Toan Ly

**Lecture 3 Exercise**

**1)** Assume each friend orders a pizza, the total number of pizza to be ordered is 7.

Since there are 4 types of pizza, and the chef only cares about the number of each type (such as 2 pizza #1, 2 pizza #2, 2 pizza #3 and 1 pizza #4), the order here doesn’t matter, for example, AABBCCD is the same as ABABCCD (assuming ABCD is pizza type).

Let’s denote pizza as an asterisk (\*), we can have this diagram to visualize the problem easier: Pizza #1 Pizza #2 Pizza #3 Pizza #4

\* \* | \* \* | \* \* | \*

Using star and bar approach, there are 7 pizza (7 asterisks \*) and 3 bars (since there are 4 types), so there are 10 total available spots. The bars can be in anywhere in the 10 spots, therefore, the total number of possible combined orders are:

**2)**

a) (i) The random variable is a

(ii) The domain is dom(a) = {1, 2, 3}

(iii) Since , it is not a valid probability distribution

b) (i) The random variable is x

(ii) The domain is dom(x) = {a, b, c}

(iii) Since it is a valid probability distribution

c) (i) Random variable is z

(ii) dom(z) = {aa, bc, de}

(iii) it is a valid probability distribution

**3)** Given:

Box 1: 3 red, 5 white (8 total) Box 2: 2 red, 5 white (7 total)

A box is chosen randomly p(box 1) = p(box 2) = 0.5

A ball chosen randomly from this box is red

p(box 1 | red ) = ?

---------------------------------------------------------------------------------------

(Bayes’ rule)

(Marginalize over box)

Plug all the above back into (1):

**4)**

(i) (Bayes’ rule)

(Marginalize over y)

(Bayes’ rule)

(Bayes’ rule)

(1)

Similarly, p(x | y, z) can be expanded using the same rule as in (1):

(2)

Plug (2) into (1):

(ii) Since y is unconditionally independent of z, p(y | z) = p(y)

Plug it into equation (1) from (i):

(iii) Since x is conditionally independent of y, given z, p(x | y, z) = p(x | z)

Plug it into equation (1) from (i):

**5)**

**A number with numbers on a white background

AI-generated content may be incorrect.**

0.49 + 0.05 + 0.46 = 1 => Valid probability distribution

A number with red lines

AI-generated content may be incorrect.

0.46 + 0.09 + 0.45 = 1 => Valid

**6)**

Therefore, A and B are not independent

**Bonus)**

Let x1, x2, x3, x4 be the number of pizza #1, #2, #3, and #4, respectively

Also, x1, x2, x3, x4 >= 0 and x1 + x2 + x3 + x4 = 7 (7 people => 7 pizza)

First, since each person has 4 options, the sample space is 47

Let x = be the variable storing the correct proportion of each pizza

Assume an ordered pizza sequence looks like \_ \_ \_ \_ \_ \_ \_ (7 slots for 7 people choices)

=> If given x1, x2, x3, x3, the number of possible pizza sequences =

=> Probability that the 7 friends choose this exact composition x1, x2, x3, x4:

Similarly, the probability that the chef chooses the correct pizza counts:

The probability that the chef matches the correct order:

(1)

From question 1, we know that there are 120 combinations of {x1, x2, x3, x4}

Therefore, to calculate the equation (1) above, we’ll loop through all 120 combinations, which will take too long, so I will use Julia to calculate the result as below:

A screenshot of a computer program

AI-generated content may be incorrect.

Therefore, the probability that the chef delivers the correct order is 0.018