§ 3.2.1 $D(\rho_1, \rho_2) \geq 0$

$tr(M(\rho_1 - \rho_2)) = tr(M\rho_1) - tr(M\rho_2)$

Assuming $D(\rho_1, \rho_2) = D(\rho_2, \rho_1)$

$= tr(M\rho_1) - tr(M\rho_2) = tr(M\rho_2) - tr(M\rho_1)$

WRONG

$tr(M\rho_1) = tr(M\rho_2)$

$(x_1 - x_2)^2 = (x_2 - x_1)^2 \not\Rightarrow x_1 - x_2 = 0$

$\because$ its the max such M. $\Rightarrow tr(M_A\rho_1) - tr(M_A\rho_2)$

$= tr(M_B\rho_2) - tr(M_B\rho_1)$

$\exists$ an M s.t.

$tr(M\rho_1) - tr(M\rho_2) > 0 \rightarrow$

$tr((M_A + M_B)\rho_1) = tr((M_B + M_A)\rho_2)$

Imagine if M_A is s.t. $tr(M_A\rho_1 - M_A\rho_2) > 0$
can you find M_B s.t. $tr(M_B\rho_2 - M_B\rho_1) > 0$?

One option is $M_B = -M_A$ but thats an issue $\because$ both M_A & $M_B > 0$.

However, $M_B = \mathbb{1} - M_A$ might work.

so that $tr((\mathbb{1} - M_A)\rho_2 - (\mathbb{1} - M_A)\rho_1)$

$= \cancel{tr(\rho_2 - \rho_1)} + tr(M_A(\rho_1 - \rho_2))$

$= tr(M_A(\rho_1 - \rho_2)) > 0$

~~seems to work.~~

symmetric ✓

SDM convex set.

Assume M is s.t.
$tr(M\rho_1) - tr(M\rho_2) < 0$

Assume M_A is s.t.
$tr(M_A\rho_1) - tr(M_A\rho_2) < 0$, find me an M_B s.t.
$tr(M_B(\rho_1 - \rho_2)) > 0$.

positive ✓

$0 < M_A < \mathbb{1}$

$0 < \alpha M_A < \alpha\mathbb{1}$

$\cancel{-\mathbb{1}}\ 0 > -\alpha M_A > -\alpha\mathbb{1}$

~~$M_A \mathbb{1}$~~

$\mathbb{1} > \mathbb{1} - \alpha M_A > \mathbb{1} - \alpha\mathbb{1}$

$tr(M_A(\rho_1 - \rho_2)) < 0 \quad tr(M_B(\rho_1 - \rho_2))$

largest

$tr(M_A(\rho_1 - \rho_2)) < 0 \quad tr(M_A(\rho_2 - \rho_1)) > 0$

$\Rightarrow tr(M_A(\rho_2 - \rho_1))$

$\rho_1 = \rho_2 \Rightarrow D(\rho_1, \rho_2) = 0$

$D(\rho_1, \rho_2) = 0$

$D(\rho_1, \rho_2) \Rightarrow \rho_1 = \rho_2 \quad \because M$ is arbitrary

Only when same

$\langle\psi| M (\rho_1 - \rho_2) |\psi\rangle$

$tr\left[\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} - \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}1/2 & 0\\ 0 & 1/2\end{pmatrix}\right]$

$tr\begin{pmatrix}-1/2 & \\ & 0\end{pmatrix} = -1/2$

Can I show $\exists$ an M s.t.

I use $\mathbb{1} - M$ again & I'll be left with

$tr(M(\rho_1 - \rho_2)) \rightarrow tr((\mathbb{1} - M)(\rho_1 - \rho_2))$

$= tr(M(\rho_2 - \rho_1))$

$tr((\mathbb{1} - M)(\rho_1 - \rho_2)) = tr(M(\rho_2 - \rho_1))$

$tr(M(\rho_1 - \rho_3)) + tr(M(\rho_3 - \rho_2))$

$= tr(M(\rho_1 - \rho_2)) + tr(M(\rho_3 - \rho_3))$

Assume $M_A, M_B \& M_C$ are s.t.

$tr(M_A(\rho_1-\rho_2))$

$tr(M_B(\rho_1-\rho_3))$

$tr(M_C(\rho_3-\rho_2))$

are max.

$\boxed{\text{Triangle}}$

Now, the claim is

$$tr(M_A(\rho_1-\rho_2)) \leq tr(M_B(\rho_1-\rho_3)) + tr(M_C(\rho_3-\rho_2))$$

$$tr(M_A(\rho_1-\rho_3)) + tr(M_A(\rho_3-\rho_2))$$

Obviously, by def^n $\leq$ & similarly for the other. So done.

$$P(X=x) = \begin{cases} 1/2 & 1 \\ 1/2 \cdot \frac{1}{2^n-1} & \text{all other strings} \end{cases}$$

$$-\frac{1}{2}\cdot\log\left(\frac{1}{2}\right) - \frac{2^n-1}{2(2^n-1)}\cdot\log\frac{1}{(2^n-1)\cdot 2}$$

$$\frac{1}{2} + \frac{2^n}{2^{n+1}}(n+1)$$

$$1+\frac{n}{2} \approx \frac{n}{2}$$

Claim: $D(\rho_1,\rho_2) = |\rho_1-\rho_2|_1$

So since $\rho_1^+ = \rho_1$ & $\rho_2^+ = \rho_2$ $\Rightarrow \rho_1-\rho_2$ can be diagonalized.

$\Rightarrow$ singular values are same as the eigenvalues

So then, since tr is invariant under change of basis; fine that's the problem.

$$\max_A tr(A(\rho_1'-\rho_2')) = tr(A'U^+\underbrace{U(\rho_1'-\rho_2')U^+}_{\text{diagonal}}U)$$

$$= tr(\underbrace{UA'U^+}_{A}(\rho_1-\rho_2)) = tr(A(\rho_1-\rho_2))$$

with $\rho_1-\rho_2$ diag. $= diag(\lambda_1 \dots \lambda_n)$

If $A>0$ then so is UAU^+.

$[M(\rho_1-\rho_2)]$

claim $\frac{1}{2}\left[|\lambda_1|+|\lambda_2|+\dots+|\lambda_n|\right]$

$$= tr\left[\begin{pmatrix} 1/2 & & \\ & -1/2 & \\ & & \ddots & 1/2 \end{pmatrix}\begin{pmatrix}\lambda_1 & & \\ & \ddots & \\ & & \lambda_n\end{pmatrix}\right] = \frac{1}{2}$$

$[\lambda_1+\lambda_2-\lambda_3 \dots + \lambda_n]$

$0<\alpha<1$

$-\frac{1}{2}<\alpha<\frac{1}{2}$

$\frac{1}{2}\mathbb{1}$ $0<\alpha<1$

$$\frac{1}{8}\begin{pmatrix}1&&\\&1&\\&&1\end{pmatrix}\begin{pmatrix}a&&\\&-b&\\&&c\end{pmatrix} = a-b+c<0$$

$$\begin{pmatrix}-1&&\\&-1&\\&&-1\end{pmatrix}\begin{pmatrix}a&&\\&-b&\\&&c\end{pmatrix} = -a+b-c$$

$$M = \begin{pmatrix}1/3&&\\&1/3&\\&&1/3\end{pmatrix}$$

$\mathbb{1}-M$ $\qquad M^+M$

λ_i are s.t.

$\sum \lambda_i = 0$

$tr([M](\rho_1-\rho_2))$

$$\begin{pmatrix}1/2&&&\\&-1/2&&\\&&\ddots&\\&&&1/2\end{pmatrix}\begin{pmatrix}\lambda_1&&\\&\ddots&\\&&\lambda_n\end{pmatrix}\Big(\quad\Big)$$

$H_{min}(X) = -\log\max_x P(X=x)$

$\max_x \sum_e P(X=x, E=e)$

$\max_x \sum_e P(X=x)P(E=e|X=x)$

$$\begin{array}{c|c} & x \\ \hline e & \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \end{array}$$

$$\frac{\mathbb{1}}{n} + \frac{1}{n}\begin{pmatrix}1&&\\&-1&\\&&\ddots\end{pmatrix}$$

$\sum_e P(E=e)\max_x P(X=x|E=e)$

max P_{guess}

$$\rho = \sum_{x,e} P(x,e)\,|x\rangle\langle x|\otimes|e\rangle\langle e|$$

$$\rho = \sum_{x,e} P(e)\, P(x|e)\; |x\rangle\langle x| \otimes |e\rangle\langle e|$$

Quiz 3.3

1. $\frac{3}{4}|0\rangle\langle 0| + \frac{1}{4}|1\rangle\langle 1|$

$$-\log\frac{3}{4} = \log\frac{4}{3} = \log 4 - \log 3 = 2 - \log 3$$

2. $\frac{1}{4}|0\rangle\langle 0|\otimes|+\rangle\langle +| + \frac{3}{4}|1\rangle\langle 1|\otimes|-\rangle\langle -|$

$$= \frac{1}{4}\mathrm{tr}(M_0 \rho_0^E) + \frac{3}{4}\mathrm{tr}((\mathbb{1}-M_0)|-\rangle\langle -|)$$

$$= \frac{3}{4} + \frac{\mathrm{tr}}{4}\left(M_0\left[\frac{\rho_0^E}{\cancel{4}} - 3\frac{\rho_1^E}{\cancel{4}}\right]\right)$$

$$= \frac{3}{4} + \frac{\mathrm{tr}}{4}\left(M_0(\rho_0^E - 3\rho_1^E)\right)$$

$$\frac{\mathrm{tr}}{4}\left(M_0 \begin{pmatrix}1 & \\ & -3\end{pmatrix}\right)$$

$$= \frac{3}{4} + \frac{4}{\cancel{4}} = 1$$

Assump^n: (i) You know the prob. dist. (joint)

(ii) You can measure your system, E.

Classically: simply measure E & then say you got e. Now find $\max_x P(X=x|E=e)$

which you know is

$$\max_x \frac{P(X=x, E=e)}{P(E=e)}.$$

Quantumly: You need to know which best measurement to make

The state now is

$$\rho_{XE} = \sum P(x)|x\rangle\langle x| \otimes \rho_x^E$$

§3.4

ρ, $\theta = 0$, $|0\rangle\langle 0|$, $|1\rangle\langle 1|$

$$\begin{array}{cc} |0\rangle\langle 0| & |+\rangle\langle +| \\ \begin{pmatrix}1 & \\ & \end{pmatrix} & \begin{pmatrix}\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}\end{array}$$

$$\frac{1}{2}\max_i(\rho M_i^{\theta=0}) + \frac{1}{2}\max_j(\rho M_j^{\theta=1})$$

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$$\cancel{|\Phi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)}$$

$\cancel{\rho_{AB}^E}$

$$\rho_{AEB} = |\Psi\rangle\langle\Psi|_{AE} \otimes |0\rangle\langle 0|_B$$

$|0\rangle|0\rangle|0\rangle + |1\rangle|1\rangle$

↓

$(|+\rangle + |-\rangle)(|+\rangle + |-\rangle)(|+\rangle + |-\rangle)$

$+ (|+\rangle - |-\rangle)(|+\rangle - |-\rangle)(|+\rangle - |-\rangle)$

+ + +   + + −   + − +   + − −   − + +   − + −   − − +   − − −

$$P_r(\text{best guess}) = \frac{1}{2}\max_\rho\left(\rho\left(M_0^{\theta=0} + M_0^{\theta=1}\right)\right)$$

$\rho > 0$ & $\mathrm{tr}(\rho) = 1$

$$= \frac{1}{2}\max_\rho\left(\rho\begin{pmatrix}3/4 & 1/2 \\ 1/2 & 1/2\end{pmatrix}\right)$$

$$\lambda_1 = \frac{1}{8}(5 + \sqrt{17})$$

$$\lambda_2 = \frac{1}{8}(5 - \sqrt{17})$$

$\frac{1}{2} + \frac{1}{2}$

$$\begin{pmatrix}\frac{3}{2} & 1/2 \\ 1/2 & 1/2\end{pmatrix}$$

$$\lambda_1 = \frac{1}{2}(2 + \sqrt{2})$$

$$\lambda_2 = \frac{1}{2}(2 - \sqrt{2})$$

$$= \frac{1}{2}\cdot\frac{1}{8}(5 + \sqrt{17})$$

$$= \frac{1}{2}\left(\frac{1}{2}(\cancel{2} + \sqrt{2})\right) \approx 0.85$$

$$\frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)$$

$$\rho_{AB} = (1-p)|\Phi\rangle\langle\Phi| + \frac{p}{4}\mathbb{1}$$

For $p = 0$, $p = 0$, $D(\ ) + 4/2$

$$\Phi = \begin{pmatrix}1/2 & 0 & 0 & 1/2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2\end{pmatrix}$$

$$\begin{pmatrix}1/2 & & & \\ & 0 & 0 & \\ & 0 & 0 & \\ & & & 1/2\end{pmatrix}$$

$$\rho_{AB} - \rho = \begin{pmatrix}-p/2 + p/4 & & & -p/2 \\ & p/4 & & \\ & & p/4 & \\ -p/2 & & & -p/2 + p/4\end{pmatrix}$$

Min entropy $H_{min} = -\log(P_{guess})$

$\rho_X = |00\rangle\langle 00|$

$= -\log(1) = 0$

$\rho_X = \frac{1}{2}|00\rangle\langle 00| + \frac{1}{2}|11\rangle\langle 11|$

Eve — Ψ

$\Theta \in \{0,1,2\}$

$(U_A \otimes \mathbb{1}_E)\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

basis transf.

$\frac{1}{\sqrt{2}}$ $|+\rangle|0\rangle + |-\rangle|1\rangle$

ING contact centre 8 am 10 pm ~~9 am~~

A ×
B ×
C]
D]

Eve — Alice

$\Psi \longrightarrow \Psi$

$\Theta \longleftarrow \Theta \in [0,1,2]$

Z X Y

guess x — X

$1/3$ $\theta = 0$ — 1

$1/3$ $\theta = 1$ — $1/2$

$1/3$ $\theta = 2$ — $1/2$

$\frac{1}{3}(1+1) = 2/3$

$|\psi\rangle = \frac{1}{\sqrt{2\sqrt{2}}}(|0\rangle + |+\rangle)$ $\frac{|0\rangle + |1\rangle}{\sqrt{2}}$

Pr (outcome corresponding to M_0)

$= tr(M_0 \rho)$

Pr(outcome is M_0 | state is ρ_0)

$= tr(M_0 \rho_0)$

Pr(outcome is M_1 | state is ρ_1)

$= tr(M_1 \rho_1)$

Prob of success

$= \frac{1}{2} tr(M_0 \rho_0)$

$+ \frac{1}{2} tr(M_1 \rho_1)$

$1/3$ $\theta = 0$ — $|\langle 0|\psi\rangle|^2 = \frac{1}{2\sqrt{2}}\left|1 + \frac{1}{\sqrt{2}}\right|^2$

$1/3$ $\theta = 1$ — $|\langle +|\psi\rangle|^2$ $= \frac{1}{2\sqrt{2}}\frac{(\sqrt{2}+1)^2}{2} = \frac{2+1+2\sqrt{2}}{4\sqrt{2}}$

$= |0.9238|^2$ $= \frac{3+2\sqrt{2}}{4\sqrt{2}}$

$1/3$ $\theta = 2$

$= 0.8535$ 0.8535

$|\langle +_y|\psi\rangle|^2 = |0.9238|^2$

$= 0.5$

$\frac{2}{3} \times 0.8535 + \frac{0.5}{3}$ $= 0.7357$ ≈ 0.736

same

$|\psi\rangle = \frac{1}{\sqrt{2+\sqrt{2}}}(|0\rangle + |0_y\rangle)$ $\boxed{0.784}$

$(|0\rangle + |+\rangle + |0_y\rangle)$

$\frac{1}{\sqrt{4+2\sqrt{2}}}(|0\rangle + \frac{1}{\sqrt{2}}(1+i)]$ $\frac{1}{\sqrt{4+2\sqrt{2}}}$

$\left(1+\frac{2}{\sqrt{2}}\right)$ $\sqrt{\frac{1}{3+2}} - i\sqrt{\frac{1}{3+2}}$

$\sqrt{\frac{3}{5}}\left(\sqrt{\frac{2}{3}}\right.$ $+ \sqrt{\frac{1}{3+2}}$

0.22844

$-\log_2 x = -1$

$\log_2 x = 1$

$x = 2$

$-\log_2 P_{guess}$

Optimal M_0 can be found as follows

$\frac{1}{2}tr(M_0\rho_0) + \frac{1}{2}tr((\mathbb{1} - M_0)\rho_1) \leq$

$= \frac{1}{2}\left[tr(M_0(\rho_0 - \rho_1))\right] + \frac{1}{2}tr\rho_1$ 1

$\frac{1}{2}\left[1 + \frac{\sum|\lambda_i|}{2}\right]$

$= \frac{1}{2}$ $\rho_0 - \rho_1$

ans: 0.8535

$= 0.854$

dE, dA

idA

$(\lambda(\frac{1}{1})\xi - \rho) > 0$