Control Charts

Process control is the active changing of the process based on the results of process monitoring. Once the process monitoring tools have detected an out-of-control situation, the person responsible for the process makes a change to bring the process back into control.

If the process is out-of-control, the process engineer looks for an assignable cause by following the out-of-control action plan (OCAP) associated with the control chart. Out-of-control refers to rejecting the assumption that the current data are from the same population as the data used to create the initial control chart limits. For classical Shewhart charts, a set of rules called the Western Electric rules, and a set of trend rules often are used to determine out-of-control

Western Electric Rules

The Western Electric rules are based on probability. For a normal distribution, the probability of encountering a point outside $\pm 3\sigma$ is 0.3%. This is a rare event. Therefore, if we observe a point outside the control limits, we conclude the process has shifted and is unstable. Similarly, other events that are equally rare can be used as flags for instability.

- Two points out of three in a row between 2σ and 3σ
- Two points out of three in a row between beyond 2σ
- Four points out of five in a row between 1σ and 2σ
- Four points out of five in a row beyond 1σ
- Eight consecutive points between 1σ and -1σ
- Eight consecutive points on one side of the center line

Trend Rules

- Six points in a row trending up or down.
- Fourteen points in a row alternating up and down.

Control Chart Sensitivity

While the Western Electric rules increase a Shewhart chart sensitivity to trends or drifts in the mean, there is a severe downside. When following the standard Shewhart "out of control" rule (i.e., signal if and only if you see a point beyond the plus or minus 3 sigma control limits) a "false alarms" will occur on average every 371 points. Adding the Western Electric rules increases the frequency of false alarms to about one in every 92 points.

X-Bar and Range Charts

If the sample size, *n*, is relatively small (say equal to or less than 10), the range can be used instead of the standard deviation of a sample to construct control charts on X-bar and the range, R. The range of a sample is simply the difference between the largest and smallest observation. The constants in the table below are used to ease computations.

n	A_2	A_3	D_3	D_4	d_2	B_3	B_4
2	1.880	2.659	0	3.267	1.128	0	3.267
3	1.023	1.954	0	2.574	1.693	0	2.568
4	0.729	1.628	0	2.282	2.059	0	2.266
5	0.5770	1.427	0	2.115	2.326	0	2.089
6	0.483	1.287	0	2.004	2.534	0.03	1.970
7	0.419	1.182	0.076	1.924	2.704	0.118	1.882
8	0.373	1.099	0.136	1.864	2.847	0.185	1815
9	0.337	1.032	0.184	1.816	2.970	0.239	1.761
10	0.308	0.975	0.223	1.777	3.078	0.284	1.716

The X-bar control chart is constructed with the following expressions.

$$\overline{\overline{X}} = \frac{\text{sum of subgroup averages}}{\text{number of subgroups}}$$

$$LCL = \overline{\overline{X}} - A_2\overline{R}$$

$$UCL = \overline{\overline{X}} + A_2\overline{R}$$

The R-bar control chart is constructed with the following expressions.

$$R = \text{Largest in subgroup - Smallest in subgroup}$$

$$\overline{R} = \frac{\text{sum of subgroup ranges}}{\text{number of subgroups}}$$

$$LCL = D_3\overline{R}$$

$$UCL = D_4\overline{R}$$

Example

Given the data in the table below, compute the control limits for the X-bar and Range charts.

Observation 1	Observation 2	Observation 3
231.6	232.5	256.2
227.2	237.5	240.4
235.4	218.0	257.4
240.4	228.5	238.4
263.0	268.2	255.0
227.8	228.9	245.7
257.8	247.4	242.8
237.0	230.9	235.8
232.8	241.8	234.6
225.6	248.8	237.7
245.2	230.4	229.8
250.6	247.7	250.8
248.1	258.2	249.6
213.7	227.9	251.7
230.0	246.4	226.4
233.9	245.3	256.9
243.7	227.6	253.7
214.6	251.9	234.7
230.4	250.0	243.1
235.6	235.1	251.1
232.7	251.5	253.8
227.9	243.7	236.5
234.8	279.8	239.5
244.5	245.1	242.9
222.4	250.5	231.4
233.9	238.0	228.9
256.5	238.3	241.0
239.5	232.0	252.0
244.0	223.2	225.6
259.3	225.4	231.9

Solution

The X-bar and R values have been computed in the table below along with X-double bar and R-bar.

Observation 1	Observation 2	Observation 3	Average	Range
231.6	232.5	256.2	240.1	24.6
227.2	237.5	240.4	235.0	13.2
235.4	218.0	257.4	236.9	39.4
240.4	228.5	238.4	235.8	11.9

263.0	268.2	255.0	262.1	13.2
227.8	228.9	245.7	234.1	17.9
257.8	247.4	242.8	249.3	15.0
237.0	230.9	235.8	234.6	6.1
232.8	241.8	234.6	236.4	9.0
225.6	248.8	237.7	237.4	23.2
245.2	230.4	229.8	235.1	15.4
250.6	247.7	250.8	249.7	3.1
248.1	258.2	249.6	252.0	10.1
213.7	227.9	251.7	231.1	38.0
230.0	246.4	226.4	234.3	20.0
233.9	245.3	256.9	245.4	23.0
243.7	227.6	253.7	241.7	26.1
214.6	251.9	234.7	233.7	37.3
230.4	250.0	243.1	241.2	19.6
235.6	235.1	251.1	240.6	16.0
232.7	251.5	253.8	246.0	21.1
227.9	243.7	236.5	236.0	15.8
234.8	279.8	239.5	251.4	45.0
244.5	245.1	242.9	244.2	2.2
222.4	250.5	231.4	234.8	28.1
233.9	238.0	228.9	233.6	9.1
256.5	238.3	241.0	245.3	18.2
239.5	232.0	252.0	241.2	20.0
244.0	223.2	225.6	230.9	20.8
259.3	225.4	231.9	238.9	33.9
Average			**240.29**	**19.88**

For a sub-group size of 3 $A_2 = 1.023$, $D_3 = 0$ and $D_4 = 2.574$. The X-bar control limits are

Lower Limit = 240.29 - 1.023(19.88) = 220.0

Upper Limit = 240.29 + 1.023(19.88) = 260.6

The R-bar control limits are

Lower Limit = 0(19.88) = 0

Upper Limit = 2.574(19.88) = 51.2

Detailed calculations are available in the Microsoft Excel file "**Chapt-6 SPC.xlsx**".

Individuals and Moving Range Charts

Control charts for individual measurements, e.g., the sample size = 1, use the moving range of two successive observations to measure the process variability. The moving range is defined as the absolute value of the first difference between two consecutive data points. Analogous to the Shewhart control chart, both the data (which are the individuals) and the moving range can be plotted.

For the control chart for individual measurements, the lines plotted are

$$LCL = \overline{X} - 3\left(\frac{\overline{MR}}{1.128}\right)$$

$$UCL = \overline{X} + 3\left(\frac{\overline{MR}}{1.128}\right)$$

where X-bar is the average of all the individuals and MR-bar is the average of all the moving ranges of two observations.

Example

Given the data in the table below, determine the control limits for the individuals control chart.

Observation Number	Observation Value
1	49.6
2	47.6
3	49.9
4	51.3
5	47.8
6	51.2
7	52.6
8	52.4
9	53.6
10	52.1

Solution

The table below shows the computed moving ranges along with X-bar and MR-bar.

Observation Number	Observation Value	Moving Range

1	49.6	
2	47.6	2
3	49.9	2.3
4	51.3	14
5	47.8	3.5
6	51.2	3.4
7	52.6	1.4
8	52.4	0.2
9	53.6	1.2
10	52.1	1.5
Average	**50.81**	**1.878**

The control limits for the individuals chart are

Lower Limit = 50.81 - 3(1.128)/1.878) = 45.8

Upper Limit = 50.81 + 3(1.128)/1.878) = 55.8

Detailed calculations are available in the Microsoft Excel file "**Chapt-6 SPC.xlsx**".

X-Bar and S Charts

While X-bar and Range charts use the range to estimate population standard deviation, X-bar and S charts use the sample standard deviation to estimate population standard deviation. In general, the range approach is satisfactory for sample sizes up to around 10. For larger sample sizes, using subgroup standard deviations is preferable. For small sample sizes, the relative efficiency of using the range approach as opposed to using standard deviations is shown in the following table.

Sample Size	Relative Efficiency
2	100%
3	99.2%
4	97.5%
5	95.5%
6	93.0%
10	85.0%

The constants in the table below are used to ease computations for the control limits of the X-bar and S charts.

n	A_2	A_3	D_3	D_4	d_2	B_3	B_4
2	1.880	2.659	0	3.267	1.128	0	3.267
3	1.023	1.954	0	2.574	1.693	0	2.568
4	0.729	1.628	0	2.282	2.059	0	2.266
5	0.5770	1.427	0	2.115	2.326	0	2.089
6	0.483	1.287	0	2.004	2.534	0.03	1.970
7	0.419	1.182	0.076	1.924	2.704	0.118	1.882
8	0.373	1.099	0.136	1.864	2.847	0.185	1815
9	0.337	1.032	0.184	1.816	2.970	0.239	1.761
10	0.308	0.975	0.223	1.777	3.078	0.284	1.716

The X-bar control chart is constructed with the following expressions.

$$\bar{\bar{X}} = \frac{\text{sum of subgroup averages}}{\text{number of subgroups}}$$

$$LCL = \bar{\bar{X}} - A_3\bar{s}$$

$$UCL = \bar{\bar{X}} + A_3\bar{s}$$

The standard deviation control chart is constructed with the following expressions.

$$\bar{s} = \frac{\text{sum of subgroup sigmas}}{\text{number of subgroups}}$$

$$LCL = B_3 \bar{s}$$

$$UCL = B_4 \bar{s}$$

Example

Given the data in the table below, compute the control limits for the X-bar and standard deviation charts.

Observation 1	Observation 2	Observation 3
231.6	232.5	256.2
227.2	237.5	240.4
235.4	218.0	257.4
240.4	228.5	238.4
263.0	268.2	255.0
227.8	228.9	245.7
257.8	247.4	242.8
237.0	230.9	235.8
232.8	241.8	234.6
225.6	248.8	237.7
245.2	230.4	229.8
250.6	247.7	250.8
248.1	258.2	249.6
213.7	227.9	251.7
230.0	246.4	226.4
233.9	245.3	256.9
243.7	227.6	253.7
214.6	251.9	234.7
230.4	250.0	243.1
235.6	235.1	251.1
232.7	251.5	253.8
227.9	243.7	236.5
234.8	279.8	239.5
244.5	245.1	242.9
222.4	250.5	231.4
233.9	238.0	228.9
256.5	238.3	241.0

239.5	232.0	252.0
244.0	223.2	225.6
259.3	225.4	231.9

Solution

The X-bar and sample standard deviation values have been computed in the table below along with X-double bar and *s*-bar.

Observation 1	**Observation 2**	**Observation 3**	**Average**	**Sample Standard Deviation**
231.6	232.5	256.2	240.1	13.95
227.2	237.5	240.4	235.0	6.94
235.4	218.0	257.4	236.9	19.74
240.4	228.5	238.4	235.8	6.37
263.0	268.2	255.0	262.1	6.65
227.8	228.9	245.7	234.1	10.03
257.8	247.4	242.8	249.3	7.68
237.0	230.9	235.8	234.6	3.23
232.8	241.8	234.6	236.4	4.76
225.6	248.8	237.7	237.4	11.60
245.2	230.4	229.8	235.1	8.72
250.6	247.7	250.8	249.7	1.73
248.1	258.2	249.6	252.0	5.45
213.7	227.9	251.7	231.1	19.20
230.0	246.4	226.4	234.3	10.66
233.9	245.3	256.9	245.4	11.50
243.7	227.6	253.7	241.7	13.17
214.6	251.9	234.7	233.7	18.67
230.4	250.0	243.1	241.2	9.94
235.6	235.1	251.1	240.6	9.10
232.7	251.5	253.8	246.0	11.58
227.9	243.7	236.5	236.0	7.91
234.8	279.8	239.5	251.4	24.74
244.5	245.1	242.9	244.2	1.14
222.4	250.5	231.4	234.8	14.35
233.9	238.0	228.9	233.6	4.56
256.5	238.3	241.0	245.3	9.82
239.5	232.0	252.0	241.2	10.10
244.0	223.2	225.6	230.9	11.38
259.3	225.4	231.9	238.9	17.99
Average			**240.29**	**10.42**

For a sub-group size of 3 $A_3 = 1.954$, $B_3 = 0$ and $B_4 = 2.568$. The X-bar control limits are

Lower Limit = 240.29 - 1.954(10.42) = 219.9

Upper Limit = 240.29 + 1.954(10.42) = 260.6

The S-bar control limits are

Lower Limit = 0(10.42) = 0

Upper Limit = 2.568(10.42) = 26.8

Detailed calculations are available in the Microsoft Excel file "**Chapt-6 SPC.xlsx**".

p Chart

The Shewhart control chart plots quality characteristics that can be measured and expressed numerically. We measure weight, height, position, thickness, etc. If we cannot represent a particular quality characteristic numerically, or if it is impractical to do so, we then often resort to using a quality characteristic to sort or classify an item that is inspected into one of two "buckets".

An example of a common quality characteristic classification would be designating units as "conforming units" or nonconforming units". Another quality characteristic criteria would be sorting units into "non defective" and "defective" categories. Quality characteristics of that type are called attributes.

Note that there is a difference between "nonconforming to an engineering specification" and "defective" -- a nonconforming unit may function just fine and be, in fact, not defective at all, while a part can be "in spec" and not function as desired (i.e., be defective). Examples of quality characteristics that are attributes are the number of failures in a production run, the proportion of malfunctioning wafers in a lot, the number of people eating in the cafeteria on a given day, etc.

Control charts dealing with the proportion or fraction of defective product are called p-charts (for proportion). The control limits for the p-chart are found with the following expressions.

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\left(1-\bar{p}\right)}{n}}$$

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}\left(1-\bar{p}\right)}{n}}$$

Example

Historically 2.1% of production has been scrap. Given a sample size of 300, compute the control limits for the p-chart.

Solution

The control limits for the p-chart are

$$LCL = 0.021 - 3\sqrt{\frac{0.021(1-0.021)}{300}} = -0.0038 \Rightarrow 0$$

$$UCL = 0.021 + 3\sqrt{\frac{0.021(1-0.021)}{300}} = 0.046$$

Detailed calculations are available in the Microsoft Excel file “**Chapt-6 SPC.xlsx**”.

c Chart

The literature differentiates between defect and defective, which is the same as differentiating between nonconformity and nonconforming units. This may sound like splitting hairs, but in the interest of clarity let's try to unravel this man-made mystery.

Consider a wafer with several chips on it. The wafer is referred to as an "item of a product". The chip may be referred to as "a specific point". There exist certain specifications for the wafers. When a particular wafer (e.g., the item of the product) does not meet at least one of the specifications, it is classified as a nonconforming item. Furthermore, each chip, (e.g., the specific point) at which a specification is not met becomes a defect or nonconformity.

So, a nonconforming or defective item contains at least one defect or nonconformity. A wafer can contain several defects but still be classified as conforming. For example, the defects may be located at noncritical positions on the wafer. If, on the other hand, the number of the so-called "unimportant" defects becomes alarmingly large, an investigation of the production of these wafers is warranted.

Control charts involving counts can be either for the total number of nonconformities (defects) for the sample of inspected units, or for the average number of defects per inspection unit. The control limits for the c-chart are found with the following expressions.

$$LCL = \bar{c} - 3\sqrt{\bar{c}}$$

$$LCL = \bar{c} + 3\sqrt{\bar{c}}$$

Example

Twenty five successive wafers are inspected and the total number of defects found is 400. Compute the control limits for the c-chart.

Solution

The control limits for the c-chart are

$$LCL = 16 - 3\sqrt{16} = 4$$

$$UCL = 16 + 3\sqrt{16} = 28$$

Detailed calculations are available in the Microsoft Excel file "**Chapt-6 SPC.xlsx**".