REPRESENTATION OF FINITE AUTOMATA WITH tnreason

RESEARCH NOTES IN THE ENEXA AND QROM PROJECTS

February 9, 2026

1 Deterministic

Literature:

- Crosswhite and Bacon (2008) Basic Representation by TT/MPS.
- **?** Usage of canonical formats (orthogonalization) to find quantum circuit representations.

Idea:

- Construct chain graphs decorated with basis encodings of state transition functions, as in Crosswhite and Bacon (2008); **?**
- Contract with basis encoding of halting states and get the boolean tensor indicating the accepted states of the automaton

Definition 1 (Deterministic finite automaton (DFA), see Def 2 in **?**). *A tuple of*

- *set of internal states* Q
- *alphabet* Σ
- *initial state* $r_0 \in Q$
- *accepted states* $F \subset Q$
- *transition function* $\delta : Q \otimes \Sigma \to Q \cup \{\varnothing\}$

is called a deterministic finite automaton (DFA).

We enumerate the set Q by an map I_Q with enumeration variable O_Q, and the alphabet Σ by a map I_Σ with an enumeration variable O_Σ. Both indicator variables will be copied for $k \in [d]$ (respectively $k \in [d+1]$) and the copies denoted by $O_{\Sigma,k}$ and $O_{Q,k}$. The acceptance tensor of order $d \in \mathbb{N}$ is the boolean tensor $\tau\left[O_{\Sigma,[d]}\right]$ indicating whether the automaton accepts the input sequence of length at most d. Its coordinates are

$$\tau\left[O_{\Sigma,[d]} = o_{\Sigma,[d]}\right] = \begin{cases} 1 & \text{if} \quad \text{The automaton accepts the input sequence } \left(I_\Sigma(O_k = o_k) \,:\, k \in [d]\right) \\ 0 & \text{else} \end{cases} .$$

Lemma 1 (Tensor representation). *Let* $(Q, \Sigma, \delta, r_0, F)$ *be a deterministic finite automaton. Then the acceptance tensor of order* $d \in \mathbb{N}$ *is the contraction*

$$\tau\left[O_{\Sigma,[d]}\right] = \left\langle \left\{\beta^\delta\left[O_{Q,k}, O_{\Sigma,k}, O_{\Sigma,k}\right] \,:\, k \in [d]\right\} \cup \left\{\epsilon_{I_Q^{-1}(r_0)}\left[O_{Q,0}\right], \epsilon_{I_Q^{-1}(F)}\left[O_{Q,d}\right]\right\} \right\rangle_{\left[O_{\Sigma,[d]}\right]} .$$

Proof. By basis calculus: Given the one-hot encoding of the input word and contracting with the tensor network (expect the basis encoding tensor of F) leaving $O_{Q,d}$ open, we get the one hot encoding of the finial state of the automaton. Contracting with the one-hot encoding of F is then 1, if and only if the final state is in the accepted states. ∎

Note that the acceptance tensor of the complement DFA is prepared when replacing the basis encoding $\epsilon_{I_Q^{-1}(F)}[O_{Q,d}]$ by the basis encoding $\epsilon_{I_Q^{-1}(\bar{F})}[O_{Q,d}]$ of the complement of the accepting states.

References

Gregory M. Crosswhite and Dave Bacon. Finite automata for caching in matrix product algorithms. *Physical Review A*, 78(1):012356, July 2008. doi: 10.1103/PhysRevA.78.012356. URL https://link.aps.org/doi/10.1103/PhysRevA.78.012356.