

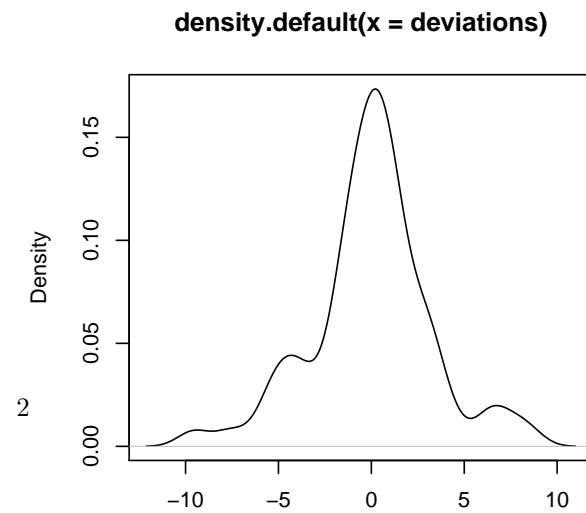
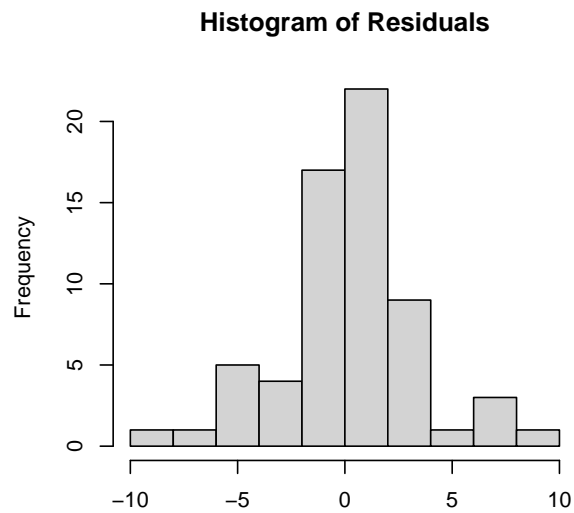
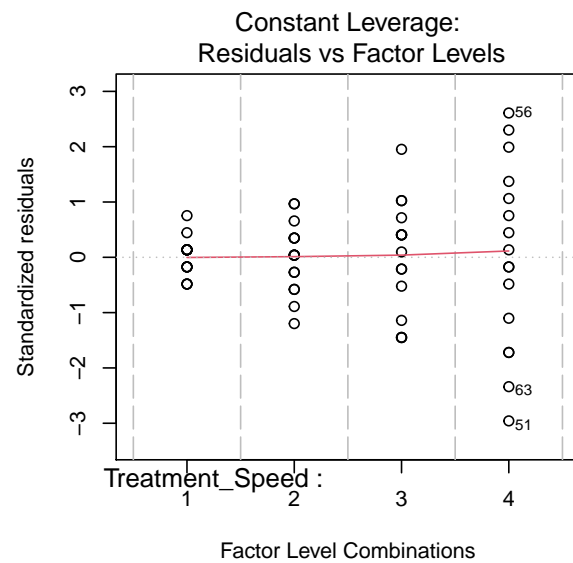
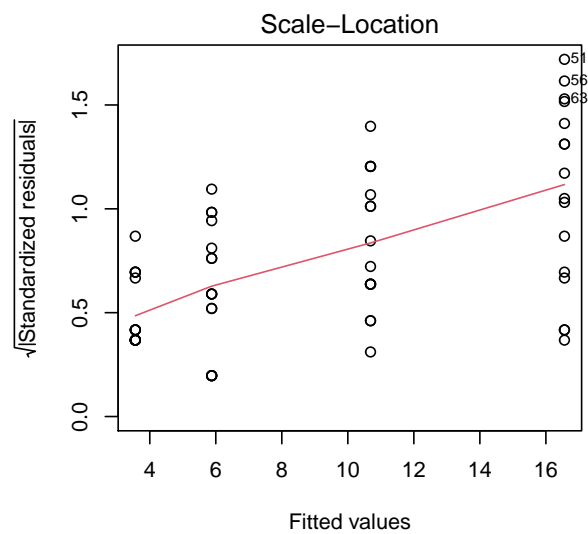
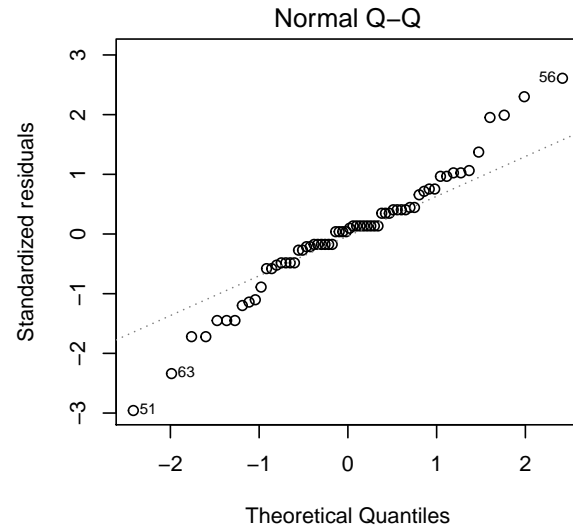
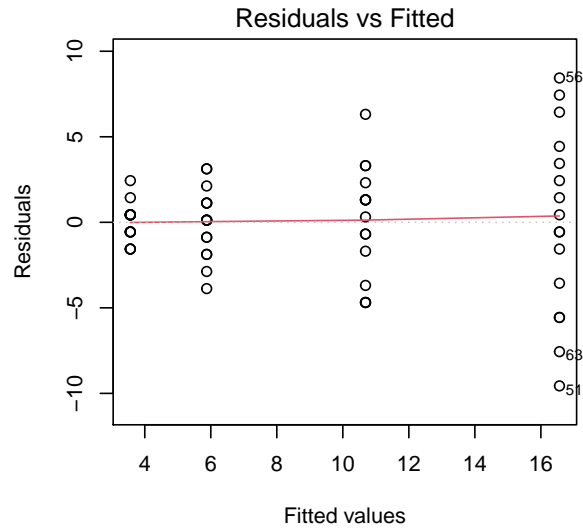
STP 531  
Applied Analysis of Variance  
Homework 3

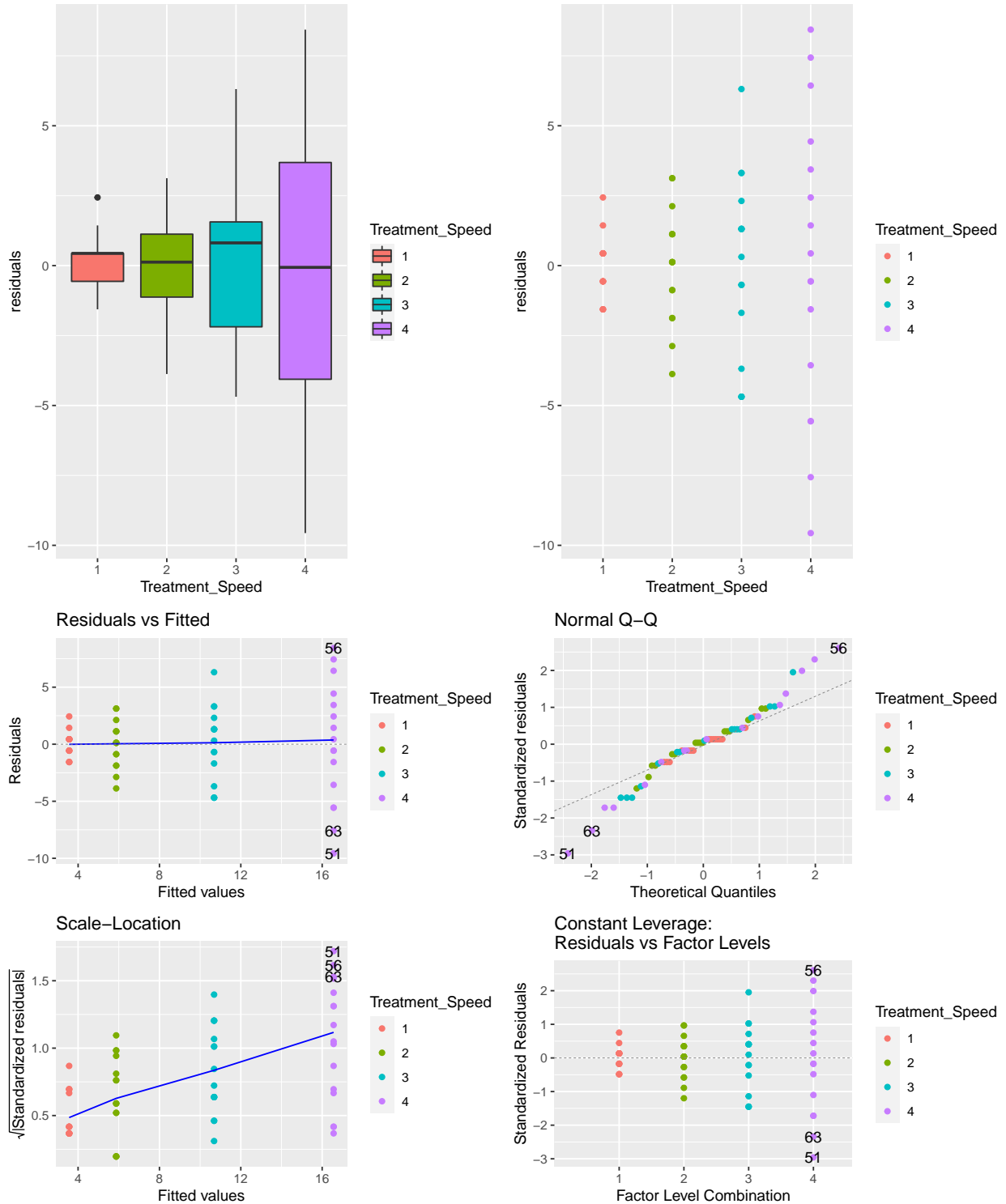
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## Question 18.7

### Part B





It does not appear that the error variances across the four treatments are the same. There's some a cone shape going on, if we arrange from treatment group 1 to group 4 sequentially.

## Part C

Brown-Forsythe Test: also recall that the Brown-Forsythe test is robust against departures from normality.

$$H_0 : \sigma_i^2 \text{ are all equal } i = 1, 2, 3, 4$$

$$H_a : \text{not all } \sigma_i^2 \text{ are equal}$$

$$F_{BF}^* = \frac{MSTR}{MSE}$$

$$MSTR = \frac{\sum n_i (\bar{d}_{i.} - \bar{d}_{..})^2}{r - 1}$$

$$MSE = \frac{\sum \sum (d_{ij} - \bar{d}_{i.})^2}{n_T - r}$$

$$\bar{d}_{i.} = \frac{\sum_j d_{ij}}{n_i}$$

$$\bar{d}_{..} = \frac{\sum \sum d_{ij}}{n_T}$$

$$d_{ij} = |Y_{ij} - \tilde{Y}_i|$$

$$F_{critical} = F(1 - \alpha; r - 1, n_T - r)$$

```
##
##  hov: Brown-Forsyth
##
## data:  Thread_Breaks
## F = 9.5416, df:Treatment_Speed = 3, df:Residuals = 60, p-value =
## 3.04e-05
## alternative hypothesis: variances are not identical
```

I didn't want to write function(s) to do the Brown-Forsythe or do it line-by-line, so I used the **hov()** in **library(HH)**. The  $p < 0.05$ , so there is evidence to reject the null hypothesis in favour of the alternative i.e., there is evidence that the factor level variances are no identical. This also supports the qualitative method in part B (the funnel shape on the residuals plot).

## Part E

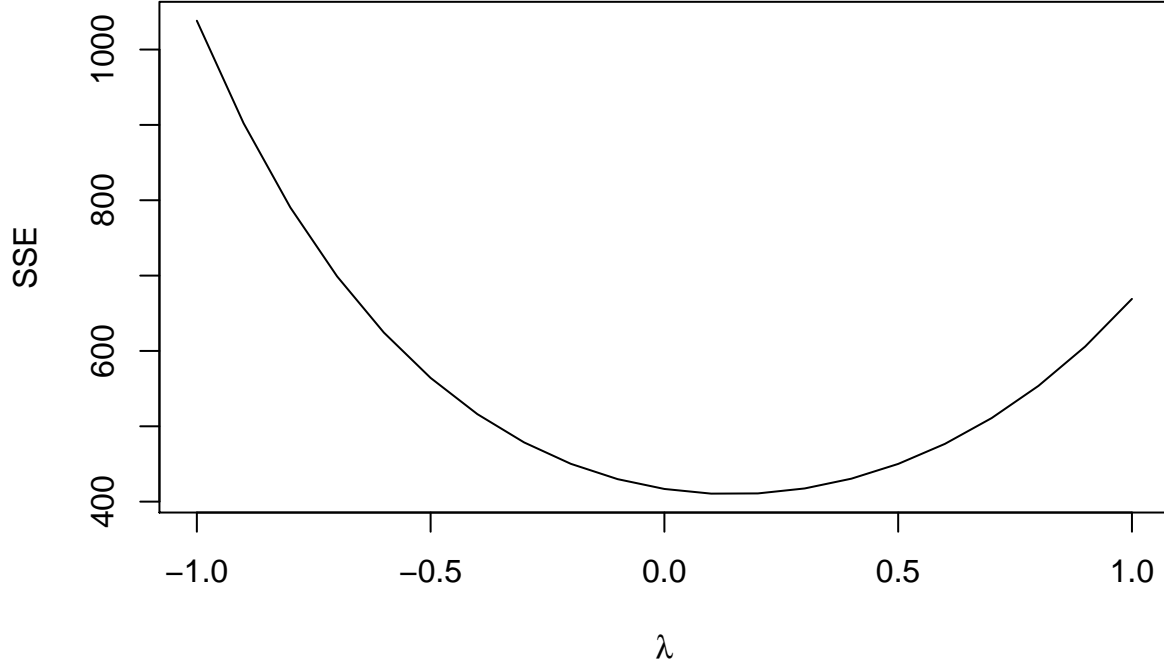


Table 1: SSE for range of lambda

	lambda	SSE
1	-1.0	1038.2614
2	-0.9	902.1834
3	-0.8	790.4288
4	-0.7	698.9339
5	-0.6	624.4079
6	-0.5	564.1934
7	-0.4	516.1552
8	-0.3	478.5914
9	-0.2	450.1625
10	-0.1	429.8359
21	0.0	416.8430
11	0.1	410.6464
12	0.2	410.9166
13	0.3	417.5165
14	0.4	430.4928
15	0.5	450.0749
16	0.6	476.6794
17	0.7	510.9218
18	0.8	553.6354
19	0.9	605.8971
20	1.0	669.0625

Yes,  $\lambda = 0$ , a logarithmic transformation, is a reasonable power transformation.

## Question 18.18

We applied a logarithmic transformation.

### Part A

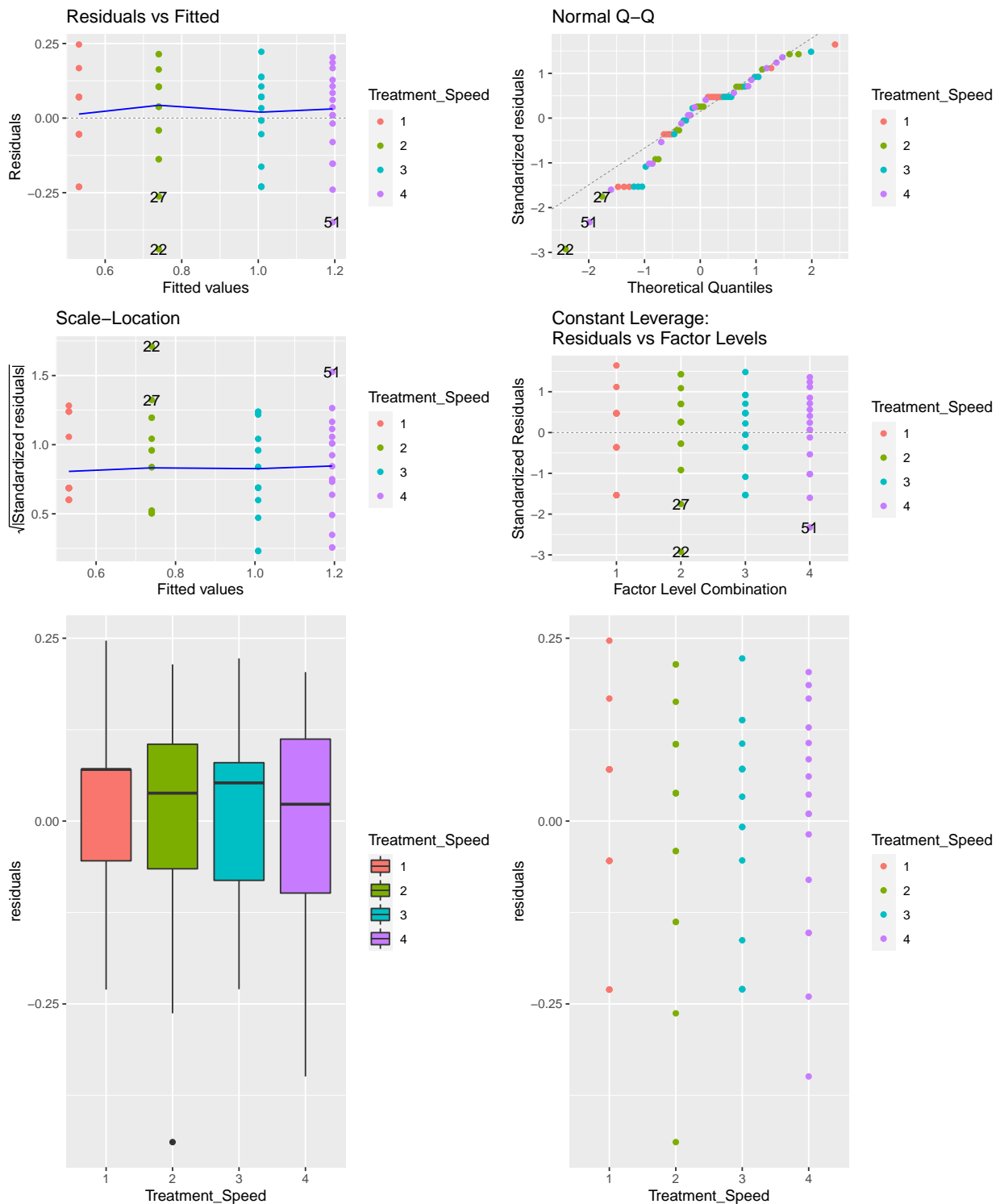
Table 2: Power Transformation Model

	Thread Breaks (Log-Transformed)
	$\log_{10}(\text{Thread\_Breaks})$
Slow	0.531*** (0.039)
Normal	0.740*** (0.039)
Fast	1.008*** (0.039)
Maximum	1.194*** (0.039)
Observations	64
R <sup>2</sup>	0.973
Adjusted R <sup>2</sup>	0.971
Residual Std. Error	0.155 (df = 60)
F Statistic	544.989*** (df = 4; 60)
<i>Note:</i>	*p<0.1; **p<0.05; ***p<0.01

Table 3: residuals

Slow	Normal	Fast	Maximum
0.0706152	0.1051177	0.0711462	0.0361987
-0.0543235	0.0381709	-0.2298838	-0.0181590
-0.2304148	-0.1379203	0.1380930	-0.3491522
-0.0543235	0.0381709	0.0711462	0.1067798
0.0706152	0.1051177	-0.0080350	-0.0803069
0.0706152	-0.4389503	-0.0537925	-0.1528576
-0.0543235	0.2142622	0.0711462	0.0098697
0.2467065	-0.0410103	0.2224139	0.2036898
0.1675252	-0.0410103	-0.1629370	-0.1528576
0.0706152	0.2142622	-0.2298838	0.1859610
-0.2304148	-0.2628591	0.0711462	0.0610223
0.0706152	0.1631097	0.0333577	0.1279691
0.0706152	0.0381709	-0.2298838	0.0098697
-0.2304148	-0.1379203	0.1059083	0.0845034
-0.0543235	0.1051177	-0.0080350	-0.2400077
0.0706152	0.0381709	0.1380930	0.1674776

## Part B



The coefficient of correlation between the ordered residuals and their expected values under normality is 0.9704474. This transformations mediated the variations in the treatment variances by a lot, qualitatively. The funnel shape is not extremely apparent, unlike the raw data residuals.



## Part C

```
##  
##  hov: Brown-Forsyth  
##  
## data:  log10(Thread_Breaks)  
## F = 0.097864, df:Treatment_Speed = 3, df:Residuals = 60, p-value =  
## 0.9609  
## alternative hypothesis: variances are not identical
```

Since  $p \gg 0.05$ , we fail to reject the null hypothesis. There is insufficient evidence to suggest that the variances among the treatment levels are different.

## Question 18.20

$$F_w^* = \frac{SSE(R) - SSE(F)}{r - 1} \div \frac{SSE(F)}{n_T - r}$$

$$SSE(F) = 60, \quad SSE(R) = 213.9541$$

$$df_F = 60, \quad df_R = 63$$

$$r = 4, \quad n_T = 64$$

$$F_{critical}^* = F(0.99; 3, 60) = 4.13, \quad (\alpha = 0.1)$$

$$H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$$

$$H_a : \text{not all } \mu_i \text{ are equal}$$

if

$$F_w^* < F(0.99; 3, 60) \approx 4.13, \quad \text{conclude } H_0$$

$$F_w^* \approx 51.32 > F_{critical}^*$$

Table 4: Weighted LS

Treatment_Speed	variances	w
1	1.195833	0.8362369
2	3.983333	0.2510460
3	10.495833	0.0952759
4	28.929167	0.0345672

There is sufficient evidence to reject the null hypothesis, that not all of the factor level means are equal. In other words, there is sufficient evidence to conclude that there is a difference between treatment levels.

## Question 18.20 (Welch Test)

```
##
##  Welch's Heteroscedastic F Test (alpha = 0.05)
## -----
##  data : Thread_Breaks and Treatment_Speed
##
##  statistic   : 49.11628
##  num df      : 3
##  denom df    : 29.74388
##  p.value     : 1.22839e-11
##
##  Result      : Difference is statistically significant.
## -----
##
##  One-way analysis of means (not assuming equal variances)
##
##  data:  Thread_Breaks and Treatment_Speed
##  F = 49.116, num df = 3.000, denom df = 29.744, p-value = 1.228e-11
Enough evidence to suggest that the means are not equal.
```

# Appendix

## Question 18.7

### Part B

```
getdata <- function(...){
  e = new.env()
  name = data(..., envir = e)[1]
  e[[name]]
}

data <- getdata("WindingSpeeds")
data <- rename(data, Thread_Breaks = 1, Treatment_Speed = 2, Observations = 3)
data$Treatment_Speed <- as.factor(data$Treatment_Speed)

# cell means model
model_1 <- lm(Thread_Breaks ~ Treatment_Speed-1, data = data)
par(mfrow = c(2,2))
plot(model_1)

deviations <- residuals(model_1)
hist(deviations, main = "Histogram of Residuals")
plot(density(deviations))

data$fitted_values <- model_1$fitted.values
data$residuals <- model_1$residuals

p1 <- data %>%
  ggplot(aes(x = Treatment_Speed, y = residuals)) +
    geom_boxplot(aes(fill = Treatment_Speed))
p2 <- data %>%
  ggplot(aes(x = Treatment_Speed, y = residuals)) +
    geom_point(aes(color = Treatment_Speed))

g1 <- ggarrange(p1, p2, nrow = 1)
g1

autoplot(model_1, colour = "Treatment_Speed")
```

### Part C

```
n_T <- dim(data)[1]

# check if cell means sizes are equal
# table(data$Treatment_Speed) # n_i = n = 16
n_i = 16

medians <- data %>%
  group_by(Treatment_Speed) %>%
  summarise(Y_tilde = median(Thread_Breaks))

medY_1 <- medians$Y_tilde[1]
medY_2 <- medians$Y_tilde[2]
```

```

medY_3 <- medians$Y_tilde[3]
medY_4 <- medians$Y_tilde[4]

library(HH)
hov(Thread_Breaks ~ Treatment_Speed, method = "bf", data = data)

```

## Part E

```

# bc <- boxcox(model_1)
# lambda <- bc$x[which.max(bc$y)] # lambda = 0 is reasonable.

attach(data)
library(ALSM)
BC <- boxcox.sse(Treatment_Speed, Thread_Breaks, l = seq(-1,1,0.2))
kable(BC[, caption = "SSE for range of lambda", format = "markdown"]) %>%
  kable_styling(position = "center")

```

## Question 18.18

### Part A

```
# lambda = 0 power transformation
model_2 <- lm(log10(Thread_Breaks) ~ Treatment_Speed-1, data = data2)
data2$residuals <- model_2$residuals

stargazer(model_2, type = "latex", header = F, no.space = T,
  covariate.labels = c("Slow", "Normal", "Fast", "Maximum"),
  dep.var.caption = "Thread Breaks (Log-Transformed)",
  title = "Power Transformation Model")

corgi <- as.data.frame(cbind(
  Slow = data2$residuals[data2$Treatment_Speed == 1],
  Normal = data2$residuals[data2$Treatment_Speed == 2],
  Fast = data2$residuals[data2$Treatment_Speed == 3],
  Maximum = data2$residuals[data2$Treatment_Speed == 4]
))

kable(corgi[, ], caption = "residuals", format = "markdown") %>%
  kable_styling(position = "center")
```

### Part B

```
autoplot(model_2, colour = "Treatment_Speed")

p3 <- data2 %>%
  ggplot(aes(x = Treatment_Speed, y = residuals)) +
    geom_boxplot(aes(fill = Treatment_Speed))
p4 <- data2 %>%
  ggplot(aes(x = Treatment_Speed, y = residuals)) +
    geom_point(aes(color = Treatment_Speed))

g2 <- ggarrange(p3, p4, nrow = 1)
g2

hamster <- qqnorm(data2$residuals, plot.it = F)
coef.cor <- cor(hamster$x, hamster$y)
```

### Part C

```
hov(log10(Thread_Breaks) ~ Treatment_Speed, method = "bf", data = data2)
```

## Question 18.20

```
variances <- data3 %>%
  group_by(Treatment_Speed) %>%
  summarise(variances = var(Thread_Breaks)) %>%
  mutate(w = 1 / variances)

kable(variances[, ], caption = "Weighted LS", format = "markdown") %>%
  kable_styling(position = "center")

w = variances$w
w = rep(w, each = 16)

# SSE_F
WLS_model <- lm(Thread_Breaks ~ Treatment_Speed-1, weights = w)
a <- anova(WLS_model)
SSE_F <- a$`Sum Sq`[2]
# SSE_R
model_H0 <- lm(Thread_Breaks ~ 1, weight =w, data = data3)
b <- anova(model_H0)
SSE_R <- b$`Sum Sq`

F_value <- ((SSE_R - SSE_F)/(4-1)) / (SSE_F/(64-4))
```

## Welch Test

```
library(onewaytests)
welch.test(Thread_Breaks ~ Treatment_Speed, data = data3)
oneway.test(Thread_Breaks ~ Treatment_Speed, data = data3)
```