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Contents

Chapter 1

Overview

1.1 Prerequisites for the chapter

Basic knowledge of physical concepts. A rough idea about qubits will be helpful; infact knowing that they reside in the space spanned by the states of any two level quantum system is sufficient.

1.2 The meat (or vegetable if you prefer)

If you've read Griffiths with any love, you would know that Earnshaw's thoerem prohibits creation of a minima using electrostatic potential. (now don't ask which Griffiths) Thus you can't trap ions conventionally.

1. Confining the ions

   (a) Production of ions: It involves heating calcium in high vacuum, then firing of high speed electrons on its vapour. Consequently some resulting ions fall inside the trap and get stuck.

   (b) Trap structure:
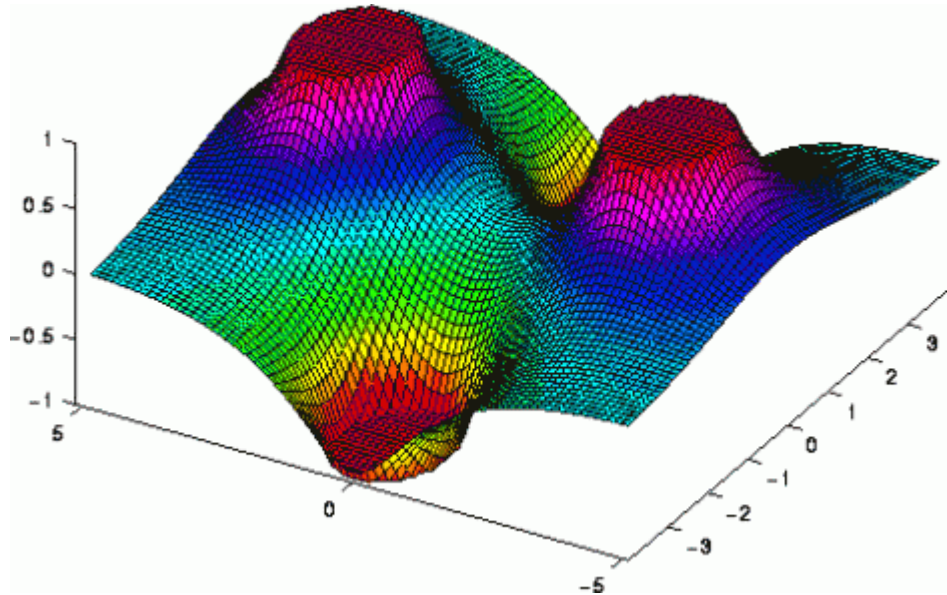
Trap structure: The idea is simple. Since you can't create a potential minima anywhere inside the boundary, you can't trap an ion. But you see the saddle point. (In the figure, the X and Y axis represent physical space and Z is the potential.) You change the potential rapidly so that the 'U' minima oscillates between being along the X and Y axis (say). (The flat circular part is the electrode's surface) This way, on an average, the saddle point sees zero average potential. If the ion was anywhere other than this point, it'll experiences oscillating forces. This effectively results in confinement of the atom. Q1. How does an oscillating force ensure confinement? Q2. What frequencies are most effective and why? Q3. How do you know only 1 ion is trapped? The next immediate question is what restricts the ion in the spatial Z axis. Well, we just pin the ion using a positive potential (by putting two electrodes along the said axis.)
(Figure was taken from [1])

2. Controlling the trapped ions

    (a) Let's say that it is obvious we need the kinetic energy of the ions to be much smaller than that it picks up upon interaction with a laser (or upon emitting a single photon). This number turns up to be about 32 K Hertz (Energy/Plank's constant). The corresponding temperature to have thermal energy lower than this, turns out to be a few micro kelvins!
    This is a serious challenge.

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(b) How do you circumvent this? Laser cooling. Oxymoron you might say but the following little thought is illuminating. If you hit an atom with light with opposite momentum, the atom slows down as the light is bounced back. So if you could get the ions to absorb light preferentially going opposite in direction to their momentum, you've found a groundbreaking method of cooling!

And yes, somebody thought of that too. You use something you're already familiar with, resonance and doppler.

    i. Resonance: This is the (in)famous phenomenon where an innocent and mild vibration can cause an otherwise stable system to have almost devastating effects, given the frequency is just right. A similar effect is observed with atoms and with remarkable sensitivity to the precision of the frequency. Light is absorbed by the atom only when the frequency is just right.

    ii. Doppler: This is as you're well aware, the principle behind the sound effect you get when a car zooms past. We exploit this and set the frequency of the laser just below the resonant frequency. This does it then.

    NOTE: This is not at all this simple. Q1. If the photon is absorbed, it must be emitted giving the atom a push again! Whats wrong? Q2. What about line broadening etc.?

3. Lets get Quantum Computing!

(a) Ions as Qbits: Every ion is one qubit. (In these experiments, we deal with 3-4 Qbits.) To process information on these qubits, we need to be able to

    i. Initialize each qubit individually: This is done by shining light on it since they're physically separated enough and as the laser light can be focussed to distances much smaller than that between adjacent ions.

    ii. Perform C-NOT between any pair of Qubits (this is explained below)

(b) Ions as tiny magnets: The ion can be thought of as a tiny magnet. We can code the 'pointing up' state to be zero and 'pointing down' to be one. So far it is the same as classical information storage. However, quantum mechanically, the magnet can 'point in other directions'. This is perhaps best understood as some sort of angle the magnet is at with the z-axis (the direction of an external magnetic field). However this is not a conventional angle. When you make a measurement, you'll get state zero or one with probabilities proportional to the projection along the relevant axis. (Its very

handwaving but gets the idea across. Do not extend this picture too much without proper formalism) This is then as though the atom is in a superposition of zero and one states and you get one of them upon measurement.

This is exploited for information processing. How we probe the ion (qubit) is by using light. Its interaction with the ion can be understood as

    i. Classically; it changes the orientation of magnet as light has (is?) electromagnetic fields. The magnet has again, a frequency dependent response and the twist magnitude can be controlled by the duration of the pulse.

    ii. Quantum(ly?); the laser mixes the zero and one states. The frequency and duration can be used to calculated precisely the mix created, enabling us to create the desired superpositions.

(c) The computer; ion string: Lets quickly review what C-NOT does. You flip the target iff the control is one. So, in our setup, the problem is to flip the state of ion B, iff ion A is in (say) state one (magnet pointing down). Note that we can't quite use the magnetic field of the ions themselves for, one they're too small and two, ion A and B may not be adjacent. Seems like a deal breaker. But these people (the real scientists) are clever. What they do goes something like this

    i. A laser light is shone on ion A such that its absorbed only in state one.

        A. An ion with magnet down (state one), resonates at a slightly different frequency compared to one with magnet up. Thus it is possible.

        B. The ion is given a little more energy (higher frequency). Why, well cause the ions are able to rattle to and fro their position in the trap. However this is in a plane perpendicular to the line of ions (the string). A vibration in this plane is transmitted to all the ions, because of their charge, viz. interaction by coulombic forces. Now as we are dealing with single ions, it so happens that this whole string vibration is also quantized! Thus we give this exact energy extra in the laser beam.

    ii. With the entire string vibrating, conditioned by if ion A is in state one, we shine a light on ion B with a little less energy. If the string is already vibrating, this would be sufficient to interact with B. This causes ion B to flip its state iff ion A is in state one.

iii. Finally, a beam of light is shone on ion A again to kill the vibration of the string.

And that's how its done! Ladies and Gentlemen, we have the ion trap quantum computer architecture.

Ofcourse, we've made far too many assumptions but the rough idea should be clear now. In the following (last) chapter, we address some of the remarks/questions you might have (such as what the hell, how do you do that etc.)

Chapter 2

Deeper in the trap

2.1 Prologue

It has been assumed that the reader has a basic knowledge of quantum mechanics (Dirac's formulation preferably) and also understands qubits and their role in information processing. The following is intended to briefly review some concepts that'll be used and to comment on some aspects specific to the architecture.

It can be shown that to produce arbitrary unitary transformations of the state of a set of qubits, it is sufficient to be able to produce arbitrary rotations in the Hilbert space of any individual qubit and to be able to carry out the controlled-rotation operation.

Although conventionally we use CNOTs instead of CROTs, the latter is more important for it is easier to implement physically and thus the changed generalization.

However, a further simplification can be made as follows. Instead of seeking CROT between any pair of qubits directly, it is sufficient to have one special qubit which can undergo CROT with any of the other. This special qubit is refered to as a 'bus' and can be used to carry information around using another gate called SWAP (does what it says, swaps the qubits). Although this can be decomposed into the aforesaid gates, in practice, this can be directly implemented physically. The major disadvantage of the bus method is that parallel operations aren't possible.

Accepting this limitation and proceeding with the 'bus' idea, we have the following minimum requirements of our quantum processor.

1. Arbitrary rotations of any single qubit

2. CROT between the bus and any other qubit (note, the other way round is not required, viz. the control is bit is the bus)

3. SWAP between the bus and any other qubit

It will be shown in the following section how all of these can be achieved on the ion trap architecture. Other than this ofcourse, we must be able to make measurements also.

Further, it is worthwhile to remark that a processor with a few qubits may be built using a single particle with sufficient degrees of freedom. The ion trap approach however is more sensible in realizing an interesting enough quantum computer, with say a hundred qubits.

2.2   Setup

2.2.1   Introduction

We discuss here an ion trap with a line of N trapped ions. Each ion has two stable and two metastable states (such as hyperfine or two zeeman levels). There's a setup to point laser beams to interact with any of the N ions. Each ion constitutes one qubit; the two internal energy eigenkets of the ion. There's also the bus qubit discussed in the previous section. The vibrational motion of the whole string of ions constitutes this qubit. This motion must be quantized for it to be usable as a qubit. This thus requires cooling the ion string well below the 'quantum limit', defined with respect to the axial vibrational frequency,

$$k_B T \ll \hbar \omega_z$$

This happens to be one of the most challenging parts of the experimental setup, after making the trap and catching the ions. (Subtleties related to the "Lambe-Dicke" regime haven't been discussed) Further, in brief, there're two known traps, the Paul (rf) and the Penning (Magnetic) and it so turns out that the former allows for tighter confinement (useful for cooling) and allows for magnetic fields to be used for other useful purposes such as splitting into zeeman levels. Further, tighter trapping leads to faster 'switching time' for quantum gates such as CROT (to be discussed in the next section).

2.2.2   Modeling the motion

The N trapped ions are modelled as a system of N point charges in a harmonic potential well of tight radial confinement, viz. $\omega_x, \omega_y \gg \omega_z$. Obviously, the parameters $\{\omega_i\}$ will be determined by the electrode geometry and potentials, which will not be discussed in this report. The corresponding hamiltonian is

$$H = \sum_{i=1}^{N} \frac{1}{2} M \left(\omega_x^2 X_i^2 + \omega_x^2 Y_i^2 + \omega_x^2 Z_i^2 + \frac{\mathbf{P}_i^2}{M^2} \right) + \sum_{i=1}^{N} \sum_{j>i} \frac{e^2}{4\pi\epsilon_0 |\mathbf{R}_i - \mathbf{R}_j|}$$

where the position and momentum are operators. Assertion: at low temperatures, for tight radial confinement, the ions all lie along the z-axis. Consequently it is reasonable to assume $|R_i - R_j| \approx |Z_i - Z_j|$ which allows for the radial and axial motion to be separated. Considering the axial motion, we first find a relevant length scale, say

$$z_s = \left(\frac{e^2}{4\pi\epsilon_0 M \omega_z^2} \right)^{1/3} \approx 10 - 100\,\mu m$$

which is of the order of separation between the ions. Solving the equation classically, one obtains the equilibrium positions of the ions. As is expected, for larger N, the outer ions tend to push the inner ions closer. Consequently, the spacing between the ions depends upon N. However, it is not so obvious that the two normal modes of oscillation about these equilibrium positions are independent of N (for small oscillations that is). Besides, even higher harmonics are nearly independent of N. The frequencies of the two normal modes are ω_z and $\sqrt{3}\omega_z$ (the list goes $\{1, \sqrt{3}, \sqrt{29/5}, 3.051, 3.671, 4.272, 4.865\ldots\}$ times ω_z). In the lowest of mode of oscillation all the ions move to and fro together. This mode thus corresponds to the harmonic motion of the centre of mass of the ion string. Since the frequencies of the higher harmonics are so different from that of the ground normal mode, it can be excited selectively, without exciting the other modes.

So far everything was classical. In cranking things up to the quantum level, we simply treat the centre of mass coordinate z_{cm} as a harmonic oscillator. We assert that the classically determined ground state excitation frequency ω_z continues to be valid even though the ion wave functions now overlap; they delicately cancel upon calculation of the centre of mass motion. Accepting this assertion, we proceed by noting that the harmonic oscillator has NM mass and frequncy ω_z. Thus the energy eigenfunctions are given as

$$\psi_n(z_{cm}) = \left(\frac{NM\omega_z}{\pi\hbar 2^{2n}(n!)^2} \right)^{1/4} H_n \left(z_{cm}\sqrt{NM\omega_z/\hbar} \right) e^{-NM\omega_z z_{cm}^2/2\hbar}$$

This gives us the extent of spatial spread of the ground state probability distribution, which is given by its standard deviation,

$$\Delta z_{cm} = \sqrt{\hbar/2NM\omega_z}$$

We certainly don't want our inter-ion distance to be comparable to Δz_{cm}, else addressing each ion individually with a laser beam would become unnecessarily hard, if not impossible. Thus we require $\Delta z_{cm} \ll \Delta z_{min}$. A numerical solution yields

$$\Delta z_{min} \approx 2.0 z_s N^{-0.57}$$

where z_s is the length we found in the beginning (this formula remains valid till $N \approx 1000$). Imposing the inequality we get

$$\frac{\omega_z}{M} \ll \frac{31 N^{1.86}}{\hbar^3} \left(\frac{e^2}{4\pi\epsilon_0} \right)^2 \approx 2.4 \times 10^{21} N^{1.86} \text{ Hz/u}$$

where u is the atomic mass unit (1.66057×10^{-27} kg). The parameters here are the frequency which we said depends on the geometry and potentials of the electrodes. The mass of the atom is also somewhat in our control (depends on which we choose, but there're other factors influencing that also). However, in practice, this condition is found to be easily satisfied. ω_z is no more than a GHz, while M ranges from 9 to 200 u. It's thus accurate to image the ions as sitting on a line, with their small wavepacket centred at their classical equilibrium location, with essentially no overlap with others. It must be stated that this condition alone doesn't guarantee that the separation is enough to allow unique addressing of the ions by laser beams.

2.3 Matrix: From paper to our loyal ions

2.3.1 The states we make

First things first. What are the states we're looking at and how long will excited states last. Lets answer them. There are two kinds of states.
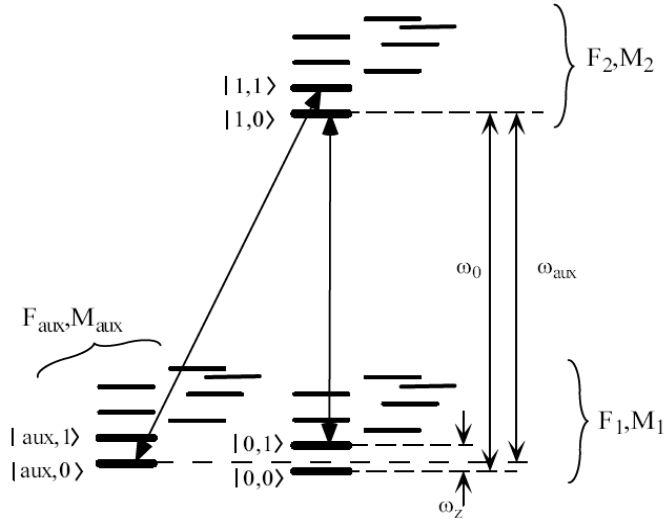
1. The first are internal energy eigenstates of the ion. Refer to the figure. (taken from [2])

(a) We write these states as $|F_1, M_1\rangle$, $|F_2, M_2\rangle$ and $|F_{aux}, M_{aux}\rangle$

(b) They are separated by ω_0 and ω_{aux} as shown.

2. The other are the excitations of the centre of mass motion.

(a) The eigenkets of the vibrational motion are written as $|\{n_i\}\rangle$ where n_i are the excitations of the various modes.

(b) Only the ground state $|0, 0, 0 \ldots\rangle$ and the first excited state $|1, 0, 0 \ldots\rangle$ are invoked for the operations

(c) This state is loosely called the phonon (used here as the bus bit)

So the states we'll be finally working with, will be the following tensor products

$$|0, 0\rangle \equiv |F_1, M_1\rangle \otimes |0, 0, 0 \ldots\rangle$$
$$|0, 1\rangle \equiv |F_1, M_1\rangle \otimes |1, 0, 0 \ldots\rangle$$
$$|1, 0\rangle \equiv |F_2, M_2\rangle \otimes |0, 0, 0 \ldots\rangle$$
$$|0, 1\rangle \equiv |F_2, M_2\rangle \otimes |1, 0, 0 \ldots\rangle$$

That answers the first question. The second is answered by the well known (but not well understood for me so far) energy time uncertainty relation, which unlike the unambiguous position momentum uncertainty arising from the commutation relation, refers to a more general class of such uncertainty relations (including ones that arise from perturbation theory). The point is that since all the levels considered here are low-lying, separated oh so mildly (hyperfine and Zeeman interactions), that the lifetime of the excited states (against spontaneous emission) is essentially infinite.

2.3.2   The gates it takes

We show how to perform the following operations.

1. *CROT* between any ion's internal state and the phonon bit, where CROT is given by

$$U_{\text{CROT}} \equiv \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{bmatrix}$$

in the computational basis, $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. So in essence, if the internal state is zero, then you do nothing. Else you apply σ_z to the phonon.

2. Carry out *arbitrary rotations* in the ion's internal state, viz. be able to apply

$$e^{-i\theta.\sigma/2} = \begin{bmatrix} cos(\theta/2) & -e^{-i\phi}sin(\theta/2) \\ e^{i\phi}cos(\theta/2) & cos(\theta/2) \end{bmatrix}$$

3. Perform a *swap* between any ion and the bus (phonon) bit. The matrix is obviously

$$U_{\text{SWAP}} \equiv \begin{bmatrix} 1 & & & \\ & 0 & 1 & \\ & 1 & 0 & \\ & & & 1 \end{bmatrix}$$

It is known that these three operations together can be used to carry out arbitrary transformations to the qubits stored.

CROT

Remember there were these $|F_{aux}, M_{aux}\rangle$ eigenstates of the ion which we didn't use in the definition of the computational basis $|0, 1\rangle, |0, 1\rangle$ etc. We now define

$$|aux, i\rangle = |F_{aux}, M_{aux}\rangle \otimes |i, 0, 0, 0 \ldots\rangle$$

where $i \in \{0, 1\}$. These states are available to us as kind of shelves. These help us make state-dependent transformations (such as controlled rotations).

A look at the figure shows that if a radiation of frequency $\omega_{aux} + \omega_z$ is applied, then only transitions between $|1, 1\rangle$ and $|aux, 0\rangle$ will occur (assuming states in unwanted levels are unoccupied).

Claim: If a 2π pulse (of the said frequency) is applied, then the state $|1, 1\rangle$ is rotated by 2π radians.

This is precisely what was required. If the state is $|1, 1\rangle$ its sign is flipped. (Note: Don't confuse the state of the computational basis with that of the auxilary state, which is in a zero phonon state)

Claim: A 2π pulse of frequency $\omega_z - (\omega_0 - \omega_{aux})$ produces a control rotation corresponding to

$$U_{\text{C}\to\text{ROT}} \equiv \begin{bmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$

Justification: This should be obvious from the previous discussion and the diagram. Effectively then, $U_{\text{C}\to\text{ROT}}$ rotates the second qubit if the first is in state zero rather than one.

Arbitrary Internal State Rotations

To rotate an ion's internal state without affecting the phonon (centre of mass motion), a pulse of frequency, without surprise, ω_0 is applied. (This part is not too clear)

Claim: If such a radiation has a phase ϕ wrt some defined origin of phase, and duration sufficient to make a $p\pi$ pulse, then the effect in the computational basis is

$$V^p(\phi) \equiv \left[\begin{array}{cc} \cos(p\pi/2) & -e^{-i\phi}\sin(p\pi/2) \\ -e^{i\phi}\cos(p\pi/2) & \cos(p\pi/2) \end{array} \right] \otimes \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right]$$

To apply such rotations successfully, it is required that experimentally, the phase is controlled at the position of the ion (and not just some other physical location, like a stable reference cavity). In this regard therefore it is much easier if the computational basis of the system are separated by radio frequencies and not optical frequencies.

Claim 2: In order to have the right phase experimentally, one needn't worry about the continuous precession with ω_0 caused by the internal Hamiltonian of each ion

Claim 3: However problems arise when different ions have different frequencies due to the residual electric and magnetic fields in the apparatus, although such problems are solvable.

Universalization: CNOT detour

We have already developed enough machinery to implement an XOR (a reversible XOR is the same as a CNOT) between the bus (phonon) and some particular ion. To recall, C NOT is defined as

$$\text{XOR} \equiv \left[\begin{array}{cccc} 1 & & & \\ & 0 & 0 & 1 \\ & 0 & 1 & 0 \\ & 1 & 0 & 0 \end{array} \right]$$

The matrix is wrong isn't it? No! We are doing a CNOT(bus,ion) as opposed to CNOT(ion,bus). Here's the sequence that does the trick

1. A $\pi/2$ pulse with frequency ω_0

$$V^{1/2}(-\pi/2) = \frac{1}{\sqrt{2}} \left[\begin{array}{cc} 1 & 1 \\ -1 & 1 \end{array} \right]_{\text{ion}} \otimes \left[\begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right]_{\text{bus}}$$

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2. CROT

3. Another $\pi/2$ with frequency ω_0 and phase π wrt the first, viz $V^{1/2}(\pi/2)$

Q1. How did they come up with this? Whats the rationale behind this?

It can be checked that the result is infact XOR(bus,ion) as given by the matrix above. So by symmetry XOR(ion,bus) should be a similar sequence with ω_0 replaced with ω_z to cause the vibration (bus) level to get affected independent of the ion. Right? Nope. Let's understand this.

The basic issue is that vibrations (the bus) aren't a two level system. And where does this bite us? Well, recall how in the implementation of the CROT we assumed that $|aux, 1\rangle$ is unoccupied, for if it were, then upon flashing $\omega_{aux} + \omega_z$ it would merrily couple to $|1, 2\rangle$ which is, surprise surprise, outside our computation Hilber space!

SWAP

Well this is by far the easiest to implement. Just consider the frequncy $\omega_0 - \omega_z$ and look at the transitions diagram. The only possible transition is between $|01\rangle$ and $|10\rangle$ (assuming ofcourse there's no population in the $|11\rangle$, $|02\rangle$ states etc.). So then that does it! We have swapped successfully between the bus and any one of the ions.

And that completes the gates part.

2.3.3   How to see without mistakes

To complete the operation of the processor, we must be able to make measurements. This is achieved with high accuracy, using what is known as 'electron shelving' or 'quantum jumps' technique. This results in whether the ion is in the $|F_1 M_1\rangle$ or $|F_2 M_2\rangle$. This is achieved by illuminating the ion with a radiation resonant with a transition from $|F_1 M_1\rangle$ to some high-lying level, whose line-width is sufficiently small so that transitions from $|F_2 M_2\rangle$ aren't excited (else what's the point eigh?). Now if the flourescence is produced, the ion state has collapsed to $|F_1 M_1\rangle$; if its not produced, then you know it is in $|F_2 M_2\rangle$. Note that you can't readout the bus directly, but since you can swap, it's not an issue now is it?

So in effect, do you realize what you've done? If this doesn't blow your mind, then you have no feelings!

2.4   Epilogue

There are some essential topics which haven't been discussed here but mentioned for the sake of completion.

1. Cooling

2. Design of the ion trap itself

3. Candidate ions

4. Errors and limitations

It is hoped that the report was informative, fun and helpful in putting the idea across. I apologize for any mistakes that might have crept in during the process of documentation.

Bibliography

[1] https://www2.physics.ox.ac.uk/research/ion-trap-quantum-computing-group/intro-to-ion-trap-qc

[2] http://xxx.soton.ac.uk/abs/quant-ph/9608011