Bohmian Mechanics and Contextuality in Phase Space

Atul Singh Arora
QCQI Group, IISER Mohali, India
ms11003@iisermohali.ac.in

Introduction

Abstract

There are atleast two known theories that describe the same physical world; Quantum Mechanics and Bohmian Mechanics (also uses the Schrödinger Equation). While the latter is not popularly known, it provides exceptional clarity about certain aspects of reality. So far, no tests are known that can distinguish these two. One fundamental difference between them is that that the latter is deterministic (in the sense that (q, p) are well defined). The purpose of this thesis is to get these theories head on; we aim to construct a theoretical situation where this type of determinism is refuted by Quantum Mechanics (using generalization of the GHZ test, contextuality etc.) and analyze it using Bohmian Mechanics. This is a step towards understanding the relation between contextuality and non-locality.

Introduction

I will first discuss Bohmian Mechanics and then go on to discuss the known standard tests of determinism (GHZ and contextuality). Thereafter I will describe how the GHZ test is explained from the Bohmian perspective and also discuss a generalization of the GHZ test to phase space (q, p). Finally, I'll show some simulation results which will be generalized in the future to perform the generalized GHZ test using BM.

Bohmian Mechanics

Background

The Quantum Mechanics that is taught, is usually the one which uses the 'Copenhagen interpretation'. This interpretation asserts that the most complete possible specification of an individual system, is in terms of ψ which yields only probabilistic results. While it can be shown to be consistent, it is worth exploring the reasons for believing this assertion. David Bohm[a] in an attempt to investigate the truth behind this, constructed a theory with 'hidden variables' (positions and momenta (q, p) of particles) that in principle completely specified the system but in practice get averaged over. He was able to show that his theory yields the same results as Quantum Mechanics in all the physical situations he considered. Such a theory is worth studying because the following are at stake.
(1) Clarity: First, the widely held notion that at the atomic level, we must give up any concievable precise description of nature, is plain false because there exists a heretic counter example. Second, deriving Classical Mechanics from QM (in it's usual interpretation) isn't possible due to the arbitrary distinction between the classical and quantum worlds. Within BM, classical mechanics can be recovered clearly.
(2) Accuracy of conclusions: The Bell test showed that there can't be hidden variable theories consistent with predictions of QM. Yet Bohmian Mechanics (Bohm's hidden variable theory) is consistent with QM; it allows the violation of Bell's inequality. The point is that we must be extremely careful about the conclusions we draw from our equations/experiments. The Bell test excludes *local* hidden variable theories, and Bohmian mechanics is explicitly *non-local*.
There are a host of interesting questions which can be raised. For instance, one could ask why position and momentum aren't simultanesouly determinable if in princple they're well defined? In the double slit then, the particle goes through one of the slits? Can one observe these trajectories? If particles have trajectories, what happens to identical particles? What happens to spins? Does the explicit non-locality entail we can communicate faster than light? Can one distinguish between Bohmian Mechanics and the usual Quantum Mechanics expeirmentally? All these questions, except the last, have been solved or atleast addressed.

Formalism

According to Bohm's original formulation of Bohmian Mechanics, a particle is associated with (1) a position and momentum (q, p), precisely and continuously defined & (2) a wave (ψ). For their description, the following are assumed:

- The ψ-field satisfies the Schrödinger equation.
- The particle momentum is restricted to $mv = p = \nabla S = \hbar\,\mathrm{Im}(\nabla(\psi/\psi))$, where $\psi = Re^{iS/\hbar}$.
- In practice, we don't control/predict precise locations of the particle; instead we have a stistical ensamble with probability densities $\rho(q) = |\psi(q)|^2$.

[a] Historically, de Broglie had formulated a similar theory and then gave it up until Bohm independently re-discovered it

Comments:
(1) Note that the observers play no fundamental role in the formalism. If $\hbar = 0$ then we recover the classical Hamilton-Jacobi equation. Unlike QM, BM has a clear classical limit.
(2) These are readily generalized for N particles. Non locality in that case becomes explicit; $p_i = \nabla_i S(q_1, q_2, \ldots, q_N)$ viz. momentum of the i^{th} particle depends on the instantaneous positions of all particles.
(3) Extension to spins: In BM, the particle only has (q, p). The spin is associated only with the wavefunction. For a spinor, say $\Psi \equiv (\psi_+, \psi_-)^T$, the generalization is that $mv = \hbar\,\mathrm{Im}((\Psi, \nabla\Psi)/(\Psi, \Psi))$ where $(.,.)$ represents inner product in the spin space $\mathbb{C}^2$.

Determinism Tests and Contextuality

The GHZ test

Objective: To show that realism is incompatible with QM.
Assume: Three particles are allowed to interact and three observers are given one particle each. The interaction is such that the following holds. There are two properties of these particles one can measure, X or Y. The outcome of the measurement is either 1 or -1.
Construction: Interestingly, for a specific initial state of these particles, if they measure $A = X \otimes Y \otimes Y$ then the outcome is guarenteed by QM to be $+1$. This also holds for $B = Y \otimes X \otimes Y$ and $C = Y \otimes Y \otimes X$. However, if $D = X \otimes X \otimes X$ is measured, then the result is -1.[a]
Hypothesis: Assume that the world is deterministic, viz. the properties had predefined values. Then if we evaluate ABC, then we know by construction that it must be $= 1$. However, it is also true that $ABC = D$ (because $Y^2 = 1$). By construction we also know that $D = -1$. Thus we arrive at $+1 = -1$.
Conclusion: This entails that our hypothesis must be wrong.

[a] The tensor has been ommited henceforth. The details have been