Olly 6.1

QED → G_{123} QFT
→ standard model gauge group

Spontaneous Symmetry Breaking

QM: { Symmetry $[U,H]=0$
$UHU^{\dagger}=H$

States: Lowest Energy State $|0\rangle$

$U|0\rangle = |0\rangle$?

	$[U,H]=0$	$U\|0\rangle = \|0\rangle$
Wigner Weyl	✓	✓
Nambu-Goldstone	✓	✗

PQFT: Perturbative Quantum Field theory
: Quantization of small oscillations of a field theory about a stable vacuum.

In QFT we start with finding the classical field configuration that minimizes the field energy.

Example:

$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$

$V(-\phi) = V(\phi)$ $\quad \mathbb{Z}_2$ $\phi = \phi^{-1}$

$\mathcal{H} = \frac{1}{2}\left((\nabla\phi)^2 + \dot\phi^2\right) + V(\phi)$

$\phi_0(x,t)$ should be s.t. $\int d^3x\,\mathcal{H}$ is a minimum

must be space time independent $\because (\nabla\phi)^2 > 0$, $\dot\phi^2 > 0$

$\Rightarrow \phi_0(x,t) = \phi_0$

& $\left.\frac{\partial V}{\partial\phi}\right|_{\phi=\phi_0} = 0$

Expand $V(\phi)$ as

$V(\phi) = \frac{1}{2}m^2\phi^2 + g\frac{\phi^3}{3} + \frac{\lambda\phi^4}{4!} + \dots$

$\because V(\phi) = V(-\phi)$

& for a renormalizable theory

$= \frac{\lambda}{4!}(\phi^2+v^2)^2 + c$

& compare terms

If the term $v^2 > 0$, then minimum $\phi_0 = 0$

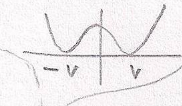
(Also $\lambda > 0$ $\because$ otherwise the theory has no ground state)

For $v^2 < 0$, we'll have $v'^2 \equiv -v^2$

so then $v' \leftrightarrow v$

$V = \frac{\lambda}{4!}(\phi^2 - v^2)^2$

This is different



$M_{vac} = \{\phi_0 = -v, \phi_0 = v\}$

$\mathbb{Z}_2 M_{vac} = M_{vac}$

But the individual soln. is not invariant under $\mathbb{Z}_2$

Quantization

Choose either $\phi_0 = +v$ or $-v$ as vacuum configuration

$\phi(x^\mu) = \phi_0 + \underbrace{\hat\phi(x^\mu)}_{\text{Quantum Field}}$

$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\lambda}{4!}(\phi^2 - v^2)^2$

$= \frac{1}{2}\partial_\mu\hat\phi\partial^\mu\hat\phi - \frac{\lambda}{24}(\cancel{\phi_0^2} + 2\phi_0\hat\phi + \hat\phi^2 \cancel{-v^2})^2$

$= \frac{1}{2}\partial_\mu\hat\phi\partial^\mu\hat\phi - \frac{\lambda}{4}(\hat\phi^4 + 4\phi_0^2\hat\phi^2 + 4\phi_0\hat\phi^3)$

without loss of generality $\phi_0 = v$

cubic!!

cubic coupling: $\frac{g}{3!}\hat\phi^3$

where $\frac{g}{3!} = \frac{\lambda}{24}\cdot 4\phi_0$

$U(1) \simeq O(2)$ $\quad \mathcal{L} = \sum_{i=1}^{2}\partial_\mu\phi_i\partial^\mu\phi_i - V(\phi_1,\phi_2)$

$\phi_i = \phi_i^*$, $i = 1,2$

& O(2) Invariance

if $V(\phi_1,\phi_2) = f(\phi_1^2+\phi_2^2) = f(\vec\phi^2)$

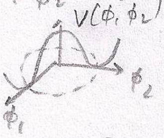
Renormalizability $\Rightarrow d = 4$,

$V(\vec\phi) = \frac{\lambda}{4!}(\vec\phi^2 - v^2)^2$

1) WW $v^2 < 0$
Minima $\vec\phi_0^2 = \phi_1^{0\,2} + \phi_2^{0\,2} = 0$

2) NG $v^2 > 0$ $\quad \phi_1^{0\,2} + \phi_2^{0\,2} = v^2$: S^1

$\mathcal{M}_v = S^1$



Eigenvalues of $M_{ij} = \frac{\partial^2 V}{\partial\phi_i\partial\phi_j}$ are +ve

$\mathcal{M}_v = \{R\begin{pmatrix} v \\ 0\end{pmatrix} ; R \in O(2)\}$

wlg we can quantize with $R = \mathbb{1}_2$

$V = (\phi^2 - v^2)^2$

$\frac{\partial V}{\partial\phi_i} = 2(\phi^2 - v^2)\phi_i = 0$

$\vec\phi^2 = v^2$

$\vec\phi = 0$

$\frac{\partial^2 V}{\partial\phi_i\partial\phi_j} = 2\phi_j\phi_i + 2(\vec\phi^2 - v^2)\delta_{ij}$

$\left.\right|_{\phi_i=0 \Leftrightarrow \vec\phi=0} = -2v^2\delta_{ij}$

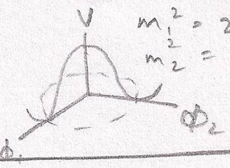
WLG $\vec\phi = \begin{pmatrix}\phi_1 \\ \phi_2\end{pmatrix} = \begin{pmatrix} v + \hat\phi_1 \\ \hat\phi_2\end{pmatrix}$

$V = \frac{\lambda}{4}(\vec\phi^2 - v^2)^2 = \left((v+\hat\phi_1)^2 + \hat\phi_2^2 - v^2\right)^2 \lambda/4$

$= (\hat\phi_1^2 + \hat\phi_2^2 + 2\hat\phi_1 v)^2 \lambda/4$

$= \left((\hat\phi_1^2 + \hat\phi_2^2)^2 + 2v\hat\phi_1(\hat\phi_1^2 + \hat\phi_2^2) + 4v^2\hat\phi_1^2\right)\frac{\lambda}{4}$

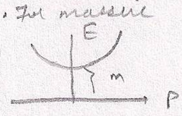
$= \frac{\hat m_1}{2}\hat\phi_1^2 + \dots$



$m_1^2 = 2\lambda v^2$

$m_2^2 = 0$

· For massive



for massless, no gap.

· All points on the circle are equivalent

Goldstone Thm:

For every continuous symmetry of the action which is not a symmetry of the groundstate, there's a massless excitation

slash

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Goldstone Thm

$O(2)$ $\vec{\phi} = \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$

$V(\vec{\phi}) = -\frac{\mu^2}{2}\vec{\phi}^2 + \frac{\lambda}{4}(\vec{\phi}^2)^2 + \text{const}$

$\epsilon = T = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$; Minima differentiate

$\mathcal{M}_V = S^1 : \{\vec{\phi}^2 = v^2\}$; $(-\mu^2 + \lambda\vec{\phi}^2)\vec{\phi} = 0$

$\Rightarrow \vec{\phi}^2 = \frac{-\mu^2}{\lambda}$ minus sign

choose one ~~photo~~point on S^1 wlg $\begin{pmatrix}v\\0\end{pmatrix} = \vec{\phi}_0$; $\vec{\phi} = \vec{\phi}_0 + \hat{\vec{\phi}}$

$\Rightarrow m_1^2 \neq 0$ $m_2^2 = 0$ → Massless Goldstone

Ex: Study Z_2 with $\phi^2 + |\phi| + |\phi|\phi^2 + \phi^4$

$R(\theta)\vec{\phi}_0 \neq \vec{\phi}_0$

$R(\theta) = e^{\theta\epsilon} = \sum \frac{\theta^n \epsilon^n}{n!}$

$\Leftrightarrow \epsilon\vec{\phi}_0 \neq 0$

NB: You could do the same with $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$ (complex) & get the same result.

$\mathcal{L}$ invariant under Global symmetry group G & ϕ transforms in Unitary representation R.

$\Phi' = U(\theta_a)\Phi$

leaves $\mathcal{L} = \partial_\mu \Phi^\dagger \partial_\mu \Phi - V(\Phi)$

$V(\Phi') = V(\Phi)$

Minimize V

$\frac{\partial V}{\partial \Phi} = 0$: choose minima

$\Phi \leftrightarrow \Phi'$

m^2 of all excitations ≥ 0

$U = e^{i\theta^a T^a}$

$T^a = T^{a\dagger}$ $d(R)\times d(R)$

$\phi_j = \frac{\rho_j + i\kappa_j}{\sqrt{2}}$

$\Psi = \begin{pmatrix}\rho_1\\ \vdots\\ \rho_d\\ \kappa_1\\ \vdots\\ \kappa_d\end{pmatrix}$ $\Phi = \begin{pmatrix}\phi_1\\ \vdots\\ \phi_d\end{pmatrix}$

$(\Phi' - \Phi)_i = \delta\phi_i = i\theta^a T^a_{ij}\Phi_j$

$\delta\Phi = i(\theta\cdot T)\Phi$

$T^a = \underbrace{R^a}_{\text{real}} + \underbrace{iI}_{\text{imaginary}}$, $d\times d$

claim: $\delta\Psi_I = \theta^a \mathcal{A}^a_{IJ}\Psi_J$

where $\mathcal{A}^a_{IJ} =$

Next, we also have

$\left.\frac{\partial V}{\partial \Psi}\right|_{\Psi = \Psi_0} = 0 \Rightarrow$ $\Psi = V(\Psi) = V(\Psi_0) + \left.\frac{\partial V}{\partial \Psi_I}\right|_{\Psi_0}\cancel{(\Psi_I - \Psi_I^0)}$

$+ \frac{1}{2}\left.\frac{\partial^2 V}{\partial\Psi_I\partial\Psi_J}\right|_{\Psi = \Psi_0}(\Psi_I - \Psi_I^0)(\Psi_J - \Psi_J^0)$

$+ \ldots$

NB: $\Rightarrow m^2_{IJ} = (\partial_I \partial_J)V|_{\Psi = \Psi_0}$

$V(\Psi + \epsilon\Delta(\Psi)) = V(\Psi) = V(\Psi) + \epsilon\Delta_I(\Psi)\frac{\partial V}{\partial \Psi_I} + O(\epsilon^2)$

$\Rightarrow \Delta_I(\Psi)\frac{\partial V(\Psi)}{\partial \Psi_I} = 0$ ($\because$ V is invariant under the transformation)

$\frac{\partial}{\partial \Psi_J}\left(\Delta_I(\Psi_I)\frac{\partial V}{\partial \Psi_I}\right) = 0$

$\left.\left(\underbrace{\partial_J \Delta_I(\Psi)\cdot\partial_I V}_{\text{extrema (minima)}} + \Delta_I\frac{\partial^2 V}{\partial\Psi_J\partial\Psi_I}\right)\right|_{\Psi = \Psi_0} = 0$

$\Rightarrow \Delta_I(\Psi)|_{\Psi = \Psi_0}$ $m^2_{IJ} = 0$

$m^2 \cdot \Delta(\Psi_0) = 0$

If $\Delta(\Psi_0) \neq 0$, $\Rightarrow$ $\Delta(\Psi_0)$ is a null eigenvector of the mass matrix

Example: For our old case, we could take $\Delta\Psi$ to be $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}v\\0\end{pmatrix} \neq 0$ & this will be a null vector of the m_{IJ} matrix.

Further, in general, you use the generators to make a vector $(T^a)(\Psi_0)$ & this will be a null vector

GT: # of (linearly indep) broken generators condition satisfied = # null eigenvectors of the mass matrix = # Goldstone bosons.

$V = \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$

$O(2)$: $J_\mu \sim \frac{\partial \mathcal{L}}{\partial\partial_\mu\Phi}(\delta\phi)$

$= \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_i)}(\delta\phi)_i$

$= (\partial_\mu\phi_i)\epsilon_{ij}\phi_j = (\partial_\mu\phi_1)\phi_2 - \ldots = (\partial_\mu\hat{\phi}_1)\hat{\phi}_2 - (v + \hat{\phi})\partial_\mu\hat{\phi}_2$

$(\delta\phi)_i = \epsilon_{ij}\phi_j$ $\phi_i = v\delta_{i1} + \hat{\phi}_i$ $= -v\,\partial_\mu\hat{\phi}_2 + O(\hat{\phi}^2)$

$\phi_1 = v + \hat{\phi}_1$

$Q = \int J_0\, d^3x$

$Q|0\rangle \neq 0$

$\Rightarrow Q J_0|0\rangle \neq 0$

$\langle 0|J_\mu|\hat{\phi}_2(p)\rangle = -v\langle 0|\partial_\mu\hat{\phi}_2\sqrt{2E_p}\,a^{2\dagger}_{\vec{p}}|0\rangle$

TODO: Complete this

Recall: $\langle p|q\rangle = (2\pi)^3\delta^3(\vec{p} - \vec{q})2E_{\vec{q}}$ | $\hat{\phi}_2 = \int\frac{d^3k}{(2\pi)^3\sqrt{2E_k}}\left(a^{2}_{\vec{k}}e^{-ik\cdot x} + a^{2\dagger}_{\vec{k}}e^{ik\cdot x}\right)$ (Not squared)

$[a_{\vec{p}}, a^\dagger_{\vec{q}}] = (2\pi)^3\delta^3(\vec{p} - \vec{q})$

$\langle 0|J_\mu(x)|\hat{\phi}_2(p)\rangle = iv\,p_\mu e^{-ip\cdot x}$

$\Rightarrow \partial_\mu\langle 0|J^\mu(x)|\hat{\phi}_2(p)\rangle = \partial_\mu(iv\,p^\mu e^{-ip\cdot x}) = p^2 v$

$\overset{!}{=} 0$

$\Rightarrow p^2 = 0 \Rightarrow$ massless particle!

SSB (Spontaneous Symmetry Breaking)

$\Phi = R^{\text{unitary}} \; d(R)$; $(\vec{\Phi}) \leftarrow 2d(R) = \Delta$

$R^a + i I^a = 7^a = 7^{a\dagger} \longrightarrow A^{aT} = -A^a$

$\delta\Phi = i\theta^a 7^a \quad I = 1, \ldots, 2d(R)$

$\mathcal{L} = |\partial\phi|^2 + \dot\phi^2$

$\left.\begin{array}{l}\vec\nabla\phi_0 = 0 \\ \dot\phi_0 = 0\end{array}\right\} \partial_I V(\Phi)\Big|_{\Phi=\Phi_0} = 0 \quad m^2_{IJ} = \dfrac{\partial^2 V}{\partial\Phi_I \partial\Phi_J}\Big|_{\Phi=\Phi_0}$

$m^2_{IJ}(\hat{\delta}^a \Phi)_J = 0$

$\quad\quad = (A^a\Phi)_J$

$m^2 \cdot v = 0$

if $(A^a \Phi_0) \neq 0$ it defines a null vector of m^2

$Q^a|0\rangle \neq 0$

$e^{iQ^a\theta^a}|0\rangle \neq |0\rangle$

$Q^a = \int d^3x \, J^a_0(\vec{x}, t)$

$\langle 0 | J^a_\mu(x) | \pi_I(p)\rangle$

$= e^{-ip\cdot x} \, p_\mu \, \dfrac{F_I^a}{(A^a\Phi_0)_I}$

$\partial\, \phi_I = A^a_{IJ}\partial_\mu \Phi_J$

$\mathcal{L} = (\partial_\mu\phi)^* \partial^\mu\phi - \lambda\left(\phi^*\phi - \frac{v^2}{2}\right)^2$

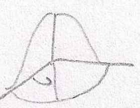
$\quad\quad (\phi_1^2 + \phi_2^2 - v^2)^2$

with $\phi = \frac{\phi_1 + i\phi_2}{\sqrt2}$

$|\partial_\mu\phi|^2 = \left|\partial_\mu\left(\frac{\hat\rho + v}{\sqrt2}\right)e^{i\frac{\chi(x)}{v}} + \left(\frac{\hat\rho+v}{\sqrt2}\right) i\left(\frac{\partial_\mu\chi}{v}\right)e^{i\chi/v}\right|^2$

$\mathcal{L} = \frac12(\partial_\mu\hat\rho)^2 + \ldots \frac{(\partial_\mu\chi)^2}{v^2}\left(\frac{\hat\rho}{v}+1\right)^2 v^2 - \frac{\lambda}{4}(\rho^2 + v^2 + 2\rho v - v^2)^2$

$\phi = \frac{(\hat\rho + v)}{\sqrt2} e^{\frac{i\hat\chi(x)}{v}}$



$\to$ goldstone mode

$\to$ radial mode.

$\vec\phi^* = \vec\phi = \begin{pmatrix}\phi_1 \\ \vdots \\ \phi_n\end{pmatrix}$

$\mathcal{L} = \frac12 \partial_\mu\vec\phi \cdot \partial^\mu\vec\phi - V(\vec\phi)$

$\quad\quad\quad V(\vec\phi) \to \frac{\lambda}{4}(\vec\phi^2 - v^2)^2$

$\vec\phi_0^2 = v^2$; $\mathcal{M}_V = S_{N-1}$

$\phi_1^{0\,2} + \phi_2^{0\,2} + \ldots \phi_N^{0\,2}$

Choose z axis along, say the vector itself. So then will have wlg

$\vec\Phi_0 = \begin{pmatrix}0\\0\\ \vdots \\ v\end{pmatrix}$ N-1 dim

$SO(N) \longrightarrow SO(N-1)$ ("little gp")

$\to$ Vacuum is invariant

generators that change the vacuum $\dfrac{N(N-1)}{2} - \dfrac{(N-1)(N-2)}{2} = \dfrac{N-1}{2}(2) = N-1$

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For $N=3 \quad SO(3) \to SO(2)$

$\vec\phi = \begin{pmatrix}\pi_1(x) \\ \pi_2(x) \\ \sigma(x) + v\end{pmatrix}$

$\frac12 \partial_\mu\vec\phi\,\partial_\mu\vec\phi = \frac12(\partial_\mu\vec\sigma)^2 + \frac12(\partial_\mu\vec\pi)^2$

$V = \frac{\lambda}{4}(\vec\pi^2 + \sigma^2 + v^2 + 2\sigma v - v^2)^2$

$= \frac{\lambda}{4}\left[(\vec\pi^2 + \sigma^2)^2 + 4\sigma^2 v^2 + 4\sigma v(\pi^2 + \sigma^2)\right]$

$m_\sigma^2 = 2\lambda v^2$

$m_\pi^2 = 0$

$O(N)$: $\vec\Phi = e^{\sum_{a=1}^{N-1}\vec\pi_a(x)A^a}\begin{pmatrix}0\\0\\v+\sigma\end{pmatrix}$

$\vec\phi\cdot\vec\phi = (v+\sigma)^2$

$V(\vec\phi) = \frac{\lambda}{4}(\sigma^2 + 2\sigma v)^2$

Teaser:

$O(4) \approx SU(2)_L \times SU(2)_R$

$3\,\pi$ fields

$1\,\sigma$ field

$\Sigma = \sigma\mathbb{1} + i\vec\tau\cdot\vec\pi$

$O(4): \Sigma \to \Sigma'$

You can show

$\Sigma' = U_L \Sigma U_R^\dagger$

$U(1)$: $e^{i\chi/v}\left(\frac{v+\rho}{\sqrt2}\right) = \phi$

$O(2)$: $\phi = R\begin{pmatrix}0\\v\end{pmatrix}$

$\phi(\vec x) = \begin{pmatrix}\cos\chi/v & -\sin\chi/v \\ \sin\chi/v & \cos\chi/v\end{pmatrix}\begin{pmatrix}0\\v\end{pmatrix}$

$= e^{\epsilon(\chi/v)}\begin{pmatrix}0 \\ v + \frac{\rho(x)}{\sqrt2}\end{pmatrix}$

Unitary Symmetry $U(2)$: $T_a, \mathbb{1}$

$\phi = \begin{pmatrix}\phi_1\\\phi_2\end{pmatrix}$ $\quad \mathcal{M}_V: \phi_0^\dagger\phi_0 = \frac{v^2}{2} = |\phi_{10}|^2 + |\phi_{20}|^2$

$= (\mathrm{Re}\,\phi_{10})^2 + (\mathrm{Im}\,\phi_{10})^2 + (\mathrm{Re}\,\phi_{20})^2 + (\mathrm{Im}\,\phi_{20})^2 \equiv S_3$

w.l.g. $\phi_0 = U\begin{pmatrix}0\\ \frac{v}{\sqrt2}\end{pmatrix}$

$V(\phi) = \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 \quad \phi_0^\dagger\phi_0 = \frac{v^2}{2}$

Generators are

$\begin{pmatrix}0&1\\1&0\end{pmatrix}, \begin{pmatrix}0&-i\\i&0\end{pmatrix}, \begin{pmatrix}1&0\\0&-1\end{pmatrix}, \begin{pmatrix}1&0\\0&1\end{pmatrix}$

$T_1\begin{pmatrix}0\\1\end{pmatrix} \neq 0$

$T_2\begin{pmatrix}0\\1\end{pmatrix} \neq 0$

$T_3 + \mathbb{1} = \begin{pmatrix}2&0\\0&0\end{pmatrix}\cdot\begin{pmatrix}0\\1\end{pmatrix} = 0$

& $\frac{T_3 - \mathbb{1}}{2} = \begin{pmatrix}0&0\\0&-1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = -\begin{pmatrix}0\\1\end{pmatrix}$

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Exam Related

$S[\psi, \partial\psi] = S[\psi', \partial\psi']$

$\psi'(x) = e^{i\theta \cdot T}\psi(x)$

$\partial_\mu \psi^\dagger \partial^\mu \psi$ is not invariant

$D_\mu = \partial_\mu + igA_\mu$

$A'_\mu = A_\mu - \frac{1}{g}\partial_\mu \theta(x)$

$\psi' = e^{i\theta(x)}\psi(x) = V\psi$

$(D\psi)' = e^{i\theta(x)}(D\psi)$

$T^a A'^a_\mu = V(A_\mu + \frac{i}{g}\partial_\mu)V^\dagger$

$D'_\mu = V D_\mu V^\dagger$

$\Rightarrow D'_\mu \psi' = V(D_\mu \psi)$

$[D_\mu, D_\nu] = igF_{\mu\nu}$

$\Downarrow$

$F'_{\mu\nu} = V F_{\mu\nu} V^\dagger$

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last from Prashansa (copy)

$U\phi_0 \neq \phi_0$

$T^a \phi_0 \neq 0$

$\Rightarrow$ Massless Scalars

$k_\mu \Pi^{\mu\nu}(k) = 0$

Higgs Mechanism:

$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \lambda(\phi^*\phi - \frac{v^2}{2})^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

Higgs model

Put $\partial_\mu \to D_\mu$

where $D_\mu = \partial_\mu + igA_\mu$

$A'_\mu = A_\mu - \frac{1}{g}\partial_\mu \theta(x)$

$\phi'(x) = e^{i\theta(x)}\phi(x)$

$\mathcal{L}(\phi', A') = \mathcal{L}(\phi, A)$

So that potential defines a vacuum manifold as $M_V : S_1 : \{|\phi| = \frac{v}{\sqrt{2}}\}$

Fast way: Put $\phi = \left(\frac{v+\rho}{\sqrt{2}}\right)e^{i\chi/v}$

$\phi' = \left(\frac{v+\rho}{\sqrt{2}}\right)e^{i\left(\frac{\chi}{v} + \theta(x)\right)}$

Choose $\theta(x) = -\frac{\chi(x)}{v}$

So our Lagrangian then becomes

$\mathcal{L} = D_\mu \phi^* D^\mu \phi - \frac{\lambda}{4}((\rho+v)^2 - v^2)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

with $|D_\mu \phi|^2 = |\partial_\mu \phi + igA_\mu \phi|^2$

$= \left|\frac{\partial_\mu \rho}{\sqrt{2}} + igA_\mu \frac{(\rho+v)}{\sqrt{2}}\right|^2$

$= \frac{1}{2}(\partial_\mu \rho)^2 + g^2 A_\mu^2 v^2 (1 + \frac{\rho}{v})^2$

Unitarity gauge: $\phi = \frac{\rho + v}{\sqrt{2}}\exp\left(\frac{i\chi}{v}\right)$

$\mathcal{L}^{quad}_{Higgs} = \frac{1}{2}(\partial_\mu \rho)(\partial^\mu \rho) - \frac{1}{4}(\partial_{[\mu}A_{\nu]})(\partial^{[\mu}A^{\nu]}) + \frac{1}{2}g^2v^2(A_0^2 - \vec{A}^2) - (2\lambda v^2)\frac{\rho^2}{2}$

$\Rightarrow m_\rho^2 = 2\lambda v^2$

$m_A^2 = g^2 v^2$

(for the vector part)

1 massless spin 1 + 2 real scalars $= 3_V + 1_S$

$\mathcal{L}_{int} = \frac{m_A^2}{2}(A_\mu A^\mu)\frac{\rho^2}{v^2} - \frac{\lambda}{4}(\rho^4 + 4\rho^3 v)$

U(1) symmetry is regulating the interaction

N [Higgs
Englert Brout
Kibble Guralnik

[Anderson
Weinberg did the final