EFFECTIVE FIELD THEORIES

A theory that is reasonably accurate in a given energy regime and is replaced by some other more complete theory at a given UV scale M.

Examples

- An engineer building a bridge don't need to care about atoms and molecules. For him Newton's Law work best.
- Chemists don't need to take into account the structure of hadrons.
- Fermi theory of electroweak interactions is an effective theory which is replaced by the SM at sufficiently high energies.

- We cannot simply ignore the high energy theory.
- It will have some (maybe small) effects on low energy physics.
- Effective Field theory gives us a procedure of how to take into account these effects.

- If we are interested in processes happening at given scale E $\ll$ M involving fields with masses much smaller than M, the heavy states with mass $\sim$ M cannot be produced.
- We can just integrate out the heavy states.

$$e^{iS_{IR}(\phi_l)} = \int \mathcal{D}\phi_h \, e^{iS_{UV}(\phi_l,\phi_h)}$$

- This integrating out procedure is not very easy.
- Note that the fields are made up of creation and annihilation operators.
- Also we need to integrate over all possible configurations of heavy fields.
- EFT provides an easy and calculable method of finding an effective theory given a theory at high energies.

EFT Procedure

- Given a UV action $S_{UV}(\varphi_L, \varphi_H)$, write down the EOM for heavy field and solve for $\varphi_H(\varphi_L)$
- So at tree level, $S_{IR}^{(0)}(\phi_l) = S_{UV}(\phi_l, \phi_h(\phi_l))$

$$S_{EFT}(\phi_l) = S_{IR}^{(0)}(\phi_l) + \Delta S(\phi_l)$$

where ΔS encodes all higher dimensional local operators up to some order, depending on the accuracy we want to achieve, and compatible with the global symmetries of the theory.

•All the unknown couplings multiplying these operators are then fixed by UV theory and the EFT. This procedure is called **Matching**.

Examples : Two Scalars

$$\mathcal{L}_{UV} = \frac{1}{2}(\partial H)^2 - \frac{1}{2}M^2H^2 + \frac{1}{2}(\partial L)^2 - \frac{1}{2}m^2L^2 - \frac{g}{2}HL^2$$

Most general effective action in light fields can be written as

$$\mathcal{L}_{IR} = \frac{1}{2}Z_L(\partial L)^2 - \frac{1}{2}\tilde{m}^2L^2 - \frac{\lambda}{4!}L^4 + \text{higher dimensional operators}$$

From UV action, the equation of motion for H fields, in the limit ($p \ll M$) is :

$$H = -gL^2/(2M^2) + \mathcal{O}(p^2/M^4)$$

Putting this back into L_{UV}

$$\mathcal{L}_{IR}^{(0)} = \frac{1}{2}(\partial L)^2 - \frac{1}{2}m^2L^2 - \frac{\lambda}{4!}L^4 + \mathcal{O}(g^2M^{-4})$$

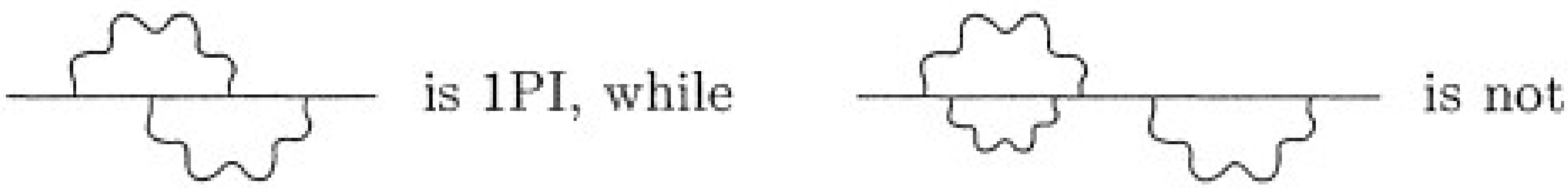
- Comparing most general EFT action and the one obtained by plugging EOM of heavy fields in UV action (Matching), we get

$$Z_L = 1 + \mathcal{O}(g^2)\,, \quad \tilde{m}^2 = m^2 + \mathcal{O}(g^2)\,, \quad \lambda = -\frac{3g^2}{M^2} + \mathcal{O}(g^4)\,.$$

- This is just tree level matching, in order to know couplings to higher order, we need to calculate IPI's in both the theories and compare them.

Calculating O(g^4) contribution to λ

- The coupling λ, appears in front of φ4, so we need to calculate $\Gamma^{(4)}$ in both the theories and compare.
- $\Gamma^{(4)}$ are IPI diagrams (the diagrams that when cut into parts don't give meaningful diagrams) with 4 external particles.



- So we calculate $\Gamma^{(4)}$ for the complete UV action (Note that the diagrams should be 1PI in light fields not in heavy ones) and the EFT action and then compare the coupling.

- So by comparing both the $\Gamma^{(4)}$ we can find the coupling λ to higher order. Notice that we need to compare both the equations at same scale (it is convenient to choose M) .
- Higher order corrections to IR mass and Z can be found by comparing $\Gamma^{(2)}$.
- The more accuracy we want, we need to compute these 1PI at higher loop orders.

Fermions and Scalars

- If we take a theory with both fermions and scalars and try to compute the low energy EFT in the case when fermions are heavy and scalars are light i.e

$$
\begin{aligned}
\mathcal{L}_{UV} &= \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \bar{\psi}(i\partial\!\!\!/ - M)\psi - g\phi\bar{\psi}\psi \\
\mathcal{L}_{IR} &= \frac{1}{2}Z_\phi(\partial\phi)^2 - \frac{1}{2}\tilde{m}^2\phi^2 + \frac{\lambda_3}{3!}\phi^3 + \frac{\lambda_4}{4!}\phi^4 + \text{h.d.o.}
\end{aligned}
$$

The tree-level integration of ψ gives $\psi = 0$, so that

$$
Z_\phi = 1 + \delta Z_\phi, \quad \tilde{m}^2 = m^2 + \delta_{m^2}\,, \quad \lambda_3 = \mathcal{O}(g^3)\,, \quad \lambda_4 = \mathcal{O}(g^4)\,.
$$

- When one calculates the higher order correction to scalar mass, it comes out to be :

$$m^2_{phys} = m^2(M) + \frac{4g^2}{16\pi^2}\Big(- 5M^2 + \frac{16}{3}m^2\Big)$$

- So we see that the physical mass of scalar at low energy comes out to be proportional to UV scale.
- So as you go up in energy, (since physical mass is fixed), your bare mass should be adjusted such that it compensates the effect of M. It is unpleasant to have in the EFT a parameter that is so sensitive to the UV physics.
- This fine tuning of bare mass is not nice for a physical theory and is termed as NATURALNESS PROBLEM

Naturalness

- Concept given by t'Hooft. It says

Dimensionless couplings $\sim O(1)$

Dimensionful couplings $\sim O(M^{[d]})$

Exceptions can arise if a symmetry is restored when a coupling (dimensionless or not) vanishes, in which case it is natural to have that coupling arbitrarily small.

e.g. In the first example g = O(M) (natural value), we get that the dimensionless coupling λ in the EFT = O(1)

- Now mass parameter has dimension 2, so according to t'hooft's naturalness, its natural value should be $\sim O(M^2)$. So it is unnatural to have light scalars.
- But in the case of fermions when m goes to zero, chiral symmetry of lagrangian is restored, so from 2nd part of naturalness definition, fermions can be arbitrarily light.

Hierarchy problems

- **Cosmological Constant Problem**

  The $[\Lambda] = 4$ So its natural value should be $O(M_{pl}^4) \sim 10^{120}$ where M_{pl} is the Planck mass. But the measured value is 0.7. This large discrepancy is called cosmological constant problem.

- **Higgs Mass**

  If SM is considered as an EFT obtained from some ultimate theory (which contains gravity also). Then Higgs being a scalar cannot be light, its mass $\sim O(M_{pl}^2)$, which is very different from the observed one. This is called SM hierarchy problem.

TO THINK

Why should we take NATURALNESS so seriously??