MED9098 Homework 2

Autumn 2023

1. Consider the following unconstrained optimization problem:

$$\text{minimize} \quad f(x) = -\sum_{i=1}^{m} \log(1 - a_i^T x) - \sum_{i=1}^{m} \log(1 - x_i)$$

where $x \in R^n$, $\operatorname{dom} f = \{x \mid a_i^T x < 1 \ (i = 1, 2, ..., m), \quad x_i < 1 \ (i = 1, 2, ..., n)\}$. Please choose the initial guess to be zeros ($x^{(0)}=0$) with the given matrix $A \in R^{m \times n}$ (m=200, n= 100). (A is given in '*hw2_prob1.mat'*)

```
f= -sum(log(1-A*x))- sum(log(1-x));

grad= A'*(1./(1-A*x)) + 1./(1-x);
hessian= A'*diag(1./(1-A*x).^2)*A + diag( 1./(1-x).^2 ) ;
```

Please check **feasibility of *x*** before line search.

```
% Check feasibility of x
t=1;
while max(A*(x + t*delta_x)) >=1 || max( (x + t*delta_x)) >=1;
      t= beta*t;
end

% Body of line search
while (-sum(log(1-A*(x + t*delta_x))) - sum(log(1-(x + t*delta_x))) ) >
(f + alpha*t*grad'*delta_x)
      t= beta*t;
end
```

a) **Optimize *x* with the gradient descent method**.

- Use backtracking line search with α=0.01, β=0.5;
- Stopping criterion: $\|\nabla f(x)\|_2 \le 10^{-3}$

Please plot $f(x^{(k)})$ - $f(x^*)$ and step length ($t^{(k)}$) for each iteration (Submit the code you generated). (30 pts)

b) **Repeat *step*(*a*) with the Newton's method**.

- Use backtracking line search with α=0.01, β=0.5;
- Stopping criterion: $\|\nabla f(x)\|_2 \le 10^{-3}$

Please plot $f(x^{(k)})$ - $f(x^*)$ and step length ($t^{(k)}$) for each iteration (Submit the code you generated). (30 pts)

2. Consider the following plan optimization problem with an **inequality constraint**

$$\text{minimize} \quad f(x) = \frac{1}{2}\|Ax - d\|_2^2$$
$$\text{subject to} \quad Ax \le d_{up}$$

where $x \in R^n$ is the fluence-map (beam intensity map), and A$\in R^{mxn}$ is the dose deposition matrix that deals with a relationship between dose and fluence-map, in which a product of A and x leads to dose distribution. $d \in R^m$ is the ideal dose distribution (prescription dose) in the target volume (assuming that it is 60 Gy in this case), and d_{up} is the dose upper bound for the given structure (assuming that it is 62 Gy in this case). Please choose the initial guess to be zeros ($x^{(0)}=0$) with the given matrix $A \in R^{mxn}$ (m=3007, n= 1568).
(A, d and d_{up} are given in '*hw2_prob2.mat*'.)

Optimize x with the gradient descent method

- Use the interior-point method with logarithmic barrier.
  (Please be careful about the feasibility of x)
- Define t to be 1 ~ 2 (t is not a step size, but a parameter for the interior-point method).
  Define Max. iteration number to > 10000.
  Declare parameters for backtracking line search: α=0.01, β=0.8
- Compare the optimizing result against that without inequality constraint in terms of 'Dose volume histogram (DVH)', and explain what you found.
  ('get_DVH_PTV.m' will provide a dose volume histogram for the optimized fluence-map x)
- Submit the code you generated.
  (40 pts)