Indian Institute of Science Education and Research Mohali

Curves and Surfaces (MTH201)

Academic Session 2012-13

Tutorial Sheet 1 **August 23 2012**

Instructions: Write main ideas / hints for solving questions in your tutorial noteook. There is no need to write full and formal solution during the tutorial session. However during off class hours you should practice writing these solutions in a formal manner. **Get the signature of your tutor after each session**.

1. Find a bijective differentiable function $f : \mathbb{R} \to \mathbb{R}$ whose inverse is not differentiable.

2. Let $f : \mathbb{R}^2 \setminus \{(0,0)\} \to \mathbb{R}$ be given by $f(x,y) = \dfrac{y(x^4 + 4x^2y^2 - y^4)}{(x^2+y^2)^2}$. Compute $\lim_{(x,y)\to(0,0)} f(x,y)$.

3. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be defined by $f(x,y,z) = x^2 - xy + yz^3 - 6z$. Find all points $(a,b,c) \in \mathbb{R}^3$ where first order partial derivatives along three axes vanish.

4. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x,y) = e^{4x-y^2}$ and compute its directional derivative at the point $(1,-2)$ in the direction of $\left(\frac{3}{4}, \frac{4}{5}\right)$.

5. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by
$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{otherwise.} \end{cases}$$
   (a) Is f continuous at $(0,0)$?
   (b) Do $\dfrac{\partial f}{\partial x}(0,0)$ and $\dfrac{\partial f}{\partial y}(0,0)$ exist? If yes, compute them.
   (c) Can you find a unit vector $u \in \mathbb{R}$ such that $D_u f(0,0)$ does not exist? If yes, find one!

6. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by
$$f(x,y) = \begin{cases} x+y, & \text{if } xy = 0 \\ 1, & \text{otherwise.} \end{cases}$$
   (a) Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist?
   (b) Is f differentiable at $(0,0)$?

7. Show, *from first principles*, that the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by $f(x,y) = (x^3 + y, x + y^3)$ is differentiable. Compute $Df_{(1,1)}$.

8. For the function $f : \mathbb{R}^2 \to \mathbb{R}^3$ given by $f(x,y) = (\sin x + y, \cos x + y, y)$ find the points $(a,b) \in \mathbb{R}^2$ where the Jacobian of f has rank 2.

9. Find all points in $\mathbb{R}^2$ where the Jacobian matrix of the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ given by
$$f(x,y) = (\cos x \cosh y, \sin x \sinh y)$$
vanishes.

10. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ be given by $f(x,y) = (x+y, x-y)$ and $g : \mathbb{R}^2 \to \mathbb{R}^3$ be given by $g(x,y) = (\sin x \cos y, x+y, x^2-y)$. Find the Jacobian matrix of $g \circ f$.
