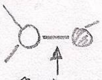
Olly 12·1



Optical Thm

Cancellation of Anomalies

Conservation of Current

$S^\dagger S = I$ $\quad \mathrm{Im}\, \mathcal{M}(a\to a) = \prod_{\{F\}} \int d\Pi_I |\mathcal{M}(a\to I)|^2$

$S = \mathbb{1} + iT$, $\quad iT = \delta(P_f - P_i)\mathcal{M}$

$\sim \dfrac{1}{p^2 - m_0^2 + M^2(p^2)}$ ← $\mathcal{M}(p\to p)$

1PI self energy

Narrow width unstable

$m^2 \to m^2 - im\Gamma$ is a good approx.

→ Invariance of S under cont. symm

⇓ Noether thm

$\partial_\mu j^\mu = 0$

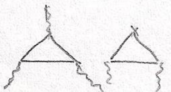
SM $\partial_\mu j^\mu_{Y,a,A} = 0$

When you evaluate

$\langle 0|T(\partial_\mu j^\mu e^{i\int S_{int}})|0\rangle$

$\bar\psi\gamma^\mu\psi \to (\bar\psi\gamma^\mu\psi A_\mu)^2$

You get diagrams like



$\partial_\mu j^\mu_a = -\dfrac{g^2}{8\pi^2} F^b_{\lambda b}\tilde F^{\lambda\sigma c} A^{abc}$

$= \mathrm{tr}(\gamma^5 t^a\{t^b, t^c\})$

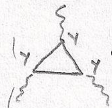
$\neq 0$ Quantum Anomaly

For vector like

$\gamma^5 = -1$ L, $+1$ R

cancels out.

Gauge Invariance Ruined/preserved



$\sim \mathrm{tr}\, Y^3 \gamma_5$

→ color, → SU(2), → Y

$\frac{6\cdot 2}{27}\cdot 9$ ← $Q_L(3,2,Y_3)$

-2 ← $L_L(1,2,-1)$

$-1\frac{1}{9}$

$u_R(3,1,4/3) \to 3\times\frac{64}{27}$

$d_R(3,1,-2/3) \to 3\times(\frac{-8}{27})$

$e_R(1,1,-2) \to -8$

$+\frac{16}{9}$

So they cancel out

$(SU(2)_L)^3$

$\mathrm{tr}\, T^a\{T^b, T^c\}$, $\{T^b, T^c\} = 2\delta^{bc}$, $\mathrm{tr}(T) = 0$!!!

(Neater version: see Prashansa's 30/3/15)

$(SU(3)_L)^3$ $\quad \mathrm{tr}\, T^A\{T^B, T^C\} \sim d^{ABC} = 0$

~~<some missing>~~ traceless

$\sim \delta^{ab}\sum_{\text{left}} Y$

$U(1)^2_Y\ \underbrace{SU(2)_L}\ \underbrace{SU(3)_c}$

$U(1)_Y (SU(2))^2_L$ $\quad = 0$ $\quad (3\times\frac{1}{3} - 1) = 0$

$L = 6(\frac{1}{3}) + 2(-1) = 0$

$R = 3(\frac{4}{3} - \frac{2}{3}) + (-2) = 0$

At tree level SM $\mathcal{L} \to$ B ✓, L_e ✓, L_μ ✓, L_τ ✓ $\quad \partial_\mu B^\mu \stackrel{?}{=} 0$, $\partial_\nu L^\nu_{e,\mu,\tau} \stackrel{?}{=} 0$

$\mathrm{tr}\, \gamma^5 B Y^2 = 3(\frac{1}{3})[-(+\frac{1}{3})^2\times 2 + (\frac{4}{3})^2 + (\frac{2}{3})^2] = 2$

$\mathrm{tr}\, \gamma^5 L Y^2 = [-2\times(-1)^2 + (-2)^2] = +2$; $\quad 2 - 2 = 0$

$\Rightarrow \partial^\mu(B_\mu - L_\mu) = 0$ $\quad$ NB!

Profound significance: Gauge B−L ; it's indeed conserved

Left · Right Symmetric Model

$SU(2)_L \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $\quad SU(2)_R \begin{pmatrix} u_R \\ d_R \end{pmatrix}$ $\quad$ Y: $4/3$, $-2/3$

$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

$Q_{em} = T_{3L} + T_{3R} + \dfrac{B-L}{2}$

Since you added B−L, you must add $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ & you must add $\begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$

$B-L = -1$, $\quad B-L = -1$

SSB $\langle\ \rangle \geq 2.5\,\mathrm{TeV} = 1000\,\mathrm{GeV}$

$SU(2)_L \times U(1)$

$\nu_R \to \nu^e_L$

$\psi^c_L = C\bar\psi^T = -i\gamma^2\psi^*_R$

LR model C-Presentation

$\begin{pmatrix} u_L \\ d_L \end{pmatrix} SU(2)_L \times \begin{pmatrix} d_L^c \\ u_L^c \end{pmatrix} SU(2)_R$ $\quad Y_3$, $-2/3$

$(2, 1, Y_3)$ $\quad (1, 2, -Y_3)$

$Q_{em} = \frac{1}{2} - \frac{1}{6} = \frac{1}{3}$

$= -\frac{2}{3}$

Path Integral Quantization

Based on Gaussian Integrals.

$\int_{-\infty}^{\infty} dx\, e^{-ax^2} = \sqrt{\frac{\pi}{a}}$

Generalise $(x_1, \dots x_N)$

$\int d^N x\, e^{-x^T A x}$ $\quad A^T = A \Rightarrow A = O^T\Lambda O$, $\Lambda = \mathrm{diag}(a_1, \dots a_N)$

$= \int \prod dx'_i\, e^{-\sum_i a_i x_i'^2}$ $\quad d^N x = d^N x' \left|\frac{\partial x}{\partial x'}\right|$

$Ox = x'$

$x = O^{-1}x' = O^T x'$

$\left|\frac{\partial x}{\partial x'}\right| = \det O^T = 1$

$= \prod_{i=1}^{N}\sqrt{\frac{\pi}{a_i}}$

$\simeq \dfrac{1}{(\det A)^{1/2}}$

$\int_{-\infty}^{\infty} dx\, e^{-x^T a x} = (\sqrt{\pi})^N (\det a)^{-1/2}$

$\int d\theta \cdot \theta = 1$

Anti commuting #

$\theta\chi = -\chi\theta$, $\theta^2 = 0$

$f(\theta) = a + b\theta$

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$$|D\langle\phi\rangle|^2 = |(\partial + i g_1 A_1 Q_1 + i g_2 A_2 Q_2)\langle\phi\rangle|^2$$

$$\left|\frac{(g_1 A_1 + 2 g_2 A_2)\, g_Z}{\sqrt{g_1^2 + (2g_2)^2}} \frac{v}{\sqrt{2}}\right|^2$$

$$g_Z^2 = g_1^2 + (2g_2)^2$$

$$m_Z^2 = (g_1^2 + 4 g_2^2) v^2$$

$$\frac{g_1}{g_Z} = c\theta, \quad \frac{2g_2}{g_Z} = s\theta$$

$$A = -s_\theta A_1 + c_\theta A_2$$

$$\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} c & -s \\ s & c \end{pmatrix}\begin{pmatrix} Z \\ A \end{pmatrix}$$

$\mathrm{tr}\, \Phi^\dagger \Phi$

$$\Phi = \begin{pmatrix} a/\sqrt{2} & b \\ c & -a/\sqrt{2} \end{pmatrix} \quad SO(6)$$

$$\Phi^\dagger = \begin{pmatrix} a^*/\sqrt{2} & c^* \\ a^* & -a^*/\sqrt{2} \end{pmatrix}$$

$$\mathrm{tr}\, \Phi^\dagger \Phi = |a|^2 + |b|^2 + |c|^2$$

G_{221}

$$\Phi'_{\alpha\dot\alpha} = U_{\alpha\beta} V^*_{\dot\alpha\dot\beta} \Phi_{\beta\dot\beta}$$

$$\Phi' = U \Phi V^\dagger$$

$$\delta\Phi = i\frac{\vec{\tau}}{2}\cdot\vec{\theta}_L \Phi - i\Phi \frac{\vec{\tau}}{2}\cdot\vec{\theta}_R$$

W_L

$\mathrm{tr}\, |D\langle\phi\rangle|^2$

$$= \frac{1}{4}\mathrm{tr}\,(\quad)^\dagger (g_L \vec{\tau}\cdot \vec{L}_\mu \langle\Phi\rangle - g_R \langle\Phi\rangle \vec{\tau}\cdot\vec{R}_\mu)^\dagger$$

W_R

Bosons $\int_{-\infty}^{\infty} dx\, e^{-xax} = \sqrt{\frac{\pi}{a}}$

$x^T A x$

$1 \to N$

$(\sqrt{\pi})^N (\det A)^{-1}$

Fermion Grassmann #s: $\theta\eta = -\eta\theta$

$\eta^2 = \theta^2 = 0$

$f(\theta) = a + b\theta$

$\int d\theta = 0, \int d\theta\, \theta = 1$

$\bar{L}_L \Phi L_R + h.c.$

$e^{-a\theta^2} = e^0 = 1$

$$\int d\theta_1 d\theta_2 e^{-a\theta_1\theta_2} = N - a\theta_1\theta_2 + \frac{(a\theta_1\theta_2)^2}{2!} + \dots$$

$\int d\theta_1 d\theta_2$

$+a$

<copy from prashansa>

$$\int \prod_{i=1}^{2N} d\theta_i\, e^{-\frac{1}{2}\theta_i A_{ij} \theta_j}$$

$\theta' T O^T A O \theta'$

$$\int \pi\, d\theta_i\, e^{-\theta_i A_{ij} \theta_j}$$

$\hookrightarrow$ Anti symmetric $2N \times 2N$

$$O^T A O = \mathrm{diag}\left(a_1\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \dots a_n\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right)$$

$$O^T = O^{-1} \quad ; \det A = (-1)^N \prod_i (a_i)^2$$

$$= \int \pi\, d\theta'_i\, e^{-\frac{1}{2}\theta^T A \theta}$$

$$= \sqrt{\det A (-1)^N}$$

$$\xi = \frac{\theta_1 + i\theta_2}{\sqrt{2}}$$

$$\int dx\, d\bar{x}\, e^{-a(x^2 + \bar{x}^2)}$$

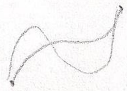
$x^* x$

$$= \sqrt{\frac{\pi}{a}}\sqrt{\frac{\pi}{a}}$$

$-x^\dagger H x$

$$\int dx\, dx^*\, e^{-x^\dagger H x} \sim (\det H)^{-1}$$

$$d\xi\, d\xi^* \sim (\det H)^{+1}$$



$x_b t_2$

$x_a t_1$

CM: $\delta S = 0$ EOM: $\frac{d}{dt}\frac{\partial L}{\partial \dot q} - \frac{\partial L}{\partial q} = 0$

$U(x_a, x_b, (t_2 - t_1) = T)$ $x(t_2) = x_b$

Schrodinger Amplitude

$(x_a, t_1) \to (x_b, t_2)$

P.I. $U(x_a, x_b, T) = \int (\mathcal{D})(x(t))\, e^{iS[x(t)]}$ Measure!

$\int_0^T dt\, L(x, \dot x) = S[x(t)]$

Functional of a path.

$x(t) = \sum a_n f_n(t)$

$f_n(t) = e^{\frac{2\pi i n t}{T}}$

$\int Dx(t)$

$\prod_n da_n \quad a_{-n} = a_n^*$

$$\int L = \int \frac{1}{2}\dot x^2 - \int \frac{1}{2}x^2$$

$$\sum_n k_n^2 |a_n|^2 - \sum_n |a_n|^2$$

$$i\hbar \frac{\partial U(x_a, x_b, T)}{\partial t} = HU$$

δt

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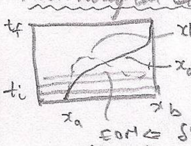
& 14·1

Path Integral Quantization



$U(x_a, x_b, t_f - t_i = T) = \int \mathcal{D}x(t)\, e^{iS[x(t)]}$

$i \frac{\partial U}{\partial t} = HU$

$S = \int_{t_i}^{t_f} L\, dt$

$\prod_{i=1}^{N} \int_{-\infty}^{\infty} dx(t_i)$ (Time Slicing)

EOM $\Leftarrow \delta S[x(t), x(t_i) = x_a, x(t_f) = x_b]$ (Newton)

$S[x_c(t) + \delta x(t)] \doteq S[x_c(t)] + \int dt \frac{\delta S}{\delta x}\Big|_{x=x_c} \delta x(t) + \frac{1}{2}\int \frac{\delta^2 S}{\delta x(t)\,\delta x(t')} \delta x(t)\delta x(t')\, dt\, dt' + \frac{\delta^3 S}{\delta x^3}\cdot \delta x^3$

Linear Perturbation

Since δx is periodic, I can fourier expand it as

$\delta x^* = \delta x(t) = \sum_n a_n e^{ik_n t}$; $k_n = \frac{2\pi i n}{T}$; $a_n = a_{-n}^*$

Functional Quantization in QFTs

$\langle \phi_b(\vec{x}) | e^{-iHT} | \phi_a(\vec{x}) \rangle$

From a to b config of ϕ in space.

$H = \int d^3x \left[\frac{1}{2}\vec{\pi}^2 + \frac{(\nabla \phi)^2}{2} + V(\phi) \right]$

Hamiltonian form

(a) $\int \mathcal{D}\phi\, \mathcal{D}\pi\, e^{-i\int_0^T d^4x (\pi\dot{\phi} - \mathcal{H})}$

(b) $U(\phi_a, \phi_b, T) = \int \mathcal{D}\phi\, e^{i\int d^4x\, \mathcal{L}(\phi, \partial_\mu \phi)}$ $\overset{\shortparallel}{\mathcal{L}}(\phi, \partial_\mu \phi)$

$x_a = x(t_{in})$

$x_b = x(t_{fin})$

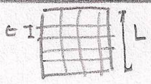
Defines QFT

Correlation f^n

$\langle \Omega | T(\phi_H(x_1) \ldots \phi(x_N)) | \Omega \rangle$

$= \lim_{T \to \infty(1-i\epsilon)} \dfrac{\int \mathcal{D}\phi\, \phi(x_1) \cdots \phi(x_N)\, e^{iS}}{\int \mathcal{D}\phi\, e^{i\int dt \int d^3x\, \mathcal{L}(\phi, \partial_\mu \phi)}}$

1) Feynman rules for a free theory $\mathcal{L}_0 = \frac{(\partial\phi)^2 - m^2\phi^2}{2}$



Discretize space-time $\{x^\mu\} \rightarrow \{x^\mu_L\}$ → Runs over the lattice

$\phi^*(x_i) = \phi(x_i) = \frac{1}{L^4}\sum_n e^{ik_n x_i} \phi(k_n)$; $\frac{1}{L^4} \sum_{\text{Dual lattice site}} \xrightarrow[\text{lt}]{\text{contin}} \int \frac{d^4x}{(2\pi)^4}$

$\tilde{\phi}^*$ <copy from abhinav

$\phi^*(k_n) = \phi(-k_n)$; $\int \mathcal{D}\phi$

k_n^μ

$\overset{\shortparallel}{\lim_{\substack{L\to\infty \\ \epsilon \to 0}}} \int \prod_{n^0, k_n^0 > 0} \mathrm{Re}\,\phi(k_n)\, \mathrm{Im}\,\phi(k_n)$

$\int d^4x\, \phi^2(x) = \sum_{n,m} \frac{1}{L^8} \iiiint_0^L d^4x\, e^{-i(k_n + k_m)\cdot x} \tilde{\phi}(k_n)\tilde{\phi}(k_m)$

$= \frac{1}{L^4}\sum_n |\phi(k_n)|^2$

$\frac{1}{2}\left((\mathrm{Re}\,\tilde{\phi}(k_n))^2 + (\mathrm{Im}\,\tilde{\phi}(k_n))^2 \right)$

$\int d^4x\, |\partial\phi|^2 = \frac{1}{L^4}\sum_n (-k_n^2)\, |\tilde{\phi}(k_n)|^2$

$S_0 = \int d^4x\, \mathcal{L}_0(\phi, \partial\phi) = -\frac{1}{L^4}\sum_{\substack{n \\ k_n^0 > 0}} (m^2 - k_n^2)\, |\tilde{\phi}(k_n)|^2$

$\boxed{\int \mathcal{D}\phi\, e^{iS_0}} = \prod_{\substack{k_n^0 > 0 \\ < 0}} \left(\sqrt{\frac{-i\pi L^4}{(m^2 - k_n^2)}} \right) = \left(\det(m^2 + \partial^2 + i\epsilon)\right)^{-1/2}$

= Functional Determinant of $(m^2 + \partial^2 + i\epsilon)$

↓ discretization + fourier analysis

$(m^2 - k_n^2 + i\epsilon)$

For correlation f^ns, we need the PI with insertions

$\int \mathcal{D}\phi\, \underbrace{\phi(x_1)\phi(x_2)}\, e^{iS[\phi]} \longrightarrow \sim e^{\frac{-i}{4L^4}\sum_{k_n^0 > 0}(m^2 - k_n^2)[R_n^2 + I_n^2]}$

$\overset{\shortparallel}{\frac{1}{L^8}\sum_{nm}} e^{-i(k_n\cdot x_1 + k_m \cdot x_2)}\, \phi(k_n)\phi(k_m)$

$\shortparallel$

$\sim (R_n + iI_n)(R_m + iI_m)$

$= (R_n R_m - I_n I_m) + i(R_n I_m)$

$k_n \neq \pm k_m$

$\int_{-\infty}^{\infty} dx\, e^{-ax^2}\cdot x^{(2n+1)} \equiv 0$ So by oddness, the double sum will collapse to a single sum

$k_n = k_m$ $\qquad k_m^0 > 0$

$\int dR_n\, dI_n (R_n^2 - I_n^2)\cdot e^{-\frac{i}{V}(m^2 - k_n^2)(R_n^2 + I_n^2)}$

If $k_n = -k_m$

$I_m = -I_n$

So we are left with $\int \mathcal{D}\phi\, \underbrace{\phi(x_1)\phi(x_2)}\, e^{iS[\phi]}$

$\overset{\shortparallel}{\frac{1}{L^8}\sum_{nm}} e^{-i(k_n\cdot x_1 + k_m\cdot x_2)}\, \phi(k_n)\phi(k_m)$

$= \frac{1}{L^4}\sum_n \frac{-i}{m^2 - k_n^2 - i\epsilon}\, e^{-ik_n\cdot(x_1 - x_2)}$

$= \int \frac{d^4k}{(2\pi)^4} \frac{i e^{-ik\cdot(x_1 - x_2)}}{k^2 - m^2 + i\epsilon} = \underbrace{\phi(x_1)\phi(x_2)}$

Similar arguments show that for a 4-pt

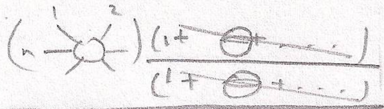
$\langle \Omega | T(\phi_1 \ldots \phi_4) | \Omega \rangle = D_F(x_1 - x_2)\, D_F(x_3 - x_4)$

$+$ combinations.

QFT : $\to \langle\Omega|T(\phi_H(x_1)\ldots\phi_H(x_n))|\Omega\rangle \to$ Heisenberg ground state

|| Interaction Picture

$$\frac{\langle 0|T(\phi_I(x_1)\ldots\phi_I(x_n)\,e^{-i\int_{-T}^{T}H_I(\phi)dt})|0\rangle}{\langle 0|T\,e^{-i\int_{-T}^{T}H_I(\phi)dt}|0\rangle} \to \text{Feynman Diagrams}$$



Path Integral

$$= \frac{\int "D\phi"\,\phi(x_1)\phi(x_2)\ldots\phi(x_n)\,e^{iS[\phi]}}{\int "D\phi"\,e^{iS[\phi]}} \quad \mathcal{L}_0 \xrightarrow[\text{discretize}]{}$$

$i(S_0[\phi]+S_{int})$ — S_{int}: treat as perturbation

Olly 15.2 (13.3 missing | wedding no 13.4)

$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{int}$
$\downarrow$ B.P.

Wick $\quad \frac{1}{2}\phi(-\partial^2-m^2)\phi$

Thus for a free field correlators based on $(-\partial^2-m^2+i\epsilon)^{-1}$

Functional Integration

$7^{\wedge}$ derivative

$$\frac{\delta g(\vec{x})}{\delta g(\vec{y})} = \delta^{(n)}(\vec{x}-\vec{y})$$

$$\frac{\delta}{\delta g(\vec{y})}\int d^{(n)}x\, g(\vec{x})\,\phi(\vec{x}) = \phi(y)$$

Obeys usual derivative properties

Leibniz Rule ; chain rule etc. follow

$\frac{\partial x^j}{\partial x^i} = \delta_i^j$

$\frac{\partial}{\partial x_i}\sum k_j x^j = k_i$

$$\left.\frac{\delta}{\delta J(x)} e^{i\int dy\, J(y)\phi(y)}\right|_{J=0} =$$

$$\left. i\phi(x)\cdot e^{i\int\ldots}\right|_{J=0} = i\phi(x)$$

$$\frac{\delta}{\delta J(x)}\int\left(\frac{\partial}{\partial y^\mu}J(y)\right)V^\mu(y)\,d^{(n)}y \overset{BP}{=} -\frac{\partial}{\partial x^\mu}V^\mu(x)$$

$$Z[J] = \int D\phi\, e^{i\int(\mathcal{L}(x)+J(x)\phi(x))d^4x}$$

$$\langle\Omega|T(\phi_H(x_1)\ldots\phi_H(x_n))|\Omega\rangle = \left.\frac{1}{Z[0]}\left(\prod_{i=1}^{n}-i\frac{\delta}{\delta J(x_i)}\,Z[J]\right)\right|_{J=0}$$

$\mathcal{L}=\mathcal{L}_0$

$$Z_{free} = \int D\phi\, e^{i(S_0+\int J\phi)}$$

$$S_0 = \int d^4x\,\frac{1}{2}\phi(\partial^2-m^2+i\epsilon)\phi$$

$$\frac{1}{2}x^Tax + b^Tx = \frac{1}{2}\left(\sqrt{a}\,x+\frac{b}{\sqrt{a}}\right)^2 - \frac{b^2}{2a}$$

$$= \frac{1}{2}(x+a^{-1}b)^T a(x+a^{-1}b) - \frac{1}{2}b^Ta^{-1}b$$

$\int_{-\infty}^{\infty}dx = \int_{-\infty}^{\infty}dx'$

$x' = x + \text{const}$

$$\int dz\,\underbrace{(\delta(x-z)K_z)}_{K_{xz}}K^{-1}_{zy}$$

$$\left(= K_x K^{-1}(x-y) = \delta(x-y)\right)$$

analogy

$$\sum_j k_{ij}(K^{-1})_{jk} = \delta_{ik}$$

$$Z_{free}[J] = \int D\phi\,\exp\frac{i}{2}\Big[\underbrace{(\phi+K^{-1}J)}_{=\phi'}K(\phi+K^{-1}J) - \int d^4x\,d^4y\,J(x)K^{-1}(x,y)J(y)\Big]$$

$D\phi$: assume shift invariant

$$Z_{free}[J] = Z_{free}[0]\,e^{-\frac{i}{2}\int J\cdot K^{-1}J}$$

$$Z[J] = e^{i\int d^4x\,\mathcal{L}_{int}\left[-i\frac{\delta}{\delta J(x)}\right]}Z_{free}[J]$$

$$\langle\Omega|T(\phi_H(x_1)\ldots\phi_H(x_n))|\Omega\rangle$$

$$= \left.\frac{\prod_{i=1}^{n}\left(-i\frac{\delta}{\delta J(x_i)}\right)Z_{free}[0]\,e^{i\int d^4x\,\mathcal{L}_{int}\left[-i\frac{\delta}{\delta J(x)}\right]}e^{-\frac{i}{2}JK^{-1}J}}{Z[0]}\right|_{J=0}$$

2 pt. correlator free

$$\langle\Omega|T(\phi_1\phi_2)|\Omega\rangle = \left.(-i)^2\frac{\delta}{\delta J_1}\frac{\delta}{\delta J_2}e^{-\frac{i}{2}JK^{-1}J}\right|_{J=0}$$

$-iK^{-1}J\,e^{()}$

$(Z(0) = Z_{free}[0]$ $\therefore$ cancelled$)$

$$= iK^{-1} = D_F(x_1-x_2) = (D_F)_{12}$$

$$\| \quad (-\partial^2-m^2+i\epsilon)^{-1}$$

$$\tilde{D}_F = \frac{i}{k^2-m^2+i\epsilon}$$

$$\langle\Omega|T(\phi_1\ldots\phi_4)|\Omega\rangle \overset{free}{=} D^F_{12}D^F_{34} + D_{13}D_{24} + \ldots$$

Gauge Theories

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} = -\frac{1}{2}\partial_\mu A_\nu(\partial^\mu A^\nu - \partial^\nu A^\mu)$$

$$\overset{BP}{=} \frac{1}{2}A_\nu\underbrace{(\partial^2\eta^{\mu\nu}-\partial^\mu\partial^\nu)}_{K^{\mu\nu}}A_\mu$$

$$\overset{mom}{=} \frac{1}{2}\tilde{A}_\nu(-k)(-k^2\eta^{\mu\nu}-\partial^\mu\partial^\nu)\tilde{A}_\mu(k)$$

(+ve $\because$ space part has right signs)

$K^{\mu\nu}$ is singular

$A_\mu = k_\mu\alpha(k)$

$\to 0 \Rightarrow$ ill defined integral $\because$ weight is non damping

$$\frac{\int DA_0\,DA_1\,DA_2\,DA_3\,e^{iS_m}}{DA_\mu}$$

$$\delta A_\mu = \frac{\partial_\mu\alpha(x)}{e}$$

$$\delta(f(x)) = \sum_i\frac{\delta(x-x_i)}{|f'(x_i)|}\quad f(x_i)=0$$

$\int dx\,\delta(f(x))\cdot|f'(x)|\,dx = \sum_i 1 = 1$ if $f(x)$ has only one root.

$$1 = \int\prod_{i=1}^{n}da_i\,\delta^n(\vec{g}(\vec{a}))\,\det\left(\frac{\partial g_i(a_i)}{\partial a_j}\right)$$

$G(A^\alpha)=0 \quad$ eg. $\partial_\mu A^\mu = \omega(x)$

$$1 \equiv \int D\alpha(x)\,\delta(G[A^\alpha])\,\det\left[\frac{\delta G[A^\alpha]}{\delta\alpha}\right]$$