IDC 201
Astrophysics assignment
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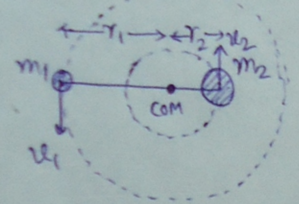
||[Question 1]||

For an eclipsing binary the observed maximum radial velocities for the two stars are 20 km/s and 5 km/s respectively. The period is 5 years. After the eclipse starts, it takes 0.3 days for intensity to fall to its minimum. The duration of the eclipse is 1.3 days. Assume that orbits are circular and also that the orbit is seen edge on.

- Find the mass of each star
- Find the radius of each star

Solution ⇒



$\Rightarrow \boxed{m_1 r_1 = m_2 r_2}$

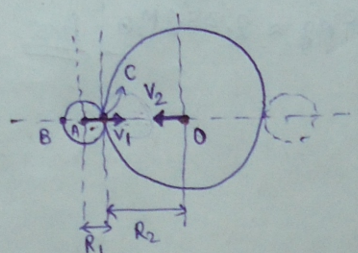
$\Rightarrow m_1 v_1 = m_2 v_2$

$\Rightarrow \frac{m_1}{m_2} = \frac{v_2}{v_1}$

$\Rightarrow \frac{m_1}{m_2} = \frac{5}{\cancel{20}\,4}$

$\Rightarrow \boxed{m_2 = 4m_1}$ —①

$\omega = \frac{v_1}{r_1} \Rightarrow \boxed{r_1 = \frac{v_1}{\omega}}$; $\boxed{r_2 = \frac{v_2}{\omega}}$



(edge-on view)

☆ We are taking our reference frame on 'm_2' ⇒

$|v_{m_1/m_2}| = (v_1 + v_2)$

☆ Time taken by 'm_1' to completely enter 'm_2' $= \frac{2R_1}{|v_{m_1/m_2}|}$
[Intensity will fall to minimum]

$\Rightarrow 0.3 \times 24 \times 3600 = \frac{2R_1}{25 \times 10^3}$

$\Rightarrow \cancel{R_1 = 13,500\ km} \Rightarrow \boxed{R_1 = 324{,}000\ km} \approx (0.46 R_\odot)$

☆ Duration of eclipse $= \frac{2(R_1+R_2)}{|v_{m_1/m_2}|}$

$\Rightarrow \frac{1.3 \times 24 \times 3600 \times 25 \times 10^3}{2} = R_1 + R_2$

$\Rightarrow \boxed{R_2 = 1{,}080{,}000\ km} \approx (1.55 R_\odot)$

☆ $\boxed{\omega = \sqrt{\frac{(m_1+m_2)G}{(r_1+r_2)^3}}}$ (for binary stars)

$\Rightarrow \omega = \sqrt{\frac{(m_1+m_2)\,G\,\omega^3}{(v_1+v_2)^3}} \Rightarrow \cancel{\omega} = \cancel{\omega}\sqrt{\frac{(m_1+m_2)G\omega}{(v_1+v_2)^3}} \Rightarrow \frac{(25\times10^3)^3}{\omega} = (6.67\times10^{-11})(m_1+m_2)$

$\Rightarrow m_1+m_2 = \frac{(25\times10^3)^3\,(5\times365\times24\times3600)}{(6.67\times10^{-11})(2\pi)} \Rightarrow \boxed{m_1+m_2 = 5.87\times10^{30}\,kg}$

$\Rightarrow 4m_1 + m_1 = 5.87 \times 10^{30}$ kg

$\Rightarrow \boxed{m_1 = 1.175 \times 10^{30} \text{ kg}} \approx 0.587 M_\odot$

* $m_2 = 4m_1$

$\Rightarrow \boxed{m_2 = 4.703 \times 10^{30} \text{ kg}} \approx 2.352 M_\odot$

* $R_1 = 324{,}000$ km $\approx 0.46 R_\odot$
* $R_2 = 1{,}080{,}000$ km $\approx 1.55 R_\odot$
* $m_1 = 1.175 \times 10^{30}$ kg $\approx 0.587 M_\odot$
* $m_2 = 4.703 \times 10^{30}$ kg $\approx 2.352 M_\odot$

Ans

[[Question 2]]

The molecular weight 'μ' is defined as the average mass of a molecule when multiple species are present. If the fraction of Hydrogen by mass is denoted by 'X' and the fraction of Helium is denoted by 'Y' then write an expression for 'μ' assuming that both species are fully ionized. How does this change if Helium is singly ionized instead?

Solution ⇒

* Let the total mass of gases is 'M'
* Mass of Hydrogen gas = MX
* Mass of Helium gas = MY
* Number of Hydrogen atoms $(X') = \frac{XM}{m_H}$
* Number of Helium atoms $(Y') = \frac{YM}{m_{He}}$

* We know that ⇒

$$\boxed{\mu = \frac{\overline{m}}{m_H}}$$

[$\overline{m}$ = weighted average of mass of individual species of molecules present.
m_H = mass of Hydrogen atom.]

$$\overline{m} = \frac{MX + MY}{2X' + 3Y'}$$

$$\Rightarrow \overline{m} = \frac{MX + MY}{\left(\frac{2XM}{m_H} + \frac{3YM}{m_{He}}\right)}$$

[$H \rightarrow H^+ + e^-$
$He \rightarrow He^+ + e^-$
$He^+ \rightarrow He^{++} + e^-$]

$$\Rightarrow \overline{m} = \frac{\cancel{M}(X+Y)}{\cancel{M}\left(\frac{2X}{m_p} + \frac{3Y}{4m_p}\right)}$$

($m_p \rightarrow$ mass of proton)
$(X+Y=1)$

$$\Rightarrow \overline{m} = \frac{1}{\frac{1}{m_p}\left(2X + \frac{3Y}{4}\right)}$$

$$\Rightarrow \boxed{\overline{m} = \frac{m_p}{\left(2X + \frac{3Y}{4}\right)}} \Rightarrow \mu = \frac{\overline{m}}{m_H (\approx m_p)} \Rightarrow \boxed{\mu = \frac{1}{\left(2X + \frac{3Y}{4}\right)}}$$

[Both are fully ionized]

Ans

* When, He is singly ionized, we have ⇒

$$\Rightarrow \overline{m} = \frac{m_p}{\left(2X + \frac{2Y}{4}\right)} \Rightarrow \frac{\overline{m}}{m_p} = \frac{1}{\left(2X + \frac{Y}{2}\right)} \Rightarrow \boxed{\mu' = \frac{1}{\left(2X + \frac{Y}{2}\right)}}$$

Ans

[Question 3]

Use the Virial theorem and show that the average pressure:

$$\boxed{\bar{P} = -\frac{1}{3}\frac{\langle E_{gr}\rangle}{V}}$$

where, 'V' is the volume of the star. Use values of $M_\odot$ & $R_\odot$ and assume that the sun is made up purely of ionized Hydrogen. Estimate '$\bar{P}_\odot$'.

Solution ⇒

* According to Virial theorem ⇒

$$\Rightarrow 2\langle E_{th}\rangle + \langle E_{gr}\rangle = 0$$

$$\Rightarrow \cancel{2}\left(\frac{3}{\cancel{2}}NkT\right) = -\langle E_{gr}\rangle$$

$$\Rightarrow \left[NkT = \frac{-\langle E_{gr}\rangle}{3}\right] \quad \text{—— (1)}$$

* Number of particles $(N) = nN_A$ [n = no. of moles; N_A = Avogadro number]

* Substituting value of 'N' in (1) ⇒

$$\Rightarrow nN_AkT = -\frac{\langle E_{gr}\rangle}{3}$$

$$\Rightarrow n(N_Ak)T = -\frac{\langle E_{gr}\rangle}{3} \qquad [N_Ak = \mathbf{R} \text{ Universal gas constant}]$$

$$\Rightarrow n\mathbf{R}T = -\frac{\langle E_{gr}\rangle}{3} \qquad [\bar{P}V = nRT]$$

$$\Rightarrow \bar{P}V = -\frac{\langle E_{gr}\rangle}{3}$$

$$\Rightarrow \boxed{\bar{P} = -\frac{1}{3}\frac{\langle E_{gr}\rangle}{V}}$$ Proved

* For the Sun, we have ⇒

$$\left[M_\odot = 2\times10^{30}\,kg \quad \& \quad R_\odot = 6.96\times10^{8}\,m\right]$$

$$\Rightarrow \bar{P}_\odot = -\frac{1}{3}\left[\frac{\left(-\frac{3GM_\odot^2}{5R_\odot}\right)}{\left(\frac{4}{3}\pi R_\odot^3\right)}\right]$$

$$\Rightarrow \boxed{\bar{P}_\odot = \frac{3GM_\odot^2}{20\pi R_\odot^4}}$$

$$\Rightarrow \bar{P}_\odot = \frac{3(6.67\times10^{-11})(2\times10^{30})^2}{20\pi(6.96\times10^{8})^4}$$

$$\Rightarrow \boxed{\bar{P}_\odot = 5.43\times10^{13}\ N/m^2}$$ Ans

[Question 4]

We believe that the main source of energy in stars in nuclear fusion. In main sequence stars this is due to conversion of Hydrogen into Helium. Conversion of four Hydrogen nuclei into a Helium nucleus in the p-p chain in low mass stars results in the release of 26.2 MeV into components other than neutrinos. In stars more massive than $4M_\odot$ the primary channel for conversion is the C-N-O cycle and here around 25 MeV is released into components other than neutrinos.

(a.) Assuming that each star converts a fixed fraction, say 15% of its mass from Hydrogen to Helium, write an expression for the life time of stars as a function of the mass & Luminosity.

(b.) Use the known parameters of the Sun to estimate its lifetime. You may assume that Helium fraction in the Sun is 0.26 & that the rest of it is in the form of Hydrogen.

(c.) If the luminosity of the stars scales in proportion with the mass as $M^{3.5}$ then find out the dependence of the lifetime of stars on the mass.

Solution ⇒

$$4H \longrightarrow He \quad 26.2\ \text{MeV} + \nu_e \quad [M_* < 4M_\odot]$$

$$\text{CNO-cycle} \quad 25\ \text{MeV} + \nu_e \quad [M_* > 4M_\odot]$$

(a.) Fraction of M_* converted from Hydrogen to Helium = 15 %

* We know that ⇒ $[L_* \propto M_*^3]$ $\left[\begin{matrix} L_* \rightarrow \text{Luminosity of star} \\ M_* \rightarrow \text{Mass of star} \end{matrix}\right]$

* $\boxed{\dfrac{L_*}{L_\odot} = \left(\dfrac{M_*}{M_\odot}\right)^3}$ $\boxed{L_\odot = 4 \times 10^{26}\ \text{J s}^{-1}}$

* Energy released per proton $= \dfrac{26.2\ \text{MeV}}{4}$

* No. of protons in stars converted to He $= \left[\dfrac{M_* \times 0.15}{m_p}\right]$

* Total energy released $= \left(\dfrac{26.2}{4} \times \dfrac{M_* \times 0.15}{m_p}\right)$ MeV

$$= (5.88 \times 10^{26}\ \text{MeV}) \times M_*$$

* Lifetime of star $= \dfrac{\text{Energy released}}{\text{Luminosity}}$

$$\Rightarrow T_* = \frac{(5.88 \times 10^{26}\ \text{MeV})\ M_* (M_\odot)^3}{(M_*)^3 L_\odot}\ s \Rightarrow \boxed{T_* = 1.8816 \times 10^{78} (M_*)^{-2}\ s}$$

$[M_* < 4M_\odot]$ Ans

* For $[M_* > 4M_\odot] \Rightarrow$

* Energy released per proton $= \frac{25}{4}$ MeV

* $\boxed{T'_* = 1.7954 \times 10^{78} (M_*)^{-2} s}$ Ans $[M_* > 4M_\odot]$

(b) No. of protons in sun converting to He $= \frac{\overbrace{(0.74 M_\odot)}^{\text{(mass of 'H' in sun)}} \times (0.15)}{m_p}$

Energy released per proton $= \frac{26.2}{4}$ MeV

$$\left[\text{Total Energy released} = \frac{0.74 M_\odot \times 0.15 \times 26.2}{4 m_p} \text{ MeV}\right]$$

* $T_\odot$ (Lifetime of the Sun) $= \left(\frac{0.74 \times 0.15 \times 26.2 \times 2 \times 10^{30} \times 10^6 \times 1.6 \times 10^{-19}}{4 \times 1.67 \times 10^{-27} \times 4 \times 10^{26}}\right)$

$\Rightarrow T_\odot = 3.4828 \times 10^{17}$ seconds

$\Rightarrow \boxed{T_\odot = 1.1044 \times 10^{10} \text{ years}}$ Ans

(c) Case: I $\Rightarrow [M_* < 4M_\odot]$

* $T_* = \frac{\text{Energy released}}{\text{Luminosity}}$ $\left[L_* = \left(\frac{M_*}{M_\odot}\right)^{3.5} L_\odot\right]$

$\Rightarrow T_* = \frac{(5.88 \times 10^{26} \times M_*) \text{MeV} (M_\odot)^{3.5}}{(M_*)^{3.5} L_\odot}$ sec

$\Rightarrow T_* = \frac{2.352 \times 10^{-13} M_* (M_\odot)^{3.5}}{(M_*)^{3.5}}$ sec

$\Rightarrow T_* = 26.6098 \times 10^{105} (M_*)^{-2.5}$ sec

$\Rightarrow \boxed{T_* = 2.66098 \times 10^{106} (M_*)^{-2.5} \text{ sec}}$ Ans

* Case: II $\Rightarrow [M_* > 4M_\odot]$

* $\boxed{T_* = 2.5391 \times 10^{106} (M_*)^{-2.5} \text{ sec}}$ Ans

[[Question 5]]

The virial theorem states that $2\langle E_{th}\rangle + \langle E_{gr}\rangle = 0$ for gravitating systems. Here 'E_{th}' is the thermal energy & can be written as $3NkT/2$, where 'N' is the total number of particles, 'k' is the Boltzmann constant & 'T' is temperature. 'E_{gr}' is the gravitational binding energy & equals $3GM^2/5R$ for an object with constant density. Use this to express the virial temperature in terms of the mass and radius of the object. Calculate virial temperature for Sun ($T_\odot$).

Solution ⇒

* According to Virial theorem ⇒

$$\Rightarrow 2\langle E_{th}\rangle + \langle E_{gr}\rangle = 0$$

$$\Rightarrow 2\left(\frac{3}{2}NkT\right) + \left(\frac{-3}{5}\frac{GM^2}{R}\right) = 0$$

$$\Rightarrow \cancel{3}NkT = \frac{\cancel{3}GM^2}{5R}$$

$$\Rightarrow \left[T = \frac{GM^2}{5RNk}\right] \text{ — (1)}$$

* We know that ⇒

$$\Rightarrow \mu = \frac{\overline{m}}{m_H}$$

$$\Rightarrow \mu m_H = \frac{M}{N}$$

$$\Rightarrow \boxed{N = \frac{M}{\mu m_H}}$$

[N = No. of particles
m_H = mass of Hydrogen
M = Total mass of object]

* Substituting value of 'N' in (1), we get ⇒

$$\Rightarrow T = \frac{GM^{\cancel{2}}\mu m_H}{5Rk\cancel{M}}$$

$$\Rightarrow \boxed{T = \frac{GM\mu m_H}{5Rk}}$$ Ans

* For the Sun, we have ⇒

$$\begin{bmatrix} M_\odot = 2\times10^{30}\ kg \\ R_\odot = 6.96\times10^{8}\ m \end{bmatrix}$$

$$\Rightarrow T_\odot = \left[\frac{(6.67\times10^{-11})(2\times10^{30})(1.67\times10^{-27})}{(5)(6.96\times10^{8})(1.38\times10^{-23})}\right] K$$

$$\Rightarrow \boxed{T_\odot = 4.6\times10^{6}\ K}$$ Ans

[[Question 6]]

Consider a "material" radial arm extending from a galactic radius of 4 kpc to 10 kpc at some initial time. Due to differential rotation, this hypothetical radial line winds up into a "material" spiral arm. Assuming a flat rotation curve, estimate the pitch angle of the spiral after 10^{10} years.

Solution ⇒

* Assuming speed for all particles to be 'v', we can write trajectory of a particle as ⇒

$$\left[\gamma(r,t) = (r\cos\omega_r t,\ r\sin\omega_r t)\right] \quad \text{where, } \left(\omega_r = \frac{v}{r}\right)$$

* [In this 'r' is constant & 't' is varying.]

* Tangent of the circle in which a particular object is rotating can be given as ⇒

$$\Rightarrow \left[\dot{\gamma}(r,t) = (-r\omega_r\sin\omega_r t,\ r\omega_r\cos\omega_r t)\right]$$

$$\Rightarrow \|\dot{\gamma}(r,t)\| = \sqrt{r^2\omega_r^2\sin^2(\omega_r t) + r^2\omega_r^2\cos^2(\omega_r t)}$$

$$\Rightarrow \boxed{\|\dot{\gamma}(r,t)\| = r\omega_r}. \qquad \left[\cdot \leftrightarrow \frac{d}{dt}\right]$$

* If we keep 't' fixed, then the curve describes the position of the objects as a function of 'r', at some instant 't'.

$$\Rightarrow \gamma(r,t) = (r\cos\omega_r t,\ r\sin\omega_r t) \qquad \left[\begin{array}{l}\text{Now, } t \to \text{fixed} \\ r \to \text{varying}\end{array}\right]$$

$$\Rightarrow \gamma(r,t) = \left[r\cos\left(\frac{vt}{r}\right),\ r\sin\left(\frac{vt}{r}\right)\right]$$

* The tangent of the curve joining all the objects can be given as ⇒

$$\Rightarrow \gamma'(r,t) = \left[\left\{+r\frac{vt}{r^2}\sin\left(\frac{vt}{r}\right) + \cos\left(\frac{vt}{r}\right)\right\}, \left\{-r\frac{vt}{r^2}\cos\left(\frac{vt}{r}\right) + \sin\left(\frac{vt}{r}\right)\right\}\right]$$

$$\Rightarrow \gamma'(r,t) = \left[\{\omega_r t\sin(\omega_r t) + \cos(\omega_r t)\}, \{-\omega_r t\cos(\omega_r t) + \sin(\omega_r t)\}\right]$$

$$\Rightarrow \|\gamma'(r,t)\| = \sqrt{\omega_r^2 t^2(\sin^2(\omega_r t) + \cos^2(\omega_r t)) + (\cos^2(\omega_r t) + \sin^2(\omega_r t))}$$

$$\Rightarrow \boxed{\|\gamma'(r,t)\| = \sqrt{\omega_r^2 t^2 + 1}}. \qquad \left[' \leftrightarrow \frac{d}{dr}\right]$$

* $\dot{\gamma}(r,t)\cdot\gamma'(r,t) = \|\dot{\gamma}(r,t)\|\|\gamma'(r,t)\|\cos\phi$

$$\Rightarrow \cos\phi = \left[\frac{-r\omega_r^2 t\sin^2(\omega_r t) - r\omega_r^2 t\cos^2(\omega_r t)}{r\omega_r\sqrt{\omega_r^2 t^2 + 1}}\right]$$

$$\Rightarrow \cos\phi = \frac{-r\omega_r^2 t}{r\omega_r\sqrt{\omega_r^2 t^2 + 1}} \Rightarrow \cos\phi = \frac{-\omega_r t}{\omega_r\sqrt{t^2 + \frac{1}{\omega_r^2}}}$$

$$\Rightarrow \cos\phi = \frac{-t}{\sqrt{t^2 + \frac{r^2}{v^2}}}$$

$$\Rightarrow \cos\phi = \frac{-vt}{\sqrt{v^2t^2 + r^2}}$$

$$\Rightarrow \boxed{\phi = \cos^{-1}\left(\frac{-vt}{\sqrt{v^2t^2+r^2}}\right)}$$ Ans

★ Here, $\begin{bmatrix} t = 10^{10} \text{ years} \\ \bar{r} = 7 \text{ kpc} \end{bmatrix}$ $[v \simeq 250 \text{ km/s}]$

$$\cancel{\Rightarrow \phi = \cos^{-1}\left(\frac{10^{10} \times 250000}{\sqrt{10^{20}(250000)^2 + (7 \times 3.08 \times 10^{16})^2}}\right)}$$

$$\cancel{\Rightarrow \phi = \cos^{-1}(0.0115)}$$

$$\Rightarrow \cancel{\phi = 79.559} \quad \cancel{\boxed{\phi = 89.34^\circ}}$$ Ans

$$★ \quad \phi = \cos^{-1}\left[\frac{-10^{10} \times 365 \times 24 \times 3600 \times 250000}{\sqrt{(10^{10} \times 365 \times 24 \times 3600)^2 (250000)^2 + (7 \times 3.08 \times 10^{19})^2}}\right]$$

$$\Rightarrow \cos\phi = \left(\frac{-7.884 \times 10^{22}}{7.884029479 \times 10^{22}}\right) \quad \cancel{\frac{7.884 \times 10^{22}}{7.884 \times 10^{22}}}$$

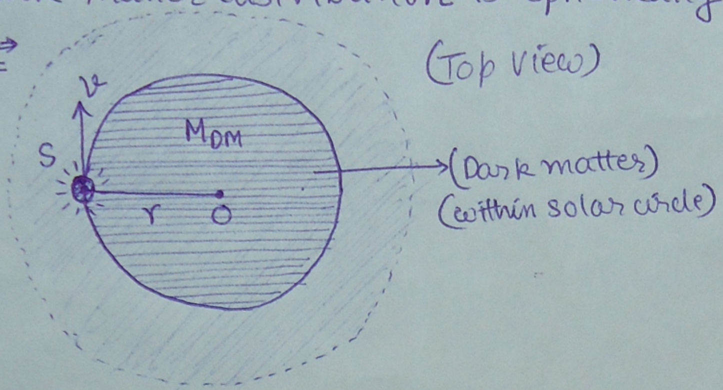
$$\Rightarrow \cos\phi = -0.99999626$$

$$\Rightarrow \cancel{\boxed{\phi = 0.1367^\circ}} \quad \boxed{\phi = 179.84^\circ}$$ Ans

[Question 7]

Given that the sun is at a distance of 8.5 kpc from the Galactic centre, and its circular speed is 240 km/s, estimate the mass of dark matter within the solar circle (assume that the dark matter distribution is spherically symmetric.)

Solution ⇒



* Assuming the dark matter distribution to be spherically symmetric we can assume the total mass of dark matter at the galactic centre 'O' [shell theorem] ⇒

$$\Rightarrow \frac{G\cancel{M_\odot} M_{DM}}{r^{\cancel{2}}} = \frac{\cancel{M_\odot} v^2}{\cancel{r}}$$

$$\Rightarrow \boxed{M_{DM} = \frac{r v^2}{G}}$$

[$r \to$ radius of solar circle
$v \to$ orbital velocity of Sun around 'O']

* Here, we have ⇒

* $r = 8.5\text{ kpc} = 8.5\,(3.08\times10^{19})\text{ m} = \boxed{2.62\times10^{20}\text{ m}}$ $(1\text{ kpc} = 3.08\times10^{19}\text{ m})$
* $v = 240\text{ km/s} = \boxed{240000\text{ m/s}}$

$$\Rightarrow M_{DM} = \frac{(2.62\times10^{17})(240000)^2\times10^3}{(6.67\times10^{-11})}$$

$$\Rightarrow M_{DM} = (2.26\times10^{38})\text{ kg}\times10^3$$

$$\Rightarrow \boxed{M_{DM} = (1.13\times10^{11})M_\odot}$$ Ans

[Question 8]

What is the mean number density of gas in ISM (Interstellar medium).

Solution ⇒

★ Assuming that ISM contains only Hydrogen, its mean number density can be given by ⇒

$$\Rightarrow \bar{\rho} = \frac{\text{number of Hydrogen atoms}}{\text{Volume of Interstellar medium } (V_{ISM})}$$

$$\Rightarrow \cancel{\bar{\rho} = \frac{(10^{10} \times 2 \times 10^{30})}{1.6}} \quad \bar{\rho} = \frac{\text{Total mass of dust}}{m_p (V_{ISM})}$$

$$\Rightarrow \bar{\rho} = \frac{10^{10} \times 2 \times 10^{30}}{(1.67 \times 10^{-27})(3.08 \times 10^{19})^3 \pi\{[(25)^2 - (15)^2]\, 0.3 + (15)^2 \times 1\}}$$

$$\Rightarrow \bar{\rho} = \frac{2 \times 10^{40}}{5.1436 \times 9.4864 \times 10^{30}\, \pi (400 \times 0.3 + 225)}$$

$$\Rightarrow \bar{\rho} = 0.378 \times 10^{-6}\ m^{-3}$$

$$\Rightarrow \boxed{\bar{\rho} = 0.378\ cc^{-1}}\ \text{Ans}$$

[Question 9]

Compute the critical density of universe $\rho_c = \frac{3H_0^2}{8\pi G}$, where $H_0 = 70\,km/s/Mpc$.

Solution ⇒

$$★\ \rho_c = \frac{3H_0^2}{8\pi G}$$

$$\Rightarrow \rho_c = \frac{3}{8}\left(\frac{70 \times 10^3}{3.08 \times 10^{22}}\right)^2 \times \frac{1}{\pi (6.67 \times 10^{-11})}$$

$$\Rightarrow \boxed{\rho_c = 2.847 \times 10^{-4}\ kg/m^3}\ \text{Ans}$$

||[Question 10]||

Find out the energy density in CMBR if the radiation has a temperature of 2.726 K. Assume a black body spectrum for the radiation. Use the answer to find out density parameter Ω_{CMBR}. Also find out the number density of photons in CMBR.

Solution⇒

★ Energy density in CMBR $= aT^4$ [a → radiation constant]

($a = 7.6\times10^{-15}$ erg $cm^{-3}K^{-4}$)

($a = 7.6\times10^{-16} J m^{-3} K^{-4}$)

$$\Rightarrow u = (7.6\times10^{-16})(2.726)^4$$

$$\Rightarrow \boxed{u = 4.1967\times10^{-14} J m^{-3}}$$ Ans

★ $$\Omega_{CMBR} = \frac{\text{Mass density}}{\rho_{critical}} = \frac{\text{Energy density}}{c^2 \rho_{critical}}$$

$$\Rightarrow \Omega_{CMBR} = \frac{4.1967\times10^{-14}}{(3\times10^8)^2(2.847\times10^{-4})}$$

$$\Rightarrow \boxed{\Omega_{CMBR} = 1.637\times10^{-27}}$$ Ans

★ No. density of photons in CMBR (n) = $\frac{\text{Energy density in CMBR}}{\text{Average energy of a photon}}$

$$\Rightarrow n = \frac{aT^4}{kT}$$

$$\Rightarrow n = \left(\frac{a}{k}\right)T^3$$

$$\Rightarrow \boxed{n = 1.1156\times10^9 m^{-3}}$$ Ans

[Question 11]

Use the definition of magnitudes to find the ~~difference in~~ ratio of flux recieved from sources where the magnitudes for these sources differ by '5'.

Solution ⇒

* The apparent magnitude is defined as ⇒

$$\boxed{m = -2.5\log_{10}(f) + C}$$ [where, $C = 2.5\log_{10}(f_0)$]

(f = flux recieved from object)

* $m_1 = -2.5\log_{10}(f_1) + C$
* $m_2 = -2.5\log_{10}(f_2) + C$
* $\Delta m = -2.5[\log_{10}(f_2) - \log_{10}(f_1)]$

⇒ $5 = -2.5\log_{10}\left(\frac{f_2}{f_1}\right)$

⇒ $-2 = \log_{10}\left(\frac{f_2}{f_1}\right)$

⇒ $\frac{f_2}{f_1} = 10^{-2}$

⇒ $\frac{f_1}{f_2} = 100$

⇒ $\boxed{f_1 = 100 f_2}$ Ans.