PHY661: Topics in CM-QM, Jan-April 2015: Assignment 2, Due March 1, 2015

Prof. Mukunda & Prof. Arvind

1. Consider the Lagrangian

$$L = \frac{1}{2}\left(q_2\dot{q}_1 - q_1\dot{q}_2\right) - \frac{1}{2}\left(q_1^2 + q_2^2\right)$$

. Calculate the Hessian and show that the system is singular. Find the constraints and pass over to the Hamiltonian description of the system.

2. Consider the Lagrangian

$$L = \frac{1}{2}\left(\dot{q}_1 + \dot{q}_2\right)^2 - V(q_1, q_2)$$

. Analyze this system as a singular system for the two choices of $V(q_1, q_2)$ namely (a) $V(q_1, q_2) = \frac{k}{2}(q_1 + q_2)^2$ and (b) $V(q_1, q_2) = \frac{k}{2}(q_1 - q_2)^2$. Pass on to the Hamiltonian description.

3. Consider Q to be a Riemann space of dim n with local coordinates q^j and metric tensor $g_{jk}(q)$. For particles in Q the Lagrangian is given by

$$L = \left(\dot{q}^2\right)^{\frac{1}{2}}, \quad \dot{q}^2 = g_{jk}(q)\dot{q}^j\dot{q}^k, \quad q^j = q^j(s), \quad \dot{q}^j = \frac{dq^j(s)}{ds}$$

and s is some evolution parameter. Analyze the system, find the constraints and pass over to a Hamiltonian description.

4. Consider a four-dimensional flat space time with metric $\eta_{\mu\nu} = (+1, -1, -1, -1)$ and coordinates x^μ with $\mu, \nu = 0, 1, 2, 3$. Let τ be the Lorentz invariant evolution parameter. For the Lagrangian

$$L = -m\left(\dot{x}^2\right)^{\frac{1}{2}} = -m\left(\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu\right)^{\frac{1}{2}} = -m\left((\dot{x}^0)^2 - \dot{x}^j\dot{x}^j\right)^{\frac{1}{2}}$$

find the constraints and pass over to the Hamiltonian description.