

Dirac and Majorana Mass

Atul Singh Arora

Indian Institute of Science Education and Research Mohali

April 16, 2015

Overview of the Talk

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

Outline

Introduction

Prerequisites

Towards a quantum theory of fields

Closing Remarks

Introduction

Motivation | Mass

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

► Inertia | Newton

Motivation | Mass

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

- ▶ Inertia | Newton
- ▶ Special Relativity | $p^2 = m^2$

Motivation | Mass

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

- ▶ Inertia | Newton
- ▶ Special Relativity | $p^2 = m^2$
- ▶ Particle Physics
 - ▶ QED (Quantum Electrodynamics)

Motivation | Mass

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

- ▶ Inertia | Newton
- ▶ Special Relativity | $p^2 = m^2$
- ▶ Particle Physics
 - ▶ QED (Quantum Electrodynamics)
 - ▶ Classical Electrodynamics gauge field: A^μ

- ▶ Inertia | Newton
- ▶ Special Relativity | $p^2 = m^2$
- ▶ Particle Physics
 - ▶ QED (Quantum Electrodynamics)
 - ▶ Classical Electrodynamics gauge field: A^μ
 - ▶ Quantize | Massless Gauge Field

- ▶ Inertia | Newton
- ▶ Special Relativity | $p^2 = m^2$
- ▶ Particle Physics
 - ▶ QED (Quantum Electrodynamics)
 - ▶ Classical Electrodynamics gauge field: A^μ
 - ▶ Quantize | Massless Gauge Field
 - ▶ Standard Model
 - ▶ Massive Gauge fields | Spontaneous Symmetry Breaking

- ▶ Inertia | Newton
- ▶ Special Relativity | $p^2 = m^2$
- ▶ Particle Physics
 - ▶ QED (Quantum Electrodynamics)
 - ▶ Classical Electrodynamics gauge field: A^μ
 - ▶ Quantize | Massless Gauge Field
 - ▶ Standard Model
 - ▶ Massive Gauge fields | Spontaneous Symmetry Breaking
 - ▶ Scalar field | Higgs

- ▶ Inertia | Newton
- ▶ Special Relativity | $p^2 = m^2$
- ▶ Particle Physics
 - ▶ QED (Quantum Electrodynamics)
 - ▶ Classical Electrodynamics gauge field: A^μ
 - ▶ Quantize | Massless Gauge Field
 - ▶ Standard Model
 - ▶ Massive Gauge fields | Spontaneous Symmetry Breaking
 - ▶ Scalar field | Higgs
 - ▶ Beyond Standard Model
 - ▶ Neutrino Oscillations

- ▶ Inertia | Newton
- ▶ Special Relativity | $p^2 = m^2$
- ▶ Particle Physics
 - ▶ QED (Quantum Electrodynamics)
 - ▶ Classical Electrodynamics gauge field: A^μ
 - ▶ Quantize | Massless Gauge Field
 - ▶ Standard Model
 - ▶ Massive Gauge fields | Spontaneous Symmetry Breaking
 - ▶ Scalar field | Higgs
 - ▶ Beyond Standard Model
 - ▶ Neutrino Oscillations
 - ▶ Neutrino Mass

- ▶ Inertia | Newton
- ▶ Special Relativity | $p^2 = m^2$
- ▶ Particle Physics
 - ▶ QED (Quantum Electrodynamics)
 - ▶ Classical Electrodynamics gauge field: A^μ
 - ▶ Quantize | Massless Gauge Field
 - ▶ Standard Model
 - ▶ Massive Gauge fields | Spontaneous Symmetry Breaking
 - ▶ Scalar field | Higgs
 - ▶ Beyond Standard Model
 - ▶ Neutrino Oscillations
 - ▶ Neutrino Mass

Prerequisites

Prerequisites

- ▶ From CM, recall
 - ▶ Lagrangian Formalism

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

Prerequisites

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

- ▶ From CM, recall
 - ▶ Lagrangian Formalism
 - ▶ Euler Lagrange equations

Prerequisites

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

- ▶ From CM, recall
 - ▶ Lagrangian Formalism
 - ▶ Euler Lagrange equations
 - ▶ Noether's Theorem relating conserved quantities and continuous symmetries

Prerequisites

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

- ▶ From CM, recall
 - ▶ Lagrangian Formalism
 - ▶ Euler Lagrange equations
 - ▶ Noether's Theorem relating conserved quantities and continuous symmetries
- ▶ From STR, I'll use
 - ▶ $\eta^{\mu\nu} = \text{diag}(1, -\vec{1})$ (NB: $\eta^T = \eta^{-1} = \eta$)

- ▶ From CM, recall
 - ▶ Lagrangian Formalism
 - ▶ Euler Lagrange equations
 - ▶ Noether's Theorem relating conserved quantities and continuous symmetries
- ▶ From STR, I'll use
 - ▶ $\eta^{\mu\nu} = \text{diag}(1, -\vec{1})$ (NB: $\eta^T = \eta^{-1} = \eta$)
 - ▶ $c = 1, \hbar = 1$

- ▶ From CM, recall
 - ▶ Lagrangian Formalism
 - ▶ Euler Lagrange equations
 - ▶ Noether's Theorem relating conserved quantities and continuous symmetries
- ▶ From STR, I'll use
 - ▶ $\eta^{\mu\nu} = \text{diag}(1, -\vec{1})$ (NB: $\eta^T = \eta^{-1} = \eta$)
 - ▶ $c = 1, \hbar = 1$
 - ▶ indices
 - ▶ i, j, k, l etc. run from 1 to 3

- ▶ From CM, recall
 - ▶ Lagrangian Formalism
 - ▶ Euler Lagrange equations
 - ▶ Noether's Theorem relating conserved quantities and continuous symmetries
- ▶ From STR, I'll use
 - ▶ $\eta^{\mu\nu} = \text{diag}(1, -\vec{1})$ (NB: $\eta^T = \eta^{-1} = \eta$)
 - ▶ $c = 1, \hbar = 1$
 - ▶ indices
 - ▶ i, j, k, l etc. run from 1 to 3
 - ▶ α, β, γ etc. run from 0 to 3

- ▶ From CM, recall
 - ▶ Lagrangian Formalism
 - ▶ Euler Lagrange equations
 - ▶ Noether's Theorem relating conserved quantities and continuous symmetries
- ▶ From STR, I'll use
 - ▶ $\eta^{\mu\nu} = \text{diag}(1, -\vec{1})$ (NB: $\eta^T = \eta^{-1} = \eta$)
 - ▶ $c = 1, \hbar = 1$
 - ▶ indices
 - ▶ i, j, k, l etc. run from 1 to 3
 - ▶ α, β, γ etc. run from 0 to 3

Prerequisites

- ▶ I'll need the 4 vector notation. Recall
 - ▶ Summation Convention $A^\alpha B_\alpha = \sum_{\alpha=0}^4 A^\alpha B_\alpha$

Prerequisites

- ▶ I'll need the 4 vector notation. Recall
 - ▶ Summation Convention $A^\alpha B_\alpha = \sum_{\alpha=0}^4 A^\alpha B_\alpha$
 - ▶ if $A^\alpha = (A^0, \vec{A})$, then $A_\alpha \equiv \eta_{\alpha\beta} A^\beta = (A^0, -\vec{A})$

Prerequisites

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

► I'll need the 4 vector notation. Recall

- Summation Convention $A^\alpha B_\alpha = \sum_{\alpha=0}^4 A^\alpha B_\alpha$
- if $A^\alpha = (A^0, \vec{A})$, then $A_\alpha \equiv \eta_{\alpha\beta} A^\beta = (A^0, -\vec{A})$
- $\lambda^\alpha_\beta, A^\alpha \rightarrow A'^\alpha = \lambda^\alpha_\beta A^\beta$

Prerequisites

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

- ▶ I'll need the 4 vector notation. Recall
 - ▶ Summation Convention $A^\alpha B_\alpha = \sum_{\alpha=0}^4 A^\alpha B_\alpha$
 - ▶ if $A^\alpha = (A^0, \vec{A})$, then $A_\alpha \equiv \eta_{\alpha\beta} A^\beta = (A^0, -\vec{A})$
 - ▶ $\lambda^\alpha_\beta, A^\alpha \rightarrow A'^\alpha = \lambda^\alpha_\beta A^\beta$
 - ▶ contracted indices don't transform (NB: $\lambda^T \eta \lambda = \eta$)

Prerequisites

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

Towards a
quantum theory
of fields

Closing Remarks

- ▶ I'll need the 4 vector notation. Recall
 - ▶ Summation Convention $A^\alpha B_\alpha = \sum_{\alpha=0}^4 A^\alpha B_\alpha$
 - ▶ if $A^\alpha = (A^0, \vec{A})$, then $A_\alpha \equiv \eta_{\alpha\beta} A^\beta = (A^0, -\vec{A})$
 - ▶ $\lambda_\beta^\alpha, A^\alpha \rightarrow A'^\alpha = \lambda_\beta^\alpha A^\beta$
 - ▶ contracted indices don't transform (NB: $\lambda^T \eta \lambda = \eta$)
- ▶ From QM, I'll need the following. Recall
 - ▶ State: $|\psi\rangle$ (or $\psi(x) = \langle x|\psi\rangle$)

Prerequisites

- ▶ I'll need the 4 vector notation. Recall
 - ▶ Summation Convention $A^\alpha B_\alpha = \sum_{\alpha=0}^4 A^\alpha B_\alpha$
 - ▶ if $A^\alpha = (A^0, \vec{A})$, then $A_\alpha \equiv \eta_{\alpha\beta} A^\beta = (A^0, -\vec{A})$
 - ▶ $\lambda_\beta^\alpha, A^\alpha \rightarrow A'^\alpha = \lambda_\beta^\alpha A^\beta$
 - ▶ contracted indices don't transform (NB: $\lambda^T \eta \lambda = \eta$)
- ▶ From QM, I'll need the following. Recall
 - ▶ State: $|\psi\rangle$ (or $\psi(x) = \langle x|\psi\rangle$)
 - ▶ Time Evolution: For H (st. $H^\dagger = H$; where $H^\dagger \equiv H^{*T}$) we have

$$H|\psi\rangle = -i\hbar \frac{\partial}{\partial t} |\psi\rangle$$

Prerequisites

- ▶ I'll need the 4 vector notation. Recall
 - ▶ Summation Convention $A^\alpha B_\alpha = \sum_{\alpha=0}^4 A^\alpha B_\alpha$
 - ▶ if $A^\alpha = (A^0, \vec{A})$, then $A_\alpha \equiv \eta_{\alpha\beta} A^\beta = (A^0, -\vec{A})$
 - ▶ $\lambda_\beta^\alpha, A^\alpha \rightarrow A'^\alpha = \lambda_\beta^\alpha A^\beta$
 - ▶ contracted indices don't transform (NB: $\lambda^T \eta \lambda = \eta$)
- ▶ From QM, I'll need the following. Recall
 - ▶ State: $|\psi\rangle$ (or $\psi(x) = \langle x|\psi\rangle$)
 - ▶ Time Evolution: For H (st. $H^\dagger = H$; where $H^\dagger \equiv H^{*T}$) we have

$$H|\psi\rangle = -i\hbar \frac{\partial}{\partial t} |\psi\rangle$$

and

$$|\psi(t)\rangle = e^{(-i\hbar)^{-1} H t} |\psi(0)\rangle$$

NB: $U \equiv e^{(-i\hbar)^{-1} H t}$ is unitary, viz. $U^\dagger = U^{-1}$

Prerequisites

- ▶ I'll need the 4 vector notation. Recall
 - ▶ Summation Convention $A^\alpha B_\alpha = \sum_{\alpha=0}^4 A^\alpha B_\alpha$
 - ▶ if $A^\alpha = (A^0, \vec{A})$, then $A_\alpha \equiv \eta_{\alpha\beta} A^\beta = (A^0, -\vec{A})$
 - ▶ $\lambda^\alpha_\beta, A^\alpha \rightarrow A'^\alpha = \lambda^\alpha_\beta A^\beta$
 - ▶ contracted indices don't transform (NB: $\lambda^T \eta \lambda = \eta$)
- ▶ From QM, I'll need the following. Recall
 - ▶ State: $|\psi\rangle$ (or $\psi(x) = \langle x|\psi\rangle$)
 - ▶ Time Evolution: For H (st. $H^\dagger = H$; where $H^\dagger \equiv H^{*T}$) we have

$$H|\psi\rangle = -i\hbar \frac{\partial}{\partial t} |\psi\rangle$$

and

$$|\psi(t)\rangle = e^{(-i\hbar)^{-1} H t} |\psi(0)\rangle$$

NB: $U \equiv e^{(-i\hbar)^{-1} H t}$ is unitary, viz. $U^\dagger = U^{-1}$

- ▶ Measurement/Observables
 - ▶ Collapse into eigenstate of operator corresponding to the measurement

Prerequisites

- ▶ I'll need the 4 vector notation. Recall
 - ▶ Summation Convention $A^\alpha B_\alpha = \sum_{\alpha=0}^4 A^\alpha B_\alpha$
 - ▶ if $A^\alpha = (A^0, \vec{A})$, then $A_\alpha \equiv \eta_{\alpha\beta} A^\beta = (A^0, -\vec{A})$
 - ▶ $\lambda^\alpha_\beta, A^\alpha \rightarrow A'^\alpha = \lambda^\alpha_\beta A^\beta$
 - ▶ contracted indices don't transform (NB: $\lambda^T \eta \lambda = \eta$)
- ▶ From QM, I'll need the following. Recall
 - ▶ State: $|\psi\rangle$ (or $\psi(x) = \langle x|\psi\rangle$)
 - ▶ Time Evolution: For H (st. $H^\dagger = H$; where $H^\dagger \equiv H^{*T}$) we have

$$H|\psi\rangle = -i\hbar \frac{\partial}{\partial t} |\psi\rangle$$

and

$$|\psi(t)\rangle = e^{(-i\hbar)^{-1} H t} |\psi(0)\rangle$$

NB: $U \equiv e^{(-i\hbar)^{-1} H t}$ is unitary, viz. $U^\dagger = U^{-1}$

- ▶ Measurement/Observables
 - ▶ Collapse into eigenstate of operator corresponding to the measurement
 - ▶ Collapse to state $|n\rangle$ with probability $|\langle n|\psi\rangle|^2$

Prerequisites

- ▶ I'll need the 4 vector notation. Recall
 - ▶ Summation Convention $A^\alpha B_\alpha = \sum_{\alpha=0}^4 A^\alpha B_\alpha$
 - ▶ if $A^\alpha = (A^0, \vec{A})$, then $A_\alpha \equiv \eta_{\alpha\beta} A^\beta = (A^0, -\vec{A})$
 - ▶ $\lambda^\alpha_\beta, A^\alpha \rightarrow A'^\alpha = \lambda^\alpha_\beta A^\beta$
 - ▶ contracted indices don't transform (NB: $\lambda^T \eta \lambda = \eta$)
- ▶ From QM, I'll need the following. Recall
 - ▶ State: $|\psi\rangle$ (or $\psi(x) = \langle x|\psi\rangle$)
 - ▶ Time Evolution: For H (st. $H^\dagger = H$; where $H^\dagger \equiv H^{*T}$) we have

$$H|\psi\rangle = -i\hbar \frac{\partial}{\partial t} |\psi\rangle$$

and

$$|\psi(t)\rangle = e^{(-i\hbar)^{-1} H t} |\psi(0)\rangle$$

NB: $U \equiv e^{(-i\hbar)^{-1} H t}$ is unitary, viz. $U^\dagger = U^{-1}$

- ▶ Measurement/Observables
 - ▶ Collapse into eigenstate of operator corresponding to the measurement
 - ▶ Collapse to state $|n\rangle$ with probability $|\langle n|\psi\rangle|^2$
- ▶ Basics of quantum harmonic oscillator using a, a^\dagger

- ▶ I'll use the following pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- ▶ Terminology from particle physics
 - ▶ Leptons: Eg. Electron, Electron Neutrino
 - ▶ Quarks

Towards a quantum theory of fields

Motivation

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ All electrons are identical

Motivation

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ All electrons are identical
- ▶ Unification of QM and STR

Motivation

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ All electrons are identical
- ▶ Unification of QM and STR
- ▶ Crisis: Can't predict the result of collision of particles

Motivation

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ All electrons are identical
- ▶ Unification of QM and STR
- ▶ Crisis: Can't predict the result of collision of particles

Targets of the new theory

- ▶ Creation and destruction

- ▶ All electrons are identical
- ▶ Unification of QM and STR
- ▶ Crisis: Can't predict the result of collision of particles

Targets of the new theory

- ▶ Creation and destruction
- ▶ Consistent with STR (high energy)

- ▶ All electrons are identical
- ▶ Unification of QM and STR
- ▶ Crisis: Can't predict the result of collision of particles

Targets of the new theory

- ▶ Creation and destruction
- ▶ Consistent with STR (high energy)
- ▶ Predict probabilities

► $(E^2 - \vec{p}^2) \psi = m^2 \psi$

- ▶ $(E^2 - \vec{p}^2) \psi = m^2 \psi$
and put $E \rightarrow -i \frac{\partial}{\partial t}$, $\vec{p} \rightarrow i \vec{\nabla}$ to get

- ▶ $(E^2 - \vec{p}^2) \psi = m^2 \psi$
and put $E \rightarrow -i \frac{\partial}{\partial t}$, $\vec{p} \rightarrow i \vec{\nabla}$ to get

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \right) \psi = m^2 \psi$$

- ▶ $(E^2 - \vec{p}^2) \psi = m^2 \psi$
and put $E \rightarrow -i \frac{\partial}{\partial t}$, $\vec{p} \rightarrow i \vec{\nabla}$ to get

$$(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2) \psi = m^2 \psi$$

- ▶ Causality

- ▶ $(E^2 - \vec{p}^2) \psi = m^2 \psi$
and put $E \rightarrow -i \frac{\partial}{\partial t}$, $\vec{p} \rightarrow i \vec{\nabla}$ to get

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2\right) \psi = m^2 \psi$$

- ▶ Causality
- ▶ Negative Energies (no stable ground state)

- ▶ $(E^2 - \vec{p}^2) \psi = m^2 \psi$
and put $E \rightarrow -i \frac{\partial}{\partial t}$, $\vec{p} \rightarrow i \vec{\nabla}$ to get

$$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2\right) \psi = m^2 \psi$$

- ▶ Causality
- ▶ Negative Energies (no stable ground state)
- ▶ Expected: t parameter, \vec{x} operator

Concept of field

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ One field for each type of particle

Concept of field

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ One field for each type of particle (Wheeler's idea)

Concept of field

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ One field for each type of particle (Wheeler's idea)
- ▶ creates and destroys particles

Concept of field

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ One field for each type of particle (Wheeler's idea)
- ▶ creates and destroys particles
- ▶ Interacting fields, interacting particles

Concept of field

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ One field for each type of particle (Wheeler's idea)
- ▶ creates and destroys particles
- ▶ Interacting fields, interacting particles

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

- ▶ Classical field | real scalar (number at every space time point)

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Classical field | real scalar (number at every space time point)
- ▶ Demand Klien Gordan, then

$$\mathcal{L} = \frac{1}{2} \left(\partial^\mu \phi \partial_\mu \phi + m^2 \phi \right)$$

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Classical field | real scalar (number at every space time point)
- ▶ Demand Klein Gordon, then

$$\mathcal{L} = \frac{1}{2} \left(\partial^\mu \phi \partial_\mu \phi + m^2 \phi \right)$$

- ▶ $\phi = \phi(t, \vec{x})$ which I assume I can write as

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Classical field | real scalar (number at every space time point)
- ▶ Demand Klien Gordon, then

$$\mathcal{L} = \frac{1}{2} \left(\partial^\mu \phi \partial_\mu \phi + m^2 \phi \right)$$

- ▶ $\phi = \phi(t, \vec{x})$ which I assume I can write as

$$\phi = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2\omega_p}} \left(a e^{i\mathbf{p}\mathbf{x}} + a^\dagger e^{-i\mathbf{p}\mathbf{x}} \right)$$

where $a = a(\vec{p})$

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Classical field | real scalar (number at every space time point)
- ▶ Demand Klien Gordon, then

$$\mathcal{L} = \frac{1}{2} \left(\partial^\mu \phi \partial_\mu \phi + m^2 \phi \right)$$

- ▶ $\phi = \phi(t, \vec{x})$ which I assume I can write as

$$\phi = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2\omega_p}} \left(a e^{i\mathbf{p}\mathbf{x}} + a^\dagger e^{-i\mathbf{p}\mathbf{x}} \right)$$

where $a = a(\vec{p})$

- ▶ π from \mathcal{L} .

- ▶ Classical field | real scalar (number at every space time point)
- ▶ Demand Klien Gordon, then

$$\mathcal{L} = \frac{1}{2} \left(\partial^\mu \phi \partial_\mu \phi + m^2 \phi \right)$$

- ▶ $\phi = \phi(t, \vec{x})$ which I assume I can write as

$$\phi = \int \frac{d^3 p}{(2\pi)^3 \sqrt{2\omega_p}} \left(a e^{i\mathbf{p}\mathbf{x}} + a^\dagger e^{-i\mathbf{p}\mathbf{x}} \right)$$

where $a = a(\vec{p})$

- ▶ π from \mathcal{L} .
- ▶ Quantum Field | $[\phi(t, \mathbf{x}), \pi(t, \mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}')$

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

► $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

► $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$

►

$$H \sim a^\dagger a + \frac{1}{2}[a(\mathbf{p}), a^\dagger(\mathbf{p})]$$

► $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$



$$H \sim a^\dagger a + \frac{1}{2}[a(\mathbf{p}), a^\dagger(\mathbf{p})]$$

► Similarity with Quantum Harmonic Oscillator

- ▶ $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$



$$H \sim a^\dagger a + \frac{1}{2}[a(\mathbf{p}), a^\dagger(\mathbf{p})]$$

- ▶ Similarity with Quantum Harmonic Oscillator
- ▶ $a^\dagger(\vec{p})|\text{vacuum}\rangle$

- ▶ $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$



$$H \sim a^\dagger a + \frac{1}{2}[a(\mathbf{p}), a^\dagger(\mathbf{p})]$$

- ▶ Similarity with Quantum Harmonic Oscillator
- ▶ $a^\dagger(\vec{p})|\text{vacuum}\rangle$
- ▶ Noether's theorem + Space-time invariance of $\mathcal{L} \rightarrow$ physical momentum and energy operators

- ▶ $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$



$$H \sim a^\dagger a + \frac{1}{2}[a(\mathbf{p}), a^\dagger(\mathbf{p})]$$

- ▶ Similarity with Quantum Harmonic Oscillator
- ▶ $a^\dagger(\vec{p})|\text{vacuum}\rangle$
- ▶ Noether's theorem + Space-time invariance of $\mathcal{L} \rightarrow$ physical momentum and energy operators
- ▶ To be a particle, it must satisfy $E^2 - \vec{p}^2 = m^2$

- ▶ $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$



$$H \sim a^\dagger a + \frac{1}{2}[a(\mathbf{p}), a^\dagger(\mathbf{p})]$$

- ▶ Similarity with Quantum Harmonic Oscillator
- ▶ $a^\dagger(\vec{p})|\text{vacuum}\rangle$
- ▶ Noether's theorem + Space-time invariance of $\mathcal{L} \rightarrow$ physical momentum and energy operators
- ▶ To be a particle, it must satisfy $E^2 - \vec{p}^2 = m^2$ and it does

- ▶ $[a(\mathbf{p}), a^\dagger(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$



$$H \sim a^\dagger a + \frac{1}{2}[a(\mathbf{p}), a^\dagger(\mathbf{p})]$$

- ▶ Similarity with Quantum Harmonic Oscillator
- ▶ $a^\dagger(\vec{p})|\text{vacuum}\rangle$
- ▶ Noether's theorem + Space-time invariance of $\mathcal{L} \rightarrow$ physical momentum and energy operators
- ▶ To be a particle, it must satisfy $E^2 - \vec{p}^2 = m^2$ and it does
- ▶ Conclusion: Parameter m is mass

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Non-interacting field

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Non-interacting field
- ▶ Observable fields must interact

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Non-interacting field
- ▶ Observable fields must interact
- ▶ QFT | perturbation theory, expanded around the non-interacting part

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Non-interacting field
- ▶ Observable fields must interact
- ▶ QFT | perturbation theory, expanded around the non-interacting part
- ▶ results in Feynman Rules

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Non-interacting field
- ▶ Observable fields must interact
- ▶ QFT | perturbation theory, expanded around the non-interacting part
- ▶ results in Feynman Rules (Prof. Mukunda story)

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Non-interacting field
- ▶ Observable fields must interact
- ▶ QFT | perturbation theory, expanded around the non-interacting part
- ▶ results in Feynman Rules (Prof. Mukunda story)
 - ▶ Decay Rates

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Non-interacting field
- ▶ Observable fields must interact
- ▶ QFT | perturbation theory, expanded around the non-interacting part
- ▶ results in Feynman Rules (Prof. Mukunda story)
 - ▶ Decay Rates
 - ▶ Scattering Cross sections

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Non-interacting field
- ▶ Observable fields must interact
- ▶ QFT | perturbation theory, expanded around the non-interacting part
- ▶ results in Feynman Rules (Prof. Mukunda story)
 - ▶ Decay Rates
 - ▶ Scattering Cross sections
- ▶ Interacting case, m no longer the mass

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Non-interacting field
- ▶ Observable fields must interact
- ▶ QFT | perturbation theory, expanded around the non-interacting part
- ▶ results in Feynman Rules (Prof. Mukunda story)
 - ▶ Decay Rates
 - ▶ Scattering Cross sections
- ▶ Interacting case, m no longer the mass
 - ▶ defined as pole of the 'full propagator'

Framework: QFT

Dirac and
Majorana Mass

Atul Singh Arora

Outline

Introduction

Prerequisites

**Towards a
quantum theory
of fields**

Closing Remarks

- ▶ Non-interacting field
- ▶ Observable fields must interact
- ▶ QFT | perturbation theory, expanded around the non-interacting part
- ▶ results in Feynman Rules (Prof. Mukunda story)
 - ▶ Decay Rates
 - ▶ Scattering Cross sections
- ▶ Interacting case, m no longer the mass
 - ▶ defined as pole of the 'full propagator' (I'll leave it at that)

The End