Zadaca 1

17. listopada 2023. 22:06

Task 1 (5+5+5+5 pts.). Use the definiton of asymptotic notation (i.e. find c and n_0) to prove the following statements:

a) $\log_2(n) = \Theta(\ln n)$

b) $3n\sqrt{n} + 6n\ln n - 4n = O(n^2)$

c) $3n\sqrt{n} + 6n\ln n - 4n = O(n^3)$

d) $\sum_{i=0}^{\lfloor \log_4 n \rfloor} 4^i = \Theta(n)$

a) $\log_2(n) = \Theta(\ln n)$

$f(n) = \Theta(g(n))$ ako $\exists c_1 > 0, c_2 > 0, n_0 > 0$ t.d.

$0 \leq c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$, $\forall n \geq n_0$

$c_1 = 1 \quad c_2 = 2 \quad n_0 = 1 \Rightarrow 0 \leq 1 \cdot \ln n \leq \log_2 n \leq 2 \cdot \ln n$ ✓

b) $3n\sqrt{n} + 6n\ln n - 4n = O(n^2)$

$f(n) = O(g(n))$ ako $\exists c > 0, n_0 > 0$ t.d.

$0 \leq f(n) \leq c \cdot g(n) \quad \forall n \geq n_0$

$c_1 = 10 \quad n_0 = 1 \Rightarrow 0 \leq 3n\sqrt{n} + 6n\ln n - 4n \leq 10 \cdot n^2$ ✓

c) $3n\sqrt{n} + 6n\ln n - 4n = O(n^3)$

$c_1 = 10 \quad n_0 = 1 \Rightarrow 0 \leq 3n\sqrt{n} + 6n\ln n - 4n \leq 10 \cdot n^3$ ✓

Task 2 (10 pts.). Explain how would you interpret the following expression:

$$8n^2 + 5n + \Theta(n \log n) = \Theta(n^2).$$

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Od svih navedenih f-ja $8n^2$ najbrže roste te ostale mogu ignorirat jer nemaju utjecaja.

ignorirat jer nema je utjecaja.

$f(n) = \Theta(g(n))$ ako $\exists c_1 > 0, c_2 > 0, n_0 > 0$ t.d.

$$0 \leq c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n), \forall n \geq n_0$$

$c_1 = 1$

$c_2 = 16$ $\Rightarrow 0 \leq 1 \cdot n^2 \leq 8n^2 + 5n + \Theta(n \log n) \leq 16 \cdot n^2$ ✓

$n_0 = 1$