Indian Institute of Science Education and Research Mohali

Curves and Surfaces (MTH201)

Academic Session 2012-13

Tutorial Sheet 5 **September 28 2012**

Instructions: Write main ideas / hints for solving questions in your tutorial noteook. There is no need to write full and formal solution during the tutorial session. However during off class hours you should practice writing these solutions in a formal manner. **Get the signature of your tutor after each session**.

1. Let $\kappa_0 : (\alpha,\beta) \to \mathbb{R}$ and $\tau_0 : (\alpha,\beta) \to \mathbb{R}$ be smooth functions and $\mathfrak{t} : (\alpha,\beta) \to \mathbb{R}^3$, $\mathfrak{n} : (\alpha,\beta) \to \mathbb{R}^3$ and $\mathfrak{b} : (\alpha,\beta) \to \mathbb{R}^3$ be smooth vector valued functions satisfying:

$$\begin{aligned} \dot{\mathfrak{t}} &= \kappa_0\, \mathfrak{n} \\ \dot{\mathfrak{n}} &= -\kappa_0\, \mathfrak{t} + \tau_0\, \mathfrak{b} \\ \dot{\mathfrak{b}} &= -\tau_0\, \mathfrak{n} \end{aligned}$$

For $s \in (\alpha,\beta)$ consider the matrix given by:

$$M(s) = \begin{pmatrix} \mathfrak{t}(s).\,\mathfrak{t}(s) & \mathfrak{t}(s).\,\mathfrak{n}(s) & \mathfrak{t}(s).\,\mathfrak{b}(s) \\ \mathfrak{n}(s).\,\mathfrak{t}(s) & \mathfrak{n}(s).\,\mathfrak{n}(s) & \mathfrak{n}(s).\,\mathfrak{b}(s) \\ \mathfrak{b}(s).\,\mathfrak{t}(s) & \mathfrak{b}(s).\,\mathfrak{n}(s) & \mathfrak{b}(s).\,\mathfrak{b}(s) \end{pmatrix}$$

Prove that $\dot{M}(s) = A_3(s)M(s) - M(s)A_3(s)$ where $A_3(s)$ is the matrix given by

$$A_3(s) = \begin{pmatrix} 0 & \kappa_0(s) & 0 \\ -\kappa_0(s) & 0 & \tau_0(s) \\ 0 & -\tau_0(s) & 0 \end{pmatrix}.$$

Here $\dot{\mathfrak{t}}$, $\dot{\mathfrak{n}}$, $\dot{\mathfrak{b}}$ and $\dot{M}$ denote the differentials with respect to s.
