

# Continuous Time Quantum Cryptography & Communication Complexity

## FRIA | research project proposal

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Atul Singh Arora

October 28, 2016



ECOLE  
POLYTECHNIQUE  
DE BRUXELLES



Promoter:

**Prof Jérémie Roland**

Centre for Quantum Information & Communication  
Université libre de Bruxelles

# Overview

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# QUANTUM INFORMATION

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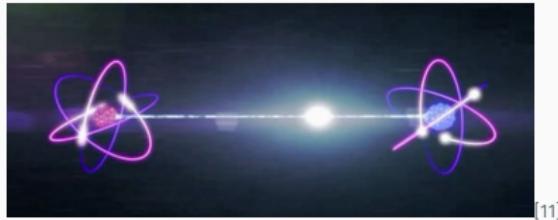
Bits, 0 and 1.

# QUANTUM INFORMATION



[7]

Bits, 0 and 1.



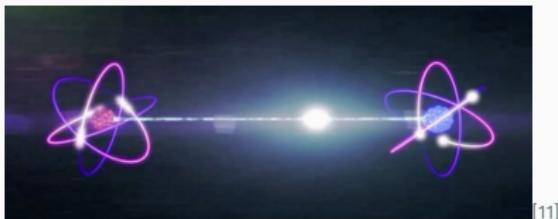
[11]

Qubits,  $|0\rangle$  and  $|1\rangle$

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Bits, 0 and 1.



Qubits,  $|0\rangle$  and  $|1\rangle$

Classical

0

1

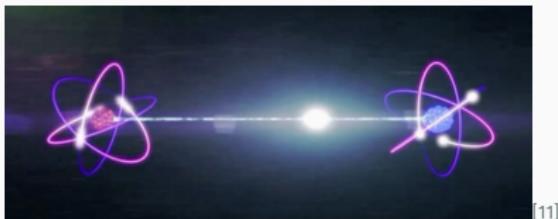
Quantum

'Forbidden'

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$\rightarrow$

Quantum

$|0\rangle$

$$\doteq \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

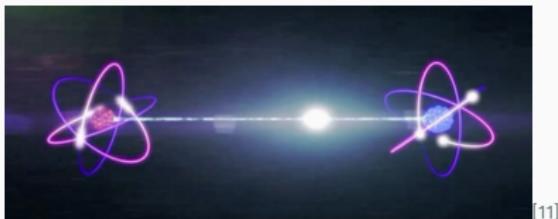
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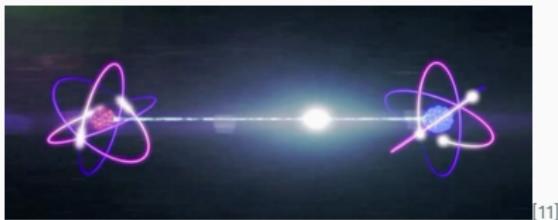
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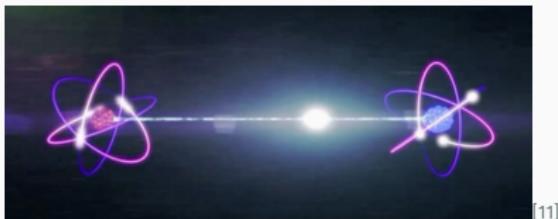
$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

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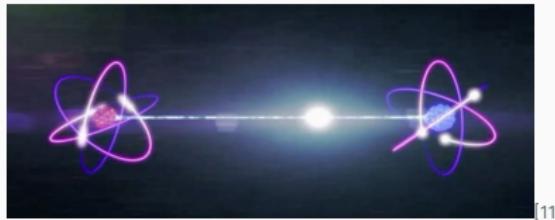
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Classical → Quantum:

e.g. Factorisation and Secure Key Distribution

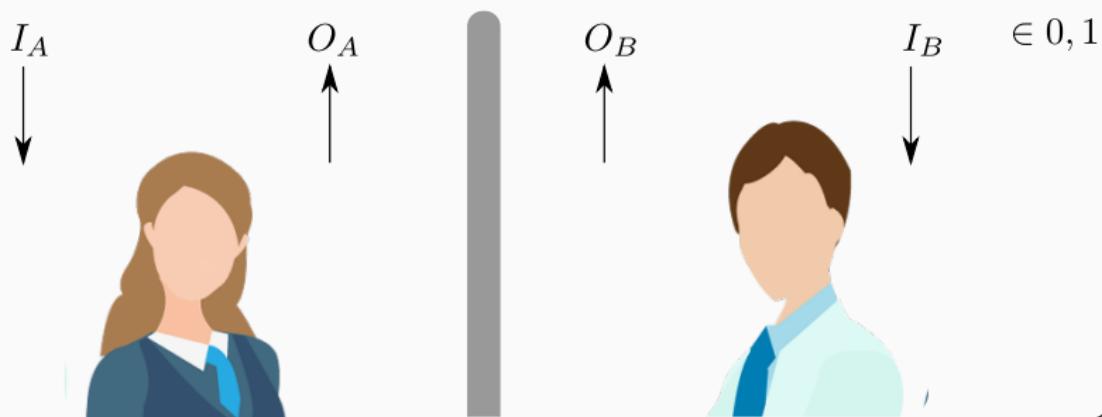
Computational Power

[20] [15]

# CHARACTERISING QUANTUM

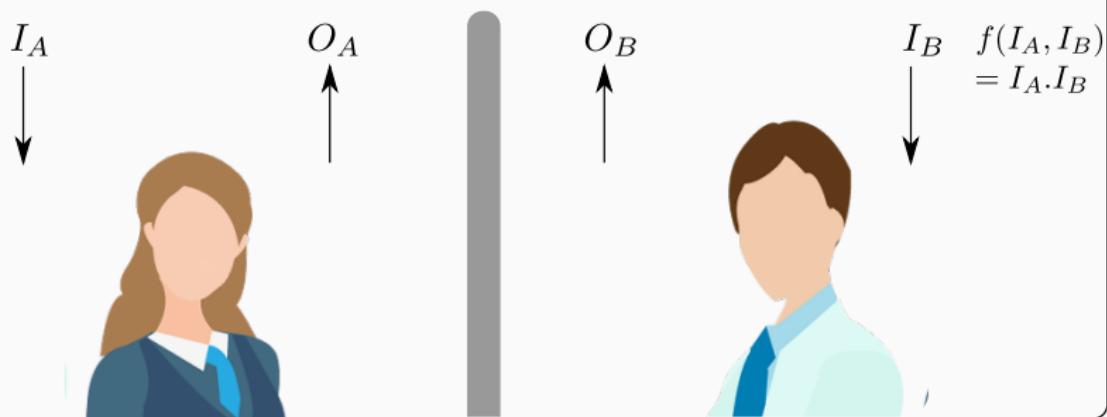
If  $f(I_A, I_B) = 1$  then **anti-correlate**  
else **correlate**

For e.g.  $f(I_A, I_B) = I_A \cdot I_B$



# CHARACTERISING QUANTUM

1	1,0	anti-correlated	0,1	1	1
0	1,0	correlated	1,0	1	0
1	1,0	correlated	1,0	0	0
0	1,0	correlated	1,0	0	0



# CHARACTERISING QUANTUM



$$\frac{3}{4} = 0.75$$



[1, 5, 6, 4]

# CHARACTERISING QUANTUM

$$\frac{1}{2} + \frac{\sqrt{2}}{4} \approx 0.85$$



[1, 5, 6, 4]

# Quantum Magic



# CHARACTERISING QUANTUM

- Non-locality: Bell Inequality

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- Non-locality: Bell Inequality
- Summer Internship: Continuous variable Bell test

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- Non-locality: Bell Inequality
- Atul Singh Arora, Ali Asadian.  
**Proposal for a macroscopic test of local realism with phase-space measurements,**  
*Physical Review A*, 2015, **92**, 062107.

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- MS Thesis: Alternative to Contextuality

## CHARACTERISING QUANTUM

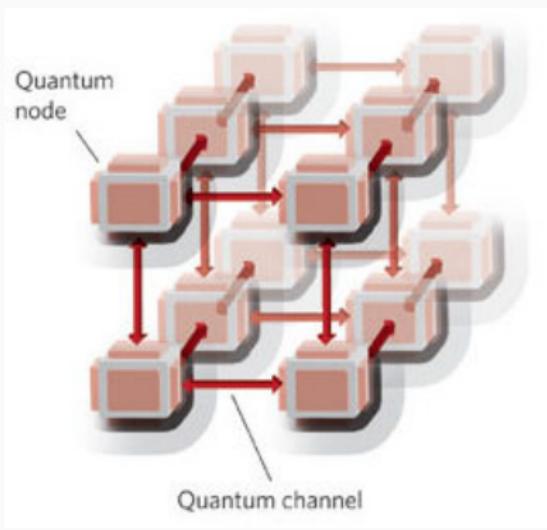
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**A non-contextual hidden variable model for quantum mechanics**  
arXiv:1607.03498, submitted to *Physical Review Letters*.

# QUANTUM NETWORKS

Objective: Harness full capacity of a quantum computer network

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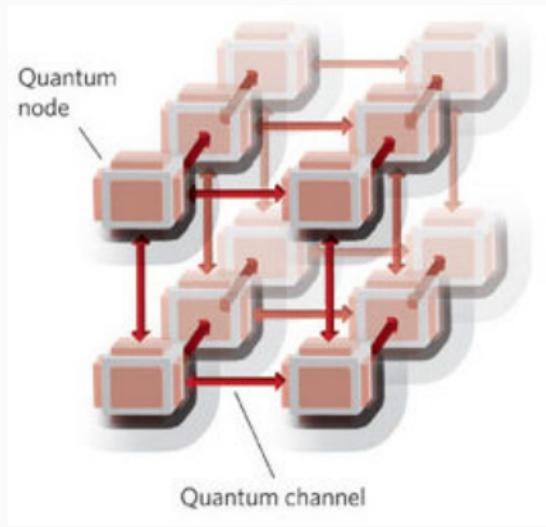
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[12]

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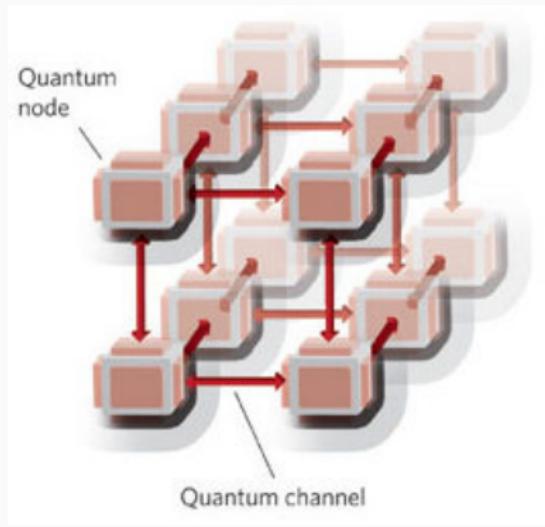
[12]

**Cryptographic Primitives** Building blocks of cryptographic algorithms

**Communication Complexity** Least bits of communication to compute a function  $f(x,y)$

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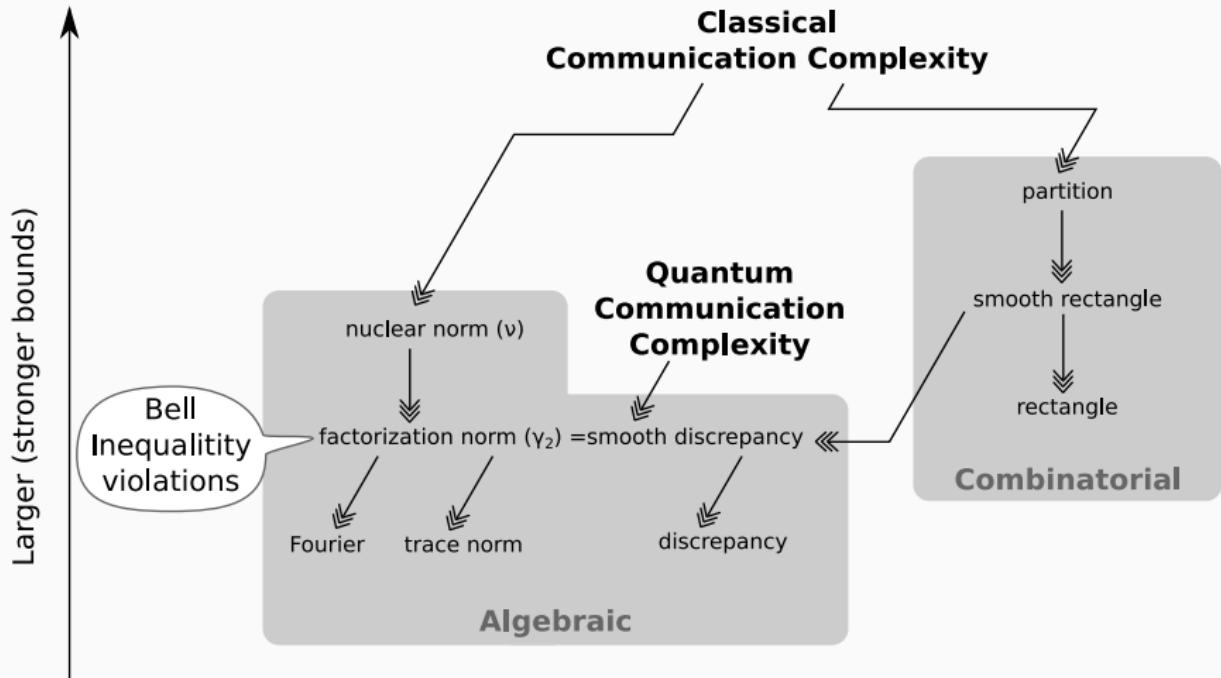
**Communication Complexity** Least bits of communication to compute a function  $f(x,y)$

Make Quantum: More secure, more efficient.

## State of the Art

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# COMMUNICATION COMPLEXITY



**Figure 1:** Schematic: Quantum Information Complexity

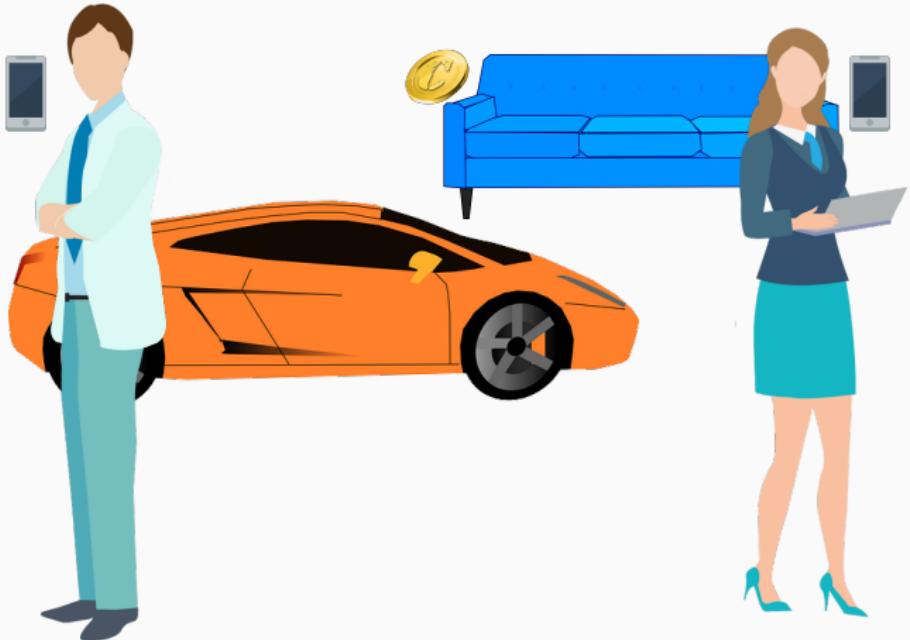
# CRYPTOGRAPHIC PRIMITIVES



[2]

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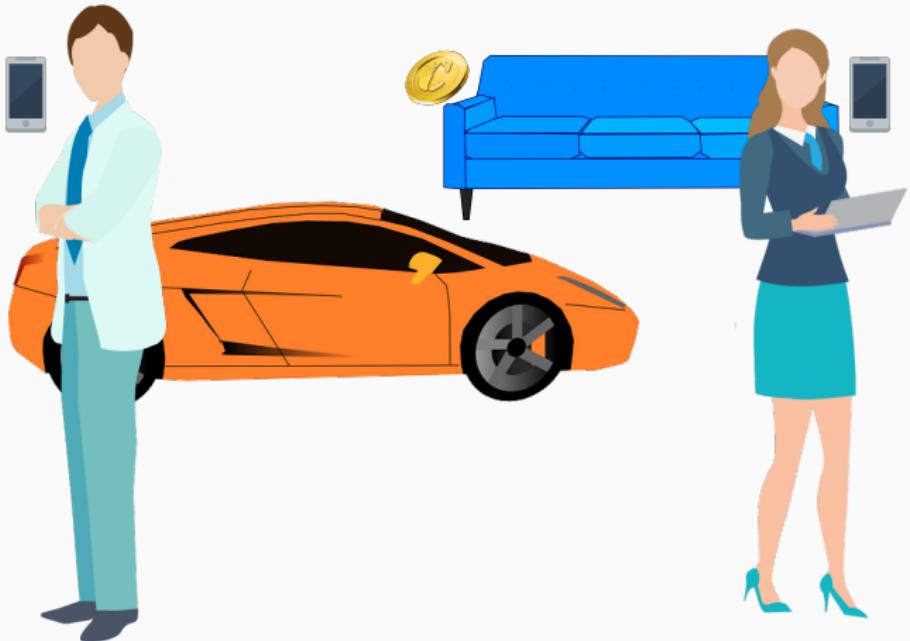
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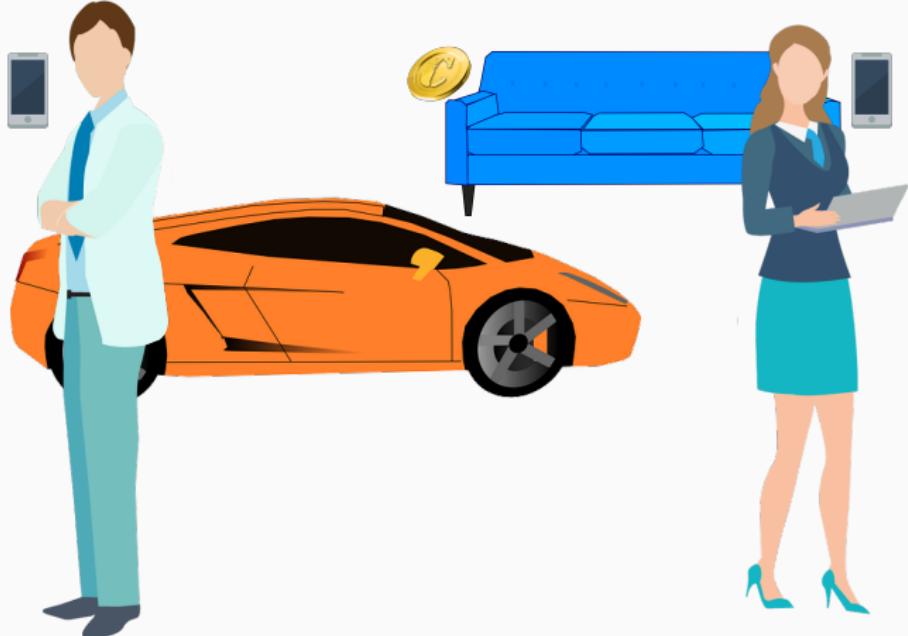


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- ‘Quantumly’, for strong coin flipping  $\epsilon > \frac{1}{\sqrt{2}} - \frac{1}{2} \approx 0.207$ .

[17]

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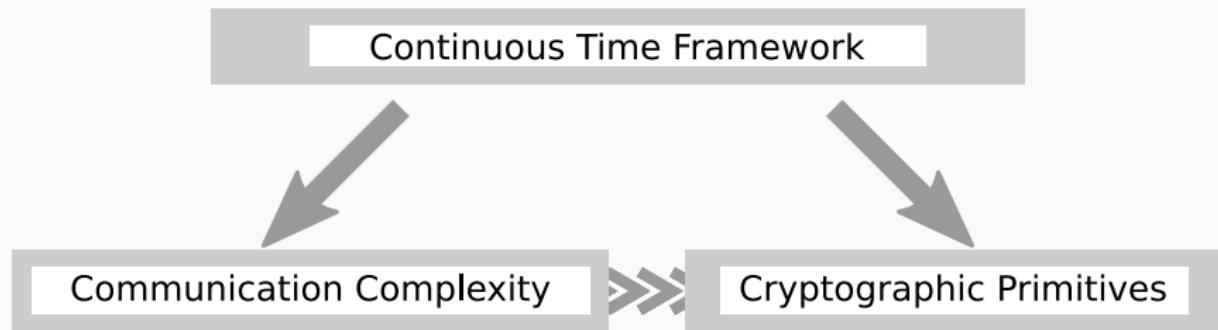
- Classically, coin flipping is not possible.
- ‘Quantumly’, for strong coin flipping  $\epsilon > \frac{1}{\sqrt{2}} - \frac{1}{2} \approx 0.207$ . [17]

- ‘Quantumly’, for weak coin flipping  $\epsilon \rightarrow 0$ ;  
best explicit protocol has  $\epsilon = \frac{1}{6}$ . [19, 14]

# The Project

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## THE SCHEME



**Figure 2:** The approach

# THE CONTINUOUS-TIME FRAMEWORK

**Discrete-Time (DT) protocols** Sequential information flow



[1, 9, 8]

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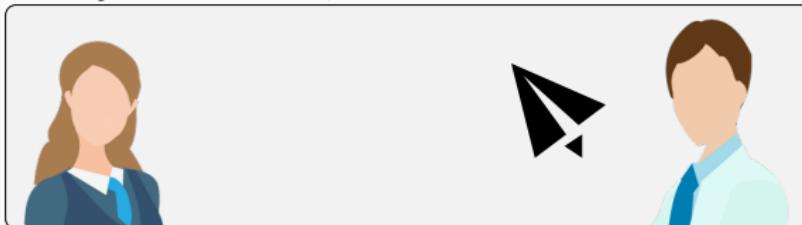
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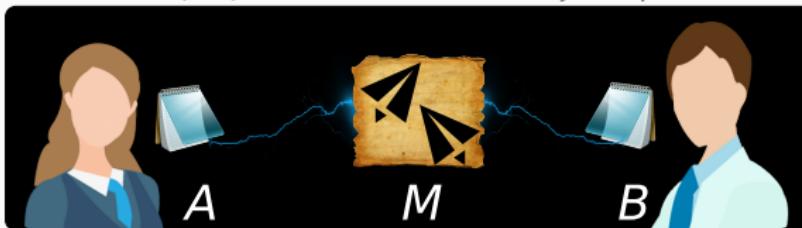
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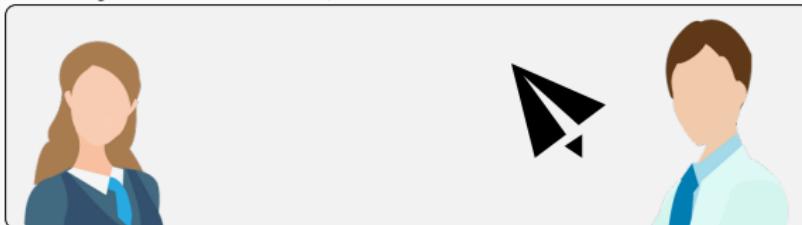
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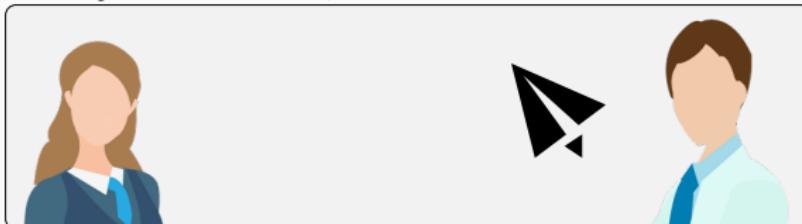
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$$\cdot H = H_A \otimes I_{MB} + H_{AM} \otimes I_B + I_A \otimes H_{MB} + I_{AM} \otimes H_B.$$

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- $H = H_A \otimes I_{MB} + H_{AM} \otimes I_B + I_A \otimes H_{MB} + I_{AM} \otimes H_B$ .
- CT protocol  $\iff$  DT protocol ( $H_{AM}$  or  $H_{BM}$  zero).

## CT COMMUNICATION COMPLEXITY (CT-CC)

**(new) Continuous Time Communication Complexity (CT-CC)** The time required to evolve the system to the desired state where  $|H_{AM}|, |H_{BM}| \leq 1$ .

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- Preliminary Results: Quantum query complexity characterised.

[16]

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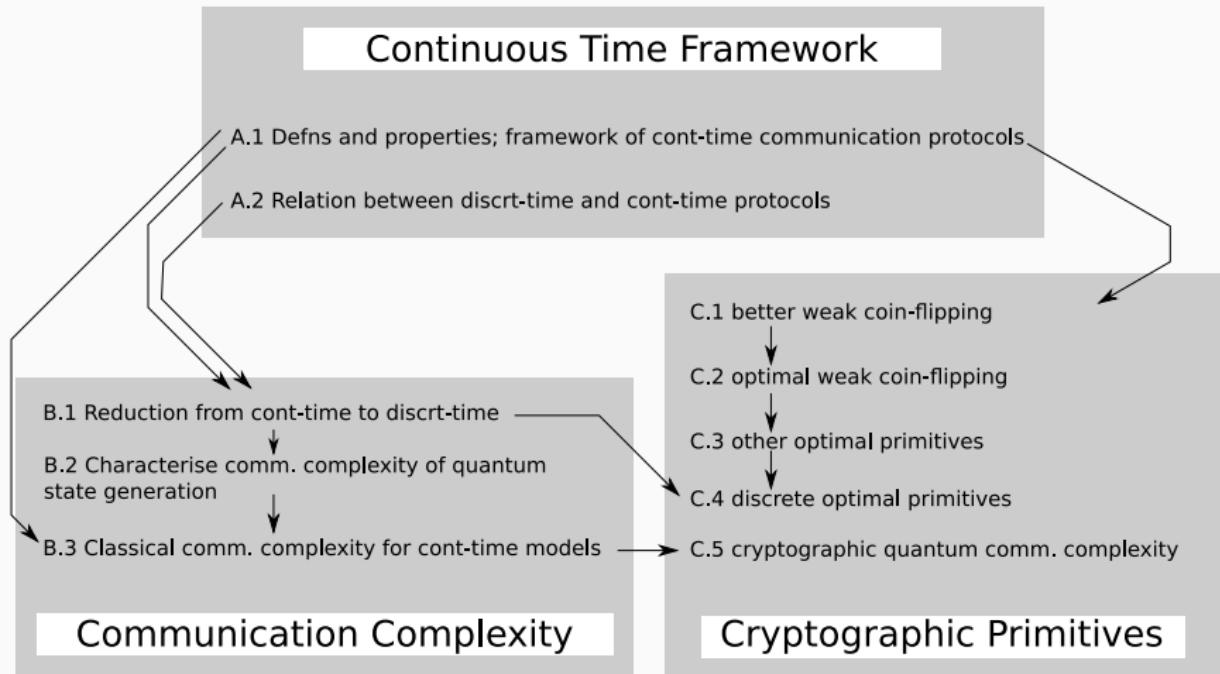
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Goal: Weak coin flipping

- Advantage: Fresh approach to open problem.
- Preliminary Result:  $\epsilon = \frac{1}{6}$  obtained by discretising.
- Insight: To break  $\frac{1}{6}$  use

$$|\Psi\rangle = \int_{t=0}^T |\psi_t\rangle |t\rangle dt$$

# EXPECTED RESEARCH FLOW



**Figure 3:** Schematic: Research Flow

# Conclusion

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- Novel and promising approach towards open and relevant problems

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Help make quantum computer networks a commercial reality.

Questions?

## REFERENCES |

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[10] Phone image.

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device-mobile-phone-icon--small--flat-iconset--paome.png.

[11] Quantum image.

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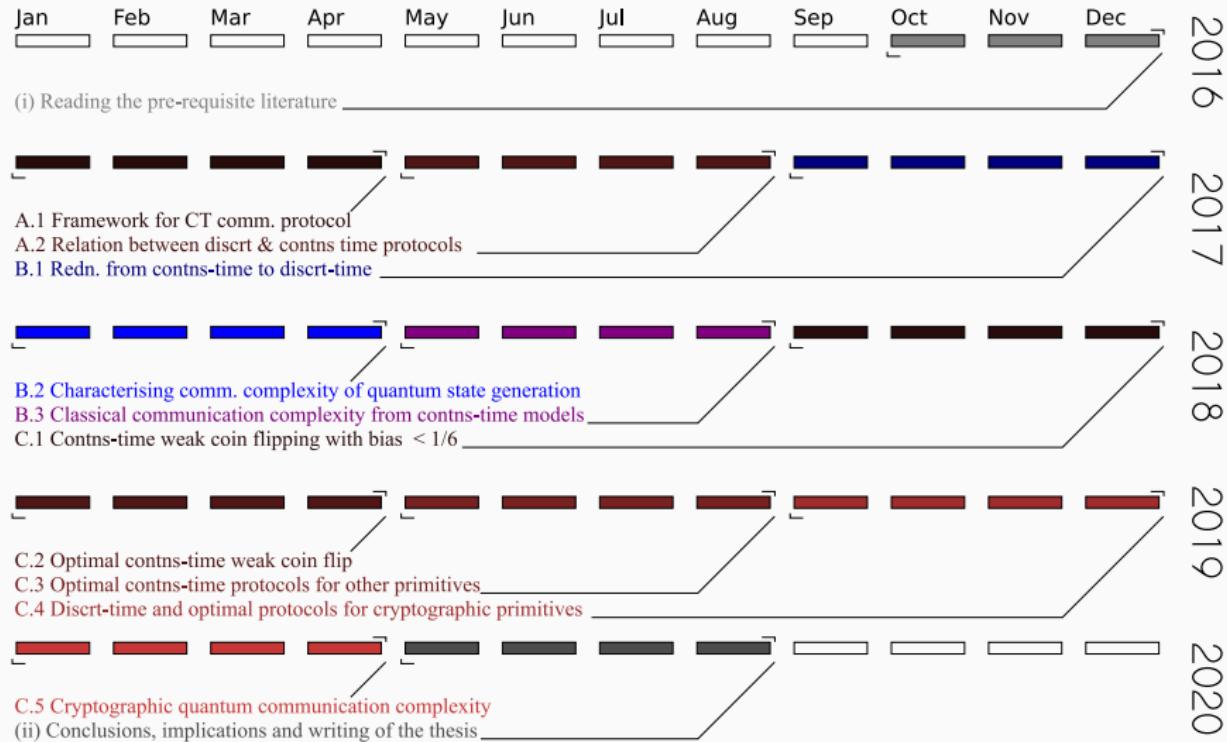
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**Quantum weak coin flipping with arbitrarily small bias**, 2007.

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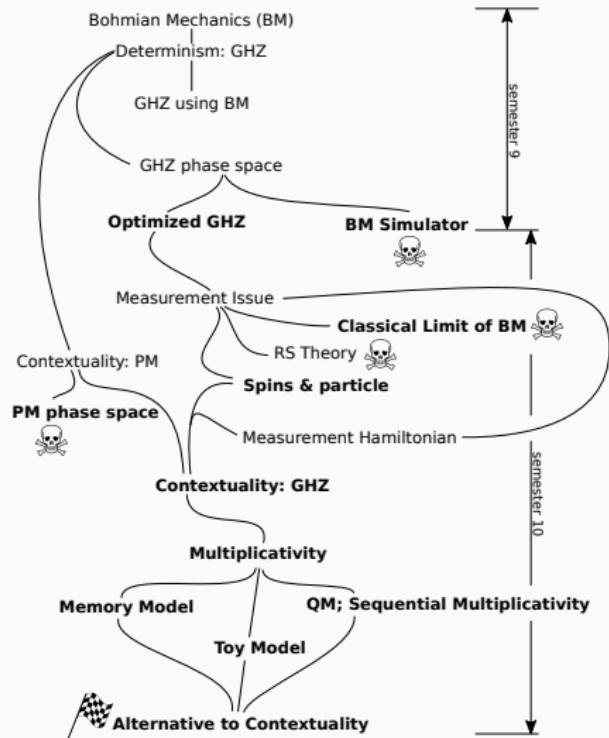
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# TENTATIVE TIMEFRAME



**Figure 4:** Schematic: Planned Schedule

# MS PROJECT



**Figure 5:** MS Project: Contextuality in a deterministic quantum theory.

## MY PUBLICATIONS

- A. S. A., Arvind.  
**A non-contextual hidden variable model for quantum mechanics**  
arXiv:1607.03498, submitted to *Physical Review Letters*.
- A. S. A., Ali Asadian.  
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