Principal component models of knee kinematics and kinetics: Normal vs. pathological gait patterns

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Abstract

Gait data were collected on a group of 29 asymptomatic elderly subjects to describe knee joint kinematics and kinetics as measured by the three components of the bone-on-bone forces, net reaction moments and relative knee angles. Each of these gait measures were considered separately in the development of Principal Component Models (PCMs) to describe the variation of the normal subjects throughout the gait cycle. The statistical similarity of patients' gait curves (waveforms) to the pattern of normal subjects' gait waveforms was assessed using the PCMs. The PCMs consider data from the entire gait cycle and detect statistically significant waveform shapes using measures of distance from normal. Osteoarthritic patients were selected from a clinical study of pre-operative and post-operative unicompartmental arthroplasty. Three cases were chosen to demonstrate the PCMs application on a waveform-by-waveform basis. In addition, the overall assessment of three patients as indicated by eight kinematic and kinetic gait measures was performed. The outcome measured by the PCMs was shown to agree with the clinical results as measured by the Knee Society Score. The PCMs were able to quantify the difference from normal with statistical significance and the structure of the models allowed for interpretation in terms of portions of the gait cycle.

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1. Introduction

The knee is one of the most common sites for osteoarthritis (OA) and accounts for more pain and disability than any other joint (Felson, 1987). While the main indication for knee arthroplasty in the treatment of OA is pain, a further objective is to improve the patient's functional ability, and more specifically, gait. This aspect of human motion has been studied intensively over the past 30 years, but its usefulness as a clinical analysis tool is still questioned by many. The interpretation of gait data is controversial and has hindered its application to clinical decision making (Brand, 1992). Much has been learned about how humans walk, but the expectation of diagnosing pathological change at a finer level has not been met. The problem is not technological since recent advances have made data collection fast and efficient with sufficient resolution to provide meaningful measurements. A significant barrier to clinical use of gait information is the successful reduction and analysis of the data (Andriacchi, 1992).

Most gait data appear as temporal waveforms representing specific joint measures throughout the gait cycle. A common way of analysing such data is by defining and extracting parameters (ranges, peak values, as well when in the gait cycle these occur) as descriptors of discrete instants or events of the gait pattern (Stauffer et al., 1977; Andriacchi et al., 1982; Schnitzer et al., 1993). Detection of abnormality reduces to finding significant differences between subject group averages of these parameters. It is often difficult to interpret the analysis of all parameters simultaneously in a way that is clinically relevant. Furthermore, the parameters may not be able to be identified easily or repeatedly when abnormal waveform shapes are involved (Chao et al., 1983; Whittle and Jefferson, 1989). Alternative analysis methods of gait waveforms include Fourier series, neural network classifiers and pattern recognition techniques (Wong et al., 1983; Lasko-McCarthey et al., 1990; Holzreiter and Kohle, 1993; Gioftsos and Grieve, 1995; Kadaba et al., 1993). All of these consider the entire gait cycle data while the latter two offer ways of classification according to the pattern of the waveforms. Principal component modelling is complementary to these techniques and emphasises comparison to a reference or normal gait pattern.

In some gait analysis studies separation of subjects into groups is done on the basis of qualitative subjective descriptions and comparisons between the overall shapes of gait waveforms (Andriacchi et al., 1982; Whittle and Jefferson, 1989; Wilson et al., 1996). This paper addresses the need for a statistically based method by which one can discriminate and classify subjects based on the entire gait waveform. Unfortunately, most standard statistical methods developed for

analysis of scalar data are not appropriate for the analysis of serial data such as gait waveforms. In analysing the net external flexion moment at the knee, Deluzio et al. (1995) implemented a principal component method that includes data from a complete stride and not just parameters extracted from the gait curves. This methodology was shown to be applicable to gait data and the analysis detected changes in the flexion moment waveform pattern associated with OA and its treatment.

An important aspect of the clinical application of gait analysis is an overall assessment of a patient's gait. In this paper Principal Component Models (PCMs) of eight kinematic (joint angles) and kinetic (forces and moments) gait measures are developed. The models utilise data from the entire gait cycle and simplify the assessment of patients through the use of measures of distance from normal. This approach, which is closely related to control theory and acceptance regions, is based on the construction of empirical models developed from a normal subject dataset which is then used to assess patient gait data relative to the normal gait pattern. Patients were selected from a unicompartmental arthroplasty clinical trial to demonstrate the application of these models to OA patient gait data both pre-operative and post-operative. Three cases are provided which illustrate the kinds of changes in curve shape that the PCMs can detect as well as the interpretation provided by the PCMs. The collective assessment of three patient's gait patterns as measured by all eight kinematic and kinetic measures is also presented to indicate the application of the models for overall gait assessment.

2. Methods

2.1. Subject selection

Thirty-seven asymptomatic volunteers who were pain-free, without any evidence or history of arthritic disease, or record of surgery to the lower limbs were considered for the normal reference dataset. These subjects were further screened by examining sagittal and frontal radiographs and eight subjects with radiographic evidence of OA (Scott et al., 1993) were then excluded. The radiographic screening was done to ensure that the normal subject dataset represented a homogeneous group free from possible osteoarthritic gait artefacts. Thus, the normal reference data set was composed of 27 elderly subjects. The mean age was 64 years (range 47 to 80) and seventeen of the subjects were females.

As part of a clinical trial the pre-operative and one year post-operative gait data were available from 13 patients. In addition, an established clinical knee
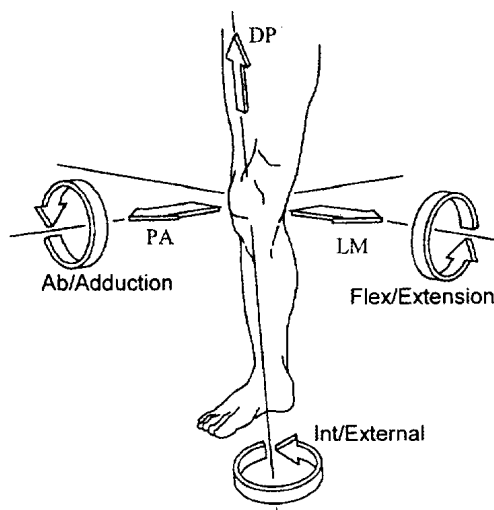
Fig. 1. *Sign Convention*. Three anatomical axes, namely the Lateral–Medial (LM), Posterior–Anterior (PA), and the Distal–Proximal (DP), are defined perpendicular to the sagittal, frontal, and coronal planes of the tibia. The relative knee angles as well as net reaction moments are measured about these axes and bone-on-bone forces are measured along the axes. The positive directions of the axes and the rotations about these axes are as indicated. Note that the naming of the axes indicates the positive direction of each axes (i.e. PA is directed positive from posterior to anterior).

rating system (KSS) (Insall et al., 1989) was administered pre-operatively and post-operatively. The patients had varied degrees of OA mainly confined to one knee compartment and received unicompartmental arthroplasties (UCA). The patient data were selected from this dataset to demonstrate the methodology.

The gait pattern was studied using a 3D gait analysis system (Costigan et al., 1992). This system utilises optoelectronic motion tracking, standardised radiographs, force plate and anthropometrics to calculate the 3D components of knee angles, net reaction moments and bone-on-bone forces (Li et al., 1993). The sign convention follows an anatomically based co-ordinate system embedded in the proximal tibia along the posterior/anterior, lateral/medial and distal/proximal directions as illustrated in Fig. 1. The novel part of this system is the incorporation of the knee alignment data and the geometry of the bones using the standardised radiographs, which allows for a more accurate determination of the bone-on-bone forces at the knee joint.

2.2. Statistical method

As a first exploratory step in the analysis of a data matrix with measurements of n persons on p variables PCA, is often used due to its potential for data

reduction and explanation (Jolliffe, 1986). The main purpose of PCA is to summarise the most important information in the data. This is accomplished by representing the persons and the variables simultaneously in a limited number of optimal components. These components are optimal in the sense that they explain a maximal amount of variance.

Mathematically, PCA consists of an orthogonal transformation which converts the p variables $X = x_1, x_2, \ldots, x_p$ (as in a time normalised gait curve sampled at each 1% from 0 to 100% of the cycle) into p new uncorrelated principal components (PC's), $Z = z_1, z_2, \ldots, z_p$. The PCs are mutually uncorrelated in the sample and are arranged in decreasing order of their sample variances. The PC model is $Z = U^t X$ where the columns of $U = u_1, u_2, \ldots, u_p$ called the loading vectors are the eigenvectors of the correlation matrix of X.

2.3. Selection of number of components

A powerful property of PCA is that if the majority of variation is explained by the first few principal components, $Z = z_1, z_2, \ldots, z_k$ where $k < p$, the remaining PCs can be dropped and the reduction in dimension is achieved. This should decrease the effect of random noise in the input data X. The residuals, $X - UZ$, are defined as the difference between the original data X and that estimated from the inverted PC model, $\hat{X} = UZ$. The residuals, therefore, should represent random noise. The number of principal components, k, needed to build a PC model which adequately describes a dataset can be found using several criteria which are based on the portion of explained variation (Jackson, 1991). As the PCMs were to be used to assess future patient data, cross-validation was chosen as it is suited to applying the model to future observations not included in the construction of the model. Cross-validation measures the predictive power of the model using a summary of the predicted residuals calculated by deleting and predicting each observation (subject) in turn from the model. The predicted residuals are compared as one adds PCs until the overall prediction is no longer significantly improved by the addition of extra PCs (Wold, 1978; Eastment and Krzanowski, 1982).

2.4. Assessment of patient data

Once a PC model has been developed to describe the gait pattern of a group of standard or reference subjects, for example a group of normal subjects, it can be used to assess the gait waveforms of OA patients. The loading vectors, U, derived from the normal subject dataset are simply applied to the individual patient gait waveform data, $Z_{patient} = U^t X_{patient}$.
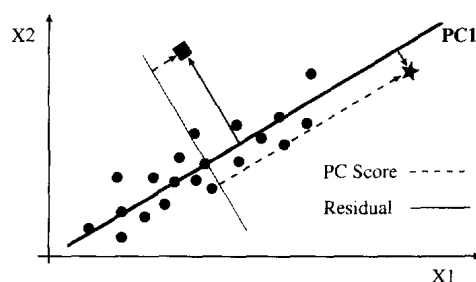
Fig. 2. *Two variable illustration of PCA.* This represents simulated data for $p = 2$ variables (x_1 and x_2) which produce a $k = 1$ PC model (the line). The dots form the reference dataset from which the PC model is calculated. The star represents an observation with a large score but a small residual whereas the square represents and observation with a large residual but a small PC score.

The PCM can be thought of geometrically as a projection of the data from the p-dimensional space defined by the original variables to a k-dimensional hyper-plane defined by the PCs. The PCs are the co-ordinates of the projected data and the residuals reflect the perpendicular distance of the observation from the hyper-plane. Large residuals indicate an outlier from the model and that the observation belongs to a population with a different underlying principal component structure from the data that formed the model. This concept is illustrated in Fig. 2 for simulated data on two variables ($p = 2$: x_1 and x_2) which are modelled by one PC ($k = 1$) in which case the hyper-plane is a line.

As in the practice of control theory, acceptance regions are derived from this normal dataset about both the PC scores and the residuals. For the PC scores, the Mahalanobis distance, T^2, is used to measure the distance of each observation (subject) from the centre of the hyper-plane defined by the PCM. The T^2 can be obtained from a weighted sum of squares of the retained principal component scores (Hotelling, 1931):

$$T^2 = z^t D^{-1} z,$$

where D is a diagonal matrix of the variances of the PCs. Confidence intervals are obtained from the distribution of T^2 values (Jackson, 1991) for the normal subjects, to which the patient gait data are then compared. If the T^2 value is significant the individual PC scores and their corresponding loading vectors are examined. The scores indicate the magnitude and direction of difference and the loading vectors indicates the portions of the gait cycle that contribute to the difference.

For the residuals, the sum of squares (SS) of the residuals, Q, is used to measure the perpendicular distance of each observation (subject) from the hyper-plane defined by the PCM. The Q value is obtained by squaring the

difference between the PCM's estimate of the patient's gait waveform and the actual waveform:

$$Q = \text{SS(res)} = (X^* - \hat{X}^*)^t (X^* - \hat{X}^*).$$

This quantity is obtained for all subjects forming the normal model and the distribution of Q values can be used to derive a confidence interval (Jackson and Mudholkar, 1979), to which the patient gait data are then compared. If the Q value is significant than the residuals are plotted to reveal the portion of the gait cycle corresponding to the large residuals.

3. Results

3.1. Principal component models

PCMs were developed for each of the three components of the knee bone-on-bone forces, net reaction moments and two components of the relative knee angles (Table 1). A PCM was developed for the knee angle measured about the DP axis of the tibia, but was discarded as it did not offer discriminatory information between the OA patients and the normals.

The number of PCs in each model was chosen through cross-validation. The eight PCMs used from two to four PCs to explain anywhere from 66 to 96% of the variation in the normal gait waveforms. The low number of PCs suggests that there is a simple underlying structure to the large variability present in the gait waveforms. The cross-validation process identified waveforms within some of the PCMs with a large residual structure. In the construction of the confidence limits these waveforms were flagged as possible outliers. However, these

Table 1

PCMs for each of the gait waveforms

	Bone-on-bone forces			Net reaction moments			Knee angle	
	PA [a]	LM	DP	PA	LM	DP	PA	LM
% variation explained	76	82	72	74	78	66	96	80
No. of PCs	4	3	3	2	3	3	2	2

The number of PCs used in each of the PCMs is shown with the corresponding total % variation explained by each of the PCMs.

[a] The letters PA, LM, and DP refer to posterior/anterior, lateral/medial and distal/proximal directions (Fig. 1).

subjects were not removed from the normal dataset as it is felt that these subjects reflect the inherent variability of gait patterns. The inclusion of potential outliers decreases the sensitivity of the PCM and the robustness of the PCs (Krzanowski, 1984; Wold et al., 1983). A larger sample of normal subjects may provide the model with greater stability (i.e. fewer outliers).

The gait data from patients were then introduced to the PCMs developed from the normal subjects. Each waveform was transformed into a set of PC scores and residuals and then compared to 95% confidence limits of the normals. Three cases have been chosen to illustrate some of the changes in curve shape that the proposed methodology can detect as well as the interpretation provided by the PC models. Each case examines the results of one gait measure for each patient.

3.2. Case A

The pre-operative and post-operative knee flexion angle of a UCA patient is shown in Fig. 3 along with the average waveform of the normal subjects. While the overall shapes of the waveforms is similar; pre-operatively the patient's degree of flexion appears to be reduced and this stiff-kneed gait is eliminated post-operatively. The PCM of the knee flexion angle was composed of two PCs. The loading vectors are the coefficients that, when applied to the gait waveforms produce the PC scores. They can be examined to reveal the underlying structure
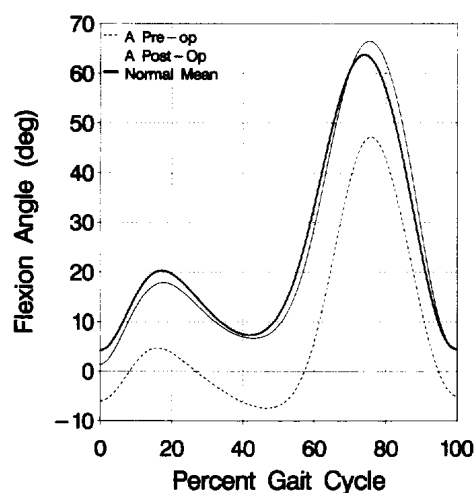

Fig. 3. Knee flexion angle for case A. The knee flexion angle waveform is shown from initial foot contact at 0% to final foot at 100% of the gait cycle. The average of the normal subjects; the dashed line is the pre-op patient data, and the thin line is 1 yr. post-op.
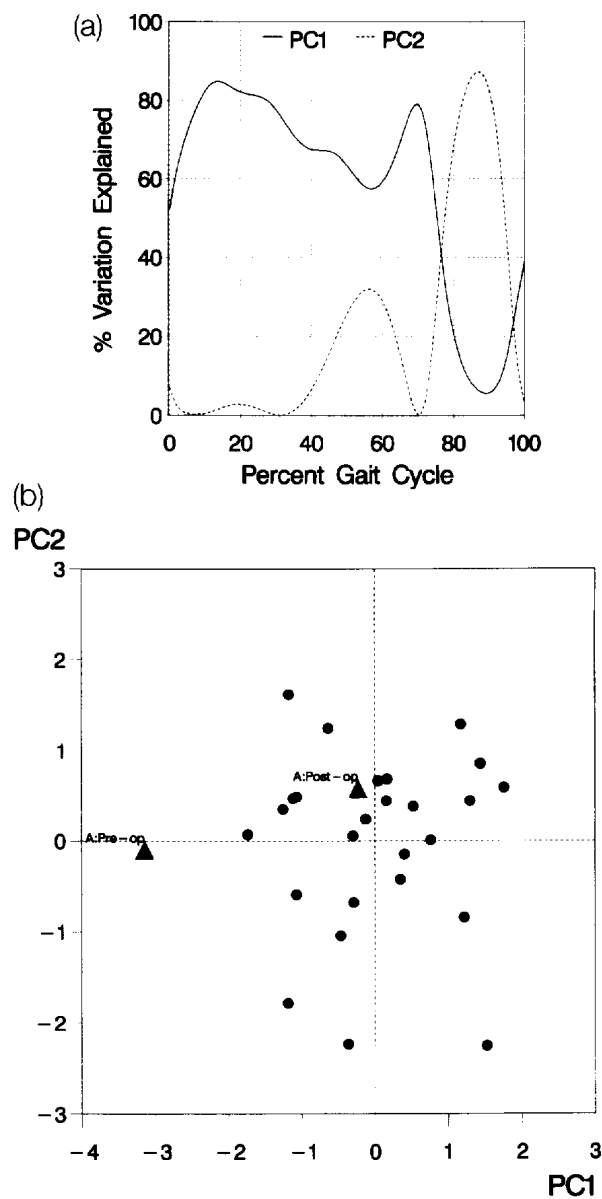
Fig. 4. *Case A PC scores and variation explained by PC 1 and 2*. (a) The variation explained by the first two PCs throughout the gait cycle. (b) The first two PC scores are shown for the normal subjects as well as the pre-operative and post-operative of case A. The Mahalanobis distance T^2 for this case was significant at the $\alpha < 0.05$ level.

of the model and the relative importance of various portions of the gait cycle to each PC in the model. Fig. 4a reveals that each PC captures complementary portions of the gait cycle with PC1 explaining the stance phase and PC2 concentrating on the swing phase. The patient data from case A were introduced to this model and while the residuals (Q value) were within normal limits the Mahalanobis distance (T^2) was significant at the $\alpha < 0.05$ level (pre-operatively). Post-operatively the patient was within normal limits. As the T^2 is just a weighted sum of squares of the PC scores a scatterplot of the PCs will reveal the cause of the significant T^2 value. Fig. 4b illustrates the patient's pre-operative and post-operative PC scores along with those from the normal subjects. Pre-operatively the PC1 score for this patient's flexion angle was significantly low while the patient falls well within the cluster of normals post-operatively. The fact that the pre-operative PC2 score is close to zero means the patient's flexion angle is within normal limits from mid to late swing. Thus, the portion of the gait cycle during which the difference arises can be identified as the stance phase.

3.3. Case B

Case B illustrates how the residuals are used to detect and interpret differences from normal. The adduction moment shown in Fig. 5 reveals patient B's
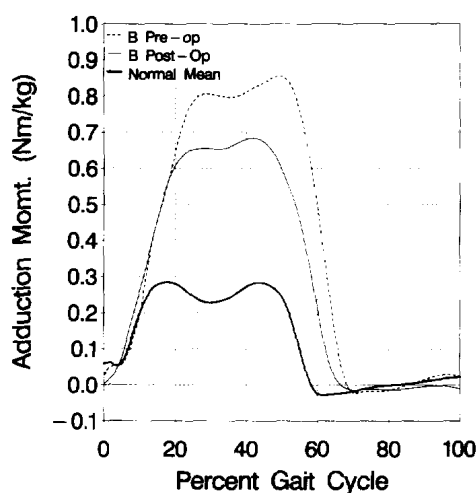

Fig. 5. *Knee Ab / adduction moment for case B*. The average of the normal subjects; the dashed line is the pre-op patient data, and the thin line is 1 yr. post-op. Along with the high adduction moment during stance the patient maintains a moment longer into the gait cycle than the normals.
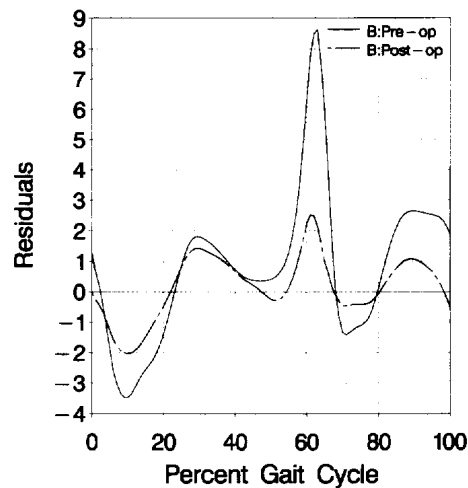
Fig. 6. *Standardised residuals for Case B.* The residuals representing the difference between the standardised gait data and that predicted by the PCM are plotted vs. the gait cycle. The sum of the squares of the residuals for the entire gait cycle for this case was significant at the $\alpha = 0.05$ level.

high adduction moment which is consistent with a varus deformity of the knee (Andriacchi and Mikosz, 1991). However, also evident in this plot is the patient's prolonged stance time. This change, reflected in the shape of the waveform, is also captured by the PCM. The PCM for the ad/abduction moment used two PCs which explained 74% of the variation in the normal subjects' waveforms. While the PC scores detect the high magnitude of the adduction moment during stance the Q value was significantly large. The residuals are defined as the difference between the actual waveform and that predicted by the PCM. This difference can be plotted throughout the gait cycle to determine the cause of the significant Q value. Fig. 6 illustrates the pre-operative and post-operative residuals at each instant of the gait cycle. The large residuals at 60–70% of the gait cycle identify the swing–stance changeover as the portion of the gait cycle contributing to the significant Q value. Smaller contributions occur earlier in the gait cycle ($\sim$ 10%) corresponding to the patient attaining peak abduction moment slower than the normal subjects. The post-operative adduction moment did not return to within normal limits.

3.4. Case C

This last case demonstrates the use of the PCM of the flexion moment to reveal subtle differences in waveform shape. The pre-operative and post-oper-
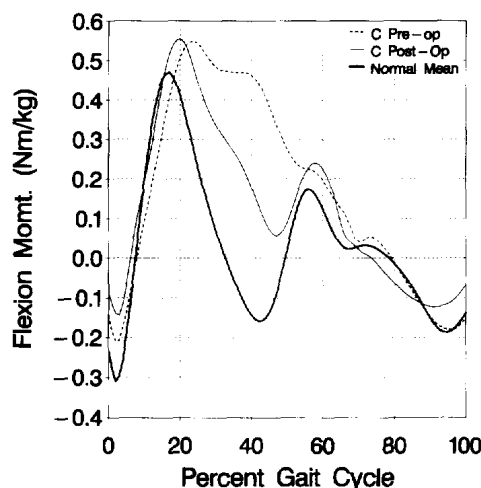
Fig. 7. *Knee net external flexion / extension moment for case C.* The average of the normal subjects; the dashed line is the pre-op patient data, and the thin line is 1 yr. post-op.

ative flexion moment for UCA patient C is shown in Fig. 7. The peak flexion moment is within 1 standard deviation of the mean for both pre-operative and post-operative; however the pre-operative flexion moment does not exhibit the biphasic pattern of the normal subjects but remains high throughout stance. This difference in waveform shape is difficult to quantify using curve parameters since the peak flexion moment is not different from normal. The PCM revealed that the patient's flexion moment was significantly different from normal pre-operative and returned to within normal limits post-operative. In this case both the T^2 and the Q values were significant. The PC scores revealed the high moment during midstance while the residuals identified the late first peak and were large at about 60% of the gait cycle as well.

3.5. Overall assessments

An overall patient gait assessment is provided by examining the PCMs of each of the kinematic and kinetic gait waveforms. Three UCA patients are presented that represent a variety of possible outcomes. Patients P1 and P3 correspond to cases B and C considered above while patient P2 is an additional case. Table 2 provides an example of an overall gait assessment of the three patients using the PCMs developed from the normal subjects. The table is simplified to provide an immediate evaluation of each gait measure; significant differences from normal can then be analysed by examining the residuals

Table 2
Patient assessments

UCA patients	KSS /200	Bone-on-bone forces			Net reaction moments			Knee angle	
		PA	LM	DP	PA	LM	DP	PA	LM
P1									
Pre	85	x	✔	x	x	x	x	x	x
Post	185	✔	✔	✔	✔	✔	✔	✔	✔
P2									
Pre	104	x	✔	✔	✔	x	✔	✔	x
Post	99	x	x	✔	x	x	x	✔	x
P3									
Pre	105	✔	x	x	x	x	x	x	x
Post	138	✔	x	x	x	✔	x	✔	✔

An x indicates significantly different from normal for either the scores or residuals at the 95% level, while ✔ indicates that waveform pattern is similar to the normal pattern. The KSS (Knee Society Score) is also shown pre-operative and post-operative.

and/or PC's. A cross indicates a significant difference from normal in either the T^2 and Q values while a check indicates that the waveform pattern is within normal limits.

Pre-operatively, patient P1 was significantly different from normal in all gait measures except the lateral/medial bone-on-bone force. Each of these gait measures returned to a normal pattern post-operatively. In contrast, patient P2 showed no improvement in the gait measures that were significant pre-operative and other gait measures that were within normal limits pre-operative became significantly different from normal post-operative. Patient P3's pre-operative gait pattern was different from the normal subjects in all gait measures except the proximal/distal bone-on-bone force. Of these the kinematic measures (knee angles) improved along with the flexion moment (LM); however the bone-on-bone forces and the moment in frontal and coronal planes (PA, DP) did not return to within normal limits.

The three patients described in Table 2 represent three different outcomes according to their gait pattern. Patient P1 improved, patient P2 got worse, and P3 improved with respect to some gait measures while others showed no improvement. These results were consistent with the clinical condition as measured by the KSS (Table 2).

4. Discussion

Principal component modelling of gait waveforms is a promising technique for the successful reduction and analysis of gait waveforms. It fulfils two

objectives of gait analysis: detection and interpretation. The first represents the ability to classify a subject as different from what is often a 'normal' population and the second (usually dependent on the first) represents the ability to explain differences in the gait data in a clinically meaningful way. The detection of difference is done on the basis of the entire gait curve using control limits, and the interpretation follows from the original gait measures combined with the portions of the gait cycle during which this difference arises.

The three cases demonstrate some of the ways in which a patient's waveform can be different from normal and the ability of the PCMs to quantify the differences. Case A illustrated that in the PCM of the knee flexion angle, the first two PCs concentrated on separate portions of the gait cycle. Such separation of PCs enables interpretation of large magnitude PC scores. However, the PCs are not always separated in this way. This is particularly true as the number of PCs in the PCM is increased. Additional PCs capture smaller portions of variability and possibly from several disjoint portions of the gait cycle, making it more difficult to isolate the portion of the gait cycle responsible for the difference from normal. In such cases control limits can be placed about estimated PC scores at each point of the gait cycle.

This practice is found in the process control field where unusual temporal events can be related to known phenomena in the process in order to determine their cause (Montgomery, 1991; Kresta et al., 1991). In biomechanics, detecting significant difference from normal and indicating the portion of the gait cycle responsible for this difference can lead clinicians in isolating the possible causes for such differences. Temporal events in the gait cycle such as the relationship between tibio-femoral contact positions, joint loading and muscle activation during gait has been described (Andriacchi et al., 1986; Sutherland et al., 1988).

Cases A to C also demonstrated some of the differences between the PC scores and the residuals. A subject with significant PC scores usually has a gait waveform that is different over the portion of the gait cycle corresponding to the specific PC as in case A and C. The first PCs tend to detect changes that inflate variances and covariances (Gnanadesikan and Kettenring, 1972). On the other hand, the residuals, which reflect the discarded or small variance PCs, are sensitive to different types of changes in waveform shape than are the retained or large variance PCs (Jolliffe, 1986; Hawkins and Fatti, 1984). These changes are usually hard to see in the original data and are related to events that violate the correlation structure of the normal dataset. Cases B and C illustrated large residuals corresponding to a waveform that is time-shifted from the normal subjects' waveforms.

As in all inferential techniques, the fundamental assumptions of 'comparable' data and 'observable' events apply for this method to work. The first assumption implies that the method is valid as long as the reference dataset of normal subjects is representative of normal gait; while, the second assumption expresses the requirement that the events one wishes to detect must be observable from the measurements being collected. This study used two levels of screening in an effort to assemble a homogenous group of normal subjects. The first level of screening allowed only those patients without symptoms or medical history related to knee joint into the gait study. The second level allowed only those patients without radiographic evidence of OA into the normal dataset. However, our analysis indicated that some of these 'normal' subjects had a pattern of gait that was different from the majority. These potential outliers may be representative of normal gait or they may be true outliers and represent a non-normal gait of unknown cause. The latter possibility is common to any new measurement tool where it is difficult to discriminate artefact from new information. Further exploration of this technique may lead to insights and understanding of what normal gait is and the causes of deviation from normal. The other possibility is that the potential outliers are in fact normal but are perceived as different due to the inability of the sample of subjects to represent the population of normals. The most likely cause of this is the low number of subjects.

The second assumption expresses the requirement that the events one wishes to detect must be observable from the measurements being collected. In the calculation of the bone-on-bone forces the assumption is made that there is no antagonistic muscular activity present. However, such activity may be an early protective mechanism for increasing stability in OA patients; such activity would increase the bone-on-bone contact forces. Due to the limitations of EMG-force relationships the effect of cocontraction is difficult to quantify (Wyss and Pollack, 1984). By considering knee kinematics and kinetics together the PCMs, should be sensitive to dynamic factors related to knee OA.

Summarising the gait assessment as in Table 2 allows an immediate assessment of the overall gait pattern. The three patients described in Table 2 represent three different outcomes according to both the gait pattern and KSS. The KSS can reveal overall changes but is unable to indicate specifically how these changes occurred. As well it is dominated by effect of pain (Insall et al., 1989) The gait assessments are able to identify the components of gait affecting the performance. The PCMs detect which gait measures are abnormal as well as the interpretation of this difference in terms of the portion of the gait cycle responsible for this difference.

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