Fuzzy Arithmetic with and without using α-cut method: A Comparative Study

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Abstract: In [2] and [6] the authors proposed a method for construction of membership function without using α-cut. They claim that the standard method of α-cut [1] fails in certain situations viz in determining square root of a fuzzy number. We would like to counter their argument by proving that α-cut method is general enough to deal with different type of fuzzy arithmetic including exponentiation, extracting nth root, taking logarithm. Infact we illustrate with examples to show that α-cut method is simpler than their proposed method.

Keywords: Fuzzy membership function, fuzzy number, alpha-cut

1. **Introduction**:

α-cut method is a standard method for performing different arithmetic operations like addition, multiplication, division, subtraction. In [2] and [6] the authors argue that finding membership function for square root of X where X is a fuzzy number, is not possible by the standard alpha-cut method. They have proposed a method of finding membership function from the simple assumption that the Dubois-Prade left reference function is a distribution function and similarly the Dubois-Prade right reference function is a complementary distribution function. In this paper we are going to show that alpha-cut method can be used for finding n^{th} root of fuzzy number and infact this method is simpler than that proposed by them. However we do acknowledge that the proposed method has more mathematical beauty than the existing alpha-cut method.

2. **Basic Concept of Fuzzy Set Theory:**

In this section, some necessary backgrounds and notions of fuzzy set theory are reviewed.

Definition 2.1: Let *X* be a universal set. Then the fuzzy subset *A* of *X* is defined by its membership function

$$\mu_A : X \to [0,1]$$

which assign a real number $\mu_A(x)$ in the interval [0, 1], to each element $x \in X$, where the value of $\mu_A(x)$ at *x* shows the grade of membership of *x* in *A*.

Definition 2.2: Given a fuzzy set *A* in *X* and any real number $\alpha \in [0, 1]$, then the α-cut or α-level or cut worthy set of *A*, denoted by ${}^{\alpha}A$ is the crisp set

$${}^{\alpha}A = \{x \in X: \mu_A(x) \geq \alpha\}$$

The strong *a* cut, denoted by ${}^{\alpha+}A$ is the crisp set

${}^{\alpha+}A = \{x \in X: \mu_A(x) > \alpha\}$

For example, let A be a fuzzy set whose membership function is given as

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{b-a}, a \leq x \leq b \\ \dfrac{c-x}{c-b}, b \leq x \leq c \end{cases}$$

To find the α-cut of A, we first set α ϵ [0,1] to both left and right reference functions of A. That is, $\alpha = \dfrac{x-a}{b-a}$ and $\alpha = \dfrac{c-x}{c-b}$.

Expressing x in terms of α we have $x = (b-a)\alpha + a$ and $x = c-(c-b)\alpha$. which gives the α-cut of A is

$$^{\alpha}A = [(b-a)\alpha + a, c-(c-b)\alpha]$$

Definition 2.3: The support of a fuzzy set *A* defined on *X* is a crisp set defined as

Supp (*A*) = {*x* Є *X*: $\mu_A(x) > 0$}

Definition 2.4: The height of a fuzzy set *A*, denoted by *h*(*A*) is the largest membership grade obtain by any element in the set.

$$h(A) = \sup_{x \in X} \mu_A(x)$$

Definition 2.5: A fuzzy number is a convex normalized fuzzy set of the real line *R* whose membership function is piecewise continuous.

Definition 2.6: A triangular fuzzy number A can be defined as a triplet [*a*, *b*, *c*]. Its membership function is defined as:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ \frac{c-x}{c-b}, b \le x \le c \end{cases}$$

Definition 2.7: A trapezoidal fuzzy number *A* can be expressed as [*a*, *b*, *c*, *d*] and its membership function is defined as:

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ 1, b \le x \le c \\ \frac{d-x}{d-c}, c \le x \le d \end{cases}$$

3. Arithmetic operation of fuzzy numbers using α-cut method:

In this section we consider arithmetic operation on fuzzy numbers using α-cut cut method considering the same problem as done in [2] and [3] and hence make a comparative study.

3.1. Addition of fuzzy Numbers:

Let X= [a, b, c] and Y= [p, q, r] be two fuzzy numbers whose membership functions are

$$\mu_X(x) = \begin{cases} \frac{x-a}{b-a}, a \le x \le b \\ \frac{c-x}{c-b}, b \le x \le c \end{cases}$$

$$\mu_Y(x) = \begin{cases} \frac{x-p}{q-p}, p \le x \le q \\ \frac{r-x}{r-q}, q \le x \le r \end{cases}$$

Then $^{\alpha}X = [(b-a)\alpha + a, c-(c-b)\alpha]$
and $^{\alpha}Y = [(q-p)\alpha + p, r-(r-q)\alpha]$
are the α –cuts of fuzzy numbers X and Y respectively. To calculate addition of fuzzy numbers X and Y we first add the α –cuts of X and Y using interval arithmetic.

$$^{\alpha}X + {}^{\alpha}Y$$

$$= [(b-a)\alpha + a, c-(c-b)\alpha] + [(q-p)\alpha + p, r-(r-q)\alpha]$$

$$= [a+p+(b-a+q-p)\alpha, c+r-(c-b+r-q)\alpha] \quad \text{..................}(3.1)$$

To find the membership function $\mu_{X+Y}(x)$ we equate to x both the first and second component in (3.1) which gives

$x = a+p+(b-a+q-p)\alpha$ and

$$x = c+r-(c+r-b-q)\alpha$$

Now, expressing α in terms of x and setting $\alpha = 0$ and $\alpha = 1$ in (3.1) we get α together with the domain of x,

$$\alpha = \frac{x-(a+p)}{(b+q)-(a+p)}, (a+p) \le x \le (b+q)$$

and

$$\alpha = \frac{(c+r)-x}{(c+r)-(b+q)}, (b+q) \le x \le (c+r)$$

which gives

$$\mu_{X+Y}(x)=\begin{cases}\dfrac{x-(a+p)}{(b+q)-(a+p)},(a+p)\le x\le(b+q)\\ \dfrac{(c+r)-x}{(c+r)-(b+q)},(b+q)\le x\le(c+r)\end{cases}$$

3.2. Subtraction of Fuzzy Numbers:

Let X= [a, b, c] and Y= [p, q, r] be two fuzzy numbers. Then $^{\alpha}X=[(b-a)\alpha+a,c-(c-b)\alpha]$ and $^{\alpha}Y=[(q-p)\alpha+p,r-(r-q)\alpha]$ are the α –cuts of fuzzy numbers X and Y respectively. To calculate subtraction of fuzzy numbers X and Y we first subtract the α –cuts of X and Y using interval arithmetic.

$$^{\alpha}X-{}^{\alpha}Y$$

$$=[(b-a)\alpha+a,c-(c-b)\alpha]$$
$$-[(q-p)\alpha+p,r-(r-q)\alpha]$$

$$=[(b-a)\alpha+a-(r-(r-q)\alpha),$$
$$c-(c-b)\alpha-((q-p)\alpha+p)]$$

$$=[(a-r)+(b-a+r-q)\alpha,$$
$$(c-p)-(c-b+q-p)\alpha]\text{............}(3.2)$$

To find the membership function $\mu_{X-Y}(x)$ we equate to x both the first and second component in (3.2) which gives
$x=(a-r)+(b-a+r-q)\alpha,$

and $x=(c-p)-(c-b+q-p)\alpha$

Now, expressing α in terms of x and setting $\alpha=0$ and $\alpha=1$ in (3.2) we get α together with the domain of x,

$$\alpha=\frac{x-(a-r)}{(b-q)-(a-r)},(a-r)\le x\le(b-q)$$

and

$$\alpha=\frac{(c-p)-x}{(c-p)-(b-q)},(b-q)\le x\le(c-p)$$

which gives

$$\mu_{X-Y}(x)=\begin{cases}\dfrac{x-(a-r)}{(b-q)-(a-r)},(a-r)\le x\le(b-q)\\ \dfrac{(c-p)-x}{(c-p)-(b-q)},(b-q)\le x\le(c-p)\end{cases}$$

3.3. Multiplication of Fuzzy Numbers:

Let X= [a, b, c] and Y= [p, q, r] be two positive fuzzy numbers. Then $^{\alpha}X=[(b-a)\alpha+a,c-(c-b)\alpha]$ and $^{\alpha}Y=[(q-p)\alpha+p,r-(r-q)\alpha]$ are the α –cuts of fuzzy numbers X and Y respectively. To calculate multiplication of fuzzy numbers X and Y we first multiply the α –cuts of X and Y using interval arithmetic.

$$^{\alpha}X*{}^{\alpha}Y$$

$$=[(b-a)\alpha+a,c-(c-b)\alpha]*$$
$$[(q-p)\alpha+p,r-(r-q)\alpha]$$

$$=[((b-a)\alpha+a)*((q-p)\alpha+p),$$
$$(c-(c-b)\alpha)*(r-(r-q)\alpha)]\text{............}(3.3)$$

To find the membership function $\mu_{XY}(x)$ we equate to x both the first and second component in (3.3) which gives

$$x=(b-a)(q-p)\alpha^2+((b-a)p+(q-p)a)\alpha+ap$$
and
$$x=(c-b)(r-q)\alpha^2-((r-q)c+(c-b)r)\alpha+cr$$

Now, expressing α in terms of x and setting $\alpha=0$ and $\alpha=1$ in (3.3) we get α together with the domain of x,

$$\alpha = \frac{-((b-a)p+q-p)a)+\sqrt{((b-a)p+q-p)a)^2-4(b-a)(q-p)(ap-x)}}{2(b-a)(q-p)}, ap \le x \le bq$$

and

$$\alpha = \frac{((r-q)c+(c-b)r)-\sqrt{((r-q)c+(c-b)r)^2-4(c-b)(r-q)(cr-x)}}{2(c-b)(r-q)}, bq \le x \le cr$$

which gives,

$$\mu_{XY}(x) = \begin{cases} \dfrac{-((b-a)p+q-p)a)+\sqrt{((b-a)p+q-p)a)^2-4(b-a)(q-p)(ap-x)}}{2(b-a)(q-p)}, & ap \le x \le bq \\ \dfrac{((r-q)c+(c-b)r)-\sqrt{((r-q)c+(c-b)r)^2-4(c-b)(r-q)(cr-x)}}{2(c-b)(r-q)}, & bq \le x \le cr \end{cases}$$

3.4. Division of Fuzzy Numbers:

Let X= [a, b, c] and Y= [p, q, r] be two positive fuzzy numbers. Then ${}^{\alpha}X = [(b-a)\alpha + a, c-(c-b)\alpha]$ and ${}^{\alpha}Y = [(q-p)\alpha + p, r-(r-q)\alpha]$ are the α –cuts of fuzzy numbers X and Y respectively. To calculate division of fuzzy numbers X and Y we first divide the α –cuts of X and Y using interval arithmetic.

$$\frac{{}^{\alpha}X}{{}^{\alpha}Y} = \frac{[(b-a)\alpha + a, c-(c-b)\alpha]}{[(q-p)\alpha + p, r-(r-q)\alpha]}$$

$$= \left[\frac{(b-a)\alpha + a}{r-(r-q)\alpha}, \frac{c-(c-b)\alpha}{(q-p)\alpha + p}\right] \quad \text{.....(3.4)}$$

To find the membership function $\mu_{X/Y}(x)$ we equate to x both the first and second component in (3.4) which gives

$$x = \frac{(b-a)\alpha + a}{r-(r-q)\alpha} \text{ and } x = \frac{c-(c-b)\alpha}{(q-p)\alpha + p}.$$

Now, expressing α in terms of x and setting $\alpha = 0$ and $\alpha = 1$ in (3.4) we get α together with the domain of x,

$$\alpha = \frac{xr-a}{(b-a)+(q-r)x}, a/r \le x \le b/q$$

and

$$\alpha = \frac{c-px}{(c-b)+q-p)x}, b/q \le x \le c/p$$

which gives

$$\mu_{X/Y}(x) = \begin{cases} \dfrac{xr-a}{(b-a)+(q-r)x}, & a/r \le x \le b/q \\ \dfrac{c-px}{(c-b)+q-p)x}, & b/q \le x \le c/p \end{cases}$$

3.5. Inverse of fuzzy number:

Let *X*= [a, b, c] be a positive fuzzy number. Then ${}^{\alpha}X = [(b-a)\alpha + a, c-(c-b)\alpha]$ is the α -cut of the fuzzy numbers *X*. To calculate inverse of the fuzzy number *X* we first take the inverse of the α -cut of X using interval arithmetic.

$$\frac{1}{{}^{\alpha}X} = \frac{1}{[(b-a)\alpha + a, c-(c-b)\alpha]}$$

$$= \left[\frac{1}{c-(c-b)\alpha}, \frac{1}{(b-a)\alpha + a} \right] ......(3.5)$$

To find the membership function $\mu_{1/X}(x)$ we equate to x both the first and second component in (3.5), which gives

$$x = \frac{1}{c-(c-b)\alpha} \text{ and } x = \frac{1}{(b-a)\alpha + a}$$

Now, expressing α in terms of x and setting $\alpha = 0$ and $\alpha = 1$ in (3.5) we get α together with the domain of x,

$$\alpha = \frac{cx-1}{x(c-b)}, \frac{1}{c} \le x \le \frac{1}{b} \text{ and}$$

$$\alpha = \frac{1-ax}{x(b-a)}, \frac{1}{b} \le x \le \frac{1}{a} \text{ which gives,}$$

$$\mu_{1/X}(x) = \begin{cases} \dfrac{cx-1}{x(c-b)}, \dfrac{1}{c} \le x \le \dfrac{1}{b} \\ \dfrac{1-ax}{x(b-a)}, \dfrac{1}{b} \le x \le \dfrac{1}{a} \end{cases}$$

3.6. Exponential of a Fuzzy number:

Let X= [a, b, c] > 0 be a fuzzy number. Then ${}^{\alpha}A = [(b-a)\alpha + a, c-(c-b)\alpha]$ is the α -cut of the fuzzy numbers A. To calculate exponential of the fuzzy number A we first take the exponential of the α -cut of A using interval arithmetic.

$$\exp({}^{\alpha}A) = \exp([(b-a)\alpha + a, c-(c-b)\alpha])$$

$$= [\exp((b-a)\alpha + a), \exp(c-(c-b)\alpha)]........(3.6)$$

To find the membership function $\mu_{\exp(X)}(x)$ we equate to x both the first and second component in (3.6) which gives

$$x = \exp((b-a)\alpha + a) \text{ and}$$

$$x = \exp(c-(c-b)\alpha)$$

Now, expressing α in terms of x and setting $\alpha = 0$ and $\alpha = 1$ in (3.6) we get α together with the domain of x,

$$\alpha = \frac{\ln(x)-a}{b-a}, \exp(a) \le x \le \exp(x) \text{ and}$$

$$\alpha = \frac{c-\ln(x)}{c-b}, \exp(b) \le x \le \exp(c)$$

which gives

$$\mu_{\exp(X)}(x) = \begin{cases} \dfrac{\ln(x)-a}{b-a}, \exp(a) \le x \le \exp(x) \\ \dfrac{c-\ln(x)}{c-b}, \exp(b) \le x \le \exp(c) \end{cases}$$

3.7. Logarithm of a fuzzy number:

Let X= [a, b, c] > 0 be a fuzzy number. Then ${}^{\alpha}X = [(b-a)\alpha + a, c-(c-b)\alpha]$ is the α -cut of the fuzzy number X. To calculate logarithm of the fuzzy number X we first take the logarithm of the α -cut of X using interval arithmetic.

$$\ln({}^{\alpha}X) = \ln([(b-a)\alpha + a, c-(c-b)\alpha])$$

$$= [\ln((b-a)\alpha + a), \ln(c-(c-b)\alpha)]........(3.7)$$

To find the membership function $\mu_{\ln(X)}(x)$ we equate to x both the first and second component in (3.7) which gives

$$x = \ln((b-a)\alpha + a) \text{ and}$$

$$x = \ln(c-(c-b)\alpha) \cdot$$

Now, expressing α in terms of x and setting $\alpha = 0$ and $\alpha = 1$ in (3.7) we get α together with the domain of x,

$$\alpha=\frac{\exp(x)-a}{b-a}, \ln(a)\le x\le \ln(b)$$ and

$$\alpha=\frac{c-\exp(x)}{c-b}, \ln(b)\le x\le \ln(c)$$

which produces

$$\mu_{\ln(X)}(x)=\begin{cases}\frac{\exp(x)-a}{b-a}, \ln(a)\le x\le \ln(b)\\ \frac{c-\exp(x)}{c-b}, \ln(b)\le x\le \ln(c)\end{cases}$$

4. Square root of fuzzy number by α-cut method

In this section we determine square root of a fuzzy number by α-cut cut method and compare with that done in [1] and [2].

Let X= [a, b, c] > 0 be a fuzzy number. Then ${}^{\alpha}X=[(b-a)\alpha+a, c-(c-b)\alpha]$ is the α-cut of the fuzzy numbers X. To calculate square root of the fuzzy number X we first take the square root of the α-cut of X using interval arithmetic.

$$\sqrt{{}^{\alpha}A}=\sqrt{[(b-a)\alpha+a, c-(c-b)\alpha]}$$
$$=\left[\sqrt{(b-a)\alpha+a}, \sqrt{c-(c-b)\alpha}\right]...(4)$$

To find the membership function $\mu_{\sqrt{X}}(x)$ we equate to x both the first and second component in (4), which gives

$$x=\sqrt{(b-a)\alpha+a}$$ and
$$x=\sqrt{c-(c-b)\alpha}$$

Now, expressing α in terms of x and setting $\alpha=0$ and $\alpha=1$ in (4) we get α together with the domain of x,

$$\alpha=\frac{x^2-a}{b-a}, \sqrt{a}\le x\le\sqrt{b}$$ and

$$\alpha=\frac{c-x^2}{c-b}, \sqrt{b}\le x\le\sqrt{c}$$

Which gives

$$\mu_{\sqrt{X}}(x)=\begin{cases}\frac{x^2-a}{b-a}, \sqrt{a}\le x\le\sqrt{b}\\ \frac{c-x^2}{c-b}, \sqrt{b}\le x\le\sqrt{c}\end{cases}$$

5. n^th root of a Fuzzy number:

Let X= [a, b, c] > 0 be a fuzzy number. Then ${}^{\alpha}X=[(b-a)\alpha+a, c-(c-b)\alpha]$ is the α-cut of the fuzzy numbers X. To calculate nth root of the fuzzy number *X* we first take the nth root of the α-cut of X using interval arithmetic.

$$({}^{\alpha}X)^{1/n}=([(b-a)\alpha+a, c-(c-b)\alpha])^{1/n}$$

$$=\left[((b-a)\alpha+a)^{1/n}, (c-(c-b)\alpha)^{1/n}\right]...(5)$$

To find the membership function $\mu_{\sqrt[n]{X}}(x)$ we equate to x both the first and second component in (5) which gives

$$x=((b-a)\alpha+a)^{1/n}$$ and

$$x=(c-(c-b)\alpha)^{1/n}$$

Now, expressing α in terms of x and setting $\alpha=0$ and $\alpha=1$ in (5) we get α together with the domain of x,

$$\alpha=\frac{x^n-a}{b-a}, \sqrt[n]{a}\le x\le\sqrt[n]{b}$$ and

$$\alpha=\frac{c-x^n}{c-b}, \sqrt[n]{b}\le x\le\sqrt[n]{c}$$

which gives

$$\mu_{\sqrt[n]{X}}(x)=\begin{cases}\frac{x^n-a}{b-a}, \sqrt[n]{a}\le x\le\sqrt[n]{b}\\ \frac{c-x^n}{c-b}, \sqrt[n]{b}\le x\le\sqrt[n]{c}\end{cases}$$

6. A comparison with the proposed method:

In this section we solve the same problems as done in [2] and [3] by α-cut method and hence make a comparative study.

6.1. Addition of fuzzy Numbers:

Let X= [1,2,4] and Y= [3,5,6] be two fuzzy numbers whose membership functions are

$$\mu_X(x)=\begin{cases} x-1, 1\le x\le 2 \\ \dfrac{4-x}{2}, 2\le x\le 4 \end{cases}$$

$$\mu_Y(x)=\begin{cases} \dfrac{x-3}{2}, 3\le x\le 5 \\ 6-x, 5\le x\le 6 \end{cases}$$

Then ${}^{\alpha}X=[1+\alpha,4-2\alpha]$ and ${}^{\alpha}Y=[2\alpha+3,6-\alpha]$ are the α–cuts of fuzzy numbers X and Y respectively. Therefore

$${}^{\alpha}X+{}^{\alpha}Y$$

$$=[1+\alpha,4-2\alpha]+[2\alpha+3,6-\alpha]$$

$$=[3\alpha+4,10-3\alpha]....................(6.1)$$

We take $x=3+4\alpha$ and $x=10-3\alpha$, Now, expressing α in terms of x by setting $\alpha=0$ and $\alpha=1$ in (6.1) we get α together with the domain of x,

$$\alpha=\frac{x-4}{3}, 4\le x\le 7 \text{ and}$$

$$\alpha=\frac{10-x}{3}, 7\le x\le 10$$

which gives

$$\mu_{X+Y}(x)=\begin{cases} \dfrac{x-4}{3}, 4\le x\le 7 \\ \dfrac{10-x}{3}, 7\le x\le 10 \end{cases}$$

6.2 Subtraction of Fuzzy Numbers:

Let X= [1,2,4] and Y= [3,5,6] be two fuzzy numbers. Then ${}^{\alpha}X=[1+\alpha,4-2\alpha]$ and ${}^{\alpha}Y=[2\alpha+3,6-\alpha]$ are the α–cuts of fuzzy numbers X and Y respectively. Therefore

$${}^{\alpha}X-{}^{\alpha}Y$$

$$=[1+\alpha,4-2\alpha]-[2\alpha+3,6-\alpha]$$

$$=[2\alpha-5,1-4\alpha].............(6.2)$$

we take $x=2\alpha-5$ and $x=1-4\alpha$
Now, expressing α in terms of x by setting $\alpha=0$ and $\alpha=1$ in (6.2) we get α together with the domain of x,

$$\alpha=\frac{x+5}{2}, -5\le x\le -3 \text{ and}$$

$$\alpha=\frac{1-x}{4}, -3\le x\le 1$$

which gives

$$\mu_{X-Y}(x)=\begin{cases} \dfrac{x+5}{2}, -5\le x\le -3 \\ \dfrac{1-x}{4}, -3\le x\le 1 \end{cases}$$

6.3 Multiplication of Fuzzy Numbers:

Let X= [1,2,4] and Y= [3,5,6] be two fuzzy numbers. Then ${}^{\alpha}X=[1+\alpha,4-2\alpha]$ and ${}^{\alpha}Y=[2\alpha+3,6-\alpha]$ are the α–cuts of fuzzy numbers X and Y respectively. Therefore,

$${}^{\alpha}X * {}^{\alpha}Y$$

$$=[1+\alpha,4-2\alpha]*[2\alpha+3,6-\alpha]$$

$$=[2\alpha^2+5\alpha+3,2\alpha^2-16\alpha+24].............(6.3)$$

We take
$x=2\alpha^2+5\alpha+3$ and $x=2\alpha^2-16\alpha+24$
Now, expressing α in terms of x by setting $\alpha=0$ and $\alpha=1$ in (6.3) we get α together with the domain of x,

$$\alpha=\frac{\sqrt{1+8x}-5}{4}, 3\le x\le 1 \text{ and}$$

$$\alpha=\frac{8-\sqrt{16+2x}}{2}, 10\le x\le 24$$

which gives,

$$\mu_{XY}(x)=\begin{cases} \dfrac{\sqrt{1+8x}-5}{4}, 3\le x\le 10 \\ \dfrac{8-\sqrt{16+2x}}{2}, 10\le x\le 24 \end{cases}$$

6.4 Division of Fuzzy Numbers:

Let X= [1,2,4] and Y= [3,5,6] be two fuzzy numbers. Then ${}^{\alpha}X = [1+\alpha, 4-2\alpha]$ and ${}^{\alpha}Y = [2\alpha+3, 6-\alpha]$ are the α –cuts of fuzzy numbers X and Y respectively. Therefore,

$$\frac{{}^{\alpha}X}{{}^{\alpha}Y} = \frac{[1+\alpha, 4-2\alpha]}{[2\alpha+3, 6-\alpha]}$$
$$= \left[\frac{1+\alpha}{6-\alpha}, \frac{4-2\alpha}{2\alpha+3}\right] ....(6.4)$$

We take

$$x = \frac{1+\alpha,}{6-\alpha} \text{ and } x = \frac{4-2\alpha}{2\alpha+3}.$$

Now, expressing α in terms of x by setting $\alpha = 0$ and $\alpha = 1$ in (6.4) we get α together with the domain of x,

$$\alpha = \frac{6x-1}{1+x}, \frac{1}{6} \le x \le \frac{2}{5}, \text{ and}$$

$$\alpha = \frac{4-3x}{2(1+x)}, \frac{2}{5} \le x \le \frac{4}{3}$$

which gives

$$\mu_{X/Y}(x) = \begin{cases} \frac{6x-1}{1+x}, \frac{1}{6} \le x \le \frac{2}{5} \\ \frac{4-3x}{2(1+x)}, \frac{2}{5} \le x \le \frac{4}{3} \end{cases}$$

6.5 Addition of a triangular and a non-triangular fuzzy number:

Let X=[2,3,4] and Y=[4,16,25] be triangular and non triangular fuzzy numbers respectively whose membership functions are given as,

$$\mu_Y(x) = \begin{cases} \frac{x-2}{2}, 2 \le x \le 4 \\ 5-x, 4 \le x \le 25 \\ 0, otherwise \end{cases} \text{ and}$$

$$\mu_Y(x) = \begin{cases} \frac{\sqrt{x}-2}{2}, 4 \le x \le 16 \\ 5-\sqrt{x}, 16 \le x \le 2 \\ 0, otherwise \end{cases}$$

Then ${}^{\alpha}X = [2+2\alpha, 5-\alpha]$ and ${}^{\alpha}Y = [(2+2\alpha)^2, (5-\alpha)^2]$ are the α –cuts of fuzzy numbers X and Y respectively. Therefore,

$${}^{\alpha}X + {}^{\alpha}Y$$
$$= [2+2\alpha, 5-\alpha] + [(2+2\alpha)^2, (5-\alpha)^2]$$
$$= [4\alpha^2 + 10\alpha + 6, \alpha^2 - 11\alpha + 30].....(6.5)$$

To find the membership function $\mu_{X+Y}(x)$ we equate to x both the first and second component in (6.5), which gives

$x = 4\alpha^2 + 10\alpha + 6$ and

$$x = \alpha^2 - 11\alpha + 30$$

Now, expressing α in terms of x and setting $\alpha = 0$ and $\alpha = 1$ in (6.5) we get α together with the domain of x,

$$\alpha = \frac{\sqrt{1+4x}-5}{4}, 6 \le x \le 20 \text{ and}$$
$$\alpha = \frac{11-\sqrt{1+4x}}{2}, 20 \le x \le 30$$

which gives

$$\mu_{X+Y}(x) = \begin{cases} \frac{\sqrt{1+4x}-5}{4}, 6 \le x \le 20 \\ \frac{11-\sqrt{1+4x}}{2}, 20 \le x \le 30 \\ 0, otherwise \end{cases}$$

By comparing the solutions of the above examples as done in [6] we have seen that the α –cuts method is simpler than the method proposed by them. Infact for performing division (and subtraction), their method is lengthy compared to α –cuts method, because they have to first perform inverse (negative) of the divisor (fuzzy number to be subtracted). In example 5 of [6] the result obtained should be

$$\mu_{X+Y}(x) = \begin{cases} \frac{\sqrt{1+4x}-5}{4}, 6 \le x \le 20 \\ \frac{11-\sqrt{1+4x}}{2}, 20 \le x \le 30 \\ 0, otherwise \end{cases}$$

instead of

$$\mu_{X+Y}(x) = \begin{cases} \frac{\sqrt{1+4x}-2}{4}, 6 \le x \le 20 \\ \frac{11-\sqrt{1+4x}}{2}, 20 \le x \le 30 \\ 0, otherwise \end{cases}$$

5. Conclusion:

In this paper we have shown that α-cut method is a method general enough to deal with all kinds of fuzzy arithmetic including nth root, exponentiation and taking log. We have solved problems using this method and compared them with the method in proposed [2] and [6]. We have seen that the alpha-cut method is simpler than the proposed method. As a passing remark we would like to mention that example 3 in [2] is confusing. Because there they started to extract nth root of X =[a, b, c], (a,b,c)>0 but ended up performing exponentiation of the fuzzy number X.

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