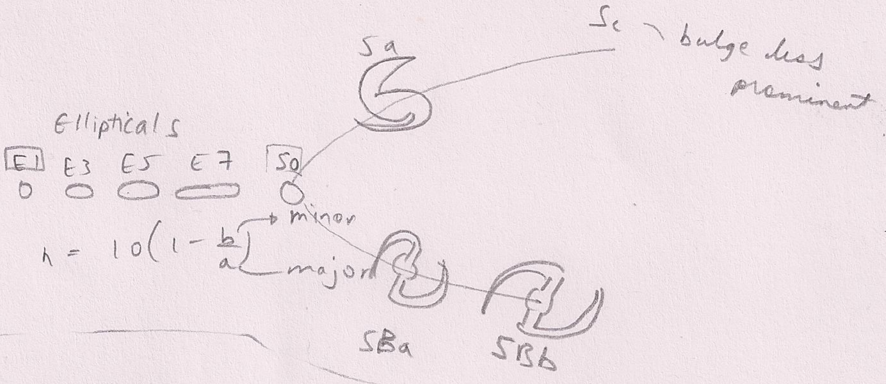
Galaxie ≡ isolated, gravitationally bound

- Diffraction point

Hubbel – – galaxy within or outside

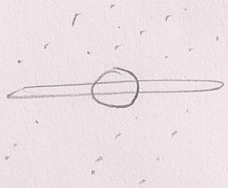


Milky : aspect ratio of bulge = 3:1

n = 3 to 1 (or 2?) –

SINGS → Penn State Uni

(a) collisional radiat decay – ns
lowd



Dark regions – massive Black
1 : 1½

– Photoionization Nebula –

– Messier Catalogue
M51 Star formation

(ASK Prashansa)

open cluster

1 AU = 10^{-5} parsec

$3 \times 10^6 M_\odot$ with radius

Sgr A*

AGC (Active Galactic Centre) spectra
HR Diagram (? ASK)
Dwarf Galaxies
Clusters of Galaxies 1 Mega Parsec

AGN

$$R_{BH} = \frac{2GM_{BH}}{c^2}$$

$$R_{ISCO} = 3R_{BH} = \frac{6GM_{BH}}{c^2}$$

Innermost Stable Circular Orbit

Accretion, slow radial drift & heat

$$E_{in} \simeq -\frac{GM_{BH}m}{R}$$

$$E_f \simeq KE - \frac{mc^2}{6} = -\frac{GM_{BH}m}{R} \simeq 0$$

E at R_{ISCO}

$$\Rightarrow \quad KE \simeq \frac{mc^2}{6}$$

$$E_{radiated} = f\,\frac{mc^2}{6} \simeq \frac{mc^2}{60}$$

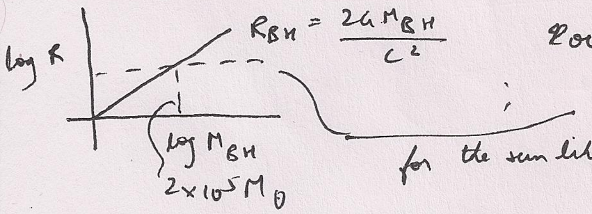
some fraction

luminosity, mass per unit time incoming

$$L = \frac{\dot{M}c^2}{60} = \frac{\epsilon M_\odot c^2}{60}$$

$$\frac{L}{L_\odot} = \frac{\epsilon M_\odot c^2}{60 L_\odot} = \frac{2\times10^{30}\times 9\times10^{16}}{\underbrace{3\times10^{7}}_{\text{for per year}}\times 60\times 4\times10^{26}}\,\epsilon\,(/\text{year}) = \frac{1}{4}\times10^{12}\left(\frac{\epsilon}{1/\text{year}}\right)\left(\frac{f}{0.1}\right)$$

Advection Dominated Accretion Flow (ADAF)



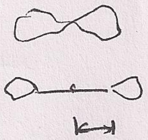
Roche limit

$$r = R_*\left(\frac{3M_{BH}}{M_*}\right)^{1/3} \simeq R_{BH}^{1/3} M_*^{2/3}$$

Quasi Stellar Sources; (a) Looks like a star, (the spectrum doesn't)
(b) Spectrum is "non-thermal"
(c) Can vary on short time scales.
(d) Very strong Ly α line

3C48, 3C273 ~ 1950s, 60s

Radio Galaxies; double lobed



as high as 1 mpc

pc = 3.26 light years.

material must've moved for all these years

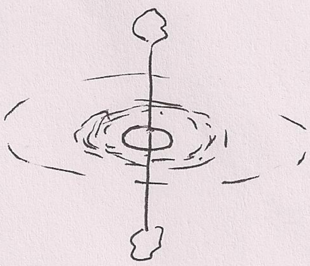
⇓

must've been stable for long enough

⇓

associated with angular momentum.

Unification model

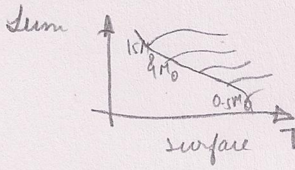


Lec#2

Stars | 0.08 - 100 $M_\odot$
Gas
Dust
graphite like crystals etc
absorb

Mass-luminosity
Mass-age

H-R



$L \propto M^{3.5}$ (exponent depends on where you are)

Main Sequence: $T \propto \frac{fM}{L} \propto M^{-2.5}$ (fraction)

$L = 4\pi R^2 \sigma T_{eff}^4 \propto M^{3.5}$

$T_{eff}^4 \propto M^{1.5} \Rightarrow T_{eff} \sim M^{3/8}$

ASK: ~~Com~~ Combine spectrum -

$O_3 : O_2$ - can tell ~~what does~~ what metallicity & temp. of the ionization gas.

- Mass f^n : - where does it come from?

Dwarf Galaxy

Clusters of Galaxy

Regions

Clusters of Galaxies

ASK: Density Parameters?

Distr of Galaxies in the Universe.

Large scale distr. of galaxies.

Lyman-α forest

ASK: redshift business

$\lambda_{em} = 1216\,Å$

$\lambda_{obs} = \lambda_{em}(1+z)$

$= 1216 \times 4.62$

$\lambda_{obs} = 1216 \times (1+3.5)$

$= 1216 \times 4.5$

~~Some~~ amount of neutral

Rec:
1) Figure relation in the limiting processes b/w eigenstate, optimized oper. & ~~non~~ near eigenstates, usual oper.
2) GHZ with expectations instead.

If I have 10^{17} / cm^2

$\boxed{\text{Neutral fraction} \sim 10^{-5}}$

Reionization.

$\boxed{\text{Standard Model of Cosmology}}$

Lec #3

Singularity — No singularity, ∃ a bounce

Radiation Dominated Era. ($k_B T >>$ rest mass energy)

Inflation — growth by $> e^{60}$ acceleration

Reheating — (whatever caused inflation, decays & yields matter) → Primordial nucleosynthesis — before stars etc.

Matter Dominated Era. ($3000 \simeq z$)

Recombination $[p + e^- \to H + \gamma]$

[Dark Ages], $z \sim 30$ First stars/galaxies, Epoch of Reionization

Cosmological const.

$$\eta = \frac{n_\gamma}{n_B} \approx 10^9$$

$$\Omega_B = 0.05$$

$z \gtrsim 6$

$z \sim 1.5$

Surface Brightness

Defⁿ: 1 pc :=

$\cancel{M_\odot}$

$L :=$ Luminosity $\frac{\text{Total energy output}}{\text{time}}$

$$S = \frac{L}{4\pi r^2}$$

Defⁿ: Apparent Magnitude $:= m = -2.5 \log_{10}\left(\frac{S}{S_{ref}}\right)$:= magnitudes

$$= -2.5 \log_{10}\left(\frac{L}{4\pi r^2 S_{ref}}\right)$$

Absolute Magⁿ $:= M = -2.5 \log_{10}\left(\frac{L}{4\pi S_{ref}(10pc)^2}\right)$

$$M_\odot \text{ (Abs mag, not mass)} = -2.5 \log_{10}\left(\frac{L_\odot}{4\pi S_{ref}(10pc)^2}\right)$$

$$M - M_\odot = -2.5 \log_{10}\left(\frac{L}{L_\odot}\right)$$

$$m = -2.5 \log_{10}\left(\frac{L}{4\pi r^2 S_{ref}} \frac{(10pc)^2}{(10pc)^2}\right) = M - 5 \log\left(\frac{r}{pc}\right) - 5$$

$$\Rightarrow m = M_\odot - 2.5 \log_{10}\left(\frac{L}{L_\odot}\right) + 5 \log_{10}\left(\frac{r}{1pc}\right) - 5$$

Example

$L_{and} = 10^{11} L_\odot$

$r_{and} = 6 \times 10^5 pc$

$M_\odot = 4.87$

$\Rightarrow m = 4.8 - 2 \times 5 \times (11) + 5 \times 5 \log 6 - 5$ TODO

$= 4.8 - 27.5 + 20$

$= 1.1$

For absolute

Defⁿ: Surface Brightness $:= I = \frac{L}{(pc)^2}$, Defⁿ: $\delta\theta = \frac{1pc}{r}$

Defⁿ: Apparent magnitude per arc second squared

$$\mu = M_\odot - 2.5 \log_{10}\left[\frac{I r^2 (\delta\theta)^2}{L_\odot}\right] + 5 \log_{10}\left(\frac{r}{pc}\right) - 5$$

$$= -2.5 \log_{10}\left[\frac{I}{L_\odot} \cdot \left(\frac{r}{pc}\right)^2 \cdot (pc)^2 (\delta\theta)^2\right] + M_\odot - 5 + 5\log_{10}\left(\frac{r}{pc}\right)$$

$$= -2.5 \log_{10}\left[\frac{I}{L_\odot/pc^2}\right] - \cancel{5 \log_{10}\left(\frac{r}{pc}\right)} - 5\log_{10}(\delta\theta) + M_\odot - 5 + \cancel{5\log_{10}\left(\frac{r}{pc}\right)}$$

$m = -2.5 \log_{10}$

Bright: $n \sim 6$
Faint: $n \sim 2$

Ellipticals / Bulges

$$I(R) = I_0 e^{-(R/R_0)^{1/n}}$$

$n = 4$ [de Vaucouler profile]

$$L = 2\pi I_0 \int_0^\infty e^{-(R/R_0)^{1/n}} R\,dR$$

$$R = R_0 u^n \Rightarrow L = 2\pi I_0 R_0^2 n \int_0^\infty u^{2n-1} du\, e^{-u}$$

$$= 2\pi I_0 R_0^2 n(2n-1)!$$

Def^n: Effective Radius $=: R_e$

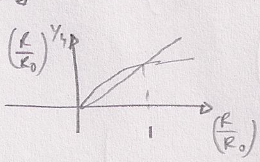
$$:= \frac{\int_0^{R_e} e^{-(R/R_0)^{1/n}} R\,dR}{\int_0^\infty e^{-(R/R_0)^{1/n}} R\,dR} = \frac{1}{2}$$

claim:
$R_e = b_n R_0$, $b_n = 2n - 0.324$

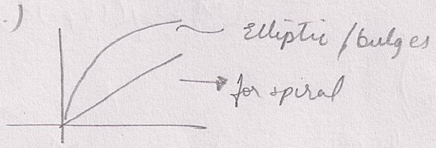
Def^n: $I_e =:$ Brightness / Intensity at radius R_e

claim: $I_0 = I_e \exp\left[-b_n\left\{\left(\frac{R}{R_e}\right)^{1/n} - 1\right\}\right]$

claim: $\mu = c_1 \times \left(\frac{R}{R_0}\right)^{1/n} + c_2$



so for c_1 large (~ 8 for eg.)



Spirals

$$I(R) = I_0 e^{-R/R_d}$$

Def^n: $R_d :=$ Disk scale Length

$$L = 2\pi \int_0^\infty I(R) R\,dR$$

$$= 2\pi \int_0^\infty I_0 e^{-R/R_d} R\,dR = 2\pi I_0 R_d^2 \underbrace{\int_0^\infty y e^{-y} dy}_{= \Gamma(2) = 1! = 1}$$

ASK

$$\mu = -2.5 \log\left[\frac{I}{L_\odot/pc^2}\right] + M_\odot + 21.57$$

$$L = 2\pi I_0 R_d^2$$

$$\mu = -2.5 \log_{10}\left[\frac{I_0 e^{-R/R_d}}{L_\odot/pc^2}\right] + M_\odot + 21.57$$

$$= -2.5 \log_{10}\left(\frac{I_0}{L_\odot/pc^2}\right) + \frac{2.5}{\ln(10)} \cdot \left(\frac{R}{R_d}\right)$$

$\mu \sim c_1 \frac{R}{R_d} + c_2$

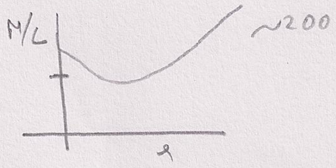
$\mu \sim c_3 \left(\frac{R}{R_0}\right)^{1/n} + c_4$ for $2 \le n \le 6$

Defⁿ: Mass to light ratio: $= \Upsilon := \frac{M/L}{M_\odot/L_\odot}$

verify what the issue was.

Claim: $L_* \propto M_*^{3.5} \Rightarrow \left(\frac{L_*}{M_*}\right)^{-1} \propto M_*^{-2.5}$

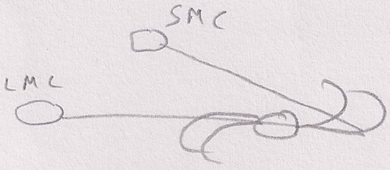
$\Rightarrow \Upsilon = \left(\frac{M_*}{M_\odot}\right)^{-2.5}$

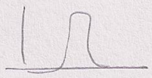


Macho.
MAssive
Compact
Halo
Objects

Faintest Stars; $\frac{M_*}{M_\odot} = 0.08 \Rightarrow \frac{M_*/L_*}{M_\odot/L_\odot} \approx 400$

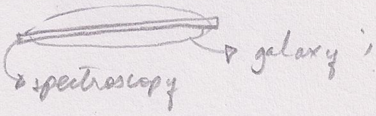
Milky way; $M_{mw}/L_{mw} \approx 20$ given $L_{mw} \simeq 10^{11} L_\odot$



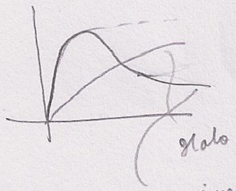


gravitational lensing to find dwarf galaxy

Experiment;



Defⁿ: $V_{circ} :=$ Implicit, $\frac{GM(<R)}{R^2} = \frac{V_{circ}^2(R)}{R} \Rightarrow \frac{M(<R)}{R} \sim V_{circ}^2(R)$



1) Bulge < 1 Kpc
2) Disk ~ 2-4 Kpc
3) Halo ~ 10 Kpc ~ least amount of light has most mass

Maximal Disk (as much mass as can be put to fit)

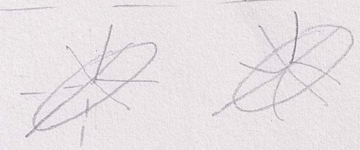
Optical/near infra-red

21 cm Hyperfine transition of Hydrogen

$\updownarrow$

1420 MHz

Spider Diagrams



ASK)

Luminosity f^n := ~~# galaxy~~

Schechter F^n

Def^n: dn := # galaxies per unit volume with luminosity $L - L + dL$

$$dn = \Phi_* \left(\frac{L}{L_*}\right)^{\alpha} e^{-L/L_*} \frac{dL}{L_*} \quad ; \quad n = \int_{L=0}^{\infty} dn$$

$$n = \Phi_* \int_0^{\infty} \left(\frac{L}{L_*}\right)^{\alpha} e^{-L/L_*} \frac{dL}{L_*} = \Phi_* \Gamma(\alpha+1)$$

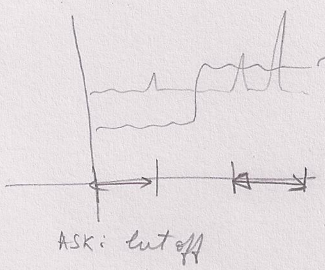
Φ_*: # density (characteristic)

$$l = \int_0^{\infty} L\, dn = \Phi_* L_* \Gamma(\alpha+2)$$

NB: n may be infinite but l will remain finite.

Def^n ASK: $$\rho_{gal} = \int_0^{\infty} \Upsilon L\, dn = \Upsilon \Phi_* L_* \Gamma(\alpha+2)$$

ρ_{gal} :: Density of the galaxy



$$M = -2.5 \log\left[\frac{L}{4\pi(10pc)^2 S_{ref}}\right]$$

$$M_\odot = -2.5 \log\left[\frac{L_\odot}{4\pi(10pc)^2 S_{ref}}\right]$$

$$M - M_\odot = -2.5 \log\left[\frac{L}{L_\odot}\right]$$

$$\Rightarrow 10^{-\frac{(M-M_\odot)}{2.5}} = \frac{L}{L_\odot} \Rightarrow L = L_\odot e^{-0.4(M-M_\odot)}$$

Then $$dn = \Phi_* \left(\frac{L_\odot}{L_*}\right)^{\alpha} 10^{-0.4\alpha(M-M_\odot)} e^{-(L_\odot/L_*)10^{-0.4(M-M_\odot)}}$$

Def^n
Fundamental Plane := The plane in the 5-d
$\left\{\begin{array}{l} \text{1) Luminosity} \\ \text{2) Vel. Disp} \\ \text{3) Effective Radius} \\ \text{4) Surface Brightness} \\ \text{5) } \Upsilon \end{array}\right\}$
space, on which galaxies are found / are observed.

Def^n: Faber Jackson rel^n := $L \propto \sigma_{\parallel}^4$ — $\sigma_{\parallel}$: line of sight dispersion.

: Kormendy rel^n :=

: Tully-Fisher := $L \propto v_{circ}^{3.5}$

(non-relativistic)

Def^n: Redshift: $Z := \frac{v}{c} = \frac{H_0 R}{c} = \frac{R}{cH_0^{-1}}$

Recall: Flux: $S := \frac{L}{4\pi R^2}$

Magnit: $m = -2.5 \log\left(\frac{S}{S_{ref}}\right) = -2.5 \log\left[\frac{L}{S_{ref}\, 4\pi R^2}\right]$

$= -2.5 \log\left[\frac{L}{4\pi\, 1(Mpc)^2\, S_{ref}}\right] + 5.0 \log\left[\frac{R}{1 Mpc}\right]$

$= M + 25 + 5 \log\left[\frac{R}{Mpc}\right]$

$\Rightarrow \frac{R}{Mpc} = 10^{0.2[m-M-25]} = 10^{0.2(dm-25)}$

Def^n: $m - M :=$ Distance modulus $=: dm$

$\Rightarrow Z = \left[\frac{cH_0^{-1}}{Mpc}\right]^{-1} 10^{0.2[m-M-25]} \Rightarrow \log_{10}(Z) = -\log_{10}\left[\frac{cH_0^{-1}}{Mpc}\right] + 0.2[m-M-25]$

$V = H_0 R$; $T \simeq H_0^{-1}$ (At large distances, the universe is homogeneous & isotropic

Weyl: "Cosmological principle": looks the same at every place & at each time

(a) global Def^n of time

b) Space is homog^n & isotropic.

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi\right]$$

synchronous

Flat space; $dl^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi$

4-d ; $dl^2 = dx^2 + dy^2 + dz^2 + dw^2$

$x^2 + y^2 + z^2 + w^2 = L^2$

$x^2 + y^2 + z^2 = r^2$

$\to 2w\,dw = -2r\,dr$

$dw = -\frac{r\,dr}{\sqrt{L^2 - r^2}}$

$dl^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2 + \frac{r^2 dr^2}{L^2 - r^2}$

$= \frac{(L^2 - r^2 + r^2)\,dr^2}{L^2 - r^2} + \ldots$

$dl^2 = \frac{dr^2}{1 - r^2/L^2} + r^2 d\theta^2 + \sin^2\theta\, r^2 d\phi$

(like 2d surface of a 3d sphere; this is curved inwards & curvature is const)

For $dl^2 = dx^2 + dy^2 + dz^2 - dw^2$

$x^2 + y^2 + z^2 - w^2 = -L^2$, the sol^n turns out to be $dl^2 = \frac{dr^2}{1 + \frac{r^2}{L^2}}$

$ds^2 = 0$ (for light)

$\times O$

$c\,dt = a(t)\frac{dr}{\sqrt{1-kr^2}} \Rightarrow \frac{dr}{\sqrt{1-kr^2}} = \frac{c\,dt}{a(t)} \Rightarrow dr = \frac{c\,dt}{a(t)} \Rightarrow r_S = \int_{t_{em}}^{t_{obs}} \frac{c\,dt}{a(t)}$

$\times S$

$$\int_{t_{em}}^{t_{obs}} \frac{c\,dt}{a(t)} = \int_{t_{em}+\Delta t_{em}}^{t_{obs}+\Delta t_{obs}} \frac{c\,dt}{a(t)}$$

another signal

$$\Rightarrow \int_{t_{em}}^{t_{em}+\Delta t_{em}} + \int_{t_{em}+\Delta t_{em}}^{t_{obs}} = \int_{t_{em}+\Delta t_{em}}^{t_{obs}} \int_{t_{obs}}^{t_{obs}+\Delta t_{obs}}$$

$$\Rightarrow \frac{c\Delta t_{em}}{a(t_{em})} = \frac{c\Delta t}{a(t_{obs})} \Rightarrow \frac{c\Delta t_{obs}}{c\Delta t_{em}} = \frac{a(t_{obs})}{a(t_{em})} \Rightarrow \frac{\lambda_{obs}}{\lambda_{em}} = \frac{a(t_{obs})}{a(t_{em})} = 1+z \quad (-1 \le z \le \infty)$$

think of wave crests.

$$r_s = \int_{t_{em}}^{t_{obs}} \frac{c\,dt}{a(t)} \quad ; \quad a(t) = a_0 + (t-t_0)\dot a(t_0) + \ldots$$

$$r_s \simeq c\int_{t_{em}}^{t_{obs}} \frac{dt}{a_0 + \dot a_0(t-t_0)} \simeq c\left[t + tt_0\frac{\dot a}{a}\Big|_{t_0} - \frac{t^2}{2}\left(\frac{\dot a}{a}\right)_{t_0} + \ldots\right] \simeq c\left[(t_0-t)\left(1+t\left(\frac{\dot a}{a}\right)_{t_0}\right)\right]$$

$$\simeq ct_0\left[\left(1-\frac{t_{em}}{t_0}\right)\left(1+\ldots\right)\right] \simeq cH_0^{-1}\left[\ldots\right]$$

Claim: $\int \frac{dr}{\sqrt{1-kr^2}} = S(r)$
$= r$ if $k=0$
$= \sin^{-1}(\sqrt{kr^2})$ if $k>0$
$= \sinh^{-1}(\sqrt{kr})$ if $k<0$.

$\vec R, a, \vec x$ ($\vec R$ → Physical, $\vec x$ → comoving)

$$\vec R = a(t)\vec x$$
$$\frac{d\vec R}{dt} = \dot a\vec x = \frac{\dot a}{a}\vec R \Rightarrow \vec V = H_0\vec R, \quad H_0 = \frac{\dot a}{a}\Big|_{t_0}$$

$R_{00} = -3\ddot a/a$ | $c=1$

$$R_{11} = -\left(\frac{\ddot a}{a} + \frac{2\dot a^2}{a^2} + \frac{2k}{a^2}\right)g_{rr}$$

$$R = -6\left[\frac{\ddot a}{a} + \frac{\dot a^2}{a^2} + \frac{k}{a^2}\right]$$

$$T_{ik} = (p+\rho)u_iu_k - p\,g_{ik}$$

$$R_{ik} - \frac{1}{2}(R+\Lambda)g_{ik} = T_{ik}8\pi G$$

$$\frac{\dot a^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{6}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{6}$$

If $k \le 0$, $\Lambda \geq 0$, $\dot a$ doesn't change sign.
→ no static universe.

(Why is pressure zero?)

$$\frac{k}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{6} = \frac{12\pi G\rho}{3} = 4\pi G\rho$$

$$\frac{4\pi G}{3}\rho = \frac{\Lambda}{6}$$

$$d(\rho a^3) + p\,d(a^3) = 0$$

$$H_0^2 + \frac{k}{a_0^2} = \frac{8\pi G\rho_0}{3} + \frac{\Lambda}{6}$$

drop for now

Critical Density

$$\rho_c = \frac{3H_0^2}{8\pi G} \quad ; \quad \Omega_0 = \frac{\rho_0}{\rho_c}$$

Density Parameter (measures density in the unit of critical density)

$$H_0^2 + \frac{k}{a_0^2} = \frac{8\pi G\rho_c}{3}\frac{\rho_0}{\rho_c} = H_0^2\Omega_0$$

$k = a_0^2H_0^2(\Omega_0 - 1)$ (c^2 for physical units)

$k=0$	if $\Omega_0 = 1$	flat	
$k>0$,	if $\Omega_0 > 1$	closed	+ve curved
$k<0$,	if $\Omega_0 < 1$	open	−ve curved

Bag, Lec#4.1

$$\frac{\dot a^2}{a^2} + \frac{K}{a^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda}{6}$$

$$\frac{\ddot a}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{6}$$

$$d(\rho a^3) + p\, d(a^3) = 0$$

If Λ is coming from some mass, then

$\frac{8\pi G \rho_\Lambda}{3} := \frac{\Lambda}{6}$ from the first

From the second,

$$\frac{4\pi G}{3}(\rho_\Lambda + 3p_\Lambda) = \frac{\Lambda}{6}$$

$$\Rightarrow \frac{\cancel{\Lambda}}{2\cdot\cancel{6}} + 3p_\Lambda \frac{4\pi G}{3} = \frac{1}{2}\frac{\Lambda}{6}$$

$$\Rightarrow p_\Lambda = -\rho_\Lambda$$

From the third, (for $T^i{}_{j;i} = 0$)

$$d(\rho_\Lambda a^3) - \rho_\Lambda d(a^3) = 0$$

$$\Rightarrow \rho_\Lambda d(a^3) + a^3 d\rho_\Lambda - \rho_\Lambda d(a^3) = 0$$

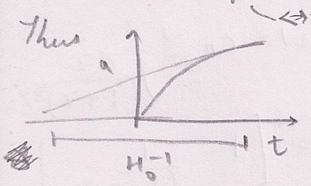
$$\Rightarrow d\rho_\Lambda = 0 \Rightarrow \rho_\Lambda = \text{const (independent of } a\text{)}$$ ~~pressure~~

$\rho \sim \rho c^2$

$p \sim \rho v^2$

basically matter + radiation deaccelerate (if Λ is ignored).

Thus $\frac{\ddot a}{a} \approx -\frac{4\pi G\rho}{3}$



then $T_{age} < H_0^{-1}$

So Λ can't be zero; QFTs may yield such matter as ρ_Λ etc

$\mathcal{L} = \frac{\dot\phi^2}{2} - V(\phi)$, $\mathcal{H} = \frac{\dot\phi^2}{2} + V(\phi)$; claim $\underset{\text{pressure}}{p} = \mathcal{L}$; $\frac{p_\phi}{\rho_\phi} = \frac{\frac{\dot\phi^2}{2} - V(\phi)}{\frac{\dot\phi^2}{2} + V(\phi)}$ if $\frac{\dot\phi^2}{2} << V(\phi)$,

then $\frac{p_\phi}{\rho_\phi} \approx -1$

"Finetuning problem"; grand unification etc, Λ is off by multiple orders of magnitudes

$$\int_0^{r_{em}} \frac{dr}{\sqrt{1 - kr^2}} = \int_{t_{em}}^{t_0} \frac{c\,dt}{a(t)}$$

$\rho_\Lambda = \text{const}$; for matter, $p \approx 0 \Rightarrow \rho_{nr} a^3 \approx \text{const}$; for radiation, put $p = \frac{\rho}{3}$, $\rho\, da^3 + a^3 d\rho + \frac{\rho}{3} da^3 = 0$

$$\Rightarrow \frac{d\rho}{\rho} = -\frac{4}{3}\frac{da^3}{a^3}$$

$$\Rightarrow \rho_\gamma a^4 = \text{const}$$

$\hookrightarrow \gamma$ 'gamma'

; recall: $\rho_c = \frac{3H_0^2}{8\pi G}$

So finally, (from 1)

$$\frac{\dot a^2}{a^2} + \frac{K}{a^2} = \frac{8\pi G}{3}\left[\rho_{nr} + \rho_\gamma + \rho_\Lambda\right]$$

$$= \frac{8\pi G}{3}\left[\rho_{nr_0}\frac{a_0^3}{a^3} + \rho_{r_0}\frac{a_0^4}{a^4} + \rho_\Lambda\right]$$

$$H_0^2 + \frac{K}{a^2} = \frac{8\pi G}{3}\rho_c\left[\frac{\rho_{nr_0}}{\rho_c} + \frac{\rho_{r_0}}{\rho_c} + \frac{\rho_\Lambda}{\rho_c}\right]$$

$$= H_0\underbrace{\left[\Omega_{nr_0} + \Omega_{\gamma_0} + \Omega_{\Lambda_0}\right]}_{\equiv\,\Omega_0}$$

$$K = a_0^2 H_0^2[\Omega_0 - 1] = a_0^2 H_0^2[\Omega_{nr} + \Omega_\gamma + \Omega_\Lambda - 1]$$

$\Omega_0 \underset{\sim}{\text{exp}} 1.02 \pm 0.03$

$$H^2 + \frac{a_0^2}{a^2}H_0^2(\Omega_0 - 1) = H_0^2\left[\Omega_{nr}\left(\frac{a_0}{a}\right)^3 + \Omega_r\left(\frac{a_0}{a}\right)^4 + \Omega_\Lambda\right]$$

$$\Rightarrow H^2 = H_0^2\left[(1-\Omega_0)\left(\frac{a_0}{a}\right)^2 + \underset{\downarrow \text{exp } 0.3}{\Omega_{nr}}\left(\frac{a_0}{a}\right)^3 + \underset{\downarrow \text{exp } 10^{-4}}{\Omega_\gamma}\left(\frac{a_0}{a}\right)^4 + \underset{\downarrow\, 0.7}{\Omega_\Lambda}\right]$$

Recall: $1 + z = \frac{a_0}{a}$

"Epoch" $\rho_{nr} \overset{?}{=} \rho_\Lambda$

$$\Omega_{nr}\left(\frac{a_0}{a}\right)^3 = \Omega_\Lambda$$

$$\Omega_{nr}(1+z)^3 = \Omega_\Lambda$$

$$1 + z = \left(\frac{\Omega_\Lambda}{\Omega_{nr}}\right)^{1/3}$$

exp: $z_{eq} = \left(\frac{0.7}{0.3}\right)^{1/3} - 1$

$\simeq 0.3$

$\rho_{nr} \overset{?}{=} \rho_\gamma \Rightarrow \Omega_{nr}(1+z)^3 = \Omega_\gamma(1+z)^4 \Rightarrow 1 + z_{eq} = \frac{\Omega_{nr}}{\Omega_\gamma} \approx 3000$

z

$\infty \leftarrow 3000$ — 0.3 — 0 — -1

Radi — Matter — Λ domin

$K=0$

1) $\Omega_r = 0$, $\Omega_\Lambda = 0 \Rightarrow \Omega_m = 1$

$$\frac{\dot{a}^2}{a^2} = H^2 = H_0^2 \left(\frac{a_0}{a}\right)^3 ;$$

$y = \frac{a}{a_0}$, $x = tH_0$

$$\frac{dy}{dx} = \frac{1}{a_0 H_0}\frac{da}{dt} \quad \Big| \quad \frac{\dot{a}^2}{a^2} = \frac{H_0^2}{y^2}\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow \frac{H_0^2}{y^2}\left(\frac{dy}{dx}\right) = \frac{H_0^2}{y^3} \quad ; \quad y^{1/2}\, dy = dx$$

$$\Rightarrow \frac{2y^{3/2}}{3} = x + \cancel{\text{const}}^{\,0} \quad \text{(from init condition)}$$

$$\Rightarrow tH_0 = \frac{2}{3}\left(\frac{a}{a_0}\right)^{3/2} \qquad t_0 = \frac{2}{3H_0}$$

$$\text{so} \quad a = a_0 \left(\frac{t}{t_0}\right)^{2/3}$$

$$\int_0^{r_{em}} dr = r(z_{em}) = c\int \frac{dt}{a} = c\int \frac{da}{a\dot{a}}$$

$$dt = \frac{dt}{da}\cdot da = \frac{da}{\dot{a}}$$

$$= c\int \frac{da}{a^2\left(\frac{\dot{a}}{a}\right)} = cH_0^{-1}\int \frac{da}{a^2 \frac{H}{H_0}}$$

$$\boxed{1+z = \frac{a_0}{a}} \quad \boxed{dz = -\frac{a_0}{a^2}\, da}$$

$$\Rightarrow \frac{da}{a^2} = -\frac{dz\, a_0}{(1+z)\, a\, a_0} = -\frac{1}{a_0}\, dz$$

so now, $r(z) = \frac{cH_0^{-1}}{a_0}\int_0^z \frac{dz}{H(z)/H_0}$

$$\boxed{\frac{H}{H_0} = (1+z)^{3/2}}$$

$$\Rightarrow r(z) = cH_0^{-1}\int \frac{dz}{(1+z)^{3/2}}$$

$$= 2cH_0^{-1}\left[1 - \frac{1}{(1+z)^{1/2}}\right]$$

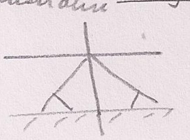
$$\lim_{z\to 0} r(z) = 2cH_0^{-1}\left[1 - \left(1 - \frac{z}{2}\right)\right]$$

$$\lim_{z\to\infty} r(z) = 2cH_0^{-1}$$

In Einstein de Sitter model

$$\begin{cases} a(t) \propto t^{2/3} \\ a_0 r(z) = 2cH_0^{-1}\left[1 - \frac{1}{\sqrt{1+z}}\right] \end{cases}$$

Illustration



Horizon Problem

$K=0,\ \Omega_\gamma = 1,\ \Omega_{nr} = \Omega_\Lambda = 0$

$H^2 = H_0^2\left(\frac{a_0}{a}\right)^4 \ ; \ \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{y^4}$

$\Rightarrow \quad y\,dy = dx$

$\frac{y^2}{2} = x + \text{const}$

$a = a_0\left(\frac{t}{t_0}\right)^{1/2}$

$t_0 = \frac{1}{2H_0}$

$K=0,\ \Omega_\Lambda = 1,\ \Omega_{nr} = \Omega_\gamma = 0$

$H^2 = H_0^2$

$\frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = 1 \ ; \ \frac{dy}{y} = dx$

$\Rightarrow \quad \ln(y) = x + \text{const}$

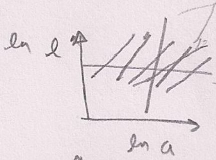
at $t = t_0$, $x = t_0 H_0$, $y = 1$

$t_0 H_0 + \text{const} = 0$

$\Rightarrow \ln(y) = (t - t_0) H_0$

$a = a_0 e^{(t-t_0)H_0}$

age is infinite, $t \to \infty$, $a \to 0$ for B

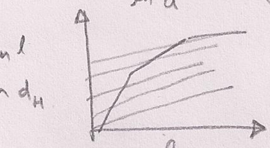


~ diff slopes now intercepts

$d_H = \frac{c}{H} \sim a^{3/2}$

eD $H^2 \sim a^{-3}$

eD rad. dominated $H^2 \sim a^{-4}$, $d_H \sim a^2$



← (ASK)

Flatness

$H^2 + \frac{K}{a^2} = \frac{8\pi G \rho}{3}$

$H^2 = H_0^2\left[(1-\Omega_0)\left(\frac{a_0}{a}\right)^2 + \Omega_0\left(\frac{a_0}{a}\right)^3\right]$

$\rho = \rho_0\left(\frac{a_0}{a}\right)^3$

From all 3,

$\frac{\rho}{\left(\frac{3H^2}{8\pi G}\right)} = \left(\frac{8\pi G}{3H_0^2}\right)\frac{H_0^2}{H^2}$ ← ASK

$= \frac{8\pi G \rho_0}{3H_0^2}\frac{H_0^2}{H^2}\left(\frac{a_0}{a}\right)^3$

$\Omega(z) = \frac{\Omega_0\left(\frac{a_0}{a}\right)^3}{H^2/H_0^2} = \frac{\Omega_0(1+z)^3}{\Omega_0(1+z)^3 + (1-\Omega_0)(1+z)^2}$

$1 - \Omega(z) = \frac{(1-\Omega_0)(1+z)^2}{(1-\Omega_0)(1+z)^2 + \Omega_0(1+z)^3}$

$= \frac{1-\Omega_0}{1-\Omega_0+\Omega_0(1+z)}$

$= \frac{1-\Omega_0}{1+\Omega_0 z}$

$\lim_{z\to\infty} 1-\Omega(z) \to 0$

"Dark matter only"

$H^2 = H_0^2\left[(1-\Omega_0)\underbrace{(1+z)^2}_{\frac{a_0}{a}} + \Omega_0\right]$

$\rho_\Lambda = \text{const}$

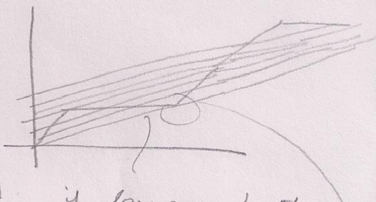
$\Omega(z) = \frac{\rho_\Lambda}{3H^2/8\pi G} = \frac{8\pi G\rho_\Lambda}{3H_0^2}\cdot\frac{H_0^2}{H^2}$

$= \frac{\Omega_0}{\left[\Omega_0 + (1-\Omega_0)(1+z)^2\right]}$

$1 - \Omega(z)$

$= \frac{(1-\Omega_0)(1+z)^2}{\left[(1-\Omega_0)(1+z)^2 + \Omega_0\right]}$

$= \frac{(1-\Omega_0)}{1-\Omega_0 + \frac{\Omega_0}{(1+z)^2}}$

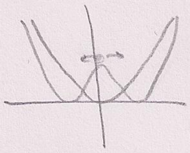


If long enough, then
(a) all sky causally connected
(b) matter → 1

(1) $\Omega_{nr} \sim 1 \to \Omega_{nr} \simeq 0.3$ $\quad P(k) = Ak^n$
(2) $n \le 1$, $n \sim 1$, $n_{obs} \simeq 0.97$
(3) Adiabatic Perturb ~ checks out.

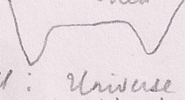
Inflation

$\mathcal{L} = \frac{\dot\phi^2}{2} - V(\phi, T)$



doesn't work

tried



reheat

issue still: Universe should be allowed to expand by e^{60}, then matter will dilute

conjecture: coupling b/w normal matter etc. – results in reheating & creation of matter

: fluctuations must become classical in GR.

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1-k^2r^2} + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi\right]$$

$R = a(t)\, r$

objective: Find the Newtonian limit of the metric ($r << R_H$)

Time must be corrected for, $\because$ of gravitational time dilation.

(uniform density)

$$T = t - t_0 + \frac{1}{2}\frac{HR^2}{c^2} + \mathcal{O}\left(\frac{R^4}{d_H^4}\right)$$

$\to$ effective Newtonian gravitational Potential

$$ds^2 = c^2\left(1 + \frac{2\Phi}{c^2}\right) dT - dR^2 - R^2 d\theta^2 - R^2 \sin^2\theta\, d\phi^2$$

$$r = \frac{R}{a} \Rightarrow dr = \frac{dR}{a} - \frac{R}{a^2}\frac{da}{dT} dT \Rightarrow a^2 dr^2 = \left[dR - RH\frac{dt}{dT} dT\right]^2$$

$$= dR^2 + R^2 H^2 \left(\frac{dt}{dT}\right)^2 dT^2 - 2RH\frac{dt}{dT} \cdot dT\, dR$$

$\Rightarrow a^2 dr^2 \simeq dR^2 + R^2 H^2 dT^2 - 2RH\, dT\, dR$ (assumed $\frac{dt}{dT} \approx 1$ $\because$ corrections are higher order)

$\Rightarrow \frac{a^2 dr^2}{1-k^2r^2} = \left[dR^2 + R^2H^2dT^2 - 2RH\,dT\,dR\right]\left[1-k^2r^2\right]^{-1}$ (only to 2nd order in $\frac{R}{d_H}$ Hubble radius)

$$\cong \left[\left(1 + \frac{kR^2}{a^2}\right) dR^2 + R^2 H^2 dT - 2RH\,dT\,dR\right]$$

Recall: $t = t_0 + T - \frac{1}{2}\frac{HR^2}{c^2}$, $\quad ; \dot{H} = \frac{d}{dt}\left(\frac{\dot a}{a}\right) = \frac{\ddot a}{a} - \frac{\dot a^2}{a^2}$

$$\Rightarrow dt = dT - \frac{HR\,dR}{c^2} - \frac{dT}{2}\frac{\ddot a}{a}\frac{R^2}{c^2}\frac{dt}{dT} + \frac{dT}{2}\frac{H^2R^2}{c^2}\frac{dt}{dT}$$

$$= dT\left[1 - \frac{1}{2}\frac{R^2}{c^2}\left(\frac{\ddot a}{a} - H^2\right)\right] - \frac{HR\,dR}{c^2}$$

$$\Rightarrow c^2 dt^2 = \frac{H^2R^2dR^2}{c^2} + c^2dT^2\left(1 - \frac{R^2}{c^2}\left(\frac{\ddot a}{a} - H^2\right)\right) - 2HR\,dR\,dT$$

Equating $ds^2 = c^2dt^2 - \frac{a^2dr^2}{1-k^2r^2}$ we have

$$ds^2 = c^2dT^2\left[1 - \frac{R^2}{c^2}\frac{\ddot a}{a}\right] - \left[1 + \frac{kR^2}{a^2} - \frac{H^2R^2}{c^2}\right]dR^2 + \ldots$$

For $R << d_H$, $R_{curv} = \frac{a}{k^{1/2}} = \frac{ac}{a_0 H_0 (\Omega_0 - 1)^{1/2}}$

So now, $\frac{2\Phi}{c^2} = \frac{-R^2}{c^2}\frac{\ddot a}{a} \Rightarrow \boxed{\Phi = -\frac{R^2}{2}\frac{\ddot a}{a}}$ $\to$ effective gravit^n pot^n

$$\Phi = -\frac{R^2}{2}\cdot\left(-\frac{4\pi G}{3}(\rho + 3p)\right)$$

$$= -\frac{R^2}{2}\left(H_0^2\left(\Omega_\Lambda - \frac{\Omega_{nr}}{2}(1+z)^3\right)\right)$$

$$= \frac{2\pi G\rho R^2}{3} - \frac{\Lambda}{12}R^2 \quad \text{(using the } \frac{\ddot a}{a} \text{ eq}^n\text{)}$$

$$\Rightarrow \ddot R = -\frac{4\pi G\rho R}{3} + \frac{\Lambda R}{6}$$

$$= -\frac{GM}{3R^2} + \frac{\Lambda R}{6}$$

$\xrightarrow{} v_0$ $\xrightarrow{} V$
A B

Geodesic

$$\frac{du^i}{ds} + \Gamma^i_{jk} u^j u^k = 0$$

$$\frac{du^i}{ds} = \frac{d^2x^i}{ds^2} = \left(\frac{\partial}{\partial x^j}\frac{\partial x^i}{\partial s}\right)\left(\frac{\partial x^j}{\partial s}\right) = u^j \frac{\partial u^i}{\partial x_j}$$

$$+ \left(\frac{d}{ds}\left(\frac{dx^i}{ds}\right)\right)''$$

$$\Rightarrow u^j \frac{du^i}{\partial x_j} + \Gamma^i_{jk} u^j u^k = 0$$

~~Ask~~ $mcu^i = p^i$

$$\Rightarrow p^j \frac{\partial p^i}{\partial x^j} + \Gamma^i_{jk} p^j p^k = 0$$

$i = 0$

$$p^j \frac{\partial p^0}{\partial x^j} + \Gamma^0_{jk} p^j p^k = 0$$

$p^2 = 0 = p^3$

So now, $p^0 \frac{\partial p^0}{\partial t} + p^1 \frac{\partial p^0}{\partial r} + \Gamma^0_{11} p^1 p^1 = 0$

(homogenity) $\Gamma^0_{11} = \frac{a\dot{a}}{1-kr^2}$

$$\text{so} \quad E\frac{dE}{dt} + \frac{a\dot{a}}{1-kr^2} p^2 = 0$$

Now the "physical momentum" $p' = p^1\sqrt{g_{rr}}$

$p^2 = p^2 \frac{a^2}{1-kr^2}$ so that

$$E\frac{dE}{dt} + \frac{\dot{a}}{a}\frac{a^2}{1-kr^2} p^2 = 0 \Rightarrow E\frac{dE}{dt} + \frac{\dot{a}}{a} p^2 = 0$$

$$E^2 = p^2 + m^2 \Rightarrow E\frac{dE}{dt} = p\frac{dp}{dt} ; \Rightarrow \frac{dp}{dt} + \frac{\dot{a}}{a} p = 0$$

$\Rightarrow pa = \text{const.}$

For radiation, $E_r = p$

$\Rightarrow E_r \propto (1+z)$ ← $mc^2 << k_BT$

Non-relativistic, $E \sim p^2$ $mc^2 >> k_B$

$\Rightarrow E_{nr} \propto (1+z)^2$

For supernova, $10^{-3} M_\odot$ inside the beam ($z=1$)

For a galaxy, $10^6 M_\odot$

More focussed ⇒ bright (ask)

$$\frac{D^2\eta^i}{d\lambda^2} = R^i_{klm} u^k u^l \eta^m$$

$$\frac{d^2\sqrt{A}}{d\lambda^2} = -\frac{4\pi G}{c^2}\rho_{AR}\sqrt{A}$$

Ask

$\rho_{AR} = T_{ij}k^i k^j$

A is the solid angle

λ is the affine parameter

A(z)

$\delta\Omega$, A, D

solid angle

$\delta\Omega = \frac{A}{D^2}$; $d\theta = \frac{L}{D}$

$$D_A := D = \frac{r(z)}{(1+z)}$$

For Einstein - Desitter,

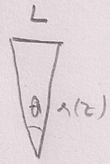
$$r(z) = \frac{2(H_0^{-1})}{(1+z)}\left[1 - \frac{1}{\sqrt{1+z}}\right]$$

$$\frac{\partial D_A}{\partial z} = 0$$

$$\Rightarrow \frac{\partial}{\partial z}\left(\frac{1}{1+z}\right) - \frac{\partial}{\partial z}\left(\frac{1}{(1+z)^{3/2}}\right) = 0$$

$$\Rightarrow -\frac{1}{(1+z)^2} + \frac{3}{2}\frac{1}{(1+z)^{5/2}} = 0$$

$1+z = \frac{9}{4} \Rightarrow z = \frac{5}{4}$



$$\theta = \frac{L\left(\frac{a_0}{a}\right)}{r(z)} = \frac{L}{D_A}$$

$$\Rightarrow D_A = \frac{a}{a_0} r(z)$$

$$S\,\delta t_{obs} = \frac{N h \nu_{em}\,\delta t_{em}}{4\pi r^2(z)}$$

$L = N h \nu_{em}$

$$S\,\delta t_{obs} = \frac{N h \nu_{em}\,\delta t_{em}}{4\pi r^2(z)}$$

$$S(\nu_{obs}) = \frac{L}{4\pi r^2(z)(1+z)^2}$$

$$D_L(z) = (1+z)\,r(z)$$

Bag Lec#5.2

$s = \frac{L}{4\pi D_L^2}$; $D_L(z) = r(z) \times (1+z)$

$S(\nu)\, d\nu_{obs} = \frac{L(\nu_{em} = \nu_{obs}(1+z))}{4\pi D_L^2}\, d\nu_e$

$S(\nu_{obs}) = \frac{L(\nu_{em})}{4\pi (1+z)\, r^2(z)}$

$m = -2.5 \log_{10}\left(\frac{S_\nu}{S_{ref}}\right) = -2.5 \log_{10}\left[\frac{L(\nu_{em})}{4\pi(1+z)\, r^2(z)\, S_{ref}}\right]$

$= -2.5 \log_{10}\left(\frac{L(\nu_{em})}{L_\odot}\right) - \ldots$

$= -2.5 \log_{10}\left(\frac{L(\nu_{em})}{L_\odot}\right) + M_\odot + 5.0 \log_{10}\left(\frac{r(z)}{1\,Mpc}\right) + 25 + 2.5 \log_{10}(1+z)$

$-2.5 \log\left[\frac{L(\nu_{em})}{L(\nu_{obs})}\right] \rightarrow$ K correction

$r(z) = cH_0^{-1} \int_0^{z_{em}} \frac{dz}{\frac{H}{H_0}}$

$\frac{H(z)}{H_0} = \Omega_\Lambda + \Omega_{nr}(1+z)^3$; $\Omega_\Lambda + \Omega_{nr} = 1$

$= \Omega_\Lambda + \Omega_{nr} + \Omega_{nr}(1+z)^3 - \Omega_{nr}$

$= 1 + \Omega_{nr}((1+z)^3 - 1)$

For small z,

$\frac{H(z)}{H_0} \approx 1 + \Omega_{nr}(1+3z-1)$

$= 1 + 3z\Omega_{nr}$

$\Rightarrow r(z) = cH_0^{-1} \int (1 - 3z\Omega_{nr})\, dz$

$= cH_0^{-1} z\left(1 - \frac{3}{2}\Omega_{nr} z\right) \Rightarrow$ for $\Omega_{nr} = 0$, $r(z) = cH_0^{-1} z$

for $\Omega_{nr} = 1$, $r(z) = cH_0^{-1} z\left(1 - \frac{3z}{2}\right)$

	z = 0.1	z = 1.0
$\Omega_{nr} = 0$	$0.1\, cH_0^{-1}$	cH_0^{-1}
$\Omega_{nr} = 1$	$0.085\, cH_0^{-1}$	$0.6\, cH_0^{-1}$

Recall: $\left(\frac{t}{t_0}\right)^{2/3} \sim a \rightarrow 1+z \approx \left(\frac{t}{t_0}\right)^{2/3}$ for $z \approx 1$, $\frac{t}{t_0} \approx \frac{1}{3}$

$H_0 = \frac{\dot{a}}{a}\Big|_{t_0}$

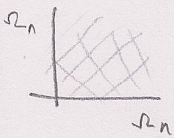
$q_0 = -\frac{\ddot{a}}{a}\Big|_{t_0} \frac{1}{H_0^2}$

$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho$

$= -\frac{1}{2} H_0^2 \Omega_{nr} (1+z)^3$

$\frac{\ddot{a}}{a}\Big|_{t_0} = -H_0^2 \frac{\Omega_{nr}}{2}$

$q_0 = \frac{1}{2}$



"standard candle"

"long enough"

"Distance Modulus" - Ask.

$$\bar{\Phi} = \frac{2}{3}\pi G(\bar{\rho}+3p)R^2 = -\frac{1}{2}\frac{\ddot{a}}{a}R^2$$

$$= \frac{2\pi G}{3}\bar{\rho}R^2 - \frac{\Lambda}{12}R^2$$

$$\ddot{\vec{R}} = -\vec{\nabla}\bar{\Phi} = -\frac{4\pi G}{3}\bar{\rho}R + \frac{\Lambda}{6}R = -\frac{GM}{R^2} + \frac{\Lambda}{6}R$$

$$\vec{R} = a(t)\vec{x}$$

$$\dot{\vec{R}} = \dot{a}\vec{x} + a\dot{\vec{x}}$$

$\dot{a}\vec{x}$: Hubble velocity; $a\dot{\vec{x}}$: peculiar velocity

$$\ddot{\vec{R}} = \ddot{a}\vec{x} + 2\dot{a}\dot{\vec{x}} + a\ddot{\vec{x}}$$

$$\ddot{a}\vec{x} + 2\dot{a}\dot{\vec{x}} + a\ddot{\vec{x}} = \frac{\ddot{a}}{a}\vec{R}$$

$$\ddot{\vec{x}} + \frac{2\dot{a}}{a}\dot{\vec{x}} = 0$$

$$\Rightarrow \frac{1}{a^2}\frac{d}{dt}\left(a^2\dot{\vec{x}}\right) = 0 \Rightarrow |\dot{\vec{x}}| \propto \frac{1}{a^2}$$

$$\vec{u} = a\dot{\vec{x}} \sim \frac{1}{a}$$

$$\bar{\Phi} = \frac{2\pi}{3}G\rho R^2 - \frac{\Lambda R^2}{12}$$

$$\rho(\vec{R},t) = \bar{\rho}(t)\left[1+\delta(\vec{R},t)\right]$$

δ: density contrast

$$-1 \le \delta \le \infty$$

$$\nabla^2\bar{\Phi} = 4\pi G\bar{\rho} - \frac{\Lambda}{2}$$

$$\nabla^2\Phi = 4\pi G\rho - \frac{\Lambda}{2}$$

$$\phi = \Phi - \bar{\Phi}$$

$$\nabla^2\phi = 4\pi G\bar{\rho}\delta$$

$$\Leftrightarrow \quad \nabla_x^2\phi = 4\pi G\bar{\rho}a^2\delta$$

$$\vec{\nabla}\Phi = \vec{\nabla}\bar{\Phi} + \vec{\nabla}\phi$$

Plug in Φ for $\ddot{\vec{R}} = -\nabla\Phi$ & you'll get

$$a\ddot{\vec{x}} + 2\dot{a}\dot{\vec{x}} = -\vec{\nabla}_R\phi$$

$$\ddot{\vec{x}} + \frac{2\dot{a}}{a}\dot{\vec{x}} = -\frac{\vec{\nabla}_x\phi}{a^2}$$

$$\vec{\nabla}_x\phi = 4\pi G\bar{\rho}a^2\delta$$

NB: δ can be negative

1) Smooth Universe

$$\bar{\Phi} = \left(\frac{2\pi G}{3}\bar{\rho} - \frac{\Lambda}{2}\right)R^2$$

$$= \left(\frac{1}{4}H_0^2\Omega_{m0} - \frac{3}{2}H_0^2\Omega_\Lambda\right)R^2$$

$$\frac{\bar{\Phi}}{c^2} = \frac{H_0^2R^2}{c^2}\frac{1}{2}\left(\frac{\Omega_{m0}}{2} - 3\Omega_\Lambda\right)$$

$$= \frac{R^2}{d_H^2}\frac{1}{2}\left(\frac{\Omega_{m0}}{2} - 3\Omega_\Lambda\right) \quad (A)$$

2) Cluster

$$\Phi = \frac{2\pi}{3}G\rho R^2 - \frac{\Lambda}{2}R^2$$

$$= \left[\frac{1}{4}H_0^2\Omega_{m0}\times 200 - \frac{3H_0^2}{2}\Omega_\Lambda\right]R^2$$

$$\frac{\Phi}{c^2} = \frac{R^2}{d_H^2}\left[50\Omega_{m0} - \Omega_\Lambda\frac{3}{2}\right] \quad (B)$$

(A) & (B) are diff by almost '10 time'.

$$\frac{\Phi}{c^2} \sim 10^{-6 \text{ or } -7}$$

So even largest fluctuation keep $\frac{\Phi}{c^2}$ small

$$\rho = \sum_i m_i\,\delta_D(\vec{x}-\vec{x}_i)$$

$$= m\sum_i \delta_D(\vec{x}-\vec{x}_i)$$

$$\rho_{\vec{k}} = \frac{m}{V}\sum_j e^{-i k\cdot x_j}$$

Also $\nabla^2\phi = 4\pi G\bar{\rho}a^2\delta$

$$-k^2\phi = 4\pi G a^2\underbrace{\bar{\rho}\delta_k}_{\rho_k} \quad \text{for } k\neq 0$$

$$\Rightarrow -k^2\phi_k = \frac{a^2 m 4\pi G}{V}\sum_j e^{-ik\cdot x_j} \quad k\neq 0$$

$$\Rightarrow \quad \phi_{\vec{k}} = \frac{-4\pi G m a^2}{Vk^2}\sum_{i=1}^{N} e^{-i\vec{k}\cdot\vec{x}_i} \quad k\neq 0$$

$$\delta_{\vec{k}} = \frac{1}{N}\sum e^{-i\vec{k}\cdot\vec{x}} \quad (k\neq 0)$$

$$\dot{\delta}_{\vec{k}} = \frac{1}{N}\sum\left(-i\vec{k}\cdot\dot{\vec{x}}_i\right)e^{-i\vec{k}\cdot\vec{x}_i}$$

$$\ddot{\delta}_{\vec{k}} = \frac{1}{N}\sum\left[-i\vec{k}\cdot\ddot{\vec{x}}_i - (\vec{k}\cdot\dot{\vec{x}}_i)^2\right]e^{-i\vec{k}\cdot\vec{x}_i}$$

$$\ddot{\delta}_{\vec{k}} + \frac{2\dot{a}}{a}\dot{\delta}_{\vec{k}} = \frac{1}{N}\sum\left[-i\vec{k}\left\{\ddot{\vec{x}}_i + \frac{2\dot{a}}{a}\dot{\vec{x}}_i\right\} - (\vec{k}\cdot\dot{\vec{x}}_i)^2\right]e^{-i\vec{k}\cdot\vec{x}_i}$$

Use now $\ddot{\vec{x}} + \frac{2\dot{a}}{a}\dot{\vec{x}} = -\frac{1}{a^2}\vec{\nabla}_x\phi$

$$\ddot{\delta}_{\vec{k}} + \frac{2\dot{a}}{a}\dot{\delta}_{\vec{k}} = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{i\vec{k}\cdot\vec{\nabla}_{x_i}\phi}{a^2} - (\vec{k}\cdot\dot{\vec{x}}_i)^2\right]e^{-i\vec{k}\cdot\vec{x}_i}$$

Padmanabhan's text; reviews, bulletin of astronomical society etc. (1992 or 93)