Angle of Repose

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1 Theory

Angle of repose is defined in the following situation. Consider physical entities which are small in size and large in number, such that when dropped from a given location, form a heap. The conical heap so formed can be characterized by the angle the inclined face of the cone makes with the horizontal. This angle is defined to be the angle of repose.

The angle of repose experimentally, is known to lie primarily between $30^o < \theta < 45^o$. This angle is also known to depend experimentally on

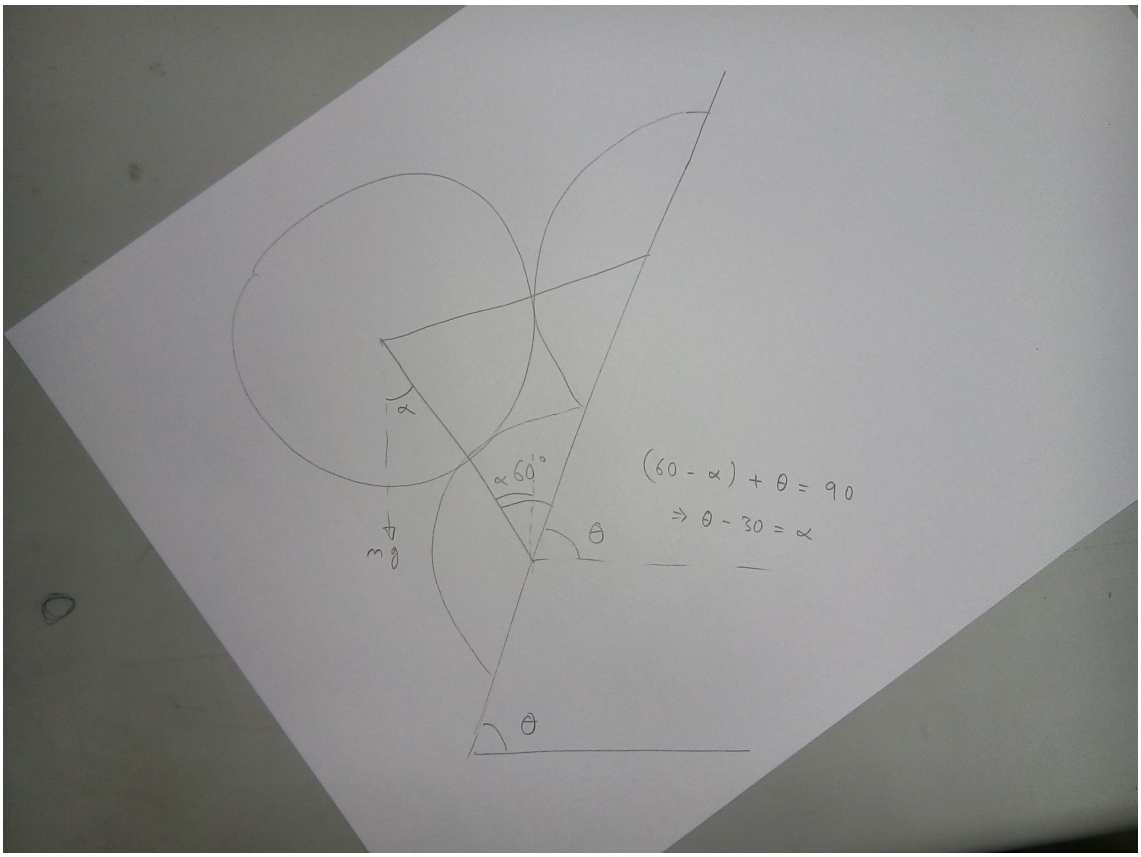
1. Coefficient of friction
2. Shape of the grain
3. Density
4. Size of the grain

We next describe two cases in which one can make simplifying assumptions and arrive at the 30^o and 45^o repose angle.

1.1 Hard Spheres

We assume that

- the particles are spherical and density of mass within the sphere is uniform
- typically, the edge layer is formed as shown in the diagram



One can see clearly from the diagram that if the angle $\alpha > 0$, the centre of mass of the sphere is on the left of the farthest contact point, thus the object at this angle will fall off the pile. Plugging this condition yields $\theta > 30^o$ will result in a slip. Thus we predict that the particles will keep contributing to the angle of repose until it hits $\sim 30^o$ after which the newly added particles will fall off.

1.2 Box sliding

We assume that

- the edge layer can be approximated effectively by an almost flat sheet
- the particles can be approximated by box

In this situation, the assumptions allow us to find the angle of repose as the condition for a box to just start sliding down an incline, which is well known to be $\tan\theta = \mu$, where μ is the coefficient of friction. Since typically is $\mu < 1$, we have typically $\theta < 45^o$.

1.3 Threshold

Dr. Yogesh suggested that in accordance with a paper titled "A sandpile experiment and its implications for self-organized criticality and characteristic earthquake" by Naoto Yoshioka, when $r/R \sim 0.02$,

a threshold is reached. Above and below this value, the angle of repose is expected to change. The motiviation slash justification is outside the scope of this report.

2 Experimental Setup

The following were used to perform the experiment.

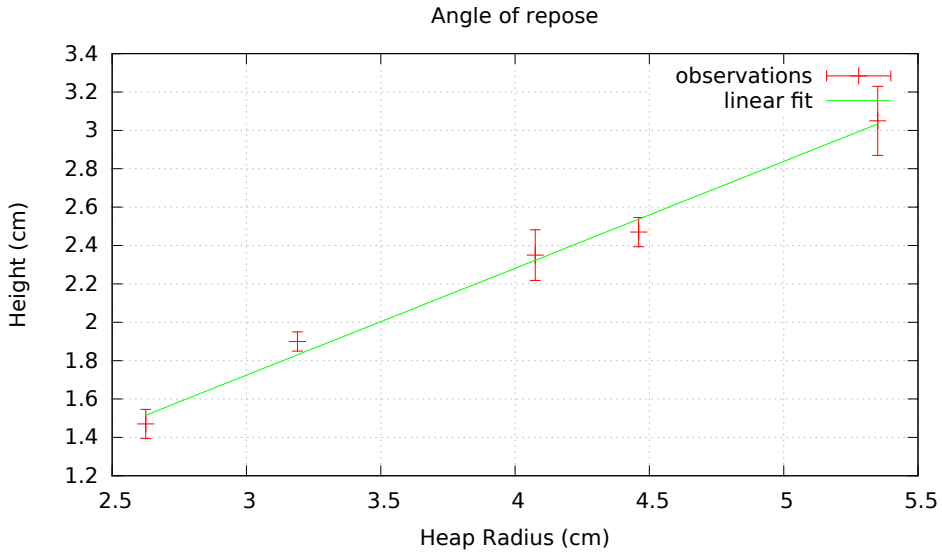
- Rai (small spherical objects (1.5 < diameter (mm) < 2.0) [corresponding critical radius of heap (in cm) $7.5 < R < 10$]
- Funnel (for dropping rai uniformly)
- Microscope (for measuring the height of the heap)
- Discs (of various sizes to make the heap of various radius)

The procedure followed was almost obvious. The particles were dropped using the funnel on the discs and the height of the resulting heap was measured. Ofcourse, it is easier said than done, but there's nothing illuminating in the process.

We took one material and repeated the experiment 3 times for each disc, and used 5 different discs.

3 Observations

Init (cm)	Final (cm)	Height (cm)	Heap		Rai diameter (mm)	
1.1	3.5	2.4	**Diameter (cm)**	8.92	Sample 1	1.6
1	3.45	2.45			Sample 2	1.7
1.1	3.65	2.55			Sample 3	2
					Sample 4	1.8
1.7	3.65	1.95	**Diameter (cm)**	6.38		
1.65	3.5	1.85				
1.65	3.55	1.9				
1.5	3.7	2.2	**Diameter (cm)**	8.15		
1.55	4	2.45				
1.55	3.95	2.4				
1.45	3	1.55	**Diameter (cm)**	5.25		
1.4	2.85	1.45				
1.4	2.8	1.4				
0	2.85	2.85	**Diameter (cm)**	10.7		
0.3	3.4	3.1				
0.25	3.45	3.2				



Final set of parameters		Asymptotic Standard Error	
m	= 0.556885	+/- 0.03021	(5.425%)
c	= 0.0538742	+/- 0.1225	(227.4%)

theta(radians)	= 0.50896	+0.02273, -0.02332
theta(degrees)	= 29.16	+1.302, -1.336

3.1 Time-line

March 30	Monday	Trying to arrive at a theoretical model | made some progress with idealized assumptions
March 31	Tuesday	Took 3 readings each with 4 different diameters for a given grain (rai)
April 4	Friday	[officially off]
April 6	Monday	Took readings with one more diameter (3 readings) and figured that the slope doesn't in fact change as was predicted by the threshold | writing the record
April 7	Tuesday	Completed writing the record and analyzing the data

4 Result

It was found that for a given material, the angle of repose was $29.1^o \pm 1.33$ independent of the radius of the heap. Explicitly, change in repose angle which was expected at $r/R \sim 0.02$ was *not* observed, where r is the radius of the particle and R that of the heap.

5 Critique

The experiment didn't have a well established theory backing it therefore it was hard to find exactly what it is that one must verify.