MS Thesis | Summary

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Contents

Introduction

The purpose of this document is to be evidence of and support the steps taken in the direction of innovating for the master's thesis.

Part I
Perfunctory Systematics of Research

1 Reg, Topic Decision, Multi-Mode Paper | August 3-8, 2015

Motivation: Construct a Bell like test for the case where the system is not two level, but d-level.

Setup: Consider a 4 mode light. One direction is $\mathbf{k}$ and the other is $\mathbf{k}'$. The annihilation operators are $a_1 = a_k^{\parallel}$ and $a_2 = a_k^{\perp}$ and similarly a_3 and a_4 are defined.
Next, we also allow some photon number conserving transformations. One possibility would be $a \to Ua$, where

Recall: Single mode light: Take a plane wave like solution for a cavity and quantize it to get quantum optics. In this you'll get a and $a^{\dagger}$ corresponding to the k and direction of E (or B) of this classical solution.
N mode light: You allow arbitrary plane waves. You'll get $a_k, a_k^{\dagger}$ but for a given direction, you can have two polarizations (which are enough to generate all polarizations)

2 Ideas, Bohmian Mechanics started | August 10-15

- [GRE] Subject GRE, vocab GRE and quant GRE were done in full swing
- [Research/ms] Made a presentation about Arvind sir's paper, discussed about uncertainty in speed of light with Bhati being the chief guest, started reading Bohmian Mechanics from Holland, thought about Bohmian Mechanics and how to make it relativistic; (figured that there must be non locality in built as it is, because even in QFT this nonlocality doesn't disappear. However, in
- [protocol f]

3 arXiv and Hamilton Jacobi for Bohmian | August 17-22

3.1 Monday | Aug 17

- [official] Summer project form
  - had to get the certificate from Ali/Otfried
  - had to get Arvind sir's signatures
- [research/summer] Started proof reading the paper
  - Had issues with the discussion section; definition $X = E_+ - E_-$ having only two values as outcomes :(

3.2 Tuesday | Aug 18

- [research/ms] Met with Arvind sir
  - Kichoo finalized his project. He's not doing Bohmian Mechanics
  - I finalized my project (as I had earlier) to Bohmian Mechanics and Contextuality
- [research/summer] Read Haridichi's paper about measuring a bounded observable using a two level system
  - Implicitly it had used results from POVMs; so had to read about POVMs from Nielsen's book
  - Couldn't still fully understand what Ali was doing, I figured that $A_i^\dagger A_i$ type of operators if they satisfy $\sum A_i^\dagger A_i = \mathbb{I}$, then one can talk about probability of getting the i^{th} input as $\left\langle A_i^\dagger A_i \right\rangle$. Ali was using $E_\pm$ and the fact that $E_+ + E_- = \mathbb{I}$ as his scheme. However, for projectors, since $P^2 = P$ and if $P^\dagger = P$, then the condition stated earlier essentially becomes $\sum P_i = \mathbb{I}$ and that makes sense. However, they aren't projectors. So I wasn't sure what was happening.
- [gre] Vocab and Quant

3.3 Wednesday | Aug 19

- [research/ms] nill :'(
- [research/summer] Put the paper on arXiv
  - Ali updated the minor modifications slightly (I still wasn't comfortable with $A^{two} = E_+ - E_-$ having only two possible values) | wasn't very happy intially but figured it today that this statement is not quite needed and had sent the update to Ali, who while made minor changes, wasn't too happy :(
  - Putting it on arxiv required some debugging etc. | moral of the story is to use PDF as images instead and not using hyperref defiend explicitily
  - Ali was happy by the end of it
- [research/official] Presentation of work at the QCQI group meet
  - Went reasonably well
- [GRE] words + little bit of quant

3.4 Thursday | Aug 20

- [official] Registration/course add/drop etc. taken care of
- [research/] The $E_\pm$ issue was understood in more detail after talking to Arvind sir
- [research/ms] Read some more things from Holland
  - Looked at the section on propogation fo the S function
  - started reading the section on Classical Statistical Mechanics
    - He talks about how in CStat Mech, we use the function $f(x,p,t)$ to describe a probability, whereas here we use the less general density $\rho(x,t)$ where the p has been specified.
    - Derives the continuity equation (by demanding the the particles be conserved)
    - He talks about some special cases; viz. specific solutions making some assumptions about the dynamics
- [GRE] words/vocab, reading section from Manhattan.
- [protocol f] Earthlings

3.5 Friday | Aug 21

- [official] KVPY report printed, printed the form again, submitted it to the dean's office
  - Went to the bank for NET
- [research/ms] Holland
  - Finally figured why $\frac{d}{dt}d\Omega = (\nabla.v)d\Omega$; just looked at Aris and it was right there. The idea was to use the jacobian to describe the change in volume and then everything follows.
  - Tried to look up papers related to Bohmian Mechanics and Contextuality (couldn't find too many papers related to this)
    - In Quantum Physics without Quantum Philosophy, 3.8.3 talks about contextuality (looks very wordy)
    - In Bohmian Mechanics and Quantum Theory, page 67 is a chapter on contextuality
  - Can I think of a way of constructing a theory that's local, but not real? Can reality then be emergent? Does it mean that the moment I say I don't assume reality, then it must mean that my theory must have some sort of measurement and then it is essentially a combination of worst of both worlds?
  - Found the following interesting overview of Bohmian Mechanics: http://philsci-archive.pitt.edu/3026/1/bohm.pdf
- [GRE] words/vocab, quant (finished the geometry section)

3.6 Saturday | Aug 22

- [official] Some silly German Research Opportunity thing
- [research/ms] Why must contextuality only be talked about in the context of spins? What about phase space contextuality? Reading Holland; following is a summary.

1. To understand the continuity equation $\rho + \nabla.(\rho v) = 0$, examples of v are taken
   (a) $v = v(x)$, a solution is obtained as
   $$\rho(x,t) = \frac{1}{v(x)} v\left[x\left(t - \int \frac{dx}{v}\right)\right] \rho_0\left[x\left(t - \int \frac{dx}{v}\right)\right]$$
   in which further assuming that $\rho(x)$ results in $\rho = A/|v(x)|$
      i. $v = v(t)$ then we get
      $$\rho(x,t) = \rho_0\left(x - \int v dt\right)$$
      which means that ρ is constant along particle trajectories
   (b) connection with Liuoville's equation
      i. $f(x,p,t)$ is defined instead of $\rho(x,t)$. Pure and mixed states are defined accordingly as $f(x,p,t) = \rho(x,t)\delta(p - \nabla S(x,t))$ being pure and the remaining as mixed.
      ii. $\frac{df}{dt} = \partial_t f + \frac{1}{m}\sum p_i \partial_{x_i} f - \sum \partial_{x_i} V \partial_{p_i} f = 0$ is the Liuoville's equation (which holds since we can show that the volume doesn't change under Hamiltonian evolution and particles inside the volume stay inside; $f(p', q', t+\delta t) = f(p,q,t)$ is essentially the statement $\frac{df}{dt} = 0$) which is linear in f.
      iii. One may project out the moment space. They define equivalent of ρ as $P(x) = \int f d^3p$, mean momentum as $\overline{p_i(x)} = \frac{\int p_i f d^3p}{P(x)}$ and $\overline{p_i p_j(x)} = \frac{\int p_i p_j f d^3p}{P(x)}$. The louvielle equation can then be expressed in terms of these spatial variables. Integrating it we get
      $$\partial_t P + \frac{1}{m}\sum \partial_{x_i}(P\overline{p_i}) = 0.$$
      To get the momentum transport equation, after multiplying the louvielle equation with p_i and integrating, we get
      $$\partial_t(P\overline{p_i}) + \frac{1}{m}\sum \partial_{x_j}(P\overline{p_i p_j}) + P\partial_{x_i}V = 0$$
      (apparently integrated by parts and assumed $f \to 0$ as $p_i \to \infty$)
      While f is constant along a phase space trajectory, the spatial density P (equivalent of ρ) is not. It's apparent from the derivation of the continuity equation; either we start with a fixed volume or a fixed number of particles, not both.

If you substitute $f = \rho\delta(p - \nabla S)$ as stated earlier, you'd get $P = \rho$, $\overline{p_i} = \partial_{x_i} S$, $\overline{p_i p_j} = \partial_{x_i} S \partial_{x_i} S$ as expected. The substitution also yields what's called a field theoretic version of Newton's Laws given by

$$\partial_t \rho + \frac{1}{m}\nabla.(\rho\nabla S) = 0$$

and

$$\left[\partial_t + \frac{1}{m}\sum \partial_{x_i} S \partial_{x_j}\right] \partial_{x_i} S = 0$$

iv. Remarks:

A. It's not obvious that if we start with a state that has well defined momentum (delta distribution) but the positions are given by $\rho(x)$, then they will continue to be well defined in momentum. This happens only exceptionally. In general, a pure state maybe sent to a mixed state. We'll see examples of these. [todo: ensure examples make sense]

B. Can we decompose any mixed ensamble into a linear combination of pure ones? The answer's no. [proof?] Say there are many solutions of the Hamilton-Jacobi equation, given by S_i. Thus, we can construct a linear combination as $f(x,p,t) = \sum P_i \rho_i(x,t)\delta\left(p - \nabla S_i(x,t)\right)$ where P_i (degenerate notation) refers to the distribution of momenta at a given point. $\sum P_i = 1$ is assumed for normalization. Claim is that this is not in general possible to decompose a state into this form. An explicit example is that of reflecting through a potential barrier (in CM) [todo: ensure the example works]

C. While this is not particularly useful in CM (the pure and mixed states), the formalism helps in comparison with QM.

(c) Pure and Mixed States

i. Illustration: We see that $f_0(x,p) = \delta(x - x_0)\delta(p - p_0)$ remains sharp
(it can be checked by inserting it in the louviel equation) to yield $f(x,p) = \delta(x - x(t, x_0, p_0))\delta(p - p(t, x_0, p_0))$
[this is expected, since you're in essence saying there's only one particle]

ii. Illustration 2: We want to see what happens to a Gaussian like state, does it spread?
We start with $\rho_0 = \frac{e^{-x^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}$ and $S_0 = px$ with σ and p constant. This form of S_0 has already been solved for and tells us $\rho = \frac{e^{-(x-vt)^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}$. There's no spreading classically! We'll see for the same initial conditions, what happens quantum mechanically.

iii. Illustration 3: What initial conditions yield a spreading Guassian? We start with the same ρ_0 but use $S_0 = \frac{m(x-x_0)^2}{2t}$, in which case the solution we saw the result is

3.7 Sunday | Aug 23

- Summarizing the work I did on saturday.

4 Reached the chapter on Bohmian Mechanics | Aug 24-29

4.1 Monday | Aug 24

- [research/ms] Worked on making notes about Bohmian Mechanics
- [GRE] words, vocab; quant
- [misc]
  - KVPY document received; had to attend a PhD defence; fixed the NET application issue (talked to Bagla sir etc.); got black and white print outs; scanned various documents
  - Found something called *SparkleShare* which is dropbox like with git under the hood. Works great. I can switch off the automated git uploading whenever I like and switch to manual git. For the usual things, I can use it like a dropbox folder :)

4.2 Tuesday | Aug 25

- [research/ms] Worked on making notes about Bohmian Mechanics from Holland
- [misc] Gave the NET document for resubmission.
- [GRE] vocab (questions), quant

4.3 Wednesday | Aug 26

- [research/ms] Started the Bohmian Mechanics part from Holland!! QCQI group meeting; I think it was Vikrams. He talked about Quantum Simulation; about tunneling.
- [misc] Registered for TOEFL
- [GRE] minor work

4.4 Thursday | Aug 27

- [research/ms] Reading Bohmian Mechanics part from Holland
- [misc] Scanning, printing etc., packing
- [GRE] word list creation on Inkscape

4.5 Friday | Aug 28 [off, travel]

- [GRE] word lists

4.6 Saturday | Aug 29 [off]

5 Midsem, Presentation, Bohmian Mechanics (made sense of insolent multi-valued integration..) | Aug 31 - Sep 6

5.1 Monday | Aug 31 [off, travel]

- [course] Philosophy Reading the book

5.2 Tuesday | Sep 1

- [course] Philosophy: Found *@voice* an application that can read out things from the phone; used gscan2pdf for converting the notes sagar had sent to OCRed PDF (which isn't very good to look at) which @voice played out :D Also made proper notes for most of the 1st chapter of the book by Benn.

5.3 Wednesday | Sep 2 [exam]

- [course] Philosophy: studied and took the exam
- [research/ms] Started making notes on the Bohmian Mechanics part, (chapter 3) from Holland; got a place in CAF to sit and study (an office if you will)
- [misc] gave away the sweets

5.4 Thursday | Sep 3

- [research/ms] Presentation: Kishor talked about LOCCs, majorization, the iff condition between them, entanglement distillation and entanglement of creation and finally how they're related to key distribution (he barely started);
  - Bohmian mechanics working on figuring that $\oint dS = nh$ and tried constructing the cases when this could happen
- [misc] Tug of war practice + Abhishek sir nomination + re arranging the room in CAF + landscape thing
- [gre] verbal GRE work done

5.5 Friday | Sep 4

- [research/ms] Presenation; mine: Talked about how to get to the Hamilton Jacobi equations and took some time. Working out the differential equations is hard. Completed till about page 4 of my notes. It went well
  - Bohmian mechanics: figured the relation with the curl theorem of $\oint dS$ and atleast it now makes a little more sense. Need to work out the details still though; the point is that I now know which curl related theorem could be used, about which I wasn't certain until now.
- [misc] Table chair thing + landscape thing + multiple protocols
- [gre] quant GRE

5.6 Saturday | Sep 5

- [research/ms] Concluded that I'll just ask sir now for some advice. What I'm doing doesn't seem to help;

  So we start with $\oint dS = \oint \nabla S.dx = \oint p.dx = \int \nabla \times p.da$. If $\oint dS = nh$, then we must have $\nabla \times p = \sum_a \Gamma_a \int_{\gamma_a} \delta(x - x_a) dx_a$ where γ_a is the nodal line. If we assume $\oint dS = nh$ holds, then can we construct some example of the same? Let's first see how $\oint dS = nh$ can be derived. If the only condition is that ψ is single valued, then we know that at any point, S' and S both yield the same ψ, where $S' = S + nh$. If one considers a loop, then say we start from a point S_a. Then after completing some distance, the change in S is given by ΔS. So the value of S starting from $S_a$will be $S_a + \Delta S$. Now if we come back to the point a, then from uniqueness of ψ, we only demand $S_a + \Delta S = S_a + nh$. If S itself was unique, then we'd say $S_a + \Delta S = S_a$. Now at this point itself I seem to have trouble. I have tacitly assumed that S is single valued when I'm evaluating the 'change is S' along the curve.

  Talked to manu for a while and made some progress, then figured it was non-sense and made some more progress. Finally, Manu found a document that helped clarify a few things. The issue was still that they had used a vector field and not a potential. And it wasn't clear to me what potential must I use in that case.

- [misc] words GRE (prashansa came to CAF) + teacher's day (abhishek sir was awarded) + dinner

5.7 Sunday | Sep 6

- [research/ms] Finally found a potential that works (looked at acheson, griffiths and an extra document that manu had found.)

  The potential is $V = k\theta$. Note how this is itself, as a function of position is multi valued and yet we never have any issues integrating this (as we'll see shortly). While V is multivalued, $\nabla V = \frac{k}{r}\hat{\theta}$ is happily single valued :) And not just that, check this; $\oint_\gamma \nabla V.dx = 2\pi$ (simply because γ is chosen to be a circle and then $dx = rd\theta\hat{\theta}$). Since in the domain of interest, everything is well defined, I can write $\oint_\gamma \nabla V.dx = \oint_\gamma dV = 2\pi$. And one can show independently (I know only a simple minded proof with discritizing the function) that $\oint dV = 0$ whenever V is single valued (or a function). So what does this example show? Various rather peculiar things. (I) that $\oint dV$ maybe non zero for a reasonable physical situation by virtue of multivaluedness of V. Yes, V is multivalued and yet we can integrate the said expression without ambiguity. (II) that there happens to be a singularity within the loop, over which the integral is non-zero. (III) The curl, $\nabla \times \nabla V \neq 0$ at the center and $= 0$ else.
  Now we've made plausible various things which would've seemed arbitrary otherwise.

- [misc] trying to get a template in which to write the thesis

6 Temerity of GRE prep, Sycophancy or Ascendency of Bohmian Mechanics | Sep 7 - 12

6.1 Monday | Sep 7

- [research/ms] Worked out an explicit example of $\nabla \times p = \sum_a \Gamma_a \int_{L^{(a)}} \delta(\mathbf{x} - \mathbf{x}^{(a)}) d\mathbf{x}^{(a)}$ by assuming that the $d\mathbf{x}^{(a)}$ term is infact a vector and $d\mathbf{a}$ is essentially $\mathbf{n}dxdy$ to see that everything fits well eventually. Then I moved to the next section and everything seemed alright. Basically the discussion of the 'quantum potential term', viz. the term which if removed would reduce the expression to a classical hamilton jacobi equation, was slightly confusing. This term is given by $Q := -(\hbar^2/2m)\nabla^2 R/R$, and yet the claim is that this essentially depends on S. The same can be said of V which depends on $x(t)$ which in turn can be determined only once S is specified (NB: S contains information about both the Hamiltonian and the momenta of the system). So in this sense, both V and Q depend on S. However, the distinction seems to be made on the following ground: if $V = V(x)$ then, ∇S is sufficient. However, for Q it seems higher derivatives will also be necessary. Why this makes any difference, I am yet to learn.

- [GRE] did 2 tests in the morning (both verbal, am still miserable at them :( )

6.2 Tuesday | Sep 8

- [research/ms] Revised work I did yesterday.

- [research/summer] started the process of submission to PRA. Figured how to change the name from Dr. to without Dr. and then am waiting for Ali; he has to tell me if I should use the arXiv link or not. Made reasonable progress at putting things up online. The main trouble was that I wasn't able to add references. Figured there was a command '\nocite{*}' that had to be inserted to fix things. Did that. In addition, I had to change the split images into a single image; else the system kept yielding errors. After that, I had to follow some minor steps but then I got a response from Otfried. He had completed proof reading the pre-print and had made several comments which had to be accommodated.

- [GRE] did 2 tests, 1 math, 1 verbal (need to find more tests now) | did vocab also :)

6.3 Wednesday | Sep 9

- [research/ms] Bohmian Mechanics; finished reading the section on uniqueness of the wavefunction and started reading further, about commutation relations and so on. Some summarizing etc. will be typed out here later. Also, prepared for the presentation with Arvind sir;
- [research/summer] glanced through the changes suggested by Otfried.
- [GRE] morning slot: didn't wake up | vocab only

6.4 Thursday | Sep 10

- [research/ms] :(Didn't do much but thought of the following. Why can't I think of a bohmian like picture, after a particle has been created for instance, according to QFT. Perhaps I should start playing with the scalar field theory and try to see how I can get bohmian trajectories into them somehow. I suppose the eventual goal would be to think of an alternate interpretation to the field interpretition. I feel that there's some better way of handling these things. What does Hamilton Jacobi translate to? What happens to the Dirac equation? In QFT we treat the dirac equation as a field and create particles off of it. What will we do here?
- [research/summer] working on fixing the paper based on Otfried's comments. Minor fixes were quick. Fixing some references took time. The issue was that one has to use {} for evaluating commands within bibtex's .bib file. Didn't know that. Also fixed some other minor things. Located the PACS numbers related to the paper, started working on a cover letter and found appropriate emails for referees.
- [GRE] morning slot: read a little about the writing comprehension etc. but was too tired, slept after breakfast | did vocab words :)

6.5 Friday | Sep 11

- [research/ms] Thought about the relationship between QM and QFT. Even tried to derive the $[q, p]$ commutation starting from $[\phi, \pi]$. Figured ofcourse that if there're no commutations in Bohmian. This makes the Field theory aspect harder. Also got Bohm's original papers and started reading them.
- [research/summer] Finalized the cover letter and finally, submitted it to PRA. My first submission :D
- [GRE] afternoon slot will be used
- [misc] Phys Majors had a meeting.

6.6 Saturday | Sep 12 [weekend]

- [GRE] GRE vocab + test
- [misc] There was a discusison on entropy/information by Raja Ram Mohan which was intriguing, although most of what he said was familiar.

6.7 Sunday | Sep 13 [weekend]

- [GRE] prep
- [misc] Onam, tug of war

Hegemony of GRE | Sep 13 - Sep 24

- [GRE] prep all the time :(

Besieging exams and beyond | Sep 25 - Sep 30

- Took the TOEFL (Sep 26) and the GRE(Sep 28)
- Relaxed for about half a day + Personal efficiency improvement | standing straight improvements; wifi linux broadcast attempts (made improvements, but not successful); other improvements

7 Reprisal of Physics | Oct 1 - 3

7.1 Thursday | Oct 1

- [misc] Room efficiency tasks | washed clothes, cleaned the room, found the old phone (with the sim), resuming tasks (calendars, emails (KVPY, deanacad etc.), PhD application issues etc.
- [research/ms] just resumed reading
- [GRE S] classical mechanics questions (Klepner and Kolenkow)

7.2 Friday | Oct 2

- [GRE S] classical mechanics primarily (Klepner and Kolenkow) + had the first help course
- [research/ms] resumed reading Bohm's paper
- [Misc.]

7.3 Saturday | Oct 3

7.4 Sunday | Oct 4

8 Indignant time contraction; Immuring Physics, Applications and Bohm | Oct 5-10

8.1 Monday | Oct 5

8.2 Tuesday | Oct 6

8.3 Wednesday | Oct 7

- [GRE S]
- [research/ms] Was trying to translate bohmian mechanics to the discrete case. Realized there'll be an issue with the grad operator but then in the case of spins, we never write the kinetic energy term! Then some discussion with Jaskaran got me realize the following rather interesting conflicting statements, at least for spins. (I) QM is non local; Essentially if you assume locality, you can show determinism must exist, using EPR type states. Now one can use Bell's inequality to show QM is non local. (II) QM is non-deterministic can be shown using contextuality arguments and coloring theorems.; Comment A: Regarding the conclusion of (I), it is weakened by the fact that one can show you in QM, you can't communicate faster than speed of light. So in this sense the non locality is, well is it there at all?[1] Comment B: Disregard (I) for the moment. Usually in starting a proof for Bell's theorem, we assume that both locality and determinism hold (whether one can be derived from another is another matter). Thus a violation entails atleast one of the assumptions is wrong. We don't know which. From (II) it seems therefore that atleast determinism is false.[2] Comment C: If only (I) were true, then constructing a theory such as Bohmian mechanics may seem strange because then non-locality is explicit, but in some sense more sensible for we don't need 'observers' to make sense of what we're saying. If (II) is true even in continuous variables, then it would seem meaningless to even imagine constructing a theory such as Bohmian Mechanics.[3] Conclusion: One needs to come up with either a contextuality test for continuous variables (Ali's work) or figure how to setup Bohmian Mechanics for discrete variables.

8.4 Thursday | Oct 8

- [GRE S]
- [research/ms] got sick of trying to figure the spin thing and decided to linearly read the papers by Bohm first and then thinking about what to do next.

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[1] Infact, one can show that the bell's inequality can be violated by 2 if the only assumption is no communication. QM does $\sqrt{2}$. So whatever this non-locality, it is certainly more restrictive than simply enforcing 'relativity'.

[2] I had been told that in contextuality, certain other assumptions have to be made (of which I'm not certain).

[3] There maybe defenses such as the observables are not really the same as the object's position/momentum etc. but more on that later

8.5 Friday | Oct 9

- [GRE S] Drude model etc.
- [research/ms] Bohm's paper; thinking about why it is that we can't talk about gravity essentially like electric fields. And then the obvious question that arises is that can there be a magnetic analogue of electric fields? | Jaskaran gave a small talk on Contextuality. He discussed various things including the KSBS inequality etc. Also briefly discussed the overview of the field and the progress. He concluded with discussing the nature paper about the Magic qbit etc.
- [Misc.]

8.6 Saturday | Oct 10

- [GRE S] griffiths, electrodynamics first chapter, classical mechanics revision
- [research/ms] Bohmian Mechanics (resumed reading the paper)
- [Misc.]

8.7 Sunday | Oct 11

- [Misc.] Philosophy exam prep
- [research/ms] Casually thought of linking Ekert and EPR protocols, primarily kichoo's idea + suggested a modification to jaskaran and kichoo's protocol

9 Emaciated resurrection of physics and inclemency of all else | Oct 12 - 16

9.1 Monday | Oct 12

- [Misc.] Philosophy exam prep, talk on Quantum Theory and SpaceTime (a person from IISER P had come and there was this student of his who in his fifth year (had in his fourth infact) published over 3 papers on the said subject! Insane), various things for the PhD application to leeds done

9.2 Tuesday | Oct 13

- [Misc.] Philosophy exam; iGuess: I figured how Stern Gerlach is analyzed in Bohmian Mechanics, also I read about how contextuality etc. it handled in bohmian mechanics. This was from Durr's book.
- [research/ms] Thought about how one could use the quantized harmonic oscillator analysis (Bohmian Mechanics) on Quantum Optics and see if it makes enough sense. Obviously this would not be about photons, but then what would be can probably be explored. I found later that he (Bohm) has already considered some such cases. This is particularly relevant if I want to apply the contextuality analysis (one that's extended to continious variables) to bohmian mechanics to see what's going wrong.

9.3 Wednesday | Oct 14 [unwell :(]

- [group meet] Bhati's talk was rather interesting, about collapsing of wavefunction etc.

9.4 Thursday | Oct 15

- [research/ms] Reading/thinking about Bohmian Mechanics;
- [sGRE] usual revision

9.5 Friday | Oct 16

- [research/ms] Could barely do anything :(
- [Misc.] Had to go get a draft done for TIFR; found the toothbrush (for Arjit); tried to get Headphones (for being able to work in the office); finally even ordered them

10 Subject GRE misconstrued as a Hiatus? | Midsem break (Oct 16 - Oct 25)

- Subject GRE prepared for well :) went home and back after the exam;

11 Demurred Obstacles | Oct 26 - Nov 1

11.1 Monday | Oct 26

- [Misc.] Couldn't get enough rest, started slow; fixing things at the office (resuming efficient work); organizing things, updating calendars etc.; there was an earthquake today; updated various things
- [research/ms] resumed reading Bohm's paper | Finalized some things:
  - (a) Bohmian Mechanics: (i) Discuss the basic formalism (ii) Discuss how measurements are done (the hardest part to explain) (iii) Explain how a position measurement may not even yield the 'true' position (iv) Talk about spins; the stern gerlach in terms of Bohmian mechanics;
  - (b) Contextuality: describe it;
  - (c) Ideas worth exploring: (i) Extension to EM fields, not with photon trajectories as the essential target but for (ii) The relation between Bohmian and Contextuality; (iii) How there maybe ontological models s.t. measurement disturbs the values of the hidden variable; this is essentially how they explain spins, or even position/momentum uncertainty etc.; the basic idea then is that how is Bell's test acceptable? Because the contextuality tests are also built like so and they get away with it by saying that contextuality is about spins, and that is not really ontological. However, now they even have continuous variable contextuality! These need to be phrased clearly.
- [research/ms] Started looking for a poster template and started filling it slightly. Also tried getting lyx to work using textext but that didn't work. They haven't updated to make it functional in the new version.

11.2 Tuesday | Oct 27

- [research/ms] Meeting with Arvind sir | described the basic idea of the poster (emailed it to him). In the next meeting with sir, we concluded that we (read Jaskaran and Kishor) write down all versions of the algorithm. Apparently my small modification itself can become the main protocol, once security can be assessed well. That part still has to be explored.
- [research/summer] Tried to look at the issues pointed out by the referee and tried addressing those I could. Wrote to Ali. He said he'd be able to handle most. I updated the technical issues (about figures etc.)
- [misc] Had the philosophy lecture; Also thought about why it is that we don't have superposition of charges, like we have superposition of magnetic moment; I mean we have states like $|\psi\rangle = \frac{|\uparrow\rangle+|\downarrow\rangle}{\sqrt{2}}$ but we don't have a neutron being in a superposition of + charge state and - charge state. I guess then this is the starting of particle physics in some sense. But this is still a curious thing, the charge for B field can be in a superposition but the charge for E field can't be.
- [health] running initiated + gym

11.3 Wednesday | Oct 28 (Unwell)

- [research/ms] Arun delivered the talk. He was insisting on the epistemic view of science. He said two things which are relevant for my work. First was the idea of a reduced density operator and its connection with Bohmian Mechanics. Detlef's book has discussed this. The next idea which is more directly relevant at the moment is that of the GHZ test. He was happily asserting that there the notion of reality must be given up. It was then obvious for me to use this as the starting point for my project. The continuous version of the GHZ test already exists and it would be fun to see how it works/fails with bohmian mechanics.

11.4 Thursday | Oct 29

- [research/summer] Updated some things that Ali had sent. There were various aspects that had to be looked at. Infact, Ali had even made a small mistake in the references (had put the wrong one). Made some minor language changes and sent it to him.
- [research/ms] Found an interesting paper that discusses how bohmian mechanics handles Bell, GHZ and more. Infact, finally I figured precisely what it is that I'll work on. Instead of looking at contextuality per say, I would instead look at the GHZ test generalized to position and momentum. The paper I found could be useful in setting up the system. In essence then I must look at the following: (1) How is measurement generalized in Bohmian Mechanics [detlef's article should suffice] (2) GHZ test in continuous variables (3) GHZ test in Bohmian mechanics
- [remarks/arvind]: Contextuality and GHZ, Nielsen doesn't prove the ancilla and POVM statement
- [misc] Got an email from the University of Leeds asking for my TOEFL scores and stating that Dr. Beige is interested in supervising my project.

11.5 Friday | Oct 30

- [research/ms] started figuring how to write the first simulation.
- [research/summer] Ali had responded. Made appropriate changes. Made some language changes. Sent it to Ali for a final glance before resubmission.
- [misc] Shopping (food) + nutrition optimization started; jelly lost her keys and found it!

11.6 Saturday | Oct 31 (unwell)

- [research/summer] Did a final review of the changes and resubmitted things to PRA.
- [research/ms] Formulated the thesis problem more clearly (in an email I wrote to Ali). Writing the small goals for the project to work on (constructed the appended topics part).
- [FYI] for lyx, to cite things, first Insert -> List .. -> BiBTEX .. and add your bibtex file. Thereafter, just use Insert -> Citation and you're done. Also, to restart numbering after a section, add \@addtoreset{section}{part} to the preamble. And finally, to add a book for citation, use this website isbt-to-bibtex.

11.7 Sunday | Nov 1

- [PhD] Applications; found this loop quantum gravity guy in France; wrote to him (he responded, as I'll find later). Have to write a letter of motivation/proposal etc. and that's about it.
- [research/ms] Found something called the relative interpretation of QM. COuldn't fully understand/appreciate it. Also learnt some GK about loop quantum gravity.

12 Voyage of Veracity, BM QM | Nov 2-9

12.1 Monday | Nov 2 (20 hours up time!)

- [research/ms] Working on the numerics aspect: Found various documents on simulating the schrodinger equation [1, 2]. They range from using RK4 to using more interesting hybrid QFT approaches. Setup the whole thing in fortran from my old chaosTerm project. Got the basics up and running. I can initialize to a Guassian now.
- [misc.] extreme shopping, project nutrition seems to have been erected successfully; also figured how to keep my schedule fixed

12.2 Tuesday | Nov 3

- [research/ms] Idea: Why is it that position/momentum can't be used to harness, essentially arbitrary quantumness? Why do we rely on formalism similar to spins for extending to the CV setting?; Next, working on the numerical simulations. It's going good. About to simulate the schrodinger equation in a very simple case. Then we'll see the bohmian trajectory for a single particle!; In practice however, I am running into issues with attempting to simulate the schrodinger equation. The point is that $\dot{\psi}$ is needed at arbitrary q and that's not possible without interpolating $\nabla^2\psi(q)$.
  Since I couldn't find anyone (surprisingly) to help me with this, I simply interpolate, using Ref [3]. However, I just realized (after having almost written the whole code) that the spline interpolation is for reals, not for complex. I think it can be readily extended, but then I'd have to mess around with spline function! :(
- [misc.] talked to dad after a long time, started making philosophy notes on the computer now

12.3 Wednesday | Nov 4

- [research/ms] Working on the interpolation; accommodating interpolation of complex by treating them as 2 reals. So basically in fortran, here's one nice way of handling things. [todo: complete this seciton]
- [research/summer] Got a response from the referees. Wrote a response (this time Ali asked me to do it straight!) and sent it to him. He suggested another change. Added that and sent it. He then wanted me to confirm what I had written. I did and reverted to him. He also read and confirmed that what I'd done was correct. He infact read it within a half hour and was even able to explain certain things I hadn't read, that were discussed in the paper. All in all, made the required changes and sent them before sleeping.

12.4 Thursday | Nov 5

- [research/summer] The paper got accepted today!
- [research/ms]

12.5 Friday | Nov 6

- [revision] Spent the day reading about Corioli's force, it was rather interesting. I was wondering if there's a quantum corioli's effect. And it turns out that there's one paper that discusses this.

12.6 Saturday | Nov 7

- [revision] Was reading about corioli's force etc. to prepare myself for presenting in front of the class, also did some coordinate systems (spherical, polar etc.) and in the actual session could only complete coordinate systems. Also made a nice feedback + attendance sheet and from the feedback it looked like they're happy with what they learnt.

12.7 Sunday | Nov 8

- [research/ms] finished putting all the pieces together and debugging all the errors to get the program to run for the first time, only to realise that RK4 can't work in principle. After evaluating $m1$, I realized that I can't do $q + m1 \times dt$ simply because $m1$ is not guarenteed to be real. I tried fixing this by taking absolute values and it still wouldn't give the right results. Spent some time in the night in trying to convert it to euler to see if something improves. No help. Eventually found a small *mistake* in the second difference formula I was using.

13 Perusing through the Tumult of the Palimpsest Code | The hegemony of constructing the exegesis of my work | Nov 9-15

13.1 Monday | Nov 9

- [research/ms] Working on trying to figure why the code's not working. Now I'm in a situation where I 'print' an object (of custom type) before sending to a function and then inside that function. They produce different results. WTH! Anyway, trying to figure this. In one attempt found a way to add procedures to types themselves (like a class). Apparently, the proof of concept code works in isolation but is'nt working with my code. Trying to figure that. Man Fortran's killing me today. So figured that. Here's the deal: when using procedures within a type, do the following:

```
type contVar
        integer :: someVariables
        contains
                procedure, pass :: contVarAllocate
end type contVar

contains
subroutine contVarAllocate(this,val)
        class(contVar) :: this
        integer :: val
        this%someVariables=val
end subroutine contVarAllocate
```

  The most important point in all of this is that you've to use the keywords *class* when you're using the datatype inside a class. The drawback is that doesn't fix the issue still :(
  Next I tried debugging using dgb. Spent quite some time, only to find that indeed before and after sending the values are different. :(
  Eventually tried writing a code with an allocatable array separately and that apparently does work alright. Alright, *figured* it. The issue was that the return type of my function was real and the value I intended to return was complex. So for some reason, that issue was rising.
  Even after resolving it, it just won't work. Neither with Euler nor with RK4. It just won't evolve properly. The norm goes crazy. Now I have no clue why it isn't working.

13.2 Tuesday | Nov 10

- [research/ms] Still can't figure what's really wrong. Checked the $\nabla^2\psi$ evaluated using the discrete difference formula and compared it to differentiating the interpolated polynomial. The latter looked better, but the difference isn't all that marked. I suppose I'd be forced to use FFT after all. I suppose the basic difficulty is that in this $\dot{\psi}$ involves a double derivative of ψ itself, whereas in all previous cases that I considered, $\dot{q}$ will usually be a function of q^m etc. Anyway, changing the approach at this stage doesn't feel feasable. I doubt anyway that this will work.

- Talked to sudeshna ma'am. While talking to her, realized that we have to keep the $\Delta t/\Delta x^2 \leq 10^{-6}$ or so for euler and 10^{-4} or so for RK4. I had made another small mistake. I was returning only the real part of $\nabla^2\psi$ initially (in the differentiation of the interpolate polynomial method).

- Talked to Abhishek sir and realized that I wasn't really using RK4. I had made a mistake in understanding how to implement. Affter the discussion figured that, fixed it and it started working! The mistake was that I was using different xs for finding $k_1, k_2, \ldots$ whereas to evaluate k_2 for example, I needed $x + 0.5\,dt\,k_1$ but $k_1$is complex in general. So I already had hints on things going wrong. But I realised after that $\psi + 0.5\,dt\,k_1$ is what I had to use because in some sense, I'm treating ψ as a functional. So, given an x, I have to find $\dot{\psi}$ at different ψs not different xs. So that fixed things. It was sad though because the interpolation code was completely useless!

13.3 Wednesday, Thursday | Nov 11, 12

- [research/ms] Numerics:
  - Got almost all basic stages to work. Was able to get the harmonic oscillator (as was done yesterday). Improved on the parameters. Then attempted to get one particle trajectory. And it worked!
  - As it turns out, interpolation was not useless but rather important. This is because I needed to find $\dot{q}$ at different qs and $\dot{q}$ is a function of $\psi(q)$. So in effect I needed both ψ and $\nabla\psi$ at arbitrary points. This worked well because I had already the interpolation code.
  - Next was to generalize this to handle an ensamble of particles. This was done by extending the interpolation code to handle arrays naturally and then the beauty of fortran kicked in and everything eventually started working smooth. Ofcourse I had made minor mistakes here and there, for example trying to add precision(real) to ψ so that I don't get a division by zero in $\dot{q}$ but that doesn't work like expected.
  - Anyway, after all of this, I realized that I need one last thing. I must start with particles distributed according to $|\psi|^2$and then evolve them. This did work finally. For this I wrote an algorithm, which was similar to the one I figured as an answer to Bagla sir's question in the computational physics course. That was a standard method of using the cumulative distribution and then inverting it. This new method was basically something that can be, unlike the former, generalized to arbitrary dimensions (at least intuitively, haven't proven this).
    * The algorithm is simply this. Start with finding the maximum of the probability density function, call it $P_{\max}$. Then construct an interval from 0 to $P_{\max}$ and successively choose a point in this interval, and move in constant steps; at some iteration, call this point h. Next, construct random numbers uniformly in the relevant range (over which the PDF is defined; viz. the relevant domain if you're fussy). Let's call it for instance x. Then check if $P(x) > h$. If it is, then you accept this point (save it in some array that'll represent the set of points which follow the said distribution). Else you reject the point. Repeat this a fixed number of times for each h. The claim is the set of points you get will satisfy the same PDF. Why it works is intuitively obvious. Start with considering $h = 0$. All points are accepted. Then as h increases, fewer and fewer points are accepted; viz. those which are more likely according to the PDF, get more representation in the final set of points. This is readily generalizable.
    * There is however a major draw back. How many particles you'll have in the final array is stochastic in this algorithm. However, this can be easily fixed. Consider the further generalization. Find h uniform randomly and then find x uniformly randomly as stated. If x is accepted, good else repeat till you have enough particles. Because of the stochastic nature of x, increasing h sequentially, or selecting it randomly, both are equivalent. The latter however allows for an obvious way of fixing the number of particles.

13.4 Friday, Saturday | Nov 13, 14 [intensity: 30 hours up out of 32 hours elapsed]

- [research/ms] Working on finalizing the poster (here's the final 1). Did various things. Read the paper that had analysed the GHZ and Bell case with Bohmian Mechanics, then looked at contextuality (the pierse mermin version) and finally looked at the phase space GHZ test. I liked the final output. Getting it printed was a chore by itself. However I was told later that there's too much text in it.

- [teaching] Took a class on Coriolis force. It was fun. Couldn't do too many examples though.

- [misc] Food improvement failed again today.

Bohmian Mechanics and Contextuality in (q,p)

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Introduction

Abstract

There are at least two known theories that describe the same physical world; Quantum Mechanics and Bohmian Mechanics (also uses the Schrödinger Equation). While the latter is not popularly known, it provides exceptional clarity about certain aspects of reality. So far, no tests are known that can distinguish these two. One fundamental difference between them is that that the latter is deterministic (in the sense that (q, p) are well defined). The purpose of this thesis is to get these theories head on; we aim to construct a theoretical situation where this type of determinism is refuted by Quantum Mechanics (using generalization of the GHZ test, contextuality etc.) and analyze it using Bohmian Mechanics. This is a step towards understanding the relation between contextuality and non-locality.

Content

I will first discuss Bohmian Mechanics and then go on to discuss the known standard tests of determinism (GHZ and contextuality). Thereafter I will describe how the GHZ test is explained from the Bohmian perspective and also discuss a generalization of the GHZ test to phase space (q, p). Finally, I'll show some simulation results which will be generalized in the future to perform the generalized GHZ test using BM.

Bohmian Mechanics [5, 4, 2]

Background

The Quantum Mechanics (QM) that is taught, is usually the one which uses the 'Copenhagen interpretation'. This interpretation asserts that the most complete possible specification of an individual system, is in terms of ψ which yields only probabilistic results. While it can be shown to be consistent, it is worth exploring the reasons for believing this assertion. David Bohm[a] in an attempt to investigate the truth behind this, constructed a theory with 'hidden variables' (positions and momenta (q, p) of particles) that in principle completely specified the system but in practice get averaged over. He was able to show that his theory yields the same results as QM in all the physical situations he considered. Such a theory is worth studying because the following are at stake.

(1) Clarity: First, the widely held notion that at the atomic level, we must give up any conceivable precise description of nature, is plain false because there exists a heretic counter example, Bohmian Mechanics (BM). Second, deriving Classical Mechanics from QM (in it's usual interpretation) isn't possible due to the arbitrary distinction between the classical and quantum worlds. Within BM, classical mechanics can be recovered clearly.

(2) Accuracy of conclusions: The Bell test showed that there can't be hidden variable theories consistent with predictions of QM. Yet BM (Bohm's hidden variable theory) is consistent with QM; it allows the violation of Bell's inequality. The point is that we must be extremely careful about the conclusions we draw from our equations/experiments. The Bell test excludes *local* hidden variable theories, and BM is explicitly *non-local*.

There are a host of interesting questions which can be raised. For instance, one could ask why position and momentum aren't simultaneously determinable if in principle they're well defined? In the double slit then, the particle goes through one of the slits? Can one observe these trajectories? If particles have trajectories, what happens to identical particles? What happens to spins? Does the explicit non-locality entail we can communicate faster than light? Can one distinguish between BM and the usual QM experimentally? All these questions, except the last, have been solved or at least addressed.

Formalism

According to Bohm's original formulation of BM, a particle is associated with (1) a position and momentum (q, p), precisely and continuously defined & (2) a wave (ψ). For their description, the following are assumed:

- The ψ-field satisfies the Schrödinger equation.
- The particle momentum is restricted to $mv = p = \nabla S = \hbar\,\mathrm{Im}(\nabla\psi/\psi)$, where $\psi = Re^{iS/\hbar}$ and Im is the imaginary part.
- In practice, we don't control/predict precise locations of the particle; instead we have a statistical ensemble with probability densities $\rho(q) = |\psi(q)|^2$.

Comments:

(1) Note that the observers play no fundamental role in the formalism. If $\hbar = 0$ then we recover the classical Hamilton-Jacobi equation. Unlike QM, BM has a clear classical limit.

(2) These are readily generalized for N particles. Non locality in that case becomes explicit; $p_i = \nabla_i S(q_1, q_2, \ldots, q_N)$ viz. momentum of the i^{th} particle depends on the instantaneous positions of all particles.

(3) Extension to spins: In BM, the particle only has (q, p). The spin is associated only with the wave-function. For a spinor, say $\Psi \equiv (\psi_+, \psi_-)^T$, the generalization is that $mv = \hbar\,\mathrm{Im}((\Psi, \nabla\Psi)/(\Psi, \Psi))$ where $(.,.)$ represents inner product in the spin space $\mathbb{C}^2$.

[a] Historically, de Broglie had formulated a similar theory and then gave it up. Later Bohm independently re-discovered it

Determinism

The GHZ test [7]

Objective: To show that realism is incompatible with QM.

Assume: Three particles are allowed to interact and three observers are given one particle each. The interaction is such that the following holds. There are two properties of these particles one can measure, X or Y. The outcome of the measurement is either 1 or -1.

Construction: Interestingly, for a specific initial state of these particles, if they measure $A = X \otimes Y \otimes Y$ then the outcome is guaranteed by QM to be +1. This also holds for $B = Y \otimes X \otimes Y$ and $C = Y \otimes Y \otimes X$. However, if $D = X \otimes X \otimes X$ is measured, then the result is -1.[a] Explicitly, this can be achieved with 3 spin half particles for example, with $|\psi\rangle = (|000\rangle - |111\rangle)/\sqrt{2}$ (where $\sigma_z|0/1\rangle = \pm|0/1\rangle$) and X, Y, Z as Pauli spin operators.

Hypothesis: Assume that the world is deterministic, viz. the properties had predefined values. Then if we evaluate ABC, then we know by construction that it must be $= 1$. However, it is also true that $ABC = D$ (because $Y^2 = 1$). By construction we also know that $D = -1$. Thus we arrive at $+1 = -1$.

Conclusion: This entails that our hypothesis must be wrong. More precisely, this implies that we can't have non-contextual determinism where the qualification "non-contextual" is subtle but necessary.

Contextuality [7, 8]

Two observables A and B are mutually compatible if the result of measuring A doesn't depend on whether B is measured (before, after, simultaneously or not measured at all). If we restrict ourselves to hidden variable models that assert that A and B have predefined values, irrespective of which compatible observable is measured, then such a theory would be termed "non-contextual" and deterministic. Kochen-Specker proved that such theories, viz. non-contextual deterministic theories are inconsistent with QM. Mermin and Peres showed this for a four-level system. Consider the following operators.

$$\begin{array}{ccc} A_{11} = \sigma_z \otimes \mathbb{I} & A_{12} = \mathbb{I} \otimes \sigma_z & A_{13} = \sigma_z \otimes \sigma_z \\ A_{21} = \mathbb{I} \otimes \sigma_x & A_{22} = \sigma_x \otimes \mathbb{I} & A_{23} = \sigma_x \otimes \sigma_x \\ A_{31} = \sigma_z \otimes \sigma_x & A_{32} = \sigma_x \otimes \sigma_z & A_{33} = \sigma_y \otimes \sigma_y \end{array}$$

Note that operators along a given row commute. This also holds for a given column and thus these are compatible. Also note that the measurement product along any row (R_K) or column (C_k) is 1, except for column three; $C_3 = -1$. Thus, QM predicts $\prod_{k=1,2,3} R_k C_k = -1$, in contrast to non-contextual models. Since no experiment yields ideal results, an inequality must be constructed. It has been shown that all non-contextual theories must satisfy $\langle \chi_{KS} \rangle = \langle R_1 \rangle + \langle R_2 \rangle + \langle R_3 \rangle + \langle C_1 \rangle + \langle C_2 \rangle - \langle C_3 \rangle \leq 4$. QM yields $\langle \chi_{KS} \rangle = 6$.

Remarks:

(1) Note that Mermin's test is state independent, unlike the GHZ test.

(2) While there's a subtle connection between the Bell test and Contextuality, the latter is more suited for testing determinism (non-contextual) because the locality assumption is not required.

[a] The tensor has been omitted henceforth. The details have been

GHZ, Spins, Phase Space and BM

In this section we'll first sketch the analysis of the GHZ situation using BM. That BM is able to explain this is not very surprising since spins are not assigned predefined values in BM, unlike (q, p). A generalization of the GHZ test to phase space is thus desired. Such a generalization has been discussed in the literature and is briefly summarized.

Spin GHZ | BM Analysis [3]

Assuming that Stern Gerlach type apparatus are used to measure the spins of the different particles, the initial state of the system maybe described as $|\Psi(r_1, r_2, r_3, t = 0)\rangle = (\psi_{+++}|000\rangle - \psi_{---}|111\rangle)/\sqrt{2}$, where r_i represents the position vector in the frame of the i^{th} observer. If the particles are assumed to be Gaussian initially and propagating (with speed v_0 for instance) along the axes of their respective SG apparatus, then one can further define $\psi_{\pm\pm\pm}$. It can be shown that[a] the time evolution of $\psi_{\pm\pm\pm}$ can be written as products of 3 single particle solutions of SG setup, which was analyzed by Bohm himself. Once $|\Psi(r_i, t)\rangle$ is known, one can evaluate the equation of motion for the three particles using the formalism of BM. If the SG apparatus are setup to measure say XYY, then it is found that (in the direction relevant to measurement), four attractors basins form: $(+++)$, $(+--)$, $(--+)$ and $(-+-)$. The product is always +1 as was predicted by QM. However, when the SG apparatus are setup to measure XXX, the trajectories are found to obey equations which posses four attractive basins: $(---)$, $(-++)$, $(+-+)$ and $(++-)$. In this case, we get -1 as the product, again in agreement with QM.

Remarks:

Non locality enters from the fact that the attractor basins depend on the settings of *all* SG apparatus. Contextuality from this perspective is essentially the statement that the results of an experiment, depend on the experiment being performed. [4]

[a] details have been skipped for brevity and clarity

Phase space GHZ [6]

Consider unitary operators X, Y and the following re-definitions; $A = X^\dagger Y Y^\dagger$, $B = Y^\dagger X^\dagger Y$, $C = Y Y^\dagger X^\dagger$ and $D = XXX$. If the following anti-commutations hold, then we'll arrive at a GHZ like situation; $\{X, Y\} = 0$ and $\{X, Y^\dagger\}$. Given this, it follows that (1) A, B, C, D all commute and (2) $ABCD = -\mathbb{I}$. Thus any simultaneous eigenstate of A, B, C, D will result in the GHZ situation. Explicitly, in phase space, for some length scale L, $X \equiv \exp(i\sqrt{\pi}x/L)$ and $Y \equiv \exp(i\sqrt{\pi}pL)$ ($\hbar$ is chosen to be 1 in this section) satisfies the aforesaid conditions. To construct the simultaneous eigenstates, observe that for

$$|\uparrow\rangle_{x_0,p_0} \equiv \frac{1}{\sqrt{2}}\Big(\sum_{k=-\infty}^{\infty} e^{i\pi 2kp_0} |x = x_0 + 2k\rangle + i \sum_{k=-\infty}^{\infty} e^{i\pi(2k+1)p_0} |x = x_0 + 2k + 1\rangle \Big),$$

$$|\downarrow\rangle_{x_0,p_0} \equiv \frac{1}{\sqrt{2}}\Big(\sum_{k=-\infty}^{\infty} e^{i\pi 2kp_0} |x = x_0 + 2k\rangle - i \sum_{k=-\infty}^{\infty} e^{i\pi(2k+1)p_0} |x = x_0 + 2k + 1\rangle \Big),$$

we yield[a] $X|\uparrow\rangle = |\downarrow\rangle$, $Y|\uparrow\rangle = i|\downarrow\rangle$ and $Z|\uparrow\rangle = |\uparrow\rangle$ and similarly for $|\downarrow\rangle$. From this, the generalization of the GHZ state is found to be $|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\uparrow\uparrow\rangle - |\downarrow\downarrow\downarrow\rangle)$.

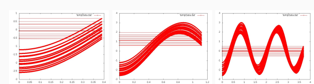
With the states and the operators defined in phase space, the GHZ test has been extended.

[a] here $x_0, p_0 \in [0, 1[$ and are numbers. Strictly one must write in place of x, $\sqrt{\pi}Lx$ and for p, $\sqrt{p}/L$.

Achievements and Outlook

Results

In addition to learning BM, surveying the literature and narrowing the problem, the first few stages of writing a BM simulator have been achieved. This is of special interest since analytic solutions to Bohmian trajectories are rarely simple. Trajectories for free evolution, squeezed state evolution under harmonic potential (shown in the figure) and a one dimensional analogue of the double slit experiment have been simulated and found to be qualitatively satisfactory. The simulator was written in Fortran and uses RK-4 along with spline interpolation for time evolution.



Immediate Goals

Numerically, extension to many particles and ability to handle spins are the essential next steps. These are required to validate the spin GHZ test as described. Theoretically, improvement of the phase space GHZ test is desired so that normalizable states can yield the paradox. The analogue of the SG apparatus for measuring p is essential for a Bohmian analysis.

Future Scope

A more ambitious goal would be to explore phase space contextuality [1] using BM to understand its relation with non locality more directly. A puzzling question is that while formally in QM, spins and (q,p) are handled rather similarly, why can't we extend BM in a manner such that spins are as 'deterministic' as (q,p)? It is worth attempting to find such a formulation or to show that it doesn't exist. This is of great interest for this answer must depend on the fundamental difference between spins and (q,p) as properties. The thesis problem is a step in this direction.

Acknowledgement and References

Acknowledgement

I thank my project guide Prof. Arvind for his timely guidance and encouragement, despite not having faith in BM. I am grateful to Dr. A. Chaudhuri and Dr. S. Sinha for their assistance with the simulation. I also acknowledge K. Bharti, R. Bhati and J. Singh (QCQI group members) for various discussions. The QCQI group talks (especially those by S. Gambhir and A. Sehrawat) have been conducive in narrowing the thesis problem. KVPY and IISER Mohali are acknowledged for funding and research education.

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Figure 1: The Poster

13.5 Sunday | Nov 15 [unwell]

- [misc] I was too exhausted by the time Sunday hit. It was worth it though, finishing all the work in time. Emailed the poster to Arvind sir.

14 The judgement day | amending dilapidated machines and minds

14.1 Monday | Nov 16 [judgement day]

- [research/ms] Presented the poster to various people. Abhishek sir was impressed and Sudeshna ma'am also seemed impressed. Explained it also to yoyo and another person who's name I don't remember. Was able to express the idea and excitement. Aulakh sir felt it was philosophy (ofcourse he didn't spend enough time understanding what's being said). KP Singh felt that since BM doesn't make any 'new' predictions, it must be useless as a theory. He didn't call it philosophy but I doubt he thinks it is of any use.

- [misc] Food improvement finally worked! Packed for leaving.

14.2 Tuesday, Wednesday | Nov 17, 18 [travel + home]

- [misc] Fixed Anika bhena's [had to open it, re-attach the keyboard, and unplug it], Attika bhena's [win 10 issues (partly fixed), applications] and mum's computer [formatted to win 8.1, explained how to transfer information]. Also got the warranty issue taken care of and returned the defective product.

- [research/summer] Got an email from PRA, requesting a response to the proof of the PRA submission.

14.3 Thursday | Nov 19 [partly travelled + working]

- [misc] reached IISER @ 10:30 or so. ate something, bathed etc. and reached CAF by 12:30; went for lunch at 1:30 and after lunch, slept. Woke up at 6, went grocery shopping and then gym + food related.

- [research/summer] Looked at the PRA proof. These guys are rather meticulous. Wrote to Ali about the PRA thing. He's in a conference he said.

14.4 Friday | Nov 20

- [PhD] Responded to Almut, University of Leeds for the offer.

- [research/summer] Responding to the PRA proof. Wrote a rough response and sent it to Ali. This happened a few times. The final version (roughly) was sent to Ali at night.

- [misc.] Got the cycle!

14.5 Saturday | Nov 21

- [research/summer] The response to the proof was submitted finally.

- [misc.] Birthday celebration! A lot of things that happened, happened really well! Had a great time.

- [course/gr] Started with Ashok Sen's GR lectures; lecture 1

14.6 Sunday | Nov 22

- [misc.] post party, sleep caught up

- [course/gr] Started with Ashok Sen's GR lectures; lecture 1

15 Aftermath, Eliciting pillory, Gleaning GR | Nov 23 - 29

15.1 Monday | Nov 23

- [misc.] sGRE scores received. Not applying to the US anymore.

- [course/gr] lecture 2 (iGuess) | also scanned and uploaded the notes

- [PhD] looking up universities where I can apply given the GRE scores; wrote to sudipta (iGuess)

- [research/ms] thought of a way of constructing more optimized operators.

15.2 Tuesday | Nov 24

- [misc.] talked to Bag about US applications and QG in general.
- [course/gr] lecture 3 (iGuess)
- [course/qg] lecture 1 (PIRSA, by Rovelli)

15.3 Wednesday | Nov 25

- [course/gr] lecture 4
- [course/qg] lecture 2 (partly)
- [PhD/exam] TIFR questions (section A almost done)

15.4 Thursday | Nov 26

- [research/summer] Received the second proof of the paper. Have to figure it out and fix it.
- [course/gr] lecture 5
- [course/phil] assembling information and question 2 done
- [PhD/exam] Working on TIFR questions, section B. Started reading griffiths (the transmission lines chapter).
- [PhD] wrote some emails (3 people in the list that Sudipta had given)

15.5 Friday | Nov 27

- [research/ms] Optimized observables for the finite wavefunctions can be given in terms the usual displacement $e^{i\hat{p}L/\hbar}$ followed by a displacement of some periodic clump in the opposite direction $e^{-if(\hat{x}_{\text{mod}NL})\hat{p}NL/\hbar}$ (check this) where f is used to define the "clump" and N is the number of 'slits' for the given eigenstate. So the full operator would perhaps be $e^{-if(\hat{x}_{\text{mod}NL})\hat{p}NL/\hbar}e^{i\hat{p}L/\hbar}$. The difficulty then is to see how one can create an operator of the form $X^{\dagger}X = 1$. This still needs to be worked out.
- [course/gr] lecture 6
- [research/summer] The proof I think is still quoting the wrong references. Other things seem to be alright.
- [PhD/exam] Worked on 2 TIFR questions; got stuck at them; had to talk to Bag.
- [course/phil] Q3 done, working on Q4.

15.6 Saturday | Nov 28

- [research/ms] Real Bell Test in (q,p) and not one that's based on the spin case. Imagine there was no spin. How would we have arrived at an inequality for local realism? I don't want to look at a subspace. Perhaps there's a more direct way which we're oblivion to at the moment.
- [course/phil] Q4 working on it.
- Discussion
  - Squeezed state & Different colours and monitor for intersection
  - Introduction
    - Field
    - Problem
  - Background
    - Review of literature
    - Summary of important techniques
  - Results
  - Conclusion
  - References
- Joseph Samuel, (? what about him)

15.7 Sunday | Nov 29

- [course/phil] philosophy revision before the exam (question 6)

16 Imperial Tumult

16.1 Monday | Nov 30

- [course/phil] HSS exam
- [misc] HDD lookup

16.2 Tuesday | Dec 1

- [PhD] Europe list construction (from Loops 2015 conference I guess)
- [research/summers] Proof work
- [research/ms] Typesetting the report

16.3 Wednesday | Dec 2

- [PhD] Sent an application to NEI (iThink) + related work
- [research/ms] Typesetting the report
- [misc] circumstantial ionization issues

16.4 Thursday | Dec 3

- [PhD] SOP writing (quantum gravity)
- [research/misc] Talk by Sudip K. Goyal (iThink) on quantum memory etc.
- [misc] cycle repair attempt (failed as I'll learn later)

16.5 Friday | Dec 4

- [PhD] SOP verification/review by Prashansa etc., application to FAU (twice?)
- [misc] Manu's presentation verification etc.; Sudipta Sir seems to be against LQG; circumstantial distinct
- [research/misc] Talk by Sudip K. Goyal, he left open a paradox, spent some time figured it;

16.6 Saturday | Dec 5

- [PhD] After extreme confusion about what to do a PhD in, applications were sent to Max Planck and Rovelli.
- [misc] today was light, objective was to fix the schedule to optimize timing and efficiency, updated the calendar for various days

16.7 Sunday | Dec 6

- [misc] Various maintenance tasks: inbox gmail issue, hdd ordering, git repository issue, updated the log for various days.

Part II
Appended Topics

1 The Thesis Problem - Non locality and Contextuality

1.1 Definition

The following has been taken from an email written to Ali Assadian.

[background]

As you know, in the GHZ test, one is able to show that determinism can't hold. However, this is done using spins. How this is handled in Bohmian Mechanics (BM) has been discussed already[5]. It is not in direct contradiction with BM because spins are not treated like (q,p). In BM, (q,p) are well defined, just that we can't observe them. However, spins in BM are only a property of the wavefunction and not postulated to have well defined 'values' like (q,p). Thus, while it is interesting to see how BM handles being deterministic and consistent with GHZ, it's not too surprising to see it work, since spins aren't assumed deterministic like (q,p).

[thesis problem]

Do you recall the GHZ paper that had been extended to continuous variables? The point was that this particular approach showed that there can't be determinism for (q,p). However, BM seems to be an example that does precisely this. How can this be? It is precisely this that I intend to explore in my thesis. If BM's predictions differ from QM, then we at once have a wonderful test to find which theory is correct. If the predictions match, as is more likely, we'll be closer to answering to atleast two important questions. (1) Which extra assumption goes into the GHZ like tests which is unaccounted and (2) How contextuality emerges from non-locality, especially in continuous variables.

[future scope]

Ofcourse, a more ambitious goal would be to look at your results [4] on contextuality in continuous variables and use BM to understand the relation of non locality with it more directly. Perhaps if I have enough time, I'll pursue that as well. The one question which I still haven't an answer to is the following: How is it that, while formally in QM, spins and (q,p) are handled very similarly, why can't we extend BM in a manner to include spins as 'deterministic' as are (q,p)? I would like to either find such a formulation or show that it doesn't exist. This is of great interest for this answer must depend on the fundamental difference between spins and (q,p) as properties. The thesis problem is a step in this direction.

1.2 Breakup

1. Numerical Analysis

   (a) Bohmian Basics

      i. Simulate a free particle. Start with say a guassian wavefunction and take the initial positions to be accordingly distributed.
      ii. Simulate a harmonic oscillator. Use the previous step, except now with a harmonic oscillator potential. The gaussian should oscillate nicely. Once that's confirmed, then check the trajectories.
      iii. Simulate a squeezed state.
      iv. Tunnelling perhaps?
      v. S,P orbitals, trajectories (interesting by themselves!)

   (b) Bohmian Advanced

      i. Simulate a two particle state.
      ii. Simulate stern gerlach
      iii. Simulate the GHZ experiment using Ref. [5]

2. Analytic Work

   (a) Bohmian Mechanics

      i. Quantum Theory of trajectories, Holland [todo: add reference] (depth-read chapter n, 50% depth-read chapter n+1)
      ii. Bohm's original paper [todo: add reference] (70% depth-read paper 1)
      iii. Measurements [6] (float-read)
      iv. Spins, Stern Gerlach [6] (float-read)

   (b) Bell, GHZ test etc. from the Bohmian perspective [5]

   (c) GHZ test in continuous variables [7] (careful-read, during summers)

   (d) Construct an experiment in BM to perform the GHZ test; check predictions

References

[1] https://jakevdp.github.io/blog/2012/09/05/quantum-python/.

[2] http://www.pa.msu.edu/~duxbury/courses/phy480/SchrodingerDynamics.pdf.

[3] http://ww2.odu.edu/~agodunov/computing/programs/book2/Ch01/spline.f90.

[4] Ali Asadian, Costantino Budroni, Frank E. S. Steinhoff, Peter Rabl, and Otfried Gühne, *Contextuality in phase space*, Phys. Rev. Lett. **114** (2015), 250403.

[5] Thomas Durt and Yves Pierseaux, *Bohm's interpretation and maximally entangled states*, Phys. Rev. A **66** (2002), 052109.

[6] Detlef Dürr, Sheldon Goldstein, and Nino Zanghì, *Quantum physics without quantum philosophy*, Springer Berlin Heidelberg, 2012.

[7] Serge Massar and Stefano Pironio, *Greenberger-horne-zeilinger paradox for continuous variables*, Phys. Rev. A **64** (2001), 062108.