## Dirac and Majorana Mass

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Dirac and Majorana Mass

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quantum theory of fields



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# Introduction

## Motivation | Mass

► Inertia | Newton

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- ► Inertia | Newton
- Special Relativity |  $p^2 = m^2$

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Towards a quantum theory of fields

- ► Inertia | Newton
- Special Relativity |  $p^2 = m^2$
- Particle Physics
  - QED (Quantum Electrodynamics)

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- ▶ Inertia | Newton
- Special Relativity |  $p^2 = m^2$
- ► Particle Physics
  - ▶ QED (Quantum Electrodynamics)
    - ightharpoonup Classical Electrodynamics gauge field:  $A^{\mu}$

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- Special Relativity |  $p^2 = m^2$
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    - $\triangleright$  Classical Electrodynamics gauge field:  $A^{\mu}$
    - Quantize | Massless Gauge Field

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    - Quantize | Massless Gauge Field
  - Standard Model
    - Massive Gauge fields | Spontaneous Symmetry Breaking

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  - Beyond Standard Model
    - Neutrino Oscillations

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- ► From CM, recall
  - ► Lagrangian Formalism

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- From CM, recall
  - ► Lagrangian Formalism
  - ► Euler Lagrange equations

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  - ► Noether's Theorem relating conserved quantities and continuous symmetries

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  - $\eta^{\mu\nu} = \text{diag}(1, -\vec{1}) \; (\text{NB: } \eta^T = \eta^{-1} = \eta)$

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• Summation Convention  $A^{\alpha}B_{\alpha}=\sum_{\alpha=0}^{4}A^{\alpha}B_{\alpha}$ 

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- I'll need the 4 vector notation. Recall
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  - State:  $|\psi\rangle$  (or  $\psi(x) = \langle x|\psi\rangle$ )

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$$H\ket{\psi}=-i\hbarrac{\partial}{\partial t}\ket{\psi}$$

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$$|\psi(t)
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NB:  $U \equiv e^{(-i\hbar)^{-1}Ht}$  is unitary, viz.  $U^{\dagger} = U^{-1}$ 

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- Measurement/Observables
  - Collapse into eigenstate of operator corresponding to the measurement

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- Measurement/Observables
  - Collapse into eigenstate of operator corresponding to the measurement
  - ightharpoonup Collapse to state  $|n\rangle$  with probability  $|\langle n|\psi\rangle|^2$
- ▶ Basics of quantum harmonic oscillator using a  $a^{\dagger}$

I'll use the following pauli matrices

$$\sigma^1 = \left(egin{array}{cc} 0 & 1 \ 1 & 0 \end{array}
ight)$$

$$\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma^3 = egin{pmatrix} 1 & 0 \ 0 & -1 \end{pmatrix}$$

- ► Terminology from particle physics
  - ▶ Leptons: Eg. Electron, Electron Neutrino
  - Quarks

# Towards a quantum theory of fields

Dirac and Majorana Mass

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All electrons are identical

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- All electrons are identical
- ▶ Unification of QM and STR

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- All electrons are identical.
- ▶ Unification of QM and STR
- Crisis: Can't predict the result of collision of particles

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- All electrons are identical
- Unification of QM and STR
- ► Crisis: Can't predict the result of collision of particles

Targets of the new theory

Creation and destruction

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- All electrons are identical
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Targets of the new theory

- Creation and destruction
- Consistent with STR (high energy)

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- All electrons are identical
- Unification of QM and STR
- Crisis: Can't predict the result of collision of particles

Targets of the new theory

- Creation and destruction
- ► Consistent with STR (high energy)
- Predict probablities

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 $(E^2 - \vec{p}^2) \psi = m^2 \psi$  and put  $E \to -i \frac{\partial}{\partial t}, \vec{p} \to i \vec{\nabla}$  to get

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$$\begin{array}{l} \bullet \ \, \left(E^2-\vec{p}^2\right)\psi=m^2\psi \\ \ \, \text{and put } E\to -i\frac{\partial}{\partial t},\vec{p}\to i\vec{\nabla} \ \, \text{to get} \\ \\ \ \, \left(\frac{\partial}{\partial t}^2-\vec{\nabla}^2\right)\!\psi=m^2\psi \end{array}$$

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 $lackbox ig(E^2-ec p^2ig)\psi=m^2\psi$  and put  $E o -irac{\partial}{\partial t},ec p o iec
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Causality

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 $(E^2 - \vec{p}^2) \, \psi = m^2 \psi$  and put  $E \to -i \frac{\partial}{\partial t}, \vec{p} \to i \vec{\nabla}$  to get

$$(rac{\partial}{\partial t}^2 - \vec{
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- Causality
- Negative Energies (no stable ground state)

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 $(E^2 - \vec{p}^2) \psi = m^2 \psi$  and put  $E \to -i \frac{\partial}{\partial t}, \vec{p} \to i \vec{\nabla}$  to get

$$(rac{\partial}{\partial t}^2-ec{
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- Causality
- ▶ Negative Energies (no stable ground state)
- ightharpoonup Expected: t parameter,  $\vec{x}$  operator

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▶ One field for each type of particle

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▶ One field for each type of particle (Wheeler's idea)

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- ► One field for each type of particle (Wheeler's idea)
- creates and destroys particles

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- One field for each type of particle (Wheeler's idea)
- creates and destroys particles
- ▶ Interacting fields, interacting particles

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Closing Remarks

 Classical field | real scalar (number at every space time point)

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- Classical field | real scalar (number at every space time point)
- ▶ Demand Klien Gordan, then

$$\mathcal{L}=rac{1}{2}\left(\partial^{\mu}\phi\partial_{\mu}\phi+m^{2}\phi
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 $lacktriangledown \phi = \phi(t,ec{x})$  which I assume I can write as

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$$\phi = \int \frac{d^3p}{\left(2\pi\right)^3 \sqrt{2\omega_p}} \left(ae^{i\mathbf{p}\mathbf{x}} + a^{\dagger}e^{-i\mathbf{p}\mathbf{x}}\right)$$

where  $a = a(\vec{p})$ 

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- $\blacktriangleright \pi$  from  $\mathcal{L}$ .
- lacksquare Quantum Field  $\mid [\phi(t,\mathsf{x}),\pi(t,\mathsf{x}')]=i\delta(\mathsf{x}-\mathsf{x}')$

 $ightharpoonup [a(\mathbf{p}), a^{\dagger}(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$ 

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$$\qquad \qquad [a(\mathbf{p}), a^{\dagger}(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$$

$$H \sim a^{\dagger}a + \frac{1}{2}[a(\mathbf{p}), a^{\dagger}(\mathbf{p})]$$

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Closing Remark

•  $[a(\mathbf{p}), a^{\dagger}(\mathbf{p}')] \sim \delta(\mathbf{p} - \mathbf{p}')$ 

 $[a(\mathbf{p}), a'(\mathbf{p}')] \sim o(\mathbf{p} - \mathbf{p}')$ 

$$H\sim a^{\dagger}a+rac{1}{2}[a(\mathbf{p}),a^{\dagger}(\mathbf{p})]$$

► Similarity with Quantum Harmonic Oscillator

Prerequisite:

Towards a quantum theory of fields

- $[a(\mathbf{p}), a^{\dagger}(\mathbf{p}')] \sim \delta(\mathbf{p} \mathbf{p}')$
- •

$$H\sim a^{\dagger}a+rac{1}{2}[a(\mathbf{p}),a^{\dagger}(\mathbf{p})]$$

- ▶ Similarity with Quantum Harmonic Oscillator
- $ightharpoonup a^{\dagger}(\vec{p}) | vacuum \rangle$

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- ► Similarity with Quantum Harmonic Oscillator
- $ightharpoonup a^{\dagger}(\vec{p}) | vacuum \rangle$
- ightharpoonup Noether's theorem + Space-time invariance of  $\mathcal{L} 
  ightharpoonup$  physical momentum and energy operators

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- $ightharpoonup [a(\mathbf{p}), a^{\dagger}(\mathbf{p}')] \sim \delta(\mathbf{p} \mathbf{p}')$

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- ▶ To be a particle, it must satisfy  $E^2 \vec{p}^2 = m^2$

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- ▶ To be a particle, it must satisfy  $E^2 \vec{p}^2 = m^2$  and it does
- ▶ Conclusion: Parameter *m* is mass

► Non-interacting field

Dirac and Majorana Mass

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Towards a quantum theory of fields

- ► Non-interacting field
- Observable fields must interact

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Prerequisites.

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## Dirac and Majorana Mass

## Atul Singh Arora

Outline

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quantum theory of fields

Closing Remarks

## The End