Olly    14·3 (no 14·2) | (missed 14·1; wedding)

PCAC (Partially Conserved Axial Current)

$\alpha_S(\mu) \to \infty$, $\mu \to \Lambda_{QCD}$ } 1) Confinement of "Quarks & Gluons" ≡ Colour
2) Condensation of $\bar{Q}Q$ pairs ⇒ breaking of "$SU(2)_A$"

Lightest Hadrons $\left\{ \begin{array}{l} m_{\pi^\pm} \approx 139.5 \text{ MeV} \\ m_{\pi^0} \approx 135 \text{ MeV} \end{array} \right\}$ $\ll m_N$, $< \Lambda_{QCD}$

3 goldstone bosons ⇐ X in Hadron Spectrum

$m_q \neq 0$

$Q \begin{pmatrix} u \\ d \end{pmatrix}$

$m_q \neq 0 \Rightarrow \pi^a$ are "pseudo-goldstone" bosons.

$j^{\mu a}_A = \bar{Q}\gamma^\mu\gamma^5\pi^a Q$

$\partial_\mu j^{\mu a}_A \sim m_a$

π decay: $\pi^- \sim (\bar{u}^{i\dagger} d_i^\dagger)|0\rangle$


(no τ, mass conservation)

$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} J^{Had+}_\mu J^{\mu -}_{Lepton}$

$(2\,\bar{d}_{LA}\gamma^\mu u_{LB} V^\dagger_{AB})$ + $\bar{u}_A V^{CKM}_{AB}\gamma^\mu d_{LB}$ + $\bar{d}_{LB}\gamma^\mu V^*_{AB} u_A$, $V^*_{AB} = (V^\dagger)_{BA}$

axial current

$2\,\bar{d}_L\gamma^\mu\left(\frac{1-\gamma_5}{2}\right)u_L$

$J_\mu^{\ a} = \bar{Q}\gamma^\mu\gamma^5\pi^a Q$ → $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} \bar{u}()d + \bar{d}()u \\ i(-\bar{u}()d + \bar{d}()u) \\ \bar{u}()u + \bar{d}()d \end{pmatrix}$ → 1, → 2

$1+i2 = \bar{u}()d$ → ⊖

$\frac{1-i2}{\sqrt{2}} = ⊕$

---

$\langle 0|J^{\mu a}_A(x)|\pi^b(p)\rangle = -i\,p^\mu f_\pi \delta^{ab} e^{-ipx}$

dim −1    Pion Decay Const.

$U(1)$

$j^\mu \sim i\phi^\dagger \overleftrightarrow{\partial_\mu}\phi$

$= vi(\langle\phi^\dagger\rangle\partial_\mu\phi - \partial_\mu\phi^\dagger\langle\phi\rangle)$

$\sim (VEV)\,\partial_\mu(GB)$ + Quadratic

$B = \int \frac{d^3K}{\Gamma}\left(a_k e^{-ik\cdot x} + a_k^\dagger e^{ikx}\right)$

$J^+_{\mu\,Had} J^{\mu -}_{lept}$

$\bar{l}_{LA}\gamma^\mu(1-\gamma_5)\nu_{LA}(f_\pi\partial_\mu\pi^-)$

$\langle l^-\bar{\nu}_e|\mathcal{L}_{eff}(x)|\pi^-(p)\rangle$

$\sim \langle l^-\bar{\nu}_e|\bar{l}_A\gamma^\mu(1-\gamma_5)\nu_B|0\rangle\langle 0|f_\pi\partial_\mu\pi^-|\pi^-(p)\rangle$

$i\mathcal{M}_F(\pi^- \to l + \bar{\nu}_e) = \int d^4x\,\langle l^-\bar{\nu}_e| - i\frac{2G_F}{\sqrt{2}}\bar{l}_L(x)\gamma^\mu\nu_L(x) f_\pi\partial_\mu\pi(x)|\pi^-(p)\rangle$ → $V^*_{ud}$

$= \frac{\pi}{\sqrt{2E_eE_\nu}} G_F f_\pi \delta^{(4)}(p-k_l-k_{\bar{\nu}})\,\bar{u}_L(\not{k}_e + \not{k}_\nu)\nu(k_{\bar{\nu}})$

$\not{k}_e \to m_e$

$\not{p}u = mu$

$\mathcal{L} = hB\bar{f}f$    $B\text{-}ff$

$T = \frac{Mh^2}{8\pi}\left(1-\frac{(m_1+m_2)^2}{M^2}\right)\left(1-\frac{(m_1-m_2)^2}{M^2}\right)$, $\left(1-\frac{(2m)^2}{M^2}\right)^{3/2}$

$h \hookrightarrow \sqrt{2}G_F f_\pi m_e$

for non anti-particle $\left(1-\frac{m_e^2}{M^2}\right)^2$

---

$T(\pi^- \to l^-\bar{\nu}_e) = \frac{m_\pi|V_{ud}|^2}{4\pi}G_F^2 f_\pi^2 m_e^2\left(1-\frac{m_e^2}{m_\pi^2}\right)^2$

$T = 26.033$ ns
$1s = 1.52 \times 10^{25}\ \text{GeV}^{-1}$
$m_\mu = 0.1056$ GeV
$m_e = 0.495$ MeV
$m_\pi = 0.13956$ GeV
$V_{ud} = 0.9745$

$\Rightarrow f_\pi = 91.1$ MeV

$\frac{\Gamma(\pi^- \to e^-\bar{\nu}_e)}{\Gamma(\pi^- \to \mu^-\bar{\nu}_\mu)} = \frac{m_e^2}{m_\mu^2}\frac{\left(1-\frac{m_e^2}{m_\pi^2}\right)^2}{\left(1-\frac{m_\mu^2}{m_\pi^2}\right)^2}$

$\left(\frac{0.5\text{ MeV}}{105.6\text{ MeV}}\right)^2$

$\approx 10^{-4}$

---

$m_{u,d} < \Lambda_{QCD}$
$SU(2)_A \Rightarrow 3$ G.B.

$m_{u,d,s} \ll \Lambda_{QCD}$

$K^0\bar{K}^0$, $K^+, K^-$ $\left\{ \begin{array}{l} SU(3)_A \to X \\ 8 \text{ G.B.} \end{array} \right.$

Light Hadrons governed by $SU(3)_F$ & $SU(3)_A$ (pert brok)

Heavy Hadrons $\frac{\Lambda_{QCD}}{m_{\text{Heavy Quark}}}$

---

<u>Pion Masses</u>

$\mathcal{L}^{m_f=2}_{QCD} = \bar{Q}i\not{D}Q - \bar{Q}mQ$, $m = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}$

EOM $i\not{D}Q = mQ$ + h.c.

$\partial_\mu j^{\mu a}_A = i\bar{Q}\gamma^5\{T^a, m\}Q$

$\langle 0|\partial_\mu j^{\mu a}_A(x)|\pi^b(p)\rangle = (-i)^2 p^2 f_\pi \delta^{ab} e^{-ip\cdot x}$

$= \langle 0|\bar{Q}i\{m, T^a\}\gamma^5 Q(x)|\pi^b(p)\rangle$

$= SU(2)_A$ inv $\cdot\ \text{tr}\{m, T^a\}T^b$

Non-perturbative

Olly 14·4

**Pion Mass**

$i\not{\partial} Q = m Q$, $m = \mathrm{Diag}(m_u, m_d)$

$\partial_\mu j_A^{\mu a} = i \bar{Q}\gamma^5 \{T^a, m\} Q$

$j_A^{\mu a} = \bar{Q}\, T^a \gamma^\mu \gamma 5 Q$

$= \bar{Q}_R T^a \gamma^\mu Q_R - \bar{Q}_L T^a \gamma^\mu Q_L$

$\langle 0 | \partial_\mu j_A^{\mu a} | \pi^b(p)\rangle = -p^2 f_\pi \delta^{ab} e^{-ip\cdot x}$

$= \langle 0 | i \bar{Q} \{T^a, m\} Q | \pi^b(p)\rangle$

$p^2 = m_\pi^2$

$= M^2 \delta^{ab}\, \mathrm{tr} \{m, T^a\} T^b$, with $\{m, T^a\}T^b = \frac{1}{3}(m_u+m_d)\delta^{ab} + i\epsilon^{abc}\,\mathrm{tr}\left(m_u\left(\frac{1+T^3}{2}\right) + m_d\left(\frac{1-T^3}{2}\right)\right)$, $\; i\epsilon^{ab3}(m_u - m_d)$

$= M^2 \delta^{ab}(m_u+m_d) + \text{small}$

$M = \sqrt{\frac{m_\pi^2 f_\pi}{m_u+m_d}} \simeq 400\ \mathrm{MeV}$

roughly $\sim \Lambda_{QCD} = \Lambda_{QCD} \times$ same #

When V: $\vec{\theta}_L = \vec{\theta}_R$ ; $\frac{\vec{\theta}_L + \vec{\theta}_R}{2} = \vec{\theta}_V$

A: $\vec{\theta}_L = -\vec{\theta}_R$ ; $\frac{\vec{\theta}_R - \vec{\theta}_L}{2} = \vec{\theta}_A$

(symmetry breaks)

Quark Mass — Inside Nucleon: effective mass at low energy in QCD vac: Constituent mass

— SM mass: Current Mass.

Gell-Mann $\sigma$ model (linear)

Introduce fields $\sigma$, $\pi^a$ ← 3 gauge bosons

$\langle\sigma\rangle$ : $SU(2)_A \to \times$

$\Sigma = \frac{\sigma + i\pi^a T^a}{\sqrt{2}}$ ; $\mathrm{tr}\,\Sigma^\dagger \Sigma = \frac{1}{2}(2\sigma^2 + 2\vec{\pi}^2)$

$N = \begin{pmatrix} p \\ n \end{pmatrix}$ ; $Q = \begin{pmatrix} u \\ d \end{pmatrix}$ ; $\mathcal{L}_{chiral} = \mathcal{L}_{\sigma\text{-model}} = \frac{1}{2}\mathrm{tr}\,\partial_\mu \Sigma^\dagger \partial^\mu \Sigma + \frac{\lambda}{2}\left(\mathrm{tr}\,\Sigma^\dagger\Sigma - f_\pi^2\right)^2$

$- g\left(\bar{Q}_L \Sigma Q_R + \bar{Q}_R \Sigma^\dagger Q_L\right)$

$\Sigma \to U_L \Sigma U_R^\dagger$ ($U$: 2x2 unitary)

$\mathrm{tr}\langle\Sigma^2\rangle = f_\pi^2 \Leftarrow$ SSB with $\langle\sigma\rangle = f_\pi$

$\langle\Sigma\rangle = \frac{\langle\sigma\rangle}{\sqrt{2}}\mathbb{1}$

$\langle\Sigma'\rangle = \frac{\langle\sigma\rangle}{\sqrt{2}} U_L U_R^\dagger$

Consequently

$\mathcal{L}_{chiral} = \frac{1}{2}\mathrm{tr}\,\partial_\mu\Sigma^\dagger\partial^\mu\Sigma + \frac{\lambda}{2}\left(\mathrm{tr}\,\Sigma^\dagger\Sigma - f_\pi^2\right)^2 - g\left(\bar{Q}_L(\langle\Sigma\rangle + \hat{\Sigma})Q_R + \bar{Q}_R \hat{\Sigma}^\dagger Q_L\right)$

$\mathcal{L}_{mass} = -g\langle\sigma\rangle(\bar{Q}_L Q_R + h.c.)$

$g\langle\sigma\rangle$ → constituent $m_Q$.

$N = qqq'$

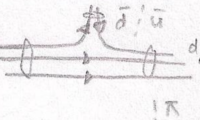
$p = uud$

$n = udd$

$\pi^+ = (\bar{d}^i u_i)$

$\pi^- = (u^i \bar{d}_i)$

$\pi^0 = \left(\frac{\bar{u}^i u_i + \bar{d}^i d_i}{\sqrt{2}}\right)$





$g_{\pi^0 pp}$

**Goldberger Treiman Relation**

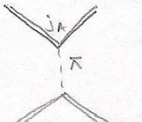
$\langle N' | \tilde{j}_A^{\mu a}(q) | N\rangle = \bar{u}(p')\left[\gamma^\mu\gamma 5 F_1^A(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}\gamma^5 F_2^A(q^2) + q^\mu \gamma 5 F_3^A(q^2)\right] T^a u(p)$

$F_1^A(0) = g_A$ ← Not fixed by any conserved charge (unlike $F_1^V(0) = Q_{em}$)

$m_q \approx 0$ $\quad q_\mu j_A^\mu(q) = 0$, $q = p' - p$ $\Rightarrow 0 = \bar{u}(p')\left((\not{p}' - \not{p})\gamma^5 F_1^A + q^2\gamma 5 F_3^A(q^2)\right)T^a u(p)$

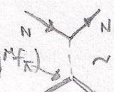
$= \bar{u}(p')\left((M_N\gamma 5 + \gamma 5 M_N) F_1^A(q^2) + q^2 \gamma 5 F_3^A(q^2)\right) u(p)$, $\; M_N\gamma 5 + \gamma 5 M_N = 2M_N\gamma 5$

If $q^2 = 0 \Rightarrow g_A = 0$ ?! unless $\pi$ propagator's pole cancels $q^2$




Since we expect a coupling exists from Quarkline heuristics

$\Delta\mathcal{L} = g_{\pi NN}\;\pi^a(x)\bar{N}\gamma^5 T^a N$




$(-i q^\mu f_\pi) \sim \frac{i}{q^2}\delta^{ab} \approx \frac{i}{q^2} g_{\pi NN}\,\bar{u}(p')\gamma 5 T^a u(p)\,(i g f_\pi)$

Match with $\bar{u}' q^\mu \gamma 5 T^a F_3^{A\mu}$ term in

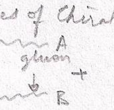
$\pi^0 \to \gamma\gamma$

$\to \gamma e^+ e^-$

$\to e^+e^- e^+e^-$

$\to e^+e^-$

Anomalies of Chiral Current




+ crossed

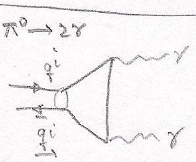
$\partial_\mu j^{\mu a} = -\frac{g^2}{8\pi}\tilde{G}^A_{\mu\nu} G^{\mu\nu B}\,\mathrm{tr}\,T^a t^A t^B$, $\; \mathrm{tr}\,T^a t^A t^B = 0$

$j_5^\mu = \bar{Q}\gamma^\mu\gamma^5 Q$

$\partial_\mu j_5^\mu = \frac{-g^2}{8\pi}\tilde{G}^A G^B\,\mathrm{tr}\,t^A t^B \neq 0$ !!, $\; \mathrm{tr}\,t^A t^B = \frac{1}{2}\delta^{AB}$

$U(1)_A$ finished by QCD.

$\pi^0 \to 2\gamma$




$\partial_\mu j^{\mu a}_{(5)} = -\frac{e^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\,\mathrm{tr}\,T^a Q_{em}^2 = \frac{-e^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}\delta^{a3}$

$a=3$: $\begin{pmatrix}1 & \\ & -1\end{pmatrix}\begin{pmatrix}4/9 & \\ & 1/9\end{pmatrix}$

$\delta^{a3}\left(\frac{1}{3}\right)\times 3$

$\langle\gamma\gamma|\partial_\mu j^{\mu a}|0\rangle \neq 0$ (!)

$\langle\gamma(p)\gamma(k)| j^{\mu a}_{(5)}(q)|0\rangle = \epsilon^*_\nu(p)\epsilon^*_\lambda(k) M^{\mu\nu\lambda}(p,k,q)$

$q_\mu$

$= \frac{-e^2}{16\pi^2}\,4\,\epsilon^*_\lambda(p)\,p_\alpha\,\epsilon^*_\nu(k)\,k_\mu\,\epsilon^{\alpha\lambda\mu\nu}$

$i q_\mu M^{\mu\nu\lambda} = \frac{e^2}{4\pi^2}\epsilon^{\alpha\mu\lambda\nu}p_\alpha k_\mu$

$= 0$ ? : Naively from current conservation

$\mathcal{L}_{eff} = A\,\pi^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$, $\; A = \frac{e^2}{4\pi^2 f_\pi}$ ??


$i A\,\epsilon^*_\nu(p)\epsilon^*_\lambda(k)\,\epsilon^{\mu\nu\alpha\beta}p_\alpha k_\beta \Rightarrow A f_\pi = \frac{e^2}{4\pi^2}$

(hint: $4\,\partial_\mu A_\nu\,\epsilon^{\mu\nu\alpha\beta}\partial_\alpha A_\beta(k)$)

$\Gamma(\pi^0 \to 2\gamma) = \frac{\alpha^2}{64\pi^3}\frac{m_\pi^3}{f_\pi^2}$

Depends on the # of colours.

$\sim 1\%$

Olly 14.5 (saturday extra)

$\pi^0 \to 2\gamma$ 0.98823 BR

$\frac{1}{f_\pi}\pi^0 F_{\mu\nu}\tilde{F}^{\mu\nu}$

$\to \gamma + e^+ + e^-$ 0.01174 $\sim O(\alpha) \sim 10^{-2}$

$\to e^+e^-e^+e^-$ $3.34\times10^{-5} \sim O(\alpha^2)$

$\to e^+ + e^-$ $6.46\times10^{-8}$

## Proton Structure from Scattering

(a) $E > 10\,GeV$ $\pi_{\text{low}}$

$N_\pi(P_2) \sim e^{-P_2/\Lambda}$

$\Lambda \sim 10^{2.5}\,MeV$

1 GeV$^{-1}$

(b) $e^- p$

$\left(\frac{d\sigma}{d\Omega}\right)^{E>>m}_{rel} = \left[\frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\right]\left(\frac{E'}{E}\right)\left(2W_1\sin^2\frac{\theta}{2} + W_2\cos^2\frac{\theta}{2}\right)$

Elastic em scattering of pt. particles — Rutherford — Recoil Factor — spin 1/2 target — spinless target

$\left(\frac{d\sigma}{d\Omega}\right)_{Mott}\left(W_2 + 2W_1\tan^2\frac{\theta}{2}\right)$

For $p^+$, $W_2 = 1$, $W_1 = \frac{Q^2}{4m^2}$

$(k-k')^2 = q^2 = -Q^2$
$= -2kk'$
$= -2EE'(1-\cos\theta)$

Elastic $e^-p \to e^-p$
Inelastic $e^-p \to e^-X$
DIS $e^-p \to e^-X$ SLAC 1967 Linear Acc.

$e$ $(15^\circ\text{-}18^\circ)$ Non-pt proton

$W_1 = G_M^2(Q^2)$ $\quad \tau = \frac{Q^2}{4M^2}$

$W_2 = \frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau}$

$G_E^p(0) = 1$ $\quad G_M^p(0) = 2.79$

$G_E^n(0) = 0$ $\quad G_M^n(0) = -1.91$

(Gordon decomposition: spin of electron; (magnetic moment))

DIS ⇒ Significant "large" angle scattering of $e^-$ at high energies $>> m_p$

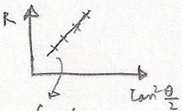
Feynman Bjorken: Proton is made of point like charged constituents "Parton Model" N partons charge $Q_i$

Hadron jet — Target parton — $e^-$ — struck parton — spectator — $p$ — spectator jet — $\xi_i$

$\sum_i \int_0^1 d\xi_i\, f_i(\xi_i)\,\sigma(\bar{e}\,i \to \bar{e}\,\xi_i)$ $\quad p_i = \xi_i$

$= \sigma(e^-p \to e^-X)$

$R = \frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{Mott}} = \frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2\tan^2\frac{\theta}{2}$



$\rho \sim \frac{1}{|Q|}$, $\;e^3 \sim \frac{1}{|Q|^3}$

$\frac{d\sigma}{d\Omega} \sim \left(\frac{\rho}{|Q|^3}\right)^2$

$G_{E,M} \sim_{\text{Dipole}} \left(1 + \frac{Q^2}{0.71\,GeV^2}\right)^{-2}$ | $R \sim Q^{-8}$ Q is large

### Lab Frame

$P = (m, 0)$
$k' = (E', \vec{k}')$
$\vec{P}' = \vec{k} - \vec{k}'$
$E_{p'}^2 - m^2 = E^2 + E'^2 - 2EE'\cos\theta$
$Q^2 = 4EE'\sin^2\frac{\theta}{2}$

$E' = \frac{E}{1 + \frac{2E}{m}\sin^2\frac{\theta}{2}}$

$\int_{-1}^1 (f_{u/p}(x) - f_{\bar{u}/p}(x)) = 2$ ; $\int (f_{d/p}(x) - f_{\bar{d}/p}(x)) = 1$

angle scattering of $e^-$ at high energies $>> m_p$

### Kinematics

Momentum Conservation at $m_e \simeq 0$; $m_p << E$

$(p+k)^2 = (p'+k')^2$ Elastic Case

$\Rightarrow \cancel{m_p^2} + \cancel{m_e^2} + 2p\cdot k = \cancel{m_p^2} + \cancel{m_e^2} + 2p'\cdot k'$

$\Rightarrow p\cdot k = p'\cdot k'$ ; $p\cdot k' = p'\cdot k$

& similarly, $(p-p')^2 = (k-k')^2$

$\Rightarrow 2m_p^2 - 2p\cdot p' = -Q^2$

General Inelastic Case

$\sum_{\text{final Hadrons}} p'_f = p' = p + k - k'$

$M_{inv}^2 = W = p'^2 = (p + \underbrace{k - k'}_{q})^2$

$= m^2 + 2p\cdot q - Q^2$

= Invariant mass squared of final Hadron System.

min. value $M_{inv}^2 = m_p^2$,

Elastic Scattering case

$\frac{Q^2}{2p\cdot q} = x = 1$ Bjorken Scaling Variable

End case

Peskin: Mandelstam for struck $\binom{e^-}{\text{parton}}$ $\hat{s}, \hat{t}, \hat{u}$ ; " " overall $\binom{e^-}{p}$ $s, t, u$ $\quad \hat{p}^M = \xi p^M$

$e^-$ parton $\xrightarrow[\gamma \text{ exchange}]{\text{Elastic}}$ $e^-$ parton

$(p_1+p_2)^2 = s$
$(p_1 - p_1')^2 = t$
$(p_1 - p_2')^2 = u$
$s + t + u = 2m_p^2 \simeq 0$
$(2E)^2$ $\quad u = -(s+t)$

$\frac{d\sigma}{d\hat{t}}\Big|_{\text{elastic}} = \frac{2\pi\alpha_{em}^2 Q_i^2}{\hat{s}^2}\left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right)$

$\frac{d\sigma}{d\hat{s}d\hat{t}}\Big|_{\text{elastic}} = \sum_i \left(\frac{2\pi\alpha_{em}^2 Q_i^2}{\hat{s}^2}\right)\left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right)\frac{\delta(x-\xi)}{(\delta(x-\xi))} \xi f_i(\xi)\, d\xi$

$\frac{d^2\sigma}{dx\,dQ^2} = 2\pi\alpha_{em}^2\sum_i f_i(x)\, z_i^2 \frac{2\pi\alpha^2}{(Q^2)^2}\left[1 + \left(1 - \frac{Q^2}{xs}\right)^2\right]$

Convert to usual guys

$\hat{s} = (k + \hat{p})^2$
$= (k + \xi p)^2$
$= m_p^2\xi^2 + 2\xi k\cdot p\xi$
$0\ (m_p \approx 0)$

$y = \frac{2p\cdot q}{2p\cdot k}$ ; $Q^2 = xys$

Callan-Gross relation ⇒ partons spin 1/2

$\boxed{\frac{Q^4}{s}\frac{d^2\sigma}{dx\,dy}(e^-p \to e^-X) = \left(\sum_i x f_i(x) z_i^2\right)\left(1 + (1-y)^2\right)}$

$Q_i \equiv z_i$

Ex: $0 \le y \le 1$

DIS

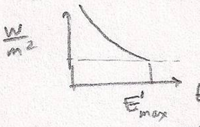
### Inelastic Case

$M_{inv}^2 > m_p^2$

Lab: $W = m^2 + 2m(E - E') - 4EE'\sin^2\frac{\theta}{2}$

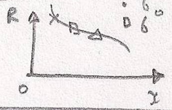
$E'_{min} = m_p \approx 0$

$\frac{W}{m^2} = 1 + \frac{2E}{m} + \text{small}$



Ex: $0 < x < 1$

$\theta$: $6^\circ$, $10^\circ$ — E: 13.5 GeV, 16.1 GeV



$R = (\ )\frac{d^2\sigma}{dx\,dQ^2} = \sum_i f_i(x_i) z_i^2$

### Structure Function

$F_1(x, Q^2) \equiv W_1 M$ $\quad \frac{d\sigma}{dE'd\Omega}\big)_{e^-p \to e^-X}$

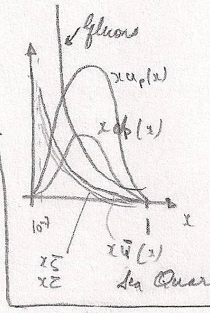
↓ pt

$\frac{Q^2}{4M}\delta\left(\nu\left(1 - \frac{Q^2}{2M\nu}\right)\right)$ $\quad \frac{p\cdot q}{m} = \frac{m(E-E')}{m}$

$F_2(x, Q^2) \equiv \nu W_2$

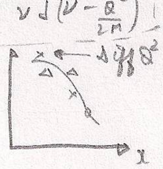
$\stackrel{pt}{=} \nu\,\delta\left(\nu - \frac{Q^2}{2M}\right)$

★ Sea is not SU(3) symmetric
★ $\bar{u}, \bar{d}$ different
★ small $\bar{c}$ observed
★ $f_{q/p}(Q^2, x)$ ← (DGL) AP diff eq's.



gluons — $x u_p(x)$ — $x d_p(x)$ — $x\bar{u}(x)$ — Sea Quark — $x\xi$ — $x\bar{z}$ — $x f_i(x)$

$F_2(x, Q^2)$



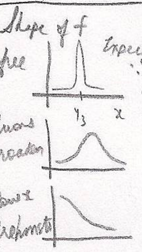
diff $Q^2$

$F_2^{\text{Parton}} = x\sum_i z_i^2 f_i(x)$

$\stackrel{\text{Parton Model}}{=} \frac{2xF_1(x, Q^2)}{F_2(x, Q^2)}$

Callan Gross Relation $\quad F_1 = \sum_i \frac{z_i^2 f(x)}{2}$

Shape of f



free — Expected ∵ u carries half the momentum — $\frac{1}{3}$ — x — gluons broaden — low x Brehmst.

Iso-spin: $f_{u/p} = f_{d/n} = u_N(x)$
$f_{d/p} = f_{u/n} = d_N(x)$
$s(x) = f_{s/p} = f_{s/n}$

$\int_0^1 x\,u(x) = 2\int_0^1 x\,d(x)$ ← Sum Rule
$u_p$ $\quad d_{down}$

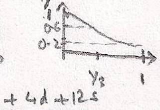
0.36

$0.36 + 0.18 = 0.54$ = fraction of nucleon momentum ⇒ 46% of momentum is not carried by the quarks

$F_{ep}^{(2)} = \frac{x}{9}[4u + d + 12\,s(x)]$ sea strange
$u = u_p$, $d = d_p$, $\bar{d} = s(x) = \bar{u}$

$F_{en}^{(2)} = \frac{x}{9}[u + 4d + 12s]$

$r = \frac{F_{en}}{F_{ep}}$



$= \frac{u + 4d + 12s}{4u + d + 12s}$

At small x, $u = 2d$
$r = 2/3$
$r = 1/4$ u down
$= 4$ d down