Indian Institute of Science Education and Research Mohali

Curves and Surfaces (MTH201)
Academic Session 2012-13

Tutorial Sheet 2 **August 31 2012**

Instructions: Write main ideas / hints for solving questions in your tutorial noteook. There is no need to write full and formal solution during the tutorial session. However during off class hours you should practice writing these solutions in a formal manner. **Get the signature of your tutor after each session**. To denote derivative of a function θ w.r.t. its parameter t we shall use both notations $\dot{\theta}$ as well as θ'.

1. Let $\vec{\mathbf{p}}$ and $\vec{\mathbf{q}}$ be two points in $\mathbb{R}^3$. Show that among all the curves which pass through these points, it is a straight line whose arc length between $\vec{\mathbf{p}}$ and $\vec{\mathbf{q}}$ is minimum.
   Steps and hints:
   (a) Straight line is: $(1-t)\vec{\mathbf{p}} + t\vec{\mathbf{q}}$. Let γ be any other parametric curve passing through p and q. Let $\vec{u}$ = unit vector in the direction $\vec{\mathbf{q}} - \vec{\mathbf{p}}$.
   (b) Compute integral of the dot product: $\int_0^1 \gamma(t)'.\vec{u}\,dt$. Isn't it same as $\int_0^1 (\gamma(t).\vec{u})'\ dt$? Isn't is the length of line between $\vec{\mathbf{p}}$ and $\vec{\mathbf{q}}$?
   (c) Use Schwarz inequality for $\gamma(t)'$ and $\vec{u}$.
   (d) No more hint!

2. A *cardioid* is the shape of the parametric curve $\gamma : \mathbb{R} \to \mathbb{R}^2$ given by

   $$\gamma(t) = (2\cos t - \cos 2t, 2\sin t - \sin 2t).$$

   (a) Draw a rough sketch of cardioid.
   (b) Determine if γ is a smooth parametric curve.
   (c) Is γ regular?
   (d) Is γ a unit speed curve?
   (e) Calculate the arc length of γ between $t = 0$ and $t = 2\pi$.

   (On the top layer of tea in cups, I have observed a cardioid pattern. I don't know why it appears? Help please!)

3. Find a unit speed reparametrization of the helix $\gamma : \mathbb{R} \to \mathbb{R}^3$ given by $\gamma(t) = (a\cos t, a\sin t, bt)$. (*Hint*: See the unit speed reparametrization proof done in class on 30/08/12. You just have to find h which is the inverse of the arc length function s.)

4. In a rat-cat game, initially a rat is at the origin of $\mathbb{R}^2$ and a cat is at $(1, 0)$. Rat starts moving along Y-axis. Condition of the game is that at any given point of time the direction of the movement of cat will be towards rat and cat will maintain a constant distance with rat. Find a parametric curve γ which gives the trajectory of cat .
