Peculiar Velocities & Red-shift Space Distortions

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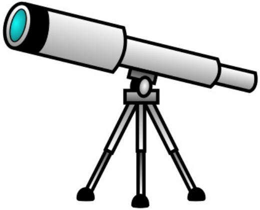
Outline

- The problem
- Background
- Approach - 5 steps
- Implications
- Conclusion

The Problem

- 
- θ, ϕ and $Z = HR/c$
- Crude approximation: $\delta^S = \delta^S(\theta, \phi, R = cZ/H)$
- Let $Z = HS/c$, where $S = R$ if we assume zero peculiar velocity.
  - $\delta^S = \delta^S(\theta, \phi, S)$ & $\delta^R = \delta^R(\theta, \phi, R)$

Background - I

- Position: $\vec{R} = a\vec{r}$.
- Velocity: $\vec{V} = \dot{a}\vec{r} + a\dot{\vec{r}}$;

$$\vec{V} = H\vec{R} + a\vec{u}. \tag{1}$$

  (where $H = \dot{a}/a$, $\vec{u} = \dot{\vec{r}}$)
- Redshift: $Z \approx V_{\mathrm{los}}/c$.
- Mass Density & Density Contrast:
  $\rho(\vec{r}, t) = \overline{\rho}(t)(1 + \delta(\vec{r}, t))$.
- Fluid approach:

$$\begin{aligned}
\frac{\partial \delta}{\partial t} + \vec{\nabla}.[(1+\delta)\vec{u}] &= 0, \\
\frac{\partial u}{\partial t} + \frac{2\dot{a}}{a}\vec{u} + (u.\nabla)u &= -\frac{1}{a^2}\vec{\nabla}\phi, \\
\nabla^2\phi &= 4\pi G a^2 \overline{\rho}\delta.
\end{aligned}$$

- Linear Limit: $\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\delta = 0$.

Background - II

- Linear Solutions: $D_{\pm}(t)$; D_+ is the growing solution.
- Interesting result:

$$d \ln D_+ / d \ln a = f(\Omega). \tag{2}$$

- General Linear Solution:

$$\delta(\vec{r}, t) = \delta_+(\vec{r}) \frac{D_+(t)}{D_+(t_i)} + \delta_-(\vec{r}) \frac{D_-(t)}{D_-(t_i)}.$$

- For growing mode initial conditions,

$$\vec{v} = -\vec{\nabla}\psi, \tag{3}$$

where $\vec{v} \equiv d\vec{r}/dD_+$ and $\psi \equiv 2a\phi/3H_0^2\Omega_{\rm nr}D_+$.

- Useful result:

$$\nabla^2\psi = \frac{\delta}{D_+} \tag{4}$$

Approach

1. Relation between $\vec{S}$ and $\vec{R}$, without neglecting peculiar velocities, but assuming linear theory.
2. Use conservation of mass in both coordinates.
3. Relation between δ^R and δ^S using the Jacobian
4. Use fourier space to eliminate v (or ψ) dependence
5. Relation between power spectra

1. Relation b/w $\vec{S}$ and $\vec{R}$

- $Z \approx \frac{\vec{V}.\hat{r}}{c}$, $\vec{S} \equiv Z\hat{r}cH^{-1}$
- $\vec{S} = (R + aH^{-1}\vec{u}.\hat{r})\hat{r}$ (using equation (1))
- Rewrite
$$\vec{u} = \frac{d\vec{r}}{dt} = \frac{dr}{dD_+}\frac{dD_+}{da}\frac{da}{dt},$$
$$\implies \vec{S} = (R + (H^{-1}\dot{a})D_+\vec{v}f.\hat{r})\hat{r}$$
(using equation (2)).
- Present time, assume $D_+(t_0) = 1$, we have
$$\vec{S} = \vec{R} + f_0(v.\hat{r})\hat{r}$$
where $f_0 \equiv f(\Omega_{\mathrm{nr}0}, \Lambda_0)$.

2. Mass Conservation, Jacobian and 3. the Relation

- $\left(1+\delta^S(\vec{S})\right) d^3\vec{S} = \left(1+\delta^R(\vec{R})\right) d^3\vec{R}$
- $d^3\vec{S} = \frac{\partial(S_x,S_y,S_z)}{\partial(R_x,R_y,R_z)} d^3\vec{R}$
  - $S\hat{r} = R(1+U/R)\hat{r}$, where $U = f_0(\vec{v}.\hat{r})$, $\Longrightarrow$ in spherical coordinates, θ and ϕ remain unchanged.
  - $d^3\vec{S} = S^2 dS \sin\theta d\theta d\phi$ can be written as $(1+U/R)^2(1+dU/dR)R^2 dR \sin\theta d\theta d\phi$
  - $J = \left(1+\frac{U}{R}\right)^2 \left(1+\frac{dU}{dR}\right)$. ‘
- Required relation:

$$1+\delta^R(\vec{R}) = \left(1+\delta^S(\vec{S})\right)\left(1+\frac{U}{R}\right)^2\left(1+\frac{dU}{dR}\right).$$

4. Fourier Transform; removing unknowns

- Single mode to simplify calculations and later sum the modes.
- $\vec{v} = \vec{v}_k e^{-i\vec{k}.\vec{R}}$, $\psi = \psi_k e^{-i\vec{k}.\vec{R}}$ and substitute in equation (3) to get $\vec{v}_k = i\vec{k}\psi_k$.
- $\delta = \delta_k e^{-i\vec{k}.\vec{R}}$, using equation (4), we get $\vec{v}_k = -i\vec{k}\delta_k/k^2$.
- Substituting for $\vec{v}$ in U,

$$U = f_0 \vec{v}_k.\hat{r}e^{-i\vec{k}.\vec{R}} = \frac{if_0\delta_k\vec{k}.\hat{r}e^{-i\vec{k}.\vec{R}}}{k^2} = \frac{if_0\delta_k\mu e^{-i\vec{k}.\vec{R}}}{k}$$

$$\frac{dU}{dR} = f_0\mu^2\delta_k e^{-i\vec{k}.\vec{R}} = f_0\mu^2\delta,$$

where $\mu = \hat{k}.\hat{r}$, the cosine of the angle between $\vec{k}$ and line of site.

- Thus, $(1+\delta^S) \approx (1+\delta^R)\left(1+f_0\delta^R\mu^2\right)^{-1}$
- To first order in δ^R, $\delta^S \approx \delta^R(1-f_0\mu^2)$.
- Substituting $\delta^S = \delta_k^S e^{-i\vec{k}.\vec{R}}$ and $\delta^R = -\delta_k^R e^{-i\vec{k}.\vec{R}}$, we have ...

5. The final relation; between power spectra

'Fouriered', Redshift Space & Real Space:

$$\delta_k^S(\vec{k}) \approx \delta_k^R(\vec{k}) \left(1 + f_0 \mu^2\right)$$

Power spectrum $= \delta_k^2$,

$$\begin{aligned} P^S(\vec{k}, \mu) &\approx \left(1 + f_0 \mu^2\right)^2 P^R(\vec{k}) \\ &= (1 + 2 f_0 \mu^2 + f_0^2 \mu^4) P^R(\vec{k}). \end{aligned} \tag{5}$$

Implications | Polar Plot

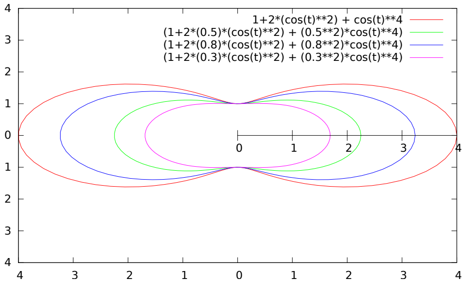


Figure: Polar plot of $P^S(\vec{k}, \mu)$, for a given $\left|\vec{k}\right|$, and $f = 0.3, 0.5, 0.8$ and 1.0. LOS is along the x-axis.

Implications | Contours

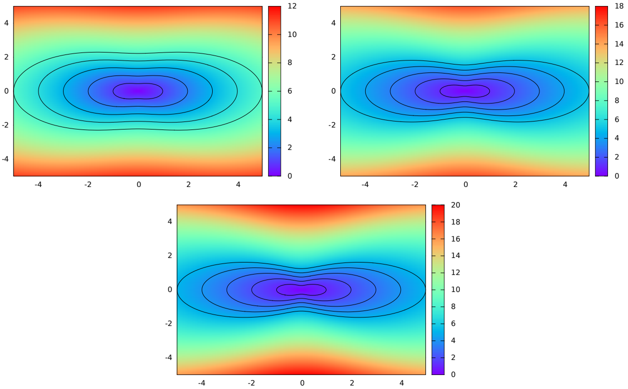


Figure: Contour plots for $f = 0.5$, 0.8 and 1.0. LOS is along the y-axis.

Implications | Observed Contours

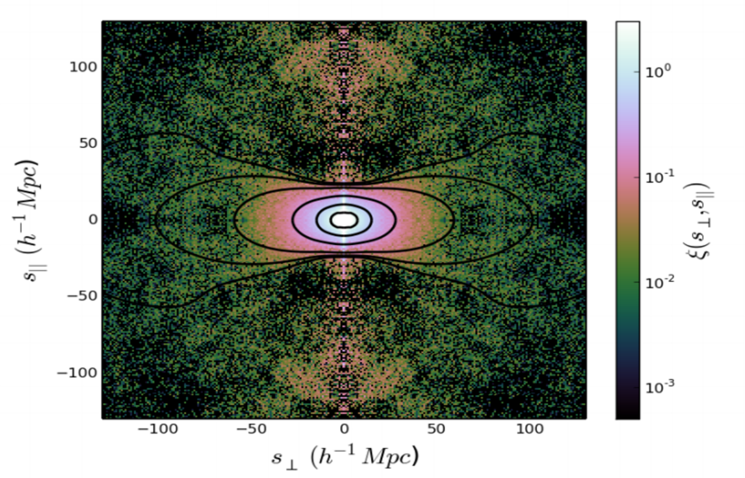


Figure: An observed contour plot of δ^S, with LOS along the y-axis.

Conclusions

- Corrected for the errors caused by neglecting the peculiar velocity, using linear theory.
- Way to estimate the value of cosmological parameters.

Questions

References

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