REPRESENTATION OF FINITE AUTOMATA WITH tnreason

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1 Deterministic Finite Automata

Literature:

- Crosswhite and Bacon (2008) Basic Representation by TT/MPS.
- Bellante et al. (2026) Usage of canonical formats (orthogonalization) to find quantum circuit representations.

Idea:

- Construct chain graphs decorated with basis encodings of state transition functions, as in Crosswhite and Bacon (2008); Bellante et al. (2026)
- Contract with basis encoding of halting states and get the boolean tensor indicating the accepted states of the automaton

Definition 1 (Deterministic finite automaton (DFA), see Def 2 in Bellante et al. (2026)). *A tuple of*

- *set of internal states Q*
- *alphabet Σ*
- *initial state $r_0 \in Q$*
- *accepted states $F \subset Q$*
- *transition function $\delta : Q \otimes \Sigma \to Q \cup \{\varnothing\}$*

is called a deterministic finite automaton (DFA).

We enumerate the set Q by an map I_Q with enumeration variable O_Q, and the alphabet Σ by a map I_Σ with an enumeration variable O_Σ. Both indicator variables will be copied for $k \in [d]$ (respectively $k \in [d+1]$) and the copies denoted by $O_{\Sigma,k}$ and $O_{Q,k}$. The acceptance tensor of order $d \in \mathbb{N}$ is the boolean tensor $\tau\left[O_{\Sigma,[d]}\right]$ indicating whether the automaton accepts the input sequence of length at most d. Its coordinates are

$$\tau\left[O_{\Sigma,[d]} = o_{\Sigma,[d]}\right] = \begin{cases} 1 & \text{if} \quad \text{The automaton accepts the input sequence } (I_\Sigma(O_k = o_k) : k \in [d]) \\ 0 & \text{else} \end{cases}.$$

Lemma 1 (Tensor representation). *Let $(Q, \Sigma, \delta, r_0, F)$ be a deterministic finite automaton. Then the acceptance tensor of order $d \in \mathbb{N}$ is the contraction*

$$\tau\left[O_{\Sigma,[d]}\right] = \left\langle \left\{\beta^\delta\left[O_{Q,k}, O_{\Sigma,k}, O_{\Sigma,k}\right] : k \in [d]\right\} \cup \left\{\epsilon_{I_Q^{-1}(r_0)}\left[O_{Q,0}\right], \epsilon_{I_Q^{-1}(F)}\left[O_{Q,d}\right]\right\} \right\rangle_{\left[O_{\Sigma,[d]}\right]}.$$

Proof. By basis calculus: Given the one-hot encoding of the input word and contracting with the tensor network (expect the basis encoding tensor of F) leaving $O_{Q,d}$ open, we get the one hot encoding of the finial state of the automaton. Contracting with the one-hot encoding of F is then 1, if and only if the final state is in the accepted states. $\qquad\square$

Note that the acceptance tensor of the complement DFA is prepared when replacing the basis encoding $\epsilon_{I_Q^{-1}(F)}[O_{Q,d}]$ by the basis encoding $\epsilon_{I_Q^{-1}(\bar{F})}[O_{Q,d}]$ of the complement of the accepting states.

2 Stochastic Finite Automata

Ideas:

- The transition function is replaced by a cpd, which is encoded by a more general directed tensor and copied to all cores of a TT.
- The result is a Markov Chain, i.e. a Bayesian Network on a chain graph.
- The contraction above then gives the probability of acceptance for a given input word.
- One can add a distribution over the input words, and contract to get a joint distribution of input words with an acceptance variable.

References

Armando Bellante, Reinis Irmejs, Marta Florido-Llinàs, María Cea Fernández, Marianna Crupi, Matthew Kiser, and J. Ignacio Cirac. Compiling Quantum Regular Language States, February 2026. URL http://arxiv.org/abs/2602.02698. arXiv:2602.02698 [quant-ph].

Gregory M. Crosswhite and Dave Bacon. Finite automata for caching in matrix product algorithms. *Physical Review A*, 78(1):012356, July 2008. doi: 10.1103/PhysRevA.78.012356. URL https://link.aps.org/doi/10.1103/PhysRevA.78.012356.