MED9098 Homework 3

Autumn 2023 (Due 12/6 18:00)

(Please submit your codes and plots with answers clearly)

1. [L1-Minimization]

(a) Please download '*hw3_prob1a.mat*' that contains A, b and *x_orig* (ground-truth of x). Consider the following L1-minimization problem

$$\text{minimize} \quad f(x) = \frac{1}{2} \| Ax - b \|_2^2 + \lambda \| x \|_1$$

where $A \in R^{m \times n}$, $x \in R^n$, and $b \in R^m$ (m=200, n= 4000)

* Implement conventional L1-minimization with soft-thresholding operator.
* Apply acceleration (FISTA) to the given optimization problem
* Show
  - reconstructed x's for the conventional and the accelerated methods with original x (*x_orig*) in two plots
  - $\|x\text{-}x\_orig\|_2$ across the iterations for both methods in one plot

(Parameters: λ =2, Stopping criterion: $\|x_k - x_{k-1}\|_2/\|x_k\|_2 < 10^{-4}$)

(20 pts)

(b) Please download '*hw3_prob1b.mat*' (for the reweighted-L1-minimization problem

$$\text{minimize} \quad f(x) = \frac{1}{2} \| Ax - b \|_2^2 + \lambda \| Wx \|_1$$

$$\text{where} \quad w_j = \frac{|x_j|}{|x_j| + \delta}$$

* Implement the reweighted L1-minimization.
  - At first, the weighting elements will be defined as 1 (behaved like conventional L1-min.). Define this as x_1 for convenience.
  - With the initial x_1, the weighting matrix will be newly defined for the reweighting process, which will yield the final reconstructed one (x_{final})
* Show the reconstructed x_1 and x_{final} with *x_orig* in two plots. (You may have to find a range of appropriate δ).

(Parameters: λ =2, Stopping criterion: $\|x_k - x_{k-1}\|_2/\|x_k\|_2 < 10^{-4}$)

(20 pts)

2. [Image Denoising]

Please download '*hw3_prob2.mat*' that includes **a noisy phantom image *f*** to be de-noised.

(a) Implement a denoising algorithm by **ADMM algorithm** with the following objective function:

$$\underset{x}{\text{minimize}} \;\; f(x) = g(x) + h_h(x) + h_v(x) = \frac{\mu}{2}\|x - f\|_2^2 + \|D_h x\|_1 + \|D_v x\|_1$$

$$\Rightarrow \underset{x, d_h, d_v, b_h, b_v}{\text{minimize}} \;\; \frac{\mu}{2}\|x - f\|_2^2 + |d_h| + |d_v| + \frac{\lambda}{2}\|d_h - D_h x - q_h\|_2^2 + \frac{\lambda}{2}\|d_v - D_v x - q_v\|_2^2$$

Please show the reconstructed image (μ=0.02, λ=0.0002, Stopping criterion: $||x_k - x_{k-1}||_2/||x_k||_2 < 10^{-4}$).

(20 pts)

(b) From the result of part (a), the image de-noising can be improved by the reweighted L1-norm as follows:

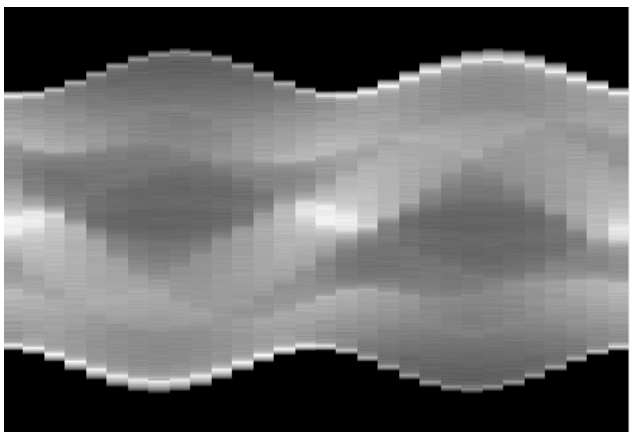
$$\underset{x}{\text{minimize}} \;\; f(x) = g(x) + h_h(x) + h_v(x) = \frac{\mu}{2}\|x - f\|_2^2 + \|W_h(D_h x)\|_1 + \|W_v(D_v x)\|_1$$

You could conduct the image reconstruction by the reweighting process with appropriate hyper-parameter setting, in which you would have to define μ, λ and δ. Please show the reconstructed image with the hyper-parameters clearly that you found.

(20 pts)

3. [CT Image Reconstruction from Few Projections (Compressed Sensing)]

From the given sinogram that has very few projection angles (30 angles), as shown below, the CT image reconstruction needs to be performed by **Chambolle-Pock algorithm**.



The objective function is defined as follow:

$$\underset{x}{\text{minimize}} \;\; F(Kx) + G(x) = \frac{\mu}{2}\|Ax - b\|_2^2 + \|D_h x\|_1 + \|D_v x\|_1 \; (x \geq 0)$$

Please download '*hw3_prob3.mat*', and implement the Chambolle-Pock algorithm. Show the reconstructed image with the following parameters:

(μ=1, Stopping criterion: $||x_k - x_{k-1}||_2/||x_k||_2 < 5 \times 10^{-4}$)

(20 pts)