Tao Nan School
Advanced Mathematics Enrichment Club (AMEC)

Rapid Calculation of Whole Numbers

(1) Find the value of 2023 – 2022 – 2021 + 2020 + 2019 – 2018 – 2017 + 2016 + 2015 – 2014 – 2013 + 2012 + 2011 – … – 5 + 4 + 3 – 2 – 1.

ANS:
(2023 – 2022 – 2021 + 2020) + (2019 – 2018 – 2017 + 2016) + (2015 – 2014 – 2013 + 2012) + 2011 – … – 5 + 4) + (3 – 2 – 1)
= 0 + 0 + 0 …….. 0 + 0
= 0

(2) Find the value of 1000 + 999 – 998 – 997 + 996 + 995 – 994 – 993 + 992 + … + 108 + 107 – 106 – 105 + 104 + 103 – 102 – 101.

ANS:
No. of terms = (1000 – 101) + 1 = 900
900 – 1 = 899
899 ÷ 4 = 224 R 3
1000 + (999 – 998 – 997 + 996) + (995 – 994 – 993 + 992) + … + 108 + (107 – 106 – 105 + 104) + (103 – 102 – 101).
= 1000 + 0 + 0 + … 0 – 100
= 900

(3) Find the value of (201.1 + 20.11 + 2.011+ 0.2011) ÷ 0.1111.

ANS:
(2011 (0.1) + 2011 (0.01) + 2011 (0.001) + 2011 (0.0001)) ÷ 0.1111
= 2011 (0.1 + 0.01 + 0.001 + 0.0001) ÷ 0.1111
= 2011 (0.1111) ÷ 0.1111
= 2011

(4) Find the value of 2010 × 20112011 – 2011 × 20102010

ANS:
2010 × 2011 × 10001 – 2011 × 2010 × 10001 = 0

(5) Find the value of 2011 × 20102010 – 2009 × 20112011.

ANS:
2011 × 2010 × 10001 – 2009 × 2011 × 10001
= 2011 × 10001 (2010 – 2009)
= 20112011

(6) Find the value of 899998 + 89998 + 8998 + 898 +88.

ANS:
899998 + 2 + 89998 + 2 + 8998 + 2 + 898 + 2 + 88 + 2 – 10
= 900000 + 90000 + 9000 + 900 + 90 – 10
= 999980

(7) Find the value of 1999 + 999 × 999.

ANS:
1999 + 999 × 999.
= 1999 + 999 (1000 – 1)
= 1999 + 999000 – 999
= 1000 + 999000
= 1000000

(8) Find the value of 9999 × 2222 + 3333 × 3334.

ANS:
3 x 3333 x 2222 + 3333 x (3333 + 1)
= 3 x 3333 x 2222 + 3333(3333) + (3333)
= (3333) [6666 + 3333 + 1]
= (3333) [10000]
= 33330000

(9) Find the value of 3333 × 5555 + 6 × 4444 × 2222.

ANS:
3(1111) × 5(1111) + 6 × 4(1111) × 2(1111)
= (1111)(1111) [15 + 48]
= (1111)(1111) [63]
= (9999) x (7777)
= (10000 – 1) (7777) = 77770000 – 7777 = 77762223

(10) Find the value of 33333 × 33333.

ANS:
99999 x 11111
= (100000 – 1) x 11111
= 1111100000 – 11111
= 11110 88889

(11) If $1^2 + 2^2 + 3^2 + \ldots + 25^2 = 5525$, find the value of $2^2 + 4^2 + 6^2 + \ldots + 50^2$.

ANS:

$(1x\ 2)^2 + (2\ x\ 2)^2 + (3\ x\ 2)^2 + \ldots + (25\ x\ 2)^2$.

$= (1)^2(2)^2 + (2)^2(2)^2 + (3)^2(2)^2 + \ldots + (25)^2(2)^2$.

$= 4\ x\ 5525$

$= 4\ x\ 25\ x\ 221$

$= 100\ x\ 221$

$= 22100$

(12) Find the value of $1 – 2 + 3 – 4 + 5 – 6 + 7 - \ldots + 2009 – 2010 + 2011$.

ANS:

Number of pairs = $2011 \div 2 = 1005R1$ → Middle number - 1006 left

Number of (2012 – 2012) pairs = $1005 \div 2 = 502R1$ → $1005 + 1007 = 2012$

$2012 – 1006 = 1006$

OR

$2011 – 2 + 1 = 2010$

$2010 \div 2 = 1005$ pairs

$1005 + 1 = 1006$

(13) Find the value of $472634^2 + 472635^2 – 472633 \times 472635 – 472634 \times 472636$.

ANS:

Let $w = 472634$

$w^2 + (w+1)^2 – (w -1) \times (w+1) – w \times (w+2)$

$= w^2 + w^2 + 2w + 1 – w^2 + 1 – w^2 – 2w$

$= 2$

(14) Find the value of

$(123456 + 234561 + 345612 + 456123 + 561234 + 612345) \div 6$.

ANS:

$(21 + 210 + 2100 + 21000 + 210000 + 2100000\ ) \div 6$

$= 21\ (1 + 10 + 100 + 1000 + 10000 + 100000) \div 6$

$= 21\ (111111) \div 6$

$= (3\ x\ 7)\ (111111) \div 6$

$= 777777 \div 2$

$= 388\ 888.5$

(15) Find the value of $(0.75 \times 42.7 – 0.573 \times 25 + 57.3) \div 3 \times 7972$.

ANS:

$(0.75\ x\ (100 – 57.3) – 57.3\ x\ 0.01\ x\ 25 + 57.3) \div 3 \times 7972$

$= (75 – 57.3\ (0.75) – 57.3\ (0.25) + 57.3) \div 3 \times 7972$

$= 75 \div 3 \times 7972$

$= 25\ x\ 7972 = 25\ x\ 4\ x\ 1993 = 199300$

(16) Find the value of $(32.8 \times 91 - 16.4 \times 92 - 1.75 \times 656) \div (0.2)^2$.

ANS:
$(656 \div 20 \times 91 - 656 \div 40 \times 92 - 656 \times 1.75) \div (0.2)^2$
$= 656\ (91/20 - 92/40 - 7/4) \div (4/100)$
$= 656\ (455/100 - 230/100 - 175/100) \div (4/100)$
$= 656\ (50/100) \div (4/100)$
$= 328 \div (4/100)$
$= 82 \times 100 = 8200$

Advanced Techniques (Optional)

(1) Find the value of $(0.1)^3 + (0.2)^3 + (0.3)^3 + \ldots + (0.9)^3$.

(2) Find the value of $1 + 2 + 2^2 + 2^3 + 2^4 + \ldots + 2^9 + 2^{10}$.

(3) Find the value of $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \ldots + 100 \times 101$.

(4) Find the value of $11 \times 12 + 12 \times 13 + 13 \times 14 + \ldots + 50 \times 51$.

(5) Find the value of $11 \times 12 \times 13 + 12 \times 13 \times 14 + 13 \times 14 \times 15 + \ldots + 100 \times 101 \times 102$.

ANS:

(1)

$(0.1)^3 [1^3 + 2^3 + 3^3 + \ldots\ldots + 9^3]$
$= (0.1)^3 [1 + 2 + 3 + \ldots\ldots + 9]^2$
$= (0.1)^3 [4.5 \times 10]^2$
$= (0.001) \times [45]^2$
$= 2.025$

(2)
$1 + 2 + 2^2 + 2^3 + 2^4 + \ldots + 2^9 + 2^{10}$
$S = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + \ldots + 2^9 + 2^{10}$
$2S = 2^1 + 2^2 + 2^3 + 2^4 + \ldots + 2^9 + 2^{10} + 2^{11}$
$2S - S = 2^{11} - 2^0$
$S = 2^{11} - 1$
$S = 2048 - 1 = 2047$

(3)

$1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \ldots + 100 \times 101.$

1 x 2 = (1 x 2 x 3) ÷ 3
2 x 3 = (2 x 3 x 4 – 1 x 2 x 3) ÷ 3
3 x 4 = (3 x 4 x 5 – 2 x 3 x 4) ÷ 3
4 x 5 = (4 x 5 x 6 – 3 x 4 x 5) ÷ 3

100 x 101 = (100 x 101 x 102 – 99 x 100 x 101) ÷ 3

SUM
= (100 x 101 x 102) ÷ 3
= 343 400

(4)

$11 \times 12 + 12 \times 13 + 13 \times 14 + 14 \times 15 + \ldots + 50 \times 51.$

11 x 12 = (11 x 12 x 13 - 10 x 11 x 12) ÷ 3
12 x 13 = (12 x 13 x 14 – 11 x 12 x 13) ÷ 3
13 x 14 = (13 x 14 x 15 – 12 x 13 x 14) ÷ 3
14 x 15 = (14 x 15 x 16 – 13 x 14 x 15) ÷ 3

50 x 51 = (50 x 51 x 52 – 49 x 50 x 51) ÷ 3

SUM
= (50 x 51 x 52 – 10 x 11 x 12) ÷ 3 = 10 (5 x 17 x 52 – 1 x 11 x 4)
= 43760

(6)
$11 \times 12 \times 13 + 12 \times 13 \times 14 + 13 \times 14 \times 15 + \ldots + 100 \times 101 \times 102.$

$11 \times 12 \times 13 = (11 \times 12 \times 13 \times 14 - 10 \times 11 \times 12 \times 13) \div 4$
$12 \times 13 \times 14 = (12 \times 13 \times 14 \times 15 - 11 \times 12 \times 13 \times 14) \div 4$
$13 \times 14 \times 15 = (13 \times 14 \times 15 \times 16 - 12 \times 13 \times 14 \times 15) \div 4$

$100 \times 101 \times 102 = (100 \times 101 \times 102 \times 103 - 99 \times 100 \times 101 \times 102) \div 4$

SUM
$= (100 \times 101 \times 102 \times 103 - 10 \times 11 \times 12 \times 13) \div 4$
= (25 x 101 x 102 x 103 – 10 x 11 x 3 x 13)
= 3 (25 x 101 x 34 x 103 – 10 x 11 x 1 x 13)
= 15 (5 x 101 x 34 x 103 – 2 x 11 x 1 x 13)
= 26523360