

RStudio: Linear Algebra

Eigenvalues and Eigenvectors

Definition (Eigenvalues)

Let A be an $n \times n$ matrix, I_n the identity $n \times n$ matrix and λI_n the diagonal with λ -scalar. Then $A - \lambda I_n$ is an $n \times n$ matrix and the determinant $|A - \lambda I_n|$ is a polynomial which we denote by $P_N(\lambda)$. $P_N(\lambda)$ is called the characteristic polynomial of A . The roots of $P_N(\lambda)$ are called eigenvalues of A .

Example

Find the eigenvalues of the matrix

$$A = \begin{pmatrix} 1 & 0 \\ 16 & 6 \end{pmatrix}.$$

Solution

We first find the characteristic polynomial.

$$\begin{aligned} P_n(\lambda) &= |A - \lambda I_n| \\ &= \begin{vmatrix} 1 - \lambda & 0 \\ 16 & 6 - \lambda \end{vmatrix} \\ &= (1 - \lambda)(6 - \lambda). \end{aligned}$$

To get the eigenvalues, set $P_n(\lambda) = 0$.

$$(1 - \lambda)(6 - \lambda) = 0$$

$$\lambda_1 = 1, \quad \lambda_2 = 6.$$

To find the eigenvalues using R, first enter the matrix

```
A<-matrix(c(1,0,16,6),nrow=2,ncol=2,byrow=TRUE)
```

Then use the command

```
eigen(A,only.value=TRUE)
```

```
## $values
## [1] 6 1
##
## $vectors
## NULL
```

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