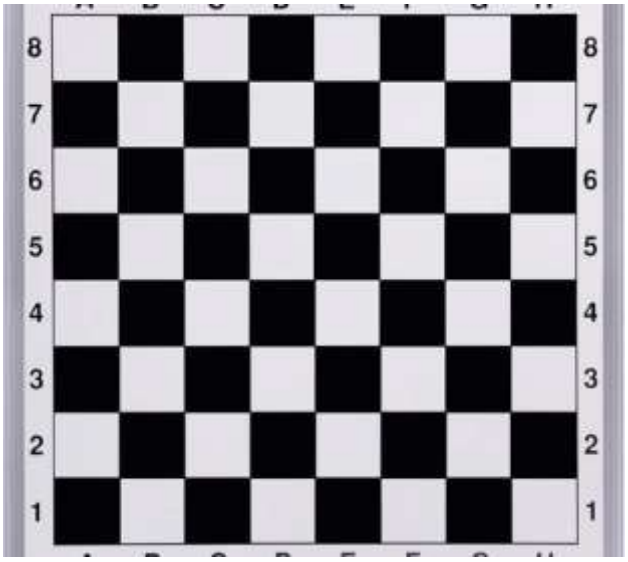
Tanay Cansin Dogan
150150119

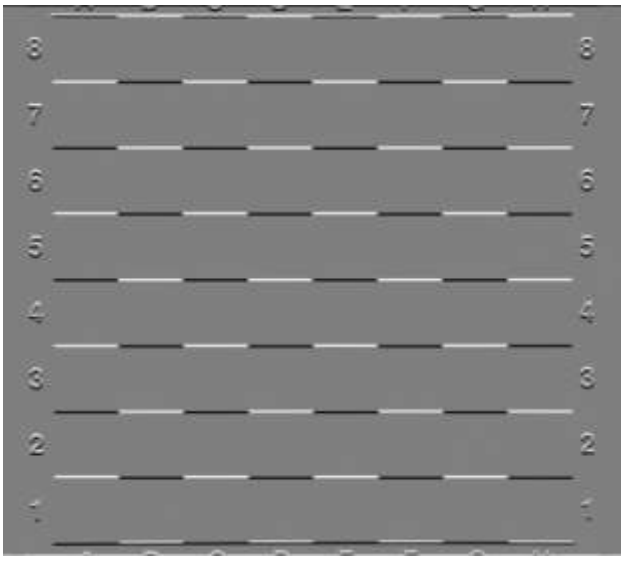
BLG 453E – Computer Vision: Homework 3

Part1:

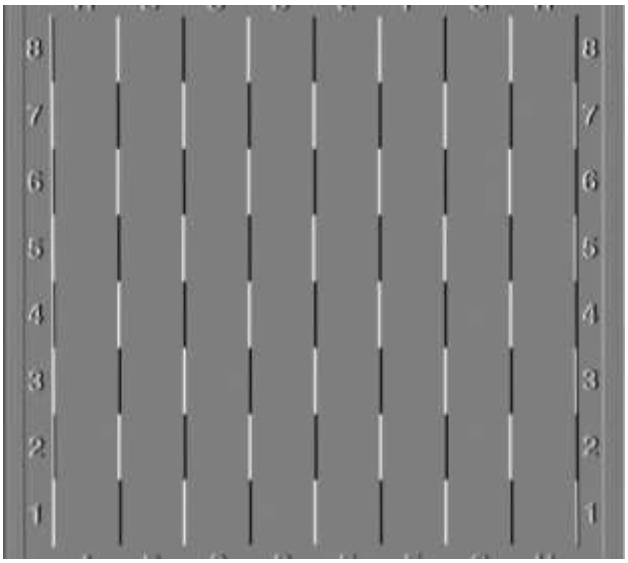
In this part we should have detected edges in the screenshot. This task accomplished by convolving sobel filters and then computing the magnitude map for the resultant vector of the responses in both directions (x and y).



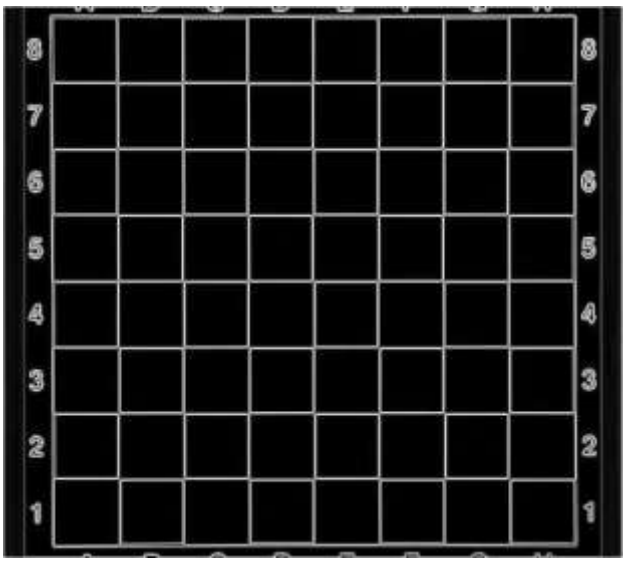
Original Image



Normalized response of sobel y filter



Normalized response of sobel x filter



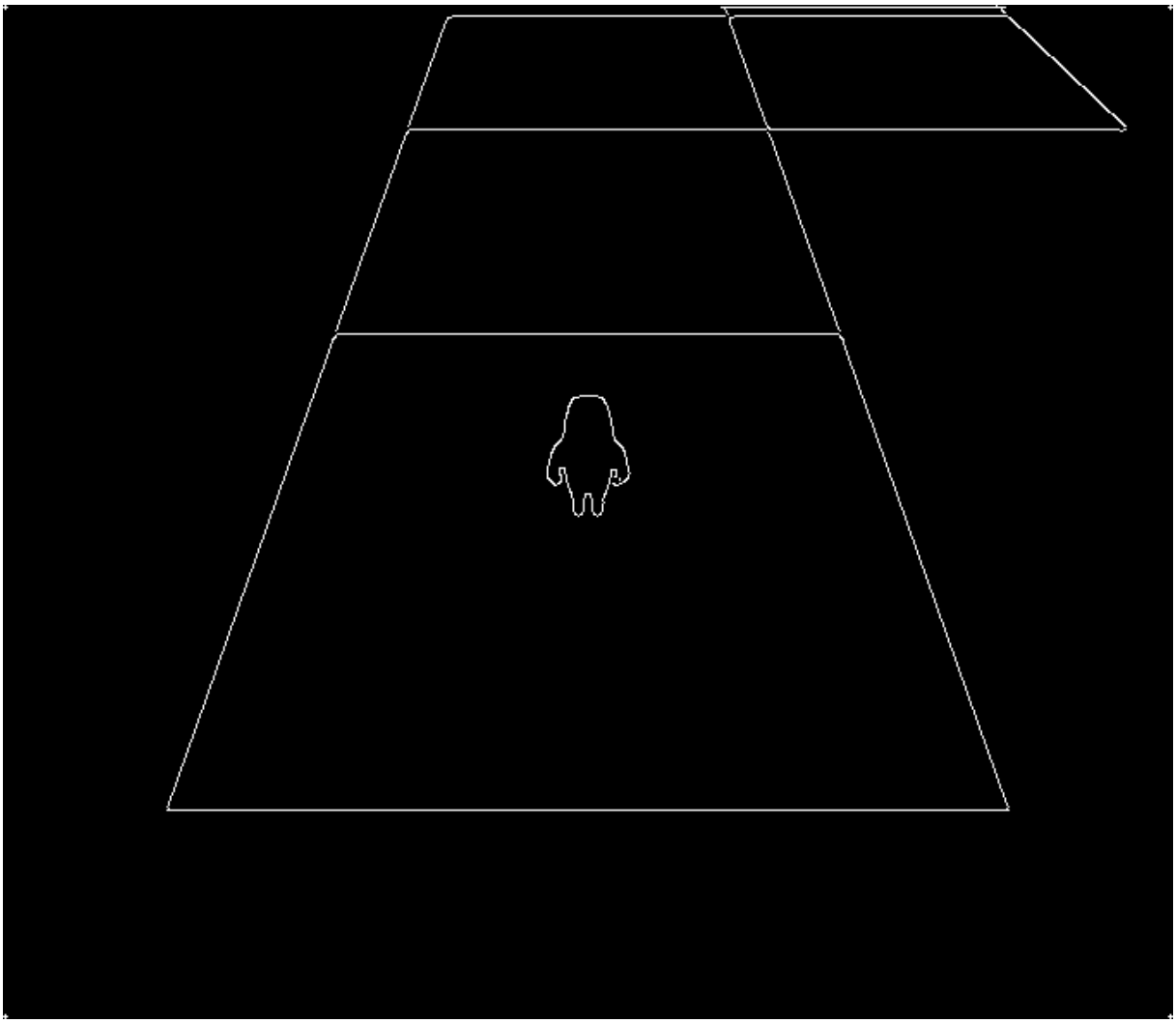
Magnitude map of the resultant response

Part2:

In this part we have used an advanced edge detection method “Canny edge detection” which is already provided by opencv library. However, before using canny algorithm we **must** smooth the image, I made smoothing operation via using 5x5 gaussian kernel with sigma = 1. In fact, image must smoothed before apply a derivation operation (eg. Sobel as in part1) but since I used a noise-free image in part1, I didn’t smooth it in part1.

Threshold values picked manually as 200, 250

Result:



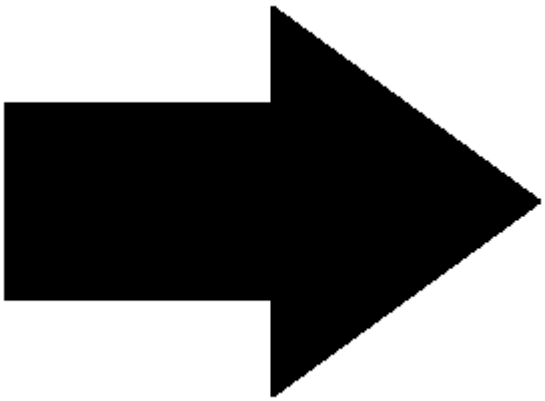
Part3:

In this part we implemented a popular corner detection method called Harris Corner Detector. In order to accomplish that we calculated gradient vector (gradient in x direction and gradient in y direction) then constructed the matrix G which called the **"Image Structure Tensor"**. Then measure the cornerness of the pixel via using **Cornerness Function**.

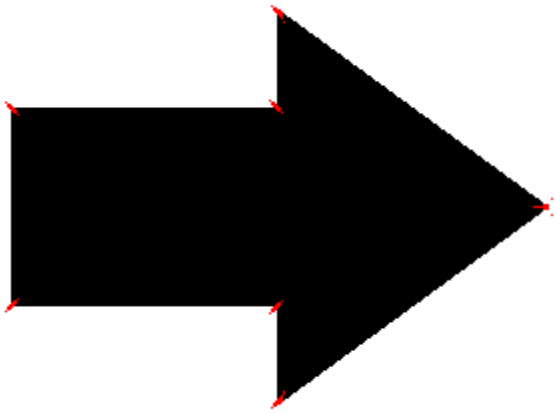
Since: det(G) = eigen1*eigen2 and

Trace(G) = eigen1 + eigen2

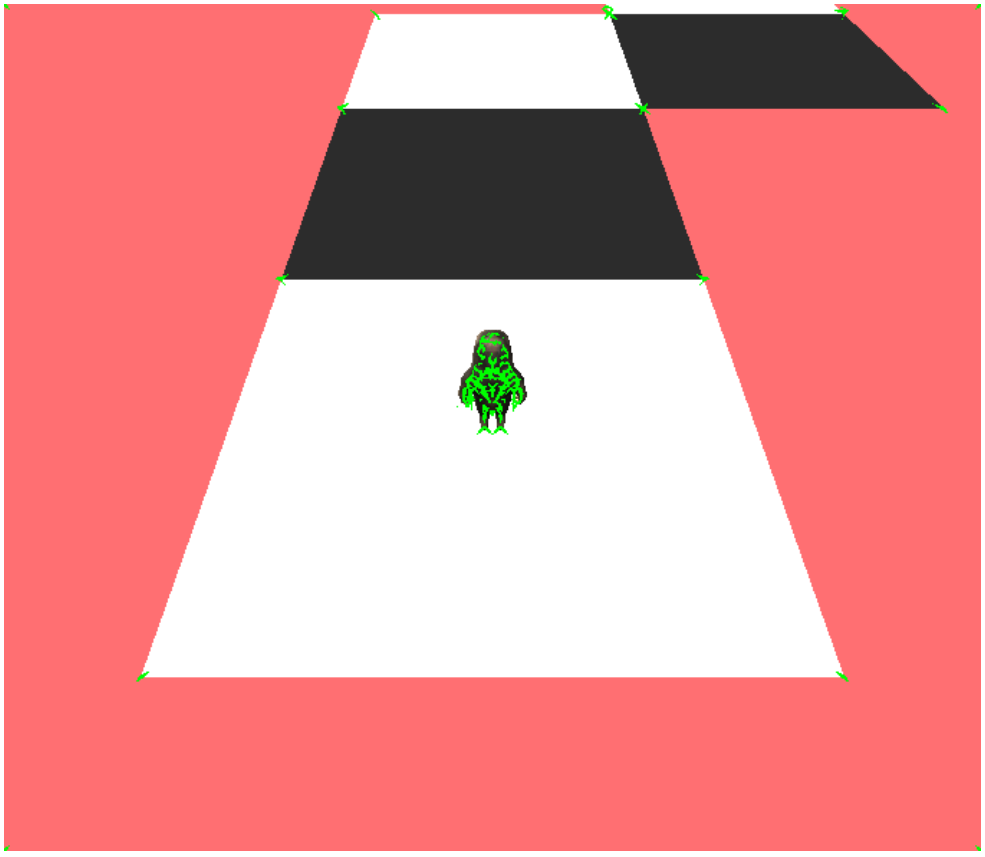
I didn't calculate the eigen values, I used determinant of matrix G and trace of matrix G Instead.



The image.



Pixels where C > 0.



Harris corner detection result on screenshot

Part4:

In this part we have find an algorithm to control the character automatically respected to the extracted features from the screenshot image. In this part I only used the canny edge map.

Warning: Due to the difference in screen resolution this part cannot yield a result as expected in your computer.

Flow of the Algorithm:

1. Midget in the decision point takes a screenshot.
2. Midget plans a path as maximum 3 tiles
    - 2.1 Midget label its location as seed point
    - 2.2 Midget sends beam in 4 main directions (up, right, down, left).
    - 2.3 Midget counts number of edges which intersect with those beams.
    - 2.4 Midget chooses a direction where #of edges > 1, except the reverse direction of previous move/path.
    - 2.5 Midget move its new seed point to the midpoint of the very end tile of that direction.
    - 2.6 If #of moves doesn't exceed 3, midget continues to search by returning to the step 2.2.
3. Midget check the plan:
    - 3.1 If: it doesn't contain any move, Then It **terminates**.
    - 3.2 Otherwise: Midget returns to the step 1.

Part5:

I have proved it on paper and scanned them. You can examine the proof from the next 2 pages.

Part5: Otsu Thresholding

Prove that: $\sigma^2_{between}(t) = \sigma^2_{total} - \sigma^2_{within}(t)$

Let: I: maximum value for intensity · eg. 255 for 8bit grayscale image
p_i: probability of intensity i
μ_j: mean value of class j
σ_j^2: variance of class j

We know:

$$\omega_1 = \sum_{i=1}^{t} p_i \quad \Big\| \quad \mu_1 = \sum_{i=1}^{t} i \cdot \frac{p_i}{\omega_1} \quad \Big\| \quad \sigma_1^2 = \sum_{i=1}^{t} (i-\mu_1)^2 \cdot \frac{p_i}{\omega_1}$$

$$\omega_2 = \sum_{i=t+1}^{I} p_i \quad \Big\| \quad \mu_2 = \sum_{i=t+1}^{I} i \cdot \frac{p_i}{\omega_2} \quad \Big\| \quad \sigma_2^2 = \sum_{i=t+1}^{I} (i-\mu_2)^2 \cdot \frac{p_i}{\omega_2}$$

$$\sigma^2 = \sum_{i=1}^{I} (i-\mu)^2 \cdot p_i \quad \Big\| \quad \mu = \omega_1 \cdot \mu_1 + \omega_2 \cdot \mu_2 \quad \Big\| \quad \omega_1 + \omega_2 = 1$$

$$\sigma^2_{between} = \omega_1 \cdot (\mu_1 - \mu)^2 + \omega_2 \cdot (\mu_2 - \mu)^2$$

$$\sigma^2_{within} = \omega_1 \cdot \sigma_1^2 + \omega_2 \cdot \sigma_2^2$$

Start with simplify: σ^2_{total}

$$1: \sum_{i=1}^{I} (i-\mu)^2 \cdot p_i = \sum_{i=1}^{I} (i^2 - 2i\mu + \mu^2) \cdot p_i =$$

$$2: \sum_{i=1}^{I} i^2 p_i - 2\mu \cdot \sum_{i=1}^{I} i \cdot p_i + \mu^2 \cdot \sum_{i=1}^{I} p_i =$$

$$3: \sum_{i=1}^{I} i^2 p_i - 2\mu \cdot \left[\sum_{i=1}^{t} i \cdot p_i + \sum_{i=t+1}^{I} i \cdot p_i \right] + \mu^2 \cdot \left[\sum_{i=1}^{t} p_i + \sum_{i=t+1}^{I} p_i \right] =$$

$$4: \sum_{i=1}^{I} i^2 \cdot p_i - 2 \cdot \mu \cdot [\omega_1 \cdot \mu_1 + \omega_2 \cdot \mu_2] + \mu^2 \cdot [\omega_1 + \omega_2] =$$

$$5: \sum_{i=1}^{I} i^2 \cdot p_i - 2 \cdot \mu \cdot \mu + 2\mu^2 \cdot 1 = \boxed{\sum_{i=1}^{I} i^2 \cdot p_i - \mu^2}$$

Then simplify: σ^2_{within}

$$6: \omega_1 \cdot \sigma_1^2 + \omega_2 \cdot \sigma_2^2 = \omega_1 \cdot \left[\frac{1}{\omega_1} \cdot \sum_{i=1}^{t} (i-\mu_1)^2 \cdot p_i\right] + \omega_2 \cdot \left[\frac{1}{\omega_2} \cdot \sum_{i=t+1}^{I} (i-\mu_2)^2 \cdot p_i\right]$$

$$7: \sum_{i=1}^{t} (i^2 - 2\mu_1 \cdot i + \mu_1^2) \cdot p_i + \sum_{i=t+1}^{I} (i^2 - 2\mu_2 \cdot i + \mu_2^2) \cdot p_i =$$

$$8: \sum_{i=1}^{t} i^2 \cdot p_i - 2\mu_1 \cdot \sum_{i=1}^{t} i \cdot p_i + \mu_1^2 \cdot \sum_{i=1}^{t} p_i + \sum_{i=t+1}^{I} i^2 \cdot p_i - 2\mu_2 \cdot \sum_{i=t+1}^{I} i \cdot p_i + \mu_2^2 \cdot \sum_{i=t+1}^{I} p_i$$

$$9: \sum_{i=1}^{I} i^2 \cdot p_i - 2 \cdot \mu_1 [\omega_1 \cdot \mu_1] + \mu_1^2 \cdot \omega_1 - 2\mu_2 \cdot [\omega_2 \cdot \mu_2] + \mu_2^2 \cdot \omega_2 =$$

$$10: \boxed{\sum_{i=1}^{I} i^2 \cdot p_i - \omega_1 \cdot \mu_1^2 - \omega_2 \cdot \mu_2^2}$$

$$11: \sigma^2_{total} - \sigma^2_{within}(t) = -\mu^2 + \omega_1 \cdot \mu_1^2 + \omega_2 \cdot \mu_2^2$$

---

finally simplify: $\sigma^2_{between} = \omega_1 \cdot (\mu_1 - \mu)^2 + \omega_2 \cdot (\mu_2 - \mu)^2 =$

$$12: \omega_1 \cdot [\mu^2 - 2\mu_1 \cdot \mu + \mu_1^2] + \omega_2 \cdot [\mu^2 - 2\mu_2 \cdot \mu + \mu_2^2] =$$

$$13: \omega_1 \mu^2 - 2\mu \cdot \omega_1 \cdot \mu_1 + \omega_1 \cdot \mu_1^2 + \omega_2 \cdot \mu^2 - 2\mu \cdot \omega_2 \cdot \mu_2 + \omega_2 \cdot \mu_2^2 =$$

$$14: \mu^2 \cdot [\omega_1 + \omega_2] - 2 \cdot \mu \cdot [\omega_1 \cdot \mu_1 + \omega_2 \cdot \mu_2] + \omega_1 \cdot \mu_1^2 + \omega_2 \cdot \mu_2^2 =$$

$$15: \mu^2 \cdot 1 - 2 \cdot \mu \cdot \mu + \omega_1 \cdot \mu_1^2 + \omega_2 \cdot \mu_2^2 = -\mu^2 + \omega_1 \cdot \mu_1^2 + \omega_2 \cdot \mu_2^2$$

proved !!!