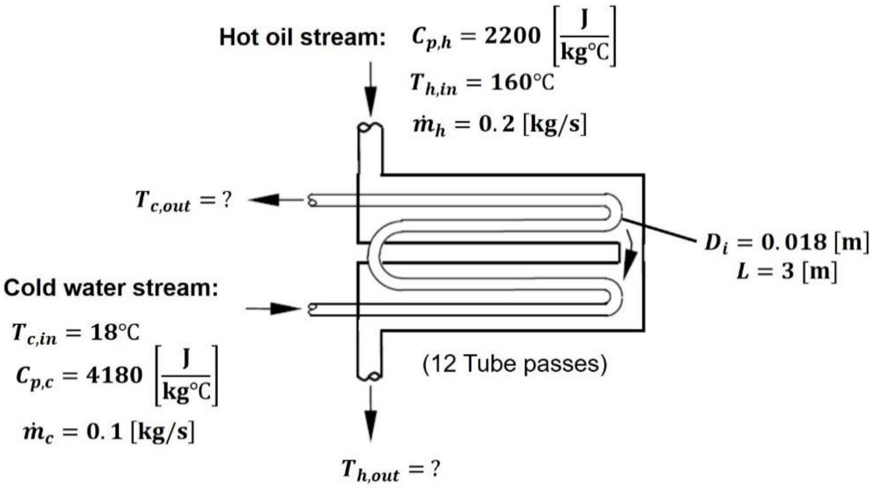
WORKED EXAMPLE

EFFECTIVENESS-NTU METHOD

Hot oil (C_p = 2200 J/kg·°C) is to be cooled by water (C_p = 4180 J/kg·°C) in a 2-shell-pass and 12-tube-pass heat exchanger. The tubes are thin-walled and are made of copper with a diameter of 1.8 cm. The length of each tube pass in the heat exchanger is L = 3 m, and the overall heat transfer coefficient is U = 340 W/m^2·°C. Water flows through the tubes at a total rate of 0.1 kg/s, and the oil through the shell at a rate of 0.2 kg/s. The water and the oil enter at temperatures of 18°C and 160°C, respectively.

Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.



SOLUTION

Objective is to determine $\dot{Q}$ and unknown outlet temperatures.

Since we don't know $\dot{Q}$ and have insufficient information to infer Q or the outlet temperatures, then this is our clue that we need to use the ε-NTU method.

We can get the heat transfer rate from (equation 23, lecture notes):

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}}$$

where,

$$Q_{\max} = C_{\text{minimum}}\left(T_{\text{h,in}} - T_{\text{c,in}}\right)$$

and

$$C_{\text{minimum}} = \min\left\{\dot{m}_{\text{c}}C_{\text{p,c}};\ \dot{m}_{\text{h}}C_{\text{p,h}}\right\}$$

With,

$$\dot{m}_c C_{p,c} = 0.1 \times 4180 = 418 \text{ [W/°C]}$$

and

$$\dot{m}_h C_{p,h} = 0.2 \times 2200 = 440 \text{ [W/°C]}$$

$$\therefore C_{minimum} = \dot{m}_c C_{p,c} = 418 \text{ [W/°C]}$$

and

$$\begin{aligned} Q_{max} &= C_{minimum}(T_{h,in} - T_{c,in}) \\ &= 418 \times (160 - 18) \\ &= 59{,}356 \text{ [W]} \end{aligned}$$

Next, we need to get ε, and noting that $\varepsilon = f(C, NTU)$, we also have to find the heat capacity ratio C from:

$$C = \frac{C_{minimum}}{C_{maximum}} = \frac{418}{440} = 0.95$$

and number of transfer units (NTU) from:

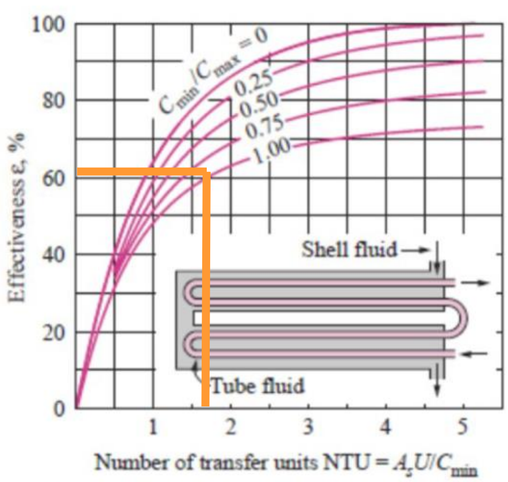
$$NTU = \frac{UA}{C_{minimum}} = \frac{340 \times 2.04}{418} = 1.659$$

where the surface area, A, was calculated from:

$$A = \pi DL \times n = \pi \times 0.018 \times 3 \times 12 = 2.04 \text{ [m}^2\text{]}$$

and the parameter $n = 12$ is the number of tube passes.

Now, from the effectiveness chart (Figure 13-26(d), Appendix C, lecture notes) for $\varepsilon = f(0.95{,}1.659)$ we get $\varepsilon \sim 0.61$.



(d) Two-shell passes and 4, 8, 12, … tube passes

The actual heat transfer rate is then:

$$Q = \varepsilon Q_{\mathrm{max}} = 0.61 \times 59356 = 36207.2 \text{ [W]}$$

The outlet temperatures then follow from:

$$Q = \dot{m}_{\mathrm{h}} C_{\mathrm{p,h}} (\Delta T_{\mathrm{h}})$$

$$\therefore T_{\mathrm{h,out}} = T_{\mathrm{h,in}} - \frac{Q}{\dot{m}_{\mathrm{h}} C_{\mathrm{p,h}}} = 160 - \frac{36207}{440} = 77.7 \text{ [°C]}$$

and, similarly:

$$T_{\mathrm{c,out}} = T_{\mathrm{c,in}} + \frac{Q}{\dot{m}_{\mathrm{c}} C_{\mathrm{p,c}}} = 18 + \frac{36207}{418} = 104.6 \text{ [°C]}$$