Curves and Surfaces (MTH201)
Academic Session 2012-13

Tutorial Sheet 10   **November 23 2012**

Instructions: Write main ideas / hints for solving questions in your tutorial noteook. There is no need to write full and formal solution during the tutorial session. However during off class hours you should practice writing these solutions in a formal manner. **Get the signature of your tutor after each session**.

1. Compute the Gaussian curvature and mean curvature of the *catenoid* $\sigma(u, \theta) = (\cosh u \cos \theta, \cosh u \sin \theta, u)$. (Observe minimal surfaces have non-positive Gaussian curvature. When is Gaussian curvature zero for minimal surfaces?)

2. Compute the Gaussian curvature of *helicoid* $\sigma(\theta, v) = (v \cos \theta, v \sin \theta, \theta)$.

3. Using the description of Gaussian curvature in terms of limits of ratio of areas show that

   (a) Gaussian curvature of a sphere of radius a is $\dfrac{1}{a^2}$.

   (b) Gaussian curvature of a cone is zero.

4. The mean curvature of a surface is 1 everywhere. Show that Gaussian curvature will not exceed 1.

5. For the surface given by the surface patch $\sigma(u, v) = (u, v, u^2 + v^2)$ determine the following:

   (a) Standard unit normal.

   (b) The first and second fundamental forms $\mathcal{F}_I$ and $\mathcal{F}_{II}$.

   (c) Gaussian curvature.

   (d) The point of maximum Gaussian curvature.

   (e) Mean curvature.

   (f) Two principal curvatures.

   (g) Elliptic points, parabolic points, hyperbolic points and flat points.