Quantum Measurements and Relativity

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Introduction

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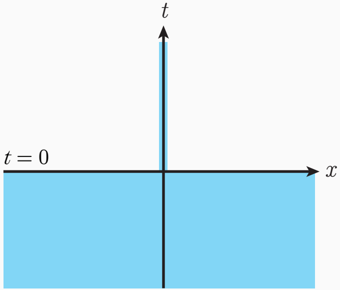

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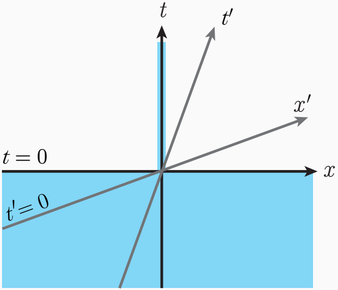

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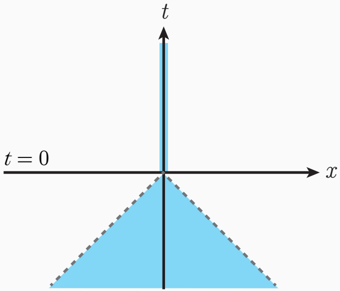

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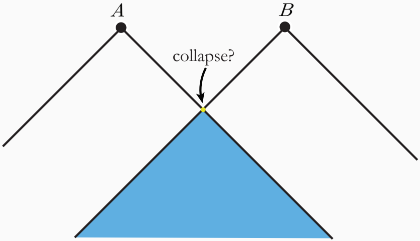

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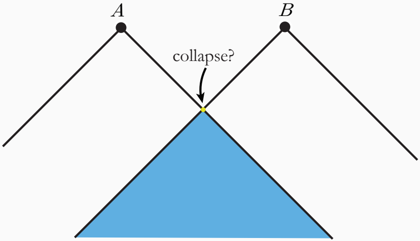

Collapse happens in the past $\implies$ A, B measures product state $\implies$ Obeys Bell's inequality $\implies$ Paradox

Relativistic Constraint

Landau and Peierls:

Theorem (Relativistic Uncertainty Relation)

Measurement of momentum cannot be instantaneous. Momentum with accuracy Δp cannot take time less than Δt such that

$$\Delta p \, \Delta t \geq \frac{\hbar}{c} \, .$$

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- Measurement of non-local operator cannot be instantaneous.

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E.g.: Measure total spin of 2 localised spin-$\frac{1}{2}$ particles with state

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- Rewrite state in total spin basis:

$$|\Psi_{\alpha\beta}\rangle = \frac{\alpha+\beta}{\sqrt{2}}\,|2,0\rangle + \frac{\alpha-\beta}{\sqrt{2}}\,|0,0\rangle\ ,$$

where

$$|0,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right) \quad \text{and} \quad |2,0\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\right)\ .$$

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- Measuring device state

$$Q_z^A + Q_z^B = 0 = P_z^A - P_z^B\,.$$

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- Do this for S_x and S_y too.
- If $S_x = S_y = S_z$, then $S^2 = 0 \implies$ verified $|0, 0\rangle$

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- The space is spanned by bases

$$|\Psi_{\alpha\beta}\rangle = \alpha\,|\uparrow\downarrow\rangle + \beta\,|\downarrow\uparrow\rangle\,; \quad \left|\Psi^{\perp}_{\alpha\beta}\right\rangle = \beta\,|\uparrow\downarrow\rangle - \alpha\,|\downarrow\uparrow\rangle\,; \quad |\uparrow\uparrow\rangle\,; \quad |\downarrow\downarrow\rangle\,.$$

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$$P(\uparrow_A) = \frac{1-2\alpha\beta}{2}\alpha^2 + \frac{1+2\alpha\beta}{2}\sum_i \left|\left\langle W_i \middle| \Psi^{\perp}_{\alpha\beta}\right\rangle\right|^2 \left[|\langle W_i|\uparrow\uparrow\rangle|^2 + \beta^2 \left|\left\langle W_i \middle| \Psi^{\perp}_{\alpha\beta}\right\rangle\right|^2 \right]$$

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- Demand probabilities to be the same

$$\implies \alpha\beta = 0 \quad \text{or} \quad \alpha = \pm\beta$$

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A and B cannot measure W instantaneously unless each eigenstate of W is $|\Psi_{\alpha\beta}\rangle$ *with* $\alpha\beta = 0$ *or* $\alpha = \pm\beta$*, up to local unitary transformation.*

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A and B cannot measure W instantaneously unless each eigenstate of W is $|\Psi_{\alpha\beta}\rangle$ *with* $\alpha\beta = 0$ *or* $\alpha = \pm\beta$*, up to local unitary transformation.*

- This is necessary but not sufficient. Why? Think of S^2.

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- Can A and B simultaneously measure such an operator?
- Take two eigenvectors to be $\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$ and $|\uparrow\uparrow\rangle$
- Can Alice and bob instantaneously measure an operator with these two states as eigenstates?

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  - $|\uparrow\uparrow\rangle$ changes to $|\uparrow\downarrow\rangle$
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  - The non-local operator has a chance of leaving the spins in the state $\frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

Sufficient Conditions

- If yes $\implies$
  - A and B prepare their spins in the state $|\uparrow\uparrow\rangle$
  - A and B measure instantaneously
  - Alice measures σ_z on her spin and she is certain to obtain $\sigma_z = 1$
  - But, what if just before the measuring the non-local operator, Bob flips his spin?
  - $|\uparrow\uparrow\rangle$ changes to $|\uparrow\downarrow\rangle$
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  - Now Alice has a chance of obtaining -1 when she measures σ_z
  - Bob can send superluminal signal to Alice! $\implies$ Alice and Bob can't measure such an operator instantaneously!

Conclusion:

A and B cannot measure W instantaneously unless each eigenstate of W is $|\Psi_{\alpha\beta}\rangle$ *with* $\alpha = \pm\beta$*, up to local unitary transformation.*

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- We have a contradiction only if observers in different frames verify incompatible account of collapse
- Which they can't!

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- No contradiction over collapse!
- In fact, disagreement is fine!
- Observers in different frames disagree over length, temporal order etc.

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- So what?

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- Is everything okay?

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Countering Collapse

- So what?
- Is everything okay?
- What about measurements?
- Relativistic causality forbids an instantaneous collapse of almost all non-local operators!

Then what happens in our experiments?