Hall Effect

Atul Singh Arora

March 2015

1 Theory

Assuming the Drude model, it can be shown that (see for example Ashcroft and Mermin) the equation of motion per electron becomes

$$\frac{dp}{dt} = -\frac{p}{\tau} + f$$

where f is the force, and τ is s.t. probability of collision of an electron in time dt is dt/τ. Let us now consider the situation depicted by the following diagram [taken from Ashcroft and Mermin]. It seems reasonable to imagine that for a fixed value of E_x, j_x will depend on B by adding to the resistance in some way. We therefore define a quantity[1], Magneto-resistance as

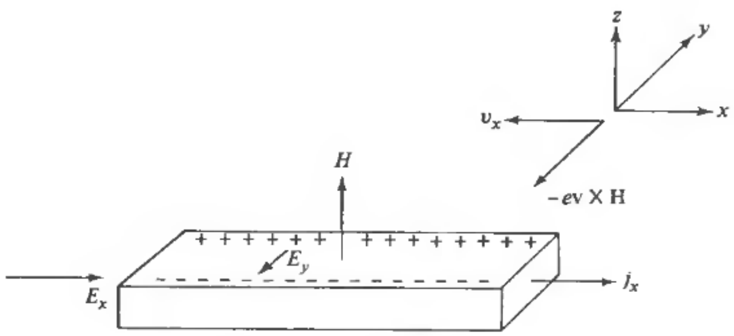
$$\rho(B) = \frac{E_x}{j_x}$$

which was experimentally found by Hall to indeed by field dependent. Next since by inspection we already know that in steady state, E_y must balance the lorentz force, which is proportional to both B and to j_x, we define a quanitity known as the Hall coeffecient

$$R_H = \frac{E_y}{j_x B}$$

At this very step, one can make a rather important point. The direction of E_y depends on whether the charge of the current mediator is positive or negative. If the charge is negative, as shown below, E_y will be negative, and therefore so will R_H. If the charge were positive, R_H will be positive.

[1] Recall: In general, we define ρ as $\mathbf{E} = \rho\mathbf{j}$. If we consider a typical case (straight wire), we get $j = I/A$, $V = EL$ and get $V = I\rho L/A$ which using ohms definition, we get the familiar $R = \rho L/A$



We now derive these quantities under the Drude model. Let's start with using the equation of motion per electron for an arbitrary E and B field as

$$\frac{d\mathbf{p}}{dt} = -e\left(\mathbf{E} + \frac{\mathbf{p}}{mc} \times \mathbf{B}\right) - \frac{\mathbf{p}}{\tau}$$

In the steady state then, for the given set of fields, we'll have

$$0 = -\,eE_x + \cancelto{-\omega_c p_y}{\left(-\frac{eB}{mc}p_y\right)} - \frac{p_x}{\tau}$$

$$0 = -\,eE_y - \cancelto{-\omega_c p_x}{\left(-\frac{eB}{mc}p_y\right)} - \frac{p_y}{\tau}$$

Recall case: Only $\mathbf{E}$ fields:

- ρ is defined as$\mathbf{E} \equiv \rho\mathbf{j}$, and $\sigma \equiv 1/\rho$
- $\mathbf{j} = -ne\mathbf{v}$ (where n is the number of electrons per unit volume)
- In the presence of only a constant $\mathbf{E}$ field, we have

  $$\mathbf{v} = -\frac{ne\mathbf{E}\tau}{m}$$

  which entails

  $$\mathbf{j} = \frac{-ne^2\tau}{m}\mathbf{E} = \sigma\mathbf{E}$$

  thus

  $$\sigma = \frac{ne^2\tau}{m}$$

Getting back to the equations, we multiply by

$$-\frac{ne\tau}{m}$$

to get

$$\begin{aligned}\sigma_0 E_x =& \omega_c \tau j_y + j_x \\ \sigma_0 E_y =& -\omega_c \tau j_x + j_y\end{aligned}$$

where $\mathbf{j} = -ne\mathbf{p}/m$. Next we assume that in steady state, j_y is zero. Plugging this in the previous equations we get

$$\begin{aligned}\sigma_0 E_x =& j_x & \implies E_x =& j_x \left(\frac{m}{ne^2\tau}\right) = j_x \rho & \implies \rho(B) = \frac{m}{ne^2\tau} \\ \sigma_0 E_y =& -\omega_c \tau j_x & \implies E_y =& -j_x \frac{\cancel{e}B}{\cancel{m}c}\cancel{\tau}\frac{\cancel{m}}{ne^{\cancel{2}}\cancel{\tau}} = j_x \left(-\frac{1}{nec}\right) B = j_x R_H B & \implies R_H = -\frac{1}{nec}\end{aligned}$$

So in accordance with our theory, $\rho(B)$ doesn't dependent on B, the field. This can be verified experimentally. Further, it depends on τ which can depend on various parameters, including temperature.

However, R_H according to the theory is rather boldly predicted to be independent of all parameters of the metal, except the electron density. This density can be calculated assuming that only the valence electrons of the metal participate in the process of metallic conduction. However, experimentally it is known that R_H does depend on the field strength B.

2 Experimental Setup

The setup is as straight forward as it gets. We have two electromagnets to produce a strong B field. Then there's a PCB that holds a crystal which has 5 probes. Now this part is slightly new. Let me define a convention for clarity. The PCB is the $x - y$ plane, the magnetic field H is along (or opposite) $\hat{z}$. Now back to the probes. Two of these probes are at horizontally opposite points, say pl, pr. The remaining three probes are along points on the vertically opposite edge, of which two are on the top edge, say $pt1$, $pt2$, and one probe on the bottom edge pb. This is a practial requirement. We want to find R_H and $\rho(B)$ for which, by definition of these quantities, we need E_y, j_x and E_x. These quantities are measured using potential differences and currents. Infact, I use a current source to pass current horizontally through the crystal. Next, we note that

$$-\frac{\partial V}{\partial x} = E_x; -\frac{\partial V}{\partial y} = E_y$$

which entails that if I wish to measure the voltage difference between two pionts on vertically opposite edges, unless the points have identical x coordinates, there'll be a potential difference along x which will be read, in addition to the vertical difference. In practice, it is nearly impossible to achieve zero difference in the x coordinate. Thus, we use two probes on the top edge and take an appropriate linear combination (done using a potentiometer) to effectively make the x coordinate of the two points zero.

So much for the five probes. Now back to the discussion of the remaining setup. We are given a gauss meter to measure the B field in a high precision ($\times 1$) and a low precision (larger range) mode ($\times 10$)[2]. We are also given a current source.

2.1 Final Formulae

We want our formulae in a form that's in terms of easier observables.

$$\begin{aligned} E_x l_x &= V_x \\ E_y l_y &= V_H \\ j_x &= I/A = I/l_y l_z \end{aligned}$$

Now recall the definitions

$$\begin{aligned} \rho(B) &= \frac{E_x}{j_x} \\ &= \frac{V_x}{l_x(I/l_y l_z)} \\ &= \frac{V_x l_y l_z}{l_x I} \\ R_H &= \frac{E_y}{j_x B} \\ &= \frac{V_H l_z \cancel{l_y}}{IB\cancel{l_y}} \\ &= \frac{V_H l_z}{IB} \end{aligned}$$

To be consistent with the manual, I define $t \equiv l_z = 0.5mm$, $b \equiv l_x = 6mm$, $l \equiv l_y = 7mm$.

2.2 Procedure

1. Calibrating the Guass Meter and the Electromagnets

    (a) Take the gauss meter sufficiently far away from the electromagnet and calibrate it to read zero in high precision (x1).

    (b) Bring the electromagnets faces sufficiently[3] close.

    (c) Now place the gauss meter between the two faces of the electromagnet, parallel to it, with the electromagnet switched off.

    (d) If the gauss meter reads a non-zero value (this may happen because of hysterisis) then

        i. Put the gauss meter in the low precision mode

[2]Note that $\times 1$ and $\times 10$ mean that whatever answer the meter reads, you multiply it by 1 and 10 respectively. This means the latter is low precision. It's easy to confuse it to mean magnification $1x$ and $10x$.

[3]such that if you place the PCB between the faces, it almost touches the PCB and the crystal on opposite ends, but doesn't infact.

ii. Turn on the electromagnet and try changing the current/voltage through the coil to get the value as close to zero as possible; if the value drops to zero in ($\times 10$, then goto high precision and repeat) [this should not work]

iii. If the previous step failed, as it should[4], in one of the coils, swap the wires (do not swap wires from distinct coils, it'll just result in reversing the direction of the large field) and try the previous step.

(e) Now keep the gauss meter away (viz. don't keep it where it has been designed to keep according to the setup)

2. Setting up the crystal, its current source and volt meter

(a) Now with the current source off, put the PCB between the electromagnet's faces such that the PCB is parallel to the faces and the crystal is in the centre. Ensure that neither the PCB nor the crystal touch the electromagnet.

(b) If the terminals aren't connected, connect them in accordance with the labels on the PCB. The reasoning should be obvious by now.

(c) To be certain, before turning on the current source, stick in the gauss meter, once behind the PCB and once in front of the crystal and ensure the field is zero. If it is not, repeat the previous set of steps.

(d) Now start the current source at some low value.

(e) Look at the reading on the voltmeter. If this were indeed measuring the Hall voltage, it should be zero since the magnetic field is zero. The reading in practice won't be zero (excluding exceptional circumstances) because of the longitudinal voltage as explained. This can

FAQ:

1. Write about how you would fix the issue: magnetic field in the centre is non zero
2. Even at zero magnetic field, the multimeter reads non-zero current (this is "hall current")

2.3 Observations

2.3.1 Time-line

March 20	Friday	Started and almost finished performing the experiment
March 23	Monday	Understood whether or not repeating is required (longitudenal voltage) | started writing the record
March 24	Tuesday	Completed the record and analyzed the data and worked on writing the record

[4] because it is designed to produce a large field. To exactly cancel the stray field, we must get the coils to produce opposite fields so that their superposition yields a small adjustable field