Bell Test in phase space (q,p)

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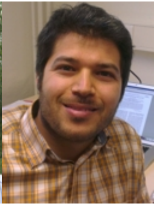
Introduction

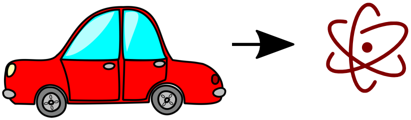
- Programme: DAAD
- Place: University of Siegen, Siegen

- People: Prof. Otfried Guehne and Dr. Ali Assadian

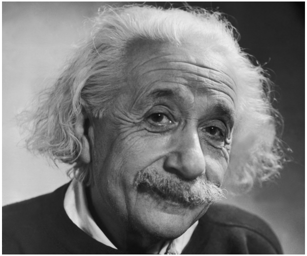
Mechanics

- Newton et. al. : 'Classical Mechanics' $\mathbf{q}, \mathbf{p}$ & forces; t



- [2][1]
- Heisenberg, Schrödinger et. al. : 'Quantum Mechanics'
  $[\hat{\mathbf{q}}, \hat{\mathbf{p}}] = i\hbar$

Questioning QM

- Einstein: not satisfied with QM
  [3]
- Beliefs
  - Locality
  - Reality
- QM must be incomplete [7]
- Suggestion: Must be a better 'locally real' theory that can describe nature

Asking Nature

- John Bell:


- Experimental: Can there be a ‘locally real’ theory that conforms with predictions of QM?[5]
- *NO!*
  - Which assumption is false? - open; progress: Contextuality
  - Can we still do better than QM? - Assume reality but not locality: DeBroglie Pilot Wave, Bohmian Mechanics
  - Does this mean $\mathbf{q}, \mathbf{p}$ don't have ‘local reality’? - Reason to believe so

Bell/CHSH Inequality [6]

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- $$\boxed{\left\langle e^{-i\hat{z}\theta/2}\hat{x}e^{i\hat{z}\theta/2} \otimes e^{-i\hat{z}\phi/2}\hat{x}e^{i\hat{z}\phi/2}\right\rangle = -\cos(\phi - \theta)}$$

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	Alice		Bob	
Convention	a_1	a_2	b_1	b_2
Angle	$\theta_1 = 0$	$\theta_2 = -\pi/2$	$\phi_1 = 3\pi/8$	$\phi_2 = 5\pi/4$

Towards Continuous Variables

- Violation
  - Entangled state
  - Measure
- Modular variables
- Displacement operators

Modular Variables

- $\hat{p}_{mod} \equiv p_{mod} \left|p\right\rangle \left\langle p\right|, \; p_{mod} \equiv p \mod (h/L)$
- $\left|\cos(\hat{p}_{mod}(L/\hbar))\right| \leq 1, \; \hat{p}_{mod} \leftrightarrow \hat{p}$
- $$\boxed{\frac{e^{i\hat{p}L/\hbar} + e^{-i\hat{p}L/\hbar}}{2} \equiv \hat{X}}$$
- $\text{LHS} = \left|\hat{X}\right|$

Displacement Operators

- CM: Momentum as a generator of infinitesimal translations
- $(1 - i\hat{p}\frac{\delta L}{\hbar})\,|x\rangle = |x + \delta L\rangle$
- $(1 - i\hat{p}\frac{L}{N\hbar})^N\,|x\rangle = |x + L\rangle$
- $\lim_{N\to\infty}(1 - i\hat{p}\frac{L}{N\hbar})^N = e^{-i\hat{p}L/\hbar}$

Approximate Eigenstates

Consider a localized state $\varphi(q) = \langle q|\varphi\rangle$ symmetric about the position $q = L/2$, where $L \equiv$length scale and $\varphi_n(q) \equiv \varphi(q - nL)$.

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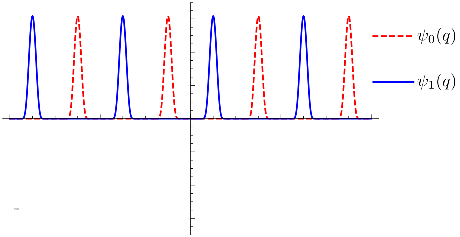
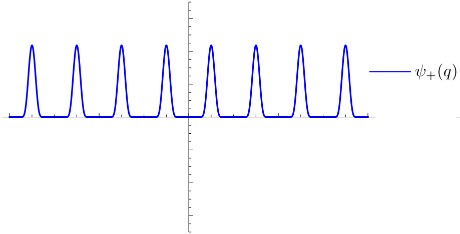
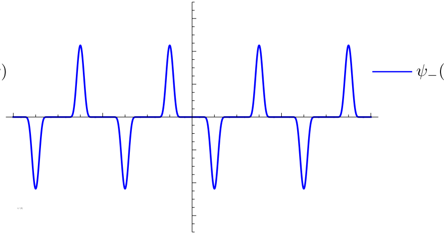
$$|\psi_0\rangle \equiv \frac{1}{\sqrt{M}} \sum_{n=-\lfloor \frac{M}{2} \rfloor}^{\lfloor \frac{M-1}{2} \rfloor} |\varphi_{2n+1}\rangle \ , \ |\psi_1\rangle \equiv \frac{1}{\sqrt{M}} \sum_{n=-\lfloor \frac{M}{2} \rfloor}^{\lfloor \frac{M-1}{2} \rfloor} |\varphi_{2n}\rangle \, .$$



Approximate Eigenstates

$$|\psi_+\rangle \equiv \frac{|\psi_0\rangle + |\psi_1\rangle}{\sqrt{2}} \quad , \quad |\psi_-\rangle \equiv \frac{|\psi_0\rangle - |\psi_1\rangle}{\sqrt{2}}. \tag{1}$$





Note

$$\langle\psi_+|\,\hat{X}\,|\psi_+\rangle = \frac{N-1}{N}$$
$$\langle\psi_-|\,\hat{X}\,|\psi_-\rangle = -\frac{N-1}{N},$$

where $N \equiv 2M$ is the number of ‘slits’.

Measurement Settings

$$\hat{U}(\phi) \equiv e^{i\hat{Z}\phi/2},$$

where $\hat{Z}$ is s.t. $\hat{Z}\,|\psi_0\rangle = |\psi_0\rangle$ and $\hat{Z}\,|\psi_1\rangle = -\,|\psi_1\rangle$.

$$\boxed{\hat{Z} \equiv \mathrm{sgn}\left(\sin\frac{\hat{\mathrm{q}}\pi}{\mathrm{L}}\right)}$$

Entangled State

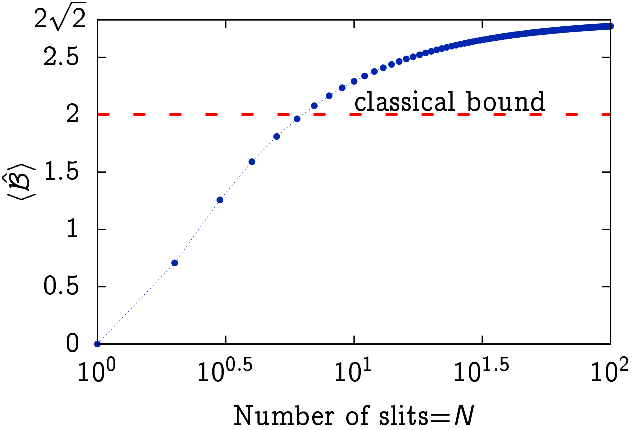
$$\boxed{|\Psi\rangle \equiv \frac{|\psi_+\rangle_1 |\psi_-\rangle_2 - |\psi_-\rangle_1 |\psi_+\rangle_2}{\sqrt{2}}} \tag{2}$$

Bell Test I

We now evaluate $\langle\hat{\mathcal{B}}\rangle$. This essentially requires terms like $\langle\hat{X}(\phi)\otimes\hat{X}(\theta)\rangle$, where $\hat{X}(\theta)\equiv\hat{U}^{\dagger}(\theta)\hat{X}\hat{U}(\theta)$.

$$\boxed{\langle\hat{X}(\phi)\otimes\hat{X}(\theta)\rangle = -\left(\frac{N-1}{N}\right)^{2}\cos(\phi-\theta)}$$

Bell Test II



$$\left|\langle\hat{\mathcal{B}}\rangle\right| = \left(\frac{N-1}{N}\right)^2 2\sqrt{2}.$$

Algebra

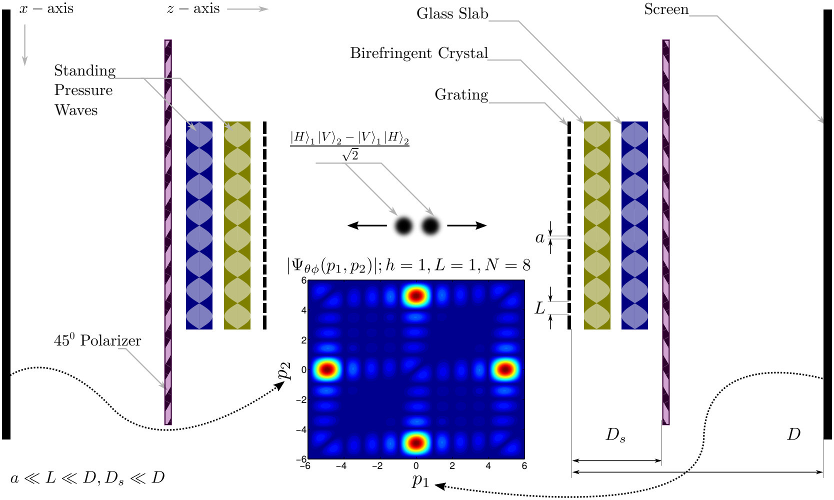
Defining $\hat{Y} = i\hat{Z}\hat{X}$ it follows that $\hat{X}, \hat{Y}, \hat{Z}$ anti commute, from which it follows that

$$\begin{aligned} [\hat{Z}, \hat{X}] &= -2i\hat{Y} \\ [\hat{X}, \hat{Y}] &= -2i\hat{Z}\hat{X}^2 = -2i\hat{X}^2\hat{Z} \\ [\hat{Y}, \hat{Z}] &= -2i\hat{X}. \end{aligned}$$

Non-local Equation of Motion

- $\hat{X}(\phi) = \hat{X}(t)$
- $$\frac{d\hat{X}}{dt} = i\hbar^{-1}[\hat{Z}, \hat{X}] = i\hbar^{-1}\left(Z(\hat{q}) - Z(\hat{q} \pm L)\right)\hat{X}$$
- Classically, $X(t) = X(t_0)$ | step potential, no force
- Scalar AB effect

Implementation



$a \ll L \ll D, D_s \ll D$

Creating the Entangled State

- $$\frac{|H\rangle_1 |V\rangle_2 - |V\rangle_1 |H\rangle_2}{\sqrt{2}} |\psi_+\rangle_1 |\psi_+\rangle_2 .$$
- $$\frac{|H\rangle_1 |V\rangle_2 |\psi_+\rangle_1 |\psi_-\rangle_2 - |V\rangle_1 |H\rangle_2 |\psi_-\rangle_1 |\psi_+\rangle_2}{\sqrt{2}} .$$
- $$|\chi_{45}\rangle |\Psi\rangle = |\nearrow\rangle_1 |\nearrow\rangle_2 \frac{|\psi_+\rangle_1 |\psi_-\rangle_2 - |\psi_-\rangle_1 |\psi_+\rangle_2}{\sqrt{2}}$$

Concluding Remarks

Conclusion

Showed q, p are not 'locally real'

Remarks

- Test
  - Parity based Bell Test for EPR states [4] | Wigner-Weyl representation (unbounded)
  - Direction: Generalize this to $d-$dimensional systems.
  - On arXiv soon
- Tools
  - lyx and emacs with LaTeX.
  - inkscape for diagrams
  - git for version control
- Felt we're well trained at IISER; was prepared in Germany.

Thanks. Questions?

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