
CMPSCI 240: Reasoning Under Uncertainty

First Midterm Exam

February 18, 2015.

Name: _____ ID: _____

Instructions:

- Answer the questions directly on the exam pages.
- Show all your work for each question. Providing more detail including comments and explanations can help with assignment of partial credit.
- If the answer to a question is a number, you may give your answer using arithmetic operations, such as addition, multiplication, “choose” notation and factorials (e.g., “ $9 \times 35! + 2$ ” or “ $0.5 \times 0.3 / (0.2 \times 0.5 + 0.9 \times 0.1)$ ” is fine).
- If you need extra space, use the back of a page.
- No books, notes, calculators or other electronic devices are allowed. Any cheating will result in a grade of 0.
- If you have questions during the exam, raise your hand.

Question	Value	Points Earned
1	10	
2	10	
3	10	
4	10	
5	10	
6	8+2 (Extra Credit)	
Total	58	

Question 1. (10 points) Indicate whether each of the following statements is TRUE or FALSE. No justification is required.

1.1 (2 points): If A and B are disjoint events then $P(A \cup B) = P(A) + P(B)$.

1.2 (2 points): If S is a set containing exactly 5 elements, then the power-set of S contains exactly 25 elements.

1.3 (2 points): For any two events A and B , $P(A \cap B) = P(A)P(B)$.

1.4 (2 points): For any two events A and B where $0 < P(B) < 1$, $P(A|B) = 1 - P(A^c|B^c)$.

1.5 (2 points): If $P(A) > 1/2$ and $P(B) > 1/2$ then $P(A \cap B) > 0$.

Question 2. (10 points) Babies are hungry 30% of the time. When a baby is hungry, they cry with probability 0.8. When a baby is not hungry, they cry with probability 0.5. Let H be the event that a baby is hungry and let C be the event that they are crying.

2.1 (4 points): Enter values for the following probabilities:

$$P(H) = \quad P(C|H) = \quad P(C^c|H) = \quad P(C|H^c) =$$

2.2 (2 points): What's the probability the baby is crying and hungry?

2.3 (2 points): What's the probability the baby is crying?

2.4 (2 points): If we can hear the baby crying, what's the probability that the baby is hungry?

Question 3. (10 points) Most debit cards have a 4-digit pin number. There are 10000 possible numbers:

$$\Omega = \{0000, 0001, 0002, \dots, 9998, 9999\}$$

3.1 (2 points): How many pin numbers are there where every digit is odd? Examples include 1353 and 7777.

3.2 (2 points): How many pin numbers are there where at least one digit is even? Examples include 1354 and 6768.

3.3 (2 points): How many pin numbers are there where every digit is different? Examples include 1354 and 6745.

3.4 (2 points): How many pin numbers are there that are palindromes, i.e., they are the same when read forwards and backwards. Examples include 1331 and 6776.

3.5 (2 points): How many pin numbers are there where every digit is strictly larger than the preceding digit? Examples include 1356 and 2459.

Question 4. (10 points) Draw two cards from a standard pack of cards *without replacement*. Recall that there are 52 cards and each card has one of four suits (hearts, clubs, spades, diamonds) and one of thirteen ranks (ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, and king). Let A_1 be the event that the *first* card you draw is an ace and let K_2 be the event that the *second* card you draw is a king. Let D be the event that the two cards are from different suits.

4.1 (4 points): Enter values for the following probabilities:

$$P(A_1) = \quad P(K_2) = \quad P(A_1 \cap K_2) = \quad P(A_1^c \cap K_2) =$$

4.2 (1 points): Are the events A_1 and K_2 independent?

4.3 (4 points): Enter values for the following probabilities:

$$P(A_1|D) = \quad P(K_2|D) = \quad P(A_1 \cap K_2|D) = \quad P(D) =$$

4.4 (1 points): Are the events A_1 and K_2 independent conditioned on the event D ?

Question 5. (10 points) The “Dicey Die” Dice Company has manufactured a loaded dice. There are six possible outcomes $\{1, 2, 3, 4, 5, 6\}$ and the probabilities of these outcomes are:

$$P(1) = 0.1, P(2) = 0.1, P(3) = 0.1, P(4) = 0.1, P(5) = 0.1, \text{ and } P(6) = 0.5.$$

5.1 (4 points): For the experiment where you roll the loaded dice once, define the events $A = \{2, 4, 6\}$ and $B = \{3, 6\}$. Enter values for the following probabilities:

$$P(A) =$$

$$P(B) =$$

$$P(A \cup B) =$$

$$P(A \cap B) =$$

5.2 (2 points): For the experiment where you roll the loaded dice three times, what’s the probability you get exactly two sixes?

5.3 (2 points): For the experiment where you roll the loaded dice twice, what’s the probability you get the same value both times?

5.4 (2 points): Suppose you roll a normal unloaded dice and the loaded dice at the same time. What’s the probability you get the same value on both dice?

Question 6. (10 points) I am thinking of a random number in the range $\{1, 2, \dots, 300\}$ and I am equally likely to be thinking of any of these three hundred numbers. Define the following events:

- A = “I’m thinking of a number that is divisible by two.”
- B = “I’m thinking of a number that is divisible by three.”
- C = “I’m thinking of a number that is divisible by five.”

For example, $C = \{5, 10, 15, \dots, 295, 300\}$, $|C| = 300/5 = 60$, and $P(C) = 1/5$.

6.1 (4 points): Enter values for the following probabilities:

$$P(A) =$$

$$P(B) =$$

$$P(A \cap B) =$$

$$P(A \cap B \cap C) =$$

6.2 (2 points): Enter values for the following probabilities:

$$P(A \cup B) =$$

$$P(A^c \cap B^c) =$$

6.3 (2 points): What’s the probability that the number I’m thinking of is divisible by exactly two of the numbers 2, 3, and 5?

6.4 (2 points): **Extra Credit:** Define the additional event

- $D = \text{"I'm thinking of a number that is divisible by seven"}$

and suppose I narrow the range of numbers I am thinking about to $\{1, 2, \dots, 100\}$. Write the event "I'm thinking of a number that isn't prime" in terms of the events A, B, C and D . What's the probability of this event?

