Young's Modulus

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1 Theory

The theory for this experiment has primarily been adapted from `http://www.animations.physics.unsw.edu.au//jw/elasticity.htm`. The graphs in this section have been taken from the same.

Consider a typical wire. Stretching this leads to a small increase in length of the wire. At the microscopic level, the average distance between the atoms, r, increases. The attractive force between the atoms balances slash resists the applied tensile force.

Similarly, shortening a metal by applying a compressive force leads to decrease in r and is again balanced by inter-atomic repulsion.

From these one can deduce that

- When $r > r_{eq}$ where r_{eq} is the interatomic distance when the wire is not under stress (will be defined precisely later), the interatomic attraction is greater than repulsion.
- When $r < r_{eq}$, the interatomic repulsion is greater than interatomic attraction.

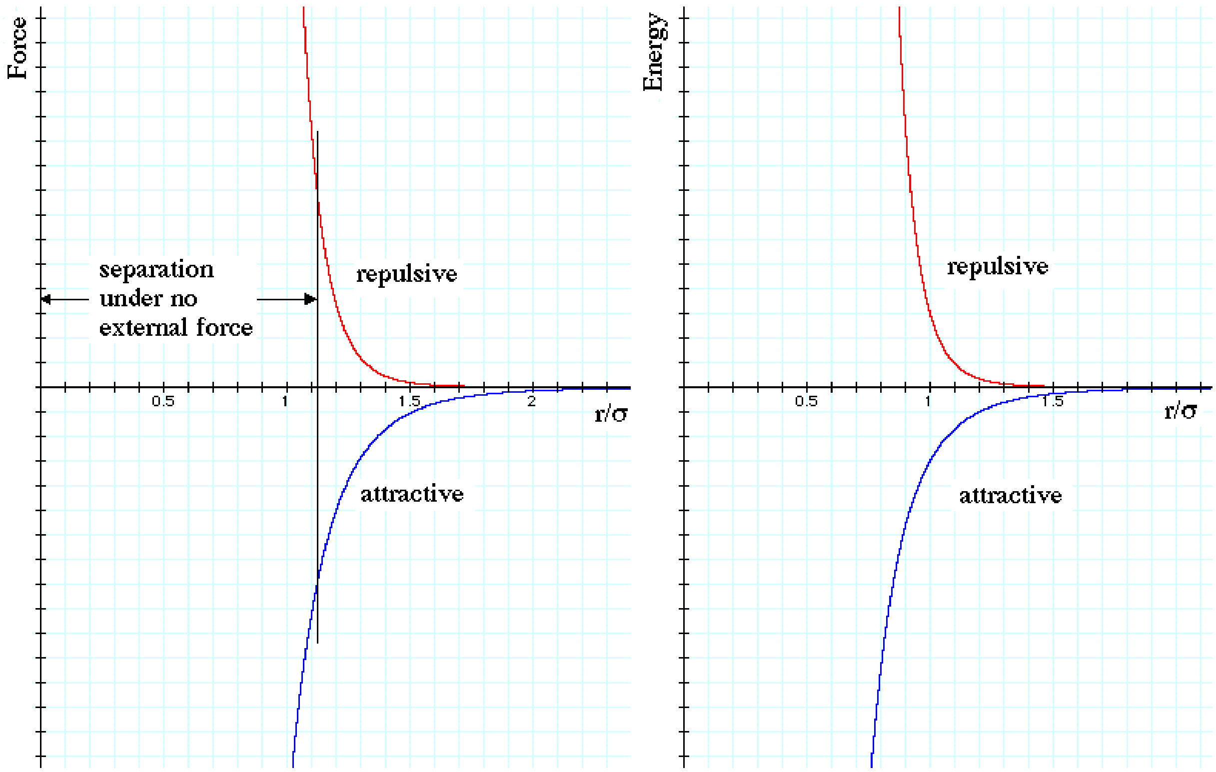
Further, from experience, we know that metals are harder to compress which suggests

- Repulsive forces grow much more rapidly with $|r - r_{eq}|$ compared to the attractive force

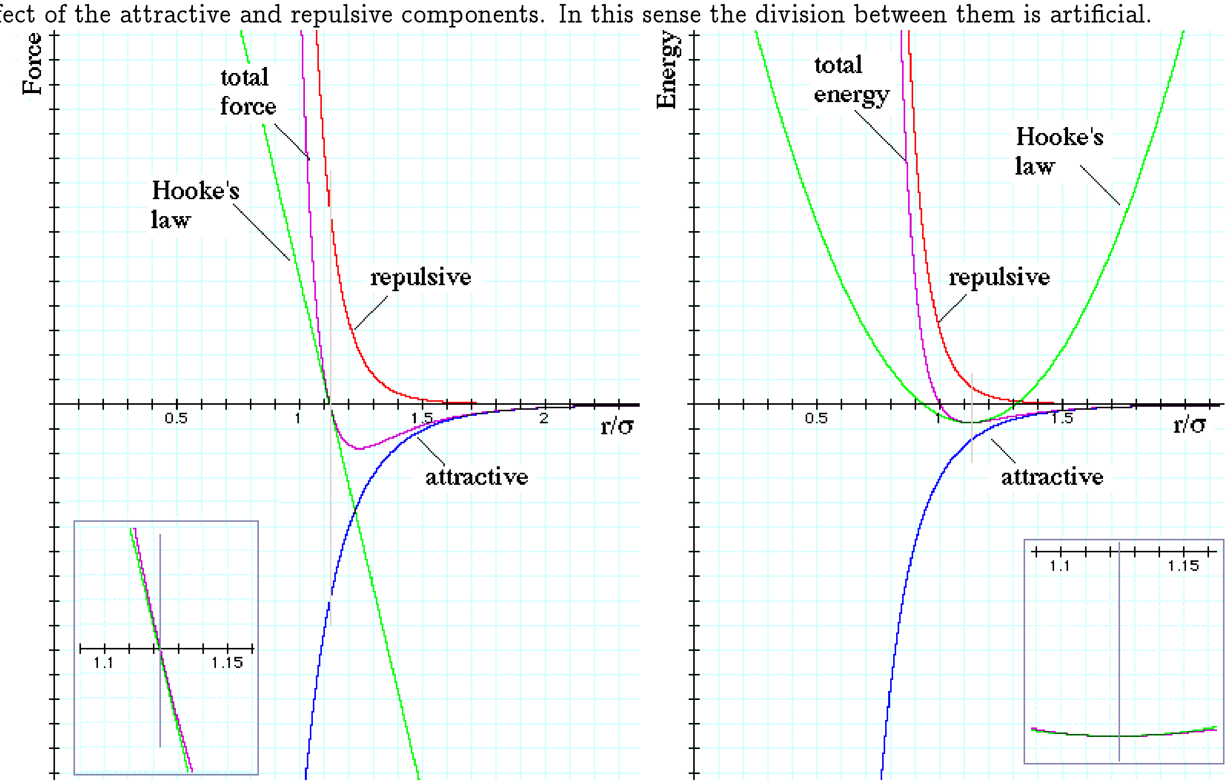
And finally, we also know that once a metal is stretched beyond a limit, it breaks and won't 'spring back together'. So we conclude

- If the interatomic distance $r > r_{max}$ (some r_{max}) then the attractive forces become effectively zero.

Using these principels, we can plot the repulsive and attractive forces as a function of r. We can also plot energy against r.



We use the convention that repulsive forces are positive and attractive are negative. Combining these graphs results in the following. One must note that what we measure eventually is the combined effect of the attractive and repulsive components. In this sense the division between them is artificial.



Note that the Hook's law (yet to be stated explicitly) is valid in the linear range of the force (and equivalently, the quadratic range of the potential). This explains the microscopic origin of Hook's law, which can be expressed in this context as

$$F \propto -(r - r_{eq})$$

Using this relation, one can write a more relavent quantity, the Young's Modulus (which is defined such that it is fixed for a given material)

$$Y = \frac{F/A}{\Delta l/l}$$

where F is the tensile force (for the wire say, that we considered to start with), A is the surface area normal to the force, Δl is the change in length from the unstressed length (viz. when $F = 0$), and l is the length when $F = 0$.

2 Experimental Setup

The setup allows one to essentially stretch a wire with known weight, as is required. Non-trivial aspects include a pulley (whose presence is not included in the calculations) and a radial dial (1 division = 0.25 cm) that measures Δl with precision. The dial also requires the wire to be looped around its knob two or three times.

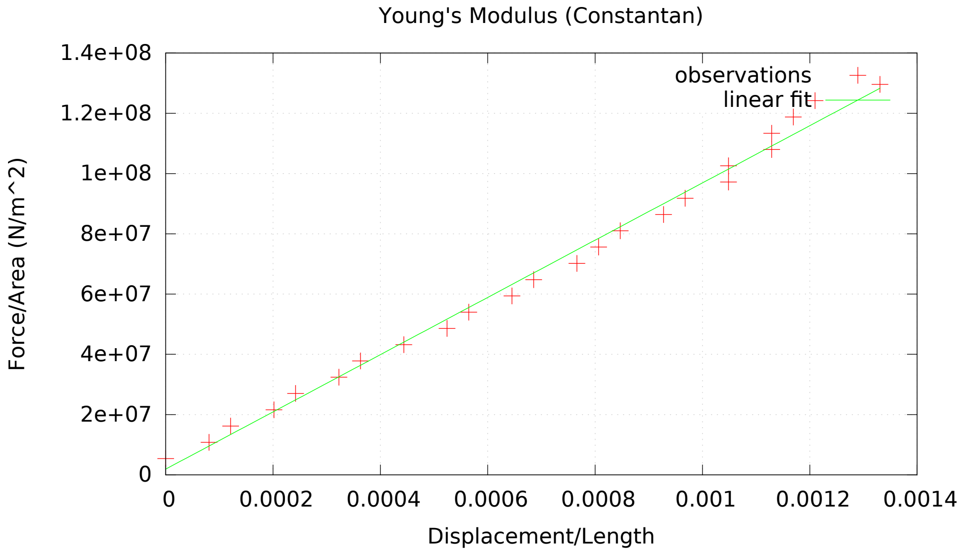
The procedure is straight forward. One measures the length of the wire (after making it taut preferably), fixes it on the setup so that its taut. The wire is looped through the displacement dial and attached to the weight through th pulley. Weight is added with as high a precision as possible. Once the small weights are over (viz. all used), the wire is removed and another wire is setup. This wire is now started with the same old weight, except this time, low precision weights are used to get the old weight. Subsequently, high precision weights are added and the procedure continued till the wire breaks. Obviously, the displacement is noted sensibly to account for consistency with the previous sets of readings. This entire procedure is repeated with a wire of a different material.

3 Observations

3.1 Time-line

March 9	Monday	Started performing the next experiment [didn't use mixed weights]
March 10	Tuesday	Performed the experiment for both Copper and Constantan using mixed weights and multiple wires
March 13	Friday	Analyzed the data and worked on writing the record

3.2 Constantan

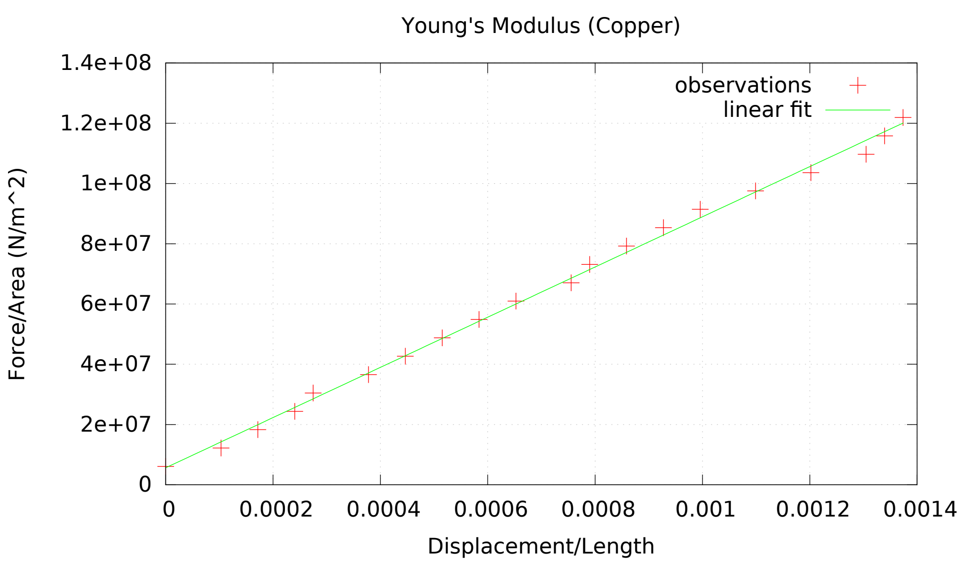


Final set of parameters		Asymptotic Standard Error	
m	= 9.50055e+10	+/− 1.708e+09	(1.798%)
c	= 1.89252e+06	+/− 1.357e+06	(71.68%)

Constanton

weight (g)	force (N)	force/area (N/m^2)	displacement (angular)	displacement (# divisions)	displacement cm	delta L/L
50	0.49	5399687.039	0	0	0	0
100	0.98	10799374.08	0.5	2	0.0125	8.06E-05
150	1.47	16199061.12	0.75	3	0.01875	0.000121
200	1.96	21598748.15	1.25	5	0.03125	0.000202
250	2.45	26998435.19	1.5	6	0.0375	0.000242
300	2.94	32398122.23	2	8	0.05	0.000323
350	3.43	37797809.27	2.25	9	0.05625	0.000363
400	3.92	43197496.31	2.75	11	0.06875	0.000444
450	4.41	48597183.35	3.25	13	0.08125	0.000524
500	4.9	53996870.39	3.5	14	0.0875	0.000565
550	5.39	59396557.42	4	16	0.1	0.000645
600	5.88	64796244.46	4.25	17	0.10625	0.000685
650	6.37	70195931.5	4.75	19	0.11875	0.000766
700	6.86	75595618.54	5	20	0.125	0.000806
750	7.35	80995305.58	5.25	21	0.13125	0.000847
800	7.84	86394992.62	5.75	23	0.14375	0.000927
850	8.33	91794679.66	6	24	0.15	0.000968
900	8.82	97194366.69	6.5	26	0.1625	0.001048
950	9.31	102594053.7	6.5	26	0.1625	0.001048
1000	9.8	107993740.8	7	28	0.175	0.001129
1050	10.29	113393427.8	7	28	0.175	0.001129
1100	10.78	118793114.8	7.25	29	0.18125	0.001169
1150	11.27	124192801.9	7.5	30	0.1875	0.00121
1200	11.76	129592488.9	8.25	33	0.20625	0.001331
1250	broke					
length	155	cm				
cross section						
scew gauge	div					
	18		1mm/50div	0.34		
	18		area(mm sq)	0.090746		
	18		area m sq	9.0746E-08		
zero error	1					

3.3 Copper



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Final set of parameters            Asymptotic Standard Error
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m               = 8.33064e+10      +/- 1.041e+09    (1.25%)
c               = 5.64506e+06      +/- 8.511e+05    (15.08%)
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Copper

weight (g)	force (N)	force/area (N/m^2)	displacement (angular)	displacement (# divisions)	displacement cm	delta L/L
50	0.49	6095740.446	0	0	0	0
100	0.98	12191480.89	0.75	3	0.01875	0.000103022
150	1.47	18287221.34	1.25	5	0.03125	0.000171703
200	1.96	24382961.78	1.75	7	0.04375	0.000240385
250	2.45	30478702.23	2	8	0.05	0.000274725
300	2.94	36574442.68	2.75	11	0.06875	0.000377747
350	3.43	42670183.12	3.25	13	0.08125	0.000446429
400	3.92	48765923.57	3.75	15	0.09375	0.00051511
450	4.41	54861664.01	4.25	17	0.10625	0.000583791
500	4.9	60957404.46	4.75	19	0.11875	0.000652473
550	5.39	67053144.9	5.5	22	0.1375	0.000755495
600	5.88	73148885.35	5.75	23	0.14375	0.000789835
650	6.37	79244625.8	6.25	25	0.15625	0.000858516
700	6.86	85340366.24	6.75	27	0.16875	0.000927198
750	7.35	91436106.69	7.25	29	0.18125	0.000995879
800	7.84	97531847.13	8	32	0.2	0.001098901
850	8.33	103627587.6	8.75	35	0.21875	0.001201923
900	8.82	109723328	9.5	38	0.2375	0.001304945
950	9.31	115819068.5	9.75	39	0.24375	0.001339286
1000	9.8	121914808.9	10	40	0.25	0.001373626
1050	broke					
length	182	cm				
cross section						
scew gauge	div					
	17		1mm/50div	0.32		
	17		area(mm sq)	0.080384		
	17		area m sq	8.0384E-08		
zero error	1					

3.4 Result

The Young's modulus for Constantan was found to be $(9.500 \pm 0.178) \times 10^{10}$ N/m^2 ($\sim$ 2% precision) and for Copper it was $(8.330 \pm 0.104)10^{10}$ N/m^2 ($\sim$ 1% precision). The breaking point of Constantan was found to occur just beyond $(1.295 \pm .0609) \times 10^8$ N/m^2 while that of Copper was $(1.219 \pm .0609) \times 10^8$ $N/m^2$where the error is caused by the smallest weight available (50 g).

4 Critique

The setup was not too shabby. The following improvements will reduce performance time and increase accuracy

- Get thinner small weights (they can be wide)
- Use a telescope to note Δl instead of the angular measurement apparatus

The angular dial has the following issues:

- the height of the dial has to be adjusted by trial
- the range of the dial is restricted

5 Acknowledgements

I thank Prashansa Gupta and Vivek Sagar for their contribution in the completion of this experiment. Further, as stated earlier, the theory for this experiment has been primarily been adapted from `http://www.animations.physics.unsw.edu.au//jw/elasticity.htm`.