Contextuality in a Deterministic Theory

January 15, 2016

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- Fundamental difference: BM is deterministic, viz. positions (q) and momenta (p) are well defined
- Aim: Construct a theoretical situation that defies determinism using QM and analyze using BM

Overview of the talk

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- Determinism: The GHZ test & Contextuality

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Bohmian Mechanics (BM)

Determinism & Contextuality

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- Immediate questions: (a) Uncertainty principle (b) Double slit, which is chosen? (c) Trajectories; observable? (d) Identical particles

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- in practice, we have a statistical ensemble with probability densities $\rho(q) = |\psi(q)|^2$

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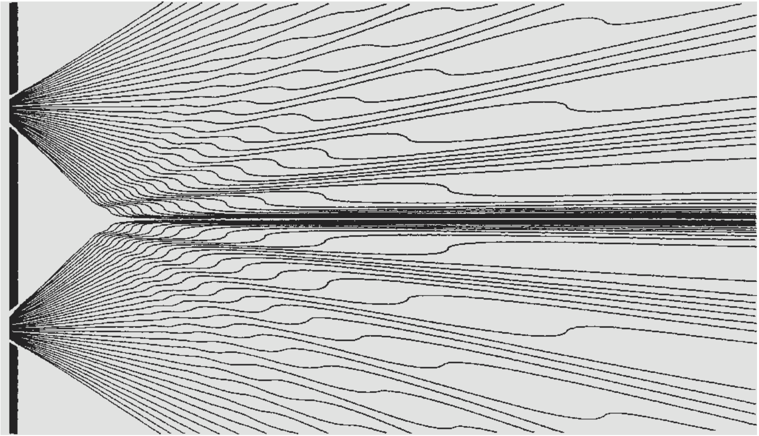
- Ready generalization to N particles. Non-locality becomes explicit; $p_i = \nabla_i S(q_1, q_2, \dots, q_N)$
- Extension to spins: Particle has (q, p). The wavefunction has the spinor, say $\Psi \equiv (\psi_+, \psi_-)^T$; the generalization is

$$p = mv = \hbar \mathrm{Im} \frac{(\Psi, \nabla \Psi)}{(\Psi, \Psi)}$$

where $(.,.)$ is the inner product in the spin space $\mathbb{C}^2$.

BM | Pictures



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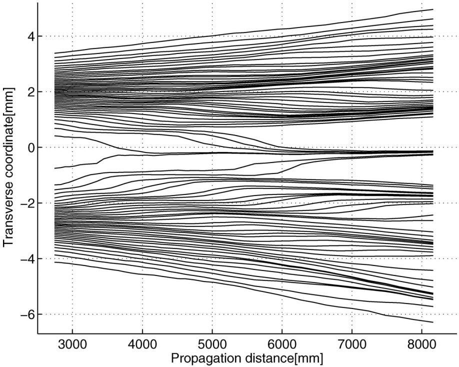

"Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer"

Bohmian Mechanics (BM)

Determinism & Contextuality

Analysis using BM

(recent) Determinism in phase space (q, p)

(in progress) Its analysis using BM

Determinism

Defn: Determinism: Observables have values regardless of whether they are measured, viz. values are predefined.

Determinism: The GHZ Test (1/2)

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- General Construction: $\hat{A} \equiv \hat{X} \otimes \hat{Y} \otimes \hat{Y}$, $\hat{B} \equiv \hat{Y} \otimes \hat{X} \otimes \hat{Y}$ and $\hat{C} \equiv \hat{Y} \otimes \hat{Y} \otimes \hat{X}$; outcome is $+1$ for each. $\hat{D} \equiv \hat{X} \otimes \hat{X} \otimes \hat{X}$ yields -1.

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- Explicit construction: $|\psi\rangle = (|000\rangle - |111\rangle)/\sqrt{2}$ ($\sigma_z |0/1\rangle = \pm |0/1\rangle$; $\hat{X}, \hat{Y}, \hat{Z}$ are Pauli spin operators

Determinism: The GHZ Test (2/2)

- Hypothesis: Assume that the world is deterministic. $\hat{A}\hat{B}\hat{C}$ must yield$= 1$. Also, $\hat{A}\hat{B}\hat{C} = \hat{D}$ (because $Y^2 = 1$). But $\hat{D}$ yields -1. Thus we get $+1 = -1$.

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- Conclusion: The hypothesis is wrong. $\implies$ can't have non-contextual determinism, where "non-contextual" is a subtle but necessary qualification.

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- 'Proof' (Mermin's):

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- NB: (1) Along a row, compatible; also along a column, (2) product along a row R_k or column C_k is 1, except for $C_3 = -1$. (Hint: $\sigma_z = -i\sigma_x\sigma_y$)

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- Remarks: (1) Mermin's test is state independent (2) More suited for testing non-contextuality, as locality is not required.

Bohmian Mechanics (BM)

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- Assmptn: (1) Stern Gerlach like measurements to measure spins; (2) $|\Psi(r_1, r_2, r_3, t = 0)\rangle = (\psi_{+++}|000\rangle - \psi_{---}|111\rangle)/\sqrt{2}$, where r_i is the position vector in the frame of the i^{th} observer. (3) Assume a Gaussian particle distribution initially, propagating along the axes of their respective SG apparatus.

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- Claim: Time evolution of $\psi_{\pm\pm\pm}$ can be written as a product of 3 single particle solutions of the SG setup. (Bohm explicitly did the latter)

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- Claim: (1) If SG are setup to measure say XYY, then four attractor basins form: $(+++)$, $(+--)$, $(--+)$ and $(-+-)$. (2) If SG are setup to measure XXX, then the basins becomes $(---)$, $(-++)$, $(+-+)$ and $(++-)$.

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- Conclusion: Consistent with QM
- Remarks: Non locality causes attractor basins to depend on settings of *all* SG apparatus. Contextuality from this perspective is essentially the statement that the results of an experiment, depend on the experiment being performed.

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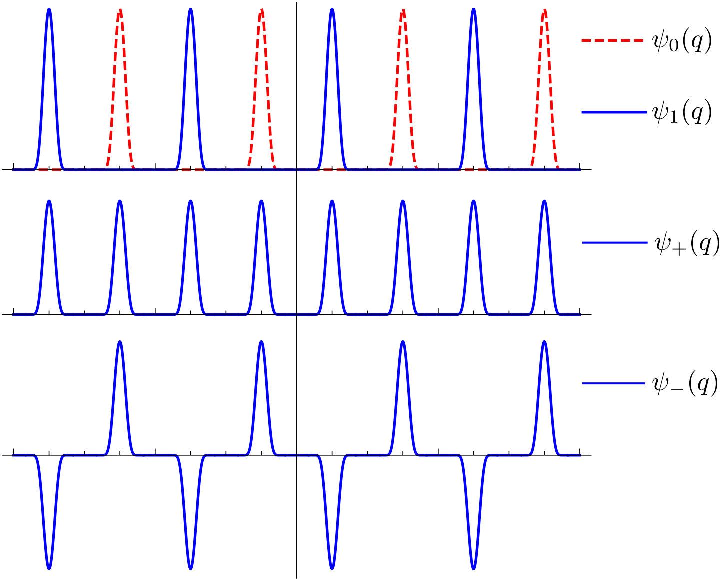
Determinism in phase space (0/3)



Determinism in phase space (1/3)

Was able to make good progress today. So here's what I did. First of all, consider the same states $|\psi_0\rangle, |\psi_1\rangle$ for $N = 8$, as those considered in my first paper. Recall that for $\hat{Z}$ as defined there, viz. $\hat{Z} = Z(\hat{q} \mod 2L)$, or more precisely as $\hat{Z} = Z(\hat{q}) = \mathrm{sgn}(\sin(\hat{q}\pi/L))$, we had $\hat{Z}|\psi_0\rangle = |\psi_0\rangle$ and $\hat{Z}|\psi_1\rangle = -|\psi_1\rangle$. In addition to this, I define $\hat{X} = e^{-i\hat{p}L/\hbar}$ (as opposed to defining it to be hermitian). Now, we know that $|\psi_\pm\rangle \equiv \frac{|\psi_0\rangle + |\psi_1\rangle}{\sqrt{2}}$ is not an eigenstate of $\hat{X}$. So we optimize the observable $\hat{X}$ to $\hat{X}' \equiv \hat{X}\hat{T}$, where $\hat{T} \equiv e^{i\hat{p}NLa(\hat{q})/2}$ where

$$a(q) = \begin{cases} 1 & 2L < q < 4L \\ 0 & \text{else} \end{cases}.$$

The idea is that you shift certain peaks to the right place, before applying the displacement operator $\hat{X}$.

Determinism in phase space (2/3)

To illustrate this, consider explicitly $|\psi_0\rangle = \left(|\varphi_{-4}\rangle + |\varphi_{-2}\rangle + |\varphi_{-1}\rangle + |\varphi_{-3}\rangle\right)/\sqrt{4}$. The operation of $\hat{T}$ is $\hat{T}|\varphi_4\rangle = |\varphi_{-5}\rangle$, $\hat{T}|\varphi_3\rangle = |\varphi_{-6}\rangle$ and $\hat{T}|\varphi_n\rangle = |\varphi_n\rangle$ for $n \in \{-4, -3, -2, -1, 1, 2\}$. It is now evident that $\hat{X}' = \hat{X}\hat{T}|\psi_0\rangle = |\psi_1\rangle$. Note also that $\hat{X}'^\dagger|\psi_0\rangle = |\psi_1\rangle$. Similarly $\hat{X}'|\psi_1\rangle = |\psi_0\rangle$ and $\hat{X}'^\dagger$ does the same. So finally, now consider $|G\rangle \equiv \left(|\psi_0\psi_0\psi_0\rangle - |\psi_1\psi_1\psi_1\rangle\right)/\sqrt{2}$. With $\hat{A} \equiv \hat{X}' \otimes \hat{Y}' \otimes \hat{Y}'^\dagger$, where $\hat{Y}' \equiv i\hat{Z}\hat{X}'$, calculations yield $\hat{A}|G\rangle = |G\rangle$. With $\hat{B} \equiv \hat{Y}'^\dagger \otimes \hat{X}' \otimes \hat{Y}'$ and $\hat{C} \equiv \hat{Y}' \otimes \hat{Y}'^\dagger \otimes \hat{X}'$ also, by symmetry we get $\hat{B}|G\rangle = |G\rangle$ and $\hat{C}|G\rangle = |G\rangle$. Now $\hat{D} \equiv \hat{A}\hat{B}\hat{C} = \hat{X}' \otimes \hat{Y}'\hat{X}\hat{Y}'^\dagger \otimes \hat{X}'$ and $\hat{E} \equiv \hat{X}' \otimes \hat{X}' \otimes \hat{X}'$ yield the paradox. If values were predefined, the value of $\hat{D}$ and $\hat{E}$ would return the same answer. However, a simple calculation yields $\hat{D}|G\rangle = |G\rangle$ (this can be seen directly by applying $\hat{A}$, $\hat{B}$ and $\hat{C}$ sequentially on $|G\rangle$ [was figured the next day]), while $\hat{E}|G\rangle = -|G\rangle$.

Determinism in phase space (3/3)

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- Counterexample: BM, a theory which is NC deterministic, in q, p.

(in progress) Its analysis using BM

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- Measuring $\hat{X}'$ is not trivial.
  - Wigner function
  - Functions of q, p directly from the particles q, p
  - Formal measurement
- Formally, any Hermitian operator can be measured using the interaction Hamiltonian, $\hat{H} = -a\hat{Q}\hat{p}_y$, where $\hat{Q}$ is the operator, and y is the coordinate of the pointer.

Conclusion

- New insight into relation between contextuality and non locality
- Fundamental difference between spins and (q, p)
- Meaning of measurement
- Pointed out and (almost) solved a paradox

The End

Questions

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