

Flow Assurance Pressure Drop Flow Patterns Slugging



1.1	Typical Subsea Development	4
2	FLUID FLOW ENERGY EQUATIONS	4
2.1	Hold-Up	5
3	FLOW REGIMES	6
3.1	Horizontal Flow Regimes	7
3.2	Vertical Flow Regimes	8
3.3	Mandhane Flow Regime Map	9
3.4	Baker Flow Regime Map	10
3.5	Taitel Dukler Flow Regime Map	14
3.6	Govier Vertical Flow Map	15
3.7	Wet Gas Pipeline Pigging and Slugcatcher	16
3.8	Eaton Wet Gas Pipeline Hold-Up	18
4	PRESSURE DROP PREDICTION	19
4.1	Pressure Drop Models Category 1	20
4.2	Pressure Drop Models Category 2	20
4.3	Pressure Drop Models Category 3	20
4.4	Homogeneous Model – Category 1	21
4.5	Lockhart and Martinelli – Category 1	21
4.6	Beggs and Brill – Category 3	23
4.7	Mechanistic Models	23
5	SLUG FLOW	24
5.1	Hydrodynamic Slugging	25
5.2	Terrain Induced Slugs	26
5.3	Severe Slugging	26
6	PIPELINE TOPOLOGY	30
7	COUPLING OF HYDROCARBON RESERVOIR AND PRODUCTION WELLS	30
7.1	Tubing Diameter	30

7.2	Gas Liquid Ratio	31
7.3	Surface Pressure	32

Flow assurance has become a critical technology area for the oil and gas industry. The simultaneous flow of gas and liquid through pipes, often referred to as multiphase flow, occurs in almost every aspect of the oil industry. Multiphase flow is present in well tubing, gathering system pipelines, and processing equipment. The use of multiphase pipelines has become increasingly important in recent years due to the development of marginal fields and deep water prospects. In many cases, the feasibility of a design scenario hinges on cost and operation of the pipeline and its associated equipment. Confidence in the flow assurance analysis – pressure loss, heat transfer, fluid chemistry, transient effects – is essential to developing a safe and operable design.

Multiphase flow in pipes has been studied for more than 50 years, with significant improvements in the state of the art during the past 15 years. The best available methods can predict the operation of the pipelines much more accurately than those available only a few years ago. The designer, however, has to know the key areas requiring analysis and which methods to use in order to deliver the most accurate results.

1.1 Typical Subsea Development

In addition to multi-phase pipelines, flow assurance also involves the design of single phase compressible and incompressible flow. A typical subsea development with multiple duties is shown in figure.

Flow assurance will involve the design and operation of;

Single phase incompressible flow –

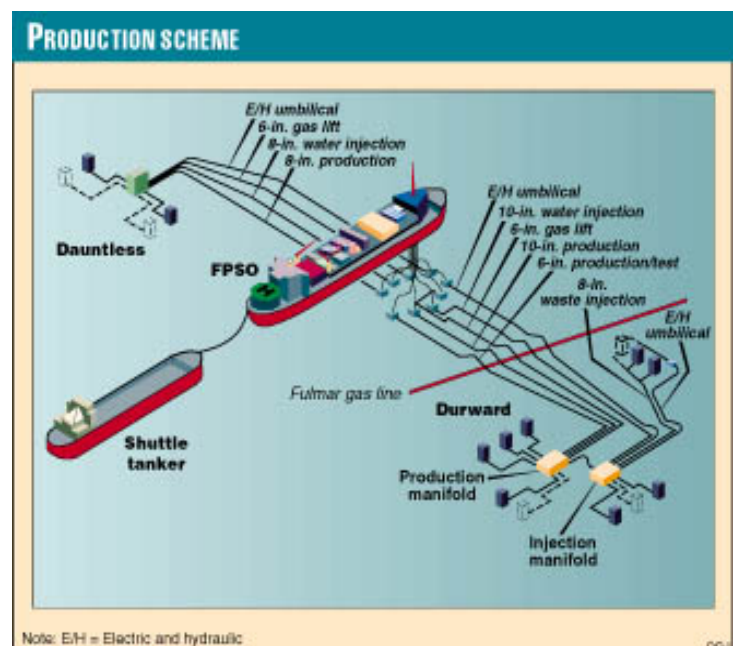
Water injection, oil export, chemical injection (methanol, corrosion inhibition etc.)

Single phase compressible flow –

Gas lift, gas export.

Multi-phase flow –

Production, test pipelines



2 Fluid Flow Energy Equations

Recall Bernoulli's equation and the lost work (friction) term.

$$\int \frac{dP}{\rho} + g \cdot \Delta X + \frac{\Delta(v)^2}{2} = -W_f - W$$

Pressure change	Elevation term	Velocity term	Friction term	Work term
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$$\Delta P_f = \frac{\rho \cdot f \cdot L \cdot v^2}{2 \cdot D}$$

For multi-phase flow it is apparent that a single density no longer applies as does a single velocity. Similarly other necessary properties viscosity, heat capacity etc. Immediately it is clear multi-phase flow will be much more complex to analyse.

2.1 Hold-Up

Liquid hold-up is the ratio of the volume of liquid in the system to the total volume. It is an important feature of multi-phase flowlines as it will affect pressure drop, slug generation, rate of pipeline cooldown and other design and operational features.

If an equal volume rate of gas and liquid are introduced into a horizontal pipeline it might be expected that the liquid

hold-up i.e. the fraction of liquid volume to the total volume, would be 50%.

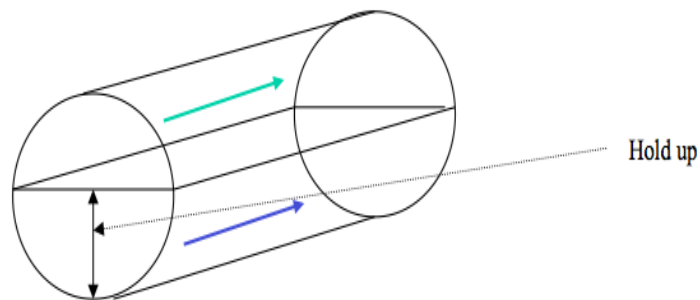
This is unlikely to be the case as the gas is much more mobile than the liquid

and will slip through the pipeline resulting in a higher than 50% liquid hold up.

The level of hold up will also be influenced by the pipeline angle. In upflow the gas will slip much more readily through the liquid resulting in higher hold-ups. In downfall the liquid will start to run downwards due to gravity hence the hold up will be smaller.

Furthermore, as the liquid viscosity is significantly greater than the gas velocity, a pressure force will give a larger gas velocity than liquid velocity. This will cause a slip between the (average) gas and liquid velocity. This slip (hold-up) is not caused by gravity, and thus is also present in horizontal pipe sections.

Terminology used in the development of multi-phase flow correlations follows;



Liquid Holdup H_l

Ratio of the volume of a pipe segment occupied by liquid to the volume of the pipe segment

$$H_l = \text{volume of liquid in a pipe segment} / \text{volume of pipe segment}$$

Value varies from 0 to 1

Gas Holdup or gas void fraction H_g

$$H_g = 1.0 - H_l$$

No-Slip Liquid Holdup – λ_l

Ratio of volume of liquid in a pipe segment divided by volume of pipe segment which would exist if the gas and liquid travelled at the same velocity (no-slippage)

$$\lambda_l = q_l / (q_l + q_g)$$

Where q_l and q_g are liquid and gas volumetric flow rates

No-Slip Gas Holdup – λ_g

$$\lambda_g = 1.0 - \lambda_l = q_g / (q_l + q_g)$$

Two-Phase Velocity/ Mixture Velocity - v_m

The velocities of the gas and liquid in the pipe are prime variables in the prediction of the behaviour of the multiphase mixture. Most multiphase flow prediction methods use the superficial gas and liquid velocities as correlating parameters. The superficial velocities are defined as the in situ volumetric flowrate of that phase divided by the total pipe cross sectional area. Or, the velocity in the pipe if the gas or liquid were flowing alone.

The following can be readily shown;

$v_m = v_{sl} + v_{sg}$ mixture velocity, v_m , is the sum of the two superficial velocities

$v_{sl} = q_l / A$ – superficial liquid velocity $v_l = q_l / (A.H_l)$ – actual liquid velocity

$v_{sg} = q_g / A$ – superficial gas velocity $v_g = q_g / (A.H_g)$ – actual gas velocity

$\lambda_l = q_l / (q_l + q_g)$, but $v_{sl} = q_l / A$ and $v_{sg} = q_g / A$, so

$$\lambda_l = v_{sl} / (v_{sl} + v_{sg}) = v_{sl} / v_m$$

Note, some researchers refer to superficial velocity as j , the volumetric flux.

3 Flow Regimes

In multiphase flow, the gas and liquid within the pipe are distributed in several fundamentally different flow patterns or flow regimes, depending primarily on the gas and liquid velocities, physical properties and the angle of inclination. Observers have labelled these flow regimes with a variety of names. Over 100 different names for the various regimes and sub-regimes have been used in the literature.

A number of workers have generated flow regime maps which can be used to determine which flow regimes are likely to occur within a particular pipeline (or parts of the pipeline) under various operating conditions. It is important to know which flow regime since the regime will determine the pressure drop along the pipe, the liquid hold-up within it and the nature of the fluid mixture which will have to be treated on arrival. Research on two phase flow regimes goes back many decades (Baker 1954). Papers by Mandhane et al (1974), Taitel and Dukler (1976) are often quoted.

With the exception of Taitel and Dukler, these researchers have used an empirical approach to interpreting experimental results. Taitel and Dukler produced a more theoretical or mechanistic approach and they compared this successfully with published results. This was a landmark advance for the science.

3.1 Horizontal Flow Regimes

Though other patterns exist, generally four horizontal regimes are defined: Bubble, Slug, Annular and Mist flow.

Plug/Bubble Flow

- Pipe is almost completely filled with liquid and the free gas phase is present in small bubbles.
- Pipe wall is always contacted by the liquid phase

Slug or Intermittent Flow

- Liquid phase with accompanying gas bubble gas

Stratified Flow

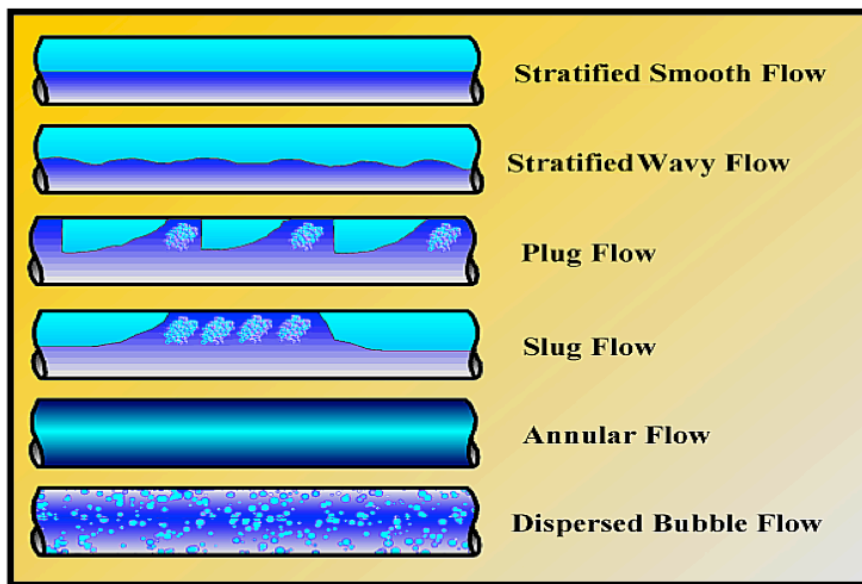
- Pipe acts as a separator
- Liquid flows along bottom with gas above

Annular Flow

- High velocity gas core
- Liquid annulus surrounds a gas core

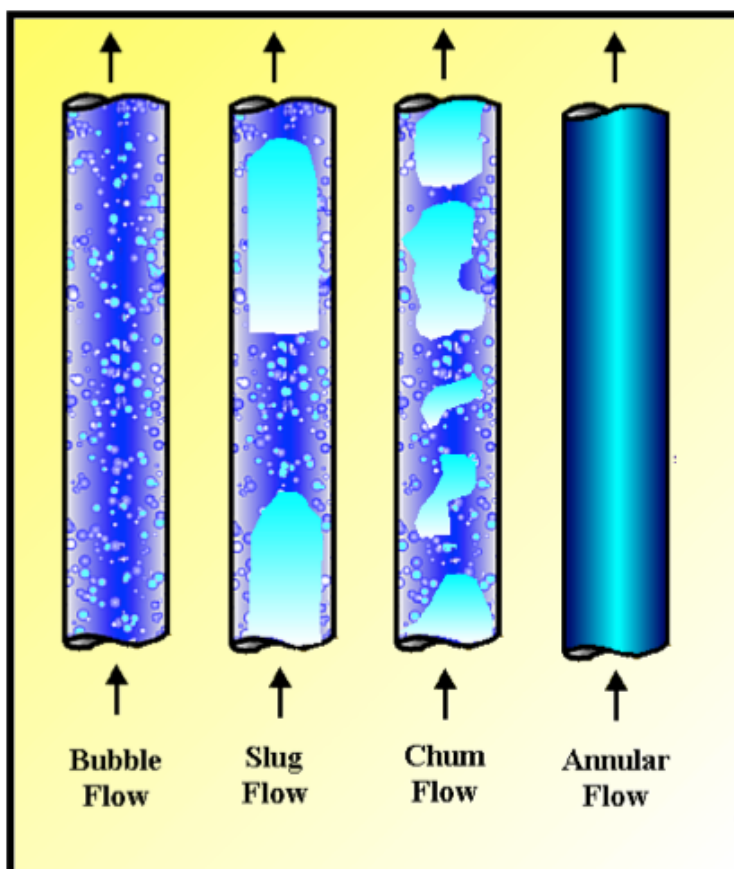
Dispersed Bubble Flow

- Bubbles of gas flow within a predominantly liquid phase
- Nearest regime to no slip conditions



3.2 Vertical Flow Regimes

For vertical flow similar regimes are defined. However, as can be seen stratified flow does not exist.



Experimental studies of flow regime transitions have shown that each of the flow regime boundaries reacts differently to changes in the system variables. The following table shows the sensitivity of the transitions to changes in the major system variables:

Transition Variable	<u>Slug to Dispersed Bubble</u>	<u>Slug to Annular</u>	<u>Slug to Stratified</u>	<u>Stratified to Annular</u>
Angle of Inclination	Small Effect	Moderate Effect	Strong Effect	Strong Effect
Gas Density	Small Effect	Strong Effect	Strong Effect	Strong Effect
Pipeline Diameter	Small Effect	Small Effect	Strong Effect	Moderate Effect
Liquid Physical Properties	Moderate Effect	Small Effect	Moderate Effect	Moderate Effect

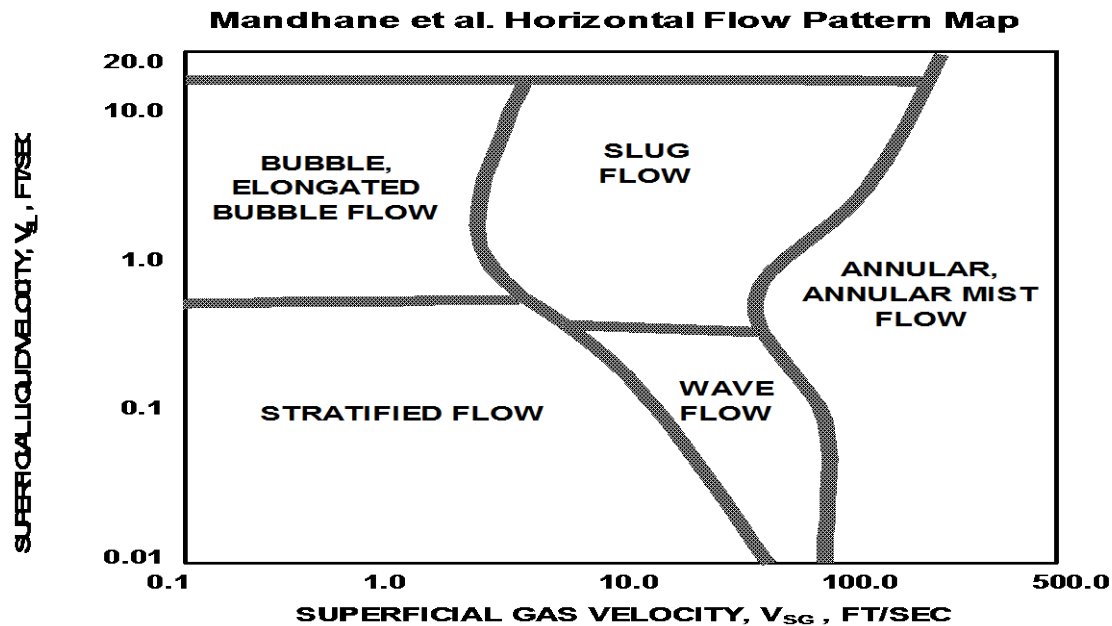
Many researchers have attempted to develop simple flow regime maps, usually using dimensionless parameters on each axis (e.g. Baker, Beggs & Brill). These methods can be inaccurate since no single parameter can model the sensitivity effects indicated above.

3.3 Mandhane Flow Regime Map

Mandhane proposed a fluid property correction to the superficial velocities, but concluded that the fluid property effects are insignificant compared to the errors in the empirical map. The map reports the flow regimes: stratified, wavy, annular mist, bubble, slug, and dispersed. Care should be taken in the interpretation of these flow maps as the regime boundaries are strongly affected by pipe inclination. Clearly, horizontal flow regime maps must not be used for vertical flow, and vertical flow regime maps must not be used for horizontal flow. The Mandhane map was developed for horizontal lines flowing air and water at near atmospheric pressure. As is seen later, inclinations in the range of 0.1-1.0 degrees can cause substantial regime boundary movement.

In addition, flow regime boundary adjustment has been observed due to fluid pressure, pipe diameter, viscosity and surface tension. The gas density increase caused by high pressure acts to move the slug-mist boundary to lower superficial gas velocities, while increased pipe diameter acts to increase the stratified wavy flow regime at the expense of the slug flow regime. In addition, foamy fluids having a high surface tension have been observed to flow in

the dispersed flow regime even though Mandhane would have predicted superficial liquid velocities too low to cause dispersed flow.



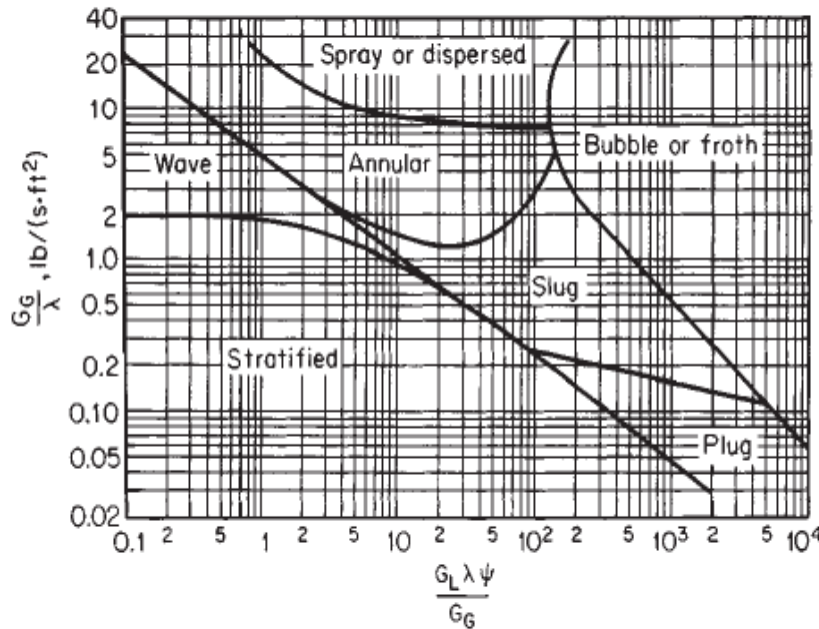
3.4 Baker Flow Regime Map

The Baker Flow Regime map incorporates physical properties not included within the Mandhane chart. In the Baker chart, G_L and G_G are the liquid and gas mass velocities, μ_L^1 is the ratio of liquid viscosity to water viscosity, μ_G^1 is the ratio of gas density to air density, ρ_L^1 is the ratio of liquid density to water density, and σ_L^1 is the ratio of liquid surface tension to water surface tension.

The reference properties are at 20°C (68°F) and atmospheric pressure, water density 1,000 kg/m³ (62.4 lbm/ft³), air density 1.20 kg/m³ (0.075 lbm/ft³), water viscosity 0.001 Pa × s, (1.0 cp) and surface tension 0.073 N/m (0.0050 lbf/ft, 73 dyne/cm). The empirical parameters λ and ψ provide a crude accounting for physical properties.

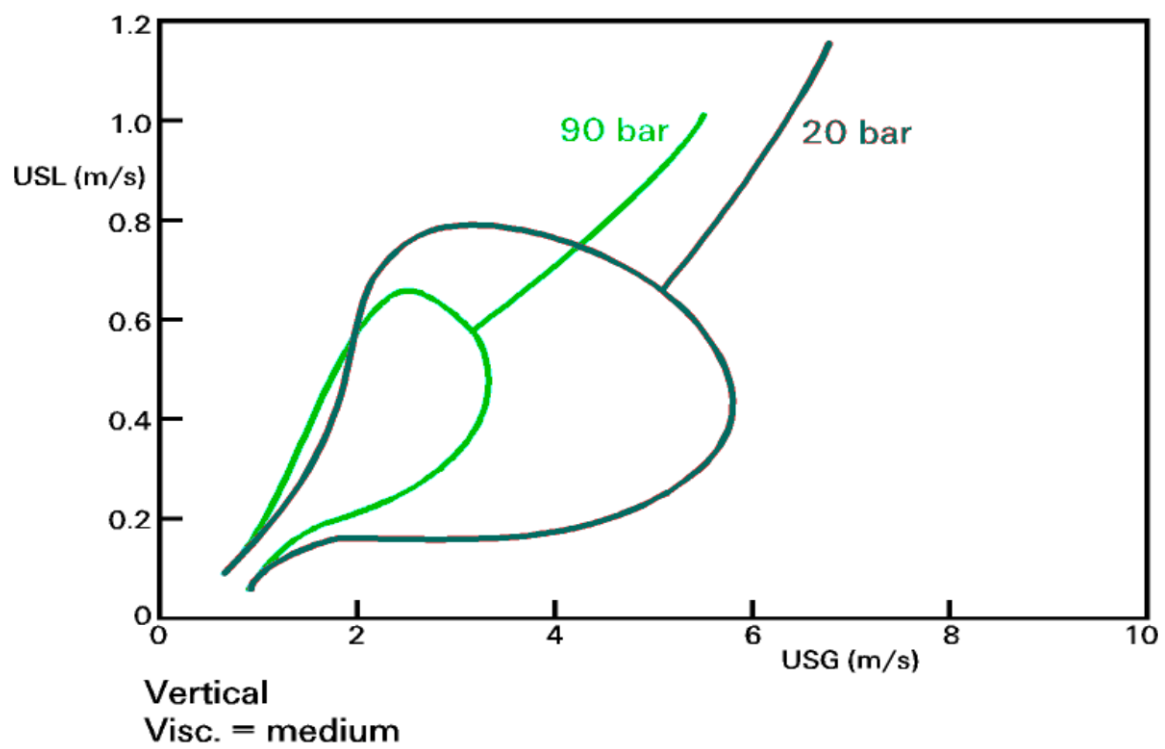
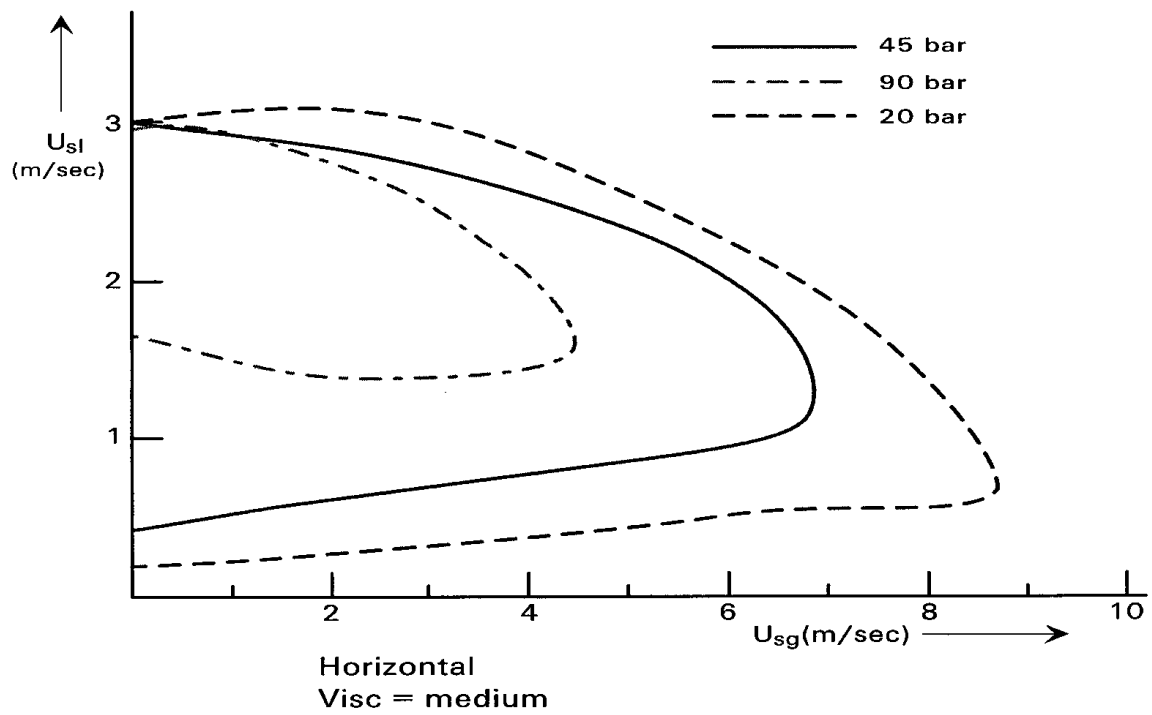
$$\lambda = (\rho'_G \rho'_L)^{1/2}$$

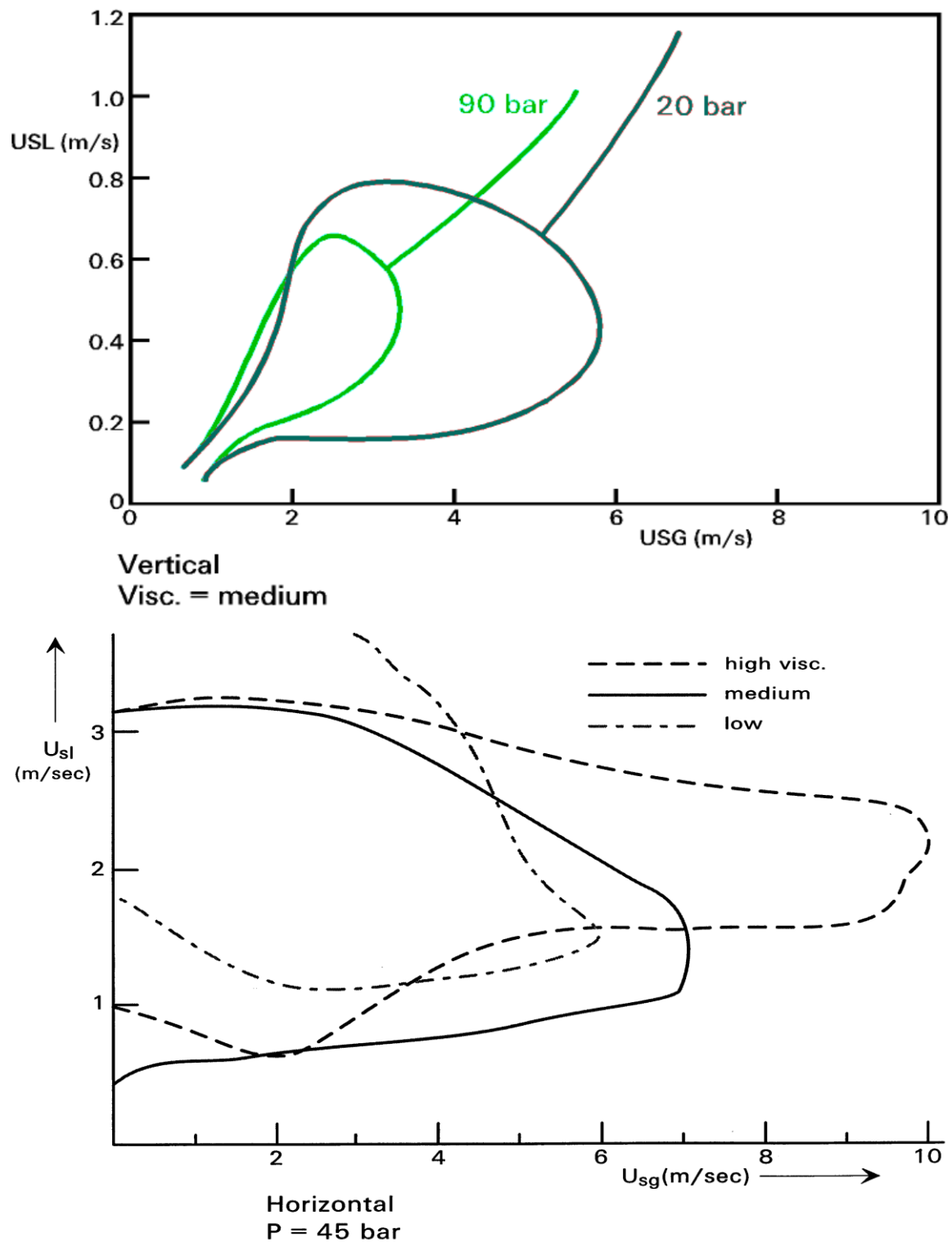
$$\psi = \frac{1}{\sigma'} \left[\frac{\mu'_L}{(\rho'_L)^2} \right]^{1/3} \quad (6-130)$$



Flow-pattern regions in cocurrent liquid/gas flow through horizontal pipes. To convert $\text{lbm}/(\text{ft}^2 \cdot \text{s})$ to $\text{kg}/(\text{m}^2 \cdot \text{s})$, multiply by 4.8824. (From Baker, Oil Gas J., 53[12], 185–190, 192, 195 [1954].)

These two methods can indicate completely differing flow regimes. The engineer now has to make a decision on which method he/she considers more accurate. The simplistic single variable axis with Mandhane would be a concern suggesting Baker to be more realistic. Most early researchers studied multi-phase flow in either horizontal or vertical with low pressure air, water and steam systems. The inherent concern was what effect will different properties have on the flow maps. Some of the effects are illustrated in the following figures. As can be seen, and probably as expected, the effects can be very significant.





Such significant effects raised concerns that flow regime models were likely to be flawed if scaled to high pressure oilfield conditions.

3.5 Taitel Dukler Flow Regime Map

Taitel and Dukler made the first attempt at understanding the underlying the physics of flow transitions. The map uses the Martinelli parameter X , the gas Froude number, Fr and the parameters T and K . The map is composed of three graphs as shown later.

The Froude number (F or Fr), is a dimensionless quantity used to indicate the influence of gravity on fluid motion. It is generally expressed as $Fr = v/(gd)^{0.5}$, in which d is depth of flow or diameter, g is the gravitational acceleration (equal to the specific weight of the water divided by its density, in fluid mechanics), v is the velocity of a small surface (or gravity) wave, and Fr is the Froude number. When Fr is less than 1, small surface waves can move upstream; when Fr is greater than 1, they will be carried downstream; and when $Fr = 1$ (said to be the critical Froude number), the velocity of flow is just equal to the velocity of surface waves.

The Martinelli parameter is

$$X^2 = \frac{(dp/dz)_{f,sl}}{(dp/dz)_{f,sg}}$$

← Pressure gradient if liquid alone flowing

← Pressure gradient if gas alone flowing

and the gas-phase Froude number is

$$F = \sqrt{\frac{\rho_g}{\rho_f - \rho_g} \frac{j_g}{\sqrt{Dg \cos \theta}}}$$

The parameter T is defined as

$$T = \left[\frac{|(dp/dz)_{sf}|}{(\rho_f - \rho_g)g \cos \theta} \right]^{1/2}$$

j_g = is the gas superficial velocity = $G \cdot x / \rho_g$

G = mass velocity $\text{kg/m}^2\text{s}$

x = mass vapour quality – vapour mass fraction

θ = pipe angle

and the parameter K is

$$K = F \left(\frac{D j_f}{\nu_f} \right)^{1/2} = F \text{Re}_{sf}^{1/2}$$

Kinematic viscosity

where the liquid-phase and gas-phase Reynolds number are

$$\text{Re}_f = \frac{G_f D}{\mu_f}$$

and

$$\text{Re}_g = \frac{G_g D}{\mu_g}$$

G is mass flux.

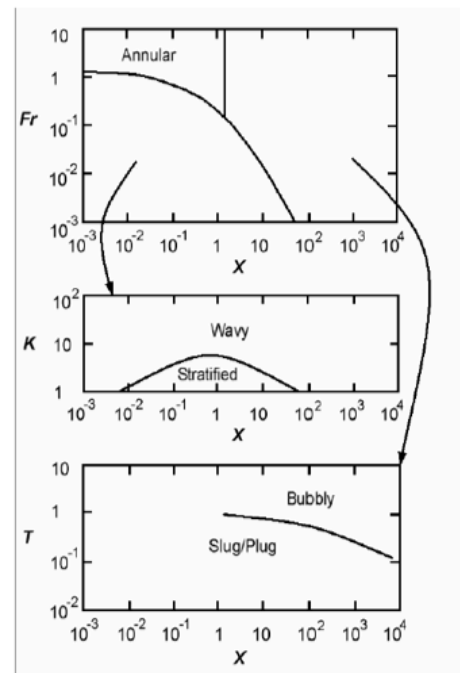
The pressure gradient of the flow for phase k (where k is either f or g) :

$$(dp/dz)_k = -\frac{2f_k G_k^2}{\rho_k D}$$

A recommended procedure of pattern determination using the Taitel and Dukler (1976) map is:

- First determine the Martinelli parameter X and F
- Using the two parameters on the top graph, if the coordinates fall in the annular flow regime, then the flow pattern is annular. Otherwise
- If the coordinates of X and Fr fall in the lower left zone of the top map, then calculate K.
 - Using K and X in the middle graph, the flow regime is identified as either stratified-wavy or as fully stratified.
- If the coordinates Fr and X fall in the right zone on the top graph, then T is calculated.
 - Using T and X in the bottom graph, the flow regime is identified as either bubbly flow or intermittent (plug or slug) flow.

A worked example is included at the end of the lecture notes.



3.6 Govier Vertical Flow Map

The correlation by Govier, et al. (Can. J. Chem. Eng., 35, 58–70 [1957]), may be used for quick estimate of flow pattern.

$$R_v = V_{sg}/V_{sl}$$

Film flow is the same as annular flow.

Froth flow is churn flow

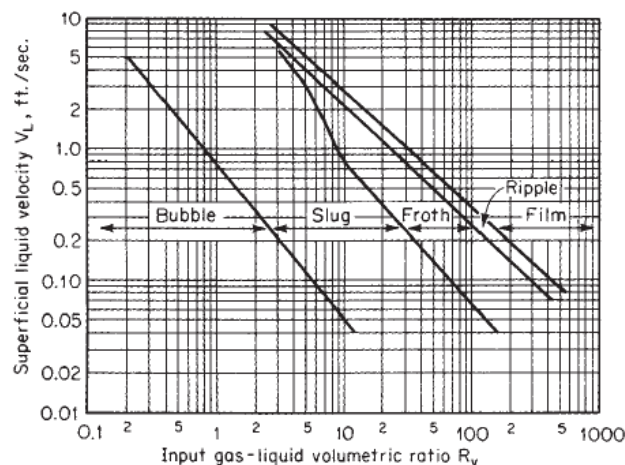


FIG. 6-29 Flow-pattern regions in cocurrent liquid/gas flow in upflow through vertical pipes. To convert ft/s to m/s, multiply by 0.3048. (From Govier, Radford, and Dunn, *Can. J. Chem. Eng.*, 35, 58-70 [1957].)

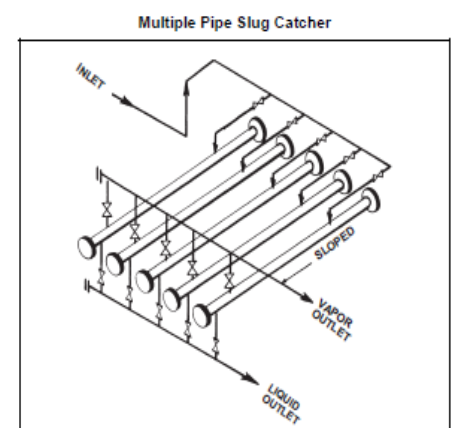
3.7 Wet Gas Pipeline Pigging and Slugcatcher

Pipelines are pigged for several reasons. If liquid water is present (holdup) in the line, it must be removed periodically in order to minimize corrosion. This water accumulates in dips in the pipeline, and these low spots are particularly susceptible to corrosion.

Pipelines are also pigged to improve pressure drop-flow rate performance. Water or hydrocarbon liquids that settle in the dips constitute constrained areas that increase pressure drop and reduce flow rates. Pigging or sphering removes these liquids and improves pipeline efficiency. The sphere will be propelled at the pipeline mixture velocity.

Pigging can also be used as a means of limiting the required slug catcher size. By pigging at frequent intervals, liquid inventory buildup in a pipeline can be reduced, and the maximum slug size can be limited. The required downstream slug catcher size must take into account pigging frequency. The amount of liquid in the pipeline is identified by calculating the liquid hold-up.

Slug catchers are devices at the downstream end or other intermediate points of a pipeline to absorb the fluctuating liquid flow rates. Slug catchers may be either a vessel or constructed of pipe. They provide residence time for vapor-liquid disengagement. Particularly for high pressure service, vessel separators may require very thick walls. In order to avoid thick wall vessels, slug catchers are frequently made of multiple pipes. Lengths of pipe many tens of meters long are

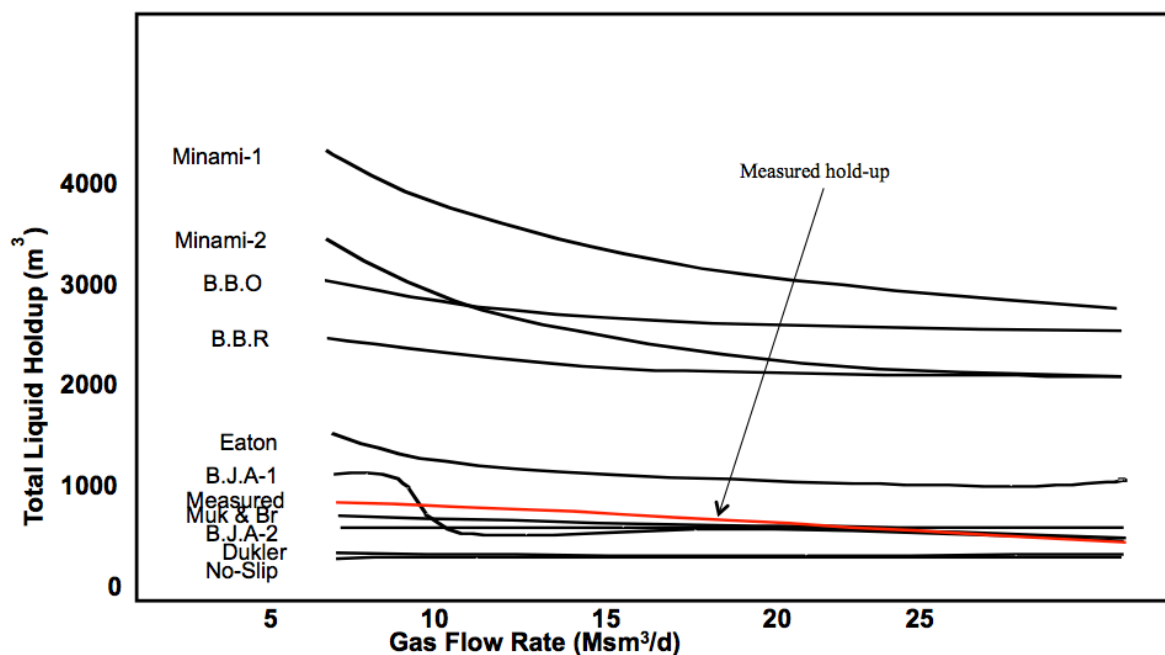


used as slender horizontal separators. The pipe is generally inclined from one to ten degrees and banks

of these slightly inclined pipes are frequently manifolded together. They are often described as finger type slug catchers.



The following figure shows the variation in predicted liquid hold-up for a large wet gas transmission line. As can be seen the correlation predictions can differ by an order of magnitude. Once again the engineer is concerned about method accuracy and flow assurance. Use of an inappropriate correlation could result in a significant under or over prediction of slugcatcher size.



3.8 Eaton Wet Gas Pipeline Hold-Up

A correlation developed by Eaton is widely used for estimating hold-up in pipe lines flowing stratified. The Eaton holdup correlation is shown in the opposite figure. In this figure, the holdup fraction is plotted directly as a function of a dimensionless group. The dimensionless groups are of the form:

$$N_e = \frac{1.84 (N_{Lv})^{0.575} \left(\frac{P_{avg}}{P_b} \right)^{0.05} (N_L)^{0.1}}{N_{gv} (N_d)^{0.0277}}$$

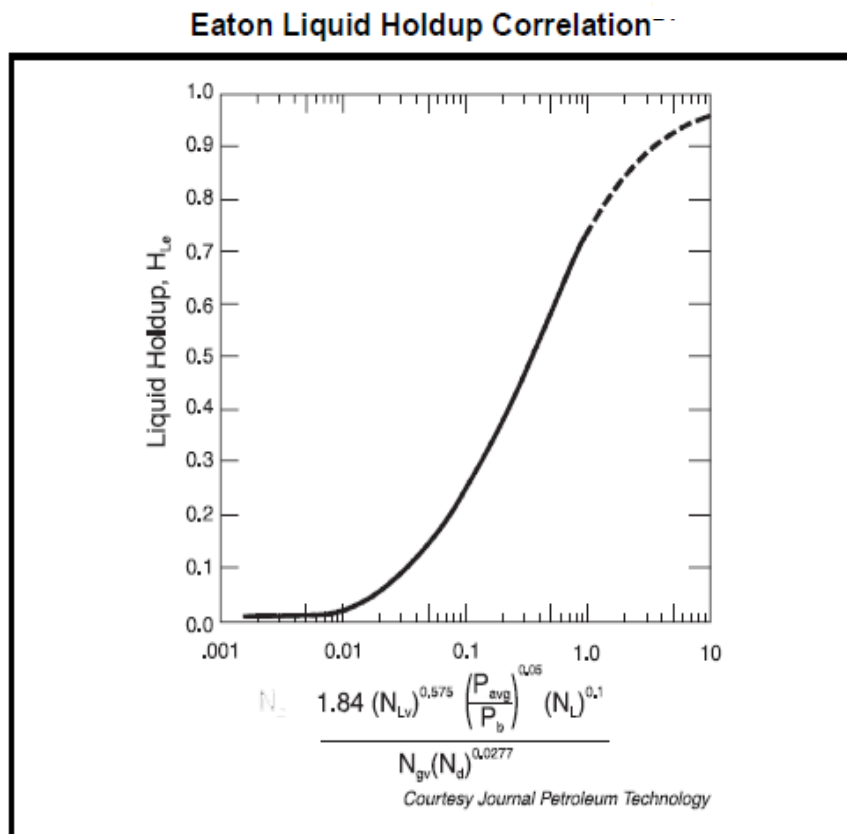
where

$$N_{Lv} = 0.0565 V_{sL} \left(\frac{\rho_L}{\sigma} \right)^{0.25}$$

$$N_{gv} = 0.0565 V_{sg} \left(\frac{\rho_L}{\sigma} \right)^{0.25}$$

$$N_d = 0.00003134 d \left(\frac{\rho_L}{\sigma} \right)^{0.50}$$

$$N_L = 0.001769 \mu_L \left(\frac{1}{\rho_L \sigma^3} \right)^{0.25}$$



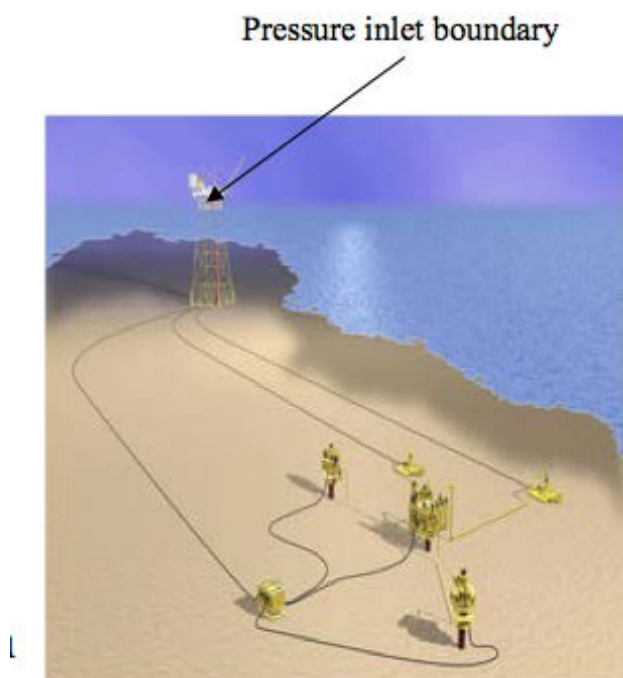
A worked example is provided at the end of the lecture notes. Note units of d are mm.

4 Pressure Drop Prediction

Accurate prediction of pressure drop in a two/multi-phase system is an essential element of flow assurance. Inaccurate analysis of pressure loss will mean that the pipeline is over or undersized which on both counts could lead to loss of production and loss of revenue together with serious operational problems.

The boundary pressure is often an existing pressure on a host installation. Hence, the sub sea architecture must be designed to ensure that the production profile can be delivered to the host pressure inlet requirements.

Using the host pressure as a boundary, calculations are undertaken to quantify required flowing wellhead pressures. These



are compared with flowing wellhead pressures established from well lift curves to ensure there is adequate wellhead pressure to deliver the production fluids.

There are three categories of flow correlations

4.1 Pressure Drop Models Category 1

- No-slip, no flow regime considered
- Mixture density and viscosity based on input average phase densities
- Gas and liquid assumed to be travelling at same velocity in pipe
- Only require a two-phase friction factor correlation
- No distinction made for different flow regimes.

Examples:

- Lockhart Martinelli
- Poettmann and Carpenter
- Baxendell and Thomas
- Fancher and Brown
- Homogeneous Flow Model

4.2 Pressure Drop Models Category 2

- Slip considered, no flow regime considered
- Gas and liquid travel at different velocity in pipe
- Requires a two-phase friction factor correlation and liquid holdup correlation
- No distinction made for different flow regimes.

Example:

- Hagedorn and Brown

4.3 Pressure Drop Models Category 3

- Slip considered, Flow Regime considered
- Requires method to predict flow regime
- Requires specific two-phase friction factor and liquid holdup correlations for each flow regime
- Acceleration pressure gradient depends on flow regime

Example:

- Duns and Ros
- Orkiszewski
- Aziz, Govier and Fogarasi

– Beggs and Brill

The correlations reflect the complexities of multi-phase flow with all work being very empirical in nature.

4.4 Homogeneous Model – Category 1

The simplest approach to the prediction of two-phase flows is to assume that the phases are thoroughly mixed and can be treated as a single-phase flow. This homogeneous model will obviously work best when the phases are inter-dispersed. This type of model utilises a two-phase friction factor calculated from a two-phase Reynold's number.

In determining two-phase Reynolds number the following equations is often used to calculate a two-phase viscosity.

$$\mu_n = \mu_l \cdot \lambda_l + \mu_g \cdot \lambda_g$$

Similarly a no slip two-phase density is utilised.

$$\rho_n = \rho_l \cdot \lambda_l + \rho_g \cdot \lambda_g$$

Recall λ_l and λ_g are the no slip liquid and gas hold up.

The two-phase Reynold's number becomes;

$$Re_n = \rho_n \cdot v_m \cdot d / \mu_n$$

Where v_m is the no slip mixture velocity.

The two – phase friction factor can be determined using the Blasius expression;

$$f_{tp} = 0.079 \cdot Re_n^{-0.25}$$

Two-frictional pressure drop becomes;

$$\Delta P_f = 2 \cdot f_{tp} \cdot \rho_n \cdot v_m^2 / d$$

4.5 Lockhart and Martinelli – Category 1

Rapid approximate predictions of pressure drop for fully developed, incompressible horizontal gas/liquid flow may be made using the method of Lockhart and Martinelli (Chem. Eng. Prog., 45, 39–48 [1949]). First, the pressure drops that would be expected for each of the two phases as if flowing alone in single-phase flow are calculated. The Lockhart-Martinelli parameter X is defined in terms of these pressure drops:

$$X = \left[\frac{(\Delta p/L)_l}{(\Delta p/L)_g} \right]^{1/2}$$

The two-phase pressure drop may then be estimated from either of the single-phase pressure drops, using;

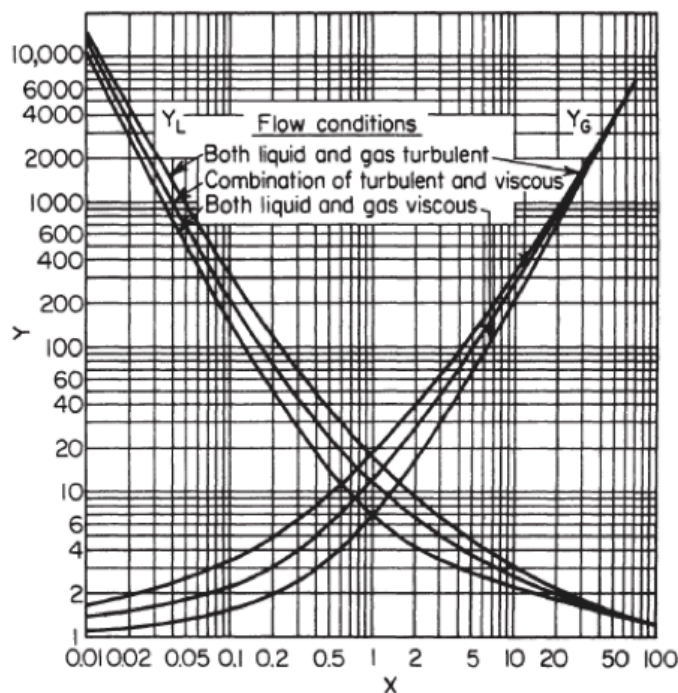
$$\left(\frac{\Delta p}{L}\right)_{TP} = Y_L \left(\frac{\Delta p}{L}\right)_L$$

$$\left(\frac{\Delta p}{L}\right)_{TP} = Y_G \left(\frac{\Delta p}{L}\right)_G$$

where Y_L and Y_G are read from the accompanying figure as functions of X . The curve labels refer to the flow regime (laminar or turbulent) found for each of the phases flowing alone. The common turbulent-turbulent case is approximated well by;

$$Y_L = 1 + \frac{20}{X} + \frac{1}{X^2}$$

Lockhart and Martinelli correlated pressure drop data from pipes 25 mm (1 in) in diameter or less within about 650 percent. In general, the predictions are high for stratified, wavy, and slug flows and low for annular flow. The correlation can be applied to pipe diameters up to about 0.1 m (4 in) with about the same accuracy.



Parameters for pressure drop in liquid/gas flow through horizontal pipes. (Based on Lockhart and Martinelli, Chem. Engr. Prog., 45, 39 [1949].)

4.6 Beggs and Brill – Category 3

This is perhaps the best known and widely used correlation. It was developed from experimental data from a small scale test facility and field data and compared well in many oilfield applications. The correlation presents different values for liquid holdup for each horizontal flow regime. Initially, the liquid holdup is calculated assuming the pipe is horizontal and then the value is corrected for the actual pipe inclination.

The correlation is modified to include transition zone between segregated and intermittent flow regimes.

A two-phase friction factor, independent of flow regime but a function of holdup is calculated.

Flow Regime Determination

The following variables are used to determine which flow regime would exist if the pipe were horizontal. Note, this flow regime is a correlating parameter and does not reflect the actual flow regime.

Froude Number: $N_{fr} = V_m^2 / (g.d)$

Liquid Velocity Number $N_{LV} = v_{sl} \cdot (\rho_l \cdot \sigma)^{0.25}$

$\lambda_l = v_{sl} / v_m$ and $\lambda_g = v_{sg} / v_m$

A worked example is included at the end of this section – for reference.

4.7 Mechanistic Models

The older correlation methods such as Beggs and Brill are now seldom used by Flow Assurance engineers. The mechanistic models developed by SINTEF are widely accepted across the industry as state of the art. The OLGAS model is recognised as an industry leader. OLGAS is based on data from the SINTEF two-phase flow laboratory near Trondheim, Norway. The test facilities were designed to operate at conditions that approximated field conditions.

- The test loop was 800m long and of 8” diameter and operated at pressures between 20 and 90 barg.
- Gas superficial velocities of up to 13 m/s, and liquid superficial velocities of up to 4 m/s were obtained.
- Different hydrocarbon liquids were used (naptha, diesel, and lube oil) in order to simulate the



range of viscosities and surface tensions. Nitrogen was used as the gas.

- Pipeline inclination angles between 1° were studied in addition to flow up or down a hill section ahead of a 50m high vertical riser.
- Over 10,000 experiments were run on this test loop during an eight year period running in both steady state and transient modes.
- OLGAS considers four flow regimes and uses a unique minimum slip criteria to predict flow regime transitions.

OLGAS has become an industry accepted tool for the analysis of steady state multi-phase systems. Also offered by the same group is OLGA – a state of the art transient multi-phase simulator. OLGA and OLGAS also have OVIP (OLGA® Verification and Improvement Project). This is a multi-client joint industry research and development program for validation and improvement of OLGA®. The OVIP program is supported by BP, Chevron, ENI, ExxonMobil, Petrobras, Statoil, Total and Shell. OVIP embodies by far the largest multiphase flow data collection ever assembled. Containing both experimental and field data, including gas/condensate pipelines, lower GOR pipelines, well, and transient data. Transient modelling is a critical aspect of flow assurance and requires highly sophisticated simulators for accurate analysis.

5 Slug Flow

Slug flow is particularly troublesome flow regime. It presents a difficult pattern for process control systems.

When liquid and gas are flowing together in a pipeline, the liquid can form slugs that are divided by gas pockets. The formation of liquid slugs can be caused by a variety of mechanisms:

1. Hydrodynamic effects (surface waves)
2. Terrain effects (dip in pipe layout)
3. Pigging
4. Startup and blow-down
5. Flow rate or pressure changes

Hydrodynamic slugs, in horizontal and near horizontal pipes, are formed by waves growing on the liquid surface for a height sufficient to completely fill the pipe. In vertical pipes the hydrodynamic slugs are associated with Taylor bubbles. The hydrodynamic slugging is difficult to prevent since it occurs over a wide range of flow conditions. The repeating

impacts of hydrodynamic slugging can cause fatigue. It is therefore important to predict the slug volume, velocity, and frequency of such slugs.

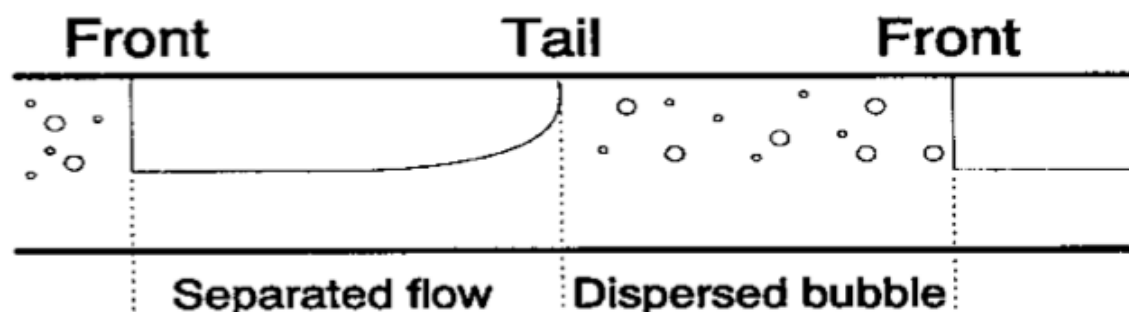
Furthermore, several hydrodynamic slugs can gather together due to terrain effects, creating larger slugs – slug growth.

A particular form of terrain slugging is created where a pipeline slopes downwards and joins a vertical pipeline (riser). In this flow phenomena liquid stalls in the riser, fills the riser and is blown out when the accompanying gas bubble has sufficient pressure to overcome the imposed hydrostatic head. The transient surges of liquid and gas associated with severe slugs can cause major problems for topside equipment – typically separators and compressors. Understanding and controlling the phenomena is essential for safe operation and to prevent shutdowns and associated lost production.

Other types of slugging are initiated by pipeline operations. Pigging of a pipeline causes most of the liquid inventory to be pushed from the line as a liquid slug ahead of the pig. Shut down of a line will drain the liquid that is left in the line down to the low points. During restart the accumulated liquid can exit the pipeline as a slug. Also, increasing or decreasing the flow rate of either gas or liquid leads to a change in liquid holdup. This can come out in the form of a slug, depending on the rate of change of flow rate.

5.1 Hydrodynamic Slugging

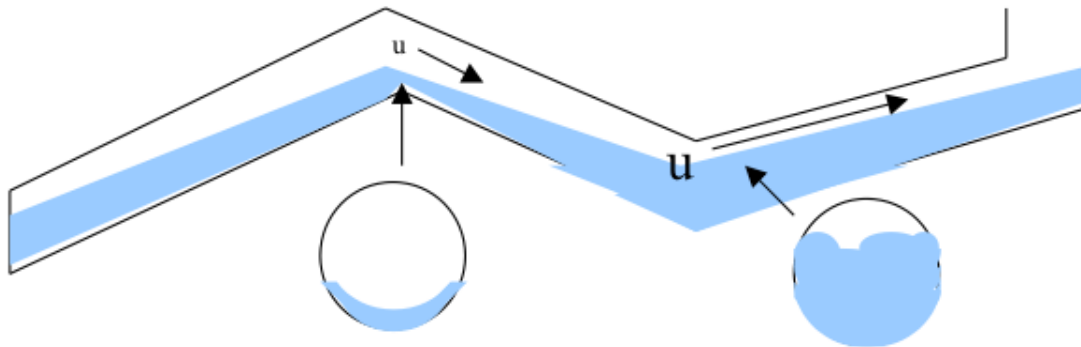
Hydrodynamic slugs occur spontaneously as a result of steady state operation in the slug flow regime (see flow regime map). These slugs occur due to an instability of the fluids gas/liquid interface causing liquid waves to reach the top of the pipe. The slugs are dependent of gas and liquid flowrates, pipe inclination, surface tension, densities and viscosities of the fluid. Hydrodynamic slugs are typically 30 – 100 pipe diameters and length.



5.2 Terrain Induced Slugs

As the pipe angle changes (up), the liquid hold-up increases reducing the area available for gas flow.

This reduced area causes the gas to accelerate creating waves on the film surface which grow to form slugs.



Slugs then grow as they pick up liquid in low spots or join with an adjacent slug.

The empirical Scott, Shoham and Brill correlation takes account of slug growth and can be used as a first approximation.

$$\ln(L_m) = -2.663 + 5.441 [\ln(d)]^{0.5} + 0.059 [\ln(V_m)]$$

where

L_m = mean slug length(ft)

d = pipe diameter (in)

5.3 Severe Slugging

Severe slugging can be seen as a special case of terrain slugging – the geometry here is a downward sloping flowline feeding a vertical riser (the a low spot being the riser base, followed by a vertical section). Low spots in the pipeline topography can lead to pooling and build up of liquid, which will eventually get swept out.

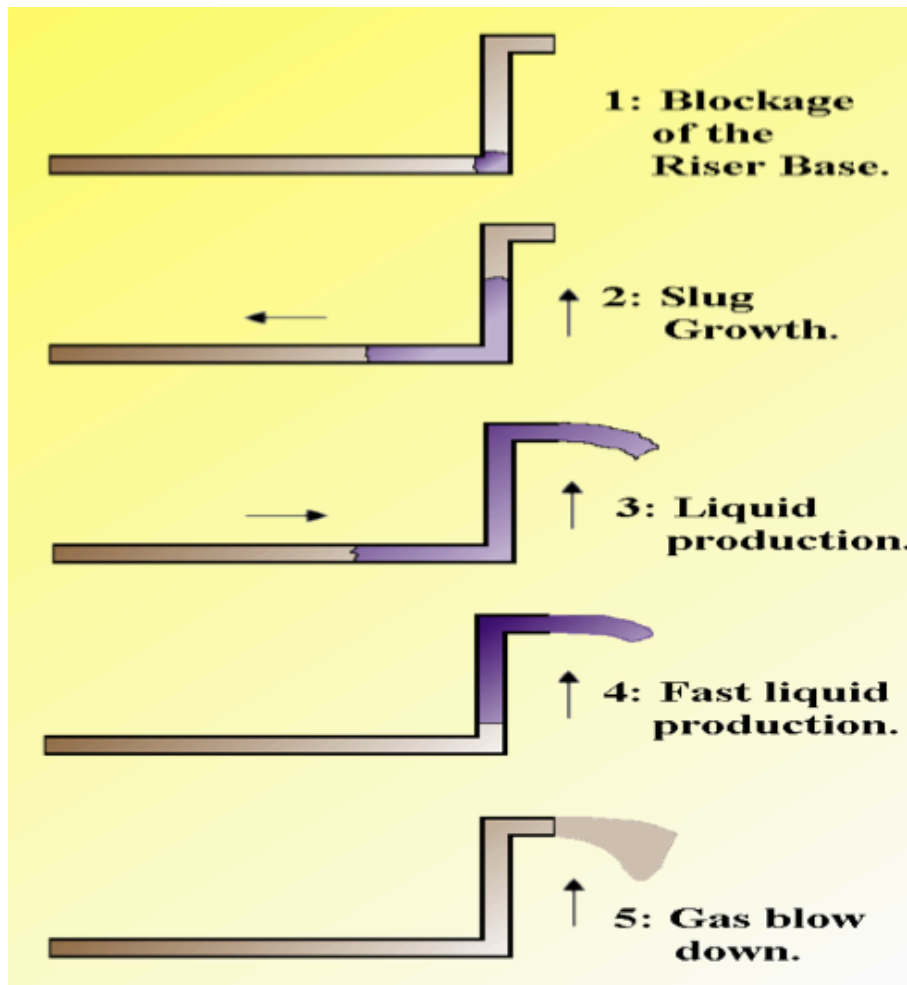
Terrain slugs differ from hydrodynamic slugs by the fact that they are influenced by the terrain undulation and can occur at flow conditions either within or outside the theoretical borders of the intermittent /slug flow regime. The pipe inclination has a strong influence on the transitions between most flow regimes. Therefore with varying undulations, slugs formed on an upward slope may be found to progress and persist further into a downstream section operating in the stratified flow pattern area of the flow map. In general, slugs tend to increase in size at upward slopes and decrease in size or even disappear on downward slopes.

For a terrain slug to be initiated a number of criteria need to be satisfied that are somewhat similar to those for severe slugging. For example, the flow in the upstream section needs to be stratified, whereas the flow in the downstream section needs to be unstable. Although often not very accurate, several correlations exist in the literature to predict the length and frequency of hydrodynamic slugging. To predict the effect of topological variations on the downstream development of these slugs a transient simulation can be made with a dynamic tool utilising a slugtracking option (such as available in OLGA). This option includes a model to insert slugs in dips, based on the local criteria. Thereafter the slug front and tail is tracked when the slug travels through the pipeline. The slug can grow or shrink, can split up in two slugs, or can merge with other slugs. Results obtained with slug tracking should be interpreted with care. This is not exact science, and there is room for improvement (both with respect to the numerical and physical modelling). Experience shows that size and frequency of the slugs that appear at the outlet with the slug tracking simulation are often dependent on user-defined parameters for slug initiation. Terrain slug can growth is very much dependent on the topography and flow regimes encountered during the slug propagation along the pipeline. Severe slugging is an extreme case of terrain induced slugging. The cycle of slugging is described.

Pipeline of downward gradient meets riser base.

1. Liquid blocks the base of the riser.
2. Riser fills with liquid if accompanying gas bubble has insufficient pressure to overcome liquid head. Riser continues to fill.
3. Once riser is full the backpressure stabilises however the accompanying gas bubble pressure continues to rise
- 4 Once the gas bubble pressure is greater than the riser back pressure the riser starts to unpack.
5. Slowly at first then accelerating as the head in the riser reduces. The accompanying gas pocket then discharges into the separator.

Severe slugging produces gas and liquid flow and pressure transients which are very difficult to manage.



Severe slugging can occur if the following 3 conditions are satisfied:

1 Liquid blockage at the riser base can only occur if;

$$(\text{dp}/\text{dt})_{\text{riser}} > (\text{dp}/\text{dt})_{\text{flowline}}$$

Here the latter pressure gradient is the rate of pressure increase in the pipeline upstream of the riser foot due to gas compression in the pipeline, whereas the former pressure gradient is the rate of pressure increase at the riser base due to an increasing liquid head caused by liquid entering the riser.

The ratio of $(\text{dp}/\text{dt})_{\text{flowline}} / (\text{dp}/\text{dt})_{\text{riser}}$ is known as the severe slugging number often labelled Π_{ss} .

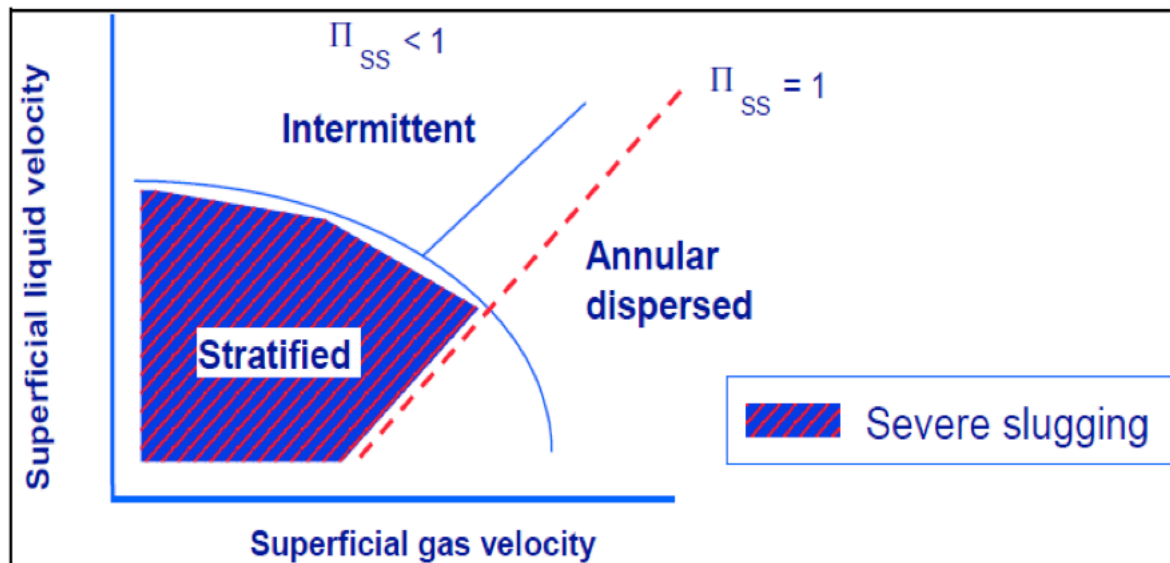
For severe slugging to occur $\Pi_{\text{ss}} < 1$ (the Boe criterion). This criterion shows that a decreasing GLR (gas liquid ratio) will enhance severe slugging. This can occur at late field life, where water break through will decrease the GLR.

2. The pipeline topography has a low point at the riser foot where liquid blockage may occur.

3. The flow line is operated either in the stratified or annular flow pattern but not in slug flow.

The criteria for severe slugging are summarized in the flow pattern map for the pipeline upstream of the riser base, as depicted in the following. Severe slugging will typically occur at turndown production, and/or at late field life when the riser is in the hydrodynamic slugging regime and the gas production is low.

When severe slugging occurs, more energy is consumed due to the regularly varying gas and liquid velocities in the line. Extra static head loss occurs when the riser is fully filled with liquid.



The possibility of severe slugging can be determined using the following procedure;

1. Assume all the liquid in the system acts to fill the riser – calculate $(dp/dt)_{\text{riser}}$. Calculate rate at which riser fills in m/s and convert to a pressure rise.
2. Calculate the no slip hold up and, assume no slip exists in the downward sloping portion of the flowline.
3. Calculate the gas volume in the downward sloping line assuming no slip.
4. Calculate the gas volume flow at average pipe line conditions .
5. Calculate $(dp/dt)_{\text{flowline}}$ the ideal gas law – rate of pressure rise will be linear with volume flow into gas volume in the sloping line section.
6. Evaluate $(dp/dt)_{\text{flowline}} / (dp/dt)_{\text{riser}}$ – less than one severe slugging possible.
7. Calculate superficial velocities and check for stratified flow or annular flow.

6 Pipeline Topology

Note that Pipeline topography can have a large influence on slugging characteristics and also on pressure loss. Accurate seabed bathymetry is very important for multi-phase analysis.

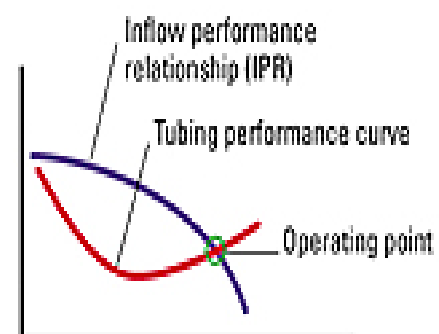
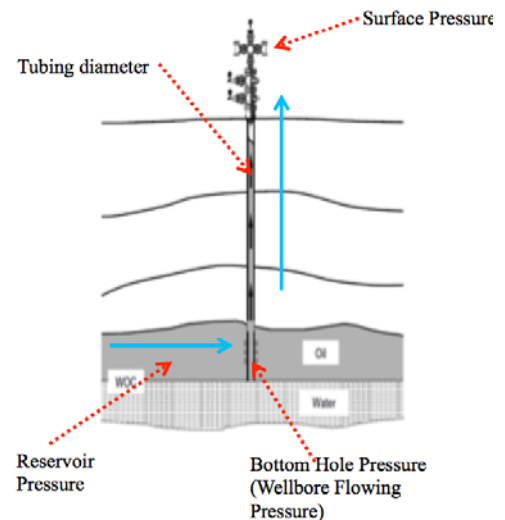
7 Coupling of Hydrocarbon Reservoir and Production Wells

A key use of fluid mechanics is the prediction of flowrates from producing wells. Here the reservoir is coupled to the well. A stable condition is reached where the rate the reservoir can deliver is matched by the resistance to flow in the well. A convenient way to analyse the reservoir and well behaviour is by the application of gradient or performance curves.

The reservoir is characterised by an Inflow Performance Relationship (IPR). This describes the flow through the rock into the well.

The well is characterised by a Tubing Performance Relationship (TPR). This describes the fluid mechanics in the well. This can change from single phase to two-phase as the fluid pressures reduces moving up the well. The fluid mechanics therefore becomes very complex.

The intersection of the IPR and TPR curves determines the rate of stable flow that can be expected from a particular well. The analysis is usually undertaken on specialist software.

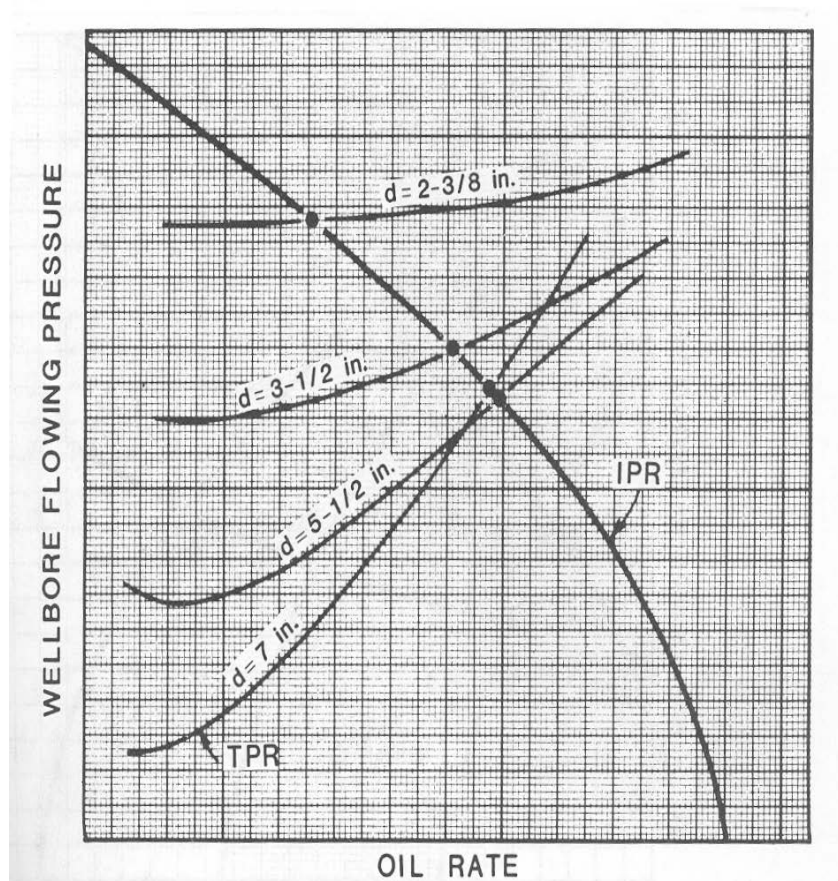


7.1 Tubing Diameter

The effect of changing tubing diameter is shown for a mixture of liquid and vapour in a well.

Increasing diameter increases the production rate until an optimal diameter is reached.

Increasing the diameter further results in a rate reduction. This is a consequence of the two components of pressure drop – friction and elevation. At higher diameters the liquid tends to ‘slump’ within the well resulting in higher mixture densities. Hence elevation/hydrostatic head increase is higher than the drop in friction losses. This effect would not be evident with single phase flow.



7.2 Gas Liquid Ratio

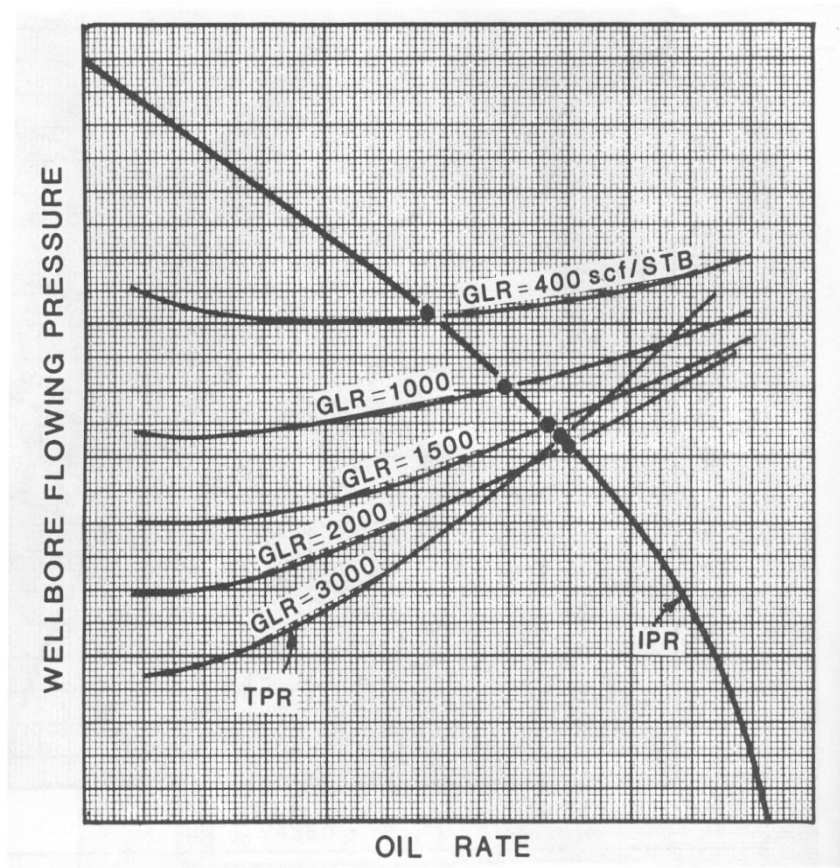
The effect of varying gas liquid ratio is shown for a mixture of liquid and vapour in a well.

The gas liquid ratio could be increased by injecting gas into the well. This is known as gas lift.

As can be seen there is an optimal GLR where further increases result in a drop off in production rate.

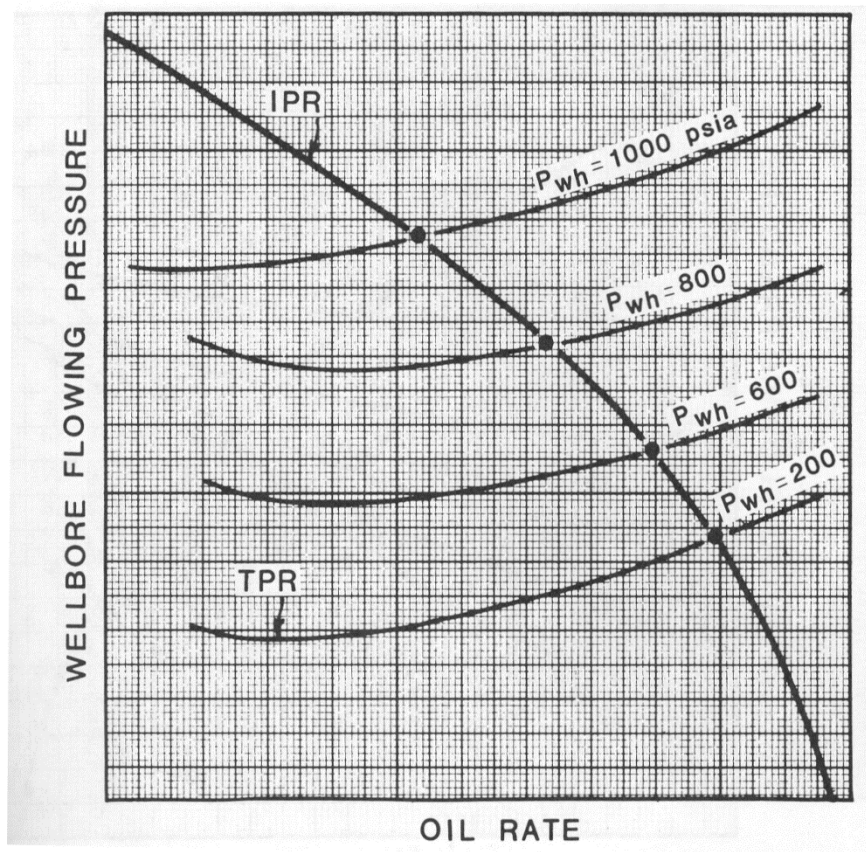
The explanation for the optimal GLR relates to the two pressure loss components; friction and elevation. Whilst injecting gas will increase the fluid velocity thus increasing the frictional losses, the reduction in mixture density and elevation losses is more significant.

However there comes a point where friction losses start to dominate and the opposite holds.



7.3 Surface Pressure

The effect of varying surface pressure is shown for a mixture of liquid and vapour in a well. Reducing surface pressure increases the production rate. As the surface pressure is reduced the back pressure reduces hence flow naturally increases. With two phase flow in the well mixture velocities will increase and mixture density reduce as the surface pressures is dropped. However, as the surface pressure drops gas volumes at surface increase resulting in large compression requirements. There is an optimum surface pressure which balances improved production rates with compression and ancillary costs.



Key Learnings

1. Superficial and mixture velocity
2. Hold – up and phase slippage
3. Flow patterns/regimes – flow regime determination
4. Slug production from pipeline sphering
5. Multi-phase pressure drop methods
6. Slugging flow – form and impact on receiving facility
7. Severe slugging mechanism
8. Severe slug criterion, Π_{ss}
9. Reservoir Well Hydraulics

Taitel Duckler Worked Example

Gas mass flux/velocity, $G_g = 5 \text{ kg/m}^2\text{s}$

Liquid " , $G_L = 495 \text{ kg/m}^2\text{s}$

Gas viscosity , $\mu_g = 1 \times 10^{-5} \text{ Pa.s}$

Liquid viscosity , $\mu_L = 2 \times 10^{-3} \text{ Pa.s}$

Gas density , $\rho_g = 20 \text{ kg/m}^3$

Liquid density , $\rho_L = 817 \text{ kg/m}^3$

Pipe diameter , $D = 0.19 \text{ m}$

Pipe angle , $\theta = 0^\circ$

Pipe friction coefficient = $0.0791 \times (N_{Re})^{-0.25}$ (for Moody friction factor).

Calculate

2.

$$\text{Liquid Reynolds Number, } N_{Re} = \frac{\rho_L v_L D}{\mu_L} = \frac{G_L D}{\mu_L}$$

$$\rho_L v_L = \frac{\text{kg}}{\text{m}^3} \times \frac{\text{m}}{\text{s}} = \frac{\text{kg}}{\text{m}^2\text{s}} = \text{mass flux}$$

$$= \frac{495 \times 0.19}{2 \times 10^{-3}}$$

$$= 47025$$

$$\text{friction coefficient, } C_f = 0.0791 (N_{Re})^{-0.25}$$

$$= 0.0791 \times (47025)^{-0.25}$$

$$= 0.0054$$

$$\text{Liquid pressure gradient } \left(\frac{dP}{dz} \right)_L = 2 C_f \frac{\rho_L v^2}{D} \quad , \quad G^2 = \rho^2 v^2$$

$$\rho v^2 = \frac{G^2}{\rho}$$

3.

$$\begin{aligned}\left(\frac{dP}{dz}\right)_L &= \frac{2 C_f G_L^2}{\rho_L D} \\ &= \frac{2 \times 0.0054 \times 495^2}{817 \times 0.19} \\ &= 17.0 \text{ Pa/m}\end{aligned}$$

$$\text{Gas Reynolds Number, } N_{Re_g} = \frac{G_g D}{\mu_g} = \frac{5 \times 0.19}{1 \times 10^{-5}}$$

$$= 95,000$$

$$\begin{aligned}\text{friction coefficient} &= 0.0791 \times (N_{Re_g})^{-0.25} \\ &= 0.0045\end{aligned}$$

$$\begin{aligned}\text{Gas pressure gradient, } \left(\frac{dP}{dz}\right)_g &= \frac{2 C_f G_g^2}{\rho_g D} \\ &= 0.059 \text{ Pa/m}\end{aligned}$$

$$\begin{aligned}\text{Lockhart Martinelli Parameter, } X &= \left[\frac{\left(\frac{dP}{dz}\right)_L}{\left(\frac{dP}{dz}\right)_g} \right]^{0.5} \\ &= 17.0\end{aligned}$$

4

$$\text{Gas phase Froude number, } F = \frac{\sqrt{\frac{\rho_g}{\rho_L - \rho_g}} \cdot \dot{V}_g}{\sqrt{D \cdot g \cdot \cos \theta}}$$

$$\dot{V}_g, \text{ superficial velocity } (V_{sg}) = \frac{G_g}{\rho_g} = \frac{5}{20} = 0.25 \text{ m/s}$$

$$F = \frac{\sqrt{\frac{20}{817-20}} \times 0.25}{\sqrt{0.19 \times 9.81 \times \cos(0)}}$$

$$F = 0.0289$$

5.

T Parameter, $T = \left[\frac{(dp/dz)_L}{(\rho_L - \rho_g) \cdot g \cdot \cos \theta} \right]^{1/2}$

$$= \left[\frac{17.0}{(817 - 20) \cdot 9.81 \cdot \cos(0)} \right]^{1/2}$$

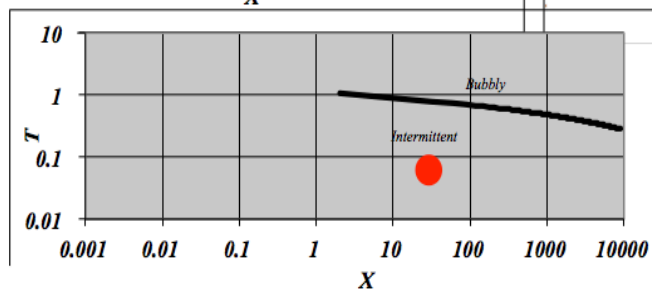
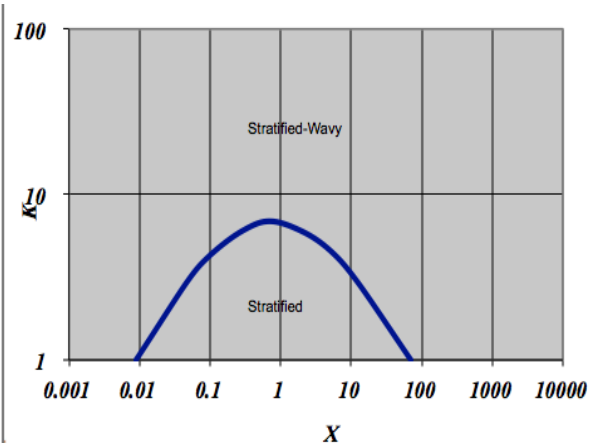
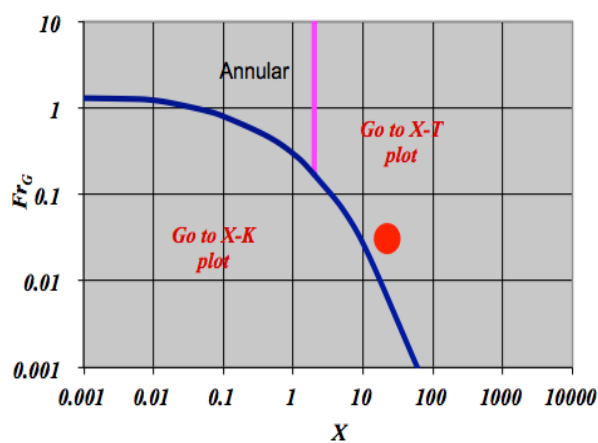
$$= 0.0466$$

K Parameter, $K = F \cdot \left(\frac{D \cdot j_f}{\nu_f} \right)^{1/2}$ $(f=L)$
 $\nu_f = \mu_L / \rho_L$

$$= F \cdot \left(D \cdot \frac{G_L}{\rho_L} \cdot \frac{\rho_L}{\mu_L} \right)^{1/2}$$

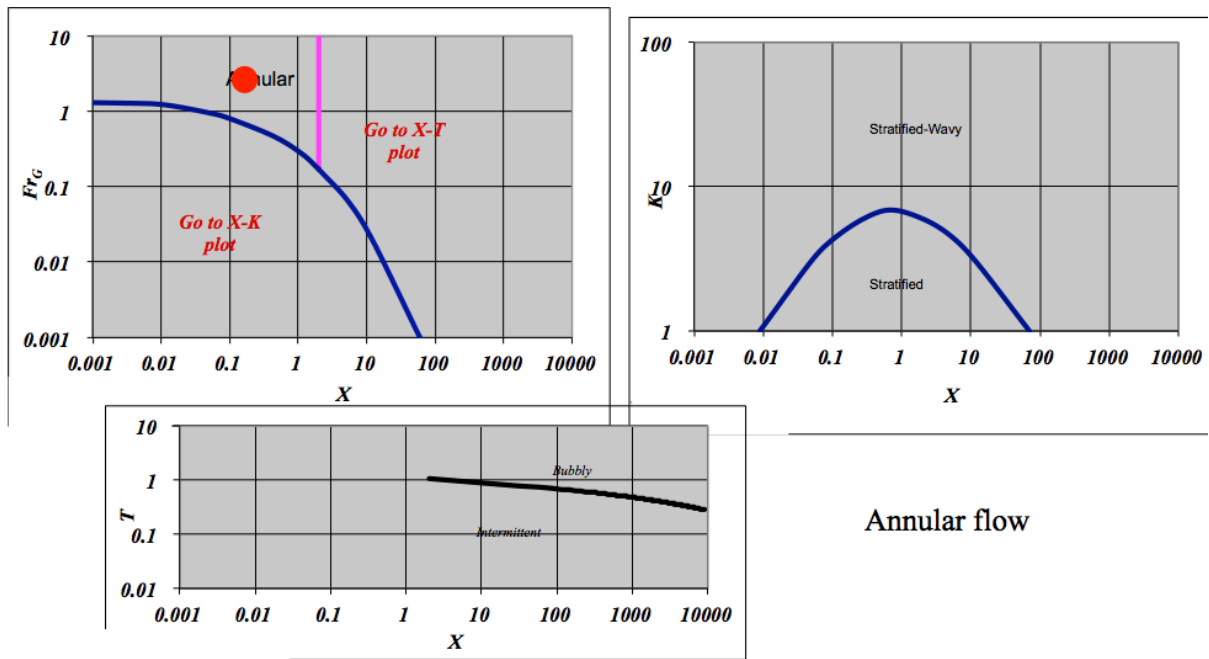
$$= F \cdot (N_{REL})^{1/2}$$

$$= 6.26$$



Intermittent or Slug
Flow

Repeat with $x=0.5$



$$\text{Lockhart Martinelli Parameter, } X = \left[\frac{\left(\frac{dP}{dz} \right)_L}{\left(\frac{dP}{dz} \right)_g} \right]^{0.5}$$

$$= 17.0$$

$$\text{Gas phase Froude number, } F = \frac{\sqrt{\frac{\rho_L}{\rho_L - \rho_g}} \cdot \dot{V}_g}{\sqrt{D \cdot g \cdot \cos \theta}}$$

$$\dot{V}_g, \text{ superficial velocity } (V_{sg}) = \frac{Q_g}{A_g} = \frac{5}{20} = 0.25 \text{ m/s}$$

$$F = \frac{\sqrt{\frac{20}{817-20}} \times 0.25}{\sqrt{0.19 \times 9.81 \times \cos(0)}}$$

$$F = 0.0289$$

$$\begin{aligned}
 \text{T Parameter, } T &= \left[\frac{(dp/dz)_L}{(\rho_L - \rho_g) \cdot g \cdot \cos \theta} \right]^{1/2} \\
 &= \left[\frac{17.0}{(817 - 20) \cdot 9.81 \cdot \cos(0)} \right]^{1/2} \\
 &= 0.0466
 \end{aligned}$$

$$\begin{aligned}
 \text{K Parameter, } K &= F \cdot \left(\frac{D \cdot j_f}{\nu_f} \right)^{1/2} \quad (f=L) \\
 &= F \cdot \left(\frac{D \cdot \frac{G_L}{\rho_L}}{\frac{\mu_L}{\rho_L}} \right)^{1/2} \quad \nu_f = \mu_L / \rho_L \\
 &= F \cdot (N_{RE_L})^{1/2} \\
 &= 6.26
 \end{aligned}$$

Beggs and Brill

The correlation presents different values for liquid holdup for each horizontal flow regime. Initially, the liquid holdup is calculated assuming the pipe is horizontal and then the value is corrected for the actual pipe inclination.

The correlation is modified to include transition zone between segregated and intermittent flow regimes.

A two-phase friction factor, independent of flow regime but a function of holdup is calculated.

Flow Regime Determination

The following variables are used to determine which flow regime would exist if the pipe were horizontal. This flow regime is a correlating parameter and does not reflect the actual flow regime.

Froude Number: $N_{fr} = V_m^2 / (g \cdot d)$

Liquid Velocity Number $N_{LV} = v_{sl} \cdot (\rho_l \cdot \sigma)^{0.25}$

$\lambda_l = v_{sl} / v_m$ and $\lambda_g = v_{sg} / v_m$

Flow regime determination

$$L_i = p_i \lambda_1^{q_i}$$

i	p_i	q_i
1	316	0.302
2	0.0009252	-2.4684
3	0.10	-1.4516
4	0.5	-6.738

L_1, L_2, L_3 and L_4 are calculated and the horizontal flow regime is identified from the following limits;

Segregated

$$\lambda_1 < 0.01 \text{ and } N_{fr} < L_1 \text{ or } \lambda_1 \geq 0.01 \text{ and } N_{fr} < L_2$$

Transition

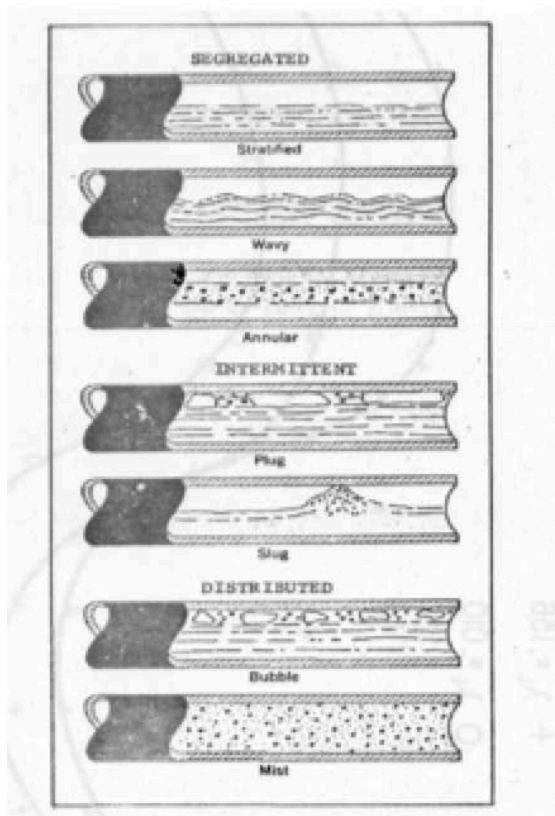
$$\lambda_1 \geq 0.01 \text{ and } L_2 \leq N_{fr} \leq L_3$$

Intermittent

$$0.01 \leq \lambda_1 < 0.4 \text{ and } L_3 < N_{fr} \leq L_1 \text{ or } \lambda_1 \geq 0.4 \text{ and } L_3 < N_{fr} \leq L_4$$

Distributed

$$\lambda_1 < 0.4 \text{ and } N_{fr} \geq L_1 \text{ or } \lambda_1 \geq 0.4 \text{ and } N_{fr} > L_4$$



When the flow falls in the transition regime the liquid holdup must be calculated using both the segregated and intermittent and interpolated with weighting factors.

Two-phase Density:

For a pipe at an angle θ from horizontal the holdup in the inclined pipe is a function of the holdup in the pipe if horizontal:

$$H_{l(\theta)} = H_{l(0)} \psi$$

$$H_{l(0)} = a \lambda_l^b / N_{fr}^c$$

Flow Pattern	a	b	c
Segregated	0.98	0.4846	0.0868
Intermittent	0.845	0.5351	0.0173
Distributed	1.065	0.5824	0.0609

Example: For segregated flow regime

$$H_{l(0)} = 0.98 \lambda_l^{0.4846} / N_{fr}^{0.0868}$$

Friction Pressure Gradient

$$(dp/dZ)_f = f_{tp} \rho_n v_m^2 / (2 g_c d)$$

where

$$\rho_n = \rho_l \lambda_l + \rho_g \lambda_g$$

$$f_{tp} = f_n (f_{tp}/f_n)$$

The non-slip friction factor is determined from the smooth pipe curve on a Moody diagram or from:

$$f_n = 1 / [2 \log(N_{Ren}/(4.5223 \log N_{Ren} - 3.8215))]^2$$

Using the Reynolds number:

$$N_{Ren} = \rho_n v_m d / \mu_n \text{ (volume average no slip viscosity and density)}$$

The ratio of the two-phase to the non-slip friction factor is calculated by

$$f_{tp}/f_n = \exp(s)$$

where

$$s = \ln(2.2 \cdot y - 1.2)$$

and

$$y = \lambda_l / [H_{L(\theta)}]^2$$

$$\psi = 1 + C [\sin(1.8 \theta) - 0.333 \sin^3(1.8 \theta)]$$

For vertical upward flow, $\theta = 90$ and hence:

$$\psi = 1 + 0.3 C$$

Where C is defined as:

$$C = (1 - \lambda_l) \ln (d \lambda_l^e N_{lv}^f N_{fg}^g)$$

Where d, e, f and g are specified for each flow regime as:

Horizontal Flow Pattern	d	e	f	g
Segregated Uphill	0.011	-3.768	3.539	-1.614
Intermittent Uphill	2.96	0.305	-0.4473	0.0978
Distributed Uphill	No Correction $C=0$ $\psi=1$, $h_l \neq f(\theta)$			
All flow patterns Downhill	4.70	-0.3692	0.1244	-0.5056

Example problem by Beggs - Brill Method.

Given the following information for a wet gas pipeline, calculate the pressure gradient due to friction. Flow is horizontal.

$$\mu_o = 1.359 \text{ cP} \quad \mu_g = 0.0233 \text{ cP}$$

$$\sigma_o = 42.608 \text{ dynes/cm}$$

$$\rho_o = 42.45 \text{ lb/ft}^3 \quad \rho_g = 13.66 \text{ lb/ft}^3$$

$$d = 16 \text{ in} = 1.333 \text{ ft}$$

$$P = 2500 \text{ psia}$$

$$T = 60^\circ \text{F}$$

$$V_{SL} = 0.264 \text{ ft/s} \quad V_{sg} = 12.867 \text{ ft/s}$$

1. Determine flow regime.

$$N_{FR} = \frac{V_m^2}{g d} \quad V_m = V_{SL} + V_{sg} = 13.131 \text{ ft/s}$$

$$= \frac{(13.131)^2}{(32.2 \times 1.33)}$$

$$= 4.02$$

$$X_L = \frac{V_{SL}}{V_m} = \frac{0.264}{13.131} = 0.020$$

$$L_1 = 316 \lambda_L^{0.302} = 96.98$$

$$L_2 = 0.0009252 \lambda_L^{-2.4684} = 14.45$$

$$L_3 = 0.10 \lambda_L^{-1.4516} = 29.25$$

$$L_4 = 0.50 \lambda_L^{-6.738} = 25.6$$

$$\left. \begin{array}{l} \lambda_L > 0.01 \quad \text{yes} \\ N_{FR} < L_2 \quad \text{yes} \end{array} \right\} \text{flow is segregated.}$$

2. Determine Liquid Hold up.

$$\begin{aligned} H_{L(o)} &= 0.98 \times \frac{\lambda_L^{0.4846}}{N_{FR}^{0.0868}} \\ &= 0.98 \times \frac{0.15}{1.128} \\ &= 0.13 \end{aligned}$$

3. Determine two-phase friction factor.

$$\begin{aligned} \text{average no slip density, } \rho_n &= 0.02 \times 42.45 + (1-0.02) \times 13.66 \\ &= 14.236 \text{ lb/ft}^3 \end{aligned}$$

$$\begin{aligned} \text{viscosity, } \mu_n &= 0.02 \times 1.4 + (1-0.02) \times 0.0233 \\ &= 0.05 \text{ cp.} \end{aligned}$$

$$\begin{aligned} N_{Ren} &= 1488 \cdot \frac{\rho_n v_m d}{\mu_n} \quad (1488 \text{ unit correction}) \\ &= 1488 \times \frac{14.236 \times 18.131 \times 1.223}{0.05} \\ &= 7.416 \times 10^6 \end{aligned}$$

$$\begin{aligned} f_n &= 1 / \left[2. \log \left(N_{Ren} / (6.5223 \cdot \log N_{Ren} - 3.8215) \right) \right]^2 \\ &= 1 / \left[2. \log (7.416 \times 10^6 / 27.25) \right]^2 \\ &= 0.00846 \end{aligned}$$

$$y = \lambda_L / (H_{L(o)})^2 = \frac{0.02}{0.13^2} = 1.183$$

$$\begin{aligned} S &= \ln (2.2 y - 1.2) \\ &= 0.338 \end{aligned}$$

$$\begin{aligned} f_{LP} &= e^S = 1.402 \\ f_n & \end{aligned}$$

$$f_{LP} = 1.402 \times 0.00846 = 0.01187$$

4. Determine frictional pressure drop

$$\left(\frac{dp}{dz}\right)_f = \frac{f_{tp} \cdot \rho_m \cdot V_m^2}{2 \cdot g \cdot d}$$

$$= \frac{0.01187 \times 14.236 \times (13.131)^2}{2 \times 32.2 \times 1.333}$$

$$= 0.34 \text{ psf/ft}$$

$$= 0.00236 \text{ psi/ft}$$

Hold-Up Eaton MethodEaton Holdup Example.

150 mm I.D. pipeline.

1200 m long.

 $\mu_L = 0.02 \text{ Pa.s.}$ liquid viscosity. $Q_L = 17 \text{ m}^3/\text{hr}$ liquid flow $Q_G = 425 \text{ m}^3/\text{hr}$ vapour flow $\rho_L = 880 \text{ kg/m}^3$ liquid density. $\rho_g = 20.8 \text{ kg/m}^3$ $\sigma = 1.5 \times 10^{-6} \text{ N/m}$

$$\left. \begin{array}{l} P_{\text{inlet}} = 2800 \text{ kPa} \\ P_{\text{out}} = 2400 \text{ kPa} \end{array} \right\} P_{\text{AVE}} = 2600 \text{ kPa}$$

 $P_b = 101.6 \text{ kPa}$

Calculate superficial velocities.

$$\text{Pipe area} = \pi \times \frac{0.15^2}{4} = 0.0177 \text{ m}^2$$

$$v_{sg} = \frac{425}{0.0177 \times 3600} = 6.67 \text{ m/s} \quad \text{gas superficial velocity}$$

$$v_{sl} = \frac{17}{0.0177 \times 3600} = 0.27 \text{ m/s} \quad \text{liquid superficial velocity}$$

Calculate dimensionless groups.

$$N_{Lv} = 0.0565 v_{sl} \left(\frac{\rho_L}{\sigma} \right)^{0.25} = 0.0565 \times 0.27 \times \left(\frac{880}{1.5 \times 10^{-6}} \right)^{0.25} = 2.3$$

$$N_{Gv} = 0.0565 v_{sg} \left(\frac{\rho_L}{\sigma} \right)^{0.25} = 57$$

$$N_d = 0.00003134 \left(\frac{\rho_L}{\sigma} \right)^{0.5} = 116$$

$$N_L = 0.001769 \mu_L \left(\frac{1}{\rho_L \sigma^3} \right)^{0.25} = 0.15$$

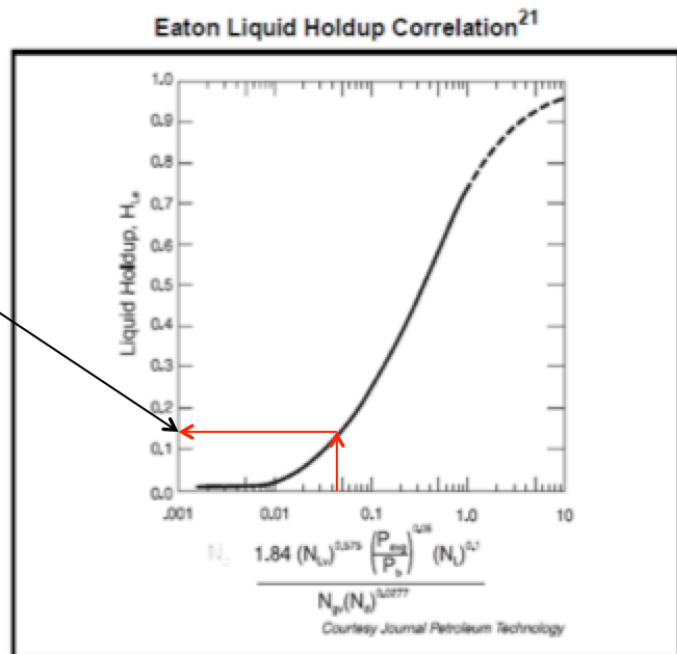
calculate Eaton's abscissa.

$$N_E = \frac{1.84 (N_{Lv})^{0.575} \cdot \left(\frac{P_{avg}}{P_b} \right)^{0.05} \cdot (N_L)^{0.1}}{N_{gv} \cdot (N_d)^{0.0277}}$$

$$= \frac{1.84 \times 2.3^{0.575} \times \left(\frac{2600}{101.6} \right)^{0.05} \times 0.15^{0.1}}{57 \times 116^{0.0277}}$$

$$= 0.0445$$

0.14 or 14% of pipeline will contain liquids.



From chart:

Holdup fraction, $H_L = 0.14$.