Dirac and Majorana Mass

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Overview of the Talk

Outline

Introduction

Prerequisites

Towards a quantum theory of fields

Dirac and Majorana Mass

Physical Relevance

Closing Remarks

Introduction

Motivation | Mass

- Inertia | Newton

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  - Basics of quantum harmonic oscillator using $a, a^\dagger$

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- Conclusion: Parameter m is mass

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&(i\partial_\mu \gamma^\mu - m)(i\partial_\nu \gamma^\nu - m) \\
=&(-\partial_\mu \partial_\nu \gamma^\mu \gamma^\nu - im\partial_\mu \gamma^\mu) - (im\partial_\nu \gamma^\nu - m^2) \\
=&(-\frac{1}{2}\partial_\mu \partial_\nu \{\gamma^\mu, \gamma^\nu\} - 2m\,[i(\partial_\mu \gamma^\mu)] + m^2) \\
=&(-\frac{1}{2}\partial_\mu \partial_\nu \{\gamma^\mu, \gamma^\nu\} - 2m^2 + m^2) \\
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- $$\begin{aligned} &(i\partial_\mu \gamma^\mu - m)(i\partial_\nu \gamma^\nu - m) \\ =&(-\partial_\mu \partial_\nu \gamma^\mu \gamma^\nu - im\partial_\mu \gamma^\mu) - (im\partial_\nu \gamma^\nu - m^2) \\ =&(-\frac{1}{2}\partial_\mu \partial_\nu \{\gamma^\mu, \gamma^\nu\} - 2m\,[i(\partial_\mu \gamma^\mu)] + m^2) \\ =&(-\frac{1}{2}\partial_\mu \partial_\nu \{\gamma^\mu, \gamma^\nu\} - 2m^2 + m^2) \\ =&(-\frac{1}{2}\partial_\mu \partial_\nu \{\gamma^\mu, \gamma^\nu\} - m^2) \end{aligned}$$

- To be Klein Gordan, $\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu}$

The Dirac Equation

- $$(i\partial_\mu\gamma^\mu - m)\psi = 0$$
- $$\begin{aligned}&(i\partial_\mu\gamma^\mu - m)(i\partial_\nu\gamma^\nu - m)\\ =&(-\partial_\mu\partial_\nu\gamma^\mu\gamma^\nu - im\partial_\mu\gamma^\mu) - (im\partial_\nu\gamma^\nu - m^2)\\ =&(-\frac{1}{2}\partial_\mu\partial_\nu\{\gamma^\mu,\gamma^\nu\} - 2m\left[i(\partial_\mu\gamma^\mu)\right] + m^2)\\ =&(-\frac{1}{2}\partial_\mu\partial_\nu\{\gamma^\mu,\gamma^\nu\} - 2m^2 + m^2)\\ =&(-\frac{1}{2}\partial_\mu\partial_\nu\{\gamma^\mu,\gamma^\nu\} - m^2)\end{aligned}$$
- To be Klein Gordan, $\{\gamma^\mu,\gamma^\nu\} = -2\delta^{\mu\nu}$
- Claim
  γ^μ are 4×4 matrices and ψ then is a $4-$component object, called a Dirac spinor.

The Dirac Equation

- Hat:

$$\sigma^\mu \equiv (1, \vec{\sigma}) \qquad \overline{\sigma}^\mu \equiv (1, -\vec{\sigma})$$

$$\gamma^\mu \equiv \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}$$

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- Claim: commutation holds

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$$\psi = \left(\begin{array}{c} \psi_L \\ \psi_R \end{array} \right)$$

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which in a compact form, I'll write as

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- $\psi^\dagger\psi$ | not Lorentz invariant
- ψ_L and ψ_R is not unitary

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- Mass term mixes the left and right spinors
- Explore: mass term that doesn't mix

Projectors, mixing of mass terms

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- In this notationa also, there's mixing

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$$\psi^T C \to i \begin{pmatrix} \psi_L^T S_L^T \sigma^2 & -\psi_R^T S_R^T \sigma^2 \end{pmatrix}$$

$$= i \begin{pmatrix} \psi_L^T \sigma^2 S_L^{-1} & -\psi_R^T \sigma^2 S_R^{-1} \end{pmatrix} = \psi^T C \lambda_{1/2}^{-1}$$

- And what will all of this do? Well, it means that

$$\psi^T C \psi \to \psi^T C \lambda_{1/2}^{-1} \lambda_{1/2} \psi = \psi^T C \psi$$

- Result: We have arrived at a Lorentz scalar!

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  - We could've arrived at the same result by simply noting that $\lambda_{1/2}^T C \lambda_{1/2} = C$
  - I can write C as a product of γ matrices as

    $$C = i \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} = -i\gamma^2\gamma^0$$

    which is easy to verify.

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- Defn: A different mass term

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\begin{aligned}
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&= -im\left(\Psi_L^T\gamma^2\gamma^0\Psi_L + \Psi_R^T\gamma^2\gamma^0\Psi_R\right) \\
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- To ensure reality, we add $-m\psi^\dagger C\psi^*$

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  - C is closely related to the charge conjugation operator

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- There're alternatives, such as 'see-saw' model

The End

?

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