Sudeshna 8.3

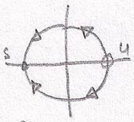
(no 8.1, 8.2)

Flows on a circle

$\dot{x} = f(x)$ | $\dot{\theta} = f(\theta)$

FP

$\dot{\theta} = \sin\theta$



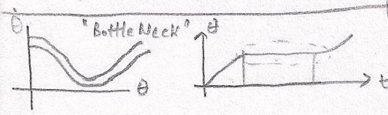
$\sin\theta > 0 \qquad \sin\theta^* = 0$

$\dot{\theta} > 0 \qquad \theta^* = 0, \pi$

$\dot{\theta} = f(\theta) = f(\theta + 2k\pi)$

$\frac{d\theta}{dt} = w \quad \theta(t) = wt + \theta_0$

$T = \frac{2\pi}{w}$



$\dot{\theta}_1 = w_1 = \frac{2\pi}{T_1}$

$\dot{\theta}_2 = w_2 = \frac{2\pi}{T_2}$

$\phi = \theta_1 - \theta_2$

$\Rightarrow \dot{\phi} = \dot{\theta}_1 - \dot{\theta}_2 = (w_1 - w_2)$

$\phi(t) = (w_1 - w_2)t + \phi_0$

Remark: ϕ is periodic

$T_\phi = \frac{2\pi}{w_1 - w_2}$

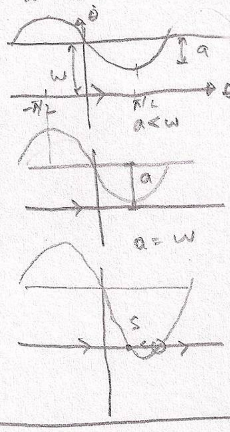
$\dot{x} = r + x^2$

$r \sim 0$

$T = \int \frac{dx}{r + x^2} \sim \frac{1}{\sqrt{r}}$

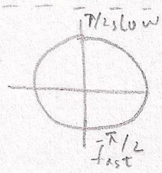
Non-uniform Oscillator

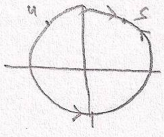
$\dot{\theta} = w - a\sin\theta$



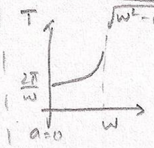
$T = \int dt = \int_0^{2\pi} \frac{d\theta}{f(\theta)} = \int_0^{2\pi} \frac{d\theta}{w - a\sin\theta} \Rightarrow$ at $a = 0$ $T = 2\pi/w$

$\Rightarrow T = \frac{2\pi}{\sqrt{w^2 - a^2}}$





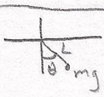
$\sin^*\theta = \frac{w}{a}$



$w \sim a$

$T = \frac{2\pi}{\sqrt{(w+a)(w-a)}}$

$= \frac{2\pi}{(2w)^{1/2}\sqrt{w-a}}$



1) Overdamped

2) Apply torque Γ

$\cancel{mL^2\ddot{\theta}} + b\dot{\theta} + mgL\sin\theta = \Gamma$

ignored; but key

Non-Dimensionalize $\rightarrow$ $\left(\frac{b}{mgL}\right)\dot{\theta} + \sin\theta = \frac{\Gamma}{mgL}$

$\dot{\theta} = \frac{d\theta}{d\tau} \cdot \frac{1}{T}$

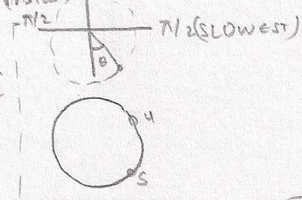
$\frac{t}{T} \approx \tau$

T → characteristic time.

I put $T = \frac{b}{mgL}$ to get

$\frac{d\theta}{d\tau} = \gamma - \sin\theta \quad$ where $\gamma = \frac{\Gamma}{mgL}$

$\gamma > 1$; $\theta^* = \frac{\pi}{2}$



Sudeshna 8.4

$\dot\theta = f(\theta)$ uniform: $\dot\theta = \omega \rightarrow \theta(t) = \omega t + \theta_0$

non-uniform: $\dot\theta = \omega + a\sin\theta \rightarrow$ sqrt scaling (?)

Phase Osc

Kurmoto 2 non-interactions: Beats $\phi = \theta_1 - \theta_2$

Rinzel (1984) Synchrony

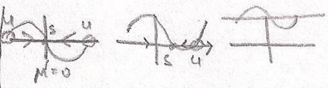
$\dot\theta = \omega$ $\quad \dot\theta = \omega + A\sin(\theta_E - \theta)$ with interaction

$\dot\theta_E = \Omega$ $\quad \dot\phi = \dot\theta_E - \dot\theta$

no interaction $(A=0)$ $\quad = (\Omega - \omega) - A\sin\phi$

$\tau = At$

$\mu = \frac{\Omega - \omega}{A} \Rightarrow \frac{d\phi}{d\tau} = \mu - \sin\phi$



$\phi^* = 0$	$0 < \mu < 1$	$\mu > 1$
stable	phase lock	phase drift

If $\mu = 0$, your external freq = natural freq

NB: $\sin\phi^* = \mu$ $\Rightarrow$ same freq as source

Proof $\Rightarrow$ as expected.

frequency of oscillation

$\mu = \sin\phi^*$

$\dot\theta = \omega + A\sin\phi$

$= \omega + A\mu$

$= \omega + \Omega - \omega = \Omega$

For phase lock

$0 < \frac{\Omega - \omega}{A} < 1 \Rightarrow 0 < \Omega - \omega < A$

$\Rightarrow \omega < \Omega < A + \omega$

Similarly

$-1 < \frac{\Omega - \omega}{A} < 0 \Rightarrow A - \omega < \Omega < \omega$

Josephson Junction "Flavour"

ψ_1	ψ_2

$i\hbar\frac{\partial\psi_1}{\partial t} = \mu_1\psi_1 + K\psi_2$

$i\hbar\frac{\partial\psi_2}{\partial t} = \mu_2\psi_2 + K\psi_1$

Ansatz

$\psi_1 = \sqrt{n_1}e^{i\theta_1}$

$\psi_2 = \sqrt{n_2}e^{i\theta_2}$

$\hbar\frac{\partial n_1}{\partial t} = -\hbar\frac{\partial n_2}{\partial t} = 2K\sqrt{n_1 n_2}\sin(\theta_1 - \theta_2)$;

$-\hbar\frac{\partial}{\partial t}(\theta_2 - \theta_1) = (\mu_2 - \mu_1)$

So finally I have (cooper pair stuff missing)

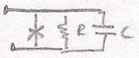
$I = I_c\sin\phi$ ✓ $\quad$ ψs are cooper pairs

$\phi = \theta_1 - \theta_2$ ✓

$I_c = 2K\sqrt{n_1 n_2}$ ✓ (almost)

$\dot\phi = \frac{2eV}{\hbar}$ $\quad V = \frac{\hbar}{2e}\dot\phi$ $\quad$ comes from $\mu_2 - \mu_1$

NB: For a const. voltage, we would have an AC current!



$I = \frac{V}{R} + C\frac{dV}{dt} + I_c\sin\phi$

$I = I_c\sin\phi + \frac{\hbar}{2eR}\dot\phi + \frac{\hbar C}{2e}\ddot\phi$

Notice the relation with

$mL^2\ddot\theta + b\dot\theta + mgl\sin\theta = \Gamma$

$\beta \ll 1 \Rightarrow$ First order like

$\beta\frac{d^2\phi}{d\tau^2} + \frac{d\phi}{d\tau} + \sin\phi = \frac{I}{I_c}$

where $\tau = \left(\frac{2eI_cR}{\hbar}\right)t$

$\beta = \frac{2eI_cR^2C}{\hbar}$

Mccumber no.

Sudeshna 9.1 (No other 9.x)

Fractals
Chapter 2, 3, 4
↓
leaning
imperfect
bifurcations

Det
Tr
1) Node S / U
2) Spirals S / U
3) Centers

2 Dim (phase space)

$\dot{x} = ax + by$

$\dot{y} = cx + dy$

$\dot{\bar{X}} = A\bar{X}$; $\bar{X} = \begin{pmatrix} x \\ y \end{pmatrix}$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

F.P. $\dot{\vec{X}} = 0 \Rightarrow \vec{X}^* = 0$

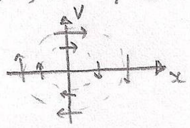
Eg. $f(x) = -kx$

$m\ddot{x} + kx = 0$

$\dot{x} = v$

$\dot{v} = -\frac{k}{m}x = -w^2 x$

$(\dot{x}, \dot{v}) = (v, -w^2 x)$



closed periodic

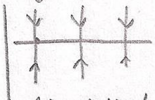
$A = \begin{pmatrix} a & 0 \\ 0 & -1 \end{pmatrix}$

$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$

$\dot{x} = ax$; $x = x_0 e^{at}$

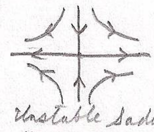
$\dot{y} = -y$, $y = y_0 e^{-t}$

$a = 0$



line of fixed points

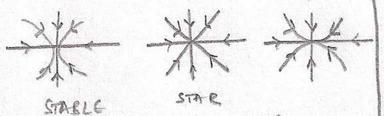
$a > 0$



Unstable Saddle Node

STABILITY
$x^* \Rightarrow$ Globally Attracting
Neutrally Stable
|||
Liapunov Stable

$a < -1$ $\quad$ $a = -1$ $\quad$ $-1 < a < 0$



STABLE NODE $\quad$ STAR

$\vec{X}(t) = e^{\lambda t}\vec{v}$

$\dot{\vec{x}}(t) = A\vec{x}$

$\lambda \cancel{e^{\lambda t}}\bar{v} = A\cancel{e^{\lambda t}}\bar{v}$; $\lambda^2 - \tau\lambda + \Delta = 0$

$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\qquad \lambda_{1,2} = \frac{\tau \pm \sqrt{\tau^2 - 4\Delta}}{2}$

$\det\begin{pmatrix} a-\lambda & b \\ c & d-\lambda \end{pmatrix} = 0$ $\qquad \Delta = ad - bc$

τ Trace $= a + d$