Olly # 7.1

Higgs Effect

SSB: $U|0\rangle \neq |0\rangle$

$e^{i T^a \theta^a}$

$\Phi(x) = e^{\frac{i T^{\bar a} \chi_{\bar a}(x)}{v}} (\phi_0 + \rho)$

SSB: $T^{\bar a}|0\rangle \neq 0$, $\bar a = 1 \ldots N$ broken generators

Local $\theta^a \to \theta^a(x)$

A^a_μ

$U(1) \approx O(2)$

$\mathcal{L} = D_\mu \phi^* D^\mu \phi - \frac{1}{4} F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{1}\left(\phi^*\phi - \frac{v^2}{2}\right)^2$

$\mathcal{M}_v = \phi^*\phi - \frac{v^2}{2}$

$\phi = e^{\frac{i\chi(x)}{v}}\left(\frac{\hat\rho + v}{\sqrt2}\right)$ | Unitarity gauge

$\mathcal{L}_{Higgs} =$

(copy, write on your own)

Vector Propagator

$\mathcal{L} \sim -\frac{1}{2}\partial_\mu A_\nu(\partial^\mu A^\nu - \partial^\nu A^\mu) + \frac{m_A^2}{2} A_\mu A^\mu$

$= \frac{1}{2} A_\nu(\partial^2 A^\nu - \partial_\mu \partial^\nu A^\mu) + \frac{m_A^2}{2} A_\mu A^\mu$

$= \frac{1}{2} A_\nu\left[(\partial^2 + m_A^2)\eta^{\mu\nu} - \partial^\mu \partial^\nu\right] A_\mu$

claim: $i\Delta^{\text{unitarity}}_{\mu\nu}(k) = \frac{-i}{k^2 - m_A^2 + i\epsilon}\left(\eta_{\mu\nu} - \frac{k_\mu k_\nu}{m_A^2}\right)$

$= \underbrace{\langle 0|T(A_\mu A_\nu)|0\rangle}_{\text{Fourier Transform}}$

Issue: as $k \to \infty$, the prop may cause issues

R_ξ gauge: $\phi = \frac{\phi_1 + i\phi_2}{\sqrt2}$ $\quad \langle\phi_1\rangle = v$, ϕ_2 = GB

$\mathcal{M}_v$: $\phi_1^2 + \phi_2^2 = \frac{v^2}{2}$

$|D_\mu\phi|^2 = \left|\frac{\partial_\mu\hat\phi_1 + i\partial_\mu\hat\phi_2}{\sqrt2} + ig\left(\frac{v + \hat\phi_1 + i\hat\phi_2}{\sqrt2}\right)A_\mu\right|^2$

$= \frac{1}{2}\left[(\partial_\mu\hat\phi_1 - g\hat\phi_2 A_\mu)^2 + (\partial_\mu\hat\phi_2 + g(v + \hat\phi_1)A_\mu)^2\right]$

$= \frac{1}{2}\left\{(\partial_\mu\hat\phi_1)^2 - 2g\hat\phi_2 A_\mu \partial^\mu\hat\phi_1 + g^2\hat\phi_2^2 A_\mu A^\mu + (\partial_\mu\hat\phi_2)^2 + g^2 v^2\left(1 + \frac{\hat\phi_1}{v}\right)^2 A_\mu A^\mu + 2g\partial_\mu\hat\phi_2 v\left(1 + \frac{\hat\phi_1}{v}\right)A^\mu\right\}$

mixes A_μ & $\hat\phi_2$ — goldstone Bosons

$\mathcal{L}_{gauge\ fix} = \frac{-1}{2\xi}(\partial_\mu A^\mu + c\hat\phi_2)^2 = \frac{-1}{2\xi}\left((\partial_\mu A^\mu)^2 + 2c\,\partial_\mu A^\mu\hat\phi_2 + c^2\hat\phi_2^2\right)$ $\quad c = -gv\xi = -m_A\xi$

can't justify at this stage

BP: $-2c\,A^\mu\partial_\mu\hat\phi_2$

$\mathcal{L}_{Gauge\ fix} = \frac{-1}{2\xi}(\partial_\mu A^\mu)^2 - \frac{\xi}{2} m_A^2\hat\phi_2^2 + m_A\hat\phi_2\partial_\mu A^\mu$

Claim:

$i D^{(A)}_{\mu\nu}(p) = \frac{-i}{p^2 - m_A^2 + i\epsilon}\left\{\eta_{\mu\nu} + \frac{(\xi - 1)p_\mu p_\nu}{p^2 - \xi m_A^2}\right\}$ — Better high energy behaviours

With $\xi \to \infty$, $\to 0$ $\qquad \downarrow \frac{-p_\mu p_\nu}{m_A^2}$

$i\Delta^{\hat\phi_2}(p) = \frac{i}{p^2 - \xi m_A^2}$

$i\Delta^{\hat\phi_1}(p) = \frac{i}{p^2 - m_{\phi_1}^2}$

$\xi = 1$, $D_{\mu\nu} \sim \frac{-i\eta_{\mu\nu}}{p^2 - m_A^2 + i\epsilon}$

$\xi = 0$, $D_{\mu\nu} \sim \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$

$\mathcal{L}_{U(1)+SSB} = -\frac{1}{4}F^2 + \frac{m_A^2}{2}A_\mu A^\mu + m_A(\partial_\mu\hat\phi_2)A^\mu + (\partial_\mu\hat\phi_2)^2 \ldots$

Interactions

$i m_A^2 \eta_{\mu\nu}$

p $\quad (m_A p_\mu)$

$e^{-ip\cdot x}\hat\phi_2$

$\Pi_{\mu\nu}(p)$

$\Pi_{\mu\nu} =$ + $\quad \mu \quad m_A p_\mu \quad \to(-m_A p_\nu) \quad \nu$

$= m_A^2\left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$

& $p_\mu \Pi^{\mu\nu} = 0$

(respects Ward Identity)

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Weinberg Salam Model of Weak Interactions

1932: Fermi theory of β decay ($n \to p + e^- + \bar{\nu}$) $\hookrightarrow$ Pauli ~1930

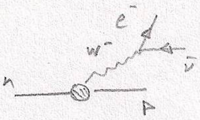
1950: QED

1955: Marshak, Sudarshan, Feynman, Gell Mann

$$\mathcal{L}_F = G_F \sum \bar{\Psi}_1 (g_V - g_A \gamma_5)\gamma^\mu \Psi_2 \; \bar{\Psi}_3 (1-\gamma_5)\gamma^\mu \Psi_4 \quad (\because \tfrac{g_V}{g_A} \sim 1)$$

$$\mathcal{L}_F = \sum_A G_A \bar{\Psi}_1 \Gamma_A \Psi_2 \bar{\Psi}_3 \Gamma_A \Psi_4 \qquad [G] = -2, \quad \Psi \to 3/2$$

Γ_A: 1, γ_5, γ_μ, $\gamma_5\gamma_\mu$, $\sigma_{\mu\nu}$



1960: Glashow (PhD) Intermediate Vector Boson Hypothesis $\quad \mu^\pm \to e^\pm + \overset{(-)}{\nu}_e + \overset{(-)}{\nu}_\mu$

1967, 68: Weinberg Salam, Georgi Glashow

$SU(2)$: 3 generators $\quad W^+, W^-, \gamma$
$\; \simeq SO(3) \qquad T^+ \quad T^- \quad T_3$

$$\left\{\begin{pmatrix} e^- \\ \mu^+ \\ \nu \end{pmatrix}\right\} \begin{matrix} -1 \\ +1 \\ 0 \end{matrix}$$

$$\langle \Phi \rangle = \left\langle \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \right\rangle = \begin{pmatrix} 0 \\ 0 \\ v \end{pmatrix} \to \text{would be massive "short ranged"}$$

didn't work

$SU(2) \times U(1)$: $W^\pm, W^3, B \xrightarrow{SSB} W^\pm, \gamma \,|\, Z$

This worked

Mass of Gauge Bosons

$\boxed{SU(2)\times U(1) \quad \Phi = \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix} \neq \Phi^*}$

$$\mathcal{L} = -\frac{1}{4}(W^2 + B^2) + |D_\mu \Phi|^2 - \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2 + \bar{L}_L i \not{D} L_L + \bar{e}_R i \not{D} e_R + (h_e \bar{L}_L \Phi e_R + h.c.)$$

$W^2 = W^a_{\mu\nu} W_a^{\mu\nu}$, $\quad B^2 = B_{\mu\nu}B^{\mu\nu}$, $\quad B_{\mu\nu} = \partial_{[\mu} B_{\nu]}$

$$W^a_{\mu\nu} = \partial_{[\mu} W^a_{\nu]} - \epsilon_{abc} W^b_\mu W^c_\nu$$

$$\boxed{D_\mu \Phi = \partial_\mu \Phi + i g T^a W^a_\mu \Phi + i g' \left(\frac{Y_\Phi}{2}\right) B_\mu \Phi} \quad \frac{Y_\Phi}{2} \to \text{Hypercharge}$$

Generators : $T^a = \dfrac{\sigma^a}{2}$

$$\mathbb{1}_2 \langle \Phi \rangle = \langle \Phi \rangle \neq 0$$

$$\frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\langle \Phi \rangle = \frac{1}{2}\frac{v}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \neq 0$$

$$\frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\langle \Phi \rangle = -\frac{1}{2}\frac{iv}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \neq 0$$

$$\frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} = \frac{1}{2}$$

Gauge Boson Mass Terms:

$$\mathcal{L}_{mass} = |D_\mu \langle\Phi\rangle|^2 = \left|\left(\partial_\mu + \frac{ig}{2} T^a W^a_\mu + \frac{ig'}{2} \mathbb{1} B_\mu\right)\begin{pmatrix} 0 \\ \frac{v}{2} \end{pmatrix}\right|^2 = \left|\frac{i}{2\sqrt{2}}\begin{pmatrix} g W_3 + g'B & g\left(\frac{W^1 - iW^2}{\sqrt{2}}\right)\sqrt{2} \\ g\left(\frac{W^1 + iW^2}{\sqrt{2}}\right)\sqrt{2} & (-g W_3 + g'B) \end{pmatrix}\begin{pmatrix} 0 \\ v \end{pmatrix}\right|^2$$

with $\frac{W^1 - iW^2}{\sqrt{2}} = W^+$, $\frac{W^1 + iW^2}{\sqrt{2}} = W^-$

$$= \frac{v^2}{8}\left[g^2(\sqrt{2} W_{+\mu})^2 + \left(\frac{-g W_{3\mu} + g' B_\mu}{\sqrt{g^2 + g'^2}}\right)^2 g_Z^2\right] = M_W^2 W_{+\mu} W^{-\mu} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu \quad \text{where}$$

to normalize; cancel, thus can be added

$$g_Z \equiv \sqrt{g^2 + g'^2}, \quad M_W^2 = \frac{g^2 v^2}{4}, \quad M_Z^2 = \frac{g_Z^2 v^2}{4}$$

Redef^n:

$$\boxed{\frac{g'}{g_Z} = \frac{g'}{\sqrt{g'^2 + g^2}} = \sin\theta_{\text{weinberg}}; \quad \frac{g}{g_Z} \equiv \cos\theta; \quad \tan\theta_W = \frac{g'}{g}}$$

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} W^3 \\ B \end{pmatrix}$$

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix}\begin{pmatrix} W_3 \\ B \end{pmatrix}, \qquad \begin{pmatrix} W_3 \\ B \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix}\begin{pmatrix} Z \\ A \end{pmatrix}$$

$$W^\pm = \frac{W^1 \mp W^2}{\sqrt{2}}, \quad W^1 = \frac{W^+ + W^-}{\sqrt{2}}, \quad W^2 = \frac{W^+ - W^-}{\sqrt{2}\,i}$$

So now
$$D_\mu = \partial_\mu + \frac{ig}{2}(W^1_\mu T^1 + W^2_\mu T^2 + W^3_\mu T^3) + \frac{ig}{2} Y B_\mu$$

with $W^1 \to \left(\frac{W^+ + W^-}{\sqrt{2}}\right)$, $W^2 \to \left(\frac{W^+ - W^-}{\sqrt{2}\,i}\right)$, $W^3 \to (c_W Z + s_W A)$, $B \to (-s_W Z + c A)$

$$= \partial_\mu + \frac{ig}{\sqrt{2}}\left(W^+_\mu \cancel{(T^1 + iT^2)} T^+ + W^-_\mu \cancel{(T^1 - iT^2)} T^-\right) + \frac{i}{2}(g c_W T^3 - g' s_W Y) Z_\mu + i\left(\underbrace{g s_W}_{\frac{gg'}{g_Z}} \frac{T^3}{2} + \underbrace{g' c_W}_{\frac{g'g}{g_Z}} \frac{Y}{2}\right) A_\mu$$

$$= \partial_\mu + \frac{ig}{\sqrt{2}}(W^+_\mu T_+ + W^-_\mu T_+) + i\left(\frac{gg'}{g_Z}\right)\left(T_3 + \frac{Y}{2}\right) A_\mu + i(g c_W T^3 - g' s_W (Q_{em} - T_3)) Z_\mu$$

$$= \partial_\mu + \frac{ig}{\sqrt{2}}(W^+ T_+ + W^- T_-) + ieQA_\mu + \underbrace{i g_Z \Big(\big(\underbrace{\frac{g c_W + g' s_W}{g_Z}}\big) T_3 - \frac{g'}{g_Z} s_W Q_{em}\Big) Z_\mu}_{c_W^2 + s_W^2 = 1}$$

$$= i g_Z Z_\mu (T_Z)$$

$$T_Z = T_3 - Q_{em}\sin^2\theta_W$$

$$Q_{em} \equiv T_3 + \frac{Y}{2}$$

$$e = g \sin\theta_W$$

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$D_\mu^{EW} = \partial_\mu + \frac{ig}{\sqrt{2}}(W_{+\mu}T^+ + W_{-\mu}T^-) + ig_Z Z_\mu T_Z + ieA_\mu Q_{em}$

Mass-Eigenstates $W^\pm, A, Z$

$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -s_{\theta_W} \\ c_W & s_W \end{pmatrix}\begin{pmatrix} W^3 \\ B \end{pmatrix}$

$M_W = \frac{gV}{2}$ $\quad \langle \Phi \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$

$M_Z = \frac{g_Z V}{2}$ $\quad \tan\theta_W = \frac{g'}{g}$

$e = g\sin\theta_W$

Ex $\mathcal{L}_{gauge} = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$

$= -\frac{1}{4}A_{\mu\nu}A^{\mu\nu} - \frac{1}{4}\partial_{[\mu}Z_{\nu]}\partial^{[\mu}Z^{\nu]} - (\#)\partial_{[\mu}W_{\nu]}\partial^{\mu}W \ldots$

W^3, B; $t_3 = 0$, $B = 0$ $\quad Q(Z) = 0$, $Q = \pm 1$

$\Phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$

	T_{3L}	$Y/2$	Q_{em}
ϕ^1	$1/2$	$1/2$	1
ϕ^2	$-1/2$	$1/2$	0

<some stuff missing :->

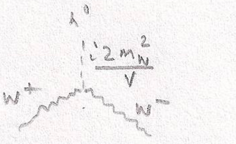
Unitarity gauge $\quad \phi^1 = 0$, $\phi^2 = \frac{v + h_0}{\sqrt{2}}$

$D_\mu\Phi = \partial_\mu\left(\frac{h_0}{\sqrt{2}} + \frac{igW_{+\mu}}{\sqrt{2}}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + \frac{ig}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}W_\mu^- + ig_Z(\overset{''}{T_{3L}}{}^{-1/2} - Q_{em}s_W^2)Z_\mu + ieQ_{em}A_\mu\right)\begin{pmatrix} 0 \\ \frac{v + h_0}{\sqrt{2}} \end{pmatrix}$

$\Rightarrow (D_\mu\Phi)_{unitarity} = \partial_\mu\frac{\begin{pmatrix} 0 \\ h_0 \end{pmatrix}}{\sqrt{2}} + \frac{ig}{2}W_+\begin{pmatrix} v + h_0 \\ 0 \end{pmatrix} - \frac{ig_Z}{2}Z_\mu\begin{pmatrix} 0 \\ \frac{v + h_0}{\sqrt{2}} \end{pmatrix}$

$= \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{igW_+(v + h_0)}{\sqrt{2}} \\ (\partial_\mu h_0 - \frac{ig_Z Z_\mu}{2})(v + h_0) \end{pmatrix}$

$D_\mu\Phi D^\mu\Phi = \overbrace{\frac{1}{2}\frac{g^2v^2}{2}}^{m_W^2}W^+W^-\underbrace{\left(1 + \frac{h_0}{v}\right)^2}_{\left(1 + \frac{2h_0}{v} + \frac{h_0^2}{v}\right)} + \frac{1}{2}\left(\partial_\mu h_0 - \frac{ig_Z vZ_\mu}{2}\left(1 + \frac{h_0}{v}\right)\right)^2$



Effective interaction of Fermion Currents

$\mathcal{L}_{fermion} = \sum_i \bar{F}_i i\not{\partial} F_i$ $\quad Q_{em} = T_{3L} + \frac{Y}{2}$ ($SU(2)_L \times U(1)$)

Single Generation: Leptons : $L_L \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} (2, -1)$ → doublet, $e_R\,(1, -2)$ → singlet ($T_{3L} = 0$, $Y = 2$), $\nu_R\,(1, 0)$ doesn't couple

$T_{3L} = 1/2$, $T_{3L} = -1/2$ $\quad$ No gauge interaction Sterile

$\begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix}$ $\quad \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}s_W^2$

$= \bar{L}\left(i\not{\partial} + \frac{ig}{\sqrt{2}}W^+T^+ + \frac{ig}{\sqrt{2}}W^-T^- + ig_Z\overset{''}{T_Z}Z + ie\overset{''}{Q_{em}}A\right)L$

$+ \bar{e}_R\left(i\not{\partial} + 0 + 0 + ig_Z(-s_W^2(-1)) + ie(-1)\not{A}\right)e_R$

$= \bar{L}i\not{\partial}L + \bar{e}_R i\not{\partial}e_R + i\left\{\frac{igW_\mu^+}{\sqrt{2}}\bar{L}T^+\gamma^\mu L + ig_Z\bar{L}_L T_Z\gamma^\mu L_L - ieA_\mu\bar{e}\gamma^\mu e + ig_Z Z_\mu\bar{e}_R\gamma^\mu T_Z e_R\right\}$ (both e_R & e_L)

$\mathcal{L}_{fermion} = \mathcal{L}_{kinetic} - eA_\mu \underset{\overset{|||}{q_i\bar{f}_i\gamma^\mu f_i}}{J^{\mu\,em}} - \frac{g}{\sqrt{2}}\left(W^+_\mu J^{(cc)}_{+\mu} + W^-_\mu J^{(cc)}_{-\mu}\right) - g_Z Z_\mu J^{(NC)\mu}$ (charged current)

$J_{+\mu} = \bar{L}_L\gamma^\mu T^+ L_L$

$= (\bar{\nu}_{eL}, \bar{e}_L)\gamma^\mu\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$

$= \bar{\nu}_{eL}\gamma^\mu e_L$

$J_{-\mu} = \bar{e}_L\gamma^\mu\nu_{eL}$

Remark: charge is indeed correct so that charge is conserved at vertices. ASK

Low Energy

Effective interaction of fermions for $q^2 << M^2_{W,Z}$

"Tree Level"

Effective Interaction: Solve EOM in approx $|\partial Heavy| << |M_H H|$ (low momentum) for W & $\mathcal{L} = m_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}m_Z^2 Z_\mu Z^\mu$

$- \frac{g}{\sqrt{2}}\left(W^+_\mu J_{+\mu} + W^-_\mu J_{-\mu}\right) - g_Z Z_\mu J^{(NC)\mu}$

+ (term dependent on momentum) + cubic & higher term

EOM: $\delta S = 0$

$\Rightarrow m_Z^2 Z_\mu - g_Z J_\mu^{NC} + \text{corr} = 0$ $\quad Z_\mu = \frac{g_Z J_\mu^{NC}}{m_Z^2}$ $\quad W^- = \frac{g}{\sqrt{2}}\frac{J_{+\mu}}{m_W^2}$

dimension

Claim: put Z_μ in the Lag & you'll see Cubic & Higher terms are supressed more by the mass

$\mathcal{L}_{eff} = \frac{1}{2}m_Z^2\left(\frac{g_Z J^{NC}}{m_Z^2}\right)^2 - \frac{g_Z^2(J^{NC})^2}{2m_Z^2} - \frac{g^2}{2m_W^2}J_{+\mu}J_-^\mu$

$= \frac{-g^2}{2m_W^2}J^{cc}_{+\mu}J_-^{cc\,\mu} - \frac{g_Z^2}{2m_Z^2}J_\mu^{NC}J^{NC\mu} + \text{small}$