

# Homework 2

## Struktura podataka i algoritmi I - I053

### Homework instructions

The submission deadline is October 25, 2023 at 9:00. You can type the tasks in  $\text{\LaTeX}$  or write them by hand and scan them. Programming tasks should be submitted as .cpp files. All files need to be submitted to Teams. You can achieve a maximum of 100 points.

**Task 1** (10 pts.). Prove the following statement:  $f(n) = \Theta(g(n))$  if and only if  $f(n) = O(g(n))$  and  $f(n) = \Omega(g(n))$ .

**Task 2** (25 pts.). Use the master theorem to asymptotically bound the following functions:

- a)  $T(n) = 16T(\frac{n}{4}) + n$ .
- b)  $T(n) = 3T(\frac{n}{6}) + n^{0.500001}$ .
- c)  $T(n) = 3T(\frac{n}{4}) + n \log n$ .
- d)  $T(n) = 4T(\frac{n}{2}) + n^2$ .
- e)  $T(n) = 4T(\frac{n}{2}) - n^2$ .

**Task 3** (10 pts.). Use the recursion tree to determine a good asymptotic upper bound on the following recurrence:

$$T(n) = 3T\left(\frac{n}{6}\right) + n^2.$$

**Task 4** (30 + 5 + 5 + 5 + 10 pts.).

- a) Implement the merge-sort algorithm that sorts a array **in-place**.
- b) What's the time complexity of the merge algorithm that merges two sorted arrays of different lengths into a sorted array?
- c) What's the time complexity of the merge-sort algorithm?
- c) What's the space complexity the merge-sort algorithm?
- e) Try generating arrays of different sizes and compare running times between selection sort and merge sort. What happens if the size of the array is 1000000?