

Quiz 3

Directions: You may not use your class notes, calculator, cell phone or any other electronic devices. You must show all your work to receive credit. Your grade will be determined not only by your final answer, but also the correctness of your work.

Name: _____

Section Number: _____

1. Calculate $\lim_{t \rightarrow 3} r(t)$, where $r(t) = \left\langle \frac{9t^{-1} - 3(t-2)^{-1}}{t-3}, \frac{t-3}{\sqrt{t^2+7}-4} \right\rangle = \langle x(t), y(t) \rangle$

$$\lim_{t \rightarrow 3} x(t) = \lim_{t \rightarrow 3} \frac{9t^{-1} - 3(t-2)^{-1}}{t-3}$$

$$= \lim_{t \rightarrow 3} \left(\frac{9}{t} - \frac{3}{t-2} \right) \times \frac{1}{t-3}$$

$$= \lim_{t \rightarrow 3} \frac{9(t-2) - 3t}{t(t-2)} \times \frac{1}{(t-3)}$$

$$= \lim_{t \rightarrow 3} \frac{9t - 18 - 3t}{t(t-2)} \times \frac{1}{t-3}$$

$$= \lim_{t \rightarrow 3} \frac{6(t-3)}{t(t-2)(t-3)} \times \frac{1}{(t-3)} = \frac{6}{3(3-2)} = 2$$

$$g(x) = \begin{cases} x^2 - c^2 & \text{if } x < 4 \\ cx + 20 & \text{if } x \geq 4 \end{cases}$$

For what value(s) of c make(s) $g(x)$ continuous?

For continuity

$$4^2 - c^2 = c(4) + 20$$

$$16 - c^2 = 4c + 20$$

$$c^2 + 4c + 4 = 0$$

$$(c+2)^2 = 0$$

$$c = -2 \text{ (twice)}$$

Pick one; $c = -2$ makes $g(x)$ continuous.

$$\lim_{t \rightarrow 3} y(t) = \lim_{t \rightarrow 3} \frac{t-3}{\sqrt{t^2+7}-4} \times \frac{\sqrt{t^2+7}+4}{\sqrt{t^2+7}+4}$$

$$= \lim_{t \rightarrow 3} \frac{(t-3)(\sqrt{t^2+7}+4)}{(t^2+7)-16}$$

$$= \lim_{t \rightarrow 3} \frac{(t-3)(\sqrt{t^2+7}+4)}{t^2-9}$$

$$= \lim_{t \rightarrow 3} \frac{(t-3)(\sqrt{t^2+7}+4)}{(t-3)(t+3)} = \frac{\sqrt{9+7}+4}{3+3} = \frac{4}{3}$$

So

$$\lim_{t \rightarrow 3} r(t) = \left\langle \lim_{t \rightarrow 3} x(t), \lim_{t \rightarrow 3} y(t) \right\rangle = \left\langle 2, \frac{4}{3} \right\rangle$$