Indian Institute of Science Education and Research Mohali

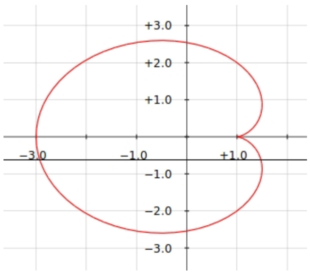
Curves and Surfaces (MTH201)
Academic Session 2012-13
Hints/Answers of Tutorial Questions

Tutorial 01 [23/08/12]

1. $f(x) = x^3$, and many more!
2. 0, what else?
3. $(a, b, c) = \left(6^{3/5}, \frac{1}{2}6^{3/5}, 6^{1/5}\right)$.
5. (a) No, f is not continuous at $(0, 0)$.

Tutorial 02 [31/08/12]

2. (a) Rough sketch is as below.



Cardioid

(b) The curve γ is smooth.

(c) Since $\|\dot{\gamma}(t)\| = 4|\sin\left(\frac{t}{2}\right)|$, it vanishes at each $t = 2n\pi$. Thus γ is not regular.

(d) γ is not unit speed.

(e) 16.

3. $\tilde{\gamma}(s) = \left(a\cos\left(\frac{s}{\sqrt{a^2+b^2}}\right), a\sin\left(\frac{s}{\sqrt{a^2+b^2}}\right), \frac{bs}{\sqrt{a^2+b^2}}\right)$.

4. $y = \pm\left(\ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}\right)$.

Hint: The tangent to the trajectory of cat will point towards rat. This will give a differential equation

$$\frac{dy}{dx} = -\frac{\sqrt{1-x^2}}{x}$$

with initial condition $y(1) = 0$.

Tutorial 03 [14/09/12]

1. The requird curve is: $\beta(t) = \left(\frac{1}{2}(t+\cos t), \frac{1}{2}(t+\sin t), 1\right)$ and its curvature is: $\kappa(t) = \frac{2|(1-\sin t+\cos t)|}{(3-2\sin t+2\cos t)^{3/2}}$.

2. Curvature attains minima at all odd integral multiples of π.

3. (a) Every point is a vertex.

(b) There are four vertices, namely $\{(a, 0), (0, b), (0, -a), (0, -b)\}$.

(c) There is one vertx at $(0, 0)$.

(d) There is no vertex.

4. $A \leftrightarrow t^2, \quad B \leftrightarrow t^2 - 4, \quad C \leftrightarrow t^2 + 1.$

Tutorial 04 [21/09/12]

1. $\dfrac{1}{2}(\cos 2t, 1 + \sin 2t)$.

2. The curvature is not planar. The equation of osculating plane at $t = 0$ is $3x + 3y - 3\sqrt{2}z = 2$.

3. (a)

$$
\begin{aligned}
\mathbb{T}(t) &= \left(-\frac{a}{\sqrt{a^2+b^2}}\sin t, \frac{a}{\sqrt{a^2+b^2}}\cos t, \frac{b}{\sqrt{a^2+b^2}}\right)\\
\mathbb{B}(t) &= \left(\frac{b}{\sqrt{a^2+b^2}}\sin t, -\frac{b}{\sqrt{a^2+b^2}}\cos t, \frac{a}{\sqrt{a^2+b^2}}\right)\\
\mathbb{N}(t) &= (-\cos t, \sin t, 0)\\
\kappa(t) &= \frac{a}{a^2+b^2}\\
\tau(t) &= \frac{b}{a^2+b^2}
\end{aligned}
$$

   (b)

$$
\begin{aligned}
\mathbb{T}(t) &= \frac{1}{\sqrt{3}}(-\sin t + \cos t, \sin t + \cos t, 1)\\
\kappa(t) &= \frac{e^{-t}\left(\sqrt{5 - \sin 2t}\right)}{3\sqrt{3}}
\end{aligned}
$$

4. **Hint**: Consider the function $f(t) = \gamma(t).\gamma(t)$. To maximize distance, you may maximize f. Now differentiate and see.

5. (b) **Hint**: Let $\mathbb{T}(t).\vec{u} = \cos\theta$, where $\vec{u}$ is a fixed unit vector. Differentiate and conclude that $\vec{u}$ is orthogonal to $\mathbb{N}(t)$, and hence in the plane of $\mathbb{B}(t)$ and $\mathbb{T}(t)$. Write $\vec{u} = a\,\mathbb{B}(t) + b\,\mathbb{T}(t)$ and differentiate once more. Use Serret-Frenet equations.

   (c) Check that $\tau = \kappa$.

Tutorial 05 [28/09/12]

1. **Hint**: Use laws of differentiation of vector valued functions and Serret-Frenet equations.

Tutorial 06 [03/10/12]

1. Condition $\tau = 0$ gives $\dddot{f} + \dot{f} = 0$. Solving one gets $f(t) = a\cos t + b\sin t + c$. One could get it directly without the calculation of τ.

3. Upto rigid motion there is only one such curve, and one such curve is the helix $\gamma(t) = \dfrac{1}{2}(\cos t, \sin t, t)$.

4.

$$
\begin{aligned}
\kappa(t) &= \frac{2\sqrt{3}t^3}{2\sqrt{2}(1+t^2+t^4)^{3/2}}\\
\tau(t) &= 0
\end{aligned}
$$

5.

$$
\begin{aligned}
\kappa(t) &= \frac{\sqrt{5 + 3\cos^2 t}}{2(1+\cos^2 t)^{3/2}}\\
\tau(t) &= \frac{5\cos t}{5 + 3\cos^2 t}\\
\mathbb{T}(t) &= \frac{1}{\sqrt{1+\cos^2 t}}(-\sin 2t, \cos 2t, \cos t)\\
\mathbb{B}(t) &= \frac{1}{\sqrt{5+3\cos^2 t}}\left(\sin t(1 + 2\cos^2 t), -2\cos^3 t, 2\right)\\
\mathbb{N}(t) &= \frac{1}{\sqrt{1+\cos^2 t}\sqrt{5+3\cos^2 t}}\left(2\sin^2 t - 2\cos^4 t, -\sin 2t\left(\frac{5}{2} + \cos^2 t\right), -\sin t\right)
\end{aligned}
$$

6.

$$\mathbb{T}(t) = \frac{1}{t^2+2}\left(2, 2t, t^2\right)$$
$$\mathbb{B}(t) = \frac{1}{t^2+2}\left(t^2, -2t, 2\right)$$
$$\mathbb{N}(t) = \frac{1}{t^2+2}\left(-2t, -t^2+2, 2t\right)$$