Olly 4.3

Gauge Theory

Yang-Mills

QED, $U(1)$ gauge theory $\xrightarrow{\text{generalise to}}$ $SU(N)$ local invariance, G

Matter $\psi_Q(x) \longrightarrow e^{iQ\theta(x)}\psi_Q(x)$

Gauge/connection $A_\mu(x) \longrightarrow A_\mu - \frac{1}{g}\partial_\mu \theta(x)$

covariant derivative

$(D_\mu\psi)' = (\partial_\mu + igQA_\mu)'\psi'$

$= e^{iQ\theta(x)} D_\mu\psi + \cancel{iQ(\partial_\mu\theta(x))\psi(x)} +$

$\cancel{iQg(-\frac{1}{g}\partial_\mu\theta(x))\psi(x)}$

Another way to look at it

$\psi(x) \longrightarrow e^{i\theta(x)}\psi(x)$

$n^\mu \frac{\partial\psi}{\partial x^\mu} = \lim_{\epsilon\to 0} \frac{\psi(x+\epsilon n^\mu) - \psi(x)}{\epsilon}$

Issue: $\psi(x+\epsilon n)$ & $\psi(x)$ transform differently

ASK $\psi(y)$ & $\psi(x)$ "

If we could introduce $U(y,x)$ s.t. under a gauge transformation,

$U'(y,x) = V(y)U(y,x)V^\dagger(y)$

where $\psi'(x) = V(x)\psi(x)$

then $(U(y,x)\psi(x))' = V(y)U(y,x)\psi(x)$

Then $n^\mu D_\mu\psi = \lim_{\epsilon\to 0} \frac{\psi(x+\epsilon n) - U(x+\epsilon n)\psi(x)}{\epsilon}$ ASK

Ex: Show $D = \partial + igA$

NB: I can expand $U(x+\epsilon n, x)$ as

$1 - ig\epsilon n^\mu A_\mu + O(\epsilon^2)$

almost obvious now.

Claim: $U(y,x) = \exp(-ig\int_x^y A_\mu(x')dx')$ works.

$[D_\mu, D_\nu]\psi_Q(x)$

$= (\partial_\mu + igQA_\mu)(\partial_\nu + igQA_\nu)\psi_Q(x)$

$- \mu \leftrightarrow \nu$

$= \cancel{\partial_{[\mu}\partial_{\nu]}\psi} + \cancel{igQA_{[\mu}\partial_{\nu]}\psi}$

$+ igQ\partial_{[\mu}A_{\nu]}\psi \longrightarrow \partial_{[\mu}A_{\nu]} + \cancel{A_{[\nu}\partial_{\mu]}\psi}$

$+ \cancel{igQ^2A_{[\mu}A_{\nu]}\psi}$

$= igQ\partial_{[\mu}A_{\nu]}\psi$ / $[D_\mu, D_\nu] = igQ$

$igQF_{\mu\nu} = \partial_{[\mu}A_{\nu]}$

Defn: $F_{\mu\nu} \equiv \partial_{[\mu}A_{\nu]}$

$(D_\mu\psi)' = V(x)(D_\mu\psi)$ ← ASK demand

$\psi'(x) = V(x)\psi(x)$ ← assumed

$D'_\mu = V(x)D_\mu V^\dagger(x)$ ← then

$A'_\mu(x) = A_\mu(x) - \frac{1}{g}\partial_\mu\theta(x)$ — ASK

$= V(x)A_\mu V^\dagger(x) - \frac{i}{g}V(x)\underbrace{\partial_\mu V^\dagger(x)}_{e^{-i\theta(x)}}$

$= V(A_\mu - \frac{1}{g}\partial_\mu)V^\dagger$ $\quad -i\partial_\mu\theta(x)$

$F'_{\mu\nu} = [D'_\mu, D'_\nu] = VF_{\mu\nu}V^\dagger$

(some part missing)

$U(y,x) = \exp -ig\int^y A_\mu(x)dx^\mu$

ASK called comparator

NB: $\oint_C A^\mu dx_\mu = \int_{\partial S = C} F^{\mu\nu} ds_{\mu\nu}$

$\mathcal{L}^{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\psi i\not{D}\psi$

$+ (D^\mu\phi)^\dagger(D_\mu\phi)$

$- m\bar\psi\psi - f(\bar\phi\phi)$

So now for $SU(N)$, G, we have

$\psi(x)' = V(x)\psi(x)$

ASK $V(x) = \exp i\theta^a(x)T^a$

T^a: generators of G in $d(R)$ dim representation R carried by ψ.

$[T^a, T^b] = if^{abc}T^c$; T^a is a $d(R)\times d(R)$ mat, $a = \{1, 2, \dots, N^2-1\}$ for $SU(N)$

$(D_\mu\psi)' = V(x)(D_\mu\psi)$

$D_\mu = \partial_\mu + igA_\mu^a T^a$

one for each θ

$(D_\mu\psi)' = (\partial_\mu + igA'_\mu)V(x)\psi(x)$

$A'_\mu = V(x)(A_\mu - \frac{i}{g}\partial_\mu)V^\dagger(x)$

$= V(x)D_\mu\psi(x) + (\partial_\mu V)\psi(x) +$

$ig(\frac{-i}{g})(V\partial_\mu V^\dagger)V\psi$

$-(\partial_\mu V)V^\dagger V$

$VV^\dagger = \mathbb{1}_{D(R)}$

$\partial_\mu(VV^\dagger) = \partial_\mu(\mathbb{1}_{D(R)}) = 0$

$\Rightarrow (\partial_\mu V)V^\dagger = -V\partial_\mu V^\dagger$

Now let's evaluate

$igT^aF^a_{\mu\nu} \equiv [D_\mu, D_\nu]\psi$

$= [\partial_\mu + igA_\mu, \partial_\nu + igA_\nu]\psi$

$= (\partial_\mu + igA_\mu)(\partial_\nu\psi + igA_\nu\psi) - \mu\leftrightarrow\nu$

$= \partial_{[\mu}\partial_{\nu]}\psi + ig(\partial_{[\mu}A_{\nu]})\psi +$

$igA_{[\nu}\partial_{\mu]}\psi + igA_{[\mu}\partial_{\nu]}\psi - g^2[A_\mu, A_\nu]\psi$

$= ig(\partial_{[\mu}A_{\nu]} + ig[A_\mu, A_\nu])\psi$

$\Rightarrow T^aF^a_{\mu\nu} = \partial_{[\mu}A_{\nu]} + ig[A_\mu, A_\nu]$

$F' = VFV^\dagger$

$\mathrm{tr}\, F'_{\mu\nu}F'^{\mu\nu} = \mathrm{tr}(\cancel{V}F_{\mu\nu}F^{\mu\nu}\cancel{V^\dagger})$

$F^a_{\mu\nu}F^{\mu\nu b}\underbrace{\mathrm{tr}(T^aT^b)}_{S_2\delta^{ab}}$

$\mathcal{L}_{YM}^{\text{kinetic}} = -\frac{1}{4}F_{\mu\nu}{}^aF^{\mu\nu a} = -\frac{1}{4}(\partial_{[\mu}A^a_{\nu]} - gf^{abc}A^b_\mu A^c_\nu)$

$(\partial^{[\mu}A^{\nu]})^a - gf^{ab'c'}A^{b'}_\mu A^{c'}_\nu)$

$\mathcal{L}_{YM} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \bar\Psi i\not{D}\Psi - h\bar\Psi\Phi\Psi$

$+ (D_\mu\Phi)^\dagger(D^\mu\Phi) - f(\Phi^\dagger\Phi, \dots)$ gauge inv

$\mathcal{L}_{\text{gauge fixing}} = -\frac{1}{2\xi}(\partial_\mu A^{\mu a})^2$

many others are possible

You can add fermions

$\mathcal{L}_{\text{Fermions}}^{\text{kinetic}} = \bar\psi i\not{D}\psi$

$i\gamma^\mu(\partial_\mu + igT^aA^a_\mu)\psi$

has all

$\mathcal{L}_{\text{scalar}} = (D_\mu\phi)^\dagger(D^\mu\phi)$

QED

U(1) $\psi \to e^{iQ\theta(x)}\psi(x)$

$\psi'(x) = V(x)\,\psi(x)$

$A' = V\left(A - \frac{i}{2}\partial\right)V^{+}$

$igQF_{\mu\nu} = [D_\mu, D_\nu]$

$F' = V(x)\,F\,V^{+}(x)$

$\mathcal{L}^{QED} =$ 🏠 (same as before)

YM

$\psi' = V(x)\psi$

$A' = A'^{a}T^{a}$

$= V\left(A - \frac{i}{g}\partial\right)V^{+}$

$D'_\mu = V D_\mu V^{+}$

$(D_\mu\psi)' = V D_\mu \psi$

$F^{a}_{\mu\nu} = \partial_{[\mu}A^{a}_{\nu]} - g f^{abc} A^{b}_{\mu} A^{c}_{\nu}$

$F' = V F V^{+}$

Olly 4.4

$$\underset{\cup}{R(G)}$$

$\psi'(x) = V(x)\psi(x)$

$V = \exp i\theta^a \tau^a \,;\, \tau^a = \begin{bmatrix} \tau^a_{(1)} & 0 & 0 \\ 0 & \tau^a_{(2)} & 0 \\ & 0 & \end{bmatrix}$

$A \equiv A^a \tau^a$

$A'_\mu = V\left(A_\mu - \frac{i}{g}\partial_\mu\right)V^+$

$D_\mu \psi = \partial_\mu \psi + i g \tau^a A^a \psi$

$\Rightarrow (D\psi)' = V(D\psi) \Rightarrow D' = VDV$

$i g F^a_{\mu\nu} \tau^a \equiv [D_\mu, D_\nu]$

$F' = VFV^+$

$F_{\mu\nu}{}^a = \partial_{[\mu} A^a_{\nu]} - g f^{abc} A^b_\mu A^c_\nu$

Dim(4) ⟺ renormalizable

$\mathcal{L}_{\text{gauge+fermi+scalar}} = \bar\Psi_\alpha (\gamma^\mu)_{\alpha\beta} (i D_\mu) \Psi_\beta$

$- \frac{1}{4}\sum_i F_{\mu\nu}{}^{ai} F^{\mu\nu a i} + (D_\mu \Phi)^+ (D^\mu \Phi)$

$- \underset{\substack{\uparrow \\ \text{Gauge Invariant}}}{V(\Phi)} - \underset{\substack{\uparrow \\ \text{Gauge}}}{\frac{1}{2\xi_a}(\partial_\mu A^{\mu a})^2} + \underset{\substack{\uparrow \\ \text{Ghost Terms}}}{C^a_\mu (D^\mu c)_a}$

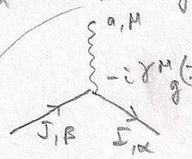
(That're needed for correct quantization; not needed at the tree level)

$- h \bar\Psi \Phi \Psi'$

Yukawa $\frac{3}{2} + 1 + \frac{3}{2}$

Feynman Rules Schematic

$(\bar\Psi)^I_\alpha \, i\gamma^\mu \left(\partial_\mu + i g (\tau^a)_I{}^J A^a_\mu\right) \Psi_{J\beta}$



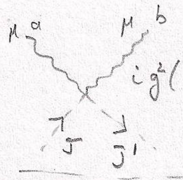
$-i\gamma^\mu g (\tau^a)_I{}^J$

Gauge Boson Propagator

$\mathcal{L} = -\frac{1}{4}\partial_{[\mu} A^a_{\nu]} \partial^{[\mu} A^{\nu] a} - \frac{1}{2\xi}(\partial_\mu A^{\mu a})^2 + \text{Interaction}$

$\overset{a}{\mu} \overset{k}{\sim\!\sim\!\sim} \overset{b}{\nu} \sim -i\delta^{ab} D^\xi_{\mu\nu}(k)$

Recall: $D^\xi_{\mu\nu}(k) = \frac{1}{k^2 - i\epsilon}\left(\eta^{\mu\nu} - (1-\xi)\frac{k^\mu k^\nu}{k^2}\right)$

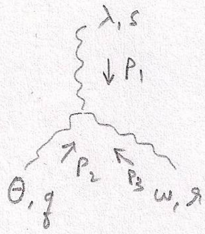


$i g^2 (\tau^a)_I{}^J (\tau^b)_{J'}{}^I$

$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a}$

$= -\frac{1}{4}\left(\partial_{[\mu} A^a_{\nu]} - g f^{abc} A^b_\mu A^c_\nu\right)\left(\partial^{[\mu} A^{\nu] a} - g f^{ade} A^{\mu d} A^{\nu e}\right)$

$= -\frac{1}{4}\left(\partial_{[\mu} A^a_{\nu]}\right)^2 + \cancel{4}\,\frac{1}{\cancel{4}}\, g\, \partial_\mu A^a_\nu f^{abc} A^{\mu d} A^{\nu e} - \frac{1}{4} g^2 f^{abc} f^{ade} A^b_\mu A^c_\nu A^{\mu d} A^{\nu e}$



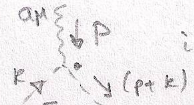
$|D_\mu \Phi_I|^2 = |\partial_\mu \phi_I + i g (\tau^a)_I{}^J A^a_\mu \phi_J|^2$

$= \partial_\mu \phi^*_I \partial^\mu \phi^I +$

$\left[(\partial_\mu \phi_I)^* i g (\tau^a)_I{}^J A^{a\mu} \phi_J - i g (\partial_\mu \phi_I)(\tau^a)_I^{J*} A^{a\mu} \phi^*_J\right]$

$+ g^2 (\tau^a)_I{}^J A^a_\mu \phi_J (\tau^a)_I^{*J} A^b_\mu \phi^*_{J'}$

Feynman Rule



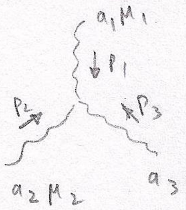
$i^3 (p+k)_\mu (\tau^a)_I{}^J$

Let these have momenta $q_1\, q_2\, q_3$ and $q_1 + q_2 + q_3 = 0$

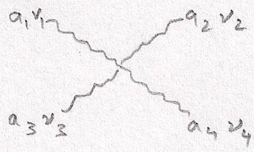
$\mathcal{L}_{FT} = -i g\, q_{1\mu} A^a_\nu(q_1) f^{abc} A^{\mu d}(q_2) A^{\nu e}(q_3)$

Vertex: $A^{\lambda s}(p_1) A^{\omega r}(p_2) A^{\theta q}(p_3)$ with A's in $\mathcal{L}$ and collect Terms.

$\rightarrow \left(-i g (p_{1\mu} \delta^\lambda_\nu \delta^{as} f^{ade} A^{\mu d}(q_2) A^{\nu e}(q_3) + q_{1\mu} A^a_\nu(q_1) f^{ade} \eta^{\lambda\mu} \delta^{ds} \delta_{q_2 p_1} + A^a_\nu(q_1) f^{ade} A^{\mu d} \delta^{es} \eta^{\mu\nu}\right) \cdot A^{\omega r}(p_2) A^{\theta q}(p_3)$



$-g f_{a_1 a_2 a_3}\left\{(p_1 - p_2)^{\mu_3} \eta^{\mu_2 \mu_1} + (p_2 - p_3)^{\mu_1} \eta^{\mu_2 \mu_3} + (p_3 - p_1)^{\mu_2} \eta^{\mu_3 \mu_1}\right\}$



$-i g^2 \left\{ f_{b a_1 a_2} f_{b a_3 a_4} \eta^{[\nu_1 \nu_3} \eta^{\nu_2] \nu_4} + f_{b a_3 a_1} f_{b a_2 a_4} \eta^{[\nu_3 \nu_2} \eta^{\nu_1] \nu_4} + \ldots \right\}$

2/2/2015.

Most general gauge invariant Renormalisable Lorentz invariant action

A_μ^a, Ψ_R, Φ

$h \Phi \bar{\Psi} \Psi$

$$\mathcal{L} = -\frac{1}{4}\sum_a f_{\mu\nu}^a F^{\mu\nu a} + \bar{\Psi} i \not{D} \Psi + (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) + \mathcal{L}_{Yuk}$$

$\underbrace{\bar{\Psi}_L \Psi_R + \Psi_L \bar{\Psi}_R}$ + a $m \bar{\Psi}\Psi$ mass term can be added

but should be gauge inv

+ you could add $\underbrace{\sum \frac{\theta}{32\pi^2} f_{\mu\nu}^a \tilde{F}^{\mu\nu a}}$ dual

Pontryagin.

Quantisation CP $\left\{ \begin{array}{l} f_{\mu\nu}^a \tilde{F}^{\mu\nu a} \sim \partial_\mu K^\mu \\ K^\mu \sim \epsilon^{\mu\nu\lambda\sigma} tr\left(A_\nu \partial_\lambda A_\sigma + \frac{2}{3} A_\nu A_\lambda A_\sigma\right) \end{array}\right.$

but doesn't have any Feynman Diag implications on $\mathcal{L}$ Eqn of Motion coz it's total div term.

Equations of Motion

We take the variation wrt $\delta\bar{\Psi}$ $\delta\phi$ $\delta\phi^\dagger$ δA_ν^a. each gives its Eqn of Mot

conjugate of each other

fermion $i\not{D}\Psi - \text{"}h\Phi\bar{\Psi}\text{"} = 0$

$-\text{"}m\phi\text{"} = 0.$

$$D^\mu D_\mu \Phi = -\frac{\partial V}{\partial \phi^\dagger}$$

$(D_\mu(\delta\Phi))^\dagger D^\mu \Phi - \frac{\partial V}{\partial \Phi}\delta\phi$

$+ (D_\mu \Phi)^\dagger (D^\mu \delta\Phi) - \frac{\partial V}{\partial \Phi^\dagger}\delta\phi^\dagger$

$f_{\mu\nu}^a = \partial_{[\mu} A_{\nu]}^a - gf$

$\int d^4x (D_\mu X) Y^\mu$

$= -\int d^4x\, X D_\mu Y^\mu.$

$$\delta S = 0 = \int d^4x -\frac{1}{4}\left(\delta(F_{\mu\nu}^a F^{\mu\nu a})\right).$$

$-\frac{2}{4}\text{"}\underbrace{\delta F_{\mu\nu}^a} F^{\mu\nu a}$

$\partial_\mu (X Y^\mu)$ gauge inv. covariant = derivative

$=$

already antisymm $2 \times \delta\left(\underbrace{\partial_\mu A_\nu}_{\text{drop}} - \frac{1}{2} g f^{abc} A_{[\mu}^b A_{\nu]}^c\right)$

$0 = (D_\mu X) Y^\mu + X D_\mu Y^\mu$

$=$

$$\partial_\mu(\delta A_\nu^a) - \frac{1}{2} g f^{abc}\left(\delta A_\mu^b A_\nu^c + A_\mu^b \delta A_\nu^c\right).$$

rename indices $c \uparrow \uparrow b$ with antisymm.

$2\left(-\frac{1}{2} g f^{abc}(\sim)\right)$ ← both give same ans ⇒ you get 2.

$= -\frac{1}{4}\int d^4x\, F^a_{\mu\nu}\,(D_\mu \delta A_\nu)^a$

Integrate by parts $\quad (D_\mu)^{ac}\,\delta A^c_\nu \quad \| \quad D_\mu \delta A^a_\nu$

$(D_\mu\phi)^a = \partial_\mu \phi^a - g f^{abc} A^b_\mu \phi^c \quad \| \quad \partial_\mu\phi^a + ig(T^b)_{ac} A^b_\mu \phi^c \quad \| \quad i f^{abc}$

$= +\int d^4x\, (D_\mu F^{\mu\nu})^a\, \delta A^a_\nu$ + variation of matter terms.

Now

$\delta(\bar\Psi\, i\not{D}\Psi) \Rightarrow \delta(\bar\Psi(i\not\partial - g\not{A})\Psi)$

$\bar\Psi(i\not\partial - g\gamma^\nu(\delta A^a_\nu)T^a)\Psi$. so we need to read off coeff of δA^a_ν.

$\int d^4x\, \delta A^a_\nu (D_\mu F^{\mu\nu a} - g J^{\nu a}_{(\Psi)}$

Now, $\delta_A (D_\mu\Phi)^\dagger(D^\mu\phi)$

$= \delta_A(\partial_\mu \Phi^\dagger - ig\Phi^\dagger A^a_\mu T^a)(\partial_\mu\Phi + igA^b_\mu T^b\Phi)$. variations gives one contribution from each bracket.

$= (D^\mu\Phi)^\dagger ig(\delta A^a_\mu)T^a\Phi - ig\Phi^\dagger T^a D^\mu\Phi\,\delta A^a_\mu$.

$= -g J^{\mu a}_\phi (\delta A^a_\mu)$

$\Rightarrow \int d^4x\, \delta A^a_\nu (D_\mu F^{\mu\nu a} - g J^{\nu a}_{(\Psi)} - g J^{\nu a}_\phi)$

$$\boxed{(D_\mu F^{\mu\nu})^a = g J^{\nu a}_{\phi+\Psi}}$$

$$J^{\nu a}_\Psi = \bar\Psi\gamma^\nu T^a\Psi + i\Phi^\dagger T^a \overleftrightarrow{D}^\nu\phi$$

Confusing

for the abelian theory

$0 = \partial_\nu\partial_\mu F^{\mu\nu} = \partial_\nu j^\nu$ only couple to a conserved current.

Now $J^{\nu a}_\Psi$ should be conserved

but $(D_\mu F^{\mu\nu})^a$ has no reason to be zero.

$\| \quad \underbrace{\partial_\mu F^{\mu\nu a}}_{\text{is zero}} - g f^{abc} A^b_\mu F^{\mu\nu c}$. came $\because$ the gauge field itself had a global variation.

so $$\boxed{\partial_\mu F^{\mu\nu a} = g\, J^{\nu a}_{tot}}$$ inhomogenous eqn with source.

$$\underset{\substack{\text{Global}\\\text{current}}}{J^{\nu a}} = f^{abc} A^b_\mu F^{\mu\nu c} + \bar{\Psi}\gamma^\nu T^a \Psi + \Phi^\dagger T^a \overleftrightarrow{D^\nu}\phi$$

Noether invariance of action under all fields $\underset{\text{global}}{X \to VX}$. If we have $\Psi' = e^{i\theta^a T^a}\Psi$ (indep of x)

$$\delta\Psi' = i\theta^a T^a \Psi + O(\theta^2)$$
$$= \theta^a(\hat{\delta}^a \Psi)$$

param we want to retain not-dependent on parameter that changes.

If $\mathcal{L}$ inv. under Noether theorem.

$$J^{\mu a} = \sum_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_i)}(\hat{\delta}^a \varphi_i) \text{ is conserved}$$

Ex: show this current $\partial_\mu J^{\mu a} = 0$ is $J^{\nu a}_{Global}$ ~~due~~ written above.

$A'_\mu = V(A_\mu - \frac{i}{g}\partial_\mu)V^\dagger$ for a local transf.

$A'_\mu = V A_\mu V^\dagger$ global transf.

What is the infinitesimal local transf?

$$(A^c_\mu + \delta A^c_\mu)T^c = A'_\mu = e^{i\theta^a T^a}\left(A^b_\mu T^b - \frac{i}{g}\partial_\mu\right)e^{-i\theta^c T^c}$$

If I wanna keep linear terms in θ

$$= i\theta^a \underbrace{[T^a, T^b]}_{if^{abc}T^c} A^b_\mu - \frac{i}{g}(-i)(\partial_\mu\theta^a)T^a + O(\theta^2).$$

$$\delta A^c_\mu = -\frac{1}{g}(D_\mu\theta)^c$$

$$(D_\mu\theta)^a = \partial_\mu\theta^a - g f^{abc} A^b_\mu \theta^c$$

$$= f^{abc}A^b_\mu \theta^c$$

$$= i(\theta.T)^{cb} A^b_\mu \quad \text{generic form}$$

same as $\varphi' = e^{i\theta T}\varphi$

$\delta\varphi = i\theta.T\varphi.$

$$\stackrel{\text{by def}^n}{=} \theta^c(\hat{\delta}A)^c$$

$$J^{\mu a} = \sum_i \frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu^b)}(\hat{\delta}^a A_\nu^b) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_i)}(\hat{\delta}^a \varphi^i) + \frac{\partial \mathcal{L}}{}$$

- scalar -

What is the homogeneous maxwell's eqn?

for any associative alg, you have this prop.

$[D_\lambda, [D_\mu, D_\nu]]$ - commutator of three op, and write down the cyclic quant.

$$[D_\lambda, [D_\mu D_\nu]] + [D_\nu, [D_\lambda, D_\mu]] + [D_\mu, [D_\nu, D_\lambda]] = 0$$

$[D_\mu D_\nu] = ig F_{\mu\nu}$

$$ig \{ D_\lambda F_{\mu\nu} + D_\nu F_{\lambda\mu} + D_\mu F_{\nu\lambda} \} = 0 \qquad \text{Bianchi Identity.}$$

$$\sum \epsilon^{\mu\nu\lambda\sigma} D_\nu F_{\lambda\sigma} = 0$$

by cyclicity under three index cycles of TASG.

take any 2 indices equal $\Rightarrow$ $\mu = \nu$: $0 + D_\mu F_{\lambda\mu} + D_\mu F_{\mu\lambda} = 0$

$\therefore$ all three have to be different. ϵ^{0123}

$$D_1 F_{23} + D_3 F_{12} + D_2 F_{31} = 0$$

Wilson loops

- geometrical way of introducing the connection.

In the Maxwell theory, you had path dep obj

$$U_p(A, x, y) = \exp -ie \int_y^x A_\mu(x)\, dx^\mu.$$

$$A_\mu' = A_\mu - \frac{1}{e}\partial_\mu \theta(x)$$

$$U(A', x, y) = V_x U_p(A, x, y) V_y^\dagger$$

$V_x = e^{i\theta(x)}$

$$U_p(A, x, y)\, \varphi(y) \sim \varphi(x).$$

In Non Abelian theory

we want the same transf.

we could replace by $T^a A^a_\mu$ - but the matrices do not commute.

path $x(s)$, s is affine parameter, y ($s=0$)

$$\exp -ie \int_0^s A(s)\, ds.$$

$A(s) \to A_\mu \frac{dx^\mu(s)}{ds}$

path is specified by

$P_y^x : x^\mu(s) \quad x^\mu(0) = y.$