Indian Institute of Science Education and Research Mohali

First Mid Semester Examination

September 4 2012 **Reg. No.** ....

MTH201 (Curves and Surfaces) **Maximum Marks: 20**

Instructions: Attempt **ALL** questions. Read the questions carefully. Write all arguments precisely and do not leave anything to the instructor's imagination.

1. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be the function defined by $f(x, y) = (\sin x \cos y, x^2 - y)$. Determine if the Jacobian $Df_{(0,0)}$ of f at $(0, 0)$ is invertible. Find a point in $\mathbb{R}^2$ where the Jacobian of f is not invertible. [2 + 2]

2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by [4]
$$f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{otherwise.} \end{cases}$$
Determine if f is continuous at $(0, 0)$.

3. Let $\gamma : (-1, 1) \to \mathbb{R}^3$ be the parametric curve given by [1 + 2]
$$\gamma(t) = \left(\frac{1}{3}(1+t)^{3/2}, \frac{1}{3}(1-t)^{3/2}, \frac{1}{\sqrt{2}}t\right).$$
   (a) Does γ pass through $(0, 0, 0) \in \mathbb{R}^3$? Justify.
   (b) Is γ a unit speed curve? Justify.

4. Compute the arc length function s of the *logarithmic spiral* $\gamma : (0, \infty) \to \mathbb{R}^2$ given by
$$\gamma(t) = (ae^{bt} \cos t, ae^{bt} \sin t),$$
[4 + 1]
where a and b are positive real constants. Further show that $b = \log_e \left(\frac{s(2)}{s(1)} - 1\right)$.

5. Let $\gamma : (\alpha, \beta) \to \mathbb{R}^3$ be a smooth curve with unit speed. Let $\dot{\gamma}$ denote the derivative and $\ddot{\gamma}$ denote the double derivative of γ with respect to t. Show that for each $t \in (\alpha, \beta)$, the vectors $\dot{\gamma}(t)$ and $\ddot{\gamma}(t)$ are orthogonal to each other. [4]
