Project Management with PERT/CPM

Why PERT/CPM ?

- PERT (program evaluation and review technique) and
- CPM (critical path method)

Are available to assist the project manager in carrying out the assigned project and related responsibilities.

PERT and CPM have been used for a variety of projects -

1. Construction of a new plant
2. Research and development of a new product
3. NASA space exploration projects
4. Movie productions
5. Building a ship
6. Government-sponsored projects for developing a new weapons system
7. Relocation of a major facility
8. Maintenance of a nuclear reactor
9. Installation of a management information system
10. Conducting an advertising campaign etc.

A PROTOTYPE EXAMPLE — THE RELIABLE CONSTRUCTION CO. PROJECT

The RELIABLE CONSTRUCTION COMPANY has just made the winning bid of $5.4 million to construct a new plant for a major manufacturer. The manufacturer needs the plant to go into operation within a year. Therefore, the contract includes the following provisions:

Job constraints

- A penalty of $300,000 if Reliable has not completed construction by the deadline 47 weeks from now.

- To provide additional incentive for speedy construction, a bonus of $150,000 will be paid to Reliable if the plant is completed within 40 weeks.

Activity list for the Reliable Construction Co. project

Activity	Activity Description	Immediate Predecessors	Estimated Duration
A	Excavate	—	2 weeks
B	Lay the foundation	*A*	4 weeks
C	Put up the rough wall	*B*	10 weeks
D	Put up the roof	*C*	6 weeks
E	Install the exterior plumbing	*C*	4 weeks
F	Install the interior plumbing	*E*	5 weeks
G	Put up the exterior siding	*D*	7 weeks
H	Do the exterior painting	*E, G*	9 weeks
I	Do the electrical work	*C*	7 weeks
J	Put up the wallboard	*F, I*	8 weeks
K	Install the flooring	*J*	4 weeks
L	Do the interior painting	*J*	5 weeks
M	Install the exterior fixtures	*H*	2 weeks
N	Install the interior fixtures	*K, L*	6 weeks

Answers to be developed for the following questions.

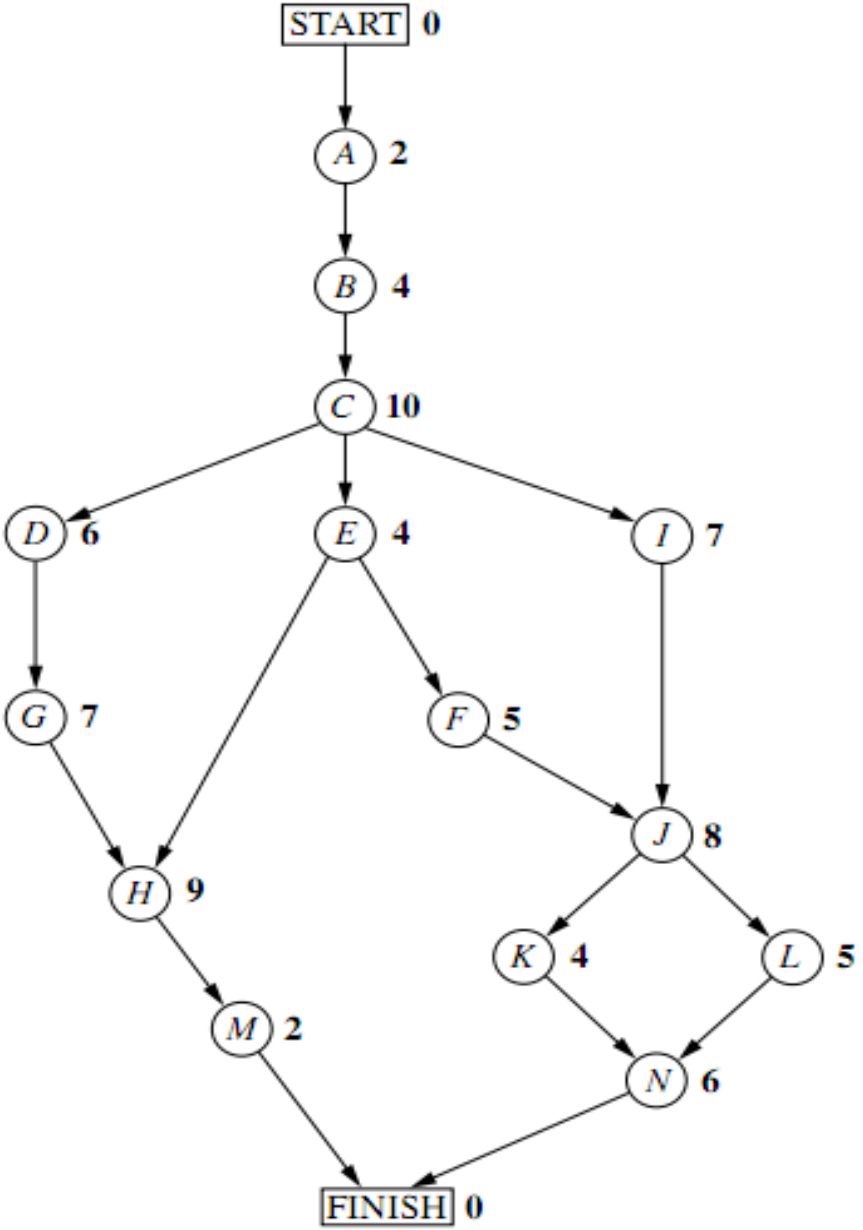
1. How can the project be displayed graphically to better visualization?
2. What is the total time required to complete the project if no delays occur?
3. When can the individual activities start and finish (at the earliest) if no delays occur?
4. When do the individual activities need to start and finish (at the latest) to meet this project completion time?
5. Which are the critical bottleneck activities where any delays must be avoided to prevent delaying project completion?
6. For the other activities, how much delay can be tolerated without delaying project completion?
7. Given the uncertainties in accurately estimating activity durations, what is the probability of completing the project by the deadline?
8. If extra money is spent to expedite the project, what is the least expensive way of attempting to meet the target completion time (40 weeks)?

Project Networks

A network used to graphically represent a project is called a project network. A project network consists of a number of nodes (typically shown as small circles or rectangles) and a number of arcs (shown as arrows) that lead from some node to another. Two types –

1. **Activity-On-Arc (AOA)**
2. **Activity-On-Node (AON)**

The project network for the Reliable Construction Co. project.



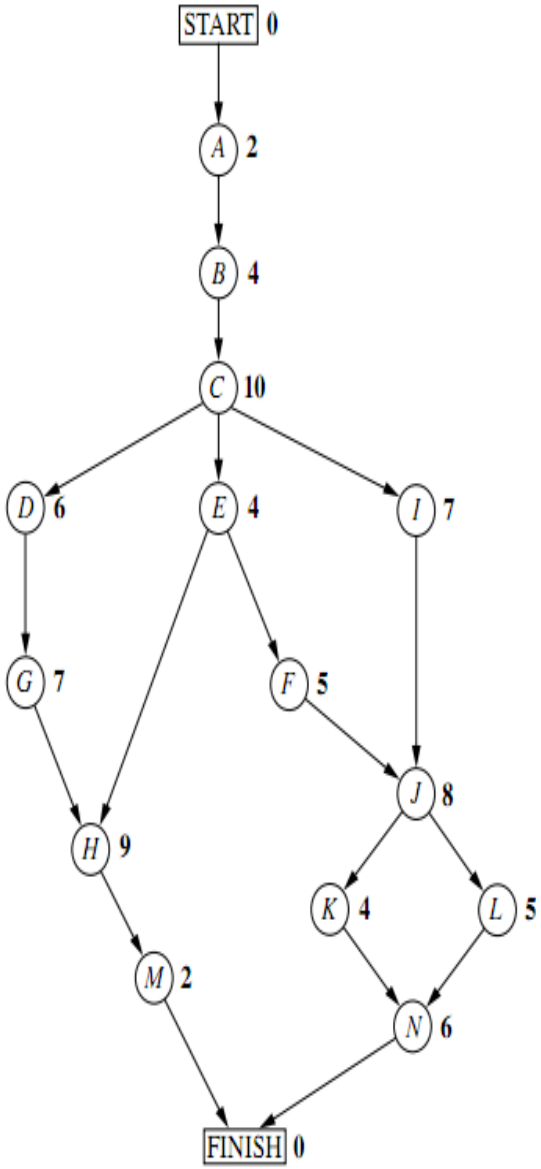
Activity Code

A. Excavate
B. Foundation
C. Rough wall
D. Roof
E. Exterior plumbing
F. Interior plumbing
G. Exterior siding
H. Exterior painting
I. Electrical work
J. Wallboard
K. Flooring
L. Interior painting
M. Exterior fixtures
N. Interior fixtures

The Critical Path

A path through a project network is one of the routes following the arcs from the START node to the FINISH node. The length of a path is the sum of the (estimated) durations of the activities on the path.

The paths and path lengths through Reliable's project network



Path	Length
START →A→B→C→D→G→H→M→ FINISH	2 + 4 + 10 + 6 + 7 + 9 + 2 = 40 weeks
START →A→B→C→E→H→M→ FINISH	2 + 4 + 10 + 4 + 9 + 2 = 31 weeks
START →A→B→C→E→F→J→K→N→ FINISH	2 + 4 + 10 + 4 + 5 + 8 + 4 + 6 = 43 weeks
START →A→B→C→E→F→J→L→N→ FINISH	2 + 4 + 10 + 4 + 5 + 8 + 5 + 6 = 44 weeks
START →A→B→C→I→J→K→N→ FINISH	2 + 4 + 10 + 7 + 8 + 4 + 6 = 41 weeks
START →A→B→C→I→J→L→N→ FINISH	2 + 4 + 10 + 7 + 8 + 5 + 6 = 42 weeks

Red color marked path is the critical path as with the maximum duration.

Scheduling Individual Activities

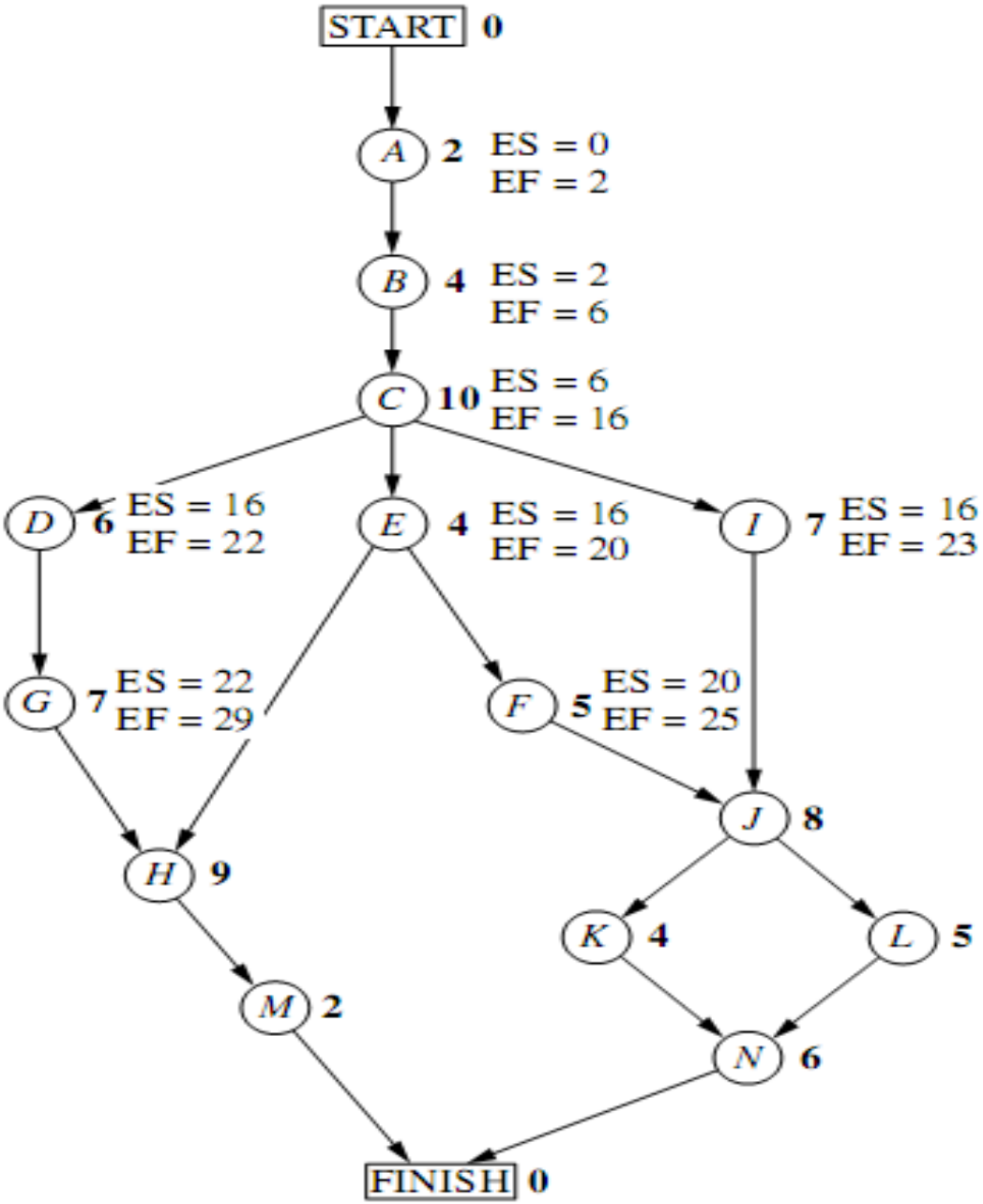
ES = Earliest Start time for a particular activity

EF = Earliest Finish time for a particular activity

LS = Latest Start time for a particular activity

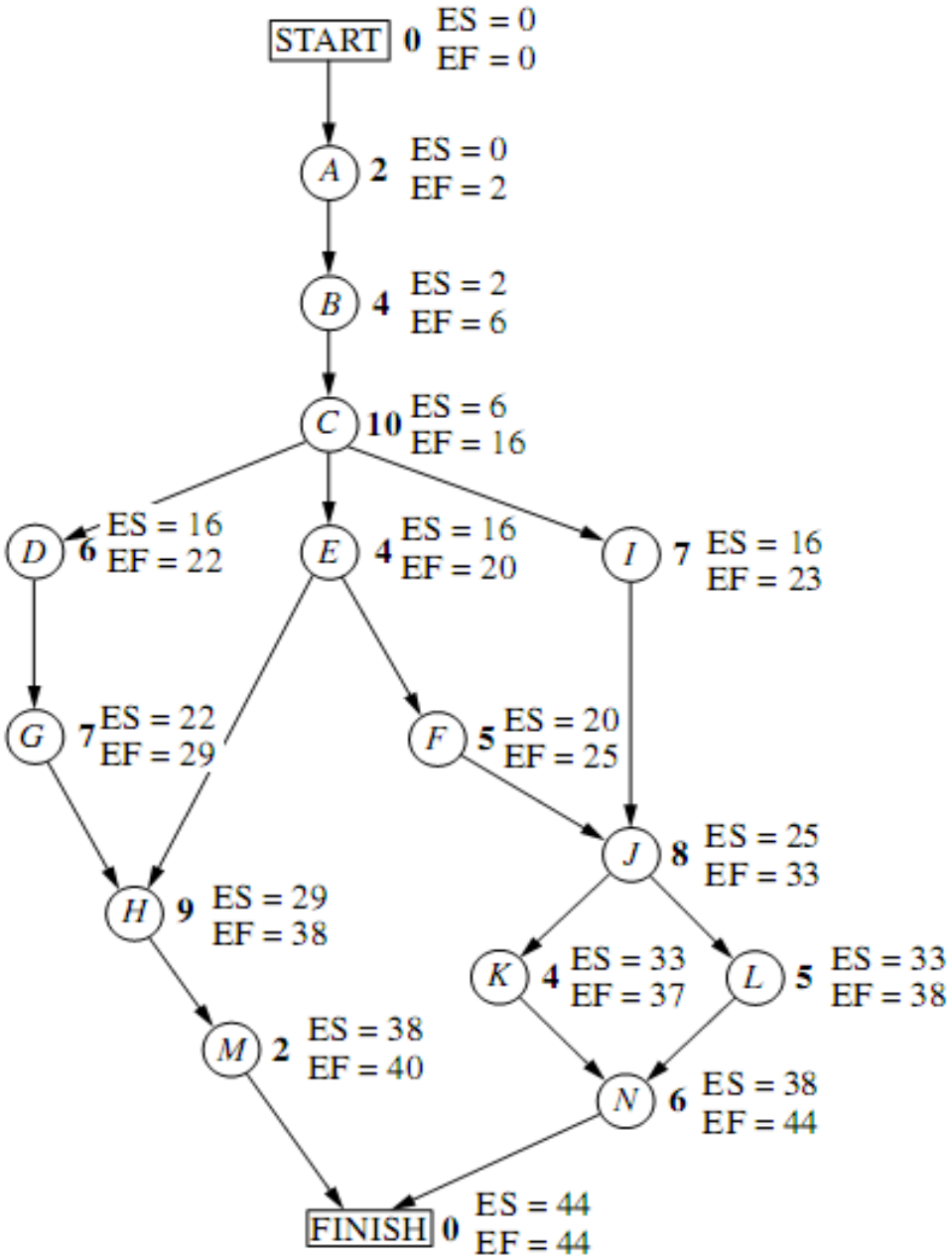
LF = Latest Finish time for a particular activity

Network Diagram



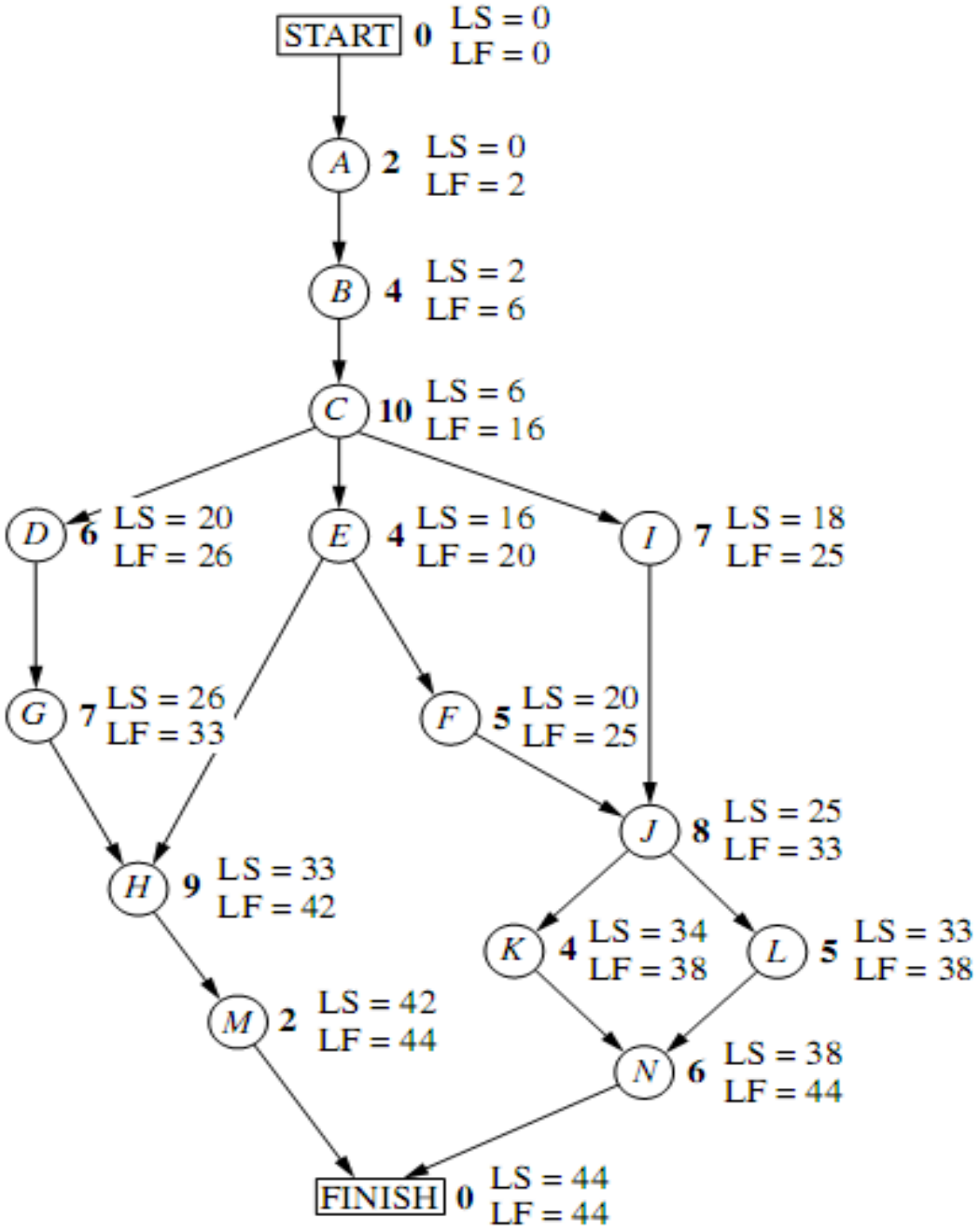
Earliest start time (ES) and earliest finish time (EF) values for the initial activities that have only a single immediate predecessor.

Network Diagram



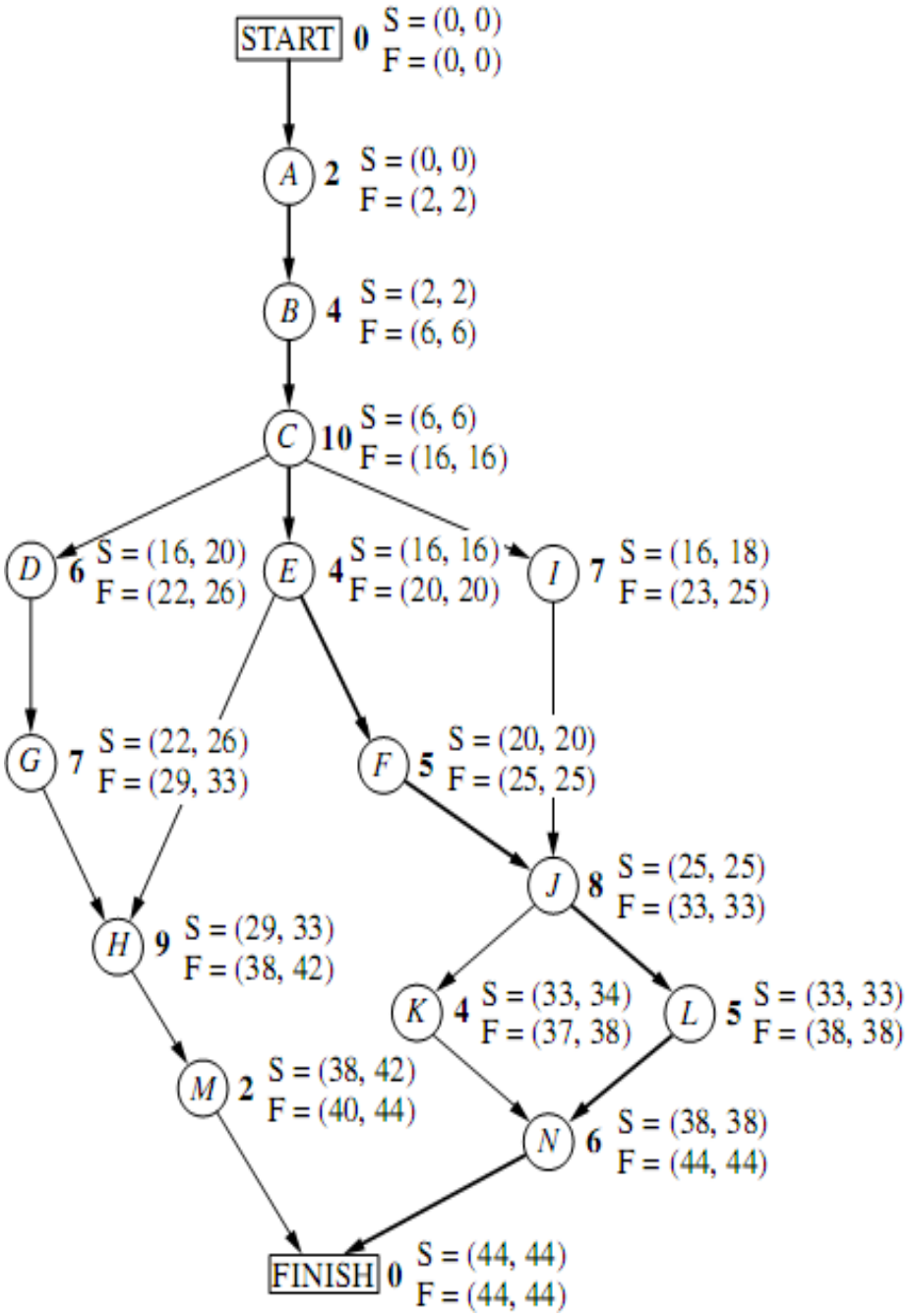
Earliest start time (ES) and earliest finish time (EF) values for all the activities (plus the START and FINISH nodes) of the Reliable Construction Co. project.

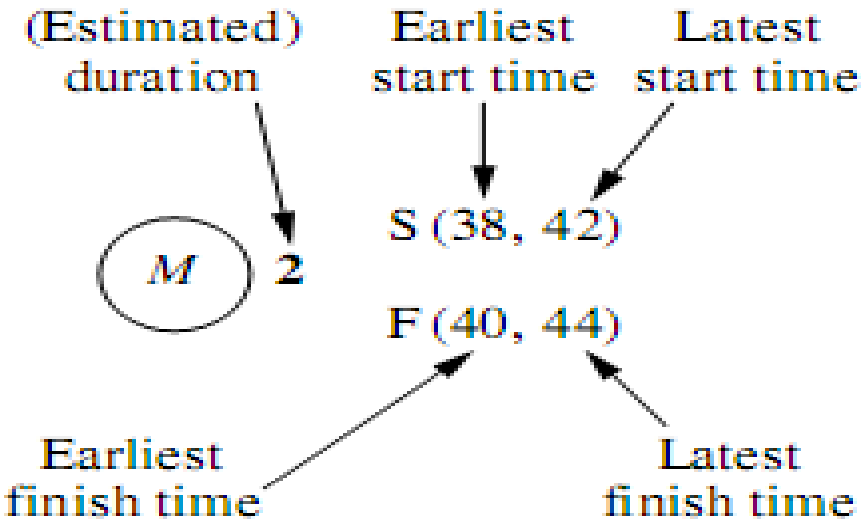
Network Diagram



Latest start time (LS) and latest finish time (LF) for all the activities (plus the START and FINISH nodes) of the Reliable Construction Co. project.

Identifying Slack in the Schedule





The complete project network showing ES and LS (in parentheses above the node) and EF and LF (in parentheses below the node) for each activity of the Reliable Construction Co. project. The darker arrows show the critical path through the project network.

Slack for Reliable's activities

Activity	Slack (LF − EF)	On Critical Path?
A	0	Yes
B	0	Yes
C	0	Yes
D	4	No
E	0	Yes
F	0	Yes
G	4	No
H	4	No
I	2	No
J	0	Yes
K	1	No
L	0	Yes
M	4	No
N	0	Yes

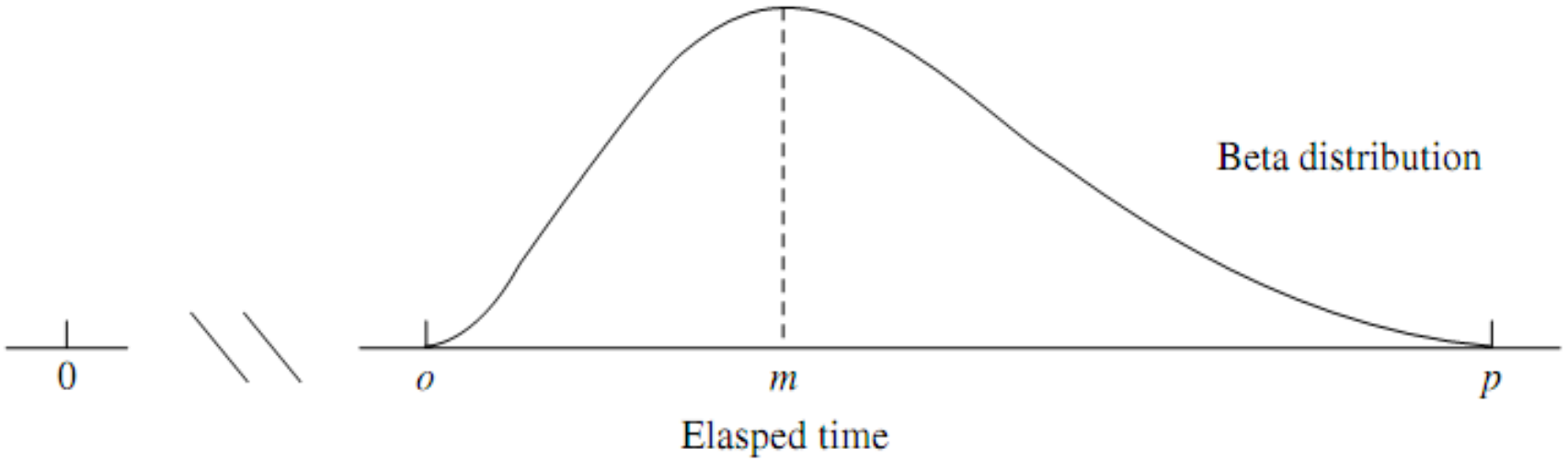
The PERT Three-Estimate Approach

- ***Most likely estimate (m)*** **= estimate of the most likely value of the duration,**
- ***Optimistic estimate (o)*** **= estimate of the duration under the most favourable conditions,**
- ***Pessimistic estimate (p)*** **= estimate of the duration under the most unfavourable conditions.**

Calculation of the mean (μ) and variance (σ^2)

$$\sigma^2 = \left(\frac{p - o}{6}\right)^2$$

$$\mu = \frac{o + 4m + p}{6}$$



Model of the probability distribution of the duration of an activity for the PERT Three-estimate approach: m = most likely estimate, o = optimistic estimate, and p = pessimistic estimate.

Expected value and variance of the duration of each activity for Reliable's project

Activity	Optimistic Estimate o	Most Likely Estimate m	Pessimistic Estimate p	Mean $\mu = \frac{o + 4m + p}{6}$	Variance $\sigma^2 = \left(\frac{p - o}{6}\right)^2$
A	1	2	3	2	$\frac{1}{9}$
B	2	$3\frac{1}{2}$	8	4	1
C	6	9	18	10	4
D	4	$5\frac{1}{2}$	10	6	1
E	1	$4\frac{1}{2}$	5	4	$\frac{4}{9}$
F	4	4	10	5	1
G	5	$6\frac{1}{2}$	11	7	1
H	5	8	17	9	4
I	3	$7\frac{1}{2}$	9	7	1
J	3	9	9	8	1
K	4	4	4	4	0
L	1	$5\frac{1}{2}$	7	5	1
M	1	2	3	2	$\frac{1}{9}$
N	5	$5\frac{1}{2}$	9	6	$\frac{4}{9}$

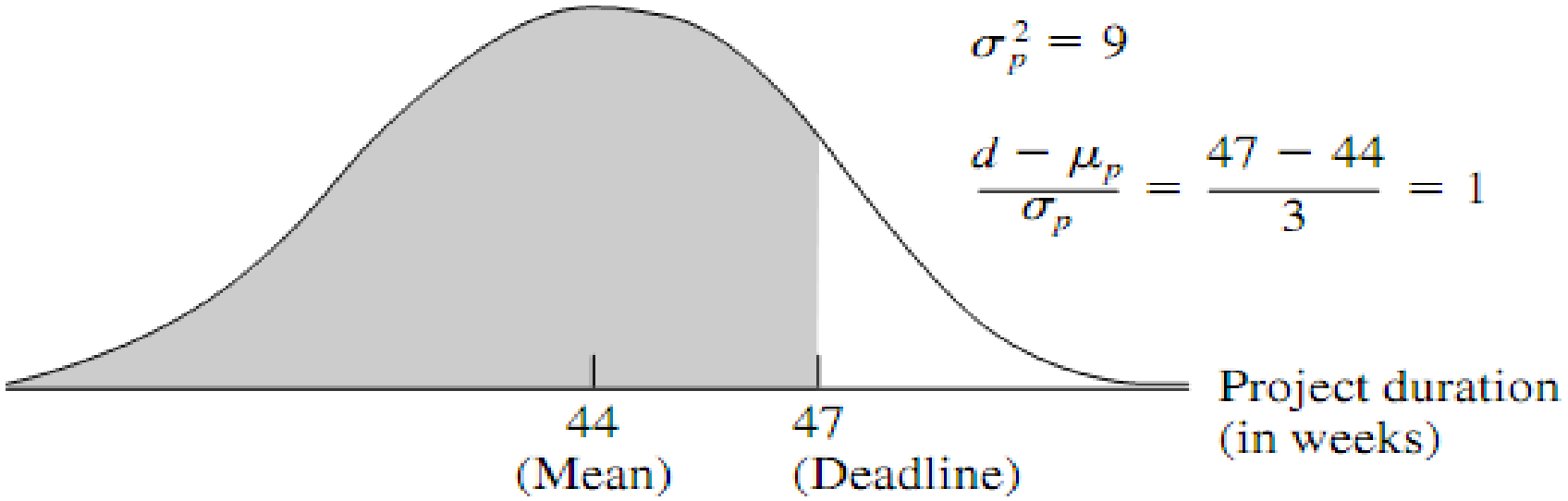
The paths and path lengths through Reliable's project network when the duration of each activity equals its pessimistic estimate

Path	Length
START→*A*→*B*→*C*→*D*→*G*→*H*→*M*→FINISH	3 + 8 + 18 + 10 + 11 + 17 + 3 = 70 weeks
START→*A*→*B*→*C*→*E*→*H*→*M*→FINISH	3 + 8 + 18 + 5 + 17 + 3 = 54 weeks
START→*A*→*B*→*C*→*E*→*F*→*J*→*K*→*N*→FINISH	3 + 8 + 18 + 5 + 10 + 9 + 4 + 9 = 66 weeks
START→*A*→*B*→*C*→*E*→*F*→*J*→*L*→*N*→FINISH	3 + 8 + 18 + 5 + 10 + 9 + 7 + 9 = 69 weeks
START→*A*→*B*→*C*→*I*→*J*→*K*→*N*→FINISH	3 + 8 + 18 + 9 + 9 + 4 + 9 = 60 weeks
START→*A*→*B*→*C*→*I*→*J*→*L*→*N*→FINISH	3 + 8 + 18 + 9 + 9 + 7 + 9 = 63 weeks

Calculation of mean and variance for Reliable's project

Activities on Mean Critical Path	Mean	Variance
A	2	$\frac{1}{9}$
B	4	1
C	10	4
E	4	$\frac{4}{9}$
F	5	1
J	8	1
L	5	1
N	6	$\frac{4}{9}$
Project duration	$\mu_p = 44$	$\sigma_p^2 = 9$

The three simplifying approximations lead to the probability distribution of the duration of Reliable's project being approximated by the normal distribution shown here. The shaded area is the portion of the distribution that meets the deadline of 47 weeks.



	A	B	C	D	E	F	G	H	I	J	K
1	Template for PERT Three-Estimate Approach										
2											
3			Time Estimates					On Mean			
4		Activity	o	m	p	μ	σ^2	Critical Path			
5		A	1	2	3	2	0.111	*		Mean Critical Path	
6		B	2	3.5	8	4	1	*			
7		C	6	9	18	10	4	*		μ =	44
8		D	4	5.5	10	6	1			σ^2 =	9
9		E	1	4.5	5	4	0.444	*			
10		F	4	4	10	5	1	*		P(T≤d)=	0.84134474
11		G	5	6.5	11	7	1			where	
12		H	5	8	17	9	4			d =	47
13		I	3	7.5	9	7	1				
14		J	3	9	9	8	1	*			
15		K	4	4	4	4	0				
16		L	1	5.5	7	5	1	*			
17		M	1	2	3	2	0.111				
18		N	5	5.5	9	6	0.444	*			
19											
20											
21			Data								
22			Results								

	F	G
5	=(C5+4*D5+E5)/6	=((E5-C5)/6)^2
6	=(C6+4*D6+E6)/6	=((E6-C6)/6)^2
7	=(C7+4*D7+E7)/6	=((E7-C7)/6)^2
8	=(C8+4*D8+E8)/6	=((E8-C8)/6)^2
9	:	:
10	:	:

	K
7	=SUMIF(H5:H18,"*",F5:F18)
8	=SUMIF(H5:H18,"*",G5:G18)
9	
10	=NORMDIST(K12,K7,SQRT(K8),1)

=NORMDIST(DeadLine, CP-Value, SquareRoot(Variance), 1)

Thank You