Quantum Phase Transition in Jaynes Cummings Hubbard Model

Kishor Bharti

Centre for Quantum Technologies, Singapore

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Radiation Hangout

Overview

1. Phase Transition
2. A few Models
   - Jaynes Cummings Model
   - Bose Hubbard Model
   - Jaynes Cummings Hubbard Model
3. Photon-blockade-induced Mott transitions and XY spin models in coupled cavity arrays
4. Quantum phase transitions of light
5. Quantum chaos and Jaynes Cummings Hubbard Model

Phase Transition

Phase and phase transition

Let

- Ω : System of interest for which we define the Hamiltonian H_Ω
- F: Free energy

We can expand free energy as

$$F(\Omega) = V(\Omega) f_b + S(\Omega) f_s + O(L^{d-2}) \tag{1}$$

Here,

- f_b : Bulk free energy density
- f_s: Surface free energy density

Phase and Phase Transition

Now one can plot the bulk free energy density as a function of the degrees of freedom of the system

Phases correspond to the regions of analyticity of bulk free energy density

- Types of Phase transition
    - First order phase transition
    - Continuous phase transition

Phase Transition

To analyze Quantum phase transition

- Write Hamiltonian for the system indicating the parameters on which it depends
- Find ground state energy E_g
- Analyze the variation of E_g with respect to the parameters of the theory

A few Models

Let us try to understand the dynamics of a free atom interacting with field. The situation could be understood at three differenet levels of complexity.

- Classical Model: The atom is modelled using a classical oscillator, while light is represented as a classical electromagnetic wave.
- Semiclassical Model: The atom is quantized (energy levels are discrete), while the light is still represented as a classical electromagnetic wave.
- Quantum Mechanical Model: Both light as well the atom are quantized.

Let us talk about the fully quantum mechanical model, since that suffices our purpose of understanding the Jaynes Cummings Hubbard model.

Jaynes Cummings Model

Let us talk about an atom with energy levels $|g\rangle$and $|e\rangle$, interacting with a single-mode electromagnetic field, given by

$$\hat{\mathbf{E}} = \mathbf{e}\left(\frac{\hbar\omega}{\epsilon_0 V}\right)\left(\hat{a} + \hat{a}^{\dagger}\right)\sin\left(kz\right). \tag{2}$$

,where symbols denote the corresponding standard meaning in literature. The hamiltonian for the model consists of atomic hamiltonian, free field hamiltonian and the interaction term. The free field hamiltonian is given by

$$\hat{H_F} = \hbar\omega(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}) \tag{3}$$

The vaccuum term in the free field hamiltonian is anyway insignificant and hence we will drop the same for future calculations.

To analyze free atom hamiltonian, let us introduce atomic transition operators

$$\hat{\sigma}_+ = |e\rangle\langle g| \tag{4}$$

and

$$\hat{\sigma}_- = |g\rangle\langle e| \tag{5}$$

We further define the inversion operator, given by

$$\hat{\sigma}_3 = |e\rangle\langle e| - |g\rangle\langle g| \tag{6}$$

If we define the energy of the atom as zero, exactly halfway between the ground and excited state, the hamiltonian for the same turns out to be

$$\hat{H_A} = \frac{1}{2}\left(E_e - E_g\right)|e\rangle\langle e| - |g\rangle\langle g| \tag{7}$$

or

$$\hat{H_A} = \frac{1}{2}\hbar\omega_0\hat{\sigma_3} \tag{8}$$

Jaynes Cummings Model

The interaction between the field and the atom is fascilitated by the dipole moment of the atom. the interaction hamiltonian, thus can be written as

$$\hat{H_I} = -\hat{\mathbf{d}} \bullet \hat{\mathbf{E}} \tag{9}$$

or

$$= -\hat{d} \left(\frac{\hbar\omega}{\epsilon_0 V} \right) \sin(kz) \left(\hat{a} + \hat{a^\dagger} \right). \tag{10}$$

Jaynes Cummings Model

Let us condense $-\left(\frac{\hbar\omega}{\epsilon_0 V}\right)\sin(kz)$ as g, which simplifies the look of the interaction Hamiltonian

$$\hat{H_I} = -\hat{d}g\left(\hat{a} + \hat{a^\dagger}\right) \tag{11}$$

Since the off diagonal elements of the dipole operator ($\hat{d}$) vanish as they are of opposite parity, the dipole operator can be written as

$$\hat{d} = d(\hat{\sigma_+} + \hat{\sigma_-}) \tag{12}$$

Thus

$$\hat{H_I} = \hbar\lambda(\hat{\sigma_+} + \hat{\sigma_-})(\hat{a} + \hat{a^\dagger}) \tag{13}$$

Jaynes Cummings Model

$$\hat{H} = \hat{H_A} + \hat{H_F} + \hat{H_I} \tag{14}$$

$$\hat{H} = \frac{1}{2}\hbar\omega_0\hat{\sigma_3} + \hbar\omega(\hat{a^\dagger}\hat{a} + \frac{1}{2}) + \hbar\lambda(\hat{\sigma_+} + \hat{\sigma_-})(\hat{a} + \hat{a^\dagger}) \tag{15}$$

Bose Hubbard Model

- Describes a system of interacting bosons on a lattice
- "Bose" refers to the fact that particles are bosonic

$$H = -t \sum_{<i,j>} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i(\hat{n}_i - 1) - \mu \sum_i \hat{n}_i \tag{16}$$

At $T = 0$

- $\frac{t}{U} << 1$: Mott insulating phase
- $\frac{t}{U} >> 1$: Superfluid phase

Jaynes Cummings Hubbard Model

- Series of optical resonators coupled through direct photon hopping
- Each resonator is doped with a two level system
-

$$H_{free} = w_d \sum_i \hat{a_k} a_k + w_0 \sum_k |e\rangle_k \langle e|_k \tag{17}$$

$$H_{int} = g \sum_k (\hat{a_k} |g\rangle_k \langle e|_k + H.C) \tag{18}$$

$$H_{hop} = -J \sum k(\hat{a_k}(a_{k+1} + H.C) \tag{19}$$

$$H = H_{free} + H_{int} + H_{hop} \tag{20}$$

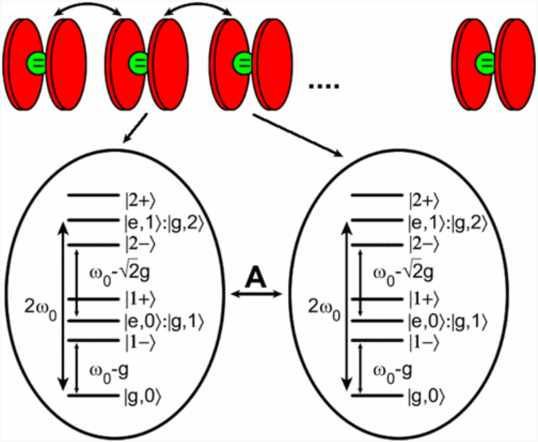
Photon-blockade-induced Mott transitions and XY spin models in coupled cavity arrays

Photon-blockade-induced Mott transitions and XY spin models in coupled cavity arrays

- Take the local part of the Hamiltonian
- Diagonalize in the basis of mixed photonic and atomic excitations
- The eigenenergies are given by

$$E_n^{\pm} = nw_d + \frac{\Delta}{2} \pm \sqrt{ng^2 + (\frac{\Delta}{2})^2} \tag{21}$$

Photon-blockade-induced Mott transitions and XY spin models in coupled cavity arrays



Photon-blockade-induced Mott transitions and XY spin models in coupled cavity arrays

- The eigenkets are

$$|n,+\rangle_k = \frac{1}{\sqrt{2}}(\sin\theta_n |g,n\rangle_k + \cos\theta_n |e,n-1\rangle_k) \tag{22}$$
$$|n,-\rangle_k = \frac{1}{\sqrt{2}}(\cos\theta_n |g,n\rangle_k - \sin\theta_n |e,n-1\rangle_k) \tag{23}$$

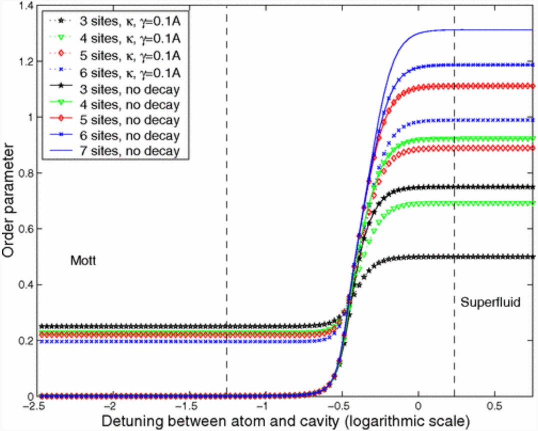
- Assume $\Delta = 0$ and expand the Hamiltonian in terms of polaritonic operators defined as

$$P_k^(\pm, n) = |g,0\rangle_k \langle n,\pm|_k \tag{24}$$

Photon-blockade-induced Mott transitions and XY spin models in coupled cavity arrays

- Consider $Jn << g\sqrt{n} << w_d$
- To understand the phase transition, we need to consider negative energy branch and the hopping term
- The on-site photonic repulsion leads to “Mott state” as ground state
- If we increase detuning, the effective nonlinearity goes down and system shifts to Superfluid regime.

Photon-blockade-induced Mott transitions and XY spin models in coupled cavity arrays



Quantum phase transitions of light

Quantum phase transitions of light

- Two dimensional array of photonic bandgap cavities
- Each cavity contains a single two level atom which is quasi resonant with the cavity mode
- The interacting photon hopping is provided by evanescent coupling between the cavities and hence we take only nearest neighbour interaction.
- Explored the dynamics of two dimensional lattice of quantum cavities
- The Hamiltonian is given by

$$\mathbb{H} = \sum_i \mathbb{H}_i^{JC} - \sum_{<i,j>} K_{i,j} a_i^\dagger a_j - \sum_i \mu_i N_i \tag{25}$$

- Altering the number of nearest neighbours does not qualitatively affect results.

We carry out a mean field approximation

- Let us introduce a superfluid order parameter $\psi =< a_i >$, which is real
- Carry out decoupling approximation
- The mean field Hamiltonian becomes

$$\mathbb{H}^{MF} = \sum_i \{\mathbb{H}_i^{JC} - zk\psi(a_i^\dagger + a_i) + zk|\psi|^2 - \mu(a_i^\dagger a_i + \sigma_i^+ \sigma_i^-)\} \quad (26)$$

Quantum phase transitions of light

- At $T = 0$, we look for the properties of the system
- Diagonalize the matrix and identify the lowest eigenvalue E_g
- Find out the converging value of E_g
- Minimize E_g with respect to ψ for different values of k, w and Δ to obtain the phase diagram
- Region with
  - $\psi = 0 \implies$ no fluctuation (Mott phase)
  - $\psi \neq 0 \implies$ Supefluid phase

Quantum chaos and Jaynes Cummings Hubbard Model

Quantum chaos and Jaynes Cummings Hubbard Model

- Case 1
  - Start with Jaynes Cummings Hamiltonian with $\Delta = \Delta_1$
  - Take a state $|\Psi\rangle$ $(t = 0)$
  - Evolve the state till $t = T$
- Case 2
  - Start with Jaynes Cummings Hamiltonian with $\Delta = \Delta_2$
  - Take a state $|\Phi\rangle$ $(t = 0)$
  - Evolve the state till $t = T$
- Calculate state fidelities at $t = 0$ and $t = T$
- Relate it to Lyapunov exponent
- Select the detunings from Mott phase, Mott and superfluid phase, Superfluid phase and look for variations

Questions?