

Analysis of Environmental Data

Regression Modeling

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Eco 602 – University of Massachusetts, Amherst – Fall 2021
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Announcements: Oct 26th

- In-Class Likelihood group self-select *should be fixed.
 - I had it associated with the wrong grouping in Moodle
- For this week
 - Tuesday
 - Finish in-class likelihood
 - Start Deck 7
 - Thursday
 - Continue Deck 7
 - In-class modeling

Regression Modeling

A Dual-Model Approach

What's in This Section?

Important take-home concepts

- What is a regression model?
- What are the 4 key assumptions?
 - Normality of the residuals
 - Homogeneity
 - Fixed x
 - Independent observations
- What is the constellation of methods?

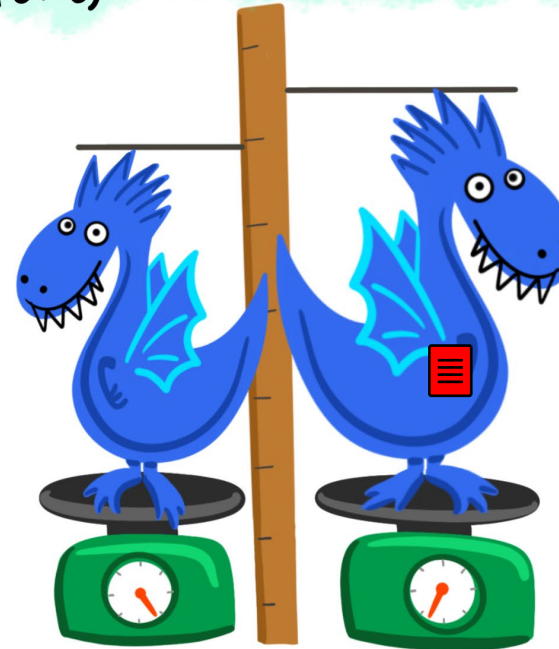
What is a Regression?

Regressions embody the dual-model concept

Regression is a modeling paradigm in which we specify a mathematical relationship between independent and dependent variables.

- A regression includes a *deterministic model* to specify the average behavior.
- It specifies a *stochastic model* to describe the variability around the average behavior.

$$\text{Weight (tons)} = 2.4 + \underline{0.3(\text{height})} + \dots$$



@allison_horst

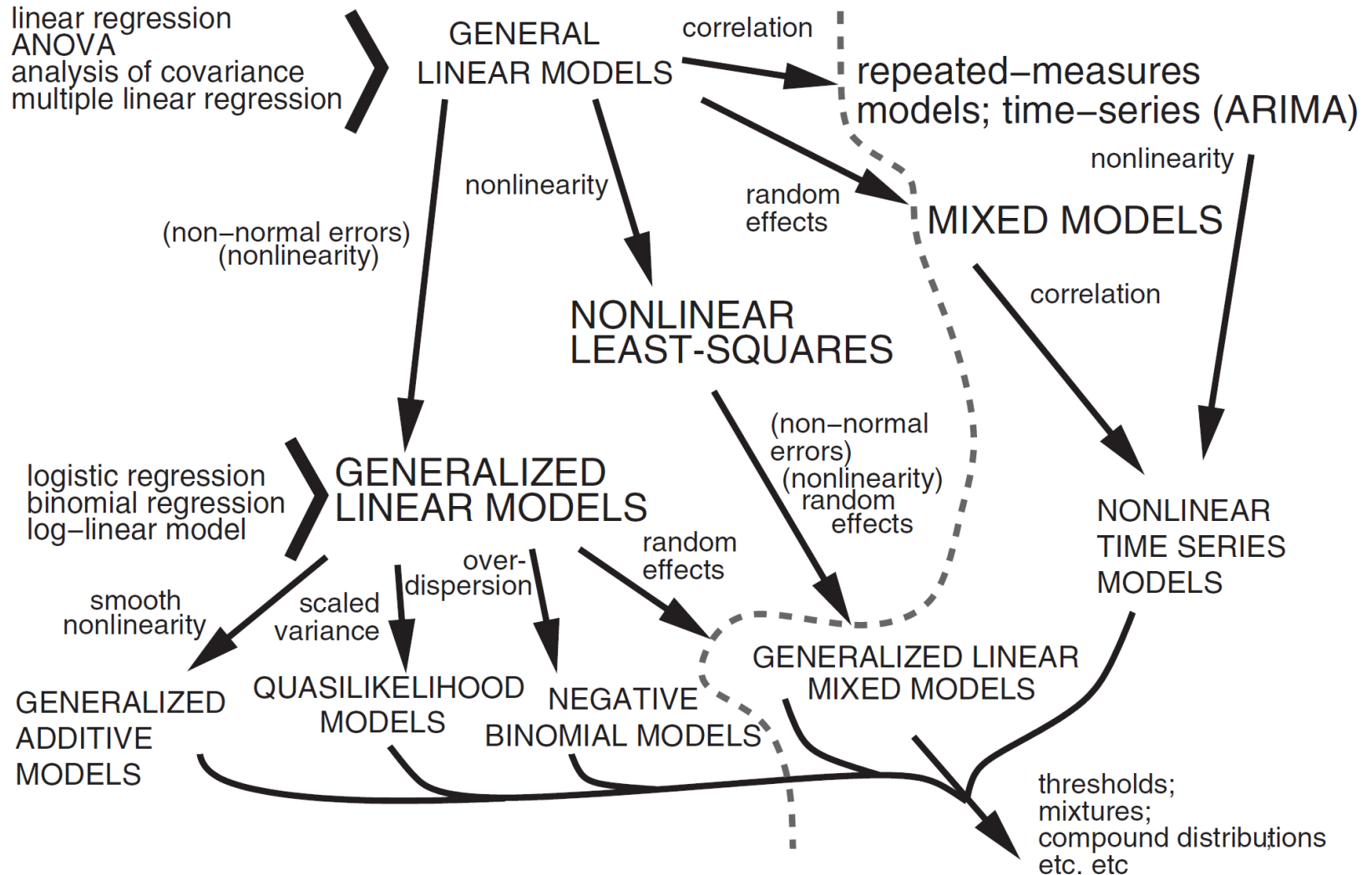
if all other variables constant, we expect a 1 foot taller dragon to weigh 0.3 tons more, on average.

Artwork by @allison_horst

Regression Acronyms: The constellation

Bolker: Ecological Models and Data in R, Figure 9.2

- There are many types of regression models including:
 - General Linear Models
 - Generalized Linear Models
 - Mixed Models
- Think of the collection as a constellation of methods
- There are a lot of similar names and acronyms?



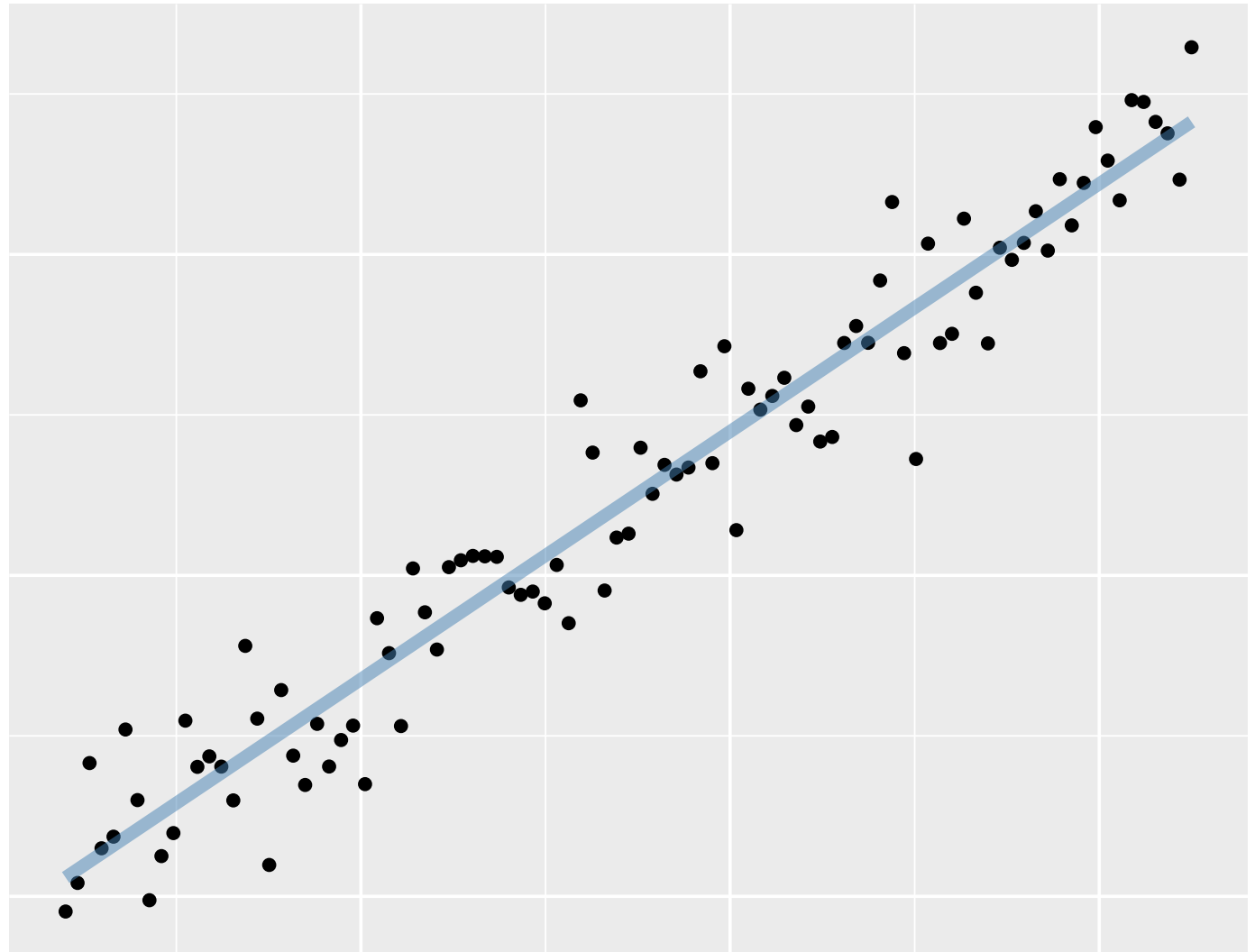
Group 1 Models

The simplest model we can fit is always a linear model!

General Linear Models form the core group of regression models.

- Other regression model paradigms are extensions of General Linear Models.

We'll spend a lot of time on this class of models, which I'll call *Group 1* models.



Group 1 Models – 4 Key Assumptions

Our Group 1 models come with some baggage... Specifically four key assumptions:

- Independent observations
- Constant variance a.k.a homoskedasticity, a.k.a. homogeneity
- Fixed x: no measurement error in our predictor variables
- Normality: normality refers to the model residuals

In addition, Group I requires that our models be *linear in the parameters* and have a response on a **continuous scale**.

The extended models can deal with different violations of these assumptions and requirements.

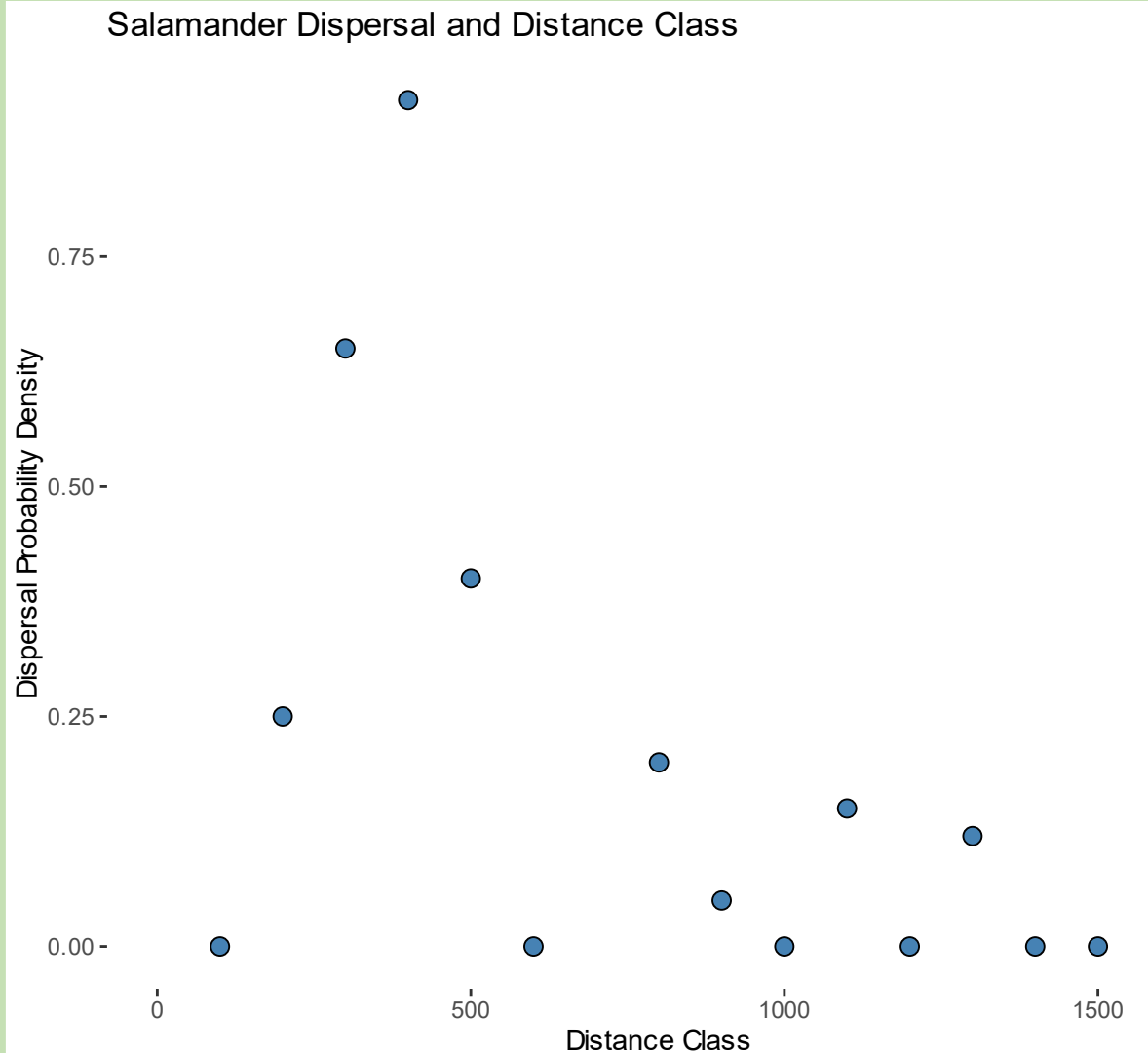
[Residual] Normality Assumption



- Under repeated sampling, data would be normally distributed *at each x* .
- Normally distributed around each *predicted value* in the *deterministic model*.
- This assumption is often misunderstood to mean that the values for each variable in a data set must be normally-distributed by themselves.
- But what is a residual?
 - The difference between a predicted and observed value

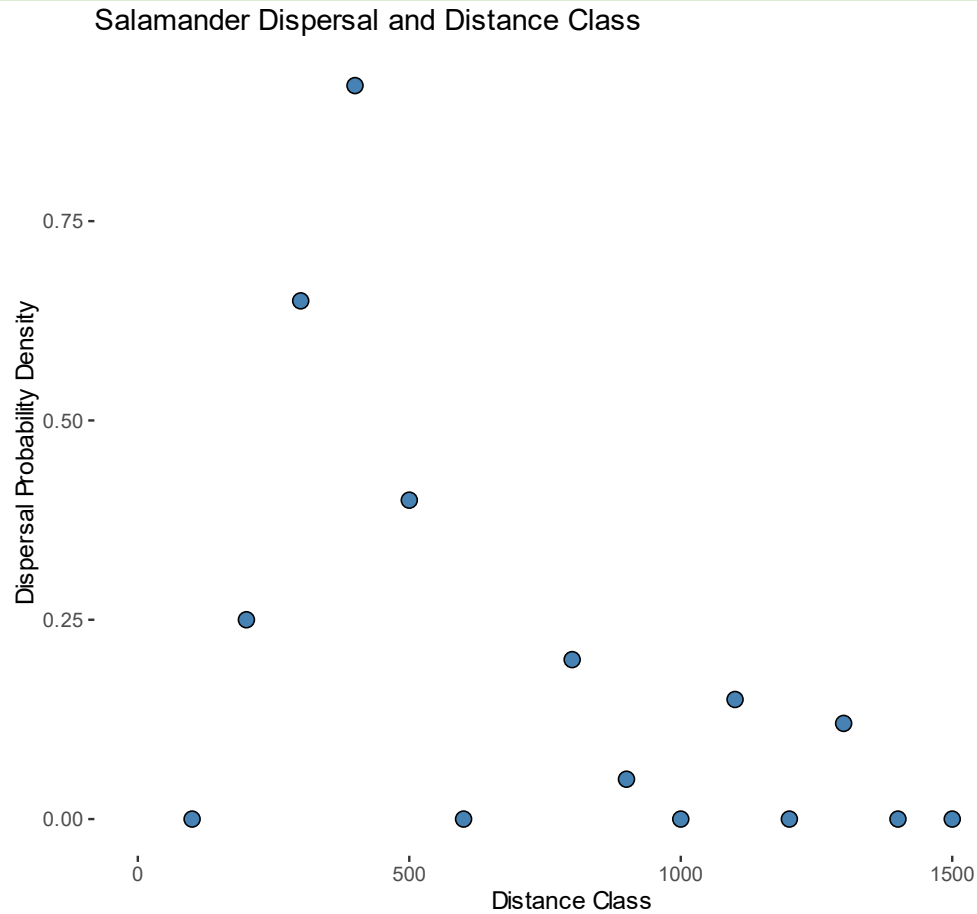
Model Residuals

- Salamander breeder dispersal data
- What kind of model should we fit?
- A Ricker curve might be a good choice

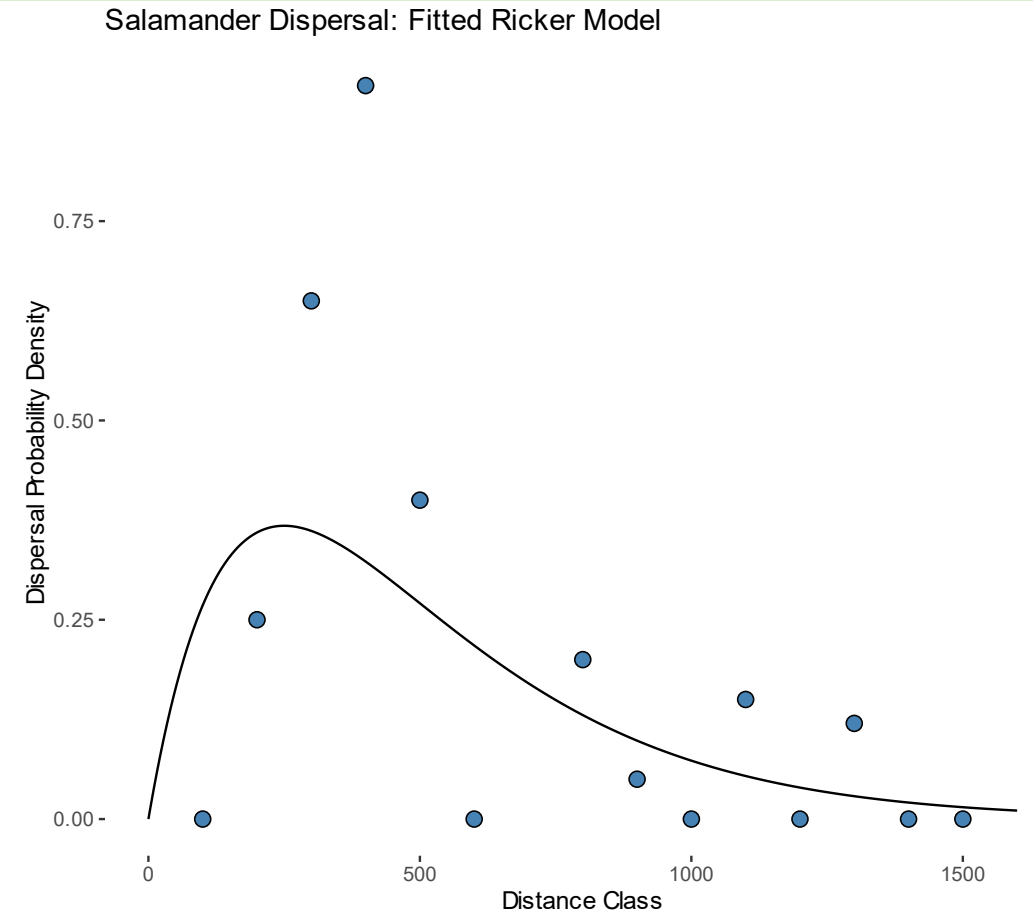


Model Residuals

The Data

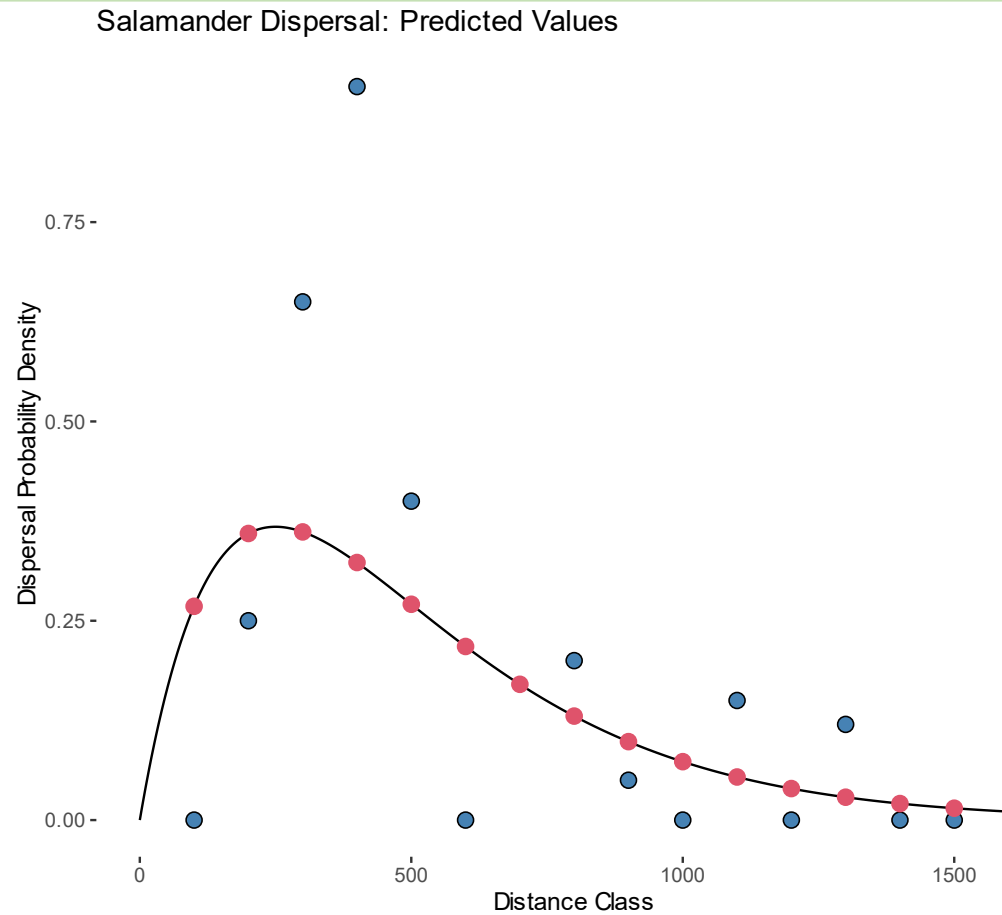


A Fitted Ricker Curve

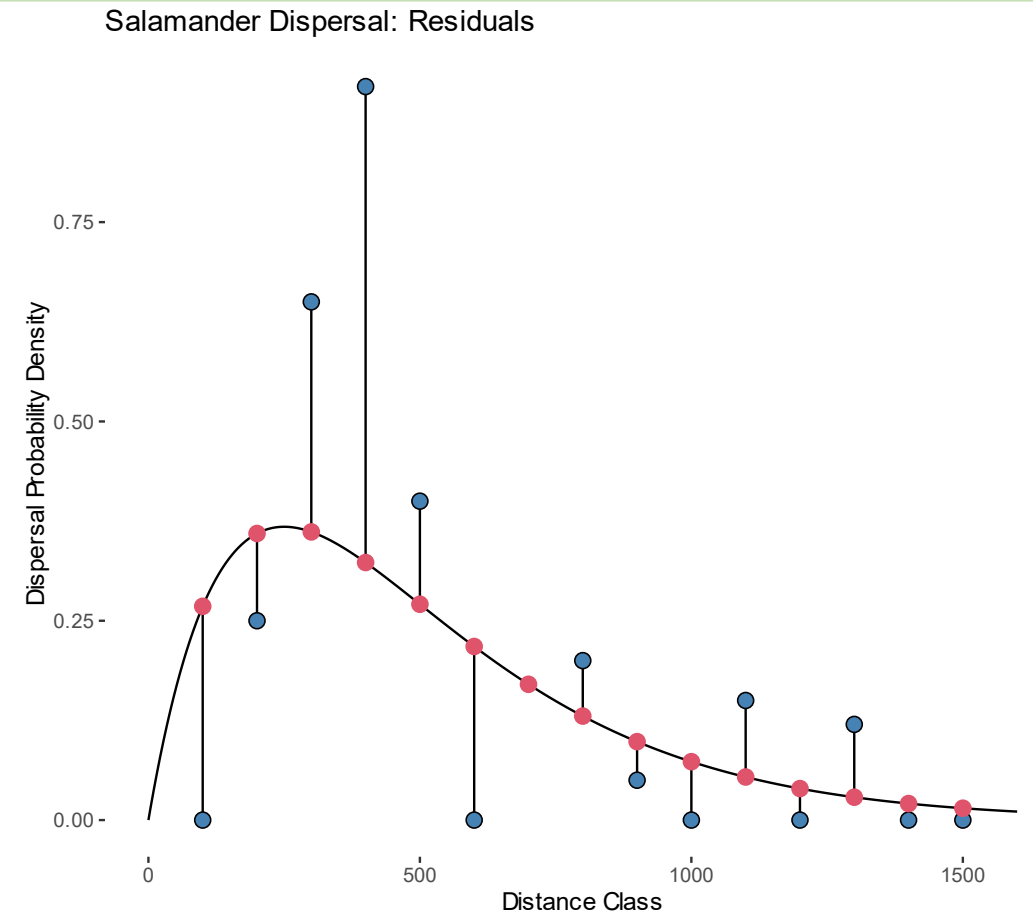


Model Residuals

The Predicted Values

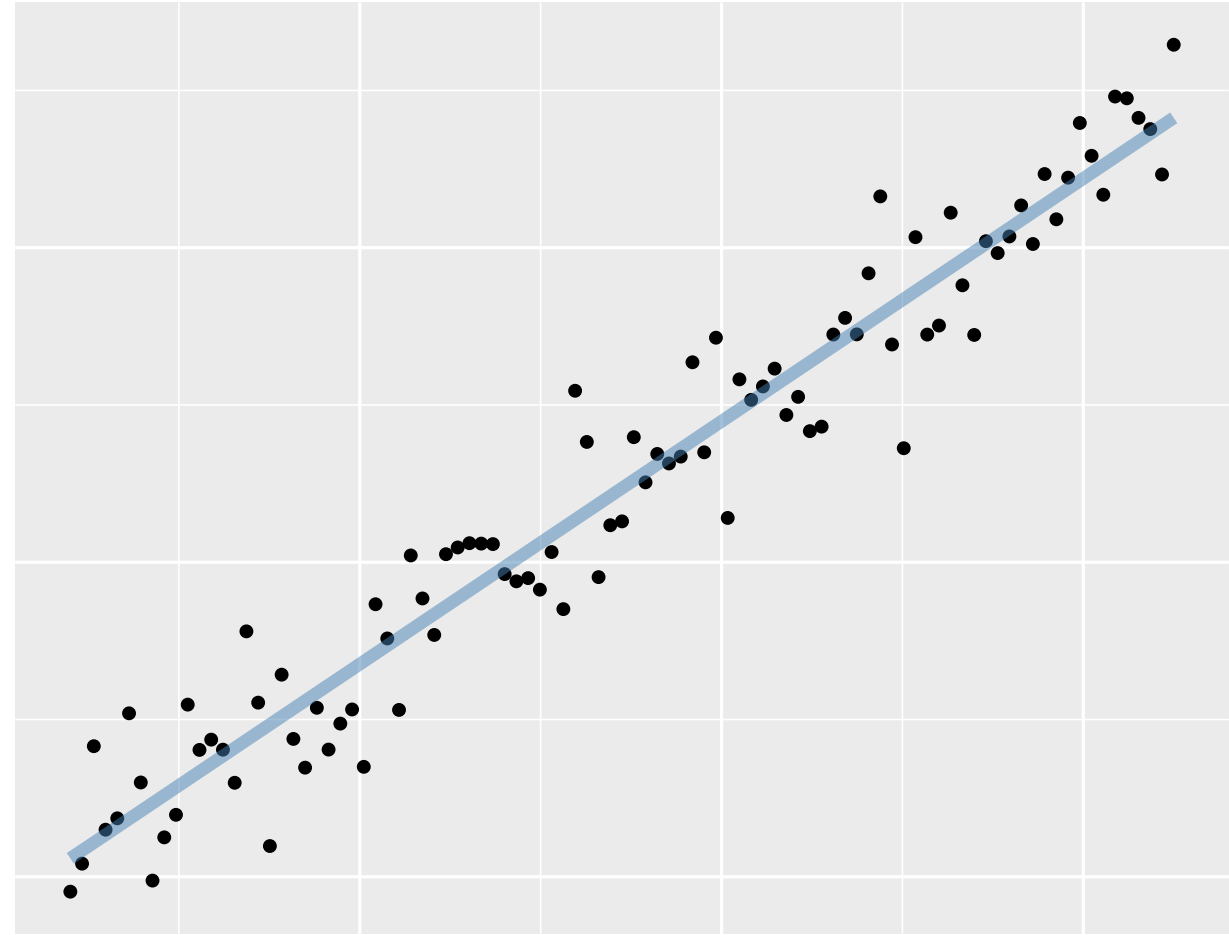


And...The Residuals!



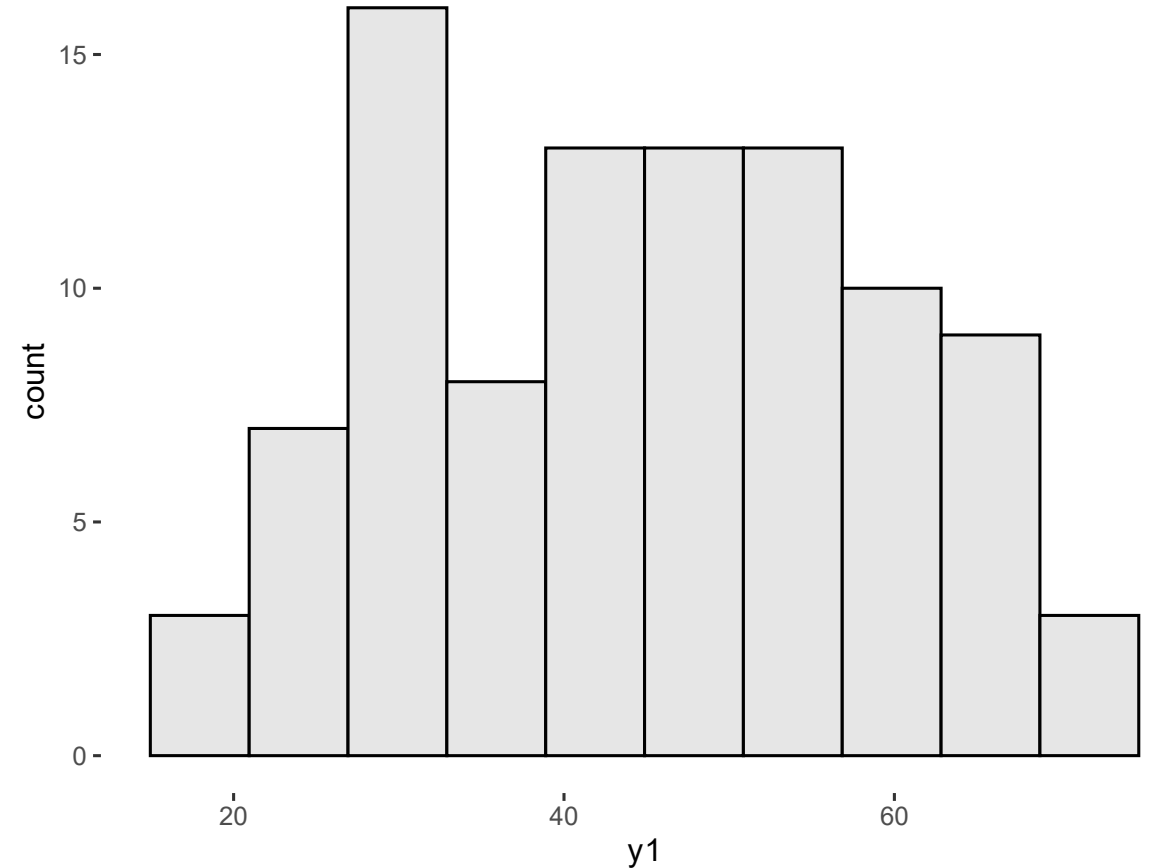
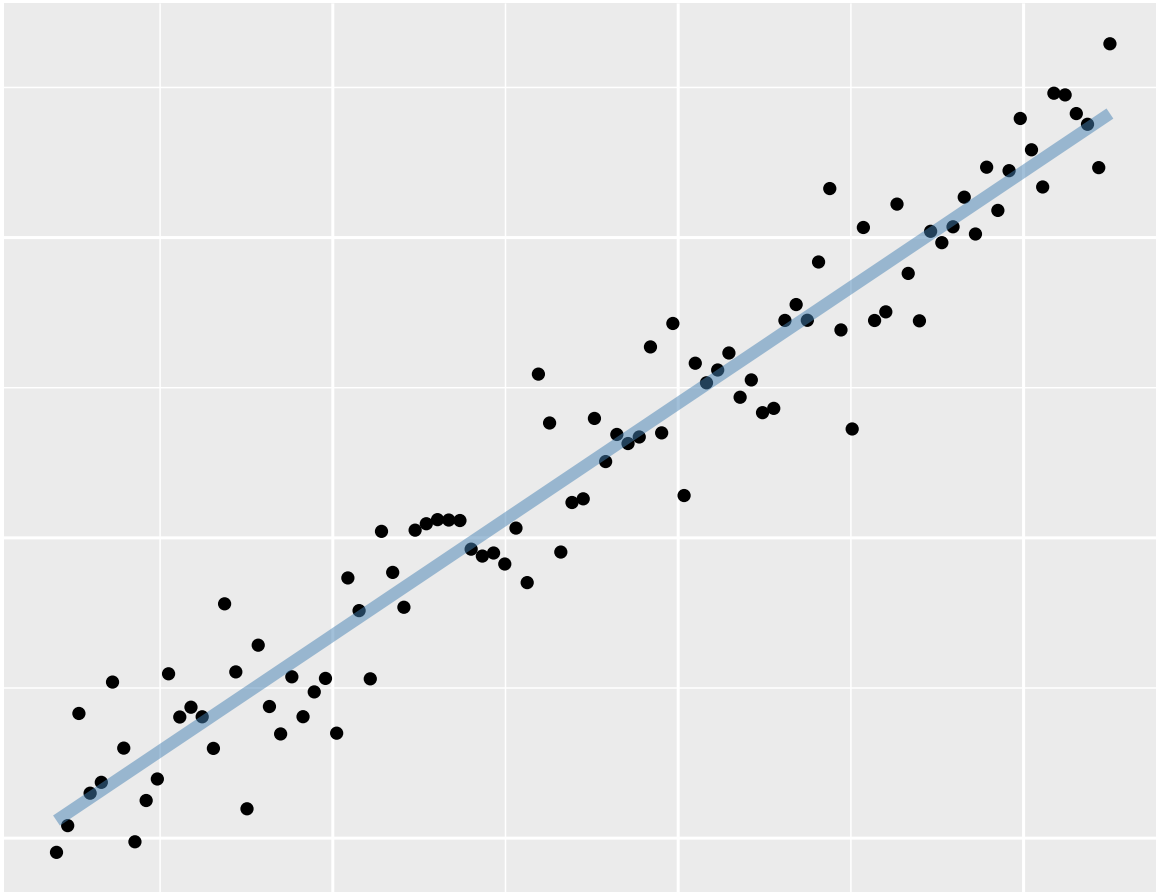
[Residual] Normality Assumption

- Group 1 models assume that residuals are Normally distributed
- This does not mean that ‘the data are normally distributed’.
 - Usually, the data points themselves aren’t Normally distributed.
 - This is a frequent point of confusion.



[Residual] Normality Assumption

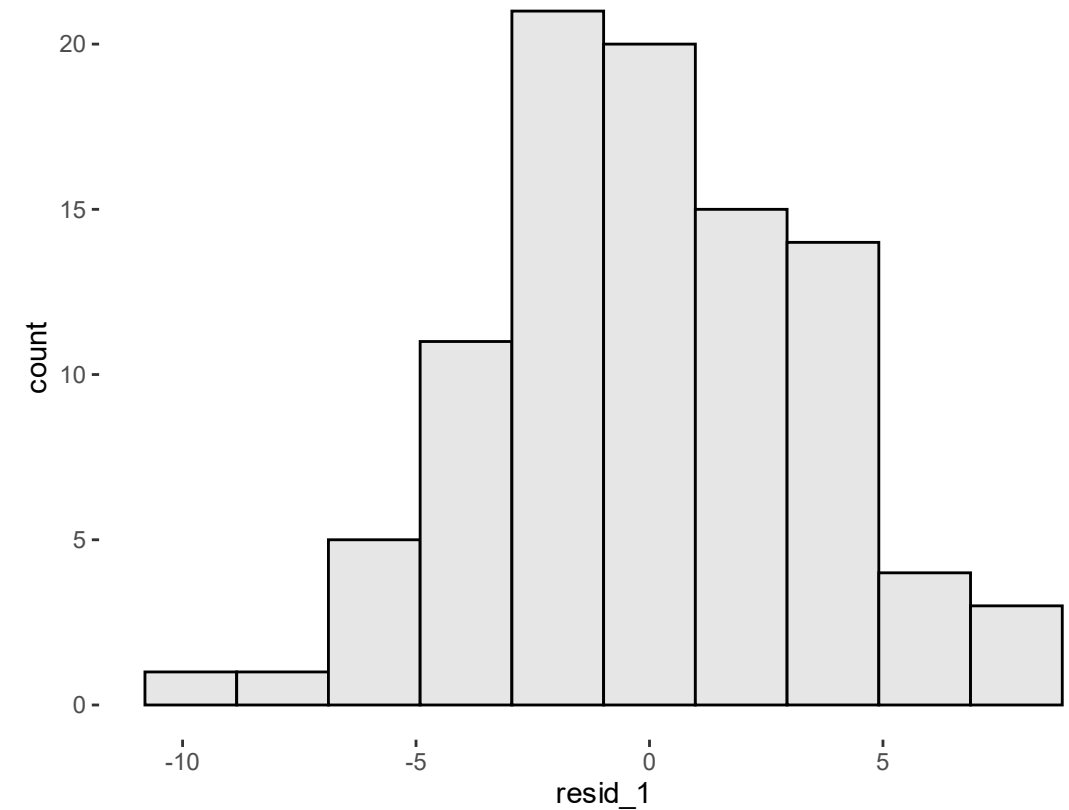
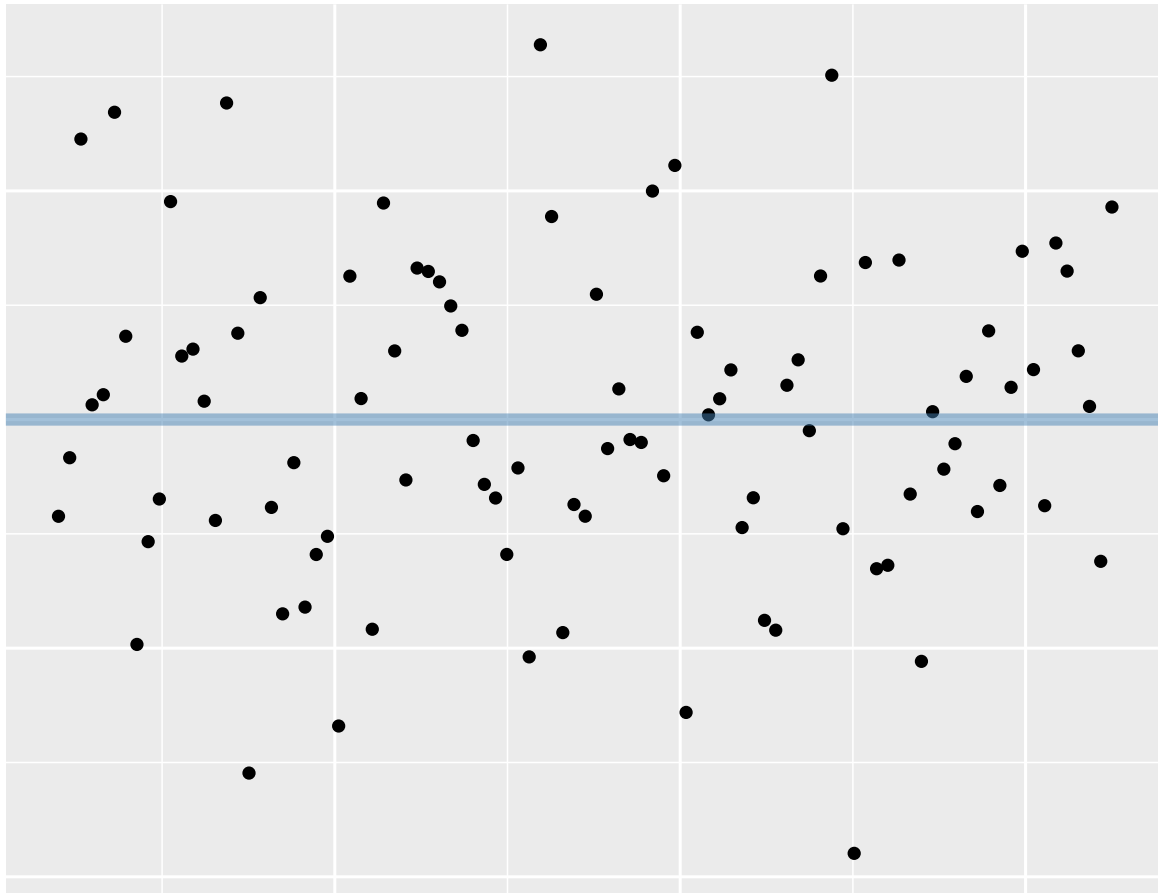
The following data look relatively well-behaved, however the histogram of the y-values suggests the distribution of values is pretty flat.
A Shapiro test provides evidence of non-normality with $p = 0.017$.



[Residual] Normality Assumption

We really care about the normality of the *residuals* from a model.

A Shapiro test on the residuals suggests normality with $p = 0.989$.

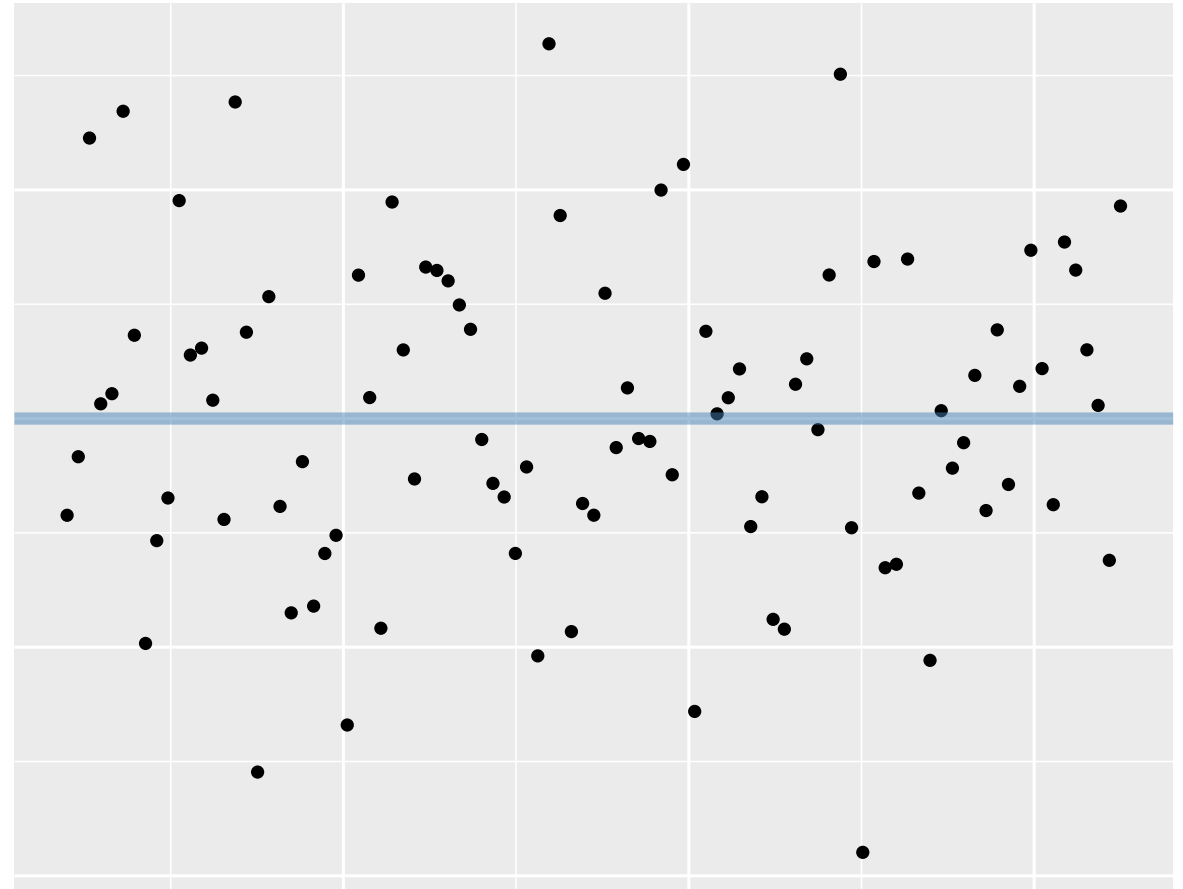


Homogeneity Assumption

The homogeneity assumption requires constant variance along the entire range of predictor values.

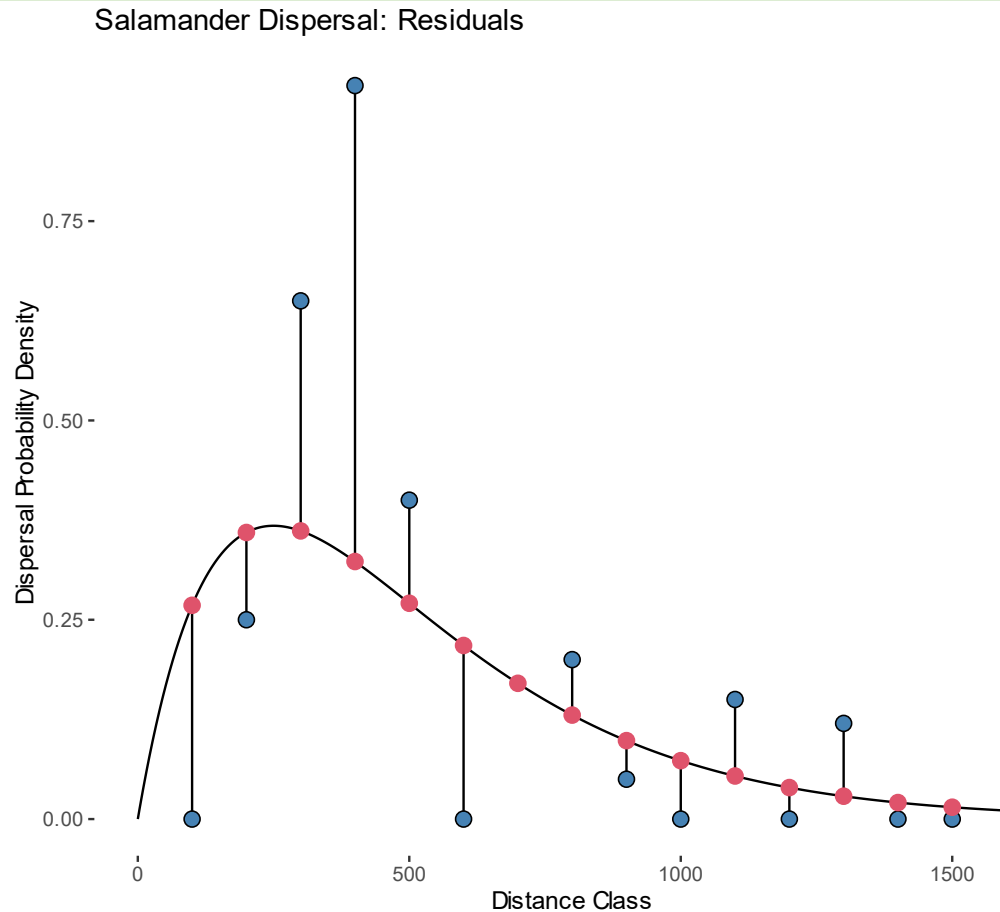
Key points of the assumption:

- The stochastic model is a Normal distribution.
- The spread parameter, σ is constant.
- In other words, the variability does not depend on the value of x

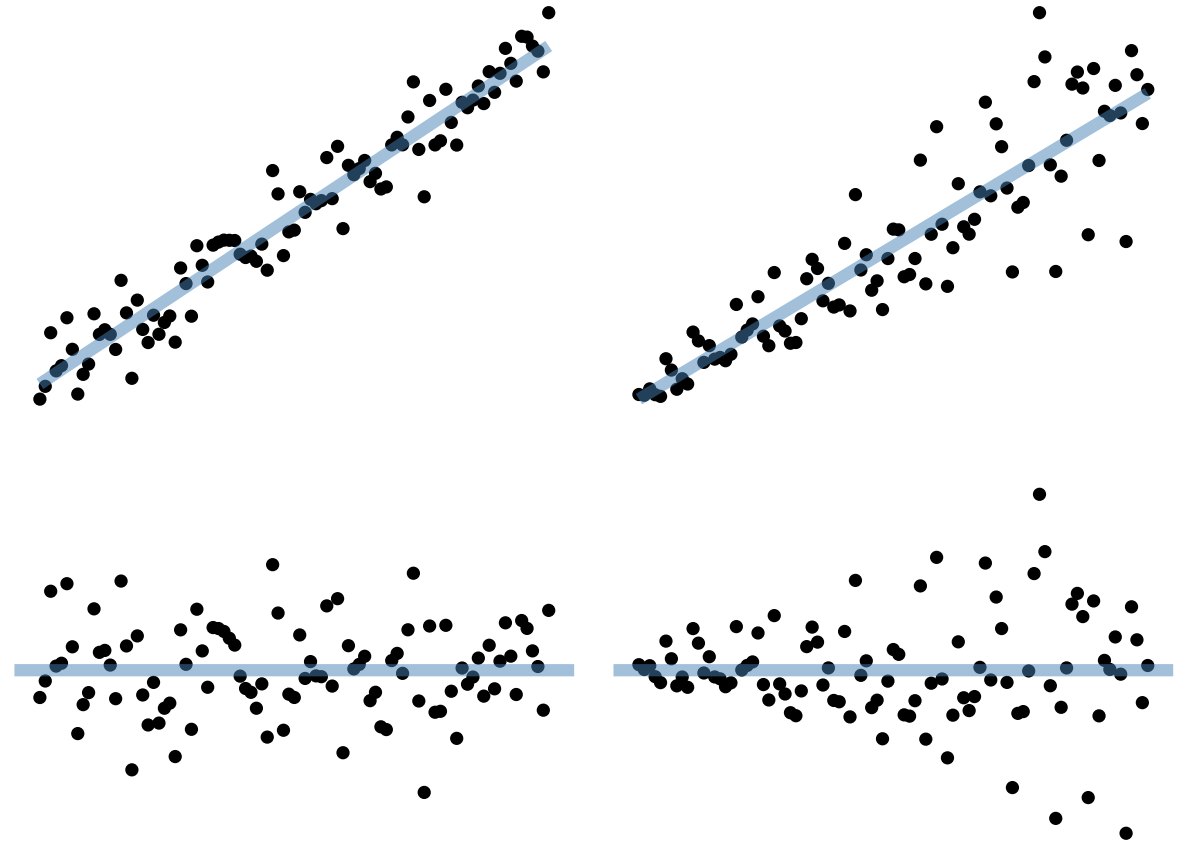


Homogeneity Assumption

Residuals of models of real data are often heterogeneous.



We don't like to see a megaphone shape



Independent Observations Assumption

Non-independence is one of the more challenging violations to deal with.

- Independent observation assumption key points:
 - Sampling is randomized.
 - Knowing something about observation x_1 gives us no information about observation x_2
- Remember independent events from probability theory?
 - The joint probability of independent events is the product of individual probabilities.
 - This is the basis for likelihood methods.
- Zuur, 2007:
 - “The independence assumption means that if an observed value is larger than the fitted value (positive residual) at a particular X value, then this should be independent of the Y value for neighboring X values.”
- Non-independence can result from:
 - Proximity in space or time
 - Hierarchical structure

Fixed X Assumption

We often forget about the fixed-x assumption.

- Perfect accuracy in measurements of explanatory variables.
- This assumption is frequently violated
- It's OK-ish if the *noise* in the predictor variables' measurement is small relative to the noise in the response.



Regression Equation

We can express the dual model compactly with a regression equation.

- The basic regression equation can be expressed in several ways:

$$y_i = \alpha + \beta_1 x_1 + \epsilon_1$$

$$y_i = \beta_0 + \beta_1 x_1 + \epsilon_1$$

$$Y \sim \text{Normal}(\alpha + \beta X, \sigma)$$

Regression parameter interpretation

- Intercept: “The value of the response when the predictor is zero”
 - The intercept often occurs outside the range of our data: it is an extrapolation.
- Slope parameters: “For each 1-unit change in x , we expect a β_1 change in the value of y (on average).”

Parameter Interpretation

- A linear regression of penguin flipper length and body mass:

$$(\textit{Flipper length}) = 136.7 + 0.015 \times (\textit{body mass})$$

```
lm(  
  flipper_length_mm ~ body_mass_g,  
  data = penguins)
```

Call:

```
lm(formula = flipper_length_mm ~ body_mass_g, data = penguins)
```

Coefficients:

(Intercept)	body_mass_g
136.72956	0.01528

Overall Model Standard Deviation

Recall the basic regression equation:

$$y_i = \alpha + \beta_1 x_1 + \epsilon_1$$

We might ask: what is the overall model standard deviation?

- By that, we mean: what is the standard deviation of the residuals:

$$sd_{model} = \sqrt{\frac{1}{n-2} \sum_{i=1}^n e_i^2}$$


Why n-2?

- We lose one degree of freedom for each parameter we estimate.
- We estimated two model parameters: α and β_1 .

A Tale of Two Tables

Model Coefficients and the ANOVA Table

Two questions we might ask of a regression model:

1. What is the *magnitude* of the relationship between predictor x_1 and response y ?
 2. How much of the variability in the model does predictor x_1 explain?
- The model coefficient table tells us the direction and magnitude of the association between predictor and response.
 - The Analysis of Variance (ANOVA)  table tells us the relative importance of the various predictors to the overall model.



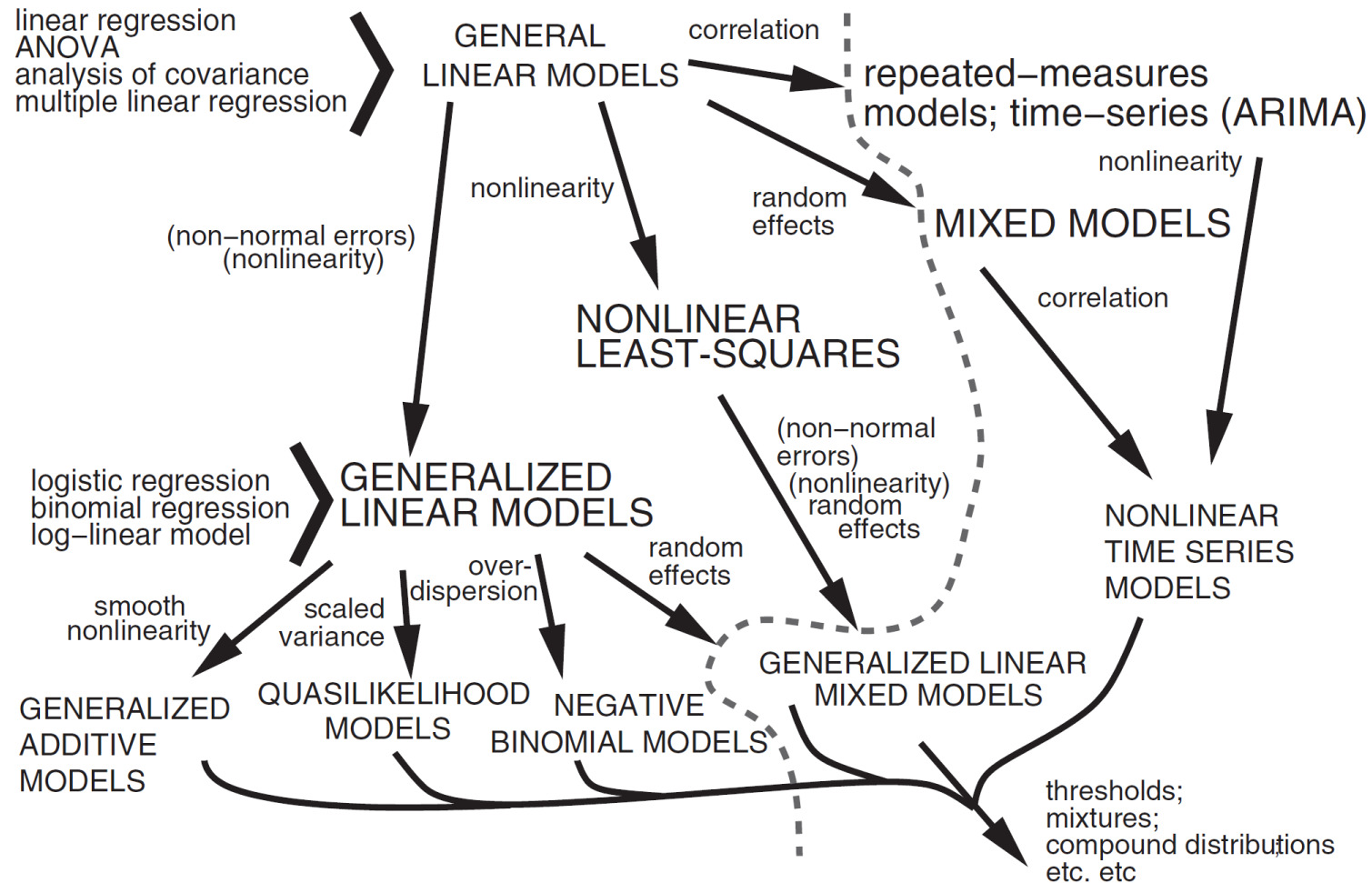
Model Diagnostics, Validation, and Selection



- How do we know that we have chosen the *best* model?
- Did we include the right predictors?
- Did our algorithm find the best parameter values?
- How well does our model fit the observed data?
- How well does our model predict new data?
- Does our data/model meet assumptions?
- Are the assumption violations *acceptably* small?



There are Many Types of Models: The Constellation



Bolker: Ecological Models and Data in R, Figure 9.2

There are Many Types of Models: The Constellation

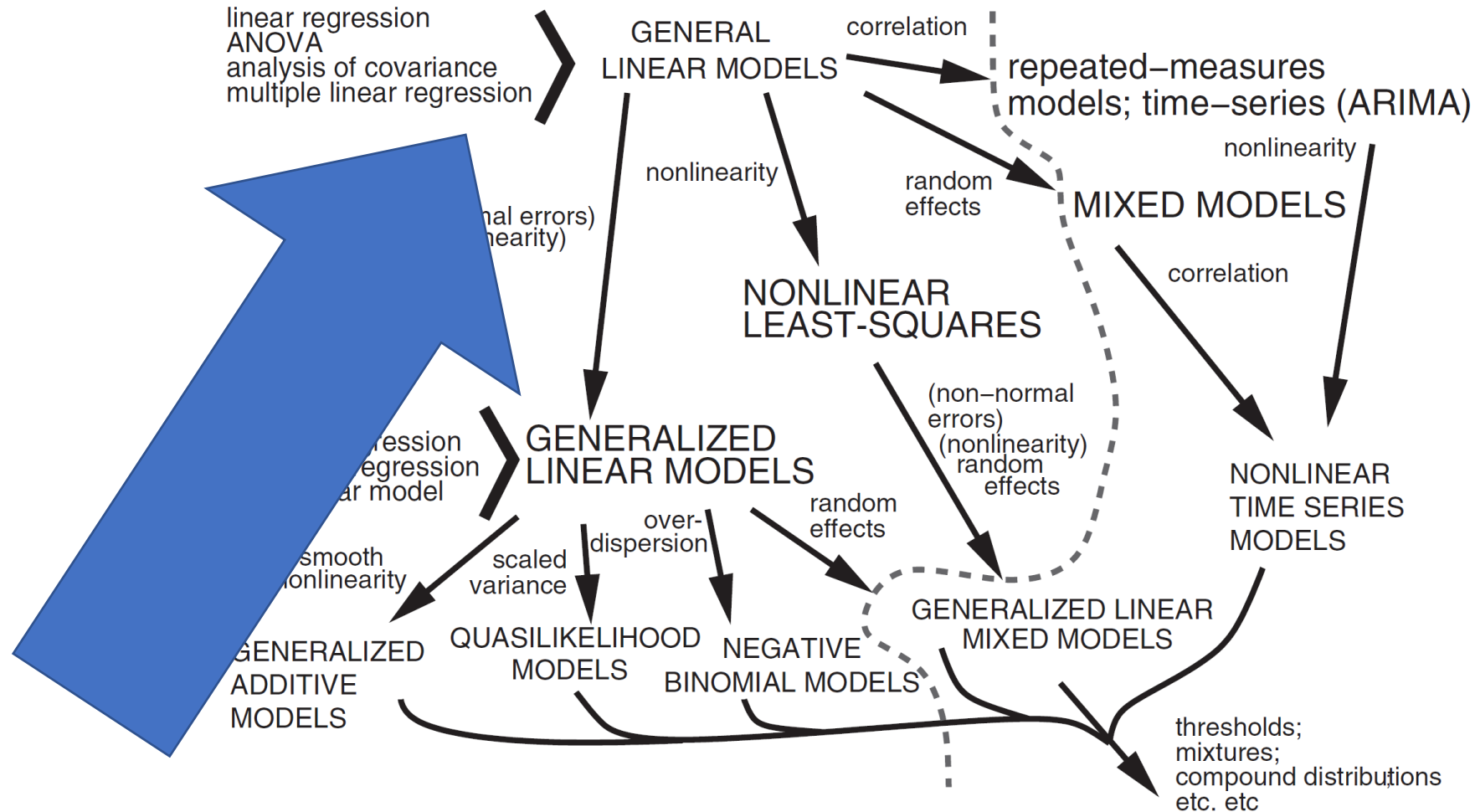
- Many of the models beyond Group 1 were developed to handle violations of one or more of the Group 1 required assumptions.
- We'll spend most of our time on Group 1 models:
 - Easiest to understand, many principles transfer to other models.
 - Easiest to implement and interpret

Key Concepts

- What is a regression model?
- What are the key assumptions?
- What is the constellation of methods?

Group 1: General Linear Models

There are Many Types of Models: The Constellation



Bolker: Ecological Models and Data in R, Figure 9.2

What's in This Section?

Take-Home Concepts

- What makes a model linear?
 - Linear in the parameters
- Categorical and continuous predictors.
- Group 1 responses are always continuous
- Key assumptions of general linear models.

Group 1: [General] Linear Models

Four key assumptions:

- Normality: normality refers to the model residuals
- Constant variance a.k.a homoskedasticity, a.k.a. homogeneity
- Independent observations
- Fixed x: no measurement error in our predictor variables

Group 1 requirements:

- Group 1 models are linear in the parameters
- Group 1 models have a single continuous response variable

Terminology

- Response: Y
- Predictor(s): X
- Intercept: α
- Slope(s): β

Group 1: Types of models

Group 1 methods are essentially variations on linear regression.

- T-Test Simple Linear Regression
- 1-Way ANOVA
- Multiple Linear Regression
- n-Way ANOVA
- ANCOVA

Group 1: general equation format

- Element-by-element form:

$$y_i = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + \dots + \beta_n x_{ni} + \epsilon_i$$

- Matrix/Vector form:

$$Y = \alpha + \beta_1 \mathbf{X}_1 + \beta_2 \mathbf{X}_2 + \dots + \beta_n \mathbf{X}_n + \epsilon$$

Linear in the Parameters

Linearity in parameters means that in the deterministic functions, the model coefficients can only have *multiplicative* relationships to the predictor variables.

- It will help to dissect some regression equations to identify variables, coefficients/parameters, and constants.

The classic simple linear regression equation: $y = \alpha + \beta x + \epsilon$

Linear in the Parameters

This model is linear in the parameters: $y = \alpha + \beta x + \epsilon$

Things to note:

- x and y correspond to our *observations*. They are not estimated.
- α and β are the model coefficients. They are the quantities we want to estimate.
- β *multiplies* the predictor variable x .
- ϵ is the residuals, i.e. the stochastic model. For Group 1 this is the Normal distribution.

Linear in the Parameters

This model is also linear in the parameters:

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \epsilon$$

Things to note:

- x and y correspond to our *observations*. They are not estimated.
- β_1 , β_2 , and β_3 multiply the variables x_1 and x_2
- We used the product of the two predictors, x_1 and x_2 as a *third* predictor.

Linear in the Parameters

This model is *not* linear in the parameters:

$$y = \alpha + \beta_1 x_1^2 + a x_2^{\beta_2} + \epsilon, \text{ Why not?}$$

- The $\beta_1 x_1^2$ is ok. We've just used the square of the first predictor. It's like a modification of a predictor. Imagine that we could create another predictor column called 'sq' in our data that contained the squares of x_1 .
 - Even though x^2 is not a linear function, the coefficient β_1 multiplies the term.
- The term $a x_2^{\beta_2}$ is *not linear in the parameters*. Why?
 - The model coefficient β_2 does not *multiply* the predictor x_2 , but rather it is an exponent.
 - The **constant** a multiplies x , but it is not a model coefficient estimated that is estimated from the data.

Linear in the Parameters

It seems weird that we can say $\beta_1 x_1^2$ is *linear* and $\alpha x_2^{\beta_2}$ is not.

- Both are nonlinear expressions.
- However, in the first term we are raising x_1 to a constant.
 - The *constant, 2*, is not estimated from the data therefore it is not a model *coefficient*.
- In the second term, we have specified a *model coefficient* as an exponent.
 - Since the coefficient does not *multiply* but rather *exponentiates* the predictor it is not *linear in the predictors*.

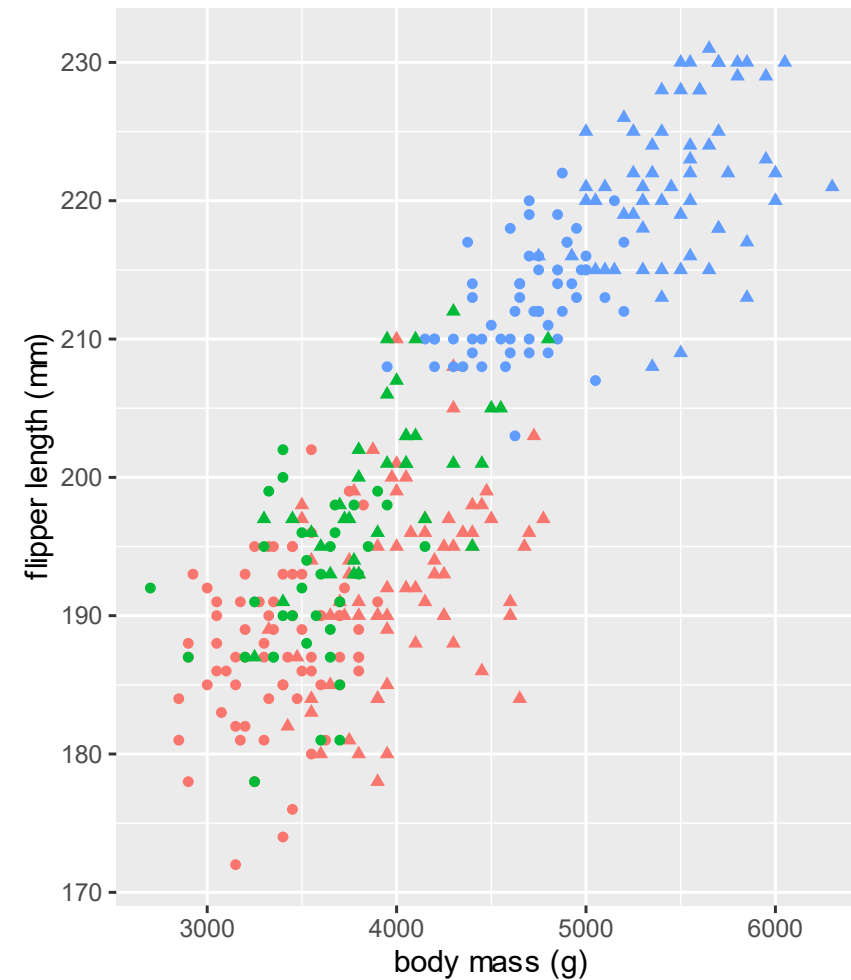
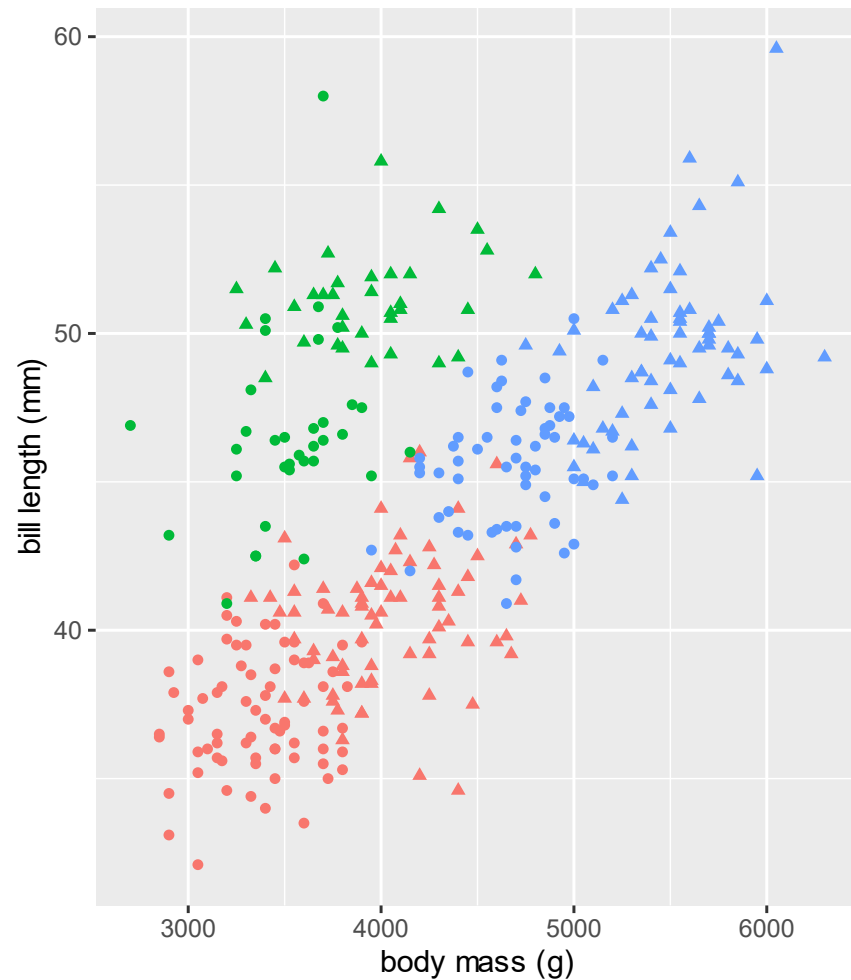
Palmer Penguin Data

We'll use the Palmer Penguin dataset to illustrate group 1 methods

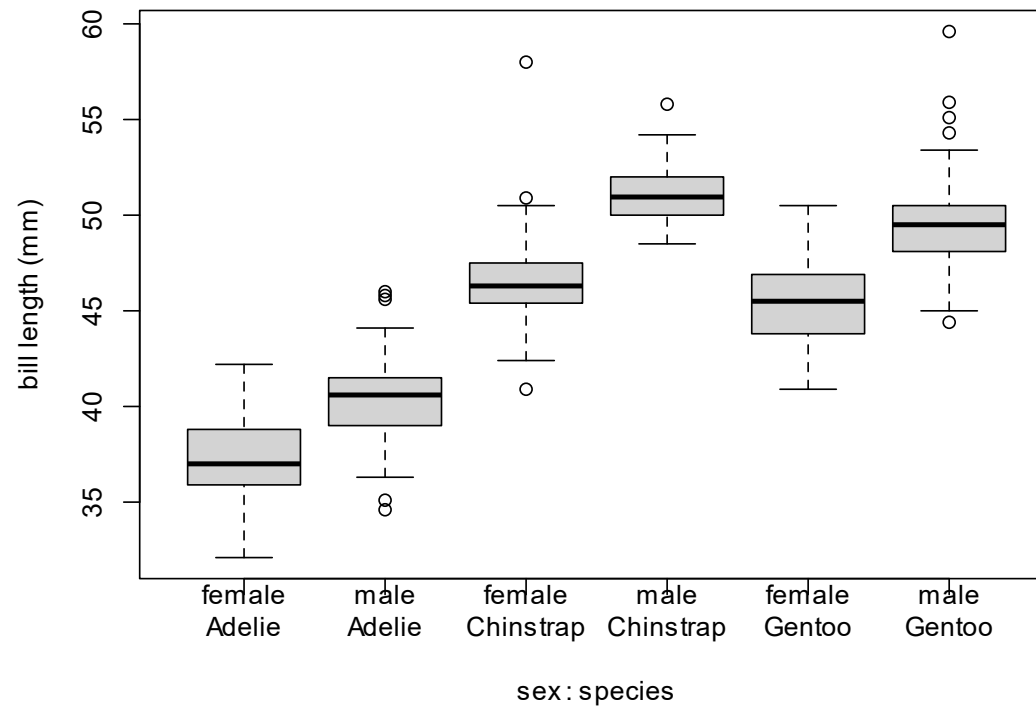
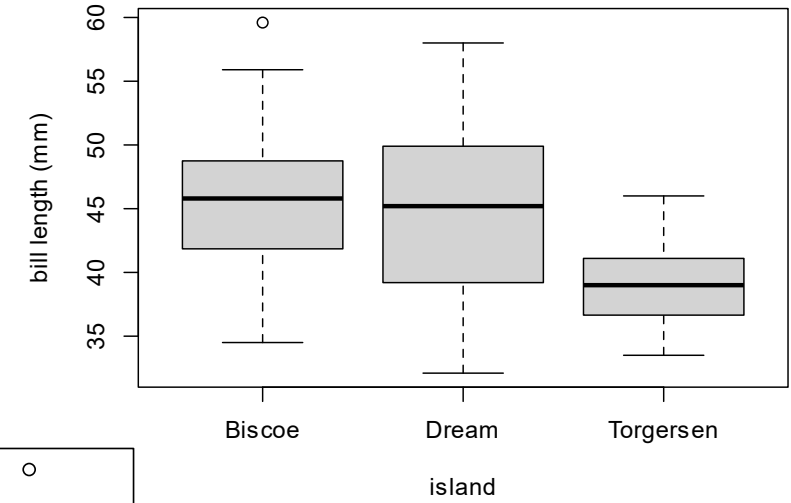
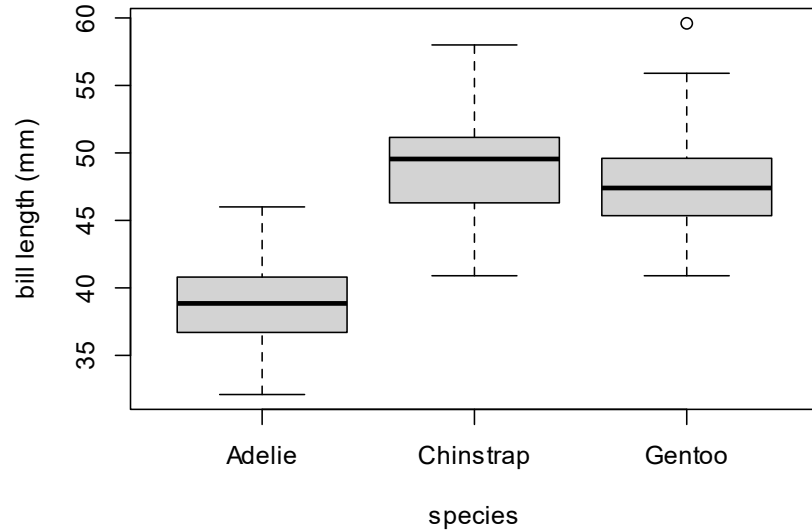
- Dr. Kristen Gorman and the Palmer Station Long Term Ecological Research (LTER) Program.
- 3 Penguin species in the Palmer Archipelago
 - size measurements: 4 continuous variables
 - species, island, and sex: categorical - nominal scale
- R package palmerpenguins

<https://education.rstudio.com/blog/2020/07/palmerpenguins-cran/>

Palmer Penguins



Palmer Penguins – Graphical Exploration



Group 1: T-tests

- **t-test**
- Simple Linear Regression
- 1-Way ANOVA
- Multiple Linear Regression
- n-Way ANOVA
- ANCOVA

T-tests are appropriate with

- One categorical predictor with 1 or 2 levels
- One continuous response

T-tests analyze the following questions:

- Is the mean of one group different from a fixed value?
- Are the means of two groups different from each other ?

An elaboration of the t-test:

- 1-way ANOVA extends t-test to 3 or more groups.

What could a t-test tell us about the penguins?

- Hint: What are the categorical predictors?



Group 1. Simple Linear Regression

- t-test
- **Simple Linear Regression**
- 1-Way ANOVA
- Multiple Linear Regression
- n-Way ANOVA
- ANCOVA

SLR Requires:

- One continuous response
- One continuous predictor
- What questions could we address in the penguin data?

Simple Linear Regression elaborations

1. Multiple linear regression: More than one continuous predictors
2. ANOVA: One categorical predictor (instead of continuous)
3. ANCOVA: Mixture of categorical and continuous predictors

Group 1: 1-Way Analysis of Variance

- t-test
- Simple Linear Regression
- **1-Way ANOVA**
- Multiple Linear Regression
- n-Way ANOVA
- ANCOVA

ANOVA: Categorical predictor, 3 or more levels

- Continuous response

Analyzes the following questions:

1. Are the group means different from one another?
- Note: ANOVA does not specify which pairs of groups are different from one another.

What could a 1-way ANOVA tell us about the penguins?

What were the categorical variables?

- Sex
- Species
- Island



ANOVA elaborations

Two or more categorical predictors: multi-way ANOVA
Categorical and continuous predictors: Analysis of Covariance (ANCOVA)

 Post ANOVA analysis: which groups are different from one another?

- Tukey Honest Significant Difference (HSD) test
 - Pairwise tests between all factor levels.
 - number of pairs gets large very quickly!
 - Correction for multiple testing: Bonferroni, etc.


Group 1: Multiple Linear Regression

- t-test
- Simple Linear Regression
- 1-Way ANOVA
- **Multiple Linear Regression**
- n-Way ANOVA
- ANCOVA

A multiple linear regression model has:

- One continuous response
- Two or more continuous predictors


The model attempts to quantify the pairwise relationships between each predictor and the response - combined effect of 2 or more predictors on the response

Multiple regression can fail with highly correlated predictors:  collinearity and multicollinearity.


Group 1: Multiple Linear Regression

- t-test
- Simple Linear Regression
- 1-Way ANOVA
- **Multiple Linear Regression**
- n-Way ANOVA
- ANCOVA

Multiple regression elaborations:

-  Mixture of categorical and continuous predictors:
 - Interaction terms: synergistic effects of two or more predictors.
 - Analysis of Covariance (ANCOVA)

What can it tell us about the penguins?

- What were the continuous predictors?
 - flipper length, bill measurements, body mass.
- Could we use these three continuous variables to predict the species? 
 - Hint: no! Group 1 methods require a continuous response!

Group 1: Multi-Way ANOVA

- t-test
- Simple Linear Regression
- 1-Way ANOVA
- Multiple Linear Regression**
- **n-Way ANOVA**
- ANCOVA

Categorical analogue of multiple regression

- Main effects
- Interactions


What could we ask with the penguin data?

- Categorical variables: island, sex

Elaboration: Mix of categorical and continuous variables Analysis of Covariance (ANCOVA)

Group 1: Analysis of Covariance

- t-test
- Simple Linear Regression
- 1-Way ANOVA
- Multiple Linear Regression**
- n-Way ANOVA
- **ANCOVA**

 ANCOVA combines categorical and continuous data:


- A mix of categorical and continuous predictors
- Continuous response

What could we ask with the penguin data?

- Categorical variables: island, sex
- Continuous variables: flipper length, bill dimensions, body mass


When do group 1 methods start to fail?

Four key assumptions:



- Independent observations
-  Constant variance a.k.a homoskedasticity, a.k.a. homogeneity
- Fixed x: no measurement error in our predictor variables
- Normality: normality refers to the model residuals

When do group 1 methods start to fail?

[Multi]Collinearity

- If two predictors are correlated they contain redundant information.
 -  How does a model know which predictor should get the credit?
- Detecting collinearity between two variables is easy: just calculate the correlation coefficients
- Multi-collinearity: complex correlational structures can exist among 3 or more variables.
 - Pearson/Spearman correlation coefficient is only for 2 variables.
 - Multicollinearity is hard to detect.
 - It causes 'unstable' coefficients: coefficients can change drastically when one observation is removed.

Key Concepts

- What makes a model linear¹
- Categorical and continuous predictors.
- Key assumptions of general linear models.

Model Coefficients and the ANOVA Table

What's in This Section?

Take-Home Concepts

- Interpreting model coefficient tables for categorical variables
- Interpreting model coefficient tables for continuous variables
- Interpreting the ANOVA table
- Intro to dummy variables

Group 1 model interpretation

Group 1 models are *linear in the parameters*

This makes the interpretation of model terms *relatively* easy.

- But note, there is still lots of complexity especially when we mix continuous and categorical terms and interaction terms.

Recall the basic equation:

$$y_i = \alpha + \beta_1 x_1 + \beta_2 x_2 + \dots + \epsilon$$

- When all of the predictor variables have a value of zero, we expect y to have a value of α , on average.
- For every 1-unit change in x_1 we expect a β_1 -unit change in y , on average.

Group 1 model summary presentations

Table of model coefficients model summary.

- This table tells us the strength of effects of predictors, overall model significance test

ANOVA table.

- This table shows the model variability attributed to each factor, factor-specific significance tests

Group 1 model interpretation

Model Coefficients

Intercept: What is the value of the response when the predictor has value zero?

Slope: What is the change in the response with each unit change in the predictor?

Standard Errors: shape of sampling distribution

F-test: overall model significance test

ANoVA Table

Degrees of freedom: Reflects the number of samples, number of factor levels, number of individuals per factor level etc.

Sum of squares: Reflects the total squared deviation from the mean explained by a source.

Mean squares: Mean SS due to a source (per DF)

F tests: Test for ratio of variability explained by a particular predictor variable

ANOVA table vs. model coefficient table

Model coefficient table tells you

1. Intercept and slope coefficients
2. Overall model significance test, correlation test

ANOVA table tells you

1. Variability explained by each factor in the model
2. Significance tests for each factor separately

1-way ANOVA

When we have a continuous response and a single categorical predictor with 2 levels we can use a t-test.

What if there are 3 or more levels?

- The t-test is not enough.
- Analysis of Variance is a generalization of the t-test for 3 or more groups.

Model Coefficient Tables: Dummy Variables

When you fit a model using a categorical predictor with n levels, the algorithm first detects all of the factor levels present in the data, then creates a set of $n - 1$ *dummy variables*.

- The dummy variables allow the model-building process to treat each factor level as if it were a separate, numerical predictor that can take on only values of zero or one.

species	speciesGentoo	speciesChinstrap
Adelie	0	0
Gentoo	1	0
Chinstrap	0	1

Model Coefficient Tables: Interpretation for Categorical Predictors

Since each factor level is treated as a predictor variable, there will be slope parameters for each.

When R builds a model, it selects one of the factor levels to serve as the *base case*.

- When the model contains only categorical variables, the base case is analogous to the *intercept* term in a model, i.e. the α .

It'll be easier to understand with an example.

1-way ANOVA: Palmer Penguins

The procedure for conducting an ANOVA in R is:

- Create a linear model fit with `lm()`.
- Use `anova()` to perform the Analysis of Variance and print the ANOVA table.

Recall that ANOVA is really a just a different way of looking at a linear model.

- To better understand the relationship, we'll focus on the model coefficient table first:

```
lm(  
  formula = body_mass_g ~ species,  
  data = penguins)
```

Call:

```
lm(formula = body_mass_g ~ species,  
    data = penguins)
```

Coefficients:

```
      (Intercept)  
            3700.66  
speciesChinstrap  
            32.43  
speciesGentoo  
            1375.35
```

Factor Base Cases

There are slopes for Chinstrap and Gentoo, but where is the Adelie coefficient?

- Recall: the *base case* is the intercept in a 1-way ANOVA.

R assigned “Adelie” to be the base case.

- Notice how R formats the factor-level coefficient names:
 - the variable name prepended to the factor level.

Interpreting the Coefficient Table

Call:

```
lm(formula = body_mass_g ~ species,  
    data = penguins)
```

Coefficients:

```
      (Intercept)  
          3700.66  
speciesChinstrap  
          32.43  
speciesGentoo  
          1375.35
```

- Mean Adelie penguin mass is 3700 grams
- Mean Chinstrap penguin mass is 3700 + 32 grams
- Mean Gentoo penguin mass is 3700 + 1375 grams

Everything is relative to the base case!

Interpreting the Coefficient Table

Call:

```
lm(formula = body_mass_g ~ species,  
    data = penguins)
```

Coefficients:

```
      (Intercept)  
          3700.66  
speciesChinstrap  
          32.43  
speciesGentoo  
          1375.35
```

- The intercept is 3700 grams: Adelie penguins weigh 3700g, on average
- The regression slope for Chinstrap is 32 grams per unit.
 - Adding one 'penguin unit' increases the penguin mass by 32 grams, on average.
- The regression slope for Gentoo slope 1375 grams
 - Adding one 'penguin unit' increases the penguin mass by 1375 grams, on average.

Everything is relative to the base case!

Interpreting the Coefficient Table

```
Call:  
lm(formula = body_mass_g ~ species,  
    data = penguins)
```

```
Coefficients:  
      (Intercept)  
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speciesGentoo  
          1375.35
```

We can obtain the mean masses of each species from the model coefficient table.

- Mean Chinstrap penguin mass
 - $3733 = 3701 + 1 \times 32 + 0 \times 1375$
- Mean Gentoo penguin mass:
 - $5076 = 3701 + 0 \times 32 + 1 \times 1375$

Dummy Variables

If we consider x_{chin} a dummy variable which is equal to 1 if the observation is a Chinstrap penguin and 0 otherwise, and likewise for x_{gentoo} we could write the regression equation symbolically as:

$$y_i = \alpha_{adelie} + \beta_{chin} \times x_{chin} + \beta_{gentoo} \times x_{gentoo}$$

What would the coefficient table and equation look like if Chinstrap penguins were lighter than Adelie penguins?

1-way ANOVA: ANOVA Table

We have examined the model coefficients and calculated the group means.

- The masses seem pretty different, but how could we assess the ANOVA *alternative* hypothesis?
 - “The body masses of penguins for *at least one* species are different from the masses of the other species”

```
## Analysis of Variance Table
##
## Response: body_mass_g
##           Df      Sum Sq  Mean Sq F value    Pr(>F)
## species      2 146864214 73432107   343.63 < 2.2e-16 ***
## Residuals 339  72443483   213698
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

1-way ANOVA: Model Coefficient Table

What can we learn from the model coefficient table?

The *intercept* and *speciesGentoo* coefficients have low p-values, but that's not exactly what we wanted to know!

- We wanted to know about the penguin species *in general*.

1-way ANOVA: ANOVA Table

The ANOVA table gives us a clue

```
Response: body_mass_g
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
species	2	146864214	73432107	343.63	< 2.2e-16 ***
Residuals	339	72443483	213698		

```
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1
```

```
'
```

The ANOVA Table

- Note how the *species* predictor is now a single line.
 - There were model coefficients for each factor level.

Response: body_mass_g

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
species	2	146864214	73432107	343.63	< 2.2e-16 ***
Residuals	339	72443483	213698		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1

■

Model Coefficients and ANOVA Provide Complementary Information

We'll cover model coefficient interpretation, and the ANOVA table details in greater depth, but for now you should notice:

- Model slope/intercept coefficients: there is one coefficient for each *factor level* of a *categorical predictor*.
- The intercept coefficient corresponds to the *base case*.
- Model coefficient table characterizes the strength and significance of individual intercept and slope coefficients.
 - It *does not* tell us about the overall significance of the categorical predictor.
- The ANOVA table evaluates the ANOVA null hypothesis.
 - It *does not* tell us *which factor levels* are different
 - The two tables each provide part of the picture.

Neither the model coefficient table nor the ANOVA table tell us if a particular pair of factor levels are *significantly* different from one another!

Model Coefficients and ANOVA Provide Complementary Information

Neither the model coefficient table nor the ANOVA table tell us whether a particular pair of factor levels are *significantly* different from one another!

- This is the realm of post-hoc testing.
 - Post-hoc testing is an analysis you perform after (post) you perform the initial analysis (hoc).
- The Tukey Honest Significant Difference is a common post-hoc method.

Key Concepts

- Interpreting model coefficient tables for categorical variables
- Interpreting model coefficient tables for continuous variables
- Interpreting the ANOVA table
- Intro to dummy variables