# APPLICATION FOR ADMISSION TO THE GRADUATE COLLEGE

ILLINOIS
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

**UIN:** **AY ID:** 6697974 **Application Type:** International

**Program Information:**

Proposed Program: Physics
Degree: Physics--PHD
Term: Fall 2016
Concentration/Specialization: None
Other Specialization: Quantum Gravity
I intend to continue for the doctoral degree: Not Applicat
Faculty Member: Thomas Faulkner, Rob Leigh
Applied Before? N
Previous Application Term:

**Applicant Information**

Last Name: Arora
Middle Name: Singh
First Name: Atul
Nick Name:
Previous Name:
Gender: Male
Date of Birth: 11/20/2015
Birth City: New Delhi
Birth State:
Birth Country: India
Country of Citizenship: India
Dual Citizenship Country:
Country of Legal Permanent Residence: India
Veteran:
Citizenship Type: International
Resident Alien Number:
Ethnicity Identification:
Race Identification(s):

**Contact Information**

E-mail Address: toAtulArora@gmail.com
2nd E-mail Address:

Mailing Address
1: 4317/3 Ansari Road
2: Darya Ganj
3:
City: New Delhi
County:
State:
Province:
Zip: 110002
Country: India
Phone (U.S):
Phone (International):
Cell Phone:

Permanent Address
1: 4317/3 Ansari Road
2: Darya Ganj
3:
City: New Delhi
County:
State:
Province:
Zip: 110002
Country: India
Phone (U.S):
Phone (International):
Cell Phone:

## Residency Information

State of Legal Residence:

Lived in State Entire Life:

When Moved to State:

Reason addresses not provided in State of Legal Residence:

## Application Fee Waiver Information

Employee Type:

Designated Program:

Allied Agency:

Allied Agency Phone:

## Financial Aid Information

I wish to receive financial aid: Y

I must receive financial aid: Y

Teaching Assistantship: 4th Choice

Research Assistantship: 3rd Choice

Fellowship: 2nd Choice

Scholarship:

## Educational History Information

Current/Expected Coursework Not Listed on Transcript:

* MS-Thesis (part II) [16 credits]
* Cosmology & Galaxy Formation [4 credits]

Institution 1: IISER Mohali

City: Mohali

State:

Country: India

Degree: Dual Degree BS-MS

Degree Translation:

Attended From: 08/2011 Attended To: 05/2016

Degree Awarded/Expected: 05/30/2016

Bachelor or Higher Degree Expected: Y

Major: Physics

Other Major:

Minor/Concentration:

Honors: (See the CV)

GPA: 9.4

Major GPA: 9.6

GPA Scale:

Non-Numerical GPA Scale:

Institution 2:

City:

State:

Country:

Degree:

Degree Translation:

Attended From: Attended To:

Degree Awarded/Expected:

Bachelor or Higher Degree Expected:

Major:

Other Major:

Minor/Concentration:

Honors:

GPA:

Major GPA:

GPA Scale:

Non-Numerical GPA Scale:

Institution 3:
City:
State:
Country:
Degree:
Degree Translation:
Attended From: Attended To:
Degree Awarded/Expected:
Bachelor or Higher Degree Expected:
Major:
Other Major:
Minor/Concentration:
Honors:
GPA:
Major GPA:
GPA Scale:
Non-Numerical GPA Scale:

Institution 4:
City:
State:
Country:
Degree:
Degree Translation:
Attended From: Attended To:
Degree Awarded/Expected:
Bachelor or Higher Degree Expected:
Major:
Other Major:
Minor/Concentration:
Honors:
GPA:
Major GPA:
GPA Scale:
Non-Numerical GPA Scale:

Institution 5:
City:
State:
Country:
Degree:
Degree Translation:
Attended From: Attended To:
Degree Awarded/Expected:
Bachelor or Higher Degree Expected:
Major:
Other Major:
Minor/Concentration:
Honors:
GPA:
Major GPA:
GPA Scale:
Non-Numerical GPA Scale:

Institution 6:
City:
State:
Country:
Degree:
Degree Translation:
Attended From: Attended To:
Degree Awarded/Expected:
Bachelor or Higher Degree Expected:
Major:
Other Major:
Minor/Concentration:
Honors:
GPA:
Major GPA:
GPA Scale:
Non-Numerical GPA Scale:

**Test Score Information**

GRE General 1: 09/28/2015 — Verbal: 157 | 74 % — Analytical: ____ | ____ %
Registration #: 3992009 — Quantitative: 161 | 80 % — Analytical Writing: 4.0 | 56 %

GRE General 2: — Verbal: ____ | ____ % — Analytical: ____ | ____ %
Registration #: — Quantitative: ____ | ____ % — Analytical Writing: ____ | ____ %

GRE Subject: 10/24/2015 — Subject: Physics
Registration #: 3992009 — Score: 730 | 57 %

GRE Writing Assessment: — Score:
Registration #:

GMAT: — Verbal: ____ | ____ % — Analytical: ____ | ____ %
Registration #: — Quantitative: ____ | ____ % — Total Score: ____ | ____ %

MAT: — Raw Score: — Overall %: — % for Intended Major: %

MCAT — Verbal Reasoning: — Physical Sciences:
Writing Sample: — Biological Sciences:

TOEFL: — Registration #: — Paper-Based Test Score:
Appointment #: — Computer-Based Test Score:
TWE Score:

TOEFL iBT 1: 09/26/2015 — Total Score: 117
Registration #: 0000000025770147 — Listening: 30 — Reading: 28 — Writing: 30 — Speaking: 29

TOEFL iBT 2: — Total Score:
Registration #: — Listening: — Reading: — Writing: — Speaking:

IELTS: — Overall Score:
Registration #: — Listening: — Academic Reading: — Academic Writing: — Speaking:



**Language Ability Information**

Native Language: Hindi

Chinese:

Russian:

French:

Spanish:

Hindi: Speak, Read, Write

Korean:

Other: Punjabi — Speak

Other: English — Speak, Read, Write

**Recommendation Information**

| | | |
|---|---|---|
| First Name: Arvind | First Name: Jasjeet | First Name: Otfried |
| Last Name: Arvind | Last Name: Bagla | Last Name: Guehne |
| Phone: | Phone: | Phone: |
| Email: arvind@iisermohali.ac.in | Email: jasjeet@iisermohali.ac.in | Email: otfried.guehne@uni-siegen.de |
| Title: Prof. | Title: Prof. | Title: Prof. |
| Employer: IISER Mohali | Employer: IISER Mohali | Employer: University of Siegen |
| Relation: Thesis Supervisor, Professor | Relation: Professor | Relation: Project Guide |
| Right to review waived? Yes | Right to review waived? Yes | Right to review waived? Yes |

## International Applicants Information

### Request for Visa Eligibility Documents

Please indicate the visa eligibility document you are requesting: I-20 (F-1)

If your DS-2019 will be issued by another organization, please indicate the organization name here:

If you selected DS-2019 (J-1), please select a Home Country Position Code:

If you are requesting an I-20 or DS-2019, please indicate the type of document that best applies to you: Initial I-20 / DS-2019 (If you are arriving in the U.S. from outside the country for

If you have requested a Change of Status I-20, do you plan to:

### Current Visa Holders

Visa Type:

Visa Expiration Date:

Do you plan to remain on your current visa for your studies at UIUC? N

### Current F and J Visa Holders

SEVIS number:

I-20 / DS-2019 Expiration Date:

If you hold a J-1 visa, please indicate if your visa is a J-1 Student or a J-1 Scholar Visa:

Are you currently on Practical Training (OPT/CPT for F-1 students) or Academic Training (for J-1 students)?

Did the University of Illinois at Urbana-Champaign (UIUC) issue your I-20 / DS-2010?

Educational Institute that issued your I-20 / DS-2019:

Institution City:

Institution State:

Institution Phone:

End of program Date:

### Dependents

Are you requesting any dependent I-20s / DS-2019s? N

If yes, how many?

**Dependent 1**

Last Name: First Name:

Middle Name: Gender: Date of Birth:

Relationship to Student: Birth City:

Birth Country: Country of Citizenship:

Country of Legal Permanent Residence:

**Dependent 2**

Last Name: First Name:

Middle Name: Gender: Date of Birth:

Relationship to Student: Birth City:

Birth Country: Country of Citizenship:

Country of Legal Permanent Residence:

**Dependent 3**

Last Name: First Name:

Middle Name: Gender: Date of Birth:

Relationship to Student: Birth City:

Birth Country: Country of Citizenship:

Country of Legal Permanent Residence:

**Dependent 4**

Last Name: First Name:

Middle Name: Gender: Date of Birth:

Relationship to Student: Birth City:

Birth Country: Country of Citizenship:

Country of Legal Permanent Residence:

**Dependent 5**

Last Name: First Name:

Middle Name: Gender: Date of Birth:

Relationship to Student: Birth City:

Birth Country: Country of Citizenship:

Country of Legal Permanent Residence:

## Primary and Secondary Education

### Primary Education

Institution 1 Name: Sardar Patel Vidyalaya
Country: India
Attended From: 05/1996
Attended to: 04/2008

Institution 2 Name:
Country:
Attended From:
Attended to:

### Secondary Education

Institution 1 Name: Sardar Patel Vidyalaya
Country: India
Attended From: 05/2008
Attended to: 04/2010

Institution 2 Name:
Country:
Attended From:
Attended to:

### Fee Payment Information

Payment Type:
Payment Amount:
Payment Status:
Payment Transaction Date:

### Applicant Signature and Confirmation of Information

The University of Illinois at Urbana-Champaign is committed to maintaining a safe environment for all members of the University community. As part of this commitment, the University requires applicants who are under current indictment or have been convicted of a crime (other than a routine traffic offense or in a juvenile proceeding) to disclose this information as a mandatory step in the application process. A previous conviction or current indictment does not automatically bar admission to the University, but does require review. Complete information must be sent by certified mail at the time of application for admission to: Review Committee, 300 Turner Student Services Building, 610 East John Street, Champaign, Illinois 61820 USA. Applicants are responsible for verifying receipt by the University and for maintaining a copy of the receipt certifying submission. Information to be submitted includes a brief explanation, location (city, state, country) of conviction or current indictment, dates, and court disposition, in English. This statement must include a grant of permission to the University for complete access to criminal records, if any. For further information on this requirement, call (217) 333-0050.

I understand that withholding pertinent information requested on this application or giving false information will make me ineligible for admission to the University or subject to dismissal. With this in mind, I certify that the statements are correct and complete, including a report of all criminal history as described above. Furthermore, I agree that if I am employed by the University of Illinois, I will adhere to and be bound by the University of Illinois Statutes, the General Rules Concerning University Organization and Procedure adopted by the Board of Trustees, and any and all other applicable rules and regulations of the University and its Board of Trustees.

Signature: ____________________________________________________  Date:

Submitted:

**CHECK WITH YOUR PROPOSED PROGRAM OF STUDY OFFICE FOR ADDITIONAL APPLICATION REQUIREMENTS. PLEASE REFER TO THE PROGRAM LISTING FOR CONTACT INFORMATION.**

was born on November 20, 1991
resides in 4317/3 Ansari Road Darya Ganj, New Delhi
☎ +91 86994 13350
✉ toAtulArora@gmail.com

# Atul Singh Arora
ətul siŋh ərəʊraː

http://github.com/toAtulArora
http://KnowledgePayback.blogspot.com



## Objective

short term Get a PhD position to explore Quantum Gravity.

general Contribute to expanding our knowledge of nature.

## Education

Present **BS-MS Dual Degree**, *Indian Institute of Science Education and Research*, Mohali, CPI: *9.4/10*.

**Semester I**: (8.5/10) Mechanics, Chemistry of elements and chemical transformations, Cellular basis of life, Symmetry, Language Skills B, Introduction to Computers, Physics Lab I, Chem Lab I, Bio Lab I

**Semester II**: (8.6/10) Electromagnetism, Atoms Molecules and Symmetry, Gene expression and development, Analysis in one variable, Hands-on electronics, History of science, Physics Lab II, Chemistry Lab II, Biology Lab II

**Semester III**: (8.8/10) Waves and optics, Spectroscopic and other physical methods, Genetics and evolution, Curves and surfaces, Introduction to Astrophysics, Workshop Training, Physics Lab III, Chemistry Lab III, Biology Lab III

**Semester IV**: (9.7/10) Thermodynamics and statistical physics, Energetics and dynamics of chemical reactions, Behaviour and ecology, Probability and statistics, Introduction to Quantum Physics, Philosophy of science, Physics Lab IV, Chemistry Lab IV, Biology Lab IV

**Semester V**: (10/10) Classical Mechanics, Quantum Mechanics, Electrodynamics, Advanced Optics Lab, Reason and Rationality

**Semester VI**: (9.6/10) Statistical Mechanics, Atomic and Molecular Physics, Quantum Computation, Advanced Electronics and Instrumentation Lab, Quantum Field Theory

**Semester VII**: (9.4/10) Solid State Physics, Nuclear and Particle Physics, Nuclear Physics Lab, Physics of Fluids, Quantum Principles and Quantum Optics, Radiative Effects and Renormalization Group in Relativistic Quantum Field Theory

**Semester VIII**: (9.5/10) Nonlinear Dynamics, Chaos and Complex Systems, Condensed matter Physics Lab, Computational Methods in Physics, Standard Model and beyond, Selected topics in classical and quantum mechanics

**Semester IX**: (10/10): Ethics, MS Thesis - Research Project I

**Semester X**: (current): Cosmology and Galaxy Formation, MS Thesis - Research Project II

2010 **CBSE 10+2**, *Sardar Patel Vidyalaya*, New Delhi, *80%*.
Physics, Chemistry, Math, Computer Science, English

2008 **CBSE X**, *Sardar Patel Vidyalaya*, New Delhi, *93%*.
Science, Maths, Social Science, English, Hindi, Information Technology

## Experience (Academic)

Summer 2015 — **Intern**, *University of Siegen*, Siegen, Germany.
I had worked under the guidance of Dr. Ali Asadian and Prof. Otfried Guehne. We proposed a test of local realism based on correlation measurements of continuum valued functions of positions and momenta, known as modular variables. The Wigner representations of these observables are bounded in phase space and therefore, the associated inequality holds for any state described by a non-negative Wigner function. This agrees with Bell's remark that positive Wigner functions, serving as a valid probability distribution over local (hidden) phase space coordinates, do not reveal non-locality. We constructed a class of entangled states resulting in a violation of the inequality and thus truly demonstrate non-locality in phase space. These states were realized through grating techniques in space-like separated interferometric setups. The non-locality is verified from the spatial correlation data that is collected from the screens.

Summer 2014 — **Intern**, *Indian Institute of Science Education and Research*, Mohali.
The objective was to device ways of using a universal quantum computer to perform simulations of quantum phenomena itself, with 'practical' resource requirements. The project involved reading of books and papers, followed by reproducing the results of a paper using a quantum computer simulator, which was written from scratch and an independent discovery of a simple quantum algorithm to simulate mixed states (this result was however already known). I was guided by Prof. Arvind and had helpful discussions with Dr. Sudipta Sarkar and Dr. Abhishek Choudhury.

Winter 2013 — **Intern**, *Indian Institute of Science Education and Research*, Mohali.
Studied Mechanics from Landau's first volume (excluding the last chapter) and covered parts of Mathematical Methods from a book on the said topic by Dennery and Krzywicki. I was guided by Prof. Jasjeet Bagla and Prof. Sudeshna Sinha.

Monsoon 2013 — **School**, *National Centre for Biological Sciences*, Bangalore.
Participated in a Monsoon School on Physics of Life where we treated selected biological phenomena with physical rigour, headed by Dr. Mukun Thattai

Summer 2013 — **Intern**, *National Physical Laboratory*, New Delhi.
Worked on setting up an experiment to study dynamics of a two dimensional magnetic dipole lattice, with Dr. Ravi Mehrotra.

Winter 2012 — **Intern**, *Indian Institute of Science Education and Research*, Mohali.
Studied Quantum Mechanics from J.J. Sakurai, under the guidance of Prof. Jasjeet Bagla and created a corresponding report.

Summer 2012 — **Intern**, *Indian Institute of Science Education and Research*, Mohali.
Studied Group Theory and Linear Algebra for understanding Symmetry, under Prof. Kapil Hari Paranjape.
A brief introductory understanding of the Knot Theory was also undertaken. LaTeX was learnt during this period, to be able to efficiently communicate via the internet.

Summer 2011 — **Intern**, *Indian Institute of Technology*, Bombay.
Worked on Image Recognition techniques using OpenCV, for Yarn Fault detection under the supervision of Prof. Anirban Guha.
This was an extension to an IIT alumni's Masters thesis. The work was done using Visual Studio, C++ and involved understanding of OpenCV and the idea behind various algorithms, to be able to solve the problem at hand.

## Publication (Academic)

2015 — A. S. A, A. Asadian. *Proposal for a macroscopic test of local realism with phase-space measurements.* Phys. Rev. A **92**, 062107

## Teaching

2015 — **TA**, Teaching Assistant. Classical Mechanics for undergraduates.

## Projects

| | |
|---|---|
| Sem VI 2014 | **Drawdio**, What is Drawdio: "Imagine you could draw musical instruments on normal paper with any pencil (cheap circuit thumb-tacked on) and then play them with your finger. The Drawdio circuit-craft lets you MacGuyver your everyday objects into musical instruments: paintbrushes, macaroni, trees, grandpa, even the kitchen sink... ". This project was originally created at the MIT Media Lab; I simply reproduced a version of this for the National Science Day, 2014. |
| Summer 2013 | **Nazar Band**, A face recognition system built using OpenCV with the aim of automating the locking and unlocking of doors, eliminating the need of keys. |
| Sem III 2012 | **Opportunity Cell Website**, Team Project, A centralized web portal for the Opportunity Cell of IISER Mohali. |
| Sem III 2012 | **Fly Count Assister**, For easing the task of counting flies (Biology experiment), this application was written in Python and used extensively. With just two buttons on the keyboard, and the voice support, the counting process was made much more efficient. |
| Sem III 2012 | **NaveenTantra**, Team Project, An Online Election system, based on a novel fraud prevention technique, created using Javascript, PHP and mySQL. |
| Summer 2012 | **Telescope**, Team Project, Newtonian Reflection Telescope for observing Transit of Venus. |
| Sem II 2012 | **Capacitive Touch Sensor**, Sensitive enough to measure changes in PicoFarads, developed for the Science Day. |
| 2010-11 | **Chatur Chaalak**, Developed with the aim of application in robotics, this project was designed to control the torque and speed of stepper motors, with precision, independently. This was implemented using C as the language and Atmel AVR as the platform. |
| 2010 | **Live GSM**, This was an attempt at controlling a phone using a microcontroller, to be able to remotely control devices, using DTMF communication protocol over voice calls. |
| Class XII 2010 | **3D Modelling and Animation**, Imitated the '21st Century FOX' animation and customized it to read 'XII class presents', for a class presentation, using the popular 3D cinema creation software, Maya. |
| Class XI-XII 2009-10 | **Space Race**, This game was developed using OpenGL to ensure cross-platform support and as a transition to the open world. Apart from the 3D-graphics, this game had Newtonian physics implemented using a point particle approach, derived from an open-source game. |
| Class XI 2009 | **Robotic Rescue Vehicle (RRV)**, It was designed using auto-mobile parts such as bicycle chains and sprockets, wiper motors, car batteries, a web-camera, and an ordinary PC, which gave it a unique look. It could be moved around wirelessly using a laptop which gave a live video feed from the robot, ideal for rescue operations. |
| Class X 2008 | **Math Project**, A calculator built using micro-controllers, to verify the property $(a+b)(a-b) = a^2 - b^2$. It was a battery operated device, with an LCD screen and used an 89S52 to process. |
| Class IX 2006 | **ALive City 2 - DirectX 9.0**, My second attempt at game making; this was developed without using any game engines, while the game itself was controlled using a USB steering wheel, built by me, based on an open-source application. |

Class VIII 2005 **Motion Detection - Image Processing**, This program was developed to save frames of a video feed, only when motion is detected, ideal for surveillance.

Class VIII 2005 **ALive City - DirectX 8.0**, My first computer graphics 3D project, a simple racing game where the player could put his/her own picture, right on the car.

Class VII 2004 **Edge Detecting Robot**, Built using stepper motors and a microprocessor, this vehicle was programmed to detect edges of a table using infra red sensors and turn to avoid falling.

Class VII 2004 **AT Keyboard Interface**, Built using the 8051 series of Microcontrollers and an LCD, this device was developed to serve as a low cost portable typing tutor for kids. It was programmed using Bascom, a basic compiler.

Class VII 2004 **School Bell Scheduler 2**, This application was re-written in Visual Basic.NET to automate ringing of school bells, given the schedule, like it's first version. It used UART for securer communication and was installed in Srijan School, Model Town, New Delhi.

Class VI 2003 **School Bell Scheduler**, A program, written in Visual Basic 6, for automating the ringing of school bells. The user simply needs to specify the schedule.

## Recognition

2016 Amongst the highest scorers in the first semester of the academic session 2015-16

2015 Awarded a Certificate of Merit for the best academic performance in the second semester of the academic session 2014-15

2015 Was awarded the DAAD WISE fellowship for a summer internship in Germany

2015 Amongst the highest scorers in the first semester of the academic session 2014-15

2014 Amongst the highest scorers in the second semester of the academic session 2013-14

2014 Awarded a Certificate of Merit for the best academic performance in the first semester of the academic session 2013-14

2012 Capacitive touch won the Best Physics Demonstration, at the Science Day 2012, organized by IISER Mohali

2011 Was awarded the KVPY fellowship, for my work on Stepper Motor control, Chatur Chaalak

2010 Was awarded the First position in Senior programming, with my Team member, in an inter-school programming competition, a part of Access, an annual Computer Symposium, Access, organized by Modern School

2010 Was selected as one of the participants for attending the Bright Green Youth, Denmark, an international climate summit for the youth, on the basis of my performance in the National Science Fair and a personal interview. In DK, our team made it to the top 14 projects

2009 The Robotic Rescue Vehicle was awarded the first position in the Delhi region and second position in the Northern region, at the National Science Fair, held at the National Science Centre, New Delhi

2005 ALive City won the first place in the open Software Display, at an inter-school Computer Symposium, Access, an annual event organized by Modern School, Barakhamba Road, New Delhi

2004 ALive City qualified the open Software Display, at the inter-school Computer Symposium, Access

2004 Displayed the Robotic Rescue Vehicle at an interschool competition and secured the third position, even though due to a component failure, the robot failed to work when it was judged

2003 Displayed the School Bell Scheduler at the National Convention 2003, Computer Society of India, IIT-Delhi

## Positions of Responsibility

2012-13 **CR**, Class Representative. Was responsible for setting up various batch-activities, Tutorial Assistance, Summer Project Talks, Origami Meets, The MS11 digest, to name a few. Various other student responsitibilities, such as infrastructure feedback, mess issues, meeting with the dean etc., were simultaneously executed.

2012-13 **WD**, Web Designer. Was responsible for developing and maintaining the Opportunity Cell website, a link to which was added in IISER's official web page.

## Languages

Native **Punjabi**

Fluent **English** *Formally studied till Sem I, BS-MS*

Fluent **Hindi** *Formally studied till class X*

## Computer Skills

Familiar OSs Windows: XP, Vista, 7, 8, 10; Linux: Ubuntu, Debian, OpenSuse, Slackware

Languages Basic, C, C++, C#, Fortran, Python, Javascript, SQL, HTML, PHP, LaTeX, Octave/Matlab, Mathematica

Applications Visual Studio, Emacs, Sublime Text, Microsoft Office (Word, Powerpoint, Outlook, OneNote, Excel), CorelDraw, Inkscape, Git, Sony Vegas, Autodesk Maya, GNU plot, SolidWorks, FL Studio, Sony Sound Forge, Cinelerra

## Extra-Curricular Activities

2015 Tug of War: IISER M, Boys Tug of War Champions.

2013 Debating: Participated in the debating event of the in-house fest, Insomnia. The home team wasn't allowed to compete beyond a stage.

2012 Music Performance: Played the Guitar and composed the percussion electronically, with two friends, at the Cultural Evening of IISER.

Current Interests Playing the Guitar, Tabla; Programming, Electronics, Robotics, Physical Fitness, Krav Maga, Taekwondo (Red I)

# Statement of Purpose

Atul Singh Arora

Fascinated by the idea that the laws of nature are discovered by people, as a child I wanted to become a scientist. Upon growing up, my interest shifted to building simple robots that can help do everyday chores. The construction involved programming, electronics and assembling mechanical parts. Upon learning physics and doing questions from books like Irodov, I became interested in physics again. It was however only after coming to IISER, my second home, that I took seriously the idea of becoming a scientist.

Initially we're taught all the basic sciences plus pure math. I developed a taste for abstract mathematics during that time. My first subject for exploration was group theory and symmetry. I also looked at knot theory at the time and was surprised to learn its relation to quantum computation and elementary physics. I learnt eventually that while mathematics was fascinating in its own right, I missed physics, the connection to reality. That *my equations describe nature*, I realised was rather important for me.

I spent the following summer constructing an experiment whose objective was to study the dynamics of spins on a lattice. Having enough experience with robotics, this project wasn't all that challenging in terms of novelty and learning, even though it took a lot of effort. By the end of it, I was convinced that while constructing physics experiments, there's not too much focus on physics itself. I learnt that I really *wish to explore theoretical physics* in my future projects.

By this time, I had chosen physics as my major. Physics had never ceased to surprise me, but with solid state physics, fluid mechanics, quantum computation, quantum field theory (QFT) and gauge theories, the standard model & beyond, the excitement pinnacled.

In my major years, I spent the first summer exploring the simulation of quantum physics on a quantum computer. This was fascinating for I had independently discovered a small simulation protocol, that extended the pure state simulation to that of a mixed state. That for me, was the first novel construction of its type. However towards the end of it, I felt that I wasn't doing physics. I wanted to work on *finding new laws of nature.*

In the next summer, I was awarded the DAAD-WISE scholarship to work in Germany. While applying, I was confused between quantum gravity and quantum optics & foundations. I chose the latter for I felt it is experimentally more accessible, that our results could at least be verified within our lifetime. I was able to make some progress and construct a new extension of the Bell test[1]. In addition to this, I learnt about Bohmian Mechanics (BM) which is a deterministic theory that describes the same phenomena that Quantum Mechanics does. While I was not disappointed with my progress and had learnt about exciting research directions such as the No-Signalling principle/PR-box and information causality, I somehow *missed the richness of the remaining physics.*

For my master's thesis, I decided to explore BM, a theory in which observers play no fundamental role. This I felt might eventually make interpretation of 'quantum spacetime' more meaningful as a concept. For the thesis though, I'm focusing on a more specific problem, viz. seeing how BM could be consistent with contextuality; more precisely, I want to see how a theory deterministic in position & momentum (q,p) can be consistent with a quantum mechanics' test that says (q,p) must be contextual, if at all they're deterministic. This would show the relation between non-locality and contextuality in the continuous variable regime, which isn't yet properly understood and is of considerable interest. The larger goal is to see how spin like discrete degrees of freedom are fundamentally different from (q,p). Perhaps this would suggest an appropriate understanding of it's extension to QFTs and quantum gravity (QG).

I haven't had any formal courses in QG and therefore the five year programme will be particularly efficacious. I have gleaned that String Theory (ST) is a promising framework for unification of all known forces, although it's still not mature enough. Consequently, both the development of ST itself and its applications to blackhole physics, for example are of braod interest to me. While I am unable to formulate a precise research problem, I hope that my past work supports my application to a PhD in this exciting field.

Needless to say, University of Illinois (UI) is an excellent place for any intellectual pursuit, where mavericks work in important and diverse fields, both within and outside QG. With various erudite researchers, such as the outstanding String Physicist, Prof. Rob Leigh and the simultaneously eclectic & perspicacious, Dr. Thomas Faulkner, who has worked on connecting ST, black holes, entanglement and quantum phases, UI dovetails all that I could ask for in a graduate school, to advance towards a career in academics & research.

[1] A. S. Arora, A. Asadian. *Proposal for a macroscopic test of local realism with phase-space measurements.* Phys. Rev. A **92**, 062107.

# Proposal for a macroscopic test of local realism with phase-space measurements

Atul S. Arora[1,2] and Ali Asadian[2]
[1] Indian Institute of Science Education & Research (IISER), Mohali, Sector 81, Mohali 140 306, India
[2] Naturwissenschaftlich-Technische Fakultät, Universität Siegen, Walter-Flex-Str. 3, D-57068 Siegen, Germany
(Dated: May-July 2015)

We propose a test of local realism based on correlation measurements of continuum valued functions of positions and momenta, known as modular variables. The Wigner representations of these observables are bounded in phase space and therefore, the associated inequality holds for any state described by a non-negative Wigner function. This agrees with Bell's remark that positive Wigner functions, serving as a valid probability distribution over local (hidden) phase space coordinates, do not reveal non-locality. We construct a class of entangled states resulting in a violation of the inequality and thus truly demonstrate non-locality in phase space. The states can be realized through grating techniques in space-like separated interferometric setups. The non-locality is verified from the spatial correlation data that is collected from the screens.





## I. INTRODUCTION

In 1935, Einstein, Podolsky, and Rosen (EPR) argued that the quantum-mechanical description of physical reality is not complete, and thus may be superseded by a more complete realistic theory which reproduces the quantum mechanical predictions, and at the same time, obeys the locality condition [1]. Bell derived an experimentally testable inequality, in his seminal 1964 paper [2], which bounds the correlations between bipartite measurements for any such local hidden-variable theory, but is violated by quantum mechanics. This was a major breakthrough towards empirical tests of quantum mechanics against theories conforming to common sense. Since then, the results constraining the permissible types of hidden variable models of quantum mechanics have attracted much attention and have been reformulated as the problem of contextual measurements by Kochen and Specker [3] and in terms of temporal correlations by Leggett and Garg [4]. Today, these concepts have mainly been formulated for intrinsic quantum degrees of freedom of microscopic particles such as spins, and tested in various experiments with photons [5], impurity spins [6, 7] or superconducting qubits [8]. All experimental observations confirmed the validity of quantum mechanics at this level.

The outstanding challenge is, however, to formulate similar tests with true phase space measurements, where non-locality is inferred directly from observing, the spatial degree of freedom, for example. This can be viewed as a natural extension to the macroscopic limit of local realism tests [9]. This is closer in spirit to the original EPR argument which uses phase space description, a natural concept in the classical world, to better address the reality and locality problems of quantum mechanics. Notably, the Wigner function associated with the entangled state used in the EPR argument, the so called EPR state, is non-negative everywhere [10]. That is why, Bell argued that EPR states do not lead to a violation of the inequalities derived from locality and hidden variable assumptions [11]. This was because non-negative Wigner functions serve as valid joint probability distributions over local *hidden* positions and momenta. Thereby, in principle, a model of local hidden variable can be attributed to such states. Banaszek and Wodkiewicz [12], however, showed that using particular measurements, namely parity, EPR states can reveal non-local features indicating that not only the state itself but the type of correlation measurements is also important in any local realism test. This opens the discussion as to which measurements are good candidates for appropriately testing local hidden variable models in phase space [13, 14]. The problem of constructing a "macroscopic" test of local hidden variable models depends on choosing proper observables whose Wigner representations satisfy the constraint imposed by the algebraic Clauser-Horne-Shimony-Holt (CHSH) [15] expression. The term "macroscopic" henceforth, will be used to refer to the measurement of a particle's phase space coordinates. This class of measurements has a clear description in classical limit and its evaluation does not involve sharp measurements that betray "quantum degrees of freedom".

The observables used here are the so called modular variables [16]. Recently, such variables have found applications in detecting certain continuous variable (CV) entangled states [17–19] and quantum information [20, 21]. Furthermore, they have been used for fundamental tests such as macroscopic realism [22], contextuality [23, 24] and even the GHZ test [25]. This strongly suggests that modular variables can be used for Bell inequality tests of local hidden variable theories as well. Recently a Bell test with discretized modular variables was proposed [26]. In the present work we put forward a Bell test with "continuous" modular variables which requires phase space measurements.

The paper is organized as follows. In Section II, we introduce our framework for the Bell test, aiming to use most "classical-like" variables and measurements. This motivates a macroscopic test of local realism [9]. In Section III we construct a Bell operator from modular variables, for which the violation is achieved only if the state is described by a negative Wigner function. We then proceed with identification of the relevant entangled state, explicitly showing the violation. Finally, in Sections IV and V we show how the entire test can be implemented in a double (multi-slit) grating setup [10]. Grating techniques have been used to experimentally demonstrate quantum matter waves [27]. We summarize by briefly discussing the outlook in Section VI and conclude in Section VII.

## II. FRAMEWORK FOR A MACROSCOPIC BELL TEST

In what follows we develop a test of local realism which complies with Bell's aforesaid argument. The central problem here is to construct a Bell-operator of the CHSH form

$$\hat{\mathcal{B}} \equiv \hat{A}_1 \otimes (\hat{A}_2 + \hat{A}'_2) + \hat{A}'_1 \otimes (\hat{A}_2 - \hat{A}'_2) \quad (1)$$

expressed in terms of suitable local CV observables $\hat{A}_i$ which must be restricted to a limited range of values to impose a well-defined classical bound. We therefore require the observables to satisfy the following properties.

(a) Eigenvalues of $\hat{A}_i$, $|a_i| \leq 1$, which for bounded observables can be achieved trivially by re-scaling. An example of this is the parity operator.
While this condition is enough to obtain a classical bound, we demand an extra constraint which is necessary for probing non-locality in phase space.

(b) The observable $\hat{A}$ corresponds to a *bounded* c-number function in phase space obtained from the Wigner-Weyl correspondence ($q \leftrightarrow \hat{q}$, $p \leftrightarrow \hat{p}$), viz.

$$|\mathcal{W}_{\hat{A}}(q,p)| \equiv \left| \int dq' e^{ipq'} \langle q - \frac{q'}{2} | \, \hat{A} \, | q + \frac{q'}{2} \rangle \right| \leq 1. \quad (2)$$

This entails,

$$|\mathcal{W}_{\hat{\mathcal{B}}}(\boldsymbol{q},\boldsymbol{p})| = |\mathcal{W}_{\hat{A}_1}(q_1,p_1)[\mathcal{W}_{\hat{A}_2}(q_2,p_2) + \mathcal{W}_{\hat{A}'_2}(q_2,p_2)] + \mathcal{W}_{\hat{A}'_1}(q_1,p_1)[\mathcal{W}_{\hat{A}_2}(q_2,p_2) - \mathcal{W}_{\hat{A}'_2}(q_2,p_2)]| \leq 2,$$

for the Wigner representation of the Bell operator where $\boldsymbol{q} \equiv (q_1, q_2)$ and $\boldsymbol{p} \equiv (p_1, p_2)$. Accordingly, for any state, including the EPR state, described by a valid (non-negative) probability distribution over phase space, the following inequality holds

$$|\langle \hat{\mathcal{B}} \rangle| = \left| \int W_{\hat{\rho}}(\boldsymbol{q},\boldsymbol{p}) \mathcal{W}_{\hat{\mathcal{B}}}(\boldsymbol{q},\boldsymbol{p}) d\boldsymbol{q} d\boldsymbol{p} \right| \leq 2, \quad (3)$$

where $W_{\hat{\rho}}$ is the Wigner quasi-probability distribution corresponding to $\hat{\rho}$ given by $W_{\hat{\rho}} = \mathcal{W}_{\hat{\rho}}/2\pi\hbar$. A violation therefore must necessarily arise from the negativity of the Wigner function describing the state.

Although formally valid, the Bell inequality expressed in terms of displaced parity operators used in Ref. [12, 28], voids the second condition; their Wigner representations are given by delta functions which are unbounded in phase space.

The measurement scheme used for evaluating the correlations, must have a clear classical limit for any reasonable "macroscopic test". This measurement strategy is in marked contrast with other approaches that use parity measurements [12]. Parity measurements unlike phase space measurements, require resolving intrinsic quantum degrees of freedom and thus have no classical analog. It has been shown that for sufficiently sharp measurements the system inevitably enters a quantum regime and no classical description is possible [29]. Thus such measurements remotely resemble "classical-like" measurements, if at all.

The binary binning of quadrature measurements has also been shown to be a possible scheme [30, 31] where entangled Schrödinger Cat states (and their appropriate generalizations) are used. Here however, to preserve features characteristic of classical dynamics, we aim to adopt a different measurement strategy which retains the continuous spectra and uses phase space exclusively.

### Phase space translation and modular variables

One particular class of bounded observables can be constructed from the quantum mechanical space translation operator, $e^{-i\hat{p}L/\hbar}$. As its name suggests, this operator displaces a particle by a finite distance $L$, which in our case will be the distance between two adjacent slits. This operator is not an observable, therefore we define a symmetric combination

$$\hat{X} \equiv \frac{e^{-i\hat{p}L/\hbar} + e^{i\hat{p}L/\hbar}}{2} = \cos(\hat{p}L/\hbar), \quad (4)$$

which is explicitly Hermitian and bounded by $\pm 1$. In fact the corresponding function $\left|\mathcal{W}_{\hat{X}}\right| = |\cos(pL/\hbar)|$ is also manifestly $\leq 1$. Further, when $\hat{X}$ is operated on $|p\rangle$, then only the modular part of $p$ is relevant to the value of the operator. Thus we may define

$$\hat{p}_{\mathrm{mod}\frac{h}{L}} \equiv (\hat{p}L/h - \lfloor \hat{p}L/h \rfloor)\frac{h}{L} \quad (5)$$

and note that measuring $\hat{p}_{\mathrm{mod}\frac{h}{L}}$ is sufficient for obtaining the value of $\hat{X} \equiv X(\hat{p}_{\mathrm{mod}\frac{h}{L}})$. Conversely, measuring $\hat{X}$ only yields $\hat{p}_{\mathrm{mod}\frac{h}{L}}$, not $\hat{p}$. The idea is to construct a Bell operator [see Eq. (1)] from $\hat{X}$ in which the different measurement settings are chosen by transforming it using suitable unitary operators.

## III. THE CONSTRUCTION

Consider a localized state $\varphi(q) = \langle q|\varphi\rangle$ symmetric about the position $q = L/2$, where $L \equiv$length scale and $\varphi_n(q) \equiv \varphi(q - nL)$. We define

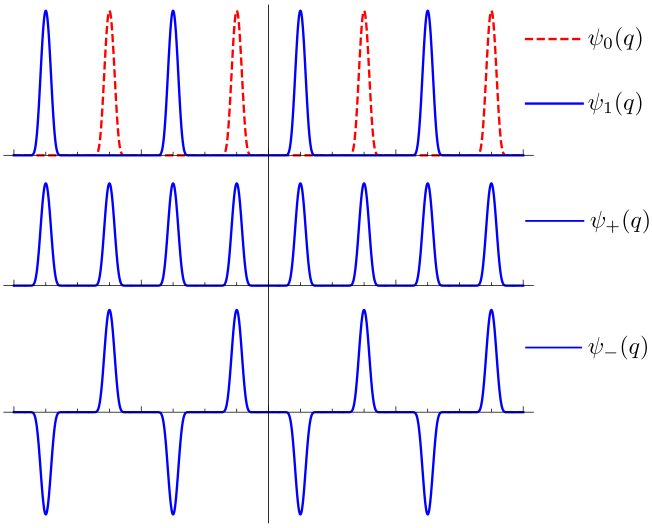


Figure 1. (Color online) Illustration of multicomponent superposition states $|\psi_\pm\rangle$ and $|\psi_0\rangle, |\psi_1\rangle$ for $N = 8$.

$$|\psi_0\rangle \equiv \frac{1}{\sqrt{M}} \sum_{n=-\lfloor \frac{M}{2} \rfloor}^{\lfloor \frac{M-1}{2} \rfloor} |\varphi_{2n+1}\rangle \ , \ |\psi_1\rangle \equiv \frac{1}{\sqrt{M}} \sum_{n=-\lfloor \frac{M}{2} \rfloor}^{\lfloor \frac{M-1}{2} \rfloor} |\varphi_{2n}\rangle \, .$$

Using these states, as illustrated in figure 1, we construct the states

$$|\psi_+\rangle \equiv \frac{|\psi_0\rangle + |\psi_1\rangle}{\sqrt{2}} \ , \ |\psi_-\rangle \equiv \frac{|\psi_0\rangle - |\psi_1\rangle}{\sqrt{2}}. \qquad (6)$$

These states were constructed with a partial translational symmetry which is appropriate to the bounded Hermitian operator $\hat{X}$ discussed earlier. These $N$-component superposition states can represent a delocalized particle after an $N$-slit grating. It follows that

$$\langle \psi_+ | \, \hat{X} \, |\psi_+\rangle = \frac{N-1}{N}$$
$$\langle \psi_- | \, \hat{X} \, |\psi_-\rangle = -\frac{N-1}{N},$$

where $N \equiv 2M$ is the number of 'slits'. Before proceeding further, we introduce a unitary operator $\hat{U}$ to implement different measurement settings. Motivated by the spins we define $\hat{U}$ by its action

$$\hat{U}(\phi) \, |\psi_0\rangle = e^{i\phi/2} \, |\psi_0\rangle \, , \quad \hat{U}(\phi) \, |\psi_1\rangle = e^{-i\phi/2} \, |\psi_1\rangle \, . \quad (7)$$

More explicitly

$$\hat{U}(\phi) \equiv e^{i\hat{Z}\phi/2},$$

where $\hat{Z}$ is s.t. $\hat{Z} \, |\psi_0\rangle = |\psi_0\rangle$ and $\hat{Z} \, |\psi_1\rangle = - |\psi_1\rangle$. We note that $\hat{Z}$ must differentiate between spatial wavefunctions $\langle q|\varphi\rangle$ and $\langle q - L|\varphi\rangle$. It is thus natural to expect

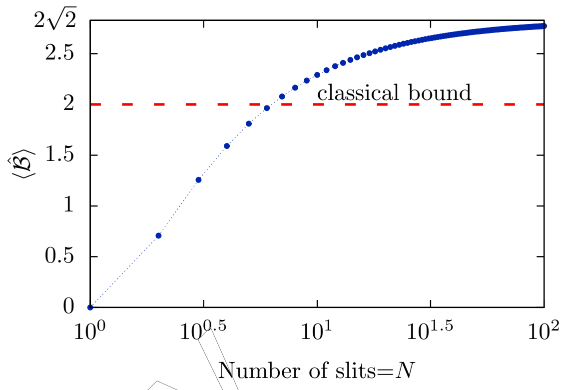


Figure 2. (Color online) Practically, the number of slits, $N$ will be finite. The plot shows $\langle \hat{\mathcal{B}} \rangle$ as a function of $N$. To get a violation, we need merely 8 slits; with 50 we almost saturate.

$\hat{Z}$ to be a function of $\hat{q}_{\text{mod}2L}$, i.e. $\hat{Z} \equiv Z(\hat{q}_{\text{mod}2L})$. For consistency then, we conclude that $Z$ must have the form of a square wave and define

$$\hat{Z} \equiv \text{sgn}\left(\sin \frac{\hat{\text{q}}\pi}{\text{L}}\right).$$

The test is performed by considering two particles and their observers, Alice and Bob; they apply the aforesaid local unitaries to define the setting, and then measure $\hat{X}$. We claim that the suitable entangled state which will yield a violation, given this scheme, is

$$|\Psi\rangle \equiv \frac{|\psi_+\rangle_1 \, |\psi_-\rangle_2 - |\psi_-\rangle_1 \, |\psi_+\rangle_2}{\sqrt{2}}. \qquad (8)$$

We now evaluate $\langle \hat{\mathcal{B}} \rangle$. This essentially requires terms like $\langle \hat{X}(\phi) \otimes \hat{X}(\theta) \rangle$, where $\hat{X}(\theta) \equiv \hat{U}^\dagger(\theta) \hat{X} \hat{U}(\theta)$. It can be shown that [see the Appendix]

$$\langle \hat{X}(\phi) \otimes \hat{X}(\theta) \rangle = -\left(\frac{N-1}{N}\right)^2 \cos(\phi - \theta). \qquad (9)$$

Thus for particular angles, i.e. $\theta'$, $\phi$, $\theta$ and $\phi'$ successively separated by $\pi/4$, we get

$$\left| \langle \hat{\mathcal{B}} \rangle \right| = \left(\frac{N-1}{N}\right)^2 2\sqrt{2}. \qquad (10)$$

The violation, i.e., $\left| \langle \hat{\mathcal{B}} \rangle \right| > 2$, requires $N > 6$; see Fig. 2. To interpret this, we must ensure that the assumptions of the framework are satisfied, viz. $\left| \mathcal{W}_{\hat{X}(\phi)} \right| \leq 1$. To that end, we note that

$$\begin{aligned} \left| \mathcal{W}_{\hat{X}(\phi)}(q,p) \right| &= \left| \frac{1}{2} \int dq' e^{ipq'/\hbar} \langle q - \frac{q'}{2} | \left( e^{-i\hat{Z}\phi/2} \right. \right. \\ &\quad \left. \left. e^{i\hat{p}\frac{L}{\hbar}} e^{i\hat{Z}\phi/2} + \text{h.c.} \right) |q + \frac{q'}{2}\rangle \right| \\ &= \left| \text{Re}\left( e^{-iZ_-(q)\phi/2} e^{ip\frac{L}{\hbar}} e^{iZ_+(q)\phi/2} \right) \right| \\ &= |\cos(pL/\hbar \pm Z_\pm(q)\phi)| \leq 1, \end{aligned} \qquad (11)$$

where $Z_\pm(q) \equiv Z[(q \pm \frac{L}{2})\mathrm{mod}2L]$ and we used the fact that $Z(q) = -Z(q+L)$ (omitting the mod$2L$).

### Passing Remarks

1. Pauli-like Commutation: While at first the definition of $\hat{Z}$ might appear arbitrary, we show that it naturally yields Pauli like algebra. We start with $[\hat{Z}, e^{i\hat{p}L/\hbar}]$. To evaluate it, we multiply the second term with $\int dq\, |q\rangle\langle q|$ and obtain $\hat{Z}e^{i\hat{p}L/\hbar}+\hat{Z}e^{i\hat{p}L/\hbar}$ where we've used $Z(\hat{q}_{\mathrm{mod}2L}) = -Z((\hat{q}\pm L)_{\mathrm{mod}2L})$.

$$[\hat{Z}, \hat{X}] = 2\hat{Z}\hat{X} = -2i\hat{Y},$$

where $\hat{Y} \equiv i\hat{Z}\hat{X}$. Here $i$ was introduced to ensure $\hat{Y}^\dagger = \hat{Y}$, since $\hat{X}^\dagger = \hat{X}$ and $\hat{Z}^\dagger = \hat{Z}$. Similarly $\{\hat{Z}, \hat{X}\} = 0$. From the definition of $Y$ and the anti-commutation, $\{\hat{Y}, \hat{X}\} = 0$ and $\{\hat{Y}, \hat{Z}\} = 0$ also follow trivially. We may point out that while $\hat{Z}^2 = 1$, it is not a sum of a 2 state projector and $\hat{X}^2 \neq 1$ in general. This manifests in the following relations.

$$\begin{aligned}[\hat{X}, \hat{Y}] &= -2i\hat{Z}\hat{X}^2 = -2i\hat{X}^2\hat{Z}\\ [\hat{Y}, \hat{Z}] &= -2i\hat{X}.\end{aligned}$$

It is apparent that the exact SU(2) algebra is not necessary to arrive at a violation.

2. Asymmetry in Z and X: Using an analogous momentum translation operator, the following can be derived from the definition of $\hat{p}$.

$$e^{i\hat{p}u}e^{i\hat{q}v} = e^{i\hbar uv}e^{i\hat{q}v}e^{i\hat{p}u}.$$

For appropriate choices of $u, v$, the translation operators can be made to commute or anti-commute. In the former case, it means that one can simultaneously measure modular position and momentum (which is in stark contrast with $\hat{x}$ and $\hat{p}$ measurements) and in the latter case, one can define Pauli matrix like commutation. Considering the operator (non-Hermitian for simplicity) $\hat{X} = e^{i\hat{p}L/\hbar}$, defining $\hat{Z} = e^{i\hat{q}2\pi/2L}$ is more natural. They also follow the desired anti-commutation $\{\hat{X}, \hat{Z}\} = 0$ and we could define $\hat{Y} = i\hat{Z}\hat{X}$ to get a more natural generalization. The question is why did $\hat{Z} = Z(\hat{q}_{\mathrm{mod}2L})$ appear in the analysis. The cause of this asymmetry hinges on the preferential treatment of position space. We could have constructed states of the form $|\psi_0\rangle = \sum_n |q+nd\rangle$ and used the natural definition of $\hat{Z}$ to obtain the violation. The issue is that this forces us to choose a countable superposition of position eigenkets as our desired state.[1] If we start with better defined and broader class of relevant states, $Z(\hat{q}_{\mathrm{mod}2L})$ appears naturally.

3. Commutation and classical limit: It is well recognized and can be shown that there is a tight relation between non-locality and non-commutativity of operators. The violation occurs for choices of settings whose corresponding observables do not commute. In our construction we can demonstrate that the source of violation can be clearly attributed to the non-commutativity between position and momentum, $[\hat{q}, \hat{p}] = i\hbar$. This would be regarded as a further illustration that our approach provides a relevant test in phase space. We show that in our case $[\hat{X}(\theta), \hat{X}(\theta')] \neq 0$. To prove that, we use $\hat{X}(\theta) = \hat{X}e^{i\hat{Z}\theta}$, $e^{i\hat{Z}\theta} = \cos\theta + i\hat{Z}\sin\theta$ and the previous results, to arrive at

$$\begin{aligned}[\hat{X}(\theta), \hat{X}(\theta')] &= 2i\sin(\theta'-\theta)\hat{Z}\hat{X}^2\\ &= 2i\hat{Z}\hat{X}^2 \neq 0,\end{aligned}$$

where the last equality holds when the angles are as defined earlier. Classically this term not only vanishes, the different measurement settings also become identical. The Heisenberg equation of motion for the displacement operator

$$\begin{aligned}\frac{d\hat{X}}{dt} &= i\hbar^{-1}[\hat{Z}, \hat{X}]\\ &= i\hbar^{-1}\Big(Z(\hat{q}_{\mathrm{mod}2L}) - Z(\hat{q}_{\mathrm{mod}2L} \pm L)\Big)\hat{X}\\ &= i\hbar^{-1}2\hat{Z}\hat{X} \qquad (12)\end{aligned}$$

where $\hat{Z}$ is the potential, shows that $\hat{X}(\theta)$ is essentially $\hat{X}$ at some later time. However, classically, since the particle experiences no force (constant potential), $X(t) = X(t_0)$. This peculiarity is the same as that of the scalar Aharonov-Bohm effect, which is exploited here for realizing different measurement settings. Manifestly then, the non-commutativity of $\hat{q}$ and $\hat{p}$ results in $\hat{X}(t) \neq \hat{X}(t_0)$ (as it follows a non-local equation of motion [32]) which is pivotal for the violation.

## IV. MEASUREMENT SCHEMES

The scheme requires us to evaluate the correlation functions such as $\langle\hat{X}(\theta) \otimes \hat{X}(\phi)\rangle$. Equivalently, the measurement settings can be chosen by applying the corresponding local unitaries on the entangled state, that is $|\Psi_{\theta\phi}\rangle = \hat{U}(\theta) \otimes \hat{U}(\phi)\,|\Psi\rangle$. Therefore, obtaining $|\langle p_1, p_2|\Psi_{\theta\phi}\rangle|^2$ is sufficient for evaluating $\langle\hat{X}(\theta)\otimes\hat{X}(\phi)\rangle = \int dp_1 dp_2 \cos(p_1 L/\hbar)\cos(p_2 L/\hbar)\,|\langle p_1, p_2|\Psi_{\theta\phi}\rangle|^2$.

It is known that in the far-field approximation [10]

$$\left|\left\langle p_1 = \frac{p_z q_1}{D}, p_2 = \frac{p_z q_2}{D}\middle|\Psi_{\theta\phi}\right\rangle\right|^2 = \frac{D^2}{p_z^2}\left|\langle q_1, q_2|\Psi^{\mathrm{screen}}_{\theta\phi}\rangle\right|^2, \qquad (13)$$

where $\left|\Psi^{\mathrm{screen}}_{\theta\phi}\right\rangle$ is the state of the system at the screen, $D$ is the distance between the gratings and the screens and $p_z$ is the $z$ component of momentum of the particle. For a photon, $p_z = h/\lambda$ while for a massive particle with

[1] Such a state is strictly not even in the Hilbert space.

mass $m$, $p_z = mD/T$, where $T$ is the time taken to arrive at the screen from the grating (see, Fig. 3). The idea is simply that the momentum distribution at the grating can be recovered by observing the spatial distribution at the screen, sufficiently far away.

## V. PHYSICAL IMPLEMENTATION

The test can be implemented in a quantum interferometric setup, using grating techniques to create multi-component superposition states, as is done in matter wave experiments for instance. We show that this scheme can be implemented using photons. We harness the two degrees of freedom of a photon, it's polarization and it's spatial degree of freedom to construct the required state. With a slightly modified setup, it is possible to do the same with spin and position for matter waves (see section IX 3). The final setup is given in figure 3. We need only consider the quantum mechanical description along the $x$-axis.

### A. Creation of the entangled state

The desired entangled state is $|\Psi\rangle$, as stated in Eq. (8). We start with noting the triviality of constructing a $|\psi_+\rangle$ state (see Eq. (6)). Consider a source that produces a state $|\gamma\rangle$ at the grating. $\langle q|\gamma\rangle$ is assumed to be a real Gaussian with $\sigma \gg 2NL$. The grating has $N$ slits of width $a \ll L$, separated by a distance $L$ (center to center). After the grating, we obtain $|\psi_+\rangle = \hat{G}\,|\gamma\rangle$, where $\hat{G}$ maybe formally defined accordingly. Similarly the $|\psi_-\rangle$ state can be constructed by using glass slabs at alternate slits, such that the phase introduced is $\pi$. In figure 3, if you consider only one particle, and disregard everything after the grating, then the setup is expected to produce a $|\psi_+\rangle$ state, right after it. To produce the desired entangled state, we start with two entangled photons, such that their polarization state can be expressed as $|\chi\rangle \equiv \frac{|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2}{\sqrt{2}}$. Their spatial description (along $x$-axis) is initially assumed to be $|\gamma\rangle_1\,|\gamma\rangle_2$ so that the post grating state is

$$\frac{|H\rangle_1\,|V\rangle_2 - |V\rangle_1\,|H\rangle_2}{\sqrt{2}}\,|\psi_+\rangle_1\,|\psi_+\rangle_2\,.$$

If we had glass slabs, whose refractive index (given some orientation) was say $\eta_H = 1$ for a horizontally polarized beam and $\eta_V = \eta \neq 1$ for vertical polarization, then we could harness the entangled polarization state to create the required spatially entangled state. Birefringent crystals have such polarization dependent refractive indices. Assume that alternating birefringent crystals have been placed after both the gratings with appropriate thickness so that the subsequent state is

$$\frac{|H\rangle_1\,|V\rangle_2\,|\psi_+\rangle_1\,|\psi_-\rangle_2 - |V\rangle_1\,|H\rangle_2\,|\psi_-\rangle_1\,|\psi_+\rangle_2}{\sqrt{2}}.$$

If the polarization state is traced out, the resultant state will be mixed, hence useless. Instead, a 45° polarizer is introduced after which (see section IX 4) the target entangled state

$$|\chi_{45}\rangle\,|\Psi\rangle = |\nearrow\rangle_1\,|\nearrow\rangle_2\,\frac{|\psi_+\rangle_1\,|\psi_-\rangle_2 - |\psi_-\rangle_1\,|\psi_+\rangle_2}{\sqrt{2}}$$

is obtained, where $|\nearrow\rangle \equiv (|H\rangle + |V\rangle)\,/\sqrt{2}$. As a remark, it maybe be stated that although to arrive at this result we assumed that $\eta_H = 1$, which is unreasonable physically, we can compensate for $\eta_H \neq 1$ by putting appropriate glass slabs at the alternate empty slits, to produce zero relative phase when the polarization is horizontal.

### B. Measurement Settings

The measurement setting is applied by local unitaries like $\hat{U}(\theta) \otimes \hat{U}(\phi)$. A local unitary can be performed by placing alternating glass slabs of widths such that Eq. (7) holds. These slabs may be placed right after the birefringent crystals, before the polarizer. The final state just after the polarizer is given by $|\Psi_{\theta\phi}\rangle = \hat{U}(\theta) \otimes \hat{U}(\phi)\,|\Psi\rangle$, where $\theta\phi$ is one of the four possible measurement settings.

### C. Effective Practical Setup

Placing glass slabs may not be suitable for fine gratings, although a similar setup maybe possible [33]. Practically we can implement the same scheme using the setup shown in figure 3. The first large slab is a Birefringent crystal $(\eta_H, \eta_V)$ while the adjacent slab is plain glass $(\eta)$. We generate longitudinal standing pressure waves so that the effective thickness at alternate grating sites are given by $d_0, d_1$ and $l_0, l_1$ for the crystal and slab respectively. The phase difference between a horizontal $|\psi_0\rangle$ and $|\psi_1\rangle$ will be given by $\eta_H(d_0 - d_1) \equiv \phi_A$; note that physically only phase differences are essential. For the vertical component, it'll be $\eta_V(d_0 - d_1)$. If we impose $\eta_V(d_0 - d_1) = \pi + \phi_A$, then we would've created[2] the state

$$e^{i\hat{Z}\frac{\phi_A}{2}}\,\frac{|\psi_+\rangle + |\psi_-\rangle}{\sqrt{2}}$$

for an incident $\frac{|H\rangle + |V\rangle}{\sqrt{2}}$ polarization state. $d_0$ and $d_1$ will be constrained by some relation depending on physical

[2] up to an overall phase

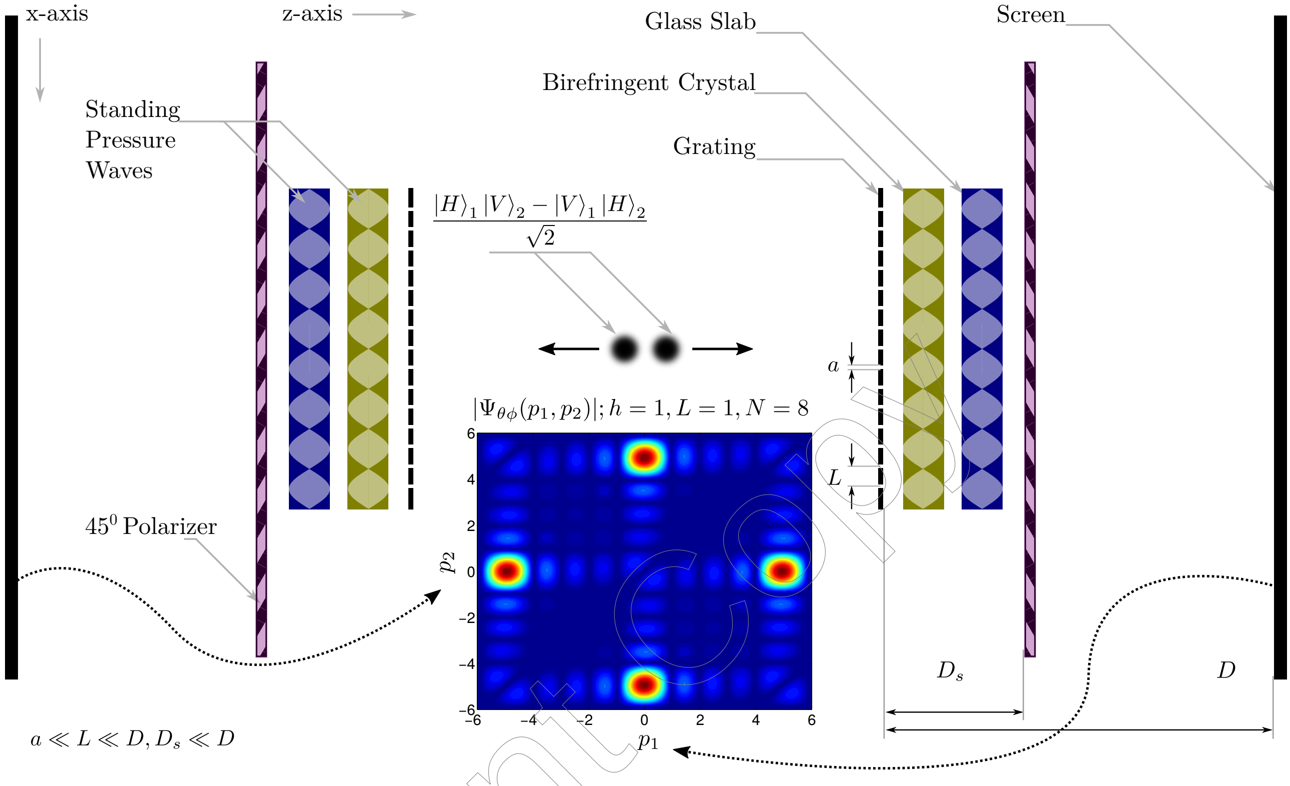


Figure 3. (Color online) The experimental setup for implementing the test. It includes the scheme for creating the necessary entangled state. See the text for further details.



properties of the crystal; they'll also depend on the amplitude of the longitudinal wave. From this and the imposed constrain, $d_0, d_1$ and the corresponding amplitude can be determined. However, we have not the freedom to change $\phi_A$. To remedy this, we use the glass slab. It will introduce an additional relative phase $\eta(l_0 - l_1) \equiv \phi_B$. Here, again $l_0, l_1$, may satisfy some constraint, but will depend on the amplitude which is adjustable. Thus, by changing this amplitude, we can set the relative phase $\phi = \phi_A + \phi_B$ arbitrarily.

Spatial light modulators maybe used to more conveniently implement the aforesaid action of the glass slab and Birefringent crystal.

Effectively therefore, this scheme allows for both creation of the entangled state and changing the measurement settings in a practical way.

## VI. DISCUSSION

It is worth adding that one can use an alternative measurement strategy giving the same violation of the inequality. That is, to measure the modular variable with two-valued POVM elements $\hat{E}_\pm$, given by

$$\hat{E}_\pm = \frac{1}{2}(\hat{\mathbb{I}} \pm \hat{X}), \tag{14}$$

satisfying $\hat{E}_+ + \hat{E}_- = \hat{\mathbb{I}}$. It follows that $\langle \hat{X} \rangle = p_+ - p_-$ where the probabilities of getting $\pm$ outcomes, $p_\pm = \langle \hat{E}_\pm \rangle$, can be determined from the observed binary statistics read out from an ancillary two-level system [34].

An interesting problem is to develop our approach to finite-dimensional systems, qudits. A class of Bell inequalities was proposed by Collins *et al.* [35], which is useful for demonstrating nonlocality in high-dimensional entangled states. For our version of Bell inequality generalized to $d$-dimensional systems can be achieved by using the discrete translation operators known as Heisenberg-Weyl or Generalized Pauli operators, i.e., $e^{-i2\pi \hat{P} l/d}$, whose action is $e^{-i2\pi \hat{P} l/d} |n\rangle = |n + l\rangle$ where $l$ describes the steps translated in discrete position space with periodic boundary conditions and $\hat{P} = \sum_{k=0}^{d-1} k |k\rangle\langle k|$ is the discrete momentum operator. From this we obtain the relevant discrete modular variable $\hat{X}_d^l = \cos(2\pi \hat{P} l/d)$. We expect that the class of $d$-dimensional entangled states which demonstrate nonlocality here will be different from those considered by Collins *et al.* [35] and Lee *et al.* [36].

It is obvious from the properties of the modular variables we use, that the violation is more pronounced for higher number of slits. One can however imagine that those entangled states created with slits fewer than the minimum number needed for obtaining a violation, must also hold non-local properties. To reveal the non-locality in this range one may need a more optimal set of observables, which involve a suitable combination of different modular variables, as opposed to the set considered here.

## VII. CONCLUSION

In the present work, we constructed a new Bell-operator in terms of phase space measurements via modular variables. In this scheme there is no possibility for bipartite system with positive definite Wigner function, formally entangled or not, to yield a violation of the inequality. Therefore, a violation of the inequality truly contradicts local (hidden) phase space models. From this perspective, our scheme is strongly different from the other approaches reported in Refs. [12, 37–39] where sharp quantum measurements with no classical analog have been used. The measurement observables in our scheme instead are very simple with a clear classical limit. The relevant entangled states used for achieving a violation of the inequality however required creation of multi-component superposition states characterized by negative Wigner function. Interestingly our scheme also involves the scalar Aharonov-Bohm effect, manifesting another type of nonlocality [32].

## VIII. ACKNOWLEDGEMENTS

The authors acknowledge discussions with O. Gühne, C. Budroni and P. Rabl. A. S. A is grateful to O. Gühne and his group for their support and hospitality during his visit to the Universität Siegen. The authors acknowledge the financial support from the German Academic Exchange Service (DAAD), the FQXi Fund (Silicon Valley Community Foundation) and also the KVPY programme, Department of Science and Technology, India; The DFG and the Austrian Science Fund (FWF) through Erwin Schrödinger Stipendium No. J3653-N27 are also acknowledged.

## IX. APPENDIX

### Claims

Here, we provide more detailed derivations of the results.

#### 1. Useful Expectation Values

$\left\langle \psi_+ \middle| \hat{X} \middle| \psi_+ \right\rangle = \frac{N-1}{N}$, $\left\langle \psi_- \middle| \hat{X} \middle| \psi_- \right\rangle = -\frac{N-1}{N}$
$\left\langle \psi_0 \middle| \hat{X} \middle| \psi_0 \right\rangle = 0$, $\left\langle \psi_1 \middle| \hat{X} \middle| \psi_1 \right\rangle = 0$
$\left\langle \psi_1 \middle| \hat{X} \middle| \psi_0 \right\rangle = \frac{\frac{N-1}{N}+\frac{N}{N}}{2} = \frac{2N-1}{2N} = \left\langle \psi_0 \middle| \hat{X} \middle| \psi_1 \right\rangle$
$\left\langle \psi_- \middle| \hat{X} \middle| \psi_+ \right\rangle = \frac{-\langle \psi_1|\hat{X}|\psi_0\rangle + \langle \psi_0|\hat{X}|\psi_1\rangle}{2} = 0 = \left\langle \psi_+ \middle| \hat{X} \middle| \psi_- \right\rangle$

$$\left\langle \Psi \middle| \hat{X}\otimes\hat{X} \middle| \Psi \right\rangle = \frac{1}{2}\Big( \left\langle \psi_- \middle| \hat{X} \middle| \psi_- \right\rangle \left\langle \psi_+ \middle| \hat{X} \middle| \psi_+ \right\rangle + \left\langle \psi_+ \middle| \hat{X} \middle| \psi_+ \right\rangle \left\langle \psi_- \middle| \hat{X} \middle| \psi_- \right\rangle \Big) = -\left(\frac{N-1}{N}\right)^2$$

#### 2. For Arbitrary $\theta_i$ and $\phi_i$

$\left\langle \hat{U}^\dagger(\phi_i)\hat{X}\hat{U}(\phi_i) \otimes \hat{U}^\dagger(\theta_i)\hat{X}\hat{U}(\theta_i) \right\rangle = -\left(\frac{N-1}{N}\right)^2 \cos(\phi_i - \theta_i)$

Proof: We start with defining $\phi \equiv \phi_i$ , $\theta \equiv \theta_i$, $\delta \equiv \phi - \theta$, $\delta' \equiv \delta/2$. Next, we note that LHS $= \left\langle \Psi' \middle| \hat{X}\otimes\hat{X} \middle| \Psi' \right\rangle$ where $|\Psi'\rangle = \hat{U}(\phi_i)\otimes\hat{U}(\theta_i)|\Psi\rangle$.

$$\begin{aligned}
|\Psi'\rangle &= \frac{e^{i\delta'}}{\sqrt{2}}\left(\frac{|\psi_+\rangle - |\psi_-\rangle}{\sqrt{2}}\right)\left(\frac{|\psi_+\rangle + |\psi_-\rangle}{\sqrt{2}}\right) \\
&\quad - \frac{e^{-i\delta'}}{\sqrt{2}}\left(\frac{|\psi_+\rangle + |\psi_-\rangle}{\sqrt{2}}\right)\left(\frac{|\psi_+\rangle - |\psi_-\rangle}{\sqrt{2}}\right) \\
&= \frac{e^{i\delta'}}{2\sqrt{2}}\left(|\psi_+\psi_+\rangle + |\psi_+\psi_-\rangle - |\psi_-\psi_+\rangle - |\psi_-\psi_-\rangle\right) \\
&\quad - \frac{e^{-i\delta'}}{2\sqrt{2}}\left(|\psi_+\psi_+\rangle - |\psi_+\psi_-\rangle + |\psi_-\psi_+\rangle - |\psi_-\psi_-\rangle\right) \\
&= \frac{e^{i\delta'} - e^{-i\delta'}}{2\sqrt{2}}|\psi_+\psi_+\rangle + \frac{e^{i\delta'} + e^{-i\delta'}}{2\sqrt{2}}|\psi_+\psi_-\rangle \\
&\quad - \left(\frac{e^{i\delta'} + e^{-i\delta'}}{2\sqrt{2}}\right)|\psi_-\psi_+\rangle - \left(\frac{e^{i\delta'} - e^{-i\delta'}}{2\sqrt{2}}\right)|\psi_-\psi_-\rangle
\end{aligned}$$

Now using section IX 1, we have

$$\begin{aligned}
\text{LHS} &= \left\langle \Psi' \middle| \hat{X} \otimes \hat{X} \middle| \Psi' \right\rangle \\
&= \frac{1}{2}\left(\frac{N-1}{N}\right)^2 \left[ \left| \frac{e^{i\delta'} - e^{-i\delta'}}{2} \right|^2 \right. \\
&\quad - \left| \frac{e^{i\delta'} + e^{-i\delta'}}{2} \right|^2 - \left| \frac{e^{i\delta'} + e^{-i\delta'}}{2} \right|^2 \\
&\quad \left. + \left| \frac{e^{i\delta'} - e^{-i\delta'}}{2} \right|^2 \right] \\
&= -\left(\frac{N-1}{N}\right)^2 \frac{1}{2}\left[2\left(\cos^2\delta/2 - \sin^2\delta/2\right)\right] \\
&= -\left(\frac{N-1}{N}\right)^2 \cos(\delta)
\end{aligned}$$

### 3. Physical implementation with electrons is also possible

If we can show that the basic components used to describe the photon setup can be translated to the electron setup, then in principle we are through. (a) Glass slab: The equivalent is the electric AB effect. We need to simply put a capacitor after the slit and the two components will pick up a phase difference. (b) Polarizer: The Stern Gerlach setup is the classic analogue. We simply block the orthogonal component. (c) Birefringent crystal: This is slightly tricky. It can be modeled by using a combination of gradient of magnetic field (as in Stern Gerlach) and a capacitor. We start with an equivalent superposition of spin states, $\frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}} |\psi_+\psi_+\rangle$. To construct the spin dependent $|\psi_-\rangle$ state, we use the magnetic field gradient to spatially separate the $|\uparrow\rangle$ and $|\downarrow\rangle$ states. We place capacitors as described at the spatial position corresponding to $|\downarrow\rangle$ say. Thereafter, we remove the magnetic field gradient and allow the beams to meet again. This will effectively act as a Birefringent crystal, since the phase difference is spin dependent.

### 4. Action of a polarizer

If we define $|\nearrow\rangle \equiv \frac{|H\rangle + |V\rangle}{\sqrt{2}}$, $|\nwarrow\rangle \equiv -\frac{|H\rangle - |V\rangle}{\sqrt{2}}$ and the $45°$ projector as $|\nearrow\rangle\langle\nearrow|$, then both $|H\rangle \to |\nearrow\rangle$ and $|V\rangle \to |\nearrow\rangle$ where of course with a probability 1/2, the photon will be lost.

### 5. More on measurement

It is essential to know what ballpark resolution is required for detecting the violation from the screen. We note

$$|\langle p_1, p_2 | \Psi_{\theta\phi}\rangle| = |\tilde{\varphi}(p_1)\tilde{\varphi}(p_2) F_{\theta\phi}(p_1, p_2)|$$

where

$$\begin{aligned}
F_{\theta\phi}(p_1,p_2) = \frac{1}{\sqrt{2}} &\sum_{n,m=-\lfloor \frac{M}{2} \rfloor}^{\lfloor \frac{M-1}{2} \rfloor} e^{i(np_1 + mp_2)L/\hbar} \\
&\Big[ -\cos(\delta')[(-1)^m - (-1)^n] \\
&+ i\sin(\delta')[1 + (-1)^{n+m}] \Big]
\end{aligned}$$

$\tilde{\varphi}(p) \equiv \langle p|\varphi\rangle$ and $\delta' = (\phi - \theta)/2$. Since the wavefunction $\varphi(q)$ was assumed sharp with respect to $L$, $|\tilde{\varphi}(p)|$ will only correspond to a broad envelope, over the range $(-Nh/2L, Nh/2L)$. Thus the main feature of $|\langle p_1, p_2, \Psi_{\theta\phi}\rangle|^2$ will be given by $|F_{\theta\phi}|$ as shown in Fig. 3. Graphically it is clear that resolving at the scale $p_{\text{typ}} = \frac{h}{L}$ should be sufficient to capture the relevant features. On the screen, this translates to a typical length, $q_{\text{typ}} = \lambda D/L$ which follows from Eq. (13) and $p_z = h/\lambda$ for a photon. This is reminiscent of typical diffraction experiments and is in units that are readily measurable.


[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[2] J. S. Bell, Physics **1**, 195 (1964).
[3] S. Kochen and E. P. Specker, Journal of Mathematics and Mechanics **17**, 59 (1967).
[4] A. J. Leggett and A. Garg, Phys. Rev. Lett. **54**, 857 (1985).
[5] A. Alain, Nature **398**, 189 (1999).
[6] G. Waldherr, P. Neumann, S. F. Huelga, F. Jelezko, and J. Wrachtrup, Phys. Rev. Lett. **107**, 090401 (2011).
[7] G. C. Knee, S. Simmons, E. M. Gauger, J. J. L. Morton, H. Riemann, N. V. Abrosimov, P. Becker, H.-J. Pohl, K. M. Itoh, M. L. W. Thewalt, G. A. D. Briggs, and S. C. Benjamin, Nat Commun **3**, 606 (2012).
[8] A. Palacios-Laloy, F. Mallet, F. Nguyen, P. Bertet, D. Vion, D. Esteve, and A. N. Korotkov, Nat Phys **6**, 442 (2010).
[9] M. D. Reid, Phys. Rev. Lett. **84**, 2765 (2000).
[10] C. Gneiting and K. Hornberger, Phys. Rev. A **88**, 013610 (2013).
[11] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, England, 1987).
[12] K. Banaszek and K. Wódkiewicz, Phys. Rev. A **58**, 4345 (1998).
[13] M. Revzen, P. A. Mello, A. Mann, and L. M. Johansen, Phys. Rev. A **71**, 022103 (2005).
[14] W. Son, J. Kofler, M. S. Kim, V. Vedral, and Č. Brukner, Phys. Rev. Lett. **102**, 110404 (2009).
[15] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt,

Phys. Rev. Lett. **23**, 880 (1969).
[16] Y. Aharonov, H. Pendleton, and A. Petersen, International Journal of Theoretical Physics **2**, 213 (1969).
[17] C. Gneiting and K. Hornberger, Phys. Rev. Lett. **106**, 210501 (2011).
[18] M. R. Barros, O. J. Farías, A. Keller, T. Coudreau, P. Milman, and S. P. Walborn, Phys. Rev. A **92**, 022308 (2015).
[19] M. A. D. Carvalho, J. Ferraz, G. F. Borges, P.-L. de Assis, S. Pádua, and S. P. Walborn, Phys. Rev. A **86**, 032332 (2012).
[20] A. Ketterer, S. P. Walborn, A. Keller, T. Coudreau, and P. Milman, arXiv:1406.6388.
[21] P. Vernaz-Gris, A. Ketterer, A. Keller, S. P. Walborn, T. Coudreau, and P. Milman, Phys. Rev. A **89**, 052311 (2014).
[22] A. Asadian, C. Brukner, and P. Rabl, Phys. Rev. Lett. **112**, 190402 (2014).
[23] A. R. Plastino and A. Cabello, Phys. Rev. A **82**, 022114 (2010).
[24] A. Asadian, C. Budroni, F. E. S. Steinhoff, P. Rabl, and O. Gühne, Phys. Rev. Lett. **114**, 250403 (2015).
[25] S. Massar and S. Pironio, Phys. Rev. A **64**, 062108 (2001).
[26] A. Ketterer, A. Keller, T. Coudreau, and P. Milman, Phys. Rev. A **91**, 012106 (2015).
[27] P. Haslinger, N. Dörre, P. Geyer, J. Rodewald, S. Nimmrichter, and M. Arndt, Nature Physics **9**, 144 (2013).
[28] S.-W. Lee, H. Jeong, and D. Jaksch, Phys. Rev. A **80**, 022104 (2009).
[29] J. Kofler and Č. Brukner, Phys. Rev. Lett. **99**, 180403 (2007).
[30] J. Wenger, M. Hafezi, F. Grosshans, R. Tualle-Brouri, and P. Grangier, Phys. Rev. A **67**, 012105 (2003).
[31] W. J. Munro, Phys. Rev. A **59**, 4197 (1999).
[32] S. Popescu, Nat Phys **6**, 151 (2010).
[33] O. J. Farías, F. de Melo, P. Milman, and S. P. Walborn, Phys. Rev. A **91**, 062328 (2015).
[34] P. Horodecki, Phys. Rev. A **67**, 060101 (2003).
[35] D. Collins, N. Gisin, N. Linden, S. Massar, and S. Popescu, Phys. Rev. Lett. **88**, 040404 (2002).
[36] S.-W. Lee and D. Jaksch, Phys. Rev. A **80**, 010103 (2009).
[37] Z.-B. Chen, J.-W. Pan, G. Hou, and Y.-D. Zhang, Phys. Rev. Lett. **88**, 040406 (2002).
[38] K. Banaszek and K. Wódkiewicz, Phys. Rev. Lett. **82**, 2009 (1999).
[39] C. Vitelli, M. T. Cunha, N. Spagnolo, F. De Martini, and F. Sciarrino, Phys. Rev. A **85**, 012104 (2012).

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| PHY411 | Nuclear Physics Lab | 4 | A |
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| PHY305 | Atomic & Molecular Physics | 4 | A |
| PHY312 | Advanced Electronics & Instrumentation Lab | 4 | B |
| PHY631 | Quantum Computation & Quantum Information | 4 | A |
| PHY646 | Field Theory | 4 | A |
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| IDC402 | Nonlinear dynamics, chaos & complex systems | 4 | A |
| IDC452 | Seminar (delivering) | 1 | B |
| PHY412 | Condensed matter physics lab | 4 | A |
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Date of Issue: **December 28, 2015**

Dapid H. Paranjape
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