Bob, again

08 September 2021 5:14 PM

$\sigma^0 \in D(HIJPQR)$

$\sigma^1 \in D(ZJR)$

$\sigma^{junk} \in D(PQR)$

channel not^n

$\Pi \longrightarrow \Xi$

$N \longrightarrow \mathcal{N}$

$\sigma_x \longrightarrow \mathcal{X}$

$$\eta_\epsilon := \max \quad \mathrm{tr}\left(\Xi^{obj} \otimes \mathcal{I}_{JR}(\sigma^2)\right)$$

$$TD(\sigma^0, \rho^{GHZ} \otimes \sigma^{junk}) \le \epsilon$$

$$TD(\mathcal{N}^{c|z}(\sigma^0), \Xi^{c|z}(\rho^{GHZ}) \otimes \sigma^{junk}) \le \epsilon$$

$$\mathrm{tr}_Z(\sigma^1) = \mathrm{tr}_{PQHI}(\sigma^0)$$

$$\sigma^2 = \sum_{c,z} \mathcal{X}^c_c \otimes \Xi^z_Z \otimes \mathcal{N}^{c|z}(|0\rangle\langle 0|_c \otimes \sigma^1)$$

$\Rightarrow \quad TD(\sigma^0_{HIJ}, \rho^{GHZ}) \le \epsilon \quad \Rightarrow \quad TD(\underbrace{\sigma^0_{HIJ}}_{\tau_0}, \rho^{GHZ}) \le \epsilon$

$\Rightarrow \quad TD(\mathrm{tr}_{PQR}\, \mathcal{N}^{c|z}(\sigma^0), \Xi^{c|z}(\rho^{GHZ})) \le \epsilon \quad \overset{\text{Claim1}}{\Rightarrow} \quad TD\left(\mathcal{M}^{c|z}(\underbrace{\sigma^0_{HIJ}}_{\tau_0}), \Xi^{c|z}(\rho^{GHZ})\right) \le \epsilon$

$\Rightarrow \quad \mathrm{tr}_Z(\tau^1) = \mathrm{tr}_{HI}(\tau^0) \quad ; \quad \tau^1 := \mathrm{tr}_R\, \sigma^1 \in D(ZJ)$

$\Rightarrow$

$$\tau^2 = \mathrm{tr}_R\, \sigma^2$$

$$= \sum_{c,z} \mathcal{X}^c_c \otimes \Xi^z_Z \otimes \mathrm{tr}_R \circ \mathcal{N}^{c|z}(|0\rangle\langle 0|_c \otimes \sigma^1)$$

$$\overset{\text{Claim1 \& 2}}{=} \sum_{c,z} \mathcal{X}^c_C \otimes \Xi^z_Z \otimes \mathcal{M}^{c|z}(|0\rangle\langle 0|_c \otimes \underbrace{\mathrm{tr}_R(\sigma^1)}_{\tau^1})$$

$$\eta^{Tr}_\epsilon \ge \mathrm{tr}\left(\Xi^{obj} \otimes \mathcal{I}_J(\tau^2)\right)$$

$$= \mathrm{tr}\left(\Xi^{obj} \otimes \mathcal{I}_J(\mathrm{tr}_R\, \sigma^2)\right)$$

$$= \mathrm{tr}\left(\Xi^{obj} \otimes \mathcal{I}_{JR}(\sigma^2)\right) = \eta_\epsilon$$

$$\Sigma \otimes \mathrm{tr}_R\, \mathcal{N}^{c|z}(\sigma^0) \overset{\text{claim1}}{=} \Sigma \otimes \mathcal{M}^{c|z}(\mathrm{tr}_R\, \sigma^0) \qquad (*)$$

$$\Sigma \otimes \mathrm{tr}_R\, \mathcal{N}^{c|z}(\sigma^1) \overset{\text{claim2 \& }(*)}{=} \Sigma \otimes \mathcal{M}^{c|z}(\mathrm{tr}_R\, \sigma^1)$$

$\Rightarrow \quad \eta_\epsilon \le \eta^{Tr}_\epsilon$ where

$\tau^0 \in D(HIJ)$

$\tau^1 \in D(ZJ)$

$$\eta^{Tr}_\epsilon := \max \quad \mathrm{tr}\left(\Xi^{obj} \otimes \mathcal{I}_J(\tau^2)\right)$$

$$TD(\tau^0, \rho^{GHZ}) \le \epsilon$$

$$TD(\mathcal{M}^{c|z}(\tau^0), \Xi^{c|z}(\rho^{GHZ})) \le \epsilon$$

$$\mathrm{tr}_Z(\tau^1) = \mathrm{tr}_{HI}(\tau^0)$$

$$\tau^2 = \sum_{c,z} \mathcal{X}^c_c \otimes \Xi^z_Z \otimes \mathcal{M}^{c|z}(|0\rangle\langle 0|_c \otimes \tau^1)$$

$\boxed{\text{Claims}}$

trace non-inc.

Claim1: $\exists\ \mathcal{M}^{c|z}$, a CPTNI map s.t.

$$\mathcal{M}^{c|z}(\sigma^0_{HIJ}) = \mathrm{tr}_{PQR}\, \mathcal{N}^{c|z}(\sigma^0).$$

Claim 2:

if

$$\mathrm{tr}_{AB}\left[\mathcal{E}_C \otimes \mathcal{E}_A \otimes \mathbb{1}_B(\sigma_{ABC})\right] = \mathrm{tr}_{AB}\left[\mathcal{E}_C \otimes \tilde{\mathcal{F}}_A \otimes \mathbb{1}_B(\sigma_{ABC})\right]$$

then

$$\mathrm{tr}_{AB}\left[\mathcal{E}_C \otimes \mathcal{E}_A \otimes \mathbb{1}(\rho_{ABC})\right] = \mathrm{tr}\left[\mathcal{E}_C \otimes \mathcal{F}_A \otimes \mathbb{1}_B(\rho_{ABC})\right]$$

granted $\rho_A = \sigma_A$.