UNIVERSITY OF ABERDEEN **ACADEMIC YEAR 2024–25**

EX3030/EM4012

Degree Examination in EX3030/EM4012 Heat, Mass, & Momentum Transfer

16th December 2024 **Time: 14:00–17:00**

PLEASE NOTE THE FOLLOWING

(i) You **must not** have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material **must not** be amended, annotated or modified in any way.

(ii) You **must not** have in your possession any material that could be determined as giving you an advantage in the examination.

(iii) You **must not** attempt to communicate with any candidate during the exam, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

(iv) You **must not** take to your examination desk any electronic devices such as mobile phones or other "smart" devices. The only exception to this rule is an approved calculator.

FURTHER REQUIREMENTS SPECIFIC TO THE SCHOOL OF ENGINEERING

a) *Candidates* **ARE** *permitted to use only an approved calculator.*

b) *Candidates* **ARE NOT** *permitted to use the Engineering Mathematics Examinations Handbook.*

c) *Candidates* **ARE NOT** *permitted to use GREEN or RED pen in their exam booklet.*

d) *Data sheets are attached to the paper.*

e) *All question papers must be submitted with the exam booklet.*

Candidates should attempt all questions. This exam contains 5 questions, each worth 20 marks. This exam will be marked using Engineering Percentages.

Question 1

Engineers in a pasta factory are drying and stretching a wide continuous sheet of pasta dough by passing it back and forth in air using a series of rollers (see Fig. 1). They decide to model the air gap between two pasta sheets to determine the stress from air friction.

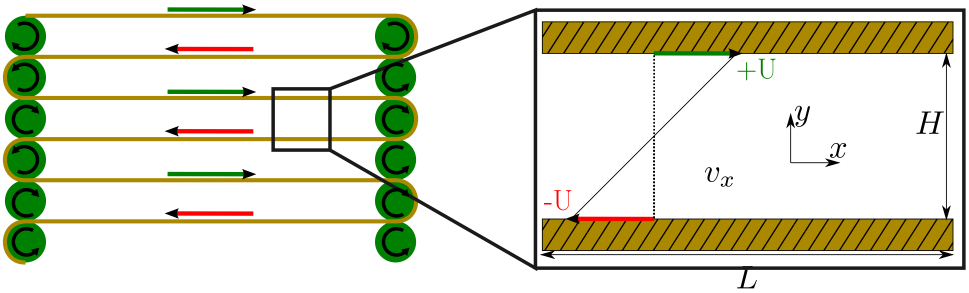


Figure 1: *An illustration of the drying system (left) and a close up of the air gap between sheets of pasta dough moving in opposite directions.*

For this simple analysis, you may assume the air is incompressible, the sheets are rectangular, parallel, horizontal, and sufficiently large to ignore the end effects of the edges and rollers. **There is also no pressure gradient driving the flow of air**. You may need to make additional assumptions. Be sure to exactly state when approximations are made, when any assumptions are needed, and exactly what terms are affected by the assumptions during your workings.

a) Start by simplifying the continuity equation for this system.

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho\, \boldsymbol{v}$$

What does your result state about the flow velocity in the x-direction? **[5 marks]**

b) Simplify the x-component of the Cauchy momentum equation:

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho\, \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla\, p + \rho\, \boldsymbol{g}$$

and derive the following expression for the flow velocity v_x as a function of the position y: **[5 marks]**

$$\frac{\partial \tau_{yx}}{\partial y} = -\mu \frac{\partial}{\partial y} \frac{\partial v_x}{\partial y} = 0$$

c) Continuing from the result of the previous question, derive the following expressions for the stress τ_{yx} and velocity v_x as a function of y using the no-slip boundary condition at the pasta surface, $v_x(y = 0) = -U$ and $v_x(y = H) = +U$. **[5 marks]**

$$\tau_{yx} = -\frac{2\,U\,\mu}{H}$$

$$v_x = 2\,U \left(\frac{y}{H} - 1\right)$$

d) Consider now a single pasta sheet in the middle of the machine which is being stressed on both sides by the friction of the air. If the sheets are 5mm thick, stretched 1 m long between rollers, with a width of 30 cm, and separated by a distance H = 0.1 m, then what is the total air stress in Newtons on a single span? You may assume the air has a dynamic viscosity of μ = 1.8 $\times$ 10^{-5} Pa s and the sheets are moving at $|U|$ = 1m s^{-1}. **[5 marks]**

[Question total: 20 marks]

Question 2

To explore the effect of using a temperature-dependent thermal conductivity, consider heat flowing through a pipe wall of inside radius R_0 and an outside radius R_1 (see Fig. 2). It is assumed that thermal conductivity varies linearly with temperature from $k_0(T = T_0)$ to $k_1(T = T_1)$ where T_0 and T_1 are the inner and outer wall temperatures respectively, giving the following form,

$$k(T) = k_0 + (T - T_0)\,\frac{k_1 - k_0}{T_1 - T_0}.$$

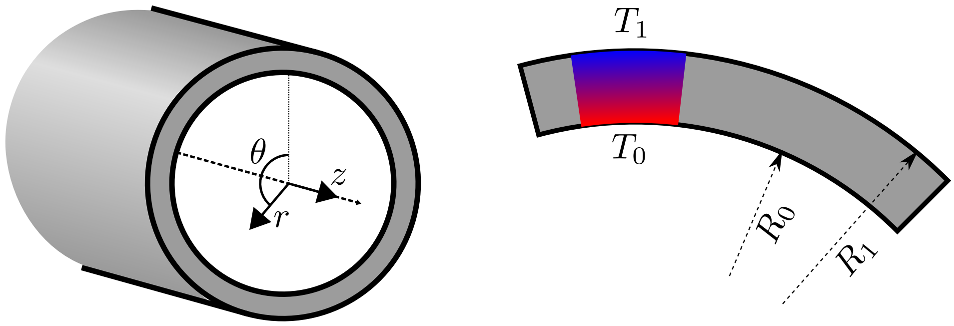


Figure 2: *A diagram of conduction through an annular(pipe) wall for Q. 2.*

a) Derive the following energy balance equation

$$\frac{\partial}{\partial r} r\, q_r = 0,$$

and state ALL assumptions required. **[5 marks]**

b) Derive the following expression for the temperature profile

$$Q_r = \frac{2\pi\, L}{\ln\left(\frac{R_0}{R_1}\right)}\frac{k_1 + k_0}{2}(T_1 - T_0),$$

where L is the length of the pipe/annulus. **[10 marks]**
Note: You may need the following identity:

$$T_1^2 - T_0^2 = (T_1 + T_0)(T_1 - T_0).$$

c) Find an expression for the heat transfer coefficient $U\,A$. Compare this expression to the standard expression for conduction in pipe walls (with constant thermal conductivity), what can you observe and when is a temperature-dependent conductivity justified? **[5 marks]**

[Question total: 20 marks]

Question 3
Consider laminar flow within a pipe. The only prior knowledge you should assume is that the pressure drop must be a function of pipe diameter D, viscosity μ, density ρ, and average velocity $\langle v_z \rangle$, i.e.,

$$\Delta p/l = f\left(D, \rho, \mu, \langle v_z \rangle\right).$$

a) Perform dimensional analysis on the pressure drop per unit length, $\Delta p/l$, and determine the relevant dimensionless groups. **[10 marks]**

b) Compare this to the exact solution, known as the Hagen-Poiseuille equation, as given below.

$$\dot{V}_z = \pi \left(\frac{-\Delta p}{l} + \rho\, g_z \right) \frac{R^4}{8\,\mu}.$$

Determine the form of the unknown function, f. **[5 marks]**

c) Comment on why dimensional analysis is so important. Also comment on why redundant dimensionless groups arise (as an example, consider the relationship between friction factor C_f and the Reynolds number). **[5 marks]**

[Question total: 20 marks]

Question 4

a) A granite sphere of 15 cm in diameter and at uniform temperature of 120°C is suddenly placed in a controlled environment where temperature is kept at 30°C. Average convective heat transfer coefficient is 350 W $m^{-2}\,K^{-1}$. Calculate the temperature of the granite sphere at a radius of 4.5 cm after 21 minutes. Given properties of granite: k = 3.2 W $m^{-1}\,K^{-1}$ and $\alpha = 13 \times 10^{-7}$ $m^2\,s^{-1}$. **[7 marks]**

b) Saturated steam is flowing at a rate of 2.3 kg s^{-1} with a pressure of 1 atm and temperature of 100°C is condensed in the shell of a shell-and-tube heat exchanger (one shell, two tube passes). Cooling water enters the tubes at 15°C with an average velocity of 3.5 m s^{-1}. The tubes are thin walled and made of copper with a diameter of 1.4×10^{-2} m and length of 0.5 m. The convective heat transfer coefficient for condensation on the outer surface of the tubes is 21.8 kW $m^{-2}\,K^{-1}$. Determine:

i) Rate of heat transfer (in W) in the heat exchanger: **[1 marks]**

ii) Overall heat transfer coefficient $\left(\text{in kW m}^{-2}\text{ K}^{-1}\right)$: **[3 marks]**

iii) Number of tubes/pass required the steam and the total surface area. If you have not solved (i) and/or (ii), you should assume U=7300 W $m^{-2}\,K^{-1}$ and/or $\dot{Q}$ = 5 MW; **[5 marks]**

iv) Outlet water temperature; **[2 marks]**

v) The maximum possible condensation rate that could be achieved with this heat exchanger using the same water flow rate and inlet temperature. **[2 marks]**

Given properties of:

- Saturated steam flow: T_{sat}=100°C and h_{fg} = 2257 kJ kg^{-1};
- Cooling water: ρ = 998 kg m^{-3}, C_p = 4181 J $kg^{-1}\,K^{-1}$, $\mu = 959\times10^{-6}$ N s m^{-2}, k = 0.606 W $m^{-1}\,K^{-1}$ and Pr = 6.62.

Also, cooling water convective heat transfer coefficient should be obtained from the Dittus-Boelter correlation:

$$Nu = 0.023\ Re^{0.8} Pr^{1/3}.$$

Also, note that the heat capacity rate $C = \dot{m}C_p$ of a fluid condensing or evaporating in a heat exchanger is infinity.

[Question total: 20 marks]

Question 5

A water purifying tablet is dropped into a bottle and comes to rest on the bottom as illustrated in Fig. 3. It quickly saturates all the water in its local vicinity to a mole fraction of 0.02 mol/mol while a large distance from the tablet we can assume the water is pure. To simplify the problem, we assume the radius of this saturated region is the same as the tablet size, which is R = 1 cm. The tablet then slowly dissolves, limited by the diffusion of material from this saturated region. The diffusion coefficient of the purifying chemical in water is 2×10^{-10} m^2 s^{-1}. The total molar density of water is $c = 55 \times 10^3$ mol m^{-3}.

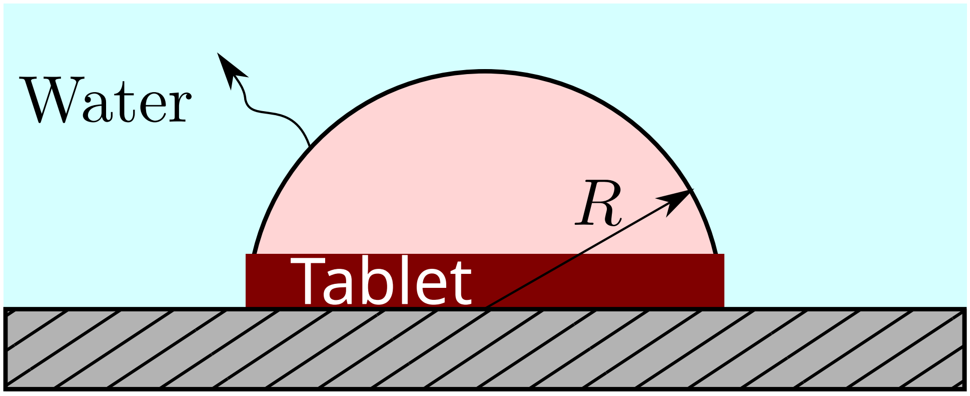


Figure 3: *The dissolving tablet just as it begins to dissolve and the resulting saturated zone of radius R.*

a) Starting from the molar balance equation,

$$\frac{\partial C_A}{\partial t} = -\nabla \cdot \boldsymbol{N}_A + \sigma_A$$

derive the following differential balance equation for the system.

$$\frac{\partial}{\partial r} r^2 N_{s,r} = 0$$

Be sure to state any and all assumptions made during your derivation. **[5 marks]**

b) Solve the differential equation to give the following result:

$$D_{sw}\, c \ln(1 - x_s) = -\frac{C_1}{r} + C_2$$

You may need the identity:

$$\int (1 - x)^{-1}\, \mathrm{d}x = -\ln(1 - x) + C$$

[5 marks]

c) Determine the unknown constants, C_1 and C_2, by using the boundary conditions of the known concentrations at $r = R$ and $r \to \infty$. Calculate the total flux of purifying chemical from the tablet into the water. You should note the surface area of a sphere is $A_{sphere} = 4\,\pi\,r^2$. **[5 marks]**

d) An experiment is performed and it is noticed that the tablet dissolves at a faster rate than predicted by the model. Explain why this might be the case and in particular describe the concept of the eddy diffusivity. **[5 marks]**

[Question total: 20 marks]

END OF PAPER

DATASHEET

General balance equations:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \, \boldsymbol{v} \qquad \text{(Mass/Continuity)} \quad (1)$$

$$\frac{\partial C_A}{\partial t} = -\nabla \cdot \boldsymbol{N}_A + \sigma_A \qquad \text{(Species)} \quad (2)$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho \, \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla p + \rho \, \boldsymbol{g} \qquad \text{(Momentum)} \quad (3)$$

$$\rho \, C_p \frac{\partial T}{\partial t} = -\rho \, C_p \, \boldsymbol{v} \cdot \nabla T - \nabla \cdot \boldsymbol{q} - \boldsymbol{\tau} : \nabla \boldsymbol{v} - p \nabla \cdot \boldsymbol{v} + \sigma_{energy} \qquad \text{(Heat/Energy)} \quad (4)$$

In Cartesian coordinate systems, ∇ can be treated as a vector of derivatives. In curvelinear coordinate systems, the directions $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\phi}$ depend on the position. For convenience in these systems, look-up tables are provided for common terms involving ∇.

Cartesian coordinates (with index notation examples)
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor.

$$\nabla s = \nabla_i s = \left[\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \nabla_i \nabla_i s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \nabla_i v_i = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\nabla \cdot \boldsymbol{\tau} = \nabla_i \tau_{ij}$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\boldsymbol{v} \cdot \nabla \boldsymbol{v} = v_i \nabla_i v_j$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_x = v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_y = v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}$$

Cylindrical coordinates
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r}\frac{\partial}{\partial r}(r\, v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{1}{r}\tau_{\theta\theta} + \frac{\partial \tau_{rz}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r}\tau_{r\theta} + \frac{\partial \tau_{\theta z}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r\, v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

Spherical coordinates
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial s}{\partial \phi}\right]$$

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial s}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 s}{\partial \phi^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{\theta\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\phi}\right) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r\, v_\theta - v_\phi^2\cot\theta}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\phi = v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r\, v_\phi + v_\theta\, v_\phi\cot\theta}{r}$$

Rectangular		Cylindrical		Spherical	
q_x	$-k\frac{\partial T}{\partial x}$	q_r	$-k\frac{\partial T}{\partial r}$	q_r	$-k\frac{\partial T}{\partial r}$
q_y	$-k\frac{\partial T}{\partial y}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$
q_z	$-k\frac{\partial T}{\partial z}$	q_z	$-k\frac{\partial T}{\partial z}$	q_ϕ	$-k\frac{1}{r\,\sin\theta}\frac{\partial T}{\partial \phi}$
τ_{xx}	$-2\,\mu\frac{\partial v_x}{\partial x} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{yy}	$-2\,\mu\frac{\partial v_y}{\partial y} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\phi\phi}$	$-2\,\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r+v_\theta\cot\theta}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{xy}	$-\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$
τ_{yz}	$-\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$	$\tau_{\theta z}$	$-\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)$	$\tau_{\theta\phi}$	$-\mu\left(\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right)$
τ_{xz}	$-\mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$	τ_{zr}	$-\mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)$	$\tau_{\phi r}$	$-\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right)$

Table 1: *Fourier's law for the heat flux and Newton's law for the stress in several coordinate systems. Please remember that the stress is symmetric, so* $\tau_{ij} = \tau_{ji}$.

Viscous models:
Power-Law Fluid:

$$|\tau_{xy}| = k\left|\frac{\partial v_x}{\partial y}\right|^n \tag{5}$$

Bingham-Plastic Fluid:

$$\frac{\partial v_x}{\partial y} = \begin{cases} -\mu^{-1}\left(\tau_{xy} - \tau_0\right) & \text{if } \tau_{xy} > \tau_0 \\ 0 & \text{if } \tau_{xy} \le \tau_0 \end{cases}$$

Dimensionless Numbers

$$\mathrm{Re} = \frac{\rho\,\langle v\rangle\,D}{\mu} \qquad \mathrm{Re}_H = \frac{\rho\,\langle v\rangle\,D_H}{\mu} \qquad \mathrm{Re}_{MR} = -\frac{16\,L\,\rho\,\langle v\rangle^2}{R\,\Delta p} \tag{6}$$

The hydraulic diameter is defined as $D_H = 4\,A/P_w$.

Single phase pressure drop calculations in pipes:
Darcy-Weisbach equation:

$$\frac{\Delta p}{L} = -\frac{C_f\,\rho\,\langle v\rangle^2}{R} \tag{7}$$

where $C_f = 16/Re$ for laminar Newtonian flow. For turbulent flow of Newtonian fluids in smooth pipes, we have the Blasius correlation:

$$C_f = 0.079\,\mathrm{Re}^{-1/4} \qquad \text{for } 2.5 \times 10^3 < \mathrm{Re} < 10^5 \text{ and smooth pipes.}$$

Otherwise, you may refer to the Moody diagram.

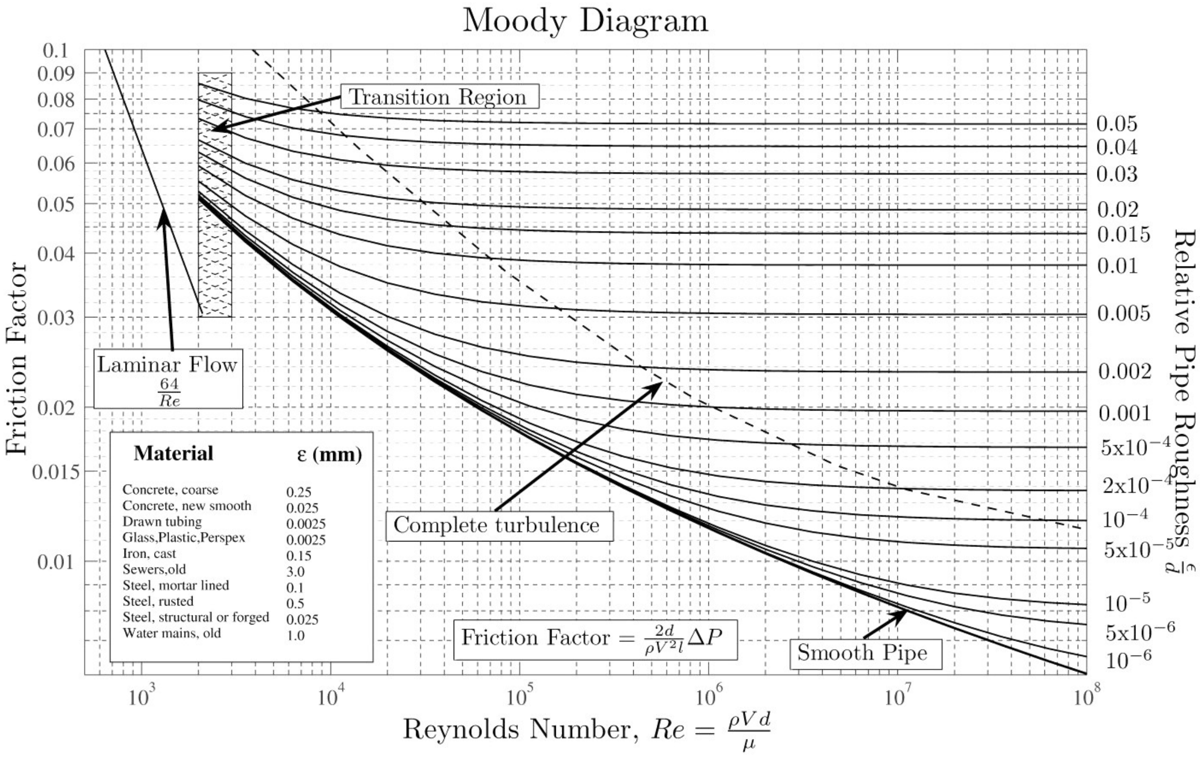


Figure 4: *The Moody diagram for flow in pipes.*

Laminar Power-Law fluid:

$$\dot{V} = \frac{n\,\pi\,R^3}{3\,n+1}\left(\frac{R}{2\,k}\right)^{\frac{1}{n}}\left(-\frac{\Delta p}{L}\right)^{\frac{1}{n}}$$

Two-Phase Flow:
Lockhart-Martinelli parameter:

$$X^2 = \frac{\Delta p_{liq.-only}}{\Delta p_{gas-only}}$$

Pressure drop calculation:

$$\Delta p_{two-phase} = \Phi^2_{liq.}\,\Delta p_{liq.-only} = \Phi^2_{gas}\,\Delta p_{gas-only}$$

Chisholm's relation:

$$\Phi^2_{gas} = 1 + c\,X + X^2$$

$$\Phi^2_{liq.} = 1 + \frac{c}{X} + \frac{1}{X^2} \qquad c = \begin{cases} 20 & \text{turbulent liquid \& turbulent gas} \\ 12 & \text{laminar liquid \& turbulent gas} \\ 10 & \text{turbulent liquid \& laminar gas} \\ 5 & \text{laminar liquid \& laminar gas} \end{cases}$$

Farooqi and Richardson expression for liquid hold-up in co-current flows of Newtonian fluids and air in horizontal pipes:

$$h = \begin{cases} 0.186 + 0.0191\,X & 1 < X < 5 \\ 0.143\,X^{0.42} & 5 < X < 50 \\ 1/(0.97 + 19/X) & 50 < X < 500 \end{cases}$$

Heat Transfer Dimensionless numbers:

$$\mathrm{Nu} = \frac{h\,L}{k} \qquad \mathrm{Pr} = \frac{\mu\,C_p}{k} \qquad \mathrm{Gr} = \frac{g\,\beta\,\rho^2\,(T_w - T_\infty)\;L^3}{\mu^2}$$

where $\beta = V^{-1}(\partial V/\partial T)$.
Heat transfer: Resistances

$$Q = U_T\,A_T\,\Delta T = R_T^{-1}\,\Delta T$$

	Conduction Shell Resistances			Radiation
	Rect.	Cyl.	Sph.	
R	$\dfrac{X}{k\,A}$	$\dfrac{\ln(R_{outer}/R_{inner})}{2\,\pi\,L\,k}$	$\dfrac{R_{inner}^{-1} - R_{outer}^{-1}}{4\,\pi\,k}$	$\left[A\,\varepsilon\,\sigma\left(T_j^2 + T_i^2\right)\left(T_j + T_i\right)\right]^{-1}$

Radiation Heat Transfer:
Stefan-Boltzmann constant $\sigma = 5.6703 \times 10^{-8}$ W m^{-2} K^{-4}.
Summation relationship, $\sum_j F_{i\to j} = 1$, and reciprocity relationship, $F_{i\to j}\,A_i = F_{j\to i}\,A_j$.
Radiation shielding factor $1/(N+1)$.

$$Q_{rad.,i\to j} = \sigma\,\varepsilon\,F_{i\to j}\,A_i\left(T_j^4 - T_i^4\right) = h_{rad.}\,A\,(T_\infty - T_w)$$

Natural Convection

Ra = Gr Pr	C	m
$< 10^4$	1.36	1/5
10^4–10^9	0.59	1/4
$> 10^9$	0.13	1/3

Table 2: *Natural convection coefficients for isothermal vertical plates in the empirical relation* $Nu \approx C\,(Gr\,Pr)^m$.

For isothermal vertical cylinders, the above expressions for isothermal vertical plates may be used but must be scaled by a factor, F (i.e., $\mathrm{Nu}_{v.cyl.} = F\,\mathrm{Nu}_{v.plate}$):

$$F = \begin{cases} 1 & \text{for } (D/H) \geq 35\,\mathrm{Gr}_H^{-1/4} \\ 1.3\left[H\,D^{-1}\,\mathrm{Gr}_D^{-1}\right]^{1/4} + 1 & \text{for } (D/H) < 35\,\mathrm{Gr}_H^{-1/4} \end{cases} \tag{8}$$

where D is the diameter and H is the height of the cylinder. The subscript on Gr indicates which length is to be used as the critical length to calculate the Grashof number.

Churchill and Chu expression for natural convection from a horizontal pipe:

$$\mathrm{Nu}^{1/2} = 0.6 + 0.387\left\{\frac{\mathrm{Gr}\,\mathrm{Pr}}{\left[1 + (0.559/\mathrm{Pr})^{9/16}\right]^{16/9}}\right\}^{1/6} \quad \text{for } 10^{-5} < \mathrm{Gr}\,\mathrm{Pr} < 10^{12} \tag{9}$$

Forced Convection:
Laminar flows:

$$\mathrm{Nu} \approx 0.332\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3} \tag{10}$$

Well-Developed turbulent flows in smooth pipes:

$$\mathrm{Nu} \approx 0.023\,\mathrm{Re}_D^{4/5}\,\mathrm{Pr}^n \tag{11}$$

where $n = 0.4$ if the fluid is being heated, and $n = 0.3$ if the fluid is being cooled.

Boiling:
Forster-Zuber pool-boiling coefficient:

$$h_{nb} = 0.00122\frac{k_L^{0.79}\,C_{p,L}^{0.45}\,\rho_L^{0.49}}{\gamma^{0.5}\,\mu_L^{0.29}\,h_{fg}^{0.24}\,\rho_G^{0.24}}\,(T_w - T_{sat})^{0.24}\,(p_w - p_{sat})^{0.75} \tag{12}$$

Mostinski correlations:

$$h_{nb} = 0.104\,p_c^{0.69}\,q^{0.7}\left[1.8\left(\frac{p}{p_c}\right)^{0.17} + 4\left(\frac{p}{p_c}\right)^{1.2} + 10\left(\frac{p}{p_c}\right)^{10}\right] \tag{13}$$

$$q_c = 3.67 \times 10^4\,p_c\left(\frac{p}{p_c}\right)^{0.35}\left[1 - \frac{p}{p_c}\right]^{0.9} \tag{14}$$

(**Note**: for the Mostinski correlations, the pressures are in units of bar)
Condensing:
Horizontal pipes

$$h = 0.72\left(\frac{k^3\,\rho^2\,g_x\,E_{latent}}{D\,\mu\,(T_w - T_\infty)}\right)^{1/4} \tag{15}$$

Lumped capacitance method:

$$\begin{aligned} \mathrm{Bi} &= \frac{h\,L_c}{\kappa} \\ L_c &= V/A \qquad\qquad \text{for Bi} < 0.1 \end{aligned}$$

$$\frac{T(t) - T_\infty}{T_0 - T_\infty} = e^{-bt} \qquad\qquad b = \frac{hA_s}{\rho V C_p}$$

1-D Transient Heat Conduction:

$$Fo = \frac{\alpha \Delta t}{(\Delta x)^2} = \tau, \qquad \alpha = \kappa\left(\rho C_p\right)^{-1}$$

$$\theta_{\mathrm{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\frac{\lambda_1 x}{L}\right), \qquad \theta_{\mathrm{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \mathbf{J_0}\left(\frac{\lambda_1 r}{r_0}\right)$$

$$\theta_{\mathrm{sph}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin\left(\frac{\lambda_1 r}{r_0}\right)}{\frac{\lambda_1 r}{r_0}}$$

$$\theta_{0,\mathrm{wall}} = \theta_{0,\mathrm{cyl}} = \theta_{0,\mathrm{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\mathrm{max}}}\right)_{\mathrm{wall}} = 1 - \theta_{0,\mathrm{wall}} \frac{\sin \lambda_1}{\lambda_1}, \quad \left(\frac{\mathcal{Q}}{\mathcal{Q}_{\mathrm{max}}}\right)_{\mathrm{cyl}} = 1 - 2\theta_{0,\mathrm{cyl}} \frac{\mathbf{J_1}(\boldsymbol{\lambda_1})}{\lambda_1}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\mathrm{max}}}\right)_{\mathrm{sph}} = 1 - 3\theta_{0,\mathrm{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

Finite-Difference Method:

$$\frac{\partial}{\partial t}(\rho\phi) + \frac{\partial}{\partial x}(\rho \boldsymbol{v}\phi) = \nabla \cdot (\Gamma \nabla \phi) + \mathcal{S} \qquad \text{(1D transport equation)}$$

$$\left(\frac{d\phi}{dx}\right)_i = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x} \quad \text{and} \quad \left(\frac{d^2\phi}{dx^2}\right)_i = \frac{\phi_{i-1} + \phi_{i+1} - 2\phi_i}{(\Delta x)^2}$$

$$T_i^{j+1} = (1 - 2\tau)\, T_i^j + \tau \left(T_{i+1}^j + T_{i-1}^j\right) + \frac{\tau\,(\Delta x)^2}{\kappa} \mathcal{S}_i^j$$

Overall Heat Transfer Coefficient:

$$\dot{\mathcal{Q}} = \frac{\Delta T}{\mathcal{R}} = UA\Delta T = U_i A_i \Delta T = U_o A_o \Delta T$$

$$\mathcal{R} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_i A_i} + \frac{\ln D_o/D_i}{2\pi\kappa L} + \frac{1}{h_o A_o}$$

Fouling Factor:

$$\mathcal{R} = \frac{1}{h_i A_i} + \frac{R_{\text{f,i}}}{A_i} + R_{\text{wall}} + \frac{R_{\text{f,o}}}{A_o} + \frac{1}{h_o A_o}$$

LMTD Method:

$$\dot{\mathcal{Q}} = UA_s \Delta T_{\text{lm}} \quad \text{with} \quad \Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$$

$$\text{Parallel flows:} \begin{cases} \Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,in}} \\ \Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,out}} \end{cases}$$

$$\text{Counter flows:} \begin{cases} \Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,out}} \\ \Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,in}} \end{cases}$$

ϵ-NTU Method:

$$\epsilon = \frac{\dot{\mathcal{Q}}}{\dot{\mathcal{Q}}_{\text{max}}}, \quad \text{with } \dot{\mathcal{Q}}_{\text{max}} = \mathcal{C}_{\text{min}}\left(T_{\text{hot,in}} - T_{\text{cold,in}}\right) \quad \text{and} \quad \mathcal{C}_{\text{min}} = Min\left\{\dot{m}_{\text{hot}} C_{p,\text{hot}}, \dot{m}_{\text{cold}} C_{p,\text{cold}}\right\}$$

$$\text{NTU} = \frac{UA_s}{C_{\text{min}}}$$

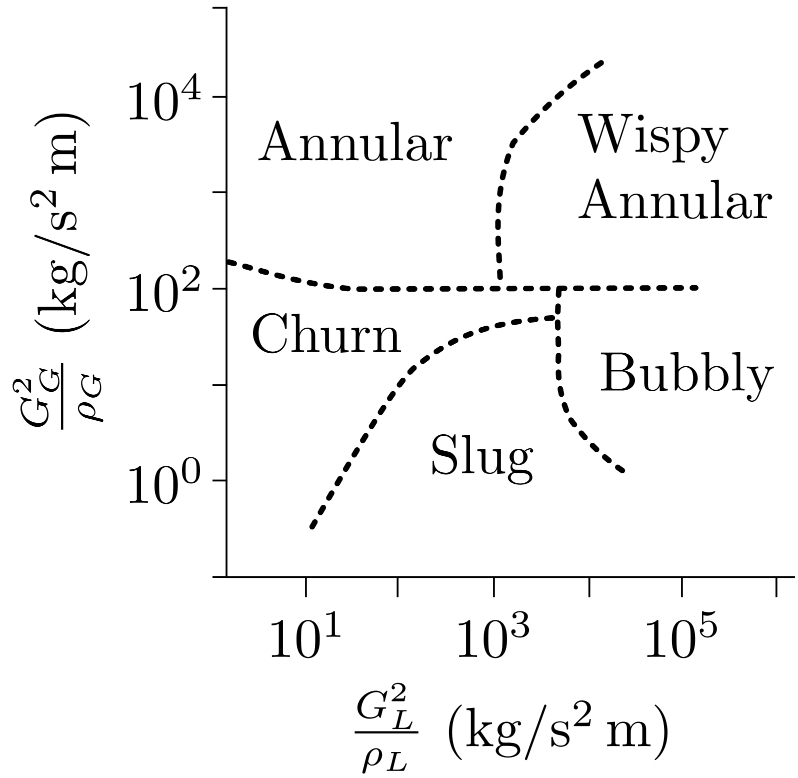


Figure 5: *Hewitt-Taylor flow pattern map for multiphase flows in vertical pipes.*

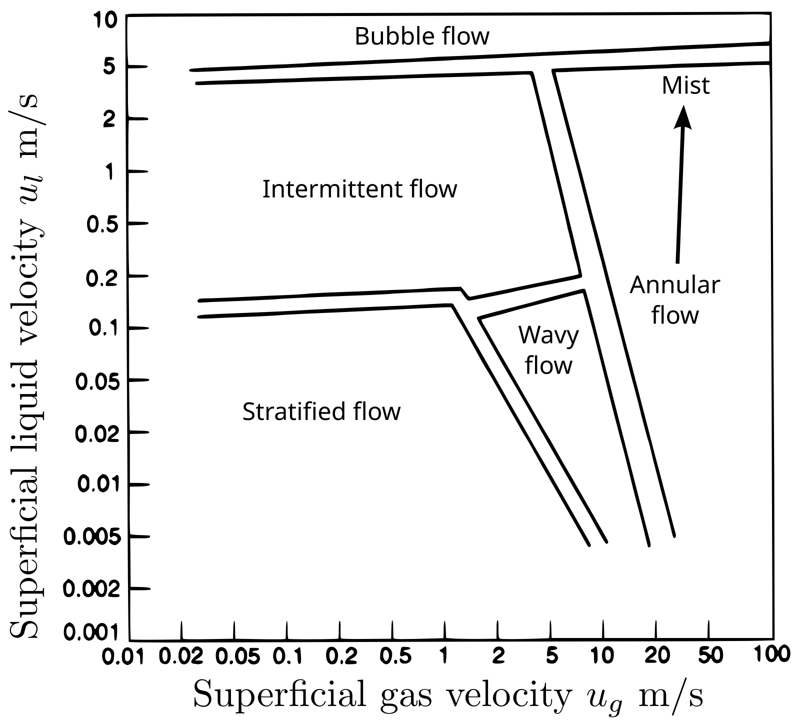


Figure 6: *Chhabra and Richardson flow pattern map for horizontal pipes.*

TABLE 4–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

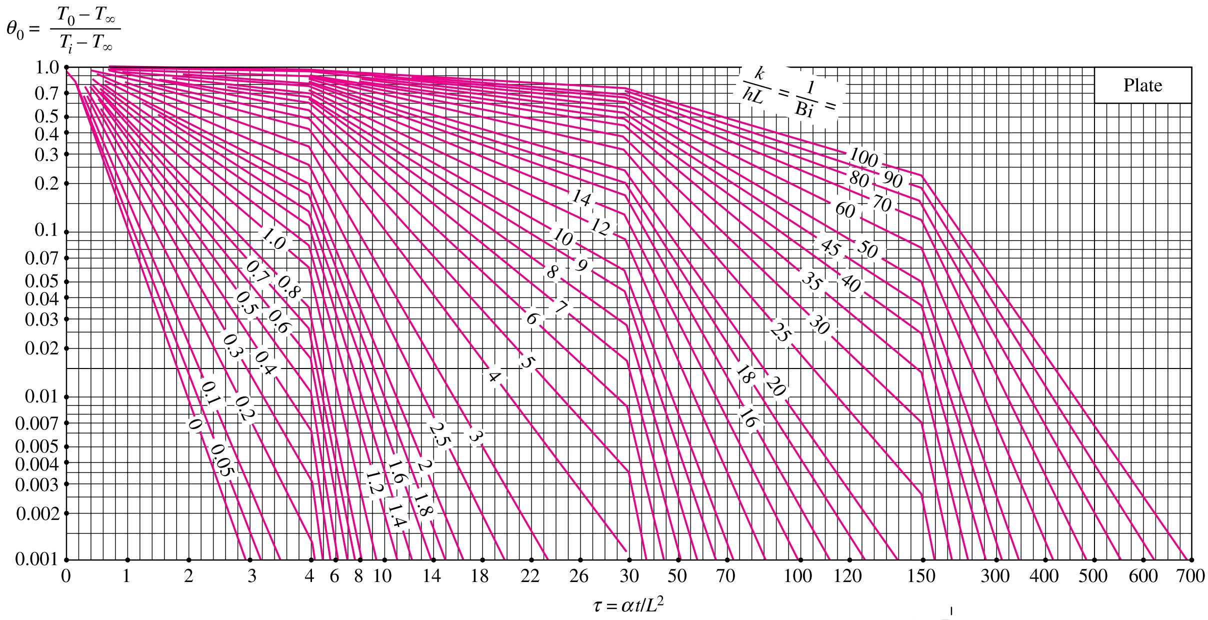
	Plane Wall		*Cylinder*		*Sphere*	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 4–3

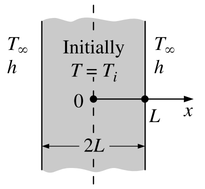
The zeroth- and first-order Bessel functions of the first kind

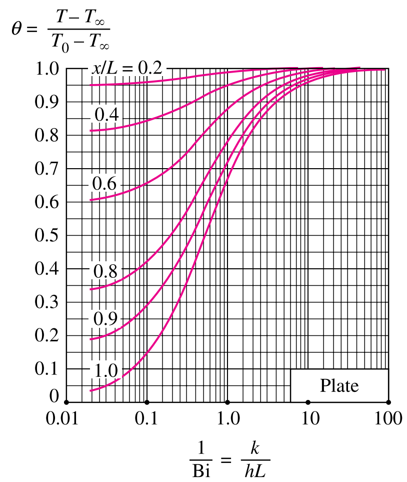
η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

Figure 7: *Coefficients for the 1D transient equations.*

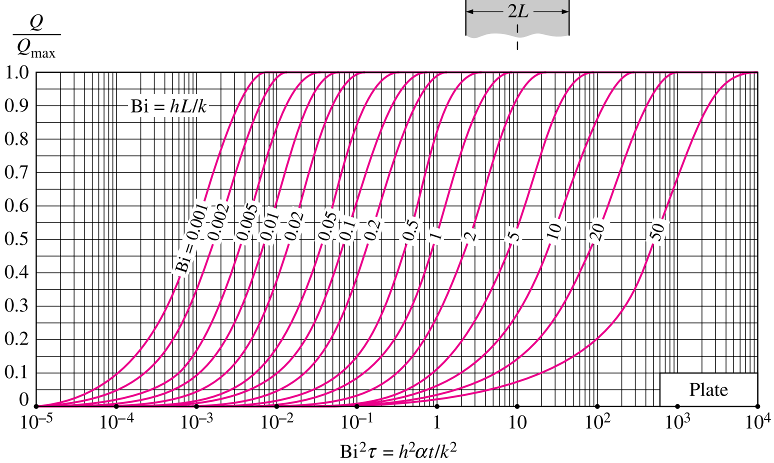


(*a*) Midplane temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





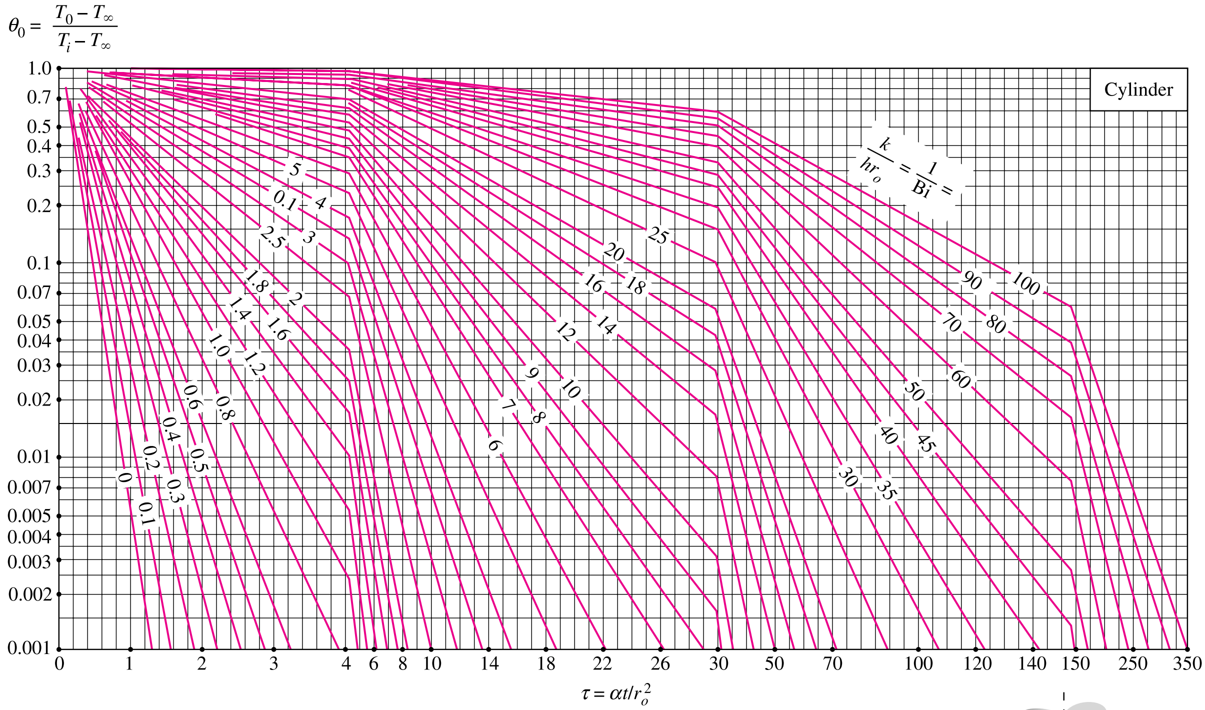
(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)



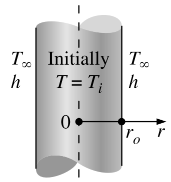
(*c*) Heat transfer (from H. Gröber et al.)

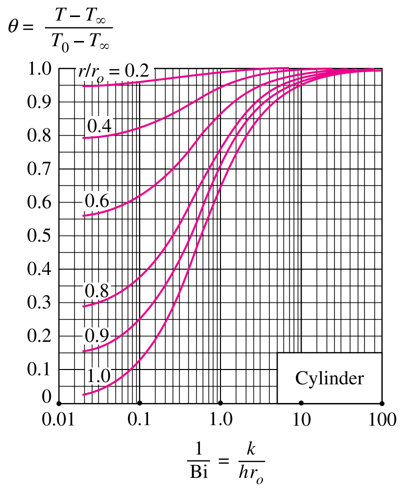
Transient temperature and heat transfer charts for a plane wall of thickness $2L$ initially at a uniform temperature T_i subjected to convection from both sides to an environment at temperature T_∞ with a convection coefficient of h.

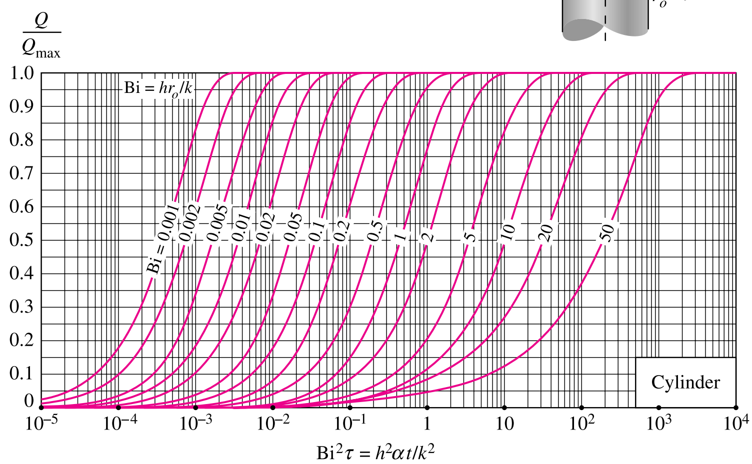
Figure 8:



(*a*) Centerline temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





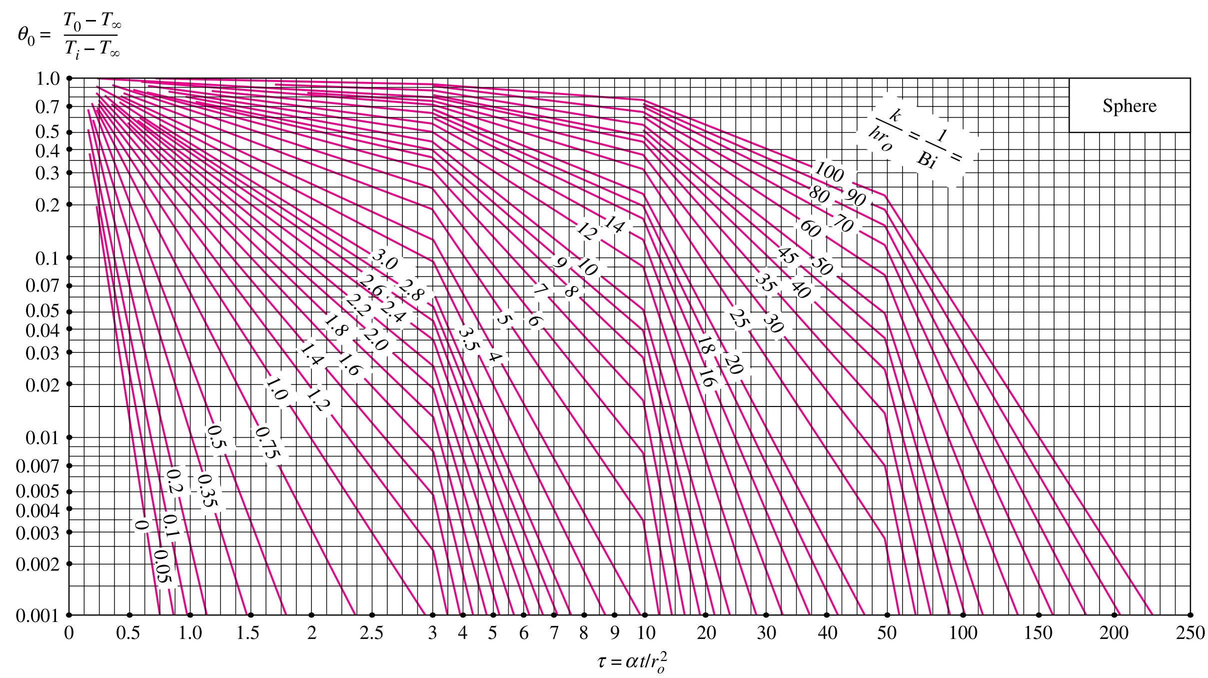


(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)

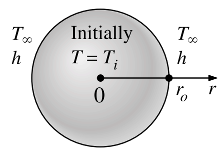
(*c*) Heat transfer (from H. Gröber et al.)

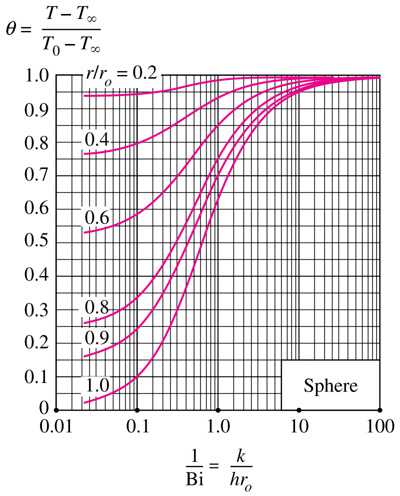
Transient temperature and heat transfer charts for a long cylinder of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

Figure 9:

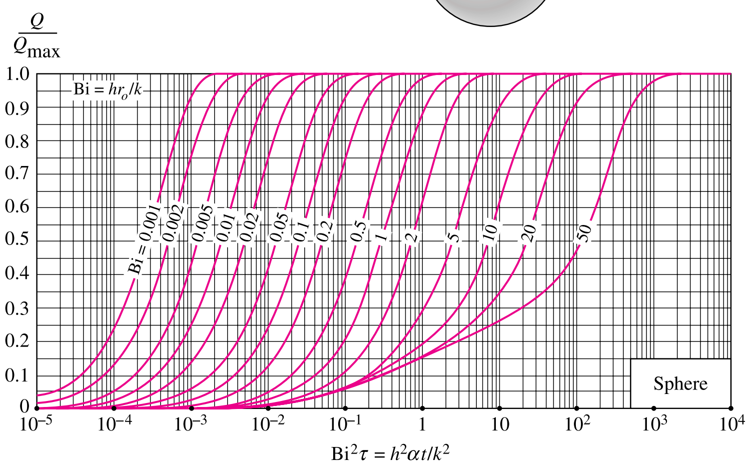


(*a*) Midpoint temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





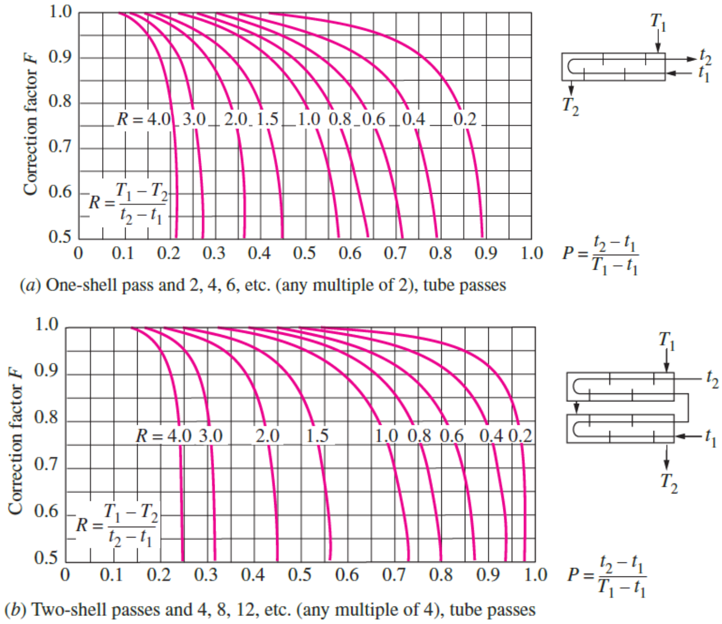
(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947,



(*c*) Heat transfer (from H. Gröber et al.)

Transient temperature and heat transfer charts for a sphere of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

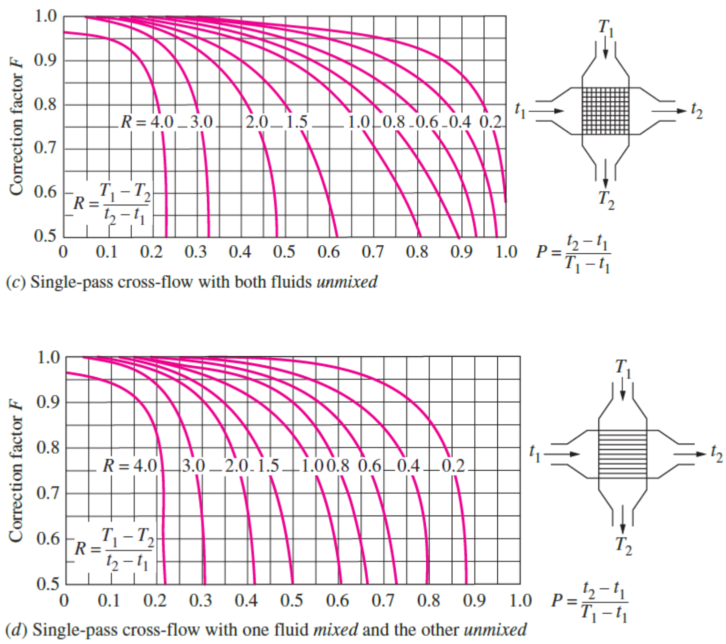
Figure 10:



Extracted from Y.A. Cengel, “Heat Transfer: A Practical Approach”, 2nd Edition.

Figure 10.8
Correction factor F charts for common shell-and-tube and cross-flow heat exchangers (from Bowman, Mueller, and Nagle, Ref. 2).

Figure 11: *Correction-factors for LMTD Method, extracted from Y. A. Cengel, “Heat transfer:A practical approach”, 2nd Ed.*



Extracted from Y.A. Cengel, “Heat Transfer: A Practical Approach”, 2nd Edition.

Figure 10.8
Correction factor F charts for common shell-and-tube and cross-flow heat exchangers (from Bowman, Mueller, and Nagle, Ref. 2).

Figure 12: *Correction-factors for LMTD Method, extracted from Y. A. Cengel, “Heat transfer:A practical approach”, 2nd Ed.*

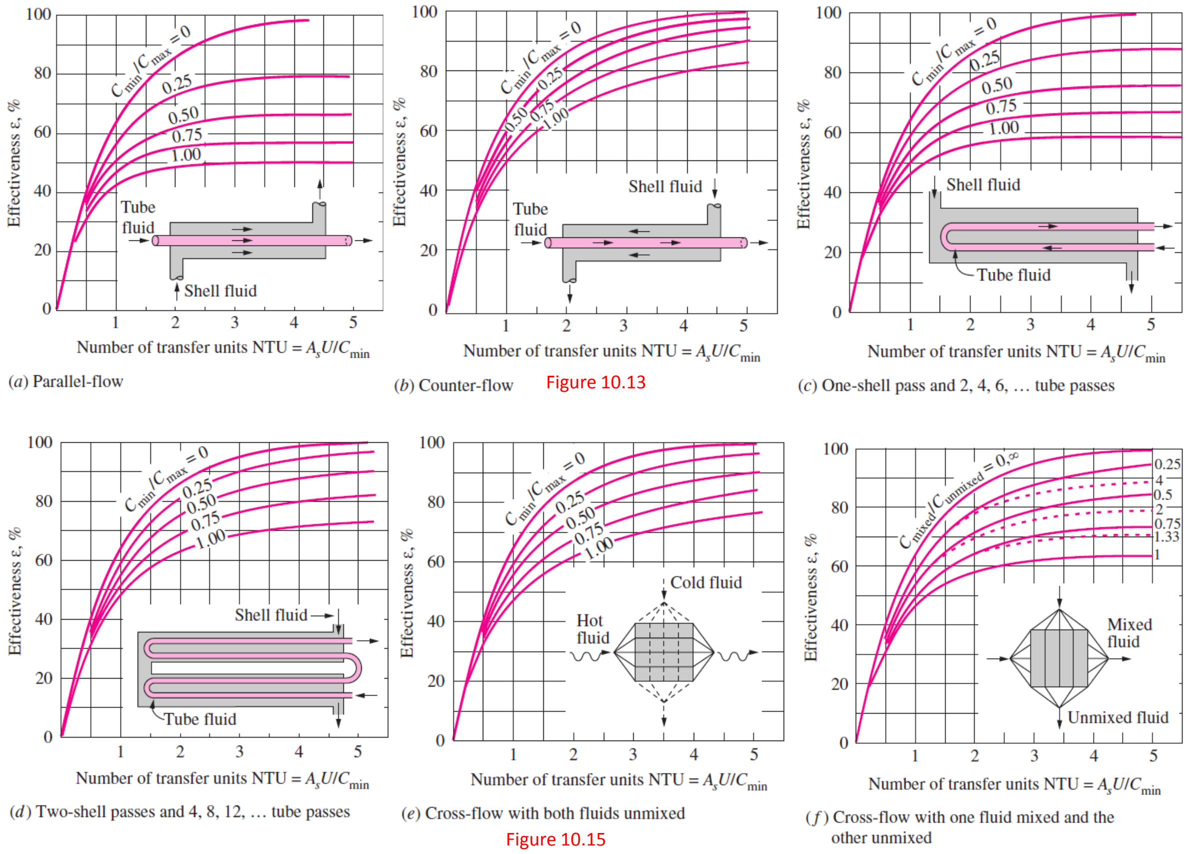
Effectiveness relations for heat exchangers: NTU $= UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	Effectiveness relation
1 *Double pipe:* Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 + c)]}{1 + c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 - c)]}{1 - c\exp[-\text{NTU}(1 - c)]}$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1 + c + \sqrt{1 + c^2}\,\dfrac{1 + \exp[-\text{NTU}\sqrt{1 + c^2}]}{1 - \exp[-\text{NTU}\sqrt{1 + c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)* Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}[\exp(-c\,\text{NTU}^{0.78}) - 1]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\{1 - c[1 - \exp(-\text{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp(-c\,\text{NTU})]\right\}$
4 *All heat exchangers with c = 0*	$\varepsilon = 1 - \exp(-\text{NTU})$

NTU relations for heat exchangers NTU $= UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	NTU relation
1 *Double-pipe:* Parallel-flow	$\text{NTU} = -\dfrac{\ln[1 - \varepsilon(1 + c)]}{1 + c}$
Counter-flow	$\text{NTU} = \dfrac{1}{c - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon c - 1}\right)$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$\text{NTU} = -\dfrac{1}{\sqrt{1 + c^2}}\ln\left(\dfrac{2/\varepsilon - 1 - c - \sqrt{1 + c^2}}{2/\varepsilon - 1 - c + \sqrt{1 + c^2}}\right)$
3 *Cross-flow (single-pass)* C_{max} mixed, C_{min} unmixed	$\text{NTU} = -\ln\left[1 + \dfrac{\ln(1 - \varepsilon c)}{c}\right]$
C_{min} mixed, C_{max} unmixed	$\text{NTU} = -\dfrac{\ln[c\ln(1 - \varepsilon) + 1]}{c}$
4 *All heat exchangers with c = 0*	$\text{NTU} = -\ln(1 - \varepsilon)$

Figure 13: *NTU relations extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*



(*a*) Parallel-flow
(*b*) Counter-flow
(*c*) One-shell pass and 2, 4, 6, … tube passes
(*d*) Two-shell passes and 4, 8, 12, … tube passes
(*e*) Cross-flow with both fluids unmixed
(*f*) Cross-flow with one fluid mixed and the other unmixed

Extracted from Y.A. Cengel, "Heat Transfer: A Practical Approach", 2nd Edition.

Figure 14: *NTU plots extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*

Diffusion Dimensionless Numbers

$$\text{Sc} = \frac{\mu}{\rho\, D_{AB}} \qquad\qquad \text{Le} = \frac{k}{\rho\, C_p\, D_{AB}}$$

Diffusion
General expression for the flux:

$$\boldsymbol{N}_A = \boldsymbol{J}_A + x_A \sum_B \boldsymbol{N}_B$$

Fick's law:

$$\boldsymbol{J}_A = -D_{AB}\, \nabla C_A$$

Stefan's law:

$$N_{s,r} = -D \frac{c}{1-x} \frac{\partial x}{\partial r}$$

Ideal Gas

$$P\, V = n\, R\, T \qquad\qquad R \approx 8.314598 \text{ J K}^{-1} \text{ mol}^{-1}$$

Geometry

$$P_{\text{circle}} = 2\,\pi\, r \qquad A_{\text{circle}} = \pi\, r^2 \qquad A_{\text{sphere}} = 4\,\pi\, r^2 \qquad V_{\text{sphere}} = \frac{4}{3}\pi\, r^3$$

$$A_{\text{cylinder}} = P_{\text{circle}}\, L \qquad V_{\text{cylinder}} = A_{\text{circle}}\, L$$