hw2_prob1

January 19, 2024

[1]:
```
import numpy as np
import scipy.io
import matplotlib.pyplot as plt
```

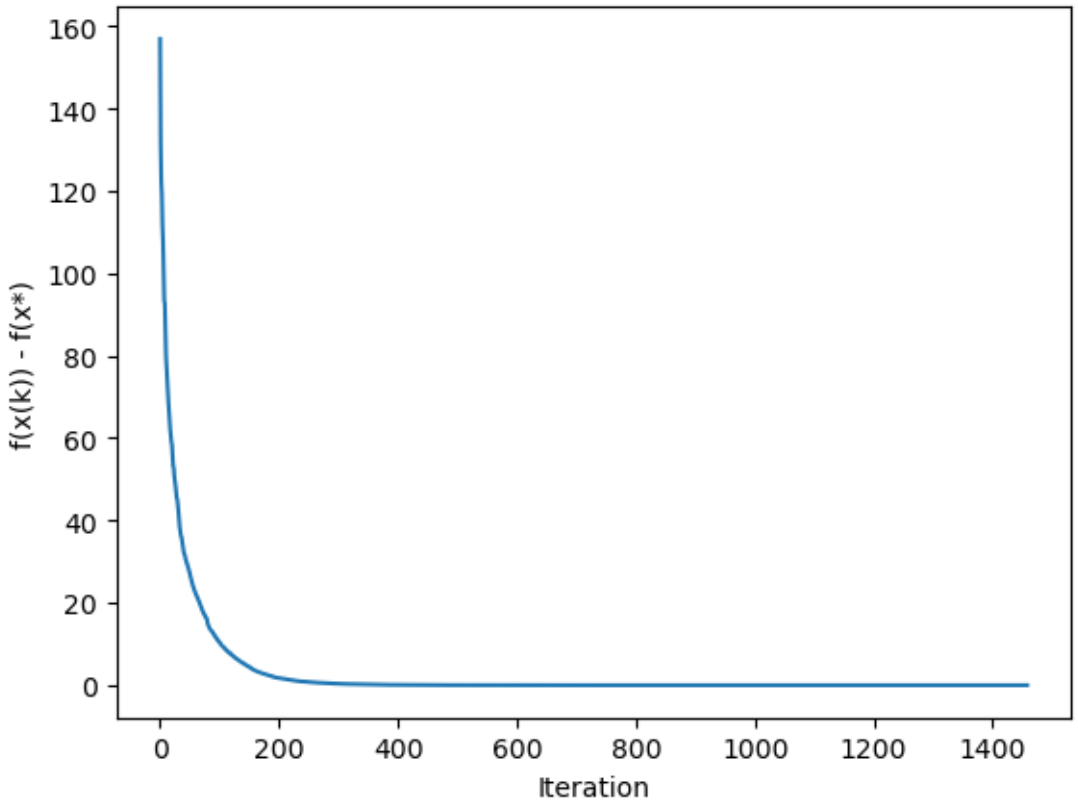
[2]:
```
# MAT
mat_path = r"../HW2_package/hw2_prob1.mat"
mat_data = scipy.io.loadmat(mat_path)

#
A = mat_data['A']
```

[3]:
```
def f(x):
    return - np.sum(np.log(1 - A @ x)) - np.sum(np.log(1 - x))
def grad(x):
    return (A.T @ np.reciprocal(1 - A @ x) + np.reciprocal(1 - x))
def hessian(x):
    return (A.T @ np.diag(np.squeeze(np.reciprocal(1- A @ x)) ** 2) @ A + np.
 ↪diag(np.reciprocal(1- x)** 2))
```

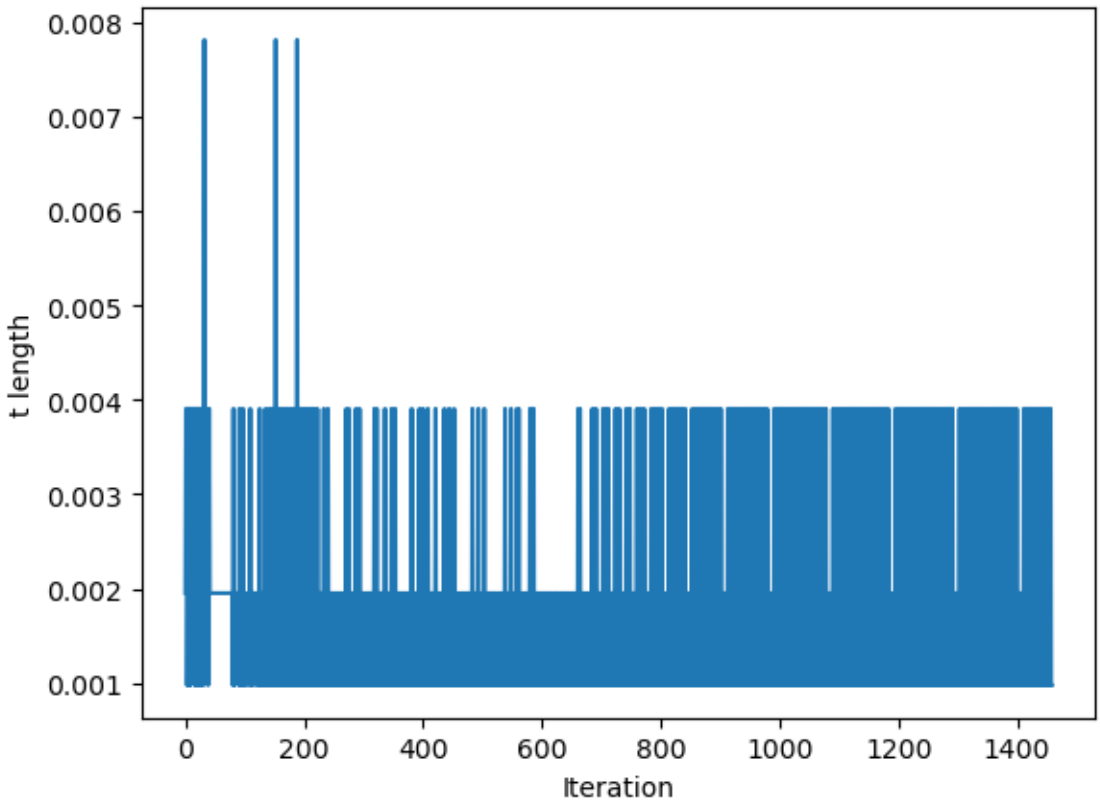
[4]:
```
# Gradient Descent
xk = np.zeros((100, 1))
alpha = 0.01
beta = 0.5
temp = 1e-3
arr_t = []
arr_x= []


while np.linalg.norm(grad(xk), 2)  > temp:
    delta_x = -grad(xk)
    t = 1

    #feasibility check
    while np.max(A @ (xk + t * delta_x)) >= 1 or np.max(xk + t * delta_x) >= 1:
        t = beta * t

    #backtracking line search
    while f(xk + t * delta_x) > (f(xk) + alpha * t * grad(xk).T @ delta_x):
```

```
        t = beta * t

    xk = xk + t * delta_x

    arr_x.append(f(xk))
    arr_t.append(t)
```

[5]:
```
plt.plot(arr_x - arr_x[-1])
plt.xlabel('Iteration')
plt.ylabel('f(x(k)) - f(x*)')

plt.show()
```



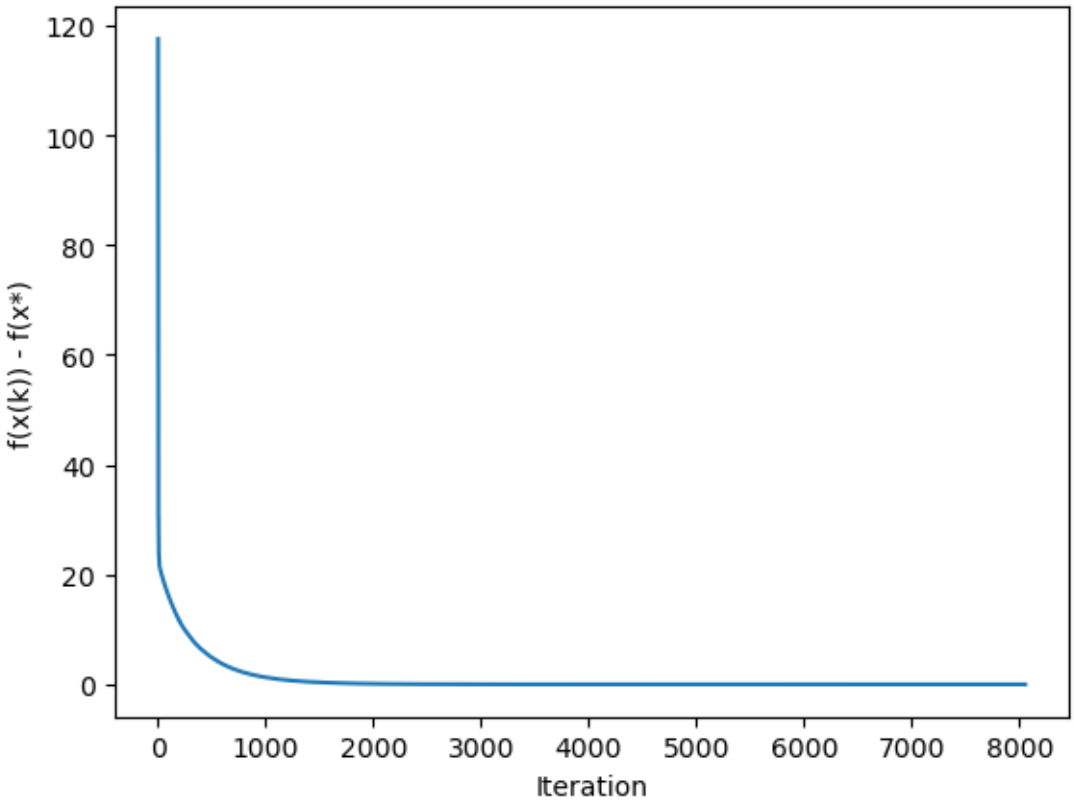
[6]:
```
plt.plot(arr_t)
plt.xlabel('Iteration')
plt.ylabel('t length')
plt.show()
```



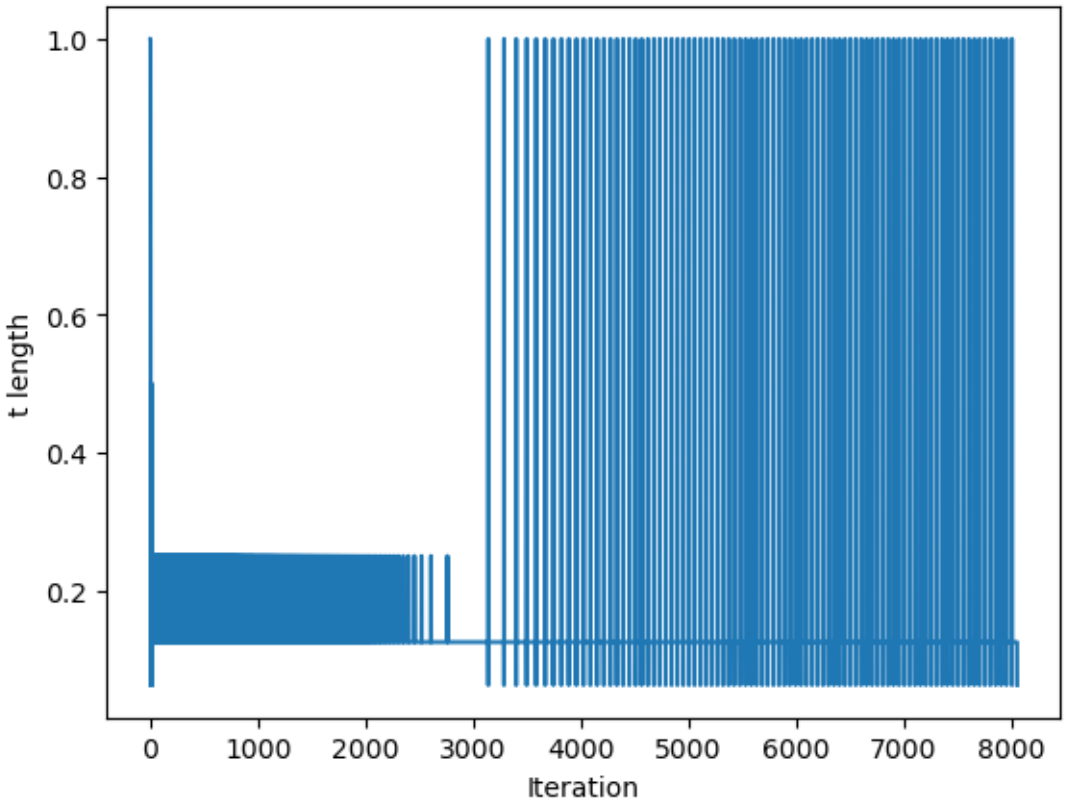
[7]:
```
# Newton's Method
xk = np.zeros((100, 1))
alpha = 0.01
beta = 0.5
temp = 1e-3
arr_t = []
arr_x= []


while np.linalg.norm(grad(xk), 2)  > temp:
    delta_x = - np.linalg.inv(hessian(xk)) @ grad(xk)
    t = 1

    #feasibility check
    while np.max(A @ (xk + t * delta_x)) >= 1 or np.max(xk + t * delta_x) >= 1:
        t = beta * t

    #backtracking line search
    while f(xk + t * delta_x) > (f(xk) + alpha * t * grad(xk).T @ delta_x):
        t = beta * t
```

```
    xk = xk + t * delta_x

    arr_x.append(f(xk))
    arr_t.append(t)
```

[8]:
```
plt.plot(arr_x - arr_x[-1])
plt.xlabel('Iteration')
plt.ylabel('f(x(k)) - f(x*)')

plt.show()
```



[9]:
```
plt.plot(arr_t)
plt.xlabel('Iteration')
plt.ylabel('t length')
plt.show()
```


1.0
0.8
0.6
0.4
0.2
t length
0
1000
2000
3000
4000
5000
6000
7000
8000
Iteration

[ ]: