UNIVERSITY OF ABERDEEN SESSION 2017–18

EX3030

Degree Examination in EX3030 Heat, Mass, & Momentum Transfer

14th December 2017 9 am – 12 pm

PLEASE NOTE THE FOLLOWING

(i) You **must not** have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material **must not** be amended, annotated or modified in any way.

(ii) You **must not** have in your possession any material that could be determined as giving you an advantage in the examination.

(iii) You **must not** attempt to communicate with any candidate during the exam, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

(iv) You **must not** take to your examination desk any electronic devices such as mobile phones or other "smart" devices. The only exception to this rule is an approved calculator.

Failure to comply with the above will be regarded as cheating and may lead to disciplinary action as indicated in the Academic Quality Handbook.

Notes:

(i) Candidates ARE permitted to use an approved calculator.

(ii) Candidates ARE NOT permitted to use the Engineering Mathematics Handbook.

(iii) Candidates ARE NOT permitted to use GREEN or RED pen in their exam booklet.

(iv) Data sheets are attached to the paper.

Candidates must attempt *ALL* questions in *BOTH* parts. All questions are worth 20 marks.

PART A: Answer using exam booklets

Question 1

In a plate heat-exchanger, water is heated by forcing it between alternating plates and heat is exchanged through the walls with a hot process stream. In order to design such an exchanger, we need to know what the relationship is between pressure drop, flow velocity, and volumetric flow-rate.

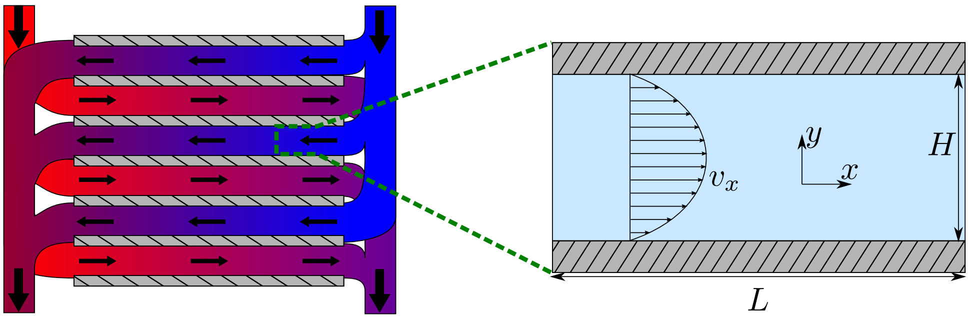


Figure 1: *A plate heat exchanger (left) and the simplification to steady state, pressure driven flow between two horizontal plates (right).*

You may neglect the effect of heat transfer on the flow. Water is incompressible and Newtonian to a good approximation. For simplicity, you can also assume that the flow is laminar.

a) Simplify the continuity equation for this system:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho\, \boldsymbol{v}$$

What does your result state about the flow velocity in the x-direction? **[4 marks]**

b) Simplify the x-component of the Cauchy momentum equation:

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho\, \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla p + \rho\, \boldsymbol{g}$$

Derive the following balance expression for the flow velocity v_x as a function of the pressure drop and position y: **[6 marks]**

$$\mu \frac{\partial^2 v_x}{\partial y^2} = \frac{\partial p}{\partial x}$$

c) Continuing from the result of the previous question, derive the following expression for the velocity v_x as a function of y using the no-slip boundary condition at the plate surfaces ($v_x = 0$ at $y = 0$ and $y = H$). **[6 marks]**

$$v_x = \frac{p_{out} - p_{in}}{2\, \mu\, L}(y^2 - H\, y)$$

d) Integrate the velocity over the plate height and width to prove the following expression for the volumetric flow of liquid through the gap as a function of pressure drop: **[4 marks]**

$$\dot{V}_x = \frac{Z\,H^3}{12\,\mu}\frac{\Delta P}{L}$$

[Question total: 20marks]

Question 2

a) A pebble-bed nuclear reactor at 69 bara is used to heat helium (4 g mol^{-1}) as part of the generation of electricity. The helium gas has a heat capacity at constant pressure of C_p = 5190 J kg^{-1} K^{-1}, a dynamic viscosity of $\mu = 5.19\times10^{-5}$ Pa s, and a thermal conductivity of k = 0.405 W m^{-1} K^{-1} and flows at 15 m s^{-1}. The pebbles have an outer radius of 3 cm which consists of a 0.5 cm coating of graphite around the radioactive core.

i) Assuming helium may be treated as an ideal gas, demonstrate that the density of the gas is 2.83 kg m^{-3}. **[3 marks]**

ii) Calculate the surface temperature of the particle if it is emitting 850 W of heat and the surrounding helium is at 900 °C. The following expression for forced convective heat-transfer around a sphere is available,

$$\mathrm{Nu}_D = 2 + 0.47\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{0.36} \quad \text{for } 3\times10^{-3} < \mathrm{Pr} < 10 \text{ and } 10^2 < \mathrm{Re}_D < 5\times10^4.$$

Radiation is neglible as all pellets have the same surface temperature, and the characteristic length used in the Reynolds and Nusselt number is the sphere diameter. **[12 marks]**

b) The James webb telescope uses a radiation shield to reduce the heat it recieves from the sun, earth, and moon (see Fig. 2). By what factor will the radiation be approximately reduced by? How realistic is this estimate (what approximations are there)? Is this an over or under estimate of the reduction in radiation? **[5 marks]**

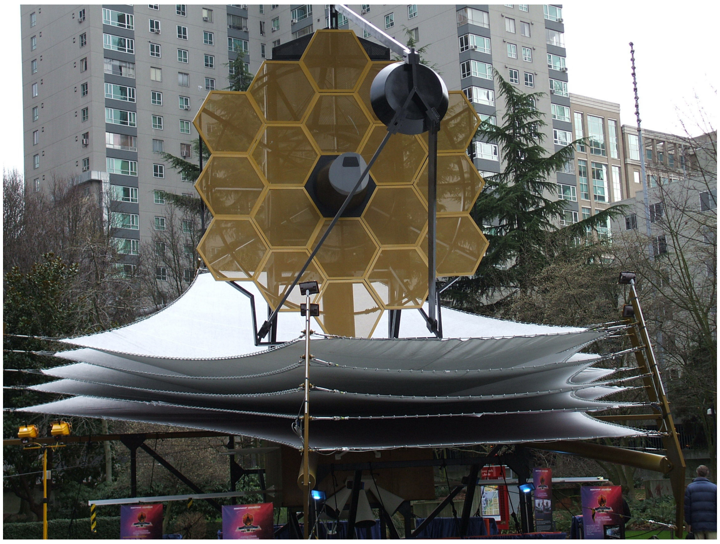


Figure 2: *A mock-up of the James Webb telescope, displaying its five-layered sun-shield.*

[Question total: 20marks]

Question 3

Consider a spherical aggregate (or ball) of bacterial cells (assumed to be homogenous) of radius R. Under certain circumstances, the oxygen metabolism rate of the bacterial cells is an almost constant reaction (zero-order) with respect to the oxygen concentration $\sigma_{O_2} = -k_{O_2}$. The diffusion of oxygen within the ball may be described by Fick's law with an effective pseudobinary diffusivity for oxygen in the bacterial medium of D_{O_2-M}. Neglect transient and convection effects because the oxygen solubility is very low in the system. Let $C_{O_2}^{(R)}$ be the oxygen mass concentration at the aggregate surface:

a) Show all of your working and state all assumptions made while demonstrating that the oxygen balance for the system,

$$\frac{\partial C_{O_2}}{\partial t} = -\nabla \cdot \boldsymbol{N}_{O_2} + \sigma_{O_2},$$

simplifies to the following expression,

$$\frac{\partial}{\partial r}\left(r^2\, N_{O_2,\,r}\right) = -k_{O_2}\, r^2.$$

[4 marks]

b) Demonstrate that the oxygen flux obeys the following relationship:

$$N_{O_2,\,r} = -k_{O_2}\, R^2\left(\frac{r}{3\,R^2} + \frac{C_1\,R}{6\,r^2}\right)$$

where C_1 is an unknown constant. **[4 marks]**

c) Demonstrate that the concentration profile obeys the following form in the limit that the O_2 concentration is small:

$$C_{O_2} = \frac{k_{O_2}\,R^2}{6\,D_{O_2-M}}\left(\frac{r^2}{R^2} - C_1\frac{R}{r}\right) + C_2$$

[4 marks]

d) Using the only available boundary condition, determine C_2 and demonstrate that the final expression for the concentration is:

$$C_{O_2} = \frac{k_{O_2}\,R^2}{6\,D_{O_2-M}}\left(\frac{r^2}{R^2} + C_1\left(1 - \frac{R}{r}\right) - 1\right) + C_{O_2}^{(R)}$$

[4 marks]

e) It is possible that the spherical bacterial ball has an oxygen-free core ($C_{O_2} = 0$ for $r \leq r_{core}$). Prove that this only happens for:

$$\frac{k_{O_2}\,R^2}{D_{O_2-M}\,C_{O_2}^{(R)}} \geq 6$$

Hints: As the concentration and diffusive flux are continuous, they both must go to zero at the core radius r_{core}. Use this to solve for C_1, then solve for r_{core} and consider what is required if $r_{core} \geq 0$. **[4 marks]**

[Question total: 20marks]

Question 4

a) A new material is to be developed for bearing balls in a new rolling-element bearing. For annealing (heat treatment) each bearing ball, a sphere of radius r_o = 5 mm, is heated in a furnace until it reaches to the equilibrium temperature of the furnace at 400°C. Then, it is suddenly removed from the furnace and subjected to a two-step cooling process.

Stage 1: Cooling in an air flow of 20°C for a period of time t_{air} until the center temperature reaches 335°C. For this situation, the convective heat transfer coefficient of air is assumed constant and equal to h = 10 W/(m^2.K). After the sphere has reached this specific temperature, the second step is initiated.

Stage 2: Cooling in a well-stirred water bath at 20°C, with a convective heat transfer coefficient of water h = 6000 W/(m^2.K).

The thermophysical properties of the material are ρ = 3000 kg/m^3, κ = 20 W/(m.K), C_p = 1000 J/(kg.K). Determine:

i) The time t_{air} required for *Stage 1* of the annealing process to be completed; **[5 marks]**

ii) The time t_{water} required for *Stage 2* of the annealing process during which the center of the sphere cools from 335°C (the condition at the completion of *Stage 1*) to 50°C. **[6 marks]**

b) Water at the rate of 68 kg/min is heated from 35 to 75°C by an oil having a specific heat of 1.9 kJ/(kg.°C). The fluids are used in a counterflow double-pipe HE, and the oil enters the exchanger at 110°C and leaves at 75°C. The overall heat-transfer coefficient is 320 W/(m^2.°C). Given heat capacity of water (at constant pressure) of 4.18 kJ/(kg.°C),

i) Calculate the HE area; **[5 marks]**

ii) Now assume that the heat exchanger is a shell-and-tube with water making one shell pass and the oil making two tube passes. Calculate the area of the new heat exchanger. Assume that the overall heat-transfer coefficient remains the same. **[4 marks]**

[Question total: 20marks]

PART B: Answer using the multiple choice sheet

All questions are worth 2 marks.

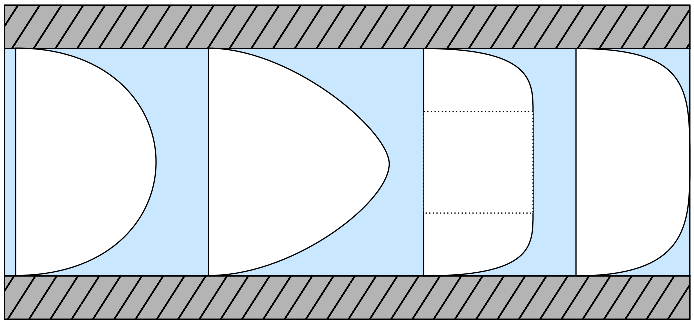


Figure 3: *Non-newtonian flow profiles compared against the newtonian flow profile.*

1) In Fig. 3, which profile corresponds to a viscoplastic fluid? **[2 marks]**

2) Which profile in Fig. 3 corresponds to a shear thickening fluid? **[2 marks]**

3) What are the value(s) of the flow index n in the Power-law model for a shear thickening fluid? **[2 marks]**

   A) $n < 0$

   B) $n < 1$

   C) $n = 1$

   D) $n > 1$

4) What is the Nusselt number for conduction through a plate of thickness L and conductivity k? **[2 marks]**

   A) $\mathrm{Nu} = 1$

   B) $\mathrm{Nu} = k/L$

   C) $\mathrm{Nu} = C\,\mathrm{Re}^n\,\mathrm{Pr}^m$

   D) $\mathrm{Nu} = L/(k\,A)$

5) The Grashof number is a ratio of what two properties? **[2 marks]**

   A) Drag and viscous forces

   B) Momentum diffusivity and thermal diffusivity

   C) Buoyancy forces and thermal diffusivity

   D) Buoyancy forces and viscous forces

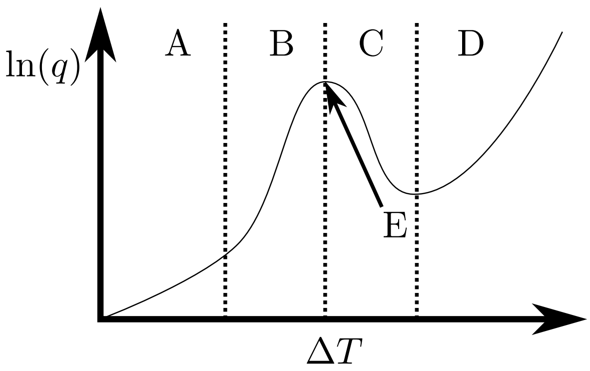


Figure 4: *A typical boiling heat flux versus driving temperature difference curve.*

6) Which region of the boiling curve in Fig. 4 is the nucleate boiling regime? **[2 marks]**

7) Which region of the boiling curve in Fig. 4 is the operation of the boiler unstable? **[2 marks]**

8) In radiation, object 1 is entirely surrounded by object 2. The area of object 1 is 15 m^2, while object 2 has a surface area of 58 m^2. What is the view-factor of object 2 from the point-of-view of object 1, i.e., $F_{1\to2}$? **[2 marks]**

   A) $F_{1\to2} \approx 0.259$
   B) $F_{1\to2} = 1$
   C) $F_{1\to2} =\approx 3.87$
   D) $F_{1\to2} = 0$

9) At what temperature should the properties used in the Prandtl number be evaluated for a pipe with a temperature drop across its length? **[2 marks]**

   A) Inlet temperature
   B) Average of the wall and bulk temperature
   C) Wall temperature
   D) Centerline temperature
   E) Average of inlet and outlet temperature

10) Two liquids flowing together in a channel, what is NOT a valid boundary condition? **[2 marks]**

   A) Stress in each phase is equal at the interface
   B) No-slip between the two phases at the interface
   C) No stress at the interface
   D) No-slip between the fluid(s) and the adjacent wall

END OF PAPER

DATASHEET

General balance equations:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho\, \boldsymbol{v} \qquad \text{(Mass/Continuity)} \quad (1)$$

$$\frac{\partial C_A}{\partial t} = -\nabla \cdot \boldsymbol{N}_A + \sigma_A \qquad \text{(Species)} \quad (2)$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho\, \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla p + \rho\, \boldsymbol{g} \qquad \text{(Momentum)} \quad (3)$$

$$\rho\, C_p \frac{\partial T}{\partial t} = -\rho\, C_p\, \boldsymbol{v} \cdot \nabla\, T - \nabla \cdot \boldsymbol{q} - \boldsymbol{\tau} : \nabla\, \boldsymbol{v} - p \nabla \cdot \boldsymbol{v} + \sigma_{energy} \qquad \text{(Heat/Energy)} \quad (4)$$

In Cartesian coordinate systems, ∇ can be treated as a vector of derivatives. In curve-linear coordinate systems, the directions $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\phi}$ depend on the position. For convenience in these systems, look-up tables are provided for common terms involving ∇.

Cartesian coordinates (with index notation examples)
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor.

$$\nabla s = \nabla_i s = \left[\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \nabla_i \nabla_i s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \nabla_i v_i = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\nabla \cdot \boldsymbol{\tau} = \nabla_i\, \tau_{ij}$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\boldsymbol{v} \cdot \nabla \boldsymbol{v} = v_i\, \nabla_i\, v_j$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_x = v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_y = v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}$$

Cylindrical coordinates

where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r}\frac{\partial}{\partial r}(r\, v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{1}{r}\tau_{\theta\theta} + \frac{\partial \tau_{rz}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r}\tau_{r\theta} + \frac{\partial \tau_{\theta z}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r\, v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

Spherical coordinates

where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial s}{\partial \phi}\right]$$

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial s}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 s}{\partial \phi^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{\theta\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\phi}\right) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r\, v_\theta - v_\phi^2\cot\theta}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\phi = v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r\, v_\phi + v_\theta\, v_\phi\cot\theta}{r}$$

Rectangular		Cylindrical		Spherical	
q_x	$-k\frac{\partial T}{\partial x}$	q_r	$-k\frac{\partial T}{\partial r}$	q_r	$-k\frac{\partial T}{\partial r}$
q_y	$-k\frac{\partial T}{\partial y}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$
q_z	$-k\frac{\partial T}{\partial z}$	q_z	$-k\frac{\partial T}{\partial z}$	q_ϕ	$-k\frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}$
τ_{xx}	$-2\,\mu\frac{\partial v_x}{\partial x} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{yy}	$-2\,\mu\frac{\partial v_y}{\partial y} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\phi\phi}$	$-2\,\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r + v_\theta\cot\theta}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{xy}	$-\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$
τ_{yz}	$-\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$	$\tau_{\theta z}$	$-\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)$	$\tau_{\theta\phi}$	$-\mu\left(\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right)$
τ_{xz}	$-\mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$	τ_{zr}	$-\mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)$	$\tau_{\phi r}$	$-\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right)$

Table 1: *Fourier's law for the heat flux and Newton's law for the stress in several coordinate systems. Please remember that the stress is symmetric, so $\tau_{ij} = \tau_{ji}$.*

Viscous models:
Power-Law Fluid:

$$|\tau_{xy}| = k\left|\frac{\partial v_x}{\partial y}\right|^n \tag{5}$$

Bingham-Plastic Fluid:

$$\frac{\partial v_x}{\partial y} = \begin{cases} -\mu^{-1}\left(\tau_{xy} - \tau_0\right) & \text{if } \tau_{xy} > \tau_0 \\ 0 & \text{if } \tau_{xy} \le \tau_0 \end{cases}$$

Dimensionless Numbers

$$\mathrm{Re} = \frac{\rho\,\langle v\rangle\,D}{\mu} \qquad \mathrm{Re}_H = \frac{\rho\,\langle v\rangle\,D_H}{\mu} \qquad \mathrm{Re}_{MR} = -\frac{16\,L\,\rho\,\langle v\rangle^2}{R\,\Delta p} \tag{6}$$

The hydraulic diameter is defined as $D_H = 4\,A/P_w$.

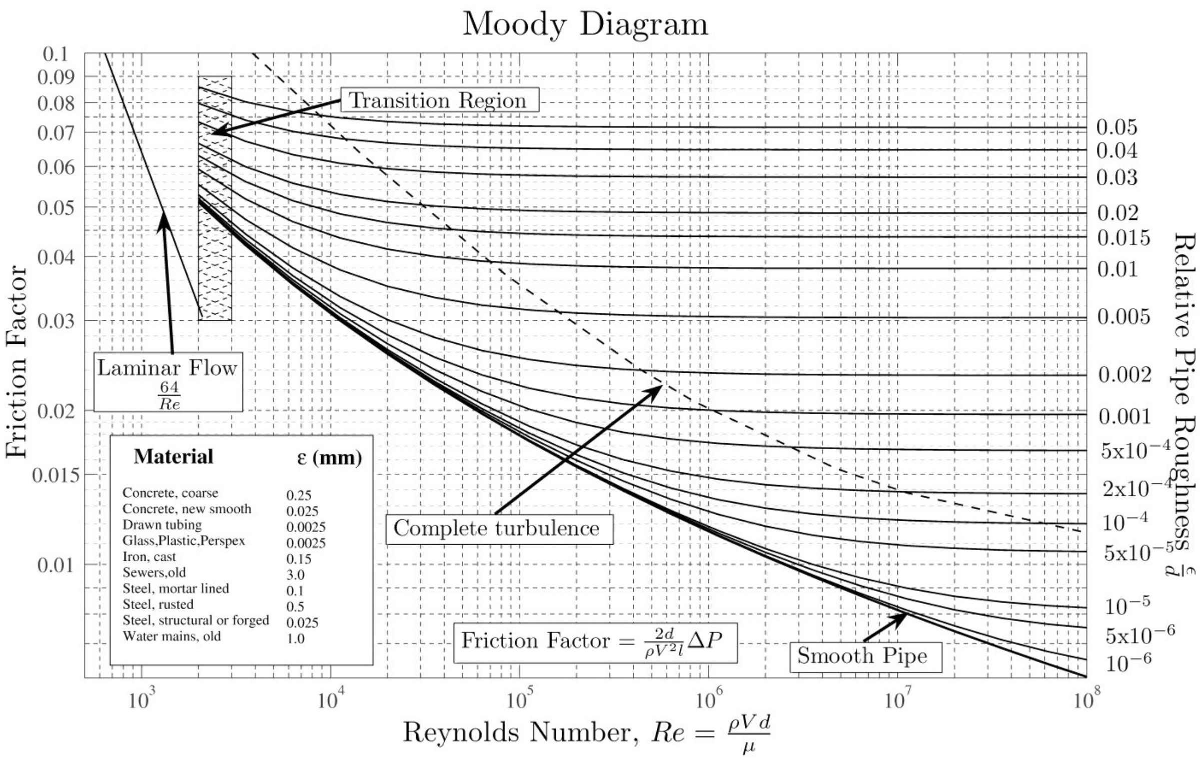
Single phase pressure drop calculations in pipes:
Darcy-Weisbach equation:

$$\frac{\Delta p}{L} = -\frac{C_f\,\rho\,\langle v\rangle^2}{R} \tag{7}$$

where $C_f = 16/Re$ for laminar Newtonian flow. For turbulent flow of Newtonian fluids in smooth pipes, we have the Blasius correlation:

$$C_f = 0.079\,\mathrm{Re}^{-1/4} \qquad \text{for } 2.5 \times 10^3 < \mathrm{Re} < 10^5 \text{ and smooth pipes.}$$

Otherwise, you may refer to the Moody diagram.



Laminar Power-Law fluid:

$$\dot{V} = \frac{n\,\pi\,R^3}{3\,n+1}\left(\frac{R}{2\,k}\right)^{\frac{1}{n}}\left(-\frac{\Delta p}{L}\right)^{\frac{1}{n}}$$

Two-Phase Flow:

Lockhart-Martinelli parameter:

$$X^2 = \frac{\Delta p_{liq.-only}}{\Delta p_{gas-only}}$$

Pressure drop calculation:

$$\Delta p_{two-phase} = \Phi^2_{liq.}\,\Delta p_{liq.-only} = \Phi^2_{gas}\,\Delta p_{gas-only}$$

Chisholm's relation:

$$\Phi^2_{gas} = 1 + c\,X + X^2$$

$$\Phi^2_{liq.} = 1 + \frac{c}{X} + \frac{1}{X^2} \qquad c = \begin{cases} 20 & \text{turbulent liquid \& turbulent gas} \\ 12 & \text{laminar liquid \& turbulent gas} \\ 10 & \text{turbulent liquid \& laminar gas} \\ 5 & \text{laminar liquid \& laminar gas} \end{cases}$$

Farooqi and Richardson expression for liquid hold-up in co-current flows of Newtonian fluids and air in horizontal pipes:

$$h = \begin{cases} 0.186 + 0.0191\,X & 1 < X < 5 \\ 0.143\,X^{0.42} & 5 < X < 50 \\ 1/\,(0.97 + 19/X) & 50 < X < 500 \end{cases}$$

Heat Transfer Dimensionless numbers:

$$\mathrm{Nu} = \frac{h\,L}{k} \qquad \mathrm{Pr} = \frac{\mu\,C_p}{k} \qquad \mathrm{Gr} = \frac{g\,\beta\,(T_w - T_\infty)\;L^3}{\nu^2}$$

where $\beta = V^{-1}(\partial V/\partial T)$.

Heat transfer: Resistances

$$Q = U_T\,A_T\,\Delta T = R_T^{-1}\,\Delta T$$

	Conduction Shell Resistances			Radiation
	Rect.	Cyl.	Sph.	
R	$\frac{X}{k\,A}$	$\frac{\ln(R_{outer}/R_{inner})}{2\,\pi\,L\,k}$	$\frac{R_{inner}^{-1} - R_{outer}^{-1}}{4\,\pi\,k}$	$\left[A\,\varepsilon\,\sigma\left(T_j^2 + T_i^2\right)\left(T_j + T_i\right)\right]^{-1}$

Radiation Heat Transfer:

Stefan-Boltzmann constant $\sigma = 5.6703 \times 10^{-8}$ W m^{-2} K^{-4}.
Summation relationship, $\sum_j F_{i\to j} = 1$, and reciprocity relationship, $F_{i\to j}\,A_i = F_{j\to i}\,A_j$.
Radiation shielding factor $1/(N+1)$.

$$Q_{rad.,i\to j} = \sigma\,\varepsilon\,F_{i\to j}\,A_i\left(T_j^4 - T_i^4\right) = h_{rad.}\,A\,(T_\infty - T_w)$$

Natural Convection

Ra = Gr Pr	C	m
$< 10^4$	1.36	1/5
10^4–10^9	0.59	1/4
$> 10^9$	0.13	1/3

Table 2: *Natural convection coefficients for isothermal vertical plates in the empirical relation* $Nu \approx C\,(Gr\,Pr)^m$.

For isothermal vertical cylinders, the above expressions for isothermal vertical plates may be used but must be scaled by a factor, F (i.e., $\mathrm{Nu}_{v.cyl.} = F\,\mathrm{Nu}_{v.plate}$):

$$F = \begin{cases} 1 & \text{for } (D/H) \geq 35\,\mathrm{Gr}_H^{-1/4} \\ 1.3\left[H\,D^{-1}\,\mathrm{Gr}_D^{-1}\right]^{1/4} + 1 & \text{for } (D/H) < 35\,\mathrm{Gr}_H^{-1/4} \end{cases}$$

where D is the diameter and H is the height of the cylinder. The subscript on Gr indicates which length is to be used as the critical length to calculate the Grashof number.

Churchill and Chu expression for natural convection from a horizontal pipe:

$$\mathrm{Nu}^{1/2} = 0.6 + 0.387\left\{\frac{\mathrm{Gr}\,\mathrm{Pr}}{\left[1 + (0.559/\mathrm{Pr})^{9/16}\right]^{16/9}}\right\}^{1/6} \qquad \text{for } 10^{-5} < \mathrm{Gr}\,\mathrm{Pr} < 10^{12}$$

Forced Convection:
Laminar flows:

$$\mathrm{Nu} \approx 0.332\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

Well-Developed turbulent flows in smooth pipes:

$$\mathrm{Nu} \approx \frac{(C_f/2)\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(C_f/2)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Boiling:
Forster-Zuber pool-boiling coefficient:

$$h_{nb} = 0.00122\frac{k_L^{0.79}\,C_{p,L}^{0.45}\,\rho_L^{0.49}}{\gamma^{0.5}\,\mu_L^{0.29}\,h_{fg}^{0.24}\,\rho_G^{0.24}}\,(T_w - T_{sat})^{0.24}\,(p_w - p_{sat})^{0.75}$$

Mostinski correlations:

$$h_{nb} = 0.104\,p_c^{0.69}\,q^{0.7}\left[1.8\left(\frac{p}{p_c}\right)^{0.17} + 4\left(\frac{p}{p_c}\right)^{1.2} + 10\left(\frac{p}{p_c}\right)^{10}\right]$$

$$q_c = 3.67\times 10^4\,p_c\left(\frac{p}{p_c}\right)^{0.35}\left[1 - \frac{p}{p_c}\right]^{0.9}$$

(**Note**: for the Mostinski correlations, the pressures are in units of bar)
Condensing:
Horizontal pipes

$$h = 0.72\left(\frac{k^3\,\rho^2\,g_x\,E_{latent}}{D\,\mu\,(T_w - T_\infty)}\right)^{1/4}$$

Lumped capacitance method:

$$\mathrm{Bi} = \frac{h\,L_c}{\kappa}$$

$$L_c = V/A \qquad \text{for Bi} < 0.1$$

$$\frac{T(t) - T_\infty}{T_0 - T_\infty} = e^{-bt} \qquad b = \frac{hA_s}{\rho V C_p}$$

1-D Transient Heat Conduction:

$$Fo = \frac{\alpha \Delta t}{(\Delta x)^2} = \tau, \qquad \alpha = \kappa \left(\rho C_p\right)^{-1}$$

$$\theta_{\text{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos\left(\frac{\lambda_1 x}{L}\right), \qquad \theta_{\text{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \mathbf{J_0}\left(\frac{\lambda_1 r}{r_0}\right)$$

$$\theta_{\text{sph}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin\left(\frac{\lambda_1 r}{r_0}\right)}{\frac{\lambda_1 r}{r_0}}$$

$$\theta_{0,\text{wall}} = \theta_{0,\text{cyl}} = \theta_{0,\text{sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\max}}\right)_{\text{wall}} = 1 - \theta_{0,\text{wall}} \frac{\sin \lambda_1}{\lambda_1}, \quad \left(\frac{\mathcal{Q}}{\mathcal{Q}_{\max}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\text{cyl}} \frac{\mathbf{J_1}(\boldsymbol{\lambda_1})}{\lambda_1}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\max}}\right)_{\text{sph}} = 1 - 3\theta_{0,\text{sph}} \frac{\sin \lambda_1 - \lambda_1 \cos \lambda_1}{\lambda_1^3}$$

Finite-Difference Method:

$$\frac{\partial}{\partial t}(\rho \phi) + \frac{\partial}{\partial x}(\rho \boldsymbol{v} \phi) = \nabla \cdot (\Gamma \nabla \phi) + \mathcal{S} \qquad \text{(1D transport equation)}$$

$$\left(\frac{d\phi}{dx}\right)_i = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta x} \quad \text{and} \quad \left(\frac{d^2\phi}{dx^2}\right)_i = \frac{\phi_{i-1} + \phi_{i+1} - 2\phi_i}{(\Delta x)^2}$$

$$T_i^{j+1} = (1-2\tau)\,T_i^j + \tau\left(T_{i+1}^j + T_{i-1}^j\right) + \frac{\tau\,(\Delta x)^2}{\kappa}\mathcal{S}_i^j$$

Overall Heat Transfer Coefficient:

$$\dot{\mathcal{Q}} = \frac{\Delta T}{\mathcal{R}} = UA\Delta T = U_iA_i\Delta T = U_oA_o\Delta T$$

$$\mathcal{R} = R_i + R_{\text{wall}} + R_o = \frac{1}{h_iA_i} + \frac{\ln D_o/D_i}{2\pi\kappa L} + \frac{1}{h_oA_o}$$

Fouling Factor:

$$\mathcal{R} = \frac{1}{h_iA_i} + \frac{R_{\text{f,i}}}{A_i} + R_{\text{wall}} + \frac{R_{\text{f,o}}}{A_o} + \frac{1}{h_oA_o}$$

LMTD Method:

$$\dot{\mathcal{Q}} = UA_s\Delta T_{\text{lm}} \quad \text{with} \quad \Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln\frac{\Delta T_2}{\Delta T_1}} = \frac{\Delta T_1 - \Delta T_2}{\ln\frac{\Delta T_1}{\Delta T_2}}$$

$$\text{Parallel flows:}\begin{cases}\Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,in}}\\ \Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,out}}\end{cases}$$

$$\text{Counter flows:}\begin{cases}\Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,out}}\\ \Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,in}}\end{cases}$$

ϵ-NTU Method:

$$\epsilon = \frac{\dot{\mathcal{Q}}}{\dot{\mathcal{Q}}_{\max}}, \quad \text{with } \dot{\mathcal{Q}}_{\max} = \mathcal{C}_{\min}\left(T_{\text{hot,in}} - T_{\text{cold,in}}\right) \quad \text{and} \quad \mathcal{C}_{\min} = Min\left\{\dot{m}_{\text{hot}}C_{p,\text{hot}}, \dot{m}_{\text{cold}}C_{p,\text{cold}}\right\}$$

$$\text{NTU} = \frac{UA_s}{C_{\min}}$$

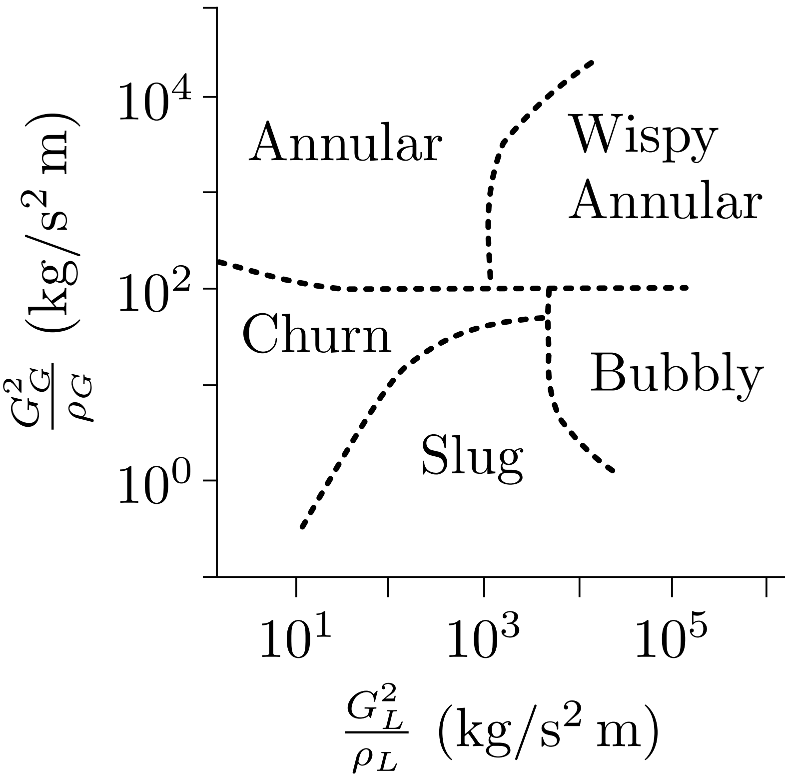


Figure 5: *Hewitt-Taylor flow pattern map for multiphase flows in vertical pipes.*

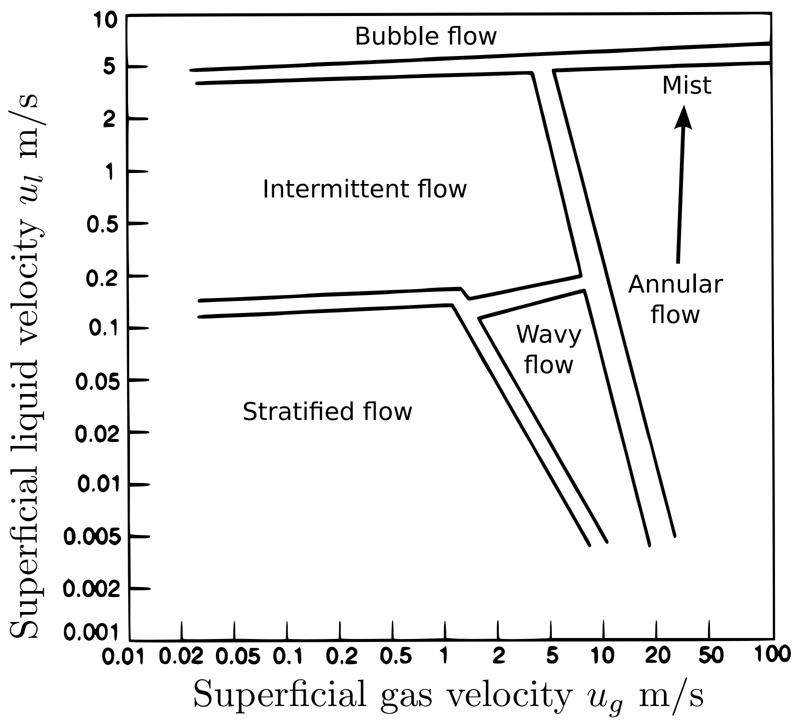


Figure 6: *Chhabra and Richardson flow pattern map for horizontal pipes.*

TABLE 4–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

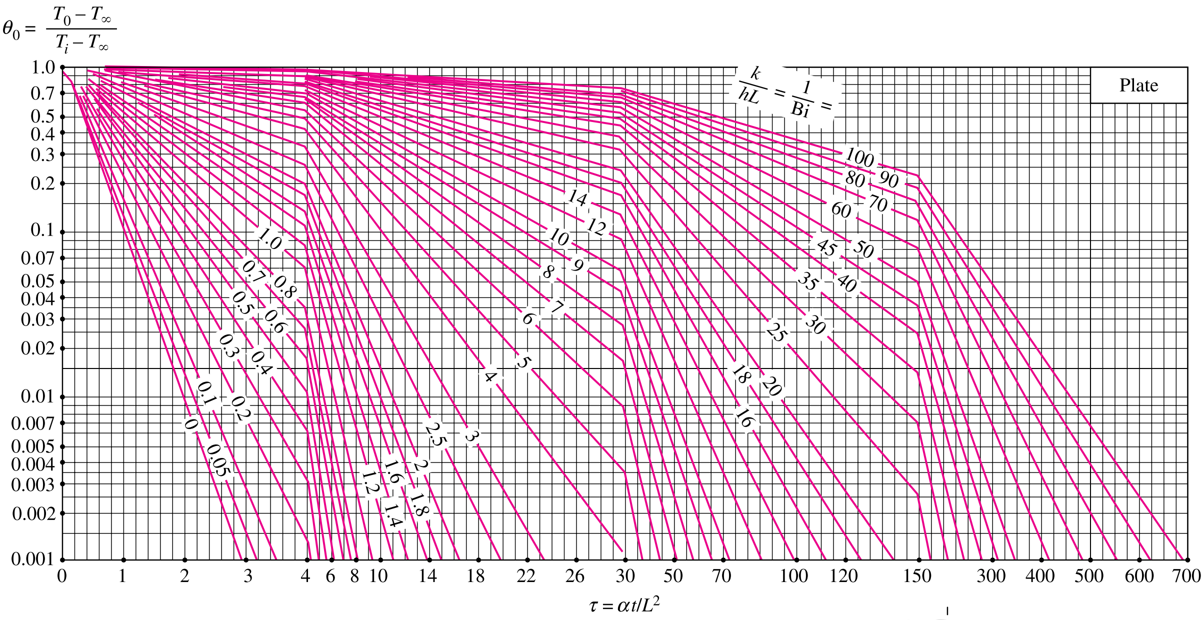
	Plane Wall		*Cylinder*		*Sphere*	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

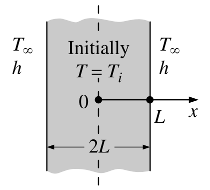
TABLE 4–3

The zeroth- and first-order Bessel functions of the first kind

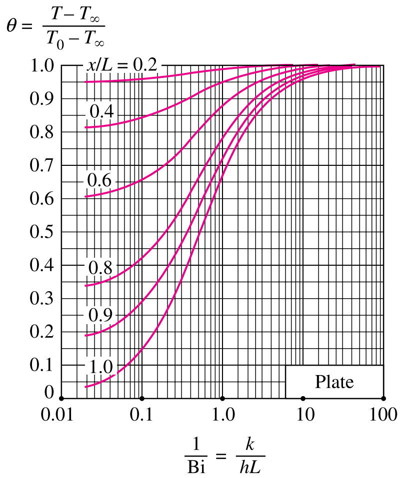
η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

Figure 7: *Coefficients for the 1D transient equations.*

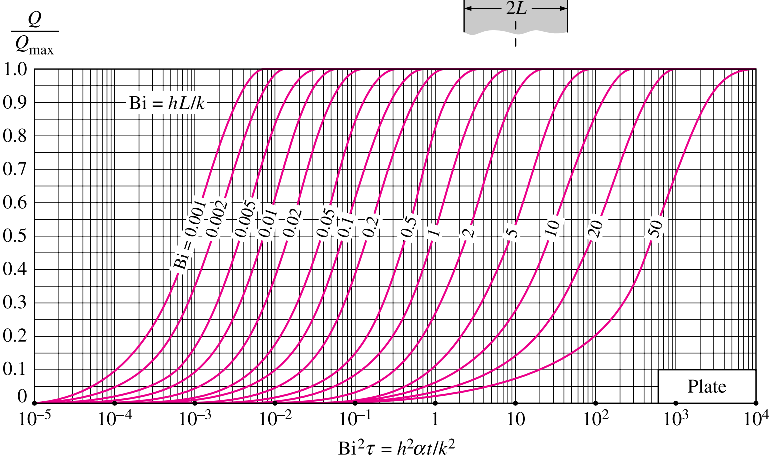




(*a*) Midplane temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)



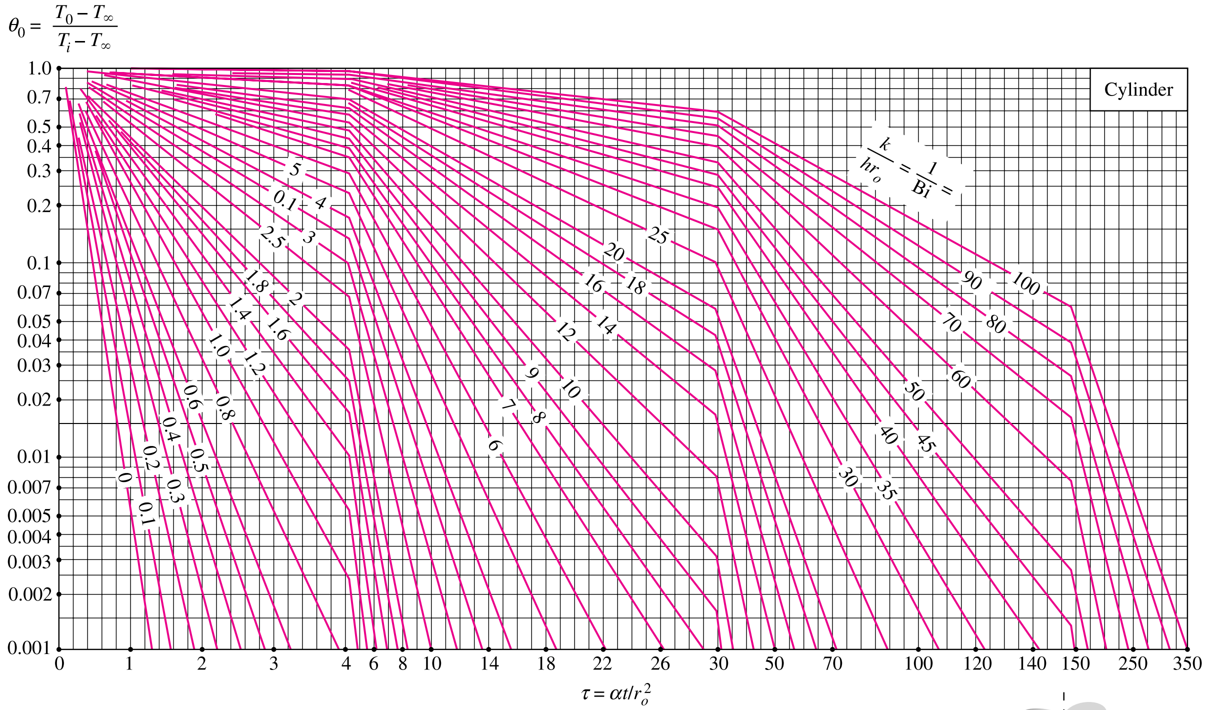
(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)



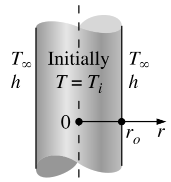
(*c*) Heat transfer (from H. Gröber et al.)

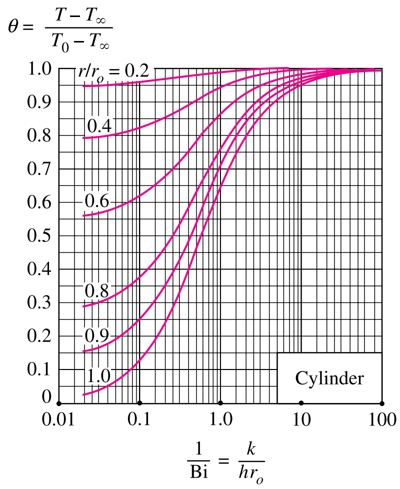
Transient temperature and heat transfer charts for a plane wall of thickness $2L$ initially at a uniform temperature T_i subjected to convection from both sides to an environment at temperature T_∞ with a convection coefficient of h.

Figure 8:

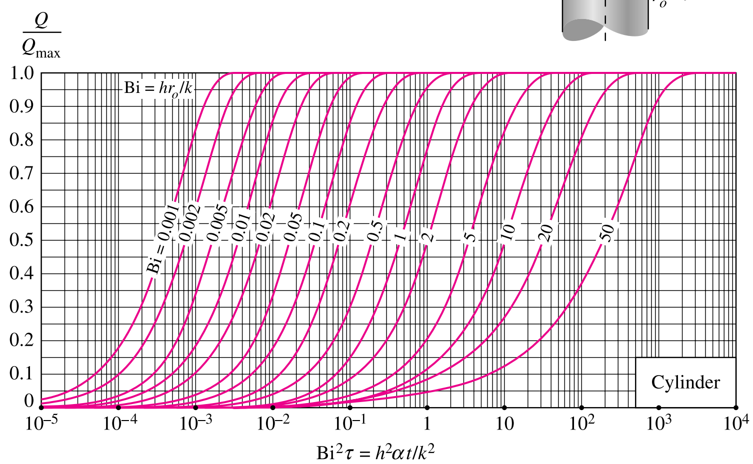


(*a*) Centerline temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





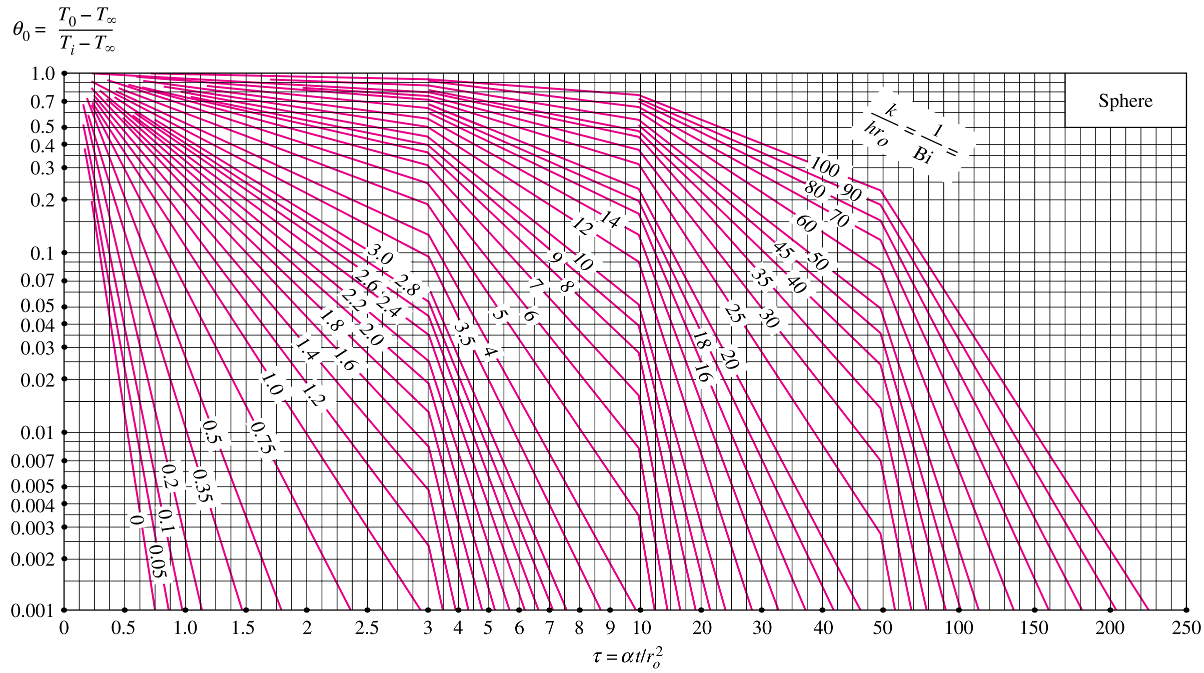
(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)



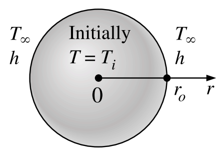
(*c*) Heat transfer (from H. Gröber et al.)

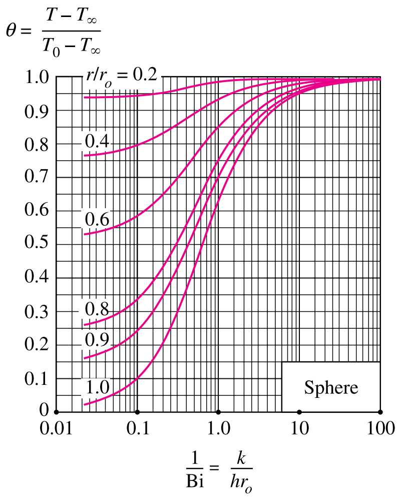
Transient temperature and heat transfer charts for a long cylinder of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

Figure 9:

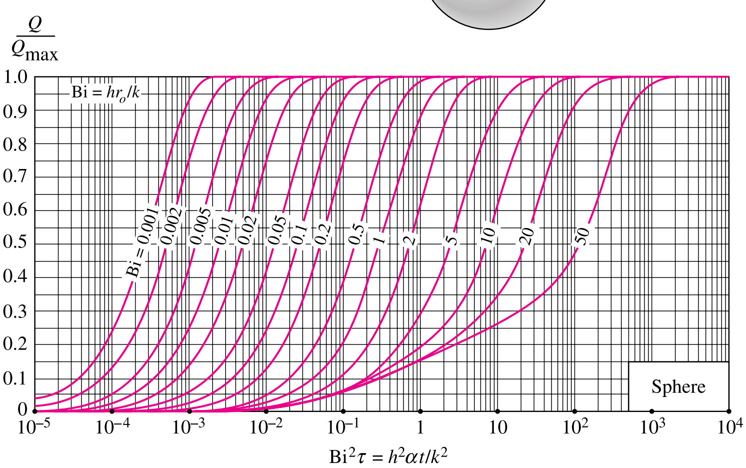


(*a*) Midpoint temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947,



(*c*) Heat transfer (from H. Gröber et al.)

Transient temperature and heat transfer charts for a sphere of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

Figure 10:

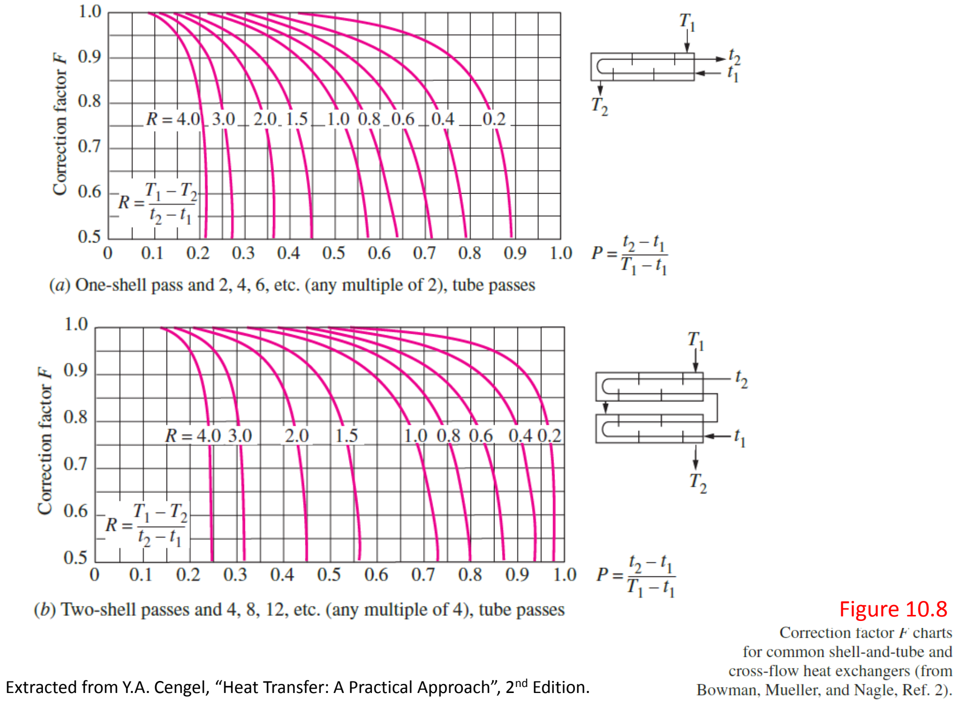


Figure 11: *Correction-factors for LMTD Method, extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*

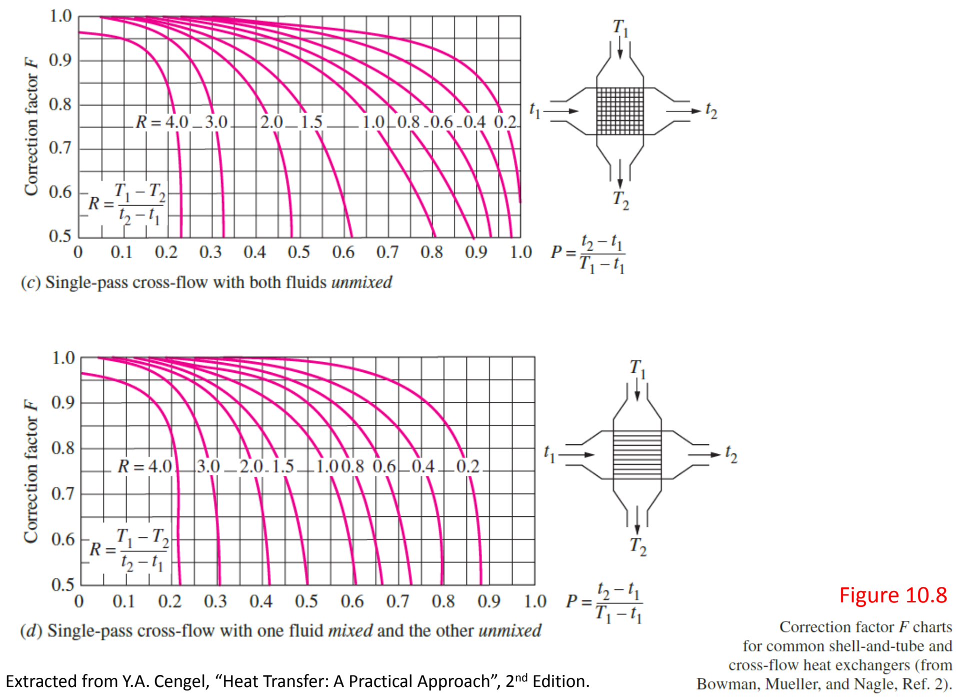


Figure 12: *Correction-factors for LMTD Method, extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.*

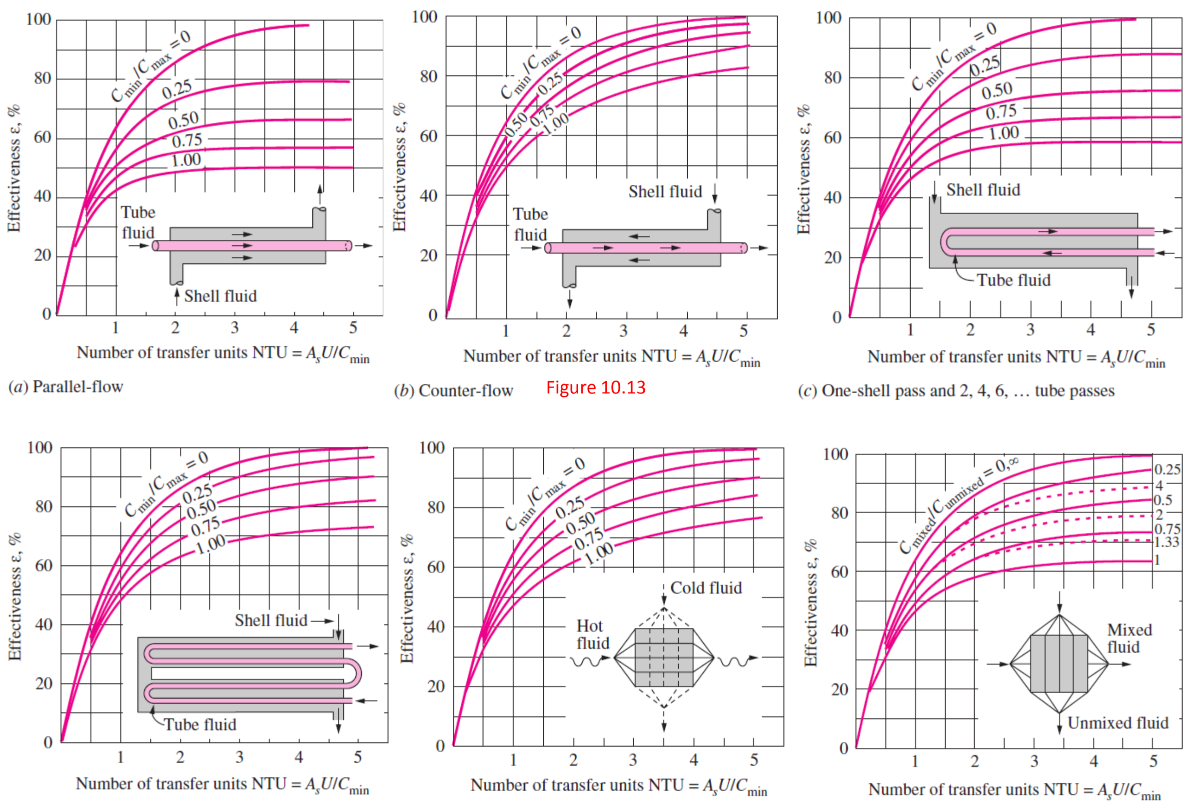
Effectiveness relations for heat exchangers: NTU = UA_s/C_{min} and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 + c)]}{1 + c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 - c)]}{1 - c\exp[-\text{NTU}(1 - c)]}$
2 *Shell and tube:*	
One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1 + c + \sqrt{1 + c^2}\,\dfrac{1 + \exp[-\text{NTU}\sqrt{1 + c^2}]}{1 - \exp[-\text{NTU}\sqrt{1 + c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)*	
Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}[\exp(-c\,\text{NTU}^{0.78}) - 1]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\{1 - c[1 - \exp(-\text{NTU})]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp(-c\,\text{NTU})]\right\}$
4 *All heat exchangers with c = 0*	$\varepsilon = 1 - \exp(-\text{NTU})$

NTU relations for heat exchangers NTU = UA_s/C_{min} and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	NTU relation
1 *Double-pipe:*	
Parallel-flow	$\text{NTU} = -\dfrac{\ln[1 - \varepsilon(1 + c)]}{1 + c}$
Counter-flow	$\text{NTU} = \dfrac{1}{c - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon c - 1}\right)$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$\text{NTU} = -\dfrac{1}{\sqrt{1 + c^2}}\ln\left(\dfrac{2/\varepsilon - 1 - c - \sqrt{1 + c^2}}{2/\varepsilon - 1 - c + \sqrt{1 + c^2}}\right)$
3 *Cross-flow (single-pass)* C_{max} mixed, C_{min} unmixed	$\text{NTU} = -\ln\left[1 + \dfrac{\ln(1 - \varepsilon c)}{c}\right]$
C_{min} mixed, C_{max} unmixed	$\text{NTU} = -\dfrac{\ln[c\ln(1 - \varepsilon) + 1]}{c}$
4 *All heat exchangers with c = 0*	$\text{NTU} = -\ln(1 - \varepsilon)$

Figure 13: *NTU relations extracted from Y. A. Cengel, “Heat transfer:A practical approach”, 2nd Ed.*



(*a*) Parallel-flow

(*b*) Counter-flow Figure 10.13

(*c*) One-shell pass and 2, 4, 6, … tube passes

(*d*) Two-shell passes and 4, 8, 12, … tube passes

(*e*) Cross-flow with both fluids unmixed

Figure 10.15

(*f*) Cross-flow with one fluid mixed and the other unmixed

Extracted from Y.A. Cengel, “Heat Transfer: A Practical Approach”, 2nd Edition.

Figure 14: *NTU plots extracted from Y. A. Cengel, “Heat transfer:A practical approach”, 2nd Ed.*

Diffusion Dimensionless Numbers

$$\mathrm{Sc} = \frac{\mu}{\rho\, D_{AB}} \qquad\qquad \mathrm{Le} = \frac{k}{\rho\, C_p\, D_{AB}}$$

Diffusion
General expression for the flux:

$$\boldsymbol{N}_A = \boldsymbol{J}_A + x_A \sum_B \boldsymbol{N}_B$$

Fick's law:

$$\boldsymbol{J}_A = -D_{AB}\,\nabla C_A$$

Stefan's law:

$$N_{s,r} = -D\frac{c}{1-x}\frac{\partial x}{\partial r}$$

Ideal Gas

$$P\,V = n\,R\,T \qquad\qquad R \approx 8.314598\ \mathrm{J\,K^{-1}\,mol^{-1}}$$

Geometry

$$P_{\text{circle}} = 2\,\pi\,r \qquad A_{\text{circle}} = \pi\,r^2 \qquad A_{\text{sphere}} = 4\,\pi\,r^2 \qquad V_{\text{sphere}} = \frac{4}{3}\pi\,r^3$$

$$A_{\text{cylinder}} = P_{\text{circle}}\,L \qquad V_{\text{cylinder}} = A_{\text{circle}}\,L$$