Question 1:

a) $\boxed{\text{True}}$

We have : $P(A|\Omega) = \dfrac{P(A\cap\Omega)}{P(\Omega)}$

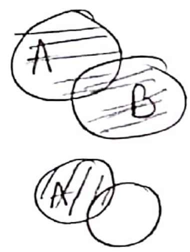
and: $P(A\cap\Omega) = P(A)$ because $A\cap\Omega = A$

$P(\Omega) = 1$

Therefore: $P(A|\Omega) = \dfrac{P(A)}{1} = P(A)$

b) $\boxed{\text{False}}$

We got : $(A\cup B)\cap A = A$



$(A\cup B)$

$(A\cup B)\cap A$

- $P(A\cup B|A) = \dfrac{P((A\cup B)\cap A)}{P(A)}$

$= \dfrac{P(A)}{P(A)} = 1 \neq P(A)$

c) False

$P(A|B) = \dfrac{P(A\cap B)}{P(B)} = 1$

$\Leftrightarrow P(A\cap B) = P(B) \Leftrightarrow B \subseteq A$

Therefore statement is false.

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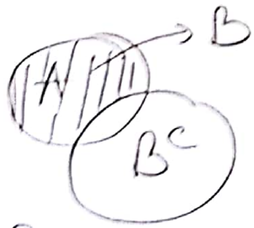
d) $\boxed{\text{True}}$

We have an example.



Because $A \cup B^c = \Omega$

Therefore B is a shaded part

$\Leftrightarrow B \subseteq A \Leftrightarrow P(B) \leq P(A)$

e) $\boxed{\text{False}}$

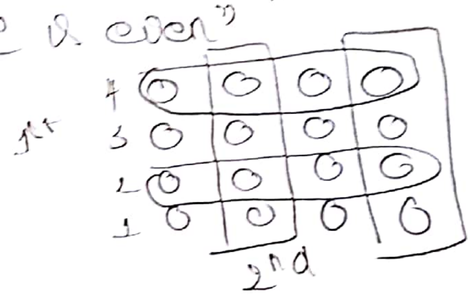
Consider the example:

Roll 4-faced dice twice, A is "first roll is even"

B is "second roll is even"

$P(A) = \frac{1}{2}$

$P(B) = \frac{1}{2}$



$P(A \cap B) = \frac{1}{4} = P(A) \cdot P(B)$

$\Rightarrow$ A and B are independent

$P(A \cup B) = \frac{3}{4} > P(A) \cdot P(B) = \frac{1}{4}$

Therefore the statement is false.

f) $P(A)P(B) = P(A) + P(B)$

$\Leftrightarrow P(A)[P(B) - 1] = P(B)$

Because $1 \geq P(B) \geq 0$ and $0 \geq P(B) - 1 \geq -1$

So $P(B)$ has to be 1

But $P(A) \cdot (1-1) = 1$ can't happen

So there is no possible range of value.

g) • If A and B are independent:

$P(A \cup B) = P(A \cap B)$

$\Leftrightarrow P(A) + P(B) - P(A \cap B) = P(A \cap B)$

$\Leftrightarrow P(A) + P(B) = 2P(A) \cdot P(B)$

$\Leftrightarrow P(A)[1 - P(B)] = P(B)[P(A) - 1]$

This only happens if $P(A) - 1 = 1 - P(B) = 0$

$\Leftrightarrow \begin{cases} P(A) = 1 \\ P(B) = 1 \end{cases}$

• If A and B are disjoint:

$P(A \cup B) = P(A \cap B) = 0$

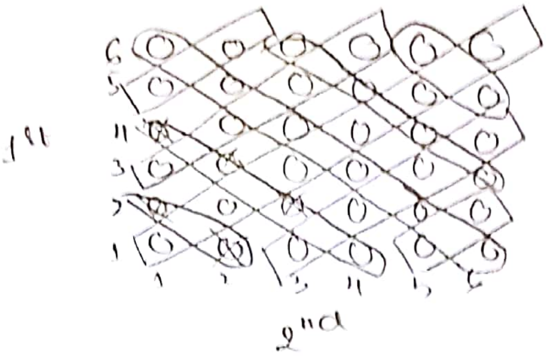
$\Leftrightarrow P(A) = P(B) = 0$

So this only happens when events A and B have no outcome.

Question 2:

$P(E_+) = \frac{1}{2}$

$P(O_+) = \frac{1}{2}$



$P(E_1) = \frac{1}{2}$; $P(O_1) = \frac{1}{2}$

$P(E_2) = \frac{1}{2}$; $P(O_2) = \frac{1}{2}$

a) Pairs of events that are disjoint:

(E_+, O_+) · because the sum of 2 rolls just can be either even or odd, so they have no share elements ⇒ they are disjoint.

(E_1, O_1) and (E_2, O_2):

because the first roll (or second roll) just can be either even or odd, so they have no share outcomes ⇒ they are disjoint

b) Because the first roll doesn't affect the value of the second roll ⇒ all the sets of first roll are mutually independent with sets of second roll

List of pairs: (E_1, O_1), (E_1, O_2) (E_2, O_1), (E_2, O_2).

Question 3 :

a) Ω & all passengers $|\Omega| = 1312$

```
def omega():
    omega = Titanic . Titanic()
    print (len (omega . passengers))
    return omega
```

b) $|S| = 1150$

```
def survived (passengers):
    s = []
    for passenger on passengers :
        if passenger . survived :
            s = s + [passenger]
    print (len(s))
    return s
S = survived (passengers)
```

c) $|F| = 322$

```
def first_class (passengers):
    f = []
    for passenger in passengers :
        if passenger . is_firstclass():
            f = f + [passenger]
    print(len(f))
    return f
f = first_class (passengers)
```

d) Probability of first class passenger survived:

$P_1 = 0.11171$

Probability of a passenger survived, given that they were given being first class:

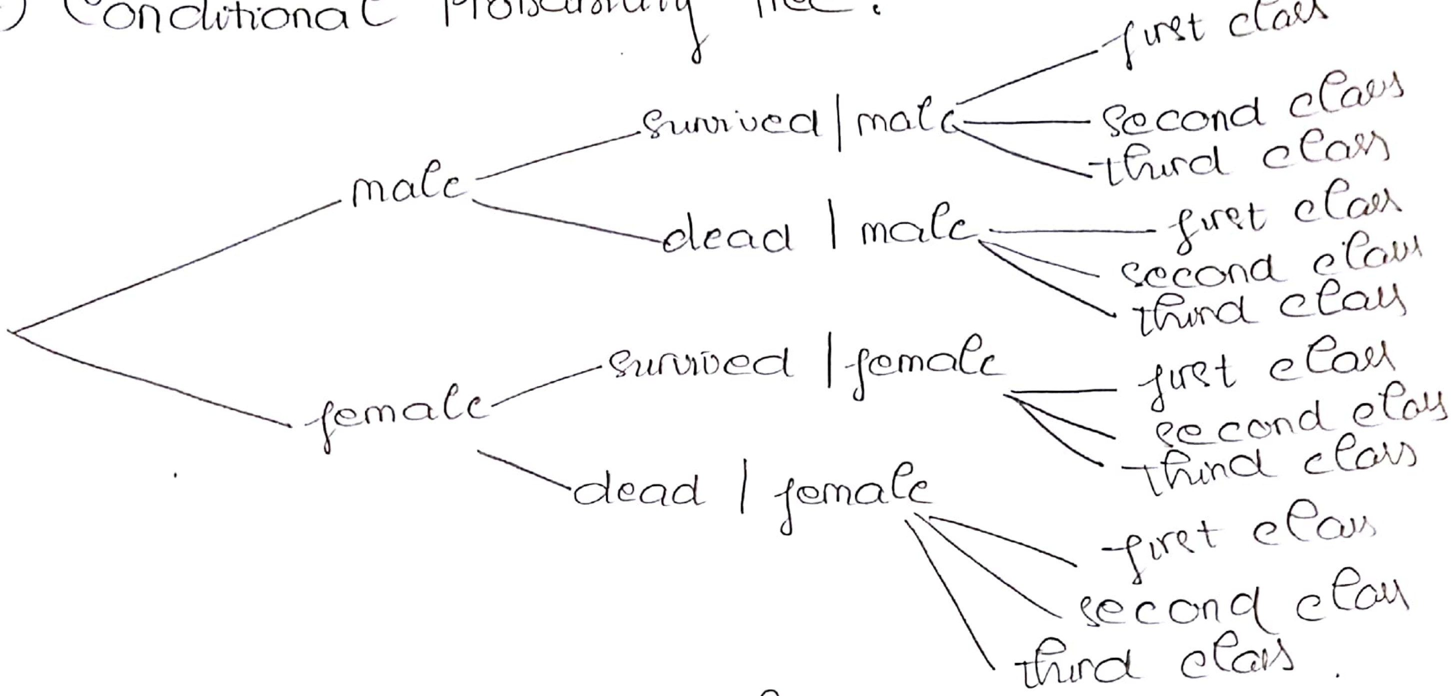
$P_2 = 0.5994$

Code:

```
def p_first_class_survivers(passengers):
    count = 0
    for passenger in passengers:
        if passenger.is_firstclass() and
              passenger.survived:
            count += 1
    print(count / len(passengers))
    return count / len(passengers)

def p_survivors_given_first_class(passengers):
    count = 0
    for passenger on passengers:
        if passenger.survived:
            count += 1
    print(count / len(passengers))
    return count / len(passengers)
p_survivor_given_first_class(f)
```

e) Conditional Probability Tree:



f) $P(f)$: probability of first class

$$= \frac{322}{1312}$$

$P(s)$. probability of survived $= \dfrac{450}{1312}$

$P(f \cap s)$: probability of first class and survived

$$\simeq 0.1471$$

$$P(f \cap s) \simeq 0.1471 \neq P(f) \cdot P(s) \simeq 0.085$$

$\Leftrightarrow$ ticket class and survival are not dependent.

g) Before looking at the data, I didn't expect any events to be independent.

So I picked the first 30 passengers and it results on my guess is correct.

Question 4:

a) There are 4 posible outcomes for each roll.
There are 3 rolls in total
So the sample space has the size of $4^3 = 64$

b) $A = \{(1,1,3), (1,2,2), (1,3,1), (2,1,2), (2,2,1), (3,1,1)\}$ where (Roll$_1$, Roll$_2$, Roll$_3$)

$|A| = 6$

$\Rightarrow P(A) = \frac{6}{64} = \frac{3}{32}$

c) A and B have no common outcome because no outcome in A has a maximum of 3 rolls greater than 3.

$\Rightarrow P(A \cap B) = 0$

$\Rightarrow P(A|B) = \frac{P(A \cap B)}{P(B)} = 0.$

d) Because first roll is 4, so the second and third rolls can be any value because the first roll has fulfilled the winning condition of Bob
So there are $4 \times 4 = 16$ outcomes for Bob to win when first roll is 4.

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e) Because the first and second roll both are 4, So it fulfills Bob's winning requirements, so third roll can be any value. 3^{rd} roll has 4 possible outcomes So there are 4 possible outcomes for Bob.

f) Consider B^c is the complement event of B where all rolls are not greater than 3.
That means each roll has 3 possible outcomes 1, 2 and 3
So $|B^c| = 3^3 = 27 \rightarrow P(B^c) = \frac{27}{64} \Rightarrow P(B) = 1 - \frac{27}{64} = \frac{37}{64}$

g) We have the possibility of no one wins is the complement of event that is the union of Bob wins and Alice wins: $P(A \cup B)^c = 1 - P(A \cup B)$

$$= 1 - [P(A) + P(B) - \underbrace{P(A \cap B)}_{0}]$$

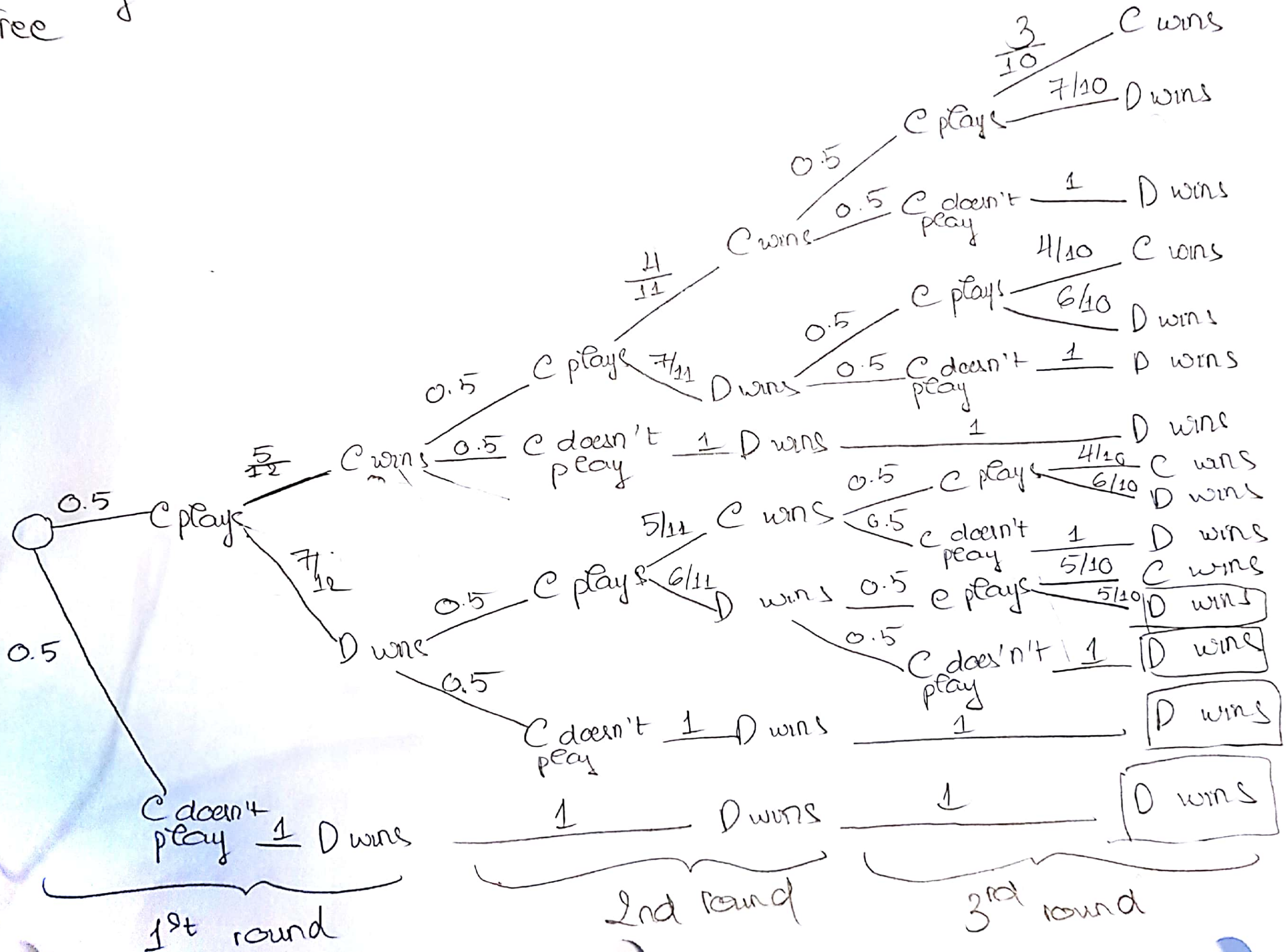
$$= 1 - P(A) - P(B)$$

$$= 1 - \frac{3}{32} - \frac{37}{64}$$

$$= \frac{21}{64}$$

h) A and B are disjoint because $P(A \cap B) = 0$
And they are not independent because $P(A) \cdot P(B) \neq 0$

Conditional Probability Tree



Question 5: a)

b) Using probability tree on a)

Probability D wins after first round:

$$0.5 \times \frac{7}{12} + 0.5 \times 1 = \frac{19}{24}$$

c) Using probability tree on a):

Probability D wins after second round given C plays first round:

$$\frac{7}{11} \times 0.5 \times \frac{5}{12} + 1 \times 0.5 \times \frac{5}{12}$$

$$+ \frac{6}{11} \times 0.5 \times \frac{7}{12} + 1 \times 0.5 \times \frac{7}{12}$$

$$= \frac{19}{24}$$

d) Using probability tree on a)

Probability D wins after third round given C plays first and second round:

$$\frac{5}{12} \times 0.5 \times \left(\frac{7}{10} \times 0.5 \times \frac{4}{11} + 1 \times 0.5 \times \frac{4}{11} + \frac{6}{10} \times 0.5 \times \frac{7}{11} + 1 \times 0.5 \times \frac{7}{11} \right)$$

$$+ \frac{7}{12} \times 0.5 \left(\frac{6}{10} \times 0.5 \times \frac{5}{11} + 1 \times 0.5 \times \frac{5}{11} + \frac{5}{10} \times 0.5 \times \frac{6}{11} + 1 \times 0.5 \times \frac{6}{11} \right)$$

$$= \frac{35}{44}$$

e) Using conditional probability tree:
D wins exactly 3 times is circled nodes:

$$P = \frac{5}{10} \times 0.5 \times \frac{6}{11} \times 0.5 \times \frac{7}{12} \times 0.5$$

$$+ 1 \times 0.5 \times \frac{6}{11} \times 0.5 \times \frac{7}{12} \times 0.5$$

$$+ 1 \times 1 \times 0.5 \times \frac{7}{12} \times 0.5$$

$$+ 1 \times 1 \times 1 \times 0.5$$

$$\simeq 0.7055$$

f) Because D wins 3 rounds, that means C doesn't win any round so her desire stays the same.
Therefore the probability is still 0.7055