Indian Institute of Science Education and Research Mohali

Curves and Surfaces (MTH201)
Academic Session 2012-13

Tutorial Sheet 3 **September 14 2012**

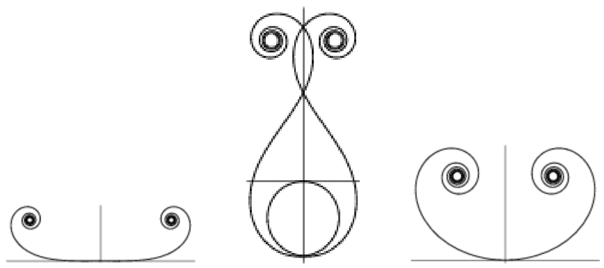
Instructions: Write main ideas / hints for solving questions in your tutorial noteook. There is no need to write full and formal solution during the tutorial session. However during off class hours you should practice writing these solutions in a formal manner. **Get the signature of your tutor after each session**.

1. Let $\gamma : (0, \pi) \to \mathbb{R}^3$ be the semicircle $\gamma(t) = (\cos t, \sin t, 0)$ and $\theta : (0, \pi) \to \mathbb{R}^3$ be the straight line $\theta(t) = (t, t, 2)$. Write the equation of the parametrised curve obtained by taking the mid points of lines joining $\gamma(t)$ and $\theta(t)$. Find the curvature of the new curve.

2. Compute the curvature function of the curve $\gamma : (0, 2\pi) \to \mathbb{R}^3$ given by
$$\gamma(t) = \left(a(t - \sin t),\ a(1 - \cos t),\ 4a \cos \frac{t}{2}\right).$$
Find all points where the curvature of γ attains a minima?

3. For a curve γ, a *vertex* of γ is a point $\gamma(t_0)$ where the curvature function attains a local maxima or a local minima. Find the curvature function, signed curvature function and vertices of:
   (a) Circle: $\gamma(t) = (a \cos t, a \sin t)$.
   (b) Ellipse: $\gamma(t) = (a \cos t, b \sin t)$, $a \neq b$.
   (c) Parabola: $\gamma(t) = (t^2, t)$.
   (d) Logarithmic Spiral: $\gamma(t) = (e^{at} \cos t, e^{at} \sin t)$.

4. Signed curvature functions of the unit speed plane curves, with the shapes A, B, C as below, in the shuffled order are known to be $\kappa(t) = t^2 - 4$, $\kappa(t) = t^2$ and $\kappa(t) = t^2 + 1$. Match these signed curvature functions with the appropriate shape and justify your answer. Assume that domain of each curve is $\mathbb{R}$.



A B C

5. Let γ be a regular smooth plane curve whose shape is given by the graph of the function $y = f(x)$. Show that the curvature of γ at the point $(x, f(x))$ is given by $\dfrac{|f''(x)|}{(1 + f'(x)^2)^{3/2}}$.

6. Let γ be a unit speed plane curve whose signed curvature $\kappa_{\text{sgn}}(s)$ at the point $\gamma(s)$ does not vanish. Let $N_{\text{sgn}}(s)$ denote the signed unit normal of γ at the point $\gamma(s)$. We define the *radius of curvature* of γ at the point $\gamma(s)$ to be the positive real number $r_\gamma(s) \overset{\text{defn.}}{:=} \dfrac{1}{|\kappa_{\text{sgn}}(s)|}$ and the *centre of curvature* of γ at $\gamma(s)$ to be the point:
$$\epsilon_\gamma(s) \overset{\text{defn.}}{:=} \gamma(s) + \frac{1}{\kappa_{\text{sgn}}(s)} N_{\text{sgn}}(s) \in \mathbb{R}^2 .$$
   (a) True or false? Justify : *There exists a point on the ellipse* $16x^2 + 25y^2 = 400$ *where the radius of curvature is* 5 *units.*
   (b) Identify the locus of centre of curvature[1] of the helix $\gamma(s) = (a \cos s, a \sin s, bs)$.
   (c) Roughly plot the evolute of the ellipse $(5 \cos t, 4 \sin t)$.
   (d) Show that evolute of the parabola $\gamma(t) = (t^2, t)$ has Cartesian equation $27y^2 = 2(2x - 1)^3$.

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[1]locus of centre of curvature of a curve γ is called the *evolute* of γ