**UNIVERSITY OF ABERDEEN SESSION 2015–16**

# EM40JK

**Degree Examination in EM40JK Thermodynamics 2**

**15th December 2015 2 pm – 5 pm**

**PLEASE NOTE THE FOLLOWING**

(i) You **must not** have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material **must not** be amended, annotated or modified in any way.

(ii) You **must not** have in your possession any material that could be determined as giving you an advantage in the examination.

(iii) You **must not** attempt to communicate with any candidate during the exam, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

(iv) You **must not** take to your examination desk any electronic devices such as mobile phones or other smart devices. The only exception to this rule is an approved calculator.

**Failure to comply with the above will be regarded as cheating and may lead to disciplinary action as indicated in the Academic Quality Handbook Section 7 and particularly Appendix 7.1**

***Notes:*** ***(i) Candidates ARE permitted to use an approved calculator.***

***(ii) Candidates ARE permitted to use the Engineering Mathematics Handbook.***

***(iii) Data sheets are attached to the paper.***

**Candidates should attempt *all three* questions from PART A AND *two* questions from *three* in PART B.**

# PART A: Answer ALL Questions

## Question 1

Using a Cartesian control volume (as illustrated in Fig. 1):

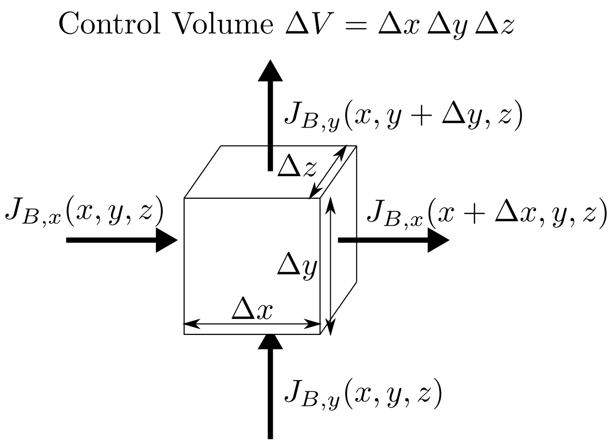



**Figure 1:** *A differential balance of flow property $B$ in cartesian coordinates.*

a) Derive the general advection-diffusion equation for a property $B$, including a source term, $\sigma_B$. [12 marks]

b) Set $B =$ mass and derive the continuity equation. [8 marks]

**Question 2**

a) The wall of a furnace comprises three layers as shown in Fig. 2. The first layer is refractory brick (whose maximum allowable temperature is $1400$°C) while the second layer is insulation (whose maximum allowable temperature is $1093$°C). The third layer is a plate of 6.35 mm thickness of steel ($k_{steel} = 45$ W/m/K). Assume that the layers are thermally bonded.

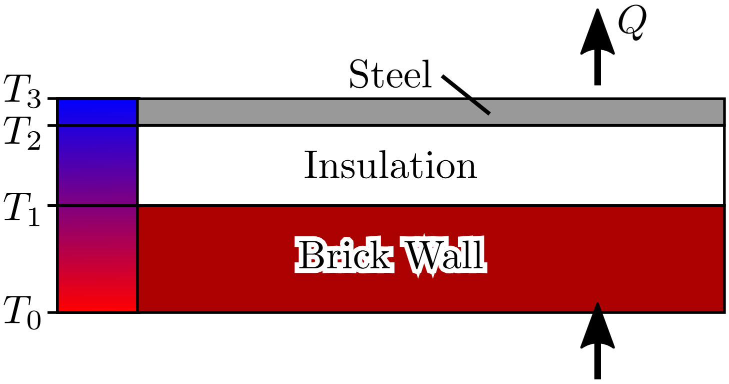



**Figure 2:** *Construction of a furnace wall.*

| Layer | $T = 37.8$°C | $T = 1093$°C |
|---|---|---|
| Brick | 3.12 W m$^{-1}$ K$^{-1}$ | 6.23 W m$^{-1}$ K$^{-1}$ |
| Insulation | 1.56 W m$^{-1}$ K$^{-1}$ | 3.12 W m$^{-1}$ K$^{-1}$ |

**Table 1:** *Thermal conductivities for Q. a.*

The temperature $T_0$ on the inside of the refractory is $1370$°C, while the temperature on the outside of the steel plate is $37.8$°C. the heat loss through the furnace wall is expected to be 15800 W/m$^2$. Determine the thickness of refractory and insulation that results in the minimum total thickness of the wall. You may use the temperature dependent thermal conductivities given in Table 1. [14 marks]

b) Define the Grashof number, Gr, and each of its terms when considering natural convection from the outer surface of a horizontal pipe suspended in air. You must also state at what conditions the properties should be evaluated at. [6 marks]

**Question 3**

a) An engineer is designing a new system in which 230 kg/h of water is heated from 35°C to 93°C by oil initially at 175°C. The mass flow rate of oil is also 230 kg/h. Two double-pipe heat exchangers are available:

- Heat Exchanger 1: Overall heat transfer coefficient $(U_1)$ of 570 W/$(\text{m}^2.^\circ\text{C})$ with superficial area $(A_1)$ of 0.47 m$^2$, and;
- Heat Exchanger 2: $U_2$ = 370 W/$(\text{m}^2.^\circ\text{C})$ and $A_2$ = of 0.94 m$^2$.

Which exchanger should be used? Why? Given $C_{p,\text{oil}}$ = 2.10 kJ/$(\text{kg}.^\circ\text{C})$ and $C_{p,\text{water}}$ = 4.18 kJ/$(\text{kg}.^\circ\text{C})$. Assume that there are no heat losses. [10 marks]

b) A plate of stainless steel (18$\%$ Cr, 8$\%$ Ni) has a thickness of 3.0 cm and is initially uniform in temperature at 500°C. The plate is suddenly exposed to a convection environment on both sides at 40°C with h = 150 W/$(\text{m}^2.^\circ\text{C})$. Calculate the times for the centre and face temperatures to reach 120°C. Given that the thermal conductivity coefficient is 16.3 W/$(\text{m}.^\circ\text{C})$, the thermal diffusivity is 0.44$\times$10$^{-5}$ m$^2$/s, and the analytical solution for the 1-D transient heat conduction for plane wall is

$$\theta = \frac{T\,(x,t) - T_\infty}{T_i - T_\infty} = A_1 \exp\left(-\lambda_1^2 \tau\right) \cos \frac{\lambda_1 x}{L}$$

where $\tau$ is the Fourier number, and $\lambda_1$ and $A_1$ are coefficient constants of the transient 1-D heat conduction equation. [10 marks]

# PART B: Answer TWO Questions From THREE

## Question 4

In a plate heat-exchanger, water is heated by forcing it between alternating plates and heat is exchanged through the walls with a hot process stream. In order to design such an exchanger, we need to know what the relationship is between pressure drop, flow velocity, and volumetric flow-rate.

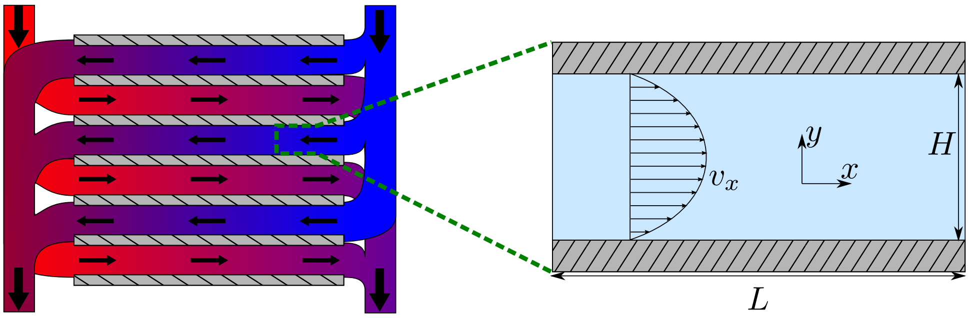



**Figure 3:** *A plate heat exchanger (left) and the simplification to steady state, pressure driven flow between two horizontal plates (right).*

You may neglect the effect of heat transfer on the flow. Water is incompressible and Newtonian to a good approximation. For simplicity, you can also assume that the flow is laminar.

a) Simplify the continuity equation for this system. What does it state about the flow velocity in the x-direction?

b) Simplify the $x$-component of the Cauchy momentum equation and derive the following balance expression for the flow velocity $v_x$ as a function of the pressure drop and position $y$:

$$\mu \frac{\partial^2 v_x}{\partial y^2} = \frac{\partial p}{\partial x}$$

c) Continuing from the result of the previous question, derive the following expression for the velocity $v_x$ as a function of $y$ using the no-slip boundary condition at the plate surfaces ($v_x = 0$ at $y = 0$ and $y = H$).

$$v_x = \frac{p_{out} - p_{in}}{2\,\mu\,L}(y^2 - H\,y)$$

d) Use an integration of the velocity over the plate height and width to prove the following expression for the volumetric flow of liquid through the gap as a function of pressure drop:

$$\dot{V}_x = \frac{Z\,H^3}{12\,\mu}\frac{\Delta P}{L}$$

**Question 5**

Consider a pot of boiling water placed on a radiant (halogen) cooking hob. As the water is boiling, the surface temperature of the pot will be approximately the boiling temperature. The pot is exposed to the atmosphere and the air/surroundings are at $20°$C.

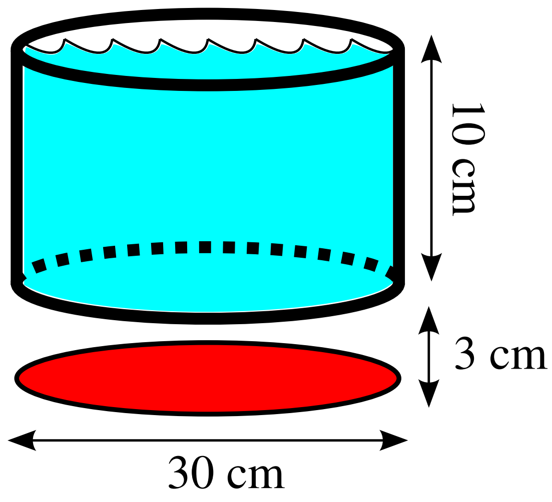



**Figure 4:** *The boiling pot problem.*

a) Calculate the natural convective heat loss from the sides of the pot given that air has a mean molar mass of $M_W \approx 29$ g/mol, a dynamic viscosity of $\mu \approx 1.8 \times 10^{-5}$ Pa s, a thermal conductivity of $k_{air} \approx 0.0257$ W m$^{-1}$ $K^{-1}$, and a Prandtl number of Pr $\approx 0.713$. [11 marks]

b) Assume that the total heat loss from the pan is $100$ W due to evaporation and radiant heat loss to surroundings. Calculate the radiant temperature of the hob/heat-source required to counteract the heat loss. You may assume the pan and heat-source are black-bodies for this calculation.

The view factor between two coaxial discs is

$$F_{1\to 2} = 0.5 \left(S - \left(S^2 - 4(r_j/r_i)^2\right)^{0.5}\right)$$

where $S = 1 + \left(1 + R_j^2\right)/R_i^2$, and the reduced radii are $R_i = r_i/L$ and $R_j = r_j/L$. Note $L$ is the gap between the discs, and $(r_i, r_j)$ are the radii of the two discs. [6 marks]

c) What fraction of the heat radiated from the heater hits the pot? [3 marks]

**Question 6**

Oil is used to lubricate two horizontal parallel plates by injecting it and allowing it to flow radially outwards from the point of injection (see Fig. 5). The fluid is flowing radially as there is a pressure difference of $P_1 - P_2$ between the inner and outer radii $r_1$ and $r_2$ respectively.

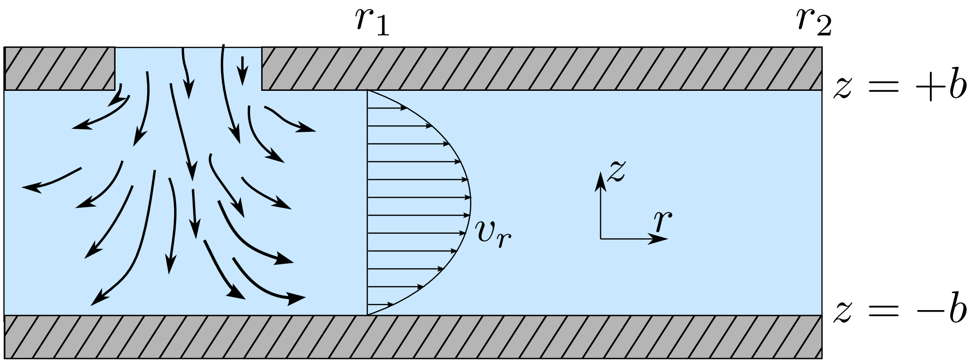



**Figure 5:** *Radial flow between two plates.*

a) Simplify the continuity equation to demonstrate that $r\, v_r$ is a function of $z$ only. [5 marks]

b) Demonstrate that the stress profile within the channel is a solution of the following equation: [10 marks]

$$\rho\, v_r \frac{\partial v_r}{\partial r} = \mu \left( 2\frac{\partial^2 v_r}{\partial r^2} + \frac{2}{r}\frac{\partial v_r}{\partial r} - \frac{2\, v_r}{r^2} + \frac{\partial^2 v_r}{\partial z^2} \right) - \frac{\partial\, p}{\partial r}$$

**Note** You must be careful during your derivation and make sure you expand each term of $\boldsymbol{\tau}$ before cancellation.

c) Using the creeping flow assumption, the following expression for the velocity profile was derived [5 marks]:

$$v_r = -r^{-1} \frac{\Delta P}{2\,\mu \ln(r_2/r_1)} \left(z^2 + C_1\, z + C_2\right)$$

Determine the integration constants $C_1$ and $C_2$, and give the final expression for the velocity profile:

## END OF PAPER

# DATASHEET

**General balance equations:**

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \boldsymbol{v} \qquad \text{(Mass/Continuity)} \quad (1)$$

$$\frac{\partial C_A}{\partial t} = -\nabla \cdot \boldsymbol{N}_A + \sigma_A \qquad \text{(Species)} \quad (2)$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla p + \rho \boldsymbol{g} \qquad \text{(Momentum)} \quad (3)$$

$$\rho C_p \frac{\partial T}{\partial t} = -\rho C_p \boldsymbol{v} \cdot \nabla T - \nabla \cdot \boldsymbol{q} - \boldsymbol{\tau} : \nabla \boldsymbol{v} - p \nabla \cdot \boldsymbol{v} + \sigma_{energy} \qquad \text{(Heat/Energy)} \quad (4)$$

In Cartesian coordinate systems, $\nabla$ can be treated as a vector of derivatives. In curvelinear coordinate systems, the directions $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$ depend on the position. For convenience in these systems, look-up tables are provided for common terms involving $\nabla$.

**Cartesian coordinates** (with index notation examples)
where $s$ is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor.

$$\nabla s = \nabla_i s = \left[\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \nabla_i \nabla_i s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \nabla_i v_i = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\nabla \cdot \boldsymbol{\tau} = \nabla_i \tau_{ij}$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\boldsymbol{v} \cdot \nabla \boldsymbol{v} = v_i \nabla_i v_j$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_x = v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_y = v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}$$

**Cylindrical coordinates**
where $s$ is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r}\frac{\partial}{\partial r}(r\, v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{1}{r}\tau_{\theta\theta} + \frac{\partial \tau_{rz}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r}\tau_{r\theta} + \frac{\partial \tau_{\theta z}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r\, v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

**Spherical coordinates**
where $s$ is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{1}{r\,\sin\theta}\frac{\partial s}{\partial \phi}\right]$$

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial s}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 s}{\partial \phi^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{\theta\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\phi}\right) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r\, v_\theta - v_\phi^2\cot\theta}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\phi = v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r\, v_\phi + v_\theta\, v_\phi\cot\theta}{r}$$

| Rectangular | | Cylindrical | | Spherical | |
|---|---|---|---|---|---|
| $q_x$ | $-k\frac{\partial T}{\partial x}$ | $q_r$ | $-k\frac{\partial T}{\partial r}$ | $q_r$ | $-k\frac{\partial T}{\partial r}$ |
| $q_y$ | $-k\frac{\partial T}{\partial y}$ | $q_\theta$ | $-k\frac{1}{r}\frac{\partial T}{\partial \theta}$ | $q_\theta$ | $-k\frac{1}{r}\frac{\partial T}{\partial \theta}$ |
| $q_z$ | $-k\frac{\partial T}{\partial z}$ | $q_z$ | $-k\frac{\partial T}{\partial z}$ | $q_\phi$ | $-k\frac{1}{r\,\sin\theta}\frac{\partial T}{\partial \phi}$ |
| $\tau_{xx}$ | $-2\,\mu\frac{\partial v_x}{\partial x} + \mu^B\,\nabla\cdot\boldsymbol{v}$ | $\tau_{rr}$ | $-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$ | $\tau_{rr}$ | $-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$ |
| $\tau_{yy}$ | $-2\,\mu\frac{\partial v_y}{\partial y} + \mu^B\,\nabla\cdot\boldsymbol{v}$ | $\tau_{\theta\theta}$ | $-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$ | $\tau_{\theta\theta}$ | $-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$ |
| $\tau_{zz}$ | $-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$ | $\tau_{zz}$ | $-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$ | $\tau_{\phi\phi}$ | $-2\,\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r + v_\theta\cot\theta}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$ |
| $\tau_{xy}$ | $-\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$ | $\tau_{r\theta}$ | $-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$ | $\tau_{r\theta}$ | $-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$ |
| $\tau_{yz}$ | $-\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$ | $\tau_{\theta z}$ | $-\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)$ | $\tau_{\theta\phi}$ | $-\mu\left(\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right)$ |
| $\tau_{xz}$ | $-\mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$ | $\tau_{zr}$ | $-\mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)$ | $\tau_{\phi r}$ | $-\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right)$ |

**Table 2:** *Fourier's law for the heat flux and Newton's law for the stress in several coordinate systems. Please remember that the stress is symmetric, so* $\tau_{ij} = \tau_{ji}$.

**Viscous models:**
Power-Law Fluid:

$$|\tau_{xy}| = k\left|\frac{\partial v_x}{\partial y}\right|^n$$

Bingham-Plastic Fluid:

$$\frac{\partial v_x}{\partial y} = \begin{cases} -\mu^{-1}\left(\tau_{xy} - \tau_0\right) & \text{if } \tau_{xy} > \tau_0 \\ 0 & \text{if } \tau_{xy} \leq \tau_0 \end{cases}$$

**Dimensionless Numbers**

$$\text{Re} = \frac{\rho\,\langle v\rangle\,D}{\mu} \qquad \text{Re}_H = \frac{\rho\,\langle v\rangle\,D_H}{\mu} \qquad \text{Re}_{MR} = -\frac{16\,L\,\rho\,\langle v\rangle^2}{R\,\Delta p} \tag{5}$$

The hydraulic diameter is defined as $D_H = 4\,A/P_w$.

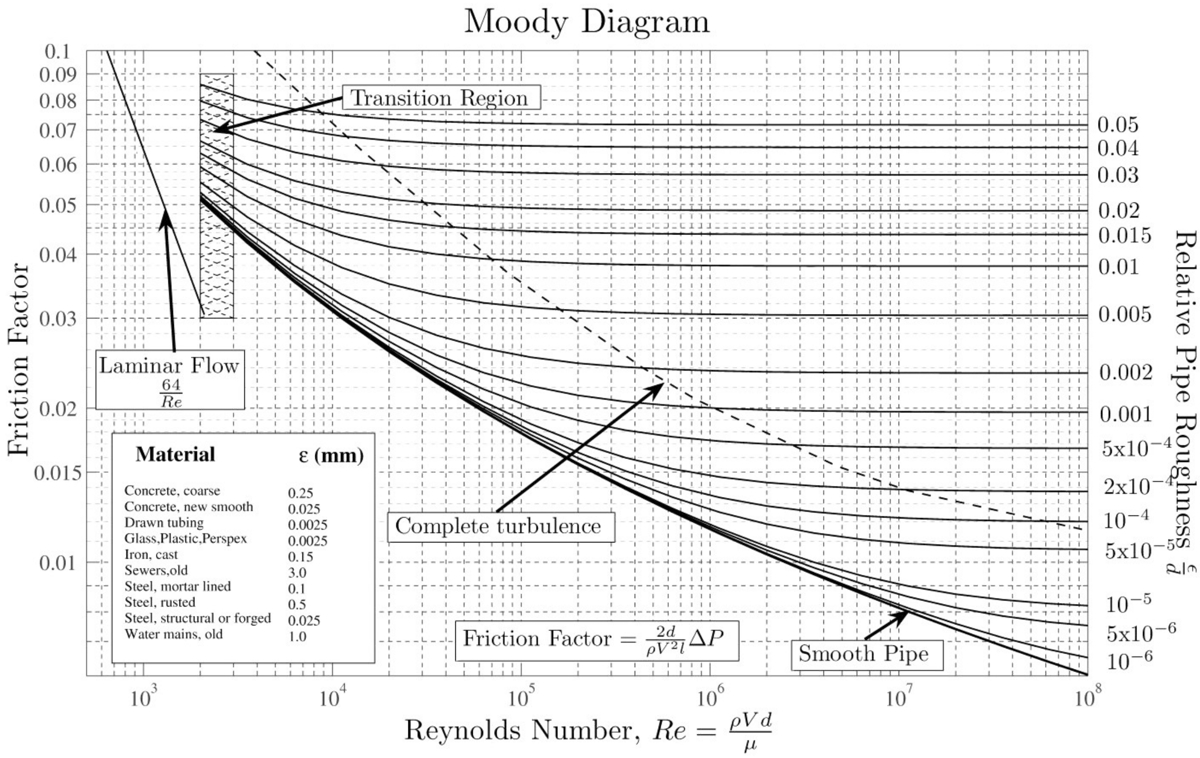
**Single phase pressure drop calculations in pipes:**
Darcy-Weisbach equation:

$$\frac{\Delta p}{L} = -\frac{C_f\,\rho\,\langle v\rangle^2}{R} \tag{6}$$

where $C_f = 16/Re$ for laminar Newtonian flow. For turbulent flow of Newtonian fluids in smooth pipes, we have the Blasius correlation:

$$C_f = 0.079\,\mathrm{Re}^{-1/4} \qquad \text{for } 2.5\times10^3 < \mathrm{Re} < 10^5 \text{ and smooth pipes.}$$

Otherwise, you may refer to the Moody diagram.




Laminar Power-Law fluid:

$$\dot{V} = \frac{n\,\pi\,R^3}{3\,n+1}\left(\frac{R}{2\,k}\right)^{\frac{1}{n}}\left(-\frac{\Delta p}{L}\right)^{\frac{1}{n}}$$

**Two-Phase Flow:**

Lockhart-Martinelli parameter:

$$X^2 = \frac{\Delta p_{liq.-only}}{\Delta p_{gas-only}}$$

Pressure drop calculation:

$$\Delta p_{two-phase} = \Phi^2_{liq.}\,\Delta p_{liq.-only} = \Phi^2_{gas}\,\Delta p_{gas-only}$$

Chisholm's relation:

$$\Phi^2_{gas} = 1 + c\,X + X^2$$

$$\Phi^2_{liq.} = 1 + \frac{c}{X} + \frac{1}{X^2} \qquad c = \begin{cases} 20 & \text{turbulent liquid \& turbulent gas} \\ 12 & \text{laminar liquid \& turbulent gas} \\ 10 & \text{turbulent liquid \& laminar gas} \\ 5 & \text{laminar liquid \& laminar gas} \end{cases}$$

Farooqi and Richardson expression for liquid hold-up in co-current flows of Newtonian fluids and air in horizontal pipes:

$$h = \begin{cases} 0.186 + 0.0191\,X & 1 < X < 5 \\ 0.143\,X^{0.42} & 5 < X < 50 \\ 1/\left(0.97 + 19/X\right) & 50 < X < 500 \end{cases}$$

**Heat Transfer:**
Stefan-Boltzmann constant $\sigma = 5.6703 \times 10^{-8}$ W/m$^2$ K$^4$

**Heat Transfer Dimensionless numbers:**

$$\text{Nu} = \frac{h\,L}{k} \qquad \text{Pr} = \frac{\mu\,C_p}{k} \qquad \text{Gr} = \frac{g\,\beta\,(T_w - T_\infty)\;L^3}{\nu^2}$$

**Resistances**

$$Q = U_T\,A_T\,\Delta T = R_T^{-1}\,\Delta T \qquad Q_{rad.} = \sigma\,\varepsilon\,A\left(T_\infty^4 - T_w^4\right) = h_{rad.}\,A\,(T_\infty - T_w)$$

| | Conduction Shell Resistances | | | Radiation |
|---|---|---|---|---|
| | Rect. | Cyl. | Sph. | |
| $R$ | $\dfrac{X}{k\,A}$ | $\dfrac{\ln\left(R_{outer}/R_{inner}\right)}{2\,\pi\,L\,k}$ | $\dfrac{R_{inner}^{-1} - R_{outer}^{-1}}{4\,\pi\,k}$ | $\left[A\,\varepsilon\,\sigma\,(T_\infty^2 + T_w^2)\,(T_\infty + T_w)\right]^{-1}$ |

**Natural Convection**

| Ra $=$ Gr Pr | $C$ | $m$ |
|---|---|---|
| $< 10^4$ | 1.36 | 1/5 |
| $10^4$–$10^9$ | 0.59 | 1/4 |
| $> 10^9$ | 0.13 | 1/3 |

**Table 3:** *Natural convection coefficients for isothermal vertical plates in the empirical relation Nu $\approx C$ (Gr Pr)$^m$.*

For isothermal vertical cylinders, the above expressions for isothermal vertical plates may be used but must be scaled by a factor, $F$:

$$F = \begin{cases} 1 & \text{for } (D/H) < 35\,\text{Gr}_H^{-1/4} \\ 1.3\left[H\,D^{-1}\,\text{Gr}_D^{-1}\right]^{1/4} + 1 & \text{for } (D/H) \geq 35\,\text{Gr}_H^{-1/4} \end{cases}$$

where $D$ is the diameter and $H$ is the height of the cylinder. The subscript on Gr indicates which length is to be used as the critical length to calculate the Grashof number.

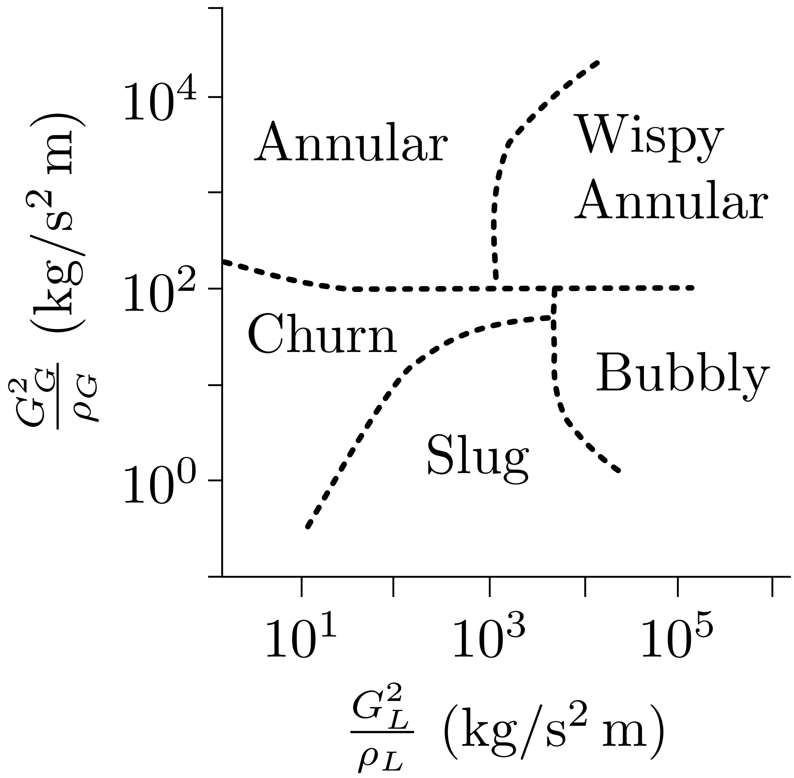



**Figure 6:** *Hewitt-Taylor flow pattern map for multiphase flows in vertical pipes.*

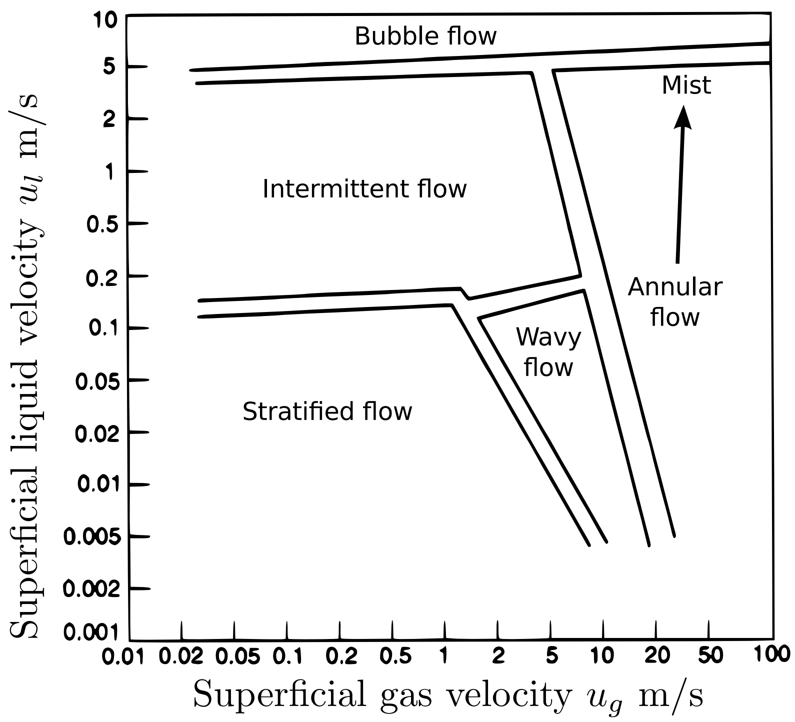



**Figure 7:** *Chhabra and Richardson flow pattern map for horizontal pipes.*

Churchill and Chu expression for natural convection from a horizontal pipe:

$$\mathrm{Nu}^{1/2} = 0.6 + 0.387 \left\{ \frac{\mathrm{Gr\,Pr}}{\left[1 + (0.559/\mathrm{Pr})^{9/16}\right]^{16/9}} \right\}^{1/6} \qquad \text{for } 10^{-5} < \mathrm{Gr\,Pr} < 10^{12}$$

**Forced Convection:**
Laminar flows:

$$\mathrm{Nu} \approx 0.332\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

Well-Developed turbulent flows in smooth pipes:

$$\mathrm{Nu} \approx \frac{(C_f/2)\mathrm{Re\,Pr}}{1.07 + 12.7(C_f/2)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

**Boiling:**
Forster-Zuber pool-boiling coefficient:

$$h_{nb} = 0.00122 \frac{k_L^{0.79}\,C_{p,L}^{0.45}\,\rho_L^{0.49}}{\gamma^{0.5}\,\mu_L^{0.29}\,h_{fg}^{0.24}\,\rho_G^{0.24}} \,(T_w - T_{sat})^{0.24}\,(p_w - p_{sat})^{0.75}$$

Mostinski correlations:

$$h_{nb} = 0.104\,p_c^{0.69}\,q^{0.7} \left[1.8\left(\frac{p}{p_c}\right)^{0.17} + 4\left(\frac{p}{p_c}\right)^{1.2} + 10\left(\frac{p}{p_c}\right)^{10}\right]$$

$$q_c = 3.67 \times 10^4\,p_c \left(\frac{p}{p_c}\right)^{0.35} \left[1 - \frac{p}{p_c}\right]^{0.9}$$

(**Note**: for the Mostinski correlations, the pressures are in units of bar)
**Condensing:**
Horizontal pipes

$$h = 0.72 \left(\frac{k^3\,\rho^2\,g_x\,E_{latent}}{D\,\mu\,(T_w - T_\infty)}\right)^{1/4}$$

**NTU method:**

$$\mathrm{NTU} = \frac{U\,A}{C_{min}} = \frac{t_{C1} - t_{C2}}{\Delta t_{ln}} \qquad\qquad R = \frac{C_{min}}{C_{max}}$$

For counter-current flow:

$$E = \frac{1 - \exp\left[-\mathrm{NTU}(1 - R)\right]}{1 - R\exp\left[-\mathrm{NTU}(1 - R)\right]}$$

For co-current flow:

$$E = \frac{1 - \exp\left[-\mathsf{NTU}(1-R)\right]}{1+R}$$

**Lumped capacitance method:**

$$\mathsf{Bi} = \frac{h\,L_c}{k}$$

$$L_c = V/A \qquad \text{for } \mathsf{Bi} < 0.1$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{h\,A_s}{\rho\,V\,C_p}t\right]$$

**Diffusion Dimensionless Numbers**

$$\mathsf{Sc} = \frac{\mu}{\rho\,D_{AB}} \qquad \mathsf{Le} = \frac{k}{\rho\,C_p\,D_{AB}}$$

**Diffusion**

General expression for the flux:

$$\boldsymbol{N}_A = \boldsymbol{J}_A + x_A \sum_B \boldsymbol{N}_B$$

Fick's law:

$$\boldsymbol{J}_A = -D_{AB}\,\nabla C_A$$

Stefan's law:

$$N_{s,r} = -D\frac{c}{1-x}\frac{\partial x}{\partial r}$$

**TABLE 4–2**

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = $hL/k$ for a plane wall of thickness $2L$, and Bi = $hr_o/k$ for a cylinder or sphere of radius $r_o$)

| | Plane Wall | | Cylinder | | Sphere | |
|---|---|---|---|---|---|---|
| **Bi** | $\lambda_1$ | $A_1$ | $\lambda_1$ | $A_1$ | $\lambda_1$ | $A_1$ |
| 0.01 | 0.0998 | 1.0017 | 0.1412 | 1.0025 | 0.1730 | 1.0030 |
| 0.02 | 0.1410 | 1.0033 | 0.1995 | 1.0050 | 0.2445 | 1.0060 |
| 0.04 | 0.1987 | 1.0066 | 0.2814 | 1.0099 | 0.3450 | 1.0120 |
| 0.06 | 0.2425 | 1.0098 | 0.3438 | 1.0148 | 0.4217 | 1.0179 |
| 0.08 | 0.2791 | 1.0130 | 0.3960 | 1.0197 | 0.4860 | 1.0239 |
| 0.1 | 0.3111 | 1.0161 | 0.4417 | 1.0246 | 0.5423 | 1.0298 |
| 0.2 | 0.4328 | 1.0311 | 0.6170 | 1.0483 | 0.7593 | 1.0592 |
| 0.3 | 0.5218 | 1.0450 | 0.7465 | 1.0712 | 0.9208 | 1.0880 |
| 0.4 | 0.5932 | 1.0580 | 0.8516 | 1.0931 | 1.0528 | 1.1164 |
| 0.5 | 0.6533 | 1.0701 | 0.9408 | 1.1143 | 1.1656 | 1.1441 |
| 0.6 | 0.7051 | 1.0814 | 1.0184 | 1.1345 | 1.2644 | 1.1713 |
| 0.7 | 0.7506 | 1.0918 | 1.0873 | 1.1539 | 1.3525 | 1.1978 |
| 0.8 | 0.7910 | 1.1016 | 1.1490 | 1.1724 | 1.4320 | 1.2236 |
| 0.9 | 0.8274 | 1.1107 | 1.2048 | 1.1902 | 1.5044 | 1.2488 |
| 1.0 | 0.8603 | 1.1191 | 1.2558 | 1.2071 | 1.5708 | 1.2732 |
| 2.0 | 1.0769 | 1.1785 | 1.5995 | 1.3384 | 2.0288 | 1.4793 |
| 3.0 | 1.1925 | 1.2102 | 1.7887 | 1.4191 | 2.2889 | 1.6227 |
| 4.0 | 1.2646 | 1.2287 | 1.9081 | 1.4698 | 2.4556 | 1.7202 |
| 5.0 | 1.3138 | 1.2403 | 1.9898 | 1.5029 | 2.5704 | 1.7870 |
| 6.0 | 1.3496 | 1.2479 | 2.0490 | 1.5253 | 2.6537 | 1.8338 |
| 7.0 | 1.3766 | 1.2532 | 2.0937 | 1.5411 | 2.7165 | 1.8673 |
| 8.0 | 1.3978 | 1.2570 | 2.1286 | 1.5526 | 2.7654 | 1.8920 |
| 9.0 | 1.4149 | 1.2598 | 2.1566 | 1.5611 | 2.8044 | 1.9106 |
| 10.0 | 1.4289 | 1.2620 | 2.1795 | 1.5677 | 2.8363 | 1.9249 |
| 20.0 | 1.4961 | 1.2699 | 2.2880 | 1.5919 | 2.9857 | 1.9781 |
| 30.0 | 1.5202 | 1.2717 | 2.3261 | 1.5973 | 3.0372 | 1.9898 |
| 40.0 | 1.5325 | 1.2723 | 2.3455 | 1.5993 | 3.0632 | 1.9942 |
| 50.0 | 1.5400 | 1.2727 | 2.3572 | 1.6002 | 3.0788 | 1.9962 |
| 100.0 | 1.5552 | 1.2731 | 2.3809 | 1.6015 | 3.1102 | 1.9990 |
| $\infty$ | 1.5708 | 1.2732 | 2.4048 | 1.6021 | 3.1416 | 2.0000 |

**TABLE 4–3**

The zeroth- and first-order Bessel functions of the first kind

| $\eta$ | $J_0(\eta)$ | $J_1(\eta)$ |
|---|---|---|
| 0.0 | 1.0000 | 0.0000 |
| 0.1 | 0.9975 | 0.0499 |
| 0.2 | 0.9900 | 0.0995 |
| 0.3 | 0.9776 | 0.1483 |
| 0.4 | 0.9604 | 0.1960 |
| 0.5 | 0.9385 | 0.2423 |
| 0.6 | 0.9120 | 0.2867 |
| 0.7 | 0.8812 | 0.3290 |
| 0.8 | 0.8463 | 0.3688 |
| 0.9 | 0.8075 | 0.4059 |
| 1.0 | 0.7652 | 0.4400 |
| 1.1 | 0.7196 | 0.4709 |
| 1.2 | 0.6711 | 0.4983 |
| 1.3 | 0.6201 | 0.5220 |
| 1.4 | 0.5669 | 0.5419 |
| 1.5 | 0.5118 | 0.5579 |
| 1.6 | 0.4554 | 0.5699 |
| 1.7 | 0.3980 | 0.5778 |
| 1.8 | 0.3400 | 0.5815 |
| 1.9 | 0.2818 | 0.5812 |
| 2.0 | 0.2239 | 0.5767 |
| 2.1 | 0.1666 | 0.5683 |
| 2.2 | 0.1104 | 0.5560 |
| 2.3 | 0.0555 | 0.5399 |
| 2.4 | 0.0025 | 0.5202 |
| 2.6 | −0.0968 | −0.4708 |
| 2.8 | −0.1850 | −0.4097 |
| 3.0 | −0.2601 | −0.3391 |
| 3.2 | −0.3202 | −0.2613 |