Indian Institute of Science Education and Research Mohali

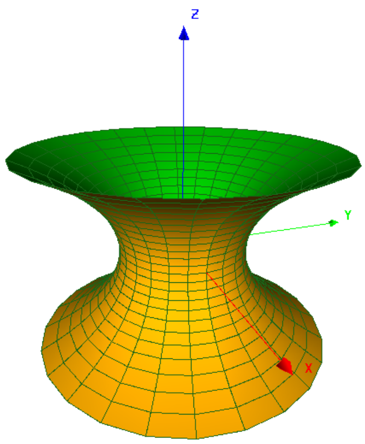
Curves and Surfaces (MTH201)

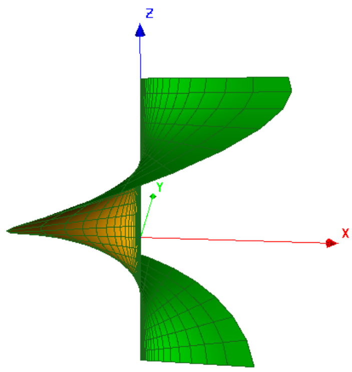
Academic Session 2012-13

Tutorial Sheet 11 **November 30 2012**

Instructions: Write main ideas / hints for solving questions in your tutorial noteook. There is no need to write full and formal solution during the tutorial session. However during off class hours you should practice writing these solutions in a formal manner. **Get the signature of your tutor after each session**.

1. Consider the catenoid C defined by the surface patch $\sigma(u, v) = (a\cosh v\cos u, a\cosh v\sin u, av)$ and the helicoid $\mathcal{H}$ defined by the surface patch $\rho(z, w) = (w\cos z, w\sin z, az)$. Show that the transformation $f : C \to \mathcal{H}$ given by $f(a\cosh v\cos u, a\cosh v\sin u, av) = (\sinh v\cos u, \sinh v\sin u, au)$ is an isometry.





2. Are all isometries equiareal? Is there an equiareal map which is not isometry? Do eqiareal maps preserve Gaussian curvature?

3. Recall the Möbius strip from Tutorial 8:

$$\sigma(t, \theta) = \left(\cos\theta\left(1 + t\cos\frac{\theta}{2}\right), \sin\theta\left(1 + t\cos\frac{\theta}{2}\right), t\sin\frac{\theta}{2}\right).$$

   Can you obtain this Möbius strip from a plane rectangular rubber sheet without stretching it?

4. Consider the surfaces $\mathcal{S}_1$ and $\mathcal{S}_2$ given $\sigma(u, v) = (u\cos v, u\sin v, \log u)$ and $\rho(u, v) = (u\cos v, u\sin v, v)$, respectively. Show that the map $f : \mathcal{S}_1 \to \mathcal{S}_2$ given by $f(\sigma(u, v)) = \rho(u, v)$ preserves Gaussian curvature but is not an isometry.
