Dirac and Majorana Mass

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Overview of the Talk

Introduction

Motivation | Mass

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Towards a quantum theory of fields

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- Conclusion: Parameter m is mass

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- $$\begin{aligned}
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=&(-\frac{1}{2}\partial_\mu\partial_\nu\{\gamma^\mu,\gamma^\nu\} - 2m\,[i(\partial_\mu\gamma^\mu)] + m^2)\\
=&(-\frac{1}{2}\partial_\mu\partial_\nu\{\gamma^\mu,\gamma^\nu\} - 2m^2 + m^2)\\
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=&(-\frac{1}{2}\partial_\mu\partial_\nu\{\gamma^\mu,\gamma^\nu\} - 2m\,[i(\partial_\mu\gamma^\mu)] + m^2)\\
=&(-\frac{1}{2}\partial_\mu\partial_\nu\{\gamma^\mu,\gamma^\nu\} - 2m^2 + m^2)\\
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\end{aligned}$$

- To be Klein Gordon, $\{\gamma^\mu,\gamma^\nu\} = -2\delta^{\mu\nu}$

The Dirac Equation

- $$(i\partial_\mu\gamma^\mu - m)\psi = 0$$
- $$\begin{aligned}&(i\partial_\mu\gamma^\mu - m)(i\partial_\nu\gamma^\nu - m)\\ =&(-\partial_\mu\partial_\nu\gamma^\mu\gamma^\nu - im\partial_\mu\gamma^\mu) - (im\partial_\nu\gamma^\nu - m^2)\\ =&(-\frac{1}{2}\partial_\mu\partial_\nu\{\gamma^\mu,\gamma^\nu\} - 2m\,[i(\partial_\mu\gamma^\mu)] + m^2)\\ =&(-\frac{1}{2}\partial_\mu\partial_\nu\{\gamma^\mu,\gamma^\nu\} - 2m^2 + m^2)\\ =&(-\frac{1}{2}\partial_\mu\partial_\nu\{\gamma^\mu,\gamma^\nu\} - m^2)\end{aligned}$$
- To be Klein Gordon, $\{\gamma^\mu,\gamma^\nu\} = -2\delta^{\mu\nu}$
- Claim
  γ^μ are 4×4 matrices and ψ then is a 4−component object, called a Dirac spinor.

The Dirac Equation

- Hat:

$$\sigma^\mu \equiv (1, \vec{\sigma}) \qquad \overline{\sigma}^\mu \equiv (1, -\vec{\sigma})$$

$$\gamma^\mu \equiv \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}$$

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- Claim: commutation holds

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- S_L and S_R are not unitary

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- $$m\overline{\psi}\psi = m \begin{pmatrix} \psi_L^\dagger & \psi_R^\dagger \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = m(\psi_R^\dagger \psi_L + \psi_L^\dagger \psi_R)$$

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- Mass term mixes the left and right spinors
- Explore: mass term that doesn't mix

Projectors, mixing of mass terms

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- In this notation also, there's mixing

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  - I can write C as a product of γ matrices as

$$C = i \begin{pmatrix} \sigma^2 & 0 \\ 0 & -\sigma^2 \end{pmatrix} = -i\gamma^2\gamma^0$$

    which is easy to verify.

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- To ensure reality, we add $-m\psi^\dagger C\psi^*$

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- There’re alternatives, such as ‘see-saw’ model

The End

?

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