Title Place Holder

Arjun Menon K, *placeholder, IEEE,* John Doe, *Fellow, OSA,* and Jane Doe, *Life Fellow, IEEE*

***Abstract*—Here we are proposing an efficient algorithm to calculate Optimum phase shift vector($\overline{\Psi}_{opt}$) for training the reflected beam form the Reflective Intelligent Surface(RIS) to a given azimuth and elevation angle (θ_r, ϕ_r). Here we assume Line of Sight(LoS) wireless channel between the Access Point(AP) and RIS. We also assume that there is no direct path between the AP and UE.**

***Index Terms*—Reflective Intelligent surface**

I. Mathematical Model

THE RIS has N_H columns and N_V rows of sub-atoms(reflectors) as shown in the Figure 1. Distance between any two reflectors in a row or in a column is $\frac{\lambda}{2}$, where λ is the wavelength of incident ray.

We assume that the positions of RIS and AP are fixed. The only variable is the direction of reflected beam from RIS towards UE in terms of (θ_r, ϕ_r), where (θ_r, ϕ_r) are the reflected azimuth and elevation angles respectively.(Ref: Figure 2). The problem being studied here is - for a given (θ_i, ϕ_i), how we can calculate the $\overline{\Psi}_{opt}$ so that the reflected beam from RIS is trained towards the desired direction (θ_r, ϕ_r) .

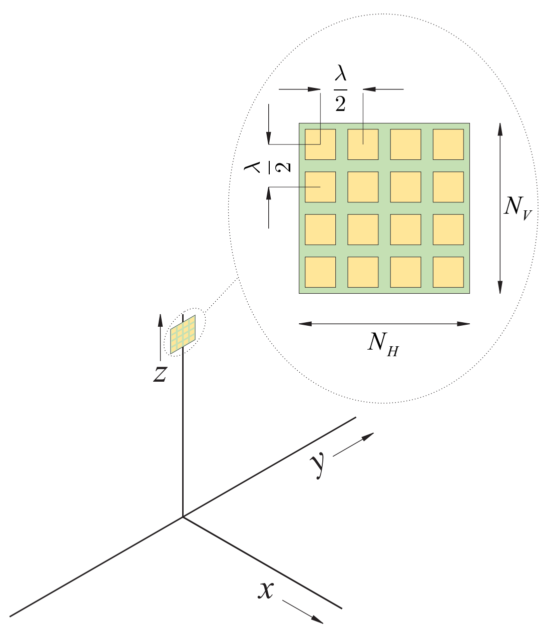


Fig. 1: The coordinate system describing the RIS with N_H columns and N_V rows.

M. Shell was with the Department of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA, 30332 USA e-mail: (see http://www.michaelshell.org/contact.html).

J. Doe and J. Doe are with Anonymous University.

Manuscript received April 19, 2005; revised August 26, 2015.

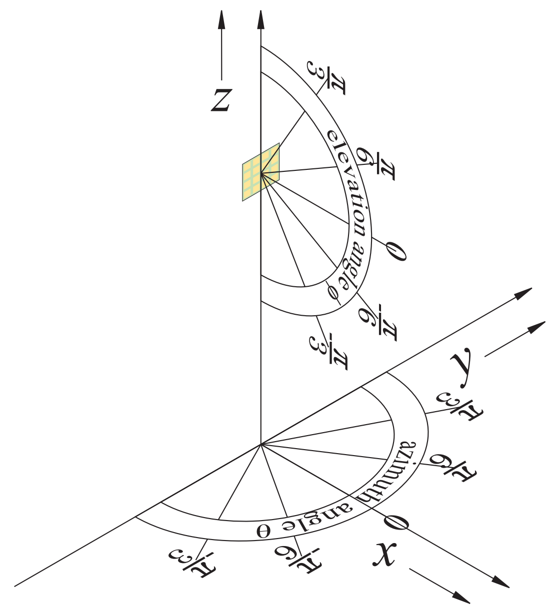


Fig. 2: Azimuth and elevation angles of incident and reflected beams.

A. Definition of azimuth and elevation angle (θ, ϕ)

Azimuth angle θ is defined as angle from positive x-axis towards positive y-axis. Elevation angle ϕ is defined as angle from positive x-axis towards positive z-axis. By default, all angle measurements are taken with RIS as origin.

B. Regarding the phase steering resolution of RIS

For a practical RIS, there will be a finite resolution for the phase shifts its reflectors can introduce. For example, if the RIS has 1-bit resolution, it can either introduce a phase-shift(ψ) of 0 or a phase-shift of π ($\psi \in \{0, \pi\}$). For an RIS of 2-bit resolution, $\psi \in \{-\frac{3\pi}{4}, -\frac{\pi}{4}, +\frac{\pi}{4}, +\frac{3\pi}{4}\}$.

C. LoS Channel Modelling

Here we assume that the distance between two sub-atoms in a row(or in a column) is $\frac{\lambda}{2}$, where λ is the wavelength. From the Figure 5, it can be observed that a Line of Sight(LoS) Channel can me modelled as follows -

$$h = e^{-i\pi \cdot n \cdot sin\theta \cdot cos\phi} \cdot e^{-i\pi \cdot m \cdot sin\phi} \cdot e^{i\varphi} \tag{1}$$

where,

- θ is the azimuth angle
- ϕ is the elevation angle
- m is the index of RIS sub-atom row wise $(m \in \{0, 1 \cdots N_H - 1\})$
- n is the index of RIS sub-atom column wise $(n \in \{0, 1 \cdots N_V - 1\})$

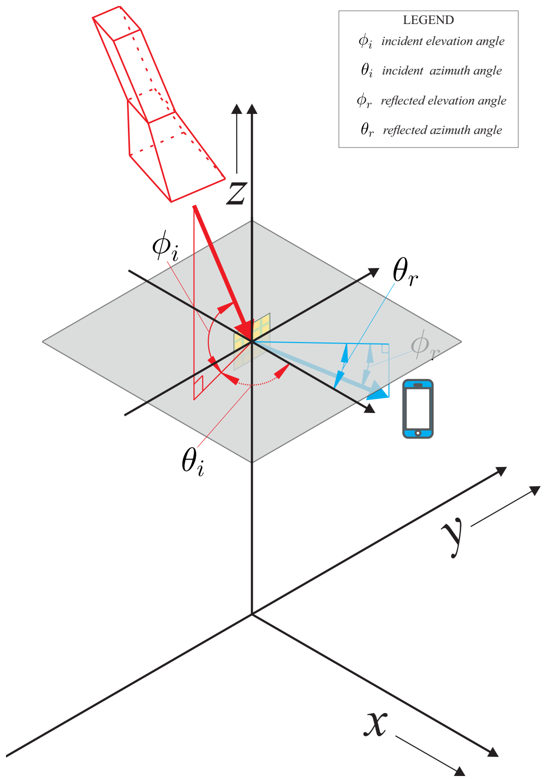


Fig. 3: Incident and reflected angles. The incident ray and incident angles are indicated by red colour and reflected ray and reflected angles are indicated by cyan color

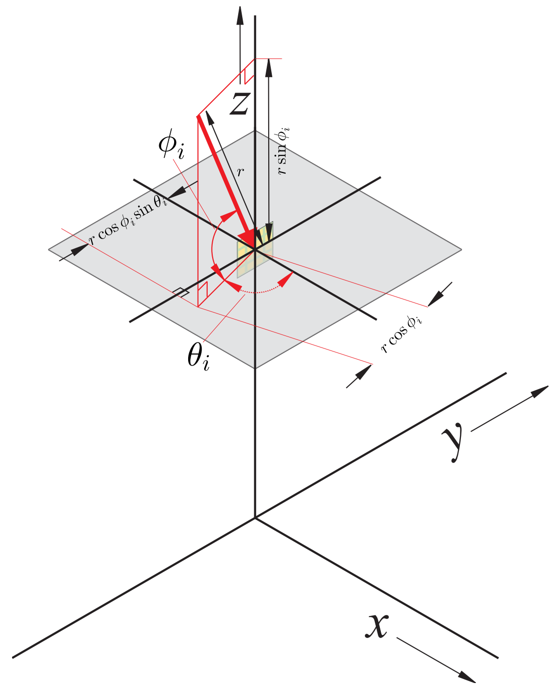


Fig. 4: Incident and reflected angles.

- φ is the initial phase

Here we are not taking free space path loss into consideration. Hence, the magnitude(envelope) of the fading coefficient has been normalised to unity.

Thus, Channel from AP to RIS will be

$$h_1 = e^{-i\pi \cdot n \cdot sin\theta_i \cdot cos\phi_i} \cdot e^{-i\pi \cdot m \cdot sin\phi_i} \cdot e^{i\varphi} \tag{2}$$

and channel from RIS to UE will be

$$h_2 = e^{-i\pi \cdot n \cdot sin\theta_r \cdot cos\phi_r} \cdot e^{-i\pi \cdot m \cdot sin\phi_r} \cdot e^{i\varphi} \tag{3}$$

II. Optimum phase shift(Ψ_{opt})calculation

From Figure 5, we can notice that for a given set of incident and reflected angles - (θ_i, ϕ_i) and (θ_r, ϕ_r), inorder to get the desred beamforming, we should introduce a phase shift of $\overline{\Psi}_{opt}$, where

$$\overline{\Psi}_{opt} = \begin{bmatrix} \psi_{03} & \psi_{13} & \psi_{23} & \psi_{33} \\ \psi_{02} & \psi_{12} & \psi_{22} & \psi_{32} \\ \psi_{01} & \psi_{11} & \psi_{21} & \psi_{31} \\ \psi_{00} & \psi_{10} & \psi_{20} & \psi_{30} \end{bmatrix} \tag{4}$$

$$\begin{aligned} \psi_{mn} &= m\pi \cos\phi_i \sin\theta_i + n\pi \sin\phi_i + m\pi \cos\phi_r \sin\theta_r + n\pi \sin\phi_r \\ &= m\pi(\cos\phi_i \sin\theta_i + \cos\phi_r \sin\theta_r) + n\pi(\sin\phi_i + \sin\phi_r) \\ &= m\pi c_{azim} + n\pi c_{azim} \end{aligned}$$

ψ_{mn} is the optimum phase shift to be given to the sub-atom located in m^{th} column and n^{th} row in the RIS, $m \in \{0, 1, ..N_H - 1\}$, $n \in \{0, 1, ..N_V - 1\}$

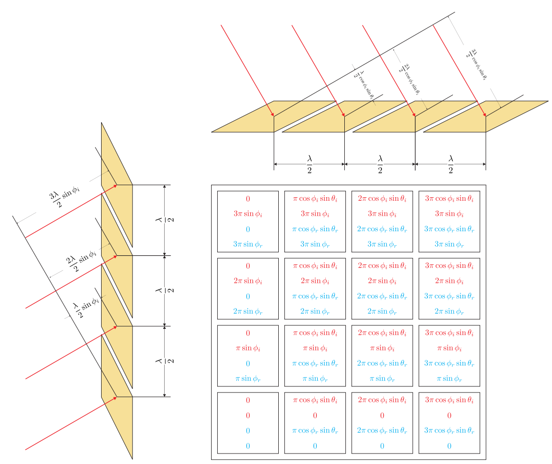


Fig. 5: The relative phase angles of incident and reflected rays in various sub-atoms of a [4x4] RIS. Red colour indicates incident ray and cyan colour indicates reflected ray. The same result can be scaled up to [N_V X N_H] RIS. Note that phase difference can be obtained from path difference by substituting $\lambda = 2\pi$

Thus, in point-wise form, we can write

$$\psi_{mn} = m\pi c_{azim} + n\pi c_{azim} \tag{5}$$

If we write this in matrix form, it will be

$$\boxed{\Psi_{opt} = \Lambda_{azim} \cdot c_{azim} + \Lambda_{elev} \cdot c_{elev}} \tag{6}$$

where,

$$c_{azim} = \sin\theta_r \cos\phi_r + \sin\theta_i \cos\phi_i \tag{7}$$

$$c_{elev} = \sin\phi_r + \sin\phi_i \tag{8}$$

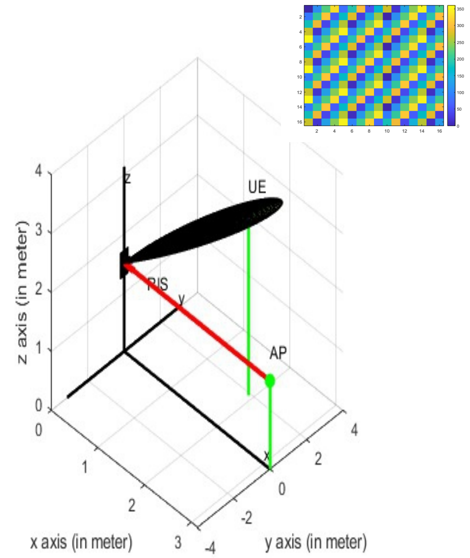


Fig. 6: Example of a beamforming that happens at $\theta_r = 45°, \phi_r = 30°$. Here AP is located at $\theta_r = 0°, \phi_r = 0°$. The corresponding RIS angle-profile for this beamforming is displayed in the inset. Here we assume that RIS has ∞ angle resolution. Size of RIS is 16×16

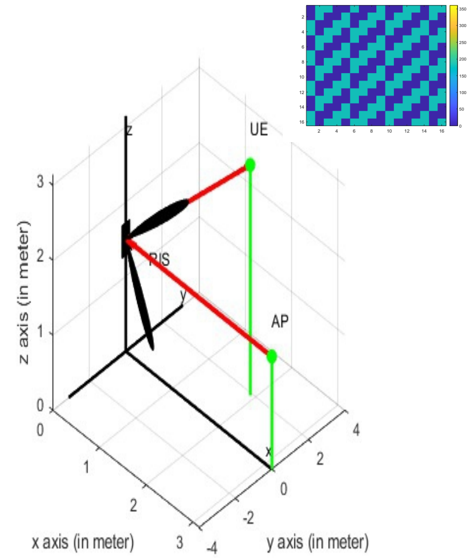


Fig. 7: In the experiment done in Figure 6, if we replace the ∞-bit resolution RIS with a 1-bit resolution RIS,this is the pattern that we get. Note the spurious lobes beside the main lobe. The reason for this poor PSLR(Peak-to-Sidelobe Ratio) is lesser angle resolution of RIS.

$$\Lambda_{azim} = \begin{bmatrix} 0 & \pi & 2\pi & \cdots & (N_H-1)\pi \\ 0 & \pi & 2\pi & \cdots & (N_H-1)\pi \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \pi & 2\pi & \cdots & (N_H-1)\pi \end{bmatrix}_{N_V \times N_H} \tag{9}$$

$$\Lambda_{elev} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \pi & \pi & \cdots & \pi \\ 2\pi & 2\pi & \cdots & 2\pi \\ \vdots & \vdots & \vdots & \vdots \\ (N_V-1)\pi & (N_V-1)\pi & \cdots & (N_V-1)\pi \end{bmatrix}_{N_V \times N_H} \tag{10}$$

III. User Equipment(UE) Search Algorithm

Since we know the Ψ_{opt} matrix, to search for the location of UE, all we need to do is plug in all possible values of (θ_r, ϕ_r) in the formula to calculate Ψ_{opt},and check the received signal power at UE. The (θ_r, ϕ_r) pair that gives the maximum received signal strength at UE, will be the direction towards UE.

When we practically implement this algorithm, we have to set a certain granularity to run this search algorithm. For instance, if we set the granularity as $\Delta = 8$, we divide the range of θ (viz. $[-\frac{\pi}{2}, \frac{\pi}{2}]$) into 8 equal parts .The same division will be done for ϕ as well. Thus, a total of $8 \times 8 = 64$ calculations of received power at UE ought to be done.

Typical values of Δ can be 8, 16, 32 or 64. The higher the value of granularity, better will be the UE tracking accuracy, but higher will be the latency.

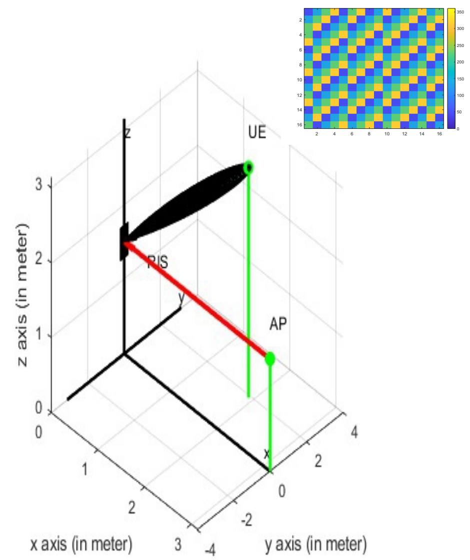
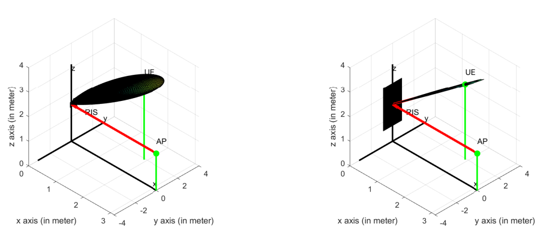


Fig. 8: In the experiment done in Figure 7, we had the problem of high PSLR value. Increasing the resolution of RIS from 1-bit to 2-bit will mitigate this problem. In this figure, the same experiment has been repeated with a 2-bit resolution RIS. Note the spurious lobes beside the main lobe has drastically reduced, and the Side-lobe suppression is close to that with ∞ resolution RIS(Ref: Figure 6).



(a) $N_V = 8, N_H = 8$ (b) $N_V = 64, N_H = 64$

Fig. 9: A demonstration of effect on beamshape, if we increase or decrease no: of sub-atoms in the RIS. It can be noticed here that if we increase the number of sub-atoms, the beam width will get narrowed down, which can cause higher data rates. The disadvantage of this is that, UE search will take more latency, as search beams became narrower, and search granularity has to increase proportionally.

Algorithm 1 UE Search Algorithm

Input: The feedback of RX signal power $rssi$.
Output: $\underset{\theta,\phi}{\operatorname{argmax}}\, rssi(\theta, \phi)$: The (θ, ϕ) pair which fetches the highest $rssi$.

$$\theta_{sweep}, \phi_{sweep} \leftarrow -\frac{\pi}{2} + \frac{\pi}{\Delta} \times \{0, 1, ...N-1\} + \frac{\pi}{2\Delta}$$

for each $\theta \in \theta_{sweep}$ **do**
  for each $\phi \in \phi_{sweep}$ **do**

$$c_{azim} \leftarrow \sin\theta_r \cos\phi_r + \sin\theta_i \cos\phi_i$$
$$c_{elev} \leftarrow \sin\phi_r + \sin\phi_i$$
$$\Psi \leftarrow \Lambda_{azim} \cdot c_{azim} + \Lambda_{elev} \cdot c_{elev}$$
$$rssi \leftarrow \left| \sum_{N_H N_V} \left(h_1 \cdot e^{i\Psi} \cdot h_2\right) \right|^2$$

  end for
end for
return $\underset{\theta,\phi}{\operatorname{argmax}}\, rssi(\theta, \phi)$