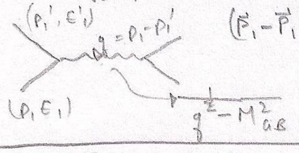
Effective Theory of Weak Interaction in terms of light low energy particles $\ll 100\,GeV$.

1 generation of Leptons $L_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}$ color $(1, 2, -\frac{1}{2})$ $m_i \, |\phi|$, Y

$e_R = (1, 1, -2)$

$$\mathcal{L} = -\frac{1}{4}(W^{a\,2}_{\mu\nu} + B^2_{\mu\nu}) + \bar{L}_L i \not{D} L_L + \bar{e}_R i \not{D} e_R$$
$$+ |D_\mu \Phi|^2 - \lambda(\Phi^\dagger\Phi - \frac{v^2}{2})^2 + \text{mass terms (of fermions)}$$

$\langle\Phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ $\quad \Phi = e^{\frac{i\eta^a T^a}{v\sqrt{2}}} \begin{pmatrix} 0 \\ \frac{v + h^0(x)}{\sqrt{2}} \end{pmatrix} \Big|_{\eta^a(x)=0}$ Unitarity gauge $\quad m_{h_0} = \sqrt{2\lambda}\, v$



$(\vec{p}_1 - \vec{p}_1{}')^2 - (E_1 - E_1{}')^2 \quad \frac{\lambda}{4}(h_0^2 + 2h_0 v)^2$

$\sqrt{p^2+m^2} \quad \frac{\lambda}{4}(4h_0^2 v^2)$

$\frac{1}{q^2 - M^2_{GB}}$

Coefficient of $\delta W^\pm, \delta Z$ in $\delta S \Rightarrow$ Eq^n of Motion

$$m_W^2(W^+\delta W^- + \delta W^+ W^-) + m_Z^2 Z\,\delta Z - \frac{g}{\sqrt{2}}(J^-\delta W^+ + J^+\delta W^-)$$
$$- g_Z J_Z \delta Z + (\text{small mom. \& non-linear correction}) = 0$$

⚠ Dimensions

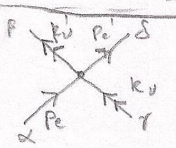
$$\left.\begin{aligned} W^\pm_\mu &= \frac{g}{\sqrt{2}}\frac{J^\pm_\mu}{m_W^2} + \dots \\ Z_\mu &= \frac{g_Z}{m_Z^2} J^Z_\mu + \dots \end{aligned}\right\} \quad \mathcal{L} \longrightarrow = -\frac{g_Z^2}{2m_Z^2} J^Z_\mu J^{Z\mu} - \frac{g^2}{2m_W^2} J^+_\mu J^{-\mu}$$

<1 (numerically he said)

$$\frac{1}{2}m_Z^2 Z^2 - g_Z Z J_Z + m_{W}^2 W^+ W^- - (W^+ \frac{g}{\sqrt{2}} J^- + h.c.)$$

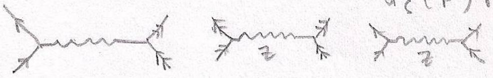
Recall: $m_W^2 = \left(\frac{gv}{2}\right)^2$; $m_Z^2 = \left(\frac{g_Z v}{2}\right)^2$

$$\Rightarrow \frac{g^2}{2m_W^2} = \frac{g_Z^2}{2m_Z^2} = \frac{2}{v^2}$$



$$-\frac{i4G_F}{\sqrt{2}}(\gamma^\mu P_L)_{\gamma\delta}(\gamma_\mu P_L)_{\beta\alpha}$$

$$i\mathcal{M}(e(p) + \nu(k) \to e(p') + \nu(k'))$$
$$= -\left(\frac{4iG_F}{\sqrt{2}}\right)\bar{u}_\nu(k)\gamma_\mu P_L u_e(p)\, \bar{u}_e(p')\gamma^\mu P_L u_\nu(k)$$



Olly 8.2 (No 8.1)

$$\mathcal{L} = -\frac{1}{4}(W^{a\,2}_{\mu\nu} + B^2_{\mu\nu}) + \bar{L}_L i\not{D} L_L + \bar{e}_R i \not{D} e_R + \frac{1}{2}m_Z^2 Z_\mu^2 + m_W^2 W^+_\mu W^{-\mu}$$
$$+ \dots\dots$$

Decoupling of Heavy Particles: $m_{heavy} \to \infty$ The amplitude with external light particles at least external heavy particles $\to 0$

$$\mathcal{L}_{eff} = -\frac{2}{v^2}(J^+_\mu J^{-\mu} + J^Z_\mu J^{Z\mu})$$
$$= -\frac{8}{v^2}(J^{cc}_\mu J^{cc\,\mu\dagger} + J^{NC}_\mu J^{NC\,\mu\dagger})$$

$\frac{G_F}{\sqrt{2}}$

Def^n: $J^{NC}_\mu \equiv 2J^Z_\mu$

$\because$ of projectors $\left(\frac{\gamma^5 - \gamma^5\gamma^\mu}{2}\right)$

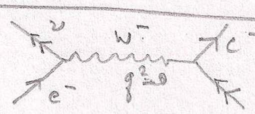
whereas $T_Z = T_{3L} - s_W^2 \dots$

Def^n: $J^{cc}_\mu \equiv 2J^+_\mu$

NB: $J^+_\mu = \bar{e}_L \gamma^\mu \nu_L = \bar{e}\gamma^\mu \underbrace{\left(\frac{1-\gamma^5}{2}\right)}_{\text{left projector}}\nu$

$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}}\left((\bar{e}\gamma^\mu(1-\gamma_5)\nu)(\bar{\nu}\gamma_\mu(1-\gamma_5)e)\right)$$
$$-\frac{G_F}{\sqrt{2}}\left(\bar{e}\gamma^\mu(1-\gamma^5)e + \dots\right.$$
$$\left. + \bar{\nu}_L\gamma^\mu(1-\gamma^5)(+\tfrac{1}{2})\nu_L\right)^2$$

Ex: do the same for



(Fermi) Effective Lagrangian for Electroweak processes at low energy $\ll 100\,\mathrm{GeV}$

1 gen of Leptons $\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}}\left(\frac{1}{2}(J^{cc}_\mu J^{\mu cc\dagger} + J^{cc\dagger}_\mu J^{\mu cc}) + J^{NC\dagger}_\mu J^\mu_{NC}\right)$

$$J^{cc}_\mu \equiv 2\,\bar{\nu}_L \gamma^\mu (1-\gamma_5) e$$

$$J^{NC}_\mu = 2\sum_f \bar{f}\gamma^\mu(\underbrace{T_3 P_L - Q_{em}\sin^2\theta_W}_{T_Z})f$$

Universal Fermi const. $\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2}$; $\langle\Phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}}\end{pmatrix}$

GLOBAL

$L \to L' = U_L^{(l)} L_L$

$l_R' = U_R^{(l)} l_R$ $\quad (U^\dagger = U^{-1})$

$\mathcal{L}$ is invariant!

$\mathcal{L}_{fermion} = \bar{L}_{LA}\, i\not{D} L_{LA} + \bar{l}_{RA}\, i\not{D}\, l_{RA} - (\bar{L}_{LA} Y^{(l)}_{AB} l_{RB}\Phi + h.c.)$

$\bar{L}_L \underset{\substack{\uparrow \\ 3\times 3 \text{ complex matrix}}}{Y^{(l)}} l_R \Phi$

$Y \to U_L Y U_R^\dagger$

$M^{(l)} \to \dots$; Y''

So now I have

$l_R = U_R^\dagger l_R'$

$L_L = U_L^\dagger L_L'$

$\bar{L}_L = \bar{L}_L' U_L$

Thm #1: $H = H^\dagger \quad H\epsilon_{(i)} = \lambda_{(i)}\epsilon_{(i)}$

$\Rightarrow \epsilon_{(i)}^\dagger \epsilon_{(j)} = \delta_{ij}$

$U = (\epsilon_{(1)}, \epsilon_{(2)}, \dots \epsilon_{(N)})$

$\Rightarrow U^\dagger U = \mathbb{1}_N$

$\Rightarrow U^\dagger H U = \mathrm{Diag}(\lambda_1 \dots \lambda_N)$

Mass of Leptons: Gauge Invariance ⊕ Renormalizability

$\langle\Phi\rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}}\end{pmatrix}$ $\begin{matrix} + \\ 0 \end{matrix}$ charge $(1,2,1)$

No direct masses are GI ⊕ R

$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \begin{matrix} 0 \\ - \end{matrix}$ $(1,2,-1)$

In the Unitary Gauge $\Phi \equiv \begin{pmatrix} 0 \\ \frac{v+h^0}{\sqrt{2}}\end{pmatrix}$

$\mathcal{L} = Y_e \bar{L}_L e_R \Phi + \cancel{h.c.}\ Y_e^* \Phi^\dagger \bar{e}_R L_L$

$\equiv \left(\frac{vY_e}{\sqrt{2}}\bar{e}_L\left(1+\frac{h^0}{v}\right)e_R + h.c.\right)$

$= -\left(m_e \bar{e}_L e_R + h.c. + \frac{m_e}{v} h \bar{e}_L e_R\right)$

You could for different generations, write

$$\mathcal{L} = -(Y_{eAB}\bar{L}_{LA} e_{RB}\Phi + Y^*_{eAB}\Phi^\dagger \bar{e}_{RB} L_{LA})$$

So $(M_L)_{AB} = \frac{Y^{(l)}_{AB} v}{\sqrt{2}}$

Physical reasoning:

No charged particle is massless, you run into trouble. ("Illustr" $m\frac{d^2x}{dt^2} = q(\vec{E} + v\times\vec{B})$)

Thm #2: Any complex mat $M \neq M^\dagger$, can be diagonalized by a bi-unitary transformation to a semi-positive diagonal form Λ, via

$V^\dagger M U = \Lambda \quad$ s.t. $\Lambda_{ii} = \Lambda^*_{ii} > 0$.

$H_1 = M^\dagger M = H_1^\dagger$

$H_2 = MM^\dagger = H_2^\dagger$

$R^\dagger(M^\dagger M)R$

claim: $M^\dagger M$ is semipositive (put $|M\psi|^2 > 0$ $\forall\psi$.

$\Rightarrow R^\dagger(M^\dagger M)R = \mathrm{Diag}(m_1^2, m_2^2, \dots, m_N^2)$

$\det(M) \neq 0$

m_i is real, I take +ve m_i wlg.

Question, does there exist a unitary L, s.t.

$LMR = \mathrm{Diag}(m_1, \dots m_N)$

given $\det M \neq 0 \Rightarrow M^{-1}$ exists

$L = \Lambda R^\dagger M^{-1}$

$L^\dagger = (M^{-1})^\dagger R\Lambda$

$LL^\dagger = \Lambda R^\dagger(M^{-1}M^{-1\dagger})R\Lambda$

$+ (M^\dagger M)^{-1} = (R\Lambda^2 R^\dagger)^{-1}$

$+ \qquad = R\Lambda^{-2}R^\dagger$

$\Rightarrow LL^\dagger = \mathbb{1}$

Lepton Mass in SM

$$\mathcal{L}_{fermion} = \underbrace{\bar{L}_A i \not{D} L_A + \bar{l}_{RA} i \not{D} l_{RA}}_{\text{Chiral}} + \underbrace{(\bar{L}_{LA} Y_{AB} \Phi l_{RB} + h.c.)}_{} + \cancel{\bar{L}_{LA} \tilde{\Phi} \nu_R}$$

$U_L^{(l)}(3) \times U_R^{(l)}(3)$ } symmetry of Kinetic Terms broken explicitly by Yukawa interaction

$\langle \Phi \rangle = \frac{v}{\sqrt{2}}$; $M^{(l)} = \frac{Y^{(l)} v}{\sqrt{2}}$ → from Φ

$L_A = (U_L^{(l)} L')_A$

$l_{RA} = (U_R^{(l)} l'_R)_A$

<copy from prashanca>

basically you show M is diagonalizable using the old thm (last class) & tre.

$m_\nu < 1eV$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}}(1+\frac{h_0}{v}) \end{pmatrix} = \begin{pmatrix} \frac{v}{\sqrt{2}}(1+\frac{h_0}{v}) \\ 0 \end{pmatrix}$$

$$\bar{L} \nu_R \tilde{\Phi}^2 = \frac{v}{\sqrt{2}}(1+\frac{h_0}{v}) \bar{\nu}_L \nu_R$$

$\nu_L \quad \nu_R$

$Y_{\nu_e} \sim 10^{-12}$

$Y_{\nu_\mu} \sim 10^{-9}$

$Y_{\nu_\tau} \sim 10^{-8}$

add to $\mathcal{L}_{fermion}$

$Y_{AB}^{(\nu)} \bar{L}_{LA} \nu_{RB} \tilde{\Phi} + h.c.$

$(1,2^*,+1) \quad (1,2,-1)$ $\phi = (1,2,1)$

$\phi^* = (1,2^*,-1)$

$$U^* = e^{-i\frac{\vec{\theta}\cdot\vec{\tau}^*}{2}} = e^{-\frac{i}{2}(\theta_1\tau_1 - \theta_2\tau_2 + \theta_3\tau_3)}$$

$$\& \quad \tilde{\Phi} \equiv \epsilon \Phi^* \Rightarrow \tilde{\Phi}' = \epsilon U^* \Phi^* = U \epsilon \Phi^* = U \tilde{\Phi}$$

$\sqrt{G_F} = \frac{1}{\sqrt{2} v}$ → $1.16 \times 10^{-5}\, GeV^{-2}$

$\Rightarrow v = 246.1\, GeV$

$m_e = 0.511 \times 10^{-3}\, GeV$

$m_\mu = 0.114\, GeV$

$m_\tau = 1.777\, GeV$

$Y_e \sim \frac{10^{-4}}{2.46 \times 10^2} = 10^{-6}$

$Y_\mu \sim 0.11 \times \frac{1}{2.46 \times 10^2} \sim 10^{-3}$

$Y_\tau \sim \frac{1.8}{2.46} \times 10^{-2}$

$$M_\nu' = U_L^{(\nu)\dagger} \frac{Y_V^{(\nu)}}{\sqrt{2}} U_R^{(\nu)}$$

$$M_l' = U_L^{(l)\dagger} \frac{Y_V^{(l)}}{\sqrt{2}} U_R^{(l)}$$

Show which in the kinetic terms (in $\not{D}$ i.e.)

$\bar{\nu}_L' U^{(\nu)\dagger} W^+ U_L^{(l)} l_L'$

$\sigma_{weak} = G_F^2 s$ ~ $10^{-10} GeV^{-4}$ (neutrino interaction)

$$\bar{\nu}_L' (1 + \frac{h_0}{v}) \Lambda^\nu \nu_R = \bar{\nu}_L'' P \Lambda^\nu \nu_R (1 + \frac{h_0}{v})$$

$$\bar{\nu}_L'' = \bar{\nu}_L' \underbrace{U_L^{(\nu)\dagger} U_L^{(l)}}_{P^\dagger} \Rightarrow \bar{\nu}_L' = \bar{\nu}_L'' P$$

so that the $\not{D}$ part remains same.

$$\mathcal{L}_{quark} = \bar{Q}_A i \not{D} Q_A + \bar{d}_R i \not{D} d_R + (\bar{Q}_{LA} Y_{AB}^{(d)} \Phi d_{RB} + h.c.)$$

$\bar{U}_R i \not{D} U_R \quad (\bar{3}, 2^*, -\frac{1}{3}) \quad (1,2,+1) \quad (3,1,-\frac{2}{3})$

$$D^{Quark} = \partial + ig W \cdot \frac{\tau}{2} + iB\frac{Y}{2} + ig_3 G^A T^A$$

$W^\pm \tau_\pm + Z \frac{\tau_3}{2}$; T^A → SU(3) generators

$$M^{(d)} = \frac{Y_V^{(d)}}{\sqrt{2}} \quad M^u = \frac{Y^{(u)} v}{\sqrt{2}} \Rightarrow M^{d'} = \Lambda^d = U_L^{(d)\dagger} M^{(d)} U_R^{(d)}$$

$$M^{u'} = \Lambda^u = U_L^{(u)\dagger} M^{(u)} U_R^{(u)}$$

(Some part maybe missing)

$C = U_L^{(u)\dagger} U_L^{(d)}$ Cabibbo Kobayashi Mas...

⇒ SM + fermion masses

⇒ No flavour changing Neutral Current ~~FCNC~~

⇒ Accidental $U(1)_B$

$m_\nu = 0 \quad U(1)_e \times U(1)_\mu \times U(1)_\tau$

Olly 9.1

SM

1) No flavour changing neutral current at tree level (FCNC)

$\mathcal{L}_{SM} = \mathcal{L}_{gauge+kinetic+Higgs} + \mathcal{L}_{Yuk}$

$Q \quad L \quad l_R \quad u_R \quad d_R$

$N_g = 3 \quad ; \quad f = U^{(f)} f'$

$\bar{Q}_A i \not{D} Q_A$

$U(3)_Q \times U_L(3) \otimes U_{l_R}(3) \otimes \cdots$

$N_g = 3$ (# of generation)

$u[\bar{L} Y_e \phi l_R] \rightarrow N_g \times N_g$ Break flavour symmetry

$+ \bar{Q} Y_d \tilde{\phi}^2 d_R$

$+ \bar{Q} Y_u \tilde{\phi} u_R$

$\rightarrow (\bar{d}_L Y^d d_R + \bar{U}_L Y^u U_R)\left(1 + \frac{h^0}{v}\right)\frac{v}{\sqrt{2}}$

$\begin{pmatrix} \nu \\ l^- \end{pmatrix}_A \quad -\frac{g_2}{\sqrt{2}} \left(\bar{\nu}_L^- W^+ l_L^-\right)$

ASK $\quad + \quad \bar{u}_L W^+ d_L^-$

don't cancel $\rightarrow U^{u_L^+} \quad U^{d_L}$

Cabbibo Kobayashi Maskawa

$V = C = U_{u_L}^{\dagger} U_{d_L}$

$\bar{U}_L i \not{D} U_L$

$-\frac{i g_2}{\sqrt{2}} C_{AB} \gamma^M \rightarrow$ off diagonal unitary

$d_{LA} \quad u_{LB}$

$O \subset U$

real $\quad$ complex

$e^A \quad e^{iH}$

$A^T = -A$

$\# \text{elements} = \frac{N(N-1)}{2}$

$\equiv$ angles

$\left(\because \begin{pmatrix} c\theta & s\theta \\ -s\theta & c\theta \end{pmatrix}\right)$

N^2 real components

Phases are

$N^2 - \frac{N(N-1)}{2} = \frac{N(N+1)}{2}$

N	# Phases
1	1
2	3
3	6

$O(N) \quad \frac{N(N-1)}{2}$ angles

$U(N) \quad '' + \frac{N(N+1)}{2}$ phases

$\mathcal{L} = m_A^u \bar{u}_{LA} u_{RA}\left(1 + \frac{h^0}{v}\right)$

$+ m_A^d \bar{d}_{LA} d_{RA}\left(1 + \frac{h_0}{v}\right)$

$-\frac{g_2}{\sqrt{2}}\left(\bar{u}_{LA} W^+ C_{AB} d_{LB} + h.c.\right)$

$-e\left(\bar{f}_L \not{A} Q_f f_L - \bar{f}_R \not{A} Q_f f_R\right)$

$g_Z Z Q_Z \left(\bar{u}_L \not{Z} U_L - \bar{u}_R \not{Z} U_R\right)$

Chiral Rephasing

$U_{LA} \rightarrow e^{i\theta_A^u} U_A$

$d_{LA} \rightarrow e^{i\theta_A^d} d_{LA}$

$C_{AB} \rightarrow e^{i(\theta_B^u - \theta_A^d)} C_{AB}$

$N + N = 2N$ # of parameters

$U_{AL} \rightarrow e^{i\theta} U_{LA}$

$d_{AL} \rightarrow e^{i\theta} d_{LA}$

$\therefore$ we can remove $2N-1$

Rephasing $U_L \, d_L$: $2N$ phases

1 phase freedom is $U(1)_B$

$2N-1$ effective rephasing freedom.

Physical $\frac{(N-2)(N-1)}{2}$ $\quad$ 1 → 0, 2 → 0, 3 → 1

$N_g = 1, \; c = 1$

$N_g = 2, \; N_\theta = 1, \; N_\varphi = 0$

$\begin{pmatrix} c\theta_c & s\theta_c \\ -s\theta_c & c\theta_c \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}$

Real + ve

$\mathcal{L}_{Yuk} = \left(m_A^u \bar{u}_{LA} u_{RA} + m_A^d \bar{d}_{LA} d_{RA}\right)\left(1 + \frac{h_0}{v}\right) + h.c.$

$N_g = 3, \; N_\theta = 3, \; N_\varphi = 1$

Standard Parametrization CKM matrix

$C = R_{23}(\theta_{23}) \begin{pmatrix} e^{-i\delta} & & \\ & 1 & \\ & & 1 \end{pmatrix} R_{13}(\theta_{13}) \begin{pmatrix} e^{i\delta} & & \\ & 1 & \\ & & 1 \end{pmatrix} R_{12}(\theta_{12})$

$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}$

Experimentally

$\lambda = \sin\theta_{12} = .23 \pm 0.003$

$\sin\theta_{13} = 0.004 \pm 0.002$

$\sin\theta_{23} = 0.04 \pm 0.005$

$\delta \approx 60° \pm .2$

$s_{13} \sim \lambda^3$

$s_{23} \sim \lambda^2$

Mod of C

$\begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & \lambda^3 \\ \lambda^2 & 1 - \frac{\lambda^2}{2} & \lambda^2 \\ \lambda^3 & \lambda^3 & 1 - \lambda^4 \end{pmatrix}$ Wolfenstein

Cabibo.l

$|V_{AB}| = \begin{pmatrix} 0.9742 & .219 & 0.004 \\ 0.22 & 0.97 & 0.04 \\ 0.004 & 0.004 & 0.999 \end{pmatrix}$

$C_{Wolfestein} = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$

$\lambda \ll 1$

$A, \rho, \eta \sim 1$

Discrete Symmetry

Lorentz invariance $\oplus$ QFT $\Rightarrow$ CPT invariance

$\mathcal{L} = \bar{\Psi} i \not{\partial} \Psi - m \bar{\Psi}\Psi$

$\Psi(x) \rightarrow \Psi'(x) = \Psi_p(x) = \eta_P \gamma^0 \Psi(t, -\vec{x})$ $\quad$ $\Lambda_P^{-1} x$

$\mathcal{L}$ is invariant $\quad (\eta_P)$

$A_M(x)\, dx^M = A'_\nu(x')\, dx'^\nu$

$A'_\nu(x') = A_M(x) \frac{\partial x^M}{\partial x'^\nu}$

$= \Lambda_\nu{}^M A_M(x)$

$A'_\nu(x) = \Lambda^M_\nu A_M(\Lambda^{-1} x)$

$x' = \nu x$

$A'_\mu(x) = (\Lambda_P)^\nu{}_\mu A_\nu(x')$

$\vec{E} = -\vec{\nabla} A_0 - \partial_0 \vec{A} \quad \vec{B} = \nabla \times A$

$A'_i(x) = -A_i(x')$

$A'_0(x) = +A_0(x') \quad x' = \Lambda x$

Olly 9.2

$$\text{“}\mathcal{L}_{SM}\text{”} = -\frac{1}{4}\underset{g,g',g_3}{F^2} + \underbrace{\bar{F}_L i\not{D} F_L}_{U(N_g)_Q \times U(N_g)_L} + \bar{f}_R i\not{D} f_R + |D\Phi|^2 - \underset{(\lambda, v)}{V(\Phi)} + \mathcal{L}_{Yuk} \qquad |m_\nu| < \text{keV}$$

Quark lepton

$$\underbrace{U_R(N_g) \times U_{\nu_R}(N_g) \times U_{d_R}(N_g)}_{\text{global}}$$

$$\mathcal{L}_{FC\ \text{terms}} = -\frac{g}{\sqrt{2}}\left(\bar{u}_L C W^+ d_L + \bar{d}_L C^+ W^- u_L\right)$$

If ν_R exist $\rightarrow -\frac{g}{\sqrt{2}}\left(\bar{\nu}_L C W^+ e_L + h.c.\right)$

$$\text{“}\mathcal{L}_{yuk}\text{”} = \text{“}\bar{F}_L Y f_R \Phi + \Phi^+ \bar{f}_R Y^+ F_L$$

$\rightarrow$ Unitarity Gauge

$$\sim\left(1 + \frac{h_0}{v}\right)\frac{v}{\sqrt{2}}$$

$$M = \frac{v}{\sqrt{2}} Y \qquad \bar{L}_L \Phi l_R \rightarrow \bar{l}_{LA} l_{RA} m_A \left(1 + \frac{h_0}{v}\right)$$

Diagonal Λ $\qquad \bar{u}_L u_R \qquad \bar{d}_L d_R$

SM: $SU(2)_L \times U(1)_Y \rightarrow$ chiral gauge theory (treats F_L, f_R differently)

$SU(3)_{color} \times U(1)_{em} \rightarrow$ vector like f_L, f_R same.

SM

3 gauge g, g', g_3

2 λ, v

9 Masses $m_u(3)$

$m_d(3)$

$m_e(3)$

4 Q Mixing CKM $(\theta_{12}, \theta_{23}, \theta_{13}, \delta_{CP})$

2+1? M_ν Δm^2_{21} Δm^2_{32}

3+(3!) Lepton Mixing $\theta^L_{12,23,13}$

23+(4?)

$m_1 = ??$

1% accuracy δ^L ??

α_1

α_2

$$\mathcal{L}_{Dirac} = \bar{\Psi}(\not{\partial} - m)\Psi$$

Parity $\Psi(x) \rightarrow \Psi_P(x) = \eta_\Psi \gamma^0 \Psi(x')$ $\quad x' = (t, -\vec{x})$

$$\gamma^5 = \begin{pmatrix} -\mathbb{1}_2 & \emptyset_2 \\ \emptyset_2 & \mathbb{1}_2 \end{pmatrix}; \quad \gamma^\mu = \begin{pmatrix} \emptyset_2 & \sigma^\mu \\ \bar{\sigma}^\mu & \emptyset_2 \end{pmatrix}$$

∂ ✓ inv

Ex: QED ✓ inv

QCD inv

$$\Psi_{L/R}(x) \xrightarrow{P} \eta_\Psi \gamma^0 \Psi_{R/L}(x')$$

$$P_L = \left(\frac{1-\gamma^5}{2}\right) = \begin{pmatrix} \mathbb{1}_2 & \emptyset \\ \emptyset & \emptyset \end{pmatrix} \qquad \sigma^\mu = (1, \sigma^i) \qquad \bar{\sigma}^\mu = (1, -\sigma^i)$$

$$A_\mu(x) \xrightarrow{P} (A_0(x'), -\vec{A}(x'))$$

$$G^A_\mu(x) \xrightarrow{P} (G^A_0(x'), -\vec{G}^A(x'))$$

$$P_L\Psi = \begin{pmatrix} \Psi_L \\ 0 \end{pmatrix}$$

$$P_R = \frac{1+\gamma^5}{2} = \begin{pmatrix} \emptyset & \emptyset \\ \emptyset & \mathbb{1}_2 \end{pmatrix} \qquad P_R\Psi = \begin{pmatrix} 0 \\ \Psi_R \end{pmatrix}$$

$$C: \Psi \xrightarrow{C} \Psi^c(x) = \eta \gamma^2 \Psi^*(x)$$

γ^2 not a square!

$$A_\mu \xrightarrow{C} -A_\mu; \quad \Phi \xrightarrow{C} +\Phi^*$$

$$\mathcal{L}_{QED/QCD} \xrightarrow{C} \mathcal{L}$$

SM: P, C maximally violated!

$$\Psi \xrightarrow{P} \eta_P \gamma^0 \Psi(x')$$

$$\downarrow C$$

$$\eta_P \eta_C \gamma^0 \gamma^2 \Psi^*(x')$$

$$\Psi_{L/R} \rightarrow \eta_P \eta_C \gamma^0 \gamma^2 \Psi^*_{L/R}$$

$\bar{F}_L i\not{D} F_L$, $\bar{f}_R i\not{D} f_R$ are CP invariant

Kinetic Gauge Terms are Yukawa Invariant

$$\mathcal{L} = y \bar{Q}_L \tilde{\phi} U_R + y^* \tilde{\phi}^* \bar{U}_R Q_{La}$$

$$= y \underbrace{\bar{Q}_{La} \epsilon_{ab} \phi^*_b}\, U_R + y^* \underbrace{\phi_b \epsilon_{ab} \bar{U}_R Q_{La}}$$

$$\downarrow CP$$

$$= y \qquad + y^*$$

CP

$$Q_L \rightarrow \eta^Q_{CP} \gamma^2 \gamma^0 Q^*_L$$

$$Q^*_L \rightarrow$$

$$u_R \rightarrow \eta^u_{CP} \gamma^2 \gamma^0 u^*_R$$

$$\phi_a \rightarrow \phi^*_a$$

$\mathcal{L} \xrightarrow{CP} \mathcal{L}' = \mathcal{L}$ iff $y^* = y$

δ_{CP} { $N_g = 3$ there is one phase in Y^u, d which is irremovable by rephasing

δ_{CP} changes under rephasing!

Rephasing invariant measure of CP violation

Jarlskog Invariant (Rephasing)

$$C^+C = CC^+ = \mathbb{1}$$

$$\Rightarrow (CC^+)_{ij} = (C^+C)_{ij} = 0 \qquad i \neq j \quad \begin{matrix} 12 \\ 13 \\ 23 \end{matrix}$$

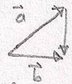
$$A_{ij} = C_{ik} C^+_{kj}; \quad C_{ik} C^*_{jk} = A_{ij} \quad i \neq j$$

$$= C_{i1} C^*_{j1} + C_{i2} C^*_{j2} + C_{i3} C^*_{j3}$$

$$= a_{ij} + b_{ij} + c_{ij} = 0 \qquad ij = ((1,2), (1,3), (2,3))$$

So now $a + b + c = 0$

$\Rightarrow$ on a plane then



CP 6 unitarity triangles

<some missing ending part>

Olly 9.3

SM: Only one complex parameter phase $e^{i\delta}$ in $V_{CKM} = C$

Celiba Jarlskog Invariant

$\mathrm{Im}(C_{ij} C_{kl} C^*_{il} C^*_{kj}) = J \sum_{mn} \epsilon_{ikm} \epsilon_{jln}$ $\quad \forall\, i,j,k,l$

$J = c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin\delta$

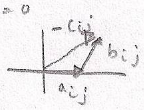
$\bar{u}'_{LA} C_{AB} d_{LB}$

$\bar{u}'_{LA} (e^{-i\theta^u_A} C_{AB} e^{i\theta^d_B}) d'_{LB}$

$(CC^\dagger)_{ij} = (C^\dagger C)_{ij} = 0 \quad i \neq j$

$C_{i1} C^\dagger_{1j} + C_{i2} C^\dagger_{2j} + C_{i3} C^\dagger_{3j} = 0$

$a_{ij} + b_{ij} + c_{ij} = 0$



$A = \frac{1}{2}|a|h = \frac{1}{2}|a||b|\sin\theta = \frac{1}{2}(\vec{a}\times\vec{b})$

$= \frac{1}{2}(a_x b_y - a_y b_x) = \frac{1}{2}(r_a I_b - r_b I_a)$

$= \frac{1}{2}\mathrm{Im}(a^* b)$

$a^* b = (r_a - iI_a)(r_b + iI_b)$

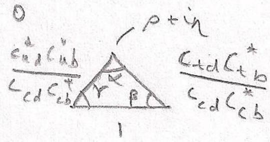
$i\,\mathrm{Im}(a^* b) = i(r_a I_b - I_a r_b)$

$\sum_i C_{i1} C^*_{i3} = (C^\dagger C)_{31}$

$C_{11} C^*_{13} + C_{21} C^*_{23} + C_{31} C^*_{33} = 0$

$\lambda \quad \lambda^2 \quad \lambda^3 \quad 1$

$\frac{C_{11} C^*_{13}}{C_{21} C^*_{23}} + 1 + \frac{C_{31} C^*_{33}}{C_{21} C^*_{23}} = 0$



Ex1: Write out the area of any two Δ & show they give the same J.

Ex2: Show area of all 6 unitarity Δs is $J/2$ (before dividing to change side to 1)

$m_H \sim \sqrt{\lambda}\, v$

	g', g, g_3 — $\alpha_e, \sin^2\theta_W, \alpha_3$ — gauge coupling
$m_H \sim \sqrt{\lambda}\, v$	$\lambda \quad v$ — G_F — masses
	$m_{u,d,l}$
Quark Mixing	$\theta^{CKM}_{12}, \theta^{CKM}_{13}, \theta_{23}, \delta_{CP}$

Neutrino Masses

Dirac Type mass

Majorana Type Mass

$\mathcal{L} \ldots \quad \mathcal{L} = \bar{\Psi}(i\partial\!\!\!/ - m)\Psi$

$\mathcal{L}_{mass} = -m\bar{\Psi}\Psi$

$= -m(\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L)$

$\Psi = \Psi_L + \Psi_R$

$\Psi_{L,R} = P_{L,R}\Psi$

Charge Conjugation Matrix

$C = i\gamma^2\gamma^0$

$= i\begin{pmatrix} 0 & \sigma^2 \\ -\sigma^2 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$= i\begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} = \begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}$

$\Psi'(x') = S_L \Psi(x)$

$\exp\frac{i}{2}\omega^{\mu\nu} J_{\mu\nu}$

$\frac{\sigma_{\mu\nu}}{2}$

$\Psi'(x) = S_L \Psi(\Lambda^{-1} x)$

$\bar{\Psi}'\Psi' = \Psi^\dagger \gamma^0 \Psi' = \Psi^\dagger S^\dagger \gamma^0 S \Psi$

$= \Psi^\dagger \gamma^0 S^{-1} S \Psi = \bar{\Psi}\Psi$

Ex: $S^T C S = C$

So I can write

$\Psi^T C \Psi$ & that's Lorentz invariant

$\mathcal{L} = \frac{1}{2}\bar{\Psi} i\partial\!\!\!/\Psi - \frac{m_{maj}}{2}(\Psi^T C\Psi + h.c.)$

This does not have $U(1)$ symmetry.

Plus, note $\Psi^T C(\Psi_L + \Psi_R) = \Psi_L^T C \Psi_L + \Psi_R^T C \Psi_R$

$\because C = i\gamma_2\gamma_0$

$\Psi = \begin{pmatrix} \chi \\ \xi \end{pmatrix}; \quad \bar{\Psi}\Psi = (\chi^\dagger, \xi^\dagger)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \chi \\ \xi \end{pmatrix} = \chi^\dagger\xi + \xi^\dagger\chi$

$= \Phi^T\xi + \xi^\dagger\Phi^*$

$SL(2,C) \approx so(1,3)$

$\delta\xi_\alpha = i(\vec{\omega}_L\cdot\vec{T})_\alpha{}^\beta \xi_\beta$

$1,2 \quad$ complex $\quad (\frac{1}{2}, 0)$

$\delta\chi_{\dot\alpha} = i(\vec{\omega}_R\cdot\vec{T})_{\dot\alpha}{}^{\dot\beta}\chi_{\dot\beta}$

Dirac: $(\frac{1}{2}, 0)\oplus(0,\frac{1}{2})$

Majorana/Weyl: $(\frac{1}{2},0)$ or $(0,\frac{1}{2})$

Majorana Spinors

$\begin{pmatrix} \chi \\ \xi \end{pmatrix} = \Psi = \Psi^c \qquad \Psi^c \equiv C\bar{\Psi}^T$

$= C\gamma^0\Psi^*$

$= i\gamma^2\Psi^*$

$= \begin{pmatrix} 0 & \epsilon \\ -\epsilon & 0 \end{pmatrix}\begin{pmatrix} \chi^* \\ \xi^* \end{pmatrix}$

$= \begin{pmatrix} \epsilon\xi^* \\ -\epsilon\chi^* \end{pmatrix}$

$\boxed{\Rightarrow \xi = -\epsilon\chi^*}$

$\Psi_{maj} = \begin{pmatrix} \chi \\ -\epsilon\chi^* \end{pmatrix}$

$\Psi^T C\Psi = (\chi^T, -\chi^\dagger \cancel{\epsilon^T})\begin{pmatrix} \epsilon & 0 \\ 0 & -\epsilon \end{pmatrix}\begin{pmatrix} \chi \\ -\epsilon\chi^* \end{pmatrix}$

$= (\chi^T, \epsilon\chi^\dagger)\begin{pmatrix} \epsilon\chi \\ -\chi^* \end{pmatrix} = \chi^T\epsilon\chi - \chi^\dagger\epsilon\chi^*$

$\delta(\chi^T\epsilon\xi) = \delta\chi_\alpha\epsilon^{\alpha\beta}\xi_\beta + \chi_\alpha\epsilon^{\alpha\beta}\delta\xi_\beta$

$= -i(\vec{\omega}_L\cdot\vec{T})^{\beta\gamma}\chi_\gamma\xi_\beta + \chi_\alpha i(\vec{\omega}_L\cdot\vec{T})^{\alpha\gamma}\xi_\gamma$

where $\xi^\alpha \equiv \epsilon^{\alpha\beta}\xi_\beta$

$= i\chi^T(-(\vec{\omega}\cdot\vec{T})^T + (\vec{\omega}\cdot\vec{T}))\xi$

$= 0$

$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} \omega_3 & \omega_- \\ \omega_+ & -\omega_3 \end{pmatrix}$

$= \begin{pmatrix} \omega_+ & -\omega_3 \\ -\omega_3 & \omega_- \end{pmatrix}$

Olly 9.4

Neutrino Masses Rot Boost $\vec{\theta}, \vec{\beta}$; $\vec{\theta} + i\vec{\beta}$

Lie Gp : $SO(1,3) \longleftrightarrow SO(4) \simeq SU(2) \times SU(2)$

(j_1, j_2) ; $d(j_1, j_2) = (2j_1+1)(2j_2+1)$

Fermion Mass Term

$SU(2) \oplus SU(2)$

$(\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \oplus (\frac{1}{2}, \frac{1}{2})$

$\Psi_L \quad \Psi_R$

$\xi_\alpha \quad \bar{\xi}_{\dot{\alpha}}$

$M = e^{\vec{\omega} \cdot \vec{T}}$ Rep of $SO(1,3)$

$SL(2, C)$

Z-component Notation for Lorentz spinors

$\epsilon^{\beta\alpha} = \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix}$ $\quad \xi \xrightarrow{LT} \xi'_\alpha = M_\alpha{}^\beta \xi_\beta$

$SL(2, C)$

$\xi^\beta \chi_\beta = \xi_\alpha \epsilon^{\beta\alpha} \chi_\beta = \xi\chi \equiv \chi\xi$

$\xi\chi$ anti-commute

$-\chi_\beta (-\epsilon^{\alpha\beta}) \xi_\alpha$

Dirac Fermion

$\psi = \begin{pmatrix} \chi \\ \bar{\xi} \end{pmatrix}$

$\psi^\dagger \gamma^0 = (\chi^\dagger, \bar{\xi}^\dagger) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = (\bar{\xi}^\dagger, \chi^\dagger)$

$\bar{\Psi}\Psi = \bar{\xi}^\dagger \chi + \chi^\dagger \bar{\xi} = \bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L = \xi\chi + \bar{\chi}\bar{\xi}$

$\bar{\chi} \quad \bar{\xi}$

$\psi^c = C\bar{\Psi}^T = C\gamma_0{}^T \psi^* = i\gamma^2\gamma^0\gamma^{0T}\psi = \begin{pmatrix} 0 & \epsilon \\ -\epsilon & 0 \end{pmatrix} \begin{pmatrix} \chi^* \\ \xi^* \end{pmatrix}$

$= \begin{pmatrix} \epsilon\xi^* \\ -\epsilon\chi^* \end{pmatrix}$

Majorana Spinor

$\psi = \psi^c = \begin{pmatrix} \chi \\ \bar{\xi} \end{pmatrix} = \begin{pmatrix} \epsilon\xi^* \\ -\epsilon\chi^* \end{pmatrix}$

$= \begin{pmatrix} \chi \\ -\epsilon\chi^* \end{pmatrix}$

$\boxed{\bar{\xi} = -\epsilon\chi^*}$

$\xi' = M\xi$; M: Recall

$(\epsilon\xi)'^* = \epsilon M^* \xi^*$

$\epsilon M^* = e^{-\vec{\omega}_* \cdot \vec{T}} \epsilon = \bar{M}'\epsilon$; $\bar{M} = e^{\vec{\omega}_* \cdot \vec{T}}$

$M^T \epsilon = e^{\vec{\omega} \cdot \vec{T}^T} \epsilon = \epsilon e^{-\vec{\omega} \cdot \vec{T}}$

Claim

$A'_{\alpha\dot{\alpha}} = M_\alpha{}^\beta \bar{M}_{\dot{\alpha}}{}^{\dot{\beta}} A_{\beta\dot{\beta}}$

$A_\mu = (\sigma_\mu)^{\alpha\dot{\alpha}} A_{\alpha\dot{\alpha}}$

$A_{\alpha\dot{\alpha}} = (\bar{\sigma}_\mu)^{\dot{\alpha}\alpha} A_\mu$

Neutrino Mass

End pt of β decay → V-A interaction ⇒ involves ν_{eL}

$m_{\nu e} + m_{\nu\mu} + m_{\nu\tau} \leq 1 eV$ } Cosmology

$(0, \nu) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = (-\nu, 0)$

3) In SM, even without introducing

<copy from prashant a>

Options for Neutrino Mass

SM: ν_R $(1, 1, 0)$ — $SU(3)$, $SU(2)_L$, $U(1)$: $M\nu_R (\nu_R^T C \nu_R) + h.c. = \mathcal{L}_{M_{\nu_R}}$

$Q = T_{3L} + \frac{Y}{2} = 0$, with $T_{3L} = 0$, $\frac{Y}{2} = 0$

$H(1, 2, +1)$ $\quad (1, 2, -1)$

Dirac: $\mathcal{L}^\nu_{mass} = l^\nu_{AB} \bar{\nu}_{RA} \langle\phi\rangle^T \epsilon L_{LB} + h.c.$ $\quad (1, 2^*, +1)$

Majorana Mass ν_R $\quad$ Big $M_{\nu_R} \gg 1 eV$ $\quad$ integrate out $M_{\nu_R} \Rightarrow \mathcal{L}_{eff} = \frac{M}{2} \nu_L \nu_L^T C \nu_L$

Small $M_{\nu_R} \lesssim 1 eV$

$\boxed{M_{\nu_L} = (m_\nu^D M_{\nu_R}^{-1} m_{\nu_L}^D)}$

$m_\nu^{Dirac} = \frac{h_{AB} V}{\sqrt{2}}$