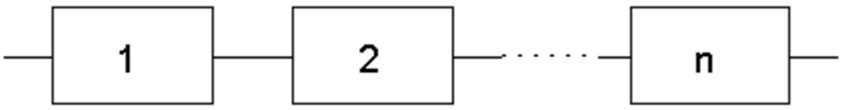
Reliability

Mathematical models

After component reliability information has been obtained, system reliability can be computed. Many systems can be modelled using reliability block diagrams. The model may be a series system, a parallel system, a standby redundant system, or a combination of the previous.

Series systems

For a series system to operate successfully, all components must operate successfully. A simple series system is shown in the figure below.



If each component in the system is independent, the reliability of a series system is:

$$R_s = \prod_{i=1}^{n} R_i$$

The system hazard function is:

$$h_s = \sum_{i=1}^{n} h_i$$

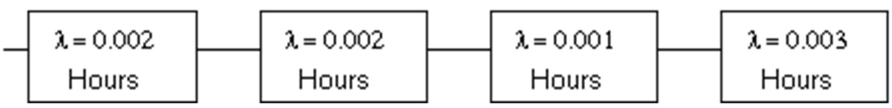
Example
Three components each with a reliability of 0.9 are placed in series. What is the reliability of the system?

Solution
The system reliability is the product of the component reliabilities.

$$R_s = 0.9^3 = 0.729$$

=0.9^3

Example
The components in the system below are exponentially distributed with the indicated failure rates. Develop an expression for the reliability of the system. What is the system reliability at time = 100 hours?



Solution

For the exponential distribution, reliability is defined as

$$R(t) = e^{-\lambda t}$$

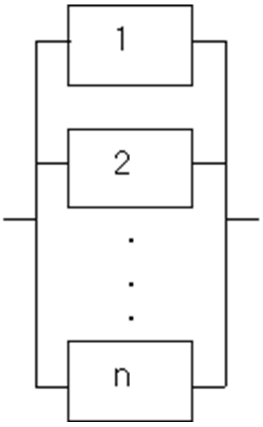
=EXP(-0.008*100)

The reliability of the system above is

$$R_s(t) = \left(e^{-0.002t}\right)\left(e^{-0.002t}\right)\left(e^{-0.001t}\right)\left(e^{-0.003t}\right) = e^{-0.008t} = 0.4493$$

Parallel systems

For a parallel system to operate successfully, at least one of the components in the system must operate successfully. A general parallel system is shown in the figure below.



The system above is an active parallel system with *n*–1 redundancy. In an active parallel system, all components are in operation when the system is in operation. An active parallel system may be a pure parallel system or a shared parallel system. In a pure parallel system, there is no change in the failure rate of the surviving components after the failure of a companion component. In a shared parallel system, the failure rates of the remaining components change when a companion component fails. An example is an automobile wheel being held on by four bolts. The load is being shared by four bolts, and when one of the bolts fails, the load on the three remaining bolts increases.

In a passive parallel system, also known as a standby redundant system, only one component functions at a given time. The remaining components do not come into service until required by the failure of other components. A switching device is usually used to transfer operation from one component to another.

The reliability of a pure, active parallel system is:

$$R_s = 1 - \prod_{i=1}^{n}\left(1 - R_i\right)$$

A parallel system consisting of components with *n* identical components with an exponential time to failure distribution has a mean time to fail of

$$MTBF_s = \sum_{i=1}^{n} \frac{1}{i\lambda}$$

where λ is the failure rate of each of the components.

Example

A system consists of three components. At time = 0, all three components are activated, and as long as at least one of the three components is functional, the system is functional. Write the expression for system reliability if each of the components has an exponential time to fail distribution with mean times to fail of 40 hours, 80 hours and 85 hours. What is the reliability at time = 25 hours?

Solution

This is a system with three components—two in active redundancy The reliability of the system is:

$$R_s(t) = 1 - \left(1 - e^{-t/40}\right)\left(1 - e^{-t/80}\right)\left(1 - e^{-t/85}\right)$$

At time = 25 hours, the reliability is:

=1-(1-EXP(-25/40))*(1-EXP(-25/80))*(1-EXP(-25/85))

$$R_s(25) = 1 - \left(1 - e^{-25/40}\right)\left(1 - e^{-25/80}\right)\left(1 - e^{-25/85}\right) = 0.9682$$

r-out-of-n systems

Another form of active redundancy is a system consisting of n components in which r of the n components must function for the system to function. An example of this is a four-engine aircraft that can maintain flight with only two of the four engines operating. If the reliability, R, of all components is equal, the reliability of an r-out-of-n system is:

$$R_s = \sum_{x=r}^{n} \binom{n}{x} R^x (1-R)^{n-x}$$

Example

A system consists of four components. If more than two of the components fail, the system fails. If the components have an exponential time to fail distribution with a failure rate of 0.000388, what is the reliability of the system at time = 300?

Solution

This is a 2-out-of-4 system; at least 2 of the 4 components must survive for the system to survive. The reliability of each component is:

$$R = e^{-0.000388(300)} = 0.89$$

=EXP(-0.000388*300)

The reliability of the system is:

$$R_s = \sum_{x=2}^{4} \binom{4}{x} R^x (1-R)^{4-x}$$

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=FACT(4)/(FACT(2)*FACT(4-2))*(0.89^2)*(1-0.89)^(4-2)+FACT(4)/(FACT(3)*FACT(4-3))*(0.89^3)*(1-0.89)^(4-3)+FACT(4)/(FACT(4)*FACT(4-4))*(0.89^4)*(1-0.89)^(4-4)
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$$
\begin{aligned}
R_s &= \left[\frac{4!}{(2!)(4-2)!}\right](0.89)^2(1-0.89)^{4-2} \\
&+ \left[\frac{4!}{(3!)(4-3)!}\right](0.89)^3(1-0.89)^{4-3} \\
&+ \left[\frac{4!}{(4!)(4-4)!}\right](0.89)^4(1-0.89)^{4-4} \\
&= 0.995
\end{aligned}
$$

Reliability Testing

There are two types of zero failure tests: Bogey testing and Bayesian testing.

Bogey Testing

Bogey testing requires the testing duration be equal to the required life. For example, if 95% reliability is required at 200,000 kilometres of service then units being tested will be removed from testing when they fail or when they complete the equivalent of 2000,000 kilometres of testing. The sample size required to demonstrate reliability of r with a confidence level of c is:

$$n = \frac{\ln(1-c)}{\ln r}$$

Example

A windshield wiper motor must demonstrate 99% reliability with 90% confidence at 2 million cycles of operation. How many motors must be tested to 2 million cycles with no failures meet these requirements?

Solution

Two hundred and thirty motors must function for 2 million cycles to demonstrate the desired level of reliability. This is shown below.

$$\frac{\ln(1-0.9)}{\ln 0.99} = 229.1$$

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=LN(1-0.9)/LN(0.99)
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Detailed calculations are available in the Microsoft Excel file "**Chapt-5 Reliability.xlsx**".

Bayesian Testing
Bogey testing is inefficient. By extending the test duration beyond the required life the total time on test can often be reduced. When the test duration is extended it is necessary to make assumptions concerning the shape of the distribution of the time to fail. This is done by assuming a Weibull distribution for time to fail, and assuming a shape parameter. The Weibull distribution can approximate many other distributions by changing the value of the shape parameter.

The assumed shape greatly affects the testing requirements. How is the shape parameter estimated? The shape parameter is governed by the physics of failure. Some phenomena are skewed right while others are skewed left or have no skewness. In general, the more the failure mode is a result of mechanical wear the larger the shape parameter will be. The shape parameter is usually stable and tends to remain constant. For example, if the shape parameter for master cylinders will be similar for master cylinders for small cars, large trucks, and for different designs because the physics of how the master cylinder fails is similar.

The best way to estimate the shape parameter is through prior testing. Many automotive companies require some testing to failure to allow the shape parameter to be determined. Keep detailed records of all tests and build a database of shape parameters. It is recommended to use the lower 90% confidence limit for the shape parameter because to the magnitude the shape parameter has on test requirements. In lack of any prior knowledge there are data sources available on the internet, or the shape parameter can be estimated based on the knowledge of the physics of failure.

Be careful when estimating the shape parameter for electronics. Many sources state the shape parameter for electronics is 1.0 because there is no mechanical wear in electronics. For electronic modules located in environmentally harsh conditions, such as under the hood of an automobile of in an airplane, fail as a result of mechanical wear. The extreme vibration, temperature cycling and, in some cases, contaminants cause mechanical failures. It is not uncommon to have shape parameters greater than 8.0 for electronic components.

The statistical properties for Bayesian testing with the Weibull distribution are based on the exponential distribution. If the parameter t follows a Weibull distribution, then the parameter t^{β} is exponentially distributed. The lower (1-α) confidence limit for reliability is

$$R_{L,\alpha}(t) = e^{\left[-\left(\frac{t}{\theta_{L,\alpha}}\right)^{\beta}\right]}$$

where

$$\theta_{L,\alpha} = \left(\frac{2\sum_{i=1}^{n} t_i^{\beta}}{\chi^2_{\alpha,d}} \right)^{1/\beta}$$

n is the number of units tested, both failed and surviving, and $\chi^2_{\alpha,d}$ is the critical value of the chi-square distribution with significance of α (0.05 for a confidence level of 95%) and d degrees of freedom. For failure truncated testing d is equal to $2r$ where r is the number of failed units. For time truncated testing d is equal to $2r+2$.

Example

Fourteen units are placed on a test stand; the first unit fails after 324 hours, the second unit fails after 487 hours of testing, a third unit failed after 528 hours of testing, and the remaining 11 units were removed from testing. Given a Weibull time to fail distribution with a shape parameter of 2.2, what is the lower 90% confidence limit for reliability at 400 hours?

Solution

Since testing was suspended at the time of the last failure, this is failure truncated testing. With failure truncated testing and 3 failures, the degrees-of-freedom for the chi-square distribution is 2(3) = 6. The critical value of the chi-square distribution given 6 degrees-of-freedom and a significance of 10% is 10.64. This value can be found from the Appendix or using the following expression in a spreadsheet.

= CHIINV (1-0.9,6)

The lower 90% confidence limit for the mean of the transformed data is

$$\theta_{L,\alpha} = \left(\frac{2(12,872,524.77)}{10.64} \right)^{1/2.2} = 797.3$$

The lower 90% confidence limit for the reliability at 400 hours is

$$R_{L,0.10}(400) = e^{\left[-\left(\frac{400}{797.3} \right)^{2.2} \right]} = 0.80311$$

Detailed calculations are available in the Microsoft Excel file "**Chapt-5 Reliability.xlsx**".