UNIVERSITY OF ABERDEEN SESSION 2014–15

Degree Examination in EG3019 Advanced Transport Processes

25th December 2014 9 am – 12 pm

Notes: *(i) Candidates ARE permitted to use an approved calculator.*

(ii) Candidates ARE permitted to use the Engineering Mathematics Handbook.

(iii) Candidates ARE permitted to use steam tables, which will be provided.

(iv) Data sheets are attached to the paper.

PLEASE NOTE THE FOLLOWING

(i) You **must not** have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material **must not** be amended, annotated or modified in any way.

(ii) You **must not** have in your possession any material that could be determined as giving you an advantage in the examination.

(iii) You **must not** attempt to communicate with any candidate during the exam, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

Failure to comply with the above will be regarded as cheating and may lead to disciplinary action as indicated in the Academic Quality Handbook

(`www.abdn.ac.uk/registry/quality/appendix7x1.pdf`) **Section 4.14 and 5.**

**Candidates must attempt *all* questions from PART A
AND *two* questions from *three* in PART B.**

PART A: Answer ALL Questions

Question 1

An incompressible polymeric fluid is to flow through 10 m of 50 mm inner-diameter piping. The flow index, n, for the fluid is 0.3 and the apparent viscosity, μ, at a shear rate of 1000 s^{-1} is 0.1 Pa s.

a) What type of fluid is this? Give a general description of its viscosity and include a sketch of the stress-rate versus strain graph and give the numerical expression for the stress τ_{xy}. [8 marks]

b) Assuming the flow is laminar, what is the frictional pressure loss if the volumetric flow rate required at the end of the pipe is 0.005 m^3 s^{-1}? [5 marks]

c) Using the Metzner-Reed Reynolds number, would you expect the flow in the pipe to be laminar or turbulent? The standard transition value for the Reynolds numer applies and you may assume a fluid density of 1500 kg m^{-3}. [4 marks]

d) How does the velocity profile in this pipe compare to one carrying a Newtonian fluid? Illustrate your answer with an appropriate diagram. [3 marks]

Question 2

a) Describe the physical interpretation of the Grashof number (Gr). Describe each of its terms and write down an equation for the temperature at which temperature-dependent terms in Gr should be evaluated. [5 marks]

b) An electric heater of 0.032 m diameter and 0.85 m long is used to heat a room. Calculate the electrical input (i.e. the sum of heat transferred by convection and radiation) to the heater when the bulk of the air in the room is at 24°C, the walls are at 12°C, and the surface of the heater is at 532°C. For convective heat transfer from the heater, assume the Nusselt number is given by

$$\mathrm{Nu} = 0.38(\mathrm{Gr})^{0.25}$$

where all properties are evaluated at the film temperature. You may assume air is an ideal gas, giving $\beta = T^{-1}$. Take the emissivity of the heater surface as $\epsilon = 0.62$ and assume that the surroundings are black. All other properties should be calculated using the steam table book. [10 marks]

Question 3

The Lockhart-Martinelli parameter, X, is a critical parameter in two-phase flow pressure-drop and liquid hold-up calculations. It is defined as the ratio of the frictional pressure drops of each phase, calculated as if each was flowing alone in the pipe.

$$X^2 = \frac{(\partial p/\partial z)_{liq.-only}}{(\partial p/\partial z)_{gas-only}}$$

a) Assuming that the pipe is smooth and that both phases are fully turbulent, derive the following expression for the Martinelli parameter

$$X_{tt} = \left(\frac{1-x}{x}\right)^{0.875} \left(\frac{\mu_{liq.}}{\mu_{gas}}\right)^{0.125} \left(\frac{\rho_{gas}}{\rho_{liq.}}\right)^{0.5}$$

[8 marks]

b) A mixture of saturated steam at 0.09 kg/s and water at 1.6 kg/s is flowing along a horizontal pipe with a diameter of 75 mm. The steam has a viscosity of $\mu_g = 0.0113 \times 10^{-3}$ N s/m^2 and density of 0.788 kg/m^3. The water has a viscosity of 0.52×10^{-3} N s/m^2 and a density of 1000 kg/m^3.

i) Determine the flow pattern inside the pipe. [2 marks]

ii) Determine the flow regime inside each phase of the pipe. [2 marks]

iii) Calculate the two phase pressure drop multiplier (for either phase). [8 marks]

iv) Calculate the pressure drop over a 12 m long smooth pipe. [5 marks]

PART B: Answer TWO Questions From THREE

Question 4

A Couette viscometer tests the viscous behaviour of a fluid using rotational shear in an annulus (see Fig. 1). The fluid is sheared by rotating the outer wall at an angular velocity of Ω_θ, giving $v_\theta(r = R) = \Omega_\theta\, R$. The inner cylinder is held stationary, giving $v_\theta(r = \kappa\, R) = 0$. There is no flow along the axis of the annulus.

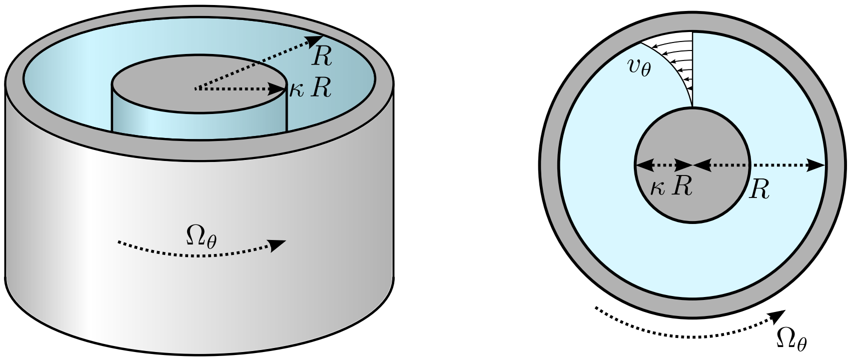


Figure 1: *A simplified diagram of a Couette viscometer.*

a) Derive the following expression by solving the continuity equation, given in Eq. (4), for this system.

$$\frac{\partial v_\theta}{\partial \theta} = 0 \tag{1}$$

Clearly state any assumptions you make. What does this tell you about the flow?

[5 marks]

b) The velocity profile of the system is given by the following expression:

$$v_\theta = \Omega_\theta\, R \frac{\frac{\kappa\, R}{r} - \frac{r}{\kappa\, R}}{\kappa - 1/\kappa} \tag{2}$$

Derive the following expression for the stress profile in the system.

$$\tau_{r\theta} = 2 \frac{\mu\, \Omega_\theta\, \kappa^2}{\kappa^2 - 1} \frac{R^2}{r^2} \tag{3}$$

[10 marks]

c) Derive the following expression for the torque exerted on the outer surface ($r = R$) to keep the fluid in motion.

$$\mathcal{T} = 4\pi\, R^2\, L \frac{\mu\, \Omega_\theta\, \kappa^2}{\kappa^2 - 1}$$

Note: The torque is the total magnitude of a tangential force, such as the viscous stress $\tau_{r\theta}$, multiplied by the radial distance at which it acts. [3 marks]

d) How can the formulas above be used to determine the viscous properties of a fluid? [2 marks]

Question 5

An electric wire of radius 0.5 mm is made of copper (electrical conductivity $k_e = 5.1 \times 10^7\ \text{ohm}^{-1}\, m^{-1}$ and thermal conductivity $k = 380\ \mathrm{W\,m^{-1}\,K^{-1}}$). It is insulated to an outer radius of 1.5 mm with plastic (thermal conductivity $k = 0.35\ \mathrm{W\,m^{-1}\,K^{-1}}$). The volumetric heat production σ, is given by $\sigma = I^2/k_e$ where I is the current density $\mathrm{A/m^2}$. The ambient air is at 38°C and the heat transfer coefficient from the outer insulated surface to the surrounding air is 8.5 $\mathrm{W\,m^{-2}K^{-1}}$.

a) Determine the maximum current in amperes that can flow through the wire if no part of the insulation may exceed 93°C. [8 marks]

b) Demonstrate that the heat flux in the copper section of the wire is given by the following expression:

$$q_r = \frac{I^2}{2\,k_e} r$$

[8 marks]

c) Solve for the temperature profile within the copper wire, assuming the outer surface of the wire is at $T_{crit.}$. [4 marks]

Question 6

Gaseous hydrogen at 10 bar and 27°C is stored in a 140 mm outer-diameter tank having a steel wall 2 mm thick and a height of 850mm. The molar concentration of hydrogen in the steel is 1.5 $\mathrm{kmol/m^3}$ at the inner surface and neglible at the outer surface, while the diffusion coefficient of hydrogen in steel is approximately $0.3 \times 10^{-12}\ \mathrm{m^2/s}$. Assume steady-state, one-dimensional conditions and a constant diffusion coefficient.

a) Derive the following expression for the flux

$$N_{H,r} = \frac{C_1}{r}$$

and derive the following expression for the concentration profile of the hydrogen in the steel wall.

$$C_{H_2} = 5.17 \times 10^4 \ln\left(\frac{0.07}{r}\right)$$

You may assume that the concentration of hydrogen within the wall is very small. [13 marks]

b) Calculate the mass flux of hydrogen through the wall using the solution to the last question. [7 marks]

END OF PAPER

DATASHEET

General balance equations (in index notation):

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \boldsymbol{v} \qquad \text{(Mass/Continuity)} \quad (4)$$

$$\frac{\partial C_A}{\partial t} = -\nabla \cdot \boldsymbol{N}_A \qquad \text{(Species)} \quad (5)$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla p + \rho \boldsymbol{g} \qquad \text{(Momentum)} \quad (6)$$

$$\rho C_p \frac{\partial T}{\partial t} = -\rho C_p \boldsymbol{v} \cdot \nabla T - \nabla \cdot \boldsymbol{q} - \boldsymbol{\tau} : \nabla \boldsymbol{v} - p \nabla \cdot \boldsymbol{v} + \sigma_{energy} \qquad \text{(Heat/Energy)} \quad (7)$$

In Cartesian coordinate systems, ∇ can be treated as a vector of derivatives. In curvelinear coordinate systems, the directions $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$ depend on the position. For convenience in these systems, look-up tables are provided for common terms involving ∇.

Cartesian coordinates (with index notation examples)
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor.

$$\nabla s = \nabla_i s = \left[\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \nabla_i \nabla_i s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \nabla_i v_i = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\nabla \cdot \boldsymbol{\tau} = \nabla_i \tau_{ij}$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\boldsymbol{v} \cdot \nabla \boldsymbol{v} = v_i \nabla_i v_j$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_x = v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_y = v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}$$

Cylindrical coordinates
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r}\frac{\partial}{\partial r}(r\, v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{1}{r}\tau_{\theta\theta} + \frac{\partial \tau_{rz}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r}\tau_{r\theta} + \frac{\partial \tau_{\theta z}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r\, v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

Spherical coordinates
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{1}{r\,\sin\theta}\frac{\partial s}{\partial \phi}\right]$$

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial s}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 s}{\partial \phi^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{\theta\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\phi}\right) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r\, v_\theta - v_\phi^2\cot\theta}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\phi = v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r\, v_\phi + v_\theta\, v_\phi\cot\theta}{r}$$

Rectangular		Cylindrical		Spherical	
q_x	$-k\frac{\partial T}{\partial x}$	q_r	$-k\frac{\partial T}{\partial r}$	q_r	$-k\frac{\partial T}{\partial r}$
q_y	$-k\frac{\partial T}{\partial y}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$
q_z	$-k\frac{\partial T}{\partial z}$	q_z	$-k\frac{\partial T}{\partial z}$	q_ϕ	$-k\frac{1}{r\,\sin\theta}\frac{\partial T}{\partial \phi}$
τ_{xx}	$-2\,\mu\frac{\partial v_x}{\partial x} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{yy}	$-2\,\mu\frac{\partial v_y}{\partial y} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\phi\phi}$	$-2\,\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r+v_\theta\cot\theta}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{xy}	$-\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$
τ_{yz}	$-\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$	$\tau_{\theta z}$	$-\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)$	$\tau_{\theta\phi}$	$-\mu\left(\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right)$
τ_{xz}	$-\mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$	τ_{zr}	$-\mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)$	$\tau_{\phi r}$	$-\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right)$

Table 1: *Fourier's law for the heat flux and Newton's law for the stress in several coordinate systems. Please remember that the stress is symmetric, so* $\tau_{ij} = \tau_{ji}$.

Viscous models:
Power-Law Fluid:

$$|\tau_{xy}| = k\left|\frac{\partial v_x}{\partial y}\right|^n$$

Bingham-Plastic Fluid:

$$\frac{\partial v_x}{\partial y} = \begin{cases} -\mu^{-1}\left(\tau_{xy} - \tau_0\right)) & \text{if } \tau_{xy} > \tau_0 \\ 0 & \text{if } \tau_{xy} \leq \tau_0 \end{cases}$$

Dimensionless Numbers

$$\mathrm{Re} = \frac{\rho\,\langle v\rangle\,D}{\mu} \qquad \mathrm{Re}_H = \frac{\rho\,\langle v\rangle\,D_H}{\mu} \qquad \mathrm{Re}_{MR} = -\frac{16\,L\,\rho\,\langle v\rangle^2}{R\,\Delta p} \tag{8}$$

The hydraulic diameter is defined as $D_H = 4\,A/P_w$.

Single phase pressure drop calculations in pipes:
Darcy-Weisbach equation:

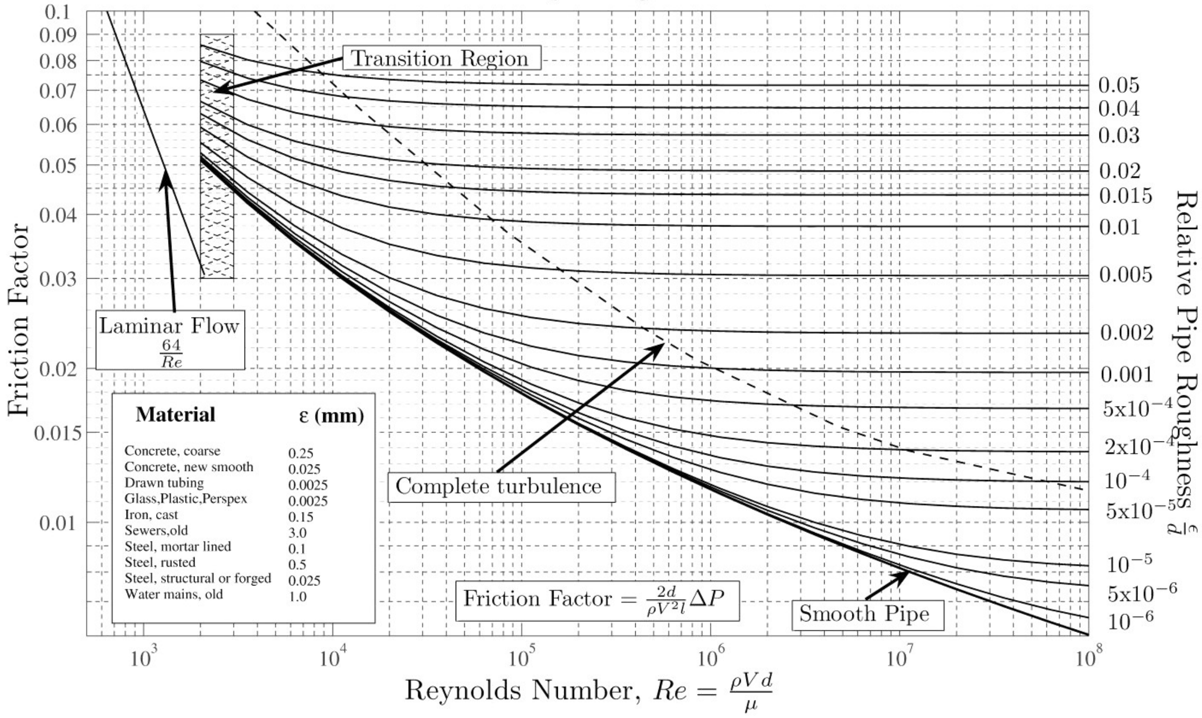
$$\frac{\Delta p}{L} = -\frac{C_f\,\rho\,\langle v\rangle^2}{R} \tag{9}$$

where $C_f = 16/Re$ for laminar Newtonian flow. For turbulent flow of Newtonian fluids in smooth pipes, we have the Blasius correlation:

$$C_f = 0.079\,\mathrm{Re}^{-1/4} \qquad \text{for } 2.5 \times 10^3 < \mathrm{Re} < 10^5 \text{ and smooth pipes.}$$

Otherwise, you may refer to the Moody diagram.

Moody Diagram



Laminar Power-Law fluid:

$$\dot{V} = \frac{n\,\pi\,R^3}{3\,n+1}\left(\frac{R}{2\,k}\right)^{\frac{1}{n}}\left(-\frac{\Delta p}{L}\right)^{\frac{1}{n}}$$

Two-Phase Flow:

Lockhart-Martinelli parameter:

$$X^2 = \frac{\Delta p_{liq.-only}}{\Delta p_{gas-only}}$$

Pressure drop calculation:

$$\Delta p_{two-phase} = \Phi^2_{liq.}\,\Delta p_{liq.-only} = \Phi^2_{gas}\,\Delta p_{gas-only}$$

Chisholm's relation:

$$\Phi^2_{gas} = 1 + c\,X + X^2$$

$$\Phi^2_{liq.} = 1 + \frac{c}{X} + \frac{1}{X^2} \qquad c = \begin{cases} 20 & \text{turbulent liquid \& turbulent gas} \\ 12 & \text{laminar liquid \& turbulent gas} \\ 10 & \text{turbulent liquid \& laminar gas} \\ 5 & \text{laminar liquid \& laminar gas} \end{cases}$$

Farooqi and Richardson expression for liquid hold-up in co-current flows of Newtonian fluids and air in horizontal pipes:

$$h = \begin{cases} 0.186 + 0.0191\,X & 1 < X < 5 \\ 0.143\,X^{0.42} & 5 < X < 50 \\ 1/\left(0.97 + 19/X\right) & 50 < X < 500 \end{cases}$$

Heat Transfer:
Stefan-Boltzmann constant $\sigma = 5.6703 \times 10^{-8}$ W/m^2 K^4

Heat Transfer Dimensionless numbers:

$$\mathrm{Nu} = \frac{h\,L}{k} \qquad \mathrm{Pr} = \frac{\mu\,C_p}{k} \qquad \mathrm{Gr} = \frac{g\,\beta\,(T_w - T_\infty)\;L^3}{\nu^2}$$

Resistances

$$Q = U_T\,A_T\,\Delta T = R_T^{-1}\,\Delta T \qquad Q_{rad.} = \sigma\,\varepsilon\,A\left(T_\infty^4 - T_w^4\right) = h_{rad.}\,A\,(T_\infty - T_w)$$

	Conduction Shell Resistances			Radiation
	Rect.	Cyl.	Sph.	
R	$\dfrac{X}{k\,A}$	$\dfrac{\ln\left(R_{outer}/R_{inner}\right)}{2\,\pi\,L\,k}$	$\dfrac{R_{inner}^{-1} - R_{outer}^{-1}}{4\,\pi\,k}$	$\left[A\,\varepsilon\,\sigma\,(T_\infty^2 + T_w^2)\,(T_\infty + T_w)\right]^{-1}$

Natural Convection

$\mathrm{Ra} = \mathrm{Gr}\,\mathrm{Pr}$	C	m
$< 10^4$	1.36	1/5
10^4–10^9	0.59	1/4
$> 10^9$	0.13	1/3

Table 2: *Natural convection coefficients for isothermal vertical plates in the empirical relation $Nu \approx C\,(Gr\,Pr)^m$.*

Churchill and Chu expression for natural convection from a horizontal pipe:

$$\mathrm{Nu}^{1/2} = 0.6 + 0.387\left\{\frac{\mathrm{Gr}\,\mathrm{Pr}}{\left[1 + (0.559/\mathrm{Pr})^{9/16}\right]^{16/9}}\right\}^{1/6} \qquad \text{for } 10^{-5} < \mathrm{Gr}\,\mathrm{Pr} < 10^{12}$$

Forced Convection:
Laminar flows:

$$\mathrm{Nu} \approx 0.332\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

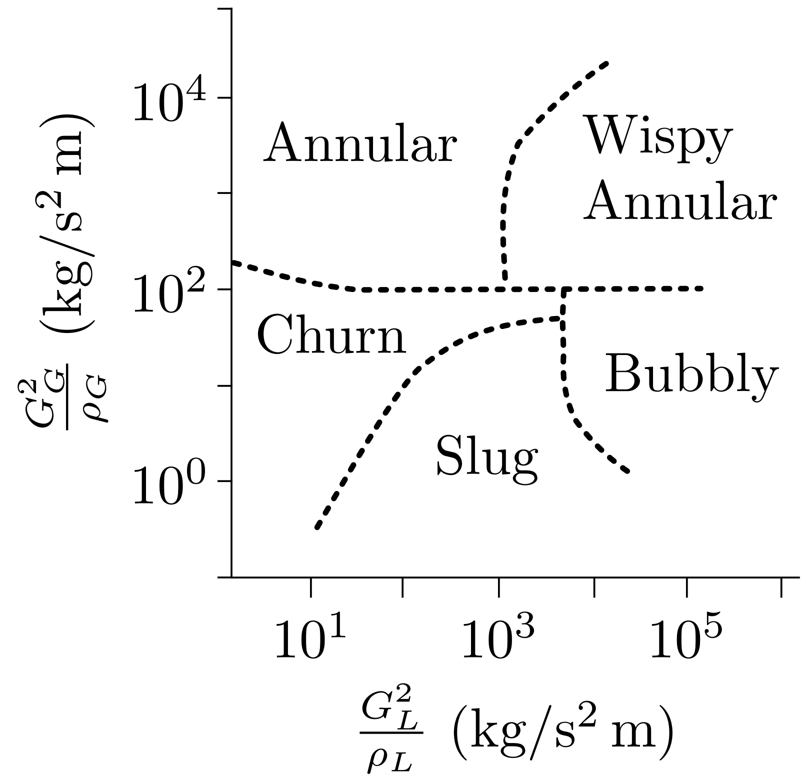


Figure 2: *Hewitt-Taylor flow pattern map for multiphase flows in vertical pipes.*

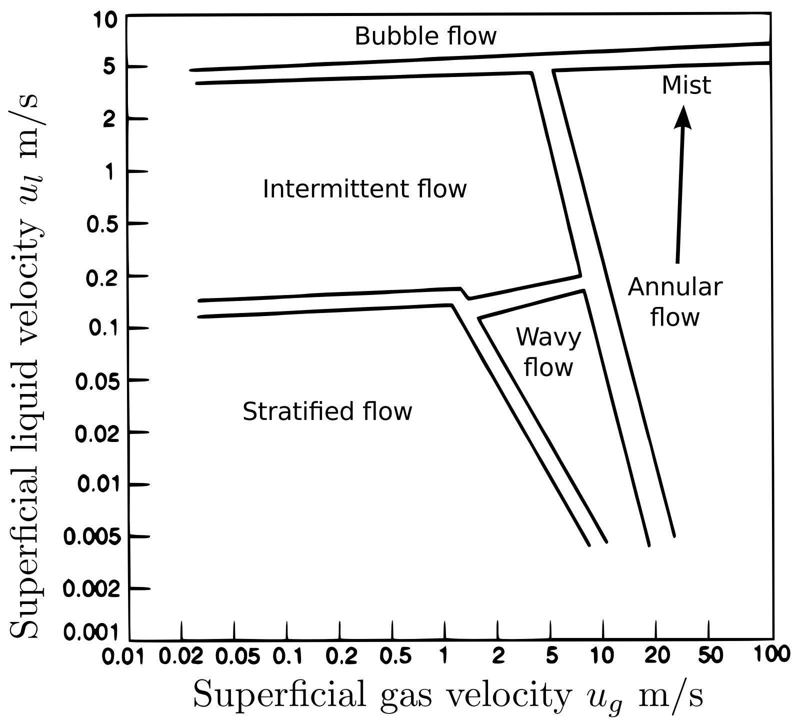


Figure 3: *Chhabra and Richardson flow pattern map for horizontal pipes.*

Well-Developed turbulent flows in smooth pipes:

$$\mathrm{Nu} \approx \frac{(C_f/2)\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(C_f/2)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Boiling:
Forster-Zuber pool-boiling coefficient:

$$h_{nb} = 0.00122 \frac{k_L^{0.79}\, C_{p,L}^{0.45}\, \rho_L^{0.49}}{\gamma^{0.5}\, \mu_L^{0.29}\, h_{fg}^{0.24}\, \rho_G^{0.24}} \left(T_w - T_{sat}\right)^{0.24} \left(p_w - p_{sat}\right)^{0.75}$$

Mostinski correlations:

$$h_{nb} = 0.104\, p_c^{0.69}\, q^{0.7} \left[1.8\left(\frac{p}{p_c}\right)^{0.17} + 4\left(\frac{p}{p_c}\right)^{1.2} + 10\left(\frac{p}{p_c}\right)^{10}\right]$$

$$q_c = 3.67 \times 10^4\, p_c \left(\frac{p}{p_c}\right)^{0.35} \left[1 - \frac{p}{p_c}\right]^{0.9}$$

(**Note**: for the Mostinski correlations, the pressures are in units of bar)
Condensing:
Horizontal pipes

$$h = 0.72 \left(\frac{k^3\, \rho^2\, g_x\, E_{latent}}{D\, \mu\, (T_w - T_\infty)}\right)^{1/4}$$

NTU method:

$$\mathrm{NTU} = \frac{U\, A}{C_{min}} = \frac{t_{C1} - t_{C2}}{\Delta t_{ln}} \qquad\qquad R = \frac{C_{min}}{C_{max}}$$

For counter-current flow:

$$E = \frac{1 - \exp\left[-\mathrm{NTU}(1 - R)\right]}{1 - R\exp\left[-\mathrm{NTU}(1 - R)\right]}$$

For co-current flow:

$$E = \frac{1 - \exp\left[-\mathrm{NTU}(1 - R)\right]}{1 + R}$$

Lumped capacitance method:

$$\mathrm{Bi} = \frac{h\, L_c}{k}$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{h\,A_s}{\rho\,V\,C_p} t\right]$$

Diffusion Dimensionless Numbers

$$\mathrm{Sc} = \frac{\mu}{\rho\,D_{AB}} \qquad\qquad \mathrm{Le} = \frac{k}{\rho\,C_p\,D_{AB}}$$

Diffusion

General expression for the flux:

$$\boldsymbol{N}_A = \boldsymbol{J}_A + x_A \sum_B \boldsymbol{N}_B$$

Fick's law:

$$\boldsymbol{J}_A = -D_{AB}\,\nabla C_A$$

Stefan's law:

$$N_{s,r} = -D \frac{c}{1-x} \frac{\partial x}{\partial r}$$