is consistent with the idea that my shock captures predominantly forward guidance rather than QE.

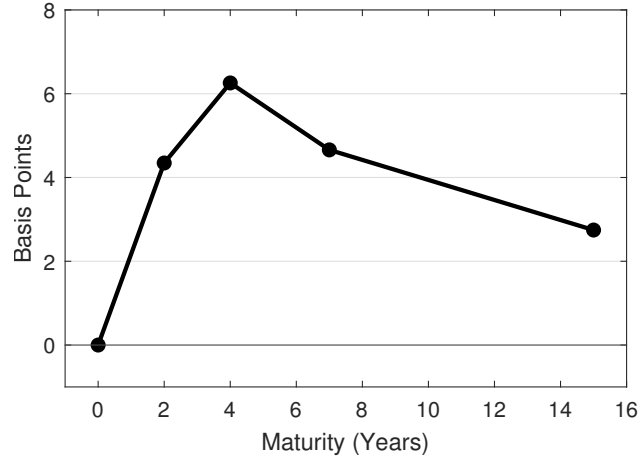
2.2 Quarterly Aggregation

Since my firm-level data is only available at the quarterly frequency, I need to construct a quarterly shock measure from the high-frequency one. In principle, one could simply add up the high-frequency shocks within a quarter if they are entirely unpredictable. However, as previous papers point out ([Miranda-Agrippino, 2016](#); [Cieslak, 2018](#); [Bauer and Swanson,](#)

³<https://www.federalreserve.gov/monetarypolicy/fomccalendars.htm>.

⁴[Campbell et al. \(2012\)](#) also drop the QE1 announcement for the same reason.

Figure 1: Loadings of Forward Guidance Shock on Yield Curve



Notes: This figure illustrates the mapping of the estimated high-frequency forward guidance shock to 30-minute changes in yields around FOMC announcements. The effects of a one standard deviation shock are shown. The yield changes are constructed from futures contracts.

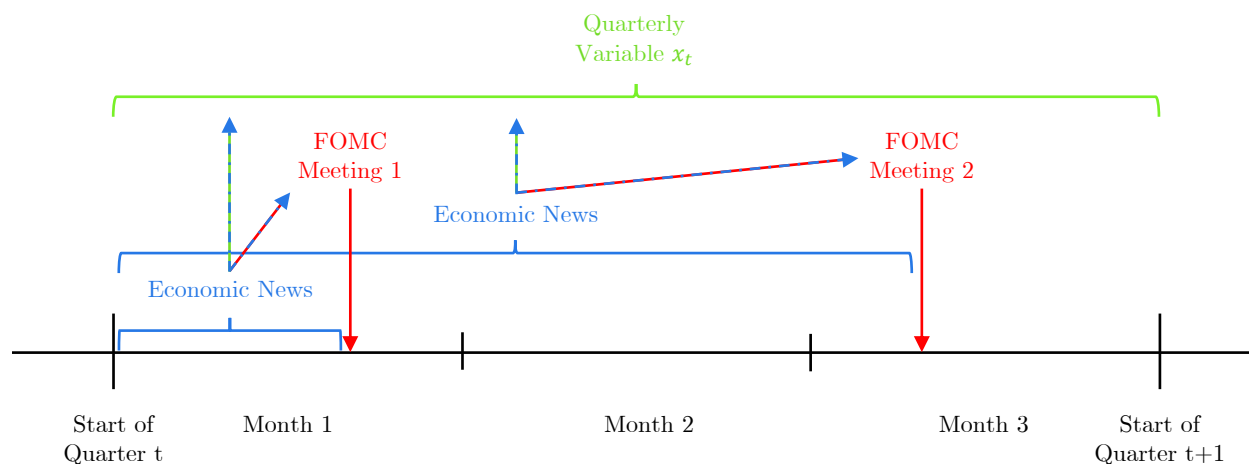
2020), these high-frequency shocks are (ex post) predictable by past macroeconomic information. Importantly, as shown by [Bauer and Swanson \(2020\)](#), the high-frequency shocks are correlated with prior macroeconomic news releases within the same quarter. This correlation potentially leads to an omitted variable bias once we study the response of macroeconomic variables to the shock, where controlling for past macroeconomic information in a quarterly setting cannot address this issue.

Figure 2 illustrates the problem. In a given quarter, public macroeconomic releases prior to the FOMC meeting can potentially lead to ex-post correlation with the high-frequency surprises for the reasons mentioned above. At the same time, these macroeconomic releases also (mechanically) affect the quarterly values of the aggregate variables. As a consequence, an omitted variable bias arises when we estimate the effect of the high-frequency surprises on aggregate variables since the induced correlation of the macroeconomic releases are attributed to the FOMC announcement.

To resolve this issue, I follow the recommendation by [Miranda-Agrippino and Ricco \(2021\)](#) and [Bauer and Swanson \(2020\)](#), and “project out” the (ex post) correlation by regressing the high-frequency shocks on macroeconomic news releases within the quarter and prior to the release (as illustrated by Figure 2). I then use the residual of this regression and sum it to a quarterly series. See Appendix B for details of the regression and construction. Figure 3 shows the resulting quarterly time series of the forward guidance shock.

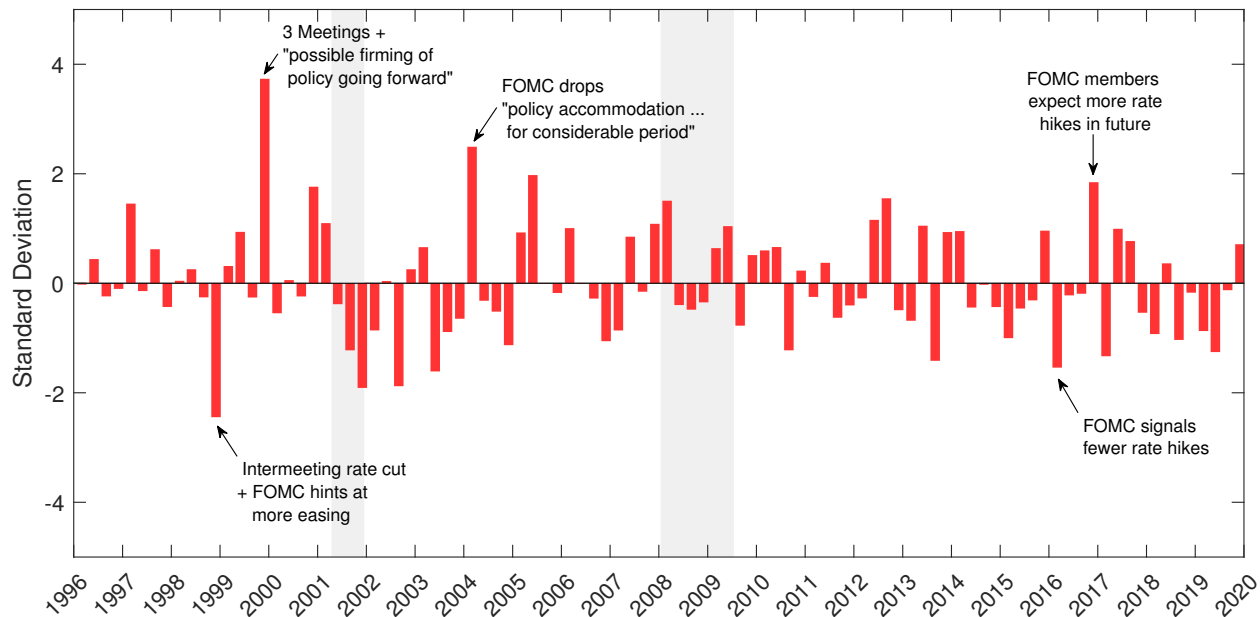
There are two things to be noted here: First, the exact cause of the predictability is

Figure 2: Omitted Variable Bias in Quarterly Aggregation of Forward Guidance Shock



Notes: This figure shows the regular timing of FOMC meetings within a quarter. It also illustrates the potential bias introduced by economic news releases prior to FOMC meetings which comes from a potential correlation between the high-frequency monetary policy surprises and these releases. See text for potential explanations for this correlation.

Figure 3: Time Series of Quarterly Forward Guidance Shock



Notes: This figure displays the time series of the quarterly forward guidance shock over the sample period. Gray bars indicate NBER recession periods.

not important for econometrically addressing the omitted variable problem.⁵ Second, this approach controls for publicly available information at the time of the monetary policy release. Hence, it does not, in principle, rule out effects through private information of the central bank.

Before moving on to the next section, I lastly want to add that throughout the paper, I interpret my shock series as direct observations of the structural forward guidance shock. Even if my shock measure were imperfectly correlated with the true structural forward guidance shock, i.e. includes measurement error, the local projections analyses throughout my paper would be still consistent. The only consequence would be the interpretation of the impulse response as a “relative” rather than an “absolute” one. In the former case, impulse responses are normalized in terms of the contemporaneous response of an endogenous variable, whereas the latter case, impulse responses are in terms of the response to a unit change of the true structural shock (Paul, 2020).

2.3 Impulse Responses of Macro Aggregates

Before turning to the firm-level data, I first investigate effect of a positive forward guidance shock on aggregate variables. To do so, I estimate the dynamic response of macroeconomic variables to my quarterly shock measure using the Jordà’s (2005) local projection method. Precisely, I estimate for each horizon h and for each variable y the following specification:

$$y_{t+h} - y_{t-1} = \alpha^{(h)} + \beta^{(h)} \varepsilon_t^{FG} + \sum_{j=1}^4 \delta_j^{(h)} X_{t-j} + \nu_{t+h}, \quad (1)$$

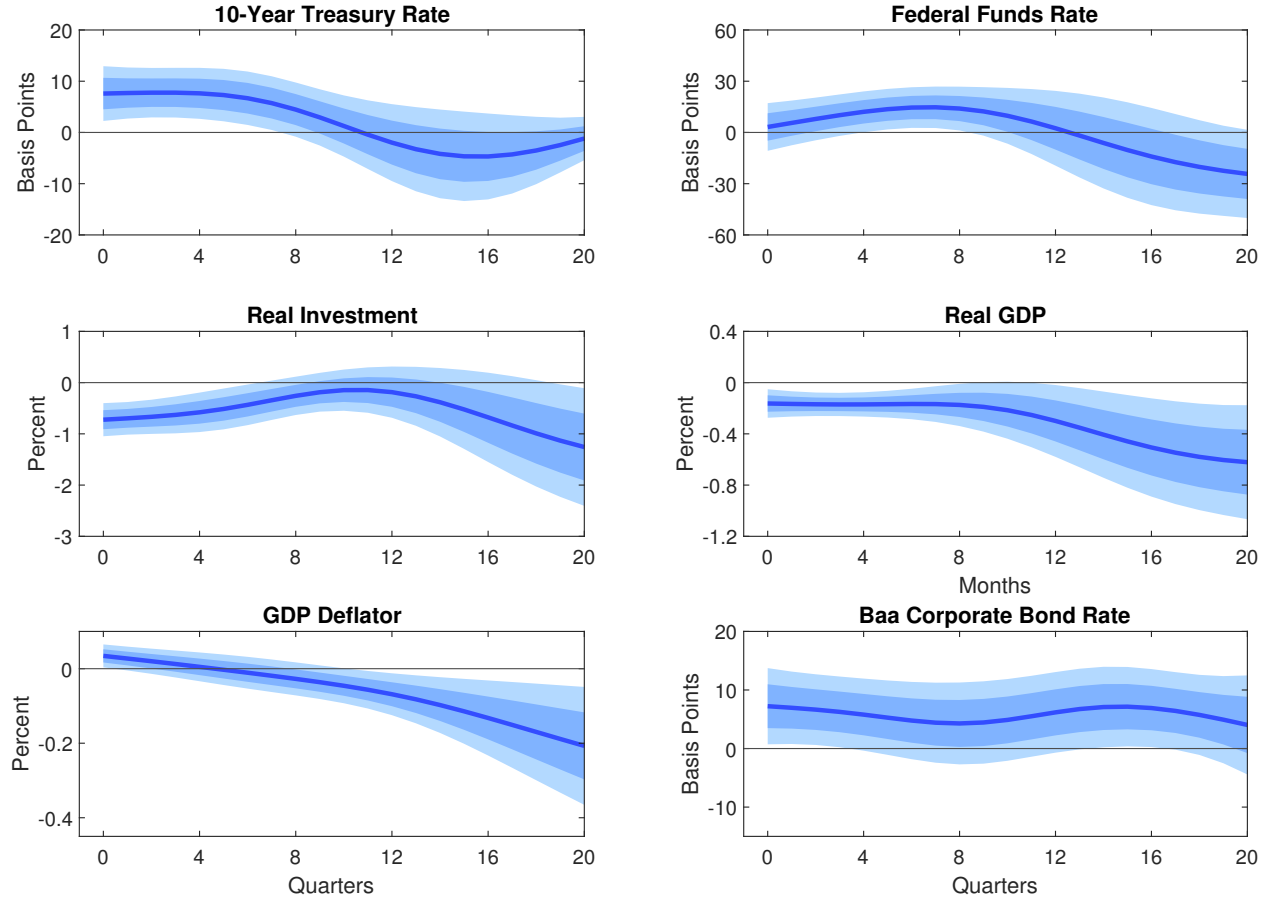
where $h = 0, 1, \dots, 20$, y_t is the outcome variable of interest, ε_t^{FG} is the forward guidance shock, and X_t is a vector of control variables. The vector of controls includes the real GDP ($100 \times \log$), real Investment ($100 \times \log$), and the GDP deflator ($100 \times \log$). Further, I include both the 10-year Treasury rate and the Federal Funds Rate to parsimoniously capture the yield curve following Eberly, Stock, and Wright (2019). Lastly, I include the Moody’s Baa corporate bond rate to measure financial conditions (Caldara and Herbst, 2019).

Local projections impose no restrictions on the dynamic effects across horizons. Combined with a small sample size as it is the case here, this can lead to highly irregular impulse responses (Ramey, 2016). To mitigate this concern, I estimate a smoothed version of (1) using the methodology by Barnichon and Brownlees (2019). In brief, the smoothing

⁵Cieslak (2018) argues in a favor of a violation of the full information rational expectations (FIRE) assumption, whereas Miranda-Agrippino (2016) favors a risk premium interpretation.

results from shrinking the local projection estimates towards a B-spline via a generalized ridge estimation.⁶ Figure 4 shows the smoothed estimates of specification (1). I construct heteroskedasticity- and autocorrelation-robust (HAR) confidence bands following the recommendation by Lazarus, Lewis, Stock, and Watson (2018). Precisely, I use Newey-West standard errors with a truncation parameter of 13, and fixed-b critical values from Kiefer and Vogelsang (2005).⁷

Figure 4: Response of Macroeconomic Aggregates to Forward Guidance Shock



Notes: This figure shows impulse responses of macroeconomic aggregates to a positive, one standard deviation forward guidance shock. The estimates are obtained from the local projection specification (1) following the methodology by Barnichon and Brownlees (2019). Dark and light bands show 68 percent and 90 percent confidence bands, respectively. The confidence bands are calculated following the recommendations by Lazarus et al. (2018).

⁶For details see Eilers and Marx (1996) or Barnichon and Brownlees (2019). Following Barnichon and Brownlees (2019), I use B-splines of order one. Further, I employ a relatively small shrinkage parameter of 20 leaving the impulse responses relative close the local projection estimates.

⁷Following Lazarus et al. (2018), the truncation parameter S is chosen such that $S = \lceil 1.3T^{1/2} \rceil$ with $T = 92$ being the (maximum) sample size.

Multiple things are noteworthy about the impulse responses in Figure 4. First, the positive forward guidance shock has the expected effect on the yield curve as the 10-year rate increases on impact, whereas the federal funds rate is only responding with a lag of multiple quarters. Further, the shock leads to a decline in real activity (real investment and GDP), a decline in prices (GDP Deflator), and a tightening of financial conditions (Baa Corporate Bond Rate). Overall, the average effect of a positive forward guidance shock is on average contractionary. My findings are roughly consistent with [Bundick and Smith \(2020\)](#) who also study the dynamic effects of forward guidance.

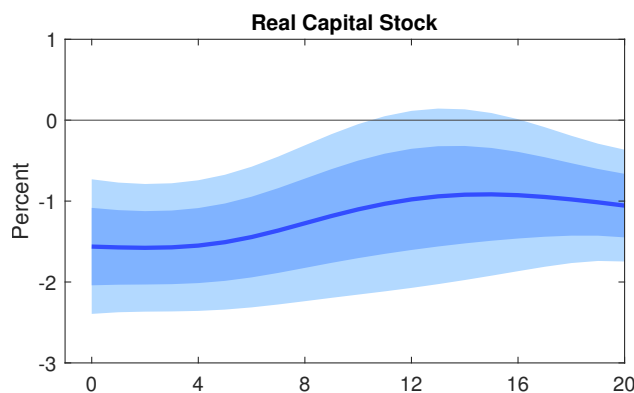
3 Firm-Level Data

For the construction of my firm-level panel, I employ the *CRSP/Compustat Merged* dataset which I further link to data by *OptionMetrics*. All data is obtained from the Wharton Research Data Services (WRDS). The sample period is from 1996Q1 to 2019Q4. The final panel contains 125,251 firm-quarter observations, i.e. on average 1305 firm observations per quarter. Appendix C provides details on the construction of the dataset. Before moving on to provide more details on the firm-level investment and uncertainty behavior in my sample, let me point out two key features of the constructed dataset which make it unique for my purposes: First, it has a relatively long time series dimension due to the early existence and the quarterly frequency. This allows me to study monetary policy in the first place.

Second, the constructed dataset does not only allow me track firms’ investment behavior and their business uncertainty, but also construct a wide range of other key characteristics emphasized. In total, I have for each firm-quarter observation the 13 following characteristics: age, dividends, EBITDA, leverage, liquidity, long-term debt dependence, price-to-cost margin, net receivables to sales, real capital stock, real sales growth, size, Tobin’s Q, uncertainty. Appendix Table C1 provides an overview of the variables. The vast number of firm characteristics allows me to isolate the role of uncertainty in the transmission. This is important since measures of uncertainty are usually correlated with measures of financial stress, both in the aggregate and at the micro level ([Stock and Watson, 2012](#); [Bloom, 2014](#); [Caldara, Fuentes-Albero, Gilchrist, and Zakrajšek, 2016](#)). To the extent that this is true on the firm-level, I need to allow the transmission to vary by other firm characteristics. The primary disadvantage of this dataset is that it only contains publicly listed U.S. firms and hence only includes a subset of the firm distribution.

Investment Throughout this analysis, I use the change in firm’s i real capital stock to measure the real investment response. As in [Jeenas \(2018\)](#), I prefer this specification compared to directly estimating the investment rate response since investment is notoriously volatile, even for the U.S. publicly traded firms ([Clementi and Palazzo, 2019](#)). This volatility makes it potentially difficult to precisely estimate systematic difference in responses in the cross-section, and in particular over longer horizons. Following a large literature, the capital stock is measured as the book value of the tangible capital stock and constructed using the perpetual inventory method. Following the timing convention in the data, $k_{i,t}$ denotes firm i ’s book value of the firm’s tangible capital stock at the end of quarter t . The series is deflated using the implied price index of gross value added in the U.S. nonfarm business sector (BEA-NIPA Table 1.3.4 Line 3). I provide further details on the capital stock construction in Appendix C.1. Figure 5 shows the impulse response of the aggregate capital stock series to a forward guidance shock, where I re-estimated specification (1) substituting the capital stock for the real investment variable. The strong impact response might be a bit surprising. However, [Clementi and Palazzo \(2019\)](#) find no evidence of investment irreversibility for U.S. publicly traded firms consistent with a stronger effect on impact.

Figure 5: Response of Compustat Capital Stock to Forward Guidance Shock

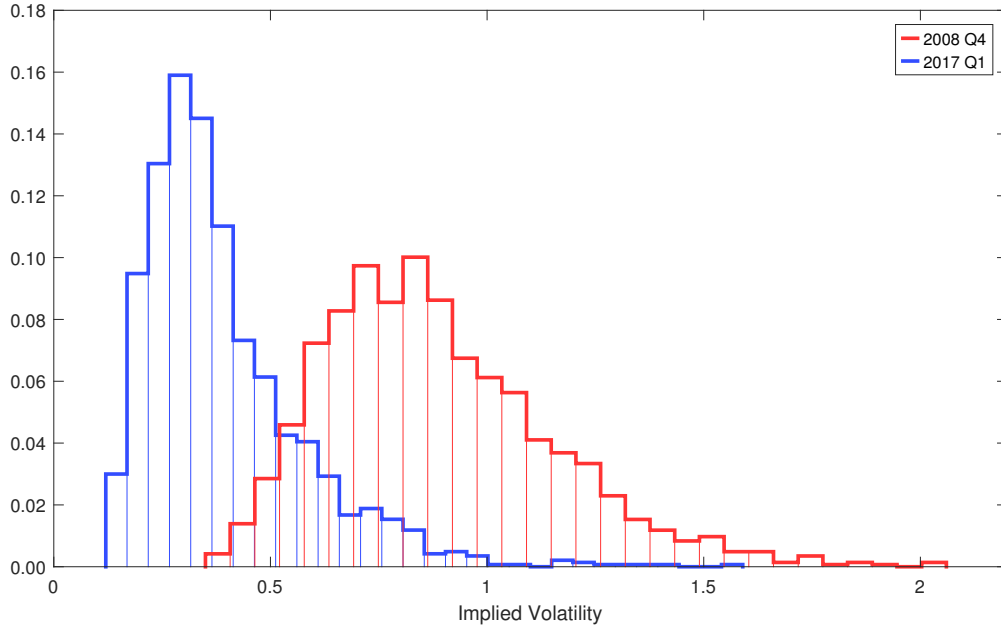


Notes: This figure shows the impulse response of the aggregated real capital stock variable constructed from Compustat to a one standard deviation forward guidance shock. The estimates are obtained from the local projection specification (1) following the methodology by [Barnichon and Brownlees \(2019\)](#). Dark and light bands show 68 percent and 90 percent confidence bands, respectively. The confidence bands are calculated following the recommendations by [Lazarus et al. \(2018\)](#).

Uncertainty To construct my baseline measure of firm-level uncertainty, I use firm’s i daily 30-day option-implied volatility of its stock price and calculate the quarterly average. The

data on implied volatility comes from OptionMetrics.⁸ Figure 6 shows the firm distribution of implied volatility for two quarters in my sample, one during an expansion period (2017Q1) and one during a recession period (2008Q4). The figure illustrates two points: First, the distributions are consistent with previous research documenting that the average level of uncertainty as well as the dispersion are countercyclical. Second, the figure illustrates that there is a substantial amount of variation in the cross-section which I exploit in the main empirical analysis in the next section.

Figure 6: Distribution of Firm-level Uncertainty



Notes: This figure shows the firm distribution of uncertainty (30-day option-implied volatility) in my sample in 2008 Q4 (red) and 2017 Q1 (blue). Note that implied volatility is measured in percentage p.a., e.g. 0.5 refers to a 50% expected yearly volatility.

Throughout the rest of the paper, I denote firm i 's uncertainty in quarter t by $uc_{i,t}$ which is the logarithm of the quarterly 30-day implied volatility ($100 \times \log$). Taking the logarithm helps not only to ease interpretation but also accounts for potential problems in the regression analyses below arising from the skewedness of the distribution (as shown in Figure 6). Appendix C.1 provides all details on the construction of baseline uncertainty measure, as well alternative measures which I later employ in my empirical robustness.

⁸For example, Alfaro, Bloom, and Lin (2018) also construct a firm-level uncertainty measure based on data from OptionMetrics.

4 Firm-Level Uncertainty and Transmission of Forward Guidance

4.1 Motivation

Before I move to the estimation of the firms' differential capital response with respect to uncertainty, I briefly motivate the empirical analysis in this subsection. The key motivation for focusing on heterogeneity in firms' responses to forward guidance is to inform an underlying mechanism. As discussed in [Nakamura and Steinsson \(2018b\)](#), estimated differential effects of an identified structural shock can distinguish between different classes of models. However, with respect to uncertainty one must be cautious about the right specification to estimate the response heterogeneity. In essence, one can treat uncertainty as risk or ambiguity (Knightian uncertainty). In the former case, probabilities are known, and an agent's subjective probability can be represented by a unique distribution. In the latter case, probabilities are unknown, and an agent's subjective probability is represented as an interval of probabilities.

Most of the literature in macroeconomics treats uncertainty as risk. For example, the option value ("wait-and-see") channel of uncertainty ([Bernanke, 1983](#); [McDonald and Siegel, 1986](#); [Bloom, 2009](#)), or the risk premium channel ([Christiano, Motto, and Rostagno, 2014](#); [Gilchrist, Sim, and Zakrajšek, 2014](#)) are mechanisms in line with this interpretation. The common prediction is that uncertainty should affect the agent's responsiveness to a shock. Hence, the differential effect γ needs to be estimated from a specification of the form of

$$k_i = \beta \varepsilon^{FG} + \gamma (\varepsilon^{FG} \times uc_i),$$

where k_i is firm's i capital, ε^{FG} is the forward guidance shock, uc_i is firm's i uncertainty, β and γ are the main and interaction effect. As shown in the previous section, the forward guidance shock has on average a contractionary effect, i.e. $\beta < 0$. Hence, $\gamma > 0$ is consistent with an attenuating channel of uncertainty, and $\gamma < 0$ is consistent with an amplifying channel of uncertainty.

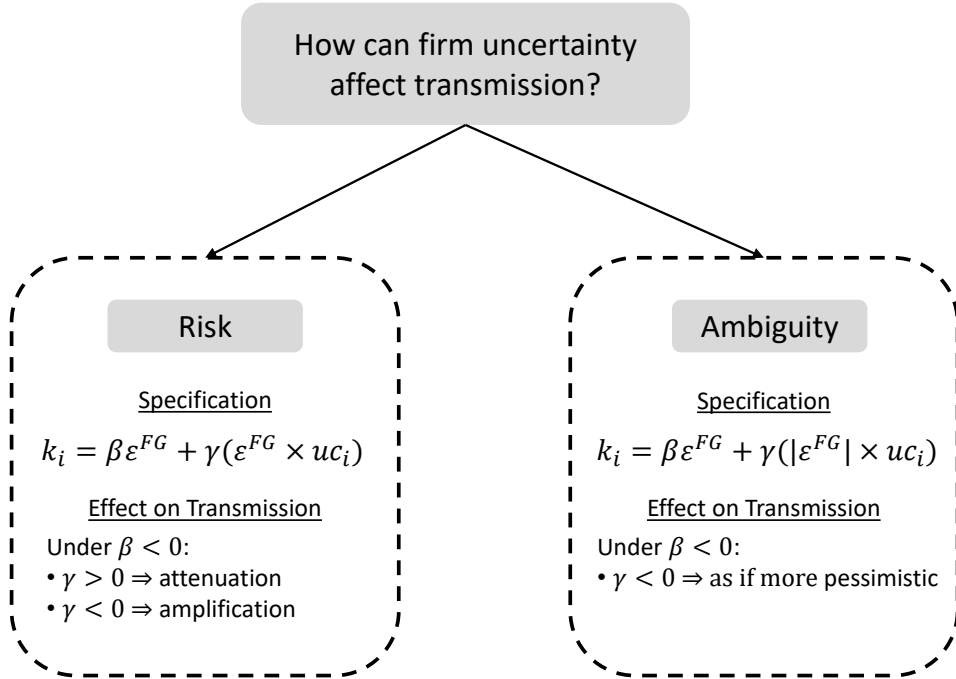
Smaller part of the papers in macroeconomics treats uncertainty as ambiguity. Following the [Ellsberg \(1961\)](#) paradox, the literature assumes agents to be ambiguity aversion which is either modeled as multiple-priors preferences ([Gilboa and Schmeidler, 1989](#); [Epstein and Schneider, 2003](#)) or robust control theory [Hansen and Sargent \(2001, 2010\)](#). The common prediction is that uncertainty should lead agents to respond to a shock "as if" they are more pessimistic. Hence, the differential effect γ needs to be estimated from a specification of the

form of

$$k_i = \beta \varepsilon^{FG} + \gamma (|\varepsilon^{FG}| \times uc_i),$$

where $|\varepsilon^{FG}|$ is now the absolute value of the forward guidance shock. Here, $\gamma < 0$ would be consistent with the pessimism effect under the natural assumption that contractionary forward guidance is seen as bad news. Figure 7 summarizes the discussion in this subsection graphically. With that motivation at hand, I can now turn to the empirical specification I estimate in the next subsection.

Figure 7: Firm-Level Uncertainty and Transmission of Forward Guidance



Notes: This figure illustrates the appropriate empirical specification when uncertainty captures either risk or ambiguity (Knightian uncertainty). See the main text for more details. Notation: k_i is firm's i capital, ε^{FG} is the forward guidance shock, uc_i is firm's i uncertainty, β and γ are the main and interaction effect.

4.2 Estimation of Response Heterogeneity

I now estimate the heterogeneity in the firms' responsiveness to forward guidance with respect to their uncertainty. To do so, I employ a panel version of the local projection approach by Jordà (2005) which allows me to assess the dynamic heterogeneity following a forward guidance shock. Following the discussion, I estimate two versions, one with the regular shock and one with its absolute value. Precisely, I run the following specification

$$\begin{aligned} \Delta_h \log(k_{i,t+h}) = & \alpha_i^{(h)} + \alpha_{n,t}^{(h)} + \alpha_{i,fq}^{(h)} + \gamma_\zeta^{(h)} (\zeta_t \times uc_{i,t-1}) + \Gamma_\zeta^{(h)} (\zeta_t \times Z_{i,t-1}) \\ & + \theta^{(h)} uc_{i,t-1} + \Phi^{(h)} Z_{i,t-1} + \nu_{i,t+h}, \quad \zeta_t \in \{\varepsilon_t^{FG}, |\varepsilon_t^{FG}|\}, \end{aligned} \quad (2)$$

where $h = 0, 1, \dots, 20$ indexes the quarters after the shock. The dependent variable is the h -period ahead cumulative growth of the capital stock, $\Delta_h \log(k_{i,t+h}) \equiv \log(k_{i,t+h}) - \log(k_{i,t-1})$. ε_t^{FG} denotes the quarterly forward guidance shock, and $uc_{i,t}$ denotes the uncertainty measure which is normalized such that it is in standard deviations relative to the mean. Further, $Z_{i,t}$ denotes a vector of firm controls which includes: age, dividends, EBITDA, leverage, liquidity, long-term debt dependence, price-to-cost margin, net receivables to sales, real capital stock, real sales growth, size, Tobin's Q. I also include multiple fixed effects: α_i denotes the firm i 's fixed effect, $\alpha_{i,fq}$ a firm's i fiscal quarter fixed effect, and $a_{n,t}$ the industry-time fixed effect of industry n .⁹ The coefficient of interest is $\gamma_\zeta^{(h)}$ which is on the interaction term of forward guidance shock and uncertainty.

Two aspects of specification (2) are worthwhile pointing out which both mitigate concerns that $\gamma_\zeta^{(h)}$ reflects rather a correlation than a causal channel: First, the specification does not only control for various firm characteristics, but also allows the effect of forward guidance to vary by each variable which is captured by $\Gamma_\zeta^{(h)}$. This is important to mitigate concerns that the findings are driven by other firm characteristics which are correlated with uncertainty.

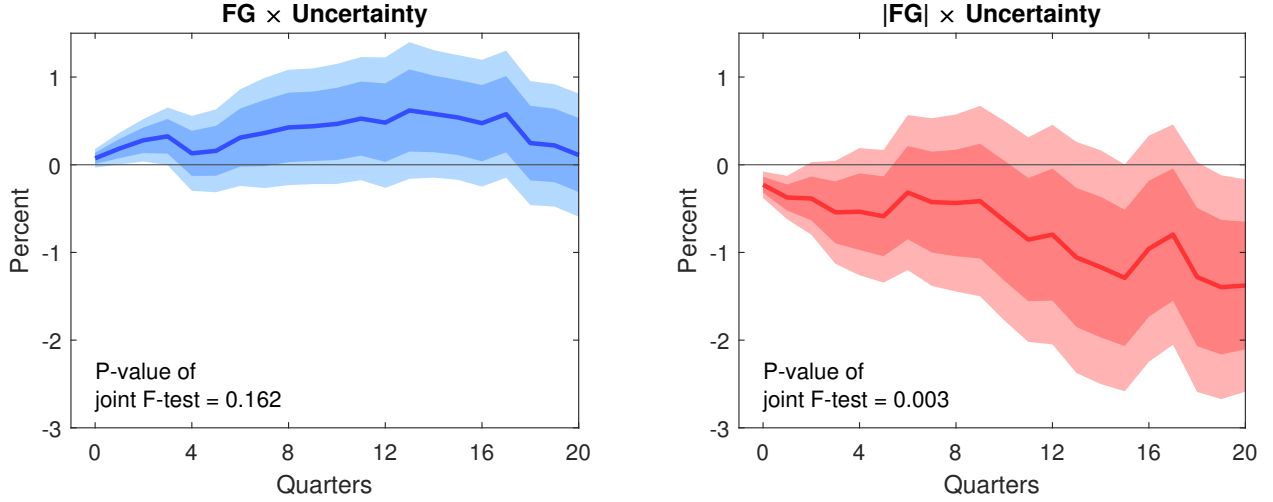
Second, I introduce a industry-time fixed effect $a_{n,t}$ which picks up aggregate effects, and means that the estimates are based on within-industry variation in the cross-section. Hence, $a_{n,t}$ makes sure that estimates of $\gamma_\zeta^{(h)}$ are not driven by industry differences nor aggregate influences such as the time-varying firm composition of the unbalanced panel.¹⁰

Figure 8 shows the results from estimating both versions of specification (2), i.e. $\gamma_{FG}^{(h)}$ and $\gamma_{|FG|}^{(h)}$ for $h = 0, 1, \dots, 20$ quarters. Both panels show the heterogeneity with respect to a one standard deviation forward guidance shock. Further, I normalize $uc_{i,t}$ such that $\gamma_\zeta^{(h)}$ corresponds to a one standard deviation increase in firm-level uncertainty relative to the mean. In the left panel, the point estimates are positive meaning that more uncertain firms are less responsive. However, the response is relatively small, and, except for quarter one and two, insignificant at the 10% percent level at all horizons. Indeed, the joint F-test that there is no effect for the first 20 quarters cannot be rejected at the 10%.

⁹I consider the following five industries in my sample: (1) agriculture, mining, and construction; (2) manufacturing; (3) transportation and communications; (4) trade; (5) services. Following prior literature, I exclude firms in the finance, insurance, and real estate, as well in utilities. Appendix C.1 provides details on the industry classification.

¹⁰For example, Andrade, Coibion, Gautier, and Gorodnichenko (2021) provide recent evidence that the industry plays crucial role for firm's decision making.

Figure 8: Heterogeneity in Responses of Capital Conditional on Uncertainty



Notes: This figures displays the dynamics of the interaction coefficients between firm-level uncertainty and the forward guidance shock from estimating both versions of specification (2). The left panel shows estimates for the forward guidance shock, i.e. $\gamma_{FG}^{(h)}$, whereas the right panel shows estimates for the absolute value of the forward guidance shock, i.e. $\gamma_{|FG|}^{(h)}$. The displayed estimates corresponds to a one standard deviation forward guidance shock and a one standard deviation increase in firm-level uncertainty relative to the mean. Dark and light bands show 68 percent and 90 percent confidence bands, respectively. Standard errors are clustered at quarter and firm level.

The right panel shows that the absolute value plays a more prominent role. More uncertain firms reduce their capital stock more regardless of the direction of the forward guidance shock, i.e. they act as if they are more pessimistic. The point estimates are statistically significant at multiple horizons and the joint F-test shows significance at the 1-percent level. Further, the magnitude of impulse response is economically sizable with a peak effect of 1.4 percent 20 quarters after the shock. To put this into perspective, moving from the 10th to the 90th percentile in the sample distribution corresponds to a 3.65 percent change in the capital stock 20 quarters after the shock. Considering that the 20-quarter median growth rate for capital stock in the sample is around 16.66 percent, the heterogeneity is not only statistically significant, but also is economically sizable.

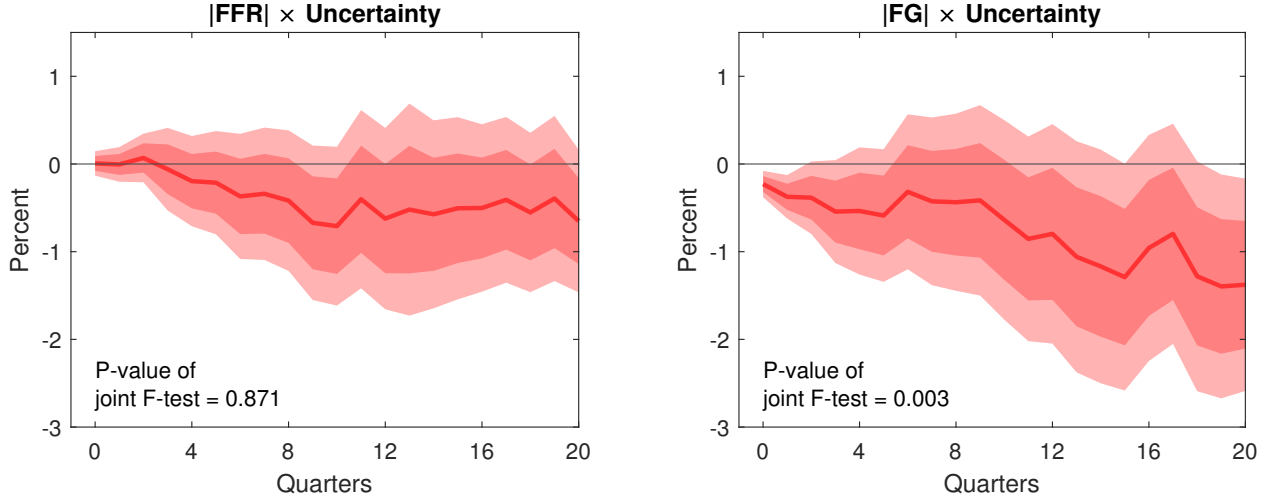
One question which arises from Figure 8 is how these two panels can be reconciled with one another. To answer that, I also estimate (2) separating positive and negative shocks. As Appendix Figure D1 shows, the heterogeneity for positive and negative shocks is in line with Figure D1, i.e. regardless of the direction of the shock, more uncertain firms adjust their capital more downward. The figure also shows that the response is stronger for expansionary shocks which explains the findings in Figure 8. That being said, the difference in magnitudes

is not statistically significant (as shown by the F-test below Figure D1).

In Appendix D, I also show that the main finding in Figure 8 — more uncertain firms adjust their capital as if they are more pessimistic following a forward guidance shock — is robust to a range of additional exercises. Specifically, I show in Figure D2 that it is robust to two different measures of uncertainty (realized volatility and 182-day implied volatility), as well as ending the sample in 2007 prior to the Great Recession and ZLB. Figure D5 displays that the result is also robust to estimating a version with the main effect and no industry-time fixed effect.

Lastly, I investigate if this heterogeneity exists following a shock to the federal funds rate as well. To do so, I re-estimate the absolute value version of specification (2) with the federal funds rate shock, which is constructed based on the other principal component of the factor estimation described in Section 2. See Appendix A for mre details. Figure 9 illustrates the results of this estimation and compares it with the one for the forward guidance shock. As the figure shows, the point estimates for the federal funds rate are statistically insignificant at all horizons and as is the joint F-test. As shown in Appendix Figure D3, this finding also holds if I estimate both specifications for the pre-ZLB period.

Figure 9: Comparison of Forward Guidance Shock with Federal Funds Rate Shock



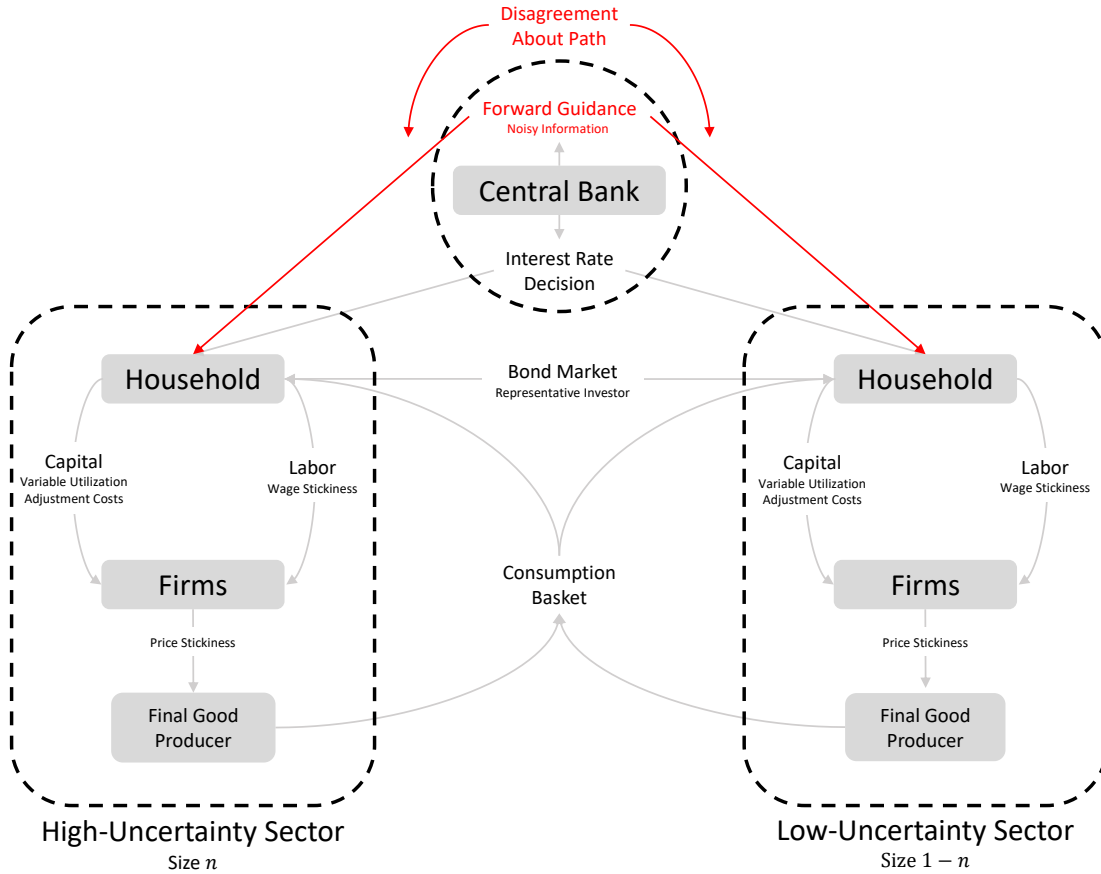
Notes: This figure displays the dynamics of the interaction coefficients between firm-level uncertainty and the federal funds rate and forward guidance shock from estimating the absolute value version of specification (2). The right panel shows estimates for the absolute value of the forward guidance shock, i.e. $\gamma_{|FG|}^{(h)}$, whereas the left panel shows estimates for the absolute value of the federal funds rate shock, i.e. $\gamma_{|FFR|}^{(h)}$. The displayed estimates corresponds to a one standard deviation shock and a one standard deviation increase in firm-level uncertainty relative to the mean. Dark and light bands show 68 percent and 90 percent confidence bands, respectively. Standard errors are clustered at quarter and firm level.

5 Two-Sector New Keynesian Model

5.1 Overview

This section outlines the key ingredients of the structural model which I use to interpret the empirical findings. The model is a medium-scale New Keynesian DSGE model with two sectors, the high- and low-uncertainty sector (henceforth H- and L-sector) of size n and $1 - n$, respectively. The H-sector is populated with a continuum of firms which are owned by the high-uncertainty household (henceforth H-household) who rents out the capital and provides the labor to the firms in the sector. The same applies to the L-sector. Both households consume a common consumption basket made out of both sector's consumption good. Figure 10 provides a graphical overview of the model.

Figure 10: Basic Skeleton of Two-Sector New Keynesian Model



Notes: This figure illustrates the basic skeleton of the structural two-sector New Keynesian Model. It shows the model's agents, interactions, and frictions.

The model includes various frictions which are standard in the literature such wage

and price stickiness, habit formation, investment adjustment costs, and variable capital utilization. These frictions are identical across sectors, and included to have more realistic model dynamics. The two sectors differ in how they interpret the forward guidance policies by the central bank, and are meant to be a parsimonious way to capture the heterogeneity observed in the empirical analysis. Overall, the skeleton of the model is borrowed from a bond economy version of a two-country currency model (e.g., [Nakamura and Steinsson, 2014](#); [Bhattarai, Lee, and Park, 2015](#)).

Below, I describe the forward guidance shock in Subsection 5.2, and the belief heterogeneity across sectors in Subsection 5.3. After that, I talk about the bond market in Subsection 5.4. Lastly, I flesh out the structure of the rest of the model in Subsection 5.5. Appendix E provides all details on the model environment and the equilibrium conditions.

5.2 Forward Guidance

The central bank sets the nominal interest R_t according to the following Taylor rule

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi} \right)^{\theta_\Pi} \left(\frac{Y_t}{Y} \right)^{\theta_Y} e^{\theta_t}, \quad (3)$$

where R_t is the nominal interest rate, and Π_t is aggregate inflation, and Y_t is aggregate output. R , Π and Y are the corresponding the steady states. Deviations from the rule are denoted by θ_t .¹¹ Log-linearizing (3) around the steady state yields

$$\hat{R}_t = \theta_\Pi \hat{\Pi}_t + \theta_Y \hat{Y}_t + \theta_t. \quad (4)$$

The policy deviation θ_t follows a news structure as in [Laséen and Svensson \(2011\)](#)

$$\theta_t = \sum_{i=0}^{15} \varepsilon_{R,t-i}^i, \quad (5)$$

with $\varepsilon_{R,t}^i \sim N(0, \sigma_R^2)$. The specification implies that the central bank can communicate deviations of up to four years, i.e. 15 quarters. As I show below, this length is needed such that I can mimic the forward guidance shock in the data.

In the model, forward guidance is a central bank announcement of a specific path of future policy deviations, where as in [Campbell, Ferroni, Fisher, and Melosi \(2019\)](#), the central bank

¹¹Note that the paper's focus is the transmission of monetary policy. Hence, I can specify the Taylor rule in terms of output rather than the output gap (output - natural output) since the natural level of output is unaffected.

cannot perfectly communicate this path. In particular, the forward guidance shock ε_t^{FG} is defined as a set of signals:

$$\varepsilon_t^{FG} = \begin{bmatrix} s_t^0 & s_t^1 & \dots & s_t^{15} \end{bmatrix},$$

where $\exists j$ such that $s_t^j \neq 0$ and $j \geq 1$. While future deviations are unobserved, i.e. for $j \geq 1$

$$s_t^j = \varepsilon_{R,t}^j + \eta_t,$$

with $\eta_t \sim N(0, \sigma_\eta^2)$, the current policy rate (and policy deviation) is always perfectly observed, i.e.

$$s_t^0 = \varepsilon_{R,t}^0.$$

Hence, conventional monetary policy is simply $\varepsilon_{R,t}^0$ in this model, and behaves in a standard manner.

5.3 Heterogeneous Beliefs

The L-household knows the informativeness of each forward guidance signal s_t^j , i.e. the true parameter σ_η . In contrast, the H-household knows only that it lies in some interval (admissible set), i.e. $\sigma_\eta \in [\underline{\sigma}, \bar{\sigma}]$. Further, the H-household cannot learn the true parameter from observing the L-household. Further, the H-household is ambiguity averse leading her to act according to her worst-case belief within the admissible set.

The H-household has a max-min-preference as axiomatized in [Gilboa and Schmeidler \(1989\)](#) and [Epstein and Schneider \(2003\)](#). Her optimization problem can be characterized as follows

$$\begin{aligned} \max_{\{\dots\}} \min_{\sigma_\eta \in [\underline{\sigma}, \bar{\sigma}]} E_0^H \sum_{t=0}^{\infty} \beta^t & \left[\frac{(C_t^h - \xi C_{t-1}^h)^{1-\sigma}}{1-\sigma} - \varrho \frac{(L_{Ht})^{1+\chi}}{1+\chi} \right] \\ \text{s.t. } P_t C_t^h + P_{Ht} I_{Ht} + \frac{1}{R_t} B_{Ht} & \leq MRS_{Ht} L_{Ht} + R_{k,Ht} u_{Ht} \bar{K}_{Ht-1} - P_{Ht} a(u_{Ht}) \bar{K}_{Ht-1} \\ & + DIV_{Ht}^p + DIV_{Ht}^w + B_{Ht-1} \\ \bar{K}_{Ht} & \leq (1 - 0.5\kappa (I_{Ht}/I_{Ht-1} - 1)^2) I_{Ht} + (1 - \delta) \bar{K}_{Ht-1}, \end{aligned} \tag{6}$$

where $E_t^H[\cdot]$ denotes the H-household's expectation conditional on information at time t , L_{Ht} is the labor supplied to the H-sector, P_{Ht} is the nominal price of the final good in the H-sector, I_{Ht} is the investment in the H-sector, B_{Ht} is the bond holding of the H-household,

MRS_{Ht} is the nominal remuneration for the supply of labor, $R_{k,Ht}$ is the nominal rental rate on capital services in the H-sector, u_{Ht} is the utilization rate which is chosen by the household, \bar{K}_{Ht} is the physical capital stock, $a(u_{Ht})$ the utilization cost, and DIV_{Ht}^p and DIV_{Lt}^w are nominal profits from the firms and unions in the H-sector, respectively.

Further, C_t^h is the H-household's consumption of the composite good defined as

$$C_t^h = \left[n^{\frac{1}{\eta}} (C_{Ht})^{\frac{\eta-1}{\eta}} + (1-n)^{\frac{1}{\eta}} (C_{Lt})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

with the corresponding aggregate price index

$$P_t = \left[n (P_t)^{1-\eta} + (1-n) (P_{Lt})^{1-\eta} \right]^{\frac{1}{\eta-1}}.$$

Both households consume the same composite good, i.e. there is no “home bias” in consumption. Eventually, all prices are eventually expressed with P_t being the numeraire.

Importantly, the ambiguity is only with respect to σ_η , and hence only affects the household's updating rule following the forward guidance signals. Following [Ilut and Schneider \(2014\)](#), I guess the worst-case belief which solves the minimization over the admissible set. Under the assumption that the H-household has a distaste for contractionary deviations compared to expansionary ones, a natural guess is that she behaves “as if” the forward guidance signal is maximal informative for a contractionary signal and minimal informative for an expansionary one.¹² Hence, following a forward guidance signal S_t , the H-household updates her expectations as follows

$$E_t^H[\theta_t] = \theta_t \quad \text{and} \quad E_t^H[\theta_{t+j}] = \psi^*(s_t^j) s_t^j + E_{t-1}^H[\theta_{t+j}],$$

where the Kalman gain $\psi^*(s_t^j)$ depends on the signal, and is given by

$$\psi^*(s_t^j) = \bar{\psi} 1(s_t^j \geq 0) + \underline{\psi} 1(s_t^j < 0).$$

Here, $\bar{\psi} = \sigma_\theta^2 / (\sigma_\theta^2 + \underline{\sigma}^2)$ and $\underline{\psi} = \sigma_\theta^2 / (\sigma_\theta^2 + \bar{\sigma}^2)$ describe the upper and lower bounds of the Kalman gain based on the admissible set.

In contrast to the H-household, the L-household has standard preferences, and solves the

¹²Although the distaste for contractionary deviations is generally hard to prove, one can simply add a term $g(\theta_t^0)$ to the utility which ensures that $\partial \sum_{t=0}^{\infty} \beta^t u(C_t^h, L_{Ht}) / \partial \theta_t^0 < 0$.

following optimization problem:

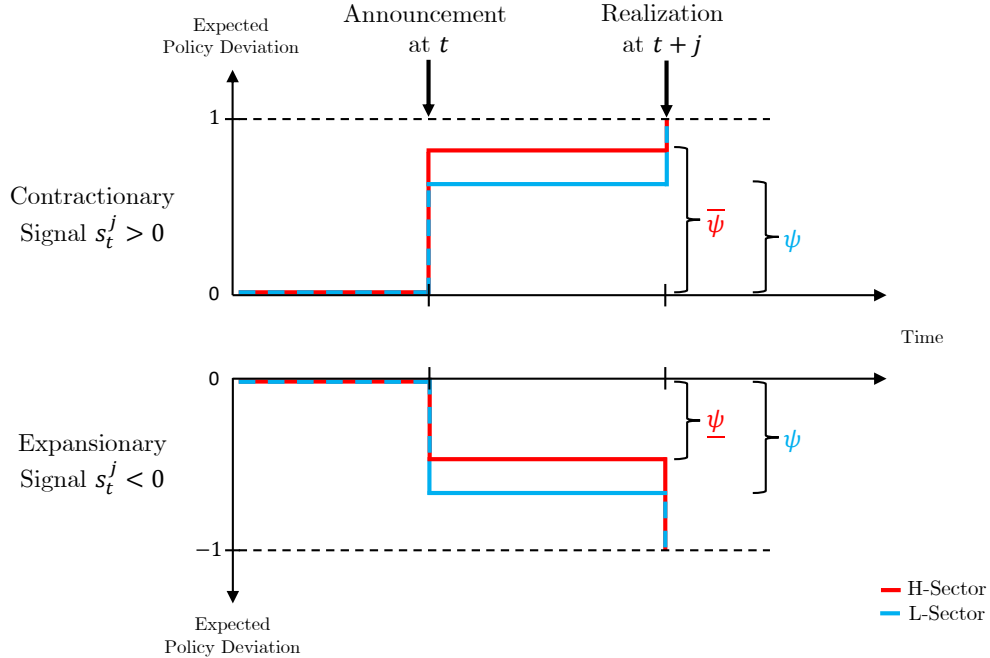
$$\begin{aligned}
\max_{\{...\}} E_0^L \sum_{t=0}^{\infty} \beta^t & \left[\frac{(C_t^l - \xi C_{t-1}^l)^{1-\sigma}}{1-\sigma} - \varrho \frac{(L_{Lt})^{1+\chi}}{1+\chi} \right] \\
\text{s.t. } P_t C_t^l + P_{Lt} I_{Lt} + \frac{1}{R_t} B_{Lt} & \leq MRS_{Lt} L_{Lt} + R_{k,Lt} u_{Lt} \bar{K}_{Lt-1} - P_{Lt} a(u_{Lt}) \bar{K}_{Lt-1} \\
& + DIV_{Lt}^p + DIV_{Lt}^w + B_{Lt-1} \\
\bar{K}_{Lt} & \leq (1 - 0.5\kappa (I_{Lt}/I_{Lt-1} - 1)^2) I_{Lt} + (1 - \delta) \bar{K}_{Lt-1},
\end{aligned} \tag{7}$$

where $E_t^L[\cdot]$ denotes the L-household's expectations conditional on information at time t , and the other variables are similarly defined as for the H-household. Following a forward guidance signal S_t , the L-household updates her expectations as follows

$$E_t^L[\theta_t] = \theta_t \quad \text{and} \quad E_t^L[\theta_{t+j}] = \psi s_t^j + E_{t-1}^L[\theta_{t+j}]$$

with the Kalman gain $\psi = \sigma_\theta^2 / (\sigma_\theta^2 + \sigma_\eta^2)$. Figure 11 illustrates the heterogeneity in the signal extraction across sectors.

Figure 11: Incorporation of Forward Guidance Signal across Sectors



Notes: This figure shows the information incorporation of a one unit forward guidance signal s_t^j in the model. The red and blue line show the expectation of policy deviation θ_{t+j} for the high- and low-uncertainty sector, respectively. The Kalman gains are denoted by ψ , $\underline{\psi}$, and $\bar{\psi}$.

To keep the heterogeneity in expectations tractable, I assume that other than the parameter σ_η and the expected path of interest rates, everything else is common knowledge, i.e. the households “agree to disagree”. Precisely, I assume that

$$E_t[X_{t+j}] = E_t^L[X_{t+j}] = E_t^H[X_{t+j}] \quad \text{for } X_{t+j} \neq \{R_{t+j}, \theta_{t+j}\}, \quad (8)$$

where $E_t[\cdot]$ denotes the common expectations operator. Further, both households disagree on expected policy deviations

$$E_t[\theta_{t+j}] = E_t^L[\theta_{t+j}] \quad \text{and} \quad E_t[\theta_{t+j}^*] = E_t^H[\theta_{t+j}], \quad (9)$$

and thus on the expected path of interest rates.

$$E_t[R_{t+j}] = E_t^L[R_{t+j}] \quad \text{and} \quad E_t[R_{t+j}^*] = E_t^H[R_{t+j}], \quad (10)$$

where θ_{t+j}^* and R_{t+j}^* denote the policy deviation and interest rate under H-household’s worst-case updating rule. From the linearized Taylor rule (4), one sees that the disagreement in the interest rate comes entirely from the disagreement in the policy deviations, i.e.

$$\begin{aligned} E_t[\hat{R}_{t+j}^*] &= \theta_\Pi E_t[\hat{\Pi}_{t+j}] + \theta_Y E_t[\hat{Y}_{t+j}] + E_t[\theta_{t+j}^*] \\ E_t[\hat{R}_{t+j}^*] - E_t[\hat{R}_{t+j}] &= E_t[\theta_{t+j}^*] - E_t[\theta_{t+j}]. \end{aligned}$$

Note that the current interest rate is always perfectly observable, i.e. $R_t^* = R_t$.

Using the common expectation operator as defined in (8)–(10), we can write both households problems (6) and (7) as

$$\begin{aligned} \max_{\{C_t^h, B_{Ht}, L_{Ht}, \bar{K}_{Ht}, I_{Ht}, u_{Ht}\}} & E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^h - \xi C_{t-1}^h)^{1-\sigma}}{1-\sigma} - \varrho \frac{(L_{Ht})^{1+\chi}}{1+\chi} \right] \\ \text{s.t.} \quad & P_t C_t^h + P_{Ht} I_{Ht} + \frac{1}{R_t^*} B_{Ht} \leq MRS_{Ht} L_{Ht} + R_{k,Ht} u_{Ht} \bar{K}_{Ht-1} - P_{Ht} a(u_{Ht}) \bar{K}_{Ht-1} \\ & + DIV_{Ht}^p + DIV_{Ht}^w + B_{t-1} \\ & \bar{K}_{Ht} \leq (1 - 0.5\kappa (I_{Ht}/I_{Ht-1} - 1)^2) I_{Ht} + (1 - \delta) \bar{K}_{Ht-1}, \end{aligned} \quad (11)$$

and

$$\begin{aligned}
& \max_{\{\dots\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^l - \xi C_{t-1}^l)^{1-\sigma}}{1-\sigma} - \varrho \frac{(L_{Lt})^{1+\chi}}{1+\chi} \right] \\
& \text{s.t. } P_t C_t^l + P_{Lt} I_{Lt} + \frac{1}{R_t} B_{Lt} \leq MRS_{Lt} L_{Lt} + R_{k,Lt} u_{Lt} \bar{K}_{Lt-1} - P_{Lt} a(u_{Lt}) \bar{K}_{Lt-1} \\
& \quad + DIV_{Lt}^p + DIV_{Lt}^w + B_{Lt-1} \\
& \quad \bar{K}_{Lt} \leq (1 - 0.5\kappa (I_{Lt}/I_{Lt-1} - 1)^2) I_{Lt} + (1 - \delta) \bar{K}_{Lt-1},
\end{aligned} \tag{12}$$

which can both be solved following standard procedure. Appendix E provides all details on the solution. Note that problems (11) and (12) are only equivalent up to a first-order to (6) and (7), respectively. However, this will not matter since the model is eventually solved with a first-order perturbation method.

5.4 Bond Market

To price the yield curve, I assume that there exists a representative investor who trades long-term bonds of maturity τ at price $Q_t^{(\tau)}$. Each bond price is assumed to be the wealth-weighted average of two bond prices in counterfactual economies, in each of which only one of the two sectors exists, i.e. $n = 1$ and $n = 0$. This assumption is consistent with various papers in finance who show that in heterogeneous asset pricing models equilibrium prices can be defined in such a manner (e.g., [Detemple and Murthy, 1994](#); [Jouini and Napp, 2007](#); [Xiong and Yan, 2010](#); [Bhamra and Uppal, 2014](#)). Broadly speaking, the idea is that an agent's wealth indicates the impact she has on the marginal pricing of the bond.

To be precise, the price of a bond with maturity τ is given by

$$Q_t^{(\tau)} = ws_{Ht} Q_{HHt}^{(\tau)} + ws_{Lt} Q_{LLt}^{(\tau)}, \tag{13}$$

where ws_{Ht} and ws_{Lt} are the wealth shares of the H-household and L-household, respectively. $Q_{HHt}^{(\tau)}$ and $Q_{LLt}^{(\tau)}$ denote bond prices in the counterfactual economies $n = 1$ and $n = 0$. The yield of maturity τ is then defined to be

$$R_t^{(\tau)} = \left(Q_t^{(\tau)} \right)^{-\frac{1}{\tau}} \text{ for } \tau \in \{1, \dots, 40\}. \tag{14}$$

The bond prices in each counterfactual economy are defined as in e.g. [Rudebusch and](#)

Swanson (2012):

$$Q_{HHt}^{(\tau)} = E_t \left[\Lambda_{HHt,t+1} (\Pi_{HHt+1})^{-1} Q_{HHt+1}^{(\tau-1)} \right] \text{ for } \tau \in \{1, \dots, 40\}, \quad (15)$$

and $Q_{HHt}^{(0)} \equiv 1$,

$$Q_{LLt}^{(\tau)} = E_t \left[\Lambda_{LLt,t+1} (\Pi_{LLt+1})^{-1} Q_{LLt+1}^{(\tau-1)} \right] \text{ for } \tau \in \{1, \dots, 40\}, \quad (16)$$

and $Q_{LLt}^{(0)} \equiv 1$,

where $\Lambda_{HHt,t+1}$ and $\Lambda_{LLt,t+1}$ are the real stochastic discount factors, and Π_{HHt+1} and Π_{LLt+1} are the aggregate inflation indexes.

Further, the wealth shares ws_{Ht} and ws_{Lt} are defined as

$$ws_{Ht} = \frac{n \text{ wea}_{Ht}}{n \text{ wea}_{Ht} + (1 - n) \text{ wea}_{Lt}}, \quad (17)$$

and

$$ws_{Lt} = \frac{(1 - n) \text{ wea}_{Lt}}{n \text{ wea}_{Ht} + (1 - n) \text{ wea}_{Lt}}, \quad (18)$$

where each household's real wealth is given by

$$\text{wea}_{Ht} = \bar{K}_{Ht} + b_{Ht}, \quad (19)$$

$$\text{wea}_{Lt} = \bar{K}_{Lt} + b_{Lt}. \quad (20)$$

Lastly, note that if I were to let both households directly trade long-term bonds, I would need to introduce a transfer from the L-household to the H-household which makes sure that the L-household's profit, which arises due to her superior information, does not keep growing. Since there is no information asymmetry with respect to the short rate, this problem does not arise in the current version of the model, where households trade the one-period bond.

5.5 Rest of Model

The rest of model is standard and follows the previous literature. The model has two labor markets, one for each sector, which are separated from each other. Except for the discounting factor, which comes from the household, both labor markets are identical. Each labor market setup follows Sims and Wu (2021) which allows for a representative household.

In each sector, there is a continuum of labor unions which buy labor from the household. The labor unions differentiate the labor, set wages subject to a Calvo pricing scheme, and sell the labor in a monopolistically competitive market to a labor packer. The labor packer uses a CES technology to create final labor which is then available for production.

The model has two production sides, one for each sector, which are separated from each other. Except for the discounting factor, which comes from the household, both production sides are identical. For each sector, there is a continuum of intermediate good firms. They rent capital and labor from the capital market, which is perfectly competitive, and the labor market, which is described in the previous paragraph. Each firm sets prices subject to a Calvo pricing scheme, and sells its goods in a monopolistically competitive market to the final good producer. The final good producer uses a CES technology to aggregate the intermediate goods to the final good in a perfectly competitive market. Each sector’s final good can then be used by the households.

5.6 Solution Method

Due to the model’s noisy information structure, I employ the equivalence result by [Chahrour and Jurado \(2018\)](#) in order to solve it. [Chahrour and Jurado \(2018\)](#) show that for a “noise representation” as described in this section, there exists an observationally equivalent “news representation” in which agents perfectly observe all future policy deviations. In practice, I solve the model under full information (news representation), and then mimic the updating structure as outlined in this section (noise representation) with the news shocks. Since the ambiguity induced non-linearity of the model arises in the updating structure, I can solve the model by a first-order perturbation around the non-stochastic steady state. Lastly, models such as the one outlined in here are usually non-stationary. To get around this issue, I impose a tiny bond holding cost ([Schmitt-Grohé and Uribe, 2003](#)).

6 Quantitative Analysis of Forward Guidance

6.1 Parameterization

In order to quantitatively employ the model, I calibrate it which is done in two steps. First, I set one of group of parameters to fixed values taken from prior papers. Second, I employ a moment matching procedure to choose the rest of the parameters such that the model is able to match empirical moments in the transmission of forward guidance.

Fixed Parameters The set of fixed parameters is shown in Table 1. As in the empirical analysis, the time frequency is quarterly in the model. Hence, the discount factor β and the depreciation rate δ are set to standard values of 0.995 and 0.025, respectively. Further, the capital share is $\alpha = 0.33$ which is also a conventional value. For the baseline case, the size of the H-sector n is chosen to be 0.5 which can be seen as a “normalization” since in the model, shifting the sector proportions is to some extent observationally equivalent to changing the amount of heterogeneity across sectors. However, this choice of n leaves equal space in both directions which is important when I shift n in Subsection 6.2 to mimic expansions and recessions. The elasticity of substitution between sectors η is set to 2 as in Nakamura and Steinsson (2014) who also model the U.S. economy as a two-sector framework.¹³

Table 1: Fixed Parameters in Baseline Model

Parameter	Value	Description	Source
β	0.995	Discount factor	Standard
δ	0.025	Depreciation rate	Standard
α	0.33	Capital share	Standard
n	0.5	Size of H-sector	“Normalization”
η	2	Elast. of substitution across sectors	Nakamura and Steinsson (2014)
θ_{Π}	1.8	Taylor rule, inflation response	Campbell et al. (2019)
θ_Y	0.4	Taylor rule, output response	Campbell et al. (2019)

Notes: This table shows the parameters which are fixed in the calibration along with their values, description, and the source from which the values are taken from. See text for more details.

Forward Guidance Shock In order to perform the moment matching, I need a model counterpart of the empirical forward guidance shock. To do so, I calibrate the forward guidance shock in the model as a sequence of signals which lead to a yield curve shift on impact consistent with the one in the empirical part. Precisely, a forward guidance shock is defined as a series of signals

$$\varepsilon_t^{FG} = \begin{bmatrix} s_t^0 & s_t^1 & \dots & s_t^{15} \end{bmatrix}$$

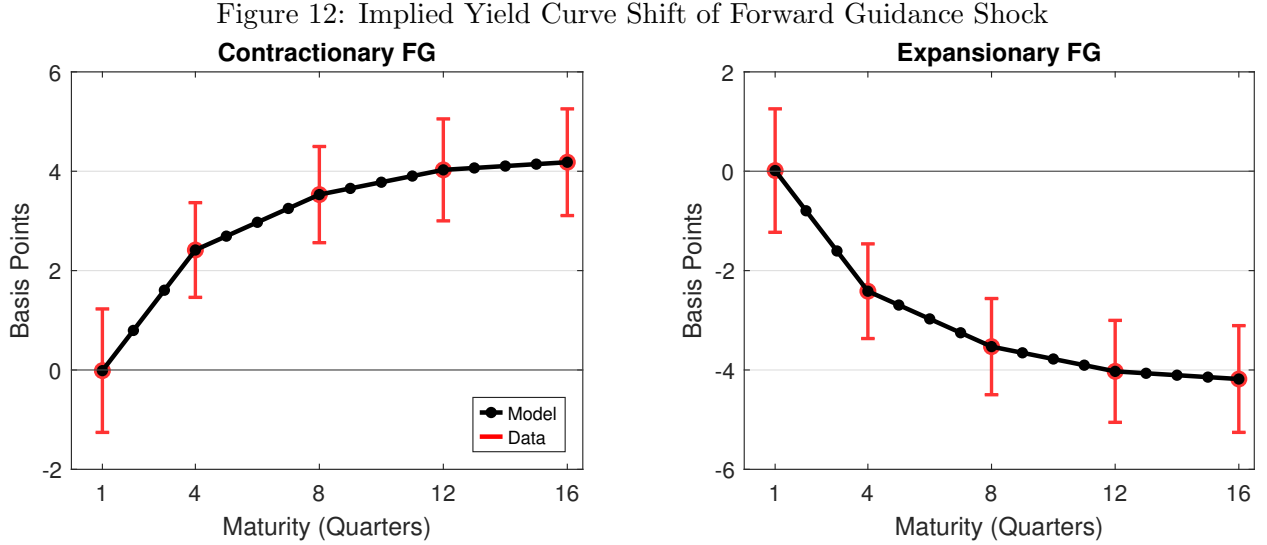
such that the impact response of yields with maturities of up to four years is consistent with the empirical counterpart:

$$IR_0(R_t^{(\tau)}) = \beta_{\tau} \quad \text{for } \tau = 1, 4, 8, 12, 16,$$

¹³Nakamura and Steinsson (2014) refer to “region” instead of “sector” in there paper.

where $IR_0(R_t^{(\tau)})$ is the impulse response in the model to a forward guidance shock at horizon 0, i.e. on impact, of the τ -quarters yield $R_t^{(\tau)}$, and β_τ is the empirical effect of the forward shock on daily changes in Treasury yields of maturity τ quarters around FOMC announcements. To obtain the empirical estimates, I employ the 3-month, 1-, 2-, 3-, and 4-year Treasury yields.

Figure 12 visualizes the fitting of the yield curve response on impact to its empirical counterpart. The red circles show the point estimates from the empirical analyses together with the 95 percent confidence intervals. The black dotted line is the yield curve in the model where yields with no empirical counterpart are linearly interpolated.



Notes: This figure displays the contemporaneous effect on the yield curve which characterizes the forward guidance shock in the model (black line). The red circles show the point estimates of empirical forward guidance shock which the model is fitted to. The red bars are the 95 percent confidence intervals. The point estimates and confidence bands are obtained from a event study regression of daily changes in Treasury yields on the shock. Heteroskedasticity-robust standard errors are employed.

Fitted Parameters With the forward guidance shock at hand, I now turn to the calibration of the remaining parameters in the model. Based on prior estimates, I define a reasonable range of values for each parameter which together span a parameter space. I then run a procedure which searches within this space the set of parameters that minimizes the distance of the following two moments: the *cross-sectional heterogeneity* in the capital response to forward guidance and the *asymmetric effect* on output. Table 2 shows the results of the moment matching procedure. In the following, I talk about the construction of these moments,

the results of Table 2, and the resulting parameter values.

Table 2: Empirical Targets

Moment	Data (%)			Model (%)
	Target	CI		
Cross-sectional Heterogeneity				
Expansionary Forward Guidance	-180	[-326	-26]	-92
Contractionary Forward Guidance	180	[26	326]	59
Asymmetric Effect	-57	[-147	31]	-10

Notes: This table displays the targeted moments in the calibration of the parameters. *Cross-sectional Heterogeneity* refers to the capital response of the high-uncertainty sector relative to the low-uncertainty one following a forward guidance shock. *Asymmetric Effect* denotes the output response to a contractionary forward guidance shock relative to expansionary one. In the *Data* column, *Target* refers to the targeted moment in the data based on the point estimate, whereas *CI* stand for the moments based on 68 percent confidence intervals. See text for details on the construction.

Based on the empirical analysis, the model should be able to match the empirical heterogeneity in the capital response following a forward guidance shock. To do so, I construct the *cross-sectional heterogeneity* both in the model and data as a summary statistic of the heterogeneity in the capital responses. In the model, it is constructed as the capital response of the high-uncertainty relative to the low-uncertainty sector as a percentage of the total capital's average response. Precisely, it is constructed as

$$het_{Model} = \frac{1}{16} \sum_{h=1}^{16} \frac{IR_h(K_H) - IR_h(K_L)}{\overline{IR(K)}}, \quad (21)$$

where $IR_h(X)$ is defined as variable X 's response τ -quarters after a forward guidance shock, and the average response is defined as $\overline{IR(X)} = \frac{1}{16} \sum_{h=1}^{16} IR_h(X)$. Note that total capital is calculated as the population weighted average of each sector's capital.

The construction of the moment is chosen such that there is a one-to-one empirical counterpart. In essence, I run a version of the main local projection specification (2) where I drop the industry-time fixed effect and include the forward guidance shock. This allows me to recover the main effect $\beta^{(h)}$ of the shock, as well as the heterogeneity with respect to uncertainty, $\gamma^{(h)}$. For details see specification (D1) in Appendix D. With these estimates at

hand, I can construct the empirical counterpart of (21) as the predicted value

$$het_{Data} = \frac{1}{16} \sum_{h=1}^{16} \frac{uc_{25}^{75} \times \gamma^{(h)}}{\bar{\beta}}, \quad (22)$$

where $\bar{\beta} = \frac{1}{16} \sum_{h=1}^{16} \beta^{(h)}$, and $uc_{25}^{75} = 1.4$ is number of standard deviations needed from the 25th to 75th percentile. In Table 2, I also report a confidence interval for het_{Data} which is based on the 68 percent confidence bands of $\gamma^{(h)}$.

The second moment of interest is the asymmetry in the effect of a contractionary and an expansionary forward guidance shock. The model displays an asymmetry between expansionary and contractionary forward guidance since the former requires stronger underlying policy deviations. This asymmetry arises since the average Kalman gain for an expansionary signal is lower than for a contractionary one.¹⁴ Hence, an increase in H-sector's ambiguity, i.e. a widening of the set $[\underline{\psi}, \bar{\psi}]$, increases the cross-sectional heterogeneity while it also induces an increased asymmetry.

The model's prediction of an asymmetric transmission is what I take to the data. To do so, I construct the *asymmetric effect* for variable X in the model as

$$asy_{Model} = \frac{1}{16} \sum_{h=1}^{16} \frac{IR_h^+(X) - IR_h^-(X)}{\overline{IR(X)}}, \quad (23)$$

where $IR_h^+(X)$ and $IR_h^-(X)$ denote variable X 's response τ -quarters after a contractionary and expansionary forward guidance shock, respectively. Further, $\overline{IR(X)}$ is constructed as $\overline{IR(X)} = \frac{1}{16} \sum_{h=1}^{16} 0.5 (IR_h^+(X) + IR_h^-(X))$. In the moment matching procedure, I only focus on the asymmetric effect on output as a summary statistic of the real transmission in the model. This is the model moment shown in Table 2.

I obtain the empirical counterpart of (23) from a local projection specification similar to (1) where allow now for differential effects of contractionary ($\beta_+^{(h)}$) and expansionary ($\beta_-^{(h)}$) shocks. For details see specification (D2) in Appendix D. With these estimates at hand, I can construct

$$asy_{Data} = \frac{1}{16} \sum_{h=1}^{16} \frac{\beta_+^{(h)} - \beta_-^{(h)}}{\bar{\beta}_{\pm}} \quad (24)$$

where $\bar{\beta}_{\pm} = \frac{1}{16} \sum_{h=1}^{16} 0.5 (\beta_+^{(h)} + \beta_-^{(h)})$. In Table 2, I also report a confidence interval for asy_{Data} which is constructed based on the 68 confidence bands of $\beta_+^{(h)}$ and $\beta_-^{(h)}$, i.e.

¹⁴The average Kalman gains are defined as $psi^{exp} = n\underline{\psi} + (1-n)\bar{\psi}$ and $psi^{con} = n\bar{\psi} + (1-n)\underline{\psi}$.

$$\frac{1}{16} \sum_{h=1}^{16} \frac{\beta_{+,16}^{(h)} - \beta_{-,84}^{(h)}}{\beta_{\pm}}, \text{ and } \frac{1}{16} \sum_{h=1}^{16} \frac{\beta_{+,84}^{(h)} - \beta_{-,16}^{(h)}}{\beta_{\pm}}.$$

In the moment matching procedure, I also impose the constraint that each sector's capital response is in the direction of the average capital response. In principle, the model is able to generate impulse response such that both sectors respond in opposite directions following a forward guidance shock. Since I do not find evidence for such responses in my empirical analysis, I impose the restriction.

Lastly, I turn to the parameter space chosen for moment matching, as well as the optimal point. Table 3 lists the parameters calibrated in the moment matching. For each parameter, the range of values comes from the previous literature on dynamic stochastic general equilibrium (DSGE) models, where I particular draw from papers which focus on monetary transmission such as [Christiano, Trabandt, and Walentin \(2010\)](#), [Campbell et al. \(2019\)](#), and [Bundick and Smith \(2020\)](#). Appendix Table E1 shows the parameter values of these papers together with my chosen range. For the non-standard Kalman gain of the L-sector ψ , the range from 0.4 to 0.6 is rather loosely based on prior evidence by [Coibion and Gorodnichenko \(2015\)](#) who calculate an implied Kalman gain of 0.46 for inflation expectations. The Kalman gain ambiguity of the H-sector ψ_{Δ} , which pins down $\underline{\psi} = \psi - \psi_{\Delta}$ and $\bar{\psi} = \psi + \psi_{\Delta}$, is set such that $\underline{\psi}$ and $\bar{\psi}$ are within the range of 0 and 1 for all possible values of ψ .

Table 3: Calibrated Parameters in Baseline Model

Parameter	Value Range	Optimal Value	Description
χ	[0.1 1]	0.411	Inverse Frisch elast.
ξ	[0.6 0.8]	0.746	Consumption habit
κ	[2 14]	3.810	Investment adjustment costs
φ	[0.01 0.5]	0.297	Elast. of capital utilization
ω_p	[0.7 0.9]	0.884	Price stickiness probability
ω_p	[0.7 0.9]	0.881	Wage stickiness probability
ϵ_p	[3 11]	5.795	Elast. of substitution w.r.t. goods
ϵ_w	[3 11]	5.684	Elast. of substitution w.r.t. labor
ψ	[0.4 0.6]	0.496	Kalman gain of L-Sector
ψ_{Δ}	[0.2 0.4]	0.264	Kalman gain ambiguity of H-Sector

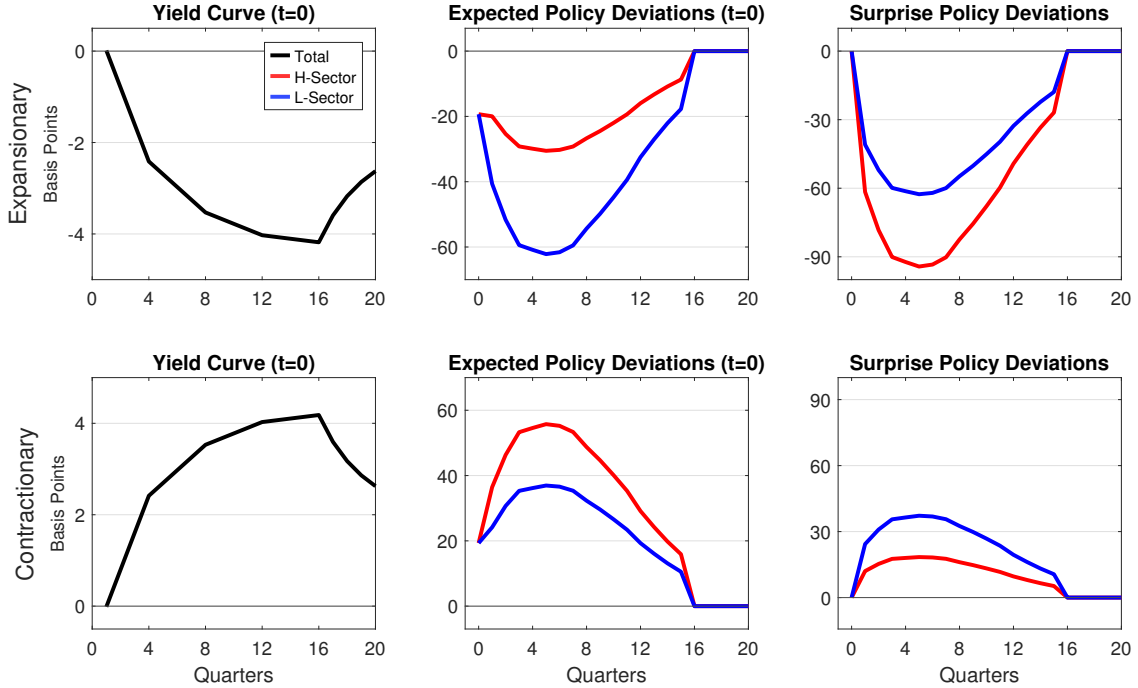
Notes: This table reports the parameters which are chosen by the moment matching exercise described above. Besides the parameters, the tables displays the range of values which are considered in the minimization procedure along with the optimal value and the parameter description.

6.2 Transmission of Forward Guidance

Belief Heterogeneity I now turn to the transmission of the forward guidance shock in the model. I start by showing the information transmission and belief heterogeneity following a shock in Figure 13. Precisely, it displays for both an expansionary and a contractionary forward guidance shock the respective yield curve movement on impact, as well as the expected policy deviations $E_t\theta_{t+j}$ and $E_t\theta_{t+j}^*$ and the realized surprises $\theta_{t+j} - E_t\theta_{t+j}$, and $\theta_{t+j}^* - E_t\theta_{t+j}^*$ following each shock. As expected, households in the H-sector update their beliefs less following an expansionary shock and more following a contractionary shock. As a consequence, the H-sector is more surprised following the shock.

Further, Figure 13 displays the asymmetry between expansionary and contractionary forward guidance. As explained above, expansionary forward guidance requires stronger policy deviations to achieve the same yield curve shift due to a lower average Kalman gain. Lastly, note that since the yield curve is only pegged up to four years, the interest rate follows the Taylor rule from four years out, i.e. there are no policy deviations.

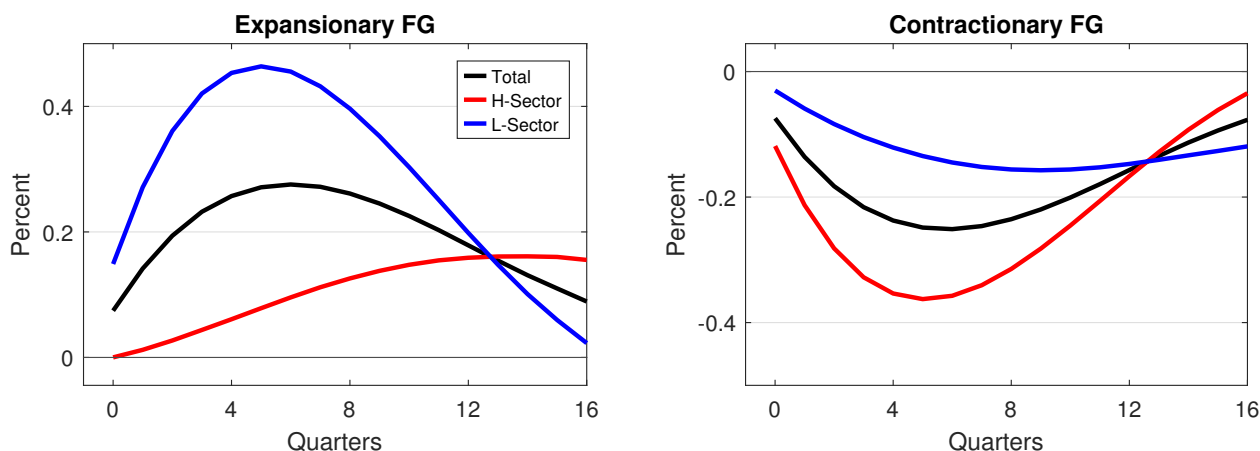
Figure 13: Belief Heterogeneity in Response to Forward Guidance Shock



Notes: This figure displays the model's impulse responses to an expansionary forward guidance shock (top row) and contractionary forward guidance shock (bottom row). The left and middle column show the contemporaneous responses of the yield curve and the expected policy deviations of each sector, respectively. The right column shows each sector's unexpected policy deviations following a shock. The red and blue lines show the responses of the high- and low-uncertainty sector.

Heterogeneity in Capital Responses I now turn now to the heterogeneity across sectors and how it relates to the heterogeneity estimated in the empirical analysis. For comparison, I focus on the capital stock. Figure 14 shows the response of each sector's capital stock, as well as the total, population-weighted capital stock to a forward guidance shock in the model. The heterogeneity can match the empirical evidence which is somewhat expected considering that a summary statistic based on the capital responses is targeted in the model calibration. The high-uncertainty sector is less responsive to an expansionary shock, and more responsive to a contractionary one. The relationship between both sectors reverses after 12-quarters. This arises since the L-sector is more surprised by the actual realizations of the policy deviations. This leads to an offsetting effect which eventually dominates after a certain time.

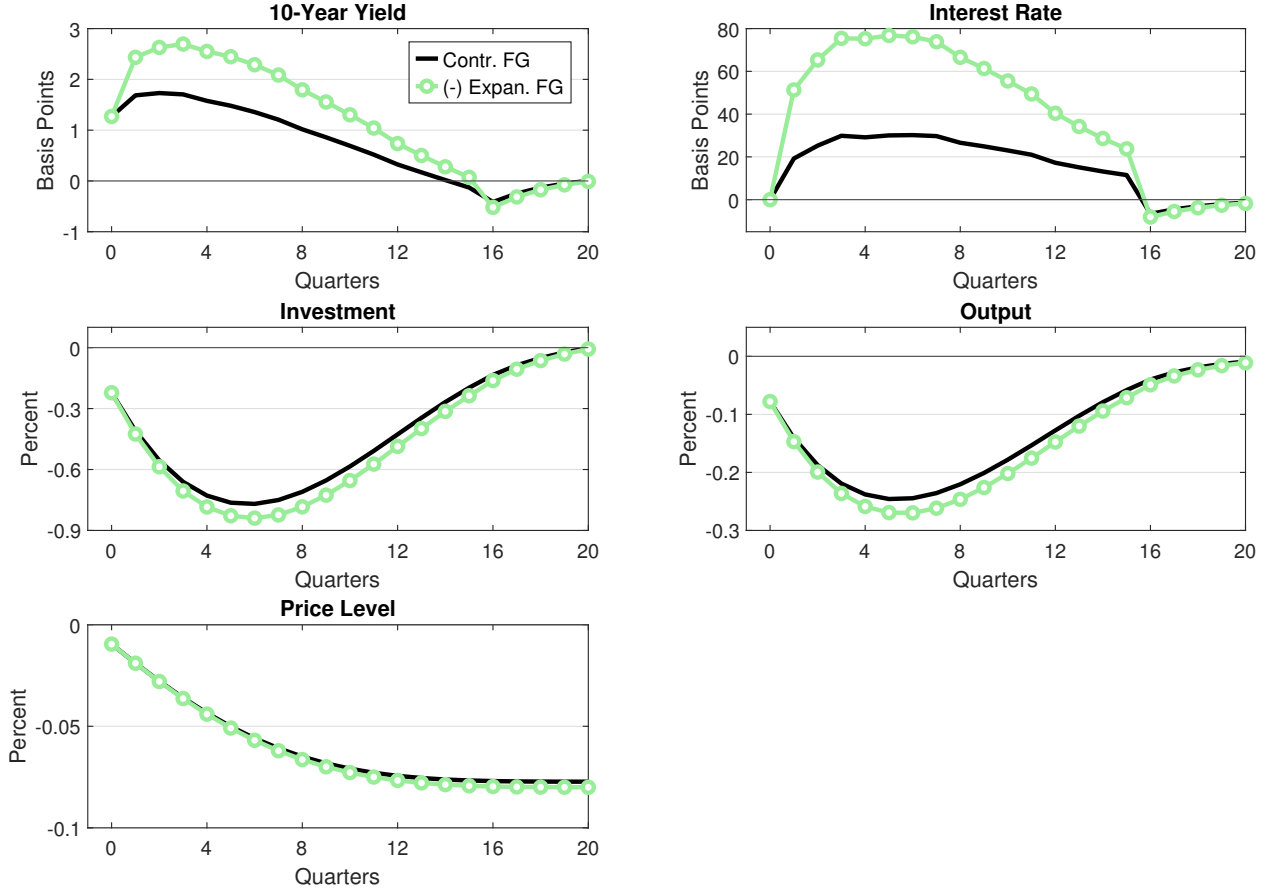
Figure 14: Heterogeneity in Capital Response to Forward Guidance Shock



Notes: This figure displays the impulse responses of capital in the model for an expansionary forward guidance shock (left) and a contractionary forward guidance shock (right). The black line depicts the response of the total, population-weighted capital stock. The blue and red line show the capital responses of the low- and high-uncertainty sector, respectively.

Aggregate Responses Figure 15 shows the impulse responses of various aggregate variables for an expansionary and contractionary shock, where the responses to the former are sign flipped for comparison. Qualitatively, the responses are as expected and consistent with the empirical evidence. Quantitatively, there is an asymmetric effect as discussed throughout the section between expansionary and contractionary forward guidance, with the former leading to stronger transmission. This asymmetry is small for real activity variables and prices, and larger for interest rates. That being said, for each variable the model asymmetry is within the confidence bands from the empirical analysis which are reported in Appendix Figure D6.

Figure 15: Impulse Responses to Forward Guidance Shock



Notes: This figure displays impulse responses to a contractionary forward guidance shock (black) and an expansionary forward guidance shock (green, circled), where the responses to the latter are flipped for comparison.

6.3 State-Dependent Effect

I now turn to the quantification of the state-dependent effect of forward guidance in the model. The transmission of forward guidance depends on the sizes of both sectors in the model. Since recessions are accompanied with an increase in uncertain firms in the data (as shown in Figure 6 in Section 3), I model recessions as an upward shift in the size of the H-sector, n .

To quantify the size of the state-dependence, I need to calibrate the average size of the H-sector in recessions and expansions, denoted with n^{Exp} and n^{Rec} , respectively. To do so, I use the distribution of my quarterly uncertainty measure uc_i as constructed in Section 3. As mentioned earlier, the baseline value of $n = 0.50$ can be thought of as a normalization

which provides sufficient room to move n in both directions.

The construction of n^{Exp} and n^{Rec} is done as follows. For the full sample, I take the 25th and 75th percentiles (denoted by $uc_{i,25}$ and $uc_{i,75}$) as the representative values of the L- and H-sector. Using the NBER business cycle dates, I can create two distributions of uc_i conditional on expansions and recessions, with means $\overline{uc}_{i,Exp}$ and $\overline{uc}_{i,Rec}$, respectively. By comparing $\overline{uc}_{i,Exp}$ with $uc_{i,25}$, I can calculate how much n needs to move in expansions, i.e. $n^{Exp} = \min(n(1 - \overline{uc}_{i,Exp}/uc_{i,25}), 0)$. Similarly, I can construct n^{Rec} by using $n^{Rec} = \max(n(1 + \overline{uc}_{i,Rec}/uc_{i,75}), 1)$. This yields $n^{Exp} = 0.435$ and $n^{Rec} = 1.00$. Table 4 details the construction.

Table 4: Calibration of Sector Sizes in Expansion and Recession

	Data			Model
	Mean	25th	75th	Size of H-Sector
Full Sample	0	-0.707	0.690	0.50
Expansion	-0.092			$0.435 = \min(0.50 \times (1 - 0.092/0.707), 0)$
Recession	0.696			$1.00 = \max(0.50 \times (1 + 0.696/0.690), 1)$

Notes: This table illustrates the calibration of the sector sizes for expansions and recession. The right panel shows the baseline size of the H-sector, as well as the construction of the sizes during expansions and recessions. These are based on the values in the left panel which reports the mean, 25th, and 75th percentile of the uncertainty variable uc_i (as constructed in Section 3) for the full sample. It also reports the means conditional on expansions and recessions during the sample. The units are standard deviations from the full sample mean. Expansion and recession quarters are taken from the NBER recession periods.

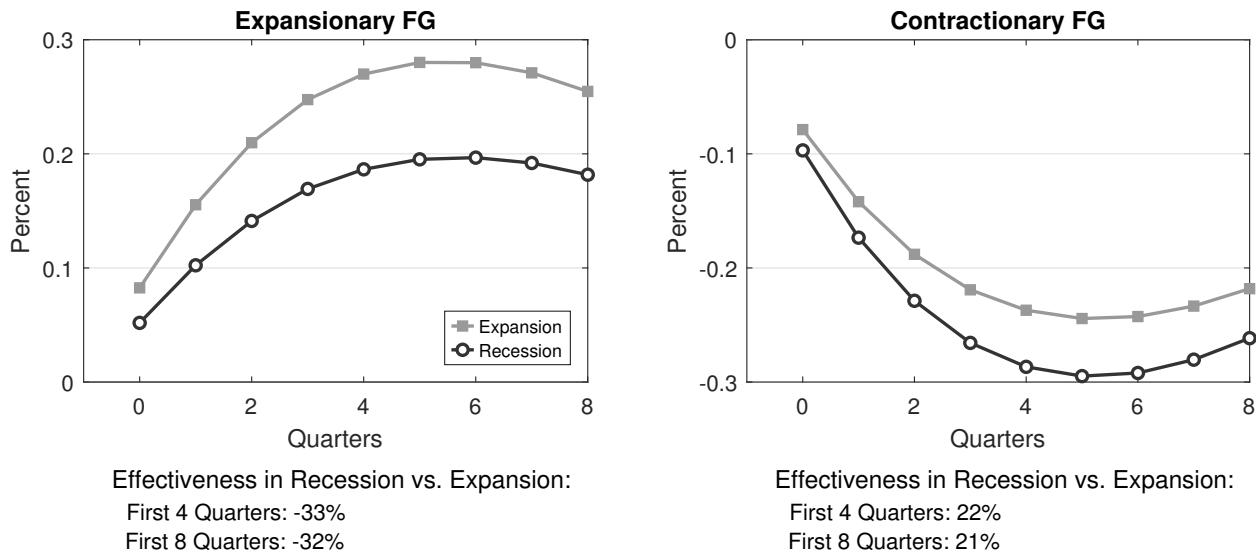
With n^{Exp} and n^{Rec} at hand, I can now study the state-dependent transmission of forward guidance in the model which I implement as follows. The central bank announces the same forward guidance policy, i.e. a set of policy deviations, as under the baseline calibration. With this policy at hand, I solve the model again for n^{Exp} and n^{Rec} , i.e. the central bank is not aware that the size of the H-sector changed.

Figure 16 shows the output response to a forward guidance shock under the expansion and recession scenario. The left panel shows that expansionary forward guidance is on average 33 percent and 32 percent less effective in recessions for the first four and eight quarters, respectively. The right panel shows that contractionary forward guidance is more effective in recessions, the state-dependence here is smaller. This comes from the fact that expansionary forward guidance consists of larger policy deviations as depicted in Figure 13.

Overall, the model shows that the implied state-dependency can be substantial, especially for expansionary forward guidance. Lastly, it should be noted the model underestimated the heterogeneity (as shown in Figure 14), which likely leads to an underestimate in the state-

dependency as well.

Figure 16: State-Dependent Response of Output to Forward Guidance Shock



Notes: This figure shows the impulse response of output to a forward guidance shock for expansions and recessions. The effectiveness is calculated as the percentage change from the impulse response in the expansion to the recession, averaged over the first four and eight quarters, respectively.

7 Conclusion

This paper provides new empirical evidence on the transmission of forward guidance and the role of firms' uncertainty. I find a substantial role of firm's uncertainty in the transmission. More uncertain firms respond as if there are more pessimistic. To rationalize my evidence, I build a New Keynesian model with a high- and low-uncertainty sector. The H-sector's ambiguity about the informativeness of forward guidance combined with its aversion to ambiguity allows me match the empirical heterogeneity. One key implication of the model is that expansionary forward guidance is less effective in recessions due to the increased share of uncertain agents.

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A Construction of High-Frequency Shocks

A.1 Data Overview

I employ intraday data on interest rate futures comes from *Thomson Reuters Tick History*. The sample period ranges from January 1996 and to December 2019. Table A1 provides an overview of the employed data. For each futures contract, I have a minute-by-minute series which includes the price of the first and last trade for a given minute. In the following, I detail the construction of the interest rate surprises from the futures contracts. Following previous papers, I use Federal Funds futures contracts to capture interest rate expectations within two quarters, Eurodollars futures to capture expectations up to from two to five quarters, and Treasury futures for expectations out to 15-years.

Table A1: Overview of Intraday Interest Rate Futures Data

Name	Symbols	RICs	Sample
Federal Funds Rate Futures first 4 contracts (monthly)	ff^1-ff^4	FFc1-FFc4	1/1996–12/2019
Eurodollar Futures second through fourth contract (quarterly)	ed^2-ed^4	EDcm2-EDcm4	1/1996–12/2019
2-Year Treasury Futures	$t2^1,t2^2$	TUc1,TUc2	1/1996–12/2019
5-Year Treasury Futures	$t5^1,t5^2$	FVc1,FVc2	1/1996–12/2019
10-Year Treasury Futures	$t10^1,t10^2$	TYc1,TYc2	1/1996–12/2019
30-Year Treasury Futures	$t30^1,t30^2$	USc1,USc2	1/1996–12/2019

Notes: This table provides an overview of the intraday data from *Thomson Reuters Tick History*. *Symbol* stands for the ticker symbol which I use throughout the paper to refer to the financial instrument. *RIC* refers to the Reuters Instrument Code, which uniquely identifies each instrument. The letters *c* and *cm* stand for continuous futures contracts.

A.2 Federal Funds Futures

For given expiry month, a federal funds rate futures contract pays out, on the last day of the expiry month, 100 minus the average (effective) federal funds rate over the expiry month. Precisely, let $p_{\zeta}^{ff^j}$ be the price at time ζ of the $(j - 1)$ month ahead federal funds futures contract. Then, the expected average federal funds rate of the $(j - 1)$ month ahead at time ζ is calculated as $ff_{\zeta}^j = 100 - p_{\zeta}^{ff^j}$.

A.2.1 Federal Funds Rate Surprise — Current Meeting

I calculate the federal funds rate meeting surprise $mp1_{\tau}$ as

$$mp1_{\tau} = \frac{m_0}{m_0 - d_0} (ff_{\tau+\Delta+}^1 - ff_{\tau-\Delta-}^1), \quad (A1)$$

where $ff_{\tau-\Delta-}^1$ and $ff_{\tau+\Delta+}^1$ are the current month's implied federal funds rates from the last trade that occurred more than 10 minutes before the FOMC announcement and the first trade that

occurred more than 20 minutes after the FOMC announcement, respectively. Further, m_0 is the total number of days in the month of announcement t , and d_0 is the day of announcement t . See [Gürkaynak \(2005\)](#) for a derivation of (A1).

Construction

1. For each ζ , calculate the implied federal funds rate, i.e. $ff_\zeta^1 = 100 - p_\tau^{ff\zeta}$.
2. Calculate $\frac{m_0}{m_0 - d_0} \left(ff_{\tau+\Delta^+}^1 - ff_{\tau-\Delta^-}^1 \right)$ for each FOMC announcement t .
3. If $m_0 - d_0 + 1 \leq 7$, i.e. the announcement occurs in the last seven days of the month, use the change in the price of next month's fed funds futures contract, i.e. $mp1_\tau = ff_\tau^2 - ff_{\tau-\Delta}^2$. This avoids multiplying by large $\frac{m_0}{m_0 - d_0}$. For example, for the FOMC announcement on January 29, 2014, we have $d_0 = 29$, $m_0 = 31$, and hence $31 - 29 + 1 = 3 < 7$.
4. I set $mp1_\tau = 0$, if there is no response in the federal funds rate futures price within 24 hours after the announcement.

A.2.2 Federal Funds Rate Surprise — Next Meeting

I calculate the revision in expectations at FOMC meeting τ about the federal funds rate change at FOMC meeting $t + 1$ as

$$mp2_\tau = \frac{m_1}{m_1 - d_1} \left[\left(ff_{\tau+\Delta^+}^{j(1)} - ff_{\tau-\Delta^-}^{j(1)} \right) - \frac{d_1}{m_1} mp1_\tau \right] \quad (\text{A2})$$

where $ff_{\tau-\Delta^-}^{j(1)}$ and $ff_{\tau+\Delta^+}^{j(1)}$ are the implied rate of the federal funds rate futures contract for the month of the next scheduled FOMC meeting from the last trade that occurred more than 10 minutes before the FOMC announcement and the first trade that occurred more than 20 minutes after the FOMC announcement, respectively. Further, m_1 is the total number of days in the month of announcement $t + 1$, and d_1 is the day of announcement $t + 1$. Usually, $j(1) = \{3, 4\}$. With a little bit of an abuse of notation, $t + 1$ refers here to the next scheduled FOMC meeting at the time of announcement t . Hence, ex-post there might be an unscheduled meeting in between those. See [Gürkaynak \(2005\)](#) for a derivation of (A2).

Construction

1. For a given FOMC announcement t , find month of next scheduled FOMC meeting, i.e. $j(1)$.
2. Calculate $\frac{m_1}{m_1 - d_1} \left[\left(ff_{\tau+\Delta^+}^{j(1)} - ff_{\tau-\Delta^-}^{j(1)} \right) - \frac{d_1}{m_1} mp1_\tau \right]$ for each announcement.
3. If $m_1 - d_1 + 1 \leq 7$, i.e. the announcement occurs in the last seven days of the month, use the change in the price of next month's fed funds futures contract, i.e. $mp2_\tau = ff_{\tau+\Delta^+}^{j(1)+1} - ff_{\tau-\Delta^-}^{j(1)+1}$.

A.3 Eurodollar Futures

Eurodollar futures are quarterly contracts which pay out 100 minus the 3-month US dollar BBA LIBOR interest rate at the time of expiration. The last trading day is the second London bank business day (typically the Monday) before the third Wednesday of the last month of the expiry quarter. Let $p_{\zeta}^{ed^j}$ be the price at ζ of the j th nearest quarterly Eurodollar futures contract (March, June, September, December), then the expiration date of $p_{\zeta}^{ed^j}$ is between j and $j - 1$ quarters in the future at any given point in time. Further, the implied rate is given by $ed_{\zeta}^j = 100 - p_{\zeta}^{ed^j}$. For a given FOMC announcement t , I calculate the difference in the implied rate of contract j

$$\Delta ed_{\tau}^j = ed_{\tau+\Delta+}^j - ed_{\tau-\Delta-}^j, \quad (\text{A3})$$

where $ed_{\tau-\Delta-}^j$ and $ed_{\tau+\Delta+}^j$ are the implied rate of the j th nearest quarterly Eurodollar futures contract from the last trade that occurred more than 10 minutes before the FOMC announcement and the first trade that occurred more than 20 minutes after the FOMC announcement, respectively.

Construction

1. For each ζ , calculate the implied rate, i.e. $ed_{\zeta}^j = 100 - p_{\zeta}^{ed^j}$.
2. For a given FOMC announcement t , calculate the difference in the implied rate of contract j , $\Delta ed_{\tau}^j = ed_{\tau+\Delta+}^j - ed_{\tau-\Delta-}^j$.

A.4 Treasury Futures

Treasury futures are quarterly contracts which obligate the seller to deliver a Treasury bond within a range of maturities to the buyer at the time of expiration. Let $p_{\zeta}^{t2^j}$ be the price at ζ of the j th nearest quarterly 2-year Treasury futures contract (March, June, September, December), then I calculate the implied yield surprise around FOMC announcement τ by dividing the price change by approximate maturity of the underlying Treasury bond and flipping the sign, i.e.

$$\Delta t2_{\tau} = - \left(p_{\tau+\Delta+}^{t2^1} - p_{\tau-\Delta-}^{t2^1} \right) / 2. \quad (\text{A4})$$

If the announcement τ is in the month of expiration (March, June, September, December), then I employ the next closest contract, i.e. $\Delta t2_{\tau} = - \left(p_{\tau+\Delta+}^{t2^2} - p_{\tau-\Delta-}^{t2^2} \right) / 2$, due to its higher liquidity [Gorodnichenko and Ray \(2017\)](#). Similarly, I calculate the implied yield changes from 5-year, 10-year, and 30-year Futures, i.e.

$$\begin{aligned} \Delta t5_{\tau} &= - \left(p_{\tau+\Delta+}^{t5^1} - p_{\tau-\Delta-}^{t5^1} \right) / 4, \\ \Delta t10_{\tau} &= - \left(p_{\tau+\Delta+}^{t10^1} - p_{\tau-\Delta-}^{t10^1} \right) / 7, \\ \Delta t30_{\tau} &= - \left(p_{\tau+\Delta+}^{t30^1} - p_{\tau-\Delta-}^{t30^1} \right) / 15, \end{aligned}$$

where I use the approximate maturities as in [Gürkaynak, Kısacıkoglu, and Wright \(2018\)](#).

A.5 Factor Estimation

Following [Gürkaynak, Sack, and Swanson \(2005\)](#), I now estimate unobserved shocks (factors) which systematically move the yield curve around FOMC announcements. To do so, I first stack up the 30-minute changes of the interest rate futures around FOMC announcements. Let \mathcal{T} be the number of FOMC announcements and n be the number of interest rate futures I use, then X denotes the $\mathcal{T} \times n$ matrix of 30-minute changes in interest rates where each row corresponds to a FOMC announcement and each column to a interest rate. Here, I have $\mathcal{T} = 201$ FOMC announcements and $n = 9$ 30-changes interest surprises where each row of X is defined as

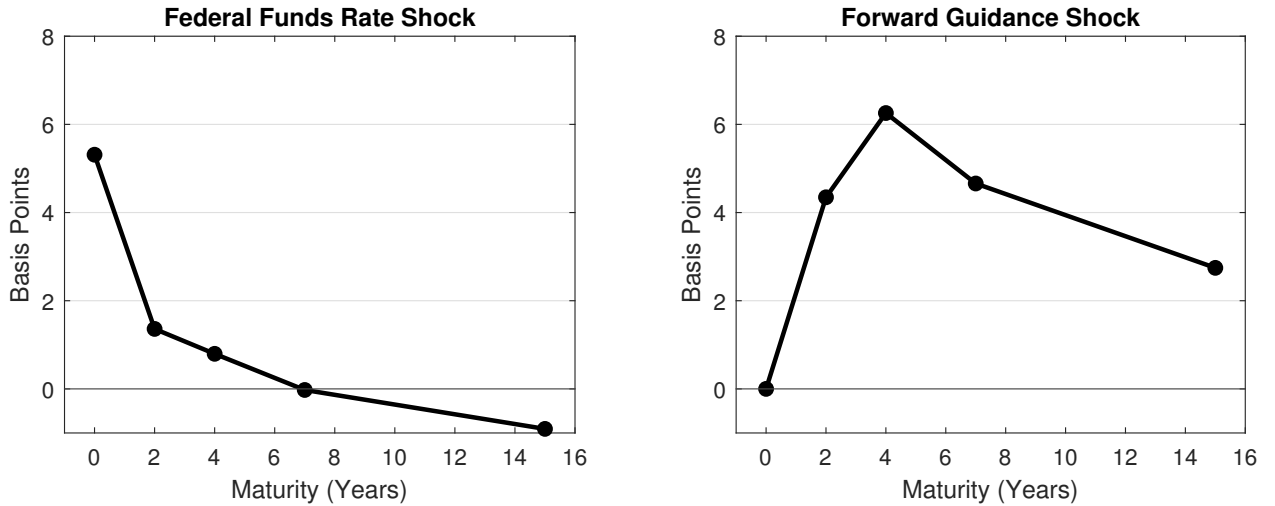
$$X_{\tau} = [mp1_{\tau}, mp2_{\tau}, \Delta ed_{\tau}^2, \Delta ed_{\tau}^3, \Delta ed_{\tau}^4, \Delta t2_{\tau}, \Delta t5_{\tau}, \Delta t10_{\tau}, \Delta t30_{\tau}]'.$$

Then this data matrix X can be thought of as the following the factor model

$$\underset{(\mathcal{T} \times n)}{X} = \underset{(\mathcal{T} \times k)}{F} \underset{(k \times n)}{\Lambda} + \underset{(\mathcal{T} \times n)}{\varepsilon}, \quad (\text{A5})$$

where F is a $\mathcal{T} \times k$ matrix of common latent factors, Λ is the $\mathcal{T} \times k$ matrix of factor loadings, and ε is the $\mathcal{T} \times n$ matrix of idiosyncratic variation in X . Following [Gürkaynak, Sack, and Swanson \(2005\)](#) and subsequent literature, I will estimate two factors, i.e. $k = 2$, by principal components and then I rotate both factors such that the second factor has no effect on the current federal funds rate. I call this factor the forward guidance shock, and the other shock the federal funds rate shock. Figure shows how both shocks map into 30-changes in the yield curve (mp2, ed omitted here).

Figure A1: Loading of High-Frequency Shocks on Yield Curve



Notes: This figure illustrates the mapping of the estimated high-frequency forward guidance shock to 30-minute changes in yields around FOMC announcements. The effects of a one standard deviation shock are shown.

A.6 Comparison to Other Shocks in Literature

I now compare my estimated shocks with previous estimates in the literature. Table ?? shows the results of this exercise. In the first row, one can see that Federal Funds Rate shock is highly correlated with $mp1$ surprise, as well as the estimates by [Gürkaynak, Sack, and Swanson \(2005\)](#) and

Swanson (2021). The second row shows that the forward guidance shock is also highly correlated with ones Gürkaynak, Sack, and Swanson (2005) and Swanson (2021). Further, it is little correlated with the LSAP shock by Swanson (2021) which confirms that my shock is not picking quantitative easing.

Table A2: Correlation with Other Shocks in Literature

	mpl	Target (GSS 05)	FFR Shock (S 21)	Path (GSS 05)	FG Shock (S 21)	LSAP Shock (S 21)
FFR Shock	0.90 (201)	0.95 (74)	0.91 (194)			
FG Shock				0.92 (74)	0.92 (194)	-0.14 (194)

Notes: This figure shows the correlations of the two estimated high-frequency shocks with similar estimated shocks by Gürkaynak, Sack, and Swanson (2005) (GSS 05), and Swanson (2021) (S 20).

B Quarterly Aggregation and Controlling for Macroeconomic News

As discussed in Section 2, I follow the recommendation by [Miranda-Agrippino and Ricco \(2021\)](#) and [Bauer and Swanson \(2020\)](#), and “project out” the (ex post) correlation of the high-frequency forward guidance shocks and macroeconomic news by estimating the following specification

$$\varepsilon_{\tau}^{FG} = \alpha + \beta news_{\tau} + \widetilde{\varepsilon_{\tau}^{FG}}, \quad (\text{B1})$$

where ε_{τ}^{FG} is the high-frequency forward guidance shocks for FOMC announcement τ , and $news_{\tau}$ is a vector of major macroeconomic news surprises released since the start of the quarter until the time of the FOMC announcement τ (as illustrated by Figure 2). I construct the surprises of the macroeconomic releases (e.g. nonfarm payrolls) employing the actual release and survey data from the Bloomberg Economic Calendar. See, for example, [Boehm and Kroner \(2020\)](#) for details on the data and construction. The selection of macro releases is motivated by previous papers ([Faust, Rogers, Wang, and Wright, 2007](#); [Rigobon and Sack, 2008](#); [Gürkaynak, Kısacıkoglu, and Wright, 2018](#)). For a given macroeconomic series, I sum up all release surprises from the start of the quarter until the time of FOMC meeting τ .

Table B1 shows the results of equation (B1). Assuming that the correlation is a violation of the FIRE assumption, and that the Fed’s interest rule has a positive coefficient on real activity and inflation, a positive (negative) regression coefficient means that Fed is more (less) responsive to the news release than expected by the private sector.¹⁵ Except for GDP releases, the private sector usually underestimates the response of the Fed to macroeconomic information consistent with [Cieslak \(2018\)](#) and [Bauer and Swanson \(2020\)](#).

Lastly, I sum up the residuals to obtain my quarterly shocks measure, i.e.

$$\varepsilon_t^{FG} = \sum_{\tau \in t} \widetilde{\varepsilon_{\tau}^{FG}}.$$

¹⁵For Initial Jobless Claims and the Unemployment Rate, the interest rule coefficient is presumably negative.

Table B1: Response of High-Frequency FG Shock to Macroeconomic News

	FG Shock (basis points)
Capacity Utilization	-0.19 (0.26)
CB Consumer Confidence	0.89*** (0.27)
Core CPI	0.09 (0.22)
Core PCE Price Index	0.52** (0.26)
Core PPI	-0.46 (0.30)
Durable Goods Orders	0.50 (0.31)
GDP	-0.56** (0.28)
Housing Starts	0.49 (0.39)
Initial Jobless Claims	-0.05 (0.12)
ISM Chicago PMI	0.69** (0.35)
ISM Mfg Index	-0.22 (0.31)
New Home Sales	-0.40 (0.26)
Nonfarm Payrolls	-0.27 (0.22)
Retail Sales (ex Auto)	0.67** (0.32)
UM Consumer Sentiment	0.38* (0.23)
Unemployment Rate	-0.25 (0.19)
Constant	0.06 (0.30)
R^2	0.18
Observations	201

Notes: This figure shows the estimation results of equation (B1). Heteroskedasticity-robust standard errors reported in parentheses. ***, **, and * indicate significance at the 1, 5, and 10 percent level. Abbreviations: Mfg — Manufacturing; CB — Chicago Board; ISM — Institute for Supply Management; PMI — Purchasing Managers Index; UM — University of Michigan.

C Construction of Firm-Level Panel

C.1 Construction of Variables

In this subsection, I provide details on the construction of the firm-level variables. Below, I describe the construction of the main variables of interest (firms' capital stock and uncertainty), the employed deflator, and the industry classification. Table C1 gives an overview of all employed firm-level variables in my analysis, their construction, and references to related papers which also use this measure. All data is obtained from the *WRDS*. Note that and colored *terms* refer to variables in the respective dataset.

Conversion to real variables All nominal variables are deflated using $X = 100 \times X/GVA$ where GVA is the implied price index of gross value added in the U.S. nonfarm business sector (BEA-NIPA Table 1.3.4 Line 3). Note that for ratios in the same period, it does not matter.

Real Capital Stock and Investment Following a large literature (see, for example, Whited, 1992; Gomes, 2001; Ottonello and Winberry, 2020), I use *Compustat* data and the perpetual inventory method to construct firm i 's real capital stock. For each firm i , I do the following steps:

1. Deflate $PPEGTQ_{i,t}$ (Property, Plant and Equipment (Gross)) and $PPENTQ_{i,t}$ (Property, Plant and Equipment (Net))
2. Find first entry of $PPEGTQ_{i,1}$ and set

$$k_{i,1} = PPEGTQ_{i,1}$$

3. Linearly interpolate $PPENTQ_{i,t}$ if $PPENTQ_{i,t-1}$ and $PPENTQ_{i,t+1}$ non-missing
4. Create capital stock:

$$k_{i,t+1} = k_{i,t} + PPENTQ_{i,t+1} - PPENTQ_{i,t}$$

and if $PPENTQ_{i,t}$ missing, calculate

$$k_{i,t+1} = k_{i,t-1} + PPENTQ_{i,t+1} - PPENTQ_{i,t-1}$$

and if $PPENTQ_{i,t-1}$ missing, calculate

$$k_{i,t+1} = k_{i,t-2} + PPENTQ_{i,t+1} - PPENTQ_{i,t-2}$$

and

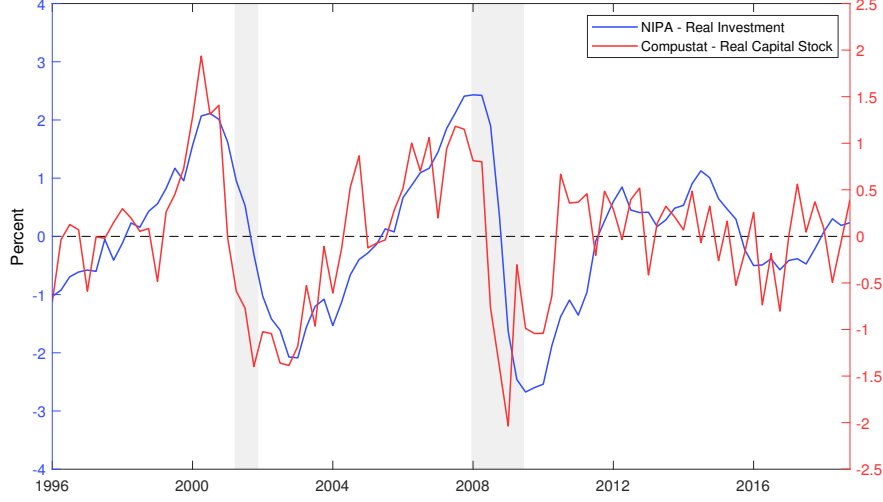
- Implementation:

- (a) Find smallest l such that $k_{i,t-l} + PPENTQ_{i,t+1} - PPENTQ_{i,t-l}$ exists.
- (b) Set $k_{i,t+1} = k_{i,t-l} + PPENTQ_{i,t+1} - PPENTQ_{i,t-l}$.

To ensure that my constructed series is reasonable and qualitatively able to mimic the investment behavior in the economy, I compare its average behavior with the one of the aggregate investment

variable from NIPA. Precisely, Figure C1 shows the cyclical component of the average firm-level investment (QoQ growth of capital stock) from Compustat and nonresidential private fixed investment from NIPA.

Figure C1: Aggregate Investment: Compustat vs. NIPA



Notes: This figure plots the QoQ growth rates of the aggregate real capital stock from Compustat and the log of real nonresidential private fixed investment from NIPA (BEA-NIPA Table 5.3.3, Line 2). The former is calculated as the average of $\Delta \log k_{i,t}$ across firms. The figure shows the cyclical component using the HP filter with smoothing parameter $\lambda = 1600$.

Uncertainty Firm's i baseline uncertainty in t is defined as

$$uc_{i,t} = 100 \times \log(ivol_{i,t}), \quad (C1)$$

and quarterly implied volatility $ivol_{i,t}$ is constructed as

$$ivol_{i,t} = \frac{1}{N_{i,t}} \sum_{n=1}^{N_{i,t}} ivol_{i,n}^{(30)}, \quad (C2)$$

where $ivol_{i,n}^{(30)}$ is the 30-day option-implied volatility of its stock price on trading day n , $N_{i,t}$ is the number of trading days in quarter t for which $ivol_{i,n}^{(30)}$ is available (daily average of put and call option *impl_volatility* with *days* = 30 from OptionMetrics).

Similarly to the baseline measure, I also construct an alternative uncertainty measure based on X-day option-implied volatility. For further horizons out, the number of available data is substantially lower. Lastly, I also create a uncertainty measure based on firm's i realized stock volatility

$$uc_{i,t}^{RV} = 100 \times \log(rvol_{i,t}), \quad (C3)$$

and realized volatility $rvol_{i,t}$ is constructed as

$$rvol_{i,t} = \sqrt{N_{i,t}} \times \sigma_{i,t}^{\text{Return}}, \quad (C4)$$

where $N_{i,t}$ is the number of trading days in quarter t for which firm i 's stock return is available and $\sigma_{i,t}^{\text{Return}}$ is the standard deviation of firm i 's ex-dividend daily stock return over quarter t (*retx* in CRSP).

Regardless of the measure, I exclude quarters if data is available for less than 20 trading days (this is done measure by measure). This ensures that each quarterly average is not driven by individual days.

Industry Classification Following [Gorodnichenko and Weber \(2016\)](#), I classify industries in the following way:

1. Agriculture, mining, and construction: $sic < 1800$
2. Manufacturing: $2000 \leq sic < 4000$
3. Transportation and Communications: $4000 \leq sic < 4900$
4. Trade: $5000 \leq sic < 6000$
5. Services: $7000 \leq sic$

Following [Jeenas \(2018\)](#) and [Clementi and Palazzo \(2019\)](#), I exclude firms in the finance, insurance, and real estate ($6000 \leq sic < 6800$), and in the utilities ($4900 \leq sic < 5000$) industry. Further, the *Services* industry consists not entirely of firms in the service industry ($7000 \leq sic < 9000$). A tiny fraction (2,927 out of 66,877) are unclassified firms ($9900 \leq sic$). There are no firms in the public administration ($9000 \leq sic < 9900$) industry in the sample.

Table C1: Overview of Firm-level Variables

Variable	Construction	Sources	References
Age	Quarters since firm's stock traded = $quarter(t) - quarter(BEGDAT_i)$	CRSP	Cloyne et al. (2018)
Dividends	$1 (DVPQ_{i,t} > 0)$	Compustat	Ottonello and Winberry (2020)
EBITDA	$100 \times \frac{SALEQ_{i,t} - COGSQ_{i,t} - XSGAQ_{i,t}^*}{GVA_t}$	Compustat, BEA	Cloyne et al. (2018)
Leverage	$\frac{\text{total debt}_{i,t}}{\text{total assets}_{i,t}} = \frac{DLCQ_{i,t} + DLTTQ_{i,t}}{ATQ_{i,t}}$	Compustat	Ottonello and Winberry (2020); Jeenas (2018)
Liquidity	$\frac{\text{cash and short-term investments}_{i,t}}{\text{total assets}_{i,t}} = \frac{CHEQ_{i,t}}{ATQ_{i,t}}$	Compustat	Jeenas (2018)
Long-term Debt Dependence	$\frac{\text{long-term debt (maturity} > \text{1year)}}{\text{total debt}} = \frac{DLTTQ_{i,t} + DD1Q_{i,t}^*}{DLCQ_{i,t} + DLTTQ_{i,t}}$	Compustat	Foley-Fisher, Ramcharan, and Yu (2016)
Price-to-cost Margin	$\frac{\text{net sales}_{i,t} - \text{costs of goods sold}_{i,t}}{\text{net sales}_{i,t}} = \frac{SALEQ_{i,t} - COGSQ_{i,t}}{SALEQ_{i,t}}$	Compustat	Gorodnichenko and Weber (2016)
Net Receivables to Sales	$\frac{\text{total receivables}_{i,t} - \text{total trade payables}_{i,t}}{\text{net sales}_{i,t}} = \frac{RECTQ_{i,t} - APQ_{i,t}}{SALEQ_{i,t}}$	Compustat	Gorodnichenko and Weber (2016)
Real Capital Stock	See text	Compustat	See text
Real Sales Growth	$100 \times \Delta \log \left(100 \times \frac{SALEQ_{i,t}}{GVA_t} \right)$	Compustat, BEA	Jeenas (2018)
Size	book value of assets = $\log(ATQ_{i,t})$	Compustat	Ottonello and Winberry (2020)
Tobin's Q	$\frac{\text{market value of assets}_{i,t}}{\text{book value of assets}_{i,t}} = \frac{ATQ_{i,t} + PRCCQ_{i,t} \times CSHOQ_{i,t} - CEQ_{i,t} + TXDITCQ_{i,t}^*}{ATQ_{i,t}}$	Compustat	Ottonello and Winberry (2020); Cloyne et al. (2018)
Uncertainty	See text	OptionMetrics	See text

Notes: This table provides an overview of all firm-level variables in the panel dataset including their precise construction, the used data sources, and selected papers which the construction is based on. Capitalized and blue marked terms stand for the original name of the variable in the corresponding dataset. * means that the construction takes the respective dataset variable only into account if it is available, otherwise it is treated as if it is zero. This is done to avoid too many missing observations which reduce the total number of observations in the analysis.

C.2 Sample Construction

CRSP/Compustat Merged Note that only firm to primary links are throughout the analysis, i.e. $LINKPRIM = P$ or $LINKPRIM = C$. Following the previous literature, I exclude the firm-quarter observation if (in order of operation):

1. The firm is not incorporated in the United States, i.e. $FIC \neq USA$
2. The firm is in finance, insurance, and real estate sectors ($sic \in [6000, 6999]$), utilities ($sic \in [4900, 4999]$)
3. A firm characteristic is missing in the data, i.e. *Age, Capital Stock, Dividends, Leverage, Long-term Debt Dependence, Price-to-cost Margin, Receivables-minus-payables to Sales, Real Sales Growth, Size, Tobin's Q*
4. The firm has a capital stock of zero (very few observations), needed for the log-specification
5. The observation is before 1996Q1 or after 2019Q4.
6. Acquisitions are larger than 5% percent of assets, i.e. $AQCCQ_{i,t}/ATQ_{i,t} > 0.05$ where $AQCCQ_{i,t}$ is constructed from $AQCY_{i,t}$.
7. Similar to Ottonello and Winberry (2020), I account for extreme values by dropping firm-quarter observations if
 - (a) Investment, i.e. $\log \Delta(k_{i,t+1})$, is in the top or bottom 0.5 percent of the distribution.
 - (b) Quarterly real sales growth is larger than 100% or smaller than -100%.
 - (c) Leverage is higher than 10.

OptionMetrics

1. I exclude the firm-quarter observations if (in order of operation):
 - (a) the firm's implied volatility *impl_volatility* is observed, for the horizon of interest ($days = 30$ for baseline), for less than 20 trading days in a quarter (Note that an average month has around 21 trading days).
2. I calculate the uncertainty measure following equations (C1) and (C2) above.

CRSP

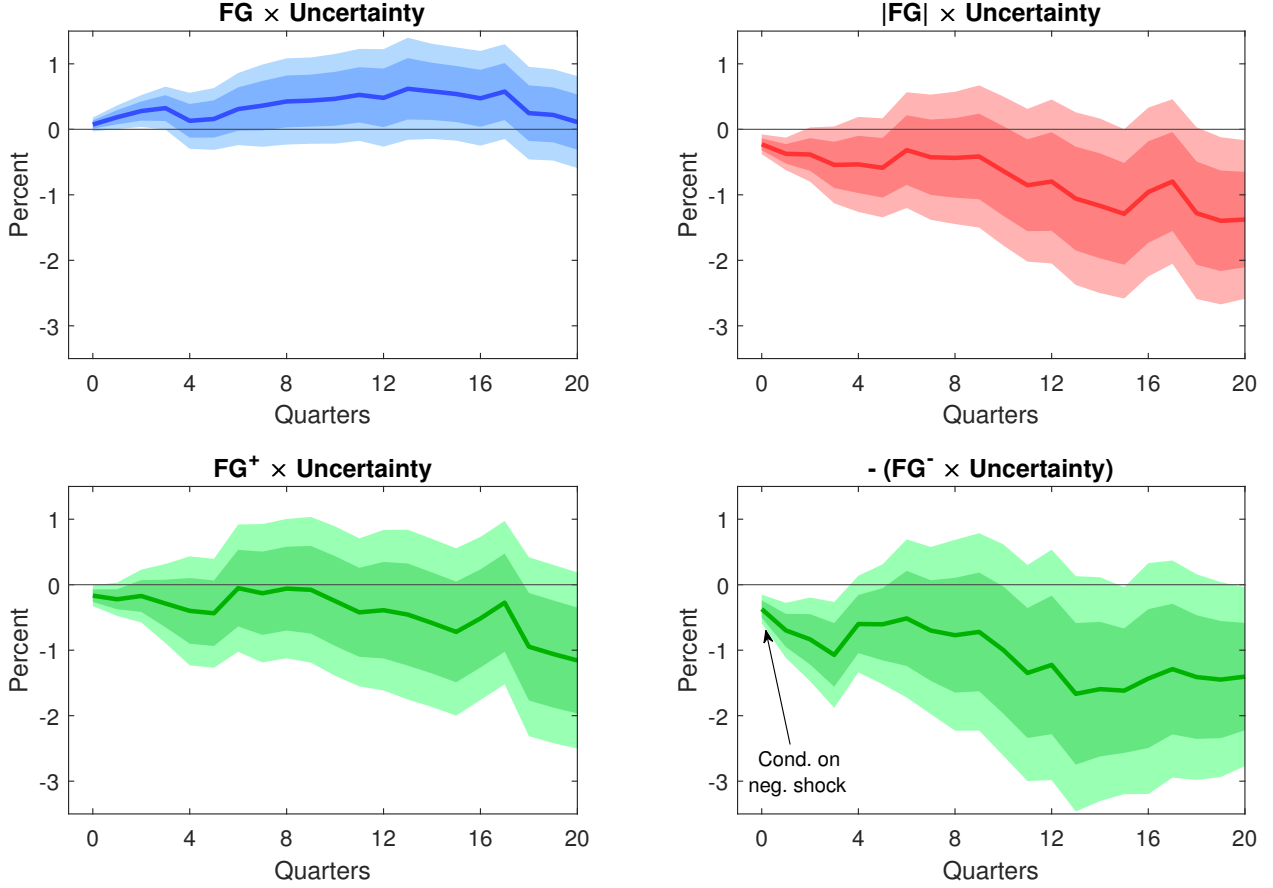
1. I exclude the firm-quarter observations if (in order of operation):
 - (a) the firm's ex-dividend daily stock return *retx* is observed for less than 20 trading days in a quarter.
2. I calculate the uncertainty measure following equations (C3) and (C4) above.

Merging the Datasets I merge the *CRSP/Compustat Merged* with *OptionMetrics* and *CRSP* data by using the permanent security identifier *PERMNO*, and the *Compustat-CRSP* and the *OptionMetrics-CRSP* linking tables from *WRDS*. For firm-quarter observations with multiple security identifiers, I follow the *Compustat-CRSP* linking table to identify the primary security identifier. After the datasets are merged, I drop all firms with less than 20 quarters observed to obtain precisely estimated firm fixed effects in my analysis (Cloyne et al., 2018).

A Note on the Operating Lease Accounting in 2019 Starting in 2019, *Compustat* changes its accounting of *PPENTQ* (See Ma (2020) for details). This change leads to a spike in the quarterly change from 2018Q4 to 2019Q1. Although I do not have access at this point to the newest data *Compustat* release which allows me to account for this, it is not problematic with respect to the analysis run in the main text. Since the accounting change is uncorrelated with monetary policy and uncertainty, it just leads to less precise estimates. In line with that intuition, I show in Appendix Figure D4 that my main result is robust to the exclusion of 2019.

D Additional Results

Figure D1: Heterogeneity in Responses of Capital — Positive & Negative Shocks



Joint F- tests for pos. & neg. shocks:

$$H_0: \gamma_{FG^+} = \gamma_{FG^-} \quad | \text{p-value} = 0.002$$

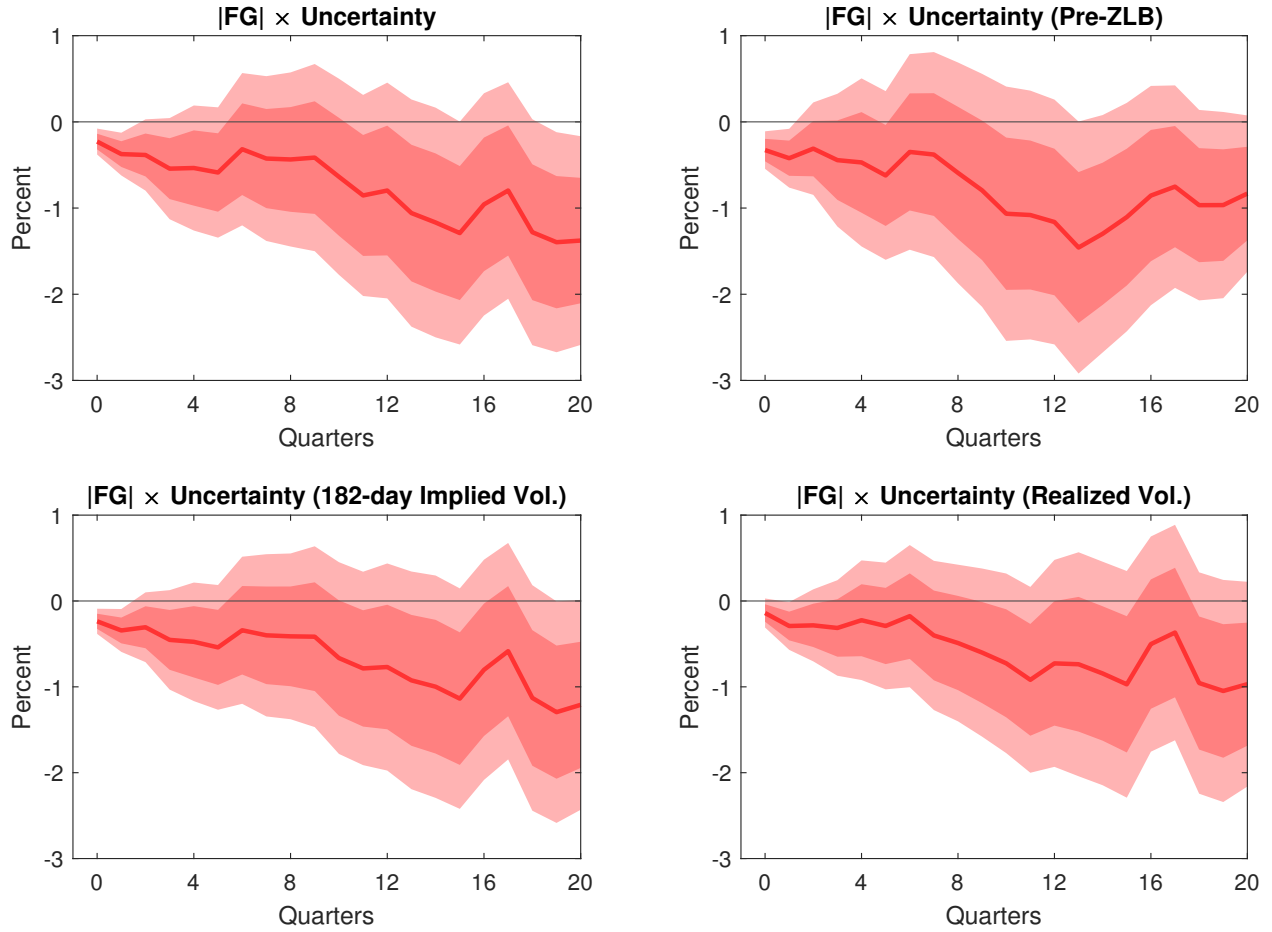
$$H_0: \gamma_{FG^+} = -\gamma_{FG^-} \quad | \text{p-value} = 0.209$$

Notes: This figure displays the dynamics of the interaction coefficients between firm-level uncertainty and the forward guidance shock from estimating different versions of specification (2). The top row shows the results of Figure 8, again. The bottom row shows the interaction terms from estimating separately positive and negative shocks, i.e. $\gamma_{FG^+}^{(h)}$ and $\gamma_{FG^-}^{(h)}$ from

$$\begin{aligned} \Delta_h \log(k_{i,t+h}) = & \alpha_i^{(h)} + \alpha_{n,t}^{(h)} + \alpha_{i,fq}^{(h)} + \sum_{\zeta_t \in \{\varepsilon_t^{FG^+}, \varepsilon_t^{FG^-}\}} \gamma_{\zeta}^{(h)} (\zeta_t \times uc_{i,t-1}) + \Gamma_{\zeta}^{(h)} (\zeta_t \times Z_{i,t-1}) \\ & + \theta^{(h)} uc_{i,t-1} + \Phi^{(h)} Z_{i,t-1} + \nu_{i,t+h}, \quad \zeta_t \in \left\{ \varepsilon_t^{FG}, \left| \varepsilon_t^{FG} \right| \right\}, \end{aligned}$$

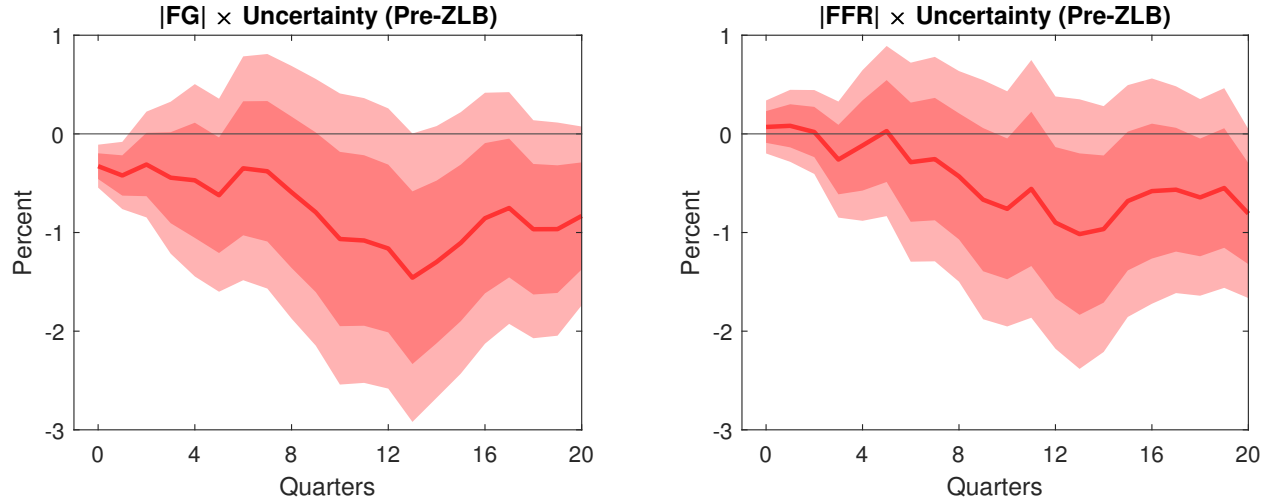
where $\varepsilon_t^{FG^+} = \max\{\varepsilon_t^{FG}, 0\}$ and $\varepsilon_t^{FG^-} = \min\{\varepsilon_t^{FG}, 0\}$. To be precise, $-\gamma_{FG^-}^{(h)}$ is shown so that the response corresponds to one conditional on a negative shock. The displayed estimates corresponds to a one standard deviation forward guidance shock and a one standard deviation increase in firm-level uncertainty relative to the mean. Dark and light bands show 68 percent and 90 percent confidence bands, respectively. Standard errors are clustered at quarter and firm level.

Figure D2: Heterogeneity in Responses of Capital — Robustness



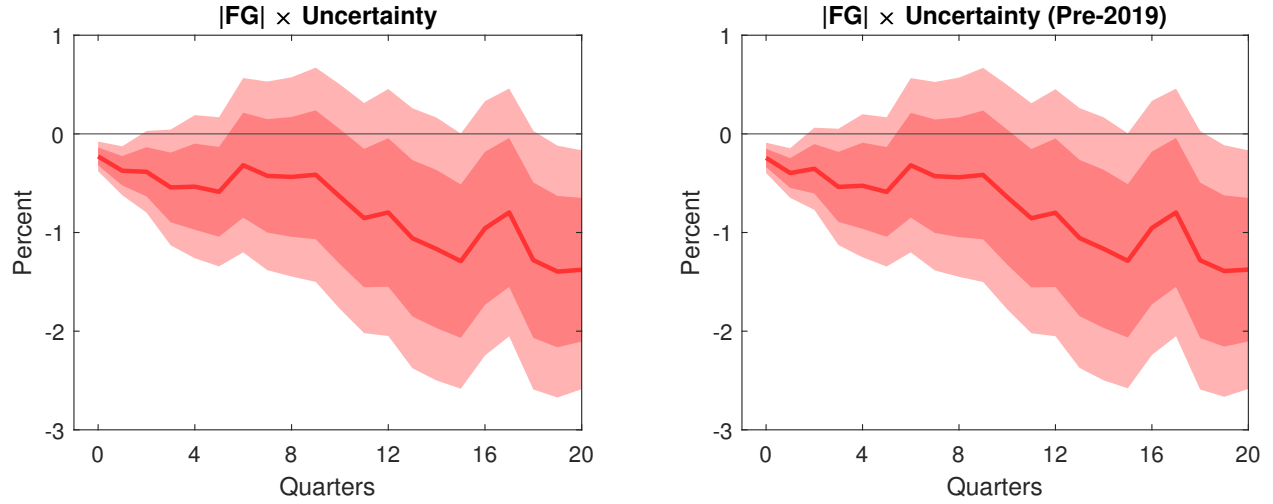
Notes: This figure provides multiple robustness checks with respect to the absolute value version of specification (2). The top left-hand panel shows the baseline estimates from Figure 8. The top right-hand panel shows the same estimates when ending the sample in 2007Q4. The bottom two panels show estimates for two alternative measures of uncertainty — 182-day implied volatility on the left and realized volatility on the right. Appendix C provides details on the construction of each measure. The displayed estimates corresponds to a one standard deviation forward guidance shock and a one standard deviation increase in firm-level uncertainty relative to the mean. Dark and light bands show 68 percent and 90 percent confidence bands, respectively. Standard errors are clustered at quarter and firm level.

Figure D3: Comparison of Forward Guidance Shock with Federal Funds Rate Shock — preZLB



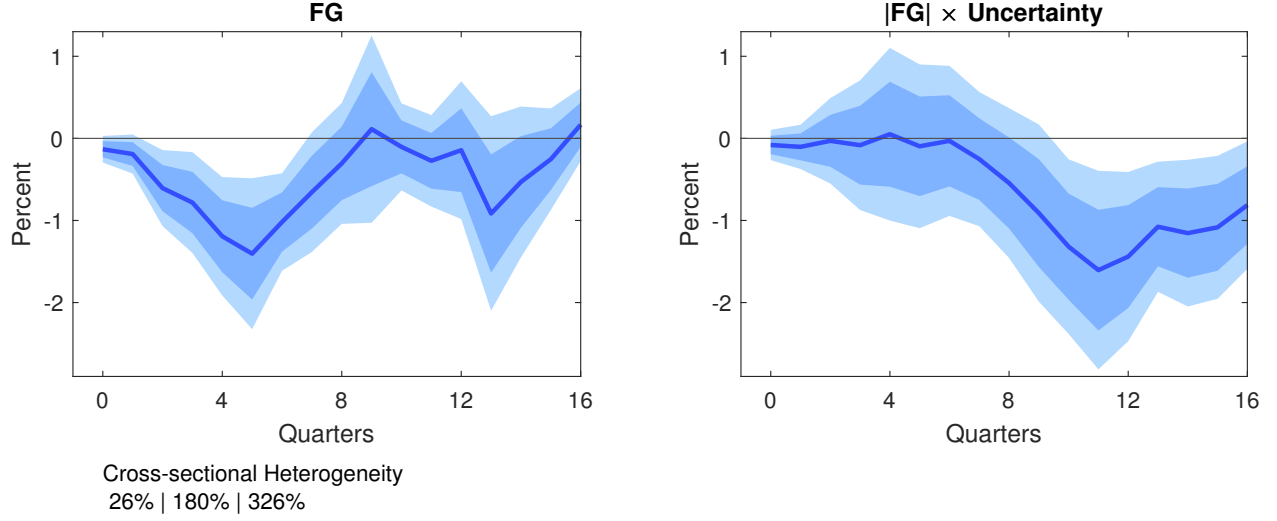
Notes: This figure displays the dynamics of the interaction coefficients between firm-level uncertainty and the federal funds rate and forward guidance shock from estimating the absolute value version of specification (2) for the pre-ZLB period, i.e. ending the sample in 2007. The left panel shows estimates for the absolute value of the forward guidance shock, i.e. $\gamma_{|FG|}^{(h)}$, whereas the right panel shows estimates for the absolute value of the federal funds rate shock, i.e. $\gamma_{|FFR|}^{(h)}$. The displayed estimates corresponds to a one standard deviation shock and a one standard deviation increase in firm-level uncertainty relative to the mean. Dark and light bands show 68 percent and 90 percent confidence bands, respectively. Standard errors are clustered at quarter and firm level.

Figure D4: Heterogeneity in Responses of Capital — pre-2019



Notes: This figure displays the dynamics of the interaction coefficients between firm-level uncertainty and the forward guidance shock from estimating the absolute value version of specification (2). The left panel shows estimates for the absolute value of the forward guidance shock, i.e. $\gamma_{|FG|}^{(h)}$ for the baseline version, whereas the right panel shows estimates for the absolute value of the forward guidance shock where last year in the sample 2019 has been excluded, i.e. $\gamma_{|FG|}^{(h)}$. The displayed estimates corresponds to a one standard deviation shock and a one standard deviation increase in firm-level uncertainty relative to the mean. Dark and light bands show 68 percent and 90 percent confidence bands, respectively. Standard errors are clustered at quarter and firm level.

Figure D5: Heterogeneity in Responses of Capital — With Main Effect

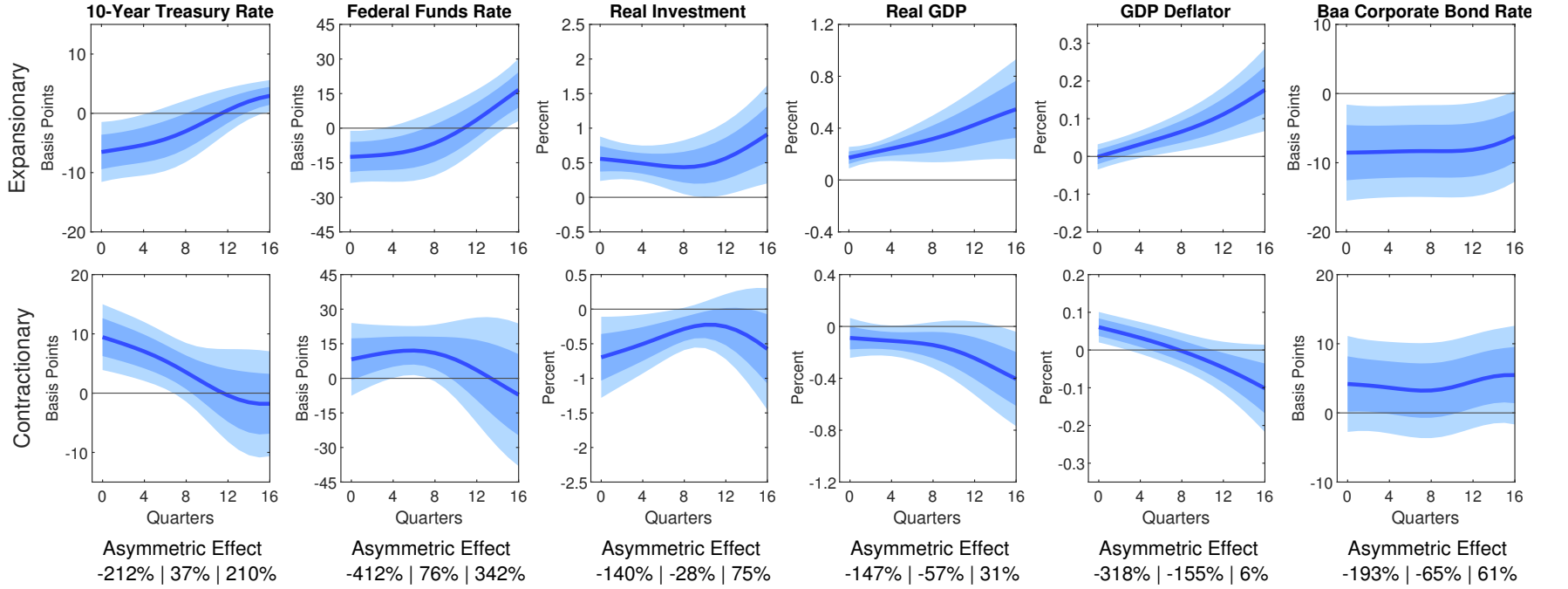


Notes: This figure displays the dynamic effect of the forward guidance shock, and the dynamics of the interaction coefficients between firm-level uncertainty and the forward guidance shock from estimating the following specification from

$$\begin{aligned} \Delta_h \log(k_{i,t+h}) = & \alpha_i^{(h)} + \alpha_{cq}^{(h)} + \alpha_{i,fq}^{(h)} + \beta^{(h)} \varepsilon_t^{FG} + \gamma^{(h)} \left(\left| \varepsilon_t^{FG} \right| \times uc_{i,t-1} \right) + \Gamma^{(h)} \left(\left| \varepsilon_t^{FG} \right| \times Z_{i,t-1} \right) \\ & + \theta^{(h)} uc_{i,t-1} + \Phi^{(h)} Z_{i,t-1} + \nu_{i,t+h}, \end{aligned} \quad (D1)$$

where $\alpha_{cq}^{(h)}$ is a calendar quarter fixed effect, and the rest of the variables is defined as in specification (2). The displayed estimates corresponds to a one standard deviation forward guidance shock and a one standard deviation increase in firm-level uncertainty relative to the mean. Dark and light bands show 68 percent and 90 percent confidence bands, respectively. Driscoll-Kraay standard errors are employed. The cross-sectional heterogeneity values are calculated as $\frac{1}{16} \sum_{h=1}^{16} \left(uc_{25}^{75} \times \tilde{\gamma}^{(h)} \right) / \bar{\beta}$, where $\bar{\beta} = \frac{1}{16} \sum_{h=1}^{16} \beta^{(h)}$, and $uc_{25}^{75} = 1.4$ is the amount of standard deviations needed from the 25th to 75th percentile. The middle value is calculated from the point estimate $\gamma^{(h)}$, whereas the left and right values are constructed from the boundaries of the 68 percent confidence band, i.e. $\tilde{\gamma}^{(h)} \in \left\{ \gamma_{16}^{(h)}, \gamma^{(h)}, \gamma_{84}^{(h)} \right\}$.

Figure D6: Response of Macro Aggregates to expansionary and contractionary FG shock



Notes: This figure shows impulse responses of macroeconomic aggregates to a negative and positive, one standard deviation FG shock. The estimates are obtained from the following local projection specification

$$y_{t+h} - y_{t-1} = \alpha^{(h)} + \beta_+^{(h)} \varepsilon_t^{FG+} + \beta_-^{(h)} \varepsilon_t^{FG-} + \sum_{j=1}^4 \delta_j^{(h)} X_{t-j} + \nu_{t+h}, \quad (D2)$$

following the methodology by [Barnichon and Brownlees \(2019\)](#). Dark and light bands show 68 percent and 90 percent confidence bands, respectively. The confidence bands are calculated following the recommendations by [Lazarus et al. \(2018\)](#). The three values under asymmetric effect are calculated as $\frac{1}{16} \sum_{h=1}^{16} \frac{\beta_{+,16}^{(h)} - \beta_{-,84}^{(h)}}{\bar{\beta}_{\pm}}$, $\frac{1}{16} \sum_{h=1}^{16} \frac{\beta_{+,84}^{(h)} - \beta_{-,16}^{(h)}}{\bar{\beta}_{\pm}}$, and $\frac{1}{16} \sum_{h=1}^{16} \frac{\beta_{+,16}^{(h)} + \beta_{-,84}^{(h)}}{\bar{\beta}_{\pm}}$, where $\bar{\beta}_{\pm} = \frac{1}{16} \sum_{h=1}^{16} 0.5 (\beta_+^{(h)} + \beta_-^{(h)})$.

E Model Appendix

In this appendix, I describe the model framework, the steady state, as well as the set of log-linearized equations which characterizes the equilibrium. Note the Appendix is based on the main text which is particularly important to understand the households' setups. Note that sector-specific variables are denoted in per capita, i.e. per person in a given sector.

E.1 Monetary Policy

The central bank sets the nominal interest R_t according to following generalized Taylor rule

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi} \right)^{\theta_\Pi} \left(\frac{Y_t}{Y} \right)^{\theta_Y} e^{\theta_t}, \quad (\text{E1})$$

where R is the steady state of the nominal interest rate, and Π_t/Π and Y_t/Y denote deviations from the steady state for inflation and output, respectively. Note that the paper's focus is the transmission of monetary policy. Hence, I can specify the Taylor rule in terms of output rather than the output gap (output - natural output) since the natural level of output is unaffected. The policy deviation θ_t follows a news structure as in [Laséen and Svensson \(2011\)](#)

$$\theta_t = \sum_{i=0}^{15} \varepsilon_{R,t-i}^i, \quad (\text{E2})$$

with $\varepsilon_{R,t}^i \sim N(0, \sigma_R^2)$. The specification implies that the central bank can communicate deviations of up to four years, i.e. sixteen quarters.

Forward Guidance I define “forward guidance” as a central bank announcement of a specific path of the interest rate over the next quarters, where as in [Campbell et al. \(2019\)](#), the central bank cannot perfectly communicate this path. In particular, forward guidance is defined as a set of signals

$$S_t = [s_t^0 \quad s_t^1 \quad \dots \quad s_t^{15}],$$

with $\exists j$ such that $s_t^j \neq 0$ and $j \geq 1$. Here, the current deviation (or rate) is perfectly observed, i.e.

$$s_t^0 = \varepsilon_{R,t}^0,$$

and future deviations are unobserved, i.e. for $j \geq 1$

$$s_t^j = \varepsilon_{R,t}^j + \eta_t,$$

with $\eta_t \sim N(0, \sigma_\eta^2)$. Note that conventional monetary policy is simply $\varepsilon_{R,t}^0$ in this model.

E.2 Households

E.2.1 Consumption and Leisure Decision

High-Uncertainty Sector As explained Section 5.3, the H-household's optimization problem can be written as

$$\begin{aligned}
& \max_{\{C_t^h, B_{Ht}, L_{Ht}, \bar{K}_{Ht}, I_{Ht}, u_{Ht}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{(C_t^h - \xi C_{t-1}^h)^{1-\sigma}}{1-\sigma} - \varrho \frac{(L_{Ht})^{1+\chi}}{1+\chi} \right] \\
& \text{s.t. } P_t C_t^h + P_{Ht} I_{Ht} + \frac{1}{R_t^*} B_{Ht} \leq MRS_{Ht} L_{Ht} + R_{k,Ht} u_{Ht} \bar{K}_{Ht-1} - P_{Ht} a(u_{Ht}) \bar{K}_{Ht-1} \\
& \quad + DIV_{Ht}^p + DIV_{Ht}^w + B_{Ht-1} \quad (\beta^t \lambda_{Ht}) \\
& \quad \bar{K}_{Ht} \leq \left(1 - \frac{\kappa}{2} \left(\frac{I_{Ht}}{I_{Ht-1}} - 1 \right)^2 \right) I_{Ht} + (1-\delta) \bar{K}_{Ht-1} \quad (\beta^t \mu_{Ht}),
\end{aligned}$$

where P_{Ht} is the nominal price of the final good in sector H , I_{Ht} is the investment in sector H , B_t is bond holding of household- H , MRS_{Ht} is the nominal remuneration for the supply of labor, $R_{k,Ht}$ real rental rate on capital services in sector H , u_{Ht} is the utilization rate, \bar{K}_{Ht} is the physical capital stock, DIV_{Ht}^p , DIV_{Ht}^w .

The Langrangian can be then written as

$$\begin{aligned}
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t & \left\{ \left[\frac{(C_t^h - \xi C_{t-1}^h)^{1-\sigma}}{1-\sigma} - \varrho \frac{(L_{Ht})^{1+\chi}}{1+\chi} \right] \right. \\
& + \lambda_{Ht} \left[MRS_{Ht} L_{Ht} + R_{k,Ht} u_{Ht} \bar{K}_{Ht-1} - P_{Ht} a(u_{Ht}) \bar{K}_{Ht-1} \right. \\
& \quad \left. + DIV_{Ht}^p + DIV_{Ht}^w + B_{Ht-1} - P_t C_t^h - P_{Ht} I_{Ht} - \frac{1}{R_t^*} B_{Ht} \right] \\
& \left. + \mu_{Ht} \left[\left(1 - \frac{\kappa}{2} \left(\frac{I_{Ht}}{I_{Ht-1}} - 1 \right)^2 \right) I_{Ht} + (1-\delta) \bar{K}_{Ht-1} - \bar{K}_{Ht} \right] \right\}.
\end{aligned}$$

The first order conditions are then

$$\begin{aligned}
C_t^h : & \left(C_t^h - \xi C_{t-1}^h \right)^{-\sigma} - \xi \beta E_t \left[\left(C_{t+1}^h - \xi C_t^h \right)^{-\sigma} \right] = P_t \lambda_{Ht} \\
B_{Ht} : & \frac{1}{R_t^*} \lambda_{Ht} = \beta E_t [\lambda_{Ht+1}] \\
L_{Ht} : & \varrho(L_{Ht})^X = MRS_{Ht} \lambda_{Ht} \\
\bar{K}_{Ht} : & \mu_{Ht} = \beta E_t [\lambda_{Ht+1} (R_{k,Ht+1} u_{Ht+1} - P_{Ht+1} a(u_{Ht+1})) + (1 - \delta) \mu_{Ht+1}] \\
I_{Ht} : & \lambda_{Ht} P_{Ht} = \mu_{Ht} \left(1 - \frac{\kappa}{2} \left(\frac{I_{Ht}}{I_{Ht-1}} - 1 \right)^2 - \kappa \left(\frac{I_{Ht}}{I_{Ht-1}} - 1 \right) \frac{I_{Ht}}{I_{Ht-1}} \right) \\
& + \beta E_t \left[\mu_{Ht+1} \kappa \left(\frac{I_{Ht+1}}{I_{Ht}} - 1 \right) \left(\frac{I_{Ht+1}}{I_{Ht}} \right)^2 \right] \\
u_{Ht} : & R_{k,Ht} = P_{Ht} a'(u_{Ht}) \\
\beta^t \lambda_{Ht} : & \dots \\
\beta^t \mu_{Ht} : & \dots
\end{aligned}$$

Let $\Lambda_{Ht,t+1} = \beta(\lambda_{Ht+1}/\lambda_{Ht})\Pi_{t+1}$ be the real stochastic discount factor, $\Xi_{Ht} = \mu_{Ht}/(\lambda_{Ht}P_t)$ Tobin's q, $mrs_{Ht} = mrs_{Ht}/P_t$ the real remuneration for the supply of labor, $r_{k,Ht} = R_{k,Ht}/P_t$ the real rental rate on capital services, $p_{Ht} = P_{Ht}/P_t$ the relative price of the final good in sector H . Then we can write the following system of equations characterizing the consumption leisure decision

$$\varrho(L_{Ht})^X = mrs_{Ht} ((J_{Ht})^{-\sigma} - \xi \beta E_t [(J_{Ht+1})^{-\sigma}]) \quad (E3)$$

$$J_{Ht} = C_t^h - \xi C_{t-1}^h \quad (E4)$$

$$1 = R_t^* E_t [\Lambda_{Ht,t+1} (\Pi_{t+1})^{-1}] \quad (E5)$$

$$\Lambda_{Ht,t+1} = \beta E_t \left[\frac{(J_{Ht+1})^{-\sigma} - \xi \beta (J_{Ht+2})^{-\sigma}}{(J_{Ht})^{-\sigma} - \xi \beta (J_{Ht+1})^{-\sigma}} \right] \quad (E6)$$

$$\Xi_{Ht} = E_t [\Lambda_{Ht,t+1} (r_{k,Ht+1} u_{Ht+1} - p_{Ht+1} a(u_{Ht+1}) + \Xi_{Ht+1} (1 - \delta))] \quad (E7)$$

$$\begin{aligned}
p_{Ht} = \Xi_{Ht} & \left(1 - \frac{\kappa}{2} \left(\frac{I_{Ht}}{I_{Ht-1}} - 1 \right)^2 - \kappa \left(\frac{I_{Ht}}{I_{Ht-1}} - 1 \right) \frac{I_{Ht}}{I_{Ht-1}} \right) \\
& + \beta E_t \left[\Lambda_{Ht,t+1} \Xi_{Ht+1} \kappa \left(\frac{I_{Ht+1}}{I_{Ht}} - 1 \right) \left(\frac{I_{Ht+1}}{I_{Ht}} \right)^2 \right]
\end{aligned} \quad (E8)$$

$$r_{k,Ht} = p_{Ht} a'(u_{Ht}) \quad (E9)$$

$$\bar{K}_{Ht} = \left(1 - \frac{\kappa}{2} \left(\frac{I_{Ht}}{I_{Ht-1}} - 1 \right)^2 \right) I_{Ht} + (1 - \delta) \bar{K}_{Ht-1}. \quad (E10)$$

Note that the budget constraint will be used in the aggregation section.

Low-Uncertainty Sector Similarly, system of equations for the L-household is given by

$$\varrho(L_{Lt})^x = mrs_{Lt} \left((J_{Lt})^{-\sigma} - \xi\beta (J_{Ht+1})^{-\sigma} \right) \quad (\text{E11})$$

$$J_{Lt} = C_t^l - \xi C_{t-1}^l \quad (\text{E12})$$

$$1 = R_t E_t \left[\Lambda_{Lt,t+1} (\Pi_{t+1})^{-1} \right] \quad (\text{E13})$$

$$\Lambda_{Lt,t+1} = \beta E_t \left[\frac{(J_{Lt+1})^{-\sigma} - \xi\beta (J_{Lt+2})^{-\sigma}}{(J_{Lt})^{-\sigma} - \xi\beta (J_{Lt+1})^{-\sigma}} \right] \quad (\text{E14})$$

$$\Xi_{Lt} = E_t [\Lambda_{Lt,t+1} (r_{k,Lt+1} u_{Lt+1} - p_{Lt+1} a(u_{Lt+1}) + \Xi_{Lt+1} (1 - \delta))] \quad (\text{E15})$$

$$\begin{aligned} p_{Lt} = & \Xi_{Lt} \left(1 - \frac{\kappa}{2} \left(\frac{I_{Lt}}{I_{Lt-1}} - 1 \right)^2 - \kappa \left(\frac{I_{Lt}}{I_{Lt-1}} - 1 \right) \frac{I_{Lt}}{I_{Lt-1}} \right) \\ & + E_t \left[\Lambda_{Lt,t+1} \Xi_{Lt+1} \kappa \left(\frac{I_{Lt+1}}{I_{Lt}} - 1 \right) \left(\frac{I_{Lt+1}}{I_{Lt}} \right)^2 \right] \end{aligned} \quad (\text{E16})$$

$$r_{k,Lt} = p_{Lt} a'(u_{Lt}) \quad (\text{E17})$$

$$\bar{K}_{Lt} = \left[1 - \frac{\kappa}{2} \left(\frac{I_{Lt}}{I_{Lt-1}} - 1 \right)^2 \right] I_{Lt} + (1 - \delta) \bar{K}_{Lt-1} \quad (\text{E18})$$

E.2.2 Consumption Basket

High-Uncertainty Sector Given the consumption C_t^h in the composite good, the H-household minimizes the cost of attaining this level, i.e.

$$\begin{aligned} & \min_{\{C_{Ht}^h, C_{Lt}^h\}} P_{Ht} C_{Ht}^h + P_{Lt} C_{Lt}^h \\ \text{s.t. } & C_t^h = \left[n^{\frac{1}{\eta}} \left(C_{Ht}^h \right)^{\frac{\eta-1}{\eta}} + (1-n)^{\frac{1}{\eta}} \left(C_{Lt}^h \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} (P_t), \end{aligned}$$

where P_t is defined as

$$P_t = [n(P_{Ht})^{1-\eta} + (1-n)(P_{Lt})^{1-\eta}]^{\frac{1}{\eta-1}}.$$

The first order conditions are given by

$$\begin{aligned} C_{Ht}^h : C_{Ht}^h &= n \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} C_t^h \\ C_{Lt}^h : C_{Lt}^h &= (1-n) \left(\frac{P_{Lt}}{P_t} \right)^{-\eta} C_t^h, \end{aligned}$$

which leads to the following sector-specific demands

$$C_{Ht}^h = n (p_{Ht})^{-\eta} C_t^h \quad (\text{E19})$$

$$C_{Lt}^h = (1 - n) (p_{Lt})^{-\eta} C_t^h \quad (\text{E20})$$

Low-Uncertainty Sector Similarly, the demand equations for the L-household are given by

$$C_{Ht}^l = n (p_{Ht})^{-\eta} C_t^l \quad (\text{E21})$$

$$C_{Lt}^l = (1 - n) (p_{Lt})^{-\eta} C_t^l \quad (\text{E22})$$

E.3 Labor Market

E.3.1 Labor Packer

High-Uncertainty Sector The labor packer minimizes the total cost subject to the constraint of meeting the total demand for labor in the production. The labor demand in the H-sector is given by nL_{Ht}^d , where L_{Ht}^d denotes the per capita demand:

$$\begin{aligned} & \min_{\{L_{Ht}(i)\}} \int_0^1 W_{Ht}(i) L_{Ht}(i) di \\ \text{s.t. } & nL_{Ht}^d \leq \left[\int_0^1 (L_{Ht}(i))^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} (W_{Ht}), \end{aligned}$$

where the wage W_{Ht} is given

$$W_{Ht} = \left(\int_0^1 W_{Ht}(i)^{1 - \varepsilon_w} di \right)^{\frac{1}{1 - \varepsilon_w}}$$

From the first order condition, we can derive the demand for each differentiated $L_{Ht}(i)$,

$$L_{Ht}(i) : L_{Ht}(i) = \left(\frac{W_{Ht}(i)}{W_{Ht}} \right)^{-\varepsilon_w} nL_{Ht}^d.$$

Low-Uncertainty Sector Similarly, we can derive

$$L_{Lt}(i) : L_{Lt}(i) = \left(\frac{W_{Lt}(i)}{W_{Lt}} \right)^{-\varepsilon_w} (1 - n) L_{Lt}^d /$$

E.3.2 Labor Unions

High-Uncertainty Sector Labor union i repackages labor from the household one-for-one for resale. Union i 's nominal profit is given by

$$DIV_{H,t}^w(i) = W_{Ht}(i) L_{Ht}(i) - MRS_{Ht} L_{Ht}(i)$$

and dividing by P_t yields real profits

$$div_{H,t}^w(i) = \frac{W_{Ht}(i)}{P_t} L_{Ht}(i) - mrs_{Ht} L_{Ht}(i)$$

Unions are subject to Calvo wage-setting, where in each period, a firm can adjust its price with a fixed probability of $1 - \omega_w$:

$$W_{Ht}(i) = \begin{cases} W_{Ht}^r(i) & \text{if } W_{Ht}(i) \text{ chosen optimally} \\ W_{Ht-1}(i) & \text{otherwise} \end{cases}.$$

Then, wage of union i in period $t + s$ when it was last able to adjust the wage in period t is given by

$$W_{Ht+s|t}(i) = W_{Ht}^r(i).$$

Finally, the profit maximization is given by

$$\begin{aligned} & \max_{\{W_{Ht}^r(i)\}} E_t \sum_{s=0}^{\infty} (\omega_w)^s \Lambda_{Ht,t+s} \left(\frac{W_{Ht}^r(i)}{P_{t+s}} - mrs_{Ht+s} \right) L_{Ht+s|t}(i) \\ & \text{s.t. } L_{Ht+s|t}(i) = \left(\frac{W_{Ht}^r(i)}{W_{Ht+s}} \right)^{-\varepsilon_w} n L_{Ht+s}^d \end{aligned}$$

Lagrangian: Plugging the constraint into the objective function, we can write

$$\begin{aligned} \mathcal{L} &= E_t \sum_{s=0}^{\infty} (\omega_w)^s \Lambda_{Ht,t+s} \left[\left(\frac{W_{Ht}^r(i)}{P_{t+s}} - mrs_{Ht+s} \right) \left(\frac{W_{Ht}^r(i)}{W_{Ht+s}} \right)^{-\varepsilon_w} n L_{Ht+s}^d \right] \\ &= n E_t \sum_{s=0}^{\infty} (\omega_w)^s \Lambda_{Ht,t+s} \left[(W_{Ht}^r(i))^{1-\varepsilon_w} P_{t+s}^{-1} W_{Ht+s}^{\varepsilon_w} L_{Ht+s}^d \right. \\ & \quad \left. - mrs_{Ht+s} \left(\frac{W_{Ht}^r(i)}{W_{Ht+s}} \right)^{-\varepsilon_w} L_{Ht+s}^d \right] \end{aligned}$$

The first-order conditions is given by

$$\begin{aligned} W_{Ht}^r : 0 &= n E_t \sum_{s=0}^{\infty} (\omega_w)^s \Lambda_{Ht,t+s} \left[(1 - \varepsilon_w) W_{Ht}^r(i)^{-\varepsilon_w} P_{t+s}^{-1} W_{Ht+s}^{\varepsilon_w} L_{Ht+s}^d \right. \\ & \quad \left. + \varepsilon_w mrs_{Ht+s} W_{Ht}^r(i)^{-\varepsilon_w - 1} W_{Ht+s}^{\varepsilon_w} L_{Ht+s}^d \right], \end{aligned}$$

The optimal reset wage is given by

$$W_{Ht}^r = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{E_t \sum_{s=0}^{\infty} (\omega_w)^s \Lambda_{Ht,t+s} [mrs_{Ht+s} P_{t+s}^{\varepsilon_w} W_{Ht+s}^{\varepsilon_w} L_{Ht+s}^d]}{E_t \sum_{s=0}^{\infty} (\omega_w)^s \Lambda_{Ht,t+s} [P_{t+s}^{\varepsilon_w - 1} W_{Ht+s}^{\varepsilon_w} L_{Ht+s}^d]}$$

which can be written recursively as

$$W_{Ht}^r = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{F_{1,Ht}}{F_{2,Ht}},$$

where

$$\begin{aligned} F_{1,Ht} &= mrs_{Ht} P_t^{\varepsilon_w} W_{Ht}^{\varepsilon_w} L_{Ht}^d + \omega_w E_t [\Lambda_{Ht,t+1} F_{1,Ht+1}], \\ F_{2,Ht} &= W_{Ht}^{\varepsilon_w} P_{t+s}^{\varepsilon_w - 1} L_{Ht}^d + \omega_w E_t [\Lambda_{Ht,t+1} F_{2,Ht+1}]. \end{aligned}$$

Defining $f_{1,Ht} = F_{1,Ht} / (P_{Ht})^{\varepsilon_w}$ and $f_{2,Ht} = F_{2,Ht} / (P_{Ht})^{\varepsilon_w - 1}$, the optimal reset wage is given by

$$w_{Ht}^r = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{f_{1,Ht}}{f_{2,Ht}} p_{Ht} \quad (\text{E23})$$

where

$$f_{1,Ht} = mrs_{Ht} (p_{Ht})^{-\varepsilon_w} (w_{Ht})^{\varepsilon_w} L_{Ht}^d + \omega_w E_t [\Lambda_{Ht,t+1} (\Pi_{Ht+1})^{\varepsilon_w} f_{1,Ht+1}] \quad (\text{E24})$$

$$f_{2,Ht} = (p_{Ht})^{1-\varepsilon_w} (w_{Ht})^{\varepsilon_w} L_{Ht}^d + \omega_w E_t [\Lambda_{Ht,t+1} (\Pi_{Ht+1})^{\varepsilon_w - 1} f_{2,Ht+1}]. \quad (\text{E25})$$

Low-Uncertainty Sector

$$w_{Lt}^r = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{f_{1,Lt}}{f_{2,Lt}} p_{Lt} \quad (\text{E26})$$

$$f_{1,Lt} = mrs_{Lt} (p_{Lt})^{-\varepsilon_w} (w_{Lt})^{\varepsilon_w} L_{Lt}^d + \omega_w E_t [\Lambda_{Lt,t+1} (\Pi_{Lt+1})^{\varepsilon_w} f_{1,Lt+1}] \quad (\text{E27})$$

$$f_{2,Lt} = (p_{Lt})^{\varepsilon_w - 1} (w_{Lt})^{\varepsilon_w} L_{Lt}^d + \omega_w E_t [\Lambda_{Lt,t+1} (\Pi_{Lt+1})^{\varepsilon_w - 1} f_{2,Lt+1}]. \quad (\text{E28})$$

E.4 Production

E.4.1 Final Good Producer

High-Uncertainty Sector The final good producer minimizes the total cost subject to the constraint of meeting the demand for the product:

$$\begin{aligned} \min_{\{Y_{Ht}(j)\}} & \int_0^1 P_{Ht}(j) Y_{Ht}(j) dj \\ \text{s.t. } & Y_{Ht} \leq \left[\int_0^1 (Y_{Ht}(j))^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} (P_{Ht}). \end{aligned}$$

where

$$P_{Ht} = \left(\int_0^1 P_{Ht}(j)^{1-\varepsilon_p} dj \right)^{\frac{1}{1-\varepsilon_p}}$$

The first order condition w.r.t $Y_{Ht}(j)$ yields the following demand curve:

$$Y_{Ht}(j) = \left(\frac{P_{Ht}(j)}{P_{Ht}} \right)^{-\varepsilon_p} n Y_{Ht}.$$

Low-Uncertainty Sector

$$Y_{Lt}(j) = \left(\frac{P_{Lt}(j)}{P_{Lt}} \right)^{-\varepsilon_p} (1 - n) Y_{Lt}.$$

E.4.2 Intermediate Goods Producers

Except for different SDFs, the firms in the high- and low-uncertainty sector an identical environment.

Cost Minimization

High-Uncertainty Sector The intermediate goods firm j 's cost minimization is given by:

$$\begin{aligned} \min_{\{L_{Ht}(j), K_{Ht}(j)\}} & W_{Ht} L_{Ht}(j) + R_{k,Ht} K_{Ht}(j) \\ \text{s.t. } & Y_{Ht}(j) \leq K_{Ht}(j)^\alpha L_{Ht}(j)^{1-\alpha} \quad (MC_{Ht}(j)). \end{aligned}$$

Lagrangian:

$$\mathcal{L} = -W_{Ht} L_{Ht}(j) - R_{k,Ht} K_{Ht}(j) + MC_{Ht}(j) \left(K_{Ht}(j)^\alpha L_{Ht}(j)^{1-\alpha} - Y_{Ht}(j) \right)$$

The first order conditions are given by

$$\begin{aligned} K_{Ht}(j) : R_{k,Ht} &= \alpha MC_{Ht}(j) \left(\frac{K_{Ht}(j)}{L_{Ht}(j)} \right)^{\alpha-1} \\ L_{Ht}(j) : W_{Ht} &= (1-\alpha) MC_{Ht}(j) \left(\frac{K_{Ht}(j)}{L_{Ht}(j)} \right)^\alpha \\ MC_{Ht}(j) : Y_{Ht}(j) &= K_{Ht}(j)^\alpha L_{Ht}(j)^{1-\alpha} \end{aligned}$$

We can rewrite them as

$$\begin{aligned} \frac{W_{Ht}}{R_{k,Ht}} &= \frac{1-\alpha}{\alpha} \frac{K_{Ht}(j)}{L_{Ht}(j)} \\ \frac{w_{Ht}}{r_{k,Ht}} &= \frac{1-\alpha}{\alpha} \frac{K_{Ht}(j)}{L_{Ht}(j)} \\ mc_{Ht}(j) &= \frac{w_{Ht}}{(1-\alpha) \left(\frac{K_{Ht}(j)}{L_{Ht}(j)} \right)^\alpha} \end{aligned}$$

Lastly, the j 's can be dropped since all intermediate goods producers are facing the same wage and capital return

$$\frac{w_{Ht}}{r_{k,Ht}} = \frac{1-\alpha}{\alpha} \frac{K_{Ht}}{L_{Ht}}, \quad (\text{E29})$$

$$mc_{Ht} = \frac{w_{Ht}}{(1-\alpha) \left(\frac{K_{Ht}}{L_{Ht}} \right)^\alpha}. \quad (\text{E30})$$

Low-Uncertainty Sector

$$\frac{w_{Lt}}{R_{k,Lt}} = \frac{1-\alpha}{\alpha} \frac{K_{Lt}}{L_{Lt}} \quad (\text{E31})$$

$$mc_{Lt} = \frac{w_{Lt}}{(1-\alpha) \left(\frac{K_{Lt}}{L_{Lt}} \right)^\alpha} \quad (\text{E32})$$

Price Setting

High-Uncertainty Sector Firm j 's nominal profits is given by

$$DIV_{Ht}^p(j) = P_{Ht}(j) Y_{Ht}(j) - W_{Ht} L_{Ht}(j) - R_{k,Ht} K_{Ht}(j),$$

and dividing by P_t yields real profits

$$div_{Ht}^p(j) = p_{Ht}(j) Y_{Ht}(j) - w_{Ht} L_{Ht}(j) - r_{k,Ht} K_{Ht}(j).$$

Using the optimality conditions from the cost minimization, we have

$$\begin{aligned} w_{Ht} L_{Ht}(j) + r_{k,Ht} K_{Ht}(j) &= (1 - \alpha) m c_{Ht}(j) K_{Ht}(j)^\alpha L_{Ht}(j)^{1-\alpha} + \alpha m c_{Ht}(j) K_{Ht}(j)^\alpha L_{Ht}(j)^{1-\alpha} \\ &= m c_{Ht} Y_{Ht}(j), \end{aligned}$$

and hence we can write the profit function as

$$\begin{aligned} div_{Ht}^p(j) &= p_{Ht}(j) Y_{Ht}(j) - m c_{Ht} Y_{Ht}(j) \\ &= \left(\frac{P_{Ht}(j)}{P_t} - m c_{Ht} \right) Y_{Ht}(j). \end{aligned}$$

Firms are subject to Calvo price-setting, where in each period, a firm can adjust its price with a fixed probability of $1 - \omega_p$:

$$P_{Ht}(j) = \begin{cases} P_{Ht}^r(j) & \text{if } P_{Ht}(j) \text{ chosen optimally} \\ P_{Ht-1}(j) & \text{otherwise} \end{cases}.$$

Then, price of firm in period $t + s$ which was able to adjust price in period t is given by

$$P_{Ht+s}(j) = P_{Ht}^r(j).$$

Finally, the profit maximization is given by

$$\begin{aligned} \max_{P_{Ht}^r(j)} E_t \sum_{s=0}^{\infty} \omega_p^s \Lambda_{Ht,t+s} \left[\left(\frac{P_{Ht}^r(j)}{P_{Ht+s}} - m c_{Ht+s} \right) Y_{Ht+s|t}(j) \right] \\ \text{s.t. } Y_{Ht+s|t}(j) = \left(\frac{P_{Ht}^r(j)}{P_{Ht+s}} \right)^{-\varepsilon_p} n Y_{Ht+s}. \end{aligned}$$

Lagrangian: Plugging the constraint into the objective function, we can write

$$\begin{aligned} \mathcal{L} &= E_t \sum_{s=0}^{\infty} (\omega_p)^s \Lambda_{Ht,t+s} \left[\left(\frac{P_{Ht}^r(j)}{P_{Ht+s}} - m c_{Ht+s} \right) \left(\frac{P_{Ht}^r(j)}{P_{Ht+s}} \right)^{-\varepsilon_p} n Y_{Ht+s} \right] \\ &= n E_t \sum_{s=0}^{\infty} (\omega_p)^s \Lambda_{Ht,t+s} \left[\left(\frac{P_{Ht}^r(j)}{P_{Ht+s}} \right)^{1-\varepsilon_p} Y_{Ht+s} - m c_{Ht+s} \left(\frac{P_{Ht}^r(j)}{P_{Ht+s}} \right)^{-\varepsilon_p} Y_{Ht+s} \right] \end{aligned}$$

FOC:

$$0 = nE_t \sum_{s=0}^{\infty} (\omega_p)^s \Lambda_{Ht,t+s} \left[(1 - \varepsilon_p) (P_{Ht}^r(j))^{-\varepsilon_p} (P_{Ht+s})^{-(1-\varepsilon_p)} Y_{Ht+s} \right. \\ \left. + \varepsilon_p mc_{Ht+s} (P_{Ht}^r(j))^{-\varepsilon_p-1} (P_{Ht+s})^{\varepsilon_p} Y_{Ht+s} \right]$$

The optimal reset price is given by

$$P_{Ht}^r = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{E_t \sum_{s=0}^{\infty} (\omega_p)^s \Lambda_{Ht,t+s} [mc_{Ht+s} (P_{Ht+s})^{\varepsilon_p} Y_{Ht+s}]}{E_t \sum_{s=0}^{\infty} (\omega_p)^s \Lambda_{Ht,t+s} [(P_{Ht+s})^{\varepsilon_p-1} Y_{Ht+s}]}$$

which be rewritten recursively as

$$P_{Ht}^r = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{X_{1,Ht}}{X_{2,Ht}},$$

where

$$X_{1,Ht} = mc_{Ht} (P_{Ht})^{\varepsilon_p} Y_{Ht} + \omega_p E_t [\Lambda_{Ht,t+1} X_{1,Ht+1}], \\ X_{2,Ht} = (P_{Ht})^{\varepsilon_p-1} Y_{Ht} + \omega_p E_t [\Lambda_{Ht,t+1} X_{2,Ht+1}].$$

System of equations: Defining $x_{1,Ht} = \frac{X_{1,Ht}}{P_{Ht}^{\varepsilon_p}}$ and $x_{2,Ht} = \frac{X_{2,Ht}}{P_{Ht}^{\varepsilon_p-1}}$, we rewrite this as

$$p_{Ht}^r = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{x_{1,Ht}}{x_{2,Ht}} p_{Ht}, \quad (\text{E33})$$

$$x_{1,Ht} = mc_{Ht} Y_{Ht} + \omega_p E_t [\Lambda_{Ht,t+1} (\Pi_{Ht+1})^{\varepsilon_p} x_{1,Ht+1}] \quad (\text{E34})$$

$$x_{2,Ht} = Y_{Ht} + \omega_p E_t [\Lambda_{Ht,t+1} (\Pi_{Ht+1})^{\varepsilon_p-1} x_{2,Ht+1}] \quad (\text{E35})$$

Low-Uncertainty Sector System of equations:

$$p_{Lt}^r = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{x_{1,Lt}}{x_{2,Lt}} p_{Lt} \quad (\text{E36})$$

$$x_{1,Lt} = mc_{Lt} Y_{Lt} + \omega_p E_t [\Lambda_{Lt,t+1} (\Pi_{Lt+1})^{\varepsilon_p} x_{1,Lt+1}] \quad (\text{E37})$$

$$x_{2,Lt} = Y_{Lt} + \omega_p E_t [\Lambda_{Lt,t+1} (\Pi_{Lt+1})^{\varepsilon_p-1} x_{2,Lt+1}] \quad (\text{E38})$$

E.5 Bond Market

As explained in Section 5.4, the price of a bond with maturity τ is given by

$$Q_t^{(\tau)} = ws_{Ht} Q_{HHt}^{(\tau)} + ws_{Lt} Q_{LLt}^{(\tau)}. \quad (\text{E39})$$

where $Q_{HHt}^{(\tau)}$ denotes the price under counterfactual economy with $n = 1$ and is given by

$$Q_{HHt}^{(\tau)} = E_t \left[\Lambda_{HHt,t+1} (\Pi_{HHt+1})^{-1} Q_{HHt+1}^{(\tau-1)} \right] \text{ for } \tau \in \{1, \dots, 40\}, \quad (\text{E40})$$

and $Q_{HHt}^{(0)} \equiv 1$.

and $Q_{LLt}^{(\tau)}$ denotes the price for $n = 0$ and given by

$$Q_{LLt}^{(\tau)} = E_t \left[\Lambda_{LLt,t+1} (\Pi_{LLt+1})^{-1} Q_{LLt+1}^{(\tau-1)} \right] \text{ for } \tau \in \{1, \dots, 40\}, \quad (\text{E41})$$

and $Q_{LLt}^{(0)} \equiv 1$.

Further, the wealth shares ws_{Ht} and ws_{Lt} are given by

$$ws_{Ht} = \frac{n wea_{Ht}}{n wea_{Ht} + (1-n) wea_{Lt}}, \quad (\text{E42})$$

and

$$ws_{Lt} = \frac{(1-n) wea_{Lt}}{n wea_{Ht} + (1-n) wea_{Lt}}, \quad (\text{E43})$$

where each household's real wealth is given by

$$wea_{Ht} = \bar{K}_{Ht} + b_{Ht}, \quad (\text{E44})$$

$$wea_{Lt} = \bar{K}_{Lt} + b_{Lt}. \quad (\text{E45})$$

Lastly, the yield of maturity τ is defined to be

$$R_t^{(\tau)} = \left(Q_t^{(\tau)} \right)^{-\frac{1}{\tau}} \text{ for } \tau \in \{1, \dots, 40\}. \quad (\text{E46})$$

E.6 Aggregation and Market Clearing

E.6.1 Labor Markets

High-Uncertainty Sector

- Market clearing: labor packer (demand) - unions/households (supply):

$$\begin{aligned} \int_0^1 \left(\frac{W_{Ht}(i)}{W_{Ht}} \right)^{-\varepsilon_w} n L_{Ht}^d di &= n L_{Ht} \\ L_{Ht}^d v_{Ht}^w &= L_{Ht}, \end{aligned} \quad (\text{E47})$$

where v_{Ht}^w measures the wage dispersion:

$$\begin{aligned} v_{Ht}^w &= \int_0^1 \left(\frac{W_{Ht}(i)}{W_{Ht}} \right)^{-\varepsilon_w} di \\ v_{Ht}^w &= (1 - \omega_w) \left(\frac{w_{Ht}^r}{w_{Ht}} \right)^{-\varepsilon_w} + \omega_w \Pi_t^{\varepsilon_w} \left(\frac{w_{Ht-1}}{w_{Ht}} \right)^{-\varepsilon_w} v_{Ht-1}^w \end{aligned} \quad (\text{E48})$$

- Market clearing: labor Packer (supply) - intermediate goods producers (demand):

$$L_{Ht}^d = \frac{1}{n} \int_0^1 L_{Ht}(j) dj.$$

- Aggregation of dividends:

$$\begin{aligned} div_{Ht}^w &= \frac{1}{n} \int_0^1 div_{Ht}^w(i) di \\ &= \frac{1}{n} \int_0^1 \frac{W_{Ht}(i)}{P_t} L_{Ht}(i) di - mrs_{Ht} \frac{1}{n} L_{Ht}(i) di \\ &= \frac{1}{n} \int_0^1 \frac{W_{Ht}(i)}{P_t} \left(\frac{W_{Ht}(i)}{W_{Ht}} \right)^{-\varepsilon_w} n L_{Ht}^d di - mrs_{Ht} L_{Ht} \\ &= L_{Ht}^d P_t^{-1} W_{Ht}^{\varepsilon_w} \int_0^1 W_{Ht}(i)^{1-\varepsilon_w} di - mrs_{Ht} L_{Ht} \\ &= w_{Ht} L_{Ht}^d - mrs_{Ht} L_{Ht}, \end{aligned}$$

where I used $\int_0^1 W_{Ht}(i)^{1-\varepsilon_w} di = W_{Ht}^{1-\varepsilon_w}$.

Low-Uncertainty Sector

- Market Clearing: labor packer (demand) - unions/household (supply):

$$L_{Lt}^d v_{Lt}^w = L_{Lt}, \quad (\text{E49})$$

where v_{Ht}^w measures the wage dispersion:

$$v_{Lt}^w = v_{Lt}^w = (1 - \omega_w) \left(\frac{w_{Lt}^r}{w_{Lt}} \right)^{-\varepsilon_w} + \omega_w \Pi_t^{\varepsilon_w} \left(\frac{w_{Lt-1}}{w_{Lt}} \right)^{-\varepsilon_w} v_{Lt-1}^w \quad (\text{E50})$$

- Market clearing: labor packer (supply) - intermediate goods producers (demand):

$$L_{Lt}^d = \frac{1}{1-n} \int_0^1 L_{Lt}(j) dj$$

- Aggregation of dividends:

$$div_{Lt}^w = w_{Lt} L_{Lt}^d - mrs_{Lt} L_{Lt}$$

E.6.2 Capital Markets

High-Uncertainty Sector

- Effective and physical capital:

$$K_{Ht} = u_{Ht} \bar{K}_{Ht} \quad (\text{E51})$$

- Market clearing: household (supply) - intermediate goods producers (Demand):

$$K_{Ht} = \frac{1}{n} \int_0^1 K_{Ht}(j) dj$$

Low-Uncertainty Sector

- Effective and physical capital:

$$K_{Lt} = u_{Lt} \bar{K}_{Lt} \quad (\text{E52})$$

- Market Clearing: household (supply) - intermediate goods producers (Demand):

$$K_{Lt} = \frac{1}{1-n} \int_0^1 K_{Ht}(j) dj$$

E.6.3 Goods Markets

High-Uncertainty Sector

- Aggregation of production function:

$$\begin{aligned} \frac{1}{n} \int_0^1 Y_{Ht}^{(d)}(j) dj &= \frac{1}{n} \int_0^1 Y_{Ht}^{(s)}(j) dj \\ \frac{1}{n} \int_0^1 \left(\frac{P_{Ht}(j)}{P_{Ht}} \right)^{-\varepsilon_p} n Y_{Ht} dj &= \frac{1}{n} \int_0^1 K_{Ht}(j)^\alpha L_{Ht}(j)^{1-\alpha} dj \\ &= \left(\frac{K_{Ht}}{L_{Ht}} \right)^\alpha \frac{1}{n} \int_0^1 L_{Ht}(j) dj \\ Y_{Ht} v_{Ht}^p &= (K_{Ht})^\alpha \left(L_{Ht}^d \right)^{1-\alpha}, \end{aligned} \quad (\text{E53})$$

where v_{Ht}^p measures the price dispersion:

$$\begin{aligned} v_{Ht}^p &= \int_0^1 \left(\frac{P_{Ht}(j)}{P_{Ht}} \right)^{-\varepsilon_p} dj \\ v_{Ht}^p &= (1 - \omega_p) \left(\frac{P_{Ht}^r}{P_{Ht}} \right)^{-\varepsilon_p} + \omega_p \left(\frac{P_{Ht-1}}{P_{Ht}} \right)^{-\varepsilon_p} v_{Ht-1}^p \\ v_{Ht}^p &= (1 - \omega_p) \left(\frac{p_{Ht}^r}{p_{Ht}} \right)^{-\varepsilon_p} + \omega_p (\Pi_t)^{\varepsilon_p} \left(\frac{p_{Ht-1}}{p_{Ht}} \right)^{-\varepsilon_p} v_{Ht-1}^p \end{aligned} \quad (\text{E54})$$

- Market clearing: firms (supply) - households (demand):

$$nY_{Ht} = nC_{Ht}^h + (1-n)C_{Ht}^l + nI_{Ht} \quad (\text{E55})$$

- Aggregation of intermediate goods producers' dividends:

$$\begin{aligned}
div_{Ht}^p &= \frac{1}{n} \int_0^1 div_{Ht}^p(j) dj \\
&= \frac{1}{n} \int_0^1 p_{Ht}(j) Y_{Ht}(j) dj - w_{Ht} \frac{1}{n} \int_0^1 L_{Ht}(j) dj - r_{k,Ht} \frac{1}{n} \int_0^1 K_{Ht}(j) dj \\
&= \frac{P_{Ht}}{P_t} Y_{Ht} \int_0^1 \left(\frac{P_{Ht}(j)}{P_{Ht}} \right)^{1-\varepsilon_p} dj - w_{Ht} L_{Ht}^d - r_{k,Ht} K_{Ht} \\
&= p_{Ht} Y_{Ht} - w_{Ht} L_{Ht}^d - r_{k,Ht} K_{Ht},
\end{aligned}$$

where I used $\int_0^1 \left(\frac{P_{Ht}(j)}{P_{Ht}} \right)^{1-\varepsilon_p} dj = P_{Ht}^{-(1-\varepsilon_p)} \int_0^1 (P_{Ht}(j))^{1-\varepsilon_p} dj = P_{Ht}^{-(1-\varepsilon_p)} P_{Ht}^{(1-\varepsilon_p)} = 1$.

Low-Uncertainty Sector

- Aggregation of production function:

$$Y_{Lt} v_{Lt}^p = (K_{Lt})^\alpha \left(L_{Lt}^d \right)^{1-\alpha} \quad (\text{E56})$$

where v_{Lt}^p measures the price dispersion:

$$\begin{aligned}
v_{Lt}^p &= \int_0^1 \left(\frac{P_{Lt}(j)}{P_{Lt}} \right)^{-\varepsilon_p} dj \\
v_{Lt}^p &= (1 - \omega_p) \left(\frac{p_{Lt}^r}{p_{Lt}} \right)^{-\varepsilon_p} + \omega_p (\Pi_t)^{\varepsilon_p} \left(\frac{p_{Lt-1}}{p_{Lt}} \right)^{-\varepsilon_p} v_{Lt-1}^p
\end{aligned} \quad (\text{E57})$$

- Market clearing: firms (supply) - households (demand):

$$(1 - n) Y_{Lt} = n C_{Lt}^h + (1 - n) C_{Lt}^l + (1 - n) I_{Lt} \quad (\text{E58})$$

- Aggregation of intermediate goods producers' dividends:

$$\begin{aligned}
div_{Lt}^p &= \frac{1}{1-n} \int_0^1 div_{Lt}^p(j) dj \\
&= \frac{1}{1-n} \int_0^1 p_{Lt}(j) Y_{Lt}(j) dj - w_{Lt} \frac{1}{1-n} \int_0^1 L_{Lt}(j) dj - R_{k,Lt} \frac{1}{1-n} \int_0^1 K_{Lt}(j) dj \\
&= \frac{P_{Lt}}{P_t} Y_{Lt} \int_0^1 \left(\frac{P_{Lt}(j)}{P_{Lt}} \right)^{1-\varepsilon_p} dj - w_{Lt} L_{Lt}^d - R_{k,Lt} K_{Lt} \\
&= p_{Lt} Y_{Lt} - w_{Lt} L_{Lt}^d - R_{k,Lt} K_{Lt}
\end{aligned}$$

E.6.4 Prices and Wages

- Law of motion for producer prices:

$$\begin{aligned}
P_{Ht} &= \left(\int_0^1 P_{Ht}(j)^{1-\varepsilon_p} dj \right)^{\frac{1}{1-\varepsilon_p}} \\
(P_{Ht})^{1-\varepsilon_p} &= (1-\omega_p) (P_{Ht}^r)^{1-\varepsilon_p} + \omega_p (P_{Ht-1})^{1-\varepsilon_p} \\
(p_{Ht})^{1-\varepsilon_p} &= (1-\omega_p) (p_{Ht}^r)^{1-\varepsilon_p} + \omega_p (\Pi_t^{-1} p_{Ht-1})^{1-\varepsilon_p},
\end{aligned} \tag{E59}$$

similary:

$$(p_{Lt})^{1-\varepsilon_p} = (1-\omega_p) (p_{Lt}^r)^{1-\varepsilon_p} + \omega_p (\Pi_t^{-1} p_{Lt-1})^{1-\varepsilon_p} \tag{E60}$$

- Law of motion for wages:

$$\begin{aligned}
W_{Ht} &= \left(\int_0^1 W_{Ht}(i)^{1-\varepsilon_w} di \right)^{\frac{1}{1-\varepsilon_w}} \\
(W_{Ht})^{1-\varepsilon_w} &= (1-\omega_w) (W_{Ht}^r)^{1-\varepsilon_w} + \omega_w (W_{Ht-1})^{1-\varepsilon_w} \\
(w_{Ht})^{1-\varepsilon_w} &= (1-\omega_w) (w_{Ht}^r)^{1-\varepsilon_w} + \omega_w (\Pi_t^{-1} w_{Ht-1})^{1-\varepsilon_w}
\end{aligned} \tag{E61}$$

similary:

$$(w_{Lt})^{1-\varepsilon_w} = (1-\omega_w) (w_{Lt}^r)^{1-\varepsilon_w} + \omega_w (\Pi_t^{-1} w_{Lt-1})^{1-\varepsilon_w} \tag{E62}$$

- Relation between producer prices:

$$\begin{aligned}
P_t &= \left[P_{Ht}^{1-\eta} + (1-n) P_{Lt}^{1-\eta} \right]^{\frac{1}{\eta-1}} \\
1 &= \left[n p_{Ht}^{1-\eta} + (1-n) p_{Lt}^{1-\eta} \right]^{\frac{1}{\eta-1}}
\end{aligned} \tag{E63}$$

- Producer price inflation:

$$\begin{aligned}
\Pi_{Ht} &= \frac{P_{Ht}}{P_{Ht-1}} \\
\Pi_{Ht} &= \frac{p_{Ht}}{p_{Ht-1}} \Pi_t
\end{aligned} \tag{E64}$$

Similarly,

$$\begin{aligned}
\Pi_{Lt} &= \frac{P_{Lt}}{P_{Lt-1}} \\
\Pi_{Lt} &= \frac{p_{Lt}}{p_{Lt-1}} \Pi_t
\end{aligned}$$

- Consumer price inflations:

$$\begin{aligned} P_t &= \left[n P_{Ht}^{1-\eta} + (1-n) P_{Lt}^{1-\eta} \right]^{\frac{1}{\eta-1}} \\ \Pi_t &= \left[n (\Pi_{Ht} p_{Ht-1})^{1-\eta} + \phi_L (\Pi_{Lt} p_{Lt-1})^{1-\eta} \right]^{\frac{1}{\eta-1}} \end{aligned} \quad (\text{E65})$$

E.6.5 Aggregate Budget Constraint

High-Uncertainty Sector

- Plug

$$\begin{aligned} div_{Ht}^p &= p_{Ht} Y_{Ht} - w_{Ht} L_{Ht}^d - r_{k,Ht} K_{Ht}, \\ div_{Ht}^w &= w_{Ht} L_{Ht}^d - mrs_{Ht} L_{Ht} \\ K_{Ht} &= u_{Ht} \bar{K}_{Ht} \end{aligned}$$

into

$$\begin{aligned} P_t C_t + P_{Ht} I_{Ht} + \frac{1}{R_t^*} B_{Ht} &= MRS_{Ht} L_{Ht} + r_{k,Ht} u_{Ht} \bar{K}_{Ht-1} - P_{Ht} a(u_{Ht}) \bar{K}_{Ht-1} \\ &\quad + DIV_{Ht}^p + DIV_{Ht}^w + B_{Ht-1} \end{aligned}$$

which yields then:

$$\frac{1}{R_t^*} b_t = (\Pi_t)^{-1} b_{t-1} + p_{Ht} Y_{Ht} - C_t^h - p_{Ht} I_{Ht} - p_{Ht} a(u_{Ht}) \bar{K}_{Ht-1} \quad (\text{E66})$$

E.7 Steady State

- Recall the following parameter choices:

$$u_k = 1 \text{ and } a(u_k) = 0$$

- We assume zero-inflation, cross-sector symmetric (no borrowing) steady state, i.e.

$$\Pi = 1 \quad (\text{E67})$$

and

$$b_H = 0.$$

- Set ϱ such that

$$L_{Ht} = L_{Lt} = 1 \quad (\text{E68})$$

- Equation (E64) becomes

$$\begin{aligned} \Pi_H &= \frac{p_H}{p_H} \Pi \\ \Pi_H &= 1 \end{aligned} \quad (\text{E69})$$

and

$$\begin{aligned}\Pi_L &= \frac{p_L}{p_L} \Pi \\ \Pi_L &= 1\end{aligned}\tag{E70}$$

- Equation (E65) becomes

$$\begin{aligned}\Pi &= \left[n (\Pi_H p_H)^{1-\eta} + \phi_L (\Pi_L p_L)^{1-\eta} \right]^{\frac{1}{\eta-1}} \\ 1 &= \left[n (p_H)^{1-\eta} + (1-n) (p_L)^{1-\eta} \right]^{\frac{1}{\eta-1}} \\ 1 - (1-n) (p_L)^{1-\eta} &= n (p_H)^{1-\eta} \\ 1 - (1-n) (p_L)^{1-\eta} &= (1 - (1-n)) (p_H)^{1-\eta}\end{aligned}$$

implying that

$$p_L = p_H = 1.\tag{E71}$$

- Equation (E59) yields

$$\begin{aligned}(p_H)^{1-\varepsilon_p} &= (1 - \omega_p) (p_H^r)^{1-\varepsilon_p} + \omega_p (\Pi^{-1} p_H)^{1-\varepsilon_p} \\ 1 &= (1 - \omega_p) (p_H^r)^{1-\varepsilon_p} + \omega_p \\ 1 - \omega_p &= (1 - \omega_p) (p_H^r)^{1-\varepsilon_p} \\ p_H^r &= 1\end{aligned}\tag{E72}$$

and from (E60), we get

$$p_L^r = 1\tag{E73}$$

- Equation (E61) becomes

$$\begin{aligned}(w_H)^{1-\varepsilon_w} &= (1 - \omega_w) (w_H^r)^{1-\varepsilon_w} + \omega_w (\Pi^{-1} w_H)^{1-\varepsilon_w} \\ (1 - \omega_w) (w_H)^{1-\varepsilon_w} &= (1 - \omega_w) (w_H^r)^{1-\varepsilon_w} \\ w_H &= w_H^r\end{aligned}\tag{E74}$$

and similarly from (E62)

$$w_L = w_L^r\tag{E75}$$

- Equation (E48) becomes

$$\begin{aligned}v_H^w &= (1 - \phi_w) \left(\frac{w_H^r}{w_H} \right)^{-\varepsilon_w} + \phi_w \Pi^{\varepsilon_w} \left(\frac{w_H}{w_H} \right)^{-\varepsilon_w} v_H^w \\ v_H^w &= (1 - \phi_w) + \phi_w v_H^w \\ v_H^w &= 1\end{aligned}\tag{E76}$$

and similarly

$$v_L^w = 1\tag{E77}$$

- Equation (E54) becomes

$$\begin{aligned}
v_H^p &= (1 - \phi_p) \left(\frac{p_H^r}{p_H} \right)^{-\varepsilon_p} + \phi_p (\Pi)^{\varepsilon_p} \left(\frac{p_H}{p_H} \right)^{-\varepsilon_p} v_H^p \\
(1 - \phi_p) v_H^p &= (1 - \phi_p) \\
v_H^p &= 1
\end{aligned} \tag{E78}$$

- Equations (E19–E22) yield

$$C_H^h = nC^h \text{ and } C_L^h = (1 - n)C^h \tag{E79}$$

$$C_H^l = nC^l \text{ and } C_L^l = (1 - n)C^l \tag{E80}$$

- Equation (E66) yields

$$\begin{aligned}
\frac{1}{R} b_H &= (\Pi)^{-1} b_H + p_H Y_H - C^h - p_H I_H - p_H a(u_H) \bar{K}_H \\
b_H &= \frac{1}{\beta - 1} (Y_H - C^h - I_H)
\end{aligned}$$

For $b_H = 0$ to hold, we need

$$Y_H - C^h - I_H = 0.$$

from equation (E55), we know that

$$Y_H = nC^h + \frac{(1 - n)}{n} nC^l + I_H,$$

and hence

$$\begin{aligned}
C^h &= nC^h + \frac{(1 - n)}{n} nC^l \\
(1 - n) C^h &= \frac{(1 - n)}{n} nC^l \\
\frac{n}{1 - n} (1 - n) C^h &= nC^l \\
C^h &= C^l.
\end{aligned}$$

- Equations (E33–E38) yield

$$\begin{aligned}
x_{1,H} &= mc_H Y_H + \omega_p [\Lambda_H (\Pi_H)^{\varepsilon_p} x_{1,H}] \\
x_{1,H} &= \frac{mc_H Y_H}{1 - \omega_p \Lambda_H}
\end{aligned}$$

$$\begin{aligned}
x_{2,H} &= Y_H + \omega_p [\Lambda_H (\Pi_H)^{\varepsilon_p - 1} x_{2,H}] \\
x_{2,H} &= \frac{Y_H}{1 - \omega_p \Lambda_H}
\end{aligned}$$

$$\begin{aligned}
p_H^r &= \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{x_{1,H}}{x_{2,H}} p_H \\
\frac{x_{1,H}}{x_{2,H}} &= \frac{\varepsilon_p - 1}{\varepsilon_p} \\
mc_H &= mc_L = \frac{\varepsilon_p - 1}{\varepsilon_p}
\end{aligned} \tag{E81}$$

- Equations (E23–E25) yield

$$\begin{aligned}
f_{1,H} &= mrs_H (p_H)^{-\varepsilon_w} (w_H)^{\varepsilon_w} L_H^d + \omega_w \Lambda_H (\Pi_H)^{\varepsilon_w} f_{1,H} \\
&= \frac{mrs_H (w_H)^{\varepsilon_w} L_H^d}{1 - \omega_w \Lambda_H}
\end{aligned} \tag{E82}$$

$$= \frac{mrs_H (w_H)^{\varepsilon_w}}{1 - \omega_w \beta} \tag{E83}$$

$$\begin{aligned}
f_{2,H} &= \frac{(w_H)^{\varepsilon_w} L_H^d}{1 - \omega_w \Lambda_H} \\
&= \frac{(w_H)^{\varepsilon_w} L_H^d}{1 - \omega_w \Lambda_H}
\end{aligned} \tag{E84}$$

$$\begin{aligned}
w_H^r &= \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{f_{1,H}}{f_{2,H}} p_H \\
mrs_H &= \frac{\varepsilon_w - 1}{\varepsilon_w} w_H
\end{aligned} \tag{E85}$$

- Equations (E29) and (E30)

$$\begin{aligned}
\frac{w_H}{r_{k,H}} &= \frac{1 - \alpha}{\alpha} \frac{K_H}{L_H}, \\
w_H &= \frac{1 - \alpha}{\alpha} K_H r_{k,H}
\end{aligned} \tag{E86}$$

$$\begin{aligned}
mc_H &= \frac{w_H}{(1 - \alpha) \left(\frac{K_H}{L_H} \right)^\alpha} \\
mc_H &= \frac{\frac{1 - \alpha}{\alpha} K_H r_{k,H}}{(1 - \alpha) (K_H)^\alpha} \\
K_H &= \left(\frac{\alpha mc_H}{r_{k,H}} \right)^{\frac{1}{1 - \alpha}}
\end{aligned} \tag{E87}$$

Due to the symmetry, we know that

$$w_L = w_H \text{ and } K_L = K_H \tag{E88}$$

- Equations (E51) and (E52)

$$\begin{aligned} K_H &= u_H \bar{K}_H \\ K_H &= \bar{K}_H \end{aligned} \quad (\text{E89})$$

and

$$K_L = \bar{K}_L \quad (\text{E90})$$

- Equations (E3–E18) yield

$$\Lambda_H = \Lambda_L = \beta \quad (\text{E91})$$

$$\begin{aligned} J_H = J_L &= (1 - \xi) C^h \\ \frac{1}{R} &= \Lambda_H \end{aligned} \quad (\text{E92})$$

$$\begin{aligned} p_H &= \Xi_H \left(1 - \frac{\kappa}{2} \left(\frac{I_H}{I_H} - 1 \right)^2 - \kappa \left(\frac{I_H}{I_H} - 1 \right) \frac{I_H}{I_H} \right) + \beta \left[\Lambda_H \Xi_H \kappa \left(\frac{I_H}{I_H} - 1 \right) \left(\frac{I_H}{I_H} \right)^2 \right] \\ \Xi_H &= \Xi_L = 1 \end{aligned} \quad (\text{E93})$$

$$\begin{aligned} \Xi_H &= [\Lambda_H (r_{k,H} u_H - p_H a(u_H) + \Xi_H (1 - \delta))] \\ r_{k,H} &= r_{k,L} = \frac{1}{\Lambda_H} - (1 - \delta) \end{aligned} \quad (\text{E94})$$

$$a'(u_H) = a'(u_L) = r_{k,H} \quad (\text{E95})$$

$$\begin{aligned} \bar{K}_H &= \left(1 - \frac{\kappa}{2} \left(\frac{I_H}{I_H} - 1 \right)^2 \right) I_H + (1 - \delta) \bar{K}_H \\ \bar{K}_H &= I_H + (1 - \delta) \bar{K}_H \\ I_H &= I_L = \delta K_H \end{aligned} \quad (\text{E96})$$

- Equations (E53) and (E64E56) yield

$$\begin{aligned} Y_H v_H^p &= (K_H)^\alpha \left(L_H^d \right)^{1-\alpha} \\ Y_H &= Y_L = (K_H)^\alpha \end{aligned} \quad (\text{E97})$$

- Equations (E39–E45)

$$Q^{(\tau)} = w_{SH} Q_{HH}^{(\tau)} + w_{SL} Q_{LL}^{(\tau)} \quad (\text{E98})$$

$$\begin{aligned} Q_{HH}^{(\tau)} &= E_t \left[\Lambda_H (\Pi_H)^{-1} Q_{HH}^{(\tau-1)} \right] \text{ for } \tau \in \{1, \dots, 40\}, \\ \text{and } Q_{HH}^{(0)} &\equiv 1. \end{aligned} \quad (\text{E99})$$

$$Q_{LL}^{(\tau)} = E_t \left[\Lambda_L (\Pi_L)^{-1} Q_{LL}^{(\tau-1)} \right] \text{ for } \tau \in \{1, \dots, 40\}, \quad (\text{E100})$$

and $Q_{LL}^{(0)} \equiv 1$.

$$\begin{aligned} ws_H &= \frac{nwea_H}{nwea_H + (1-n)wea_L} \\ &= n \end{aligned} \quad (\text{E101})$$

$$\begin{aligned} ws_L &= \frac{(1-n)wea_L}{nwea_H + (1-n)wea_L} \\ &= 1-n \end{aligned} \quad (\text{E102})$$

$$wea_H = wea_L = \bar{K}_H + b_H, \quad (\text{E103})$$

- For $L_H = L_L = 1$, we need

$$\varrho = mrs_H (1 - \xi\beta) (J)^{-\sigma}$$

E.8 Log-linearized Equilibrium

Log-linear deviations from steady state are defined as follows

$$\hat{X}_t \equiv \frac{X_t - X}{X},$$

where X_t is an arbitrary variable. Note that the wage and price dispersions vanish, i.e.

$$\hat{v}_{Ht}^p = 0 \text{ and } \hat{v}_{Lt}^p = 0$$

and

$$\hat{v}_{Ht}^w = 0 \text{ and } \hat{v}_{Lt}^w = 0.$$

As consequence, equations (E47) and (E49) become

$$\hat{L}_{Ht}^d = \hat{L}_{Ht} \text{ and } \hat{L}_{Lt}^d = \hat{L}_{Lt}.$$

Overall, the following 60 variables characterize the equilibrium

$$\begin{aligned} &\hat{C}_t^h, \hat{C}_{Ht}^h, \hat{C}_{Lt}^h, \hat{J}_{Ht}, \hat{L}_{Ht}, \hat{I}_{Ht}, \hat{K}_{Ht}, \hat{\bar{K}}_{Ht}, \hat{u}_{Ht}, \hat{\Lambda}_{Ht,t+1}, \hat{\Xi}_{Ht}, \widehat{mrs}_{Ht}, \\ &\hat{C}_t^l, \hat{C}_{Ht}^l, \hat{C}_{Lt}^l, \hat{J}_{Lt}, \hat{L}_{Lt}, \hat{I}_{Lt}, \hat{K}_{Lt}, \hat{\bar{K}}_{Lt}, \hat{u}_{Lt}, \hat{\Lambda}_{Lt,t+1}, \hat{\Xi}_{Lt}, \widehat{mrs}_{Lt} \\ &\widehat{mc}_{Ht}, \hat{r}_{k,Ht}, \hat{w}_{Ht}, \widehat{mc}_{Lt}, \hat{r}_{k,Lt}, \hat{w}_{Lt}, \hat{R}_t, \hat{R}_t^* \\ &\hat{x}_{1,Ht}, \hat{x}_{2,Ht}, \hat{Y}_{Ht}, \hat{p}_{Ht}^r, \hat{p}_{Ht}, \hat{f}_{1,Ht}, \hat{f}_{2,Ht}, \hat{w}_{Ht}^r \\ &\hat{x}_{1,Lt}, \hat{x}_{2,Lt}, \hat{Y}_{Lt}, \hat{p}_{Lt}^r, \hat{p}_{Lt}, \hat{f}_{1,Lt}, \hat{f}_{2,Lt}, \hat{w}_{Lt}^r \\ &\hat{\Pi}_t, \hat{\Pi}_{Ht}, \hat{\Pi}_{Lt}, \hat{Y}_t, b_{Ht}, b_{Lt} \\ &\hat{Q}_t^{(\tau)}, \hat{R}_t^{(\tau)}, \widehat{wea}_{Ht}, \widehat{wea}_{Lt}, \widehat{ws}_{Ht}, \widehat{ws}_{Lt}, \end{aligned}$$

where the process for θ_t and θ_t^* are as defined in Subsection and are taken as given. Further, the variables from the counterfactual economies such as $\hat{Q}_{HHt}^{(\tau)}$, $\hat{Q}_{LLt}^{(\tau)}$, $\hat{\Lambda}_{HHt,t+1}$, $\hat{\Pi}_{HHt+1}$, $\hat{\Lambda}_{LLt,t+1}$, $\hat{\Pi}_{LLt+1}$ are taken as given here. These are pinned down by two very similar sets of equations which are solved simultaneously. The following 60 equations characterize the log-linearized equilibrium of the model.

E.8.1 Household (20 equations)

$$\hat{C}_{Ht}^h = -\eta \hat{p}_{Ht} + \hat{C}_t^h \quad (\text{E104})$$

$$\hat{C}_{Lt}^h = -\eta \hat{p}_{Lt} + \hat{C}_t^h \quad (\text{E105})$$

$$\hat{C}_{Ht}^l = -\eta \hat{p}_{Ht} + \hat{C}_t^l \quad (\text{E106})$$

$$\hat{C}_{Lt}^l = -\eta \hat{p}_{Lt} + \hat{C}_t^l \quad (\text{E107})$$

$$\chi \hat{L}_{Ht} = \widehat{mrs}_{Ht} - \frac{\sigma}{1-\xi\beta} \left(\hat{J}_{Ht} - \xi\beta E_t \left[\hat{J}_{Ht+1} \right] \right) \quad (\text{E108})$$

$$\hat{J}_{Ht} = \frac{1}{1-\xi} \left(\hat{C}_t^h - \xi \hat{C}_{t-1}^h \right) \quad (\text{E109})$$

$$0 = \hat{R}_t^* + E_t \left[\hat{\Lambda}_{Ht,t+1} \right] - E_t \left[\hat{\Pi}_{t+1} \right] \quad (\text{E110})$$

$$E_t \left[\hat{\Lambda}_{Ht,t+1} \right] = -\frac{\sigma}{1-\xi\beta} \left(\left(E_t \left[\hat{J}_{Ht+1} \right] - \xi\beta E_t \left[\hat{J}_{Ht+2} \right] \right) - \left(\hat{J}_{Ht} - \xi\beta E_t \left[\hat{J}_{Ht+1} \right] \right) \right) \quad (\text{E111})$$

$$\hat{\Xi}_{Ht} = E_t \left[\hat{\Lambda}_{Ht,t+1} \right] + (1-\beta(1-\delta)) E_t \left[\hat{r}_{k,Ht+1} \right] + \beta(1-\delta) E_t \left[\hat{\Xi}_{Ht+1} \right] \quad (\text{E112})$$

$$\hat{p}_{Ht} = \hat{\Xi}_{Ht} + \kappa\beta E_t \left[\hat{I}_{Ht+1} \right] - \kappa(1+\beta) \hat{I}_{Ht} + \kappa \hat{I}_{Ht-1} \quad (\text{E113})$$

$$\hat{R}_{k,Ht} = \hat{p}_{Ht} + \vartheta \hat{u}_{Ht} \quad (\text{E114})$$

$$\hat{K}_{Ht} = \delta \hat{I}_{Ht} + (1-\delta) \hat{K}_{Ht-1} \quad (\text{E115})$$

$$\chi \hat{L}_{Lt} = \widehat{mrs}_{Lt} - \frac{\sigma}{1-\xi\beta} \left(\hat{J}_{Lt} - \xi\beta E_t \left[\hat{J}_{Lt+1} \right] \right) \quad (\text{E116})$$

$$\hat{J}_{Lt} = \frac{1}{1-\xi} \left(\hat{C}_t^l - \xi \hat{C}_{t-1}^l \right) \quad (\text{E117})$$

$$0 = \hat{R}_t + E_t \left[\hat{\Lambda}_{Lt,t+1} \right] - E_t \left[\hat{\Pi}_{t+1} \right] \quad (\text{E118})$$

$$E_t \left[\hat{\Lambda}_{Lt,t+1} \right] = -\frac{\sigma}{1-\xi\beta} \left(\left(E_t \left[\hat{J}_{Lt+1} \right] - \xi\beta E_t \left[\hat{J}_{Lt+2} \right] \right) - \left(\hat{J}_{Lt} - \xi\beta E_t \left[\hat{J}_{Lt+1} \right] \right) \right) \quad (\text{E119})$$

$$\hat{\Xi}_{Lt} = E_t \left[\hat{\Lambda}_{Lt,t+1} \right] + (1-\beta(1-\delta)) E_t \left[\hat{r}_{k,Lt+1} \right] + \beta(1-\delta) E_t \left[\hat{\Xi}_{Lt+1} \right] \quad (\text{E120})$$

$$\hat{p}_{Lt} = \hat{\Xi}_{Lt} + \kappa\beta E_t \left[\hat{I}_{Lt+1} \right] - \kappa(1+\beta) \hat{I}_{Lt} + \kappa \hat{I}_{Lt-1} \quad (\text{E121})$$

$$\hat{R}_{k,Lt} = \hat{p}_{Lt} + \vartheta \hat{u}_{Lt} \quad (\text{E122})$$

$$\hat{K}_{Lt} = \delta \hat{I}_{Lt} + (1-\delta) \hat{K}_{Lt-1} \quad (\text{E123})$$

E.8.2 Labor Market (6 equations)

$$\hat{w}_{Ht}^r = \hat{f}_{1,Ht} - \hat{f}_{2,Ht} + \hat{p}_{Ht} \quad (\text{E124})$$

$$\begin{aligned} \hat{f}_{1,Ht} = & (1 - \omega_w \beta) \left(\widehat{mrs}_{Ht} - \varepsilon_w \hat{p}_{Ht} + \varepsilon_w \hat{w}_{Ht} + \hat{L}_{Ht} \right) \\ & + \omega_w \beta \left[E_t \left[\hat{\Lambda}_{Ht,t+1} \right] + \varepsilon_w E_t \left[\hat{\Pi}_{Ht+1} \right] + E_t \left[\hat{f}_{1,Ht+1} \right] \right] \end{aligned} \quad (\text{E125})$$

$$\begin{aligned} \hat{f}_{2,Ht} = & (1 - \omega_w \beta) \left((1 - \varepsilon_w) \hat{p}_{Ht} + \varepsilon_w \hat{w}_{Ht} + \hat{L}_{Ht} \right) \\ & + \omega_w \beta \left(E_t \left[\hat{\Lambda}_{Ht,t+1} \right] + (\varepsilon_w - 1) E_t \left[\hat{\Pi}_{Ht+1} \right] + E_t \left[\hat{f}_{2,Ht+1} \right] \right). \end{aligned} \quad (\text{E126})$$

$$\hat{w}_{Lt}^r = \hat{f}_{1,Lt} - \hat{f}_{2,Lt} + \hat{p}_{Lt} \quad (\text{E127})$$

$$\begin{aligned} \hat{f}_{1,Lt} = & (1 - \omega_w \beta) \left(\widehat{mrs}_{Lt} - \varepsilon_w \hat{p}_{Lt} + \varepsilon_w \hat{w}_{Lt} + \hat{L}_{Lt} \right) \\ & + \omega_w \beta \left[E_t \left[\hat{\Lambda}_{Lt,t+1} \right] + \varepsilon_w E_t \left[\hat{\Pi}_{Lt+1} \right] + E_t \left[\hat{f}_{1,Lt+1} \right] \right] \end{aligned} \quad (\text{E128})$$

$$\begin{aligned} \hat{f}_{2,Lt} = & (1 - \omega_w \beta) \left((\varepsilon_w - 1) \hat{p}_{Lt} + \varepsilon_w \hat{w}_{Lt} + \hat{L}_{Lt} \right) \\ & + \omega_w \beta \left(E_t \left[\hat{\Lambda}_{Lt,t+1} \right] + (\varepsilon_w - 1) E_t \left[\hat{\Pi}_{Lt+1} \right] + E_t \left[\hat{f}_{2,Lt+1} \right] \right). \end{aligned} \quad (\text{E129})$$

E.8.3 Production (10 equations)

$$\hat{w}_{Ht} - \hat{R}_{k,Ht} = \hat{K}_{Ht} - \hat{L}_{Ht} \quad (\text{E130})$$

$$\widehat{mc}_{Ht} = \hat{w}_{Ht} - \alpha \left(\hat{K}_{Ht} - \hat{L}_{Ht} \right) \quad (\text{E131})$$

$$\hat{w}_{Lt} - \hat{R}_{k,Lt} = \hat{K}_{Lt} - \hat{L}_{Lt} \quad (\text{E132})$$

$$\widehat{mc}_{Lt} = \hat{w}_{Lt} - \alpha \left(\hat{K}_{Lt} - \hat{L}_{Lt} \right) \quad (\text{E133})$$

$$\hat{p}_{Ht}^r = \hat{x}_{1,Ht} - \hat{x}_{2,Ht} - \hat{p}_{Ht} \quad (\text{E134})$$

$$\hat{x}_{1,Ht} = (1 - \omega_p \beta) \left(\widehat{mc}_{Ht} + \hat{Y}_{Ht} \right) + \omega_p \beta \left[E_t \left[\hat{\Lambda}_{Ht,t+1} \right] + \varepsilon_p E_t \left[\hat{\Pi}_{Ht+1} \right] + E_t \left[\hat{x}_{1,Ht+1} \right] \right] \quad (\text{E135})$$

$$\hat{x}_{2,Ht} = (1 - \omega_p \beta) \hat{Y}_{Ht} + \omega_p \beta \left[E_t \left[\hat{\Lambda}_{Ht,t+1} \right] + (\varepsilon_p - 1) E_t \left[\hat{\Pi}_{Ht+1} \right] + E_t \left[\hat{x}_{2,Ht+1} \right] \right] \quad (\text{E136})$$

$$\hat{p}_{Lt}^r = \hat{x}_{1,Lt} - \hat{x}_{2,Lt} - \hat{p}_{Lt} \quad (\text{E137})$$

$$\hat{x}_{1,Lt} = (1 - \omega_p \beta) \left(\widehat{mc}_{Lt} + \hat{Y}_{Lt} \right) + \omega_p \beta \left[E_t \left[\hat{\Lambda}_{Lt,t+1} \right] + \varepsilon_p E_t \left[\hat{\Pi}_{Lt+1} \right] + E_t \left[\hat{x}_{1,Lt+1} \right] \right] \quad (\text{E138})$$

$$\hat{x}_{2,Lt} = (1 - \omega_p \beta) \hat{Y}_{Lt} + \omega_p \beta \left[E_t \left[\hat{\Lambda}_{Lt,t+1} \right] + (\varepsilon_p - 1) E_t \left[\hat{\Pi}_{Lt+1} \right] + E_t \left[\hat{x}_{2,Lt+1} \right] \right] \quad (\text{E139})$$

E.8.4 Monetary Policy (3 equations)

$$\hat{R}_t = \theta_\Pi \hat{\Pi}_t + \theta_Y \hat{Y}_t + \theta_t \quad (\text{E140})$$

$$\hat{R}_t^* = \theta_\Pi \hat{\Pi}_t + \theta_Y \hat{Y}_t + \theta_t^* \quad (\text{E141})$$

$$\hat{Y}_t = n \hat{Y}_{Ht} + (1 - n) \hat{Y}_{Lt} \quad (\text{E142})$$

E.8.5 Bond Market (6 equations)

$$\hat{Q}_t^{(\tau)} = w s_H \left(\widehat{w s}_{Ht} + \hat{Q}_{HHt}^{(\tau)} \right) + w s_L \left(\widehat{w s}_{Lt} + \hat{Q}_{LLt}^{(\tau)} \right), \quad (\text{E143})$$

where $\hat{Q}_{HHt}^{(\tau)}$ and $\hat{Q}_{LLt}^{(\tau)}$ are taken as given

$$\begin{aligned} \hat{Q}_{HHt}^{(\tau)} &= E_t \left[\hat{\Lambda}_{HHt,t+1} \right] - E_t \left[\hat{\Pi}_{HHt+1} \right] + E_t \left[\hat{Q}_{HHt+1}^{(\tau-1)} \right] \text{ for } \tau \in \{1, \dots, 40\}, \\ \text{and } \hat{Q}_{HHt}^{(0)} &\equiv 0. \end{aligned}$$

$$\begin{aligned} \hat{Q}_{LLt}^{(\tau)} &= E_t \left[\hat{\Lambda}_{LLt,t+1} \right] - E_t \left[\hat{\Pi}_{LLt+1} \right] + E_t \left[\hat{Q}_{LLt+1}^{(\tau-1)} \right] \text{ for } \tau \in \{1, \dots, 40\}, \\ \text{and } \hat{Q}_{LLt}^{(0)} &\equiv 0. \end{aligned}$$

$$w e a_H \widehat{w e a}_{Ht} = \bar{K}_H \hat{K}_{Ht} + b_{Ht} \quad (\text{E144})$$

$$w e a_L \widehat{w e a}_{Lt} = \bar{K}_L \hat{K}_{Lt} + b_{Lt} \quad (\text{E145})$$

$$\widehat{w s}_{Ht} = w s_L (\hat{w}_{Ht} - \hat{w}_{Lt}) \quad (\text{E146})$$

$$\widehat{w s}_{Lt} = w s_H (\hat{w}_{Lt} - \hat{w}_{Ht}) \quad (\text{E147})$$

$$\hat{R}_t^{(\tau)} = -\frac{1}{\tau} \hat{Q}_t^{(\tau)} \text{ for } \tau \in \{1, \dots, 40\}. \quad (\text{E148})$$

E.8.6 Aggregation and Market Clearing (15 equations)

$$\hat{K}_{Ht} = \hat{u}_{Ht} + \hat{K}_{Ht-1} \quad (\text{E149})$$

$$\hat{K}_{Lt} = \hat{u}_{Lt} + \hat{K}_{Lt-1} \quad (\text{E150})$$

$$\hat{Y}_{Ht} = \alpha \hat{K}_{Ht} + (1 - \alpha) \hat{L}_{Ht} \quad (\text{E151})$$

$$Y_H \hat{Y}_{Ht} = C_H^h \hat{C}_{Ht}^h + \left(\frac{1-n}{n} \right) C_H^l \hat{C}_{Ht}^l + I_H \hat{I}_{Ht} \quad (\text{E152})$$

$$\hat{Y}_{Lt} = \alpha \hat{K}_{Lt} + (1 - \alpha) \hat{L}_{Lt} \quad (\text{E153})$$

$$Y_L \hat{Y}_{Lt} = \frac{n}{1-n} C_L^h \hat{C}_{Lt}^h + C_L^l \hat{C}_{Lt}^l + I_L \hat{I}_{Lt} \quad (\text{E154})$$

$$\hat{p}_{Ht} = (1 - \omega_p) \hat{p}_{Ht}^r + \omega_p \left(-\hat{\Pi}_t + \hat{p}_{Ht-1} \right) \quad (\text{E155})$$

$$\hat{p}_{Lt} = (1 - \omega_p) \hat{p}_{Lt}^r + \omega_p \left(-\hat{\Pi}_t + \hat{p}_{Lt-1} \right) \quad (\text{E156})$$

$$\hat{w}_{Ht} = (1 - \omega_w) \hat{w}_{Ht}^r + \omega_w (-\hat{\Pi}_t + \hat{w}_{Ht-1}) \quad (\text{E157})$$

$$\hat{w}_{Lt} = (1 - \omega_w) \hat{w}_{Lt}^r + \omega_w (-\hat{\Pi}_t + \hat{w}_{Lt-1}) \quad (\text{E158})$$

$$0 = n\hat{p}_{Ht} + (1 - n)\hat{p}_{Lt} \quad (\text{E159})$$

$$\hat{\Pi}_{Ht} = \hat{p}_{Ht} - \hat{p}_{Ht-1} + \hat{\Pi}_t \quad (\text{E160})$$

$$\hat{\Pi}_t = n\hat{\Pi}_{Ht} + (1 - n)\hat{\Pi}_{Lt} \quad (\text{E161})$$

$$\beta b_{Ht} = b_{Ht-1} + Y_H (\hat{p}_{Ht} + \hat{Y}_{Ht}) - C^h \hat{C}_t^h - I_H (\hat{p}_{Ht} + \hat{I}_{Ht}) - R_{k,H} K_H \hat{u}_{Ht} \quad (\text{E162})$$

$$b_{Lt} = -\frac{n}{1 - n} b_{Ht} \quad (\text{E163})$$

E.8.7 Digression: Elimination of wage- and price-dispersion

- Linearizing (E59) leads to

$$\begin{aligned} \hat{p}_{Ht} &= (1 - \omega_p) \hat{p}_{Ht}^r + \omega_p (-\hat{\Pi}_t + \hat{p}_{Ht-1}) \\ \hat{p}_{Ht}^r &= \frac{1}{1 - \omega_p} (\hat{p}_{Ht} - \omega_p (-\hat{\Pi}_t + \hat{p}_{Ht-1})) \end{aligned}$$

which then is used for the linearization of (E54)

$$\begin{aligned} v_H^p \hat{v}_{Ht}^p &= -\varepsilon_p (1 - \omega_p) \left(\frac{p_H^r}{p_H} \right) (\hat{p}_{Ht}^r - \hat{p}_H) \\ &\quad + \omega_p (\Pi)^{\varepsilon_p} \left(\frac{p_H}{p_H} \right)^{-\varepsilon_p} v_H^p (\varepsilon_p \hat{\Pi}_t - \varepsilon_p \hat{p}_{Ht-1} + \hat{p}_{Ht} + v_{Ht-1}^p) \\ &= -\varepsilon_p (1 - \omega_p) (\hat{p}_{Ht}^r - \hat{p}_{Ht}) \\ &\quad + \omega_p (\varepsilon_p \hat{\Pi}_t - \varepsilon_p \hat{p}_{Ht-1} + \hat{p}_{Ht} + v_{Ht-1}^p) \\ &= -\varepsilon_p \omega_p (\hat{p}_{Ht} + \hat{\Pi}_t - \hat{p}_{Ht-1}) \\ &\quad - \varepsilon_p \omega_p (-\hat{p}_{Ht} - \hat{\Pi}_t + \hat{p}_{Ht-1}) + \omega_p v_{Ht-1}^p \\ \hat{v}_{Ht}^p &= \omega_p v_{Ht-1}^p \end{aligned}$$

We are approximating around steady state with $v_H^p = 1$. Hence, we are starting from $\tilde{v}_{H0}^p = 0$ and hence we have

$$\hat{v}_{Ht}^p = 0.$$

and similarly

$$\hat{v}_{Lt}^p = 0.$$

- Equation (E61) becomes

$$\begin{aligned}
(1 - \varepsilon_w) \hat{w}_{Ht} &= (1 - \omega_w) (1 - \varepsilon_w) (\hat{w}_{Ht}^r) + \omega_w (1 - \varepsilon_w) \left(-\hat{\Pi}_t + \hat{w}_{Ht-1} \right) \\
\hat{w}_{Ht} &= (1 - \omega_w) \hat{w}_{Ht}^r + \omega_w \left(-\hat{\Pi}_t + \hat{w}_{Ht-1} \right) \\
\hat{w}_{Ht}^r &= \frac{1}{1 - \omega_w} \left(\hat{w}_{Ht} - \omega_w \left(-\hat{\Pi}_t + \hat{w}_{Ht-1} \right) \right)
\end{aligned}$$

then is used for the linearization of (E48)

$$\begin{aligned}
v_{Ht}^w &= (1 - \omega_w) \left(\frac{w_{Ht}^r}{w_{Ht}} \right)^{-\varepsilon_w} + \omega_w \Pi_t^{\varepsilon_w} \left(\frac{w_{Ht-1}}{w_{Ht}} \right)^{-\varepsilon_w} v_{Ht-1}^w \\
\hat{v}_{Ht}^w &= -\varepsilon_w (1 - \omega_w) (\hat{w}_{Ht}^r - \hat{w}_{Ht}) \\
&\quad + \omega_w \left(\varepsilon_w \hat{\Pi}_t - \varepsilon_w (\hat{w}_{Ht-1} - \hat{w}_{Ht}) + \hat{v}_{Ht-1}^w \right) \\
\hat{v}_{Ht}^w &= -\varepsilon_w \omega_w \left(\hat{w}_{Ht} + \hat{\Pi}_t - \hat{w}_{Ht-1} \right) \\
&\quad + \varepsilon_w \omega_w \left(\hat{w}_{Ht} + \hat{\Pi}_t - \hat{w}_{Ht-1} \right) + \omega_w \hat{v}_{Ht-1}^w \\
\hat{v}_{Ht}^w &= \omega_w \hat{v}_{Ht-1}^w
\end{aligned}$$

and hence by similar reasoning as above, we have

$$\hat{v}_{Ht}^w = 0 \text{ and } \hat{v}_{Lt}^w = 0.$$

E.9 Fitted Parameters

Table E1: Fitted Parameters — Previous Papers

Parameter	CTW 2010	CFFM 2019	BS 2020	SW 2021	Value Range	Description
χ	0.12*	0.589*	0.5	1	[0.1 1]	Inverse Frisch elast.
ξ	0.77*	0.78*	0.613*	0.7	[0.6 0.8]	Consumption habit
κ	14.30*	5.354*	2.336*	2	[2 14]	Investment adjustment costs
φ	0.30*	0.474*	0.01*	0.33	[0.01 0.5]	Elast. of capital utilization
ω_p	0.62*	0.831*	0.886*	0.75	[0.7 0.9]	Price stickiness probability
ω_w	0.75	0.914*	—	0.75	[0.7 0.9]	Wage stickiness probability
ϵ_p	$\frac{1.2}{1.2-1} = 6^*$	$\frac{1.5}{1.5-1} = 3$	6	11	[3 11]	Elast. of substitution w.r.t. goods
ϵ_w	$\frac{1.01}{1.01-1} = 101$	$\frac{1.5}{1.5-1} = 3$	—	11	[3 11]	Elast. of substitution w.r.t. labor

Notes: This table reports the values from previous papers for the fitted parameters, i.e. the ones which are determined in the moment matching calibration. The values are coming from [Christiano, Trabandt, and Walentin \(2010\)](#) (CTW 2010), [Campbell et al. \(2019\)](#) (CFFM 2019), [Bundick and Smith \(2020\)](#) (BS 2020), and [Sims and Wu \(2021\)](#) (SW 2021). The table also reports the value range over which the moment matching procedure searches over. * denotes that the taken parameter value is estimated in the corresponding paper. For CTW 2010, the posterior mean is reported, and for CFFM 2019 and BS 2020 the posterior mode is reported.