Olly 10.3

(No 10.1, 2 exams)

Neutrino Mass

$i\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} = \epsilon \,;\ \epsilon^2 = -\mathbb{1}$

$$C = -i\gamma^2\gamma^0 = -i\begin{pmatrix}0 & \sigma^2\\ -\sigma^2 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}-\epsilon & 0\\ 0 & \epsilon\end{pmatrix}$$

$$\psi^c = C\bar{\Psi}^T = -i\gamma^2 \cancel{\gamma^0}\gamma^{0T}\psi^* = \begin{pmatrix}0 & -\epsilon\\ \epsilon & 0\end{pmatrix}\begin{pmatrix}\psi_L\\ \psi_R\end{pmatrix}^*$$

$$\begin{pmatrix}\xi\\ \epsilon\chi^*\end{pmatrix}^c \to \begin{pmatrix}\xi^*\\ \epsilon\chi\end{pmatrix}$$

$$\begin{pmatrix}\xi\\ \epsilon\chi^*\end{pmatrix} = \begin{pmatrix}\cancel{-\epsilon^2}\chi\\ \epsilon\xi^*\end{pmatrix} \qquad \xi = \chi$$

ξ, χ are 2 component left handed spinors transform under $SO(1,3) \simeq SL(2,\mathbb{C})$ as $\xi'_\alpha = M_\alpha{}^\beta \xi_\beta$

$$\xi\chi \equiv \xi^\alpha\chi_\alpha = \epsilon^{\alpha\beta}\xi_\beta\chi_\alpha = -\chi^T\epsilon\,\xi \quad (= \chi\xi)$$

Anti-commuting spinors

$$(\xi\chi)' = -\chi^T M^T\epsilon M\,\xi = -\chi^T\epsilon M^{-1}M\xi$$

$$M^T\epsilon = e^{(i\vec{\omega}-\vec{\beta})\cdot\vec{T}^*}\epsilon = \epsilon\, e^{-(i\vec{\omega}-\vec{\beta})\cdot\vec{T}} = \epsilon M^{-1}$$

$\chi^T\epsilon\, i\cancel{\partial}\epsilon\chi^*$

B.P. $-\chi^T\epsilon\, i\overleftarrow{\cancel{\partial}}\epsilon\chi^* = +\chi^\dagger\epsilon\, i\cancel{\partial}^T\epsilon\chi$

$\dagger\ \epsilon\sigma^{iT}\epsilon$

$i=2$; $\epsilon(-\sigma^2)\epsilon = -\epsilon^2\sigma^2 = \sigma^2$

$\cancel{\partial}^T = \partial_0 + \partial_i\sigma^{iT}$

$\psi_{Dirac} = \begin{pmatrix}\xi\\ \epsilon\chi^*\end{pmatrix}$

$\psi_M = \begin{pmatrix}\xi\\ \epsilon\xi^*\end{pmatrix}$

$\psi_{Weyl;L} = P_L\begin{pmatrix}\psi_u\\ \psi_d\end{pmatrix} = \begin{pmatrix}\psi_u\\ 0\end{pmatrix}$ (ξ)

$$\psi^\dagger\gamma^0\psi = (\xi^\dagger, -\chi^T\epsilon)\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\begin{pmatrix}\xi\\ \epsilon\chi^*\end{pmatrix} = \xi^\dagger\epsilon\chi^* - \chi^T\epsilon\xi = \bar{\xi}^{\dot\alpha}\bar{\chi}_{\dot\alpha} + \chi^\alpha\xi_\alpha$$

$\dagger\ \xi^{*}_{\dot\alpha} = \bar{\xi}_{\dot\alpha}$

$\chi^\beta = \epsilon^{\beta\alpha}\chi_\alpha$

$\chi_\alpha\epsilon^{\alpha\beta}\xi_\beta = -\chi\xi$

$\bar{\xi}^{\dot\alpha}\epsilon_{\dot\alpha\dot\beta}\bar{\chi}^{\dot\beta}$

$\dagger\ (\epsilon^{\alpha\beta})^* = \epsilon_{\dot\alpha\dot\beta}$

$$\mathcal{L}_D = \bar{\Psi}i\cancel{\partial}\Psi - m\bar{\Psi}\Psi = \Psi^\dagger\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}i\begin{pmatrix}0 & \partial\\ \bar{\partial} & 0\end{pmatrix}\Psi - m(\xi\chi + \bar{\xi}\bar{\chi})$$

$$(\xi^\dagger, -\chi^T\epsilon)\, i\begin{pmatrix}\bar{\partial} & 0\\ 0 & \partial\end{pmatrix}\begin{pmatrix}\xi\\ \epsilon\chi^*\end{pmatrix}$$

$$= \xi^\dagger i\bar{\cancel{\partial}}\xi - \chi^T\epsilon\, i\cancel{\partial}\epsilon\chi^*$$

$= \bar{\xi}^{\dot\alpha}(i\bar{\partial})_{\dot\alpha}{}^{\alpha}\xi_\alpha +$ <missing>

Recall γ^μ, σ^μ, $\bar{\sigma}^\mu$

$$\epsilon\begin{Bmatrix}\mathbb{1}\\ \sigma^1\\ \sigma^3\end{Bmatrix}\epsilon = \begin{Bmatrix}-\mathbb{1}\\ +\sigma^1\\ +\sigma^3\end{Bmatrix} \qquad \because \epsilon^2 = -1,\ \{\epsilon, \sigma^{1,3}\} = 0$$

$$\Rightarrow\ + \begin{Bmatrix}+\mathbb{1}\\ -\sigma^1\\ -\sigma^2\\ -\sigma^3\end{Bmatrix}$$

$$\mathcal{L}^{2comp} = \bar{\xi}i\bar{\cancel{\partial}}\xi - \frac{m}{2}(\xi\xi + \bar{\xi}\bar{\xi})$$

Neutrino Mass ≠ 0: Option 1: Dirac Mass: ν_R $(1,1,0)$

$\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}$ $\begin{pmatrix}\phi_+\\ \phi_0\end{pmatrix}$

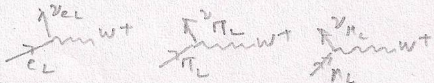
$$\mathcal{L}^{SM} = \underbrace{\bar{\nu}_R i\cancel{\partial}\nu_R + \bar{L}_L i\cancel{\partial}L_L}_{\bar{\nu}i\cancel{\partial}\nu} - \bar{\nu}_{RA}Y^{(\nu)}_{AB}L_{LaB}\epsilon^{ab}\langle\phi_b\rangle + \bar{L}_{La}\nu_R Y^{(\nu)}\epsilon^{ab}\phi_b^*$$

$$\langle\phi_b\rangle = \begin{pmatrix}0\\ \frac{v}{\sqrt{2}}\end{pmatrix}$$

$$m^{Dirac}_{\nu AB} = +\frac{Y^\nu_{AB}v}{\sqrt{2}} = \frac{Y^\nu_{AB}}{\sqrt{2}}(2.46\times10^{11}\,eV) \approx 50\times10^{-3}\,eV$$

$\therefore |Y^\nu| < 10^{-13}$

$\bar{\nu}_L \cancel{w^+} e_L \to \bar{\nu}'_L U^{(\nu)}_L W^+ U^{(e)}_L e'_L$



PMNS

$$\mathcal{L}^{SM} = \bar{\nu}_R i\cancel{\partial}\nu_R + \bar{\nu}_L i\cancel{\partial}\nu_L + \bar{e}_L i\cancel{\partial}e_L - \frac{g_2}{\sqrt{2}}\left((\bar{\nu}_{LA}P_{AB})W^+ l_{LB} + h.c.\right)$$

$\underbrace{P_{AB}W^+ l_{LB}}_{\bar{\nu}'_{LB}}$

$-$ other flavour diag gauge terms

$-(\bar{l}_{LA}m^{(l)}_A l_{eA} + h.c.)$

$-\left((\bar{\nu}'_L P^\dagger)_A m^{(\nu)}_A \nu_{RA} + \underbrace{\bar{\nu}_R\{}_{h.c.}\right)$

$$|\nu'_A, t\rangle = U_{AB}|\nu_B, t\rangle = \sum_B\left(U_{AB}e^{-ip^0_B t}|\nu_B, 0\rangle\right) = U_{AB}(t)|\nu_B, 0\rangle$$

$\dagger\ p^0_A = \sqrt{|\vec{p}|^2 + m_{\nu A}^2}$

Majorana mass: ν_R has a large Majorana mass

$\bar{\nu}_R\nu_L$

$$\mathcal{L} = \mathcal{L}^{(Dirac)}_{SM+\nu_R} - \frac{1}{2}\underbrace{M_{\nu c}}_{Large}(\nu^c_L\nu^c_L + \bar{\nu}^c_L\bar{\nu}^c_L)$$

$$= -\nu^c_{LA}Y^{(\nu)}_{AB}(L^T_B\epsilon\phi)$$

EoM: $-Y^\nu_{AB}L^T_B\epsilon\phi - M_{\nu cA}\nu^c_{LA} \approx 0 \Rightarrow \nu^c_A = \frac{-1}{M_A}Y^\nu_{AB}L^T_B\epsilon\phi +$ Momentum Dependent Terms.

$$+\frac{1}{2}M_{\nu A}\left(+\frac{1}{M_A}Y^\nu_{AB}(L^T_B\epsilon\phi)\right)\left(+\frac{1}{M_A}Y^\nu_{AB'}L^T_{B'}\epsilon\phi\right)$$

$$+\left(\frac{1}{M_A}Y^\nu_{AB}L^T_B\epsilon\phi\right)Y_{AB'}L^T_{B'}\epsilon\phi - \nu^c_A Y^\nu_{AB}(L^T_B\epsilon\phi)$$

$$\to \mathcal{L}_{SM} + \mathcal{L}_{weinberg\ op}$$

$$\mathcal{L}_{weinberg\ op} = \frac{1}{2}M_A^{-1}(Y^\nu_{AB}L_B\phi)_A(Y^\nu_{AB}L_B\phi)_A$$

$$\xrightarrow{\langle\phi\rangle} \frac{1}{2}\nu^T_{LA}\left(Y^{\nu T}\cdot\frac{1}{M}Y_\nu\right)_{AB}\nu_{LB}$$

$$\left(m^{\nu_L}_{maj}\right)_{AB} = -\frac{1}{2}Y^T_{AB'}\frac{1}{M^{\nu c}_{B'}}Y_{B'B}$$

$m^\nu = m^{\nu T} \Rightarrow\ = P^T\Lambda_\nu P$ (P Unitary)

Olly 10.4

Parameters of SM: $g, m_f, \theta, \delta_{CP}, v, \lambda, \boxed{\theta_{strong}}$

$v \to M_{gb}$, $\lambda \to m_H$

Neutrino Oscillations

Scattering cross-sections

Decay rate

Stable

$e^-, (\nu_e, \nu_\mu, \nu_\tau)$

$\tau_p > 10^{34}$ year

$\tau_{universe} \sim 10^{10}$ yr.

$$(J_L + J_H)^2 = J_L^2 + J_H^2 + J_L J_H$$

$$\mathcal{M} = \langle f | (J_L^2 + 2 J_L J_H + J_H^2) | i \rangle$$
$$= \langle lept | J_L^2 | i_{lept} \rangle$$
$$= \langle had | J_{had} | had \rangle$$

$[\sigma] = -2$

EM: $\sigma_{EM} \sim \dfrac{\alpha^2}{E^2}$

$$\sim \frac{\alpha^2}{s}$$

$$\sim \frac{10^{-4}}{(s/GeV^2)} GeV^{-2}$$

$$\sim \frac{10^{-4 -26}}{s/GeV^2} cm^2$$

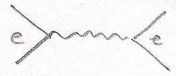
$$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt{2}} \left(J_\mu^+ J^{\mu -} + J_\mu^0 J^{\mu 0} \right)$$

$$J_\mu^- = 2 \sum_{A=1}^{3} \bar{\nu}_{LA} \gamma^\mu l_A + 2 \bar{u}_{LA} V_{AB}^{CKM} \gamma^\mu d_B$$

Discussion of experimental consequence $\leftarrow$ $= J_{\mu\, exp}^- + J_\mu^{hadron/quark}$

$\Rightarrow$ Hadron $\leftrightarrow$ Quark Conversion

1) Expt 2) Numerical Calc 3) Modelling Effective theory



$1\, GeV^{-1} = 1.2 \times 10^{-13}$ cm

$= 4 \times 10^{-24}$ sec

$1 = \hbar = 6.6 \times 10^{-34}$ J sec

$\Gamma = 1$ GeV

$\tau \sim 10^{-24}$ sec

$\Gamma_Z = 2.5$ GeV

$\sigma_W \sim G_F^2 E^2$

$-2 \qquad -4$

$10^{-36} E^2 = \dfrac{10^{-30}}{E^2}$

$E^4 \sim 10^4$

$E \sim 10$ GeV

$[\Gamma] = [\tau^{-1}] = m$

$\tau \sim \dfrac{1}{\alpha m} \sim 10^{-15} - 10^{-20}$ sec

$\tau_\mu \approx 2 \times 10^{-6}$ sec

$m_\mu = 0.1$ GeV

$\sigma_{weak} = G_F^2 \left(\dfrac{E}{1\, GeV} \right)^2 GeV^2$

$G_F \sim 1.16 \times 10^{-5}\, GeV^{-2}$

$\sigma_W \sim 10^{-10} (GeV^{-2}) \left(\dfrac{E}{GeV} \right)^2$

$\downarrow$

10^{-26}

$\sim 10^{-36}\, cm^2 \left(\dfrac{E^2}{GeV^2} \right)$

$\pi^0 \to 2\gamma$

$\tau_{\pi^0} \sim 10^{-17}$ sec

μ^- (105 MeV) $\longrightarrow e^- + \gamma + \bar{\nu}_e + \nu_\mu$

$\longrightarrow e^- + \bar{\nu}_e + \nu_\mu$ ($\approx 100\%$)

$\dfrac{1}{\tau} \sim \Gamma_W \sim G_F^2 m^5$

$+1 \qquad -4$ weak

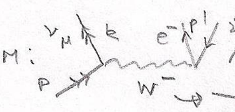
$\Gamma = 10^{-10} GeV^{-4+5} \left(\dfrac{m}{GeV} \right)^5$

$\tau \sim \dfrac{10^{10}\, GeV^{-1}}{(m/GeV)^4}$ (2000)

$10^{13}\ 10^{-24}$

10^{-11} sec

SM:



$W^- \to \dfrac{1}{q^2 - m_W^2} \approx \dfrac{-1}{m_W^2}$

Eff SM:

$\sim \left(\dfrac{G_F}{\sqrt{2}} \right)$

$$f_i = (2\pi)^4 i \delta^4 (\Sigma p_f - \Sigma p_i) \frac{\mathcal{M}_{fi}}{\sqrt{2 E_i V}}$$

$$\Pi \frac{1}{2 E_f V}$$

Decay rate:

$$\Gamma = \frac{1}{2E_i} \int_{\Omega\, phase} \prod_f \frac{d^3 \vec{p}_f}{(2\pi)^3 2E_f} (2\pi)^4 \delta^4 (p_{in} - \Sigma p_f) |i \mathcal{M}_{fi}|^2$$

$$\frac{1}{2} \Sigma |i \mathcal{M}_{fi}|^2 = \frac{1}{2} \sum_{spin} \left| -\frac{i G_F}{\sqrt{2}} \bar{u}_{e_\lambda}(p') \cdot \gamma^\lambda (1-\gamma_5) v_{\nu_{e s'}}(k') \cdot \bar{u}_{\nu_{\mu s}}(k) \cdot \gamma_\lambda (1-\gamma_5) u_{\mu_s}(p) \right|^2$$

$$\overline{|\mathcal{M}_{fi}|^2} = \frac{G_F^2}{4} \mathrm{tr} \left[(\not{p}' + m_e) \gamma^\lambda (1-\gamma_5) \not{k}' \gamma_\rho (1 - \gamma_5) \right] \mathrm{tr} \left[\not{k} \gamma_\lambda (1-\gamma_5) (\not{p} + m_\mu) \gamma^\rho (1 - \gamma_5) \right]$$

$$\Gamma_{tot} = \frac{G_F^2}{\pi^4 m_\mu} \int \frac{d^3 p'}{2 p'_0} \frac{d^3 k}{2 k_0} \frac{d^3 k'}{2 k'_0} \delta^4 (p_i - \sum_f p_f) \; ; \quad p \cdot k' \quad k \cdot p'$$

$$I = \int d^4 p' \, \delta(p'^2 - m_e^2) \, p^\lambda p'^\rho \, I_{\lambda\rho}(p - p') \; ; \quad I_{\lambda\rho}(q) = \int d^4 k \, \delta(k^2) \, d^4 k' \, \delta(k'^2) \, \delta^4 (q - k - k') k'_\lambda k_\rho$$

$$p - p' = k + k'$$

$$= A q^2 \eta_{\lambda\rho} + B q_\lambda q_\rho \leftarrow \eta^{\lambda\rho}, \; q^\lambda q^\rho$$

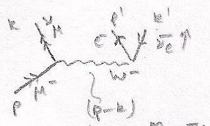
Olly 11.1

$\tau^- \to \nu_\tau + \begin{cases} e^- + \bar\nu_e \\ \mu^- + \bar\nu_\mu \end{cases}$

$\mu^- \to \nu_\mu + e^- + \bar\nu_\mu$

105.6 MeV

$m_{\pi\pm,0} \sim 135$ MeV



$-\frac{i g_{\mu\nu}}{m_W^2}$; $m_W \approx 80000$ MeV

$\mathcal{L}_{eff} = -\frac{G_F}{\sqrt2}\left(\bar\nu_\mu \gamma^\lambda(1-\gamma_5)\mu\ \bar e_L \gamma_\lambda (1-\gamma_5)\nu_e + \dots\right)$

$\frac{1}{2}\sum_{spins}|M_{fi}|^2 \Rightarrow \Gamma(\mu^- \to e^- + \bar\nu_e + \nu_\mu) = \frac{G_F^2}{8\pi^5 m_\mu} I$

$I_{\lambda\rho}(q) = \int d^4k\, \delta(k^2)\, d^4k'\, \delta(k'^2)\, \delta^4(q-k-k')\, k_\lambda k'_\rho$

<1 step missing>

$\begin{pmatrix} 4Aq^2 + Bq^2 \\ 4(q^2)^2 + B(q^2)^2 \end{pmatrix} = \int \frac{d^3k}{2k_0}\frac{d^3k'}{2k_0'}\delta^4(q-k-k') \cdot \begin{pmatrix} k\cdot k' \\ q\cdot k\ q\cdot k' \end{pmatrix}$

$q = p - p' = k + k'$

Frame: $q\cdot k = k^2 + k\cdot k' = q\cdot k'$

$\overset{\shortparallel}{0} \quad \Rightarrow k_0 = k_0' = |\vec k|$

Frame: $\vec k + \vec k' = 0$

$\int \frac{d^3k'}{2k_0'}\delta^3(\vec q - \vec k - \vec k')(k, k')^n = (2k_0^2)^n / 2k_0$

$\Gamma = \frac{G_F^2}{2^4\pi^4 m_\mu}\int\frac{d^3p'}{2p_0'}(q^2 p\cdot p' + 2 q\cdot p\ q\cdot p')$

$p = (m_\mu, 0) \leftarrow$ rest frame

$p' = (E_e, \vec p')$

$p\cdot p' = m_\mu E_e$

$q^2 = m_\mu^2 + m_e^2 - 2m_\mu E_e$

Isotropic

$= \int dE_e \frac{d\Gamma}{dE_e}$ ✓ differential decay rate

$\frac{1}{\Gamma}\frac{d\Gamma}{dE_e} dE_e =$ Prob of observing e^- in range $(E_e, E_e + dE_e)$

$(105)^2\ (.5)^2$

$q\cdot p = m_\mu^2 - m_\mu E_e$

$\Gamma = \frac{G_F^2}{2^4\pi^4}\int \frac{d^3\vec p'}{2E_e}\left((m_\mu^2 + \cancel{m_e^2} - 2m_\mu E_e)\frac{m_\mu E_e}{m_\mu} + 2(m_\mu - E_e)(m_\mu E_e - m_\mu^2)\right)$

$= \frac{G_F^2}{12\pi^3}\int_{E_{min}}^{E_{max}} dE \sqrt{E^2 - m_e^2}\{m_\mu E(3m_\mu - 4E_e)\}$

$E = |\vec p'| = |\vec k + \vec k'| = \sqrt{\vec k^2 + \vec k'^2 + 2\vec k\cdot\vec k'}$

Max for colinear

$E_{max} = k_0 + k_0'$

$\Gamma = \frac{G_F^2 m_F^5}{192\pi^3}$

$= \left[1 - \frac{\alpha}{2\pi}\left(\pi^2 - \frac{25}{4}\right) - \frac{8m_e^2}{m_\mu^2} + \frac{3}{5}\frac{m_\mu^2}{m_W^2} + \dots\right]$

$= [1 + (-4203.85 - 187.12 + 1.05) \times 10^{-4} + \dots]$

$G_F^{(\mu)} = \frac{1}{2v^2} = 1.1635 \times 10^{-10}$ GeV^{-2}

$v = 246.1$ GeV

Muon

$\mu^- \to \nu_\mu e^- \bar\nu_e \simeq 100\%$

(include $e^- \bar\nu_e \nu_\mu \oplus \gamma$, $E_\gamma < 10$ MeV)

$\nu_\mu e^- \bar\nu_e \gamma$ ($\gamma > 10$ MeV)

Branching Ratio $(1.4 \pm 0.4\%)$

$e^- \bar\nu_e \nu_\mu e^+ e^- \ (3.4 \pm 0.4) \times 10^{-5}$

$m_\tau = 1776.82 \pm 0.16$ MeV

$\Gamma_\tau = (2.906 \pm 0.01) \times 10^{-13}$ sec

$\left(\frac{m_\mu}{m_\tau}\right)^2 = \left(\frac{0.105}{10.776}\right)^5 \approx 7 \times 10^{-7}$ | gives 1.4×10^{-12} sec lifetime

$\tau^- \to e^- \bar\nu_e \nu_\tau$ 17.9%

$\to \mu^- \bar\nu_\mu \nu_\tau$ 17.4%

$\to \pi^- \nu_\tau$ 10.8% $\to J^\mu_{had}(\bar\nu_\tau \gamma_\mu (1-\gamma_5)\tau)$

$(\bar u d)$

$\to \pi^- \pi^0 \nu_\tau$ 25-52% $\quad \langle \pi^- | \bar d \gamma^\mu (1-\gamma_5) u | 0 \rangle$

$2\pi^0$ 9.3% $\quad J_\mu \sim \langle \phi^+ \rangle \partial^\mu \phi_+ \dots$

$3\pi^0$ 1.05% $\quad f_\pi(\partial\pi)$

$\pi^- \bar K^0 \nu_\tau$ 1% $\quad$ 93.41 MeV

$e^- \bar\nu_e \nu_\tau \gamma$

$\tau^- \to \mu^- \gamma \\ e^-\gamma \} 10^{-8}$

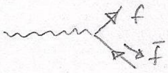
$\mu^- \to e^-\gamma$ Branching Ratio $< 2.4 \times 10^{-12}$

$e^- e^+ e^-$ $< 10^{-12}$

$e\, 2\gamma$ $< 7.2 \times 10^{-12}$

$\mathcal{L} = -g_Z \bar f T_Z \gamma^\mu f Z_\mu$

$\to (T_{3L} - Q_F \sin^2\theta_W)$



$m_Z = 91.3876$ GeV

$\Gamma_Z = 2.4952 \pm 0.0026$ GeV

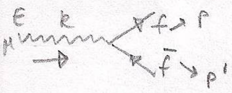
Olly 11.2

Gauge Boson Decay

$$\mathcal{L}_{int} = -\frac{g_Z}{2}\bar{f}\gamma^\mu(a_f - b_f\gamma_5) f Z_\mu + \dots$$

$a_f = \frac{g_V}{2}$, $b_f = \frac{g_A}{2} = T_{3L}$

$t_{3L} - Q_{em}\sin^2\theta_W$



$$\mathrm{Tr}\, \not{p}\gamma^\mu \not{p}'\gamma^\nu\gamma_5 = -4i p_\lambda p'_\sigma \epsilon^{\mu\nu\lambda\sigma} = 0$$

$k^\mu = (M_Z, 0)$

$p^\mu = \left(\frac{M_Z}{2}, \vec{p}\right)$

$p'^\mu = \left(\frac{M_Z}{2}, -\vec{p}\right)$

f	a_f	b_f	$a_f^2 + b_f^2$	T_Z
ν	$1/2$	$1/2$	$1/2$	✓ ×3
l	$-1/2 + 2s_W^2$	$1/2$	$1/4$	84.01 ×3
u	$1/2 - \frac{4}{3}s_W^2$	$1/2$	.29	300.3 ×2
d	$-1/2 + \frac{2}{3}s_W^2$	$-1/2$	.37	383.1 ×3

$$\langle|i\mathcal{M}_{fi}|^2\rangle = \langle| -\frac{ig_Z}{2}\bar{u}(p)\gamma^\mu(a_f - b_f\gamma_5)v(p')\epsilon_\mu(k)|^2\rangle$$

avg Z polarization
sum $f\bar{f}$ spins

$$= \frac{g_Z^2}{4}\langle\epsilon_\mu(k)\epsilon_\nu^*(k)\rangle \cdot \mathrm{Tr}\left[(\not{p} + m_f)\gamma^\mu(a_f - b_f\gamma_5)(\not{p}' + m_f)\gamma^\nu(a_f - b_f\gamma_5)\right]$$

$$\langle\epsilon_\mu(k)\epsilon_\nu^*(k)\rangle = \frac{1}{3}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}\right)$$

$$\left(\because \frac{1}{3}\sum_\lambda \epsilon_\mu^\lambda(k)\epsilon_\nu^{\lambda *}(k)\right)$$

$$= \frac{g_Z^2}{3}(a_f^2 + b_f^2)\left(p\cdot p' + 2\frac{k\cdot p\, k\cdot p'}{M_Z^2}\right)$$

$$\Gamma(Z\to f\bar{f}) = \frac{g_Z^2 M_Z}{48\pi}(a_f^2 + b_f^2)$$

$\Gamma_Z = 2495.2 \pm 2.6$ MeV

$\Gamma(Z \to \text{invisible}) = 499.4 \pm 1.7$ MeV

no EM charge

$n_\nu^{light} = 2.984 \pm 0.01$

$$\frac{G_F M_Z^3}{6\sqrt{2}\pi}\left[g_V^2 + g_A^2\right]$$

$g_V^2 = 2a_f^2$, $g_A^2 = 2b_f^2$

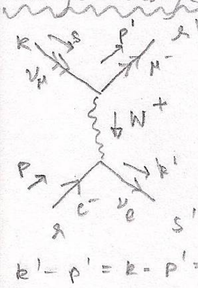
1 lepton

Ex: $W^+ \to e + \nu_e$

$$\text{Quarks} = 3\left(1 + \frac{\alpha_s}{\pi} + 1.409\frac{\alpha_s^2}{\pi} + \dots\right)$$

\# colors

Charge Exchange Scattering



$\nu_\mu + e^- \to \nu_e + \mu^-$

$$i\mathcal{M}_{fi} = \bar{u}(p')\left(-\frac{g}{\sqrt{2}}i\gamma^\lambda P_L\right)u(p)\cdot i\Delta^W_{\lambda\sigma}(q)\cdot \bar{u}(k')\left(-\frac{g}{\sqrt{2}}i\gamma^\sigma P_L\right)u(p)$$

unitarity

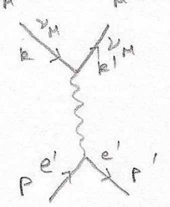
$$\frac{-i}{q^2 - m_W^2 + i\epsilon}\left[\eta_{\lambda\sigma} - \frac{q_\lambda q_\sigma}{m_W^2}\right] \to 0$$

$$= \frac{i\eta_{\lambda\sigma}}{m_W^2}$$

$k' - p' = k - p' = q$

$p + k - p' = k'$

$\nu_\mu e^- \to \nu_\mu e^-$



$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}|\bar{\mathcal{M}}|^2\left\{\frac{(s-m_\mu^2)^2}{(s-m_e^2)^2}\right\}^{1/2} \underset{m_e=0}{=} \frac{G_F^2}{4\pi^2 s}(s - m_\mu^2)^2$$

$$\sigma_{tot} = \frac{G_F^2}{\pi}\frac{(s-m_\mu^2)^2}{s} \xrightarrow{s \gg m_\mu} \frac{G_F^2 s}{\pi} \sim 10^{-10}\,\mathrm{GeV}^{-2}\left(\frac{s}{\mathrm{GeV}^2}\right)$$

$$\sim 10^{-36}\,\mathrm{cm}^2\left(\frac{s}{\mathrm{GeV}^2}\right)$$

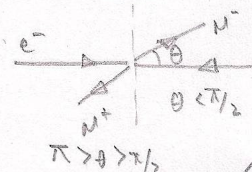
$$\ll \sigma_{hadron} \approx 10^{-23}\,\mathrm{cm}^2$$

$k_\nu^{0(1)} = \omega^{(1)}$

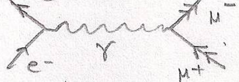
$$\sigma_{tot} = \frac{G_F^2 m_e \omega}{2\pi}\left[(1 - 2s_W^2)^2 + \frac{1}{3}4s_W^4\right]$$

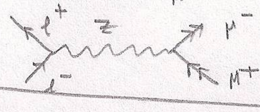
$$|i\mathcal{M}_{fi}|^2 = 16G_F^2\left\{(-1 + 2s_W^2)^2(k\cdot p)^2 + 4s_W^2(k'\cdot p)^2 + 2s_W^2(1 - 2s_W^2)m_e^2 k\cdot k'\right\}$$

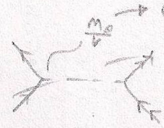
Forward-Backward Assymetry



$$A = \frac{\int_0^{\pi/2} d\theta \sin\theta \frac{d\sigma}{d\Omega} - \int_{\pi/2}^{\pi} \text{''}}{\text{''} + \text{''}}$$







$\frac{m_e}{v} \to \frac{0.005}{256\,\mathrm{GeV}} \to 0$

$$A_{FB} = \frac{3a_2}{8\cdot 1 + a_1} \quad ; \quad a_2 = 4f_Z^2 g_A^2 + 8f_Z^2 g_V^2 g_A^2$$

$$= 2f_Z g_A^2 + f_Z^2(g_V^2 + g_A^2) \quad ; \quad f_Z = \frac{s}{(\sin 2\theta_W)(s - M_Z^2)} \xrightarrow{s = \left(\frac{M_Z}{2}\right)^2} -.4$$

Unstable Particles

$\tau_{proper} = \frac{t}{\gamma}$; $E = m\gamma$

Heuristic: Rate $= \Gamma = \frac{1}{\tau}$

$|\phi|^2 \sim e^{-t_{proper}\Gamma} = e^{-\Gamma\frac{mt}{E}}$

Unstable particle, $\phi \sim e^{-\frac{m\Gamma}{2E}t} e^{ip\cdot x}$ of 4-mom p^μ $\quad (Et - \vec{p}\cdot\vec{x})$

$(\partial_t^2 - \vec{\nabla}^2) e^{-i(Et - \vec{p}\cdot\vec{x}) - \frac{m\Gamma t}{2E}}$

$= (-E^2 + \vec{p}^2) e^{-(\ \ )} = -m^2 e^{(\ \ )}$

$-(E - i\frac{m\Gamma}{2E})^2 + \vec{p}^2 \quad \Gamma \ll m$

$-(E^2 - i m\Gamma + \mathcal{O}(\Gamma^2)) + \vec{p}^2$

From $\mathcal{L}$ you get $\phi(\Box^2 - m^2)\phi = 0$ & the propagator is inverse of the kinetic op.

<copy from prashansa>

$i\Delta = \frac{i}{p^2 - m^2 + i\epsilon} \longrightarrow \frac{i}{p^2 - m^2 + im\Gamma}$

Qly 11.4 (No 11.3)

Optical Theorem

$S = T\{e^{i\int d^4x \mathcal{L}_{int}}\} = \mathbb{1} + iT$

$S^\dagger S = \mathbb{1} = \mathbb{1} + i(T - T^\dagger) + T^\dagger T$

$(\mathbb{1} - iT^\dagger)(\mathbb{1} + iT)$

$(\mathbb{1} + iT)(\mathbb{1} - iT)$

$\Rightarrow i\underbrace{(T - T^\dagger)}_{2i\,\mathrm{Im}(T)} = -T^\dagger T$

$\Rightarrow i(T^\dagger - T) = -TT^\dagger$

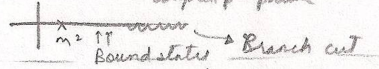
$(i\mathcal{M}_F)(2\pi)^4\delta^{(4)}$ momentum conservation

it corresponds to a branch cut.

With scalars, bound states of smallest mass are 2 scalars, & with EM, you can have e^- & γ bound state. In EM it can be shown that this "branch cut" indeed occurs at $(m_e + m_\gamma)^2$ electron, photon

$i\Delta(p) = \int e^{ip\cdot x}\langle\Omega| T\{\phi(x)\phi(0)\}|\Omega\rangle$

complex p^2 plane



$\langle a| i(T - T^\dagger)|b\rangle = -\sum_I \langle a|T|I\rangle\langle I|T^\dagger|b\rangle$

$\{i\} = \{\vec{p}_1, \ldots \vec{p}_{n_i}\}$

$d\Pi = \prod_{j=1} \frac{d^3p_j}{(2\pi)^3 2E_j}$

$= -\sum_{\{I\}}\int d\Pi_I \langle a|T|I\rangle\langle I|T^\dagger|b\rangle$

$\langle a|T|I\rangle = (2\pi)^4 M_{I\to a}\delta^{(4)}(P_I - P_a)$; $\langle I|T^\dagger|b\rangle = (2\pi)^4 M^*_{b\to I}\delta^{(4)}(P_b - P_I)$

$= (2\pi)^4 M_{b\to I} M^*_{a\to I}\delta^{(4)}(P_a - P_b)\delta^{(4)}(P_I - P_a)$

$\Rightarrow i\langle a|\mathcal{M} - \mathcal{M}^\dagger|b\rangle = -\sum_I \int d\Pi_I \mathcal{M}_{b\to I}\mathcal{M}^*_{a\to I}(2\pi)^4\delta^4(P_b - P_I)$

$b = a$

Cutkosky Rule

$\Leftarrow$ $2i\,\mathrm{Im}(\mathcal{M}_{a\to a}) = i(\langle a|\mathcal{M}|a\rangle - \langle a|\mathcal{M}|a\rangle^*) = -\sum_I \int d\Pi_I |\mathcal{M}(a\to I)|^2 (2\pi)^4\delta^{(4)}(P_a - P_I)$

$i2i\,\mathrm{Im}\,\mathcal{M}_{aa} = 2E_{cm}P_{cm}\sigma_{tot}$

normalization $2E_aE_I|v_a - v_I|\sigma_{tot}$

Full propagator

 $= \frac{i}{p^2 - m_0^2 - M^2(p^2)}$

$\mathcal{M}^2(p^2)$

LSZ

$\int e^{ipx}\langle\Omega|T(\phi(x)\phi(0))|\Omega\rangle = (i\sqrt{Z})^2$(Amputated S-matrix Element)

$\mathcal{M}_F(p\to p) = -Z M^2(p^2)$

(a) Stable Particle: $\mathrm{Im}\,\mathcal{M}(p\to p) = \sum_f \int d\pi |M(p\to f)|^2$ $\quad 0 \because$ stable

$p^2 - m_0^2 - \mathrm{Re}(M^2(m^2)) - \underbrace{\mathrm{Im}(M^2(m^2))}_{\text{as known}} = 0$

b) Unstable $\quad p^2 - m_0^2 - \mathrm{Re}(M^2(m^2)) - \mathrm{Im}(M^2(m^2)) = 0$

 $= \frac{i}{p^2 - m_0^2 - M^2(p^2)} = \frac{iZ}{p^2 - m^2 - iZ\,\mathrm{Im}\,M^2(p^2)}$

If $\mathrm{Im}\,M^2(m^2)$ is small then near the pole $p^2 = m^2$, we can approx.

$\Gamma = -\frac{Z}{m}\mathrm{Im}\,M^2(m^2)$

$\sigma \sim \left|\frac{1}{s - m^2 + im\Gamma}\right|^2 \sim \frac{1}{(s - m^2)^2 + m^2\Gamma^2}$