UNIVERSITY OF ABERDEEN SESSION 2019–20

EM40JN

Degree Examination in EM40JN Heat and Momentum Transfer

5th December 2019 9 am – 11 am

PLEASE NOTE THE FOLLOWING

(i) You **must not** have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material **must not** be amended, annotated or modified in any way.

(ii) You **must not** have in your possession any material that could be determined as giving you an advantage in the examination.

(iii) You **must not** attempt to communicate with any candidate during the exam, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

(iv) You **must not** take to your examination desk any electronic devices such as mobile phones or other "smart" devices. The only exception to this rule is an approved calculator.

Failure to comply with the above will be regarded as cheating and may lead to disciplinary action as indicated in the Academic Quality Handbook.

Notes:

(i) Candidates ARE ONLY permitted to use APPROVED calculators.

(ii) Candidates ARE NOT permitted to use the Engineering Mathematics Handbook.

(iii) Candidates ARE NOT permitted to use GREEN or RED pen in their exam booklet.

(iv) Data sheets are attached to the paper.

Candidates must attempt *ALL* questions. Question 1 and 2 are worth 33 marks each. Question 3 is worth 34 marks.

Question 1

a) A twin-tube counterflow heat exchanger (Fig. 1) operates with air flow rates of 0.003 kg s^{-1} for both hot and cold streams. The cold stream enters the heat exchanger at 280 K and must be heated to 340 K using hot air that enters the domain at 360 K. The average pressure of the airstreams is 1 atm and the maximum allowable pressure drop for the cold airstream is of 10 kPa. The tube walls may be assumed to act as fins, each with an efficiency of 100%. Determine:

(i) the effectiveness and Number of Transfer Units (NTU); **[5 marks]**

(ii) the tube diameter D and L that satisfy the prescribed heat transfer and pressure drop requirements. In case you have not managed to solve (a), you may assume $\epsilon = 0.7$; **[15 marks]**

(iii) the heat transfer coefficient. In case you have not managed to solve (a) and (b), you should assume $D = 9 \times 10^{-3}$ m and $L = 2.6$ m. **[5 marks]**

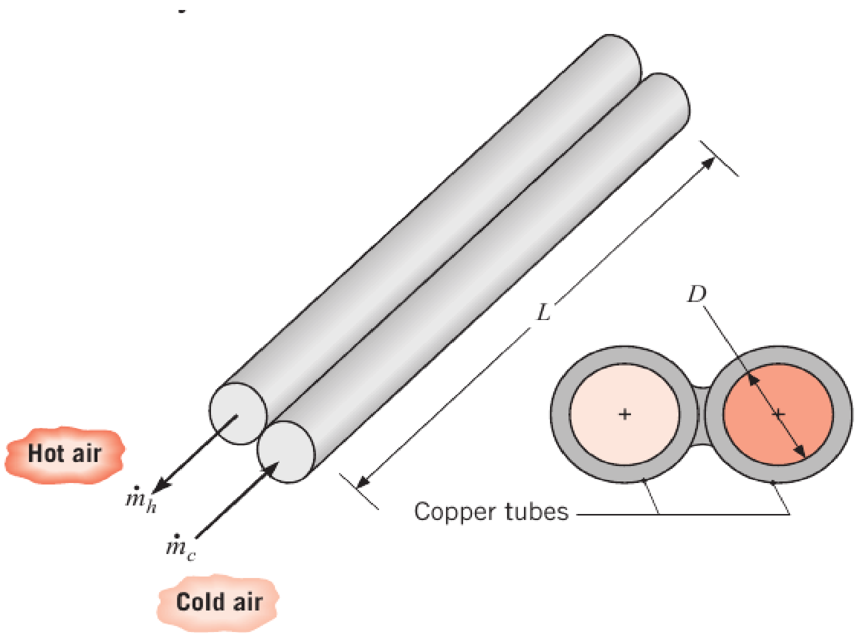


Figure 1: *Twin-tube counterflow heat exchanger.*

Given:

- Thermophysical properties of air: ρ = 1.128 kg m^{-3}; C_p = 1.007 kJ (kg K)$^{-1}$; $\mu = 18.93 \times 10^{-6}$ $m^2 s^{-1}$; κ = 0.0270 W (m K)$^{-1}$; Pr = 0.7056;
- Tube Reynolds number: $Re_D = 4\dot{m}(\pi D \mu)^{-1}$, where $\dot{m}$ is the mass flow rate;
- Colburn correlation for turbulent and fully developed flow: $Nu_D = hD\kappa^{-1} = 0.023 Re_D^{0.8} Pr^{1/3}$;

- Pressure drop for fully developed flow: $\Delta P = f \rho u_m^2 L (2D)^{-1}$, where $u_m \left(= \frac{4\dot{m}}{\rho \pi \frac{D^2}{4}} \right)$ is the mean flow velocity, and $f \left(= [(0.79 \ln (Re_D) - 1.64)]^{-2}, \text{ for } 3000 \leq Re_D \leq 5 \times 10^6 \right)$ is the Moody friction factor;
- The effectiveness relation for this heat exchanger is $NTU = \epsilon/(1-\epsilon)$.

b) Steel balls of 12 mm in diameter are annealed by heating to 1150 K and then slowly cooled to 400 K in an air environment $\left(T_\infty = 325 \text{ K and h} = 20 \text{ W m}^{-2}\text{K}^{-1}\right)$. Estimate the time required for the cooling process. Given thermophysical properties of steel: $\kappa = 40$ W(m K)$^{-1}$, $\rho = 7800$ kg m^{-3} and $C_p = 600$ J(kg K)$^{-1}$.

[8 marks]

[Question total: 33 marks]

Question 2

Oil is flowing up an annulus (see Fig. 2) around the central "production" pipe. The relationship between pressure drop and volumetric flow-rate is required for design calculations.

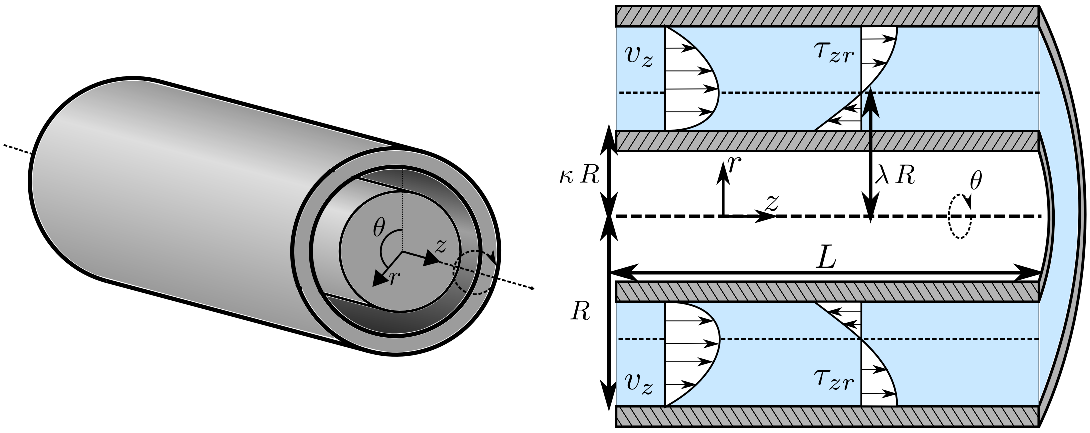


Figure 2: *An annular flow geometry.*

a) Demonstrate that the continuity equation simplifies to the following expression.

$$\frac{\partial v_z}{\partial z} = 0$$

State your assumptions and interpretation of this result. **[6 marks]**

b) Simplify the Cauchy momentum balance equation to yield the following result.

$$0 = -\frac{1}{r}\frac{\partial}{\partial r}(r\,\tau_{rz}) - \frac{\partial p}{\partial z} + \rho\, g_z$$

[6 marks]

c) Integrate the equation to express it in terms of the pressure drop over the length of the annulus. Give reasons why the stress term τ_{rz} is independent of z.

$$\frac{\Delta p}{L} = -\frac{1}{r}\frac{\partial}{\partial r}(r\,\tau_{rz}) + \rho\, g_z$$

[6 marks]

d) Solve the above equation for the stress profile in an annulus using the assumed boundary condition that the stress is zero at a critical radius $r = \lambda R$. Prove that it is the following expression:

$$\tau_{rz} = \frac{1}{2}\left(\rho\, g_z - \frac{\Delta p}{L}\right)\left(r - \frac{\lambda^2 R_0^2}{r}\right)$$

Note: The critical radius λR is the location of the maximum velocity, and will be determined once the viscous model is inserted. **[6 marks]**

e) Derive the velocity profile by assuming the fluid is Newtonian. Try to rearrange the result of the integration into the following convenient form:

$$v_z = -\frac{R^2}{4\mu}\left(\rho\, g_z - \frac{\Delta p}{L}\right)\left(\frac{r^2}{R^2} - 2\lambda^2 \ln\left(\frac{r}{R}\right) + C\right)$$

[7 marks]

f) Using the no slip boundary condition at $r = R$ and $r = \kappa R$, solve for the unknown constants C and λ in the above equation and generate the final expression. **[2 marks]**

[Question total: 33 marks]

Question 3

a) Describe the physical interpretation of the Grashof number for natural convection. Describe each of its terms and write down an equation for the temperature at which temperature-dependent terms in Gr should be evaluated. **[9 marks]**

b) A black-body car is left in direct sunlight at midday which (at the lattitude of the UK) can be approximated as a constant heat flux $q_{sun} = 1000$ W m^{-2}. The car's surface temperature reaches steady state with its surroundings and is approximately constant. The car has a surface area of 26 m^2 but only 8 m^2 are exposed to sunlight.

   i) Assuming that the ambient temperature is 15°C and that radiation is the only heat transfer mechanism, calculate the surface temperature of the car. Is the estimate realistic? **[7 marks]**

ii) Using the previous estimate for the surface temperature, estimate the heat flux due to natural convection and comment on its magnitude. You may approximate the sides of the car as a vertical wall 12 m wide and 1.5 m high. You may assume the following properties of air at these conditions. State why natural convection from the top of the car is insignificant when compared to the sides. **[15 marks]**

ρ (kg m^{-3})	k (W m^{-1} K^{-1})	μ (kg m^{-1} s^{-1})	C_p (kJ kg^{-1} K^{-1})	Avg. Mol. Weight (g mol^{-1})
1.225	0.026	1.827×10^{-5}	1.007	29

iii) Discuss how you might improve the accuracy of the calculations, and what the effect of setting the car in motion will be. **[3 marks]**

[Question total: 34 marks]

END OF PAPER

DATASHEET

General balance equations:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho\, \boldsymbol{v} \qquad \text{(Mass/Continuity)} \quad (1)$$

$$\frac{\partial C_A}{\partial t} = -\nabla \cdot \boldsymbol{N}_A + \sigma_A \qquad \text{(Species)} \quad (2)$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho\, \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla p + \rho\, \boldsymbol{g} \qquad \text{(Momentum)} \quad (3)$$

$$\rho\, C_p \frac{\partial T}{\partial t} = -\rho\, C_p\, \boldsymbol{v} \cdot \nabla T - \nabla \cdot \boldsymbol{q} - \boldsymbol{\tau} : \nabla \boldsymbol{v} - p \nabla \cdot \boldsymbol{v} + \sigma_{energy} \qquad \text{(Heat/Energy)} \quad (4)$$

In Cartesian coordinate systems, ∇ can be treated as a vector of derivatives. In curvelinear coordinate systems, the directions $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\phi}$ depend on the position. For convenience in these systems, look-up tables are provided for common terms involving ∇.

Cartesian coordinates (with index notation examples)
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor.

$$\nabla s = \nabla_i s = \left[\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \nabla_i \nabla_i s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \nabla_i v_i = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\nabla \cdot \boldsymbol{\tau} = \nabla_i \tau_{ij}$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\boldsymbol{v} \cdot \nabla \boldsymbol{v} = v_i \nabla_i v_j$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_x = v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_y = v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}$$

Cylindrical coordinates

where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r}\frac{\partial}{\partial r}(r\, v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{1}{r}\tau_{\theta\theta} + \frac{\partial \tau_{rz}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r}\tau_{r\theta} + \frac{\partial \tau_{\theta z}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r\, v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

Spherical coordinates

where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial s}{\partial \phi}\right]$$

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial s}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 s}{\partial \phi^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{\theta\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\phi}\right) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r\, v_\theta - v_\phi^2\cot\theta}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\phi = v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r\, v_\phi + v_\theta\, v_\phi\cot\theta}{r}$$

Rectangular		Cylindrical		Spherical	
q_x	$-k\frac{\partial T}{\partial x}$	q_r	$-k\frac{\partial T}{\partial r}$	q_r	$-k\frac{\partial T}{\partial r}$
q_y	$-k\frac{\partial T}{\partial y}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$
q_z	$-k\frac{\partial T}{\partial z}$	q_z	$-k\frac{\partial T}{\partial z}$	q_ϕ	$-k\frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}$
τ_{xx}	$-2\,\mu\frac{\partial v_x}{\partial x} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{yy}	$-2\,\mu\frac{\partial v_y}{\partial y} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\phi\phi}$	$-2\,\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r+v_\theta\cot\theta}{r}\right) +\mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{xy}	$-\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$
τ_{yz}	$-\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$	$\tau_{\theta z}$	$-\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)$	$\tau_{\theta\phi}$	$-\mu\left(\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right)$
τ_{xz}	$-\mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$	τ_{zr}	$-\mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)$	$\tau_{\phi r}$	$-\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right)$

Table 1: *Fourier's law for the heat flux and Newton's law for the stress in several coordinate systems. Please remember that the stress is symmetric, so* $\tau_{ij} = \tau_{ji}$.

Viscous models:
Power-Law Fluid:

$$|\tau_{xy}| = k\left|\frac{\partial v_x}{\partial y}\right|^n \tag{5}$$

Bingham-Plastic Fluid:

$$\frac{\partial v_x}{\partial y} = \begin{cases} -\mu^{-1}\left(\tau_{xy} - \tau_0\right) & \text{if } \tau_{xy} > \tau_0 \\ 0 & \text{if } \tau_{xy} \leq \tau_0 \end{cases}$$

Dimensionless Numbers

$$\mathrm{Re} = \frac{\rho\,\langle v\rangle\,D}{\mu} \qquad \mathrm{Re}_H = \frac{\rho\,\langle v\rangle\,D_H}{\mu} \qquad \mathrm{Re}_{MR} = -\frac{16\,L\,\rho\,\langle v\rangle^2}{R\,\Delta p} \tag{6}$$

The hydraulic diameter is defined as $D_H = 4\,A/P_w$.

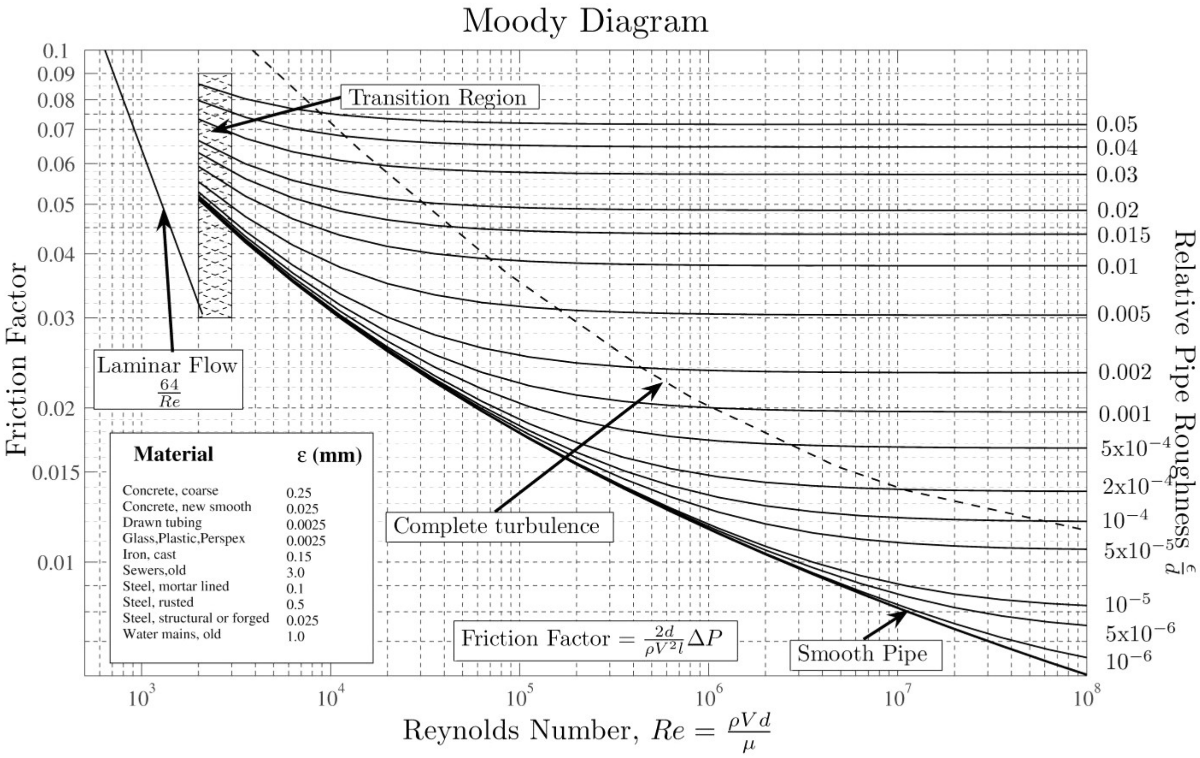
Single phase pressure drop calculations in pipes:
Darcy-Weisbach equation:

$$\frac{\Delta p}{L} = -\frac{C_f\,\rho\,\langle v\rangle^2}{R} \tag{7}$$

where $C_f = 16/Re$ for laminar Newtonian flow. For turbulent flow of Newtonian fluids in smooth pipes, we have the Blasius correlation:

$$C_f = 0.079\,\mathrm{Re}^{-1/4} \qquad \text{for } 2.5 \times 10^3 < \mathrm{Re} < 10^5 \text{ and smooth pipes.}$$

Otherwise, you may refer to the Moody diagram.



Laminar Power-Law fluid:

$$\dot{V} = \frac{n\,\pi\,R^3}{3\,n+1}\left(\frac{R}{2\,k}\right)^{\frac{1}{n}}\left(-\frac{\Delta p}{L}\right)^{\frac{1}{n}}$$

Two-Phase Flow:
Lockhart-Martinelli parameter:

$$X^2 = \frac{\Delta p_{liq.-only}}{\Delta p_{gas-only}}$$

Pressure drop calculation:

$$\Delta p_{two-phase} = \Phi^2_{liq.}\,\Delta p_{liq.-only} = \Phi^2_{gas}\,\Delta p_{gas-only}$$

Chisholm's relation:

$$\Phi^2_{gas} = 1 + c\,X + X^2$$

$$\Phi^2_{liq.} = 1 + \frac{c}{X} + \frac{1}{X^2} \qquad c = \begin{cases} 20 & \text{turbulent liquid \& turbulent gas} \\ 12 & \text{laminar liquid \& turbulent gas} \\ 10 & \text{turbulent liquid \& laminar gas} \\ 5 & \text{laminar liquid \& laminar gas} \end{cases}$$

Farooqi and Richardson expression for liquid hold-up in co-current flows of Newtonian fluids and air in horizontal pipes:

$$h = \begin{cases} 0.186 + 0.0191\,X & 1 < X < 5 \\ 0.143\,X^{0.42} & 5 < X < 50 \\ 1/\,(0.97 + 19/X) & 50 < X < 500 \end{cases}$$

Heat Transfer Dimensionless numbers:

$$\mathrm{Nu} = \frac{h\,L}{k} \qquad \mathrm{Pr} = \frac{\mu\,C_p}{k} \qquad \mathrm{Gr} = \frac{g\,\beta\,(T_w - T_\infty)\;L^3}{\nu^2}$$

where $\beta = V^{-1}(\partial V/\partial T)$.
Heat transfer: Resistances

$$Q = U_T\,A_T\,\Delta T = R_T^{-1}\,\Delta T$$

	Conduction Shell Resistances			Radiation
	Rect.	Cyl.	Sph.	
R	$\dfrac{X}{k\,A}$	$\dfrac{\ln(R_{outer}/R_{inner})}{2\,\pi\,L\,k}$	$\dfrac{R_{inner}^{-1} - R_{outer}^{-1}}{4\,\pi\,k}$	$\left[A\,\varepsilon\,\sigma\left(T_j^2 + T_i^2\right)\left(T_j + T_i\right)\right]^{-1}$

Radiation Heat Transfer:
Stefan-Boltzmann constant $\sigma = 5.6703 \times 10^{-8}$ W m^{-2} K^{-4}.
Summation relationship, $\sum_j F_{i\to j} = 1$, and reciprocity relationship, $F_{i\to j}\,A_i = F_{j\to i}\,A_j$.
Radiation shielding factor $1/(N+1)$.

$$Q_{rad.,i\to j} = \sigma\,\varepsilon\,F_{i\to j}\,A_i\left(T_j^4 - T_i^4\right) = h_{rad.}\,A\,(T_\infty - T_w)$$

Natural Convection

Ra = Gr Pr	C	m
$< 10^4$	1.36	1/5
10^4–10^9	0.59	1/4
$> 10^9$	0.13	1/3

Table 2: *Natural convection coefficients for isothermal vertical plates in the empirical relation* $Nu \approx C\,(Gr\,Pr)^m$.

For isothermal vertical cylinders, the above expressions for isothermal vertical plates may be used but must be scaled by a factor, F (i.e., $\mathrm{Nu}_{v.cyl.} = F\,\mathrm{Nu}_{v.plate}$):

$$F = \begin{cases} 1 & \text{for } (D/H) \geq 35\,\mathrm{Gr}_H^{-1/4} \\ 1.3\left[H\,D^{-1}\,\mathrm{Gr}_D^{-1}\right]^{1/4} + 1 & \text{for } (D/H) < 35\,\mathrm{Gr}_H^{-1/4} \end{cases}$$

where D is the diameter and H is the height of the cylinder. The subscript on Gr indicates which length is to be used as the critical length to calculate the Grashof number.

Churchill and Chu expression for natural convection from a horizontal pipe:

$$\mathrm{Nu}^{1/2} = 0.6 + 0.387\left\{\frac{\mathrm{Gr}\,\mathrm{Pr}}{\left[1 + (0.559/\mathrm{Pr})^{9/16}\right]^{16/9}}\right\}^{1/6} \qquad \text{for } 10^{-5} < \mathrm{Gr}\,\mathrm{Pr} < 10^{12}$$

Forced Convection:
Laminar flows:

$$\mathrm{Nu} \approx 0.332\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

Well-Developed turbulent flows in smooth pipes:

$$\mathrm{Nu} \approx \frac{(C_f/2)\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(C_f/2)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Boiling:
Forster-Zuber pool-boiling coefficient:

$$h_{nb} = 0.00122\frac{k_L^{0.79}\,C_{p,L}^{0.45}\,\rho_L^{0.49}}{\gamma^{0.5}\,\mu_L^{0.29}\,h_{fg}^{0.24}\,\rho_G^{0.24}}\,(T_w - T_{sat})^{0.24}\,(p_w - p_{sat})^{0.75}$$

Mostinski correlations:

$$h_{nb} = 0.104\,p_c^{0.69}\,q^{0.7}\left[1.8\left(\frac{p}{p_c}\right)^{0.17} + 4\left(\frac{p}{p_c}\right)^{1.2} + 10\left(\frac{p}{p_c}\right)^{10}\right]$$

$$q_c = 3.67 \times 10^4\,p_c\left(\frac{p}{p_c}\right)^{0.35}\left[1 - \frac{p}{p_c}\right]^{0.9}$$

(**Note**: for the Mostinski correlations, the pressures are in units of bar)
Condensing:
Horizontal pipes

$$h = 0.72\left(\frac{k^3\,\rho^2\,g_x\,E_{latent}}{D\,\mu\,(T_w - T_\infty)}\right)^{1/4}$$

Lumped capacitance method:

$$\text{Bi} = \frac{h\,L_c}{\kappa}$$
$$L_c = V/A \qquad\qquad \text{for Bi} < 0.1$$

$$\frac{T(t) - T_\infty}{T_0 - T_\infty} = e^{-bt} \qquad\qquad b = \frac{hA_s}{\rho V C_p}$$

1-D Transient Heat Conduction:

$$Fo = \frac{\alpha \Delta t}{(\Delta x)^2} = \tau, \qquad \alpha = \kappa\left(\rho C_p\right)^{-1}$$

$$\theta_{\text{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\frac{\lambda_1 x}{L}\right), \qquad \theta_{\text{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \mathbf{J_0}\left(\frac{\lambda_1 r}{r_0}\right)$$

$$\theta_{\text{sph}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin\left(\frac{\lambda_1 r}{r_0}\right)}{\frac{\lambda_1 r}{r_0}}$$

$$\theta_{0,\text{wall}} = \theta_{0,\text{cyl}} = \theta_{0,\text{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\max}}\right)_{\text{wall}} = 1 - \theta_{0,\text{wall}} \frac{\sin \lambda_1}{\lambda_1}, \quad \left(\frac{\mathcal{Q}}{\mathcal{Q}_{\max}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\text{cyl}} \frac{\mathbf{J_1}\left(\boldsymbol{\lambda_1}\right)}{\lambda_1}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\max}}\right)_{\text{sph}} = 1 - 3\theta_{0,\text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

Finite-Difference Method:

$$\frac{\partial}{\partial t}(\rho\phi) + \frac{\partial}{\partial x}(\rho \boldsymbol{v} \phi) = \nabla \cdot (\Gamma \nabla \phi) + \mathcal{S} \qquad \text{(1D transport equation)}$$

$$\left(\frac{d\phi}{dx}\right)_i = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x} \quad \text{and} \quad \left(\frac{d^2\phi}{dx^2}\right)_i = \frac{\phi_{i-1} + \phi_{i+1} - 2\phi_i}{(\Delta x)^2}$$

$$T_i^{j+1} = (1 - 2\tau)\, T_i^j + \tau\left(T_{i+1}^j + T_{i-1}^j\right) + \frac{\tau\,(\Delta x)^2}{\kappa}\mathcal{S}_i^j$$

Overall Heat Transfer Coefficient:

$$\dot{\mathcal{Q}} = \frac{\Delta T}{\mathcal{R}} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

$$\mathcal{R} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln D_o/D_i}{2\pi\kappa L} + \frac{1}{h_o A_o}$$

Fouling Factor:

$$\mathcal{R} = \frac{1}{h_i A_i} + \frac{R_{\text{f,i}}}{A_i} + R_{\text{wall}} + \frac{R_{\text{f,o}}}{A_o} + \frac{1}{h_o A_o}$$

LMTD Method:

$$\dot{\mathcal{Q}} = UA_s\Delta T_{\text{lm}} \quad \text{with} \quad \Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln\frac{\Delta T_2}{\Delta T_1}} = \frac{\Delta T_1 - \Delta T_2}{\ln\frac{\Delta T_1}{\Delta T_2}}$$

$$\text{Parallel flows:} \begin{cases} \Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,in}} \\ \Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,out}} \end{cases}$$

$$\text{Counter flows:} \begin{cases} \Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,out}} \\ \Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,in}} \end{cases}$$

ϵ-NTU Method:

$$\epsilon = \frac{\dot{\mathcal{Q}}}{\dot{\mathcal{Q}}_{\text{max}}}, \quad \text{with } \dot{\mathcal{Q}}_{\text{max}} = \mathcal{C}_{\text{min}}\left(T_{\text{hot,in}} - T_{\text{cold,in}}\right) \quad \text{and} \quad \mathcal{C}_{\text{min}} = Min\left\{\dot{m}_{\text{hot}} C_{p,\text{hot}}, \dot{m}_{\text{cold}} C_{p,\text{cold}}\right\}$$

$$\text{NTU} = \frac{UA_s}{C_{\text{min}}}$$

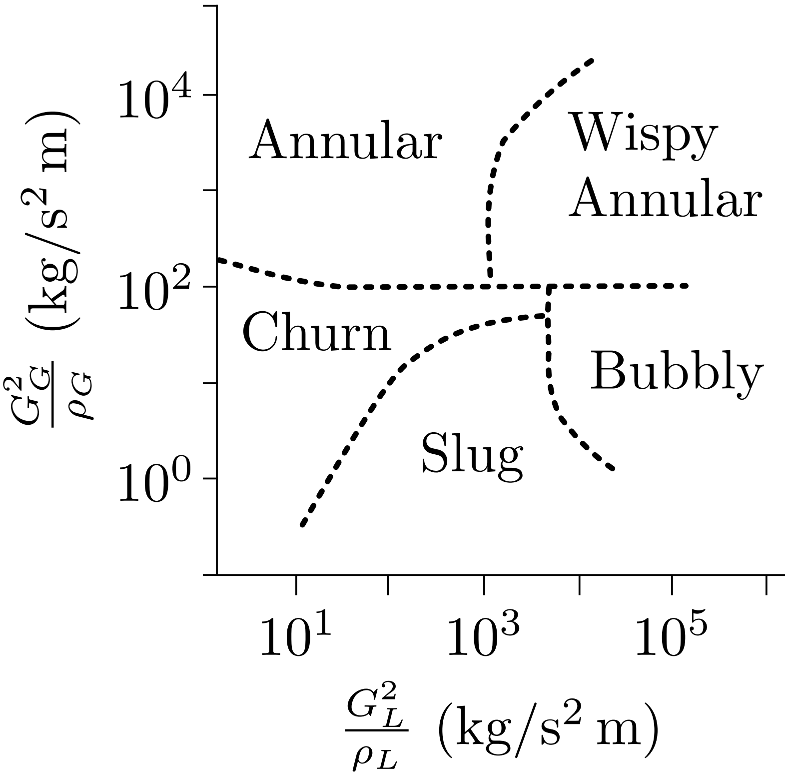


Figure 3: *Hewitt-Taylor flow pattern map for multiphase flows in vertical pipes.*

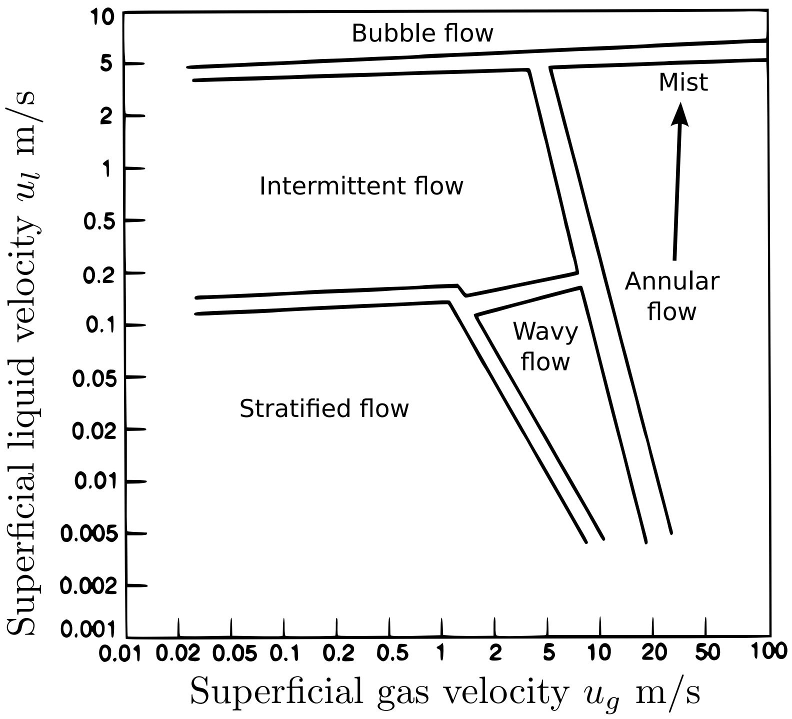


Figure 4: *Chhabra and Richardson flow pattern map for horizontal pipes.*

TABLE 4–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

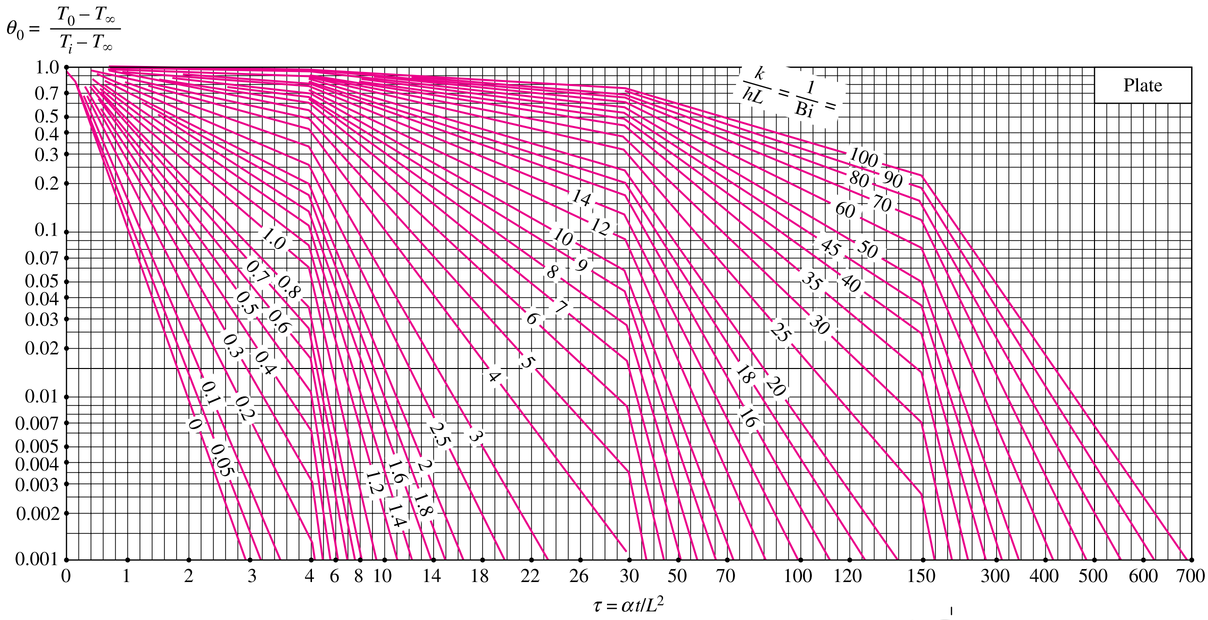
	Plane Wall		*Cylinder*		*Sphere*	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 4–3

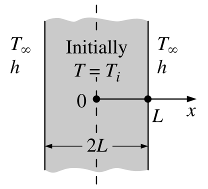
The zeroth- and first-order Bessel functions of the first kind

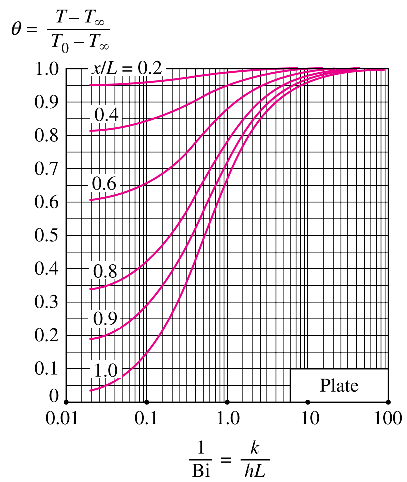
η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

Figure 5: *Coefficients for the 1D transient equations.*

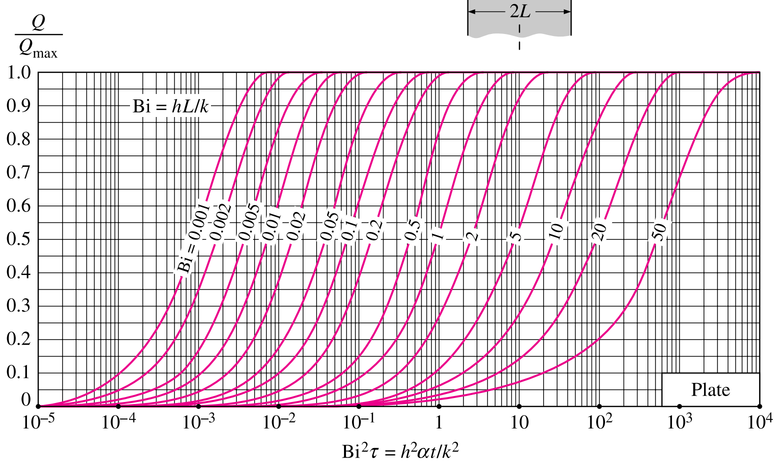


(*a*) Midplane temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





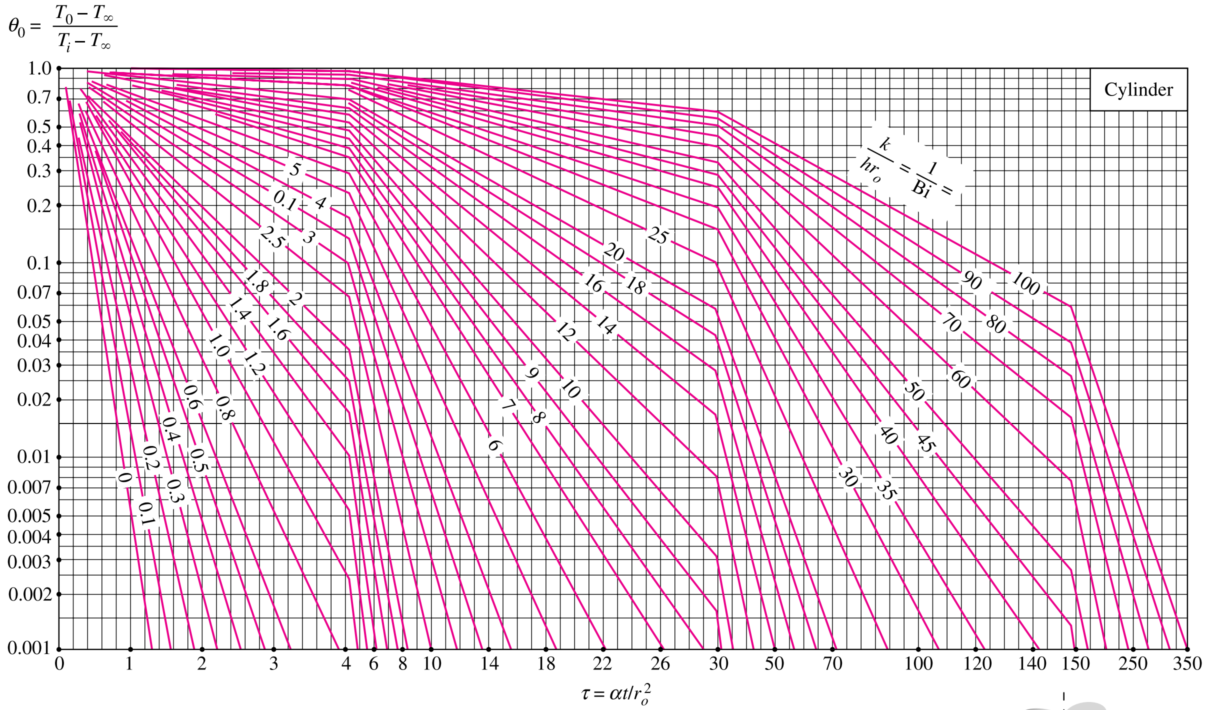
(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)



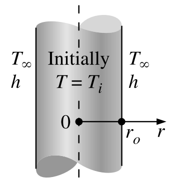
(*c*) Heat transfer (from H. Gröber et al.)

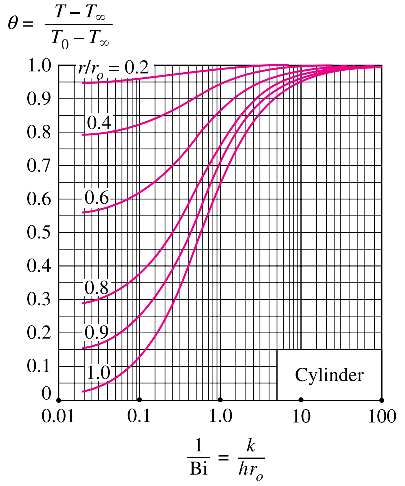
Transient temperature and heat transfer charts for a plane wall of thickness $2L$ initially at a uniform temperature T_i subjected to convection from both sides to an environment at temperature T_∞ with a convection coefficient of h.

Figure 6:

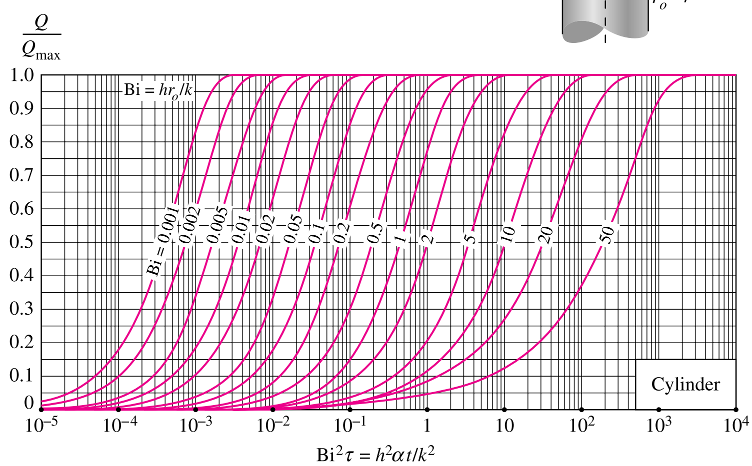


(*a*) Centerline temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





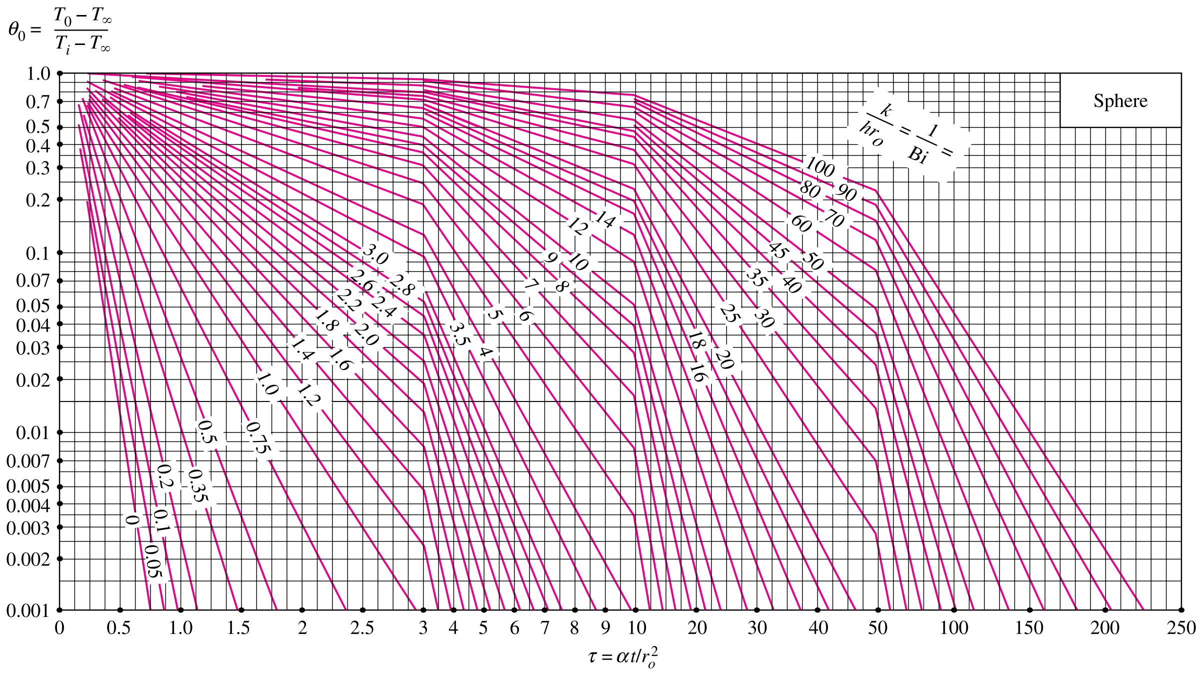
(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)



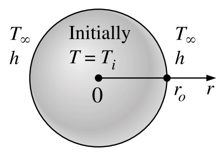
(*c*) Heat transfer (from H. Gröber et al.)

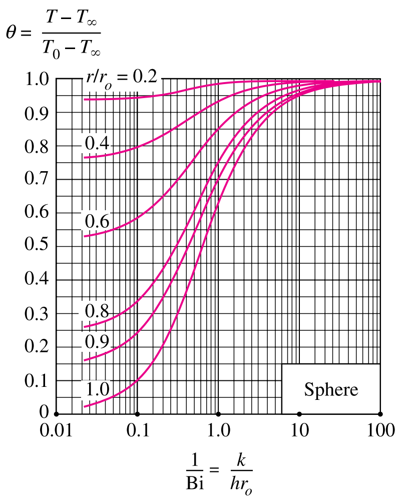
Transient temperature and heat transfer charts for a long cylinder of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

Figure 7:

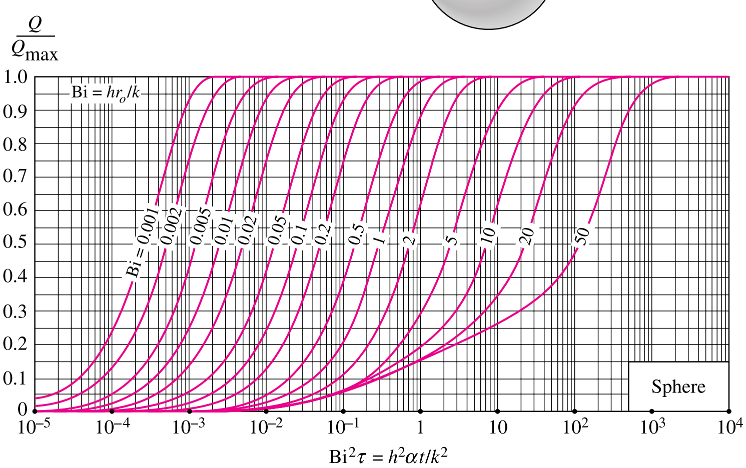


(*a*) Midpoint temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





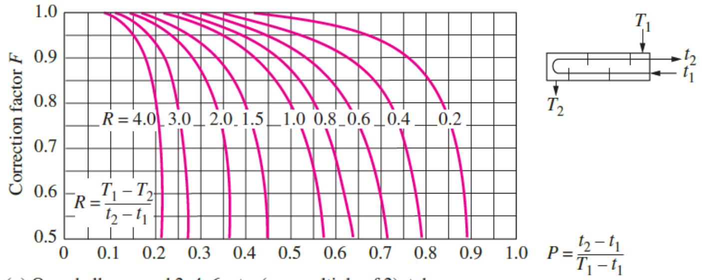
(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947,



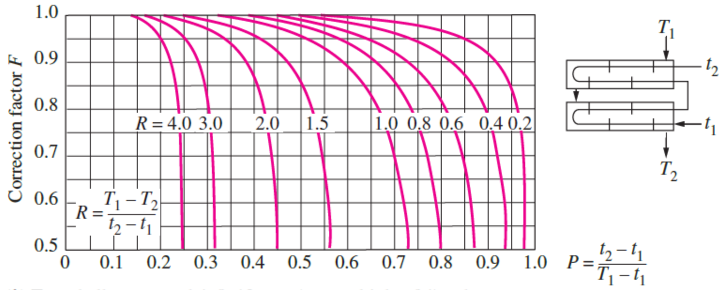
(*c*) Heat transfer (from H. Gröber et al.)

Transient temperature and heat transfer charts for a sphere of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

Figure 8:



(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



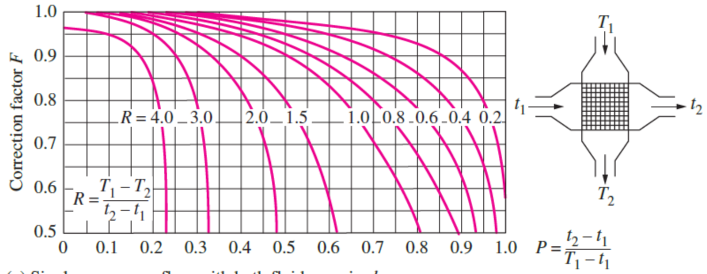
(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes

Figure 10.8

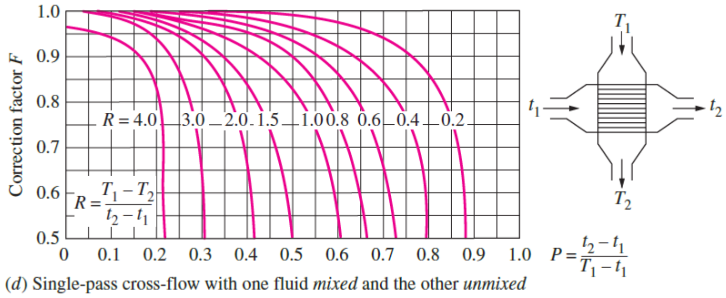
Correction factor *F* charts for common shell-and-tube and cross-flow heat exchangers (from Bowman, Mueller, and Nagle, Ref. 2).

Extracted from Y.A. Cengel, "Heat Transfer: A Practical Approach", 2nd Edition.

Figure 9: *Correction-factors for LMTD Method, extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*



(*c*) Single-pass cross-flow with both fluids *unmixed*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

Figure 10.8

Correction factor *F* charts for common shell-and-tube and cross-flow heat exchangers (from Bowman, Mueller, and Nagle, Ref. 2).

Extracted from Y.A. Cengel, "Heat Transfer: A Practical Approach", 2nd Edition.

Figure 10: *Correction-factors for LMTD Method, extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*

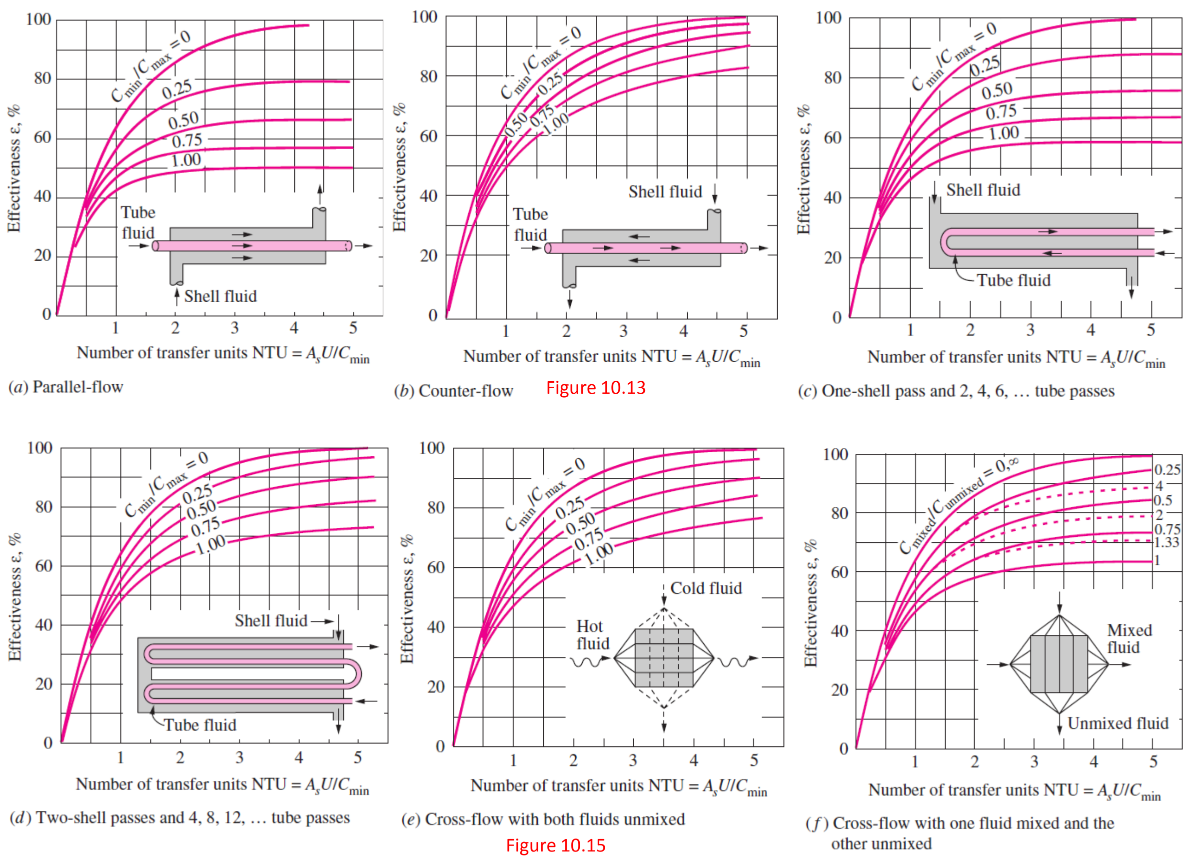
Effectiveness relations for heat exchangers: NTU $= UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	Effectiveness relation
1 *Double pipe:* Parallel-flow	$\varepsilon = \dfrac{1-\exp[-\text{NTU}(1+c)]}{1+c}$
Counter-flow	$\varepsilon = \dfrac{1-\exp[-\text{NTU}(1-c)]}{1-c\exp[-\text{NTU}(1-c)]}$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1+c+\sqrt{1+c^2}\,\dfrac{1+\exp[-\text{NTU}\sqrt{1+c^2}]}{1-\exp[-\text{NTU}\sqrt{1+c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)* Both fluids unmixed	$\varepsilon = 1-\exp\left\{\dfrac{\text{NTU}^{0.22}}{c}\left[\exp(-c\,\text{NTU}^{0.78})-1\right]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{c}(1-\exp\{1-c[1-\exp(-\text{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1-\exp\left\{-\dfrac{1}{c}[1-\exp(-c\,\text{NTU})]\right\}$
4 *All heat exchangers with c = 0*	$\varepsilon = 1-\exp(-\text{NTU})$

NTU relations for heat exchangers NTU $= UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	NTU relation
1 *Double-pipe:* Parallel-flow	$\text{NTU} = -\dfrac{\ln[1-\varepsilon(1+c)]}{1+c}$
Counter-flow	$\text{NTU} = \dfrac{1}{c-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon c-1}\right)$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$\text{NTU} = -\dfrac{1}{\sqrt{1+c^2}}\ln\left(\dfrac{2/\varepsilon-1-c-\sqrt{1+c^2}}{2/\varepsilon-1-c+\sqrt{1+c^2}}\right)$
3 *Cross-flow (single-pass)* C_{max} mixed, C_{min} unmixed	$\text{NTU} = -\ln\left[1+\dfrac{\ln(1-\varepsilon c)}{c}\right]$
C_{min} mixed, C_{max} unmixed	$\text{NTU} = -\dfrac{\ln[c\ln(1-\varepsilon)+1]}{c}$
4 *All heat exchangers with c = 0*	$\text{NTU} = -\ln(1-\varepsilon)$

Figure 11: *NTU relations extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*



Extracted from Y.A. Cengel, "Heat Transfer: A Practical Approach", 2nd Edition.

Figure 12: *NTU plots extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*

Diffusion Dimensionless Numbers

$$\text{Sc} = \frac{\mu}{\rho\, D_{AB}} \qquad\qquad \text{Le} = \frac{k}{\rho\, C_p\, D_{AB}}$$

Diffusion
General expression for the flux:

$$\boldsymbol{N}_A = \boldsymbol{J}_A + x_A \sum_B \boldsymbol{N}_B$$

Fick's law:

$$\boldsymbol{J}_A = -D_{AB}\, \nabla C_A$$

Stefan's law:

$$N_{s,r} = -D \frac{c}{1-x} \frac{\partial x}{\partial r}$$

Ideal Gas

$$P\, V = n\, R\, T \qquad\qquad R \approx 8.314598 \text{ J K}^{-1} \text{ mol}^{-1}$$

Geometry

$$P_{\text{circle}} = 2\, \pi\, r \qquad A_{\text{circle}} = \pi\, r^2 \qquad A_{\text{sphere}} = 4\, \pi\, r^2 \qquad V_{\text{sphere}} = \frac{4}{3} \pi\, r^3$$

$$A_{\text{cylinder}} = P_{\text{circle}}\, L \qquad V_{\text{cylinder}} = A_{\text{circle}}\, L$$