Generalized Aharnov Bohm Effect in Photons

3rd June 2015

1 Introduction

Abstract

It is common to attribute the Aharonov Bohm (AB) effect to the Gauge and Local Phase coupling of the EM field and the wavefunction. The effect is peculiar because there's no force being applied on the system (no EM field), yet there is an observable change (in the interference pattern). To show this effect for photons, we can no longer use the field as classical and the particle as quantum; it is however unnecessary to construct a complete field theoretic description with gauge interactions. We show (almost trivially) that even for arbitrary potentials, which apparently have nothing to do with gauge transformations, such an effect can be observed. In particular we demonstrate that for a photon in the cat state, we can obtain observable changes, without application of any force. The underlying mechanism is shown to be identical to that of the electric AB effect.

Organization of the document

I'll first describe the relation of gauges and local phase. I'll then relate this to the electric AB effect and explain the change in the interference pattern. Finally, I'll describe how the same effect can be understood rather trivially in terms of a step potential. I will describe next the difficulty with constructing an experiment to observe the same effect in photons. Finally, I'll describe how one such possible setup.

2 Gauge and Local Phase

Here we review how from the Hamiltonian of a particle interacting with EM fields, it becomes necessary to couple to the gauge freedom of the potentials and the local phase of the wavefunction.

Recall: In quantum mechanics, the transformation $\psi \to e^{i\lambda}\psi$ has no observable effect, given λ is constant. This follows because expectation of no observable depends on λ.

NB: If however*, $\lambda = \lambda(x)$ (called local), then manifestly, quantities like $\langle p^i \rangle = \langle -i\hbar\partial^i \rangle$ depend on λ.

Remark: Indeed, as will be shown, when we introduce electromagnetic interactions, then the generator of translations, $P^i = -i\hbar\partial^i$ becomes a gauge dependent quantity and shouldn't be observable (if our theory is correct). The kinematic momentum however, an observable by experience, $p^i = \pi^i \equiv m\partial_0 x^i$, is gauge invariant; will be proved.

W Concl: Global phase yields an equivalent description
Local phase yields an inequivalent description

Let us see how this conclusion is not accurate. The flaw in the logic will be pointed out in the end. For the moment, let us introduce electromagnetic interactions and analyse. We start with deriving the Hamiltonian for the EM case, from the Lagrangian. This is done to avoid any confusion with various definitions of the momentum.

Recall 1: $F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ & 0 & -B_z & B_y \\ & & 0 & -B_x \\ & & & 0 \end{bmatrix}$; $F^{\mu\nu} = \partial^{[\mu}A^{\nu]}$;
$A^\mu = (V, \mathbf{A})^\mu$; $\mathbf{A}$ is the vector potential for $\mathbf{B}$ and V is the scalar potential for $\mathbf{E}$
Clearly, $F^{\mu\nu}$ is invariant under $A^\mu \to A^\mu + \partial^\mu\lambda$, where $\lambda = \lambda(x^\mu)$

Recall 2: A^μ is required for the Lagrangian/Hamiltonian† formalism to get as the EOM, Lorentz force equation: $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
The following is the required Lagrangian:

$$S = \int -mds - eA_\mu ds^\mu$$

*Notation: $x^\mu = (t, \mathrm{x})^\mu$, $\mu \in \{0,1,2,3\}$, $i \in \{1,2,3\}$, $\partial^i \equiv \partial/\partial x_i$
†Units are s.t. $c = 1$ (for simplicity)

The momentum we quantize, is one that generates infinitesimal translations, which is this.

$$\frac{\partial S}{\partial x} = \frac{\partial L}{\partial v} = P = p + eA$$

where $p^\mu = (E, \mathbf{p})$ for the particle (obtained by considering $S = \int -m ds$).

NB:

$$\mathbf{p} = \frac{m\mathbf{v}}{\sqrt{1-\mathbf{v}^2}}$$

Recall: In the Hamiltonian picture, we have coordinates, x_i, P_i. Further,

$$\begin{aligned} x_i &\rightarrow x_i + \epsilon \partial g/\partial P^i \\ P_i &\rightarrow P_i + \epsilon \partial g/\partial x^i \end{aligned}$$

so if we want $x_i \rightarrow x_i + \epsilon$ but P to be invariant (from definition of generator of infinitesimal translations), we must have $g = P_i$.

Recall 2: $H = v.P - L$

NB: The relativistic Hamiltonian is then

$$H = v.P - L = \frac{m}{\sqrt{1-\mathbf{v}^2}} + eV$$

Combining the expression for P and H, we have

$$\begin{aligned} H &= \sqrt{m^2 + (\mathbf{P} - e\mathbf{A})^2} + eV \\ &\approx \frac{1}{2m}(\mathbf{P} - e\mathbf{A})^2 + eV \end{aligned}$$

Now we can go from classical mechanics to quantum mechanics.

Remark: In QM, $A = A(\hat{\mathbf{x}})$, the Hamiltonian is symmetrized in $\mathbf{P}$ and $\mathbf{A}$.

Impose: $[P_i, x_j] = -i\hbar\delta_{ij}$

Recall: $i\hbar\partial_0\psi = \hat{H}\psi$ so that

$$i\hbar\partial_0\psi = \left[\frac{1}{2m}(-i\hbar\nabla - e\mathbf{A})^2 + eV\right]\psi$$

where we've used the fact that P is the generator of infinitesimal translations.

Now with this, we are ready to investigate the repercussions of gauge transformations. Gauge transformations must yield equivalent situations.

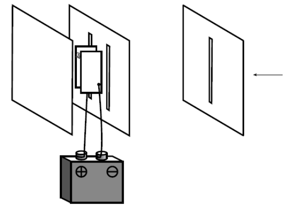

Figure 1: Electric AB effect setup

Assume: Gauge transformation is defined as $A^\mu \rightarrow A'^\mu = A^\mu + \partial^\mu\lambda$ and $\psi \rightarrow \psi'$

NB: We must have $i\hbar\partial_0\psi = \hat{H}\psi \iff i\hbar\partial_0\psi' = \hat{H}'\psi'$ which is guaranteed for $\psi' = e^{ie\lambda/\hbar}\psi$

NB 2: $\partial_0 \langle \mathbf{x} \rangle = \frac{i}{\hbar}\langle [H, \mathbf{x}] \rangle = \frac{1}{m}\langle \psi | (\mathbf{P} - e\mathbf{A}) | \psi \rangle = \frac{1}{m}\langle \mathbf{p} \rangle$ is gauge invariant. This follows from noting $\mathbf{p}' e^{i\lambda(x)}\psi = e^{i\lambda(x)}\mathbf{p}\psi$ holds, $\implies \psi'^*\mathbf{p}'\psi' = \psi^*\mathbf{p}\psi$

Remark: The flaw in the argument from which we made the wrong conclusion was this. Gauge dependence in the Hamiltonian wasn't considered; this resulted in ambiguity of gauge dependence of observables. Indeed if the Hamiltonian is gauge independent, then the conclusion must hold.

Remark 2: In this sense, it is known that quantum mechanics has a closer relation to the potentials. They can't be despensed. The following discussion will conclusively show that the potential is indeed physical as opposed to being a tool for calculational simplicity, as in classical electrodynamics.

Remark 3: One final remark is in order. To preserve the principle of locality (viz. forces can be mediated by fields only where they are present, i.e. fields (not particles) can't act at a distance) , one must introduce $\mathbf{A}$. Else in Faraday's Law of induction, we must accept that the changing magnetic field, acts at a distance. This was Maxwell's point and he therefore regarded $\mathbf{A}$ as more physical than $\mathbf{B}$.

3 Electric AB Effect

Consider: A capacitor is placed between the double slits as shown in figure 1.

The battery is connected only when the electron is close to one of the plates of the capacitors*

*This is done to avoid the complications caused due to fringing

NB: Outside the capacitor, $\mathbf{E}$ is zero
The plates have a charge* $\pm CV$.

Working: (a) When the particle, say electron, passes, say slit one (chosen to be the $+$ plate), because of Coulombic interaction, the plates are pushed apart.
(b) Similarly for slit two, the plates are pulled together.
(c) Since the $\mathbf{E}$ field is zero where the electron is present, therefore it feels no force.
(d) Hence we can find which slit the electron passed through, *without influencing* the electron.

Issue: Newton's third law?

Paradox: It is known that knowing which way the electron went (viz. which slit it passed through) must result in disappearance of the interference. In this case then, however it was shown that the electron is not influenced, therefore putting a capacitor can't have any effect on the interference pattern. We have arrived at a contradiction with the predictions of Quantum Mechanics (which is accurate).

Aim: Here we content ourselves by showing that the electron is influenced by the $\mathbf{E}$ field and the conclusion that $\mathbf{E} = 0 \implies$ 'no observable effect' is wrong.

4 Electric AB Effect simplified

To understand the underlying principle, let us begin with the simplest situation

Consider: $\hat{H} = \hat{H}_0 + \phi$, where ϕ is a real constant

NB: $|\psi(t)\rangle = e^{i\phi t}\left(e^{i\hat{H}_0 t}|\psi\rangle\right) = e^{i\phi t}|\psi_0(t)\rangle$, where $|\psi_0(t)\rangle$ is implicitly defined, $\Phi(t) = \phi t$

Remark: In classical mechanics, adding a constant to the Hamiltonian leads to no difference in the physical situation it describes. In quantum mechanics, evidently, this appears in the evolved state as an overall phase. It must therefore be unobservable by the same arguments used for λ in the previous sections.†

*where C is capacitance and V is the voltage applied

†Note that even though Φ is a function of time, it doesn't depend on position. Therefore no observable constructed using $\hat{x}$, $\hat{p}$ can capture this.

Consider: (a) $\hat{H} = \hat{H}_0 + \tilde{\phi}(x)$, where $\tilde{\phi}$ is now a step function;

$$\tilde{\phi}(x) \equiv \phi H(x) = \begin{cases} \phi & x > 0 \\ \frac{\phi}{2} & x = 0 \\ 0 & x < 0 \end{cases}, \text{ with } \phi \in \mathbb{R}$$

(b) ψ_R is s.t. $\forall\, x < 0\, \psi_R = 0$
(c) ψ_L is s.t. $\forall\, x > 0$, $\psi_L = 0$
(d) $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\psi_L\rangle + |\psi_R\rangle\right)$

Remark: This is a generalization of the situation where a particle is on the right side in the real line. In this domain the potential $\phi(x)$ is again a constant and mustn't affect the classical dynamics.

NB: Similarly, we have $|\psi_R(t)\rangle = e^{i\Phi}|\psi_{R0}(t)\rangle$ and $|\psi_L(t)\rangle = |\psi_{L0}(t)\rangle$, thus

$$|\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|\psi_{L0}(t)\rangle + e^{i\Phi}|\psi_{R0}(t)\rangle\right)$$

Remark: Consider classically, two particles, one on either side of the origin (along the real line), and they are under some potential. If one adds to this potential, $\tilde{\phi}(x)$, no change in the dynamics is expected.

Remark 2: Quantum Mechanically, if one particle is in a superposition as described, then there is an observable effect of adding $\tilde{\phi}(x)$

Remark_3: We were not required to investigate the gauge properties to arrive at this conclusion; we didn't have to assume electro-magnetic interaction

So far, we have not taken any specific physical situation. However, if we are able to show that the electric AB effect is essentially described by this very situation, then we would've explained the AB effect without stressing on the gauge freedom.