# CMPSCI 240 Spring 2018: Homework 1

Due: 5pm February 9, 2018

## Instructions for submission:

**Everyone:** Write each question on a separate page. Failure to do so may result in grading delays. You may use multiple pages for a single question. Do not write your name on any page of your submission.

- Any question with your name on it will result in automatic failure for that question. If your name appears on every page, it will result in failure of the assignment.
- If the graders cannot read a solution, they will mark it as zero *and will not re-grade it*. Therefore, make sure you check that your submission is legible before the deadline.
- Solutions will be released at 10am Monday morning. *Do not post on Piazza about homework solutions before then.*

**If you have consented to using Gradescope:** Submit a typed or legible scanned copy *without your name on any page* to Gradescope.

- If you submit your homework on Gradescope but have not consented to using Gradescope, the graders will not mark your homework, which could result in an incomplete or failing grade for the course. If you have questions about this, contact Yash Chandak (ychandak@cs.umass.edu).

**If you have not consented to using Gradescope:** Contact Yash Chandak (ychandak@cs.umass.edu) for a token to write on every page. Submit your work to the homework drop box in Computer Science main office before it closes at 5pm.

- Be sure to write your token on every page; homeworks may be separated to parallelize grading.

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**Q1.** (*21pts*) Let $A$, $B$, and $C$ be three sets. For each of the statements a)-e), state whether the statement is true or false. If it is true, provide a proof. If it is false, provide an example where the statement fails to hold. For the statements f) and g), answer as indicated.

**a)** $\mathbb{P}(A \mid \Omega) = \mathbb{P}(A)$.

**b)** $\mathbb{P}(A \cup B \mid A) = \mathbb{P}(A)$.

**c)** If $\mathbb{P}(A \mid B) = 1$, then $A \subseteq B$.

**d)** If $\mathbb{P}(A \cup B^c) = 1$, then $\mathbb{P}(B) \leq \mathbb{P}(A)$.

**e)** If $A$ and $B$ are independent, then $\mathbb{P}(A \cup B) \leq \mathbb{P}(A)\mathbb{P}(B)$.

**f)** For what range of values does $\mathbb{P}(A)\mathbb{P}(B) = \mathbb{P}(A) + \mathbb{P}(B)$ hold? Justify your answer.

**g)** Can $\mathbb{P}(A \cup B) = \mathbb{P}(A \cap B)$ be true if $A$ and $B$ are independent? Disjoint? If it can be true, provide an example. If it cannot, provide a proof.

**Q2.** (*12pts*) Consider a sample space of the outcomes of two rolls of a regular die. Define the following events:

$E_+$**:** the sum of the rolls is even

$O_+$**:** the sum of the rolls is odd

$E_1$**:** the first roll is even

$O_1$**:** the first roll is odd

$E_2$**:** the second roll is even

$O_2$**:** the second roll is odd

**a)** List all the pairs of events that are disjoint and justify your answers by reasoning about the sets and/or their probabilities.

**b)** List all pairs of events that are independent and justify your answers by reasoning about the sets and/or their probabilities.

**Q3.** (*25pts*) For this problem, you will will be using the Titanic survival data set. We have provided some starter code in Python and the data. All of the names in teletype font refer to functions in `starter.py`.

**a)** (*3pts*) In this context, what is $\Omega$? $|\Omega|$? Turn in your code for `omega` here.

**b)** (*3pts*) Let the event $S$ mean that a passenger survived. What is $|S|$? Turn in your code for `survived` here.

**c)** (*3pts*) Let the event $F$ mean that a passenger traveled first class. What is $|F|$? Turn in your code for `first_class` here.

**d)** (*3pts*) What is the probability that a first class passenger survived? What is the probability that a passenger survived, given that they were first class? Turn in your code for `p_first_class_survivors` and `p_survivor_given_first_class` here.

**e)** (*3pts*) Draw the conditional probability tree such that the leaves give you the joint probability of ticket class, sex, and survival.

**f)** (*3pts*) Are ticket class and survival independent? Justify your answer.

**g)** (*7pts*) Before looking at the data, would you have expected any of the events (e.g., ticket class and sex) to be independent? Select an arbitrary 30 outcomes and analyze whether ticket class and survival are independent. Does your answer change? Write a paragraph on what you observe and what you think is happening, and report on how you chose those 30 outcomes.

**Q4.** (*24pts*) Alice and Bob are playing a game with three four-sided fair dice. Alice wins if the sum of the three dice is 5. Bob wins if the maximum of the three dice is greater than 3. Show your work and/or argumentation in all cases. *Note: you can solve this problem without combinatorics.*

**a)** What is the size of the sample space?

**b)** Let $A$ denote the event that Alice wins. What is $\mathbb{P}(A)$?

**c)** Let $B$ denote the event that Bob wins. What is $\mathbb{P}(A \mid B)$?

**d)** Assume the first die shows a 4. How many possible winning outcomes are there for Bob?

**e)** Assume either the first or the second die shows a 4. How many possible winning outcomes are there for Bob?

**f)** What is $\mathbb{P}(B)$?

**g)** What is is the probability that no one wins?

**h)** Are $A$ and $B$ disjoint, independent, both, or neither? Justify your answer.

**Q5.** (*18pts*) Cibele and Dave are playing a game with 12 cards, numbered 1 through 12. Each round they pick one of the cards. Cibele wins the round if the card is greater than 7 and Dave wins otherwise. Each time a round is played, the card is removed. Furthermore, Cibele is tired and needs to grade exams, so she only has a 50% chance of wanting to play before each round. If Cibele doesn't play, Dave wins. Once Cibele declines to play, she never plays again. Dave would like to play three rounds.

**a)** Draw the conditional probability tree for this scenario.

**b)** What's the probability that Dave wins after the first round (remember, Cibele has to decide to play first)?

**c)** What's the probability that Dave wins after the second round, given that Cibele and Dave played a first round?

**d)** What's the probability that Dave wins after the third round, given that Cibele and Dave already played two rounds?

**e)** What's the probability that Dave wins in exactly three rounds?

**f)** Now assume that Cibele is looking for a rewarding distraction. Every time she wins, her desire to play another round goes up by 50%. What is the probability that Dave wins in exactly three rounds now?