Dirac and Majorana Mass

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Abstract

The concept of mass is intuitively linked with inertia. It is first encountered in a quantitatively precise formulation, in the work of Newton. With Einstein's special relativity, it acquires a new meaning ($p^2 = m^2$) (and ofcourse again, with identification of gravitational mass and inertial mass; general relativity). In particle physics (the study of 'fundamental' particles) historically, it has been hard and consequently interesting to formulate a consistent theory of particles interacting in the way we know they do from experiments, and implement their masses, taking inspiration from one of the most successful theory of physics, Quantum Electrodynamics (quantum generalization of classical electrodynamics). Assuming the knowledge of STR and QM, I'll motivate the concept of fields and particles and briefly describe how it all fits to describe fundamental particles. Thereafter, I'll take the example of a Klein Gordan Field (obtained by replacing E with $-i\hbar\frac{\partial}{\partial t}$ and $\vec{p}$ with $i\hbar\vec{\nabla}$ in $p^2 = m^2$) to explain the origin and definition of mass. Then I'll move to the main topic, that of Dirac and Majorana Mass, starting with the Dirac Equation. I'll describe the physical significance of this investigation, viz. why it is a candidate in describing the mass of neutrinos (a particle which was suspected to exist, assuming conservation of energy holds, by observing the spectrum of beta decay) and hint on alternatives.

1 Prerequisites

From CM,

- Definition of L, the Lagrangian

- Euler Lagrange equations and their derivation

- Noether's Theorem that relates conserved quantities and continuous symmetries of the Lagrangian

From STR, I'll use the following conventions

- $\eta^{\mu\nu} = \text{diag}(1, -\vec{1})$ (NB: $\eta^T = \eta^{-1} = \eta$)

- $c = 1$, $\hbar = 1$

- indices

- i, j, k, l etc. run from 1 to 3

- α, β, γ etc. run from 0 to 3

and need the four vector notation. Recall

- the Einstein summing convention, $A^\alpha B_\alpha = \sum_{\alpha=0}^{4} A^\alpha B_\alpha$ (contractions)

- if $A^\alpha = (A^0, \vec{A})$ then $A_\alpha \equiv \eta_{\alpha\beta} A^\beta = (A^0, -\vec{A})$

- Under a Lorentz transformation λ_β^α, $A^\alpha \to A'^\alpha = \lambda_\beta^\alpha A^\beta$; objects with all indices contracted are invariant under lorentz transformations (NB: $\lambda^T \eta \lambda = \eta$)

From QM, I'll need the postulates. Recall

- System state: State of a system is described by a ket (or a wavefunction) $|\psi\rangle$ (or $\psi(x) = \langle x|\psi\rangle$)

- Time evolution: Given a Hermitian operator H (viz. $H^\dagger = H$; where $H^\dagger \equiv H^{*T}$),

$$H |\psi\rangle = -i\hbar \frac{\partial}{\partial t} |\psi\rangle$$

When H has no time dependence, then $|\psi(t)\rangle = e^{(-i\hbar)^{-1} Ht} |\psi(0)\rangle$. NB: $U \equiv e^{(-i\hbar)^{-1} Ht}$, is unitary, viz. $U^\dagger = U^{-1}$ (OR, TODO: more generally, a unitary operator)

- Measurement/Observables:

  - Upon being measured, the system collapses into one of the eigenstates of the operator corresponding to the measurement

  - The probability that the system will collapse to the state $|n\rangle$, is given by $|\langle n|\psi\rangle|^2$, where $|n\rangle$ is the nth eigenstate of the system.

and I'll need the basics of the quantum harmonic oscillator developed using the a and $a^\dagger$ notation.

I'll further use the usual conventions for the pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Terminology from particle physics maybe helpful

- Leptons [TODO: Complete this part]

- Quarks

2 Towards a quantum theory of fields

2.1 Motivation

One can ask for a theory that links QM and STR, solely on grounds that QM is built on top of classical mechanics which we know is bound to fail when we goto higher speeds (thus energies). Practically though that's not such a big issue. However, when one looks at collision of particles, but can't make any predictions about the products, except being able to veto products on whose mass is greater than the centre of mass energy (solely on the energy momentum relation based on STR), there's a real crisis. The idea is to build a theory that can automatically handle the creation and destruction of particles in a way that's fully consistent with STR and at the same time, predict the results of colliding high energy particles.

2.2 Known Issues

One may guess that instead of writing $H_{\text{classical}}\psi = -i\hbar\frac{\partial}{\partial t}\psi$, if we write the relativistic version as $\left(E^2 - \vec{p}^2\right)\psi = m^2\psi$ and replace $E \to -i\frac{\partial}{\partial t}$, $\vec{p} \to i\vec{\nabla}$, we get

$$\Box\psi = m^2\psi$$

where $\Box = \frac{\partial}{\partial t}^2 - \vec{\nabla}^2$, it might just solve the problem. As it turns out, this attempt fails miserably, leading to issues in causality (events in the past are affected by the future), negative energies (this means there's no energy minima, which implies there's no stable ground state) among others.

One can actually predict that things will get inconsistent simply by observing that in QM, time and position are treated very differently; one is a parameter, the other's an operator. STR seamlessly mixes the two. This meant that we need to either make time an operator (which again is known to not work) or demote position from an operator to a parameter. All these hint to interpreting the object ψ differently.

2.3 Concept of field

This idea almost sounds like it's been taken from a fairy tale, even though we're familiar with a similar notion. It is that of a field. Fields were first encountered in the context of electromagnetism. In the beginning, we use it as a device for calculating forces, and once we reach the Maxwell equations, we find that these fields can exist without sources that we thought they were an artifact of, to start with.

In this context, there's a field for every fundamental[1] particle. This field can create particles out of thin air and destroy them too. These fields can talk to each other and consequently the particles can interact. If we can construct a framework in which if we write a relativistically covariant (or invariant?) Lagrangian describing the fields and their interaction, it produces for us probabilities of all possible

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[1] I know I haven't defined the word 'fundamental' yet, but its intent and meaning will become obvious as we proceed.

processes that involve these particles, then, then that does it in principle. It'll only remain to write the right Lagrangian (which has taken years to pin down exactly and is known as the Standard Model[2])

2.4 Framework: QFT

Let us start with a classical field. Consider the simplest case, a real scalar field. Imagine a number being assigned to every location in space and time. If I demand that the field must satisfy the Klien Gordan equation[3], I can write a Lagrangian[4]

$$\mathcal{L} = \frac{1}{2} \left(\partial^\mu \phi \partial_\mu \phi + m^2 \phi \right)$$

by noting that the equation of motion produced by this $\mathcal{L}$ is the Klien Gordan equation itself. Note that in general, $\phi = \phi(t, \vec{x})$. If I further assume that it is sufficiently well behaved, I can write

$$\phi = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2\omega_p}} \left(a e^{i\mathbf{px}} + a^\dagger e^{-i\mathbf{px}} \right)$$

with $a = a(\vec{p})$ and the second term added explicitly to ensure ϕ is real[5]. I can also evaluate π, the conjugate momentum from $\mathcal{L}$. If I impose now, the commutation (equal time) $[\phi(t, \mathbf{x}), \pi(t, \mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}')$, (note, we just converted our classical field to a quantum field) it can be shown that $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$. Further, it can be shown that the Hamiltonian for the given Lagrangian, turns out to be

$$H \sim a^\dagger a + \frac{1}{2} [a(\mathbf{p}), a^\dagger(\mathbf{p})]$$

This now has striking similarities to the quantum harmonic oscillator. This motivates existence of states such as $a^\dagger(\vec{p}) |\text{vacuum}\rangle$ which can be rigerously shown to be particles indeed. From the space-time translation invariance, one can arrive[6], at the physical momentum and energy operators. If the aforesaid state indeed represents a particle, it must satisfy $E^2 - \vec{p}^2 = m^2$. And infact, it so turns out that applying the momentum operator and the hamiltonian operator and finding the eigenvalues of this state produces exactly that. Thus we conclude that this state indeed represents a particle that has a definite energy and momentum. Further, we now know that the parameter m that appeared in the Lagrangian was indeed the mass.

Obviously, the example we took had only one field which interacts with nothing. Any observable field must interact to be observable. QFT is developed then as a perturbation theory, expanded around the non-interacting part, such as the one just described. This results in what are known as Feynman rules. These rules can be used to evaluate quantities such as decay rates and scattering cross sections, which can be measured experimentally.

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[2]Except ofcourse neutrino masses and oscillations
[3]where note now that m is only a parameter to start with
[4]density to be precise
[5]$\frac{d^3 p}{(2\pi)^3 \sqrt{2\omega_p}}$ is a Lorentz invariant measure which I won't discuss here
[6]using Noether's theorem

In the discussion of mass, it must be mentioned that when interactions are introduced, the term m in the Lagrangian, no longer represents mass. In that case, it is defined as the pole of the 'full propogator'. I'll leave it at that.

3   Dirac and Majorana Mass

3.1   The Dirac Equation

Let us now consider the famous Dirac equation and the particles its field produces. I must begin with motivating the Dirac equation, because it can be overwhelming at first. The idea was to factor the Klien Gordan equation. So I start with writing an equation of the form

$$(i\partial_\mu \gamma^\mu - m)\psi = 0$$

where γ^μ are to start with, some parameters (we think of them as numbers to start with). Now if I multiply this equation with itself (excluding the ψ part), I'll have

$$(i\partial_\mu \gamma^\mu - m)(i\partial_\nu \gamma^\nu - m)$$
$$=(-\partial_\mu \partial_\nu \gamma^\mu \gamma^\nu - im\partial_\mu \gamma^\mu) - (im\partial_\nu \gamma^\nu - m^2)$$
$$=(-\frac{1}{2}\partial_\mu \partial_\nu \{\gamma^\mu, \gamma^\nu\} - 2m\left[i(\partial_\mu \gamma^\mu)\right] + m^2)$$
$$=(-\frac{1}{2}\partial_\mu \partial_\nu \{\gamma^\mu, \gamma^\nu\} - 2m^2 + m^2)$$
$$=(-\frac{1}{2}\partial_\mu \partial_\nu \{\gamma^\mu, \gamma^\nu\} - m^2)$$

Now if I want this to become Klien Gordan, it's trivial to conclude that $\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu}$. Now without getting into more detail, I'll just claim that the parameters γ^μ are chosen to be 4×4 matrices and ψ then is a $4-$component object, called a Dirac spinor. It is useful to work in a specific basis (choice of γ) which for this discussion I'll take out of a hat to be

$$\sigma^\mu \equiv (1, \tilde{\sigma}) \qquad \overline{\sigma}^\mu \equiv (1, -\vec{\sigma})$$
$$\gamma^\mu \equiv \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}$$

One can check that the anti-commutation is indeed what we wanted.

3.2 The Dirac Lagrangian

ψ has interesting transformation properties which I won't derive but state as and when I need them. I claim that if I write my four component ψ as

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

then under a lorentz boost β and a rotation θ,

$$\psi_L \rightarrow e^{-i\theta \cdot \frac{\sigma}{2} - \beta \cdot \frac{\sigma}{2}} \psi_L = S_L \psi_L$$
$$\psi_R \rightarrow e^{-i\theta \cdot \frac{\sigma}{2} + \beta \cdot \frac{\sigma}{2}} \psi_R = S_R \psi_R$$

which in a compact form, I'll write as

$$\psi \rightarrow \Lambda_{1/2} \psi$$

We need this to understand how must we write a Lagrangian for the Dirac equation. We must first realize that the Lagrangian is a scalar. We must understand therefore how to create scalars out of these objects we call Dirac spinors. A first guess at writing a real Lagrangian could be to write $\psi^\dagger \psi$. However, this object is not Lorentz invariant. This is easy to see by simply noting that the transformation of ψ_L and ψ_R is not unitary. I claim that

$$\psi^\dagger \gamma^0 \rightarrow \psi^\dagger \gamma^0 \Lambda_{1/2}^{-1}$$

So now I simply define

$$\overline{\psi} \equiv \psi^\dagger \gamma^0$$

so that

$$\overline{\psi} \rightarrow \overline{\psi} \Lambda_{1/2}^{-1}$$

Obviously now, $\overline{\psi}\psi$ is Lorentz invariant. I claim further that $\overline{\psi}\gamma^\mu\psi$ transforms as a four-vector. Armed with these, I can write the following Lorentz invariant quantity

$$\mathcal{L}_{\text{Dirac}} = \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

I am allowed to call it the Dirac Lagrangian, if it's equation of motion is the Dirac equation itself. This is indeed so; recall

$$\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

which in this case, (treating ψ and $\overline{\psi}$ as independent) for $\overline{\psi}$ yeilds the Dirac equation in ψ.

Since this discussion is focussed on the mass term, let me expand it for you.

$$m\overline{\psi}\psi = m \begin{pmatrix} \psi_L^\dagger & \psi_R^\dagger \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = m(\psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R)$$

The objective of this exercise was to show that in the Dirac equation, the mass term mixes the left and right spinors. We develop some more formalism and then see whether we can write a different Lorentz invariant mass term that doesn't mix the left and right parts.

3.3 Projectors, mixing of mass terms

If from the full spinor, I wanted to obtain only the left handed part, I could do the following

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$$

As it turns out, if I define $\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$, then in our basis it becomes $\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. Note then that the left hand projector can be written simply as

$$P_L = \frac{1 - \gamma^5}{2}$$

Similarly, its almost immediate that

$$P_R = \frac{1 + \gamma^5}{2}$$

Further, in our basis, it's obvious from explicit matrices corresponding to P_L and P_R that

$$P_L + P_R = 1_{4\times4}$$

and that

$$P_L P_R = 0; \ P_R P_L = 0$$

I claim that $\{\gamma^5, \gamma^\mu\} = 0$ which allows me to show that

$$\gamma^\mu P_L = \gamma^\mu \frac{(1 - \gamma^5)}{2} = \frac{(1 + \gamma^5)}{2}\gamma^\mu = P_R\gamma^\mu$$

Finally I define

$$\Psi_L \equiv P_L\psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$$

and

$$\Psi_R \equiv P_R\psi = \begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$$

This allows me to write

$$\psi = 1_{4 \times 4} \psi = \left(P_L + P_R\right) \psi = \Psi_L + \Psi_R$$

along with the hermitian conjugate

$$\psi^\dagger = \psi^\dagger 1_{4 \times 4} = \psi^\dagger \left(P_L + P_R\right) = \Psi_L^\dagger + \Psi_R^\dagger$$

plus for clarity

$$\overline{\psi} = \psi^\dagger \gamma^0 = \psi^\dagger 1_{4 \times 4} \gamma^0 = \psi^\dagger \left(P_L + P_R\right) \gamma^0 = \Psi_L^\dagger \gamma^0 + \Psi_R^\dagger \gamma^0 = \overline{\Psi}_L + \overline{\Psi}_R$$

and that in turn allows me to expand the mass term

$$
\begin{aligned}
m \overline{\psi} \psi = m \left(\overline{\Psi}_L + \overline{\Psi}_R\right) \left(\Psi_L + \Psi_R\right) = \ & m \left(\overline{\Psi}_L + \overline{\Psi}_R\right) \left(\Psi_L + \Psi_R\right) \\
= \ & m \left(\overline{\Psi}_L + \overline{\Psi}_R\right) \left(\Psi_L + \Psi_R\right) \\
= \ & m \left(\overline{\Psi}_L \Psi_R + \overline{\Psi}_R \Psi_L\right)
\end{aligned}
$$

It is easy to get confused between the equation we derived in the previous subsection

$$m \overline{\psi} \psi = m(\psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R)$$

but both are correct and equivalent. The latter is written in the 2-component form, while the former is in the 4-component notation. And while I'm still discussing notation, let me define for a four vector B^μ

$$\slashed{B} \equiv \gamma^\mu B_\mu$$

So in this notation, I can write the Lagrangian as

$$
\begin{aligned}
\mathcal{L}_{\text{Dirac}} = \ & \overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi \\
= \ & \left(\overline{\Psi}_L + \overline{\Psi}_R\right) \left(i\slashed{\partial} - m\right) \left(\Psi_L + \Psi_R\right) \\
= \ & \overline{\Psi}_L i\slashed{\partial} \Psi_L + \overline{\Psi}_R i\slashed{\partial} \Psi_R - m \left(\overline{\Psi}_L \Psi_R + \overline{\Psi}_R \Psi_L\right)
\end{aligned}
$$

Note how the kinetic term has the left and right handed spinor separated, but the mass term is mixing the spinor components. We now have enough machinery to explore mass terms that don't mix the spinor components.

3.4   Majorana Mass

I start with defining the following matrix

$$C = i \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix}$$

Now I go back to the transformations of the Dirac spinor. If I write ψ^T as

$$\psi^T = \left(\begin{array}{cc} \psi_L^T & \psi_R^T \end{array} \right)$$

then under a lorentz boost β and a rotation θ,

$$\psi_L^T \rightarrow \psi_L^T e^{-i\theta \cdot \frac{\sigma^{\mathbf{T}}}{2} - \beta \cdot \frac{\sigma^{\mathbf{T}}}{2}} = \psi_L^T S_L^T$$
$$\psi_R^T \rightarrow \psi_R^T e^{-i\theta \cdot \frac{\sigma^T}{2} + \beta \cdot \frac{\sigma^T}{2}} = \psi_R^T S_R^T$$

which in a compact form, according to prior definitions is

$$\psi^T \rightarrow \psi^T \Lambda_{1/2}^T$$

Noting from the explicit Pauli matrices,

$$\sigma^1 = \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right)$$

$$\sigma^2 = \left(\begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right)$$

$$\sigma^3 = \left(\begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right)$$

we see that

$$\left(\sigma^1\right)^T = \sigma^1$$
$$\left(\sigma^2\right)^T = -\sigma^2$$
$$\left(\sigma^3\right)^T = \sigma^3$$

Further, we can use the fact that $\{\sigma^i, \sigma^k\} = 2\delta^{ik}$ to see that

$$\left(\sigma^1\right)^T \sigma^2 = -\sigma^2 \sigma^1$$
$$\left(\sigma^2\right)^T \sigma^2 = -\sigma^2 \sigma^2$$
$$\left(\sigma^3\right)^T \sigma^2 = -\sigma^2 \sigma^3$$

and this is fantastic, why? Well, consider the object $\psi_L^T \sigma^2$. We'll have

$$\psi_L^T \sigma^2 \rightarrow \psi_L^T e^{-i\theta \cdot \frac{\sigma^{\mathbf{T}}}{2} - \beta \cdot \frac{\sigma^{\mathbf{T}}}{2}} \sigma^2 = \psi_L^T \sigma^2 e^{-\left(-i\theta \cdot \frac{\sigma}{2} - \beta \cdot \frac{\sigma}{2}\right)} = \psi_L^T \sigma^2 S_L^{-1}$$

and by the same reasoning, we have

$$\psi_R^T \sigma^2 \rightarrow \psi_R^T e^{-i\theta \cdot \frac{\sigma^T}{2} + \beta \cdot \frac{\sigma^T}{2}} \sigma^2 = \psi_R^T \sigma^2 e^{-\left(-i\theta \cdot \frac{\sigma}{2} + \beta \cdot \frac{\sigma}{2}\right)} = \psi_R^T \sigma^2 S_R^{-1}$$

We now want to write this in the compact notation. For this, we note that

$$\psi^T C = i \begin{pmatrix} \psi_L^T & \psi_R^T \end{pmatrix} \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} = i \begin{pmatrix} \psi_L^T \sigma^2 & -\psi_R^T \sigma^2 \end{pmatrix}$$

so that the transformation is

$$\psi^T C \to i \begin{pmatrix} \psi_L^T S_L^T \sigma^2 & -\psi_R^T S_R^T \sigma^2 \end{pmatrix} = i \begin{pmatrix} \psi_L^T \sigma^2 S_L^{-1} & -\psi_R^T \sigma^2 S_R^{-1} \end{pmatrix} = \psi^T C \lambda_{1/2}^{-1}$$

And what will all of this do? Well, it means that

$$\psi^T C \psi \to \psi^T C \lambda_{1/2}^{-1} \lambda_{1/2} \psi = \psi^T C \psi$$

so that means we have arrived at a Lorentz scalar! Before proceeding, I'll make two comments.

1. We could've arrived at the same result by simply noting that $\lambda_{1/2}^T C \lambda_{1/2} = C$

2. I can write C as a product of γ matrices as

$$C = i \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} = -i\gamma^2 \gamma^0$$

which is easy to verify.

So let us use this term finally to define a different mass term. I write

$$\begin{aligned} \mathcal{L}_{\text{Majorana Mass}} \sim & \quad m\psi^T C \psi \\ =& -im \left(\Psi_L^T + \Psi_R^T \right) \gamma^2 \gamma^0 \left(\Psi_L + \Psi_R \right) \\ =& -im \left(\Psi_L^T \gamma^2 \gamma^0 \Psi_L + \Psi_R^T \gamma^2 \gamma^0 \Psi_R \right) \\ =& \quad m \left(\Psi_L^T C \Psi_L + \Psi_R^T C \Psi_R \right) \end{aligned}$$

so here I have arrived at a mass term which does *not* mix the left and right spinors! I'd point out that this object ofcourse is not guaranteed to be real. To ensure that, we must add its hermitian conjugate $(-m\psi^\dagger C \psi^*)$. To conclude this subsection, I'd mention that

- there are what's called a majorana fermion, which is such that ψ equals it's own 'conjugate'. This results in reduction of degrees of freedom from 4 to 2

- C is closely related to the charge conjugation operator

and state some results

- The Lagrangian with the 'kinetic part' is

$$\begin{aligned} \mathcal{L}_{\text{Majorana}} =& \psi_L^\dagger i\bar{\sigma}.\partial \psi_L + \frac{im}{2} \left(\psi_L^T \sigma^2 \psi_L - \psi_L^\dagger \sigma^2 \psi_L^* \right) \\ =& \quad i\bar{\Psi}_L \slashed{\partial} \Psi_L - \frac{m}{2} \left(\Psi_L^T C \Psi_L + \Psi_L^\dagger C \Psi_L^* \right) \end{aligned}$$

- Corresponding Euler Lagrange

$$i\overline{\sigma}.\partial\psi_L - im\sigma^2\psi_L^* = 0$$

which implies that the Klien Gordan is satisfied and since it is derived from a Lorentz invariant lagrangian, it too is Lorentz invariant.

4 Physical Relevance

In the physical world, we don't observe 'right handed'[7] neutrinos. Even the 'left handed' neutrino was thought to be massless for a long period of time. The standard model consequently has been created to have the following term

$$\mathcal{L} = \overline{\nu}_L i\slashed{\partial}\nu_L$$

However, lately experiments have shown that the 'left handed' neutrinos also have mass. To implement them, we first of all need a way of handling the fact that there's just one part of the full spinor, viz. only 2 degrees of freedom. Plus the Dirac mass as we know it, necessarily mixes the left and right handed part. In this regard, atleast in some sense, the Majorana mass and Majorana spinor are possible candidates to describe neutrinos.

For completeness, I must mention that there are other alternatives to describe the mass of neutrinos, such as the 'see-saw' model.

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[7]in the sense we have just discussed