UNIVERSITY OF ABERDEEN SESSION 2015–16

EX3030

Degree Examination in EX3030 Heat, Mass, and Momentum Transfer

15th December 2015 2 pm – 5 pm

PLEASE NOTE THE FOLLOWING

(i) You **must not** have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material **must not** be amended, annotated or modified in any way.

(ii) You **must not** have in your possession any material that could be determined as giving you an advantage in the examination.

(iii) You **must not** attempt to communicate with any candidate during the exam, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

(iv) You **must not** take to your examination desk any electronic devices such as mobile phones or other smart devices. The only exception to this rule is an approved calculator.

Failure to comply with the above will be regarded as cheating and may lead to disciplinary action as indicated in the Academic Quality Handbook Section 7 and particularly Appendix 7.1

Notes: ***(i) Candidates ARE permitted to use an approved calculator.***

(ii) Candidates ARE permitted to use the Engineering Mathematics Handbook.

(iii) Data sheets are attached to the paper.

Candidates should attempt *all three* questions from PART A AND *two* questions from *three* in PART B.

PART A: Answer ALL Questions

Question 1

Using a Cartesian control volume (as illustrated in Fig. 1):

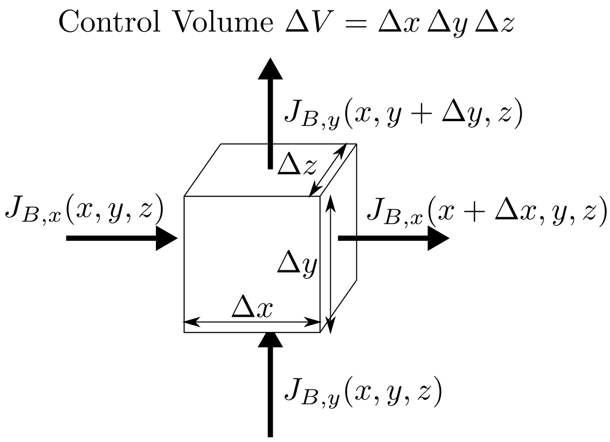


Figure 1: *A differential balance of flow property B in cartesian coordinates.*

a) Derive the general advection-diffusion equation for a property B, including a source term, σ_B. [12 marks]

b) Set $B =$ mass and derive the continuity equation. [8 marks]

Question 2

The Lockhart-Martinelli parameter, X, is a critical parameter in two-phase flow pressure-drop and liquid hold-up calculations. It is defined as the ratio of the frictional pressure drops of each phase, calculated as if each was flowing alone in the pipe.

$$X^2 = \frac{(\partial p/\partial z)_{liq.-only}}{(\partial p/\partial z)_{gas-only}}$$

a) Assuming that the pipe is smooth and that both phases are fully turbulent, derive the following expression for the Martinelli parameter

$$X_{tt} = \left(\frac{1-x}{x}\right)^{0.875}\left(\frac{\mu_{liq.}}{\mu_{gas}}\right)^{0.125}\left(\frac{\rho_{gas}}{\rho_{liq.}}\right)^{0.5}$$

Extra hint: You may need the Darcy-Weissbach equation and a suitable expression for the friction factor (see the datasheet). [4 marks]

b) A mixture of saturated steam at 0.09 kg/s and water at 1.6 kg/s is flowing along a horizontal pipe with an internal diameter of 75 mm. The steam has a viscosity of $\mu_g = 0.0113 \times 10^{-3}$ N s/m^2 and density of 0.788 kg/m^3. The water has a viscosity of 0.52×10^{-3} N s/m^2 and a density of 1000 kg/m^3.

i) Determine the flow pattern inside the pipe. [3 marks]

ii) Determine the flow regime inside each phase of the pipe. [4 marks]

iii) Calculate the two phase pressure drop multiplier (for either phase). [6 marks]

iv) Calculate the pressure drop over a 12 m long smooth pipe. [3 marks]

Question 3

a) An engineer is designing a new system in which 230 kg/h of water is heated from 35°C to 93°C by oil initially at 175°C. The mass flow rate of oil is also 230 kg/h. Two double-pipe heat exchangers are available:

- Heat Exchanger 1: Overall heat transfer coefficient (U_1) of 570 W/(m^2.°C) with superficial area (A_1) of 0.47 m^2, and;
- Heat Exchanger 2: U_2 = 370 W/(m^2.°C) and A_2 = of 0.94 m^2.

Which exchanger should be used? Why? Given $C_{p,\text{oil}}$ = 2.10 kJ/(kg.°C) and $C_{p,\text{water}}$ = 4.18 kJ/(kg.°C). Assume that there are no heat losses. [10 marks]

b) A plate of stainless steel (18% Cr, 8% Ni) has a thickness of 3.0 cm and is initially uniform in temperature at 500°C. The plate is suddenly exposed to a convection environment on both sides at 40°C with h = 150 W/(m^2.°C). Calculate the times for the centre and face temperatures to reach 120°C. Given that the thermal conductivity coefficient is 16.3 W/(m.°C), the thermal diffusivity is 0.44×10^{-5} m^2/s, and the analytical solution for the 1-D transient heat conduction for plane wall is

$$\theta = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 \exp\left(-\lambda_1^2 \tau\right) \cos \frac{\lambda_1 x}{L}$$

where τ is the Fourier number, and λ_1 and A_1 are coefficient constants of the transient 1-D heat conduction equation. [10 marks]

PART B: Answer TWO Questions From THREE

Question 4

Consider a spherical aggregate (or ball) of bacterial cells (assumed to be homogenous) of radius R. Under certain circumstances, the oxygen metabolism rate of the bacterial cells is an almost constant reaction (zero-order) with respect to the oxygen concentration $\sigma_{O_2} = -k_{O_2}$. The diffusion of oxygen within the ball may be described by Fick's law with an effective pseudobinary diffusivity for oxygen in the bacterial medium of D_{O_2-M}. Neglect transient and convection effects because the oxygen solubility is very low in the system. Let $C_{O_2}^{(R)}$ be the oxygen mass concentration at the aggregate surface:

a) Demonstrate that the oxygen balance for the system:

$$\frac{\partial C_{O_2}}{\partial t} = -\nabla \cdot \boldsymbol{N}_{O_2} + \sigma_{O_2}$$

simplifies to:

$$\frac{\partial}{\partial r}\left(r^2 N_{O_2,r}\right) = -k_{O_2} r^2$$

Show all of your working and state all assumptions made. [4 marks]

b) Demonstrate that the oxygen flux obeys the following relationship:

$$N_{O_2,r} = -k_{O_2} R^2 \left(\frac{r}{3R^2} + \frac{C_1 R}{6r^2}\right)$$

where C_1 is an unknown constant. [4 marks]

c) Demonstrate that the concentration profile obeys the following form in the limit that the O_2 concentration is small:

$$C_{O_2} = \frac{k_{O_2} R^2}{6 D_{O_2-M}}\left(\frac{r^2}{R^2} - C_1 \frac{R}{r}\right) + C_2$$

[4 marks]

d) Using the only available boundary condition, determine C_2 and demonstrate that the final expression for the concentration is:

$$C_{O_2} = \frac{k_{O_2} R^2}{6 D_{O_2-M}}\left(\frac{r^2}{R^2} - C_1\left(1 + \frac{R}{r}\right) - 1\right) + C_{O_2}^{(R)}$$

[4 marks]

e) It is possible that the spherical bacterial ball has an oxygen-free core ($C_{O_2} = 0$ for $r \leq r_{core}$). Prove that this only happens for:

$$\frac{k_{O_2} R^2}{D_{O_2-M} C_{O_2}^{(R)}} \geq 6$$

Hints: As the concentration and diffusive flux are continuous, they both must go to zero at the core radius r_{core}. Use this to solve for C_1, then solve for r_{core} and consider what is required if $r_{core} \geq 0$. [4 marks]

Question 5

Consider a pot of boiling water placed on a radiant (halogen) cooking hob. As the water is boiling, the surface temperature of the pot will be approximately the boiling temperature. The pot is exposed to the atmosphere and the air/surroundings are at $20°$C.

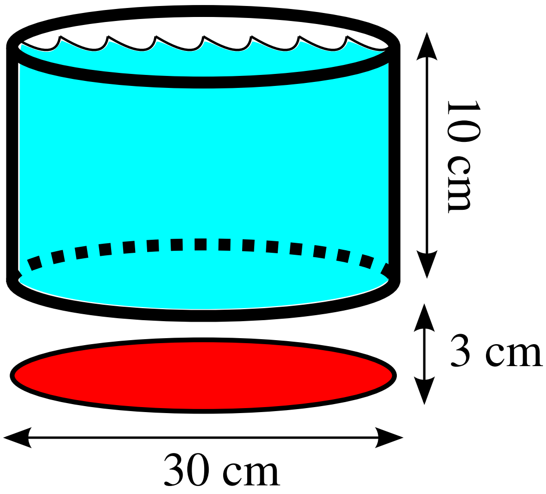


Figure 2: *The boiling pot problem.*

a) Calculate the natural convective heat loss from the sides of the pot given that air has a mean molar mass of $M_W \approx 29$ g/mol, a dynamic viscosity of $\mu \approx 1.8 \times 10^{-5}$ Pa s, a thermal conductivity of $k_{air} \approx 0.0257$ W m^{-1} K^{-1}, and a Prandtl number of Pr ≈ 0.713. [11 marks]

b) Assume that the total heat loss from the pan is 100 W due to evaporation and radiant heat loss to surroundings. Calculate the radiant temperature of the hob/heat-source required to counteract the heat loss. You may assume the pan and heat-source are black-bodies for this calculation.

The view factor between two coaxial discs is

$$F_{1\to 2} = 0.5\left(S - \left(S^2 - 4(r_j/r_i)^2\right)^{0.5}\right)$$

where $S = 1 + \left(1 + R_j^2\right)/R_i^2$, and the reduced radii are $R_i = r_i/L$ and $R_j = r_j/L$. Note L is the gap between the discs, and (r_i, r_j) are the radii of the two discs. [6 marks]

c) What fraction of the heat radiated from the heater hits the pot? [3 marks]

Question 6

Oil is used to lubricate two horizontal parallel plates by injecting it and allowing it to flow radially outwards from the point of injection (see Fig. 3). The fluid is flowing radially as there is a pressure difference of $P_1 - P_2$ between the inner and outer radii r_1 and r_2 respectively.

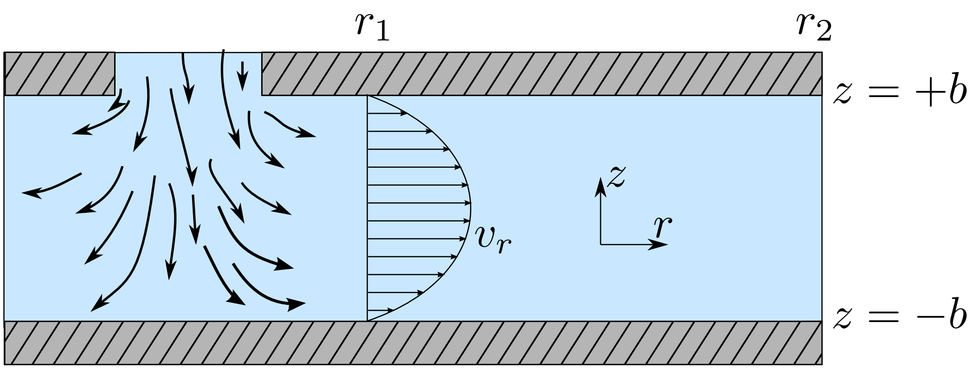


Figure 3: *Radial flow between two plates.*

a) Simplify the continuity equation to demonstrate that $r\,v_r$ is a function of z only. [5 marks]

b) Demonstrate that the stress profile within the channel is a solution of the following equation: [10 marks]

$$\rho\, v_r \frac{\partial v_r}{\partial r} = \mu \left(2\frac{\partial^2 v_r}{\partial r^2} + \frac{2}{r}\frac{\partial v_r}{\partial r} - \frac{2\, v_r}{r^2} + \frac{\partial^2 v_r}{\partial z^2} \right) - \frac{\partial\, p}{\partial r}$$

Note You must be careful during your derivation and make sure you expand each term of $\boldsymbol{\tau}$ before cancellation.

c) Using the creeping flow assumption, the following expression for the velocity profile was derived [5 marks]:

$$v_r = -r^{-1}\frac{\Delta P}{2\,\mu \ln(r_2/r_1)}\left(z^2 + C_1\, z + C_2\right)$$

Determine the integration constants C_1 and C_2, and give the final expression for the velocity profile:

END OF PAPER

DATASHEET

General balance equations:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho\,\boldsymbol{v} \qquad \text{(Mass/Continuity)} \quad (1)$$

$$\frac{\partial C_A}{\partial t} = -\nabla \cdot \boldsymbol{N}_A + \sigma_A \qquad \text{(Species)} \quad (2)$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho\,\boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla\,p + \rho\,\boldsymbol{g} \qquad \text{(Momentum)} \quad (3)$$

$$\rho\,C_p \frac{\partial T}{\partial t} = -\rho\,C_p\,\boldsymbol{v} \cdot \nabla\,T - \nabla \cdot \boldsymbol{q} - \boldsymbol{\tau} : \nabla\,\boldsymbol{v} - p\,\nabla \cdot \boldsymbol{v} + \sigma_{energy} \qquad \text{(Heat/Energy)} \quad (4)$$

In Cartesian coordinate systems, ∇ can be treated as a vector of derivatives. In curve-linear coordinate systems, the directions $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$ depend on the position. For convenience in these systems, look-up tables are provided for common terms involving ∇.

Cartesian coordinates (with index notation examples)
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor.

$$\nabla s = \nabla_i s = \left[\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \nabla_i \nabla_i s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \nabla_i v_i = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\nabla \cdot \boldsymbol{\tau} = \nabla_i\,\tau_{ij}$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\boldsymbol{v} \cdot \nabla \boldsymbol{v} = v_i\,\nabla_i\,v_j$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_x = v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_y = v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}$$

Cylindrical coordinates
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r},\ \frac{1}{r}\frac{\partial s}{\partial \theta},\ \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r}\frac{\partial}{\partial r}(r\, v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{1}{r}\tau_{\theta\theta} + \frac{\partial \tau_{rz}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r}\tau_{r\theta} + \frac{\partial \tau_{\theta z}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r\, v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

Spherical coordinates
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r},\ \frac{1}{r}\frac{\partial s}{\partial \theta},\ \frac{1}{r\,\sin\theta}\frac{\partial s}{\partial \phi}\right]$$

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial s}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 s}{\partial \phi^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta}+\tau_{\phi\phi}}{r}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{\theta\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\phi}\right) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r\, v_\theta - v_\phi^2\cot\theta}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\phi = v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r\, v_\phi + v_\theta\, v_\phi\cot\theta}{r}$$

Rectangular		Cylindrical		Spherical	
q_x	$-k\frac{\partial T}{\partial x}$	q_r	$-k\frac{\partial T}{\partial r}$	q_r	$-k\frac{\partial T}{\partial r}$
q_y	$-k\frac{\partial T}{\partial y}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$
q_z	$-k\frac{\partial T}{\partial z}$	q_z	$-k\frac{\partial T}{\partial z}$	q_ϕ	$-k\frac{1}{r\,\sin\theta}\frac{\partial T}{\partial \phi}$
τ_{xx}	$-2\,\mu\frac{\partial v_x}{\partial x} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{yy}	$-2\,\mu\frac{\partial v_y}{\partial y} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\phi\phi}$	$-2\,\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r + v_\theta\cot\theta}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{xy}	$-\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$
τ_{yz}	$-\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$	$\tau_{\theta z}$	$-\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)$	$\tau_{\theta\phi}$	$-\mu\left(\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right)$
τ_{xz}	$-\mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$	τ_{zr}	$-\mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)$	$\tau_{\phi r}$	$-\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right)$

Table 1: *Fourier's law for the heat flux and Newton's law for the stress in several coordinate systems. Please remember that the stress is symmetric, so* $\tau_{ij} = \tau_{ji}$.

Viscous models:
Power-Law Fluid:

$$|\tau_{xy}| = k\left|\frac{\partial v_x}{\partial y}\right|^n$$

Bingham-Plastic Fluid:

$$\frac{\partial v_x}{\partial y} = \begin{cases} -\mu^{-1}\left(\tau_{xy} - \tau_0\right)) & \text{if } \tau_{xy} > \tau_0 \\ 0 & \text{if } \tau_{xy} \le \tau_0 \end{cases}$$

Dimensionless Numbers

$$\text{Re} = \frac{\rho\,\langle v\rangle\,D}{\mu} \qquad \text{Re}_H = \frac{\rho\,\langle v\rangle\,D_H}{\mu} \qquad \text{Re}_{MR} = -\frac{16\,L\,\rho\,\langle v\rangle^2}{R\,\Delta p} \tag{5}$$

The hydraulic diameter is defined as $D_H = 4\,A/P_w$.

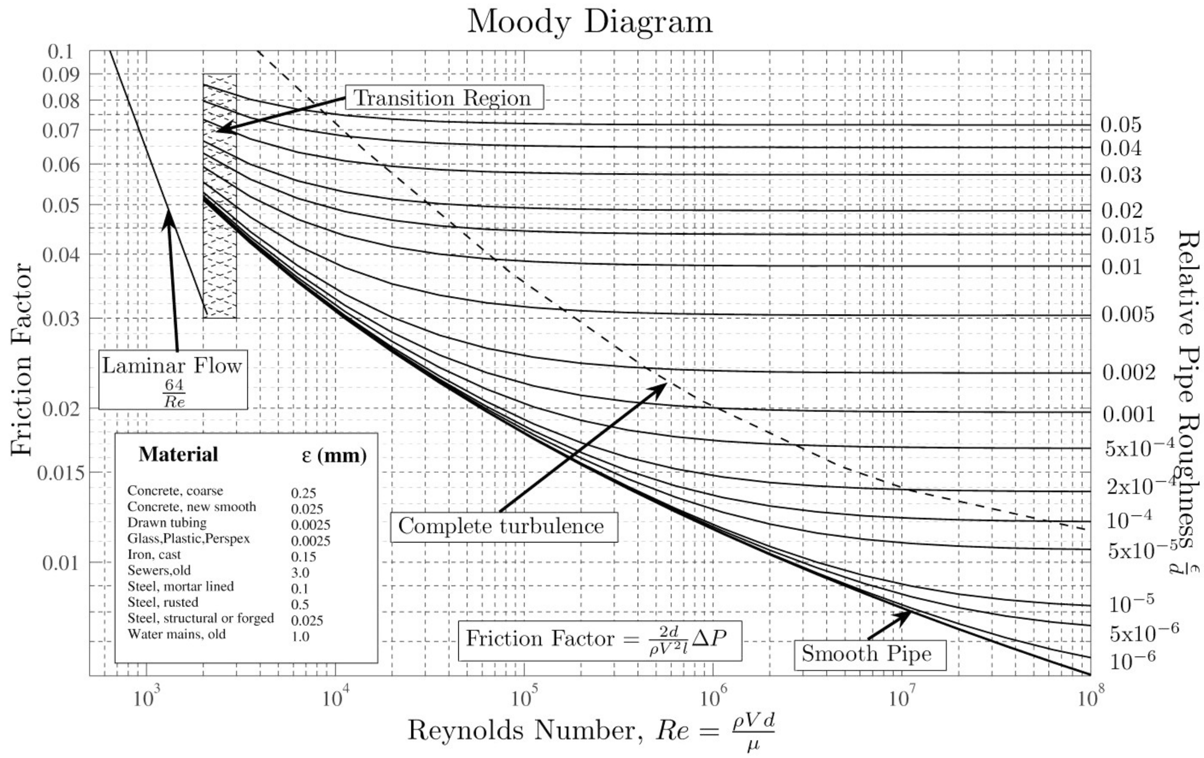
Single phase pressure drop calculations in pipes:
Darcy-Weisbach equation:

$$\frac{\Delta p}{L} = -\frac{C_f\,\rho\,\langle v\rangle^2}{R} \tag{6}$$

where $C_f = 16/Re$ for laminar Newtonian flow. For turbulent flow of Newtonian fluids in smooth pipes, we have the Blasius correlation:

$$C_f = 0.079\,\mathrm{Re}^{-1/4} \qquad \text{for } 2.5\times 10^3 < \mathrm{Re} < 10^5 \text{ and smooth pipes.}$$

Otherwise, you may refer to the Moody diagram.



Laminar Power-Law fluid:

$$\dot{V} = \frac{n\,\pi\,R^3}{3\,n+1}\left(\frac{R}{2\,k}\right)^{\frac{1}{n}}\left(-\frac{\Delta p}{L}\right)^{\frac{1}{n}}$$

Two-Phase Flow:

Lockhart-Martinelli parameter:

$$X^2 = \frac{\Delta p_{liq.-only}}{\Delta p_{gas-only}}$$

Pressure drop calculation:

$$\Delta p_{two-phase} = \Phi^2_{liq.}\,\Delta p_{liq.-only} = \Phi^2_{gas}\,\Delta p_{gas-only}$$

Chisholm's relation:

$$\Phi^2_{gas} = 1 + c\,X + X^2$$

$$\Phi^2_{liq.} = 1 + \frac{c}{X} + \frac{1}{X^2} \qquad c = \begin{cases} 20 & \text{turbulent liquid \& turbulent gas} \\ 12 & \text{laminar liquid \& turbulent gas} \\ 10 & \text{turbulent liquid \& laminar gas} \\ 5 & \text{laminar liquid \& laminar gas} \end{cases}$$

Farooqi and Richardson expression for liquid hold-up in co-current flows of Newtonian fluids and air in horizontal pipes:

$$h = \begin{cases} 0.186 + 0.0191\,X & 1 < X < 5 \\ 0.143\,X^{0.42} & 5 < X < 50 \\ 1/\left(0.97 + 19/X\right) & 50 < X < 500 \end{cases}$$

Heat Transfer:
Stefan-Boltzmann constant $\sigma = 5.6703 \times 10^{-8}$ W/m^2 K^4

Heat Transfer Dimensionless numbers:

$$\text{Nu} = \frac{h\,L}{k} \qquad \text{Pr} = \frac{\mu\,C_p}{k} \qquad \text{Gr} = \frac{g\,\beta\,(T_w - T_\infty)\;L^3}{\nu^2}$$

Resistances

$$Q = U_T\,A_T\,\Delta T = R_T^{-1}\,\Delta T \qquad Q_{rad.} = \sigma\,\varepsilon\,A\left(T_\infty^4 - T_w^4\right) = h_{rad.}\,A\left(T_\infty - T_w\right)$$

	Conduction Shell Resistances			Radiation
	Rect.	Cyl.	Sph.	
R	$\dfrac{X}{k\,A}$	$\dfrac{\ln\left(R_{outer}/R_{inner}\right)}{2\,\pi\,L\,k}$	$\dfrac{R_{inner}^{-1} - R_{outer}^{-1}}{4\,\pi\,k}$	$\left[A\,\varepsilon\,\sigma\,(T_\infty^2 + T_w^2)\,(T_\infty + T_w)\right]^{-1}$

Natural Convection

$\text{Ra} = \text{Gr}\,\text{Pr}$	C	m
$< 10^4$	1.36	1/5
10^4–10^9	0.59	1/4
$> 10^9$	0.13	1/3

Table 2: *Natural convection coefficients for isothermal vertical plates in the empirical relation Nu* $\approx C\,(\text{Gr}\,\text{Pr})^m$.

For isothermal vertical cylinders, the above expressions for isothermal vertical plates may be used but must be scaled by a factor, F:

$$F = \begin{cases} 1 & \text{for } (D/H) < 35\,\text{Gr}_H^{-1/4} \\ 1.3\left[H\,D^{-1}\,\text{Gr}_D^{-1}\right]^{1/4} + 1 & \text{for } (D/H) \geq 35\,\text{Gr}_H^{-1/4} \end{cases}$$

where D is the diameter and H is the height of the cylinder. The subscript on Gr indicates which length is to be used as the critical length to calculate the Grashof number.

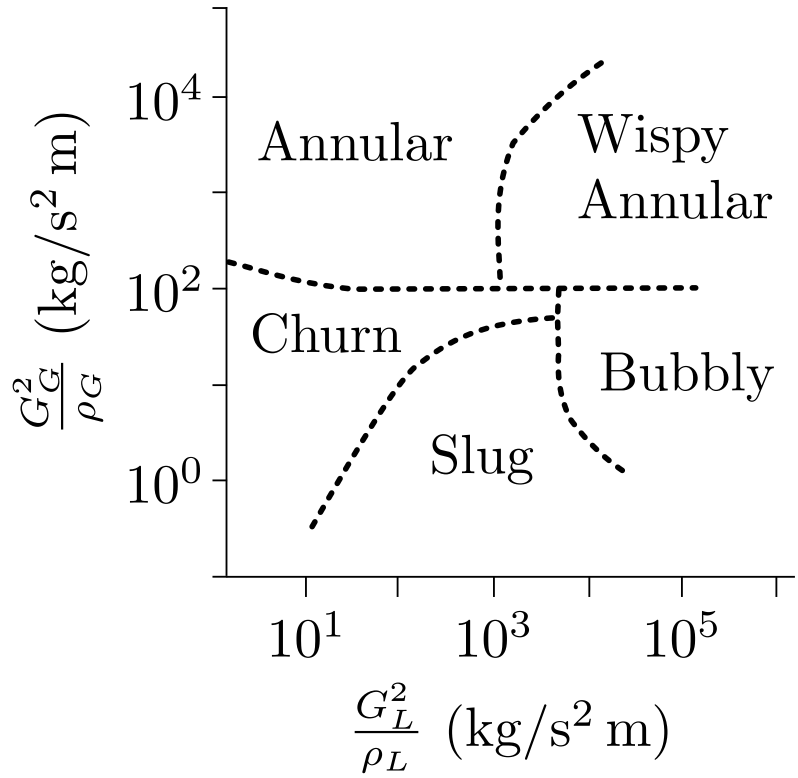


Figure 4: *Hewitt-Taylor flow pattern map for multiphase flows in vertical pipes.*

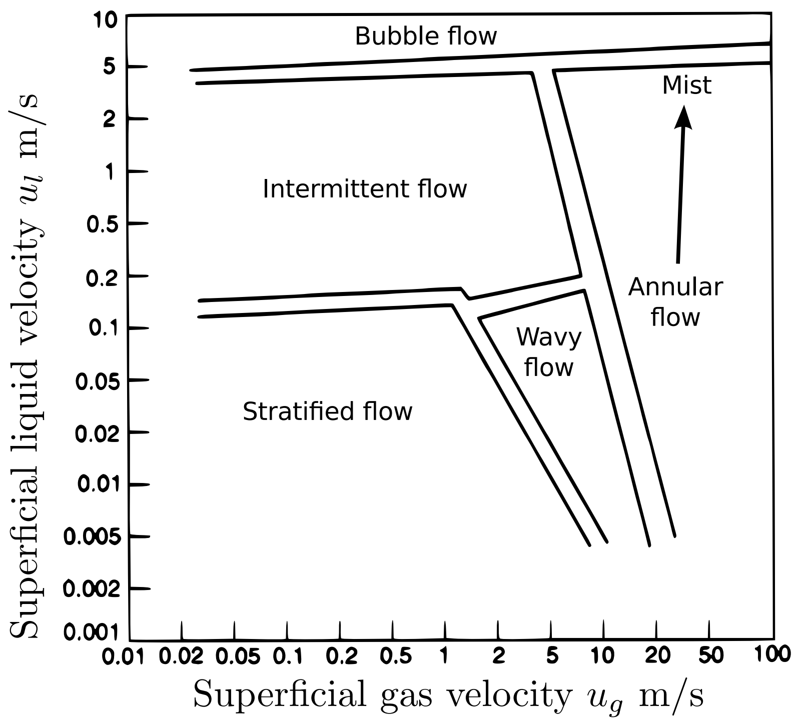


Figure 5: *Chhabra and Richardson flow pattern map for horizontal pipes.*

Churchill and Chu expression for natural convection from a horizontal pipe:

$$\mathrm{Nu}^{1/2} = 0.6 + 0.387 \left\{ \frac{\mathrm{Gr}\,\mathrm{Pr}}{\left[1 + (0.559/\mathrm{Pr})^{9/16}\right]^{16/9}} \right\}^{1/6} \qquad \text{for } 10^{-5} < \mathrm{Gr}\,\mathrm{Pr} < 10^{12}$$

Forced Convection:
Laminar flows:

$$\mathrm{Nu} \approx 0.332\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

Well-Developed turbulent flows in smooth pipes:

$$\mathrm{Nu} \approx \frac{(C_f/2)\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(C_f/2)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)} \left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Boiling:
Forster-Zuber pool-boiling coefficient:

$$h_{nb} = 0.00122 \frac{k_L^{0.79}\,C_{p,L}^{0.45}\,\rho_L^{0.49}}{\gamma^{0.5}\,\mu_L^{0.29}\,h_{fg}^{0.24}\,\rho_G^{0.24}} \left(T_w - T_{sat}\right)^{0.24} \left(p_w - p_{sat}\right)^{0.75}$$

Mostinski correlations:

$$h_{nb} = 0.104\,p_c^{0.69}\,q^{0.7} \left[1.8 \left(\frac{p}{p_c}\right)^{0.17} + 4 \left(\frac{p}{p_c}\right)^{1.2} + 10 \left(\frac{p}{p_c}\right)^{10}\right]$$

$$q_c = 3.67 \times 10^4\,p_c \left(\frac{p}{p_c}\right)^{0.35} \left[1 - \frac{p}{p_c}\right]^{0.9}$$

(**Note**: for the Mostinski correlations, the pressures are in units of bar)
Condensing:
Horizontal pipes

$$h = 0.72 \left(\frac{k^3\,\rho^2\,g_x\,E_{latent}}{D\,\mu\,(T_w - T_\infty)}\right)^{1/4}$$

NTU method:

$$\mathrm{NTU} = \frac{U\,A}{C_{min}} = \frac{t_{C1} - t_{C2}}{\Delta t_{ln}} \qquad\qquad R = \frac{C_{min}}{C_{max}}$$

For counter-current flow:

$$E = \frac{1 - \exp\left[-\mathrm{NTU}(1 - R)\right]}{1 - R\exp\left[-\mathrm{NTU}(1 - R)\right]}$$

For co-current flow:

$$E = \frac{1 - \exp\left[-\mathsf{NTU}(1 - R)\right]}{1 + R}$$

Lumped capacitance method:

$$\mathsf{Bi} = \frac{h\,L_c}{k}$$

$$L_c = V/A \qquad \text{for Bi} < 0.1$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{h\,A_s}{\rho\,V\,C_p} t\right]$$

Diffusion Dimensionless Numbers

$$\mathsf{Sc} = \frac{\mu}{\rho\,D_{AB}} \qquad \mathsf{Le} = \frac{k}{\rho\,C_p\,D_{AB}}$$

Diffusion

General expression for the flux:

$$\boldsymbol{N}_A = \boldsymbol{J}_A + x_A \sum_B \boldsymbol{N}_B$$

Fick's law:

$$\boldsymbol{J}_A = -D_{AB}\,\nabla C_A$$

Stefan's law:

$$N_{s,r} = -D \frac{c}{1 - x} \frac{\partial x}{\partial r}$$

TABLE 4–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

	Plane Wall		*Cylinder*		*Sphere*	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 4–3

The zeroth- and first-order Bessel functions of the first kind

η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613