The recommended questions you should solve for each week are:

- Tutorial 1: Q. 1 to Q. 11.
- Tutorial 2: Q. 13 and Q. 14.
- Tutorial 3: Q. 15 and Q. 18.
- Tutorial 4: Q. 26 and Q. 31.
- Tutorial 5: Q. 27, Q. 28, and Q. 33.
- Tutorial 6: Q. 35 and Q. 46.
- Tutorial 9: Q. 52, Q. 53, and Q. 57.
- Tutorial 10: Q. 56, Q. 58, and Q. 60.
- Tutorial 11: Q. 65, Q. 67, and Q. 68.

All other questions are provided for additional practice and should help you to explore all aspects of the course.
Fully worked solutions are given, but you should attempt the problems without the solutions, its the only way to know what you don't know!
Where marks are given, these are indicative of the *relative* weighting each part of a question might have. Please note, the number of questions in an exam (and exam durations) have changed over the years, so the overall marks for a question may now be different to what is reported here.
All past exam questions are collected in this document.

Questions

Question 1
Your house is 18°C inside when it is 4°C outside. If your walls are 20 cm thick and have a thermal conductivity of 0.03 W m^{-1} K^{-1}, calculate the heat lost per unit area of wall.
Notes: The heat transfer rate per unit area of wall, q (W m^{-2}), is given by:

$$q = U\,\Delta T$$

where U (W m^{-2} K^{-1}) is the heat transfer coefficient, and ΔT is the driving temperature difference. For solid, rectangular walls $U = k/L$, where k is the thermal conductivity and L is the wall thickness.

[Question end]

Question 2
Model your house as a box 10 m×10 m×10 m and calculate the heat transfer from its side walls, is this estimate reasonable? What natural effects are missing from this model?
Notes: For simple heat transfer, the total heat transfer Q (W) is given by:

$$Q = q\,A = U\,A\,\Delta T \tag{1}$$

[Question end]

Question 3
What is the pressure at the bottom of the Mariana Trench (the deepest part of the ocean)?
Note: Its depth is 10.911 km and you may assume the density of water is roughly constant at $\rho = 1000$ kg m^{-3}.
Bernoulli's equation is

$$\frac{1}{2}\rho_1\, v_1^2 + p_1 + \rho_1\, g\, h_1 = \frac{1}{2}\rho_2\, v_2^2 + p_2 + \rho_2\, g\, h_2 \tag{2}$$

[Question end]

Question 4
Assuming that blood has a density of 1060 kg m^{-3}, what is the maximum height your heart can lift your blood, given that a typical driving pressure of the heart is 100 mmHg (0.13 bar)?
Note: You can use Bernoulli's equation and as you're looking for the maximum height, you may treat the blood as stationary at both ends.

[Question end]

Question 5
Write the following expressions in index notation, and state whether the answer is a scalar, vector, or matrix.

a) $\boldsymbol{a} + \boldsymbol{b}$

b) $\boldsymbol{ab}$

c) $\boldsymbol{c} \cdot \boldsymbol{ab}$

d) $\boldsymbol{a} \cdot \boldsymbol{A}$

e) $\boldsymbol{A} \cdot \boldsymbol{b}$

f) $\boldsymbol{a}^2$

g) $\boldsymbol{A}^2 \cdot \boldsymbol{b}$

h) $\boldsymbol{abc} \cdot \boldsymbol{A} \cdot \boldsymbol{d}$

i) $\nabla \cdot \boldsymbol{bc}$

[Question end]

Question 6
Given $\boldsymbol{a} = [1, 2, 3]$ and $\boldsymbol{b} = [4, 5, 6]$, calculate the following

a) $\boldsymbol{a} + \boldsymbol{b}$

b) $4\,\boldsymbol{a}$

c) $\boldsymbol{a} \cdot \boldsymbol{b}$

d) $\boldsymbol{a}^2$

e) $\nabla \cdot \boldsymbol{b}$

f) $\nabla \boldsymbol{b}$

[Question end]

Question 7
Write the following expressions in vector notation.

a) $a_l\, b_j$

b) $a_k\, b_k$

c) $b_j\, A_{ij}\, a_i$

d) $a_i\, b_j\, c_k$

e) $a_i\, b_j\, a_i$

f) $a_i\, (\partial b_j / \partial r_i)$

[Question end]

Question 8
The del operator ($\nabla = \partial/\partial r_i$) is a "vector" version of the derivative. Like the normal derivative operation, it has a product rule. Prove the following identity:

$$\nabla \cdot \boldsymbol{ab} = \boldsymbol{b}\nabla \cdot \boldsymbol{a} + \boldsymbol{a} \cdot \nabla \boldsymbol{b}$$

Hint: Use index notation, treat a_i and b_i as functions of x, y, z, and use the normal product rule!

[Question end]

Question 9
Using index notation, prove the following vector calculus identity:

$$\nabla^2 f\, g = f\, \nabla^2 g + 2(\nabla f) \cdot (\nabla g) + g\, \nabla^2 f$$

[5 marks]

Note: You must treat f and g as functions of x, y, z.

[Question total: 5 marks]

Question 10
Solve the following integration and differentiation problems:

a) $\int r\, \mathrm{d}r\, r$

b) $\int\int \theta\, \mathrm{d}\theta\, \mathrm{d}r$

c) $\int_A^B y^{-1} \mathrm{d}y$

d) $\int x \sin x\, \mathrm{d}x$ (hint: by parts)

e) $\nabla \cdot \boldsymbol{r}$ where $\boldsymbol{r} = [x, y, z]$

f) $\nabla \boldsymbol{r}$ where $\boldsymbol{r} = [x, y, z]$

[Question end]

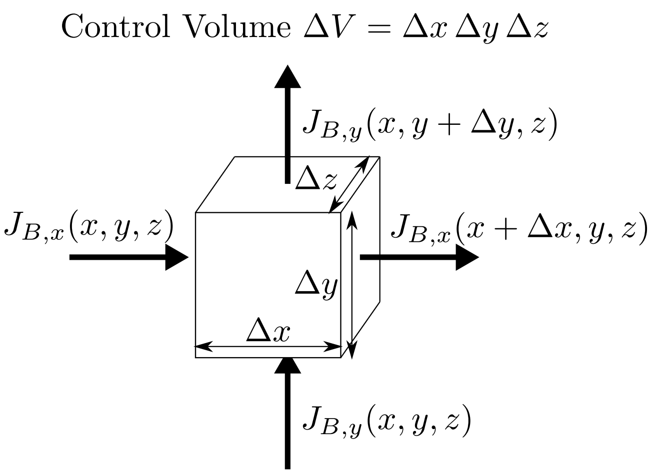


Figure 1: A differential balance of flow property B in cartesian coordinates.

Question 11
Solve the following integration problem by using a variable substitution of $\eta = y/H$.

$$\int_0^H \left(A\frac{y^2}{H^2} + B\frac{y}{H}\right) \mathrm{d}y$$

[Question end]

Question 12
2015 Exam Question Using a Cartesian control volume (as illustrated in Fig. 1):

a) Derive the general advection-diffusion equation for a property B, including a source term, σ_B. **[12 marks]**

b) Set B = mass and derive the continuity equation. **[8 marks]**

[Question total: 20 marks]

Question 13
Using index notation:

a) Write down the continuity equation (Eq. (19)).

b) Write down the Cauchy momentum equation.

[Question end]

Question 14
In a plate heat-exchanger, water is heated by forcing it between alternating plates and heat is exchanged through the walls with a hot process stream. In order to design such an exchanger, we need to know what the relationship is between pressure drop, flow velocity, and volumetric flow-rate.
You may neglect the effect of heat transfer on the flow. Water is incompressible and Newtonian to a good approximation. For simplicity, you can also assume that the flow is laminar.

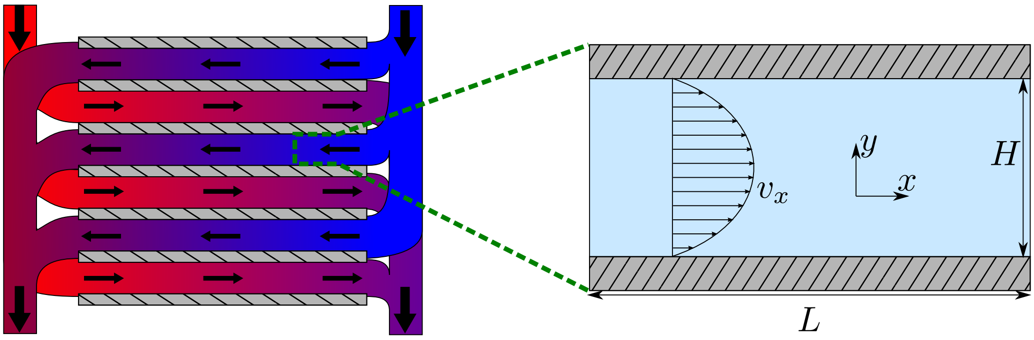


Figure 2: A plate heat exchanger (left) and the simplification to steady state, pressure driven flow between two horizontal plates (right).

a) Simplify the continuity equation for this system:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho\, \boldsymbol{v}$$

What does your result state about the flow velocity in the x-direction? **[4 marks]**

b) Simplify the x-component of the Cauchy momentum equation:

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho\, \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla p + \rho\, \boldsymbol{g}$$

Derive the following balance expression for the flow velocity v_x as a function of the pressure drop and position y: **[6 marks]**

$$\mu \frac{\partial^2 v_x}{\partial y^2} = \frac{\partial p}{\partial x}$$

c) Continuing from the result of the previous question, derive the following expression for the velocity v_x as a function of y using the no-slip boundary condition at the plate surfaces ($v_x = 0$ at $y = 0$ and $y = H$). **[6 marks]**

$$v_x = \frac{p_{out} - p_{in}}{2\,\mu\,L}(y^2 - H\,y)$$

d) Integrate the velocity over the plate height and width to prove the following expression for the volumetric flow of liquid through the gap as a function of pressure drop: **[4 marks]**

$$\dot{V}_x = \frac{Z\,H^3}{12\,\mu}\frac{\Delta P}{L}$$

e) **Extra credit:** Assume that somehow, the top plate is set in motion with a velocity u_{plate} in the x-direction. Derive the following new expression for the velocity between the plates:

$$v_x = \frac{p_{out} - p_{in}}{2\,\mu\,L}(y^2 - H\,y) + \frac{y}{H}\,u_{plate}$$

[Question total: 20 marks]

Question 15
A plate heat exchanger is used to heat water inside a condensing reboiler (a modern central heating boiler). Water flows through both sides of the exchanger. The exchanger consists of 8 channels (4 per side) each with a gap of 1 mm between the plates. Plates may be modelled as 30cm long in the direction of flow and 10 cm wide.

a) If the water pressure drops by 0.06 bar across one side of the exchanger, what is the resultant volumetric flow of water? You may assume an effective viscosity of $\mu \approx 0.5$ mPa s and a density of $\rho = 1000$ kg m^{-3}.

b) State all of the assumptions that have you made in this estimate.

c) Is this likely to be an over or under-estimation of the flow rate?

[Question end]

Question 16
Water is overflowing a dam and down an inclined slope (see Fig. 3). The surface of the dam can be idealised as a rectangular plane which is symmetric in the z-direction, and (for now) only laminar flow is being considered.

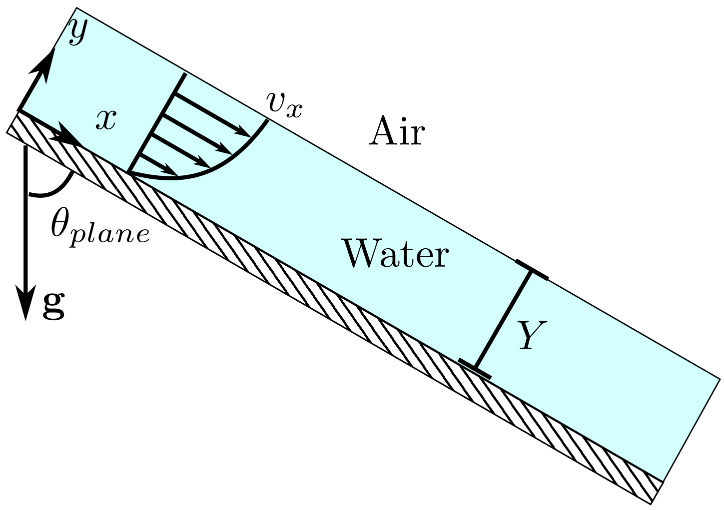


Figure 3: Water flowing down an inclined plane.

a) Simplify the continuity equation for this system and state any assumptions you make. **[6 marks]**

b) Derive the following results from the Cauchy momentum equation and the general form of Newton's law of viscosity: **[10 marks]**

$$\frac{\partial \tau_{yx}}{\partial y} = \rho\, g_x \qquad\qquad \tau_{yx} = -\mu \frac{\partial v_x}{\partial y}.$$

c) Define your boundary conditions and derive the following expression for the velocity profile, **[9 marks]**

$$v_x = \frac{\rho\, g_x}{\mu}\left(Y\,y - \frac{y^2}{2}\right)$$

d) Use an integration of the velocity over the flow area to determine the following expression for the volumetric flow rate, **[6 marks]**

$$\dot{V}_x = \frac{\rho\, g_x\, Y^3\, Z}{3\,\mu}.$$

e) Provide an expression for the maximum flow velocity. **[2 marks]**

[Question total: 33 marks]

Question 17

Consider pressure-driven flow along a horizontal pipe, as illustrated in Fig. 4.

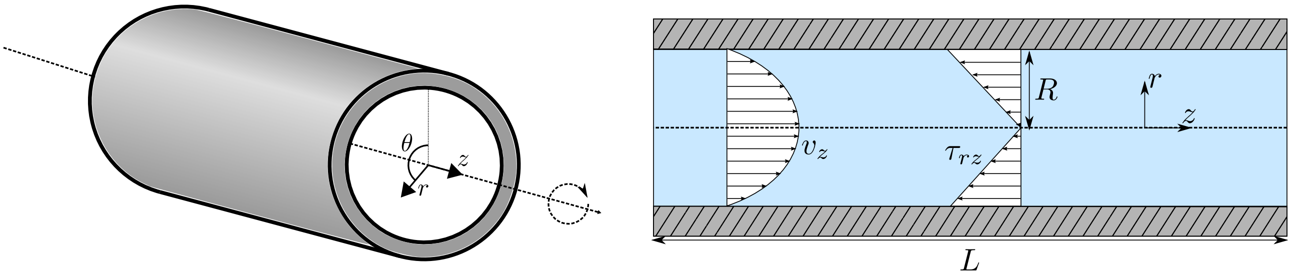


Figure 4: An illustration of pipe flow.

a) Simplify the continuity equation for this system, what does it tell you about the flow? Remember to make your assumptions and their effects clear. **[6 marks]**

b) Derive the following differential equation from the Cauchy momentum equation.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right) = -\frac{\partial p}{\partial z}$$

Remember to make your assumptions and their effects clear. **[7 marks]**

c) Determine the following expression for the stress profile.

$$\tau_{rz} = -\frac{\Delta p}{2\,L}r$$

[3 marks]

d) Demonstrate that the velocity profile is as given below.

$$v_z = \frac{\Delta p}{4\,\mu\,L}\left(r^2 - R^2\right)$$

[4 marks]

[Question total: 20 marks]

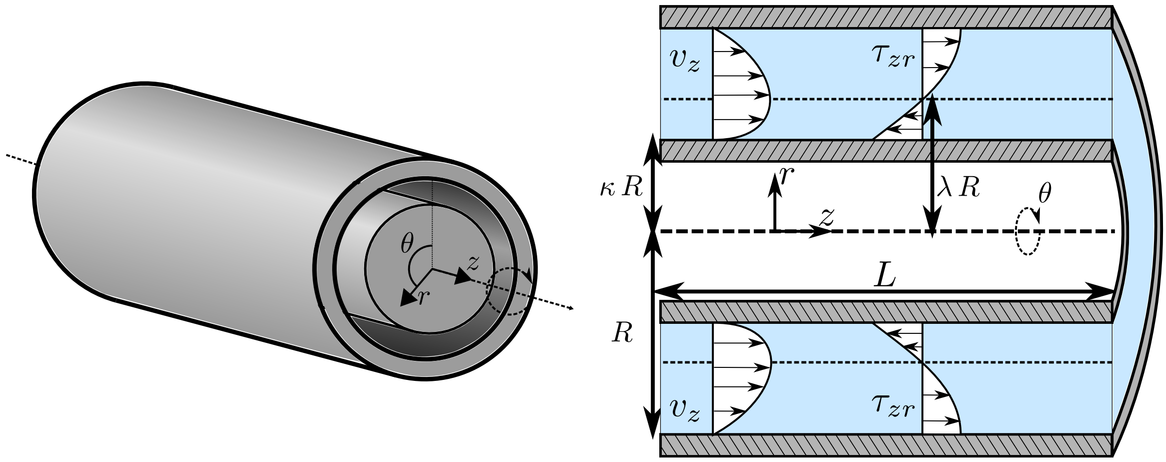


Figure 5: An annular flow geometry.

Question 18

An annulus (see Fig. 5) is a very common flow configuration where a fluid is flowing between two concentric pipes. Real examples of annuli include oil and gas wells and concentric-tube heat-exchangers in air conditioners. A "completed" oil-well may consist of up to 3 annuli around the central "production" pipe. We need design equations to calculate the relationship between pressure drop and volumetric flow-rate.

Assuming we have a steady-state, laminar, incompressible, and well-developed flow inside an annulus:

a) Demonstrate that the continuity equation simplifies to the following expression.

$$\frac{\partial v_z}{\partial z} = 0$$

State your interpretation of this expression.

b) Simplify the Cauchy momentum balance equation to yield the following result.

$$0 = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\tau_{rz}\right) - \frac{\partial p}{\partial z} + \rho\, g_z$$

c) Integrate the equation to express it in terms of the pressure drop over the length of the annulus. Give reasons why the stress term τ_{rz} is independent of z.

$$\frac{\Delta p}{L} = -\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\tau_{rz}\right) + \rho\, g_z$$

d) Solve the above equation for the stress profile in an annulus using the assumed boundary condition that the stress is zero at a critical radius $r = \lambda\, R$. Prove that it is the following expression:

$$\tau_{rz} = \frac{1}{2}\left(\rho\, g_z - \frac{\Delta p}{L}\right)\left(r - \frac{\lambda^2\, R_0^2}{r}\right)$$

Note: The critical radius $\lambda\, R$ is the location of the maximum velocity, and will be determined once the viscous model is inserted.

e) Solve for the velocity profile by assuming the fluid is Newtonian. Try to rearrange the result of the integration into the following convenient form:

$$v_z = -\frac{R^2}{4\,\mu}\left(\rho\,g_z - \frac{\Delta p}{L}\right)\left(\frac{r^2}{R^2} - 2\,\lambda^2\,\ln\left(\frac{r}{R}\right) + C\right)$$

f) Using the no slip boundary condition at $r = R$ and $r = \kappa\,R$, solve for the unknown constants C and λ in the above equation and generate the final expression.

[Question end]

Question 19

An evaporative cooler is sketched in Fig. 6. The process functions by first pumping water up a vertical pipe and then allowing it to flow down the exterior of the pipe. The properties of the external film flow are essential for the design of such a cooler.

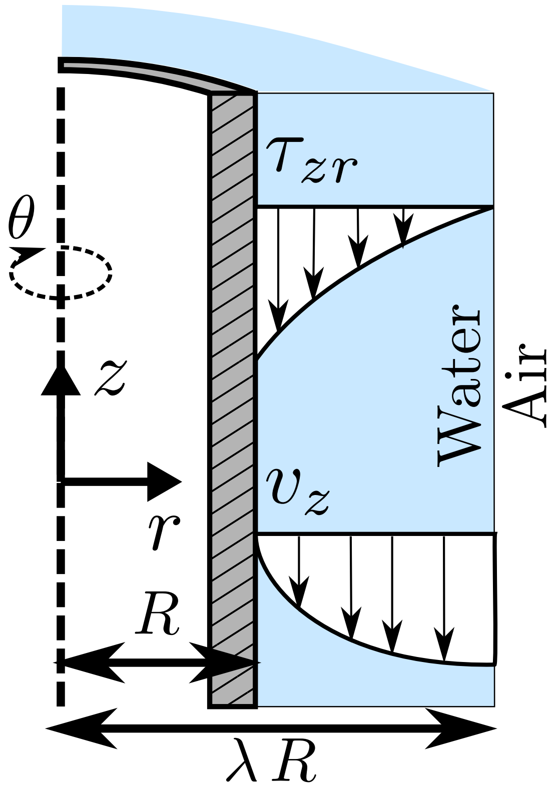


Figure 6: A sketch of the evaporative cooler

a) Simplify the continuity equation for this system. What are your assumptions and what does your result tell you about the flow along the pipe? **[5 marks]**

b) Derive the following equation for the stress profile from the general momentum balance equation (Eq. (21)). State any additional assumptions you make.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\tau_{rz}\right) = \rho\,g_z$$

[6 marks]

c) Solve the equation for the stress profile to obtain the following velocity profile for the flow.

$$v_z = \frac{\rho\, g\, R^2}{4\,\mu}\left(1 - \left(\frac{r}{R}\right)^2 + 2\,\lambda^2 \ln\left(\frac{r}{R}\right)\right)$$

[9 marks]

[Question total: 20 marks]

Question 20
Example exam question
A Couette viscometer tests the viscous behaviour of a fluid using rotational shear in an annulus (see Fig. 7). The fluid is sheared by rotating the outer wall at an angular velocity of Ω_θ, giving $v_\theta(r = R) = \Omega_\theta\, R$. The inner cylinder is held stationary, giving $v_\theta(r = \kappa\, R) = 0$. There is no flow along the axis of the annulus.

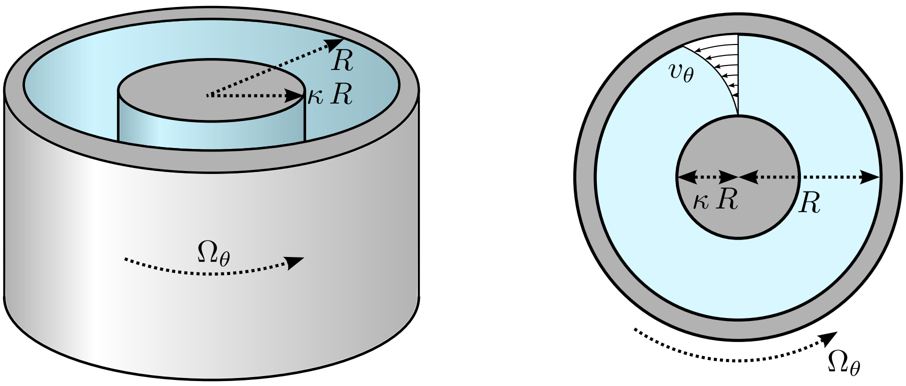


Figure 7: A simplified diagram of a Couette viscometer.

a) Derive the following expression by solving the continuity equation, given in Eq. (19), for this system.

$$\frac{\partial v_\theta}{\partial \theta} = 0 \tag{3}$$

Clearly state any assumptions you make. What does this tell you about the flow? **[5 marks]**

b) The velocity profile of the system is given by the following expression:

$$v_\theta = \Omega_0\, R \frac{\frac{\kappa\, R}{r} - \frac{r}{\kappa\, R}}{\kappa - 1/\kappa} \tag{4}$$

Derive the following expression for the stress profile in the system.

$$\tau_{r\theta} = 2\frac{\mu\, \Omega_0\, \kappa^2}{\kappa^2 - 1}\frac{R^2}{r^2} \tag{5}$$

[10 marks]

c) Derive the following expression for the torque exerted on the outer surface ($r = R$) to keep the fluid in motion.

$$\mathcal{T} = 4\pi\, R^2\, L \frac{\mu\, \Omega_0\, \kappa^2}{\kappa^2 - 1}$$

where L is the length of the viscometer.
Note: The torque is the total magnitude of a tangential force, such as the viscous stress $\tau_{r\theta}$, multiplied by the radial distance at which it acts. **[3 marks]**

d) The torque is measured during the operation of the viscometer. How are the viscous properties of the flow determined? **[2 marks]**

[Question total: 20 marks]

Question 21
Example exam question with marks:
Coil-tubing is being removed from an oil and gas well. This may be modelled as a cylindrical rod, radius R_1, moving upwards along the axis of a vertical cylindrical tube with inner radius R_2, at velocity, U (see Fig. 8). Water flows freely in the annular gap between the rod and the tube wall.

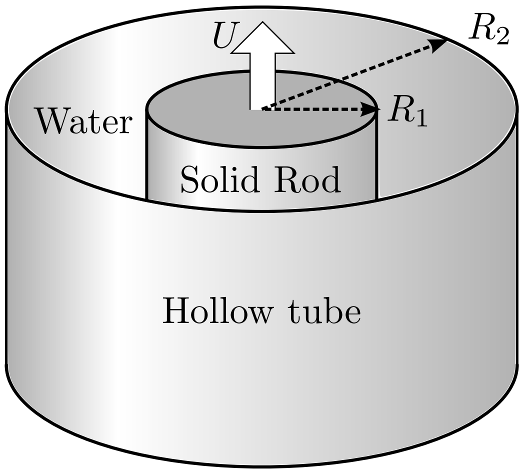


Figure 8: Flow of water within a vertical annulus.

Note: You may ignore the effects of pressure gradients in this question.

a) Define the coordinate system you will use and the boundary conditions of the flow. **[3 marks]**

b) Simplify the continuity equation for this system. What are your assumptions and what does your result tell you about the flow along the annulus? **[4 marks]**

c) Derive the following balance equation for the momentum. You may assume that water is a Newtonian fluid, the flow is well developed, at steady state, and that any effect of pressure can be ignored. **[5 marks]**

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,\tau_{rz}) = \rho\, g_z$$

d) Derive the following expression for the velocity profile of the fluid within the tube. **[4 marks]**

$$v_z = -\frac{\rho\, g_z\, r^2}{4\,\mu} + \frac{C_1}{\mu}\ln r + C_2$$

where C_1 and C_2 are unknown integration constants.

e) Using the boundary conditions, solve for the constants C_1 and C_2. **[2 marks]**

f) After using the boundary conditions to solve for the constants C_1 and C_2, the velocity profile was determined to be

$$v_z = \frac{\rho\, g_z\, R_1^2}{4\,\mu}\left(1 - \frac{r^2}{R_1^2} + \left(\lambda^2 - 1\right)\frac{\ln\left(r/R_1\right)}{\ln\lambda}\right) + U\left(1 - \frac{\ln\left(r/R_1\right)}{\ln\lambda}\right)$$

where $\lambda = R_2/R_1$. What is the average velocity of water in the annulus?
Note: You may need the integration identity

$$\int x\,\ln(x)\,\mathrm{d}x = \frac{x^2}{2}\left(\ln(x) - \frac{1}{2}\right)$$

g) Given a flow system with dimensions of R_1 = 10 mm and R_2 = 11 mm, at what speed, U, does the rod need to be moved upwards so that there is no net upwards or downwards flow of the fluid? Water has a viscosity of $\mu = 8.9 \times 10^{-4}$ Pa s and a density of ρ = 1000 kg m^{-3}. The z-component of gravity is given by $g_z = -9.81$ m s^{-2}. The average flow velocity in the annulus is given by

$$\langle v_z\rangle = \frac{\rho\, g_z\, R_1^2}{8\,\mu}\left(1 + \lambda^2 - \frac{\lambda^2 - 1}{\ln\lambda}\right) + \frac{U}{\lambda^2 - 1}\left(\frac{\lambda^2 - 1}{2\,\ln\lambda} - 1\right)$$

where $\lambda = R_2/R_1$. **[2 marks]**

h) Given a flow system with dimensions of R_1 = 50 mm and R_2 = 51 mm, at what speed, U, does the rod need to be moved upwards so that there is no net upwards or downwards flow of the fluid? Water has a viscosity of $\mu = 8.9 \times 10^{-4}$ Pa s and a density of ρ = 1000 kg m^{-3}. The z-component of gravity is given by $g_z = -9.81$ m s^{-2}. **[2 marks]**

[Question total: 22 marks]

Question 22
Example exam question (2015)
Oil is used to lubricate two horizontal parallel plates by injecting it and allowing it to flow radially outwards from the point of injection (see Fig. 9). The fluid is flowing radially as there is a pressure difference of $P_1 - P_2$ between the inner and outer radii r_1 and r_2 respectively.

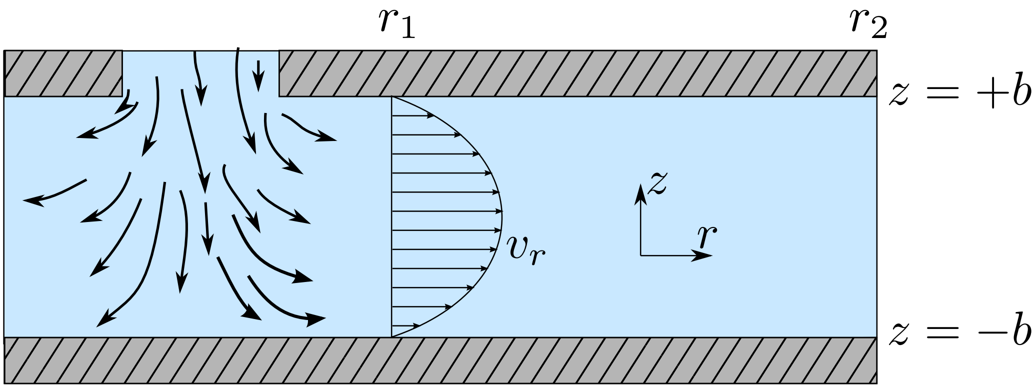


Figure 9: Radial flow between two plates.

a) Simplify the continuity equation to demonstrate that $r\,v_r$ is a function of z only. **[5 marks]**

b) Demonstrate that the stress profile within the channel is a solution of the following equation: **[10 marks]**

$$\rho\, v_r \frac{\partial v_r}{\partial r} = \mu\left(2\frac{\partial^2 v_r}{\partial r^2} + \frac{2}{r}\frac{\partial v_r}{\partial r} - \frac{2\,v_r}{r^2} + \frac{\partial^2 v_r}{\partial z^2}\right) - \frac{\partial\, p}{\partial r}$$

Note You must be careful during your derivation and make sure you expand each term of τ before cancellation.

c) Using the creeping flow assumption, the following expression for the velocity profile was derived **[5 marks]**:

$$v_r = -r^{-1}\frac{\Delta P}{2\,\mu\ln(r_2/r_1)}\left(z^2 + C_1\,z + C_2\right)$$

Determine the integration constants C_1 and C_2, and give the final expression for the velocity profile:

[Question total: 20 marks]

Question 23
Example exam question (2016)

A wire-coating die consists of a cylindrical wire of radius, $\kappa\,R$, moving horizontally at a constant velocity, v_{wire}, along the axis of a cylindrical die of radius, R. You may assume the pressure is constant within the die (it is not pressure driven flow) but the flow is driven by the motion of the wire (it is “axial annular Couette flow”). Neglect end effects and assume an isothermal system.

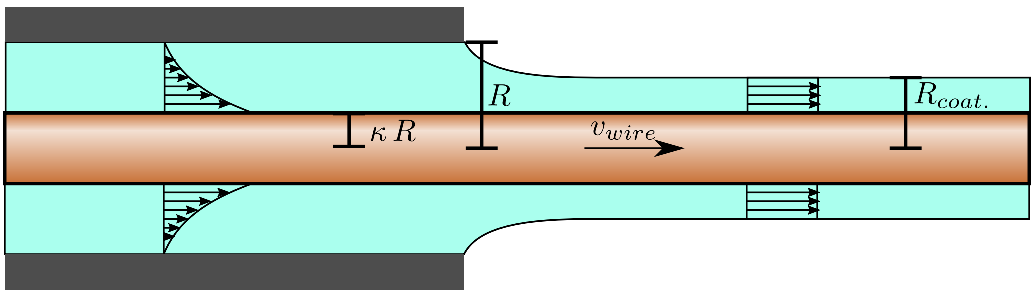


Figure 10: A diagram of a wire coating die for Q. 23.

a) State the two relevant boundary conditions for the flow within the die and how they arise. **[2 marks]**

b) The stress profile for an annular system is of the following form

$$\frac{1}{r}\frac{\partial}{\partial r} r\,\tau_{rz} = -\frac{\partial p}{\partial z} + \rho\,g_z.$$

Derive the following expression for the flow profile

$$v_z = \frac{v_{wire}}{\ln\kappa}\ln\left(\frac{r}{R}\right).$$

[9 marks]

c) Derive the following expression for the volumetric flow-rate of liquid through the die

$$\dot{V}_z = -\pi\, R^2\, v_{wire} \left(\kappa^2 + \frac{1-\kappa^2}{2\ln\kappa}\right).$$

[5 marks]

Note: You will need the integration identity

$$\int x\,\ln(x)\,\mathrm{d}x = \frac{x^2}{2}\left(\ln(x) - \frac{1}{2}\right).$$

d) Derive an expression for the outer radius of the coating, $R_{coat.}$, far away from the die exit. **[4 marks]**

[Question total: 20 marks]

Question 24
A solid wire is being used to carry electrical current (see Fig. 11).

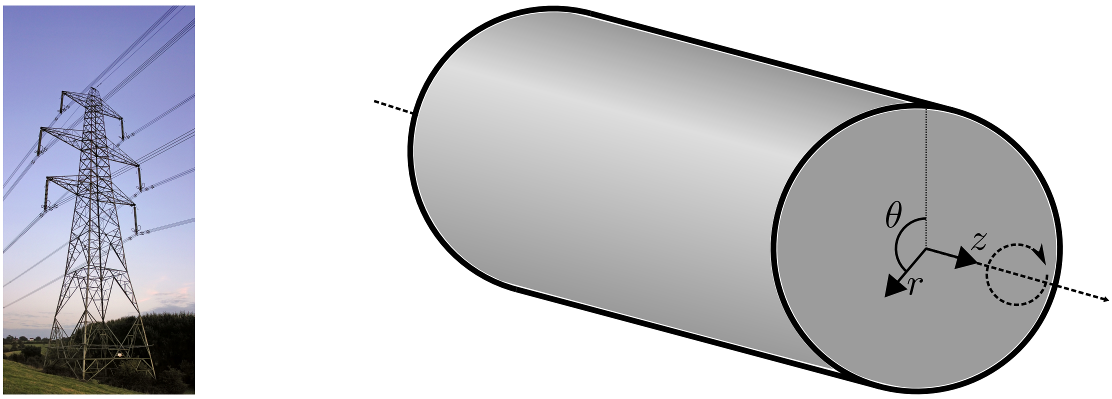


Figure 11: A solid wire.

a) Simplify the differential energy balance equation, state any assumptions you make, and derive the temperature profile in the wire. You may assume that heat is generated constantly within the volume of the wire at the following rate:

$$\sigma_{energy}^{current} = \frac{I^2}{k_e}$$

b) Discuss if the assumptions you have made are realistic.

c) How might the surface boundary condition be improved?

[Question end]

Question 25
Example exam question
An electric wire of radius 0.5 mm is made of copper (electrical conductivity $k_e = 5.1 \times 10^7$ ohm^{-1} m^{-1} and thermal conductivity $k = 380$ W m^{-1} K^{-1}). It is insulated to an outer radius of 1.5 mm with plastic (thermal conductivity $k = 0.35$ W m^{-1} K^{-1}). The volumetric heat production σ, is given by $\sigma = I^2/k_e$ where I is the current density A/m^2. The ambient air is at 38°C and the heat transfer coefficient from the outer insulated surface to the surrounding air is 8.5 W m^{-2}K^{-1}.

a) Determine the maximum current in amperes that can flow through the wire if no part of the insulation may exceed 93°C. **[8 marks]**

b) Demonstrate that the heat flux in the copper section of the wire is given by the following expression:

$$q_r = \frac{I^2}{2\,k_e} r$$

[8 marks]

c) Solve for the temperature profile within the copper wire, assuming the outer surface of the wire is at $T_{crit.}$. **[4 marks]**

[Question total: 20 marks]

Question 26

Again consider that we have a cylindrical wire of length L and radius R, generating heat at a rate of I^2/k_e per unit volume. Using a simple (not differential!) energy balance over the whole volume of the wire, what is the total heat generated Q? Compare this to the expression for the heat flux $q(r)$ evaluated at the surface of the wire ($r = R$) which you derived in Q. 24.

[Question end]

Question 27

In the lectures, we've derived the following integrated expressions for heat transfer in a plate and a pipe:

$$Q_x = \frac{k}{X} A\left(T_{inner} - T_{outer}\right) \qquad Q_r = \frac{2\,\pi\,L\,k}{\ln\left(R_{outer}/R_{inner}\right)}\left(T_{in} - T_{out}\right) \tag{6}$$

An equivalent equation is required for spherical geometries.

a) What single assumption was made in the derivation energy balance equation (see Eq. (22))?

b) Simplify the energy balance equation, Eq. (22), to the following expression:

$$\frac{\partial}{\partial r} r^2\,q_r = 0$$

Clearly state any assumptions you make along the way.

c) Solve for the following equation for the heat flux in spherical shells.

$$q_r = \frac{k}{r^2\left(R_{inner}^{-1} - R_{outer}^{-1}\right)}\left(T_{inner} - T_{outer}\right)$$

d) Demonstrate that the resistance to heat transfer, for a spherical shell is given by the following expression:

$$R = \frac{1}{UA} = \frac{R_{inner}^{-1} - R_{outer}^{-1}}{4\,\pi\,k}$$

Note: You will need to derive the expression for the overall heat flux, Q_r, and then isolate the $R = 1(UA)$ term.

[Question end]

Question 28

A spherical nuclear pellet, with an outer radius of 6 cm, is designed to produce 1kW of heat through fission. The heat transfer from the pellet is limited by a 5 mm pyrolytic graphite coating on the surface, which has a thermal conductivity of 240 W m^{-1} K^{-1}. Underneath the graphite is a 1 mm layer of Silicon Carbide reinforcement, which has a thermal conductivity of 4 W cm^{-1} K^{-1}. As the pellet is cooled by forced convection using a gas, the external convective heat transfer coefficient is around 100 W m^{-2} K^{-1}. If the ambient temperature is 150°C, calculate the surface temperature at the interface between the core and the Silicon Carbide.

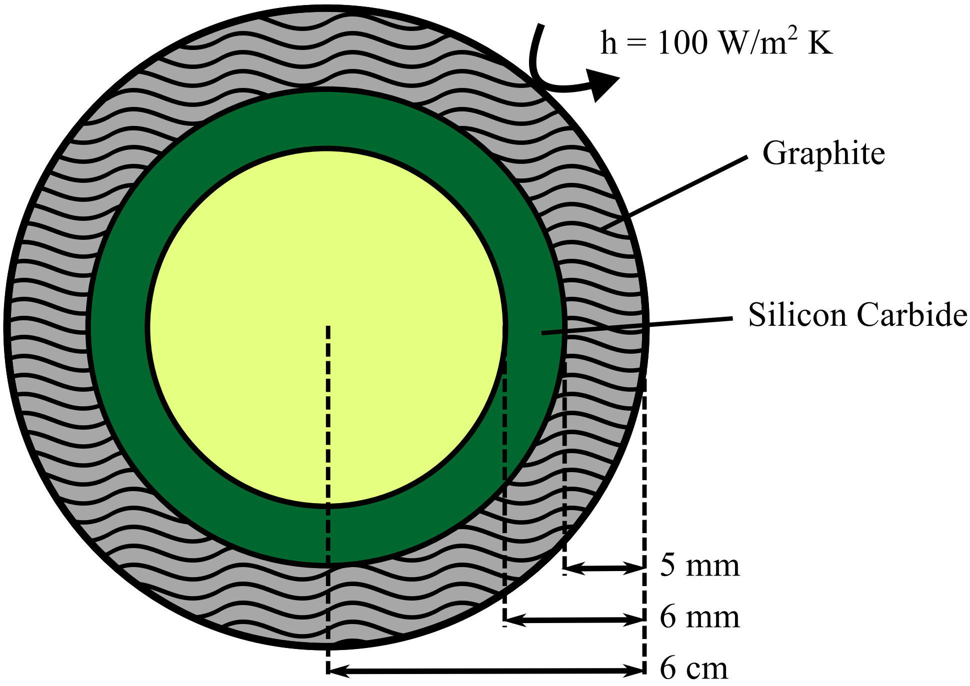


Figure 12: The nuclear pellet described in Q. 28.

[Question end]

Question 29

The temperature profile inside a nuclear fuel rod is needed as part of the design calculations for a reactor. The rod is a cylinder with a radius, R, and is assumed to be composed of a homogeneous fuel which is producing heat with the following profile:

$$\sigma_{heat} = \sigma^0 \left(1 + b\left(\frac{r}{R}\right)^2\right) \tag{7}$$

a) What assumption has been made to derive the energy balance equation below?

$$\rho\, C_p \frac{\partial T}{\partial t} = -\rho\, C_p\, \boldsymbol{v} \cdot \nabla\, T - \nabla \cdot \boldsymbol{q} - \boldsymbol{\tau} : \nabla\, \boldsymbol{v} - p \nabla \cdot \boldsymbol{v} + \sigma_{energy}$$

[2 marks]

b) Simplify the energy balance equation to the following expression:

$$\frac{1}{r}\frac{\partial}{\partial r}(r\, q_r) = \sigma_{energy}$$

Clearly state any assumptions you use. **[8 marks]**

c) Derive the expression below for the heat flux from the simplified energy balance.

$$q_r = \sigma^0 \left(\frac{r}{2} + b\frac{r^3}{4\,R^2}\right) \tag{8}$$

Clearly state any assumptions you use. **[5 marks]**

d) Derive the following expression for the temperature profile.

$$T - T_0 = \frac{\sigma^0}{k}\left(\frac{R^2 - r^2}{4} + b\frac{R^4 - r^4}{16\,R^2}\right) \tag{9}$$

You will need to select an appropriate boundary condition and give the meaning of the constant T_0. **[5 marks]**

[Question total: 20 marks]

Question 30

To explore the effect of using a temperature-dependent thermal conductivity, consider heat flowing through an annular (pipe) wall of inside radius R_0 and an outside radius R_1. It is assumed that thermal conductivity varies linearly with temperature from $k_0(T = T_0)$ to $k_1(T = T_1)$ where T_0 and T_1 are the inner and outer wall temperatures respectively.

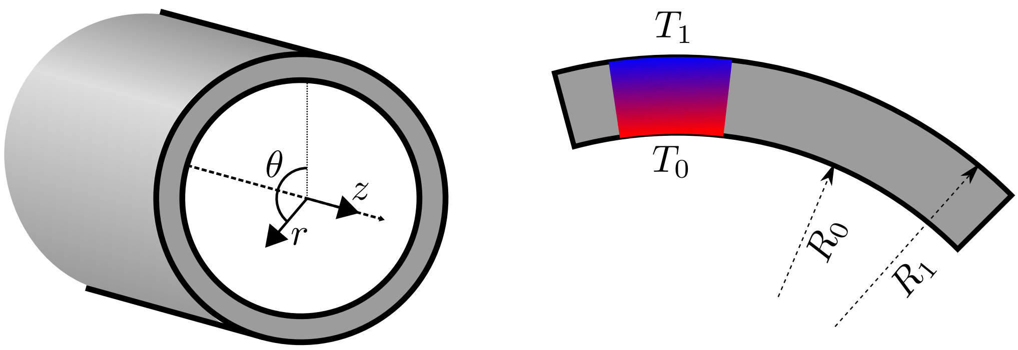


Figure 13: A diagram of conduction through an annular(pipe) wall for Q. 30.

a) Derive the following energy balance equation

$$\frac{\partial}{\partial r} r\, q_r = 0,$$

and state ALL assumptions required. **[7 marks]**

b) Derive the following expression for the temperature profile

$$Q_r = \frac{2\pi\, L}{\ln\left(\frac{R_0}{R_1}\right)}\frac{k_1 + k_0}{2}(T_1 - T_0),$$

where L is the length of the pipe/annulus. **[10 marks]**
Note: You will need the following identity:

$$T_1^2 - T_0^2 = (T_1 + T_0)(T_1 - T_0).$$

c) Compare this expression to the standard expression for conduction in pipe walls (with constant thermal conductivity), what can you observe? **[3 marks]**

[Question total: 20 marks]

Question 31
Consider the flow of a Newtonian liquid between two plates, similar to Q.14, but now both plates are maintained two different temperatures. We will attempt to take into account the effect of temperature on the flow profile.

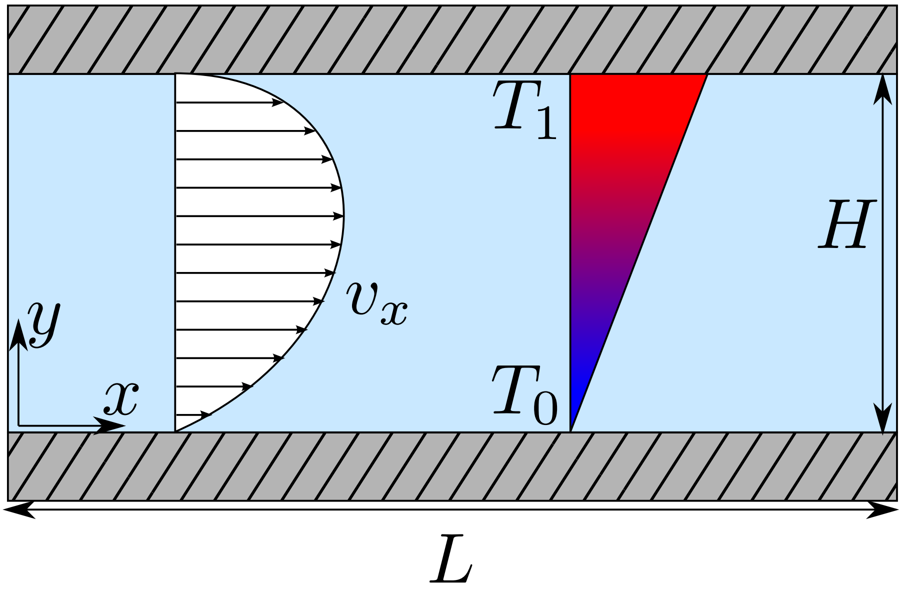


Figure 14: Flow through parallel plates.

You may assume that the viscosity, μ, of the liquid depends on temperature T according to the following relationship:

$$\mu(T) = \frac{\mu_0}{1 + \beta(T - T_0)}$$

where T_0 is a reference temperature, and μ_0 and β are empirical constants. The fluid flows under the influence of a pressure gradient $\Delta P/L$ between two flat plates, as shown in Fig. 14. The walls are at temperatures T_0 and T_1, where T_0 is the reference temperature, and $T_1 > T_0$.

a) Temporarily ignoring the motion of the fluid ($\boldsymbol{v} \approx \vec{0}$), demonstrate that the temperature can be taken to be a linear function of position:

$$T \approx T_0 + (T_1 - T_0)\frac{y}{H}$$

b) Derive the stress profile and prove that it is equal to the expression below. Compare this stress profile to the stress profile for flow between two plates, and for film flow on a plate. What is unique about the stress profile?

$$\tau_{yx} = \frac{\Delta p\, H}{L}\left(\frac{y}{H} + C_1\right)$$

c) Assuming the temperature profile is indeed linear, derive the following velocity profile for this system.

$$v_x(y) = -\frac{\Delta p\, H^2}{L\,\mu_0}\frac{y}{H}\left[\beta(T_1 - T_0)\frac{y^2}{3\,H^2} + (1 + C_1\,\beta\,[T_1 - T_0])\frac{y}{2\,H} + C_1\right] \qquad (10)$$

where C_1 is a dimensionless integration constant which you must determine.

d) Determine the flow-rate to pressure drop relationship.

e) Calculate the x-component of the force of the fluid on the bottom surface $y = 0$ per unit area of the plate and compare it to the value on the top surface.

[Question end]

Question 32

Perform dimensional analysis on a pendulum of length l, mass m, under gravity g to better understand the period of oscillation, t. How does the pendulum period change with changes in its mass?

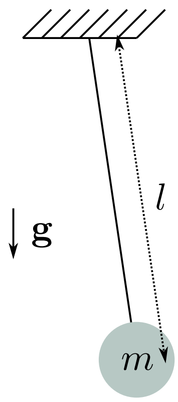


Figure 15: A pendulum with mass m, length l, in gravity of g.

[Question end]

Question 33
Example exam question (2016)

Consider laminar flow within a pipe. The only prior knowledge you should assume is that the pressure drop must be a function of pipe diameter D, viscosity μ, density ρ, and average velocity $\langle v_z \rangle$, i.e.,

$$\Delta p/l = f\left(D, \rho, \mu, \langle v_z \rangle\right).$$

a) Perform dimensional analysis on the pressure drop per unit length, $\Delta p/l$, and determine the relevant dimensionless groups. **[12 marks]**

b) Compare this to the exact solution, known as the Hagen-Poiseuille equation, as given below.

$$\dot{V}_z = \pi \left(\frac{-\Delta p}{l} + \rho\, g_z\right) \frac{R^4}{8\,\mu}.$$

Determine the form of the unknown function, f. **[5 marks]**

c) Comment on why dimensional analysis is so important. Also comment on why redundant dimensionless groups arise (as an example, consider the relationship between friction factor C_f and the Reynolds number). **[3 marks]**

[Question total: 20 marks]

Question 34
Example exam question (2014)
Carry out a dimensional analysis on the forced convection heat transfer coefficient, h, to determine which are the fundamental dimensionless numbers involved. You may assume the following general dependence

$$h = f\left(d,\, \mu,\, k,\, \langle v\rangle,\, \rho,\, C_p\right)$$

where d is the channel diameter (m), μ is the viscosity (Pa s), k is the thermal conductivity (W m^{-1} K^{-1}), $\langle v\rangle$ is the mean flow velocity (m s^{-1}), ρ is the mass density (kg m^{-3}), and C_p is the specific heat capacity at constant pressure (kJ kg^{-1} K^{-1}). **[10 marks]**

[Question total: 10 marks]

Question 35
Calculate the dimensionless heat transfer coefficient (Nu) for conductive heat transfer through rectangular walls. **Note:** You will need to rephase the conductive resistance as a heat transfer coefficient h.

[Question end]

Question 36
Example exam question (2012)
The heat loss from a pipe which is carrying a hot process fluid must be estimated to evaluate if additional insulation is economically justified.

a) Starting from the general expression for steady-state conduction in cylindrical shells:

$$\frac{\partial}{\partial r} r\, q_r = 0 \tag{11}$$

Derive the following expression for the heat flux in a cylindrical wall:

$$q_r = \frac{k}{r\,\ln\left(R_{outer}/R_{inner}\right)}\left(T_{inner} - T_{outer}\right) \tag{12}$$

[8 marks]

b) Derive the following expression for the heat transfer resistance for conduction in a cylindrical wall.

$$R = \frac{\ln\left(R_{outer}/R_{inner}\right)}{2\,\pi\,L\,k} \tag{13}$$

[3 marks]

c) The pipe carrying the process fluid has an inner diameter of 15 cm and a length of 50 m. The process fluid, flowing at 1 kg s^{-1}, has a density of 800 kg m^{-3}, a viscosity of 2×10^{-3} Pa s, a heat capacity of 1.2 kJ kg^{-1} K^{-1}, and a thermal conductivity of 0.15 W m^{-1} K^{-1}.

   i) Is the flow inside the pipe turbulent? **[2 marks]**

   ii) Demonstrate that the forced convection heat transfer coefficient on the inside of the pipe is approximately $h \approx 57$ W m^{-2} K^{-1}. **[2 marks]**

iii) The pipe has a carbon-steel wall which is 1 cm thick and has a thermal conductivity of 43 W m^{-1} K^{-1}. The pipe is also insulated using a 1 cm layer of rock wool, which has a thermal conductivity of 0.045 W m^{-1} K^{-1}. The external heat transfer coefficient, which includes radiation and natural convection, is estimated to be 5 W m^{-2} K^{-1}. Determine the overall heat flux through the pipe if the process fluid is at 80°C and the surroundings are at 10°C. **[5 marks]**

[Question total: 20 marks]

Question 37
Example exam question

a) Chilled water flowing through brass tubes of 0.0126 m inside diameter and 0.0018 m thickness cools a stream of air flowing outside of the tube. The film coefficients for the air and water flows are 176 Wm^{-2} K^{-1} and 5660 W m^{-2} K^{-1} respectively and thermal conductivity of the brass is 102 W m^{-1} K^{-1} (see Fig. 16).

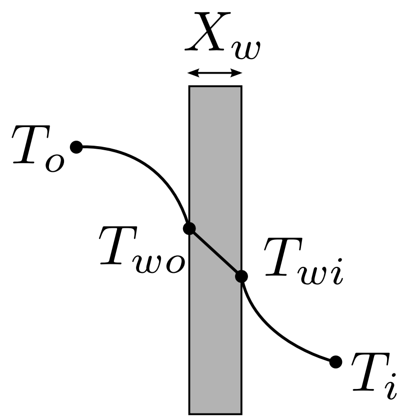


Figure 16: The temperature profile through the pipe wall.

i) Calculate overall heat transfer resistance $R_{total} = (UA)^{-1}_{total}$. **[6 marks]**

ii) State what is the limiting heat resistance (i.e., what is the controlling heat transfer mechanism). **[2 marks]**

iii) Calculate heat transferred per metre length of tube at the point where the bulk temperatures of the air and water streams are 326°C and 15°C respectively. **[2 marks]**

b) The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature-time history of a sphere fabricated from pure copper. The diameter of sphere is 17 mm. The sphere is at 86°C before it is inserted into an airstream having a temperature of 22°C. A thermocouple on the outer surface of the sphere indicates 62°C at 116 seconds after the sphere is inserted into the airstream.
Note: The properties of copper at 347K are ρ = 8933 kg m^{-3}, C_p = 389 J kg^{-1} K^{-1}, and k = 398 W m^{-1} K^{-1}.

i) Calculate the heat transfer coefficient by assuming that the lumped capacitance method is valid. **[7 marks]**

ii) Show that the Biot number supports the application of the lumped capacitance method. **[3 marks]**

[Question total: 20 marks]

Question 38
Example exam question
An electric heater of 0.032 m diameter and 0.85 m in length is used to heat a room. Calculate the electrical input (i.e. the sum of heat transferred by convection and radiation) to the heater when the bulk of the air in the room is at 24°C, the walls are at 12°C, and the surface of the heater is at 532°C. For convective heat transfer from the heater, assume the heater is a horizontal cylinder and the Nusselt number is given by

$$\mathrm{Nu} = 0.38(\mathrm{Gr})^{0.25}$$

where all properties are evaluated at the film temperature. You may assume air is an ideal gas, giving $\beta = T^{-1}$. Take the emissivity of the heater surface as $\epsilon = 0.62$ and assume that the surroundings are black. All other properties should be calculated using the table provided (see Table 1). **[10 marks]**

T (K)	μ (kg m^{-1} s^{-1})	k (W m^{-1} K^{-1})	ρ (kg m^{-3})
550	2.849×10^{-5}	4.357×10^{-2}	0.6418
600	3.017×10^{-5}	4.661×10^{-2}	0.5883
700	3.332×10^{-5}	5.236×10^{-2}	0.5043
800	3.624×10^{-5}	5.774×10^{-2}	0.4412
900	3.897×10^{-5}	6.276×10^{-2}	0.3922

Table 1: Physical properties of air at 1 atm for Q.38.

[Question total: 10 marks]

Question 39
A pebble-bed nuclear reactor at 69 bara is used to heat helium (4 g mol^{-1}) as part of the generation of electricity. The helium gas has a heat capacity at constant pressure of C_p = 5190 J kg^{-1} K^{-1}, a dynamic viscosity of $\mu = 5.19 \times 10^{-5}$ Pa s, and a thermal conductivity of k = 0.405 W m^{-1} K^{-1} and flows at 15 m s^{-1}. The pebbles have an outer radius of 3 cm which consists of a 0.5 cm coating of graphite around the radioactive core.

a) Assuming helium may be treated as an ideal gas, demonstrate that the density of the gas is 2.83 kg m^{-3}. **[3 marks]**

b) Calculate the surface temperature of the particle if it is emitting 850 W of heat and the surrounding helium is at 900 °C. The following expression for forced convective heat-transfer around a sphere is available,

$$\mathrm{Nu}_D = 2 + 0.47\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{0.36} \qquad \text{for } 3 \times 10^{-3} < \mathrm{Pr} < 10 \text{ and } 10^2 < \mathrm{Re}_D < 5 \times 10^4.$$

Radiation is neglible as all pellets have the same surface temperature, and the characteristic length used in the Reynolds and Nusselt number is the sphere diameter. **[12 marks]**

[Question total: 15 marks]

Question 40
Example exam question
A single-pass, counter-flow shell-and-tube heat exchanger is required to operate as an oil cooler with 316 tubes of internal diameter 0.016 m, outer diameter 0.018 m, and length 5.6 m. The oil flows in the tube side entering at a mass flow rate of 32 kg s^{-1} at a temperature of 136°C. Cooling water in the shell side enters at a mass flow rate of 33 kg s^{-1} at a temperature of 10°C. The shell side heat transfer coefficient is 850 W m^{-2} K^{-1}; and the specific heat capacity of water is 4.187 kJ kg^{-1} K^{-1}. The Nusselt number is approximately related to the Reynolds and Prandtl numbers as follows

$$\mathrm{Nu} = 0.025\,\mathrm{Re}^{3/4}\,\mathrm{Pr}^{2/5} \tag{14}$$

and the following property values apply: specific heat capacity of oil: 3.42 kJ kg^{-1} K^{-1}; density of oil: 900 kg m^{-3}; dynamic viscosity of oil: 1.5×10^{-3} kg m^{-1} s^{-1}; thermal conductivity of oil: 0.15 W m^{-1} K^{-1}; thermal conductivity of the steel pipe wall: 54 W m^{-1} K^{-1}. Calculate:

a) The number of transfer units. **[12 marks]**

b) The effectiveness of the heat exchanger. **[3 marks]**

[Question total: 15 marks]

Question 41
Example exam question (2015)
Consider a pot of boiling water placed on a radiant (halogen) cooking hob. As the water is boiling, the surface temperature of the pot will be approximately the boiling temperature. The pot is exposed to the atmosphere and the air/surroundings are at 20°C.

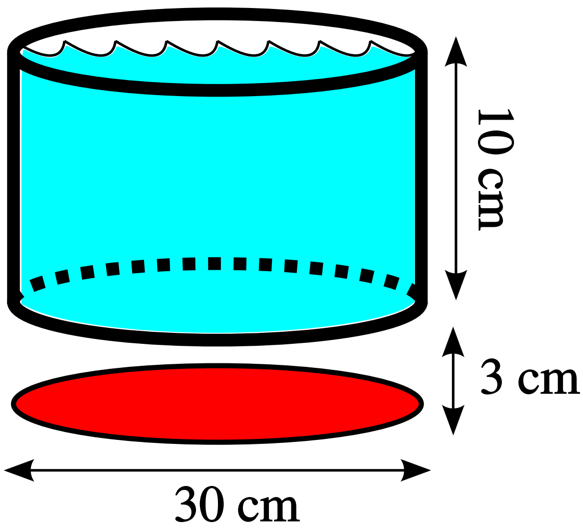


Figure 17: The boiling pot problem.

a) Calculate the natural convective heat loss from the sides of the pot given that air has a mean molar mass of $M_W \approx 29$ g mol^{-1}, a dynamic viscosity of $\mu \approx 1.8 \times 10^{-5}$ Pa s, a thermal conductivity of $k_{air} \approx 0.0257$ W m^{-1} K^{-1}, and a Prandtl number of Pr ≈ 0.713. **[11 marks]**

b) Assume that the total heat loss from the pan is 100 W due to evaporation and radiant heat loss to surroundings. Calculate the radiant temperature of the hob/heat-source required to counteract the heat loss. You may assume the pan and heat-source are black-bodies for this calculation.

The view factor between two coaxial discs is

$$F_{1\to 2} = 0.5\left(S - \left(S^2 - 4(r_j/r_i)^2\right)^{0.5}\right)$$

where $S = 1 + \left(1 + R_j^2\right)/R_i^2$, and the reduced radii are $R_i = r_i/L$ and $R_j = r_j/L$. Note L is the gap between the discs, and (r_i, r_j) are the radii of the two discs. **[6 marks]**

c) What fraction of the heat radiated from the heater hits the pot? **[3 marks]**

[Question total: 20 marks]

Question 42
Consider an unshielded thermometer placed in a room (see Fig. 18). The walls of the house are poorly insulated and the internal surfaces are at a temperature of 5°C. If the thermometer reads 20°C and all surfaces have an emissivity of 0.9, what is the real temperature of the air? You may assume a rough estimate of the natural convective coefficient as $h \approx 10\ \mathrm{W\,m^{-2}\,K^{-1}}$.

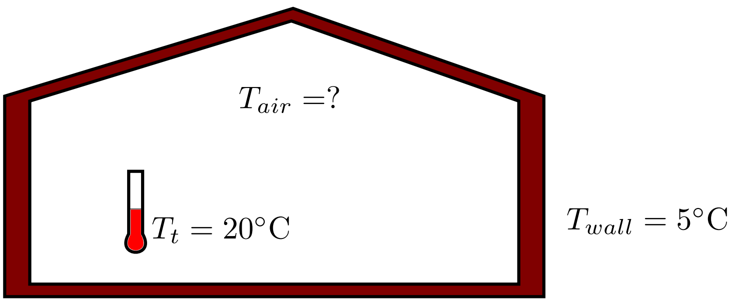


Figure 18: An unshielded thermometer in a room with cold walls.

[Question end]

Question 43
The James webb telescope uses a radiation shield to reduce the heat it recieves from the sun, earth, and moon (see Fig. 19). By what factor will the radiation be approximately reduced by? How realistic is this estimate (what approximations are there)? Is this an over or under estimate of the reduction in radiation? **[5 marks]**

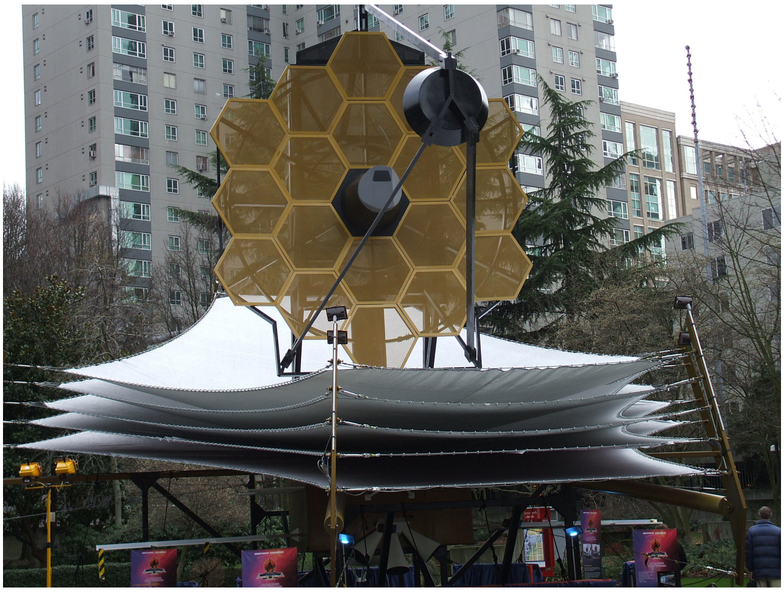


Figure 19: A mock-up of the James Webb telescope, displaying its five-layered sunshield.

[Question total: 5 marks]

Question 44

What are the reciprocity relationship and the summation rule with respect to radiative heat transfer? How are these useful?

[Question end]

Question 45

Example exam question

A 10 m pipe with a outer-radius of r_{pipe} = 2.5 cm is to be insulated using a layer of insulation with a thermal conductivity of k = 0.18 W m^{-1} K^{-1}. You may assume that the external convective heat transfer coefficient of the insulation is constant at h = 5 W m^{-2} K^{-1} and that these two mechanisms are the only significant heat transfer resistances.

a) Write down the heat transfer equation for this system showing how the overall heat transfer rate Q depends on k, r_{pipe}, the outer radius of the pipe insulation $r_{ins.}$, the pipe length L, and the temperature difference ΔT between the pipe wall and the ambient air. **[4 marks]**
Note: The resistance to heat transfer in a cylindrical shell is:

$$R = \frac{\ln\left(r_{outer}/r_{inner}\right)}{2\,\pi\,k\,L}$$

b) Calculate the heat transfer rate for three thicknesses of insulation where the outer radius of the insulation is $r_{ins.}$ = 3.0 cm, 3.6 cm, and 4.2 cm). The surface temperature of the pipe is 400°C and ambient conditions are at 10°C. **[3 marks]**

c) Explain why you observe a maximum in the heat transfer rate. **[3 marks]**

[Question total: 10 marks]

Question 46

The potential heat loss from a distillation column to the environment must be calculated to determine if lagging (insulation) on the column is required. The proposed design of a distillation column can be modelled to a rough approximation as a 12 m high cylinder with a diameter of 0.7 m. Convection and radiation are assumed to be the limiting heat transfer processes, so it can be assumed the column surface is at the internal operating temperature of 60°C. The minimum ambient air temperature should be used for the calculations in order to design for a worst-case scenario

Aberdeen ambient temperature range: -10°C to 20°C.

Emissivity of oxidised steel: $\varepsilon \approx 0.657$

T (°C)	ρ (kg m^{-3})	C_p (kJ kg^{-1} K^{-1})	k (W m^{-1} K^{-1})	ν ($\times 10^{-6}$ m^2 s^{-1})	Pr
-50	1.534	1.005	0.0204	9.55	0.725
0	1.293	1.005	0.0243	13.30	0.715
20	1.205	1.005	0.0257	15.11	0.713
40	1.127	1.005	0.0271	16.97	0.711
60	1.067	1.009	0.0285	18.90	0.709

Table 2: Properties of Air

a) Describe the physical interpretation of the Grashof number for natural convection. Describe each of its terms and write down an equation for the temperature at which temperature-dependent terms in Gr should be evaluated. **[5 marks]**

b) Show that the thermal expansion coefficient, defined as

$$\beta = \frac{1}{V}\frac{\partial V}{\partial T}$$

reduces to the following expression for an ideal gas

$$\beta_{ig.} = \frac{1}{T}$$

Hint: Use the ideal gas equation! **[2 marks]**

c) Calculate the Grashof number and determine the convective flow regime. State any assumptions you make. Remember to use the correct temperature for calculating the properties of the flow! **[5 marks]**

d) Calculate the convective heat transfer coefficient for the column surface. Can you use the expression for vertical plates directly? **[9 marks]**

e) Calculate the total heat lost to the environment including radiation. Compare the two losses. **[5 marks]**

f) Do you think this heat loss justifies adding insulation or lagging to the outside of the column? **[1 marks]**

g) Due to strict new environmental legislation, it is decided that the maximum acceptable heat loss to the environment is 10 kW. Roughly calculate the maximum acceptable surface temperature. **[3 marks]**

h) Comment on what steps would be required to improve the accuracy of the surface temperature calculation in Q. g. **[2 marks]**

[Question total: 32 marks]

Question 47
Write down the expressions for the Prandtl number. Define every term and describe the physical interpretation of the dimensionless numbers.

[Question end]

Question 48
Example exam question (2015)
The wall of a furnace comprises three layers as shown in Fig. 20. The first layer is refractory brick (whose maximum allowable temperature is 1400°C) while the second layer is insulation (whose maximum allowable temperature is 1093°C). The third layer is a plate of 6.35 mm thickness of steel (k_{steel} = 45 W m^{-1} K^{-1}). Assume that the layers are thermally bonded.

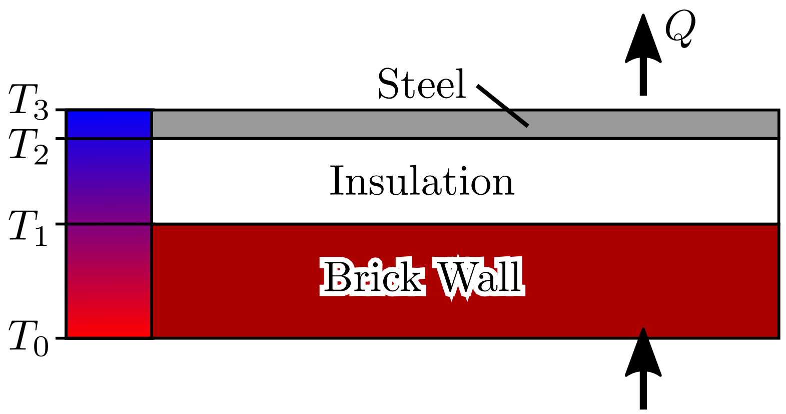


Figure 20: Construction of a furnace wall.

Layer	T = 37.8°C	T = 1093°C
Brick	3.12 W m^{-1} K^{-1}	6.23 W m^{-1} K^{-1}
Insulation	1.56 W m^{-1} K^{-1}	3.12 W m^{-1} K^{-1}

Table 3: Thermal conductivities for Q. 48.

The temperature T_0 on the inside of the refractory is 1370°C, while the temperature on the outside of the steel plate is 37.8°C. the heat loss through the furnace wall is expected to be 15800 W m^{-2}. Determine the thickness of refractory and insulation that results in the minimum total thickness of the wall. You may use the temperature dependent thermal conductivities given in Table 3. **[14 marks]**

[Question total: 14 marks]

Question 49
In prilling towers, molten fertilizer slurry is dripped to form frozen spherical pellets called prills. As a first approximation to understanding the heat transfer from the falling prills, consider a heated sphere of radius, R, and fixed surface temperature, T_R, suspended in a large, motionless body of fluid.

a) Set up the differential equation describing the temperature, T, in the surrounding fluid as a function of r, the distance from the center of the sphere. The thermal conductivity, k, of the fluid is considered constant. **[14 marks]**

b) Integrate the differential equation and use these boundary conditions to determine the integration constants: at $r = R$, $T = T_R$; and at $r = \infty$, $T = T_\infty$. **[8 marks]**

c) From the temperature profile, obtain an expression for the heat flux at the surface. Equate this result to the heat flux given by “Newton’s law of cooling” and show that a dimensionless heat transfer coefficient (known as the Nusselt number) is given by,

$$\mathrm{Nu} = \frac{h\,D}{k} = 2,$$

in which D is the sphere diameter. **[12 marks]**

[Question total: 34 marks]

Question 50
Example exam question
A black-body car is left in direct sunlight at midday which (at the lattitude of the UK) can be approximated as a constant heat flux q_{sun} = 1000 W m^{-2}. The car’s surface temperature reaches steady state with its surroundings and is approximately constant. The car has a surface area of 26 m^2 but only 8 m^2 are exposed to sunlight.

a) Assuming that the ambient temperature is 15°C and that radiation is the only heat transfer mechanism, calculate the surface temperature of the car. Is the estimate realistic? **[5 marks]**

b) Using the previous estimate for the surface temperature, estimate the heat flux due to natural convection and comment on its magnitude. You may approximate the sides of the car as a vertical wall 12 m wide and 1.5 m high. You may assume the following properties of air at these conditions. State why natural convection from the top of the car is insignificant when compared to the sides. **[8 marks]**

ρ (kg m^{-3})	k (W m^{-1} K^{-1})	μ (kg m^{-1} s^{-1})	C_p (J mol^{-1} K^{-1})	Avg. Mol. Weight (g mol^{-1})
1.225	0.026	1.827×10^{-5}	29.19	29

c) Discuss how you might improve the accuracy of the calculations, and what the effect of setting the car in motion will be. **[2 marks]**

[Question total: 15 marks]

Question 51
The wall of a furnace was measured to be at a temperature of T_w = 60 °C when the ambient air temperature is at T_∞ = 10 °C. The wall is 3 m high, 5 m wide, and has a surface emissivity of ε = 0.7. The properties of air are given in the table below.

μ	1.78×10^{-5} Pa s	ρ	1.2 kg m^{-3}
k	0.02685 W m^{-1} K^{-1}	C_p	1.005 kJ kg^{-1} K^{-1}

a) Determine the convective flow regime of the air, noting that the critical Grashof number is Gr $\approx 4 \times 10^8$.

b) Calculate the heat lost through the furnace wall. Remark on the relative magnitudes of the two heat transfer mechanisms involved.

END OF EG40JK QUESTIONS

[Question end]

Question 52

A new type of one-coat spray paint is being developed which flows to precisely the minimum thickness required for a uniform coat. To achieve this property, the paint must effectively be a Bingham plastic.

a) Balance the total gravitational force ($\rho\, g_y$) against the viscous force on a vertical plate to derive the following force balance for the stress at the wall surface:

$$\tau_{boundary} = Y\rho\, g_y$$

b) Assuming that the paint has a density of 900 kg m^{-3}, what yield stress (τ_0) is needed to ensure the paint has a maximum static thickness of 2 mm?

[Question end]

Question 53

When manufacturing a plastic toy, a polypropylene melt with a density of 739 kg m^{-3} is to be extruded through a pipe with a length of 1 m and a diameter of 2.5 cm into a die. A shear rate of 1000 s^{-1} is expected at the die lips and experiments at this shear rate have measured an apparent viscosity of 10 N s m^{-2}.

a) A Power-Law model with an exponent of $n = 0.35$ is thought to be a suitable model for the viscous behaviour. Assuming this is true, determine the consistency coefficient k and write down the rheological stress-strain equation for the fluid. **[3 marks]**

b) What is the type of this fluid and how will it respond to increasing rates of shear? Describe this using the concept of the apparent viscosity. **[2 marks]**

c) Sketch two graphs to illustrate the differences between the velocity profile of this fluid and a Newtonian fluid, and between this fluid and a Bingham plastic fluid. **[5 marks]**

d) Derive the following expression for the Reynolds number in Power-Law fluids.

$$\mathrm{Re}_{MR} = \frac{8\,\rho\,\langle v\rangle^{2-n}\,R^n}{k}\left(\frac{n}{3\,n+1}\right)^n$$

Hint: the Metzner-Reed Reynolds number is defined through the friction factor relation,

$$C_f = \frac{16}{\mathrm{Re}_{MR}}.$$

The volumetric flow equation for a laminar power-law fluid is available in the datasheet (see Eq. (23)). **[7 marks]**

e) If a volumetric flow rate of 0.1 m^3 h^{-1} is required, determine if the flow is laminar in the pipe and calculate the pressure drop. **[3 marks]**

[Question total: 20 marks]

Question 54

A non-Newtonian fluid flows through a 20 m length pipe with a diameter of 25 mm. Its apparent viscosity is 0.1 N s m^{-2} at a shear rate of 1000 s^{-1} and its density is estimated to be 1600 kg m^{-3}.

a) If the flow index n is 0.33, show that the consistency k is 10 if the Power Law model applies. Give the rheological equation for the fluid. **[3 marks]**

b) What type of fluid is this and how will it respond to increasing rates of shear? **[3 marks]**

c) If a flow-rate of 1 m^3 hr^{-1} is required, show that the flow would be laminar and calculate the pressure drop. **[5 marks]**
Note: The definition of the Metzner-Reed Reynolds number for Power-Law fluids in pipes is given by

$$\mathrm{Re}_{MR} = 8\left(\frac{n}{6\,n+2}\right)^{n}\frac{\rho\left\langle v\right\rangle^{2-n}D_H^n}{k}$$

d) Roughly sketch the flow profile for this fluid comparing it to the sketch of a Newtonian fluid and a Bingham-plastic fluid. Explain the differences between the profiles. **[3 marks]**

[Question total: 14 marks]

Question 55

An incompressible polymeric fluid is to flow through 10 m of 50 mm inner-diameter piping. The flow index, n, for the fluid is 0.3 and the apparent viscosity, μ, at a shear rate of 1000 s^{-1} is 0.1 Pa s.

a) What type of fluid is this? Give a general description of its viscosity and include a sketch of the stress-rate versus strain graph and give the numerical expression for the stress τ_{xy}. **[8 marks]**

b) Assuming the flow is laminar, what is the frictional pressure loss if the volumetric flow rate required at the end of the pipe is 0.005 m^3 s^{-1}? **[5 marks]**

c) Using the Metzner-Reed Reynolds number, would you expect the flow in the pipe to be laminar or turbulent? The standard transition value for the Reynolds numer applies and you may assume a fluid density of 1500 kg m^{-3}. **[4 marks]**

d) How does the velocity profile in this pipe compare to one carrying a Newtonian fluid? Illustrate your answer with an appropriate diagram. **[3 marks]**

[Question total: 20 marks]

Question 56

Consider the flow profile of a incompressible, Newtonian fluid through a horizontal annnulus (see Fig. 21).
The velocity profile was derived in Q.18, and is given by the following equation.

$$v_z = -\frac{\Delta p\,R^2}{4\,L\,\mu}\left(\frac{r^2}{R^2} - \frac{\kappa^2-1}{\log\kappa}\log\left(\frac{r}{R}\right) - 1\right)$$

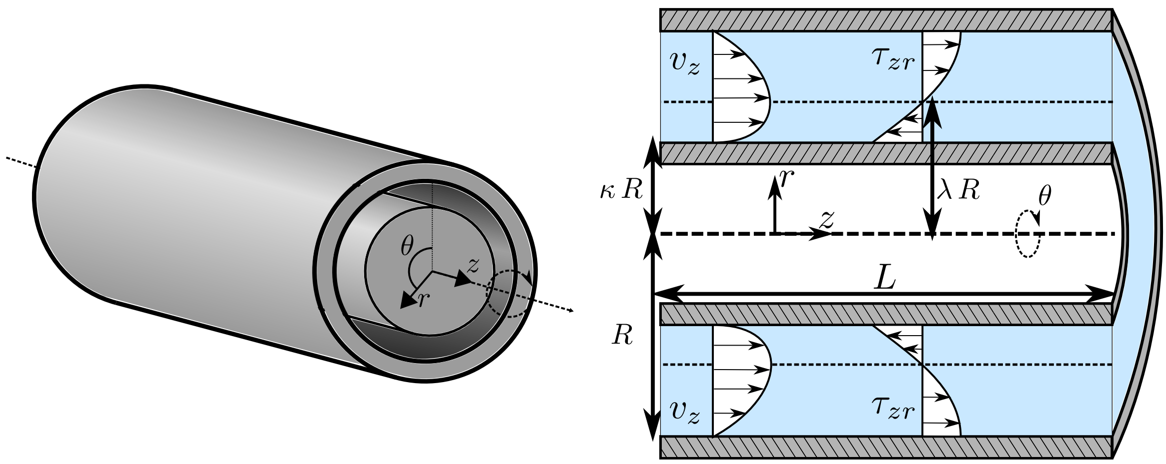


Figure 21: Axial flow in an annulus (pipe in pipe).

a) Derive the following expression for the volumetric flow rate as a function of pressure drop.

$$\dot{V}_z = \frac{\pi\,\Delta p\,(1-\kappa^2)\,R^4}{8\,L\,\mu}\left[1+\kappa^2+\frac{\left(1-\kappa^2\right)}{\log\kappa}\right]$$

Hint: You may need the following identity obtained from integration by parts.

$$\int x\,\log(x)\,\mathrm{d}x = \frac{x^2\,\log(x)}{2} - \frac{x^2}{4} + C$$

b) Derive the following expression for the mean flow velocity, $\langle v_z\rangle$.

$$\langle v_z\rangle = \frac{\Delta p\,R^2}{8\,L\,\mu}\left[1+\kappa^2+\frac{\left(1-\kappa^2\right)}{\log\kappa}\right]$$

c) One method to generalise the definition of the Reynolds number is to use a hydraulic diameter, $D_H = 4\,A_{flow}/P_w$, in place of the diameter:

$$\mathrm{Re}_H \equiv \frac{\rho\,\langle v_z\rangle\,D_H}{\mu}$$

Use this definition to calculate the following expression for the Reynolds number of a incompressible, Newtonian fluid through a horizontal annnulus:

$$\mathrm{Re}_H \equiv \frac{2\,\rho\,\langle v_z\rangle\,R\,(1-\kappa)}{\mu}$$

d) Describe (not derive) how Metzer-Reed generalised the definition of the Reynolds number (what did they do to fix the definiton of Re)?

$$\mathrm{Re}_{MR} = -\frac{8\,\rho\,\langle v\rangle^2\,P_w\,L}{A_{flow}\,\Delta p}$$

Using this approach, derive the following expression for the Metzner-Reed Reynolds number of a incompressible, Newtonian fluid through a horizontal annnulus.

$$\mathrm{Re}_{MR} = -\frac{2\,\rho\,\langle v\rangle\,R}{\mu}\left[\frac{1+\kappa^2}{1-\kappa}+\frac{1+\kappa}{\log\kappa}\right]$$

e) Comment on the two definitions of the Reynolds numbers and discuss which is “better”?

[Question end]

Question 57

Two immiscible incompressible Newtonian fluids flow co-currently in a horizontal plane channel, as shown in Fig. 22. The density and viscosity of fluid 1 are ρ_1 and μ_1, respectively; the density and viscosity of fluid 2 are ρ_2 and μ_2. This is the simplest example of **multiphase flow**, and is one of the few systems with an analytical solution. Each phase must be solved separately (two Continuity/Cauchy equations) as they only interact with each other through their boundary conditions. Assuming the two fluids are liquids (not liquid gas), we can apply a no-slip condition between the two phases at $y = h$ (the velocities of the two phases are equal). We also know that the stresses are equal at the interface from Newton's third law of motion.

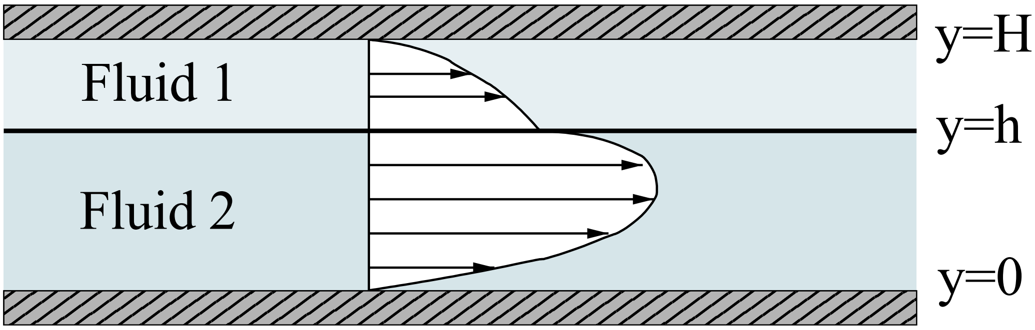


Figure 22: Flow of two immiscible fluids in a planar channel.

a) Derive the following expressions for the velocity distributions in each fluid.

$$v_{x,1}(y) = -\frac{\Delta p H^2}{2\mu_1 L}\left(1-\frac{y}{H}\right)\left[1+A_1+\frac{y}{H}\right] \tag{15}$$

$$v_{x,2}(y) = \frac{\Delta p H^2}{2\mu_2 L}\left(\frac{y}{H}\right)\left(\frac{y}{H}+A_1\right) \tag{16}$$

where

$$A_1 = -\left[1+\left(\frac{\mu_1}{\mu_2}-1\right)\frac{h^2}{H^2}\right]\left[1+\left(\frac{\mu_1}{\mu_2}-1\right)\frac{h}{H}\right]^{-1} \tag{17}$$

Clearly state what assumptions you make along the way.

b) Derive the volumetric flow rate of each phase and give the ratio of the two flow rates. The answer is:

$$\begin{aligned}\frac{\dot{V}_1}{\dot{V}_2} = -\frac{3\mu_2}{\mu_1}\frac{H^3}{h^3}&\left[(1+A_1)\left(1-\frac{h}{H}\right)-\frac{A_1}{2}\left(1-\frac{h^2}{H^2}\right)-\frac{1}{3}\left(1-\frac{h^3}{H^3}\right)\right]\\ &\times\left[1+A_1\frac{3}{2}\frac{H}{h}\right]^{-1}\end{aligned} \tag{18}$$

c) Compare the expression above for the ratio of the volumetric flows, to the ratio of the channel occupied by the flow ($(H-h)/h$). Why do these differ? What does this imply for gas-liquid systems?

[Question end]

Question 58
Exam question (2011 and 2014)
The Lockhart-Martinelli parameter, X, is a critical parameter in two-phase flow pressure-drop and liquid hold-up calculations. It is defined as the ratio of the frictional pressure drops of each phase, calculated as if each was flowing alone in the pipe.

$$X^2 = \frac{(\partial p/\partial z)_{liq.-only}}{(\partial p/\partial z)_{gas-only}}$$

a) Assuming that the pipe is smooth and that both phases are fully turbulent, derive the following expression for the Martinelli parameter

$$X_{tt} = \left(\frac{1-x}{x}\right)^{0.875}\left(\frac{\mu_{liq.}}{\mu_{gas}}\right)^{0.125}\left(\frac{\rho_{gas}}{\rho_{liq.}}\right)^{0.5}$$

Extra hint: You may need the Darcy-Weissbach equation and a suitable expression for the friction factor (see the datasheet). **[4 marks]**

b) A mixture of saturated steam at 0.09 kg s^{-1} and water at 1.6 kg s^{-1} is flowing along a horizontal pipe with an internal diameter of 75 mm. The steam has a viscosity of $\mu_g = 0.0113 \times 10^{-3}$ N s m^{-2} and density of 0.788 kg m^{-3}. The water has a viscosity of 0.52×10^{-3} N s m^{-2} and a density of 1000 kg m^{-3}.

i) Determine the flow pattern inside the pipe. **[3 marks]**

ii) Determine the flow regime inside each phase of the pipe. **[4 marks]**

iii) Calculate the two phase pressure drop multiplier (for either phase). **[6 marks]**

iv) Calculate the pressure drop over a 12 m long smooth pipe. **[3 marks]**

[Question total: 20 marks]

Question 59
Example exam question
A mixture of 0.15 kg s^{-1} saturated steam and 1.6 kg s^{-1} water is flowing along a horizontal pipe with an inner diameter of 88.9 mm. At the conditions in the pipe, the steam has a viscosity of $\mu_g = 0.0108 \times 10^{-3}$ N s m^{-2} and density of 0.774 kg m^{-3}. The water has a viscosity of 0.51×10^{-3} N s m^{-2} and a density of 998 kg m^{-3}.

a) Determine the flow pattern inside the pipe. How does this horizontal flow pattern differ from the equivalent vertical flow pattern? **[5 marks]**

b) Determine the flow regime for both phases of the flow. **[3 marks]**

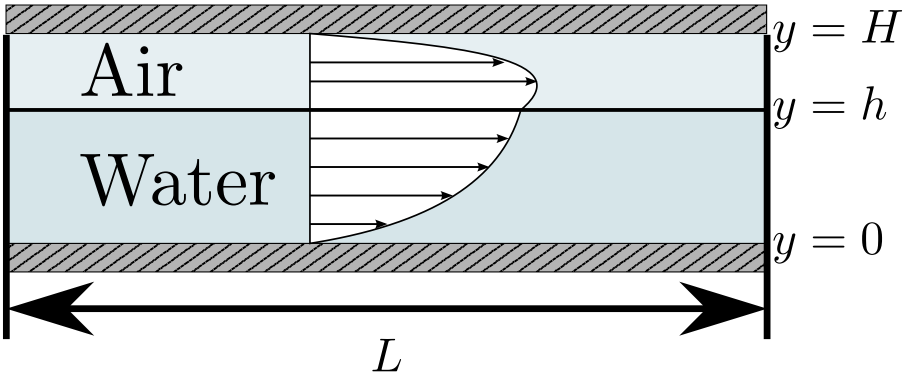
c) Calculate the two-phase pressure drop multiplier (for either phase). **[5 marks]**

d) Assuming the Farooqi and Richardson correlation holds for this system, calculate the liquid hold-up and estimate the true average velocities of the gas and liquid phases. **[5 marks]**

e) Estimate the average density of the fluid using the liquid hold-up. **[2 marks]**

[Question total: 20 marks]

Question 60
Example exam question with marks
Consider the segregated horizontal flow of water and air between two plates of width Z = 50 cm and length L, spaced H = 5 cm apart. The two fluid phases flow at a rate of $\dot{V}_{water}$ = 10 l min^{-1} and $\dot{V}_{air}$ = 45 l min^{-1}.



At the conditions in the channel, water has a density of ρ_{water} = 985 kg m^{-3} and a viscosity of μ_{water} = 0.51 × 10^{-3} Pa s. Air has a density of ρ_{air} = 1.14 kg m^{-3} and a viscosity of μ_{air} = 1.89 × 10^{-5} Pa s.

a) Demonstrate that the no-slip liquid hold-up for this system is $h \approx 0.91$ cm. Comment on how realistic this estimation is. **[4 marks]**

b) Define and calculate the *superficial* velocity, u, and the *actual* velocity, $\langle v \rangle$, for the each phase, assuming the no-slip liquid holdup estimation is correct. **[5 marks]**

c) The Reynolds number for single-phase flow in a pipe is defined as:

$$\mathrm{Re} = \frac{\rho \, \langle v \rangle \, D}{\mu}$$

   i) Define and calculate the superficial Reynolds number for the water phase. You should note that the characteristic length for flow between two plates is 2 H. **[3 marks]**

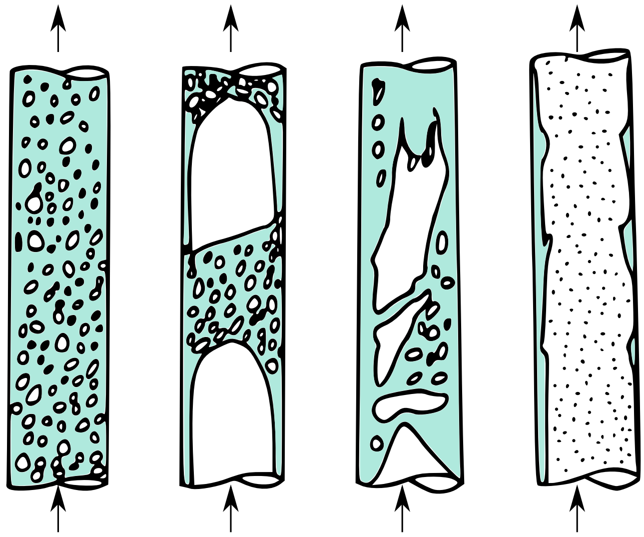
   ii) Define and calculate the Reynolds number for the actual liquid phase using the hydraulic diameter. You can neglect the effect of the air phase (ignore its wetted perimeter). Is this definition consistent? **[4 marks]**

   iii) Comment on the difference between the two results, including the limitations of these expressions. Is one estimate better than the other? **[3 marks]**

d) Assuming the no-slip liquid hold-up is correct, use the Chhabra-Richardson flow map to calculate the flow-regime. **[3 marks]**

e) Assuming the liquid hold-up remains constant, at what liquid flow-rate does the flow turn intermittent? Why is this flow regime generally avoided? **[3 marks]**

f) Consider the following vertical flow patterns.



i) Name each flow pattern, and identify which you might class as intermittent flow.

ii) Which is the most desirable flow pattern if the pressure drop is to be minimised and why?

[Question total: 25 marks]

Question 61

Consider condensing heat transfer:

a) The film thickness is a critical design parameter of condensing heat transfer. Explain how the thickness of the liquid layer affects the heat transfer and what the optimal conditions are for maximising the condensing rate.

b) Discuss dropwise and film condensation. Which is most likely and why is dropwise condensation more favourable?

[Question end]

Question 62
Example exam question with marks

a) i) Sketch the pool boiling curve (heat-flux/transfer-coefficient versus excess temperature), identify the key boiling regimes and describe the conditions in each. **[8 marks]**

ii) On your pool boiling curve, indicate the location of the critical heat flux and describe advantages and the danger of operating an electrical boiler at this point. **[2 marks]**

iii) Why are electrical boilers vunerable to burnout near the critical heat flux when compared to boilers which use condensing steam as a heat source? **[2 marks]**

b) A kettle-type re-boiler operating at a pressure of 0.3 bar is used to boil a fluid of orthodichlorobenzene at a temperature of 120 °C. The properties of the mixture are given in the table below.

μ_L	0.45×10^{-3} Pa s	μ_G	0.01×10^{-3} Pa s
ρ_L	1170 kg m^{-3}	ρ_G	1.31 kg m^{-3}
k_L	0.11 W m^{-1} K^{-1}	$C_{p,L}$	1.25 kJ kg^{-1} K^{-1}
p_c	41 bar	Boiling point	136 °C

i) Assuming 40 m^2 of surface area is available for boiling and neglecting the geometry, calculate the heat transferred due to pure nucleate boiling. **[6 marks]**

ii) Estimate the critical heat flux and determine if the reboiler is operating in a safe region. **[4 marks]**

[Question total: 22 marks]

Question 63

Fick's law is often modified to the following form:

$$N_{A,x} = -(D_{AB} + E_D)\frac{\partial C_A}{\partial x}$$

What is the parameter E_D and what does it represent?

[Question end]

Question 64

Consider the dimensionless Lewis number:

$$\mathrm{Le} = \frac{k}{\rho\, C_p\, D_{AB}}$$

What two transport processes are compared through this number and what does the limit $\mathrm{Le} \to \infty$ correspond to?

[Question end]

Question 65

Gaseous hydrogen at 10 bar and 27°C is stored in a 140 mm outer-diameter tank having a steel wall 2 mm thick and a height of 850mm. The molar concentration of hydrogen in the steel is 1.5 kmol m^{-3} at the inner surface and neglible at the outer surface, while the diffusion coefficient of hydrogen in steel is approximately 0.3×10^{-12} m^2 s^{-1}. What is the rate of mass loss of hydrogen by diffusion per square meter of tank wall? Assume steady-state, one-dimensional conditions.

a) Assuming the curvature of the tank is negligible (you can use rectangular coordinates), show that the molar flux of hydrogen is constant through the wall.

$$N_{H_2,z} = N_{H_2,0}$$

b) Noting that the concentration of hydrogen in the steel wall is very low $x_{H_2} \ll 1$, determine the concentration profile of hydrogen in the wall.

c) Calculate the total mass flow rate of hydrogen transported through the side walls of the vessel (consider just the cylindrical sides).

d) It is determined that the effect of curvature must be included in the estimation of the mass flux (we must use a cylindrical geometry). Derive the following expression for the flux

$$N_{H,r} = \frac{C_1}{r}$$

and derive the following expression for the concentration profile of the hydrogen in the steel wall.

$$C_{H_2} = 5.17 \times 10^4 \ln\left(\frac{0.07}{r}\right)$$

e) Calculate the mass flux of hydrogen through the wall using the solution to the last question.

[Question end]

Question 66
Example exam question
Helium gas at 100 bar and 20°C is stored in a 140 mm outer-diameter vessel with a pyrex wall 4 mm thick and a height of 850 mm. The molar concentration of helium in the pyrex is 35 mol m^{-3} at the inner surface and negligible at the outer surface, while the diffusion coefficient of helium in pyrex is approximately 0.2×10^{-12} m^2 s^{-1}.

a) Assuming the curvature of the tank is negligible (you can use rectangular coordinates) and steady-state, one-dimensional conditions, show that the molar flux of helium is constant through the wall. **[3 marks]**

$$N_{He,z} = N_{He,0}$$

b) The concentration of helium in the pyrex wall is very low $x_{He} \ll 1$, allowing the use of the simple form of Fick's law. Determine the concentration profile of helium in the wall. **[4 marks]**

c) Calculate the total mass flow-rate of helium transported through the side walls of the vessel (consider just the cylindrical sides). **[3 marks]**

[Question total: 10 marks]

Question 67
To maintain a pressure close to 1 atm, an industrial pipeline containing ammonia gas is vented to ambient air. Venting is achieved by tapping the pipe and inserting a 3 mm diameter tube, which extends for 20 m into the atmosphere. With the entire system operating at 25 °C and 1 bar, the ideal gas equation of state predicts a total molar concentration of 40.9 mol m^{-3}. Equimolar counter-diffusion can be assumed, and both the concentration of air in the pipeline and the concentration of ammonia in the atmosphere can be considered negligible. The diffusion coefficient of ammonia through air is approximately 2×10^{-5} m^2 s^{-1}.

a) Determine the mass rate of ammonia (17 g mol^{-1}) lost in to the atmosphere, $\mathbf{N}_A$, in kg/h and the mass rate of contamination of the pipe with air (29 g mol^{-1}), $\mathbf{N}_B$, in the same units. **[12 marks]**

b) A new high-tech membrane, which is impermeable to air, is installed at the bottom of the pipe to prevent air polluting the pipeline. The *air* within the tube is now **stationary** and the mole fraction of ammonia at the surface of the membrane is $x_A(z = 0) = 0.9$. Resolve the problem again to determine the flux of ammonia.
Note: Stefan's law (in mole fractions for ideal gases) is given by the following

$$N_{A,z} = -D_{AB} \frac{C_T}{1 - x_A} \frac{\partial x_A}{\partial z}$$

[8 marks]

[Question total: 20 marks]

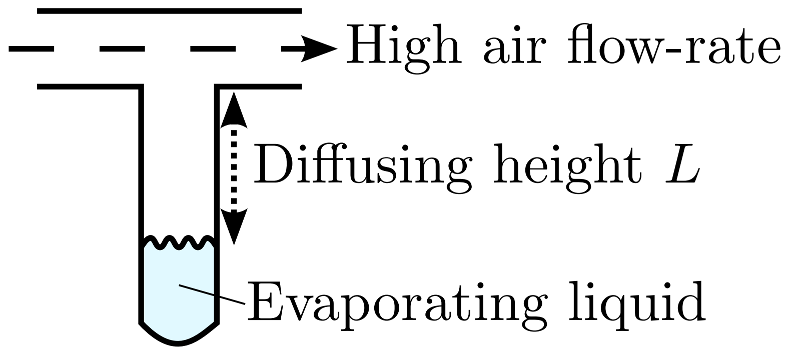


Figure 23: A winklemann experiment.

Question 68

A Winkelmann apparatus is used to measure the diffusivity of a substance, A, in air. It is sketched in Fig. 23. To perform the experiment, a quantity of liquid A is placed at the bottom of a test tube. The liquid evaporates to a vapour mole fraction of $x_{A,sat}$ at the liquid surface (which is determined in a separate equilibrium experiment). The vapourised A then diffuses up the tube where it is removed by a steady flow of air. As A is removed, the liquid level in the tube drops and by monitoring it's rate of change the total diffusive flux can be calculated. We can assume the diffusion profile is at steady state if the rate of evaporation is slow. We also assume the vapours of air and A form an ideal gas, so density is constant inside the tube.

a) Derive the following differential balance equation governing the diffusion of mass in the system. Remember to state any assumptions you make.

$$\frac{\partial}{\partial z}N_{A,z} = 0$$

[5 marks]

b) Write down the boundary conditions of the system and state which class of diffusion problem this is. **[3 marks]**

c) Derive Stefan's law, given below, from the general expression for the diffusive flux. **[4 marks]**

$$N_{A,z} = -D_{A,air}\frac{C}{1-x_A}\frac{\partial x_A}{\partial z}$$

d) Derive the following expression for the mole fraction profile x_A in the system. **[8 marks]**

$$x_A = 1 - \left(1 - x_{A,sat}\right)^{1-z/L}$$

using the identity

$$\frac{\partial N_{A,z}}{\partial z} = 0$$

e) The derivative of the mole fraction in position is

$$\frac{\partial x_A}{\partial z} = \frac{\ln\left(1-x_{A,sat}\right)\left(1-x_{A,sat}\right)^{1-z/L}}{L}$$

Derive the following expression for the flux of A, $N_{A,z}$, at any location in the tube. **[3 marks]**

$$N_{A,z} = -D_{A,air}\frac{C}{L}\ln\left(1-x_{A,sat}\right)$$

f) The mysterious ingredient 7X in a popular drinks beverage evaporates to a mole fraction of 0.02 in air at standard temperature and pressure (20 °C and 1 atm). In a Winklemann experiment, the level is dropping at a rate of 1 mm min^{-1} when the diffusing height is 5 cm. Determine the diffusion coefficient of 7X through air. You may assume the vapours of 7X and air form an ideal gas and that liquid 7X has a density of 18 kmol m^{-3}. **[5 marks]**

[Question total: 28 marks]

Question 69

a) Define the Schmidt number, what does this dimensionless number tell you about the transport processes in a fluid? **[2 marks]**

b) A hemispherical lump of sugar, initially of radius R = 0.005 m, is dropped into a cup of tea, quickly coming to rest on the bottom of the cup as shown in Fig. 24. The sugar lump then slowly dissolves into the tea. The diffusion coefficient of sugar in tea is 4×10^{-10} m^2 s^{-1}. The saturation mole fraction of sugar in tea is 0.1 and the total molar density of the system is $c = 55 \times 10^3$ mol m^{-3}.

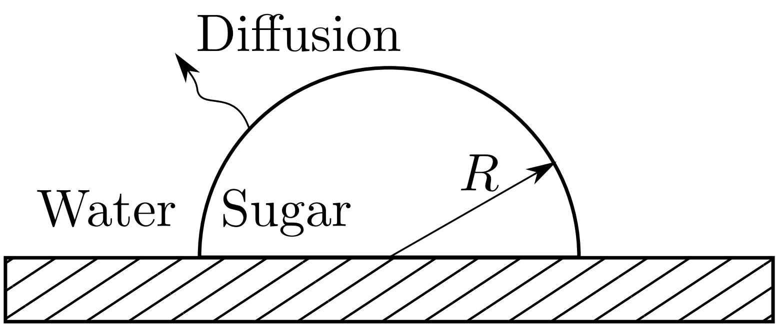


Figure 24: The lump of dissolving sugar.

i) Derive the following differential balance equation for the system.

$$\frac{\partial}{\partial r} r^2 N_{s,r} = 0$$

[5 marks]

ii) Determine the boundary conditions. **[2 marks]**

iii) Assuming the tea is stagnant, derive the following expression for the variation of the sugar mole fraction in the water.

$$x_s = 1 - 0.9^{0.005/r}$$

You may need the identity:

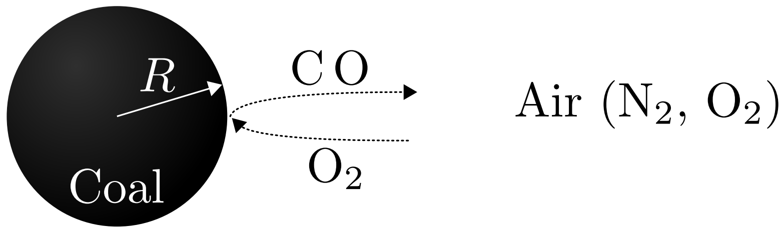
$$\int (1 - x)^{-1}\,\mathrm{d}x = -\ln(1 - x) + C$$

[11 marks]

[Question total: 20 marks]

Question 70
Consider a spherical coal particle undergoing combustion. Combustion of solids is typically limited by the rate at which oxygen can get to the combusting surface. As the reaction is oxygen limited, we assume that as soon as oxygen reaches the coal surface it is instantly converted to carbon monoxide (CO).

$$2\,C + O_2 \to 2\,CO$$



You can assume that there is no oxygen at the surface of the coal particle $x_{O_2}(r = R) = 0$, and a oxygen mole fraction of 21% at a large distance from the particle $x_{O_2}(r \to \infty) = 0.21$. You can also assume steady state conditions, a constant temperature and pressure, and that all gases are ideal gases and mixtures.

a) Specify and simplify the balance equation for the oxygen in this system.

b) Derive the following expression for the oxygen flux.

$$N_{O_2,r} = -\frac{D\,C_T}{1 + x_{O_2}}\frac{\partial x_{O_2}}{\partial r}$$

c) Using the expression for the oxygen flux and the balance for the oxygen flux, solve for the concentration profile of oxygen around the particle.

d) What can you use the information you've derived for?

[Question end]

Question 71
Consider a spherical aggregate (or ball) of bacterial cells (assumed to be homogenous) of radius R. Under certain circumstances, the oxygen metabolism rate of the bacterial cells is an almost constant reaction (zero-order) with respect to the oxygen concentration $\sigma_{O_2} = -k_{O_2}$. The diffusion of oxygen within the ball may be described by Fick's law with an effective pseudobinary diffusivity for oxygen in the bacterial medium of D_{O_2-M}. Neglect transient and convection effects because the oxygen solubility is very low in the system. Let $C_{O_2}^{(R)}$ be the oxygen mass concentration at the aggregate surface:

a) Show all of your working and state all assumptions made while demonstrating that the oxygen balance for the system,

$$\frac{\partial\, C_{O_2}}{\partial t} = -\nabla \cdot \boldsymbol{N}_{O_2} + \sigma_{O_2},$$

simplifies to the following expression,

$$\frac{\partial}{\partial r}\left(r^2\, N_{O_2,\,r}\right) = -k_{O_2}\, r^2.$$

[4 marks]

b) Demonstrate that the oxygen flux obeys the following relationship:

$$N_{O_2,r} = -k_{O_2}\,R^2\left(\frac{r}{3\,R^2} + \frac{C_1\,R}{6\,r^2}\right)$$

where C_1 is an unknown constant. **[4 marks]**

c) Demonstrate that the concentration profile obeys the following form in the limit that the O_2 concentration is small:

$$C_{O_2} = \frac{k_{O_2}\,R^2}{6\,D_{O_2-M}}\left(\frac{r^2}{R^2} - C_1\frac{R}{r}\right) + C_2$$

[4 marks]

d) Using the only available boundary condition, determine C_2 and demonstrate that the final expression for the concentration is:

$$C_{O_2} = \frac{k_{O_2}\,R^2}{6\,D_{O_2-M}}\left(\frac{r^2}{R^2} + C_1\left(1 - \frac{R}{r}\right) - 1\right) + C_{O_2}^{(R)}$$

[4 marks]

e) It is possible that the spherical bacterial ball has an oxygen-free core ($C_{O_2} = 0$ for $r \leq r_{core}$). Prove that this only happens for:

$$\frac{k_{O_2}\,R^2}{D_{O_2-M}\,C_{O_2}^{(R)}} \geq 6$$

Hints: As the concentration and diffusive flux are continuous, they both must go to zero at the core radius r_{core}. Use this to solve for C_1, then solve for r_{core} and consider what is required if $r_{core} \geq 0$. **[4 marks]**

[Question total: 20 marks]

Example multiple choice questions for EX3030

2017-18 paper

1) In Fig. 25, which profile corresponds to a viscoplastic fluid? **[2 marks]**

2) Which profile in Fig. 25 corresponds to a shear thickening fluid? **[2 marks]**

3) What are the value(s) of the flow index n in the Power-law model for a shear thickening fluid? **[2 marks]**

   A) $n < 0$
   B) $n < 1$
   C) $n = 1$
   D) $n > 1$

4) What is the Nusselt number for conduction through a plate of thickness L and conductivity k? **[2 marks]**

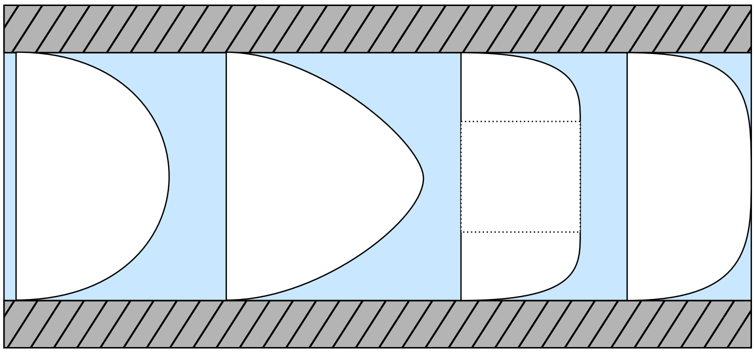


Figure 25: Non-newtonian flow profiles compared against the newtonian flow profile.

A) Nu = 1
B) Nu = k/L
C) Nu = $C\,\mathrm{Re}^n\,\mathrm{Pr}^m$
D) Nu = $L/(k\,A)$

5) The Grashof number is a ratio of what two properties? **[2 marks]**

A) Drag and viscous forces
B) Momentum diffusivity and thermal diffusivity
C) Buoyancy forces and thermal diffusivity
D) Buoyancy forces and viscous forces

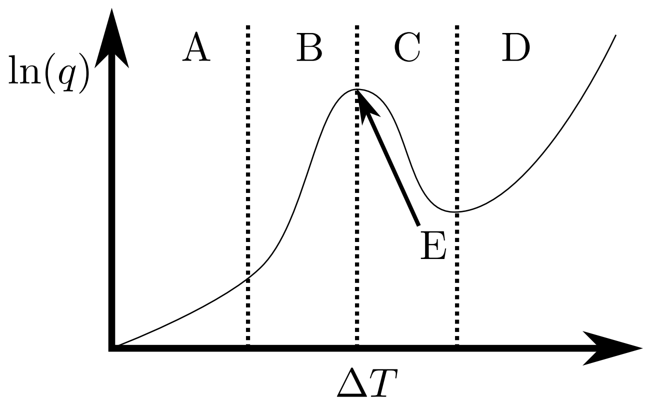


Figure 26: A typical boiling heat flux versus driving temperature difference curve.

6) Which region of the boiling curve in Fig. 26 is the nucleate boiling regime? **[2 marks]**

7) Which region of the boiling curve in Fig. 26 is the operation of the boiler unstable? **[2 marks]**

8) In radiation, object 1 is entirely surrounded by object 2. The area of object 1 is 15 m^2, while object 2 has a surface area of 58 m^2. What is the view-factor of object 2 from the point-of-view of object 1, i.e., $F_{1\to2}$? **[2 marks]**

   A) $F_{1\to2} \approx 0.259$
   B) $F_{1\to2} = 1$
   C) $F_{1\to2} = \approx 3.87$
   D) $F_{1\to2} = 0$

9) At what temperature should the properties used in the Prandtl number be evaluated for a pipe with a temperature drop across its length? **[2 marks]**

   A) Inlet temperature
   B) Average of the wall and bulk temperature
   C) Wall temperature
   D) Centerline temperature
   E) Average of inlet and outlet temperature

10) Two liquids flowing together in a channel, what is NOT a valid boundary condition? **[2 marks]**

   A) Stress in each phase is equal at the interface
   B) No-slip between the two phases at the interface
   C) No stress at the interface
   D) No-slip between the fluid(s) and the adjacent wall

DATASHEET

General balance equations:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho\, \boldsymbol{v} \qquad \text{(Mass/Continuity)} \qquad (19)$$

$$\frac{\partial C_A}{\partial t} = -\nabla \cdot \boldsymbol{N}_A + \sigma_A \qquad \text{(Species)} \qquad (20)$$

$$\rho \frac{\partial \boldsymbol{v}}{\partial t} = -\rho\, \boldsymbol{v} \cdot \nabla \boldsymbol{v} - \nabla \cdot \boldsymbol{\tau} - \nabla p + \rho\, \boldsymbol{g} \qquad \text{(Momentum)} \qquad (21)$$

$$\rho\, C_p \frac{\partial T}{\partial t} = -\rho\, C_p\, \boldsymbol{v} \cdot \nabla T - \nabla \cdot \boldsymbol{q} - \boldsymbol{\tau} : \nabla \boldsymbol{v} - p \nabla \cdot \boldsymbol{v} + \sigma_{energy} \qquad \text{(Heat/Energy)} \qquad (22)$$

In Cartesian coordinate systems, ∇ can be treated as a vector of derivatives. In curvelinear coordinate systems, the directions $\hat{\boldsymbol{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\phi}$ depend on the position. For convenience in these systems, look-up tables are provided for common terms involving ∇.

Cartesian coordinates (with index notation examples)
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor.

$$\nabla s = \nabla_i s = \left[\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \nabla_i \nabla_i s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \nabla_i v_i = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

$$\nabla \cdot \boldsymbol{\tau} = \nabla_i \tau_{ij}$$

$$[\nabla \cdot \boldsymbol{\tau}]_x = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_y = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

$$\boldsymbol{v} \cdot \nabla \boldsymbol{v} = v_i \nabla_i v_j$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_x = v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_y = v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z}$$

Cylindrical coordinates
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{\partial s}{\partial z}\right]$$

$$\nabla^2 s = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r}\frac{\partial}{\partial r}(r\, v_r) + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rr}) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{1}{r}\tau_{\theta\theta} + \frac{\partial \tau_{rz}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r}\tau_{r\theta} + \frac{\partial \tau_{\theta z}}{\partial z}$$

$$[\nabla \cdot \boldsymbol{\tau}]_z = \frac{1}{r}\frac{\partial}{\partial r}(r\, \tau_{rz}) + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z\frac{\partial v_r}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r\, v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_z = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}$$

Spherical coordinates
where s is a scalar, $\boldsymbol{v}$ is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\nabla s = \left[\frac{\partial s}{\partial r}, \frac{1}{r}\frac{\partial s}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial s}{\partial \phi}\right]$$

$$\nabla^2 s = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial s}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial s}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 s}{\partial \phi^2}$$

$$\nabla \cdot \boldsymbol{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(v_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_r = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{rr}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{r\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\theta = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\tau_{\theta\theta}\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial \tau_{\theta\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot\theta}{r}\tau_{\phi\phi}$$

$$[\nabla \cdot \boldsymbol{\tau}]_\phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\, \tau_{r\phi}\right) + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} + \frac{2\cot\theta}{r}\tau_{\theta\phi}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r\, v_\theta - v_\phi^2\cot\theta}{r}$$

$$[\boldsymbol{v} \cdot \nabla \boldsymbol{v}]_\phi = v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r\, v_\phi + v_\theta\, v_\phi\cot\theta}{r}$$

Rectangular		Cylindrical		Spherical	
q_x	$-k\frac{\partial T}{\partial x}$	q_r	$-k\frac{\partial T}{\partial r}$	q_r	$-k\frac{\partial T}{\partial r}$
q_y	$-k\frac{\partial T}{\partial y}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$	q_θ	$-k\frac{1}{r}\frac{\partial T}{\partial \theta}$
q_z	$-k\frac{\partial T}{\partial z}$	q_z	$-k\frac{\partial T}{\partial z}$	q_ϕ	$-k\frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}$
τ_{xx}	$-2\,\mu\frac{\partial v_x}{\partial x} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{rr}	$-2\,\mu\frac{\partial v_r}{\partial r} + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{yy}	$-2\,\mu\frac{\partial v_y}{\partial y} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\theta\theta}$	$-2\,\mu\left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	τ_{zz}	$-2\,\mu\frac{\partial v_z}{\partial z} + \mu^B\,\nabla\cdot\boldsymbol{v}$	$\tau_{\phi\phi}$	$-2\,\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r+v_\theta\cot\theta}{r}\right) + \mu^B\,\nabla\cdot\boldsymbol{v}$
τ_{xy}	$-\mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$	$\tau_{r\theta}$	$-\mu\left(r\frac{\partial}{\partial r}\left(\frac{v_\theta}{r}\right) + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)$
τ_{yz}	$-\mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)$	$\tau_{\theta z}$	$-\mu\left(\frac{1}{r}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z}\right)$	$\tau_{\theta\phi}$	$-\mu\left(\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{v_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi}\right)$
τ_{xz}	$-\mu\left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)$	τ_{zr}	$-\mu\left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r}\right)$	$\tau_{\phi r}$	$-\mu\left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{v_\phi}{r}\right)\right)$

Table 4: Fourier's law for the heat flux and Newton's law for the stress in several coordinate systems. Please remember that the stress is symmetric, so $\tau_{ij} = \tau_{ji}$.

Viscous models:
Power-Law Fluid:

$$|\tau_{xy}| = k\left|\frac{\partial v_x}{\partial y}\right|^n \tag{23}$$

Bingham-Plastic Fluid:

$$\frac{\partial v_x}{\partial y} = \begin{cases} -\mu^{-1}\left(\tau_{xy} - \tau_0\right) & \text{if } \tau_{xy} > \tau_0 \\ 0 & \text{if } \tau_{xy} \leq \tau_0 \end{cases}$$

Dimensionless Numbers

$$\mathrm{Re} = \frac{\rho\,\langle v\rangle\,D}{\mu} \qquad \mathrm{Re}_H = \frac{\rho\,\langle v\rangle\,D_H}{\mu} \qquad \mathrm{Re}_{MR} = -\frac{16\,L\,\rho\,\langle v\rangle^2}{R\,\Delta p} \tag{24}$$

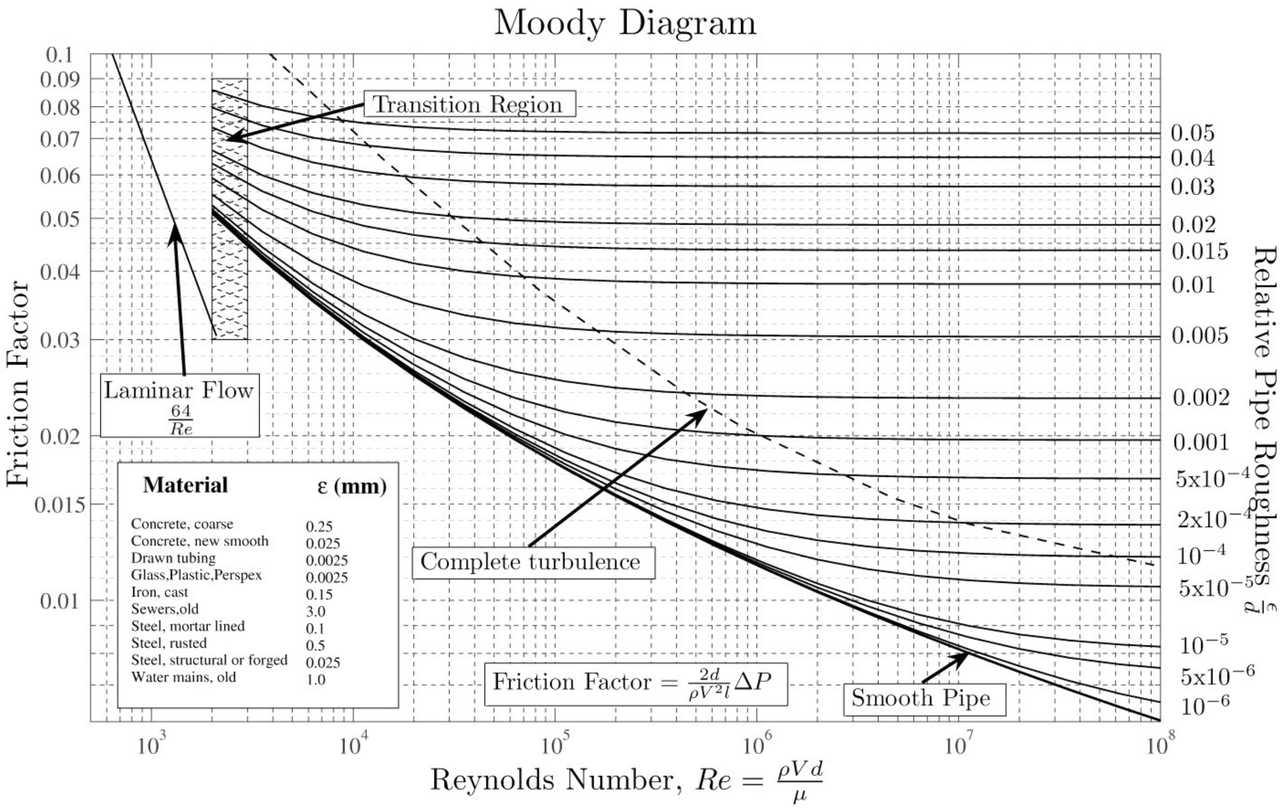
The hydraulic diameter is defined as $D_H = 4\,A/P_w$.
Single phase pressure drop calculations in pipes:
Darcy-Weisbach equation:

$$\frac{\Delta p}{L} = -\frac{C_f\,\rho\,\langle v\rangle^2}{R} \tag{25}$$

where $C_f = 16/Re$ for laminar Newtonian flow. For turbulent flow of Newtonian fluids in smooth pipes, we have the Blasius correlation:

$$C_f = 0.079\,\mathrm{Re}^{-1/4} \qquad \text{for } 2.5\times10^3 < \mathrm{Re} < 10^5 \text{ and smooth pipes.}$$

Otherwise, you may refer to the Moody diagram.



Laminar Power-Law fluid:

$$\dot{V} = \frac{n\,\pi\,R^3}{3\,n+1}\left(\frac{R}{2\,k}\right)^{\frac{1}{n}}\left(-\frac{\Delta p}{L}\right)^{\frac{1}{n}}$$

Two-Phase Flow:
Lockhart-Martinelli parameter:

$$X^2 = \frac{\Delta p_{liq.-only}}{\Delta p_{gas-only}}$$

Pressure drop calculation:

$$\Delta p_{two-phase} = \Phi^2_{liq.}\,\Delta p_{liq.-only} = \Phi^2_{gas}\,\Delta p_{gas-only}$$

Chisholm's relation:

$$\Phi^2_{gas} = 1 + c\,X + X^2$$

$$\Phi^2_{liq.} = 1 + \frac{c}{X} + \frac{1}{X^2} \qquad c = \begin{cases} 20 & \text{turbulent liquid \& turbulent gas} \\ 12 & \text{laminar liquid \& turbulent gas} \\ 10 & \text{turbulent liquid \& laminar gas} \\ 5 & \text{laminar liquid \& laminar gas} \end{cases}$$

Farooqi and Richardson expression for liquid hold-up in co-current flows of Newtonian fluids and air in horizontal pipes:

$$h = \begin{cases} 0.186 + 0.0191\,X & 1 < X < 5 \\ 0.143\,X^{0.42} & 5 < X < 50 \\ 1/(0.97 + 19/X) & 50 < X < 500 \end{cases}$$

Heat Transfer Dimensionless numbers:

$$\mathrm{Nu} = \frac{h\,L}{k} \qquad \mathrm{Pr} = \frac{\mu\,C_p}{k} \qquad \mathrm{Gr} = \frac{g\,\beta\,(T_w - T_\infty)\,L^3}{\nu^2}$$

where $\beta = V^{-1}(\partial V/\partial T)$.

Heat transfer: Resistances

$$Q = U_T\,A_T\,\Delta T = R_T^{-1}\,\Delta T$$

	Conduction Shell Resistances			Radiation
	Rect.	Cyl.	Sph.	
R	$\frac{X}{k\,A}$	$\frac{\ln(R_{outer}/R_{inner})}{2\,\pi\,L\,k}$	$\frac{R_{inner}^{-1} - R_{outer}^{-1}}{4\,\pi\,k}$	$\left[A\,\varepsilon\,\sigma\left(T_j^2 + T_i^2\right)\left(T_j + T_i\right)\right]^{-1}$

Radiation Heat Transfer:

Stefan-Boltzmann constant $\sigma = 5.6703 \times 10^{-8}$ W m^{-2} K^{-4}.

Summation relationship, $\sum_j F_{i\to j} = 1$, and reciprocity relationship, $F_{i\to j}\,A_i = F_{j\to i}\,A_j$. Radiation shielding factor $1/(N+1)$.

$$Q_{rad.,i\to j} = \sigma\,\varepsilon\,F_{i\to j}\,A_i\left(T_j^4 - T_i^4\right) = h_{rad.}\,A\,(T_\infty - T_w)$$

Natural Convection

$\mathrm{Ra} = \mathrm{Gr}\,\mathrm{Pr}$	C	m
$< 10^4$	1.36	1/5
10^4–10^9	0.59	1/4
$> 10^9$	0.13	1/3

Table 5: Natural convection coefficients for isothermal vertical plates in the empirical relation $\mathrm{Nu} \approx C\,(\mathrm{Gr}\,\mathrm{Pr})^m$.

For isothermal vertical cylinders, the above expressions for isothermal vertical plates may be used but must be scaled by a factor, F (i.e., $\mathrm{Nu}_{v.cyl.} = F\,\mathrm{Nu}_{v.plate}$):

$$F = \begin{cases} 1 & \text{for } (D/H) \geq 35\,\mathrm{Gr}_H^{-1/4} \\ 1.3\left[H\,D^{-1}\,\mathrm{Gr}_D^{-1}\right]^{1/4} + 1 & \text{for } (D/H) < 35\,\mathrm{Gr}_H^{-1/4} \end{cases}$$

where D is the diameter and H is the height of the cylinder. The subscript on Gr indicates which length is to be used as the critical length to calculate the Grashof number.
Churchill and Chu expression for natural convection from a horizontal pipe:

$$\mathrm{Nu}^{1/2} = 0.6 + 0.387\left\{\frac{\mathrm{Gr}\,\mathrm{Pr}}{\left[1 + (0.559/\mathrm{Pr})^{9/16}\right]^{16/9}}\right\}^{1/6} \qquad \text{for } 10^{-5} < \mathrm{Gr}\,\mathrm{Pr} < 10^{12}$$

Forced Convection:
Laminar flows:

$$\mathrm{Nu} \approx 0.332\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

Well-Developed turbulent flows in smooth pipes:

$$\mathrm{Nu} \approx \frac{(C_f/2)\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(C_f/2)^{1/2}\left(\mathrm{Pr}^{2/3} - 1\right)}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

Boiling:
Forster-Zuber pool-boiling coefficient:

$$h_{nb} = 0.00122\frac{k_L^{0.79}\,C_{p,L}^{0.45}\,\rho_L^{0.49}}{\gamma^{0.5}\,\mu_L^{0.29}\,h_{fg}^{0.24}\,\rho_G^{0.24}}\,(T_w - T_{sat})^{0.24}\,(p_w - p_{sat})^{0.75}$$

Mostinski correlations:

$$h_{nb} = 0.104\,p_c^{0.69}\,q^{0.7}\left[1.8\left(\frac{p}{p_c}\right)^{0.17} + 4\left(\frac{p}{p_c}\right)^{1.2} + 10\left(\frac{p}{p_c}\right)^{10}\right]$$

$$q_c = 3.67 \times 10^4\,p_c\left(\frac{p}{p_c}\right)^{0.35}\left[1 - \frac{p}{p_c}\right]^{0.9}$$

(**Note**: for the Mostinski correlations, the pressures are in units of bar)
Condensing:
Horizontal pipes

$$h = 0.72\left(\frac{k^3\,\rho^2\,g_x\,E_{latent}}{D\,\mu\,(T_w - T_\infty)}\right)^{1/4}$$

Lumped capacitance method:

$$\mathrm{Bi} = \frac{h\,L_c}{\kappa}$$

$$L_c = V/A \qquad \text{for Bi} < 0.1$$

$$\frac{T(t)-T_\infty}{T_0-T_\infty}=e^{-bt} \qquad b=\frac{hA_s}{\rho V C_p}$$

1-D Transient Heat Conduction:

$$Fo=\frac{\alpha\Delta t}{(\Delta x)^2}=\tau, \qquad \alpha=\kappa\left(\rho C_p\right)^{-1}$$

$$\theta_{\text{wall}}=\frac{T(x,t)-T_\infty}{T_i-T_\infty}=A_1e^{-\lambda_1^2\tau}\cos\left(\frac{\lambda_1 x}{L}\right), \qquad \theta_{\text{cyl}}=\frac{T(r,t)-T_\infty}{T_i-T_\infty}=A_1e^{-\lambda_1^2\tau}\mathbf{J_0}\left(\frac{\lambda_1 r}{r_0}\right)$$

$$\theta_{\text{sph}}=\frac{T(r,t)-T_\infty}{T_i-T_\infty}=A_1e^{-\lambda_1^2\tau}\frac{\sin\left(\frac{\lambda_1 r}{r_0}\right)}{\frac{\lambda_1 r}{r_0}}$$

$$\theta_{0,\text{wall}}=\theta_{0,\text{cyl}}=\theta_{0,\text{sph}}=\frac{T_0-T_\infty}{T_i-T_\infty}=A_1e^{-\lambda_1^2\tau}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\text{max}}}\right)_{\text{wall}}=1-\theta_{0,\text{wall}}\frac{\sin\lambda_1}{\lambda_1}, \qquad \left(\frac{\mathcal{Q}}{\mathcal{Q}_{\text{max}}}\right)_{\text{cyl}}=1-2\theta_{0,\text{cyl}}\frac{\mathbf{J_1}\left(\lambda_1\right)}{\lambda_1}$$

$$\left(\frac{\mathcal{Q}}{\mathcal{Q}_{\text{max}}}\right)_{\text{sph}}=1-3\theta_{0,\text{sph}}\frac{\sin\lambda_1-\lambda_1\cos\lambda_1}{\lambda_1^3}$$

Finite-Difference Method:

$$\frac{\partial}{\partial t}(\rho\phi)+\frac{\partial}{\partial x}(\rho\boldsymbol{v}\phi)=\nabla\cdot(\Gamma\nabla\phi)+\mathcal{S} \qquad \text{(1D transport equation)}$$

$$\left(\frac{d\phi}{dx}\right)_i=\frac{\phi_{i+1}-\phi_{i-1}}{2\Delta x} \quad \text{and} \quad \left(\frac{d^2\phi}{dx^2}\right)_i=\frac{\phi_{i-1}+\phi_{i+1}-2\phi_i}{(\Delta x)^2}$$

$$T_i^{j+1}=(1-2\tau)\,T_i^j+\tau\left(T_{i+1}^j+T_{i-1}^j\right)+\frac{\tau\,(\Delta x)^2}{\kappa}\mathcal{S}_i^j$$

Overall Heat Transfer Coefficient:

$$\dot{\mathcal{Q}}=\frac{\Delta T}{\mathcal{R}}=UA\Delta T=U_iA_i\Delta T=U_oA_o\Delta T$$

$$\mathcal{R}=R_i+R_{\text{wall}}+R_o=\frac{1}{h_iA_i}+\frac{\ln D_o/D_i}{2\pi\kappa L}+\frac{1}{h_oA_o}$$

Fouling Factor:

$$\mathcal{R}=\frac{1}{h_iA_i}+\frac{R_{\text{f,i}}}{A_i}+R_{\text{wall}}+\frac{R_{\text{f,o}}}{A_o}+\frac{1}{h_oA_o}$$

LMTD Method:

$$\dot{Q} = UA_s \Delta T_{\text{lm}} \quad \text{with} \quad \Delta T_{\text{lm}} = \frac{\Delta T_2 - \Delta T_1}{\ln \frac{\Delta T_2}{\Delta T_1}} = \frac{\Delta T_1 - \Delta T_2}{\ln \frac{\Delta T_1}{\Delta T_2}}$$

$$\text{Parallel flows:} \begin{cases} \Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,in}} \\ \Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,out}} \end{cases}$$

$$\text{Counter flows:} \begin{cases} \Delta T_1 = T_{\text{hot,in}} - T_{\text{cold,out}} \\ \Delta T_2 = T_{\text{hot,out}} - T_{\text{cold,in}} \end{cases}$$

ϵ-NTU Method:

$$\epsilon = \frac{\dot{Q}}{\dot{Q}_{\text{max}}}, \quad \text{with } \dot{Q}_{\text{max}} = C_{\text{min}} \left(T_{\text{hot,in}} - T_{\text{cold,in}} \right) \quad \text{and} \quad C_{\text{min}} = Min \left\{ \dot{m}_{\text{hot}} C_{p,\text{hot}}, \dot{m}_{\text{cold}} C_{p,\text{cold}} \right\}$$

$$\text{NTU} = \frac{UA_s}{C_{\text{min}}}$$

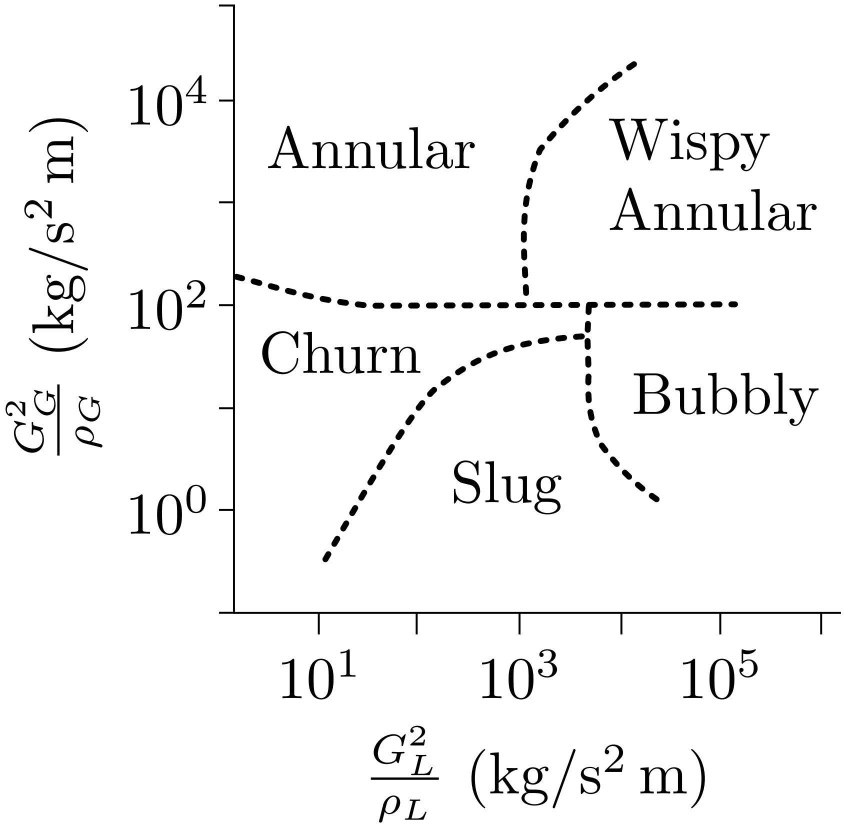


Figure 27: Hewitt-Taylor flow pattern map for multiphase flows in vertical pipes.

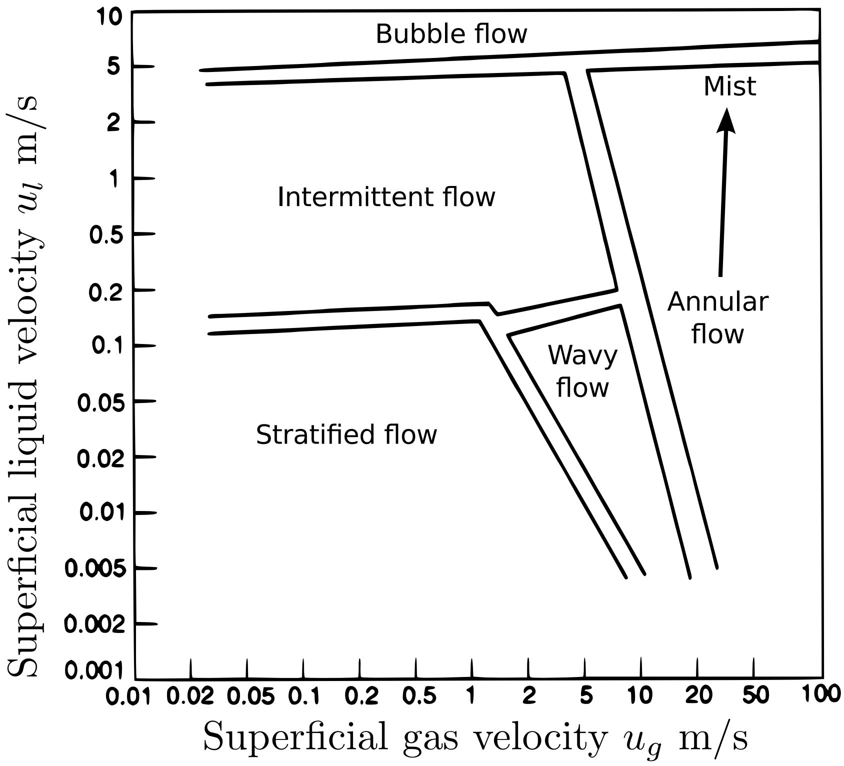


Figure 28: Chhabra and Richardson flow pattern map for horizontal pipes.

TABLE 4–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

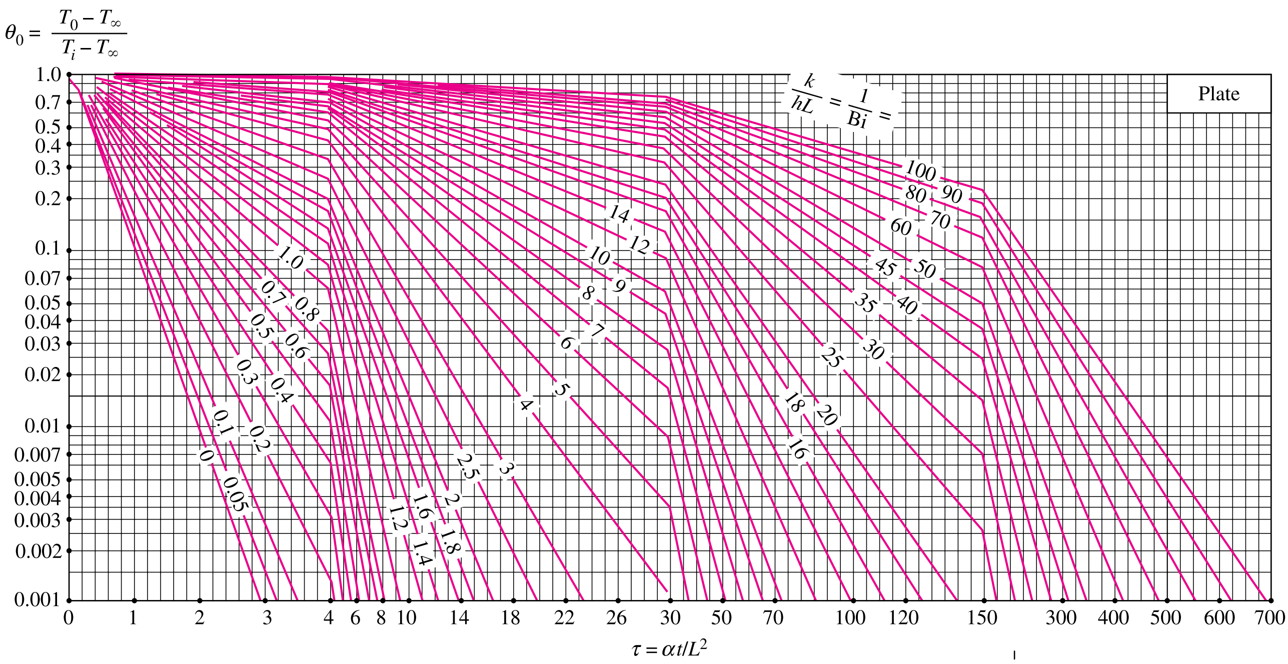
	Plane Wall		*Cylinder*		*Sphere*	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

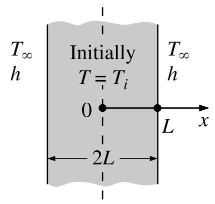
TABLE 4–3

The zeroth- and first-order Bessel functions of the first kind

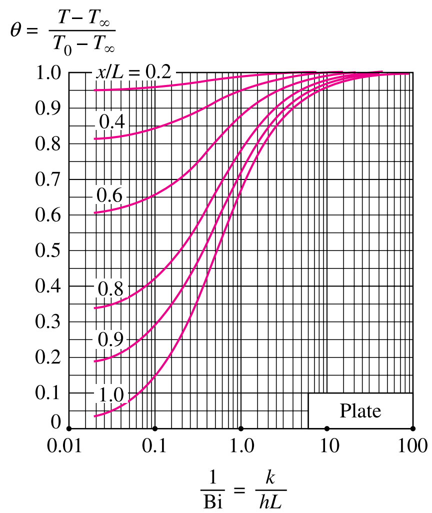
η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

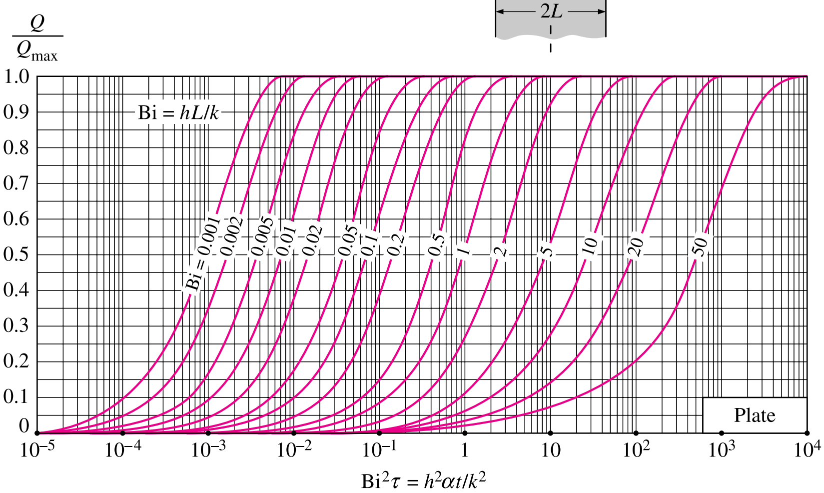
Figure 29: Coefficients for the 1D transient equations.





(*a*) Midplane temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)



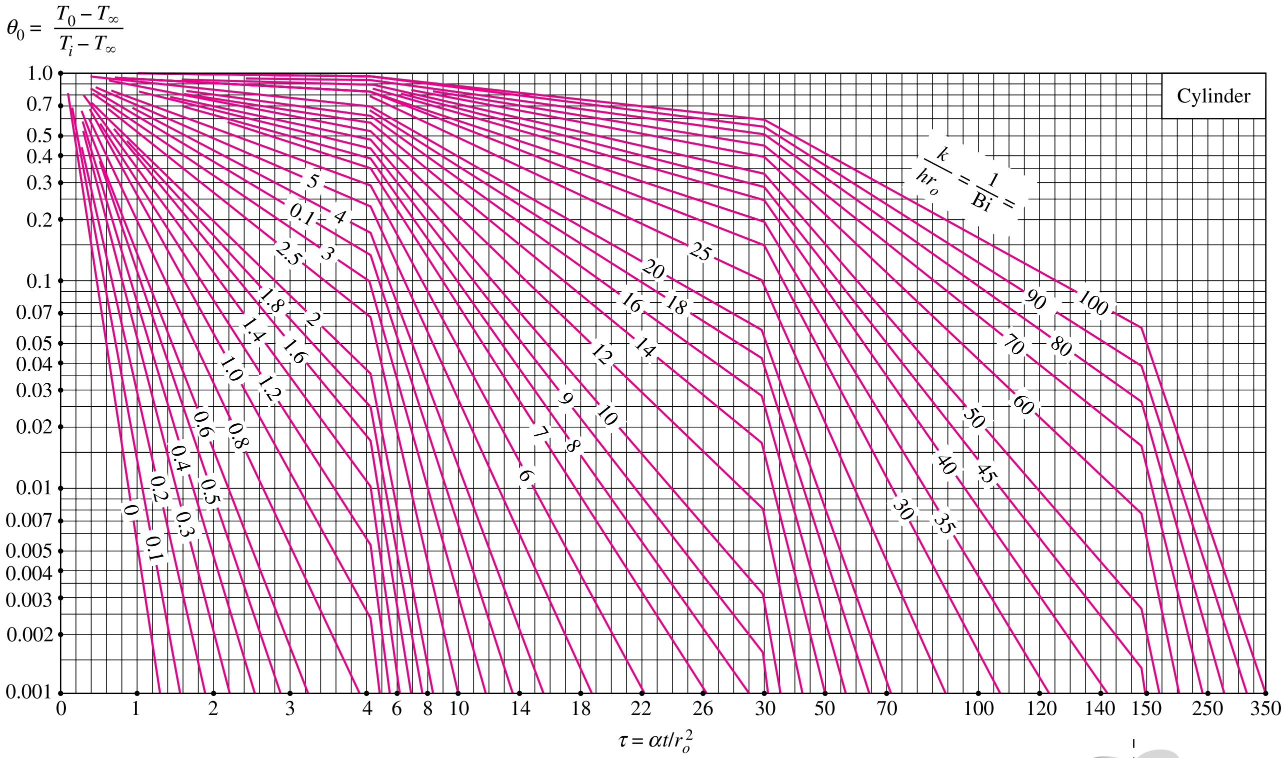


(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)

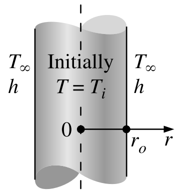
(*c*) Heat transfer (from H. Gröber et al.)

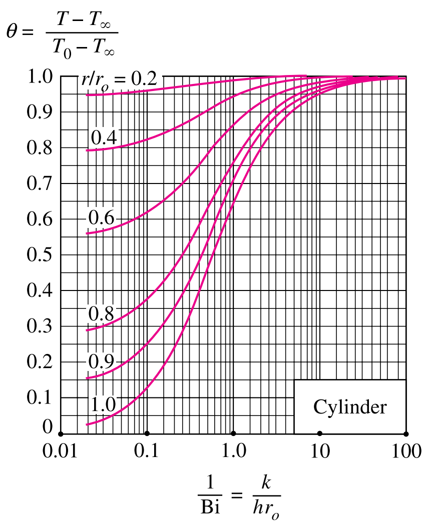
Transient temperature and heat transfer charts for a plane wall of thickness $2L$ initially at a uniform temperature T_i subjected to convection from both sides to an environment at temperature T_∞ with a convection coefficient of h.

Figure 30:

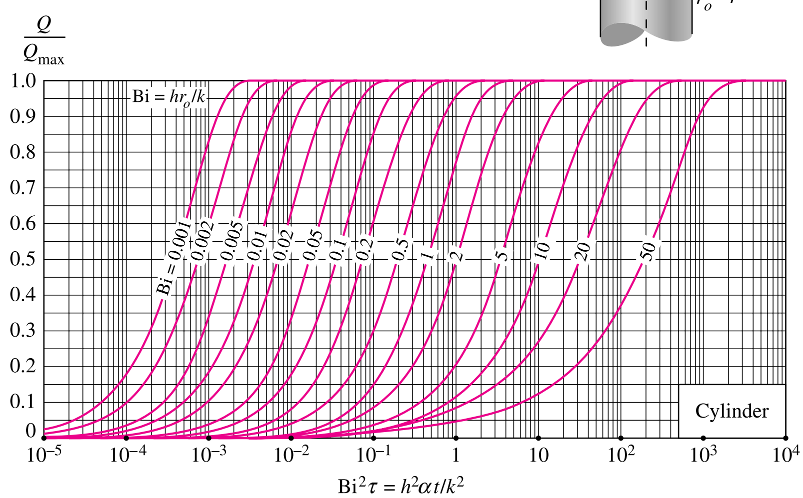


(*a*) Centerline temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)





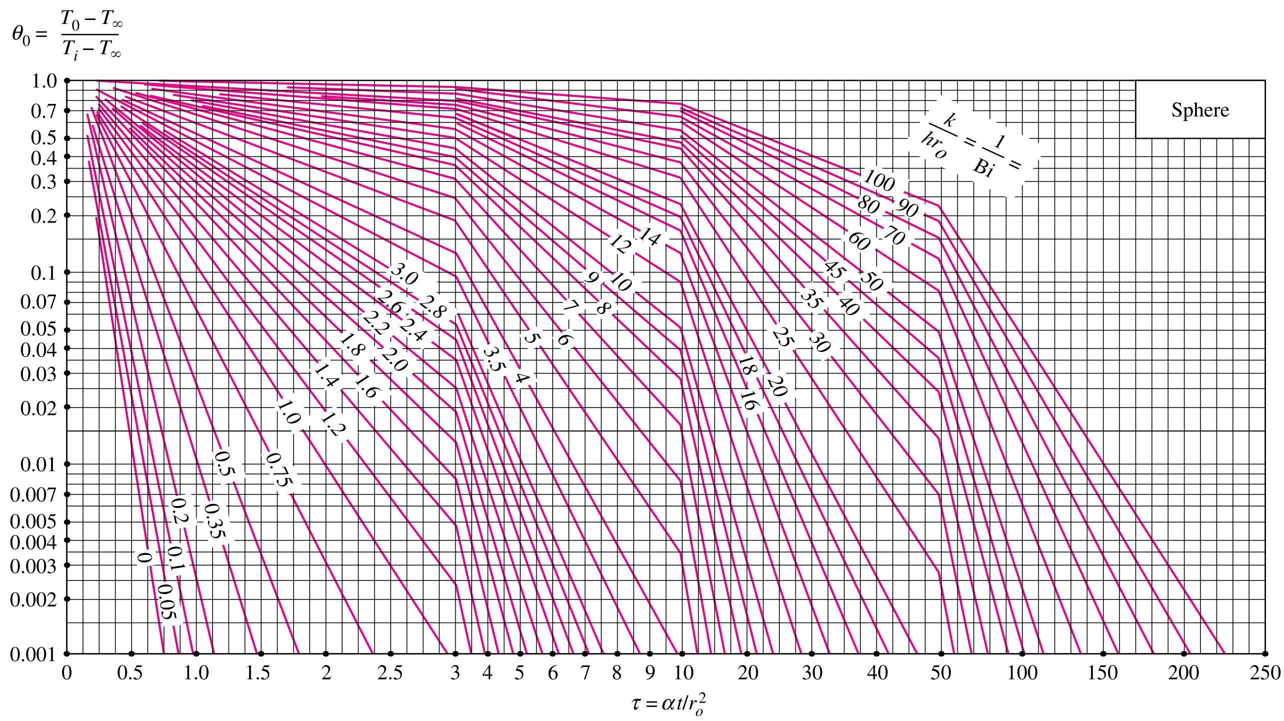
(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)



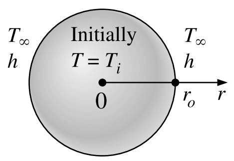
(*c*) Heat transfer (from H. Gröber et al.)

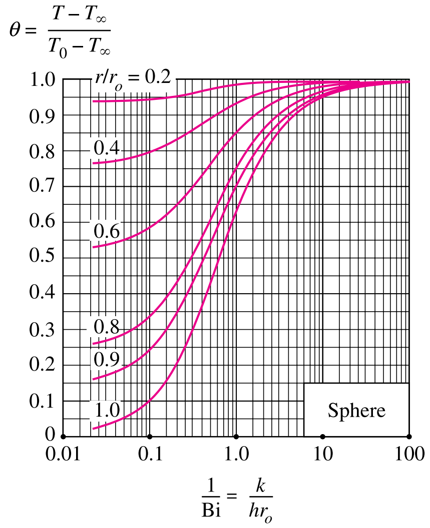
Transient temperature and heat transfer charts for a long cylinder of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

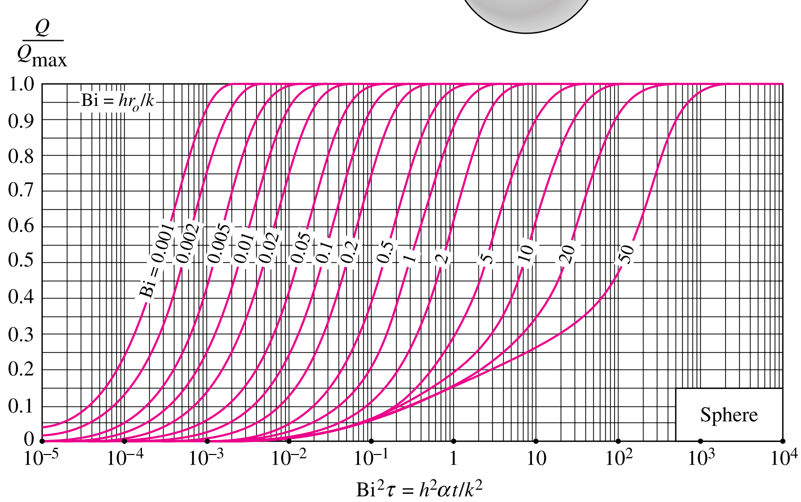
Figure 31:



(*a*) Midpoint temperature (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947, pp. 227–36. Reprinted by permission of ASME International.)







(*b*) Temperature distribution (from M. P. Heisler, "Temperature Charts for Induction and Constant Temperature Heating," *Trans. ASME 69*, 1947,

(*c*) Heat transfer (from H. Gröber et al.)

Transient temperature and heat transfer charts for a sphere of radius r_o initially at a uniform temperature T_i subjected to convection from all sides to an environment at temperature T_∞ with a convection coefficient of h.

Figure 32:

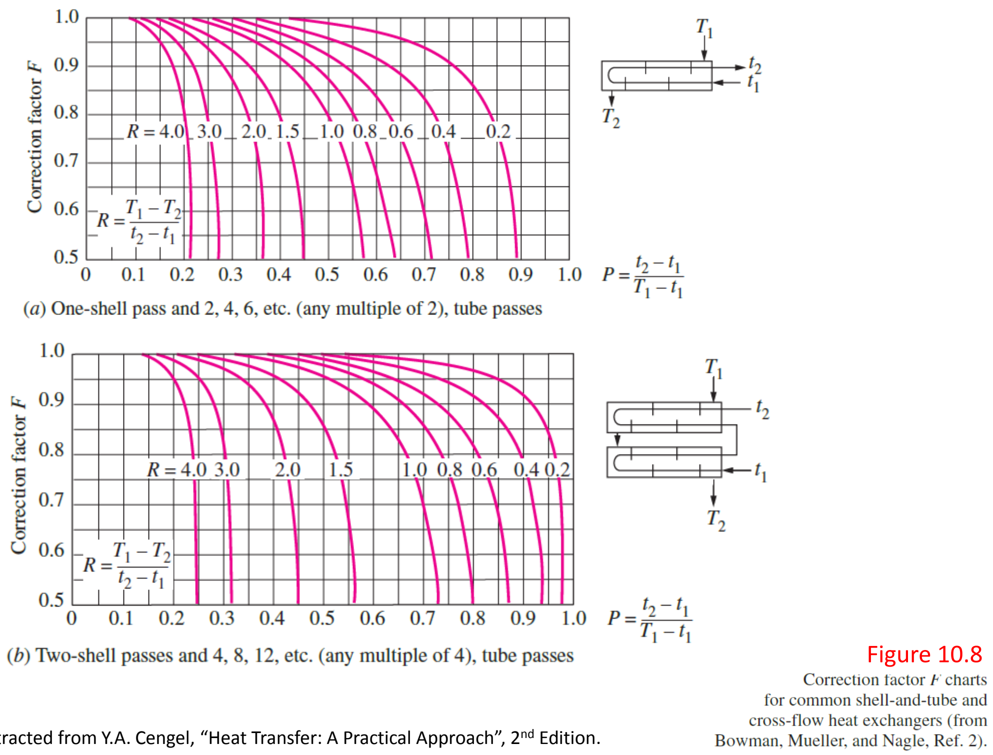


Figure 33: Correction-factors for LMTD Method, extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.

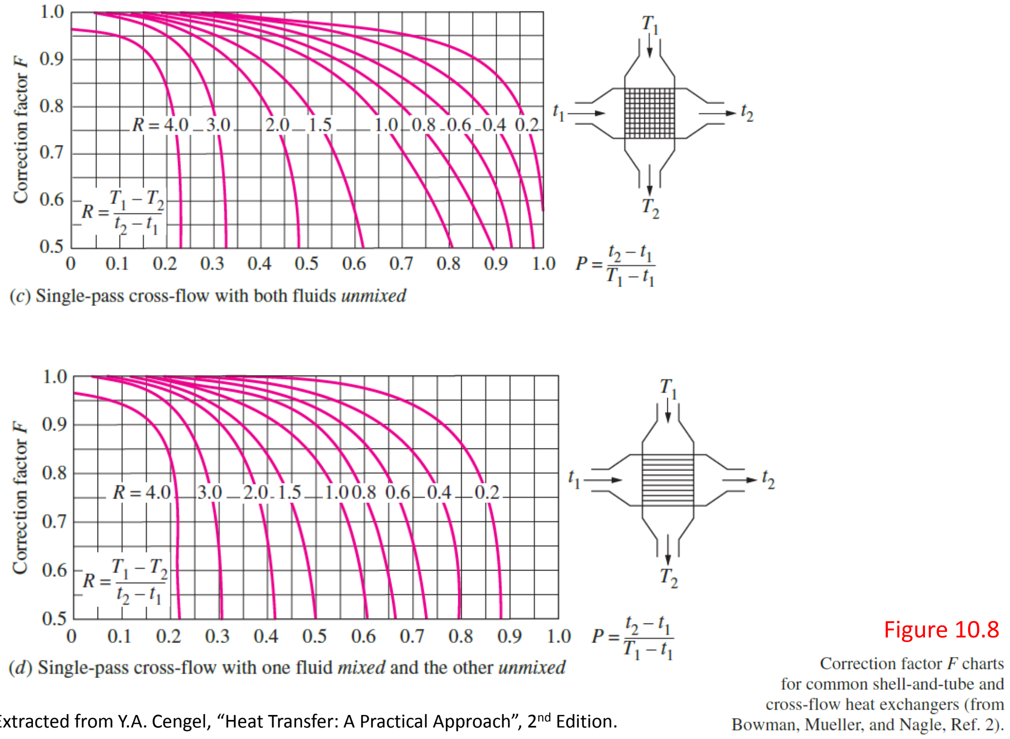


Figure 34: Correction-factors for LMTD Method, extracted from Y. A. Cengel, "Heat transfer:A practical approach", 2nd Ed.

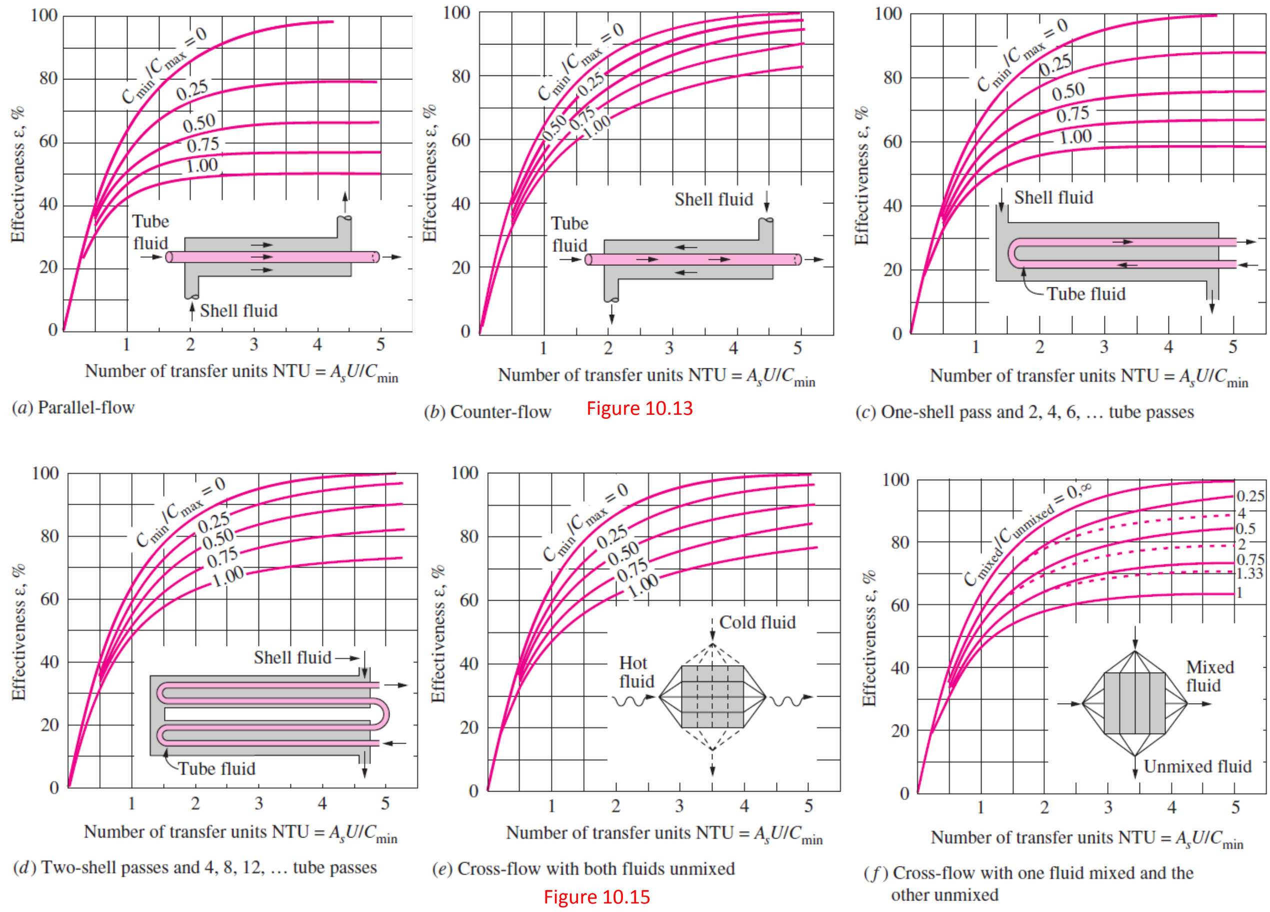
Effectiveness relations for heat exchangers: $NTU = UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	Effectiveness relation
1 *Double pipe:* Parallel-flow	$\varepsilon = \dfrac{1-\exp[-NTU(1+c)]}{1+c}$
Counter-flow	$\varepsilon = \dfrac{1-\exp[-NTU(1-c)]}{1-c\exp[-NTU(1-c)]}$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1+c+\sqrt{1+c^2}\,\dfrac{1+\exp[-NTU\sqrt{1+c^2}]}{1-\exp[-NTU\sqrt{1+c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)* Both fluids unmixed	$\varepsilon = 1-\exp\left\{\dfrac{NTU^{0.22}}{c}[\exp(-c\,NTU^{0.78})-1]\right\}$
C_{max} mixed, C_{min} unmixed	$\varepsilon = \dfrac{1}{c}(1-\exp\{1-c[1-\exp(-NTU)]\})$
C_{min} mixed, C_{max} unmixed	$\varepsilon = 1-\exp\left\{-\dfrac{1}{c}[1-\exp(-c\,NTU)]\right\}$
4 *All heat exchangers with $c = 0$*	$\varepsilon = 1-\exp(-NTU)$

NTU relations for heat exchangers $NTU = UA_s/C_{min}$ and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	NTU relation
1 *Double-pipe:* Parallel-flow	$NTU = -\dfrac{\ln[1-\varepsilon(1+c)]}{1+c}$
Counter-flow	$NTU = \dfrac{1}{c-1}\ln\left(\dfrac{\varepsilon-1}{\varepsilon c-1}\right)$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$NTU = -\dfrac{1}{\sqrt{1+c^2}}\ln\left(\dfrac{2/\varepsilon-1-c-\sqrt{1+c^2}}{2/\varepsilon-1-c+\sqrt{1+c^2}}\right)$
3 *Cross-flow (single-pass)* C_{max} mixed, C_{min} unmixed	$NTU = -\ln\left[1+\dfrac{\ln(1-\varepsilon c)}{c}\right]$
C_{min} mixed, C_{max} unmixed	$NTU = -\dfrac{\ln[c\ln(1-\varepsilon)+1]}{c}$
4 *All heat exchangers with $c = 0$*	$NTU = -\ln(1-\varepsilon)$

Figure 35: NTU relations extracted from Y. A. Cengel, “Heat transfer:A practical approach”, 2nd Ed.



Extracted from Y.A. Cengel, “Heat Transfer: A Practical Approach”, 2nd Edition.

Figure 36: NTU plots extracted from Y. A. Cengel, “Heat transfer:A practical approach”, 2nd Ed.

Diffusion Dimensionless Numbers

$$\mathrm{Sc} = \frac{\mu}{\rho\, D_{AB}} \qquad \mathrm{Le} = \frac{k}{\rho\, C_p\, D_{AB}}$$

Diffusion

General expression for the flux:

$$\boldsymbol{N}_A = \boldsymbol{J}_A + x_A \sum_B \boldsymbol{N}_B$$

Fick's law:

$$\boldsymbol{J}_A = -D_{AB}\, \nabla C_A$$

Stefan's law:

$$N_{s,r} = -D \frac{c}{1-x} \frac{\partial x}{\partial r}$$

Ideal Gas

$$P\, V = n\, R\, T \qquad R \approx 8.314598 \text{ J K}^{-1} \text{ mol}^{-1}$$

Geometry

$$P_{\text{circle}} = 2\, \pi\, r \qquad A_{\text{circle}} = \pi\, r^2 \qquad A_{\text{sphere}} = 4\, \pi\, r^2 \qquad V_{\text{sphere}} = \frac{4}{3} \pi\, r^3$$

$$A_{\text{cylinder}} = P_{\text{circle}}\, L \qquad V_{\text{cylinder}} = A_{\text{circle}}\, L$$