# MS Thesis | Summary

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## Introduction

The purpose of this document is to

## Contents



## 1 Reg, Topic Decision, Multi-Mode Paper | August 3-8, 2015

Motivation: Construct a Bell like test for the case where the system is not two level, but d-level.

Setup: Consider a 4 mode light. One direction is $\mathbf{k}$ and the other is $\mathbf{k}'$. The annihilation operators are $a_1 = a_k^{\parallel}$ and $a_2 = a_k^{\perp}$ and similarly $a_3$ and $a_4$ are defined. Next, we also allow some photon number conserving transformations. One possibility would be $a \to Ua$, where

Recall: Single mode light: Take a plane wave like solution for a cavity and quantize it to get quantum optics. In this you'll get $a$ and $a^\dagger$ corresponding to the $k$ and direction of $E$ (or $B$) of this classical solution. N mode light: You allow arbitrary plane waves. You'll get $a_k, a_k^\dagger$ but for a given direction, you can have two polarizations (which are enough to generate all polarizations)

## 2 Ideas, Bohmian Mechanics started | August 10-15

- [GRE] Subject GRE, vocab GRE and quant GRE were done in full swing
- [Research/ms] Made a presentation about Arvind sir's paper, discussed about uncertainty in speed of light with Bhati being the chief guest, started reading Bohmian Mechanics from Holland, thought about Bohmian Mechanics and how to make it relativistic; (figured that there must be non locality in built as it is, because even in QFT this nonlocality doesn't disappear. However, in
- [protocol f]

## 3 arXiv and Hamilton Jacobi for Bohmian | August 17-22

### 3.1 Monday | Aug 17

- [official] Summer project form
  - had to get the certificate from Ali/Otfried
  - had to get Arvind sir's signatures
- [research/summer] Started proof reading the paper
  - Had issues with the discussion section; definition $X = E_+ - E_-$ having only two values as outcomes :(

### 3.2 Tuesday | Aug 18

- [research/ms] Met with Arvind sir
  - Kichoo finalized his project. He's not doing Bohmian Mechanics
  - I finalized my project (as I had earlier) to Bohmian Mechanics and Contextuality
- [research/summer] Read Haridichi's paper about measuring a bounded observable using a two level system
  - Implicitly it had used results from POVMs; so had to read about POVMs from Nielsen's book
  - Couldn't still fully understand what Ali was doing, I figured that $A_i^\dagger A_i$ type of operators if they satisfy $\sum A_i^\dagger A_i = \mathbb{I}$, then one can talk about probability of getting the $i^{th}$ input as $\left\langle A_i^\dagger A_i \right\rangle$. Ali was using $E_\pm$

and the fact that $E_+ + E_- = \mathbb{I}$ as his scheme. However, for projectors, since $P^2 = P$ and if $P^\dagger = P$, then the condition stated earlier essentially becomes $\sum P_i = \mathbb{I}$ and that makes sense. However, they aren't projectors. So I wasn't sure what was happening.

- [gre] Vocab and Quant

## 3.3 Wednesday | Aug 19

- [research/ms] nill :'(
- [research/summer] Put the paper on arXiv
  - Ali updated the minor modifications slightly (I still wasn't comfortable with $A^{two} = E_+ - E_-$ having only two possible values) | wasn't very happy intially but figured it today that this statement is not quite needed and had sent the update to Ali, who while made minor changes, wasn't too happy :(
  - Putting it on arxiv required some debugging etc. | moral of the story is to use PDF as images instead and not using hyperref defiend explicitily
  - Ali was happy by the end of it
- [research/official] Presentation of work at the QCQI group meet
  - Went reasonably well
- [GRE] words + little bit of quant

## 3.4 Thursday | Aug 20

- [official] Registration/course add/drop etc. taken care of
- [research/] The $E_\pm$ issue was understood in more detail after talking to Arvind sir
- [research/ms] Read some more things from Holland
  - Looked at the section on propogation fo the S function
  - started reading the section on Classical Statistical Mechanics
    * He talks about how in CStat Mech, we use the function $f(x, p, t)$ to describe a probability, whereas here we use the less general density $\rho(x, t)$ where the $p$ has been specified.
    * Derives the continuity equation (by demanding the the particles be conserved)
    * He talks about some special cases; viz. specific solutions making some assumptions about the dynamics
- [GRE] words/vocab, reading section from Manhattan.
- [protocol f] Earthlings

## 3.5 Friday | Aug 21

- [official] KVPY report printed, printed the form again, submitted it to the dean's office
  - Went to the bank for NET
- [research/ms] Holland
  - Finally figured why $\frac{d}{dt}d\Omega = (\nabla.v)d\Omega$; just looked at Aris and it was right there. The idea was to use the jacobian to describe the change in volume and then everything follows.
  - Tried to look up papers related to Bohmian Mechanics and Contextuality (couldn't find too many papers related to this)
    * In Quantum Physics without Quantum Philosophy, 3.8.3 talks about contextuality (looks very wordy)
    * In Bohmian Mechanics and Quantum Theory, page 67 is a chapter on contextuality
  - Can I think of a way of constructing a theory that's local, but not real? Can reality then be emergent? Does it mean that the moment I say I don't assume reality, then it must mean that my theory must have some sort of measurement and then it is essentially a combination of worst of both worlds?
  - Found the following interesting overview of Bohmian Mechanics: `http://philsci-archive.pitt.edu/3026/1/bohm.pdf`
- [GRE] words/vocab, quant (finished the geometry section)

## 3.6 Saturday | Aug 22

- [official] Some silly German Research Opportunity thing
- [research/ms] Why must contextuality only be talked about in the context of spins? What about phase space contextuality?
  Reading Holland; following is a summary.

1. To understand the continuity equation $\rho + \nabla.(\rho v) = 0$, examples of $v$ are taken
   (a) $v = v(x)$, a solution is obtained as
   $$\rho(x,t) = \frac{1}{v(x)} v\left[x\left(t - \int \frac{dx}{v}\right)\right]\rho_0\left[x\left(t - \int\frac{dx}{v}\right)\right]$$
   in which further assuming that $\rho(x)$ results in $\rho = A/|v(x)|$
      i. $v = v(t)$ then we get
      $$\rho(x,t) = \rho_0\left(x - \int v dt\right)$$
      which means that $\rho$ is constant along particle trajectories
   (b) connection with Liuoville's equation

i. $f(x,p,t)$ is defined instead of $\rho(x,t)$. Pure and mixed states are defined accordingly as $f(x,p,t)=\rho(x,t)\delta(p-\nabla S(x,t))$ being pure and the remaining as mixed.

ii. $\frac{df}{dt}=\partial_t f+\frac{1}{m}\sum p_i\partial_{x_i}f-\sum\partial_{x_i}V\partial_{p_i}f=0$ is the Liuoville's equation (which holds since we can show that the volume doesn't change under Hamiltonian evolution and particles inside the volume stay inside; $f(p',q',t+\delta t)=f(p,q,t)$ is essentially the statement $\frac{df}{dt}=0$) which is linear in $f$.

iii. One may project out the moment space. They define equivalent of $\rho$ as $P(x)=\int f d^3p$, mean momentum as $\overline{p_i(x)}=\frac{\int p_i f d^3p}{P(x)}$ and $\overline{p_ip_j(x)}=\frac{\int p_ip_j f d^3p}{P(x)}$. The louvielle equation can then be expressed in terms of these spatial variables. Integrating it we get

$$\partial_t P+\frac{1}{m}\sum\partial_{x_i}(P\overline{p_i})=0.$$

To get the momentum transport equation, after multiplying the louvielle equation with $p_i$ and integrating, we get

$$\partial_t(P\overline{p_i})+\frac{1}{m}\sum\partial_{x_j}(P\overline{p_ip_j})+P\partial_{x_i}V=0$$

(apparently integrated by parts and assumed $f\to 0$ as $p_i\to\infty$)
While $f$ is constant along a phase space trajectory, the spatial density $P$ (equivalent of $\rho$) is not. It's apparent from the derivation of the continuity equation; either we start with a fixed volume or a fixed number of particles, not both.
If you substitute $f=\rho\delta(p-\nabla S)$ as stated earlier, you'd get $P=\rho$, $\overline{p_i}=\partial_{x_i}S$, $\overline{p_ip_j}=\partial_{x_i}S\partial_{x_i}S$ as expected. The substitution also yields what's called a field theoretic version of Newton's Laws given by

$$\partial_t\rho+\frac{1}{m}\nabla.(\rho\nabla S)=0$$

and

$$\left[\partial_t+\frac{1}{m}\sum\partial_{x_i}S\partial_{x_j}\right]\partial_{x_i}S=0$$

iv. Remarks:

A. It's not obvious that if we start with a state that has well defined momentum (delta distribution) but the positions are given by $\rho(x)$, then they will continue to be well defined in momentum. This happens only exceptionally. In general, a pure state maybe sent to a mixed state. We'll see examples of these. [todo: ensure examples make sense]

B. Can we decompose any mixed ensamble into a linear combination of pure ones? The answer's no. [proof?] Say there are many solutions of the Hamilton-Jacobi equation, given by $S_i$. Thus, we can construct a linear combination as $f(x,p,t)=\sum P_i\rho_i(x,t)\delta\left(p-\nabla S_i(x,t)\right)$ where $P_i$ (degenerate notation) refers to the distribution of momenta at a given point. $\sum P_i=1$ is assumed for normalization. Claim is that this is not in general possible to decompose a state into this form. An explicit example is that of reflecting through a potential barrier (in CM) [todo: ensure the example works]

C. While this is not particularly useful in CM (the pure and mixed states), the formalism helps in comparison with QM.

(c) Pure and Mixed States

i. Illustration: We see that $f_0(x,p)=\delta(x-x_0)\delta(p-p_0)$ remains sharp
(it can be checked by inserting it in the louviel equation) to yield $f(x,p)=\delta(x-x(t,x_0,p_0))\delta(p-p(t,x_0,p_0))$ [this is expected, since you're in essence saying there's only one particle]

ii. Illustration 2: We want to see what happens to a Gaussian like state, does it spread?
We start with $\rho_0=\frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}$ and $S_0=px$ with $\sigma$ and $p$ constant. This form of $S_0$ has already been solved for and tells us $\rho=\frac{e^{-(x-vt)^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}$. There's no spreading classically! We'll see for the same initial conditions, what happens quantum mechanically.

iii. Illustration 3: What initial conditions yield a spreading Guassian? We start with the same $\rho_0$ but use $S_0=\frac{m(x-x_0)^2}{2t}$, in which case the solution we saw the result is

### 3.7 Sunday | Aug 23

- Summarizing the work I did on saturday.

# 4 Aug 24-29

## 4.1 Monday | Aug 24

- [research/ms] Worked on making notes about Bohmian Mechanics

- [misc] Found something called *SparkleShare* which is dropbox like with git under the hood. Works great. I can switch off the automated git uploading whenever I like and switch to manual git. For the usual things, I can use it like a dropbox folder :)