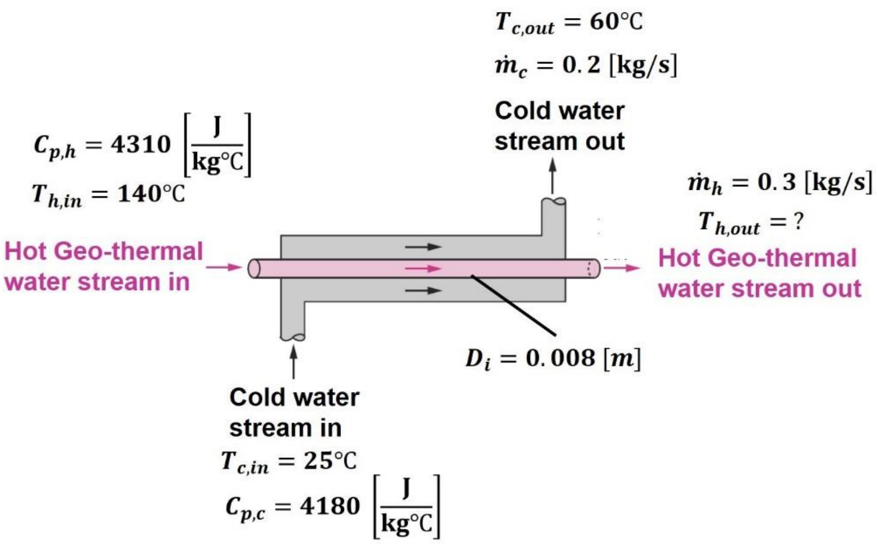
WORKED EXAMPLE

LOG MEAN TEMPERATURE DIFFERENCE METHOD

A double-pipe parallel-flow heat exchanger is to heat water ($C_{p,c}$ = 4180 J/kg·°C) from 25°C to 60°C at a rate of $\dot{m}_c$ = 0.2 kg/s. The heating is to be accomplished by geothermal water ($C_{p,\mathrm{h}}$ = 4310 J/kg·°C) available at 140°C at a mass flow rate of $\dot{m}_{\mathrm{h}}$ = 0.3 kg/s. The inner tube is thin-walled and has a diameter of 8 mm and the overall heat transfer coefficient of the heat exchanger is U = 550 W/m²·°C.

Determine the length of the heat exchanger required to achieve the desired heating.



SOLUTION

The objective is to find L, the length of the heat exchanger.

The length has a controlling influence on the surface area of the heat exchanger:

$$A = \pi D L$$

and the area, in turn, is important in the overall heat transfer, through:

$$\dot{Q} = UA\Delta T$$

Heat transfer from the cold stream is:

$$\begin{aligned}\dot{Q}_{\mathrm{c}} &= \dot{m}_{\mathrm{c}} C_{\mathrm{p,c}}\left(T_{\mathrm{c,out}} - T_{\mathrm{c,in}}\right) \\ &= 0.2 \times 4180 \times (60 - 25) \\ &= 29{,}260\ [\mathrm{W}]\end{aligned}$$

Similarly, heat transfer from the hot stream is:

$$\dot{Q}_{\mathrm{h}} = \dot{m}_{\mathrm{h}} C_{\mathrm{p,h}} \left(T_{\mathrm{h,in}} - T_{\mathrm{h,out}}\right)$$

We know the heat transferred from the hot stream must be equal to the heat transferred to the cold stream:

$$\therefore \dot{m}_{\mathrm{c}} C_{\mathrm{p,c}} \left(T_{\mathrm{c,out}} - T_{\mathrm{c,in}}\right) = \dot{m}_{\mathrm{h}} C_{\mathrm{p,h}} \left(T_{\mathrm{h,in}} - T_{\mathrm{h,out}}\right)$$

Since we don't know $T_{\mathrm{h,out}}$, we can rearrange to find it:

$$\begin{aligned} T_{\mathrm{h,out}} &= T_{\mathrm{h,in}} - \frac{\dot{m}_{\mathrm{c}}}{\dot{m}_{\mathrm{h}}} \frac{C_{\mathrm{p,c}}}{C_{\mathrm{p,h}}} \left(T_{\mathrm{c,out}} - T_{\mathrm{c,in}}\right) \\ &= 140 - \frac{0.2}{0.3} \times \frac{4180}{4310} \times (60 - 25) \\ &= 117.4\ [°\mathrm{C}] \end{aligned}$$

Now we can estimate ΔT_{m} as (equation 15 from notes):

$$\Delta T_{\mathrm{m}} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

Since this is a parallel flow heat exchanger:

$$\Delta T_1 = T_{\mathrm{h,in}} - T_{\mathrm{c,in}} = 140 - 25 = 115\ [°\mathrm{C}]$$

and

$$\Delta T_2 = T_{\mathrm{h,out}} - T_{\mathrm{c,out}} = 117.4 - 60 = 57.4\ [°\mathrm{C}]$$

$$\therefore \Delta T_{\mathrm{m}} = \frac{115 - 57.4}{\ln\left(\frac{115}{57.4}\right)} = 82.9\ [°\mathrm{C}]$$

Now we can get the surface area from $Q = UA\Delta T_{\mathrm{m}}$, as:

$$\begin{aligned} A &= \frac{Q}{U \Delta T_{\mathrm{m}}} \\ &= \frac{29{,}260}{550 \times 82.9} \\ &= 0.642\ [\mathrm{m}^2] \end{aligned}$$

Finally, we can get the required length of the heat exchanger from $A = \pi D L$, as:

$$L = \frac{A}{\pi D} = \frac{0.642}{\pi \times 0.008} = \mathbf{25.5\ [m]}$$