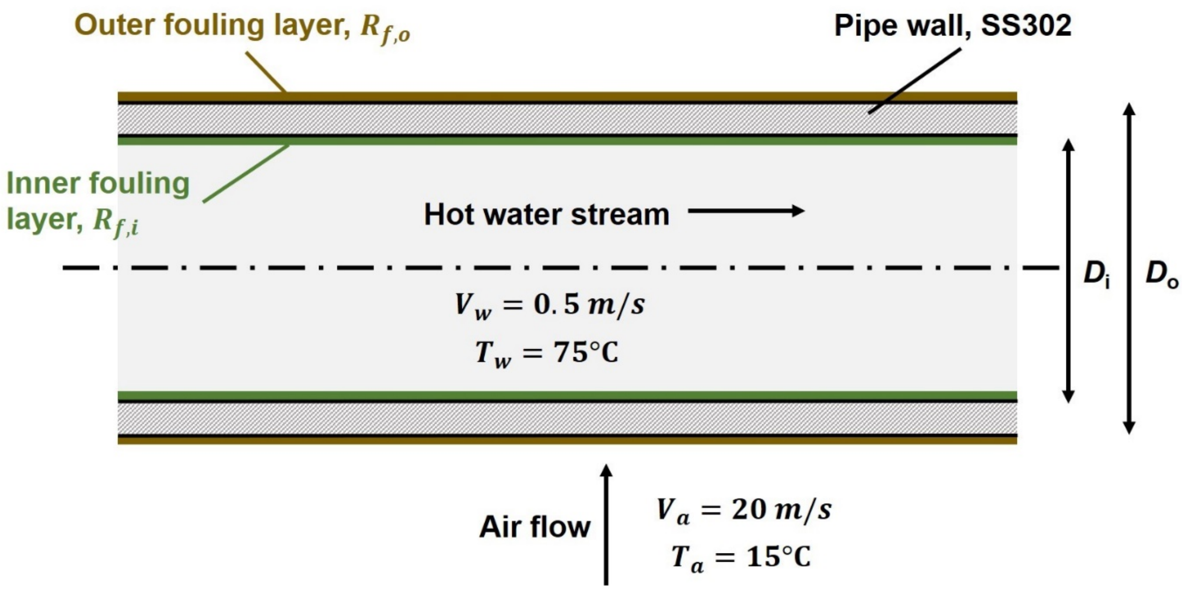
WORKED EXAMPLE

OVERALL HEAT TRANSFER COEFFICIENT

A type-302 stainless steel tube of inner and outer diameters D_i = 22 mm and D_o = 27 mm, respectively, is used in a cross-flow heat exchanger. The fouling factors, R_f, for the inner and outer surfaces are estimated to be 0.0004 and 0.0002 m²·K/W, respectively.

Determine the overall heat transfer coefficient based on the outside area of the tube, U_o.

Given the outer convection heat transfer coefficient has a value of h_o = 104 W/m²·K , the thermal conductivity of the pipe wall is k_{steel} = 15.1 W/m·K, the density and viscosity of the water are ρ = 974.8 kg/m³ and μ = 3.746× 10^{-4} Ns/m² , respectively, the Prandtl number is Pr = 2.354 and the thermal conductivity of the water is k_w = 0.668 W/m·K.



SOLUTION

Our objective is to find U_0

We can start by writing our expression for the overall heat transfer coefficient (equation 12 from the lecture notes):

$$\frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{R_{f,i}}{A_i} + R_{\text{wall}} + \frac{R_{f,o}}{A_o} + \frac{1}{h_o A_o}$$

From the equation above, we know the following:

Fouling factors:

$$R_{f,i} = 0.0004 \ [\mathrm{m^2K/W}]$$
$$R_{f,o} = 0.0002 \ [\mathrm{m^2K/W}]$$

Surface areas:

$$A_i = \pi D_i L = \pi \times 0.022 \times 1 = 0.06912 \ [\mathrm{m^2}]$$
$$A_o = \pi D_o L = \pi \times 0.027 \times 1 = 0.08482 \ [\mathrm{m^2}]$$

(Note, as we were not told the length of the pipe, we have taken $L = 1$ so the results will be on a 'per metre' basis.)

Resistance through the wall:

$$R_{\mathrm{wall}} = \frac{\ln(D_o/D_i)}{2\pi L k_{\mathrm{steel}}} = \frac{\ln(27/22)}{2\pi \times 1 \times 15.1} = 2.159 \times 10^{-3} \ [\mathrm{m^2K/W}]$$

However, we don't know the inner convection heat transfer coefficient h_i...

To resolve this we can use the Nusselt number relation below:

$$Nu = \frac{h_i D_i}{k} \begin{cases} 4.36 \ \ (\text{laminar flow}) \\ 0.023\mathrm{Re}^{0.8}\mathrm{Pr}^{\mathrm{n}} \ \ (\text{turbulent flow}) \end{cases}$$

where the expression $Nu = 0.023\mathrm{Re}^{0.8}\mathrm{Pr}^{n}$ is known as the Dittus-Boelter equation, which is an empirical relation for estimating the Nusselt number in turbulent flow.

Now we need to check if the flow is laminar or turbulent:

$$Re = \frac{\rho u D_i}{\mu} = \frac{974.8 \times 0.5 \times 0.022}{3.746 \times 10^{-3}} = 28{,}624$$

Therefore, since Re>>2000 the flow in the pipe is turbulent and we can get Nu from:

$$\begin{aligned} Nu &= 0.023\mathrm{Re}^{0.8}\mathrm{Pr}^{n} \\ &= 0.023 \times 28624^{0.8} \times 2.354^{0.3} \\ &= 109.31 \end{aligned}$$

(note, we have used $n = 0.3$ since the water in the pipe is being cooled, if the water was being heated then we would use $n = 0.4$)

We can now get the unknown h_i from:

$$h_i = \frac{Nu \cdot k_\mathrm{w}}{D_i} = \frac{109.31 \times 0.668}{0.022} = 3{,}320\ [\mathrm{W/m^2K}]$$

Now returning to the original equation for U_o we can input all known values to get:

$$\frac{1}{U_o A_o} = \frac{1}{3320 \times 0.06912} + \frac{0.0004}{0.06912} + 2.159 \times 10^{-3} + \frac{0.0002}{0.08482} + \frac{1}{104 \times 0.08482}$$

$$\frac{1}{U_o} = 0.12802516 \times 0.08482$$

$$U_o = \mathbf{92.1\ [W/m^2K]}$$