projectSiegen | Summary

20th May 2015

1 May 7-13 [week 0 and 1]

1.1 Impressions

- I started with reading about Modular Variables from 'Dynamical quantum non-locality', a perspective paper from Nature Physics. The paper basically talks about how modular variables capture the essence of a quantum state in a way that no other variable we've considered so far does. It relates this to non-locality in quantum mechanics. This non-locality however is not the one usually considered in the context of say the singlet state. The point they make is that this non-locality arises from the equations of motion (Heisenberg picture), which are non-local themselves (since operators are involved). I found the paper is simple, subtle and interesting.

- Next I started reading a paper titled 'Quantum interference experiments, modular variables and weak measurements' from IOP Science. This I was told is an elaboration of the perspective paper. However, I didn't complete this for I hadn't frozen the topic yet.

- I talked to a person named 'Roope' at the group and his work came across as rather fascinating. I was impressed by his work; it is related to 'measurement equivalent of mixed state'. You make a certain kind of measurment with a certain classical probability. With this type of measruement operators, he was able to show that for compatiblity, commutation is not the best criterion. Of course compatability means that the two measurement can be done simultaneously without affecting each other. He gave a good example from his paper 'Joint Measureablity of Generalized Measruements Implies Classicality', PRL to illustrate the points. His main result was unification of the concept of steering with that of his test of joint measurement, viz. compatibility.

- Modular variables are discussed quite neatly in the book by Aharonov et. al., titled 'Quantm Paradoxes: Quantum Theory for the Perplexed'. I read the first few pages, which are a delight to read (about how paradoxes help, classification of paradoxes etc.). I read the main chapter related to modular variables. I still have some small doubts which I'll clarify soon. Other than that, I have a good basic idea of the concept.

- I talked to 'Costentino' who basically told me that his work revolves around looking at Bells inequalities in more complicated systems. I didn't find that particularly attractive. I also talked to 'Nicolai' and he told me he works on finding interesting states. The kind of states he/they look for are such that the sub systems (partially traced) are separable but the entire state is 'genuinely entangled' (this is the region of state space excluding separable and after tracing entangled states). He talked about witnesses and an algorithm to find an optimal witness and an optimal state correspondingly, recursively, starting from a random initial state matrix. This was ok, but again, not very appealing to me. And last person for the (that) day, I talked to 'Marius' and he told me about how he studies bell's inequalities in decaying particles, and his system of choice was Kions. Again, it maybe non trivial and hard, but didn't come accross as worth pursuing.

- Discussed various topics with Roope including
  - How contextuality is expected from usual understanding of QM, but it is essential to characterize the quantum feature of the system (later Dr. Guehne explained how in Bell's case, local hidden variable is the assumption, whereas here, the assumption is non-contextuality [since we're talking about only one system])
  - Discussed how one can exponentiate an unbounded operator. He seemed to have some big mathematical machinary, but I am still not convinced that it is necessary.
  - I wasn't able to understand how to prove (especially after the new definition of exponentiation) $[e^A, e^B] = 0 \implies [A, B] = 0$. We tried some things, but they didn't help much.
  - Implementing causality etc.

- Maria: She works on hypergraph states. The idea is to represent quantum states as graphs. One can show the limits of usual graphs of this form. The limit is that one can represent say the GHZ state, but not a related less entagled state (forgot the name). She had made a lot of new progress in barely 3 months. She found states which maximally violate the bell inequality in 3 qubits, which weren't known earlier. She was able to even derive conditions on probablities of certain outcomes which consequently rule out various local hidden variable models. In the course of doing this, she'd made various conjectures and proven them, by looking at patterns in certain calculations. I found her work rather interesting and impressive, given that she did it all in barely 3 months (and that she is/was a computer science student), but not something I'd pursue.

- Frank: He started with explaining various usual quantum optics subtopics, such as action of a beam splitter in terms of creation anhilation operators, how its action is similar to that of CNOT, how it can easily convert superposition to entanglement, how a coherent state would pass through it etc. Then he mentioned what's called a P distribution, (stands for P something and Sudarshan distribution) which is given for a state ρ as $P_\rho(\alpha)$, with $\alpha \in C$, and s.t. $\rho = \int P_\rho(\alpha) \left|\alpha\right\rangle \left\langle\alpha\right|$. This he said is more useful in characterizing quantum properties. Then he discussed how like in the one qubit case, we have a bloch sphere, we can construct similar objects with more qubits also. Then he talked about relating $\left|\alpha\right\rangle = D(\alpha)\left|0\right\rangle$ with creating states from the extremum state, like $\left|j,j\right\rangle$ by applying similarly constructed D equivalents, except in this case with $J_\pm$ instead of a, $a^\dagger$. He claimed that the states in the orbit of this group (group of transformations), don't span the full space. He said that mixed states so produced can then be mapped to entangled state like was done with the beam splitter. The details he said are involving. This also looked interesting, but again, not the direction in which I'd like to work at the moment.

1.2 Return of Ali

- Read chapter five with a microscope, the way I usually read. I came accross various difficulties consequently. One of them was the fact that they seem to have shown that if we assume a periodic potential and apply perturbation, then the angle of diffraction must correspond to that of a maxima. Else the probability of an electron having any other momenta vanishes. Spent a lot of time reading the first section properly, but it isn't quite worth it, it seemed.
- Ali had returned and I sat and explained to him what it is that I had read in the days he wasn't here. He said he'll be away next week again for another conference. In any case, I explained to him the theme of chapter 4. It had to do with the Aharonov Bohm effect; this relates the gauge freedom in $(V, \vec{A})$ with that of ψ to be overall gauge invariant. This consequently shows up as a phase in an interference experiment. The important thing again is that the interaction is non-local (the EM field is zero where the electrons are present).
  - Next I tried to explain to him the details as given the book, but it was soon concluded that it is not a very good idea to continue in that direction
  - He then described his work on exploiting the relation of the form $D(x)D(p) = e^{i\text{some phase}}D(x+ip)$ to get non local effects. He talked about how plotting wigner functions help visualize the interference patterns etc. and then went on to talk about doing bell type tests on these. He has written Phys Rev papers on these topics.
  - Finally, he showed me some work that he had started working on, but hadn't completed which he feels is new and can be a good project for me to extend/establish.
- Finally, I was convinced that to understand various settings, I must be able to visualize, which requires MATLAB. Unfortunately or fortunately, MATLAB wasn't available on the computer and neither was Marius
- Then I thought of some ideas related to observing nonlocal effects etc. No good though.

2 May 18 - 23 [week 2]

2.1 Monday | Multiple Clarifications and Ambiguities

- The contextuality assumption is not equivalent to the assumption of determinism [this clarification was due to Otfried]
  - The idea is that there can be deterministic contextuality. My claim was that if this is so, then it must violate causality. The counter was based on an explicit construction. The idea was simply that the contextuality can be based on the past and doesn't have to rely on the future.
  - Let's quickly understand this. Assume the following matrix of observables, such that along a column all observables can be measured simultanesouly. Similarly, along the row all observables can be measured simultanesouly.

$$\begin{array}{ccc} A & B & C \\ a & b & c \\ \alpha & \beta & \gamma \end{array}$$

    * My argument was that if the assignment is deterministic and contextual, then say A has a value depending on the context. However, say I just measure A and nothing else at the moment. Since the value of A had to depend on the context, which I haven't picked yet, therefore it is equivalently the statement that the value of A depended on a future event. Which is in clear violation of causality.

    * This is countered by constructing an explicit example. I say that say I picked B next. Eventually I pick C. Now the values of B and C will depend on the fact that A had been chosen to start with. The context is created by the past, not the future. Apparently something called Mealy machines can be constructed which use memory of the kind, which measurement was made and violate the inequality set by assuming non-contextuality
  - Conclusion: Non contextuality is a stronger condition than determinism, which is violated. Which means that we still haven't ruled out deterministic contextual theories.

- The claim that there're no observables that measure the phase in the expression

$$|\psi\rangle = |\psi_1\rangle + e^{i\phi} |\psi_2\rangle$$

  subjected to $\langle\psi_1| |\psi_2\rangle = 0$ was challenged by the following [this was due to Zanna]

  - One could take each of these through a fibre optic and measure its phase wrt another reference laser and find the phase difference OR one could shine a third light $|\psi_3\rangle$ and look at the interference it produces. $|\psi_3\rangle$ is assumed to s.t. $\langle\psi_i| |\psi_3\rangle \neq 0$ for $i \in \{1, 2\}$.
  - This is a serious issue because it had been proven that no moment of x and p can be dependent on ϕ and therefore (ignoring for the moment infinite series etc.) ϕ should not be observable.

- I further realized a crisis in the understanding of the word *non local dynamics*. I initiated reading (fortunately or unfortunately) the paper titled 'Quantum interference experiments, modular variables and weak measruements'. It does seem to have more detail, although I am not certain how relevant and how precise.

- Other than this, I talked to Zanna and she told me she works on something called Quantum Metrology. This is built on the idea that under a Hamiltonian, an entangled state evolves a lot faster and therefore is a lot more sensitive to the external environment. Consequently, this can be used to amplify detection! The catch is that these entangled states are also highly sensitive to noise because of the same reason. One work around is that you have say $2n$ qubits and let n of them evolve under the Hamiltonian whose properties you want to measure (could be a magnetic field say for instance), viz. $H + H_{\text{nosie}}$ and let the other n evolve (without the magnetic field), viz. under H_{noise}. You can then subtract the results (not being precise but that's the idea) and get a neater signal.

2.2 Tuesday

- Objective was to plot the wigner function for various possible states. To this end, I read (like one should) sections 3.5, 3.6, 3.7 and glanced through 3.8 from Gerry and Knight. I covered topics such as

  - $\int |\alpha\rangle \langle\alpha| d^2\alpha = 1$
  - $\hat{F} = \frac{1}{\pi^2} \int d^2\beta \int d^2\alpha e^{-1/2(|\beta|^2+|\alpha|^2)}.F(\beta^*, \alpha) |\beta\rangle \langle\alpha|$
  - P-distribution, Q-distribution, Wigner distribution etc. and various things related to their derivations which aren't worth typing incomplete
  - Glanced through the idea of why and how all these distributions are related

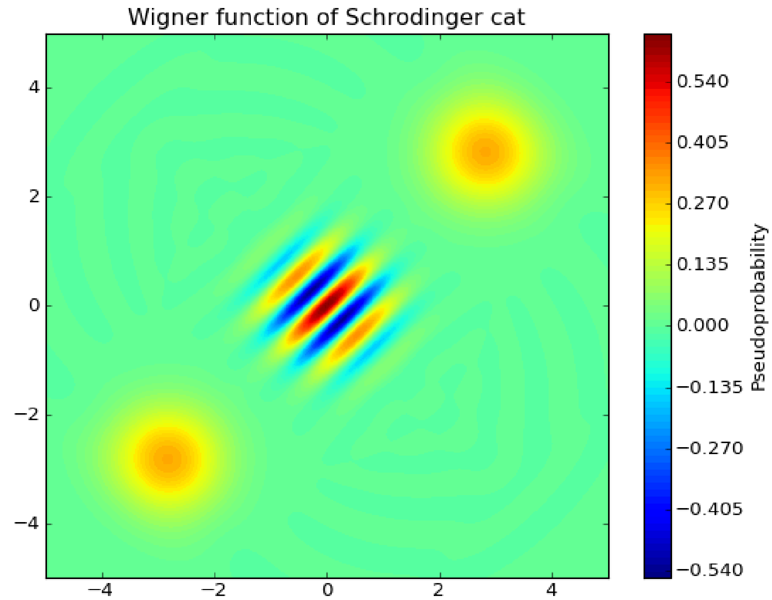
- I attempted getting an expression for the wigner function for the state $|\alpha\rangle$ and what I finally obtained, didn't seem to be real.

  - This I realized while attempting to plot the wigner function using MATLAB. Here's the code for it.

```
%— 05/19/2015 03:14:58 PM —%
a=linspace(1+i,100+i,100)
clear
lambda=linspace(-100+i,100+i,1000);
alpha=5+0i
f=e^(-0.5*( lambda*conj(lambda) + conj(lambda)*alpha + 3*conj(alpha)*lambda))
f=exp(-0.5*( lambda*conj(lambda) + conj(lambda)*alpha + 3*conj(alpha)*lambda))
a=linspace(-1,1,100)
b=0.5*a
b=exp(a)
b=exp(0.5*a)
f=exp(lambda)
f=exp((-0.5)*( lambda*conj(lambda) + conj(lambda)*alpha + 3*conj(alpha)*lambda))
f=exp((-0.5)*(lambda*conj(lambda)))
```

```
f=exp(lambda*conj(lambda)))
f=exp(lambda*conj(lambda))
b=f*f
b=f.*f
f=exp(-0.5*( lambda.*conj(lambda) + conj(lambda).*alpha + 3*conj(alpha).*lambda))
lambda=linspace(-10+i,10+i,1000);
f=exp(-0.5*( lambda.*conj(lambda) + conj(lambda).*alpha + 3*conj(alpha).*lambda))
f=exp(-0.5*( lambda.*conj(lambda) + conj(lambda).*alpha - conj(alpha).*lambda))
w=fft(f)
plot(w)
plot3(mod(w),real(w),imag(w))
plot3(abs(w),real(w),imag(w))
plot3(abs(w),real(lambda),imag(lambda))
plot3(real(w),real(lambda),imag(lambda))
plot3(im(w),real(lambda),imag(lambda))
plot3(imag(w),real(lambda),imag(lambda))
w=fft(f)
plot3(imag(w),real(lambda),imag(lambda))
plot3(1,real(lambda),imag(lambda))
plot3(ones(1000),real(lambda),imag(lambda))
plot(real(lambda),imag(lambda))
x=linspace(-10,10,1000)
y=linspace(-10i,10i,1000)
r_d=linspace(-10,10,1000);
x=repmat(r_d,1000);
x=repmat(r_d,1000,1);
y=repmat(r_d',1,1000);
real(lambda)=x
lambda=x
lambda=x+iy
lambda=x+i*y
lambda=x+i*y;
lambdaMat=x+i*y;
lambda=reshape(lambdaMat,1,1000000)
lambda=reshape(lambdaMat,1,1000000);
f=exp(-0.5*( lambda.*conj(lambda) + conj(lambda).*alpha - conj(alpha).*lambda))
f=exp(-0.5*( lambda.*conj(lambda) + conj(lambda).*alpha - conj(alpha).*lambda));
w=fft(f)
commandhistory
```

- I found that plotting these and other distributions has been made rather simple
  - http://qutip.googlecode.com/svn/doc/2.0.0/html/examples/basic/ex-10.html



where their state was $|\psi\rangle = |\alpha\rangle - |{-\alpha}\rangle$ with $\alpha = 2 + i2$

2.3 Wednesday

- Continued fine reading of chapter 5 from Aharnov's book. It has started to make some sense finally. Although I am still not clear about the doubts from last time, the new sections (with the modifications such as $e^{i\hat{p}L/\hbar}$ as opposed to $e^{ipL/\hbar}$) have started making sense. I also seem to have found a flaw in one of the calculations, where it is shown that $e^{ipL/\hbar}f(x) = f(x+L)$ and so on. The important point I learnt this time however, was the following statement: p_{mod} (defined always with $e^{i\hat{p}L/\hbar}$ at the back of the mind) is completely uncertain iff (if and only if) $\left\langle e^{in\hat{p}L/\hbar}\right\rangle = 0\,\forall\, n \in \mathcal{N}$. If one thinks of $\hat{p}$ as a number, and the expectation as these numbers occurring with classical probabilities, then one can easily prove this. In any case, if taken as a definition also (because it is rather sensible: it is possible that $\langle\hat{p}\rangle \neq 0$ but $\left\langle e^{in\hat{p}L/\hbar}\right\rangle = 0 \iff p_{mod} = 0$) then one can show quite easily that detecting the particle at one hole, leads to loss of information about the modular variable! This is at the very least now makes some sense in doing.
  In my CT I thought about how it is possible for p_{mod} to evolve with time, viz. $\left\langle e^{in\hat{p}L/\hbar}\right\rangle$ to evolve with time, but when looked at in the Schrodinger picture, since the wavefunction $|\psi\rangle$ doesn't change (assume the potential to be locally constant), then how will $\left\langle e^{in\hat{p}L/\hbar}\right\rangle$ evolve?
- Next I tried plotting the wigner function for the state $\rho = |\alpha\rangle\langle\alpha|$
  - I used the method where $W(\gamma) = \frac{1}{(2\pi\hbar)^2}\int d^2\lambda e^{(\lambda\gamma^* - \gamma\lambda^*)/2}\mathrm{tr}\left(\rho D(\lambda)\right)$
  - I converted this to two fourier transforms as $W(\gamma_1 - i\gamma_2) = \frac{1}{(2\pi\hbar)^2}\int d\lambda_1 d\lambda_2 e^{i(\lambda_2\gamma_1 + \lambda_1\gamma_2)}\mathrm{tr}\left(\rho D(\lambda)\right)$
  - I proved that analytically, W must be real in general.
  - I evaluated analytically the expression for $\mathrm{tr}\left(\rho D(\lambda)\right)$ for the given coherent state
  - I attempted plotting using this script

```
max=40
half=max/2
alpha=-1.5i;
r_d=linspace(-2*pi,2*pi,max);
x=repmat(r_d,max);
x=repmat(r_d,max,1);
y=repmat(r_d',1,max);
lambda=x+1i*y;
% f=exp(-0.5*( lambda.*conj(lambda) + conj(lambda).*alpha - conj(alpha).*lambda));
% f=exp(-abs(lambda - alpha).^2);
f=exp(-0.5*(abs(alpha).^2) -0.5*(abs(alpha + lambda).^2) - conj(alpha).*(lambda+alpha));
subplot(2,2,1);
contourf(abs(f),10);
```

```
subplot(2,2,2);
contourf(angle(f),10);

w=fft2(f);
subplot(2,2,3);
contourf(abs(w([half:max 1:half],[half:max 1:half])),10)
subplot(2,2,4);
contourf(angle(w([half:max 1:half],[half:max 1:half])),10)
% contourf(angle(w(1:100,1:100)),10)

% subplot(2,2,3);
% contourf(abs(w(1:100,1:100)),10)
% subplot(2,2,4);
% contourf(angle(w(1:100,1:100)),10)

% plot3(w)



% lambdaMat=x+i*y;
% lambda=reshape(lambdaMat,1,1000000)
% lambda=reshape(lambdaMat,1,1000000);
% f=exp(-0.5*( lambda.*conj(lambda) + conj(lambda).*alpha - conj(alpha).*lambda))
% f=exp(-0.5*( lambda.*conj(lambda) + conj(lambda).*alpha - conj(alpha).*lambda));
% w=fft(f)
```

However, numerically W was not evaluating to be even close to real

- I then calculated the trace for $|\psi\rangle = |\alpha\rangle + |\beta\rangle$ and attempted plotting again with an improved script, still the results weren't correct. I didn't get the kind of interference I am expected to
- I proved analytically that W for the coherent state case must be real explicitly.

- Some results I used to cross check my answers:
  - Fourier transform of shifted guassian: `http://www.thefouriertransform.com/applications/gaussian.php`
  - Analytic expression for W for cat states: Gerry and Knight
- Tried thinking about some new things. [TODO: Complete this]