Indian Institute of Science Education and Research Mohali

Curves and Surfaces (MTH201)
Academic Session 2012-13

Tutorial Sheet 8 **November 02 2012**

Instructions: Write main ideas / hints for solving questions in your tutorial noteook. There is no need to write full and formal solution during the tutorial session. However during off class hours you should practice writing these solutions in a formal manner. **Get the signature of your tutor after each session**.

1. Show that the map $\mathcal{L} : S^2 \to S^1 \times (-1, 1)$ given by $\mathcal{L}(\cos\theta\cos\phi, \cos\theta\sin\phi, \sin\theta) = ((\cos\phi, \sin\phi), \sin\theta)$ is equiareal.

2. Show that the surface given by

$$\sigma(t, \theta) = \left(\cos\theta\left(1 + t\cos\frac{\theta}{2}\right), \sin\theta\left(1 + t\cos\frac{\theta}{2}\right), t\sin\frac{\theta}{2}\right)$$

   is a ruled surface. Identify the surface.

3. A unit speed helix $\gamma(t) = (a\cos t, a\sin t, bt)$ is wrapped on the cylinder $\sigma(\theta, v) = (a\cos\theta, a\sin\theta, v)$. Compute the normal curvature of helix.

4. Compute the geodesic curvature of the circle of radius 1 embedded in the sphere of radius 2.

5. Let $\sigma : U \to \mathbb{R}^3$ be a surface patch with $S = \sigma(U)$. The map $\mathcal{G} : S \to S^2$ given by $\mathcal{G}(p) = N_\sigma(p)$ is called the *Gauss map*. Find the Gauss map for the cone $\sigma(\theta, v) = (v\cos\theta, v\sin\theta, v)$ and estimate the area of the image of $\mathcal{G}$ on the sphere.
