CMPSCI 240: Reasoning Under Uncertainty
First Midterm Exam

February 13, 2013.

Name: _____ ID: _____

Instructions:

- Answer the questions directly on the exam pages.
- Show all your work for each question. Giving more detail including comments and explanations can help with assignment of partial credit.
- If the answer to a question is a number, you may give your answer using arithmetic operations, such as addition, multiplication, and factorial (e.g., "$9 \times 35! + 2$" or "$0.5 \times 0.3/(0.2 \times 0.5 + 0.9 \times 0.1)$" is fine). "Choose" notation must be expanded in terms of arithmetic operations in final answers to receive full points.
- If you need extra space, use the back of a page.
- No books, notes, calculators or other electronic devices are allowed. Any cheating will result in a grade of 0.
- If you have questions during the exam, raise your hand.

Question	Value	Points Earned
1	10	
2	10	
3	10	
4	10	
5	10	
6 (Extra Credit)	10	
Total	50	

Question 1. (*10 points*) Indicate whether each of the following statements is TRUE or FALSE. No justification is required.

1.1 (*2 points*): *For any two events A and B,*

$$(A \cup B)^c = A^c \cap B^c$$

1.2 (*2 points*): *For any three events $A, B,$ and C where A and B are disjoint and $0 < P(C) < 1$,*

$$P(A \cup B|C) = P(A|C) + P(B|C)$$

1.3 (*2 points*): *For any two events A and B where $0 < P(B) < 1$,*

$$P(A|B) = 1 - P(A|B^c)$$

1.4 (*2 points*): *For any three events $A, B,$ and C,*

$$P(A \cap B \cap C) = P(A)P(B)P(C)$$

1.5 (*2 points*): *For any two events A and B,*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Question 2. (*10 points*) Suppose you throw a fair 12-sided dice to get a value from the set

$$\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} .$$

Consider the events

$$A = \{1, 2, 3, 4, 11, 12\} \quad , \quad B = \{3, 4, 5, 6, 11, 12\} \quad , \quad C = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

2.1 (*1 points*): *What is the value of* $P(A)$*?*

2.2 (*1 points*): *What is the value of* $P(B)$*?*

2.3 (*2 points*): *What is the value of* $P(A \cap B)$*?*

2.4 (*1 points*): *Are the events* A *and* B *independent?*

2.5 (*1 points*): *What is the value of* $P(A|C)$*?*

2.6 (*1 points*): *What is the value of* $P(B|C)$*?*

2.7 (*2 points*): *What is the value of* $P(A \cap B|C)$*?*

2.8 (*1 points*): *Are the events* A *and* B *independent conditioned on* C*?*

Question 3. (*10 points*) Suppose you perform an experiment where the sample space is

$$\Omega = \{o_1, o_2, o_3, o_4, o_5, o_6\}$$

and the probability rule satisfies:

$$P(\{o_1\}) = 1/2 \quad , \quad P(\{o_2\}) = 1/4 \quad , \quad P(\{o_3\}) = 1/8 \quad , \quad P(\{o_4\}) = 1/16 \quad , \quad P(\{o_5\}) = 1/32$$

Define the events $A = \{o_1, o_2\}$, $B = \{o_2, o_3\}$, and $C = \{o_1, o_2, o_3, o_4, o_5\}$.

3.1 (*2 points*): *What is the value of* $P(\{o_6\})$*?*

3.2 (*2 points*): *What is the value of* $P(A)$*?*

3.3 (*2 points*): *What is the value of* $P(A \cup B)$*?*

3.4 (*2 points*): *What is the value of* $P(A \cap B)$*?*

3.5 (*2 points*): *What is the value of* $P(A \cap B|C)$*?*

Question 4. (*10 points*) Every Tuesday night I go to one of three restaurants: Amherst Chinese, Freshside, or Paradise of India. I go to Amherst Chinese with probability 0.3. I go to Freshside with probability 0.6. I go to Paradise of India with probability 0.1. If I go to Amherst Chinese, I'll eat rice with probability 1. If I go to Freshside, I'll eat rice with probability 0.3. If I go to Paradise of India, I'll eat rice with probability 0.4. Define the events:

$$C = \text{“go to Amherst Chinese”} \quad , \quad F = \text{“go to Freshside”}$$

$$I = \text{“go Paradise of India”} \quad , \quad R = \text{“eat rice”}$$

4.1 (*2 points*): *Enter the values for the following probabilities:*

$$P(C) = \qquad P(F) = \qquad P(I) = \qquad P(R|C) = \qquad P(R|F) = \qquad P(R|I) =$$

4.2 (*2 points*): *What is the probability that I'll go to Freshside and eat rice?*

4.3 (*2 points*): *What is the probability that I'll go to Paradise of India and not eat rice?*

4.4 (*2 points*): *What is the probability that I'll eat rice?*

4.5 (*2 points*): *If you know I ate rice, what is the probability that I went to Amherst Chinese?*

Question 5. (*10 points*) There are 97 students in the class: 36 are in the first discussion section, 37 are in the second discussion section, and 24 are in the third discussion section. Suppose I write the name of each student on a piece of paper and place all the pieces in a hat. I then randomly pick 2 names out of the hat (without replacement).

5.1 (*3 points*): *What's the probability they are both in the first discussion section?*

5.2 (*3 points*): *What's the probability they are both in the same discussion section?*

5.3 (*2 points*): *What's the probability they are in different discussion sections?*

5.4 (*2 points*): *If I keep on picking names out of the hat, what's the probability that the last name is from the third discussion section?*

Question 6. (*10 points*) ***Extra Credit:*** In the Spring 2013 offering of CMPSCI 245 "Reasoning about Certainty" there are six students: Amit, Bob, Charlie, Diane, Ely, and Fiona. Amit, Bob, and Charlie are juniors and Diane, Ely, and Fiona are seniors. In each lecture, some subset of the students are present. Let $S = \{\text{Amit, Bob, Charlie, Diane, Ely, Fiona}\}$. Hint: You'll be able to solve parts 2, 3, 4, and 5 by enumerating all the possibilities (and will get full credit if you do it correctly) but there are more efficient ways.

6.1 (*2 points*): *How many subsets of S are there? Remember to include the empty set and S.*

6.2 (*2 points*): *How many subsets are there with exactly one junior?*

6.3 (*2 points*): *How many subsets are there with exactly three students?*

6.4 (*2 points*): *How many subsets are there with the same number of juniors and seniors?*

6.5 (*2 points*): *How many subsets are there where there are strictly more seniors than juniors?*