

UNIVERSITY OF ABERDEEN SESSION 2014–15**Degree Examination in EG40JK Thermodynamics 2****25th December 2014****9 am – 12 pm**

- Notes:*
- (i) Candidates ARE permitted to use an approved calculator.*
 - (ii) Candidates ARE permitted to use the Engineering Mathematics Handbook.*
 - (iii) Candidates ARE permitted to use steam tables, which will be provided.*
 - (iv) Data sheets are attached to the paper.*

PLEASE NOTE THE FOLLOWING

- (i) You **must not** have in your possession any material other than that expressly permitted in the rules appropriate to this examination. Where this is permitted, such material **must not** be amended, annotated or modified in any way.
- (ii) You **must not** have in your possession any material that could be determined as giving you an advantage in the examination.
- (iii) You **must not** attempt to communicate with any candidate during the exam, either orally or by passing written material, or by showing material to another candidate, nor must you attempt to view another candidate's work.

Failure to comply with the above will be regarded as cheating and may lead to disciplinary action as indicated in the Academic Quality Handbook

(www.abdn.ac.uk/registry/quality/appendix7x1.pdf) Section 4.14 and 5.

**Candidates must attempt *all* questions from PART A
AND *two* questions from *three* in PART B.**

PART A: Answer ALL Questions

Question 1

- a) Chilled water flowing through brass tubes of 0.0126 m inside diameter and 0.0018 m thickness cools a stream of air flowing outside of the tube. The film coefficients for the air and water flows are $176 \text{ W/m}^2 \text{ K}$ and $5660 \text{ W/m}^2 \text{ K}$ respectively and thermal conductivity of the brass is $102 \text{ W/m}^2 \text{ K}$ (see Fig. 1).

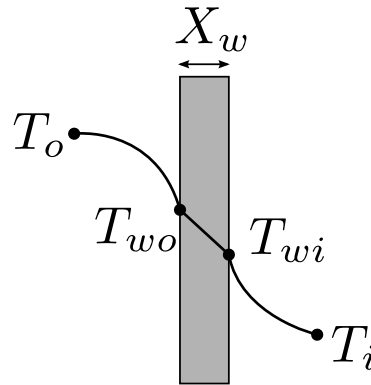


Figure 1: *The temperature profile through the pipe wall.*

- i) Calculate overall heat transfer resistance $R_{total} = (UA)_{total}^{-1}$. [6 marks]
 - ii) State what is the limiting heat resistance (i.e., what is the controlling heat transfer mechanism). [2 marks]
 - iii) Calculate heat transferred per metre length of tube at the point where the bulk temperatures of the air and water streams are 326°C and 15°C respectively. [2 marks]
- b) The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature-time history of a sphere fabricated from pure copper. The diameter of sphere is 17 mm. The sphere is at 86°C before it is inserted into an airstream having a temperature of 22°C . A thermocouple on the outer surface of the sphere indicates 62°C at 116 seconds after the sphere is inserted into the airstream. **Note:** The properties of copper at 347K are $\rho = 8933 \text{ kg/m}^3$, $C_p = 389 \text{ J kg}^{-1} \text{ K}^{-1}$, and $k = 398 \text{ W m}^{-1} \text{ K}^{-1}$.
- i) Calculate the heat transfer coefficient by assuming that the lumped capacitance method is valid. [7 marks]
 - ii) Show that the Biot number supports the application of the lumped capacitance method. [3 marks]

Question 2

- a) Describe the physical interpretation of the Grashof number (Gr). Describe each of its terms and write down an equation for the temperature at which temperature-dependent terms in Gr should be evaluated. [5 marks]
- b) An electric heater of 0.032 m diameter and 0.85 m long is used to heat a room. Calculate the electrical input (i.e. the sum of heat transferred by convection and radiation) to the heater when the bulk of the air in the room is at 24°C, the walls are at 12°C, and the surface of the heater is at 532°C. For convective heat transfer from the heater, assume the Nusselt number is given by

$$Nu = 0.38(Gr)^{0.25}$$

where all properties are evaluated at the film temperature. You may assume air is an ideal gas, giving $\beta = T^{-1}$. Take the emissivity of the heater surface as $\epsilon = 0.62$ and assume that the surroundings are black. All other properties should be calculated using the steam table book. [10 marks]

Question 3

- a) Carry out a dimensional analysis on the heat transfer coefficient, h , to determine which are the fundamental dimensionless numbers involved. You may assume the following general dependence

$$h = f(d, \mu, k, \langle v \rangle, \rho, C_p)$$

where d is the channel diameter (m), μ is the viscosity (Pa s), k is the thermal conductivity (W/ m K), $\langle v \rangle$ is the mean flow velocity (m/s), ρ is the mass density (kg / m³), and C_p is the specific heat capacity at constant pressure (kJ/kg K). [10 marks]

- b) A single-pass, counter-flow shell-and-tube heat exchanger is required to operate as an oil cooler with 316 tubes of internal diameter 0.016 m, outer diameter 0.018 m, and length 5.6 m. The oil flows in the tube side entering at a mass flow rate of 32 kg/s at a temperature of 136°C. Cooling water in the shell side enters at a mass flow rate of 33 kg/s at a temperature of 10°C. The shell side heat transfer coefficient is 850 W/m²K; and the specific heat capacity of water is 4.187 kJ/kg K. The Nusselt number is approximately related to the Reynolds and Prandtl numbers as follows

$$Nu = 0.025 Re^{3/4} Pr^{2/5} \quad (1)$$

and the following property values apply: specific heat capacity of oil: 3.42 kJ/kg K; density of oil: 900 kg/m³; dynamic viscosity of oil: 1.5×10^{-3} kg/m s; thermal conductivity of oil: 0.15 W/m K; thermal conductivity of the steel pipe wall: 54 W/m K. Calculate:

- i) The number of transfer units. [12 marks]
- ii) The effectiveness of the heat exchanger. [3 marks]

PART B: Answer TWO Questions From THREE

Question 4

A Couette viscometer tests the viscous behaviour of a fluid using rotational shear in an annulus (see Fig. 2). The fluid is sheared by rotating the outer wall at an angular velocity of Ω_θ , giving $v_\theta(r = R) = \Omega_\theta R$. The inner cylinder is held stationary, giving $v_\theta(r = \kappa R) = 0$. There is no flow along the axis of the annulus.

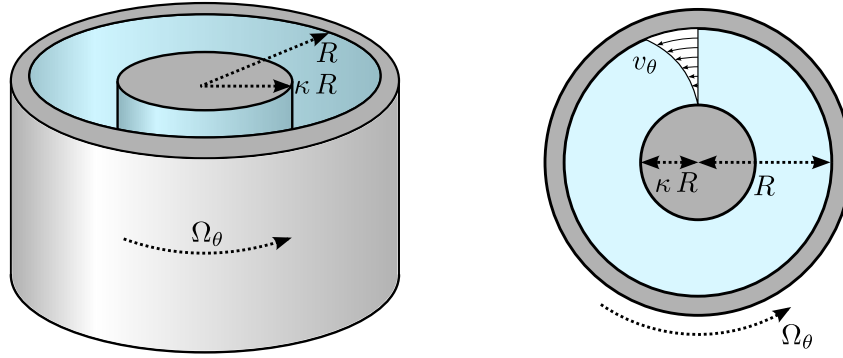


Figure 2: A simplified diagram of a Couette viscometer.

- a) Derive the following expression by solving the continuity equation, given in Eq. (6), for this system.

$$\frac{\partial v_\theta}{\partial \theta} = 0 \quad (2)$$

Clearly state any assumptions you make. What does this tell you about the flow? [5 marks]

- b) The velocity profile of the system is given by the following expression:

$$v_\theta = \Omega_\theta R \frac{\frac{\kappa R}{r} - \frac{r}{\kappa R}}{\kappa - 1/\kappa} \quad (3)$$

Derive the following expression for the stress profile in the system.

$$\tau_{r\theta} = 2 \frac{\mu \Omega_\theta \kappa^2 R^2}{\kappa^2 - 1} \frac{1}{r^2} \quad (4)$$

[10 marks]

- c) Derive the following expression for the torque exerted on the outer surface ($r = R$) to keep the fluid in motion.

$$\mathcal{T} = 4\pi R^2 L \frac{\mu \Omega_\theta \kappa^2}{\kappa^2 - 1}$$

Note: The torque is the total magnitude of a tangential force, such as the viscous stress $\tau_{r\theta}$, multiplied by the radial distance at which it acts. [3 marks]

- d) How can the formulas above be used to determine the viscous properties of a fluid?
[2 marks]

Question 5

An electric wire of radius 0.5 mm is made of copper (electrical conductivity $k_e = 5.1 \times 10^7 \text{ ohm}^{-1} \text{ m}^{-1}$ and thermal conductivity $k = 380 \text{ W m}^{-1} \text{ K}^{-1}$). It is insulated to an outer radius of 1.5 mm with plastic (thermal conductivity $k = 0.35 \text{ W m}^{-1} \text{ K}^{-1}$). The volumetric heat production σ , is given by $\sigma = I^2/k_e$ where I is the current density A/m^2 . The ambient air is at 38°C and the heat transfer coefficient from the outer insulated surface to the surrounding air is $8.5 \text{ W m}^{-2}\text{K}^{-1}$.

- a) Determine the maximum current in amperes that can flow through the wire if no part of the insulation may exceed 93°C . [8 marks]
- b) Demonstrate that the heat flux in the copper section of the wire is given by the following expression:

$$q_r = \frac{I^2}{2k_e}r$$

[8 marks]

- c) Solve for the temperature profile within the copper wire, assuming the outer surface of the wire is at $T_{crit.}$ [4 marks]

Question 6

Consider the flow of a Newtonian liquid between two plates where both plates are maintained at two different temperatures.

The viscosity μ of the liquid depends on temperature T according to the following relationship:

$$\mu(T) = \frac{\mu_0}{1 + \beta(T - T_0)}$$

where T_0 is a reference temperature, and μ_0 and β are empirical constants. The fluid flows under the influence of a pressure gradient $\Delta P/L$ between two flat plates, as shown in Fig. 3. The walls are at temperatures T_0 and T_1 , where T_0 is the reference temperature, and $T_1 > T_0$.

- i) Derive the stress profile and prove that it is equal to the expression below.

$$\tau_{yx} = -\frac{\Delta p H}{L} \left(\frac{y}{H} + C_1 \right)$$

Remember to state any assumptions that you make. [6 marks]

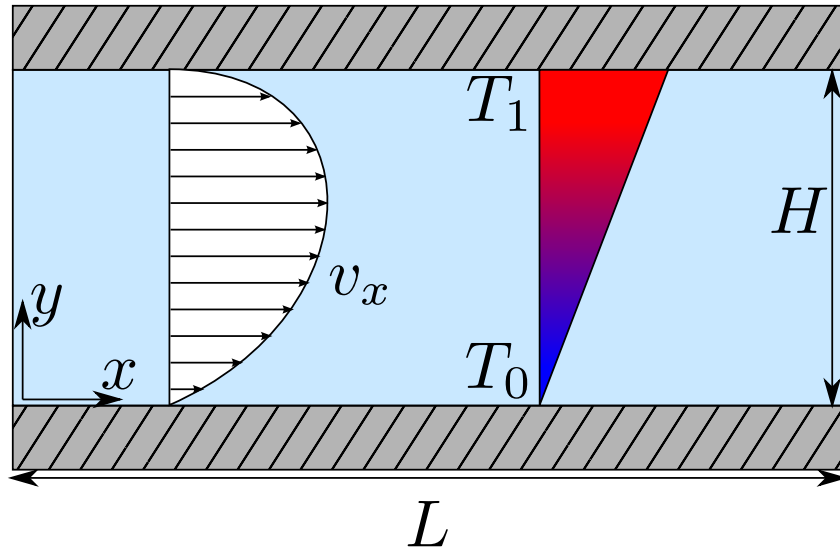


Figure 3: Flow through two parallel plates held at different temperatures.

- ii) Assuming the temperature profile is linear and has the following form:

$$T \approx T_0 + (T_1 - T_0) \frac{y}{H}$$

derive the following velocity profile for this system.

$$v_x(y) = \frac{\Delta p H^2}{L \mu_0} \frac{y}{H} \left[\beta(T_1 - T_0) \frac{y^2}{3 H^2} + (1 + C_1 \beta [T_1 - T_0]) \frac{y}{2 H} + C_1 \right] \quad (5)$$

where C_1 is a dimensionless integration constant which you must determine. [7 marks]

- iii) Determine the flow-rate to pressure drop relationship. [7 marks]

END OF PAPER

DATASHEET

General balance equations (in index notation):

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot \rho \mathbf{v} \quad (\text{Mass/Continuity}) \quad (6)$$

$$\frac{\partial C_A}{\partial t} = -\nabla \cdot \mathbf{N}_A \quad (\text{Species}) \quad (7)$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\rho \mathbf{v} \cdot \nabla \mathbf{v} - \nabla \cdot \boldsymbol{\tau} - \nabla p + \rho \mathbf{g} \quad (\text{Momentum}) \quad (8)$$

$$\rho C_p \frac{\partial T}{\partial t} = -\rho C_p \mathbf{v} \cdot \nabla T - \nabla \cdot \mathbf{q} - \boldsymbol{\tau} : \nabla \mathbf{v} - p \nabla \cdot \mathbf{v} + \sigma_{energy} \quad (\text{Heat/Energy}) \quad (9)$$

In Cartesian coordinate systems, ∇ can be treated as a vector of derivatives. In curvilinear coordinate systems, the directions $\hat{\mathbf{r}}$, $\hat{\boldsymbol{\theta}}$, and $\hat{\boldsymbol{\phi}}$ depend on the position. For convenience in these systems, look-up tables are provided for common terms involving ∇ .

Cartesian coordinates (with index notation examples)

where s is a scalar, \mathbf{v} is a vector, and $\boldsymbol{\tau}$ is a tensor.

$$\begin{aligned} \nabla s &= \nabla_i s = \left[\frac{\partial s}{\partial x}, \frac{\partial s}{\partial y}, \frac{\partial s}{\partial z} \right] \\ \nabla^2 s &= \nabla_i \nabla_i s = \frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} \\ \nabla \cdot \mathbf{v} &= \nabla_i v_i = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\ \nabla \cdot \boldsymbol{\tau} &= \nabla_i \tau_{ij} \\ [\nabla \cdot \boldsymbol{\tau}]_x &= \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \\ [\nabla \cdot \boldsymbol{\tau}]_y &= \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \\ [\nabla \cdot \boldsymbol{\tau}]_z &= \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} \\ \mathbf{v} \cdot \nabla \mathbf{v} &= v_i \nabla_i v_j \\ [\mathbf{v} \cdot \nabla \mathbf{v}]_x &= v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \\ [\mathbf{v} \cdot \nabla \mathbf{v}]_y &= v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \\ [\mathbf{v} \cdot \nabla \mathbf{v}]_z &= v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \end{aligned}$$

Cylindrical coordinates

where s is a scalar, \mathbf{v} is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\begin{aligned}
\nabla s &= \left[\frac{\partial s}{\partial r}, \frac{1}{r} \frac{\partial s}{\partial \theta}, \frac{\partial s}{\partial z} \right] \\
\nabla^2 s &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial s}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 s}{\partial \theta^2} + \frac{\partial^2 s}{\partial z^2} \\
\nabla \cdot \mathbf{v} &= \frac{1}{r} \frac{\partial}{\partial r} (r v_r) + \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial z} \\
[\nabla \cdot \boldsymbol{\tau}]_r &= \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rr}) + \frac{1}{r} \frac{\partial \tau_{r\theta}}{\partial \theta} - \frac{1}{r} \tau_{\theta\theta} + \frac{\partial \tau_{rz}}{\partial z} \\
[\nabla \cdot \boldsymbol{\tau}]_\theta &= \frac{1}{r} \frac{\partial \tau_{\theta\theta}}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + \frac{2}{r} \tau_{r\theta} + \frac{\partial \tau_{\theta z}}{\partial z} \\
[\nabla \cdot \boldsymbol{\tau}]_z &= \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rz}) + \frac{1}{r} \frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} \\
[\mathbf{v} \cdot \nabla \mathbf{v}]_r &= v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} + v_z \frac{\partial v_r}{\partial z} \\
[\mathbf{v} \cdot \nabla \mathbf{v}]_\theta &= v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z \frac{\partial v_\theta}{\partial z} \\
[\mathbf{v} \cdot \nabla \mathbf{v}]_z &= v_r \frac{\partial v_z}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_z}{\partial \theta} + v_z \frac{\partial v_z}{\partial z}
\end{aligned}$$

Spherical coordinates

where s is a scalar, \mathbf{v} is a vector, and $\boldsymbol{\tau}$ is a tensor. All expressions involving $\boldsymbol{\tau}$ are for symmetrical $\boldsymbol{\tau}$ only.

$$\begin{aligned}
\nabla s &= \left[\frac{\partial s}{\partial r}, \frac{1}{r} \frac{\partial s}{\partial \theta}, \frac{1}{r \sin \theta} \frac{\partial s}{\partial \phi} \right] \\
\nabla^2 s &= \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial s}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial s}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 s}{\partial \phi^2} \\
\nabla \cdot \mathbf{v} &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (v_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} \\
[\nabla \cdot \boldsymbol{\tau}]_r &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tau_{rr}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\tau_{r\theta} \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial \tau_{r\phi}}{\partial \phi} - \frac{\tau_{\theta\theta} + \tau_{\phi\phi}}{r} \\
[\nabla \cdot \boldsymbol{\tau}]_\theta &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tau_{r\theta}) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\tau_{\theta\theta} \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial \tau_{\theta\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} - \frac{\cot \theta}{r} \tau_{\phi\phi} \\
[\nabla \cdot \boldsymbol{\tau}]_\phi &= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \tau_{r\phi}) + \frac{1}{r} \frac{\partial \tau_{\theta\phi}}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial \tau_{\phi\phi}}{\partial \phi} + \frac{\tau_{r\theta}}{r} + \frac{2 \cot \theta}{r} \tau_{\theta\phi} \\
[\mathbf{v} \cdot \nabla \mathbf{v}]_r &= v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} \\
[\mathbf{v} \cdot \nabla \mathbf{v}]_\theta &= v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2 \cot \theta}{r} \\
[\mathbf{v} \cdot \nabla \mathbf{v}]_\phi &= v_r \frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r} \frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi + v_\theta v_\phi \cot \theta}{r}
\end{aligned}$$

Rectangular		Cylindrical		Spherical	
q_x	$-k \frac{\partial T}{\partial x}$	q_r	$-k \frac{\partial T}{\partial r}$	q_r	$-k \frac{\partial T}{\partial r}$
q_y	$-k \frac{\partial T}{\partial y}$	q_θ	$-k \frac{1}{r} \frac{\partial T}{\partial \theta}$	q_θ	$-k \frac{1}{r} \frac{\partial T}{\partial \theta}$
q_z	$-k \frac{\partial T}{\partial z}$	q_z	$-k \frac{\partial T}{\partial z}$	q_ϕ	$-k \frac{1}{r \sin \theta} \frac{\partial T}{\partial \phi}$
τ_{xx}	$-2\mu \frac{\partial v_x}{\partial x} + \mu^B \nabla \cdot \mathbf{v}$	τ_{rr}	$-2\mu \frac{\partial v_r}{\partial r} + \mu^B \nabla \cdot \mathbf{v}$	τ_{rr}	$-2\mu \frac{\partial v_r}{\partial r} + \mu^B \nabla \cdot \mathbf{v}$
τ_{yy}	$-2\mu \frac{\partial v_y}{\partial y} + \mu^B \nabla \cdot \mathbf{v}$	$\tau_{\theta\theta}$	$-2\mu \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) + \mu^B \nabla \cdot \mathbf{v}$	$\tau_{\theta\theta}$	$-2\mu \left(\frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r} \right) + \mu^B \nabla \cdot \mathbf{v}$
τ_{zz}	$-2\mu \frac{\partial v_z}{\partial z} + \mu^B \nabla \cdot \mathbf{v}$	τ_{zz}	$-2\mu \frac{\partial v_z}{\partial z} + \mu^B \nabla \cdot \mathbf{v}$	$\tau_{\phi\phi}$	$-2\mu \left(\frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{v_r + v_\theta \cot \theta}{r} \right) + \mu^B \nabla \cdot \mathbf{v}$
τ_{xy}	$-\mu \left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right)$	$\tau_{r\theta}$	$-\mu \left(r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right)$	$\tau_{r\theta}$	$-\mu \left(r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right)$
τ_{yz}	$-\mu \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y} \right)$	$\tau_{\theta z}$	$-\mu \left(\frac{1}{r} \frac{\partial v_z}{\partial \theta} + \frac{\partial v_\theta}{\partial z} \right)$	$\tau_{\theta\phi}$	$-\mu \left(\frac{\sin \theta}{r} \frac{\partial}{\partial \theta} \left(\frac{v_\phi}{\sin \theta} \right) + \frac{1}{r \sin \theta} \frac{\partial v_\theta}{\partial \phi} \right)$
τ_{xz}	$-\mu \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} \right)$	τ_{zr}	$-\mu \left(\frac{\partial v_r}{\partial z} + \frac{\partial v_z}{\partial r} \right)$	$\tau_{\phi r}$	$-\mu \left(\frac{1}{r \sin \theta} \frac{\partial v_r}{\partial \phi} + r \frac{\partial}{\partial r} \left(\frac{v_\phi}{r} \right) \right)$

Table 1: Fourier's law for the heat flux and Newton's law for the stress in several coordinate systems. Please remember that the stress is symmetric, so $\tau_{ij} = \tau_{ji}$.

Viscous models:

Power-Law Fluid:

$$|\tau_{xy}| = k \left| \frac{\partial v_x}{\partial y} \right|^n$$

Bingham-Plastic Fluid:

$$\frac{\partial v_x}{\partial y} = \begin{cases} -\mu^{-1} (\tau_{xy} - \tau_0) & \text{if } \tau_{xy} > \tau_0 \\ 0 & \text{if } \tau_{xy} \leq \tau_0 \end{cases}$$

Dimensionless Numbers

$$\text{Re} = \frac{\rho \langle v \rangle D}{\mu} \quad \text{Re}_H = \frac{\rho \langle v \rangle D_H}{\mu} \quad \text{Re}_{MR} = -\frac{16 L \rho \langle v \rangle^2}{R \Delta p} \quad (10)$$

The hydraulic diameter is defined as $D_H = 4 A / P_w$.

Single phase pressure drop calculations in pipes:

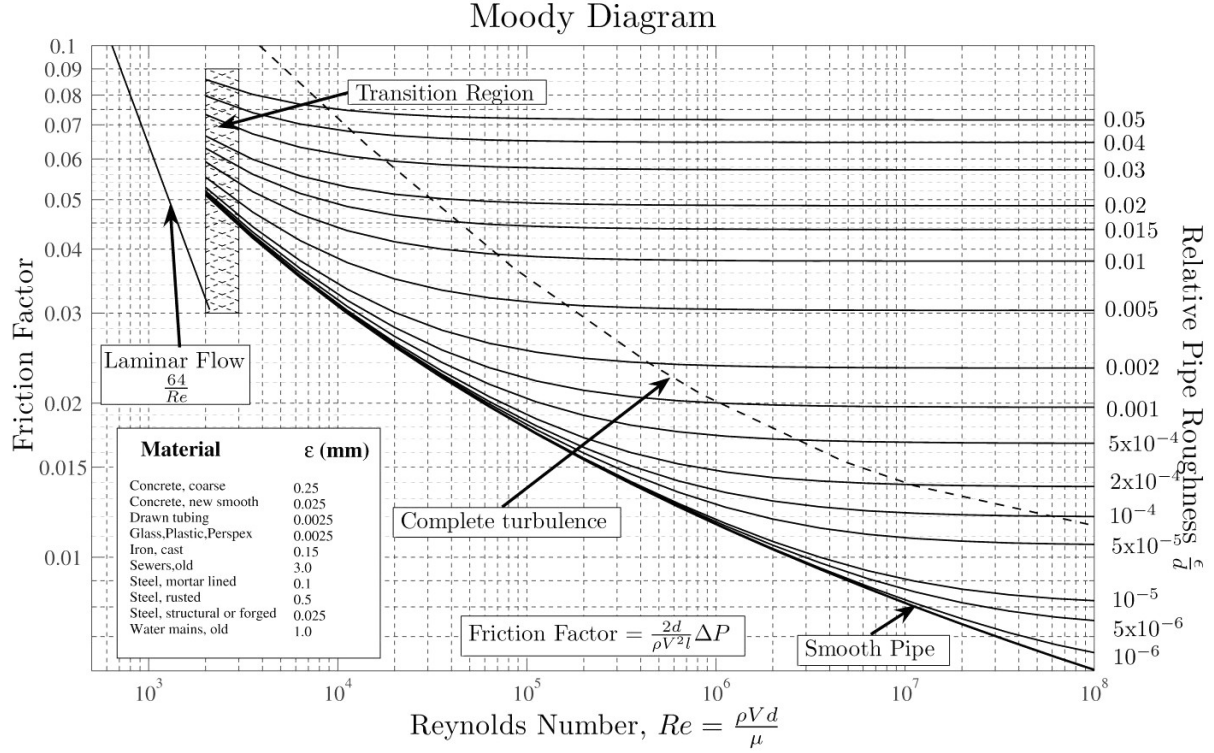
Darcy-Weisbach equation:

$$\frac{\Delta p}{L} = -\frac{C_f \rho \langle v \rangle^2}{R} \quad (11)$$

where $C_f = 16/Re$ for laminar Newtonian flow. For turbulent flow of Newtonian fluids in smooth pipes, we have the Blasius correlation:

$$C_f = 0.079 Re^{-1/4} \quad \text{for } 2.5 \times 10^3 < Re < 10^5 \text{ and smooth pipes.}$$

Otherwise, you may refer to the Moody diagram.



Laminar Power-Law fluid:

$$\dot{V} = \frac{n \pi R^3}{3n + 1} \left(\frac{R}{2k} \right)^{\frac{1}{n}} \left(-\frac{\Delta p}{L} \right)^{\frac{1}{n}}$$

Two-Phase Flow:

Lockhart-Martinelli parameter:

$$X^2 = \frac{\Delta p_{liq.-only}}{\Delta p_{gas-only}}$$

Pressure drop calculation:

$$\Delta p_{two-phase} = \Phi_{liq.}^2 \Delta p_{liq.-only} = \Phi_{gas}^2 \Delta p_{gas-only}$$

Chisholm's relation:

$$\Phi_{gas}^2 = 1 + cX + X^2$$

$$\Phi_{liq.}^2 = 1 + \frac{c}{X} + \frac{1}{X^2} \quad c = \begin{cases} 20 & \text{turbulent liquid \& turbulent gas} \\ 12 & \text{laminar liquid \& turbulent gas} \\ 10 & \text{turbulent liquid \& laminar gas} \\ 5 & \text{laminar liquid \& laminar gas} \end{cases}$$

Farooqi and Richardson expression for liquid hold-up in co-current flows of Newtonian fluids and air in horizontal pipes:

$$h = \begin{cases} 0.186 + 0.0191 X & 1 < X < 5 \\ 0.143 X^{0.42} & 5 < X < 50 \\ 1 / (0.97 + 19/X) & 50 < X < 500 \end{cases}$$

Heat Transfer:

Stefan-Boltzmann constant $\sigma = 5.6703 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

Heat Transfer Dimensionless numbers:

$$\text{Nu} = \frac{h L}{k} \quad \text{Pr} = \frac{\mu C_p}{k} \quad \text{Gr} = \frac{g \beta (T_w - T_\infty) L^3}{\nu^2}$$

Resistances

$$Q = U_T A_T \Delta T = R_T^{-1} \Delta T \quad Q_{rad.} = \sigma \varepsilon A (T_\infty^4 - T_w^4) = h_{rad.} A (T_\infty - T_w)$$

	Conduction Shell Resistances			Radiation
	Rect.	Cyl.	Sph.	
R	$\frac{X}{k A}$	$\frac{\ln(R_{outer}/R_{inner})}{2 \pi L k}$	$\frac{R_{inner}^{-1} - R_{outer}^{-1}}{4 \pi k}$	$[A \varepsilon \sigma (T_\infty^2 + T_w^2) (T_\infty + T_w)]^{-1}$

Natural Convection

$\text{Ra} = \text{Gr Pr}$	C	m
$< 10^4$	1.36	1/5
$10^4 - 10^9$	0.59	1/4
$> 10^9$	0.13	1/3

Table 2: Natural convection coefficients for isothermal vertical plates in the empirical relation $\text{Nu} \approx C (\text{Gr Pr})^m$.

Churchill and Chu expression for natural convection from a horizontal pipe:

$$\text{Nu}^{1/2} = 0.6 + 0.387 \left\{ \frac{\text{Gr Pr}}{[1 + (0.559/\text{Pr})^{9/16}]^{16/9}} \right\}^{1/6} \quad \text{for } 10^{-5} < \text{Gr Pr} < 10^{12}$$

Forced Convection:

Laminar flows:

$$\text{Nu} \approx 0.332 \text{Re}^{1/2} \text{Pr}^{1/3}$$

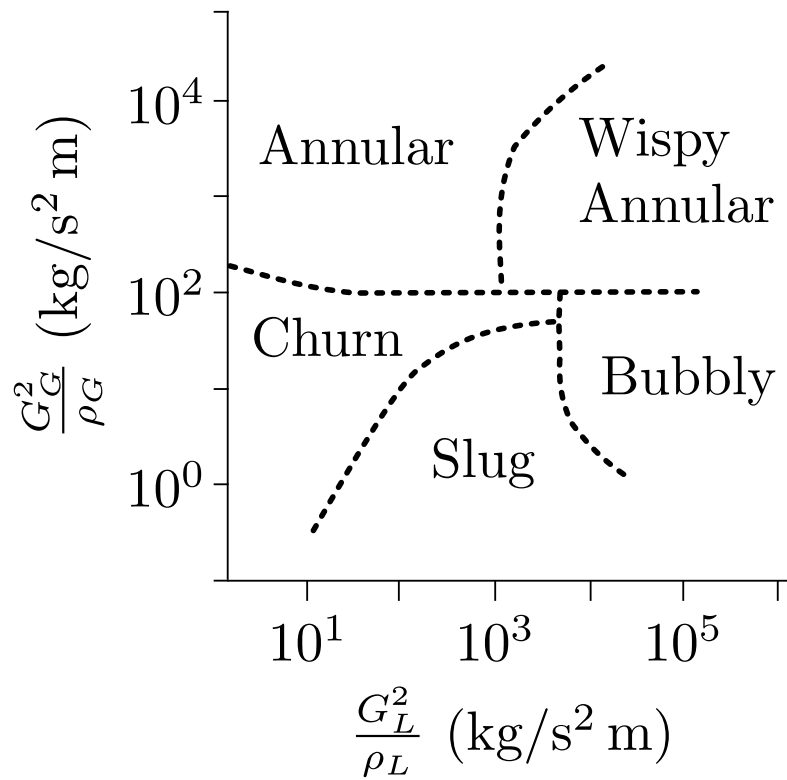


Figure 4: *Hewitt-Taylor flow pattern map for multiphase flows in vertical pipes.*

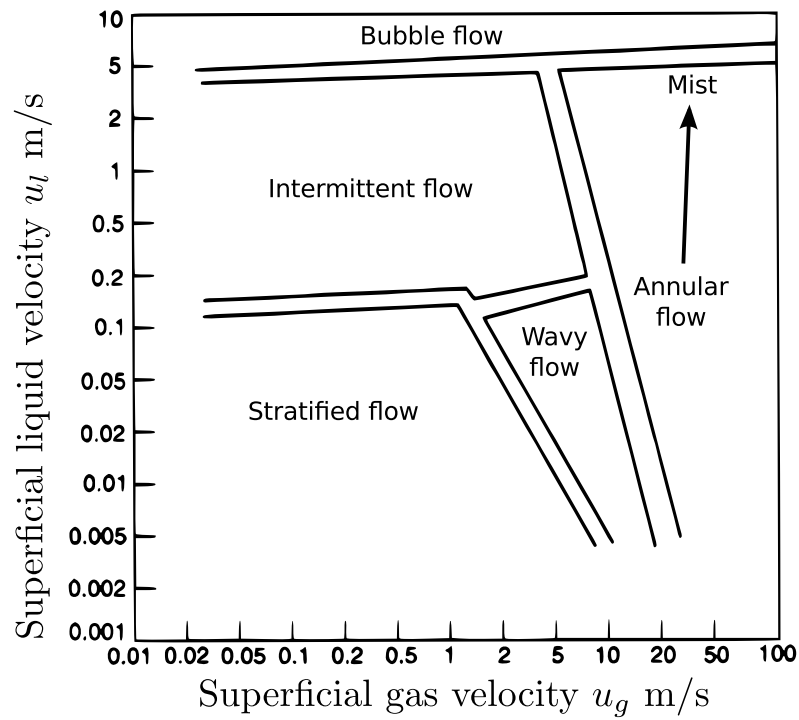


Figure 5: *Chhabra and Richardson flow pattern map for horizontal pipes.*

Well-Developed turbulent flows in smooth pipes:

$$\text{Nu} \approx \frac{(C_f/2)\text{Re Pr}}{1.07 + 12.7(C_f/2)^{1/2} (\text{Pr}^{2/3} - 1)} \left(\frac{\mu_b}{\mu_w} \right)^{0.14}$$

Boiling:

Forster-Zuber pool-boiling coefficient:

$$h_{nb} = 0.00122 \frac{k_L^{0.79} C_{p,L}^{0.45} \rho_L^{0.49}}{\gamma^{0.5} \mu_L^{0.29} h_{fg}^{0.24} \rho_G^{0.24}} (T_w - T_{sat})^{0.24} (p_w - p_{sat})^{0.75}$$

Mostinski correlations:

$$h_{nb} = 0.104 p_c^{0.69} q^{0.7} \left[1.8 \left(\frac{p}{p_c} \right)^{0.17} + 4 \left(\frac{p}{p_c} \right)^{1.2} + 10 \left(\frac{p}{p_c} \right)^{10} \right]$$

$$q_c = 3.67 \times 10^4 p_c \left(\frac{p}{p_c} \right)^{0.35} \left[1 - \frac{p}{p_c} \right]^{0.9}$$

(**Note:** for the Mostinski correlations, the pressures are in units of bar)

Condensing:

Horizontal pipes

$$h = 0.72 \left(\frac{k^3 \rho^2 g_x E_{latent}}{D \mu (T_w - T_\infty)} \right)^{1/4}$$

NTU method:

$$\text{NTU} = \frac{U A}{C_{min}} = \frac{t_{C1} - t_{C2}}{\Delta t_{ln}} \quad R = \frac{C_{min}}{C_{max}}$$

For counter-current flow:

$$E = \frac{1 - \exp[-\text{NTU}(1 - R)]}{1 - R \exp[-\text{NTU}(1 - R)]}$$

For co-current flow:

$$E = \frac{1 - \exp[-\text{NTU}(1 - R)]}{1 + R}$$

Lumped capacitance method:

$$\text{Bi} = \frac{h L_c}{k}$$

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp \left[-\frac{h A_s}{\rho V C_p} t \right]$$

Diffusion Dimensionless Numbers

$$\text{Sc} = \frac{\mu}{\rho D_{AB}} \qquad \text{Le} = \frac{k}{\rho C_p D_{AB}}$$

Diffusion

General expression for the flux:

$$\mathbf{N}_A = \mathbf{J}_A + x_A \sum_B \mathbf{N}_B$$

Fick's law:

$$\mathbf{J}_A = -D_{AB} \nabla C_A$$

Stefan's law:

$$N_{s,r} = -D \frac{c}{1-x} \frac{\partial x}{\partial r}$$