MED9098 Homework 1

Autumn 2023

(Due 10/11 18:00)

1. The runtime T of an algorithm depends on its input data, which is characterized by three key parameters: k, m, and n (given the problem data). A simple and standard model that shows how T scales with k, m, and n has the form

$$\hat{T} = \alpha k^{\beta} m^{\gamma} n^{\delta}$$

where $\alpha, \beta, \gamma, \delta \in R$ are constants that characterize the approximate runtime model. Now suppose you are given measured runtimes for ***N* executions** of the algorithm, with different sets of input data. For each data record, you are given T_i (the runtime), and the parameters k_i, m_i, and n_i.

We wish to find values of α, β, γ, and δ for which our model fits our measurements (T_i). We define the fitting cost as

$$J = \sum_{i=1}^{N} (\log(\hat{T}_i / T_i))^2$$

where $\hat{T}_i = \alpha k_i^{\beta} m_i^{\gamma} n_i^{\delta}$ is the runtime predicted by our model, using the given parameter values. This fitting cost can be (loosely) interpreted in terms of relative or percentage fit. Please find constants $\alpha, \beta, \gamma, \delta$ that minimize J. (The file **hw1_prob1.csv (hw1_prob1.mat)** contains k, m, n, T, N)

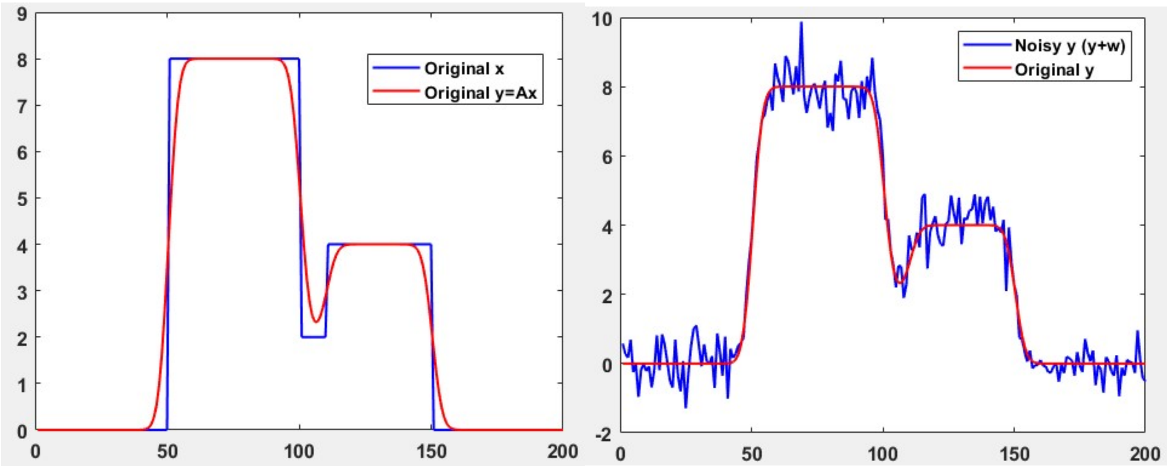
a) Explain how you design $Ax=b$. (Think of what factors are given as measurements, and what variables are unknown. Also, consider how to construct matrix A from the given information). (20 pt)

b) Give the values of $\alpha, \beta, \gamma, \delta$ that minimize J, and the corresponding value of J. (20 pt)

2. We have a discrete signal $x \in R^n$, and a measurement $y \in R^n$, associated with a square matrix $A \in R^{nXn}$:

$$y_i = \sum_{i=1}^{n} a_{ij} x_j + w_i \qquad \Rightarrow \qquad y = Ax + w = y_{orig} + w$$

The matrix A plays a role as turning the signal x into y (smoothing x), which was perturbed by noise signal w. (The file **hw1_prob2.csv (hw1_prob2.mat)** contains *A, x, y, y_orig* (before perturbation)).



a) Using inverse of A and pseudo-inverse (least-squares) methods, please find x and compare those with the original discrete signal x. (5 pt)

b) Do not panic while obtaining the frustrated results from part (a). Let us take a look at a different approach based on the singular value decomposition (SVD). Take singular value decomposition on A. Plot the diagonal components (singular values of A), and show the 4 input (right) singular vectors (vector v). Explain what you observe in relation to the role of matrix A (smoothing). (15 pt)

c) Recall [Low rank approximation] of SVD in Lecture 3.

(i) Approximate A with the r largest singular values and corresponding singular vectors.

(ii) Do the least squares method to find x

(iii) Repeat (i)-(ii) while varying r from 1 to 50

Find the value of r that minimizes the norm between reconstructed x and original x. Plot the reconstructed x with original x. (20 pt)

d) Recall [Tychonov Regularization] in Lecture 4.

$$\| Ax - b \|_2^2 + \mu \| x \|_2^2$$

Find the value of μ that minimizes the norm between reconstructed x and original x. Plot the reconstructed x with original x. (Hint: Optimal μ that I found was less than 0.1) (20 pt)