Indian Institute of Science Education and Research Mohali

Curves and Surfaces (MTH201)
Academic Session 2012-13

Tutorial Sheet 9 **November 16 2012**

Instructions: Write main ideas / hints for solving questions in your tutorial noteook. There is no need to write full and formal solution during the tutorial session. However during off class hours you should practice writing these solutions in a formal manner. **Get the signature of your tutor after each session**.

1. With your knowledge of principal curvatures, argue that one can not draw straight lines on a sphere. (Have you ever seen a proof of this 'obvious' fact?)

2. Find examples of surfaces for which $\kappa_1 = \kappa_2$ at every point.

3. Find example of a surface for which $\kappa_1 = 0$ and κ_2 assumes all positive real values.

4. Compute the Gaussian curvature of the *torus* given by

$$\sigma(\phi, \theta) = ((R + r\cos\phi)\cos\theta, (R + r\cos\phi)\sin\theta, r\sin\phi),$$

where $0 < r < R$. Can you identify points on a torus which have negative Gaussian curvature? Identify the two principal vectors at $\phi = \frac{\pi}{2} = \theta$.

5. A *helicoid* is the surface given by the surface patch:

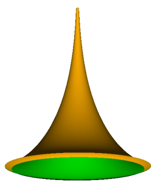
$$\sigma(\theta, v) = (v\cos\theta, v\sin\theta, a\theta).$$

Show that at every point of the helicoid one has $\kappa_1 + \kappa_2 = 0$.

6. A *pseudosphere* is the surface obtained by revolving a tractrix

$$\gamma(u) = \left(u, 0, \sqrt{1-u^2} - \ln\frac{1+\sqrt{1-u^2}}{u}\right)$$

about z-axis. Show that at every point the Gaussian curvature of a pseudosphere is -1.



A pseudosphere
