Indian Institute of Science Education and Research Mohali

Curves and Surfaces (MTH201)
Academic Session 2012-13

Tutorial Sheet 4 **September 21 2012**

Instructions: Write main ideas / hints for solving questions in your tutorial noteook. There is no need to write full and formal solution during the tutorial session. However during off class hours you should practice writing these solutions in a formal manner. **Get the signature of your tutor after each session**.

1. Write a unit speed parametrised equation of the osculating circle of the parabola $y = x^2$ at $(0, 0)$.

2. Determine if the curve $\gamma(t) = \left(\frac{1}{3}(1+t)^{3/2}, \frac{1}{3}(1-t)^{3/2}, \frac{1}{\sqrt{2}}t\right)$ is planar. Write the equation of the osculating plane of γ at $t = 0$.

3. Calculate the Frenet frame $\{\mathbb{T}, \mathbb{N}, \mathbb{B}\}$, curvature κ and torsion τ for the following space curves:
   (a) $\gamma(t) = (a\cos\ t, a\sin\ t, bt)$.
   (b) $\gamma(t) = (e^t\cos\ t, e^t\sin\ t, e^t)$.

4. Let $\gamma : \mathbb{R} \to \mathbb{R}^3$ be a unit speed curve and $t_0 \in \mathbb{R}$ be a point of local maxima of the distance of $\gamma(t)$ from origin. Show that $\kappa(t_0) \geq \dfrac{1}{\|\gamma(t_0)\|}$.

5. A curve $\gamma : \mathbb{R} \to \mathbb{R}^3$ is called a *cylindrical helix* (or a *generalized helix*) if there exists a constant vector $\vec{u}$ such that $\mathbb{T}(t).\vec{u}$ is constant (i.e. is the tangent makes a fixed angle with $\vec{u}$).
   (a) Show that the curve $\gamma(t) = (a\cos t, a\sin t, bt)$ is a cylindrical helix.
   (b) Show that a curve γ with nowhere vanishing curvature κ and non-zero torsion τ is a cylindrical helix if and only if the ratio $\dfrac{\tau}{\kappa}$ is constant.
   (c) Determine if the curve $\gamma(t) = \left(\frac{1}{3}(1+t)^{3/2}, \frac{1}{3}(1-t)^{3/2}, \frac{1}{\sqrt{2}}t\right)$ is a cylindrical helix.
   (d) Show that the twisted cubic $\gamma(t) = (at, bt^2, t^3)$ is a cylindrical helix if and only if $4b^4 = 9a^2$.
