Thermoelectric Effect

Atul Singh Arora

February 2015

1 Theory

1.1 A simple model

It is observed that if a temperature gradient is applied (to what? a metal) there is a flow of current. This may be stated as

$$E = Q\nabla T$$

where $Q \equiv$ Thermopower, T is the temperature (a function of position), and E is the electric field created by the temperature gradient. This effect is known as the seeback effect. [1] Let us develop the idea under the assumptions of the Drude model. Consider a 1 dimensional case. Since by assumption, in the Drude model, the velocity of electrons depends on the local temperature, we may write an expression for the velocity of electrons due to the temperature gradient as

$$v_Q = \frac{1}{2}[v(x - v\tau) - v(x + v\tau)] = -\tau\frac{dv}{dx} = -\tau\frac{d}{dx}\left(\frac{v^2}{2}\right)$$

where v is the local average velocity of the particle and τ is the average time between collisions; We can generalize this readily to the 3d case by putting $v^2 \to v_x^2$ and nothing that

$$< v_i^2 >= \frac{1}{3}v^2;\ i \in \{x, y, z\}$$

so that we have (using the 3d generalization of $\frac{dv^2}{dx} = \frac{dv^2}{dT}\frac{dT}{dx}$),

$$v_Q = -\frac{\tau}{6}\frac{dv^2}{dT}(\nabla T)$$

Now we know from the theory of EM and Newton that the velocity due to the uniform electric field (we haven't stated yet the origin of this field) will be

$$v_E = -\frac{eE\tau}{m}$$

[1] It maybe stated for completeness that there's a peltier effect, which creates a temperature gradient upon application of a potential difference. However this is not considered here.

At equilibrium then, we can assume that $v_E + v_Q = 0$ (why?). This finally yields a linear relation between E and ∇T as

$$E = \left(-\frac{1}{3e}\frac{d}{dT}\frac{mv^2}{2}\right)\nabla T = \left(-\frac{c_v}{3ne}\right)\nabla T = Q\nabla T;\; Q \equiv \left(-\frac{c_v}{3ne}\right)$$

where we are not fully justified in the last substitution. Finally, we can use $E = -\nabla V$ to write

$$V = \alpha T;\; \alpha = -Q$$

As a remark it must be mentioned that it is possible for v_Q to be $= 0$ while there is still heat flow in accordance with the temperature difference at equilibrium.

2 Experimental Verification

One might find it reasonable to think that the simplest way of measuring seeback effect would be to just stick in voltmeter probes to two points in a metal that're maintained at distinct temperatures. Reasonable as it sounds, there's a serious flaw; the voltmeter itself consists of metallic parts which will produce their own seeback effect due to the temperature difference at the probes. So then one may think it can be rectified by heating a metallic block at the middle and measuring the voltage difference at the equitemperature sides. Well, a little though reveals that this would be zero.[2] However, the idea has merit. If we put two different metal blocks' edges in contact (it'll be both a thermal and electrical contact) and maintain it at temperture T_C and measure voltage off of the open edges (with open edges of both metals maintained) at temperture T_E, we'll get a non-zero voltage (unless the metals have the same seeback cofficient). Let's calculate this potential difference. We assume that $T_C > T_E$; $\Delta T = T_c - T_E > 0$, potential at the cold edge of metal one and two is V_1 and V_2 respectively, and potential at the middle (common edge) is V, so that

$$\begin{aligned}
V - V_1 &= \alpha_1 \Delta T, & V - V_2 &= \alpha_2 \Delta T \\
\implies V_2 - V_1 &= (\alpha_2 - \alpha_1)\,\Delta T & & \\
\implies \Delta V &= \alpha \Delta T, & \alpha &= (\alpha_2 - \alpha_1)\,,\; \Delta V = V_2 - V_1
\end{aligned}$$

This is a neat result. So all that I am now required to do is to keep ΔT sufficiently large, so that I can measure ΔV reliably using my voltmeter.

[2] Use $\Delta V = \alpha \Delta T$; its trivial then

2.1 The Setup

That setup should be then as described earlier. Well, almost because in our lab we have a slightly different setup. As shown in the diagram, across the circuit, we can write

$$\alpha_1(T_{oc} - T_c) + \alpha_2(T_o - T_{oc}) + \Delta V + \alpha_2(T_{oe} - T_o) + \alpha_1(T_e - T_{oe}) = 0$$

if we assume that $T_{oe} \approx T_{oc}$, which it physically might very well be, we have

$$\Delta V = \alpha_1(T_c - T_e)$$

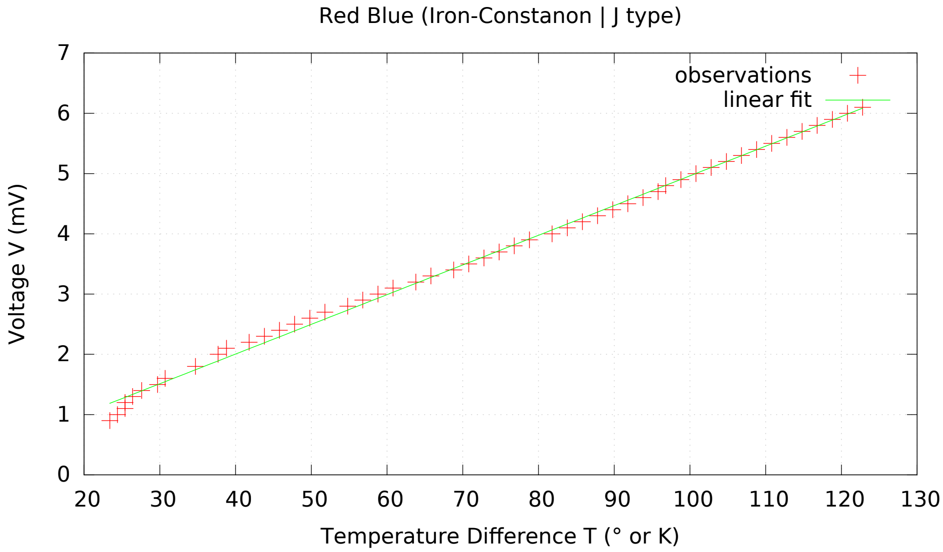
which peculiarly enough, depends only on the seeback coefficient of metal two. This maybe wrong because if it were so simple to find the seeback coefficient of one metal in isolation, Ashcroft won't say use a superconductor instead of the second metal (a superconductor doesn't exhibit seeback (cofficient is zero))

2.2 Observations

2.2.1 Time-line

Feb 16	Monday	[finished the previous experiment] Started reading the theory
Feb 17	Tuesday	[Holiday | shivratri]
Feb 20	Friday	Reading the theory done | Started performing the experiment; cooling method caused errors (to be repeated)
Feb 23	Monday	Took 2 sets of readings (the latter wasn't linear, plausibly because of accelerated cooling) | worked on the record alongside
Feb 24	Tuesday	Repeated for the third time; got the right result | concluded cooling rapidly is causing errors | understood a missing piece of the theory, why we need a thermo-'couple'
Feb 27	Friday	[Lab Exam] | finishing the record

2.2.2 Iron-Constanon

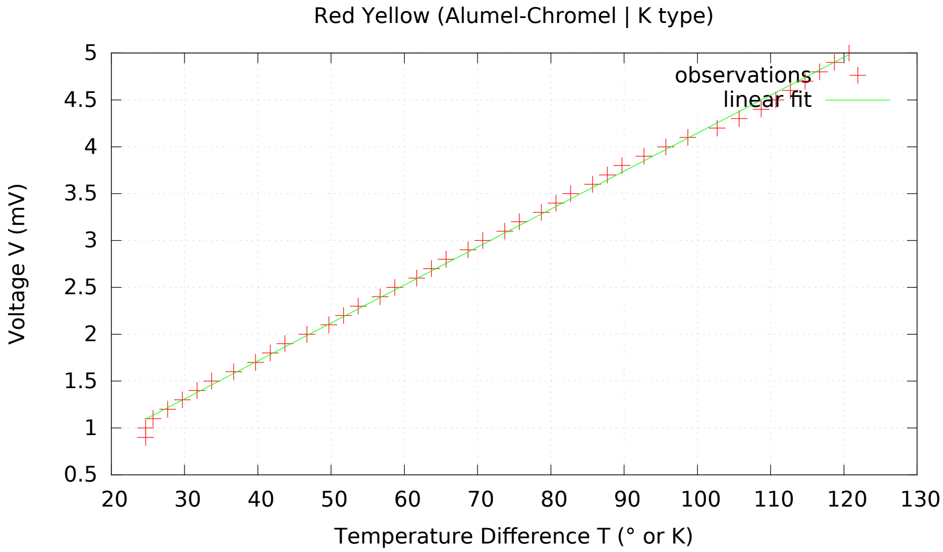


Final set of parameters		Asymptotic Standard Error	
m	= 0.0492904	+/− 0.0003823	(0.7756%)
c	= 0.0330264	+/− 0.02983	(90.33%)

Temp 1	Temp 2	Temp Difference	Voltage
23	−0.4	23.4	0.9
24	−0.4	24.4	1
25	−0.4	25.4	1.1
25	−0.4	25.4	1.2
26	−0.4	26.4	1.3
27	−0.6	27.6	1.4
29	−0.7	29.7	1.5
30	−0.7	30.7	1.6
34	−0.7	34.7	1.8
37	−0.7	37.7	2
38	−0.8	38.8	2.1
41	−0.8	41.8	2.2
43	−0.8	43.8	2.3
45	−0.8	45.8	2.4
47	−0.8	47.8	2.5
49	−0.8	49.8	2.6
51	−0.8	51.8	2.7

54	−0.8	54.8	2.8
56	−0.8	56.8	2.9
58	−0.8	58.8	3
60	−0.8	60.8	3.1
63	−0.8	63.8	3.2
65	−0.8	65.8	3.3
68	−0.8	68.8	3.4
70	−0.8	70.8	3.5
72	−0.8	72.8	3.6
74	−0.8	74.8	3.7
76	−0.8	76.8	3.8
78	−0.8	78.8	3.9
81	−0.8	81.8	4
83	−0.8	83.8	4.1
85	−0.8	85.8	4.2
87	−0.8	87.8	4.3
89	−0.8	89.8	4.4
91	−0.8	91.8	4.5
93	−0.8	93.8	4.6
95	−0.8	95.8	4.7
96	−0.8	96.8	4.8
98	−0.8	98.8	4.9
100	−0.8	100.8	5
102	−0.8	102.8	5.1
104	−0.8	104.8	5.2
106	−0.8	106.8	5.3
108	−0.8	108.8	5.4
110	−0.8	110.8	5.5
112	−0.8	112.8	5.6
114	−0.8	114.8	5.7
116	−0.8	116.8	5.8
118	−0.8	118.8	5.9
120	−0.8	120.8	6
122	−0.8	122.8	6.1

2.2.3 Alumel-Chromel



Final set of parameters		Asymptotic Standard Error	
m	= 0.0405008	+/− 0.0002697	(0.666%)
c	= 0.0952692	+/− 0.02065	(21.68%)

Temp A	Temp B	Temp Diff	Voltage (mV)
24	−0.7	24.7	0.9
24	−0.7	24.7	1
25	−0.7	25.7	1.1
27	−0.7	27.7	1.2
29	−0.7	29.7	1.3
31	−0.7	31.7	1.4
33	−0.7	33.7	1.5
36	−0.7	36.7	1.6
39	−0.7	39.7	1.7
41	−0.7	41.7	1.8
43	−0.7	43.7	1.9
46	−0.7	46.7	2
49	−0.7	49.7	2.1
51	−0.7	51.7	2.2
53	−0.7	53.7	2.3
56	−0.7	56.7	2.4
58	−0.7	58.7	2.5

61	−0.7	61.7	2.6
63	−0.7	63.7	2.7
65	−0.7	65.7	2.8
68	−0.7	68.7	2.9
70	−0.7	70.7	3
73	−0.7	73.7	3.1
75	−0.7	75.7	3.2
78	−0.7	78.7	3.3
80	−0.7	80.7	3.4
82	−0.7	82.7	3.5
85	−0.7	85.7	3.6
87	−0.7	87.7	3.7
89	−0.7	89.7	3.8
92	−0.7	92.7	3.9
95	−0.7	95.7	4
98	−0.7	98.7	4.1
102	−0.7	102.7	4.2
105	−0.7	105.7	4.3
108	−0.7	108.7	4.4
110	−0.7	110.7	4.5
112	−0.7	112.7	4.6
114	−0.7	114.7	4.7
116	−0.7	116.7	4.8
118	−0.7	118.7	4.9
120	−0.7	120.7	5

2.3 Results

It was found that $\Delta V \propto \Delta T$ to a good (1%) precision. For Iron-Constenon, the seeback cofficient was found to be $0.0492904 \pm 0.0003823 (0.7756\%)$ mV/K while for Alumel-Chromel, it was $0.0405008 \pm 0.0002697 (0.666\%)$ mV/K.

3 Critique

The setup was reasonably good and the results obtained were in good agreement with the predictions. The manual didn't discuss the mechanism behind the effect in much detail. No modification seemed necessary. Only issue is that the reasoning given for the setup provided maybe wrong and perhaps a method could be devised to test that more stringently.

4 Acknowledgments

I would like to acknowledge the contribution of my teammates, Prashansa Gupta and Vivek Sagar who played a key role in helping me understand the theory and contributed to performance of the experiment. Like before, Ashcroft and Mermin's book on Solid State Physics was referred to for the corresponding theory.