Problem Set

Astronomy & Astrophysics (IDC 201)

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1. Write down and solve Friedmann equations to find the scale factor as a function of time for the following cases:
   (a) $k = 0, \Omega_{nr} = 1$.
   (b) $k = 0, \Omega_{\gamma} = 1$.
   (c) $k = 0, \Omega_{\Lambda} = 1$.
   (d) $k = 0, \Omega_{nr} + \Omega_{\Lambda} = 1$.
   (e) $k = 1, \Omega_{\gamma} = 0 = \Omega_{\Lambda}$.
   (f) $k = -1, \Omega_{\gamma} = 0 = \Omega_{\Lambda}$.
2. Find the comoving distance r as a function of redshift z for Einstein-deSitter universe ($k = 0, \Omega_{nr} = 1$).
3. A face on disk galaxy is located at a distance of 1 Mpc from us. You can assume that the stars are distributed uniformly in a disk of radius 20 kpc in this galaxy and that there are 10^{11} stars in the disk of this galaxy. We can observe the galaxy with an angular resolution of 0.1". What is the average number of stars in each pixel of an image if each pixel has the same size as the resolution? Repeat the calculation for a galaxy located at a distance of 10 Mpc. [Info: Distance determination requires us to measure flux received from bright stars, but if there are more than one star in the resolution element then there will clearly be an error in our estimate.]
4. Compute the critical density of the universe $\rho_c = 3H_0^2/8\pi G$ where $H_0 = 70$ km/s/Mpc.
5. CMBR temperature varies as $T \propto (1+z)$ where z is the redshift. Find out the functional form of temperature on the sky if we are moving through space with a speed of 600 km/s.
6. Find out the energy density in CMBR if the radiation has a temperature of 2.726 K. Assume a black body spectrum for the radiation. Use the answer to find out density parameter Ω_{cmbr}. Also find out the number density of photons in CMBR.
7. Density parameter of Baryons is $\Omega_B = 0.045$. If the universe is made up of Hydrogen (76% by mass) and Helium (24% by mass) then find out the average number density of electrons. Compute the electron to photon ratio for the universe.
8. If the cosmological constant has density parameter $\Omega_{\Lambda} = 0.74$ and non-relativistic matter contributes $\Omega_{nr} = 0.26$ then find out the red shift at which both contribute equally to energy density of the universe.

9. Use the density parameter for CMBR and for non-relativistic matter to find out the redshift at which we expect matter-radiation equality? How does this change if we include the three species of Neutrinos with a Fermi-Dirac distribution at a temperature of 1.9 K with radiation? You may assume neutrinos to be massless for this calculation.

10. Find out the redshift where typical CMBR photos can:

    (a) Ionize Hydrogen atoms.

    (b) Ionize Helium atoms.

    (c) Lead to generation of electron-positron pairs.

    (d) Lead to destruction of a Deuterium nucleus.