Problem Set 1

Astronomy & Astrophysics (IDC 201)

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IISER Mohali

1. Use the definition of magnitudes to find the difference in flux received from sources where the magnitudes for these sources differ by 5.

2. Use the relation between magnitude and distance, and the fact that the Sun has a magnitude of -26 whereas brightest stars have a magnitude of 0 to estimate the distance to these stars. You may assume that all stars have the same absolute magnitude.

3. Starting with the definition of magnitudes, rewrite the expression in terms of the luminosity of the source and its distance. If two objects have the same luminosity but magnitude differing by 2 then what is the difference in distance?

4. Show that in absence of absorption by intervening material, the difference of apparent magnitude in two bands is same as the difference of absolute magnitudes in these wave bands.

5. For an eclipsing binary the observed maximum radial velocities for the two stars are 20 km/s and 5 km/s respectively. The period is 5 years. After the eclipse starts, it takes 0.3 days for intensity to fall to its minimum. The duration of the eclipse is 1.3 days. (Note: The minimum intensity will persist until the eclipsing star is fully blocking the eclipsed star.) Assume that orbits are circular and also that the orbit is seen edge on.

    - Find the mass of each star.
    - Find the radius of each star.

6. The molecular weight μ is defined as the average mass of a molecule when multiple species are present. If the fraction of Hydrogen by mass is denoted by X and the fraction of Helium is denoted by Y then write an expression for μ assuming that both the species are fully ionized. How does this change if Helium is singly ionized instead?

7. The virial theorem states that $2\langle E_{th}\rangle + \langle E_{gr}\rangle = 0$ for gravitating systems. Here E_{th} is the thermal energy and can be written as $3NkT/2$, where N is the total number of particles, k is the Boltzmann constant and T is the temperature. E_{gr} is the gravitational binding energy and equals $3GM^2/5R$ for an object with constant density. Use this to express the *virial* temperature in terms of the mass and radius of the object. Calculate the temperature for the Sun $T_{\odot}$.

8. Use the Virial theorem and show that the average pressure:

$$\bar{P} = -\frac{1}{3}\frac{E_{gr}}{V}$$

where V is the volume of the star. Use values of $M_\odot$ and $R_\odot$ and assume that the Sun is made up purely of ionized Hydrogen. Estimate $\bar{P}_\odot$.

9. Assuming that the virial temperature of stars is almost constant in stars that are converting Hydrogen to Helium in the core, estimate the average gas pressure as a function of the mass of stars.

10. You are given that we recieve an energy flux $f_0 = 1.4$ W/m^2 from the Sun. If the Sun subtends a solid angle Ω in the sky then show that the flux at the solar surface is $\pi f_0/\Omega$. If the Sun subtends an angle $0.57°$ then use the Stefan-Boltzmann law and calculate T_{eff} for the Sun. Can we do this exercise if we only know the flux at one frequency instead of the integrated flux?

11. The angular diameters are known for very few stars. The temperature can still be derived using stellar fluxes at two different frequencies. Explain how this can be done? When will this method fail?

12. For a model star, assume that the density is independent of r. The star is made up of fully ionized Hydrogen which can be considered as a non-relativistic ideal gas. Integrating stellar structure equations with the boundary condition that the pressure vanishes at $r = R_*$, find $P(r)$ and use this to find $T(r)$.

13. Consider a situation where radiation pressure is very large inside a star. Then:

$$\bar{P} = \bar{P}_{gas} + \bar{P}_{rad} = -\frac{1}{3}\frac{E_{gr}}{V}$$

   - Write down the average pressure in terms of the average density, mass of the star and some constants.
   - If the radiation and gas pressures are equal then show that

$$\bar{P}_{rad} = \left(\frac{3}{a}\right)^{1/3}\left(\frac{k\rho}{\mu m_H}\right)^{4/3}$$

   - If radiation pressure becomes stronger than the gas pressure then the star approaches instability. Find out the expression for maximum mass of a star. Estimate the numerical value in units of the solar mass.

14. We believe that the main source of energy in stars is nuclear fusion. In main sequence stars this is due to conversion of Hydrogen into Helium. Conversion of four Hydrogen nuclei into a Helium nucleus in the p-p chain in low mass stars results in the release of

26.2 MeV into components other than neutrinos. In stars more massive than 4 $M_\odot$ the primary channel for conversion is the C-N-O cycle and here around 25 MeV is released into components other than neutrinos.

- Assuming that each star converts a fixed fraction, say 15% of its mass from Hydrogen to Helium, write an expression for the life time of stars as a function of the mass and Luminosity. You may assume that the luminosity does not change with time during the Hydrogen burning phase.
- Use the known parameters of the Sun to estimate its lifetime. You may assume that Helium fraction in the Sun is 0.26 and that the rest of it is in the form of Hydrogen.
- If the luminosity of stars scales in proportion with the mass as $M^{3.5}$ then find out the dependence of the life time of stars on the mass.
- The age of the universe is 13.7 billion years and it is believed that stars started forming when the universe was about 1 billion years old. What is the mass of the heaviest star from the first generation that can still be seen in the main sequence? You may assume $Y = 0.24$ for such a star.