Information Causality as a Physical Principle

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  - The set of measurable non-local operators shrinks.

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- So it is sensible to take a step back and look at Special Theory of Relativity and Quantum Theory at its basic level

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  - The laws of physics are invariant (i.e. identical) in all inertial systems (non-accelerating frames of reference)
  - The speed of light in a vacuum is the same for all observers, regardless of the motion of the light source.
- Do we have any "physical statement based structure" for Quantum theory?

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- Efforts have been made in the past to understand Quantum theory in "**Axiomatic manner**"
    - Nonlocality as an Axiom for Quantum theory
- Till now, no one (including the authors of this paper) has succeeded.

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- The striking features of quantum mechanics have often provided the much needed hope to the scientists to come up with a physical principle behind its formalism

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  - The answer was found to be negative
- There is a broad class of theory which mimics typical quantum features like "intrinsic randomness", “no-cloning” and so on
- So what is the specificity of quantum theory?

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- Generalizes no-signalling principle
- Is respected by both Classical and Quantum physics and violated by "more that quantum" no-signalling theories

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- The bits communicated are classical (and not quantum bits)

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Alice and Bob can share only no signalling resources. We will call these resources as no-signalling boxes.

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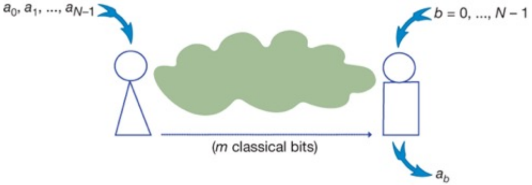

M Pawłowski *et al.* *Nature* **461**, 1101-1104 (2009) doi:10.1038/nature08400

nature

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here, $I(a_K : \beta | b = K)$ is the Shannon mutual information between a_K and β

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But this intuitive description is not theory-independent.

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Claim: Information causality is respected by both classical and quantum physics. Other no-signalling theories violate information causality.

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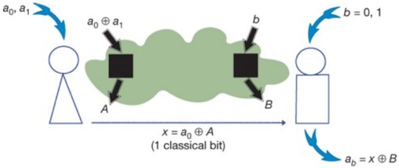
Van Dam's protocol[17].



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- Bob computes his guess $\beta = x \oplus B = a_0 \oplus A \oplus B$

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- The Classical correlations are bounded by $S \leq S_C = 3$
- Quantum correlations violate this limit up to $S \leq S_Q = 2 + \sqrt{2}$

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$$E > E_Q \implies I \nleq 1 \tag{7}$$

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- Why this principle is not so appealing?
  - Doesn't seem to distinguish between "degenerate theories"
  - Tries to derive Quantum Mechanics within its framework

What can be possible attempts in this direction?