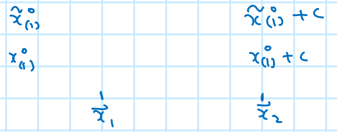
Lecture 9 | Gravitational Redshift + EM

01 October 2017 01:00 PM

Gravitational Redshift



Consider: a clock sitting on the point $\vec{x}_1$ in space.

$\Rightarrow \frac{dx^i}{dt} = 0.$

Assume: Time independent metric $g_{\mu\nu} = g_{\mu\nu}(\vec{x})$
(I think we're taking the non-relativistic limit $\Rightarrow$ weak field approx.) Nope!

Recall: For a physical particle, $\frac{ds^2}{du^2} < 0$ & since u is an affine parameter $\frac{ds}{du} = \text{const}$.

This means I can always rescale to get $\frac{ds^2}{du^2} = -1$ (for time like)

$\Rightarrow du^2 = -ds^2 = d\tau^2$. So I can write

$$-\frac{ds^2}{du^2} = \frac{ds^2}{d\tau^2} = -g^{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau} = 1$$

(In the weak field limit, one ignores $\frac{dx^i}{du}$ compared to $\frac{dx^0}{du}$. That's what I think is being done here but I need to check this)

$$-g^{00}\left(\frac{dx^0}{d\tau}\right)^2 = 1 \quad \Rightarrow \quad \Delta x^0 = \Delta \tau_{(1)}\left(-g^{00}(\vec{x}_{(1)})\right)^{-1/2}$$

(ticks) a const not speed of light, I

Story: The clock emits two signals, at $x^0_{(1)}$ & $\tilde{x}^0_{(1)} + dx^0$. Say it took 'ϵt length of time' for the first signal to reach point $\vec{x}_2$. Since $g^{\mu\nu}$ is time independent, the second signal will also take as long (see the figure)
The interval between the two signals at $\vec{x}_2$ will again be dx^0

I now want to find the proper time b/w two clock ticks, as seen by an observer sitting on the point $\vec{x}_{(2)}$
Why bother? Because the clock of observer 2 is ticking according to his proper time.

Calc: $\Delta x^{(0)} = \Delta \tau_{(2)}\left(-g^{00}(\vec{x}_{(2)})\right)^{-1/2}$ I wrote the eq^n for a clock sitting at $\vec{x}_{(2)}$ observing the time difference of the signals that reached $\vec{x}_{(2)}$.

$$\Rightarrow \Delta\tau_{(1)}\, g^{00}(\vec{x}_{(1)})^{-1/2} = \Delta\tau_{(2)}\, g^{00}(\vec{x}_{(2)})^{-1/2}$$

$$\Rightarrow \frac{\Delta\tau_{(1)}}{\Delta\tau_{(2)}} = \left[\frac{g^{00}(\vec{x}_{(1)})}{g^{00}(\vec{x}_{(2)})}\right]^{1/2}.$$

It seems that sir didn't actually make the weak field approximation but he does it now.

Weak Field: $g^{00} = \eta^{00} + h^{00}$ where $h^{00} = -\frac{2\phi}{c^2}$ & $\eta^{00} = -1$

$$\text{so} \quad \frac{\Delta\tau_{(1)}}{\Delta\tau_{(2)}} = \left[\frac{1 + \frac{2\phi(\vec{x}_{(1)})}{c^2}}{1 + \frac{2\phi(\vec{x}_{(2)})}{c^2}}\right]^{1/2} \simeq 1 + \frac{1}{c^2}\left[\phi(\vec{x}_{1}) - \phi(\vec{x}_{2})\right]$$

If $\phi(\vec{x}_{(1)}) \geq \phi(\vec{x}_{(2)})$, then $\Delta\tau_{(1)} \geq \Delta\tau_{(2)}$.

The clock at $\vec{x}_{(1)}$ will be slower than the one at $\vec{x}_{(2)}$

I am, however, still a little confused about why we're using "proper time".
What happens if I take the clock at $\vec{x}_{(2)}$ & bring it to $\vec{x}_{(1)}$?
Shouldn't their proper times be the same?
↳ clock already at $x_{(1)}$ & the one I got from $\vec{x}_{(2)}$?