Self Critique: Test for dynamical non-locality

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18th May 2015

Abstract

Quantum corelations are non local and yet can't be used to send non local signals. Local hidden variable theories were ruled out as possible theories to explain the correlations. Tests such as Bell's, CHSH etc. have been key to this. However, these tests are not sensitive to another type of non locality. This enters quantum mechanics through the dynamics. Here we attempt to construct such tests which relax the local realism assumptions and probe the aforesaid kind of non locality.

1 Preliminary Thoughts

What I have therefore come to believe is that I must not attempt to do the usual impossible, such as action at a distance or equivalently, signalling faster than speed of light. I must therefore come up with a situation which I know is impossible classically, yet is not a violation of action at a distance. Examples of such cases are already known, such as quantum correlations (bell's inequality), non-commutativity of operators leading to contextuality etc. So perhaps then the question then is to formulate the basis for a new test in quantum mechanics. One that tests something of the aforesaid variety. So the question then changes from non locality to that of non classicality. One such feature which exists is the Aharnov Bohm effect. The close relation between a gauge field and the wavefunction. However, I already know that the effect that is observed shouldn't happen in the first place. Still if one insists on thinking about it further, one can investigate exactly what it is that this Aharnov Bohm effect is doing, which we were not able to accomplish using usual unitary operators. Could these infact be something we haven't looked at earlier and be usable in quantum computation in some non-trivial way? Well, after some thought the following became clear. What is interesting in the Aharnov Bohm effect is not that $\psi \to \psi_1 + e^{i\phi}\psi_2$ (because the analogous can be done on qubits using unitaries). It is the fact that this effect is happening due to a field which is never in contact with the particle. Therefore in my opinion, we have various tests for type 1 non locality, which arises from the structure of the Hilbert space (such as states like $|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$). These include quantum correlation and contextuality etc. However, one must investigate how to test for type 2 non locality, viz. non locality in the dynamics of quantum mechanics. An explicit example of this, is the time evolution of the displacement operator (Heisenberg picture).

So in an attempt to create a test for type 2 non locality, I must start with thinking about the assumptions on which to base the test. The tests I have seen so far, seem to be based on certain classical assumptions about locality and realism. They then derive under those assumptions, certain relations between measurable quantities. These then they show, according to quantum mechanics must be violated under certain situations. Ofcourse, then experiments are done and victorious emerges our quantum theory. I could think about various more basic questions, but at the moment, I must restrict myself to the easiest (relatively readily solvable) I can formulate. Recall that I had thought about paradoxes in QM if you apply STR and reverse sequence of events because simultaneity we'd just learnt was relative. Then there were issues with randomness and how it must be thought of as a local phenomenon and so on.

Let me therefore attempt to list down some of the assumptions. This is already getting tricky. Let's start naive. (call these set B)

1. All measurables of the system have a well defined value prior to measurement
2. All interactions are local (even if there're hidden variables) [so far these are the same as those assumed by Bell]

NB1: From usual observations themselves we know that 1 doesn't hold. The failure of Bell test then implies 2 must be false.

NB2: The weaker the assumption, stronger will be the repercussions of its violation

In addition to these I assume

1. Related to evolution of the system: The measurables are a function of time and of the hidden variable, which itself maybe assumed to be time dependent.
   (a) Idea: At most there're local hidden variables that are allowed to evolve with time
   (b) Idea: Assume nothing about the cause of the dynamics. Just assume some time dependence
   (c) Idea: Then show that Aharnov Bohm shows type 2 non locality

Stage 2 will be to come up with a neater set of assumptions, that don't assume the Bell conditions, but still assume something that can only happen in the quantum case. My guess would be to not assume B1 and B2. This will suddenly become hard. But I think that is the point. I must be able to show that it was possible for there to be a theory which has the quantum correlations that QM shows (type 1 non locality) and yet its dynamical equations are local, unlike those of QM (type 2).

It would then be sensible to say that QM has features which not restricted to type 1 non locality, but also has type 2. One approach could be to attempt constructing a theory with the Hilbert space structure, but with strictly local time evolution. Then I can look at the weakest assumption I must make to do this and on those grounds, make a prediction. Then find a QM example that violates it.

2 Rethinking

2.1 Assumptions of the Test

What do I really wish to show. I want to show that there exists another type of non locality, namely dynamic non locality. Let's analyse the following

Claim: If I start with (all or some of) the assumptions of Bell, then I would have proved nothing.

Justfctn: Since QM already violates Bell inequality, therefore it means that atleast one of the assumptions in B must be false. If my test is based on those assumptions, then it would not be reasonable to assume that QM will necessarily pass it. Further, if my objective is to show a feature in addition to quantum correlations, then my assumptions mustn't rule out the latter, from the start.

Against: One can however un assume the Bell result. In that case, this new test will be another way of ruling out non local theories. It is similar in spirit with the Non-contextuality result. It's independent of Bell's results, and yet rules out assumption B1 of Bell's. [this last line is actually not correct. The non-contextuality means again one of the assumptions is false. Not necessarily both]

Remark: *If my test is based on (all or some of) those assumptions, then it would not be reasonable to assume that QM will necessarily satisfy it.* A violation therefore will not clarify the cause; whether it is based on quantum correlations or on non local dynamics.

Infact: It seems that if I can construct a test based on only the assumptions of
1. Contextuality and 2. Locality of interaction,
but not assume predefined state of all measurables, then perhaps one can conclude more about precisely which assumption is violated by QM.

2.2 Shades of Non Locality

[I'm hungry, but will write the idea quickly] Bell had to assume locality in his theory because he had considered two particles. In the non-contextuality thing, kroker shpecker (TODO: fix the names) have only one system, therefore they didn't have to assume locality. It appears to me, that I must go another step in this direction. I must assume a single particle/system, and assume locality of some sort within that framework. This will be interesting because it is in stark contrast with bell type locality.

The point really is that non locality now is not about two different particles. I want to see the non locality that exists for a single particle itself.

3 Criticism

There is essentially nothing that has been shown. All of these are trivial consequences of the axioms of quantum mechanics. Note that this doesn't hold for Bell's work. Bell at that time concluded a debate about local realism. What is the point of doing all of this in today's context?

4 Understanding Ali's work

4.1 First Attempt

With the density matrix picture in the back of our minds, we construct a more general framework. This must be able to easily incorporate a wider set of theories with only one constraint; interactions between particles is local. Explicitly then, we assume

- Space like separated particles can have no instantaneous effect on each other

NB: We have assumed nothing about all measurables being simultaneously well defined.

Now one must be careful. We define the following:

1. λ represents the 'state of the system'.
   This may include inaccessible hidden variables. For the case of QM, we'll let $\lambda = (q, p)$

2. Λ is the set that contains all possible states of the system; $\lambda \in \Lambda$ by definition

3. $\mu(\lambda, t)$ is a function generated by the theory, that contains information about all that can be predicted about the system
   This function must satisfy the following

   (a) Normalisation: $\mu(\lambda, t) = \int \mu(\lambda, t) d\lambda = 1$

   (b) ~~Non-negativity:~~ $\cancel{\mu(\lambda, t) > 0}$
       ~~With these, one can (not mathematically strictly perhaps) take~~ $\cancel{\mu}$ ~~to be a probability density~~

   (c) For two systems 1, 2, we define $P_{12}(t) \equiv \int \mu_1(\lambda, t)\mu_2(\lambda, t) d\lambda$
       μ in general therefore should be s.t.
       $$0 \leq P_{12} \leq 1$$
       for all μ that the theory allows.