This document contains the tutorial questions for the 15-credit Separation Processes 1 course delivered at the University of Aberdeen.

The recommended questions you should solve for each week are marked **Recommended** and are linked below:

- Tutorial 1: Q. 1.3.

- Tutorial 2: Q. 2.5.

- Tutorial 3: Q. 3.7.

- Tutorial 4: Q. 4.9.

- Tutorial 5: Q. 5.14, Q. 5.18, and Q. 5.19.

- Tutorial 6: Q. 6.23.

- Tutorial 7: Q. 7.24 and Q. 7.31.

- Tutorial 8: Q. 8.36 and Q. 8.37.

- Tutorial 9: Q. 9.38.

Any other questions are past exam questions, class examples, or others and are provided for additional practice and should help you to explore all aspects of the course.

Fully worked solutions are available but you should attempt the problems without the solutions, its the only way to find out what you don't know! w

Where marks are given, these are indicative of the *relative* weighting each part of a question might have. Please note, the number of questions in an exam (and exam durations) have changed over the years, so the overall marks for a question may now be different to what is reported here.

Contents

Part I
Questions

1 Week 1: Single-stage evaporators

Q.1.1 **Question 1.1**
A company is investigating the concentration of orange using the process outlined in Fig. 1. The evaporator must be able to process 1000 kg h^{-1} of raw orange juice which has a solids concentration of 7.08% w/w into a 50% w/w solids stream. For this rough design, you may assume that all streams have the same thermodynamic properties as pure water. The raw orange juice is supplied to the evaporator at 20 °C and the evaporator operates at 1 atm(a).
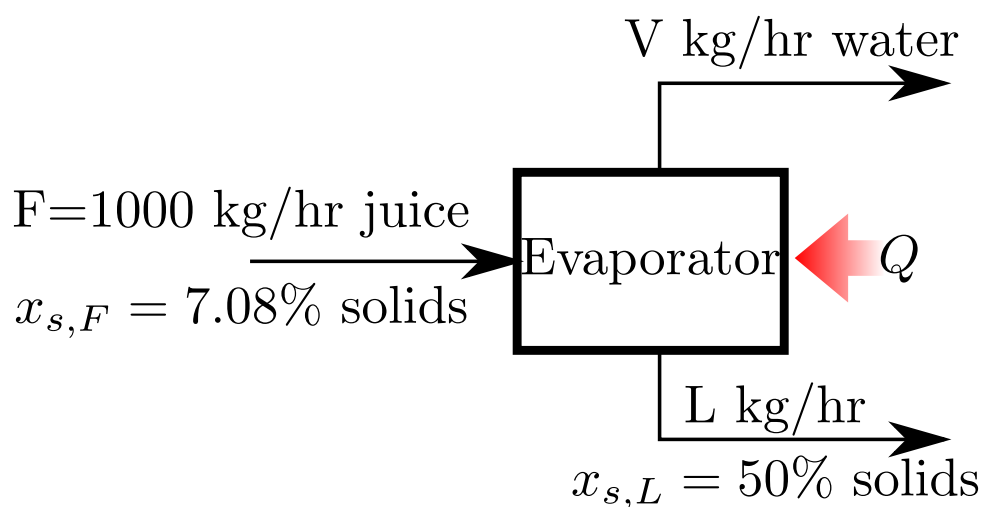

Figure 1: Orange juice concentrator.

a) Determine the flow-rate of all streams.   **[3 marks]**
   Solution:
   Performing a mass balance in solids we have:

$$x_{s,F}\, F = x_{s,L}\, L$$
$$L = \frac{x_{s,F}\, F}{x_{s,L}}$$
$$= 141.6 \text{ kg h}^{-1}$$

From the overall mass balance:

$$F = L + V$$
$$V = F - L$$
$$= 1000 - 141.6$$
$$= 858.4 \text{ kg h}^{-1}$$

b) Determine the duty of the evaporator.   **[6 marks]**
   Solution:
   As the evaporator is operating at 1 atm(a), and all streams are assumed to have the same thermodynamic properties as water, we can state that the outlet temperature is 100 °C.

We must calculate the specific enthalpy for each stream. Fortunately, our steam tables allow us to directly look this up. **Note:** Doing this implicitly assumes that the enthalpy of the liquid phase has no pressure dependence (which is not a bad approximation at these pressure changes) and that there is no superheat of the gas.

Fortunately, the temperatures of the evaporator exactly coincide with the temperatures of the entries in the steam table (so no linear interpolation is required) and we have:

$$h_F = 83.79 \text{ kJ kg}^{-1}$$
$$h_L = 419 \text{ kJ kg}^{-1}$$
$$h_V = 2688 \text{ kJ kg}^{-1}$$

An enthalpy balance over the exchanger gives:

$$h_F F + Q = h_L L + h_V V$$
$$Q = h_L L + h_V V - h_F F$$
$$= 419 \times 141.6 + 2688 \times 858.4 - 83.79 \times 1000$$
$$\approx 2283000 \text{ kJ h}^{-1}$$
$$\approx 634 \text{ kW}$$

c) The evaporator may be operated at 0.725 bar(g). Update your calculations for this operating condition. Comment if this is a more favourable operating point.   **[6 marks]**
Solution:
The mass balance remains the same, only the enthalpy balance changes.

The pressure 0.725 bar(g)=1.738 bar(a) is in between two points on our steam table, so for accuracy we will use linear interpolation on all properties. The boiling temperature at this pressure is **115.5** °C. The enthalpies are

$$h_F = 83.79 \text{ kJ kg}^{-1} \qquad \text{(unchanged)}$$
$$h_L \approx 484.875 \text{ kJ kg}^{-1}$$
$$h_V \approx 2717.75 \text{ kJ kg}^{-1}$$

The energy balance gives

$$Q = h_L L + h_V V - h_F F$$
$$= 484.875 \times 141.6 + 2717.75 \times 858.4 - 83.79 \times 1000$$
$$\approx 2318000 \text{ kJ h}^{-1}$$
$$\approx 643.83 \text{ kW}$$

Operating at higher pressures elevates the boiling point, requiring a higher duty to boil the same amount of water. Lower pressures is more economical.

If the student solves (incorrectly) at 0.725 bar(a):
The pressure **0.725** bar(a) is in between two points on our steam table, so for accuracy we will use linear interpolation on all properties. The boiling temperature at this pressure is **90.9** °C. The enthalpies are

$$h_F = 83.79 \text{ kJ kg}^{-1} \qquad \text{(unchanged)}$$
$$h_L \approx 380.65 \text{ kJ kg}^{-1}$$
$$h_V \approx 2671 \text{ kJ kg}^{-1}$$

The energy balance gives

$$Q = h_L L + h_V V - h_F F$$
$$= 380.65 \times 141.6 + 2671 \times 858.4 - 83.79 \times 1000$$
$$\approx 2263000 \text{ kJ h}^{-1}$$
$$\approx 628.6 \text{ kW}$$

This is a small saving on the operating costs of the evaporator. It might not be significant enought to justify the cost of maintaining a vaccuum depending on how this is achieved. Lower pressures may be more beneficial.

d) Size the evaporator assuming that the process is operated at 1 atm(a). You may assume that 1.5 bar(a) steam is available for heating the evaporator and the overall heat transfer coefficient is 1.4 kW m^{-2} K^{-1}.   **[3 marks]**
Solution:
Steam at 1.5 bar(a) has a boiling point of 111 °C. Using the standard heat transfer expression:

$$Q = U A \left(T_{steam} - T_{evap.} \right)$$
$$A = \frac{Q}{U \left(T_{steam} - T_{evap.} \right)}$$
$$= \frac{634}{1.4 \left(111 - 100 \right)}$$
$$= 41.17 \text{ m}^2$$

e) How much steam is required to operate this evaporator?   **[2 marks]**
Solution:
The latent heat of vaporisation of the steam at 1.5 bar is $h_{fg} = 2243$ kJ kg^{-1}, thus:

$$S = \frac{Q}{h_{fg}} = \frac{634}{2243} = 0.2827 \text{ kg s}^{-1} \approx 1018 \text{ kg h}^{-1}$$

It should be noted that this is higher than the vapour production rate ($V = 858.4$ kg h^{-1}) due to the subcooling of the feed. If the feed was at its boiling point (and the steam and feed have similar thermodynamic properties), then we might expect $S \approx V$.

[Question total: 20 marks]

Q.1.2   Question 1.2
In the production of sea salt, seawater is first evaporated to form a concentrated brine before then being passed to a crystallizer. The seawater feed to the evaporator can be modelled as a 4% w/w NaCl-water mixture and is available at a temperature of 9 °C. The brine must reach a concentration of 37% w/w NaCl before leaving the evaporator, otherwise the crystallizer will be unable to function.

a) An evaporator is needed to process 100 kg of sea salt per hour. Perform mass balances over the evaporator to determine the required flow rates of seawater, brine, and water vapour.   **[5 marks]**
Solution:
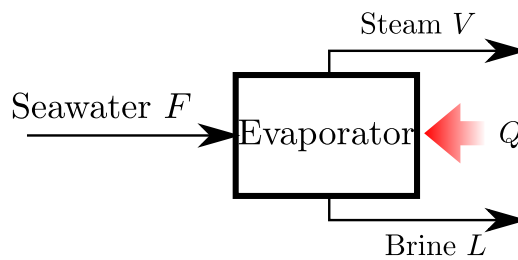
[1/5] We always assume that pure solvent is recovered in the vapour stream for evaporators.✓₁ The mass balance equations for the evaporator are then given by

$$F = L + V \qquad\qquad x_{s,F}\, F = x_{s,L}\, L = \dot{M}_{salt}$$

[1/5] ✓₁We know the amount of salt flowing through the system to be $\dot{M}_{salt} = 100$ kg h⁻¹, so we have

$$F = \frac{\dot{M}_{salt}}{x_{s,F}} = \frac{100}{0.04} = 2500 \text{ kg h}^{-1} \qquad\qquad L = \frac{\dot{M}_{salt}}{x_{s,L}} = \frac{100}{0.37} \approx 270 \text{ kg h}^{-1}$$

[2/5] ✓₂We can then calculate the vapour flow-rate from the total mass balance

$$V = F - L \approx 2230 \text{ kg h}^{-1}$$

[1/5] ✓₁

b) For maximum efficiency, the evaporator is operated at an absolute pressure of 0.48 bar. Calculate the operating temperature of the evaporator, stating any assumptions you make, and estimate the boiling point rise of the solution. You may need the Dühring data provided in Fig. 2. **[4 marks]**
Solution:

From the steam tables, the boiling temperature of water at a pressure of 0.48 bar is
[1/4] $T_{water,sat.} = 80.3°$C.✓₁We assume that the evaporator is well mixed and the outlet concentra-
[1/4] tion reflects the operating conditions.✓₁Using the boiling temperature of water at 0.48 bar (353 K) as the x-coordinate for the Dühring chart (Fig. 2) and reading off from the out-let concentration line (37% w/w) we have a boiling temperature of around **360 K** for the
[1/4] mixture✓₁. This corresponds to a boiling point rise of $360 - 353 = 7$ K.✓₁
[1/4]

c) Why is it convenient to talk in terms of a boiling point rise for mixtures at a certain con-centration? **[2 marks]**
Solution:

The boiling point rise of a solution at a given concentration is largely independent of pres-
[1/2] sure. ✓₁This is apparent from the parallel lines of the Dühring chart.✓₁
[1/2]

d) Calculate the duty of the evaporator using your steam tables. You may neglect the effect of NaCl concentration on the specific enthalpy. Comment on the superheat of the vapour. **[4 marks]**

Solution:

The vapour is superheated as it is pure steam produced from a mixture which is boiling 7°C higher than water would boil at the same pressure. We will ignore this small superheat as
[1/4] it is negligible when compared to the latent heat of vaporisation.✓₁

From the steam tables, water at 9°C has a specific enthalpy of $h_F = 37.8$ kJ kg⁻¹. At a pressure of 0.48 bar, saturated water has an enthalpy of $h_L = 336$ kJ kg⁻¹ and saturated
[2/4] steam has an enthalpy of $h_V = 2644$ kJ kg⁻¹.✓₂

Performing an energy balance over the evaporator, we have

$$h_F\, F + Q = h_L\, L + h_V\, V$$
$$37.8 \times 2500 + Q = 336 \times 270 + 2644 \times 2230$$
$$Q \approx 5.89 \times 10^6 \text{ kJ h}^{-1} \approx 1.64 \text{ MW}$$

[1/4]   ✓
          1

e) Saturated steam is available at 1.5 bar, calculate the steam economy of the evaporator.
                                                                        [3 marks]
Solution:

[1/3]   Using the steam tables, steam at 1.5 bar has a latent heat of vaporisation of $h_{fg} = 2243 \text{ kJ kg}^{-1}$.✓ The
          mass flow-rate of steam required to provide the heat to the evaporator is

$$S = \frac{Q}{h_{fg}} = \frac{5.89 \times 10^6}{2243} = 2626 \text{ kg hr}^{-1} = 0.729 \text{ kg s}^{-1}$$

[1/3]   ✓
          1

The steam economy is the amount of steam raised/evaporated compared to the steam
used/condensed.

$$\frac{V}{S} = \frac{2230}{2626} \approx 0.85$$

[1/3]   ✓
          1

f) Calculate the heat transfer area required if the overall heat transfer coefficient is $U = 1600 \text{ W m}^{-2} \text{ K}^{-1}$.   **[2 marks]**
Solution:

[1/2]   The temperature difference driving the heat transfer is the temperature of the evaporator
          (360 K, 87°C) versus the temperature of the 1.5 bar steam (111°C).✓

Using the standard heat transfer equation the area of the evaporator may be calculated.

$$Q = U\, A\, (T_S - T_{evap})$$
$$1.64 \times 10^6 = 1600\, A\, (111 - 87)$$
$$A = 42 \text{ m}^2$$

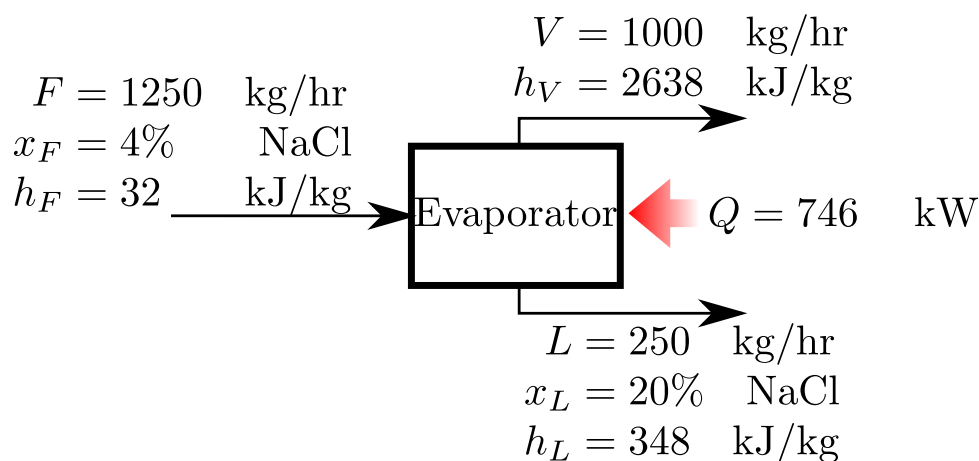**[1/2]**   ✓
          1

[Question total: 20 marks]

Q.1.3   Question 1.3   **Recommended**

An evaporator is required to produce 1000 kg h^{-1} of fresh water from seawater. The sea-
water can be approximated as a 4% w/w solution of NaCl and is available at 7°C. Due to
scale concerns, the sea water is to be released at 20% w/w concentration. The change in
enthalpy due to the addition of NaCl to water is assumed to be constant ($h_{sol} = 65 \text{ kJ kg}^{-1}$
dissolved NaCl) at 0°C). The heat capacity of the solution can be assumed to be the same
as for pure water and Dühring data is given in Fig. 2.

$$V = 1000 \quad \text{kg/hr}$$
$$h_V = 2638 \quad \text{kJ/kg}$$

$$F = 1250 \quad \text{kg/hr}$$
$$x_F = 4\% \qquad \text{NaCl}$$
$$h_F = 32 \qquad \text{kJ/kg}$$

Evaporator $Q = 746 \quad \text{kW}$

$$L = 250 \quad \text{kg/hr}$$
$$x_L = 20\% \quad \text{NaCl}$$
$$h_L = 348 \quad \text{kJ/kg}$$

a) Explain why evaporators are generally operated at a reduced pressure.
   Solution:
   The boiling temperature of a mixture is lowered at reduced pressures. This means that less sensible heat is required to get the feed mixture to its boiling temperature, reducing the evaporator duty required for generating the same amount of vapour.

b) Calculate the missing flow rates *F* and *L* in the process diagram. Remember to state any assumptions that you make.
   Solution:
   We always assume that pure solvent is recovered in the vapour stream for evaporators. The mass balance equations for the evaporator are then given by

   $$F = L + V \qquad\qquad x_{s,F}\, F = x_{s,L}\, L$$
   $$F = L + 1000 \qquad\qquad 0.04\, F = 0.2\, L$$

   Rearranging the NaCL mass balance we have

   $$F = \frac{0.2}{0.04}\, L$$

   Inserting this into the total mass balance, we have

   $$\frac{0.2}{0.04}\, L = L + 1000$$
   $$L = 1000 / \left(\frac{0.2}{0.04} - 1 \right) = 250 \text{ kg h}^{-1}$$

   The feed flow-rate is then easily calculated to be $F = L + V = 1000 + 250 = 1250$ kg h^{-1}.

c) It is decided to operate the evaporator at a pressure of 0.42 bar. Calculate the operating temperature of the evaporator at this pressure and the boiling point rise.
   Solution:
   To use the Dühring plot, we first need to look up the boiling point of the pure solvent (water) at the corresponding pressure. The students need to look at their steam tables for saturated water. At a pressure of 0.42 bar, water has a convenient boiling point of **77.1°C** or 350.1K. This temperature is the horizontal axis of the Dühring plot.

   We often assume for evaporators that the liquid in the evaporator is well-mixed and at equilibrium. Thus the operating concentration of the evaporator is the output concentration (20%).

   Locating this point on the graph, we estimate the boiling temperature of the solution to be approximately ≈ **355**K. The boiling point rise of the solution is then **355 − 350 = 5**K.

Boiling point of water (°F)
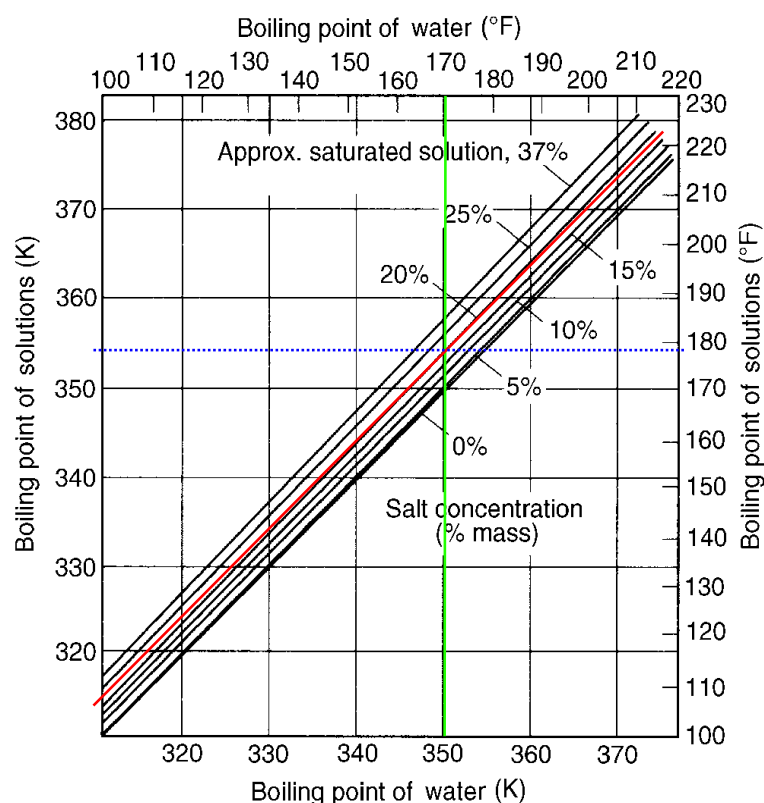
d) Determine the specific enthalpies of every stream. You may ignore the small superheat of the vapour stream (assume it is at it's saturation pressure).
Note: Use reference enthalpies of $h_{\text{water}} = 0$ and $h_{\text{NaCl}} = 0$ at 0°C.
Solution:
The enthalpy of the vapour phase can be directly looked up in the steam tables for saturated steam at 0.42 bar, yielding $h_V = 2638$ kJ kg^{-1}.

The enthalpy of a mixture can be calculated with the following thermodynamic "path." We first mix the two streams at the reference temperature (0°C), and then heat the mixture to the required temperature.

$$h_{mixture}(T) = (1-x)\,h_{\text{water}}\underbrace{(T=0)}_{0} + x\,h_{\text{NaCl}}\underbrace{(T=0)}_{0} + x\,h_{sol.} + \int_0^T C_{p,mixture}(T')\,dT'$$

However, the heat capacity of the mixture is the same as pure water, so we have

$$h_{mixture}(T) = x\,h_{sol.} + \int_0^T C_{p,water}(T')\,dT'$$

There are two options here to calculate the second term. The student can either assume a constant value for the heat capacity $C_{p,water} = 4.18$ kJ kg^{-1} K^{-1}, or they can use the steam tables.

For the feed stream, we have $x_F\,h_{sol.} = 0.04 \times 65 = 2.6$ kJ kg^{-1}. To this we add the enthalpy of water from the steam tables (water at 7°C has an enthalpy of **29.4** kJ kg^{-1}) yielding a feed-stream specific-enthalpy of approximately $h_F = 32$ kJ kg^{-1}.

For the liquid stream, we have $x_L\,h_{sol.} = 0.2 \times 65 = 13$ kJ kg^{-1}. To this we add the enthalpy of water from the steam tables (water at 355 K/82°C has an enthalpy of **335**kJ kg^{-1}) yielding $h_L = 348$ kJ kg^{-1}.

Alternatively, the student might opt to use a constant heat capacity for the water ($C_p = 4.18$ kJ kg^{-1} K^{-1}). This will yield enthalpies of $h_F = 2.6 + 4.18\,(7-0) = 32$ kJ kg^{-1} and

h_L = 13 + 4.18 × (82 − 0) ≈ 356 kJ kg^{-1}. This is not exactly wrong, but it involves a different approximation to using the steam tables. Both methods are an approximation of the mixtures real heat capacity.

e) Calculate the duty, Q, of the evaporator.
Solution:
The duty of the evaporator is given by a heat balance:

$$Q = L\,h_L + V\,h_V - F\,h_F$$
$$= 250 \times 348 + 1000 \times 2638 - 1250 \times 32$$
$$= 2685 \text{ MJ h}^{-1} = 746 \text{ kW}$$

f) Saturated steam is available at 1.6 bar for heating the evaporator. Determine the amount of steam required and the steam economy.
Solution:
From the steam tables, the enthalpy of vapourisation of steam at 1.6 bar is h_{fg} = 2221 kJ kg^{-1}.

The amount of steam required, S, is again given by an energy balance

$$S = \frac{Q}{h_{fg}} = \frac{2685 \times 10^3}{2221} = 1209 \text{ kg hr}^{-1}$$

The steam economy is given by the ratio of vapour raised to steam condensed, which yields 1000/1209 = 0.83.

g) Determine the heat transfer area required for a heat transfer coefficient of U = 1600 W m^{-2} K^{-1}.

Solution:
From the steam tables, the temperature of the steam is 113.3°C. Taking the heat transfer equation and rearranging for the area, we have:

$$A = \frac{Q}{U\,\Delta T} = \frac{746 \times 10^3}{1600\,(113.3 - 82)} = 14.9 \text{ m}^2$$
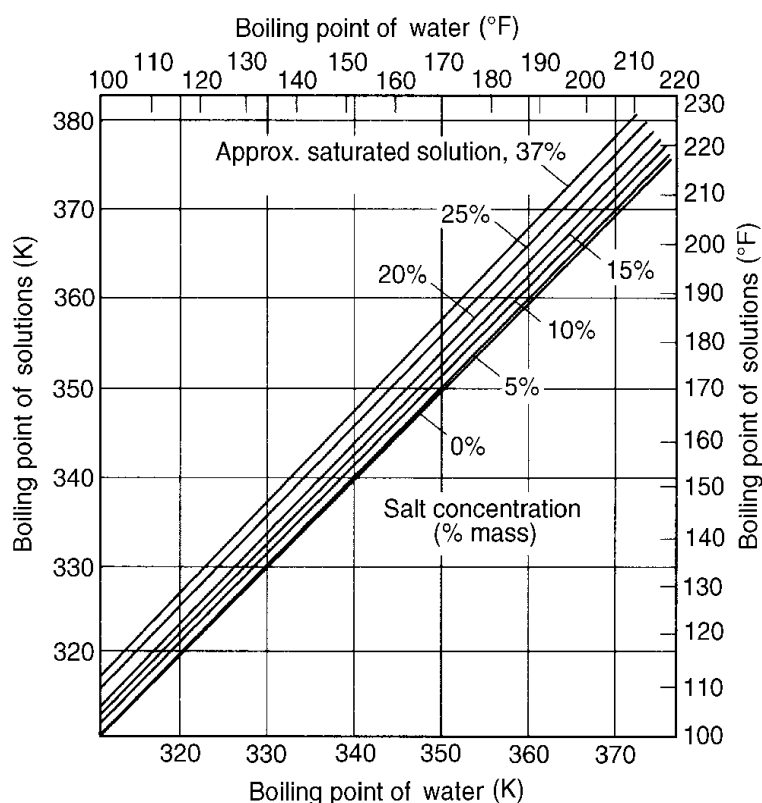
Figure 2: Boiling points for NaCl solutions, taken from Fig. 14.1, pg. 772 C&R Vol. 2.

[Question end]

Q.1.4   Question 1.4

An evaporator is used to concentrate 4536 kg h^{-1} of a 20 % w/w solution of NaOH in water entering at 60 °C (140 °F) to a product of 50% w/w solids. The pressure of the saturated steam used is 172.4 kPa and the pressure in the vapour space of the evaporator is 11.7 kPa. The overall heat-transfer coefficient is 1560 W m^{-2} K^{-1}.
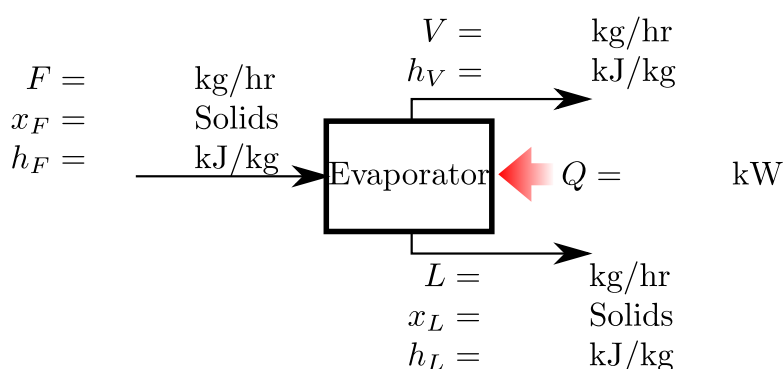
a) Explain why the evaporator is operated at a reduced pressure.   **[2 marks]**
   Solution:

   The boiling temperature of a mixture is lowered at reduced pressures. This means that less sensible heat is required to get the feed mixture to its boiling temperature, reducing the evaporator duty required for generating the same amount of vapour.

b) Determine the stream flow-rates remembering to clearly state any assumptions you make.
                                                                              [3 marks]

Solution:

Assuming that all solids leave in the liquid stream, we can perform a solids balance to yield:

$$L = \frac{x_F\,F}{x_L} = \frac{0.2 \times 4536}{0.5} \approx 1814 \text{ kg h}^{-1}$$

Performing an overall balance for the vapour stream, we have

$$V = F - L = 4536 - 1814 = 2722 \text{ kg h}^{-1}$$

c) Calculate the boiling point rise and operating temperature of the evaporator. Why is it convenient to discuss a boiling point rise rather than an absolute temperature? A Dühring chart for NaOH-water mixtures is available in Fig. 4. **[4 marks]**

Solution:

To determine the boiling point rise (BPR) we need to use the Dühring chart. This requires the boiling temperature of pure water at the same pressure as input. The pressure of the evaporator is 11.7 kPa, which roughly corresponds to a boiling temperature of pure water at 50 °C. From the chart, the boiling point of the 50 mol% mixture at this pressure is then 90 °C. This is the operating temperature of the evaporator and the boiling point rise is $90 - 50 = 40$ °C.

d) Determine the duty of the evaporator. You must clearly state any other approximations you make. An enthalpy-concentration diagram for aqueous NaOH mixtures is available in Fig. 6. **[8 marks]**

Solution:

The enthalpy of the feed stream can be directly read from the diagram, $h_F = h(140\,°\text{F}, 20\%) \approx 200$ kJ kg^{-1}.

The product stream is leaving at the boiling point of the evaporator, so this may be read off $h_L = h(194\,°\text{F}, 50\%) \approx 515$ kJ kg^{-1}. A small approximation is made and the 200 °F line is used.

The vapour produced by the evaporator is superheated steam (as we've **assumed** all solids leave in the liquid stream). We should not ignore this superheat as it is equal to the boiling point rise which is not small at 40 °C. **The steam tables have the same reference as the enthalpy diagram, so we can directly use them to determine the vapour enthalpy.** From the steam tables, saturated steam at 50 °C has an enthalpy of 2591 kJ kg^{-1}. Looking at the properties of steam over the temperature range $50 \to 90$ °C, we can **assume** a roughly constant steam heat capacity of $C_P \approx 1.94$ kJ kg^{-1} K^{-1}. The vapour enthalpy is then $h_V = 2591 + 1.94 \times 40 \approx 2669$ kJ kg^{-1}.

The duty is the difference in the enthalpy of these streams.

$$\begin{aligned} Q &= h_V\,V + h_L\,L - h_F\,F \\ &= 2669 \times 2722 + 515 \times 1814 - 200 \times 4536 \\ &= 7292000 \text{ kJ hr}^{-1} \\ &= 2026 \text{ kW} \end{aligned}$$

e) Determine the evaporator area, mass flow-rate of steam required, and the steam economy. **[3 marks]**

Solution:

Looking in the steam tables, the latent heat of vapourisation of steam at 172.4 kPa is 2216 kJ kg^{-1}. If the duty is $Q = 2026$ kW, this means that the steam required is $2026/2216 = 0.91$ kg s$^{-1} = 3291$ kg hr^{-1}.

The steam economy is the quantity of steam produced over the steam used. Thus, the steam economy is 2722/3291 = 83%.

The saturation temperature of steam at 172.4 kPa is 115 °C. Thus the driving force for heat transfer is ΔT = 115 − 90 = 25 °C. Calculating the heat transfer area required, we have $Q/U\Delta T$ = 2026000/(1560 × 25) = 51.9 m².

<div align="center">

[Question total: 20 marks]

</div>

2   Week 2: Multi-stage evaporators

Q.2.5   Question 2.5   **Recommended**

A triple effect evaporator in a forward feed configuration is to be used to concentrate a 10% w/w NaOH solution entering the evaporator at 37.8°C, up to a concentration of 50% w/w. Steam is available at 58.6 kPa (gauge). The operating pressure of the final stage is 6.76 kPa. The feed rate is 13608 kg h^{-1}. The heat transfer coefficients are U_1 = 6246 W m^{-2} K^{-1}, U_2 = 3407 W m^{-2} K^{-1}, and U_3 = 2271 W m^{-2} K^{-1}. The constraint on the design is that all effects must have the same area. Calculate the surface area per effect, steam consumption and steam efficiency.

Solution:

Steam Conditions

For steam at 58.6 kPa (gauge), the absolute pressure is P_S = 58.6+101.3 = 159.9 kPa≈ 1.6 bar. From the steam tables we obtain a saturation temperature of T_S = 113.3 °C ≈ 386.45 K.

Final Effect Conditions

From the steam tables, the boiling point of pure water at P_3 = 6.75 kPa is 38.4 °C. This is the saturation temperature of the pure water vapour (V_3) leaving the final effect, but it is not the operating temperature, T_3, due to the boiling point rise. From the Dühring chart, the boiling point of a x_3 = 50% NaOH mixture at the boiling temperature of water at P_3 = 6.75 kPa is T_3 = 78°C = 351.15 K (BPR_3 = 39.6 K) (see Fig. 4).

Mass Balances

We now solve for L_3 by using the NaOH mass balance.

$$x_F\, F = x_3\, L_3$$
$$0.1 \times 13608 = 0.5 \times L_3$$
$$L_3 \approx 2722 \text{ kg h}^{-1}$$

We can now solve for the total vapour flow-rate using the total mass balance.

$$V_1 + V_2 + V_3 = F - L_3$$
$$= 13608 - 2722 = 10886 \text{ kg h}^{-1}$$

As a preliminary estimate to start our calculations, we allocate an equal amount of produced vapour to each stage.

$$V_1 \approx V_2 \approx V_3 \approx 10886/3 = 3629 \text{ kg h}^{-1}$$

Using these estimates for the vapour flow-rates, we can use a mass balance around each evaporation stage to calculate the liquid flow-rates.

$$L_1 = F - V_1 = 13608 - 3629 = 9979 \text{ kg h}^{-1}$$
$$L_2 = L_1 - V_2 = 9979 - 3629 = 6350 \text{ kg h}^{-1}$$
$$L_3 = L_2 - V_3 = 6350 - 3629 = 2721 \text{ kg h}^{-1} \qquad \checkmark$$

The final mass balance was just to check that we hadn't made a numerical error. It is out by 1 kg h^{-1} from the previous calculation of L_3 due to a rounding of the vapour flow-rates.

Now we can calculate an estimate for the operating concentration of each evaporator using a NaOH mass balance around each effect.

$$x_1 = x_F \, F / L_1 = 0.1 \times 13608/9979 \approx 0.136$$
$$x_2 = x_1 \, L_1 / L_2 = 0.136 \times 9979/6350 \approx 0.214$$
$$x_3 = x_1 \, L_1 / L_2 = 0.214 \times 6350/2722 \approx 0.499 \qquad \checkmark$$

Again, the final mass balance is performed just to check our calculations, and the slight deviance from the actual value is caused by rounding in the vapour flow-rates.

Boiling Point Rise

With the concentrations estimated we can now calculate the operating temperatures of the effects, but we have one difficulty remaining.

Our Dühring chart requires the boiling temperature of pure water, *at the operating pressure*, to calculate the boiling temperatures of the solutions. But we won't know the operating pressure until we know the boiling point of the solution! We must use some trial and error or another approximation. . .

If we take a look at the Dühring chart, we can see the iso-concentration lines are almost parallel. This indicates that the boiling point *rise* is relatively independent of pressure! This is a common assumption that lets us calculate the BPR, without knowing the operating pressure (this was also assumed in the BPR data used in the in-class sugar example).

The evaporators operate somewhere between the final effect temperature $T_3 = 78$ °C and the steam temperature $T_S = 113.3$ °C. Taking a rough operating temperature of ~ 90 °C, we find the following boiling point rises, $BPR_1(90 \text{ °C}, 13.6\%) \approx 5$ K and $BPR_2(90 \text{ °C}, 21.4\%) \approx 8$ K.

Temperature Drops

Using the formula in the notes, the total temperature drop available for heat transfer is given by

$$\sum_i \Delta T_i = T_{steam} - T_3 - \text{BPR}_1 - \text{BPR}_2$$

$$= 113.3 - 78 - 5 - 8 = 22.3 \text{ °C or K}$$

We can estimate the temperature drops in each stage using the other formula:

$$\Delta T_j \approx \frac{1}{U_j} \frac{\sum_i \Delta T_i}{\sum_i 1/U_i}$$

The term on the right is constant for all calculations.

$$\frac{\sum_i \Delta T_i}{\sum_i 1/U_i} = \frac{22.3}{\frac{1}{6246} + \frac{1}{3407} + \frac{1}{2271}} \approx 24945 \text{ W m}^{-2}$$

Solving for the approximate temperature drops, we have

$$\Delta T_1 \approx \frac{1}{U_1} \frac{\sum_i \Delta T_i}{\sum_i 1/U_i} = \frac{1}{6246} 24945 \approx 4 \, K$$
$$\Delta T_2 \approx \frac{1}{U_2} \frac{\sum_i \Delta T_i}{\sum_i 1/U_i} = \frac{1}{3407} 24945 \approx 7.3 \, K$$
$$\Delta T_3 \approx \frac{1}{U_3} \frac{\sum_i \Delta T_i}{\sum_i 1/U_i} = \frac{1}{2271} 24945 \approx 11 \, K$$

Again, we note that the above values do not include sensible heat effects. In particular, the input stream is probably far from its boiling point and extra heating will be needed in the first effect. We then adjust the temperature drops slightly to increase the heat transferred in the first stage, while preserving the total drop to give

$$\Delta T_1 \approx 5.5\,K \qquad\qquad \Delta T_2 \approx 7\,K \qquad\qquad \Delta T_3 \approx 9.8\,K$$

Operating Temperatures
We can calculate the operating temperatures from the temperature drops, remembering to take into account the boiling point rises.

$$T_1 = T_{steam} - \Delta T_1 = 386.45 - 5.5 = 381\ \text{K}$$
$$T_2 = T_1 - \text{BPR}_1 - \Delta T_2 = 381 - 5 - 7 = 369\ \text{K}$$
$$T_3 = T_2 - \text{BPR}_2 - \Delta T_3 = 369 - 8 - 9.8 = 351.2\ \text{K} \qquad \checkmark$$

Again, the final calculation is only done to check the calculations, as T_3 is already known. We are now ready to solve the energy balances for the system.

Energy Balances
At this point we need to verify the estimated vapour and liquid flow rates. These were used to estimate the concentrations and the temperatures of the evaporators. As such, we "forget" that we have estimates for F, L_1, L_2, V_1, V_2, and V_3 (but we still have L_3 and the total amount of vapour produced).

We can set up the energy and mass balances over each effect to provide three simultaneous equations to solve. For the first effect, we look up the enthalpy of the feed, $h_F = 130$ kJ kg^{-1} (@311 K=100 °F, 0.1 conc.), and liquid stream, $h_{L,1} = 395$ kJ kg^{-1} (@381 K=226 °F, 0.136 conc.), using Fig. 6. The vapour stream is superheated by $BPR_1 = 5$ K from a saturation temperature of 102.8 °C, which yields an enthalpy of $h_{V,1} = 2670 + 2\,BPR_1 = 2680$ kJ kg^{-1} from the steam tables (this includes a rough estimate of the superheat enthalpy, which is a combination of the approximate ≈ 2 kJ kg^{-1} K^{-1} heat capacity of steam and the boiling point rise).

$$h_F\,F + Q = h_{V,1}\,V_1 + h_{L,1}\,L_1 \qquad\qquad \leftarrow (L_1 = F - V_1)$$
$$h_F\,F + Q = h_{V,1}\,V_1 + h_{L,1}\,(F - V_1)$$
$$130 \times 13608 + Q = 2680\,V_1 + 395\,(13608 - V_1)$$

For the second effect, we look up the enthalpy of the liquid stream, $h_{L,2} = 340$ kJ kg^{-1} (@369 K=204 °F, 0.214 conc.). The vapour stream is superheated by $BPR_2 = 8$ K from a saturation temperature of 88 °C, which yields an enthalpy of $h_{V,2} = 2657 + 2\,BPR_2 = 2673$ kJ kg^{-1} from the steam tables (including a rough estimate of the superheat enthalpy). The vapour from the first effect will heat the second effect by yielding its enthalpy of the superheat and its enthalpy of vapourisation. This is given by $h_{V,1,f\rightarrow g} = 2251 + 2\,BPR_1 = 2261$ kJ kg^{-1}.

$$h_{L,1}\,L_1 + h_{V,1,f\rightarrow g}\,V_1 = h_{V,2}\,V_2 + h_{L,2}\,L_2 \qquad\qquad \leftarrow (L_2 = L_1 - V_2)$$
$$h_{L,1}\,L_1 + h_{V,1,f\rightarrow g}\,V_1 = h_{V,2}\,V_2 + h_{L,2}\,(L_1 - V_2) \qquad\qquad \leftarrow (L_1 = F - V_1)$$
$$h_{L,1}\,(F - V_1) + h_{V,1,f\rightarrow g}\,V_1 = h_{V,2}\,V_2 + h_{L,2}\,(F - V_1 - V_2)$$
$$395\,(13608 - V_1) + 2261\,V_1 = 2673\,V_2 + 340\,(13608 - V_1 - V_2)$$

For the final effect, we look up the enthalpy of the liquid stream, $h_{L,3} = 470$ kJ kg^{-1} (@369 K=172 °F, 0.5 conc.). The vapour stream is superheated by $BPR_3 = 39.6$ K from a saturation temperature of 38.4 °C, which yields an enthalpy of $h_{V,3} = 2572 + 2\,BPR_3 = 2651$ kJ kg^{-1} from the steam tables (including a rough estimate of the superheat enthalpy). The vapour from the second

effect will heat the second effect by yielding its enthalpy of the superheat and its enthalpy of vapourisation. This is given by $h_{V,2,f\to g} = 2288 + 2\,BPR_2 = 2304$ kJ kg^{-1}.

$$h_{L,2}\,L_2 + h_{V,2,f\to g}\,V_2 = h_{V,3}\,V_3 + h_{L,3}\,L_3 \qquad\qquad \leftarrow (L_2 = L_1 - V_2)$$
$$h_{L,2}\,(L_1 - V_2) + h_{V,2,f\to g}\,V_2 = h_{V,3}\,V_3 + h_{L,3}\,L_3 \qquad\qquad \leftarrow (L_1 = F - V_1)$$
$$h_{L,2}\,(F - V_1 - V_2) + h_{V,2,f\to g}\,V_2 = h_{V,3}\,V_3 + h_{L,3}\,L_3$$
$$340\,(13608 - V_1 - V_2) + 2304\,V_2 = 2651\,V_3 + 470 \times 2722$$

In summary, our balance equations are

$$130 \times 13608 + Q = 2680\,V_1 + 395\,(13608 - V_1)$$
$$395\,(13608 - V_1) + 2261\,V_1 = 2673\,V_2 + 340\,(13608 - V_1 - V_2)$$
$$340\,(13608 - V_1 - V_2) + 2304\,V_2 = 2651\,V_3 + 470 \times 2722$$
$$V_1 + V_2 + V_3 = 10886$$

We now face a common problem in process engineering. As the systems we design become more and more complex, we have to solve an ever increasing number of simultaneous equations like the four above. Fortunately, the energy and mass balances around many unit operations are sets of **linear** equations, and this is a standard problem to solve.

Solving linear sets of equations

The fastest way to solve these equations is simply substitution! You have already done this, its familiar to you. The notes below show how to solve it using computational tools.

You may perform full **Gaussian elimination** or **matrix inversion** to solve sets of simultaneous equations, and these approaches are particularly important when you have too many variables to work by hand.

First, it is convenient to arrange your equations so all unknowns are present and you know the constants in front of every term:

$$
\begin{array}{rrrrcr}
+1\,Q & -2285\,V_1 & +0\,V_2 & +0\,V_3 & = & +3606120 \\
+0\,Q & +2206\,V_1 & -2333\,V_2 & +0\,V_3 & = & -748440 \\
+0\,Q & -340\,V_1 & +1964\,V_2 & -2651\,V_3 & = & -3347380 \\
+0\,Q & +1\,V_1 & +1\,V_2 & +1\,V_3 & = & +10886
\end{array}
$$

Then it is easy to see that this system may be written as a matrix and two vectors:

$$
\begin{pmatrix}
+1 & -2285 & +0 & +0 \\
+0 & +2206 & -2333 & +0 \\
+0 & -340 & +1964 & -2651 \\
+0 & +1 & +1 & +1
\end{pmatrix}
\cdot
\begin{pmatrix}
Q \\ V_1 \\ V_2 \\ V_3
\end{pmatrix}
=
\begin{pmatrix}
+3606120 \\ -748440 \\ -3347380 \\ +10886
\end{pmatrix}
$$

Now it is in matrix form you can use Gaussian elimination to solve this. Another method is found by multiplying both sides from the left with the inverse of the matrix to yield:

$$
\begin{pmatrix}
Q \\ V_1 \\ V_2 \\ V_3
\end{pmatrix}
=
\begin{pmatrix}
+1 & -2285 & +0 & +0 \\
+0 & +2206 & -2333 & +0 \\
+0 & -340 & +1964 & -2651 \\
+0 & +1 & +1 & +1
\end{pmatrix}^{-1}
\cdot
\begin{pmatrix}
+3606120 \\ -748440 \\ -3347380 \\ +10886
\end{pmatrix}
$$

It is convenient to get a computer to do the matrix inversion and multiply for you. This is demonstrated in the example Matlab code below. In symbolic mathematics packages such as

Mathematica or Maxima, it is less work to just get the package to directly solve the original list of equations. Examples are given below:

Matlab (or Octave) solution (via the matrix/vector operations):

Matrix=[1 -2285 0 0; 0 +2206 -2333 0; 0 -340 +1964 -2651; 0 1 1 1]
Constants = [+3606120; -748440; -3347380; +10886]
Solutions = inv(Matrix)*Constants

Direct Mathematica/Wolfram Alpha solution:

Solve[{130*13608+q==2680*x+395*(13608-x),
395*(13608-x)+2261*x==2673*y+340*(13608-x-y),
340*(13608-x-y)+2304*y==2651*z+470*2722,x+y+z==10886}, {x,y,z,q}]

Click here to run this command on Wolfram Alpha.

Direct Maxima (free Mathematica-like program) solution:

float(solve([130*13608+Q=2680*V1+395*(13608-V1),
395*(13608-V1)+2261*V1=2673*V2+340*(13608-V1-V2),
340*(13608-V1-V2)+2304*V2=2651*V3+470*2722, V1+V2+V3=10886], [Q,V1,V2,V3]));

Back to the evaporator design

The set of linear simultaneous equations (once solved) yields the following results:

$$Q = 11830 \text{ MJ h}^{-1} \approx 3.29 \text{ MW}$$
$$V_1 = 3600$$
$$V_2 = 3725$$
$$V_3 = 3560$$

Comparing these vapour flow-rates to the initial assumption of $V_1 \approx V_2 \approx V_3 \approx 3629 \text{ kg h}^{-1}$, it is clear that the initial guess was very accurate in this case, and we will not need to iterate the calculations.

Evaporator Area

Finally, we need to calculate the required area for heat transfer in each effect and check that these areas are close to one another. The amount of heat transferred in each stage is given by

$$Q_1 = Q = 3.29 \text{ MW}$$
$$Q_2 = h_{V,1,f\rightarrow g} \, V_1 = 2261 \times 3725 = 8422 \text{ MJ h}^{-1} = 2.34 \text{ MW}$$
$$Q_3 = h_{V,2,f\rightarrow g} \, V_2 = 2304 \times 3560 = 8202 \text{ MJ h}^{-1} = 2.28 \text{ MW}$$

The heat transfer areas are then

$$A_1 = \frac{Q_1}{U_1 \, \Delta T_1} = \frac{3.29 \times 10^6}{6246 \times 5.5} = 95.77 \text{ m}^2$$
$$A_2 = \frac{Q_2}{U_2 \, \Delta T_2} = \frac{2.34 \times 10^6}{3407 \times 7} = 98.12 \text{ m}^2$$
$$A_3 = \frac{Q_3}{U_3 \, \Delta T_3} = \frac{2.28 \times 10^6}{2271 \times 9.8} = 102.45 \text{ m}^2$$

The mean area is $A_m = (A_1 + A_2 + A_3)/3 = 98.78 \text{ m}^2$ and none of the above areas differ from this amount by more than 10%, so we can consider our design complete!

Steam Consumption/Economy

The heat transferred in the first effect is supplied by steam. Using our steam tables we can calculate the latent heat of vapourisation for 1.6 bar steam to be $h_{S,f\to g}$ = 2221 kJ kg^{-1}. Therefore, the amount of steam consumed, S, is given by

$$S = \frac{Q_1}{h_{S,f\to g}} = \frac{11830000}{2221} = 5326 \text{ kg h}^{-1}$$

The total amount of vapour produced is $V_1 + V_2 + V_3 = 10886$ kg h^{-1}, yielding a steam efficiency of $10886/5326 = 2.04$.
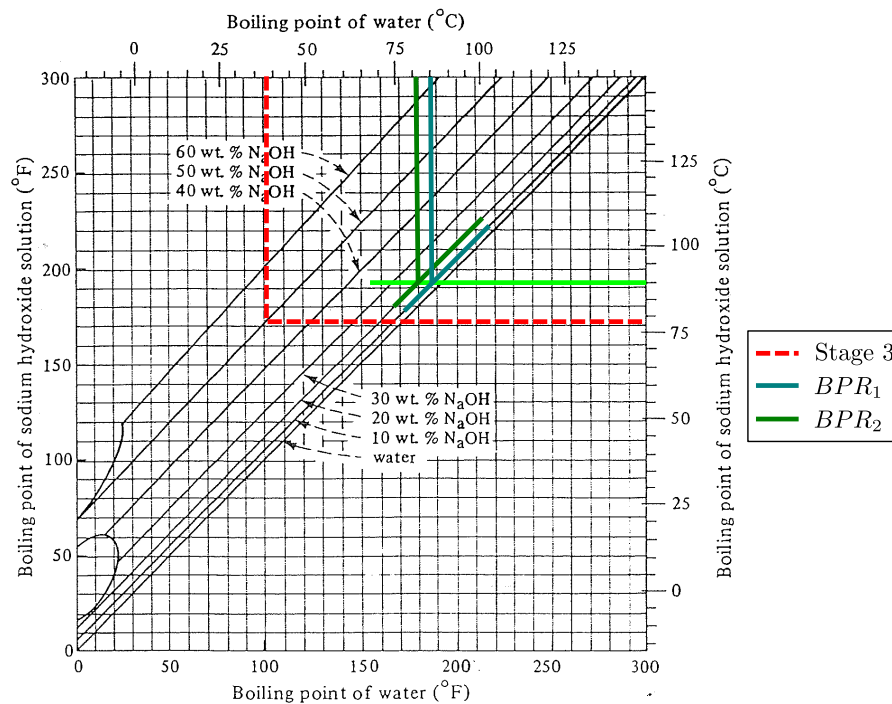

FIGURE 8.4-2. *Dühring lines for aqueous solutions of sodium hydroxide.*

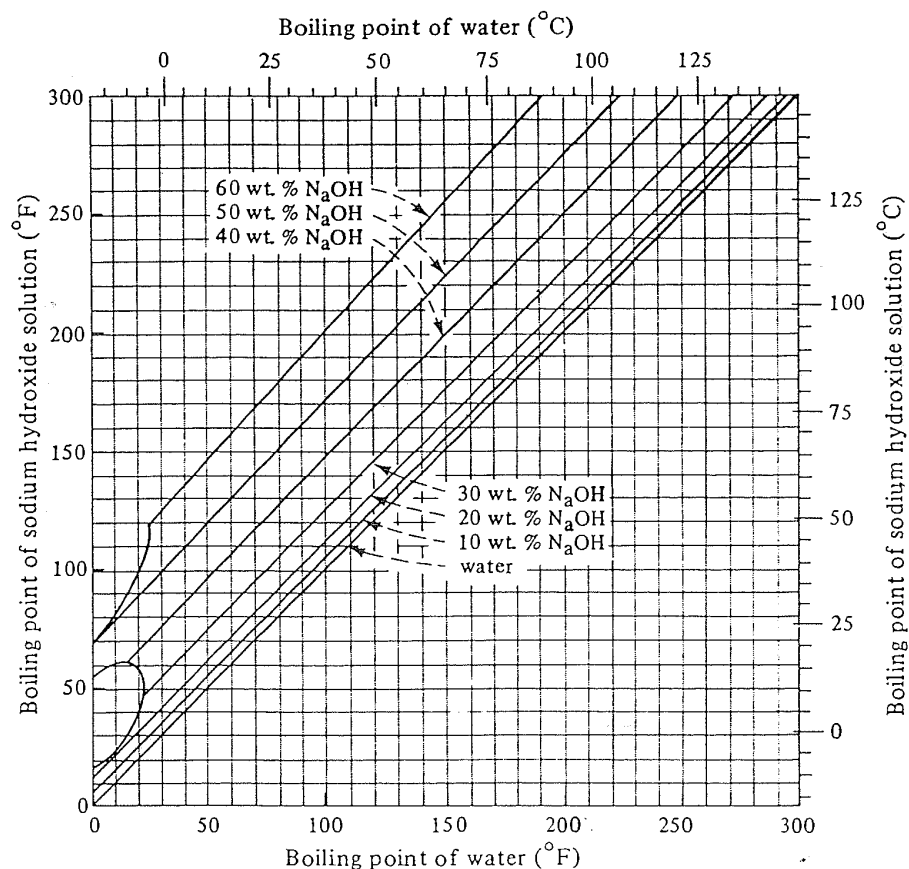Figure 3: Calculations for the boiling points of each stage.

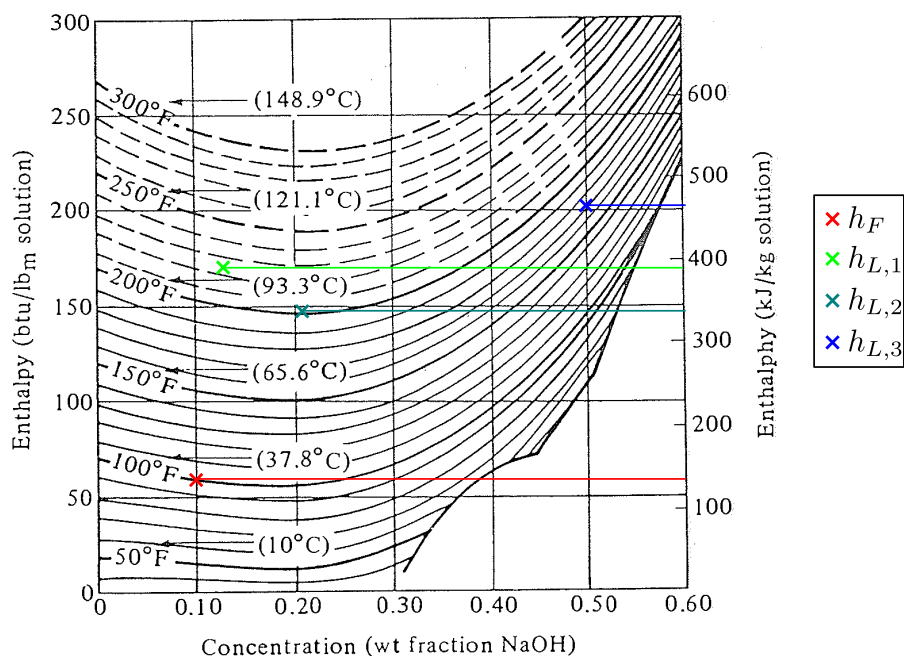Figure 4: Boiling points for NaOH solutions for use in Q. 2.5.

FIGURE 8.4-3. *Enthalpy–concentration chart for the system NaOH–water.* [*Reference state liquid water at 0°C (273 K) or 32°F.*] [*From W. L. McCabe, Trans. A.I.Ch.E.,* **31,** *129 (1935). With permission.*]

Figure 5: Calculations for the enthalpies of the NaOH solutions in the system.
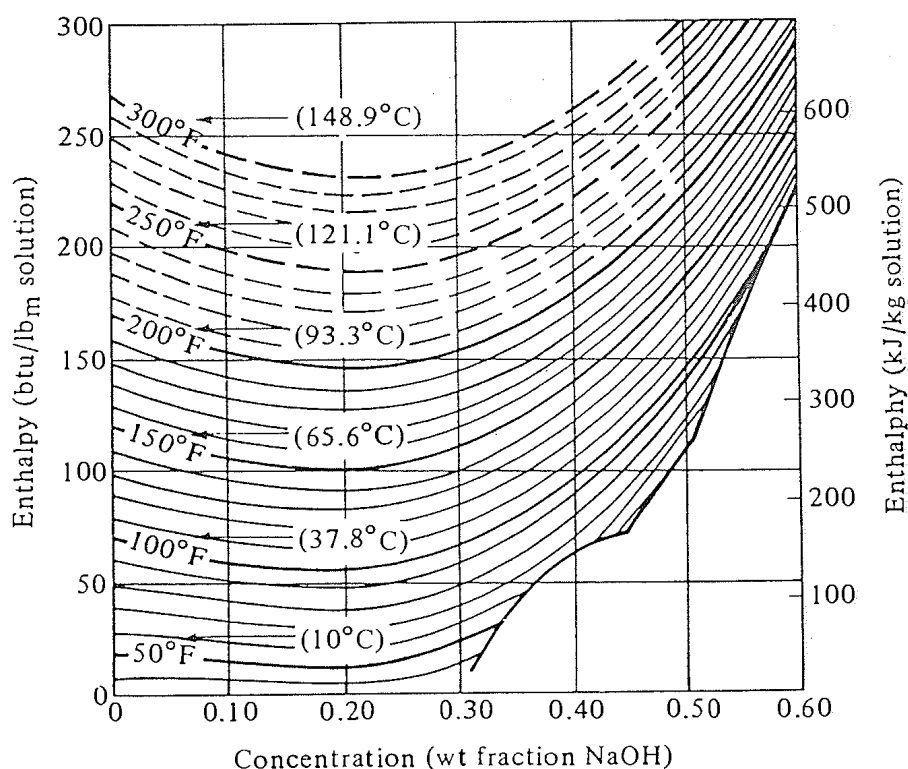
Figure 6: Enthalpies of NaOH solutions for use in Q. 2.5.

[Question end]

3 Week 3: Single-stage absorption

Q.3.6 Question 3.6

What is the concentration of O_2 in 25°C water under atmospheric conditions?
Note: Henry's constant for O_2 in water at 25°C is 4.259×10^4 atm.
Solution:
The concentration of O_2 in air is $\approx$ **21%** thus the partial pressure of O_2 is 0.21 atm.

$$p_{O_2} = \mathcal{H}_A\, x_{O_2}$$
$$x_{O_2} = p_{O_2}/\mathcal{H}_A = 0.21/4.259 \times 10^4 = 4.80 \times 10^{-6} \text{ mol } \%$$

[Question end]

Q.3.7 Question 3.7 **Recommended**

A gas mixture at 2.026×10^5 Pa total pressure containing air and SO_2 is contacted in a single-stage equilibrium mixer with pure water at 293 K. The partial pressure of SO_2 in the original gas is 1.52×10^4 Pa. The inlet gas is flowing at a total of 5.70 kmol s^{-1} and the inlet water is flowing at a total of 2.20 kmol s^{-1}. The exit gas and liquid leaving are in equilibrium.

a) Derive the following balance equation from a mass balance over a single stage contactor. Remember to state your assumptions and show your workings. A small diagram with labelled streams would also be appreciated.

$$L' \frac{x_{SO_2,0}}{1 - x_{SO_2,0}} + V' \frac{y_{SO_2,2}}{1 - y_{SO_2,2}} = L' \frac{x_{SO_2,1}}{1 - x_{SO_2,1}} + V' \frac{y_{SO_2,1}}{1 - y_{SO_2,1}}$$

Solution:
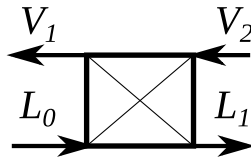
Taking a mass balance in SO_2, we have

$$L_0\, x_{SO_2,0} + V_2\, y_{SO_2,2} = L_1\, x_{SO_2,1} + V_1\, y_{SO_2,1} = M\, x_{SO_2 M} \tag{1}$$

We now must make two assumptions. We assume that air is insoluble in water and that no water evaporates during the contact. This implies that the vapour streams (SO_2-air) and liquid streams (SO_2-water) are binary mixtures. The concentrations of air and water are then given by $y_{air} = 1 - y_{SO_2}$ and $x_{water} = 1 - x_{SO_2}$ respectively.

The amount of air flowing through the system is written as

$$V' = V_1\, y_{air,1} = V_1\left(1 - y_{SO_2,1}\right)$$

Rearranging for V_1 and we have $V_1 = V'/\left(1 - y_{SO_2,1}\right)$. We can do something similar for V_2, L_0, and L_1 to yield

$$V_1 = \frac{V'}{1 - y_{SO_2,1}} \qquad\qquad V_2 = \frac{V'}{1 - y_{SO_2,2}}$$

$$L_0 = \frac{L'}{1 - x_{SO_2,0}} \qquad\qquad L_1 = \frac{L'}{1 - x_{SO_2,1}}$$

We substitute these into Eq. (1) to give the final answer

$$L'\frac{x_{SO_2,0}}{1 - x_{SO_2,0}} + V'\frac{y_{SO_2,2}}{1 - y_{SO_2,2}} = L'\frac{x_{SO_2,1}}{1 - x_{SO_2,1}} + V'\frac{y_{SO_2,1}}{1 - y_{SO_2,1}}$$

b) Calculate the flow-rates and compositions of the outlet phases using the data in Fig. 7.
   Answer: $x_{SO_2,1} = 0.00495$, $y_{SO_2,1} = 0.0733$, $L_1 = 2.21$ kmol s^{-1}, $V_1 = 5.69$ kmol s^{-1}
   Solution:
   We need to use our balance equation for absorbers.

$$L'\frac{x_{SO_2,0}}{1 - x_{SO_2,0}} + V'\frac{y_{SO_2,2}}{1 - y_{SO_2,2}} = L'\frac{x_{SO_2,1}}{1 - x_{SO_2,1}} + V'\frac{y_{SO_2,1}}{1 - y_{SO_2,1}}$$

The concentration of SO_2 in the inlet gas is given by

$$y_{SO_2,2} = \frac{1.52 \times 10^4}{2.026 \times 10^5} \approx 0.075$$

The molar flow-rate of air in the inlet stream is then

$$V' = y_{air,2}\, V_2 = \left(1 - y_{SO_2,2}\right) V_2 = \left(1 - 0.075\right) \times 5.7 \approx 5.27 \text{ kmol s}^{-1}$$

As the liquid inlet is pure water, we have $x_{SO_2,0} = 0$ and $L_0 = L' = 2.20$ kmol s^{-1}. Filling out the known values in our balance equation yields

$$2.20\frac{\cancel{0}^{\,0}}{1 - 0} + 5.27\frac{0.075}{1 - 0.075} = 2.20\frac{x_{SO_2,1}}{1 - x_{SO_2,1}} + 5.27\frac{y_{SO_2,1}}{1 - y_{SO_2,1}} \tag{2}$$

We now need the equilibrium data to provide another equation so that we can solve for the concentrations. However, we have an issue, the available data is only partly represented by Henry's law. We will use Henry's law but we must then check that our solution lies in the region where it is valid.

Taking care to note that the Henry's law given in the graph is for the partial pressure in **Atm**, the vapour phase mole fraction is given by

$$y_{SO_2,1} = \frac{H}{P} x_{SO_2,1} = 29.6 \frac{1.013 \times 10^5}{2.026 \times 10^5} x_{SO_2,1}$$
$$= 14.8 \, x_{SO_2,1}$$

Substituting this expression for $y_{SO_2,1}$ into Eq. (2), yields

$$5.27 \frac{0.075}{1 - 0.075} = 2.20 \frac{x_{SO_2,1}}{1 - x_{SO_2,1}} + 5.27 \frac{14.8 \, x_{SO_2,1}}{1 - 14.8 \, x_{SO_2,1}}$$
$$0.4273 = 2.20 \frac{x_{SO_2,1}}{1 - x_{SO_2,1}} + 78.00 \frac{x_{SO_2,1}}{1 - 14.8 \, x_{SO_2,1}}$$

We can solve this equation using Matlab/Octave or Mathematica/Maxima. However, we should be able to solve this by hand (or at least try). Multiplying both sides by $(1 - x_{SO_2,1})(1 - 14.8 \, x_{SO_2,1})$ gives

$$0.4273(1 - x_{SO_2,1})(1 - 14.8 \, x_{SO_2,1}) = 2.20 \, x_{SO_2,1}(1 - 14.8 x_{SO_2,1}) + 78 \, x_{SO_2,1}(1 - x_{SO_2,1})$$
$$0.4273(1 - x_{SO_2,1} - 14.8 \, x_{SO_2,1} + 14.8 \, x_{SO_2,1}^2) = x_{SO_2,1} \left[2.20(1 - 14.8 \, x_{SO_2,1}) + 78(1 - x_{SO_2,1}) \right]$$
$$0.4273(1 - 15.8 \, x_{SO_2,1} + 14.8 \, x_{SO_2,1}^2) = x_{SO_2,1} \left(80.2 - 110.56 \, x_{SO_2,1} \right)$$
$$0.4273 - 86.95 \, x_{SO_2,1} + 116.88 \, x_{SO_2,1}^2 = 0$$

We can now use the quadratic formula to find the roots of the equation

$$(x_{SO_2,1} - 0.00495)(x_{SO_2,1} - 0.740) = 0$$

The smallest root is the most likely value, thus $x_{SO_2,1} = 0.00495$, and $y_{SO_2,1} = 14.8 \times 0.00495 \approx 0.0733$. Checking the graph, we can see that Henry's law is still valid at these concentrations, so we will not need to iterate our calculations.

Double checking the question, it asks us to calculate the amounts of the outlet phases which can be obtained like so

$$L_1 = \frac{L'}{1 - x_{SO_2,1}} = \frac{2.2}{1 - 0.00495} \approx 2.21 \text{ kmol s}^{-1}$$
$$V_1 = \frac{V'}{1 - y_{SO_2,1}} = \frac{5.27}{1 - 0.0733} \approx 5.69 \text{ kmol s}^{-1}$$
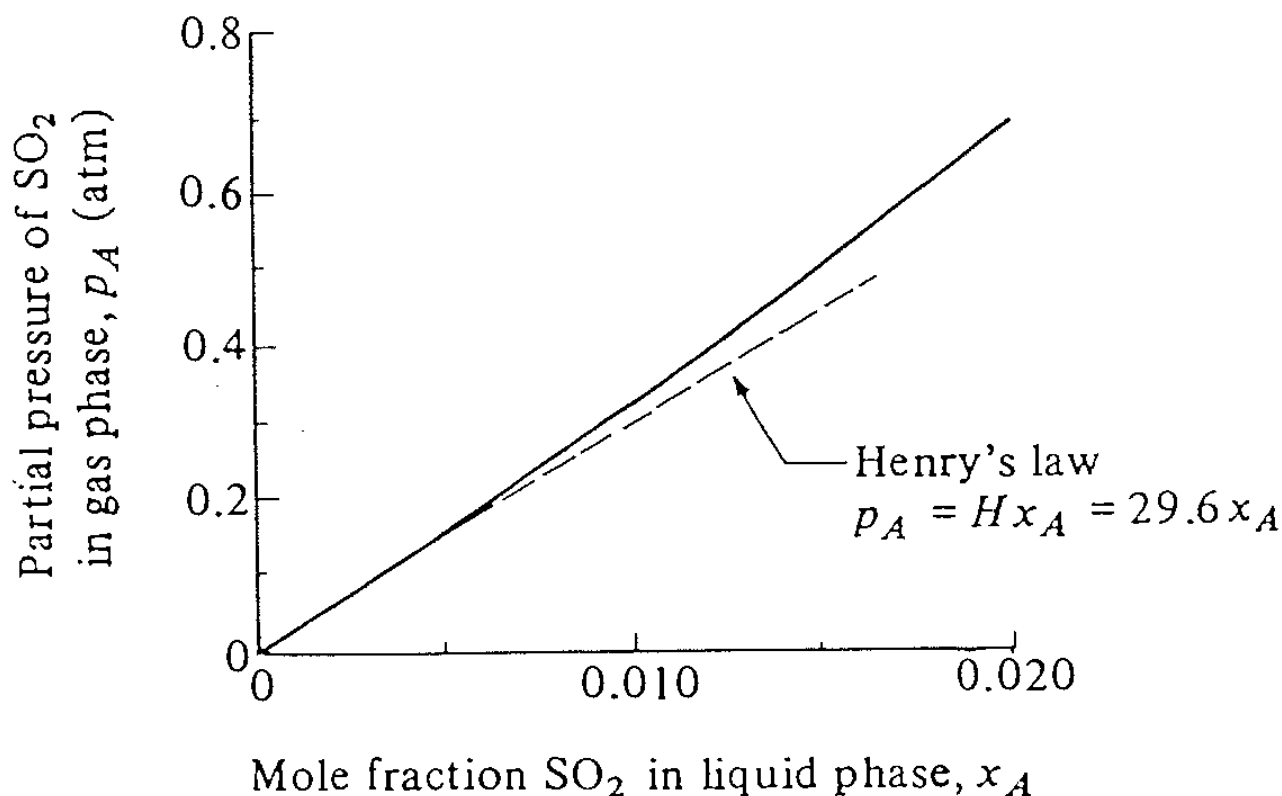
Figure 7: Equilibrium plot for water-SO2 system at 20°C.

[Question end]

Q.3.8 Question 3.8

A gas mixture at 1.0 atm(a) pressure containing air and CO_2 is contacted in a single-stage mixer continuously with pure water at 293 K. The two exit gas and liquid streams reach equilibrium as they are discharged. The inlet gas flow rate is 100 kg mol h^{-1}, with a mole fraction of CO_2 of 0.20. The liquid flow rate entering is 300 kg mol water h^{-1}. Calculate the amounts and compositions of the two outlet phases. You may assume that water does not vaporize (i.e., no water in the gas phase) and use the Henry's law constant for CO_2 at 293 K in water as $H = 1.438 \times 10^8$ Pa. **[20 marks]**

Solution:

The inert water flow is $L' = L_0 = 300$ mol h^{-1}. The inert air flow is $V' = V_2(1 - y_2) =$
[3/20] $100(1 - 0.20) = 80$ mol h^{-1}.$_3$ Considering the general absorption balance,

$$L'\frac{x_{CO_2,0}}{1 - x_{CO_2,0}} + V'\frac{y_{CO_2,2}}{1 - y_{CO_2,2}} = L'\frac{x_{CO_2,1}}{1 - x_{CO_2,1}} + V'\frac{y_{CO_2,1}}{1 - y_{CO_2,1}}$$

$$300\frac{0}{1 - 0} + 80\frac{0.20}{1 - 0.20} = 300\frac{x_{CO_2,1}}{1 - x_{CO_2,1}} + 80\frac{y_{CO_2,1}}{1 - y_{CO_2,1}}$$

$$(1 - y_{CO_2,1})(1 - x_{CO_2,1}) = 15\,x_{CO_2,1}(1 - y_{CO_2,1}) + 4y_{CO_2,1}(1 - x_{CO_2,1})$$

[4/20] $_4$ Using Henry's law for the two streams in equilibrium $y_{CO_2,1} = H\,x_{CO_2,1}/P = 1.438 \times 10^8\,x_{CO_2,1}/1.013 \times$
[2/20] $10^5 = 1.42 \times 10^3\,x_{CO_2,1}$ $_2$ and inserting it into the balance:

$$(1 - 1420\,x_{CO_2,1})(1 - x_{CO_2,1}) = 15\,x_{CO_2,1}(1 - 1420\,x_{CO_2,1}) + 4 \times 1420\,x_{CO_2,1}(1 - x_{CO_2,1})$$

[2/20] $_2$ Grouping terms

$$1 - 1421\,x + 1420\,x^2 = 15\,x - 21300\,x^2 + 5680\,x - 5680\,x^2$$
$$1 - 7116\,x + 28400\,x^2 = 0$$

[2/20] $\checkmark_2$ Solving this in the quadratic formula:

$$x = \frac{7116 \pm \sqrt{7116^2 - 4 \times 28400 \times 1}}{2 \times 28400}$$
$$= 0.12528169\ldots \pm 0.125141082\ldots$$
$$= 0.2504 \text{ or } 1.406 \times 10^{-4}$$

[3/20] $\checkmark_3$ The second root is the required solution. One reason for this, is that the second root is unphysical as it leads to a $y_{CO_2,1} > 1$ (i.e., $y_{CO_2,1} = 1.42 \times 10^3\, x_{CO_2,1} \approx 355.57$). Although these liquid concentrations might be physically reached under extreme circumstances, these high liquid concentrations are outside the applicability of Henry's law. The correct root gives

[2/20] $\checkmark_2$ $y_{CO_2,1} = 1.42 \times 10^3\, x_{CO_2,1} \approx 0.2$.
Calculating the exit flow rates:

$$L_1 = \frac{300}{1 - 1.41 \times 10^{-4}} = 300 \text{ kg mol h}^{-1} \qquad V_1 = \frac{80}{1 - 0.2} = 100 \text{ kg mol h}^{-1}$$

[2/20] $\checkmark_2$ As the liquid solution is so dilute, there is no real change in the flow rates.

[Question total: 20 marks]

4   Week 4: Multi-stage absorption

Q.4.9   **Question 4.9**   **Recommended**
It is decided to deodorise fresh cream by stripping the cream with pure steam. The cream enters the stripper carrying 0.0015 mole fraction of a odorous taint, and it is desired to reduce this to 0.0003 mole fraction to prevent the consumer detecting it. The absorber will need to process 100 kmol h^{-1} of fresh cream and the VLE curve for the taint in cream is presented in Fig. 9.
Note: Real taints in cream are usually in the ppm.

a) Determine the theoretical minimum steam flow rate required to strip the taint from the stream.   **[9 marks]**
   Solution:
   We will sketch a diagram of a general multi-stage absorber to help us remember the numbering of the streams.
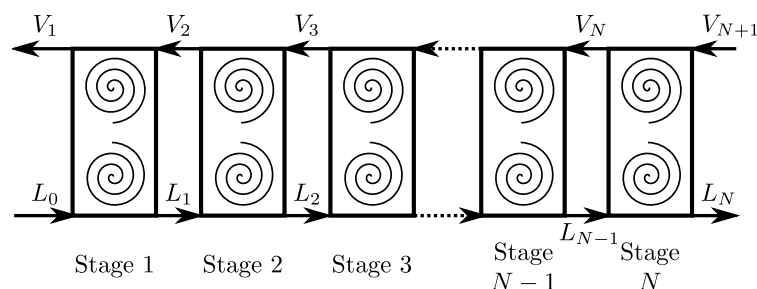
The inlet flow of tainted cream is $L_0 = 100$ kmol h^{-1} with a taint concentration of $x_{A,0} = 0.0015$. The pure cream content of that flow (inert liquid) is $L' = L_0(1 - x_{A,0}) = 100(1 - 0.0015) = 99.85$ kmol h^{-1} and the outlet cream has a loading of $x_{A,N} = 0.0003$. The inlet (pure) steam concentration is $y_{A,N+1} = 0$ (therefore the inert vapour flow-rate is $V' = V_{N+1}\left(1 - y_{A,N+1}{}^{0}\right) = V_{N+1}$).

We nearly have all of the information for the inlet and outlet streams of the absorber. To calculate the minimum steam/inert-vapour flow-rate that we could use to carry out the absorption, we need to find what is the maximum concentration we can make the vapour.

The maximum concentration of taint in the liquid phase is $x_{A,0} = 0.0015$. If we look at the VLE data, the equilibrium concentration of taint in the steam at this liquid concentration is $y_{max} = 0.128$. This is the maximum theoretical loading of the steam which we can physically achieve. We need to use this value as the output concentration $y_{A,1} = y_{max} = 0.128$ to determine the minimum feasible liquid flow-rate. We now need to find an equation where we can use these values to calculate V_{N+1}.

We can perform a mass balance over any collection of stages from stage i to stage j ($i < j$) and generate the following equation

$$\text{In} = \text{Out} \tag{3}$$

$$L'\frac{x_{A,i}}{1 - x_{A,i}} + V'\frac{y_{A,j+1}}{1 - y_{A,j+1}} = L'\frac{x_{A,j}}{1 - x_{A,j}} + V'\frac{y_{A,i+1}}{1 - y_{A,i+1}} \tag{4}$$

At the start, we almost know all of the information of the two ends of the absorber, so we set $i = 0$ and $j = N$ to get

$$L'\frac{x_{A,0}}{1 - x_{A,0}} + V'\frac{y_{A,N+1}}{1 - y_{A,N+1}} = L'\frac{x_{A,N}}{1 - x_{A,N}} + V'\frac{y_{A,1}}{1 - y_{A,1}}$$

Substituting the known values into this balance equation, we have

$$99.85\frac{0.0015}{1 - 0.0015} + V'_{min}\frac{\cancelto{0}{0}}{\cancel{1 - 0}} = 99.85\frac{0.0003}{1 - 0.0003} + V'_{min}\frac{0.128}{1 - 0.128}$$

$$V'_{min} \approx 0.818 \text{ kmol h}^{-1}$$

This is the absolute minimum amount of steam we could use to achieve the desired separation, but it would need an inifinite number of ideal trays to achieve this!

b) From experience, it is known that the optimal steam rate is approximately twice the minimum flow rate. Determine the number of theoretical trays required to carry out the stripping. **[9 marks]**
   Solution:
   The actual steam flow-rate is $V' = 2\,V'_{min} = 1.636$ kmol h^{-1}.

   Now we know the real flow rates in the system we can use the graphical stepping method to calculate the number of theoretical stages in the absorber. We need to use the operating line equation and the VLE data to calculate how many stages are needed. We will do this using the graphical method, as outlined in the lecture notes.

   We need to generate several points on the operating line to be able to plot it. The first point on the operating line is given by the fully specified end of the evaporator ($x_{A,N} = 0.0003$, $y_{A,N+1} = 0$).

We will evaluate another point at the inlet liquid concentration $x_{A,0} = 0.0015$, but we will need to use the balance equation, to solve for $y_{A,1}$ (the value calculated in the previous question was only valid for the minimum vapour flow-rate $V' = V'_{min}$, but we now have $V' = 2\,V'_{min}$).

$$L'\frac{x_{A,0}}{1-x_{A,0}} + V'\frac{y_{A,N+1}}{1-y_{A,N+1}} = L'\frac{x_{A,N}}{1-x_{A,N}} + V'\frac{y_{A,1}}{1-y_{A,1}}$$

$$99.85\frac{0.0015}{1-0.0015} + 1.636\underset{y_{A,1}\,\approx\,0.0684}{\cancelto{0}{\frac{0}{1-0}}} = 99.85\frac{0.0003}{1-0.0003} + 1.636\frac{y_{A,1}}{1-y_{A,1}}$$

$$y_{A,1} \approx 0.0684$$

We will add intermediate points to check if the operating line is curved. We need to plot Eq. (3). We've already solved for both ends of the evaporator, so it is our choice whether to set $i = 0$ **or** $j = N$ (leaving the other one free, so we have something to plot). We will use $i = 0$, as this gives an equation similar to the one plotted in the last tutorial.

$$L'\frac{x_{A,0}}{1-x_{A,0}} + V'\frac{y_A}{1-y_A} = L'\frac{x_A}{1-x_A} + V'\frac{y_{A,1}}{1-y_{A,1}}$$

$$99.85\frac{0.0015}{1-0.0015} + 1.636\frac{y_A}{1-y_A} = 99.85\frac{x_A}{1-x_A} + 1.636\frac{0.0684}{1-0.0684}$$

We now choose two intermediate points, $x_A = 0.0011$ and $x_A = 0.0007$. Solving the above equation gives $y_A(x_A = 0.0011) = 0.0467$ and $y_A(x_A = 0.0007) = 0.0239$. This curve is now determined to a precision well beyond what the graph paper allows. Using just three points for the operating line would have been sufficient in this case and we might have predicted this when we observed the small concentrations of taint in the system.

We can then immediately step the absorber solution. We start the stepping from the tower top ($x_{A,0} = 0.0015$,). We do this as it is the easiest way to determine the size of fractional stage at the bottom.

Performing the stepping, we find we need almost exactly two theoretical stages.

c) A tray tower is to be used to carry out the stripping. Plates are available with an overall column efficiency of 28%. Determine the number of real plates required to perform the stripping. **[2 marks]**

Solution:
The number of real trays needed is the number of theoretical stages divided by the plate efficiency $2/0.28 = 7.14$. As we can only construct whole trays, the tower design must incorporate 8 trays.

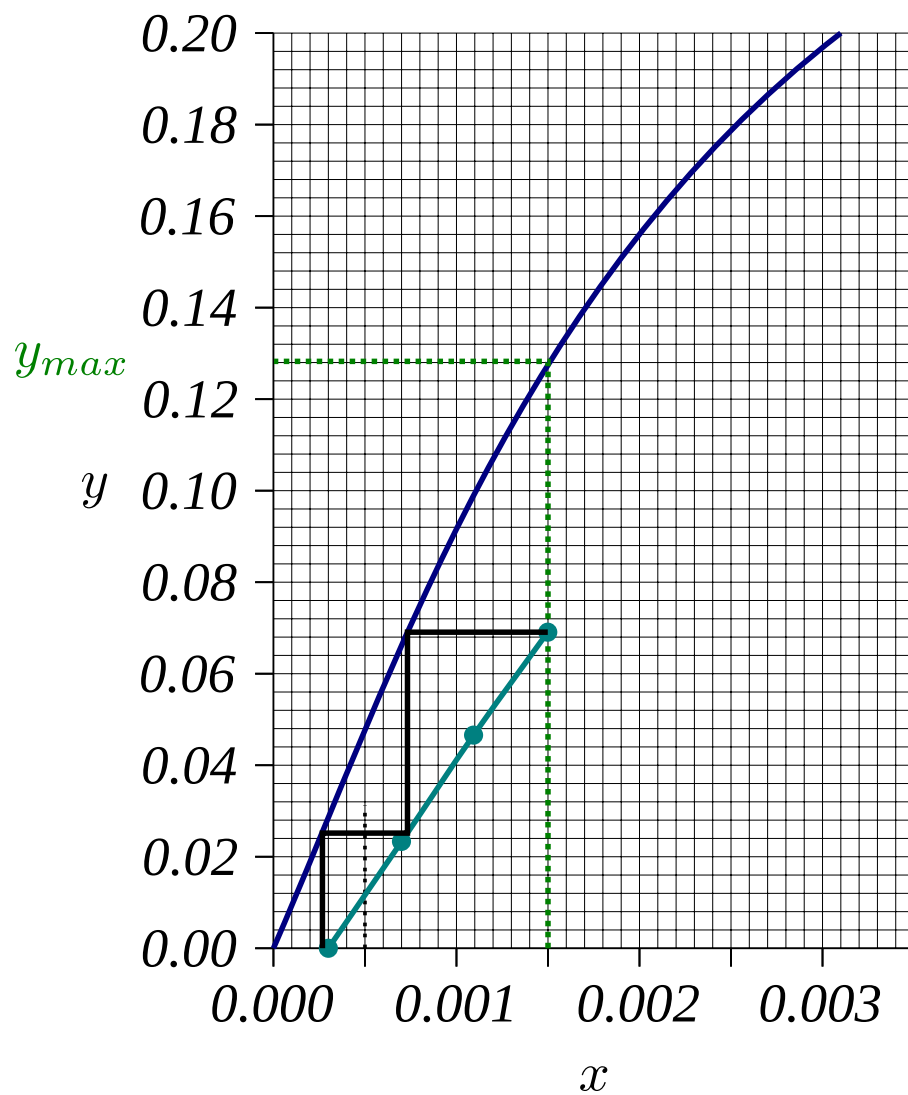Figure 8: VLE data (mole fraction in vapour *y* versus mole fraction in liquid *x*) for an odorous taint in cream, including the design calculation.

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.
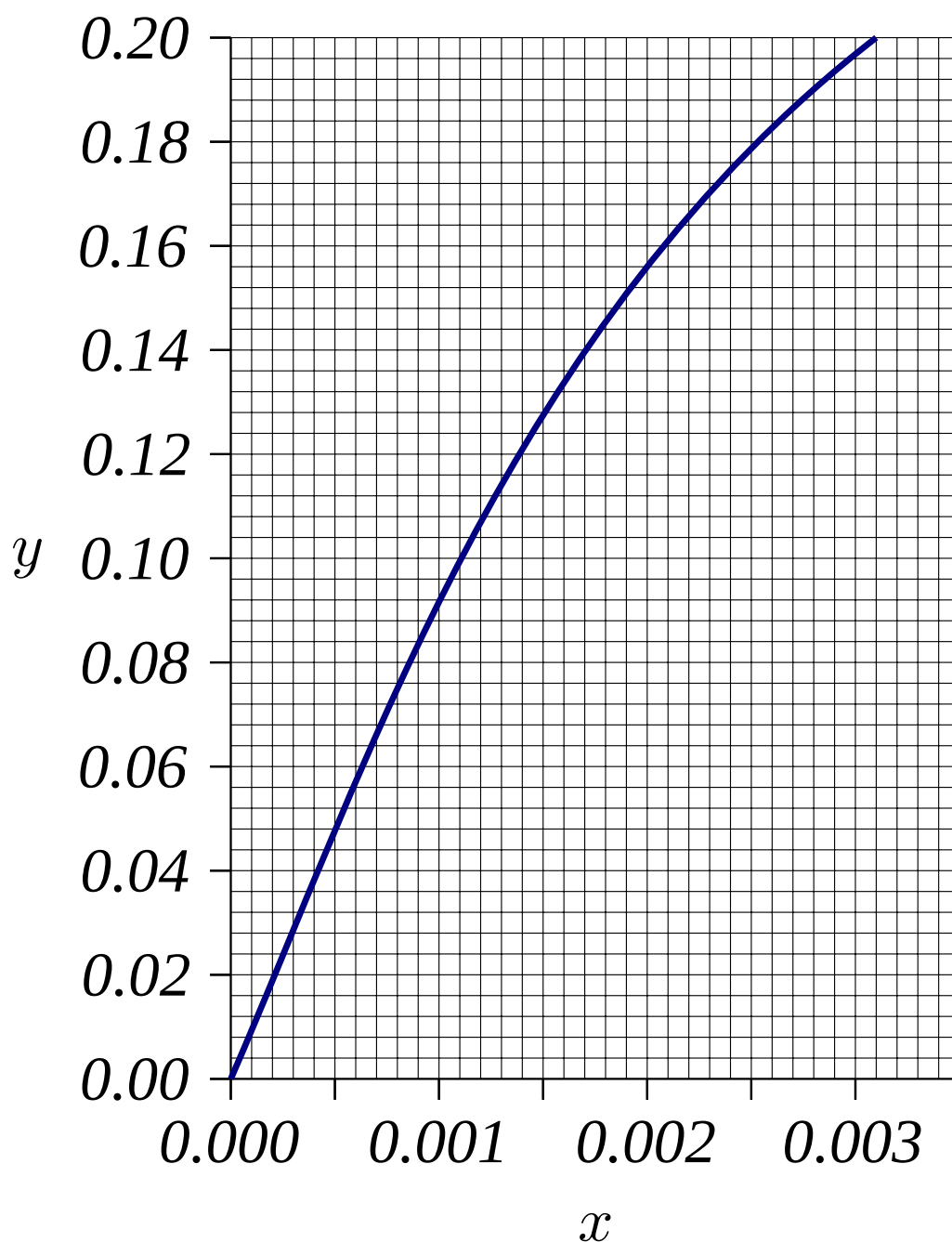
Student ID:

Figure 9: VLE data (mole fraction in vapour y versus mole fraction in liquid x) for an odorous taint in cream. For use in Q. 4.9

[Question total: 20 marks]

Q.4.10 Question 4.10

a) Derive the operating line equation for an absorber, given below. Clearly state any assumptions that you make.   **[8 marks]**

$$\frac{y_{A,n+1}}{1 - y_{A,n+1}} = \frac{L'}{V'}\frac{x_{A,n}}{1 - x_{A,n}} + \frac{y_{A,1}}{1 - y_{A,1}} - \frac{L'}{V'}\frac{x_{A,0}}{1 - x_{A,0}}$$

Solution:
The primary assumption is that both phases remain as binary mixtures. This requires that only one component is exchanged between the two contacting phases. This implies there is an inert gas and an inert liquid component with a constant flow-rates of V' and L' respectively.

We now perform a mass balance in the absorbed component over the column, from one end to some arbitrary stage n:
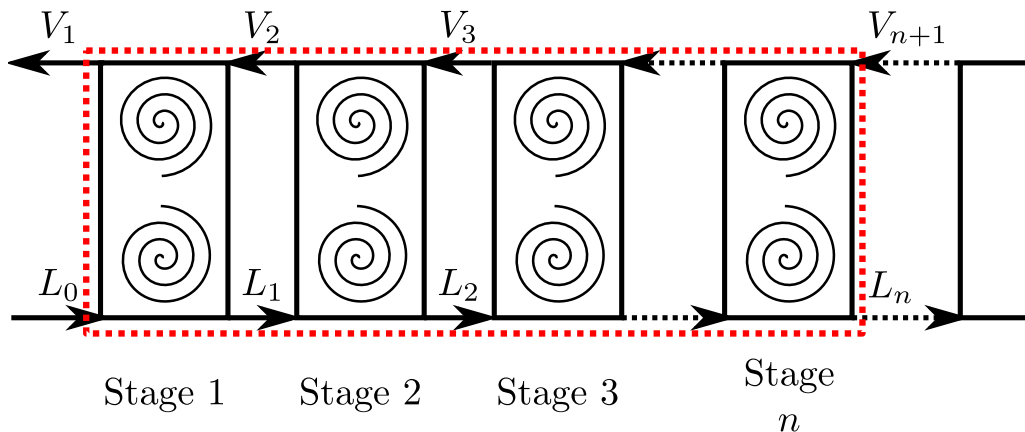
To yield:

$$x_0\,L_0 + y_{n+1}\,V_{n+1} = x_n\,L_n + y_1\,V_1$$

Using the binary assumption and x and y as the mole fractions of the absorbed component in each phase, we can express the flow-rates of some arbitrary stream n as follows:

$$L_n = L'/(1 - x_n) \qquad\qquad V_n = V'/(1 - x_n)$$

where L' is the inert liquid flow rate and V' is the inert gas flow rate.

Substituting this rule into the mass balance gives

$$x_0\,L_0 + y_{n+1}\,V_{n+1} = x_n\,L_n + y_1\,V_1$$
$$\frac{x_0}{1 - x_0}L' + \frac{y_{n+1}}{1 - y_{n+1}}V' = \frac{x_n}{1 - x_n}L' + \frac{y_1}{1 - y_1}V'$$

Rearranging to obtain the final result:

$$\frac{y_{n+1}}{1 - y_{n+1}} = \frac{x_n}{1 - x_n}\frac{L'}{V'} + \frac{y_1}{1 - y_1} - \frac{x_0}{1 - x_0}\frac{L'}{V'}$$

b) Simplify the operating line equation to a linear form using the assumption of low absorbent concentrations. Discuss how these conditions are compatible with the use of Henry's law.

[4 marks]

Solution:

In the limit that the gas and liquid concentrations of the absorbed component are very low, the fractional terms can be simplified.

$$\frac{y_{n+1}}{1 - y_{n+1}} = \frac{x_n}{1 - x_n}\frac{L'}{V'} + \frac{y_1}{1 - y_1} - \frac{x_0}{1 - x_0}\frac{L'}{V'}$$

$$y_{n+1} = \frac{L'}{V'}(x_n - x_0) + y_1$$

This approximate form of the operating line equation is much more convenient to work with, as it is a straight line on an x-y plot.

This form of the operating line equation can be combined with the use of Henry's law, as it only applies in the same limit of low liquid concentrations ($x \to 0$).

c) A gas mixture of air and SO_2 comes into contact with pure water at 293 K and 4 bar in a single stage equilibrium mixer. The partial pressure of SO_2 in the inlet gas is 0.1 bar and the total inlet gas flow-rate is 5.13 kmol s^{-1}. The inlet flow rate of water is 2 kmol s^{-1} and the outlets are in equilibrium. Using the Henry's law expression below, calculate the compositions and flow rates of all streams.

$$P_A = \mathcal{H}_A\, x_A$$

where $\mathcal{H}_{SO_2}$ = 29.98 bar at 293 K.   **[8 marks]**

Solution:

We start with the balance equation from the previous questions (or datasheet) and set $n = 1$ to mark that it is a balance over a single stage.

$$\frac{x_0}{1 - x_0}L' + \frac{y_2}{1 - y_2}V' = \frac{x_1}{1 - x_1}L' + \frac{y_1}{1 - y_1}V'$$

The concentration of the inlet gas is

$$y_2 = \frac{0.1}{4} = 0.025$$

The molar flow rate of air is

$$V' = (1 - y_2)V_2 = (1 - 0.025)5.13 \approx 5 \text{ kmol s}^{-1}$$

We can substitute all of this information into the general balance equation to get:

$$L'\frac{x_0}{1 - x_0}{}' + V'\frac{y_2}{1 - y_2} = L'\frac{x_1}{1 - x_1} + V'\frac{y_1}{1 - y_1}$$

$$2\frac{0}{1 - 0} + 5\frac{0.025}{1 - 0.025} = 2\frac{x_1}{1 - x_1} + 5\frac{y_1}{1 - y_1}$$

$$0.1282 = 2\frac{x_1}{1 - x_1} + 5\frac{y_1}{1 - y_1}$$

We need one more equation to solve this expression and, as the outputs are in equilibrium, this is provided by the Henry's law data.

$$y_1 = \frac{P_{SO_2, x_1}}{P} = \frac{29.98\, x_1}{4} = 7.495\, x_1$$

Low concentration limit solution

The student may choose to use the low concentration limit of the balance equation as guided by the previous question (but must state that they are using this limit). In this case, we have

$$0.1282 = 2\,x_1 + 5\,y_1$$

Substituting in the Henry's law expression

$$0.1282 = 2\,x_1 + 5 \times 7.495\,x_1$$
$$x_1 \approx 0.00325$$
$$y_1 = 0.0244$$

General solution

Inserting the Henry's law expression into the balance, we have:

$$0.1282 = 2\frac{x_1}{1 - x_1} + 5\frac{7.495\,x_1}{1 - 7.495\,x_1}$$

Multiplying both sides by $(1 - x_1)(1 - 7.495\,x_1)$:

$$0.1282(1 - x_1)(1 - 7.495\,x_1) = 2\,x_1(1 - 7.495\,x_1) + 5 \times 7.495\,x_1(1 - x_1)$$

Expanding:

$$0.1282 - 1.089\,x_1 + 0.9609\,x_1^2 = 2\,x_1 - 14.99\,x_1^2 + 37.48\,x_1 - 37.48\,x_1^2$$

Grouping on the right.

$$0 = -53.43\,x_1^2 + 40.57\,x_1 - 0.1282$$

Using the quadratic equation (available in the data sheet), we have

$$x_1 = \frac{-40.57 \pm \sqrt{40.57^2 - 4 \times 53.43 \times 0.1282}}{2 \times -53.43}$$
$$= 0.3797 \pm 0.3765$$
$$= [0.7562,\ 0.0032]$$

The correct value is $x_1 = 0.0032$ as Henry's law is not typically valid at high liquid concentrations and the absorbent is starting from a low concentration. The output vapour concentration is given by:

$$y_1 = \frac{29.98\,x_1}{4} = 0.02398$$

Finally, we calculate the outlet vapour and liquid flow rates

$$L_1 = \frac{L'}{1 - x_1} \qquad\qquad\qquad V_1 = \frac{V'}{1 - y_1}$$
$$= \frac{2}{1 - 0.0032} \qquad\qquad\qquad = \frac{5}{1 - 0.02398}$$
$$\approx 2.006 \text{ kmol s}^{-1} \qquad\qquad = 5.123 \text{ kmol s}^{-1}$$

[Question total: 20 marks]

Q.4.11 Question 4.11

A 30 kmol h^{-1} gas stream produced from coal contains a light oil vapour (2 mol%) which is scrubbed out using an absorption column. The light oil vapour is washed out of the gas by contacting the gas with a 13 kmol h^{-1} non-volatile heavy oil absorbent.
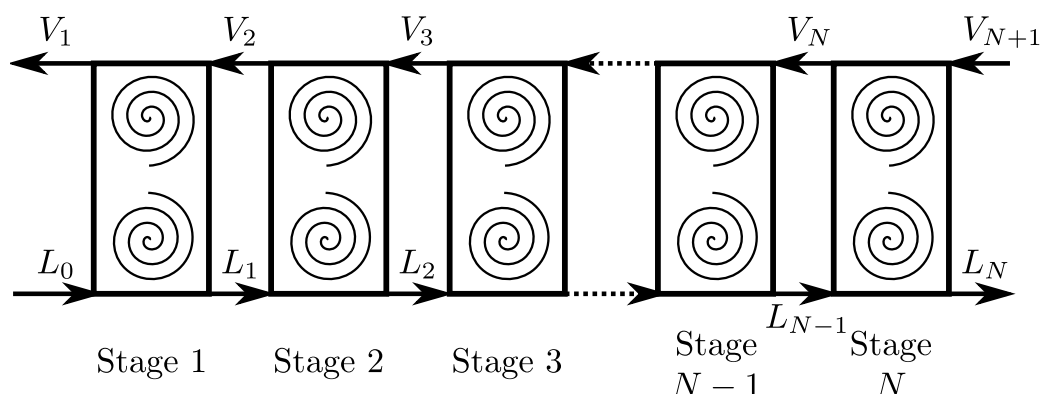

Figure 10: *
Figure: The stream numbering within a multi-stage absorber

The heavy oil absorbent initially contains 0.005 mol% of absorbed light oil and it absorbs 90% of the light oil in the inlet gas. The equilibrium between the light oil vapour and the absorbed light oil is given by $y = 0.275\,x$. An X-Y diagram is provided in Fig. 11 and the HETP is 1.4 m stage^{-1}. Determine the height of packing required to carry out the specified absorption in a counter-current column.   **[20 marks]**

Solution:

We plot the design graph in the variables $X = x/(1 - x)$, $Y = y/(1 - y)$ to obtain a straight operating line. The X-Y diagram has already been prepared for us, so we only need consider the operating line.

$$\frac{y_{A,n+1}}{1 - y_{A,n+1}} = \frac{L'}{V'}\frac{x_{A,n}}{1 - x_{A,n}} + \frac{y_{A,1}}{1 - y_{A,1}} - \frac{L'}{V'}\frac{x_{A,0}}{1 - x_{A,0}}$$

$$Y_{n+1} = \frac{L'}{V'}(X_n - X_0) + Y_1$$

The non-volatile stream flow rates are

$$V' = V_{N+1}(1 - y_{N+1}) = 30(1 - 0.02)$$
$$= 29.4 \text{ kmol h}^{-1}$$
$$L' = L_0(1 - x_0) = 13(1 - 0.00005)$$
$$\approx 13 \text{ kmol h}^{-1}$$

The amount of light oil entering the absorber is $0.02 * 30 = 0.6$ kmol h^{-1}. 90% of this is absorbed into the heavy oil ($0.6 \times 0.9 = 0.54$ kmol h^{-1}). The outlet vapour concentration is then $y_1 = (0.6 - 0.54)/(30 - 0.54) \approx 0.002$ mol%. The outlet liquid concentration is $x_N = 0.54/(13 + 0.54) \approx 0.04$

Changing to the convenient X and Y variables, we have

$$Y_1 = \frac{0.002}{1 - 0.002} \approx 0.002 \qquad\qquad X_0 = \frac{0.00005}{1 - 0.00005} \approx 0$$

$$Y_{N+1} = \frac{0.02}{1 - 0.02} \approx 0.02 \qquad\qquad X_N = \frac{0.04}{1 - 0.04} \approx 0.042$$

We can now plot the straight operating lines on a VLE chart as a line between (0,0.002) to (0.042,0.02).
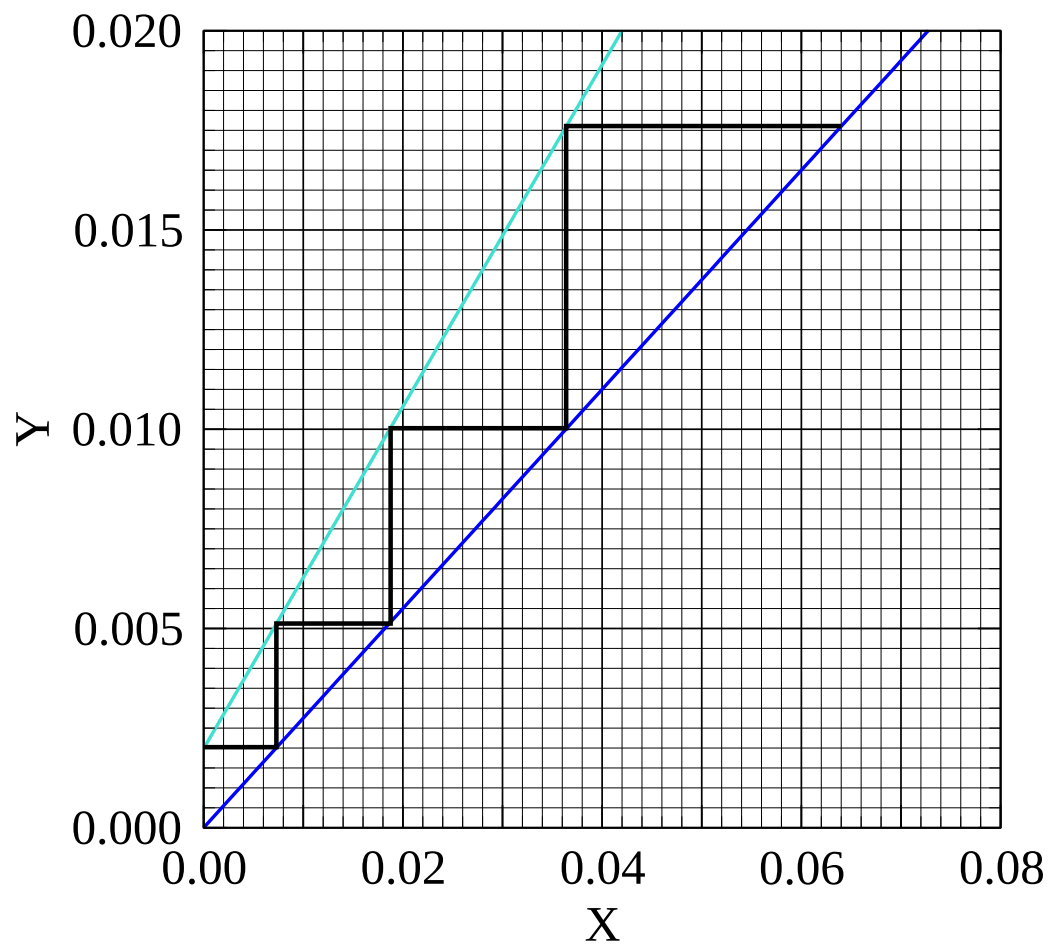
There are 3.2 theoretical stages in the column. This corresponds to $1.4 \times 3.2 \approx 4.5$ m of packing.

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

Student ID:

Figure 11: A *X-Y* graph for Q. 4.11.

[Question total: 20 marks]

Q.4.12 Question 4.12

Air is dried by contacting it with a solution of sulphuric acid in a counter-current absorber column. The air enters the absorber with 2.7 mol% of water and leaves with 0.8 mol% of water. Absorbent is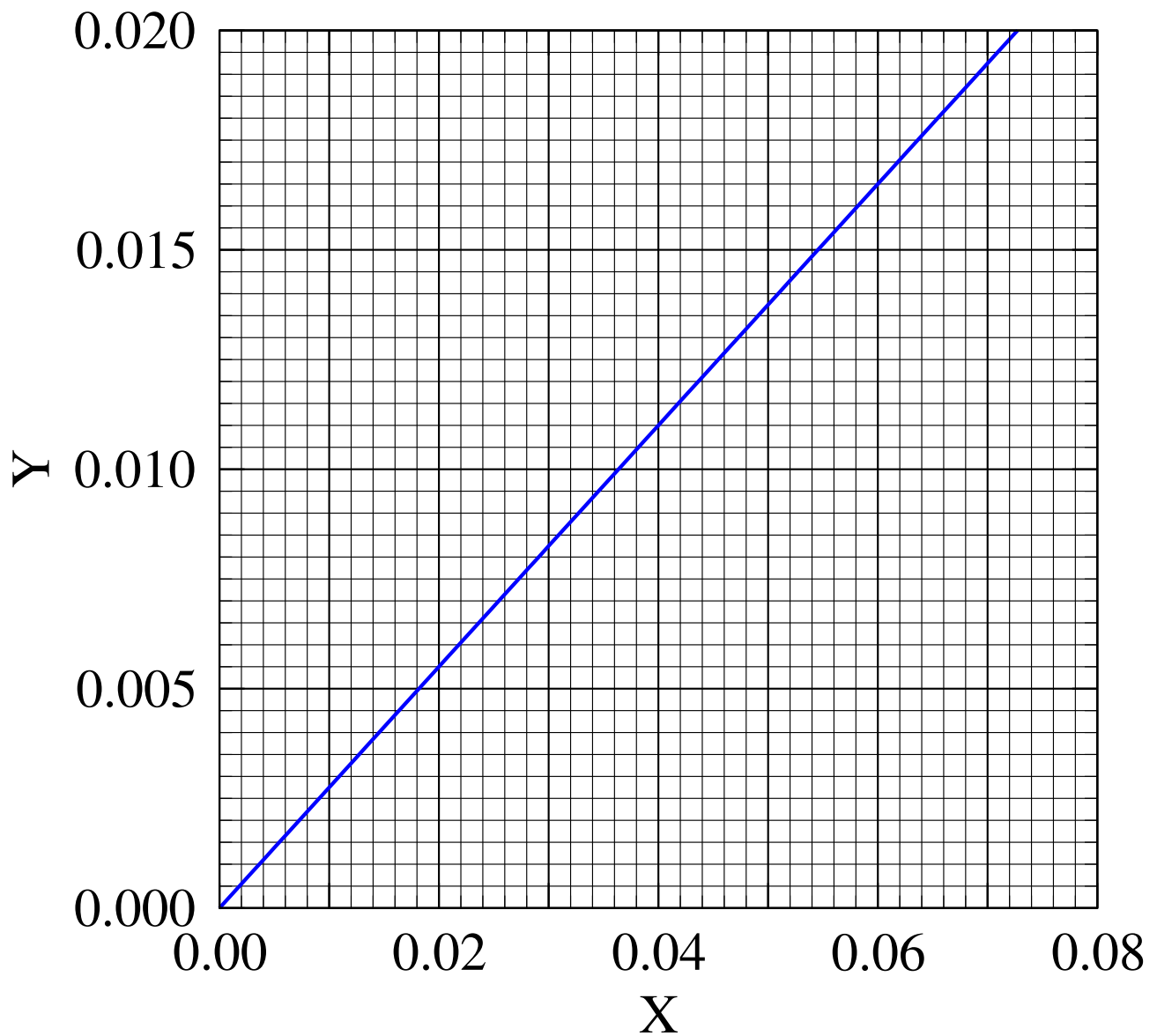 supplied to the column at a concentration of 67 mol% H_2SO_4 and 33 mol% water, and exits the column at a concentration of 40 mol% H_2SO_4. The column is operating at atmospheric pressure (760 mmHg) and at a temperature of 25°C. You may assume that water is the only component transferred between the phases (H_2SO_4 remains in the liquid phase and no air dissolves).
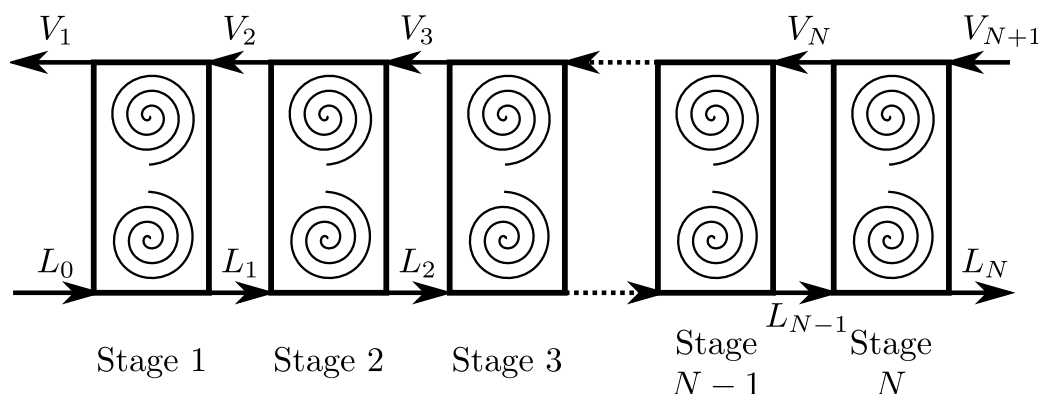
Figure 12: The stream numbering within a multi-stage absorber

a) Determine the ratio of the flow rate of dry air to the flow rate of H_2SO_4.   **[8 marks]**

Solution:

Here we need to perform mass balances over the column to solve for the inert flow rates L' and V'. The additional unknowns are the flow-rates of each inlet/outlet stream, or in the notation of the diagram L_0, L_N, V_1 and V_{N+1}.

We know the inlet and outlet concentrations of water, which are:

$$x_0 = 0.33 \qquad\qquad x_N = 0.6$$
$$y_1 = 0.008 \qquad\qquad y_{N+1} = 0.027$$

We know that the inert components, L' and V', remain in their respective phases. Using x and y as the mole fractions of water in each phase, we can express the flow-rates of each stream as follows:

$$L_0 = L'/(1 - x_0) \qquad\qquad L_N = L'/(1 - x_N)$$
$$V_1 = V'/(1 - y_1) \qquad\qquad V_{N+1} = V'/(1 - y_{N+1})$$

A balance over the total mass entering and exiting the column gives:

$$L_0 + V_{N+1} = V_1 + L_N$$

$$\frac{L'}{1 - x_0} + \frac{V'}{1 - y_{N+1}} = \frac{V'}{1 - y_1} + \frac{L'}{1 - x_N}$$

$$V'\left(\frac{1}{1 - y_{N+1}} - \frac{1}{1 - y_1}\right) = L'\left(\frac{1}{1 - x_N} - \frac{1}{1 - x_0}\right)$$

Inserting the known values, we have

$$V'\left(\frac{1}{1-y_{N+1}} - \frac{1}{1-y_1}\right) = L'\left(\frac{1}{1-x_N} - \frac{1}{1-x_0}\right)$$

$$V'\left(\frac{1}{1-0.027} - \frac{1}{1-0.008}\right) = L'\left(\frac{1}{1-0.6} - \frac{1}{1-0.33}\right)$$

$$0.01968\,V' = 1.007\,L'$$

$$\frac{V'}{L'} = 51.17$$

$$\frac{L'}{V'} = 0.0195$$

Alternatively, we might work with the species mass balance:

$$\frac{y_{n+1}}{1-y_{n+1}} = \frac{L'}{V'}\frac{x_n}{1-x_n} + \frac{y_1}{1-y_1} - \frac{L'}{V'}\frac{x_0}{1-x_0}$$

$$\frac{0.027}{1-0.027} = \frac{L'}{V'}\frac{0.6}{1-0.6} + \frac{0.008}{1-0.008} - \frac{L'}{V'}\frac{0.33}{1-0.33}$$

$$0.02775 = \frac{L'}{V'}1.5 + 0.00806 - \frac{L'}{V'}0.493$$

$$\frac{V'}{L'} = \frac{1.5 - 0.493}{0.02775 - 0.00806} = \frac{1.007}{0.0197} = 51.12$$

You must be careful as the question asks for dry air and H_2SO_4, not the ratio of any stream flowrates.

b) Using the equilibrium data provided in Table 2, determine the number of equilibrium stages required to perform the absorption. Plot your graph in x-y coordinates on Fig. 13 and ensure that you adequately capture the curvature of the operating line (or use X-Y coordinates). **[12 marks]**
Solution:
To start, we convert the data from partial pressures to mole fractions using Dalton's law:

$$y_{H_2SO_4} = \frac{P_{H_2SO_4}}{P}$$

We know the column is operating at atmospheric pressure, which is $P = 760$ mmHg, so we can generate the final column of Table 2.

Here we note that the concentration in both phases is too high to allow the use of the straight operating line. We must either use a curved operating line or transform to the X-Y coordinates. The solution here uses the first method and plots a curved operating line.

We already have two points on the operating line from the inlet and outlet stream conditions.

$$\begin{array}{cc} (x_0, \quad y_1) & (x_N, \quad y_{N+1}) \\ (0.33, \quad 0.008) & (0.6, \quad 0.027) \end{array}$$

We need to generate at least another two points to capture the curvature of the operating line. The operating line equation for an absorber is given in the data sheet (assuming

Table 1: Partial pressure data for water over aqueous solutions of sulphuric acid at 25°C, along with gas concentrations calculated for a gas mixture at atmospheric pressure.

Liquid Water conc. x_{H_2O} (mol%)	Partial Pressure of Water (mmHg)	Gas Water conc. y_{H_2O} (mol%)
80	20.8	0.0274
70	17.8	0.0234
60	13.5	0.0178
50	8.45	0.0111
40	3.97	0.00522
30	1.03	0.00136
20	0.124	0.000163
10	0.00765	0.0000101

$A = H_2O$) as:

$$\frac{y_{n+1}}{1 - y_{n+1}} = \frac{L'}{V'} \frac{x_n}{1 - x_n} + \frac{y_1}{1 - y_1} - \frac{L'}{V'} \frac{x_0}{1 - x_0}$$

$$\frac{y_{n+1}}{1 - y_{n+1}} = \frac{1}{51.17} \times \frac{x_n}{1 - x_n} + \frac{0.008}{1 - 0.008} - \frac{1}{51.17} \times \frac{0.33}{1 - 0.33}$$

$$\frac{y_{n+1}}{1 - y_{n+1}} = 0.01954 \times \frac{x_n}{1 - x_n} - 0.001561$$

We will select two convenient x locations, between x_0 and x_N, to evaluate for values of y. I have chosen $x = 0.4$ and $x = 0.5$ for convenience when plotting to give:

$$(0.4, 0.01134) \qquad\qquad (0.5, 0.01766)$$

When plotting these values (see below) the curvature is obvious and the student may choose to do more points, but these will not affect the calculations significantly if a smooth line was hand drawn between the VLE and operating line points.
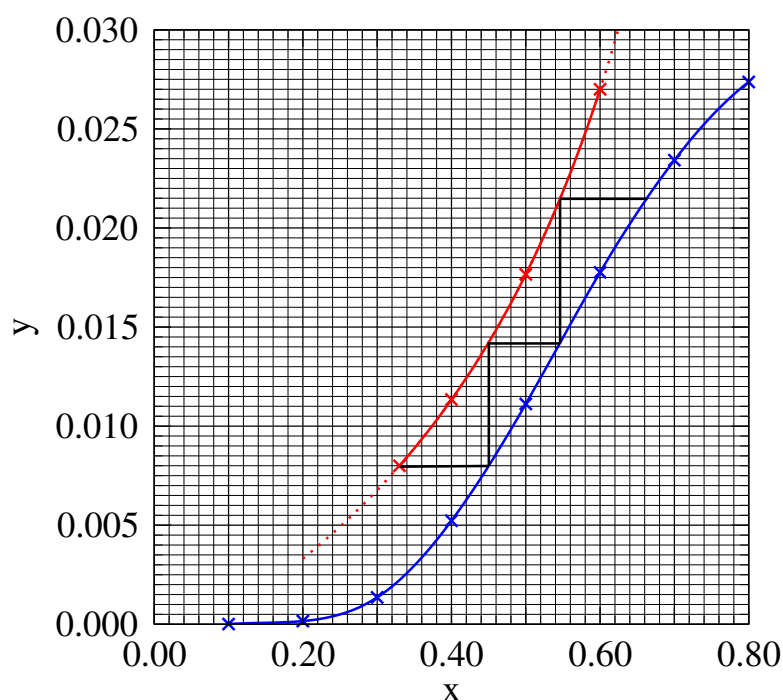
From the graphical method, I estimate that approximately $\approx$ **2.5** theoretical stages are needed.

Alternatively, we can perform the calculation in transformed coordinates X/Y, but as the supplied graph is in x and y this is not done here.

Table 2: Partial pressure data for water over aqueous solutions of sulphuric acid at 25°C.

Liquid Water conc. (mol%)	Partial Pressure of Water (mmHg)
80	20.8
70	17.8
60	13.5
50	8.45
40	3.97
30	1.03
20	0.124
10	0.00765

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

Student ID:
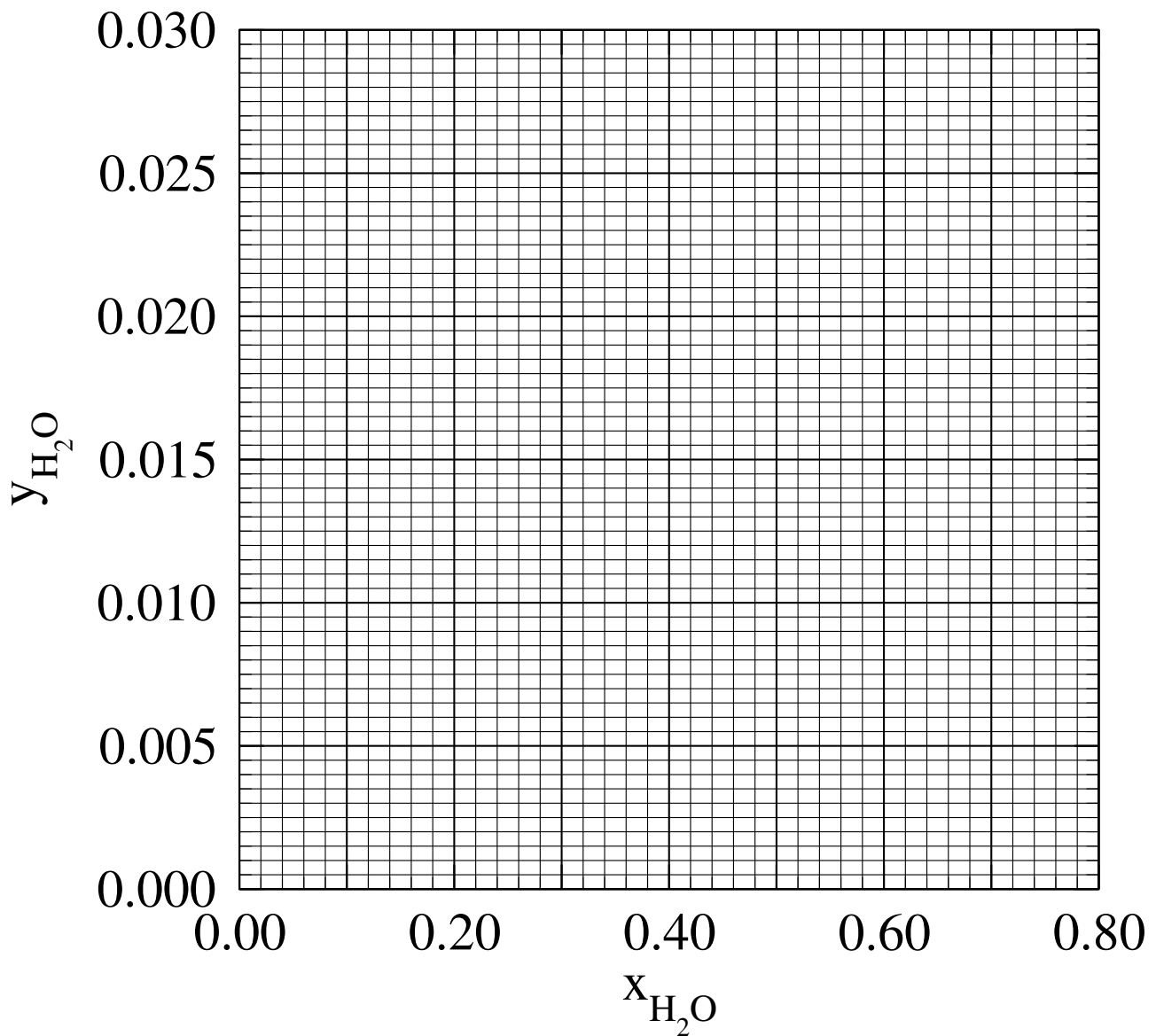
Figure 13: Blank graph for the air dryer in Q. 4.12.

[Question total: 20 marks]

Q.4.13 Question 4.13

A straw oil used to absorb benzene (C_6H_6) from coke-oven gas is to be stripped with pure steam in a sieve plate column at atmospheric pressure to recover the dissolved benzene. Equilibrium conditions at the operating temperature are approximated by Henry's law such that, when the oil phase contains 10 mol% C_6H_6, the C_6H_6 pressure above the oil is 5.07 kPa. The oil may be considered non-volatile. The oil enters containing 8 mol% benzene, 75% of which is to be recovered. The steam leaves containing 3 mol% C_6H_6.

**Note:** If you perform a graphical design you will need to submit your graph paper.

a) Determine the flowrates of the non-volatile components (steam and straw oil) using a basis of 100 mol for the liquid feed.   **[4 marks]**

Solution:

[2/4] Using a basis of 100 mol straw-oil/benzene mixture input, 8 mols of benzene enter, thus 6 mols are recovered in the steam and the inert liquid (straw oil) flowrate is $L' = 92$ mols. ✓₂ If the steam leaves at 3 mol%, then the output gas flowrate is $6/0.03 = 200$ mol and the inert

[2/4] gas (steam) flowrate is $V' = 194$ mol. ✓₂

b) How many theoretical stages are required?   **[14 marks]**

Solution:

The inlet/outlet conditions are as follows,

$$x_{N+1} = 0.08 \qquad\qquad y_N = 0.03$$

$$x_0 = \frac{2}{94} \approx 0.0213 \qquad\qquad y_1 = 0$$

[1/14] ✓₁

Entering these into the operating line and using the conditions at the steam entrance/liquid exit,

$$\frac{y_{A,n+1}}{1 - y_{A,n+1}} = \frac{L'}{V'}\frac{x_{A,n}}{1 - x_{A,n}} + \frac{y_{A,1}}{1 - y_{A,1}} - \frac{L'}{V'}\frac{x_{A,0}}{1 - x_{A,0}}$$

$$= \frac{92}{194}\frac{x_{A,n}}{1 - x_{A,n}} + \frac{0}{1 - 0} - \frac{92}{194}\frac{\frac{2}{94}}{1 - \frac{2}{94}}$$

$$= 0.474\frac{x_{A,n}}{1 - x_{A,n}} - 0.0103$$

[2/14] ✓₂

[1/14] As the concentrations are less than 10%, the operating line should be reasonably straight. ✓₁ To
[1/14] be certain, two additional points are used to explore its curvature. ✓₁ Selecting two points equispaced between the end points,

$$x = 0.04 \qquad\qquad y = 9.36 \times 10^{-3}$$

$$x = 0.06 \qquad\qquad 1.96 \times 10^{-2}$$

[2/14] ✓₂

The Henry's coefficient can be determined from the given point,

$$5.07 \times 10^3 = 0.1\,\mathcal{H}$$

$$\mathcal{H} = 5.07 \times 10^4 \text{ Pa}$$

The corresponding x-y relationship is,

$$y_n = \frac{5.07 \times 10^4}{1.013 \times 10^5} x_n = 0.500\, x_n$$

[2/14] $\overset{\checkmark}{2}$
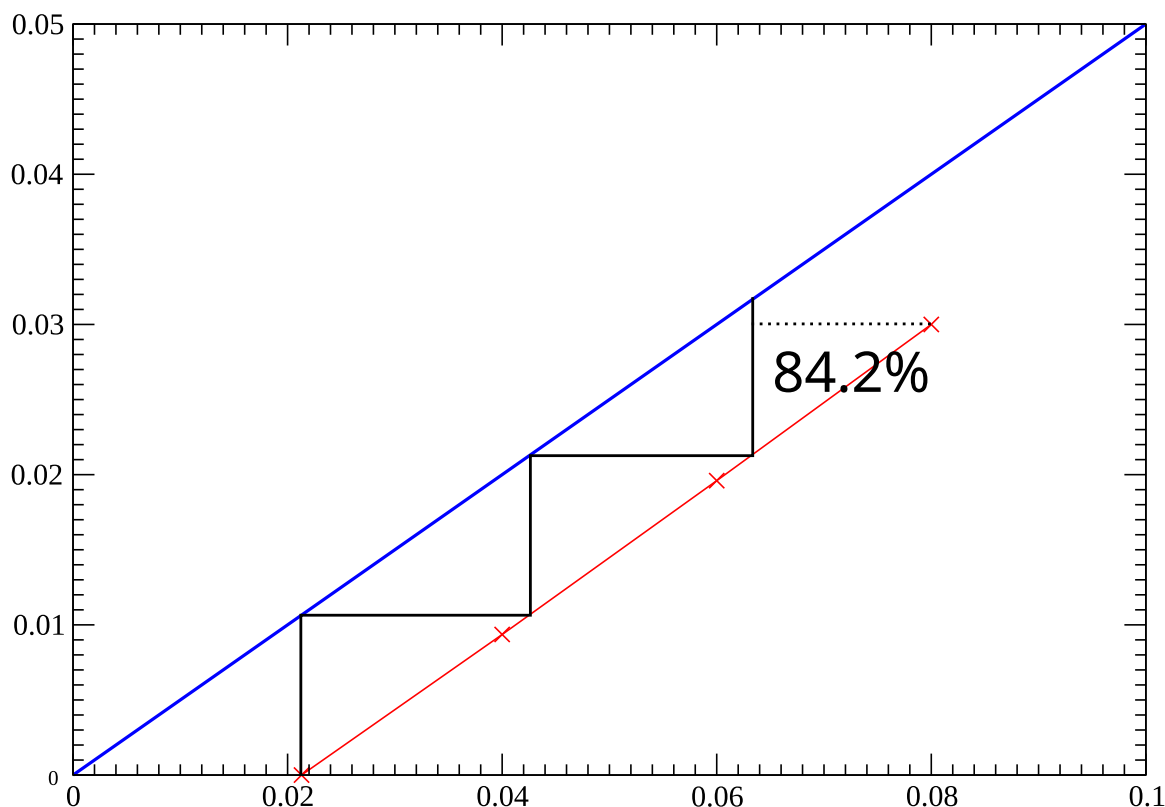
[4/14]

$\overset{\checkmark}{4}$

[1/14]   The number of theoretical stages is 2.842.$\overset{\checkmark}{1}$

c) A plate column with an overall efficiency of 62% is available, determine the number of plates required.   **[2 marks]**
   Solution:

[2/2]   The number of stages is $2.842/0.62 = 4.58 = 5$ plates.$\overset{\checkmark}{2}$

[Question total: 20 marks]

5   Week 5: Single-stage distillation

Q.5.14   Question 5.14   **Recommended**

Show that for a binary mixture having a constant relative volatility, α, the mole fraction of the Most Volatile Vomponent (MVC) in the vapour phase, y_A, is related to the mole fraction of the MVC in the liquid phase, x_A, by the following equation: **[10 marks]**

$$y_A = \frac{\alpha\, x_A}{1 + (\alpha - 1)x_A}$$

State clearly any assumptions made.
Solution:
Assuming the mixture is ideal, we can use Raoult's law to define the relationship between the partial pressure and the vapour pressure:

$$p_A(T) = x_A P_A^0(T)$$

[2/10] $\checkmark \atop 2$ where $p_A(T)$ is the partial pressure of component A at temperature T. We know that the sum of the partial pressures is the total pressure:

$$P = p_A + p_B$$

[2/10] $\checkmark \atop 2$ And that Dalton's law for an ideal gas mixture gives the relationship between the vapour concentration y and the partial pressure:

$$y = p_A/P$$

[2/10] $\checkmark \atop 2$ Combining these expressions, we have

$$y = \frac{p_A}{P}$$
$$= \frac{x P_A^0}{p_A + p_B}$$
$$y = \frac{x P_A^0}{x P_A^0 + (1 - x) P_B^0}$$

[2/10] $\checkmark \atop 2$ Dividing the top and bottom of the fraction by P_B^0, we have

$$y = \frac{x P_A^0/P_B^0}{x P_A^0/P_B^0 + (1 - x)}$$

But we have $\alpha = P_A^0/P_B^0$, which gives

$$y = \frac{x \alpha}{1 + (\alpha - 1)x}$$

[2/10] $\checkmark \atop 2$ which is the expression required.

[Question total: 10 marks]

Q.5.15 Question 5.15
A 20 mol% benzene and 80 mol% toluene mixture is heated then passes through a controlled pressure-reducing valve, causing the stream to flash such that 20% of the feed is vapourised.

a) Determine the composition of the liquid and vapour given that the relative volatility of the system is $\alpha \approx 2.5$. **[8 marks]**
   Solution:
   A mass balance over a flash drum gives

$$F x_F = x L + y V$$

We have $V = 0.2 F$ and $L = 0.8 F$. Inserting this and $x_F = 0.2$, we have:

$$y = 1 - 4 x$$

As the exit streams are in equilibrium, we use the relative volatility VLE relationship:

$$1 - 4x = \frac{\alpha x}{1 + (\alpha - 1)x}$$

$$1 - 4x = \frac{2.5x}{1 + 1.5x}$$

$$(1 + 1.5x)(1 - 4x) = 2.5x$$

$$1 - 5x - 6x^2 = 0$$

Using the quadratic equation, we have $x = -1$ or $x = 1/6$. The positive value is the real root, which gives $y = 1/3$.

b) How might a higher degree of separation be achieved?   **[2 marks]**
   Solution:
   Simply using multi-stage distillation (batch or continuous) would allow a higher top-product concentration, and/or a lower bottom-product concentration.

[Question total: 10 marks]

Q.5.16 Question 5.16

Practice using Raoult's law and Dalton's law by setting up a calculation for VLE data (a spreadsheet or MATLAB program will do). Plot "smooth curve" data for benzene-toluene binary mixtures (including a x-y line using

a) Data you find in the literature.

b) Data calculated using a constant relative volatility of 2.4.

c) Data from the following temperature and vapour pressure values (1 atm).

T (°C)	$P^0_{benzene}$	$P^0_{toluene}$
80.1	101.32	-
85	116.9	46.0
90	135.5	54.0
95	155.7	63.3
100	179.2	74.3
105	204.2	86.0
110.6	240.0	101.32

d) Calculate values of y for $x \in [0.195, 0.372, 0.647, 0.800, 0.910, 0.019]$.

e) Explore the effect of relative volatility on the x-y plot. What is the significance of:

   i) Decreasing relative volatility.
   ii) A relative volitility of one.
   iii) A relative volatility less than one.
   iv) Relative volatilities of zero and infinity.

Solution:
No solution, just practice calculations

[Question end]

Q.5.17 Question 5.17

A liquid mixture of 80 mol% *n*-hexane and 20 mol% benzene enters a long and narrow pipeline at 3 bara and 100 mol s^{-1} and quickly reaches the ambient conditions of 50°C. Antoine data are available for both components in Table 3.

a) Describe the assumptions required to use Raoult's law and justify its use for the hexane-benzene mixture.   **[3 marks]**

Solution:

Raoult's law assumes that the mixture is ideal (each component has identical interactions with each other component). For this to be approximately correct, the components must be similar at a molecular level (homologous). benzene and hexane are simple hydrocarbons with similar molecular weights, therefore it is likely that Raoult's law applies.

b) Using Raoult's law, calculate the allowable pressure drop before the mixture separates into two phases.   **[5 marks]**

Solution:

The saturation pressure of an ideal mixture is given by the sum of the partial pressures, which are in turn given by Raoult's law

$$P = P_{benzene} + P_{hexane} = x_{benzene}\, P_{sat,benzene} + x_{hexane}\, P_{sat,hexane}$$

Evaluating the pure saturation pressures at **50°C**, we have

$$P_{sat,hexane} = 402.4 \text{ torr} = 0.536 \text{ bar}$$
$$P_{sat,benzene} = 271.4 \text{ torr} = 0.362 \text{ bar}$$

The mixture saturation pressure is then

$$P = 0.8 \times 0.536 + 0.2 \times 0.362 = 0.501 \text{ bar} = 376.2 \text{ torr}$$

Thus, if there is a pressure drop of > 2.5 bar, the mixture will divide into two phases.

c) Calculate the relative volatility of *n*-hexane at 50°C and describe the three categories of relative volatility.   **[4 marks]**

Solution:

The relative volatility is the ratio of the saturation partial pressures. There are two choices for the relative volatility depending on which component is the relative one.

$$\alpha = \frac{P_{sat,hexane}}{P_{sat,benzene}} = \frac{0.536}{0.362} \approx 1.48 \approx 0.675^{-1}$$

The three possible categories of relative volatility are:

- $\alpha_{A/B} > 1$, the component **A** is the Most Volatile Component (MVC) and will generally appear in greater concentrations in the vapour phase than in the liquid phase.

- $\alpha_{A/B} < 1$, the opposite is true and **B** is the MVC.

- $\alpha_{A/B} = 1$, when the relative volatility is unity, the saturation vapour pressures of the two components are equal. Distillation of such a mixture is ineffective as the vapour produced from a partial boiling has the same concentration as the original liquid. This is also known as an azeotropic mixture.

d) At the end of the pipeline, the pressure suddenly drops to 0.45 bara, causing the temperature to drop and 15 mol% of the stream to vapourise. Assuming the exit vapour and liquid phases are in equilibrium, calculate the composition and flow rate of the two phases. You should perform mass balances over the pipeline and assume that the relative volatility is constant.   **[8 marks]**

Solution:

We have $V = 0.15 F = 15$ mol s^{-1} giving $L = 85$ mol s^{-1}. Performing another mass balance over the pipeline yields

$$x_F\, F = x\, L + y\, V$$
$$80 = 85\, x + 15\, y$$
$$y = \frac{80}{15} - \frac{85}{15}\, x$$

From the data sheet we can obtain the VLE relationship for constant relative volatility:

$$y = \frac{\alpha\, x}{1 + (\alpha - 1)x}$$

The relative volatility will not have changed much over this pressure drop, thus we will use the same value $\alpha = 1.48$. Using the mass balance to eliminate y from the VLE equation, we have

$$\frac{80}{15} - \frac{85}{15}\, x = \frac{1.48\, x}{1 + (1.48 - 1)x}$$
$$\frac{80}{15}(1 + 0.48\, x) - \frac{85}{15}\, x(1 + 0.48\, x) - 1.48\, x = 0$$
$$-2.72\, x^2 - 4.587\, x + 5.333 = 0$$

Solving this using the quadratic formula which is also in the data sheet, we have

$$x = -\frac{4.587}{2 \times 2.72} \pm \frac{\sqrt{4.587^2 + 4 \times 2.72 \times 5.33}}{2 \times 2.72}$$
$$= -0.843 \pm 1.634$$

The only physical root is $x = 0.791$. The vapour concentration is then

$$y = \frac{80}{15} - \frac{85}{15}\, x = 0.851$$

Component	A	B	C	T_{min} (°C)	T_{max} (°C)
n-hexane	6.91058	1189.640	226.280	-30	170
benzene	6.87987	1196.760	219.161	8	80

Table 3: $\log_{10} P_{sat}(T) = A - B/(T + C)$ where T is in Celsius and P_{sat} is in torr. Please note that 760 torr=1 atm=1.013 bar.

[Question total: 20 marks]

Q.5.18 Question 5.18   **Recommended**

VLE data is needed for a proposed separation of isopropanol (AKA propan-2-ol or 2-propanol) and water at 1 atm. Experimental data is available in Table 4, but a smooth x-y relationship is required for the design calculations.

i) Use Raoult's law to calculate the T-x-y VLE curve for the system and compare it to the experimental data. Expressions for the pure component vapour pressures are given in Table 5. Explain any deviations you observe in terms of the chemical structure of the components.

   General Hint:
   You need to solve the following equation for the temperature T which corresponds to the operating pressure:

   $$P(T, x_A, x_B, ...) = \sum_i P_i^{sat}(T) \, x_i \tag{5}$$

   Once this is done, you can then work out the corresponding vapour concentration using the following expression:

   $$y_A(T, x_A, x_B, ...) = \frac{P_A^{sat}(T) \, x_A}{\sum_i P_i^{sat}(T) \, x_i}$$

   You will need to do this repeatedly for a range of liquid concentrations to plot a smooth curve. Double check the units of all parameters and please note that the log function in the Antoine expression is base 10.

   Excel Hint:
   To solve the equation above using excel, set the liquid concentrations, then use goal seek to vary the temperature to match the calculated pressure to the operating pressure. Ensure your temperature is initially set to a reasonable value (non-zero, positive) before running goal seek, otherwise it might not converge.

   Matlab Hint:
   Use the *fsolve* function to numerically solve the equation $P(T, x) - P_{target} = 0$ by varying the boiling temperature T. Ensure your initial guess for the temperature is reasonable (non-zero, positive).

ii) Use modified Raoult's law to calculate the T-x-y curve and compare to the experimental data. Comment on the agreement and its suitability for design calculations.

   The Margules equations for the activity coefficients are:

   $$\ln \gamma_A = x_B^2 \left(\mathcal{A}_{AB} + 2 \, x_A \left[A_{BA} - A_{AB} \right] \right)$$
   $$\ln \gamma_B = x_A^2 \left(\mathcal{A}_{BA} + 2 \, x_B \left[A_{AB} - A_{BA} \right] \right)$$

   where the coefficients for isopropanol(A)-water(B) are $A_{AB} = 2.3319$ and $A_{BA} = 0.8976$, taken from Table 13-2 of Perry's 7th Ed.

Table 4: Experimental VLE data for Isopropanol-water mixtures at 1 atm.

T °C	82.2	81.48	80.70	80.37	80.23	80.11	80.16	80.15	80.31	80.38	80.67
x	100.00	95.35	87.25	80.90	76.50	69.55	66.05	64.60	55.90	51.45	44.60
y	100.00	93.25	83.40	77.45	73.70	69.15	67.15	66.45	62.55	60.75	59.20

T °C	80.90	81.28	81.29	81.23	81.62	81.75	81.58	81.99	82.32	82.70	84.57
x	38.35	29.80	29.75	28.35	24.50	19.35	18.95	16.65	12.15	10.00	5.70
y	57.00	55.10	55.40	55.30	53.90	53.20	53.75	52.15	51.20	50.15	45.65

T °C	88.05	93.40	95.17	100.0
x	3.65	1.60	1.15	0.00
y	36.55	21.15	16.30	0.00

Table 5: Antoine coefficients for isopropanol and water (taken from Table 13-4 in Perry's 7th ed.).

Species	A	B	C	Range
2-Propanol	8.87829	2010.320	252.636	$-26 \rightarrow +83$ °C
Water	8.07131	1730.630	233.426	$+1 \rightarrow +100$ °C

$\log_{10} P^{sat} = A - B/(T + C)$ where P^{sat} is in units of torr and T is in units of Celcius.

[Question end]

Q.5.19 Question 5.19   **Recommended**

You are in the business development (BD) unit of Nirvana Compounds Ltd, a company that specialises in producing volatile organic compounds (VOCs). They have asked you, as their top process engineer, to perform a study into the efficiency of batch and flash distillation when used to separate a benzene-toluene mixture. This product will then be sold under the TeenSpirit volatile compounds brand.

An equimolar mixture of toluene and benzene is produced from an existing process and is available for separation.

a) What is the outlet concentration of the vapour and liquid streams from a flash drum? The flash drum is operated so that only 20% of the feed stream is vapourised. x-y VLE data is available in Fig. 16. **[7 marks]**

Solution:

To design a flash drum, we only need to plot the operating line on the VLE chart and to solve for the intersection with the equilibrium line.

The operating line equation is given by a mass balance of volatile component over the flash drum.

$$x_f F = x L + y V$$

$$y \frac{V}{F} = x_f - x \frac{L}{F}$$

[1/7] The feed mixture is equimolar, so $x_F = 0.5$. We note that the vapour stream is equal to
[1/7] $V = 0.2 F$, from the total mass balance we have $F = V + L$, so we have $L = 0.8 F$. If we
[1/7] substitute these in above, we have

$$x_f F = x L + y V$$

$$0.2 y = 0.5 - 0.8 x$$

$$y = 2.5 - 4 x$$

[1/7] $\checkmark_1$ We need to generate two points to plot the straight line. I choose ($x = 0.375$, $y = 1$) and
[2/7]   ($x = 0.625$, $y = 0$) as these coincide with the axis and are therefore easy to plot.$\checkmark_2$ The outlet
     concentrations, read from where the equilibrium line and operating line cross, are $x = 0.457$
[1/7]   and $y = 0.676$.$\checkmark_1$

b) What is the average vapour concentration and the final liquid concentration if the same
feed is supplied to a single-stage batch still instead? You should assume the same pro-
portion of vapour is produced as in the flash drum. Integrated VLE data is available in
Fig. 17 for this calculation.   **[8 marks]**
Note: You may find the following expression useful

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_0^{x_{final}} (y - x)^{-1}\, dx - \int_0^{x_{initial}} (y - x)^{-1}\, dx$$

Solution:
We know that $L_{final}/L_{initial} = 0.8$, as 20% of the feed liquid is vapourised.

We somehow need to calculate the outlet concentration of the still using the given equation

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_0^{x_{final}} (y - x)^{-1}\, dx - \int_0^{x_{initial}} (y - x)^{-1}\, dx$$

[2/8]   We note that $\ln(L_{final}/L_{initial}) = -0.223$,$\checkmark_2$ but the right hand side is impossible to numerically
calculate, as we only have graphical VLE data.

However, we know the initial concentration $x_{initial}$, so we can calculate the first term on the
right hand side by looking up $x = x_{initial} = 0.5$ to find

$$\int_0^{x_{initial}} (y - x)^{-1}\, dx \approx 4.1$$

[2/8] $\checkmark_2$ Therefore, the Rayleigh equation becomes

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_0^{x_{final}} (y - x)^{-1}\, dx - \int_0^{x_{initial}} (y - x)^{-1}\, dx$$

$$-0.223 = \int_0^{x_{final}} (y - x)^{-1}\, dx - 4.1$$

$$\int_0^{x_{final}} (y - x)^{-1}\, dx = 4.1 - 0.223 = 3.88$$

We can look this value up in the chart to calculate the exit concentration and we find
[2/8] $x_{final} = 0.45$.$\checkmark_2$

We can then find the average vapour concentration from the mass balance

$$x_{initial}\, L_{initial} = x_{final}\, L_{final} + y_{avg}\, V$$
$$0.5\, L_{initial} = 0.45\, L_{final} + y_{avg}\, V$$

Again, we know that $L_{final} = 0.8\, L_{initial}$, and that $V = 0.2\, L_{initial}$, So we have

$$0.5\, L_{initial} = 0.45 \times 0.8\, L_{initial} + y_{avg}\, 0.2\, L_{initial}$$
$$0.5 = 0.45 \times 0.8 + 0.2\, y_{avg}$$
$$y_{avg} = 0.7$$

[2/8] $\checkmark_2$ So a batch distillation process would produce $x = 0.45$ and $y = 0.7$ concentration outputs.
A flash distillation process would produce $x = 0.457$ and $y = 0.676$ outputs, but would be
much cheaper to run and build.

c) A young maverick process engineer laughs at your use of graphical methods, and says you should just have used a relative volatility of $\alpha \approx 2.45$ and the formula given in the lecture notes. Does this give reasonable results when compared to the graphical solution?

[5 marks]

Solution:

From the notes, we have

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = (\alpha - 1)^{-1}\ln\left(\frac{x_{final}(1 - x_{initial})}{x_{initial}(1 - x_{final})}\right) + \ln\left(\frac{1 - x_{initial}}{1 - x_{final}}\right)$$

Substituting everything we know to to calculate the final concentration, we have

$$\ln(0.8) = (2.45 - 1)^{-1}\ln\left(\frac{x_{final}(1 - 0.5)}{0.5(1 - x_{final})}\right) + \ln\left(\frac{1 - 0.5}{1 - x_{final}}\right)$$

We can take this equation and solve it in Matlab or excel to determine the exit concentration.

However, just to stay in shape (mathematically), we will try to find a solvable form. It will not result in a quadratic equation so we cannot directly solve it. But we can express it in an iterable form like so

$$\ln(0.8) = 1.45^{-1}\ln\left(\frac{x_{final}}{1 - x_{final}}\right) + \ln\left(\frac{0.5}{1 - x_{final}}\right)$$

$$\ln(0.8) = \ln\left[\left(\frac{x_{final}}{1 - x_{final}}\right)^{1/1.45}\frac{0.5}{1 - x_{final}}\right]$$

$$0.8 = \left(\frac{x_{final}}{1 - x_{final}}\right)^{1/1.45}\frac{0.5}{1 - x_{final}}$$

$$x_{final} = 1 - 0.625\left(\frac{x_{final}}{1 - x_{final}}\right)^{1/1.45}$$

However, when I tried to use this expression, it turned out to be highly unstable! We would have to resort to some more complicated root finding technique (e.g., Newton-Raphson).

I obtain $x_{final} = 0.452$ using Maxima. This is quite close to the graphically calculated value so it appears the maverick is right. However, their method is actually more complicated to solve and requires more work with unstable equations and doubt about finding the correct root to the equation!

d) You decide to teach the maverick a lesson, literally, and tell them when is it acceptable to use a constant relative volatility and how this constant volatility is calculated?

Solution:

You can use a constant relative volatility if the relative volatility varies no more than 15% over the operating conditions of the distillation equipment.

If this is true, the average relative volatility over this region should be used in the calculations.
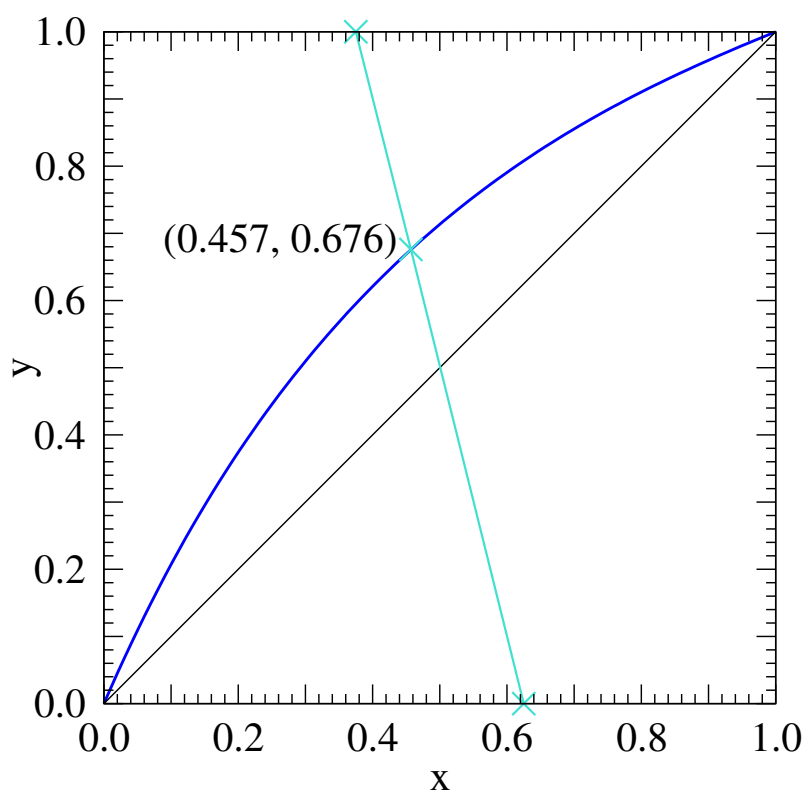
Figure 14: VLE data for benzene(x/y)-toluene mixtures, including the solution for the flash drum.
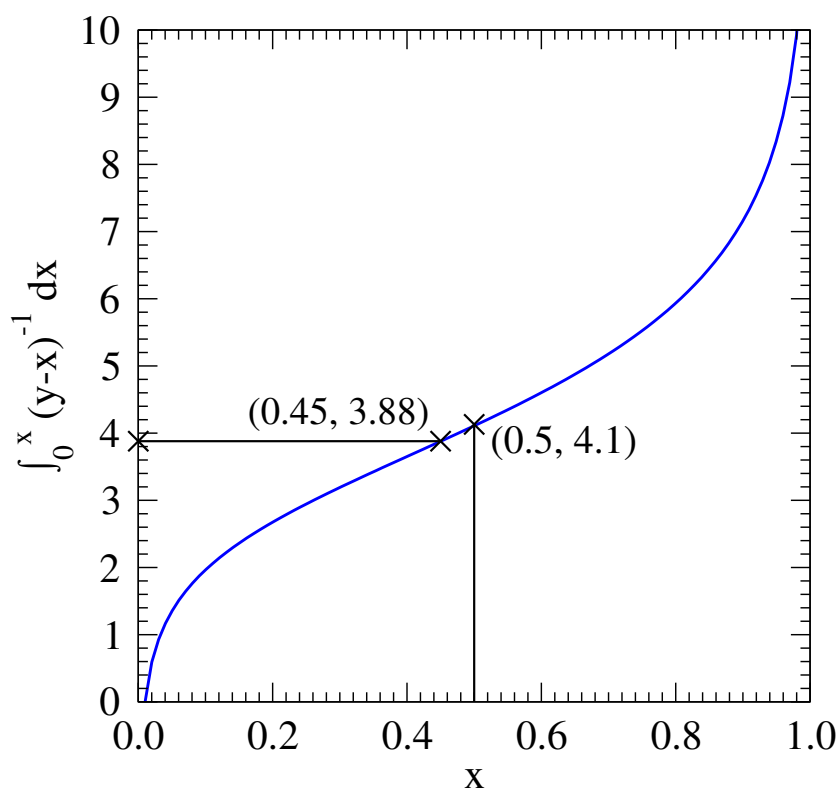

Figure 15: Integrated VLE data for benzene(x/y)-toluene mixtures, including the solutions for the batch distiller.

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.
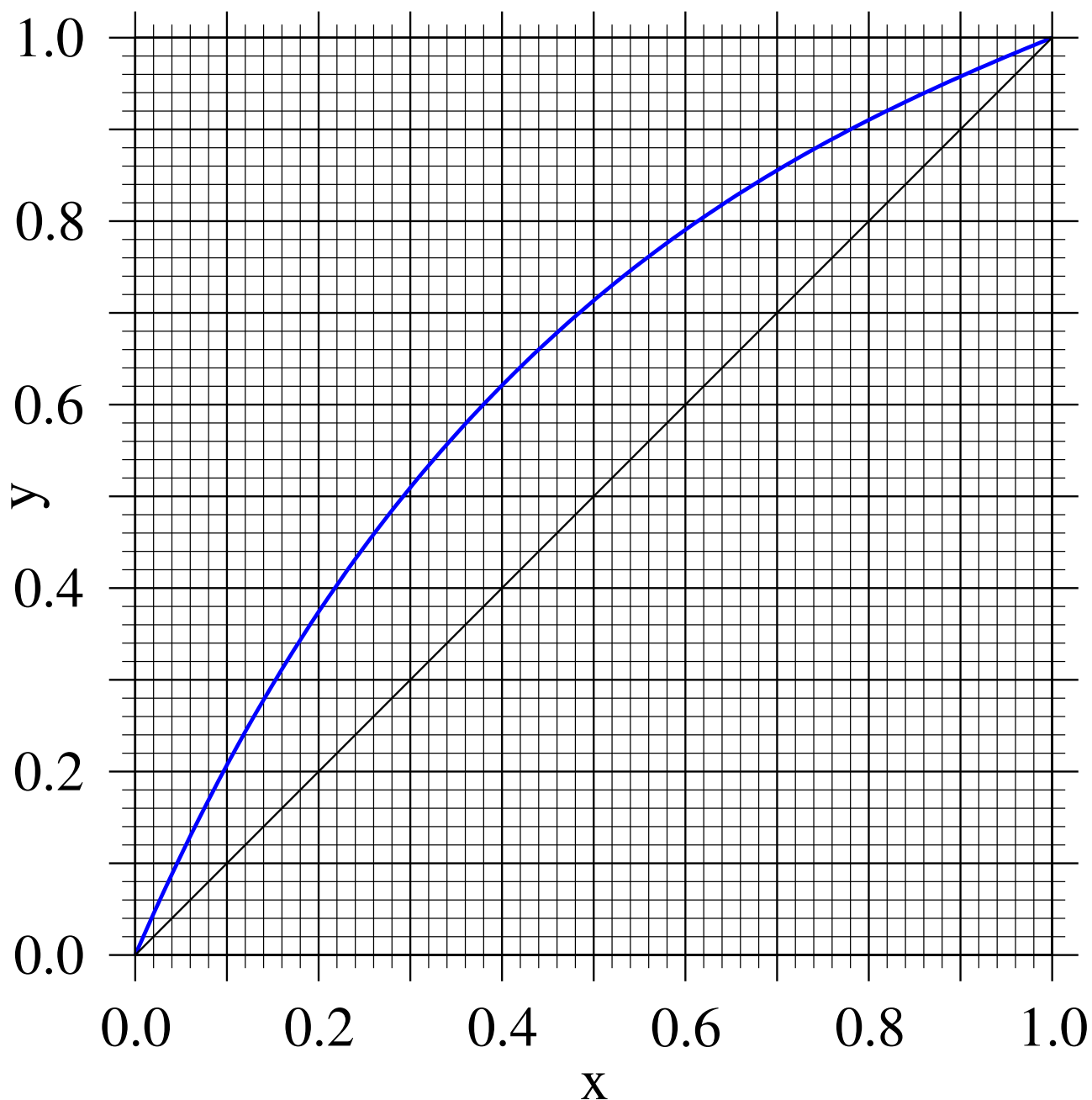
Student ID:

Figure 16: VLE data for benzene(x/y)-toluene mixtures. For use in Q. 5.19, 5.20, and 7.30.

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.
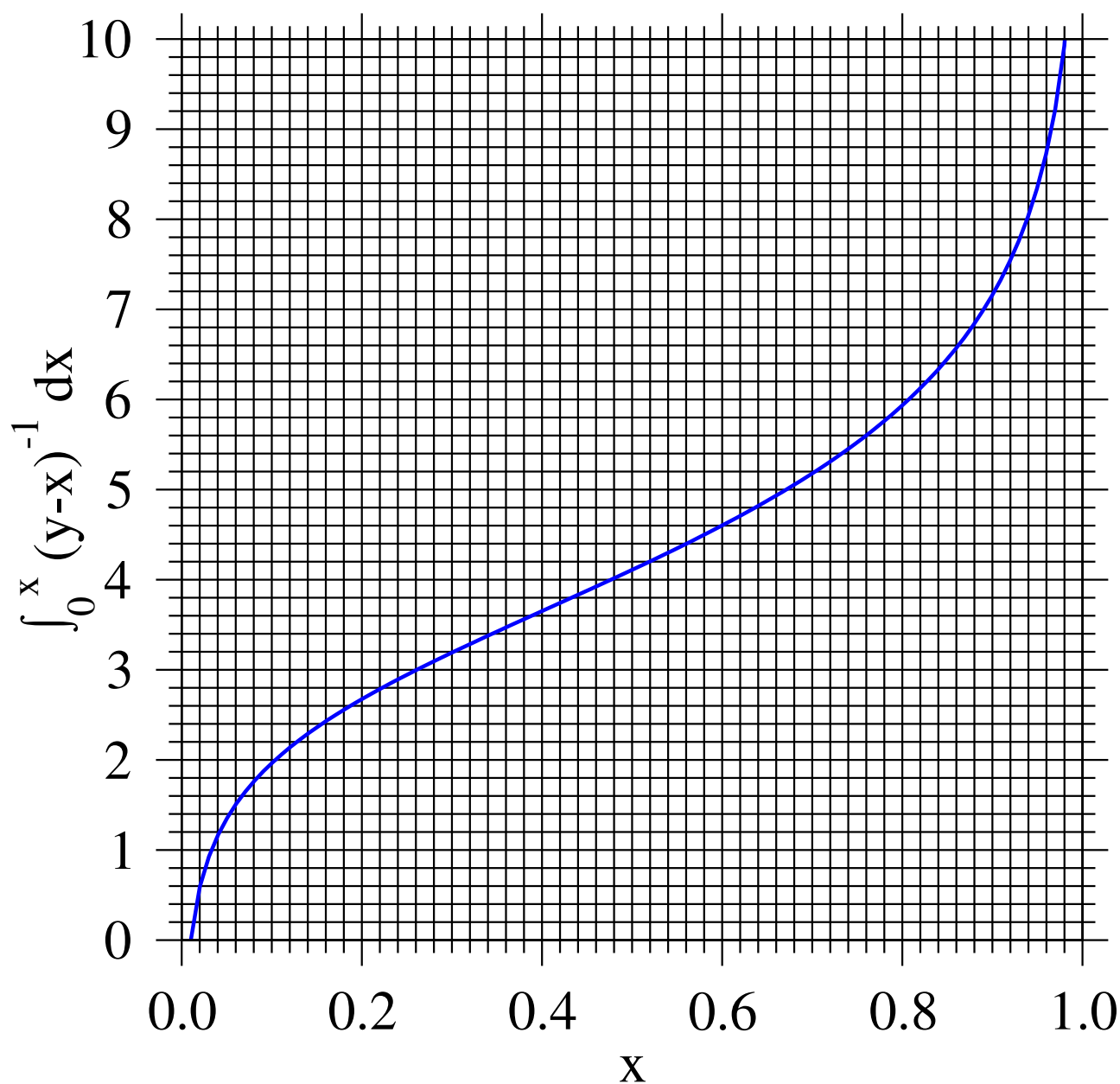
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Figure 17: Integrated VLE data for benzene(x/y)-toluene mixtures. For use in Q. 5.19 and 5.20.

[Question total: 20 marks]

Q.5.20 Question 5.20

A 10,000 kmol benzene-toluene mixture is sent for batch distillation at atmospheric pressure. Integrated VLE data is available in Fig. 17 and standard VLE data is available in Fig. 16.

a) Derive the following equation, known as Rayleigh's equation, for the concentration in the evaporator.   **[10 marks]**

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_{x_{initial}}^{x_{final}} \frac{dx}{y - x}$$

where $L_{initial}$ and L_{final} are the initial and final molar amounts of liquid within the still, x is the concentration of the liquid, and y is the concentration of the produced vapour.

Solution:

Assume the still is in operation. We have L moles of liquid in the still with a concentration of x. In a small instant of time, dL moles of liquid boil to vapour of concentration y. This

[3/10]   reduces the amount of liquid by dL, and the concentration of the liquid by dx. $\overset{\checkmark}{3}$

Performing a mass balance between these two instants of time, we have

$$x\,L = (x - dx)\,(L - dL) + y\,dL$$

[1/10] $\overset{\checkmark}{1}$ We can cancel the $dx\,dL$ term as it is small$\overset{\checkmark}{1}$, and we have

[1/10]

$$x\,L = x\,L - L\,dx - x\,dL + y\,dL$$

[1/10] $\overset{\checkmark}{1}$ Performing some rearrangement, we have

$$\frac{dL}{L} = \frac{dx}{y - x}$$

[2/10] $\overset{\checkmark}{2}$ Finally, integrating both sides from the start to the end of the batch distillation we have

$$\int_{L_{initial}}^{L_{final}} \frac{dL}{L} = \int_{x_{initial}}^{x_{final}} \frac{dx}{y - x}$$

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_{x_{initial}}^{x_{final}} \frac{dx}{y - x}$$

[2/10] $\overset{\checkmark}{2}$

b) The distillation is begun and the initial vapour concentration is 60 mol% benzene. Determine the initial liquid concentration in the still.   **[2 marks]**

Solution:

[2/2]   Using Fig. 16, the liquid in equilibrium with a $y = 0.6$ mixture is $x = 0.38$. $\overset{\checkmark}{2}$

c) Distillation is continued until the produced vapour concentration drops to 40 mol% benzene. Determine the amount of liquid remaining in the still.   **[6 marks]**

Solution:

At 40 mol% vapour, the equilibrium liquid concentration is now 22 mol%. To use Rayleigh's equation we need to evaluate the integral between these two liquid concentration limits.

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_{x_{initial}}^{x_{final}} \frac{dx}{y - x}$$

Rewriting the integral so that the pre-integrated graph might be used gives

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_0^{x_{final}} \frac{dx}{y - x} - \int_0^{x_{initial}} \frac{dx}{y - x}$$

[1/6] ✓₁ Reading from the graph, we have $\int_0^{x_{final}}(y - x)^{-1}dx \approx 2.8$ and $\int_0^{x_{initial}}(y - x)^{-1}dx \approx 3.6$.
[2/6] ✓₂ Calculating the final concentration in the still

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_{x_{initial}}^{x_{final}} \frac{dx}{y - x}$$
$$L_{final} = 10\,000\exp(2.8 - 3.6)$$
$$\approx 4493 \text{ kmol}$$

[3/6] ✓₃

d) Calculate the average concentration of the collected distillate.   **[2 marks]**
Solution:
The total amount of distillate produced is obtained through an overall mass balance:

$$L_{distillate} = L_{initial} - L_{final}$$
$$\approx 5507 \text{ kmol}$$

[1/2] ✓₁ Performing a benzene mass balance over the distillation gives

$$y_{avg} = \frac{x_{initial}\, L_{initial} - x_{final}\, L_{final}}{L_{distillate}}$$
$$= \frac{0.38 \times 10000 - 0.22 \times 4493}{5507}$$
$$\approx 0.511$$

[1/2] ✓₁

[Question total: 20 marks]

Q.5.21 Question 5.21

An undergraduate is investigating the distillation of a 40 mol% isopropanol and 60 mol% water mixture in the Chemical Engineering laboratory. The distillation unit has had its column removed and is being operated as a single-stage unit.

a) Derive the following equation, known as Rayleigh's equation, for the concentration in the evaporator.   **[10 marks]**

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_{x_{initial}}^{x_{final}} \frac{dx}{y - x}$$

where $L_{initial}$ and L_{final} are the initial and final molar amounts of liquid within the still, x is the concentration of the liquid, and y is the concentration of the produced vapour.
Solution:
Assume the still is in operation. We have L moles of liquid in the still with a concentration of x. In a small instant of time, dL moles of liquid boil to vapour of concentration y. This reduces the amount of liquid by dL, and the concentration of the liquid by dx

Performing a mass balance between these two instants of time, we have

$$x\,L = (x - dx)\,(L - dL) + y\,dL$$

We can cancel the $dx\,dL$ term as it is small, and we have

$$x\,L = x\,L - L\,dx - x\,dL + y\,dL$$

Performing some rearrangement, we have

$$\frac{dL}{L} = \frac{dx}{y - x}$$

Finally, integrating both sides from the start to the end of the batch distillation we have

$$\int_{L_{initial}}^{L_{final}} \frac{dL}{L} = \int_{x_{initial}}^{x_{final}} \frac{dx}{y - x}$$

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_{x_{initial}}^{x_{final}} \frac{dx}{y - x}$$

b) Using the integrated VLE data in Fig. 18, calculate the final concentration in the batch still and average vapour concentration if 50 mol% of the mixture is evaporated.   **[8 marks]**
Solution:
Starting with Rayleigh's equation

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_{x_{initial}}^{x_{final}} \frac{dx}{y - x}$$

$$\ln(0.5) = \int_{x_{initial}}^{x_{final}} \frac{dx}{y - x}$$

We can split the integral into two parts

$$\ln(0.5) = \int_{x_{initial}}^{x_{final}} \frac{dx}{y - x}$$

$$= \int_{0}^{x_{final}} \frac{dx}{y - x} - \int_{0}^{x_{initial}} \frac{dx}{y - x}$$

We know that $x_{initial} = 0.40$, looking up the value of the integral in Fig. 18, we have

$$\int_{0}^{x_{initial}} \frac{dx}{y - x} \approx 1.4$$

Using this to find the value of the other integral

$$\int_{0}^{x_{final}} \frac{dx}{y - x} = \ln(0.5) + 1.4 \approx 0.71$$

Looking for this integral value on the chart, we can see that $x_{final} \approx 0.25$. Using a mass balance, we know that

$$L_{initial}\, x_{initial} = L_{final}\, x_{final} + L_{distillate}\, x_{distillate}$$

$$L_{initial}\, x_{initial} = 0.5\, L_{initial}\, x_{final} + 0.5\, L_{initial}\, x_{distillate}$$

$$x_{distillate} = 2\, x_{initial} - x_{final} \approx 0.55$$

c) Consider the integrated VLE data in Fig. 18, why does the graph diverge in the middle of the plot? Why can you not use batch distillation to distill from low concentrations to high concentrations?

Hint: The VLE behaviour of isopropanol-water mixtures is similar to ethanol-water systems. **[2 marks]**

Solution:

The divergence in the middle of the plot is due to the presence of an azeotrope. It is impossible to use a single batch or continuous distillation process to distill concentrations across the azeotrope.
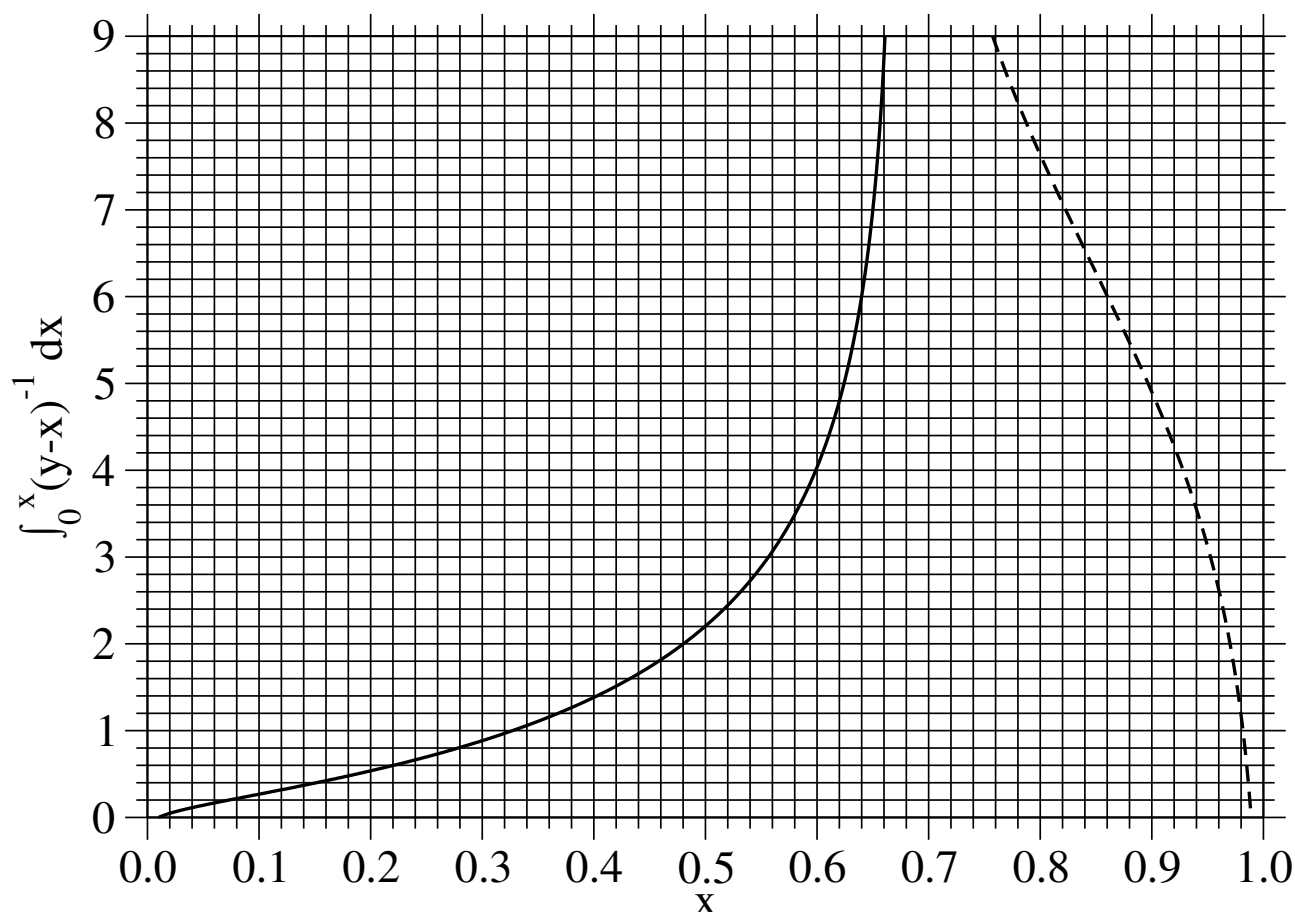

Figure 18: Integrated VLE data for the Isopropanol-water system in Q. 5.21. **Note:** The solid and dashed curves are distinct and no single-stage equilibrium process connects them.

[Question total: 20 marks]

Q.5.22   Question 5.22

For mixtures with an almost constant relative volatility, Rayleigh's equation may be solved to give

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = (\alpha - 1)^{-1} \ln\left(\frac{x_{final}(1 - x_{initial})}{x_{initial}(1 - x_{final})}\right) + \ln\left(\frac{1 - x_{initial}}{1 - x_{final}}\right)$$

a) Define the relative volatility and describe the three cases, $\alpha > 1$, $\alpha = 1$, and $\alpha < 1$. **[3 marks]**

Solution:
The relative volatility is defined as the ratio of the saturation vapour pressures of two components.

$$\alpha = \frac{P_{sat.,A}}{P_{sat.,B}}$$

For $\alpha > 1$, the component A is the Most Volatile Component (MVC) and will appear in greater concentrations in the vapour phase than the liquid phase. For $\alpha < 1$, the opposite is true and B is the MVC. When the relative volatility is unity ($\alpha = 1$), the vapour pressures are equal. Distillation of such a mixture is ineffective as the vapour produced from a partial boiling has the same concentration as the original liquid. Mixtures which have $\alpha = 1$ are known as azeotropic.

b) A poteen producer wants to distill a 2.4 kmol mixture of ethanol (3.5 mol%) and water using Rayleigh distillation to produce a liqueur. Assuming an effective relative volatility of $\alpha \approx 7$, determine the concentration of the liqueur if 0.6 kmol of distillate is produced.

[7 marks]

Solution:
The most convenient form of the Rayleigh equation comes about from removing the logarithms.

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = (\alpha - 1)^{-1} \ln\left(\frac{x_{final}(1 - x_{initial})}{x_{initial}(1 - x_{final})}\right) + \ln\left(\frac{1 - x_{initial}}{1 - x_{final}}\right)$$

$$\left(\frac{L_{final}}{L_{initial}}\right)^{\alpha-1} = \frac{x_{final}}{x_{initial}}\left(\frac{1 - x_{initial}}{1 - x_{final}}\right)^{\alpha}$$

Substituting in the known values and rearranging to an iterative form:

$$\left(\frac{1.8}{2.4}\right)^6 = \frac{x_{final}}{0.035}\left(\frac{1 - 0.035}{1 - x_{final}}\right)^7$$

$$x_{final} = 0.035\left(\frac{1.8}{2.4}\right)^6\left(\frac{1 - x_{final}}{1 - 0.035}\right)^7$$

$$x_{final} = 0.008\left(1 - x_{final}\right)^7$$

Guessing at an initial solution of $x_{final} = 0.035$, we can use the above formula to iterate to the result (students will probably have used the solve function on their calculator before reaching here, but this route is available).

Iteration	0	1	2	3	4
x_{final}	0.035	0.00623	0.00766	0.00758	0.00758

The average concentration of the distillate can be found from a mass balance.

$$x_{distillate} = \frac{x_{initial}\,L_{initial} - x_{final}\,L_{final}}{L_{initial} - L_{final}}$$

$$x_{distillate} = \frac{0.035 \times 2.4 - 0.00758 \times 1.8}{0.6} \approx 0.117$$

The liqueur has a concentration of 11.7 mol%.

[Question total: 10 marks]

6   Week 6: Multi-stage distillation

Q.6.23   Question 6.23 **Recommended**

A saturated liquid feed of 200 kmol h^{-1} of a heptane-Ethyl Benzene mixture containing 42% heptane and 58% ethyl benzene is to be fractionated at 101.32 kPa to give a distillate containing 97% heptane and a bottoms containing 1.1% heptane. The reflux ratio used is $R = 2.5$ and vapour-liquid equilibrium data for the mixture is given in Table 6 below.

a) Calculate the number of theoretical trays required to perform this distillation **and** the feed tray number. A blank VLE graph is given in Fig. 20.
Answer: 10.5 stages (but may be around 12 depending on your accuracy).
Solution:

We must generate a VLE curve to use in the McCabe-Thiele graphical method for designing binary distillation columns. To do this we must plot the data in Table 6. Ideally, this data should be interpolated smoothly, as the "real" VLE curves are typically smooth. To generate the smooth VLE plot in Fig. 19, cubic splines were fitted to the data points. This curvature may also be approximated by careful hand drawing.

The blank VLE chart (Fig. 19) is provided for the convenience of students wishing to repeat the remainder of the calculations.

We will also repeat the calculations for a VLE chart where straight lines have been drawn between the VLE data points (see Fig. 22). This is incorrect and you should use a smoothed line in your own calculations, but it is performed to illustrate the effect that interpolation can have on the final result.

The first step of the McCabe-Thiele method is to plot the enrichment section operating line. This is given by the straight line passing through the points
($x = 0$, $y = x_D/(R + 1) = 0.97/(2.5 + 1) = 0.277$)
and
($x = x_D = 0.97$, $y = x_D = 0.97$).

The next line to be drawn is the q-line. The first point on the q-line is given by
($x = x_F = 0.42$, $y = x_F = 0.42$).
We need to generate another point from the q-line equation.

$$y = x\frac{q}{q-1} - \frac{x_F}{q-1}$$

However, the feed is a saturated liquid which means $q = 1$. This results in a **vertical** q-line and we don't need to plot another point, just to draw a vertical line.

Finally, we need to plot the stripping section operating line. We only need one extra data point as one is provided by the intersection of the enrichment section operating line and the q-line. This extra point is located at
($x = x_W = 0.011$, $y = x_W = 0.011$).

We can plot these operating lines on the VLE charts and step to solve for the number of stages. If we use a smoothed VLE graph (Fig. 21), we obtain around 10.5 ideal stages (9.5 ideal column stages and a re-boiler) and a feed stage of 6 (stage 5 in the column if a re-boiler is used). If we compare to the solution obtained with linearly interpolated data, we obtain 11.5 stages and a feed tray of 7. This is a significant difference and care must be taken with the VLE data when designing difficult separations (close VLE and operating lines).

b) Calculate the flow-rates of the top and bottom product.
Solution:

We can obtain these values without solving the distillation design. All we need are the two

mass balances over the whole column

$$F = D + W \qquad\qquad x_F\, F = x_D\, D + x_W\, W$$
$$200 = D + W \qquad\qquad 0.42 \times 200 = 0.97\, D + 0.011\, W$$

Solving these two equations, we obtain $D = 85.3$ kmol h and $W = 114.7$ kmol h.

Temperature (K)	x	y	Temperature (K)	x	y
409.3	0.000	0.000	383.8	0.485	0.730
402.6	0.080	0.230	376.0	0.790	0.904
392.6	0.250	0.514	371.5	1.000	1.000

Table 6: Boiling temperature and VLE data for a heptane-ethyl benzene mixture at 101.32 kPa. Concentrations are reported in the mole fraction of the most volatile component (heptane).
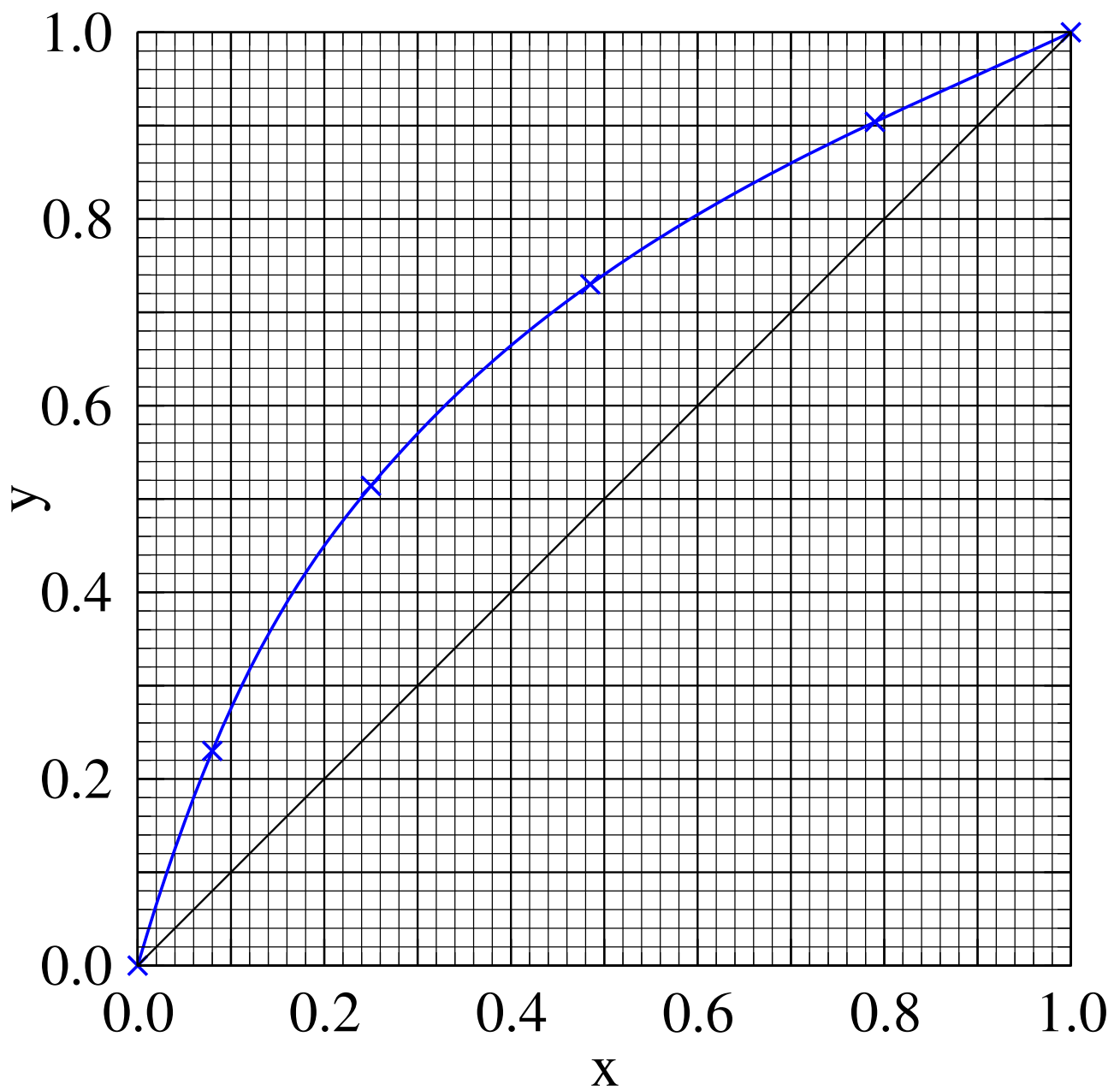
Figure 19: For use in Q. 6.23 if students have trouble plotting their own smooth VLE curve..

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.
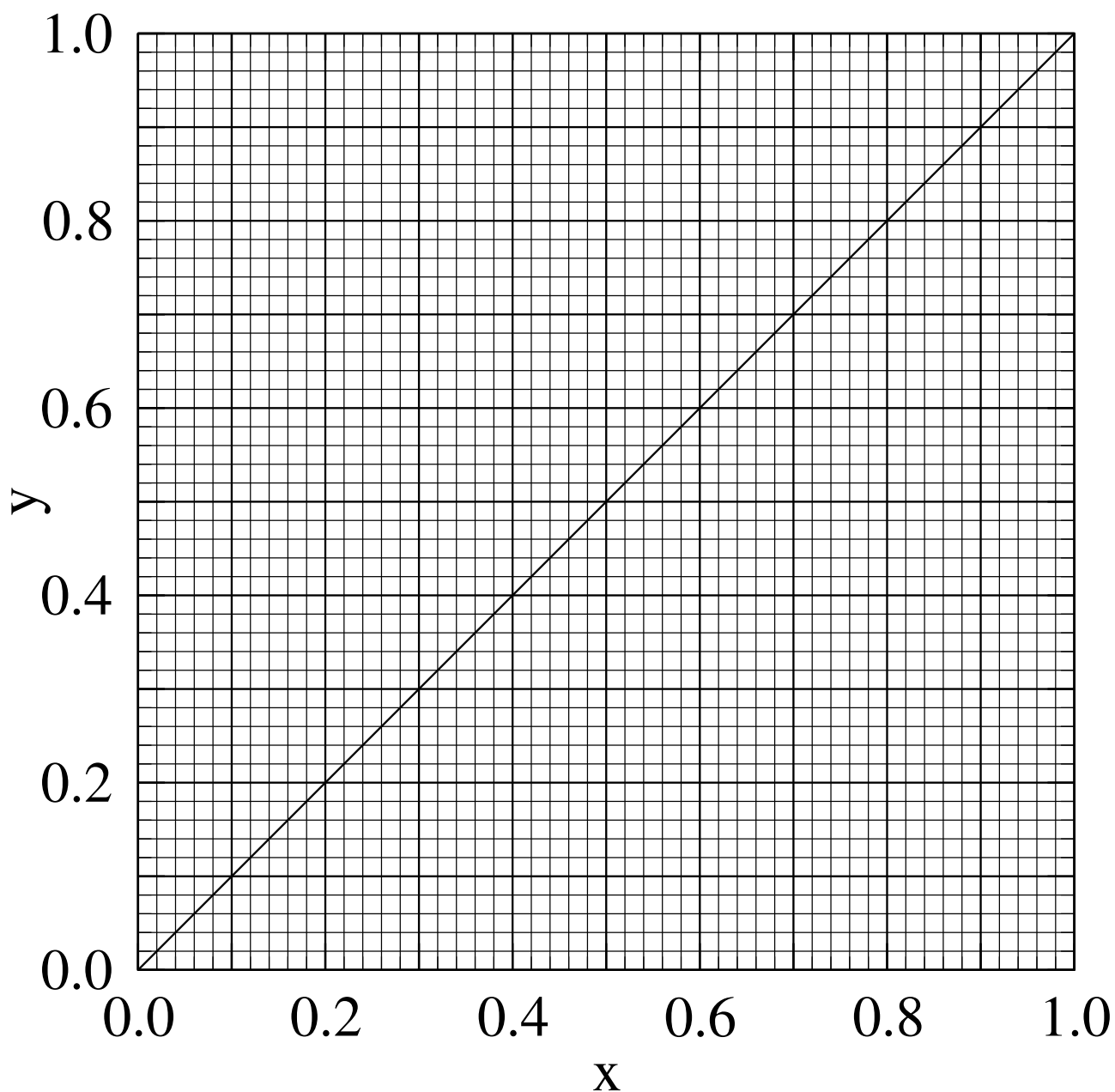
Student ID:

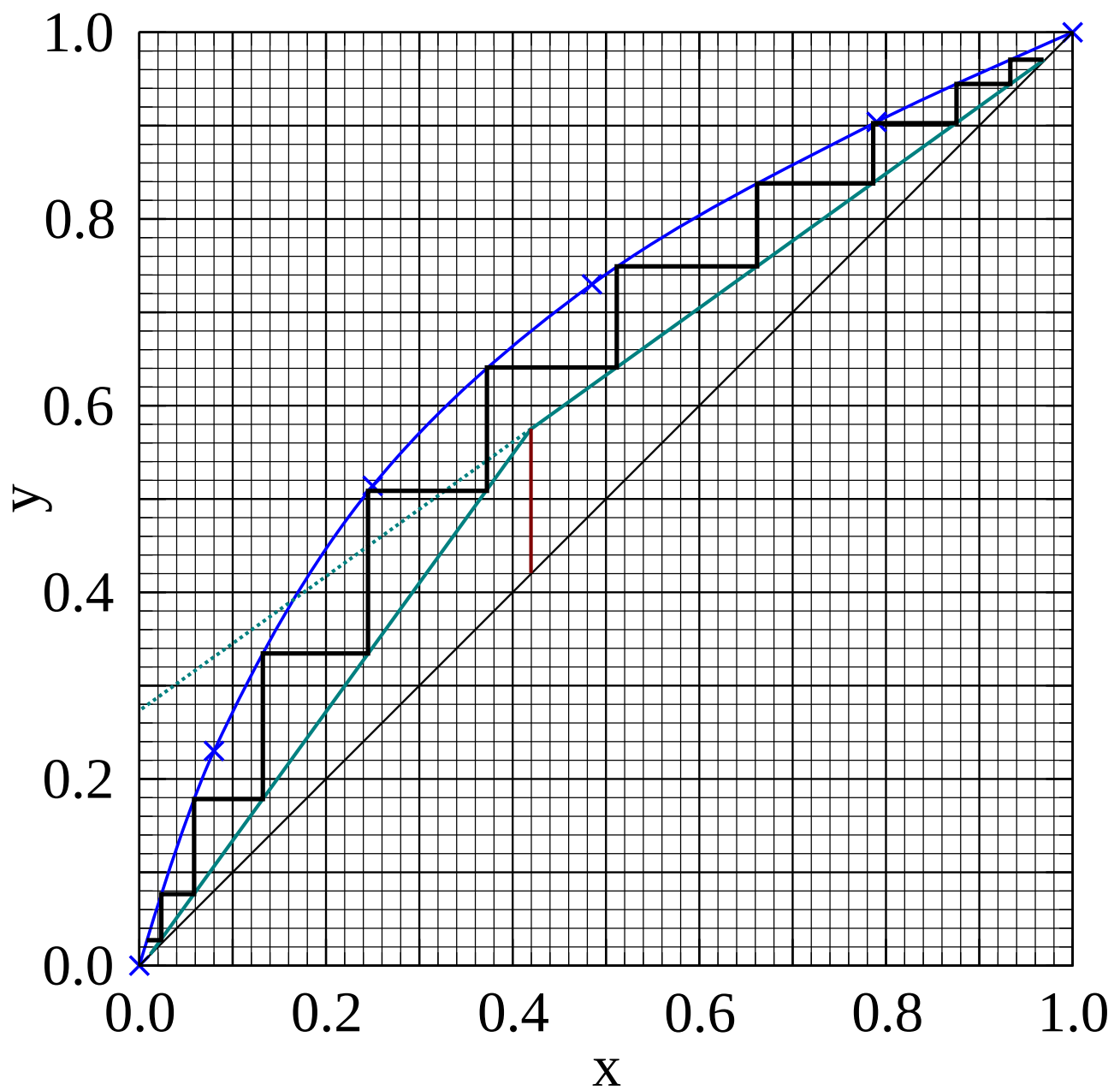Figure 20: A blank chart for plotting VLE/operating-line data on. For use in Q. 6.23.

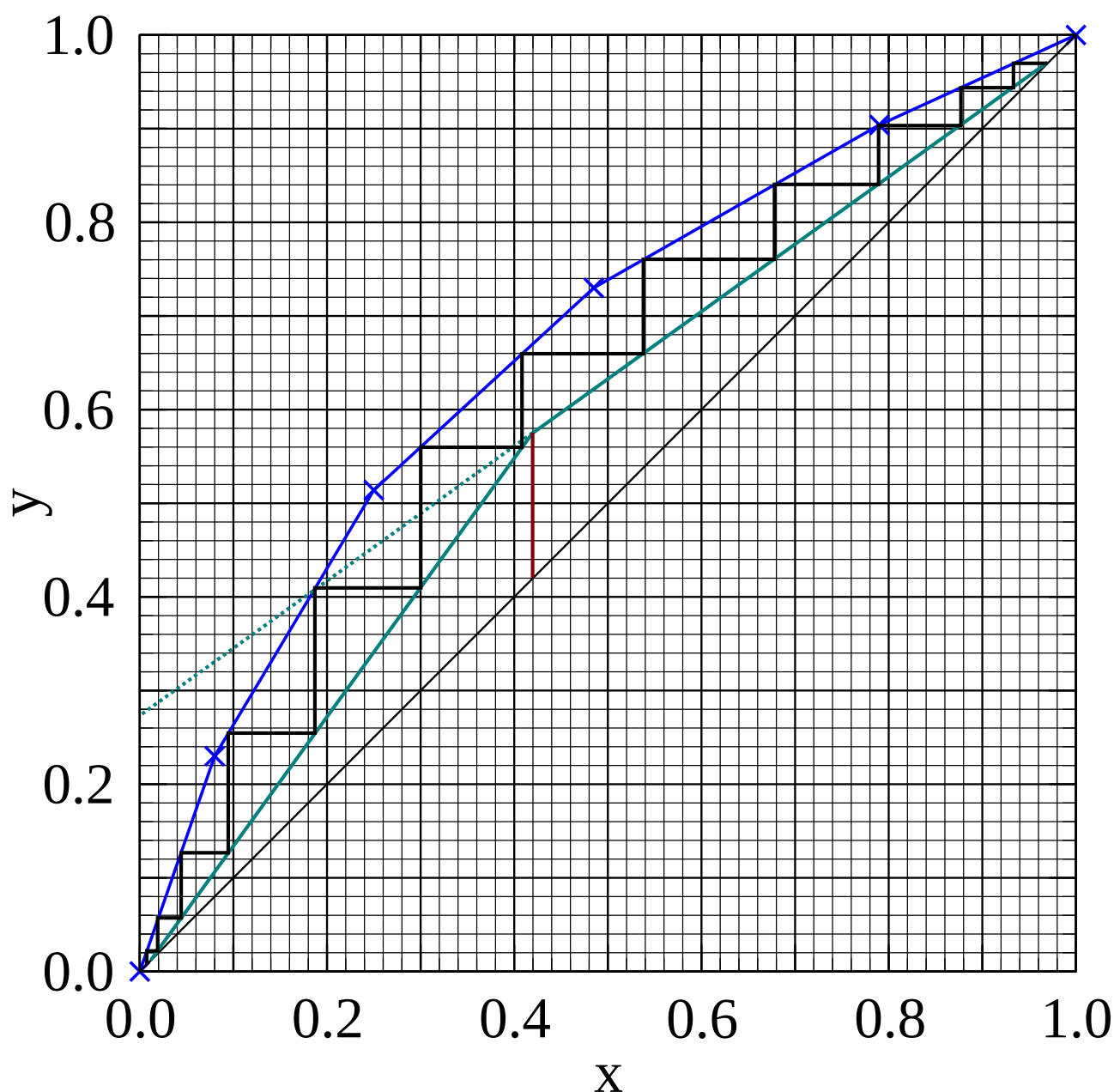Figure 21: A completed VLE chart for the McCabe-Thiele design in Q. 6.23.

Figure 22: A completed VLE chart for the McCabe-Thiele design in Q. 6.23 which incorrectly uses straight lines to interpolate between the VLE data points.

[Question end]

7 Week 7: Multi-stage distillation (azeotropes, partial columns, efficiencies)

Q.7.24 Question 7.24   **Recommended**

Derive the following operating line equation for the enriching section of a column using a mass balance.

$$y_n = \frac{R}{R+1}x_{n+1} + \frac{x_D}{R+1}$$

State any assumptions you make and discuss their range of validity.   **[10 marks]**
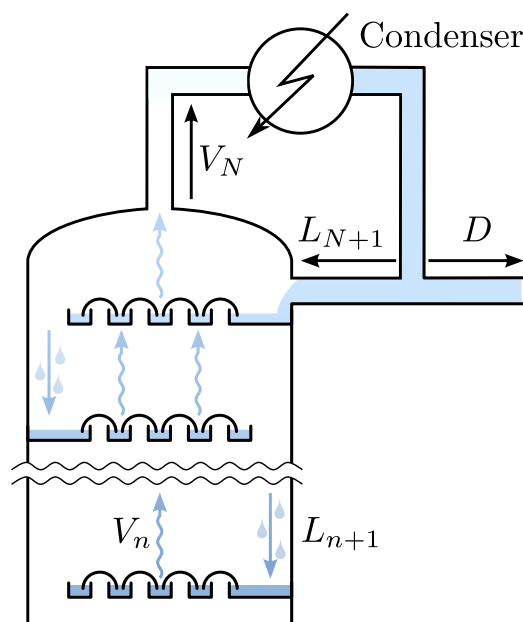
Figure 23: A diagram of the streams in the enrichment section of a distillation column.

Solution:

If the assumption of **constant molar overflow** holds, the liquid and vapour flow-rates are constant in each section (stripping or enrichment) of the column.

$$L_n = L_{n+1} \qquad\qquad V_n = V_{n+1}$$

This assumption is true when the latent heat of vapourisation does not vary significantly within the column (around 10% is acceptable, see C&R Vol. 2) **AND** no vapour condenses due to heat lost from the column surface.

Defining the reflux ratio, we have

$$L_{N+1} = R\,D \qquad\qquad V_N = (R+1)\,D$$

Assuming constant molar overflow, for some arbitrary tray, n, we have

$$L_n = L_{N+1} \qquad\qquad V_n = V_N$$

Now, performing a species mass balance from just above a tray, n, up to and including the distillate stream yields the following expression.

$$y_n\,V_n = x_{n+1}\,L_{n+1} + x_D\,D$$
$$y_n\,V_N = x_{n+1}\,L_{N+1} + x_D\,D$$
$$y_n\,(R+1)\,D = x_{n+1}\,R\,D + x_D\,D$$
$$y_n = x_{n+1}\frac{R}{R+1} + \frac{x_D}{R+1}$$

[Question total: 10 marks]

Q.7.25 Question 7.25

Calculate the *minimum* number of ideal stages required in a distillation column to separate ethanol-water mixtures to a purity of 5 mol% bottoms and 60 mol% distillate. Why is the feed concentration unimportant in this limit? VLE data for ethanol-water mixtures is available in

Fig. 24.   **[8 marks]**

Solution:
The minimum number of theoretical stages are required when the reflux ratio is infinite ($R \to \infty$). Physically, this is only achievable when no distillate is collected from the column. As the column cannot actually separate the feed while remaining in this limit, the feed concentration is unimportant.

This independence can be mathematically proven by taking the limit of the enrichment operating line equation.

$$y_n(R \to \infty) = \lim_{R \to \infty} \left[x_{n+1} \frac{R}{R+1} + \frac{x_D}{R+1} \right]$$

$$= x_{n+1} \lim_{R \to \infty} \frac{R}{R+1}^{\,1} + \lim_{R \to \infty} \frac{x_D}{R+1}^{\,0}$$

$$= x_{n+1}$$
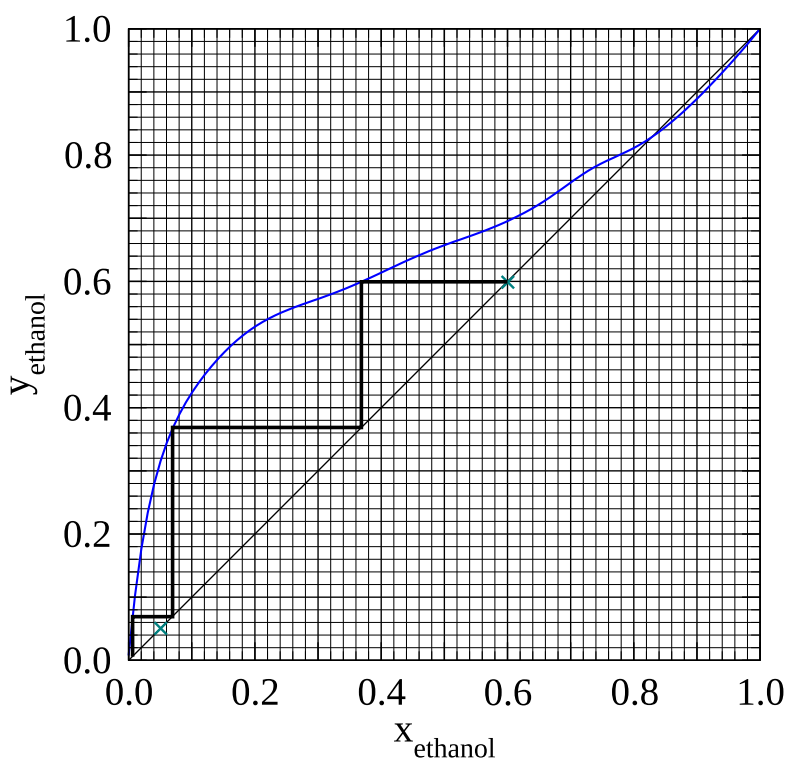
The minimum number of theoretical stages for the specified distillation is ≈ 2.3.

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.
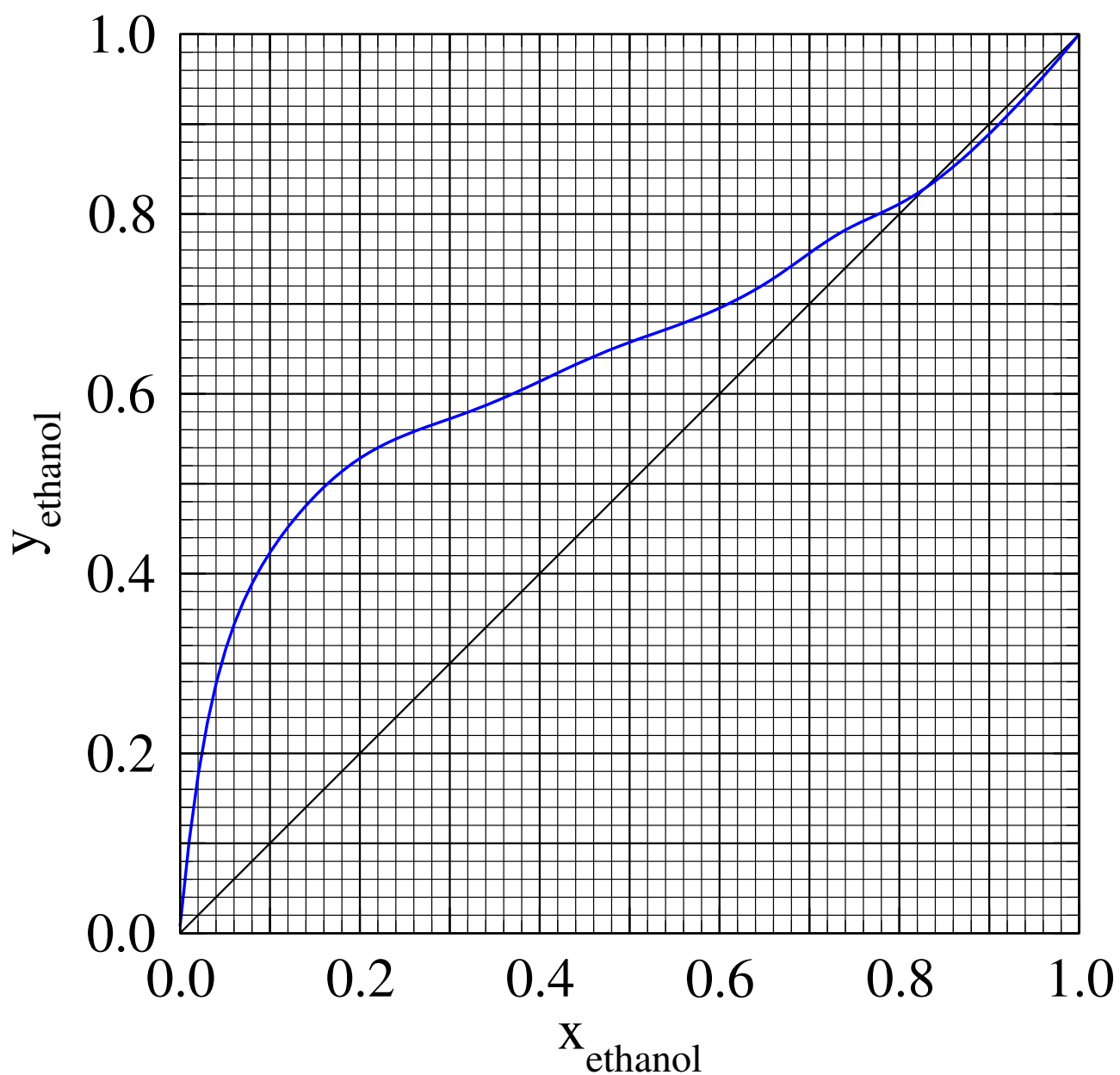
Student ID:

Figure 24: VLE diagram for ethanol-water mixtures at a pressure of 1 atm. This figure is required for Q. 7.25.

[Question total: 8 marks]

Q.7.26   Question 7.26

a) Derive the following operating line equation for the stripping section of a column using a mass balance.

$$y_m = x_{m+1}\frac{L_m}{V_m} - x_W\frac{W}{V_m}$$

State any assumptions you make and discuss their range of validity. A diagram including a suggested labelling of streams is available in Fig. 25.   **[5 marks]**
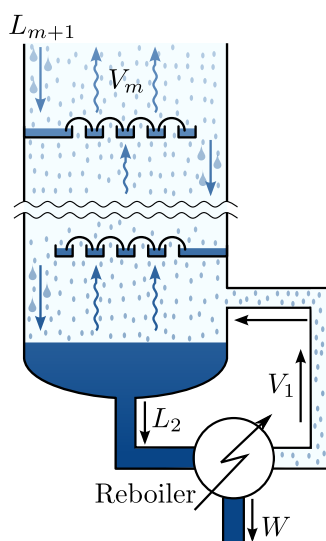

Figure 25: A diagram of the streams in the stripping section of a distillation column.

Solution:

If the assumption of **constant molar overflow** holds, the liquid and vapour flow-rates are constant in each section (stripping or enrichment) of the column.✓₁

[1/5]

$$L_m = L_{m+1} \qquad\qquad V_m = V_{m-1}$$

This assumption is true when the latent heat of vapourisation does not vary significantly within the column (around 10% is acceptable, see C&R Vol. 2) **AND** no vapour condenses due to heat lost from the column surface.✓₂

[2/5]

Performing a species mass balance from just above a tray, m, down to and including the bottoms product stream yields the following expression.

$$x_{m+1}\,L_{m+1} = y_m\,V_m + x_W\,W$$

$$y_m = x_{m+1}\frac{L_{m+1}}{V_m} - x_W\frac{W}{V_m}$$

[1/5]   ✓₁Assuming constant molar overflow yields

$$y_m = x_{m+1}\frac{L_m}{V_m} - x_W\frac{W}{V_m}$$

[1/5]   ✓₁

b) Explain why the stripping section operating line equation is inconvenient to use when compared to the enrichment operating line if the reflux ratio and the q value are selected as the design variables. **[3 marks]**

Solution:

The stripping section operating line is inconvenient to use as it is in terms of the flowrates in

[1/3] the lower section of the column. These ratios of flowrates may be rearranged into terms involving q and the reflux ratio but the functional form will not be as simple as the enrichment

[1/3] section.

It is simpler to instead plot the enrichment operating line, q-line, and solve for one point

[1/3] on the stripping line to plot the enrichment line.

c) Determine the location of one point on the stripping operating line. **[2 marks]**

Solution:

If we set $x_{m+1} = x_W$ and use the stripping section operating line, we have

$$y_m = x_W \frac{L_m - W}{V_m}$$

[1/2] An overall mass balance on the bottom section of the column yields

$$V_m = L_m - W$$

Therefore

$$y_m = x_W \frac{V_m}{V_m}$$

[1/2] And the point (x_W, x_W) lies on the stripping operating line.

d) Define the relative volatility and describe the three cases, $\alpha > 1$, $\alpha = 1$, and $\alpha < 1$. **[3 marks]**

Solution:

The relative volatility is defined as the ratio of vapour pressures of two components.

$$\alpha = \frac{P_A}{P_B}$$

[1/3]

For $\alpha > 1$, the component A is the Most Volatile Component (MVC) and will appear in greater concentrations in the vapour phase than component B. For $\alpha < 1$, the opposite is true and B is the MVC. When the relative volatility is unity ($\alpha = 1$), the vapour pressures are equal. Distillation of such a mixture is ineffective as the vapour produced from a partial boiling has the same concentration as the original liquid. Azeotropes are an example of

[2/3] mixtures where $\alpha = 1$.

e) A knockout drum is used to remove condensate from a 8 kmol s^{-1} methanol vapour stream. The vapour stream enters the knockout drum at a temperature of 72°C and contains 5 mol% water. Determine the exit concentration of the vapour and liquid streams if 0.5 kmol s^{-1} of condensate is collected in the drum.

You may assume methanol has a constant relative volatility of $\alpha = 3.9$ when compared with water. **[7 marks]**

Solution:

If $F = 8$ kmol s^{-1} of feed vapour enter the drum and $L = 0.5$ kmol s^{-1} condense then

[1/7] $V = 7.5$ kmol s^{-1} exit the drum in the vapour phase.

Performing a mass balance over the knockout (flash) drum in the methanol component yields

$$y_F\, F = y\, V + x\, L$$
$$0.95 \times 8 = 7.5\, y + 0.5\, x$$
$$y = 1.013 - 0.06667\, x$$

[2/7] $\overset{\checkmark}{2}$ The equilibrium relationship for the system is given by Eq. (11). Substituting in the relative volatility yields

$$y = \frac{3.9\, x}{1 + 2.9\, x}$$

[1/7] $\overset{\checkmark}{1}$ Substituting in the mass balance

$$(1.013 - 0.06667\, x)(1 + 2.9\, x) = 3.9\, x$$
$$1.013 - 1.02897\, x - 0.193343\, x^2 = 0$$

Solving this using the quadratic formula yields two roots

$$x \approx -6.17 \qquad\qquad\qquad x \approx 0.85$$

[2/7] $\overset{\checkmark}{2}$ Only positive values of x are physical, therefore the liquid stream concentration is $x \approx 0.85$. Calculating the vapour concentration gives

$$y = 1.013 - 0.06667 \times 0.85 = 0.956$$

[1/7] $\overset{\checkmark}{1}$

[Question total: 20 marks]

Q.7.27 Question 7.27

a) Consider the VLE data for ethanol-water mixtures which are presented in Fig. 26.

   i) What VLE "feature" is present at high mole fractions of ethanol ($x_{ethanol} \approx 0.9$) and what difficulties will it present when trying to distill high purity ethanol from dilute feedstock? **[3 marks]**

   Solution:

[1/3] At high concentrations of ethanol an azeotrope appears. $\overset{\checkmark}{1}$ Graphically, an azeotrope is where the VLE line contacts or crosses the $45°$ line. Physically, at the azeotrope con-
[1/3] centration the equilibrium vapour and liquid phases have equal concentrations. $\overset{\checkmark}{1}$ This implies that distillation will be unable to effect a separation, as the relative volatility is
[1/3] 1. $\overset{\checkmark}{1}$

   ii) Describe one method for how such a feature might be broken/overcome?

                                      [3 marks]

   Solution:

   To break an azeotrope, there are several methods available (students describe one of them):

- Using two distillation columns, pressure swing may be used to break the azeotrope. The first column brings the solution close to the azeotrope concentration. This mixture is then passed to a second column which operates at a different pressure. The concentration of the azeotrope will be shifted by the pressure change and distillation can continue to higher concentrations.

- By adding a third component to the azeotropic mixture, extractive or azeotropic distillation may be performed. The third component changes the VLE behaviour of the mixture and may form a new ternary azeotrope or effectively wash one of the components out of the column, allowing the distillation to be continued. The third component usually appears in either the top or the bottom product and must be recovered and recycled.
- Alternative separation techniques may be used which exploit other thermodynamic differences in the components. Examples are absorption and liquid-liquid extraction, which exploit the difference in solubility of the components instead of the relative volatility.
- One component may be removed chemically by reacting it to form a solid or gas phase which is simple to remove from the liquid azeotrope.

b) Using a mass balance, derive the following operating line equation for a flash drum.

$$y = (x_F - x\,q)\,/\,(1 - q)$$

where $q = L/F$ is the fraction of un-vapourised feed, x_F is the feed concentration, x is the outlet liquid concentration, and y is the outlet vapour concentration. **[6 marks]**

Solution:
Performing mass balances over a flash drum, we have

$$F = L + V \qquad\qquad\qquad x_F\,F = x\,L + y\,V$$

[2/6] $\overset{\checkmark}{2}$ Dividing the second equation by F, and inserting the first equation we have

$$
\begin{aligned}
x_F &= x\,\frac{L}{F} + y\,\frac{V}{F} \\
&= x\,\frac{L}{F} + y\,\frac{F - L}{F} \\
&= x\,q + y\,(1 - q) \\
y &= (x_F - x\,q)\,/\,(1 - q)
\end{aligned}
$$

[4/6] $\overset{\checkmark}{4}$

c) A flash drum, operating at 1 atm, is used to distill a 60 mol% ethanol and 40 mol% water mixture at a flow rate of 150 kmol h^{-1}. The heat exchanger before the flash drum is set so that 80 mol% of the feed will vapourise in the drum. Graphically determine the outlet concentrations of the vapour and liquid streams using the VLE data for ethanol-water mixtures available in Fig. 26. **[8 marks]**
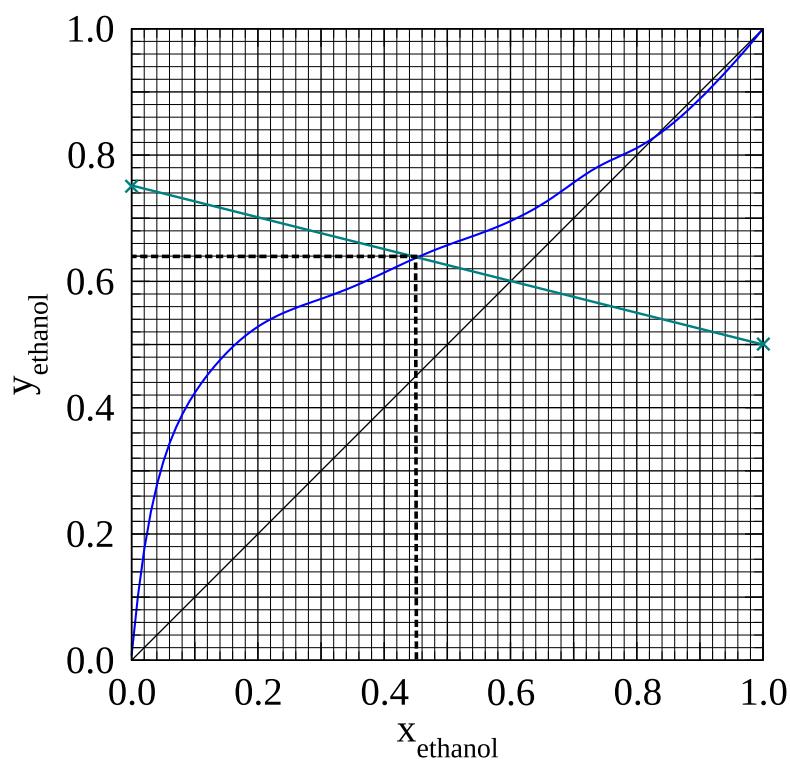
Solution:

[1/8] If 80 mol% of the feed is vapourised, we have $q = 1 - 0.8 = 0.2$. $\overset{\checkmark}{1}$ The operating line equation becomes

$$
\begin{aligned}
y &= (0.6 - 0.2\,x)\,/0.8 \\
&= 0.75 - 0.25\,x
\end{aligned}
$$

[1/8]
[1/8] $\overset{\checkmark}{1}$ To plot this line on the VLE chart, we need to evaluate two points. The first is $y(x = 0) = 0.75$ and the second is $y(x = 1) = 0.5$. $\overset{\checkmark}{1}$

[4/8]

[1/8]

$\overset{\checkmark}{4}$ From the intercept of the two lines, we have a vapour concentration of $y = 0.64$ and a liquid concentration of $x = 0.45.\overset{\checkmark}{1}$
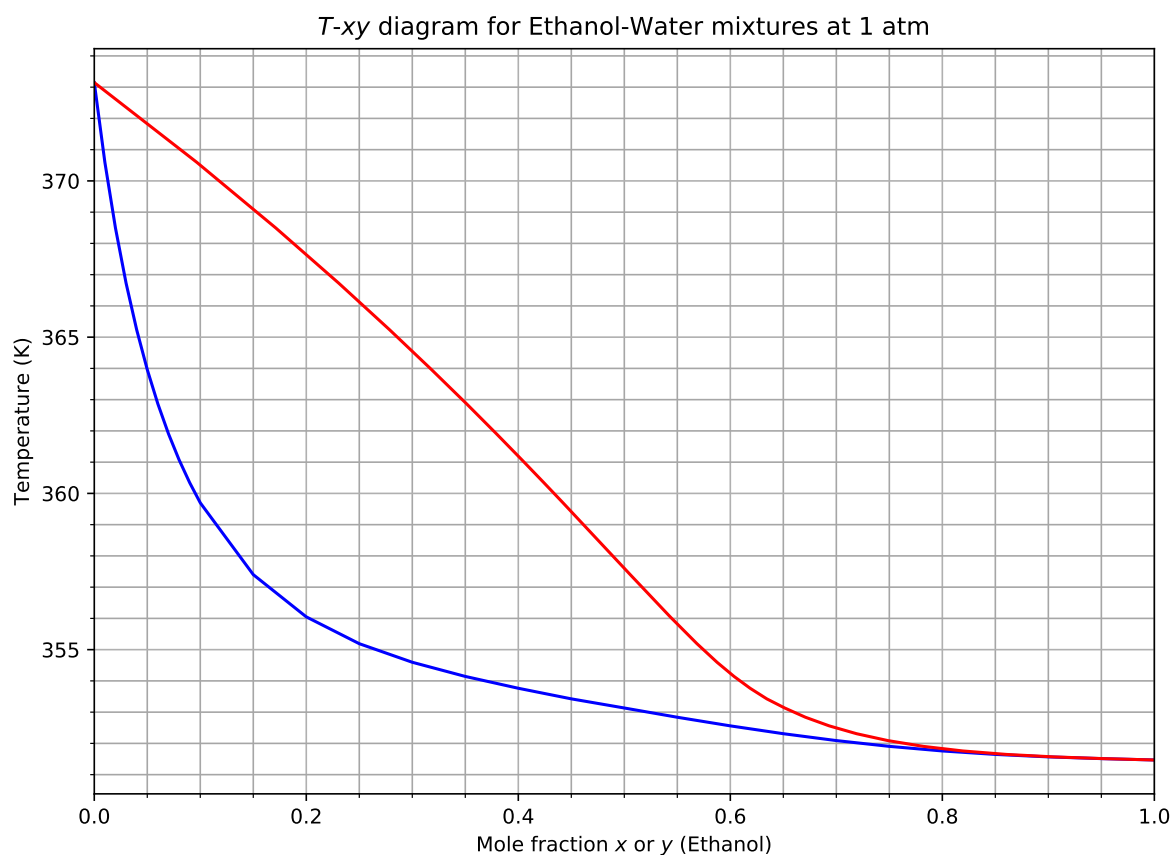

Figure 26: The *T-xy* diagram for a ethanol-water mixture at atmospheric pressure. Data was calcuated using the Margules activity coefficients (taken from Perry's) and Antoine coefficients from the Dortmund database. For use in Q. 7.27.

[Question total: 20 marks]

Q.7.28 Question 7.28

a) Give the definition of the q parameter in multi-stage distillation and identify the state of the feed for the following q-values, $q < 0$, $q = 0$, $q = 0.5$, $q = 1$, and $q > 1$.   **[5 marks]**
Solution:
Superficially, the **q**-value is the mole fraction of liquid in the feed once it enters the column; however, this definition only applies to saturated or partially vapourised systems ($0 \leq q \leq 1$).

The **q**-value is precisely the ratio of specific enthalpy difference between the feed stream and its saturated vapour enthalpy when compared to the enthalpy of vapourisation. From the datasheet, we have

$$q = \frac{h_{F,sat.vapour} - h_F}{h_{fg}}$$

- Sub-cooled liquid feed ($q > 1$).
- Saturated liquid feed ($q = 1$).
- Equimolar vapour and liquid phases ($q = 0.5$).
- Saturated vapour feed ($q = 0$).
- Super-heated vapour feed ($q < 0$).

b) Derive the following equation describing the q-line.

$$y = x\frac{q}{q - 1} - \frac{x_F}{q - 1}$$

The following diagrams are provided to aid you in labelling your streams.   **[13 marks]**
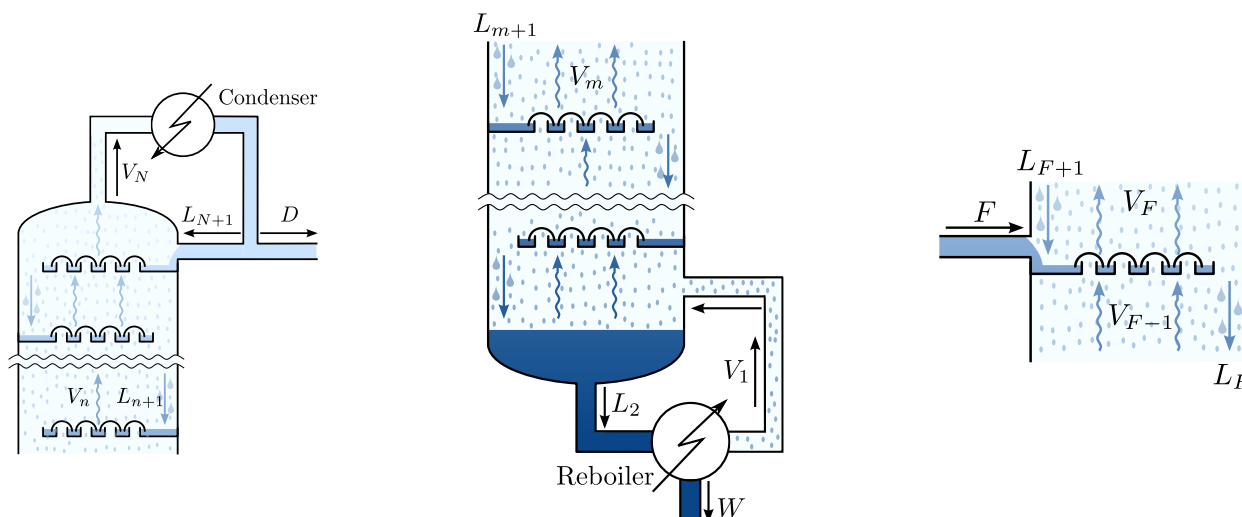
Solution:
Performing a mass balance around the feed tray, we can generate a relationship between the column flow-rates in the enriching and stripping sections of the column.

$$L_F = L_{F+1} + q\,F$$
$$V_F = V_{F-1} + (1-q)F$$

where we have defined q as the molar flow rate change of the liquid stream over the feed plate, relative to the feed stream flow rate. Assuming constant molar overflow allows us to write the above balances in terms of the liquid and vapour flow-rates in both sections of the column.

$$L_m = L_n + q\,F$$
$$V_n = V_m + (1-q)F$$

Performing a balance over the top and bottom sections of the column, we have

$$y_m\,V_m = x_{m+1}\,L_m - x_W\,W$$
$$y_n\,V_n = x_{n+1}\,L_n + x_D\,D$$

These operating lines must meet at the feed tray ($y = y_m = y_n$, $x = x_{m+1} = x_{n+1}$), as the feed tray is in **both** sections of the column. Subtracting one operating equation from the other, we obtain

$$y(V_m - V_n) = x\,(L_m - L_n) - x_W\,W - x_D\,D$$

Using the mass balances over the feed tray and some rearrangement, we can rewrite the equation to eliminate these flow rates

$$y = x\frac{q}{q-1} - \frac{x_W\,W + x_D\,D}{F\,(q-1)}$$

Replacing the top part of the right fraction using the species mass balance over the column yields the final result.

$$y = x\frac{q}{q-1} - \frac{x_F}{(q-1)}$$

c) The molar enthalpies of saturated liquid and saturated vapour are 20 MJ kmol^{-1} and 50 MJ kmol^{-1} respectively. Determine the values of "q" and the gradient of the "q-line" given that the molar enthalpy of the feed is:

    i) 18 MJ kmol^{-1}.   **[2 marks]**
    Solution:
    $q = 1.067$, gradient$=15.925$

    ii) 20 MJ kmol^{-1}.   **[2 marks]**
    Solution:
    $q = 1.0$, gradient$=\infty$

    iii) 52 MJ kmol^{-1}.   **[2 marks]**
    Solution:
    $q = -0.067$, gradient$=0.0628$

[Question total: 24 marks]

Q.7.29 Question 7.29

A mixture of 40 mol% benzene and 60 mol% toluene is fed to a distillation column for purification. The bottoms product is specified at 10 mol% benzene and a minimum of 95 mol% benzene is required in the top product. 50 mol% of the feed flashes to vapour upon entering the column. VLE data is provided in Fig. 29.

a) Determine the minimum reflux ratio for this separation. **[3 marks]**

Solution:

As this VLE curve is quite regular, the first pinch point occurs at the intersection of the VLE, operating line and *q*-line. The *q*-line is inclined at an angle of $-45°$ as half the feed stream evaporated on entering the column. Sketching the *q*-line we can determine the intersection of the enrichment operating line with the y-axis for the pinched operation.
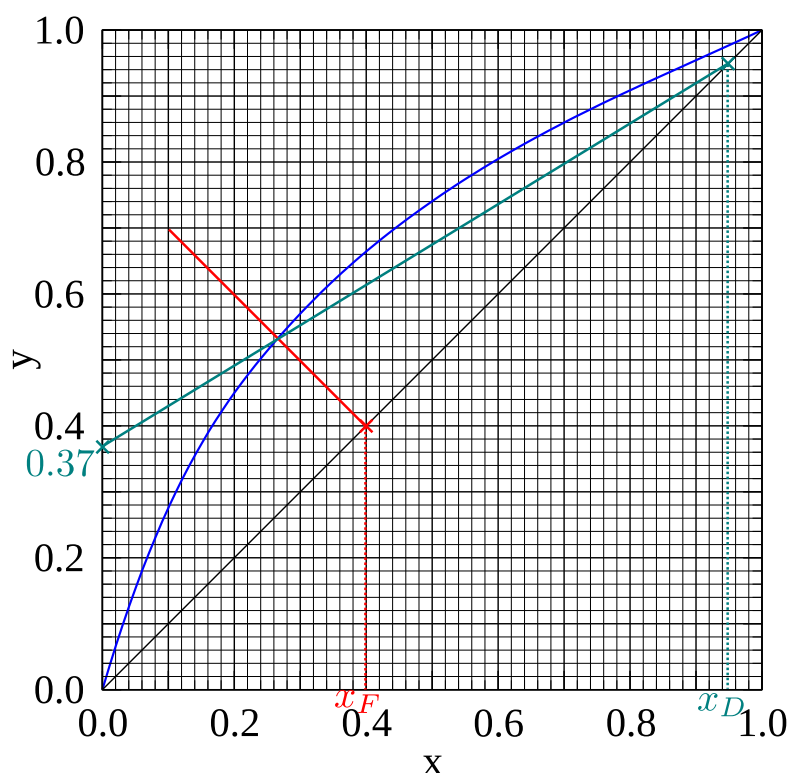
The minimum reflux ratio is

$$0.37 = \frac{0.95}{R_{min} + 1}$$

$$R_{min} = 0.95/0.37 - 1 \approx 1.57$$
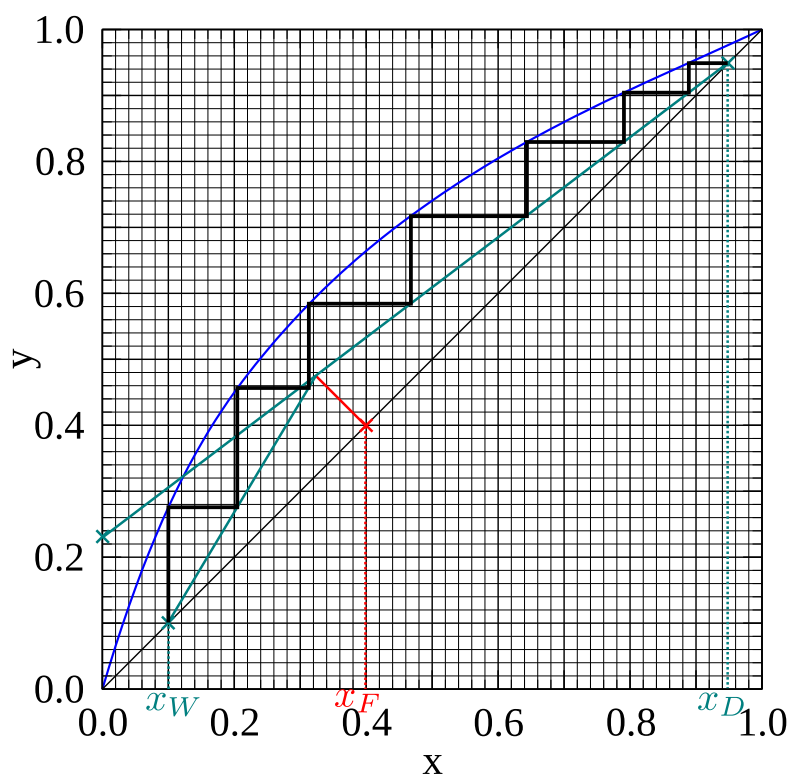
b) Calculate the number of theoretical stages required to perform the distillation at twice the minimum reflux ratio ($R = 2\,R_{min}$). **[5 marks]**

Solution:

The operating reflux ratio is $R = 2\,R_{min} = 3.14$. The new intersection of the stripping operating line with the y-axis is

$$y_{strip.}(x = 0) = \frac{x_D}{R + 1} = \frac{0.95}{3.14 + 1} \approx 0.23$$

Performing the McCabe-Thiele calculation yields almost exactly 7 theoretical stages (6 ideal trays and a reboiler).

c) Calculate the number of real stages, using a Murphree tray efficiency of 0.5 below the feed and 0.75 above the feed. **[10 marks]**
Solution:
To utilise the Murphree tray efficiency, we need to plot an effective VLE line at a fraction of the distance between the operating line and the VLE line equal to the efficiency. Here we have efficiencies for above and below the feed so the line will have a discontinuity at the feed. It is best to start stepping from the reboiler (see Fig. 27) in this case, as this avoids any ambiguity arising from stepping down from the distillate. Stepping from the top is incorrect (see Fig. 28)

d) Calculate and comment on the overall tray efficiency of the column.   **[2 marks]**
Solution:
The previous calculations yielded 6 ideal trays and 10 real trays, giving an overall efficiency of $E_O = 6/10 = 60\%$. It should be noted that the bottoms/tops concentrations differ in the ideal and Murphree cases.

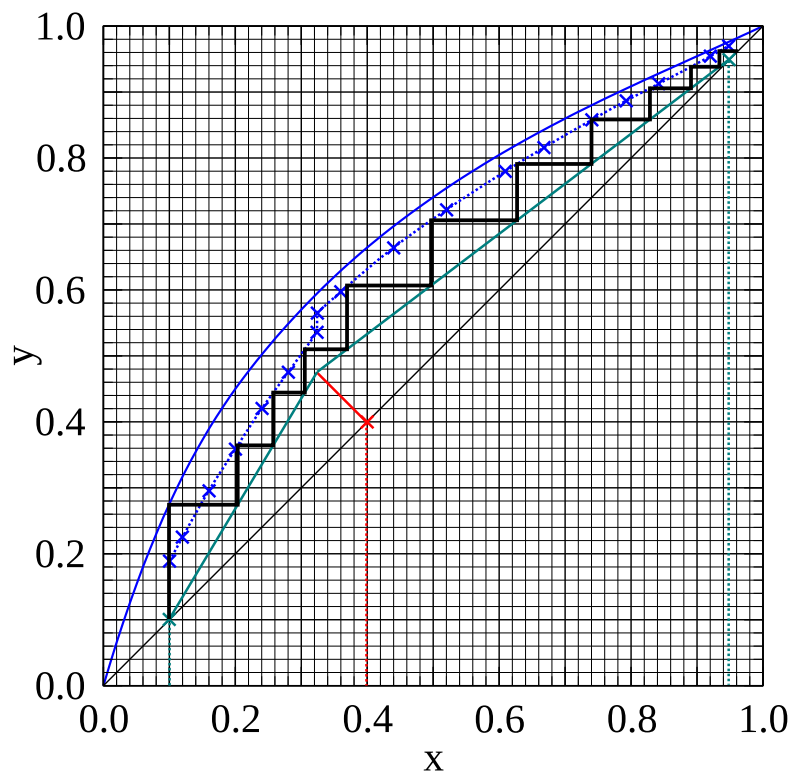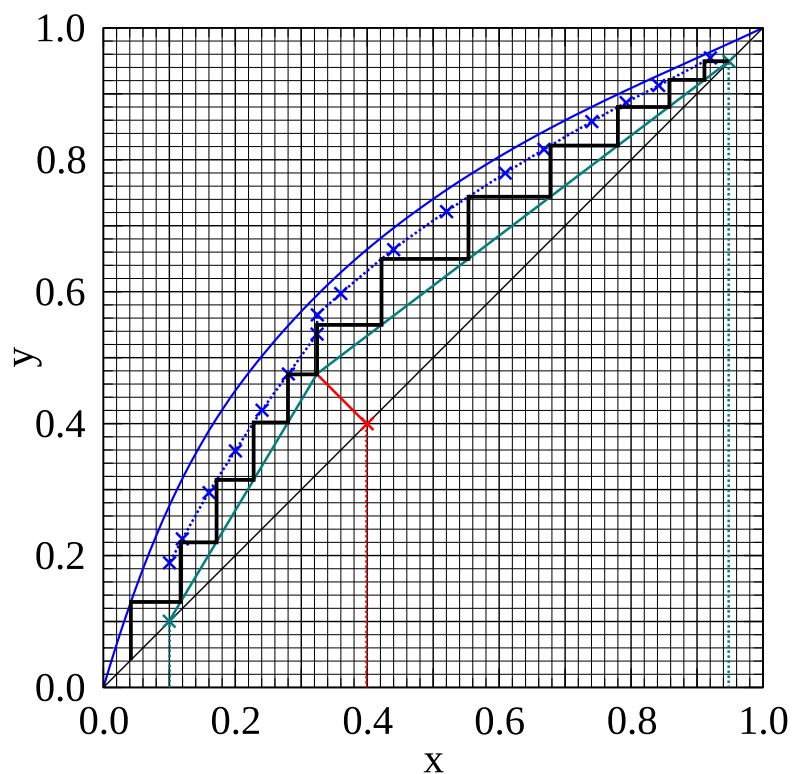Figure 27: Stepping from the bottom (10 real trays plus one ideal reboiler stage)

Figure 28: Stepping from the top, encountering the discontinuity (11 real trays plus one ideal reboiler stage)

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

Student ID:
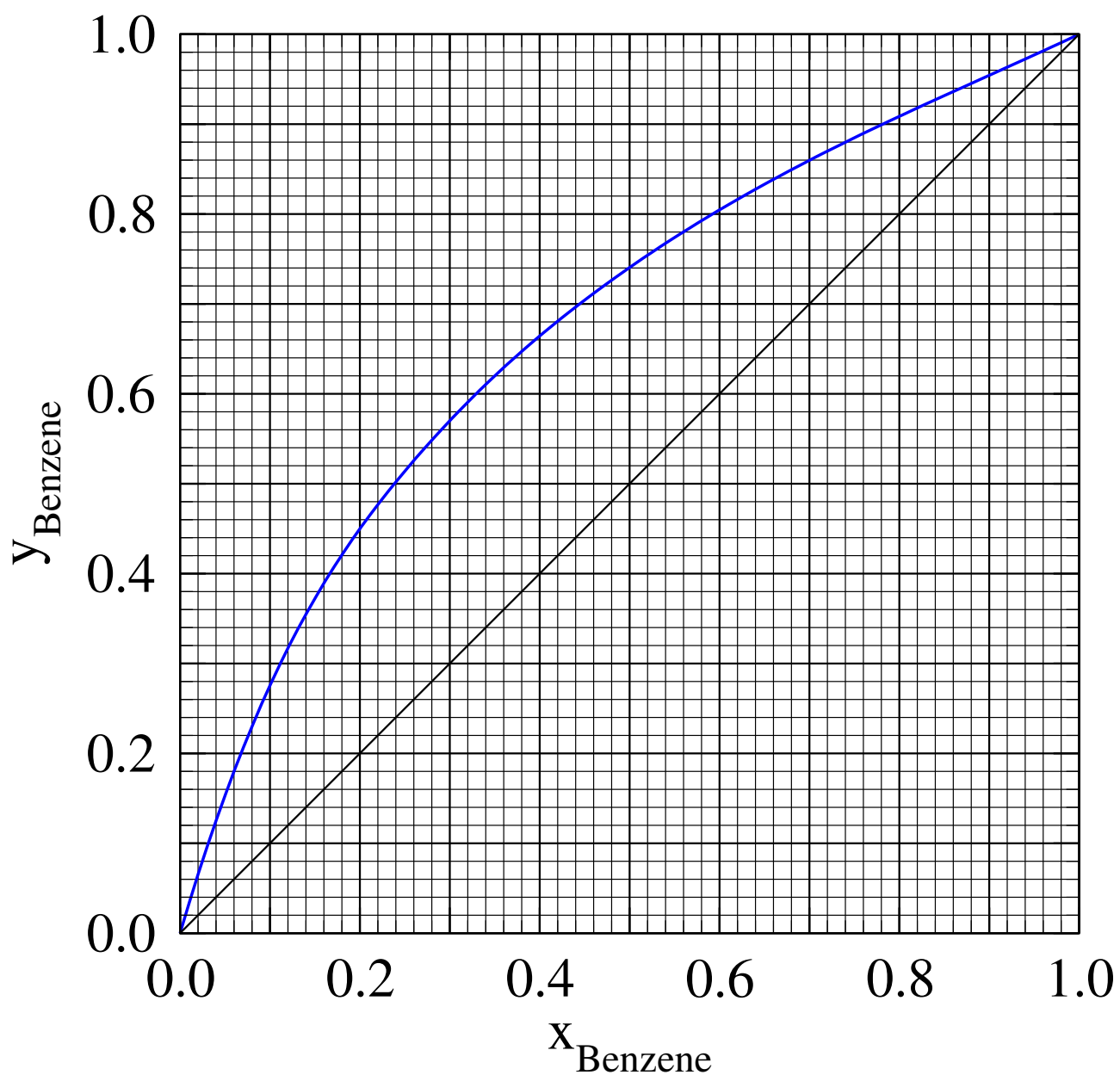
Figure 29: VLE diagram for benzene-toluene mixtures at 1 atm. This figure is required for Q. 7.29.

[Question total: 20 marks]

Q.7.30 **Question 7.30**

A continuous distillation column is being used to separate benzene from toluene. There are six plates above the feed plate and the column is operated with a reflux ratio of 3. When the feed composition is 40 mol% benzene, the top product has a composition of 97 mol% benzene. VLE data for the system is provided in Fig. 16.

a) What range of q-values will yield the top-product specification?   **[10 marks]**
  Solution:

[2/10] The intercept of the enrichment operating line is at $x_D/(R+1) \approx 0.24$. ⚡Plotting the enrichment line and performing the stepping from the top for the first 7 stages yields the following construction.
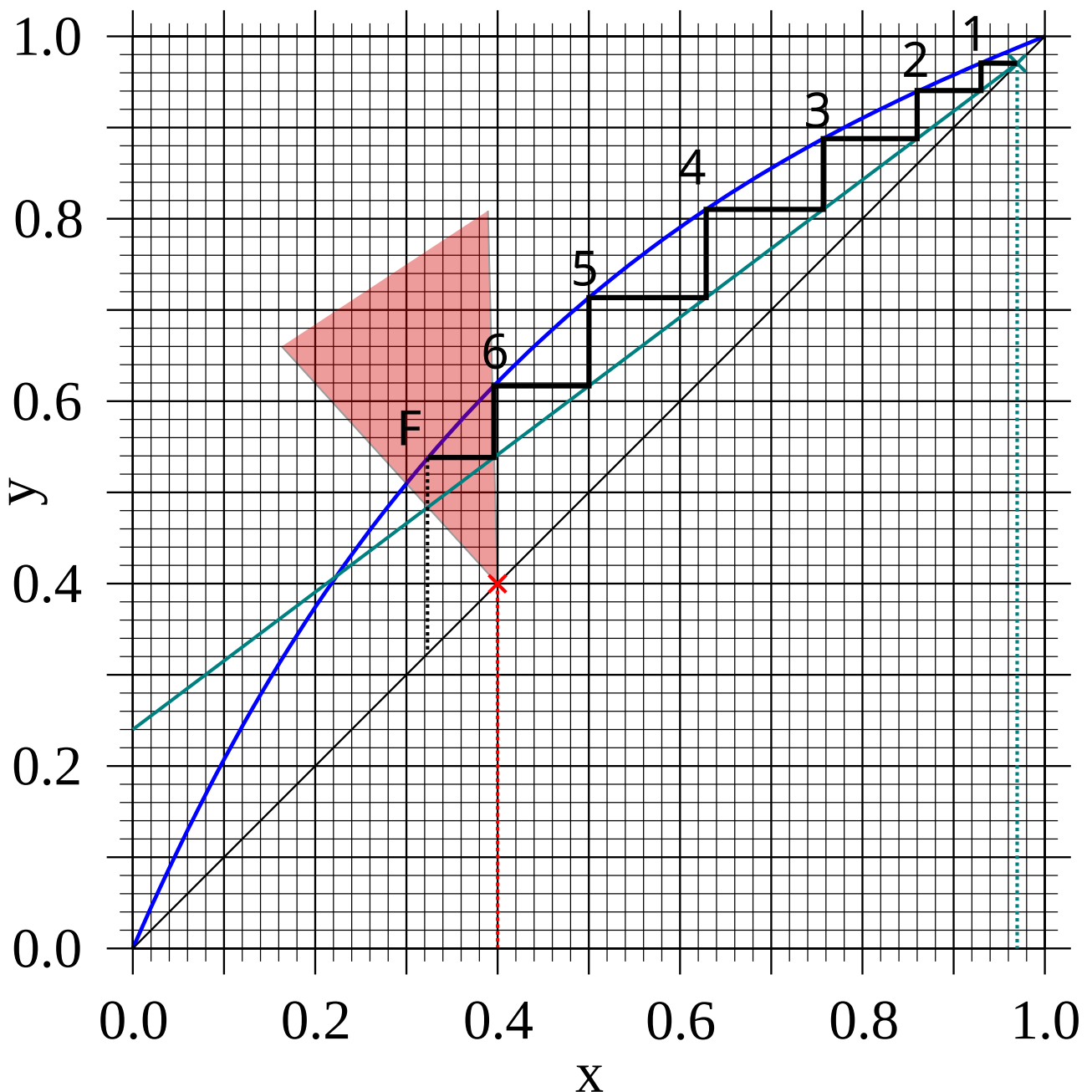
The q-line must lie in the red triangle. (The next part is not required, but is carried out for illustration) Taking the two edge cases we have

Intercept at left edge of feed tray (qmin=0.5)   Intercept at right edge of feed tray (qmax=
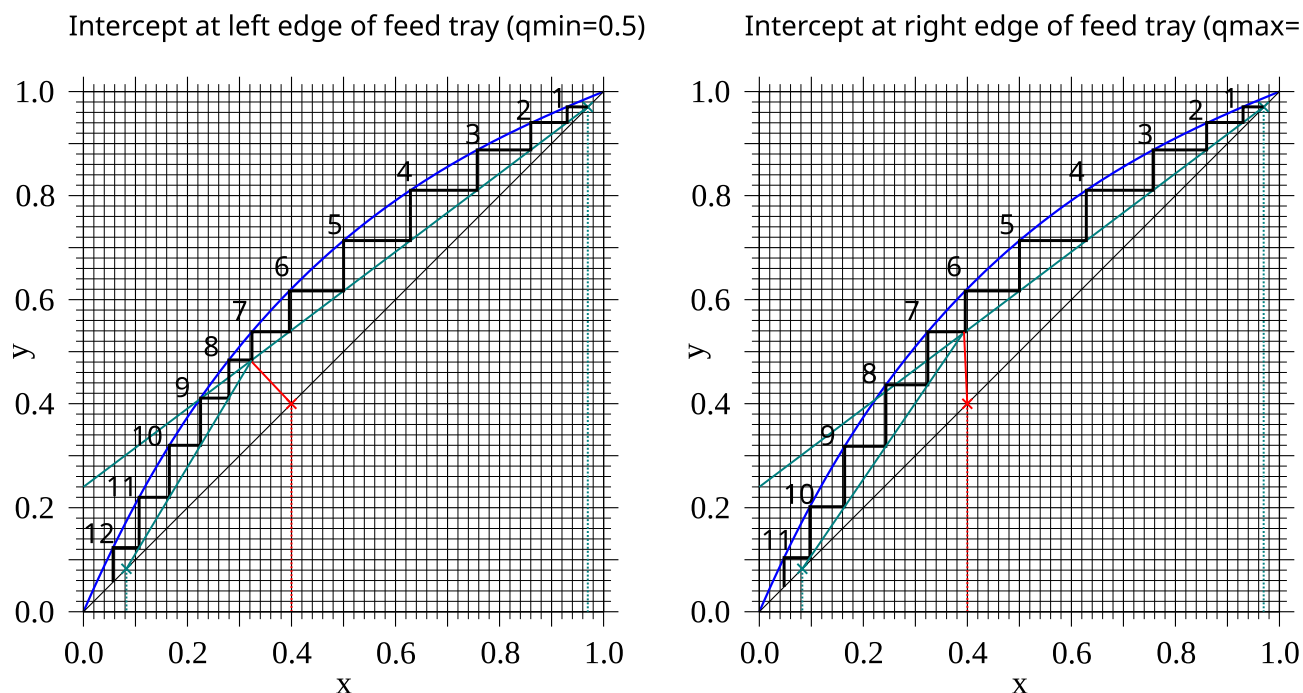
Thus the allowable range of values is $q \in [0.5, 1.0]$.

b) Discuss what the optimum q-value for the column is and how it affects the costs of construction and operation. **[5 marks]**
   Solution:

[1/5]   The optimal condition of the feed is the one that produces the minimum number of trays.[1] This would be the q-line which minimises the slope of the stripping section operating line, or $q = 1$
[1/5]   in this case.[1]

This shifts some of the duty on the feed preheater to the reboiler but otherwise does not affect the column operating costs. It will increase the size of the reboiler needed, but this
[3/5]   may be offset by using a smaller feed preheater.[3]

c) Using the optimum q-value, calculate how many plates/trays are required below the feed tray. You may assume that a partial reboiler is used and the bottoms product is 8 mol% benzene. **[3 marks]**
   Solution:

The solutions performed above indicate that a liquid feed ($q = 1$) would produce the minimum number of stages. This choice results in roughly 10.3 ideal stages in total, 3.3 below
[3/3]   the feed, so 2.3 trays plus an ideal reboiler.[3]

As an additional note (not examined), in some texts it is suggested that the optimal q-value is where the q line intercepts with the VLE line at the location of the feed tray. Performing the calculation with this value results in a worse result, as shown below:

Even intercept
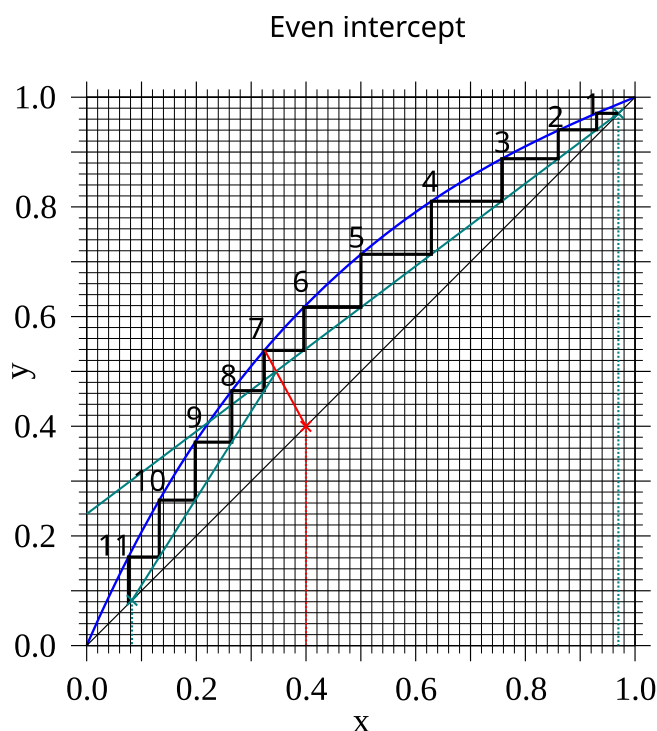
d) Why would it be impractical to operate the column with a fully vapourised feed ($q = 0$)?

[2 marks]

Solution:

A fully vapourised feed would result in a pinch occuring on the enrichment operating line

[2/2]   before the feed stage was reached. ✓
                                                                    2

[Question total: 20 marks]

**Q.7.31 Question 7.31**   **Recommended**

Note: This question is a lighthearted problem on a *ficitional* compound and was written around the time of the Avatar film's release (although the compound has featured in a number of other popular science fiction works). It is used to demonstrate that pinch points on operating lines may appear in a number of places and you cannot immediately tell which operating line will pinch first.

The mining company RDA wants to perform a separation of the volatile component unobtainium from water (VLE curve provided in Fig. 33, print at least three copies as you will need them!). A 44% unobtainium-water feed is available which will need to be concentrated to 90% unobtainium before shipment. To meet the lax environmental regulations, the water in the bottoms product must reach a concentration of 96% water before being discarded. The column will be operated at atmospheric pressure, and the feed stream will flash to equal moles of vapour and liquid at these conditions. A thermosyphon (total) reboiler is used and therefore it does not function as a stage of distillation.

a) Determine the minimum reflux ratio, R_{min}, needed for this separation. Clearly mark your pinch point.

Hint: Pinches can occur on either operating line!

Solution:

To find the pinch points, we find the operating lines with the largest angles from the $45°$ line which only just intercept the VLE lines before reaching the q-line.

To start, we draw the q-line and the two known points of the operating lines ($y = x = x_D$ and $y = x = x_W$) on the graph. Then, we draw the largest angle operating lines which

just intercept the VLE line without crossing it. The operating line for the section with the lowest intercept on the q-line is the limiting section and contains the true pinch point. In this case, it is the stripping section operating line. The solution is presented in Fig. 30.
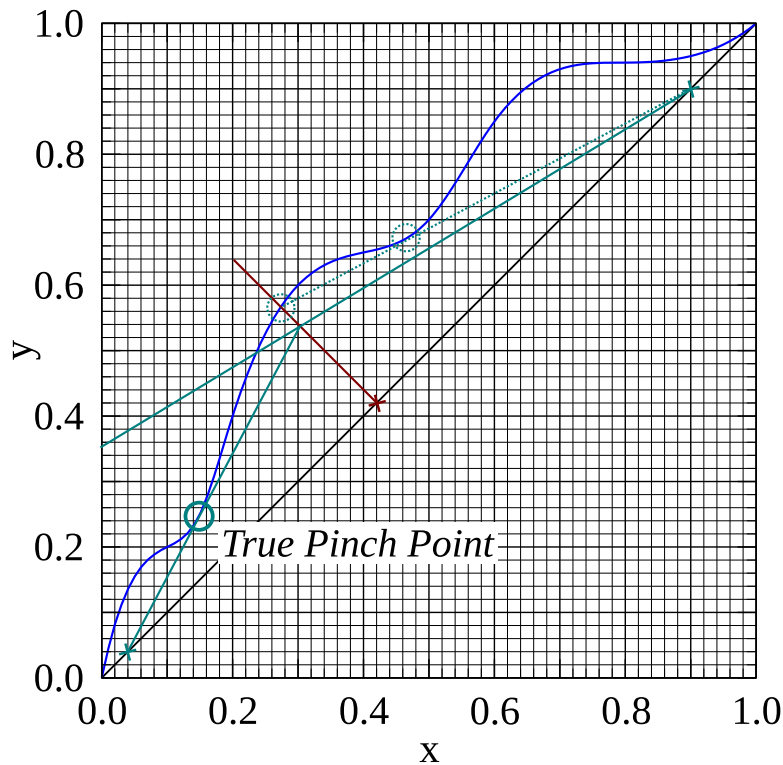
Figure 30: Determination of the minimum reflux ratio by identifying the pinch points on the graph. There are three possible pinch points in this system, but the first pinch to appear occurs on the stripping section. The enrichment section (dotted line) pinches occur after the stripping section pinch has been passed.

Once we have the operating lines for the minimum reflux ratio case, we need to calculate the actual value of R. The intercept of the enrichment operating line with the y-axis is given by

$$y_{enrich.}^{(pinch)}(x = 0) = \frac{x_D}{R_{min} + 1}$$

which can be rearranged to give

$$R_{min} = \frac{x_D}{y_{enrich.}^{(pinch)}(x = 0)} - 1 = \frac{0.9}{0.355} - 1 \approx 1.54$$

b) The economic reflux ratio is around $R = 1.5 \times R_{min}$. Determine the required number of theoretical stages to perform the separation at this value of R.
Solution:
The operating reflux ratio is

$$R = 1.5\, R_{min} = 1.5 \times 1.54 = 2.31$$

The enrichment operating line intercepts the y-axis at

$$y_{enrich.}(x = 0) = \frac{x_D}{R + 1} = \frac{0.9}{2.31 + 1} \approx 0.27$$

We can then plot the operating lines on the chart and solve for the number of theoretical stages. The solution is presented in Fig. 31 and comes to around 8.8 theoretical stages.
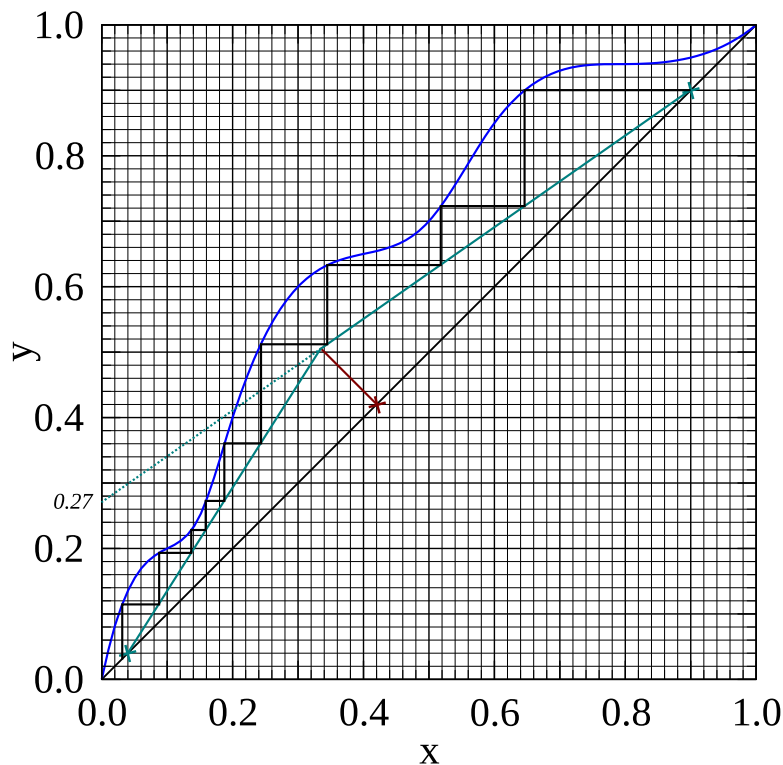
Figure 31: Determination of the number of theoretical stages for a reflux ratio of $R = 2.31$.

c) The bubble-cap trays selected for carrying out the distillation have a Murphree tray efficiency of 50%. Calculate the real number of trays required to perform the distillation.
Solution:
The Murphree tray efficiency is given by the following relation:

$$E_M = \frac{y_n - y_{n-1}}{y_n^* - y_{n-1}}$$

The denominator term, $y_n^* - y_{n-1}$, is actually the vertical distance between the VLE line and the operating line. The numerator term, $y_n - y_{n-1}$ is the separation actually achieved. Thus, the Murphree tray efficiency allows you to draw an "effective" VLE line, which I call the Murphree line, at a fraction of the distance between the VLE line and the operating line equal to the tray efficiency. By selecting a number of points to calculate the Murphree line at, we can perform a distillation design (See Fig. 32). Here, we have a requirement of at least 18 real trays.

0
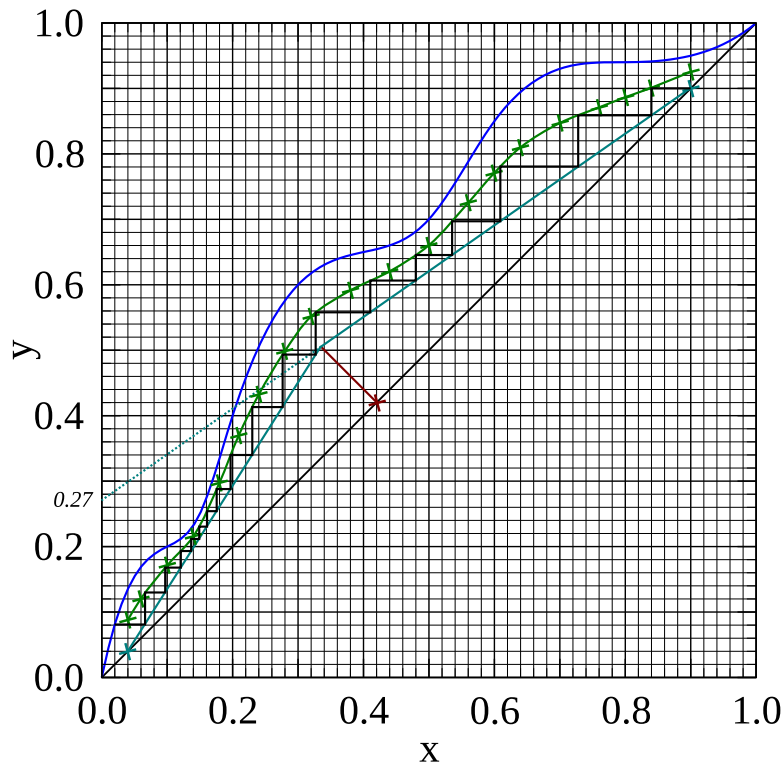
Figure 32: A distillation design which uses the Murphree tray efficiency.

d) What is the overall column tray efficiency? Compare this to the Murphree tray efficiency.
Solution:
The overall column tray efficiency is given by the ratio of theoretical to ideal stages.

$$E_O = \frac{N}{N_{real}} \approx \frac{8.8}{18} \approx 0.49$$

This estimate is close to the Murphree tray efficiency, showing that for a quick estimate the Murphree tray efficiency may be used as a rough overall efficiency. The deviation between E_O and E_M here is primarily due to the (required) rounding of the number of real stages to a whole integer.

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

Student ID:
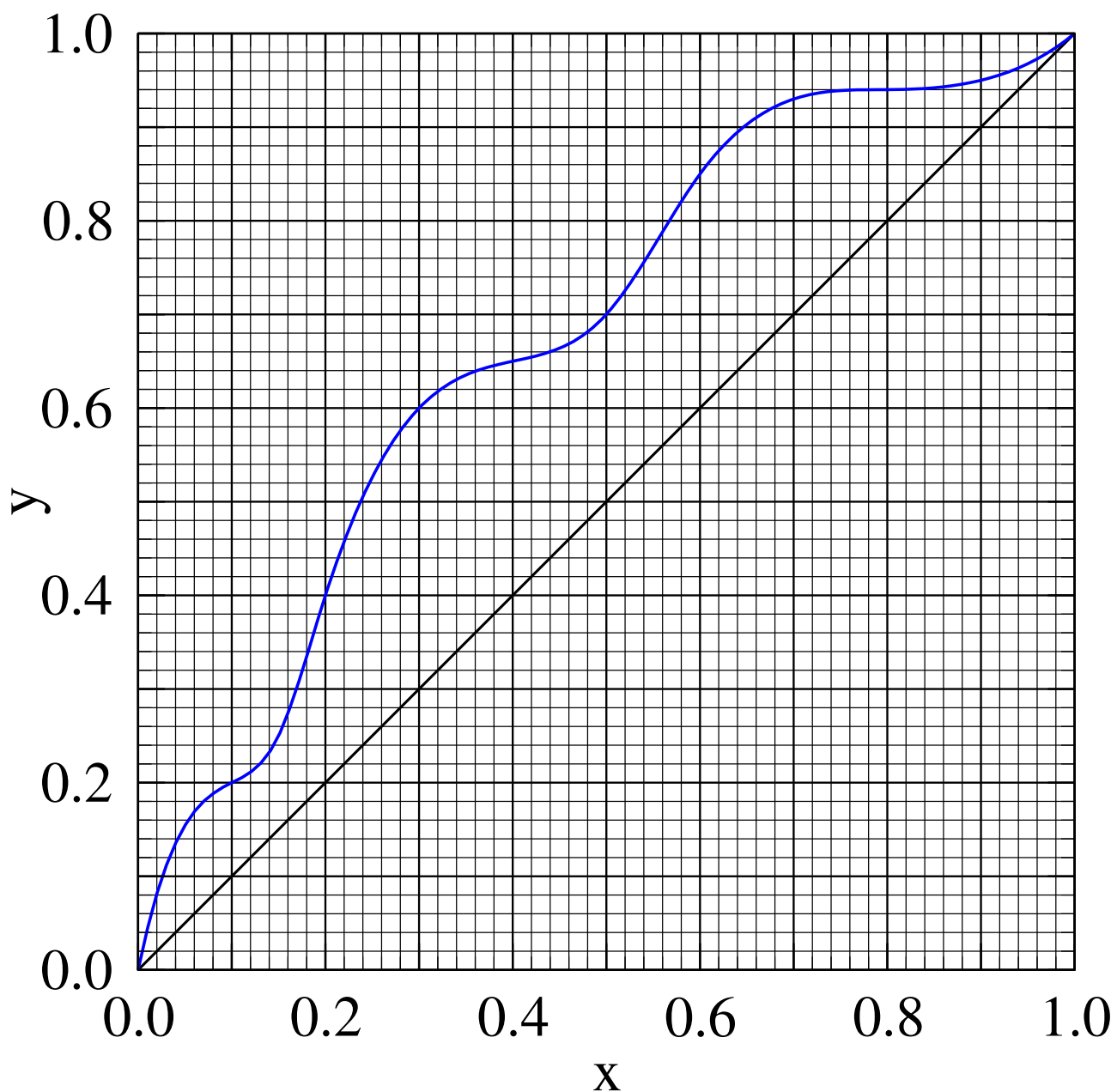
Figure 33: The VLE data for a unobtainium-water mixture at atmospheric pressure. Unobtainium is more volatile than water. For use in Q. 7.31.

[Question end]

Q.7.32 Question 7.32

A hot product stream, containing 40 mol% of a volatile lipid, from a reactor producing soap is to be continuously distilled in a plate tower until it reaches a purity of 90 mol% of lipid and a waste stream containing less than 10 mol% of lipid. As the lipid mixture increases in concentration it begins to foam and the column plate efficiencies begin to drop. The theoretical and the effective "Murphree" VLE lines for the column are plotted in Fig. 34.

a) The column is operating at a reflux ratio of $R = 1.5 \times R_{min}$. Using the effective VLE line, determine the operating reflux ratio of the column if the feed mixture is supplied to the column as a saturated liquid. **[5 marks]**

Solution:

A saturated liquid feed results in a vertical q-line. We then draw the q-line and the shallowest enrichment operating line along with the steepest stripping lines possible.
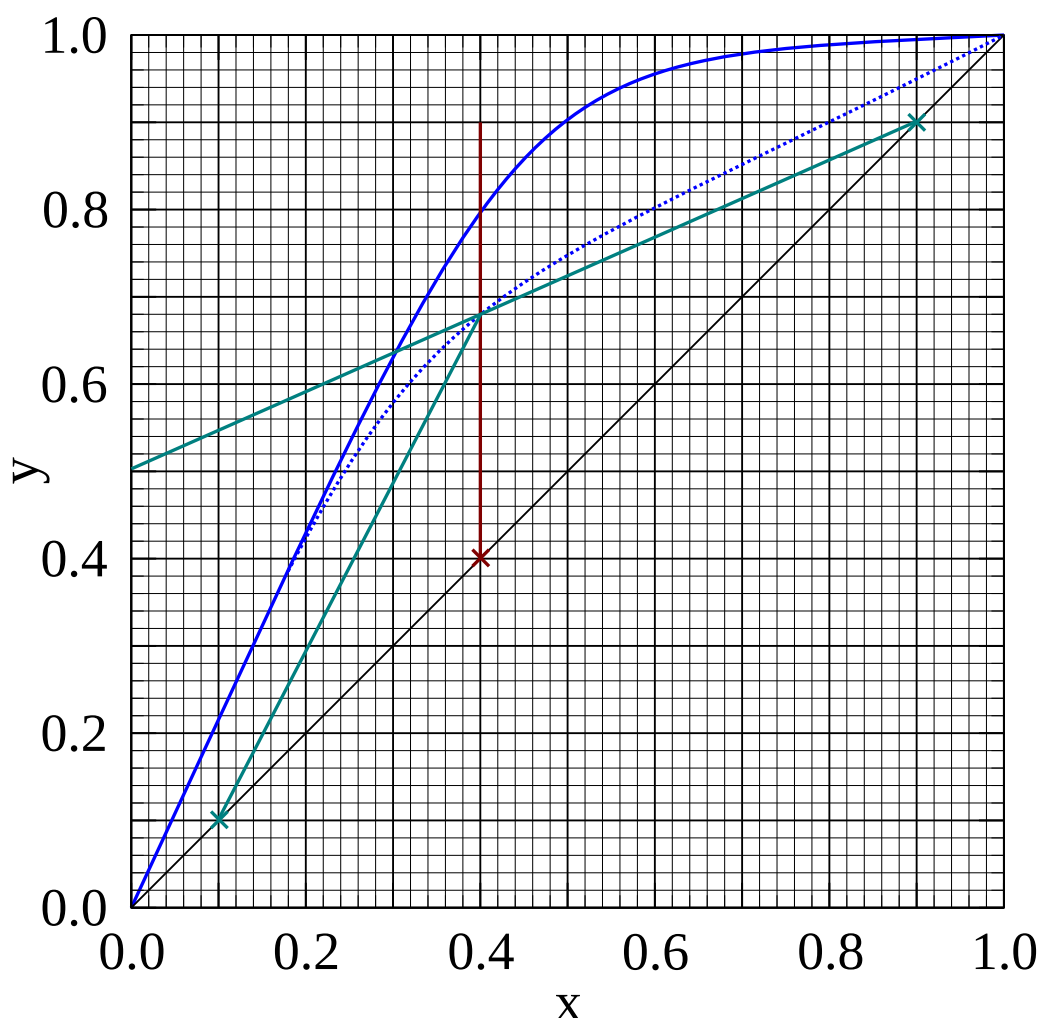
In this case, both intersect at the same point. The enrichment operating line intersects the y-axis at $y = 0.5$. We can calculate the minimum reflux ratio as:

$$y = \frac{x_D}{R_{min} + 1}$$

$$0.5 = \frac{0.9}{R_{min} + 1}$$

$$R_{min} = \frac{0.9}{0.5} - 1 = 0.8$$

Therefore the operating reflux ratio is

$$R = 1.5\,R_{min} = 1.2$$

b) Determine the number of stages in the column required to perform the distillation. Assume that the column is fitted with a partial reboiler and a total condenser.   **[8 marks]**
Solution:
The new intersection of the enrichment line with the y-axis is:

$$y = \frac{x_D}{R+1} = \frac{0.9}{1.2+1} = 0.4091$$

Performing the McCabe-Thiele method, we have

Tray 4 & 5 Efficiency Calculation
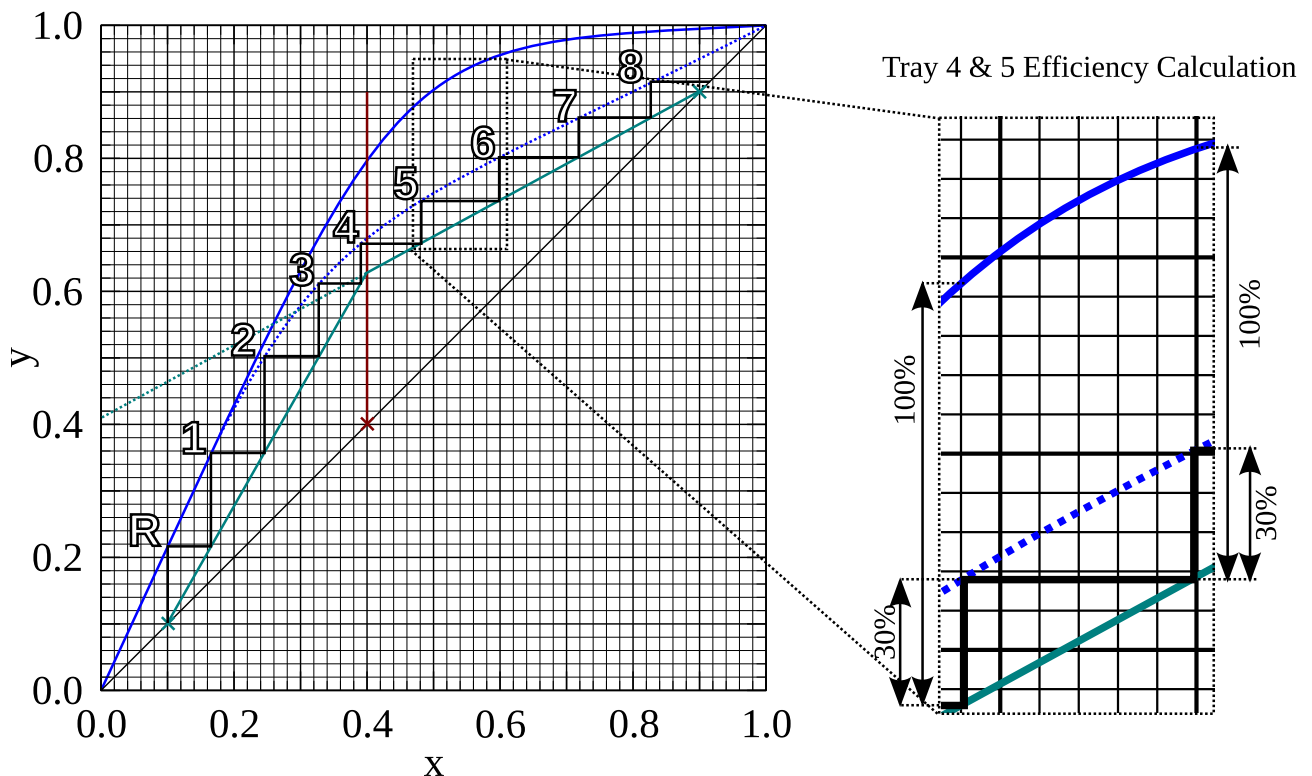
There are 8 plate stages plus a reboiler stage (we do not count fractional stages as these are real stages). The feed tray is tray 4, as it spans the two operating lines.

c) Define the Murphree plate efficiency, and calculate its values for the two stages above the feed plate.   **[7 marks]**
Solution:
The Murphree plate efficiency is defined as follows:

$$E_M = \frac{y_n - y_{n-1}}{y_n^* - y_{n-1}}$$

where y_n is the real concentration of vapour coming off stage n, and y_n^* is the equilibrium vapour concentration. It is therefore the fraction of how effective a single tray is, when compared to the performance of an ideal stage.

The feed plate is tray 4, so we want the Murphree tray efficiency for trays 5 and 6. Measuring on the graph, tray 4 and 5 have a tray efficiency of $\approx$ **30%**. A diagram of the method is available in the previous question's solution.

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

Student ID:
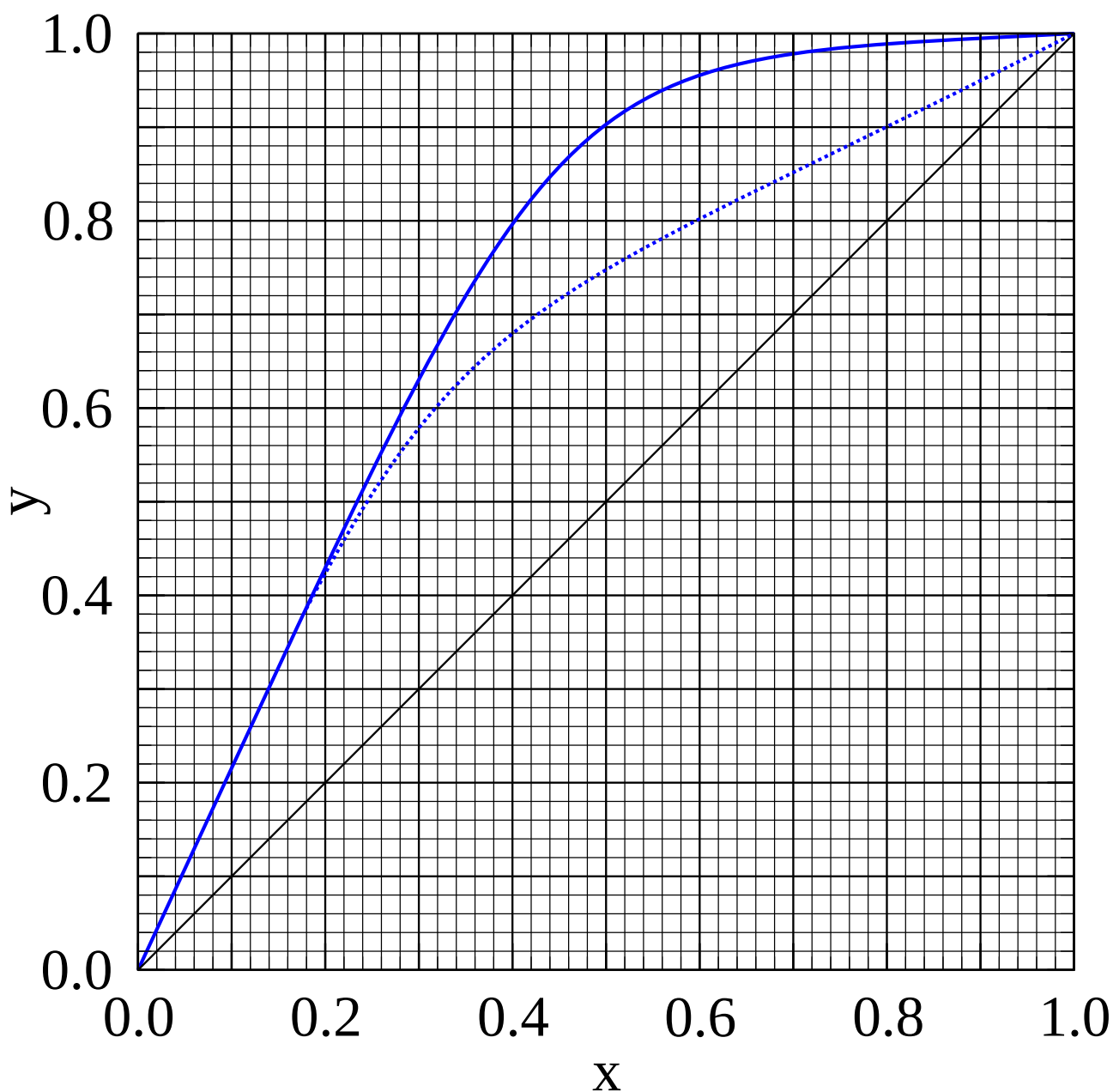
Figure 34: VLE diagram for the lipid mixture. This figure is required for Q. 7.32.

[Question total: 20 marks]

Q.7.33 Question 7.33

A solvent vapour is to be scrubbed from an air stream before the treated air can be released to the atmosphere. Absorption using clean water in a plate column is recommended. The solvent vapour-air mixture enters the column at a concentration of 2 kmol solvent vapour per 100 kmol solvent-free air. The maximum allowable loss to the atmosphere is 0.1 kmol solvent vapour per 100 kmol solvent-free air. A liquid rate of 1.3 times the minimum is employed, and the Murphree plate efficiency is known to be 50% based on the gas phase. Equilibrium data are represented by $Y = 0.5 X^2$, where Y and X are the molar ratios of the solvent in the gas and liquid phases respectively.

a) Show graphically the meaning of the Murphree plate efficiency.   **[5 marks]**
   Solution:
   It is the fraction of equilibrium acheieved, and should be represented using a diagram like this:
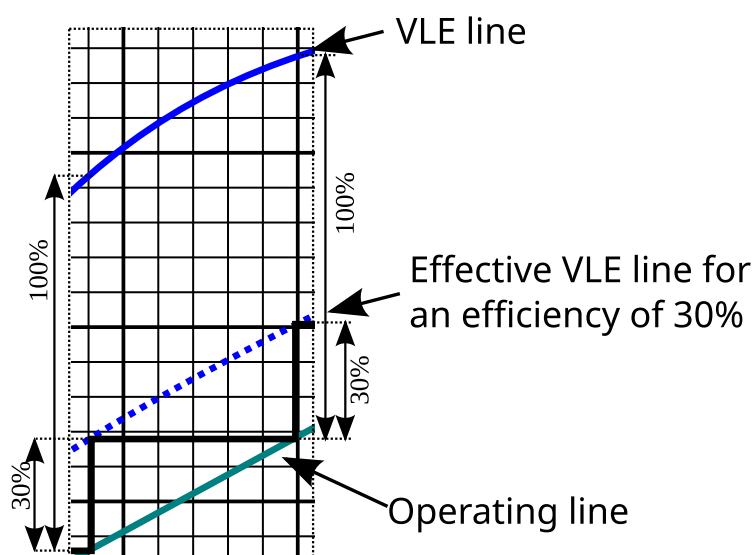
b) Determine the number of real plates required. A blank X-Y graph is provided in Fig. 35.
                                                                                          [10 marks]

   Solution:
   Using the following stream numbering convention:
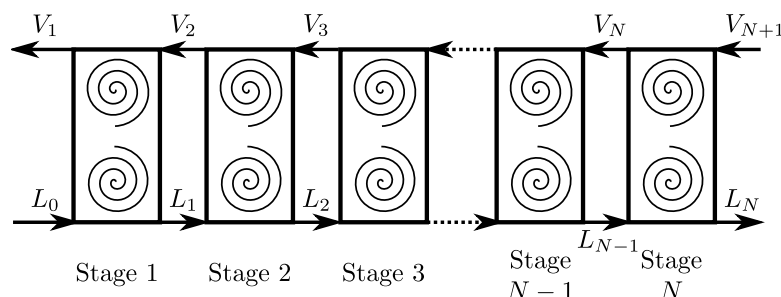
From the problem statement, we have $X_0 = 0$, $Y_1 = 0.1/100 = 0.001$, and $Y_{N+1} = 2/100 = 0.02$. Noting that under conditions of the minimum flow rate, X_N is in equilibrium with Y_{N+1}. The VLE relationship gives:

$$Y_{N+1} = 0.5 X_N^2$$

$$X_N = \sqrt{2 Y_{N+1}} \approx 0.2$$

The unknowns are now the inert liquid and vapour flow rates, V_N and V_N. Taking the standard mass balance on the solvent:

$$\frac{y_{A,n+1}}{1 - y_{A,n+1}} = \frac{L'}{V'} \frac{x_{A,n}}{1 - x_{A,n}} + \frac{y_{A,1}}{1 - y_{A,1}} - \frac{L'}{V'} \frac{x_{A,0}}{1 - x_{A,0}}$$

Writing this in terms of Y and X we have:

$$\frac{L'}{V'} = \frac{Y_{A,n+1} - Y_{A,1}}{X_{A,n} - X_{A,0}} = \frac{0.02 - 0.001}{0.2 - 0} \approx 0.095$$

This is at the minimum liquid flowrate. The column is operated at 1.3 times the minimum giving an actual flow ratio of $L'/V' \approx 0.1235$.

Using this, and the fact that $X_{A,0} = 0$, we can calculate the true exit concentration X_N, again using the mass balance:

$$X_{A,n} = (Y_{A,n+1} - Y_{A,1}) \left(\frac{L'}{V'}\right)^{-1} = (0.02 - 0.001)/0.1235 \approx 0.154$$

We now have two points to plot the operating line, which are $(X_N, Y_{N+1}) = (0.154, 0.02)$ and $(X_0, Y_1) = (0, 0.001)$.

Performing the graphical solution below (black line), we have 6 real plates required (no fractional plates when using a murphree efficiency).
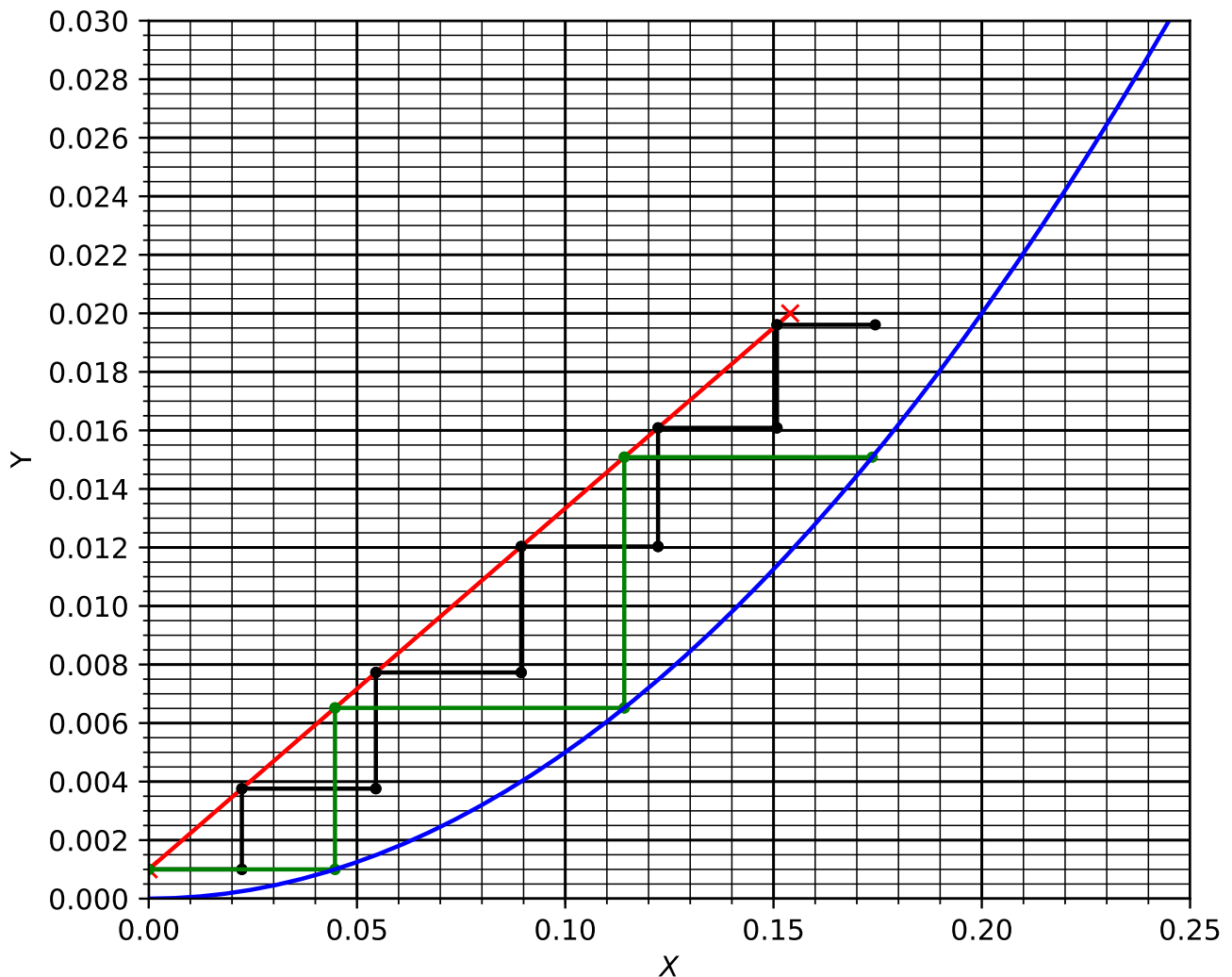
c) Determine the overall column efficiency.   **[5 marks]**
   Solution:
   Performing the same stepping, but using an efficiency of 100% (green lines above), we have a
   requirement of $\approx$ **2.67** theoretical stages. This gives an overall efficiency of **2.67/6** $\approx$ **0.445**.

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

Student ID:
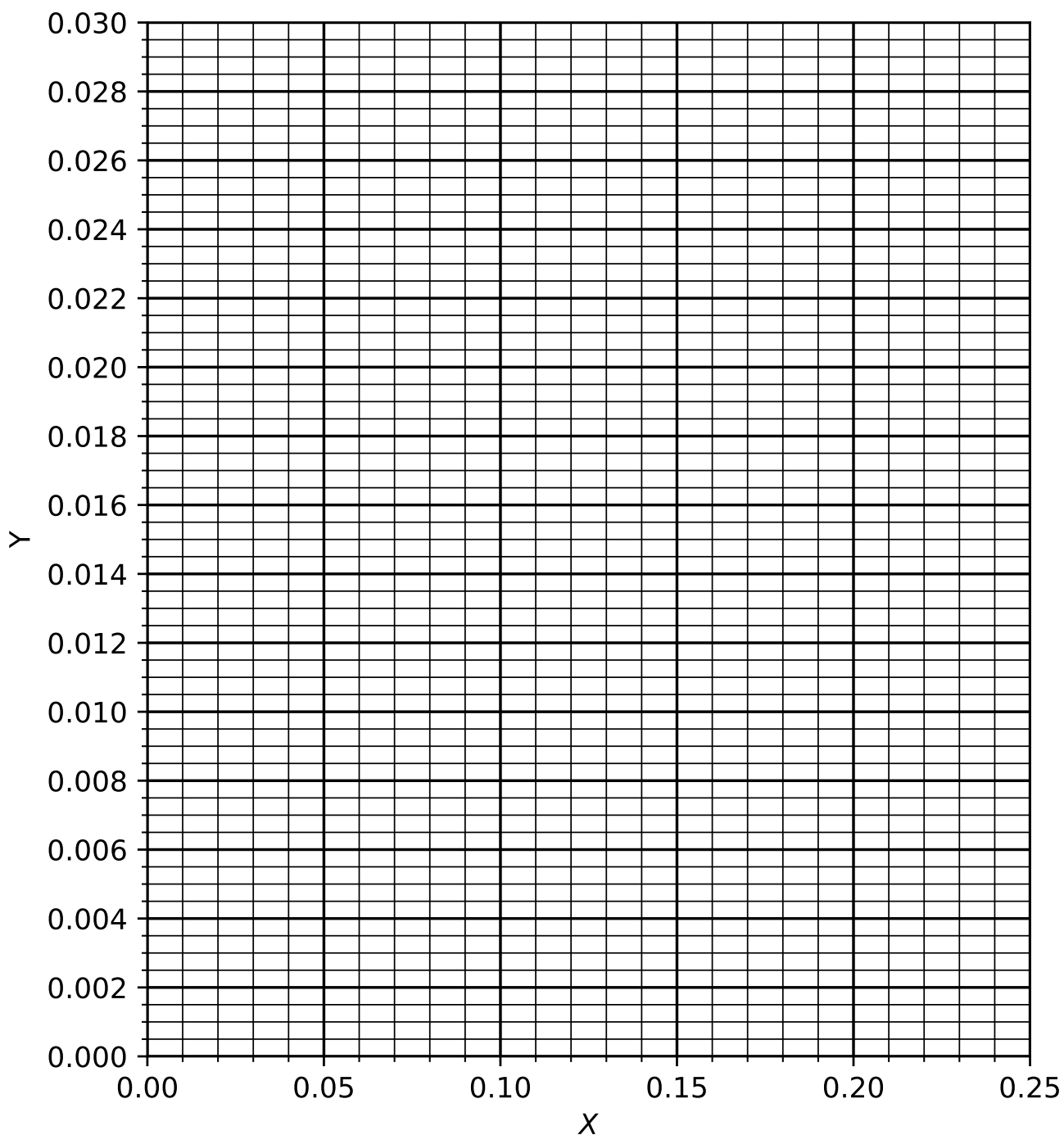
Figure 35: A blank X-Y graph for use in Q. 7.33.

[Question total: 20 marks]

Q.7.34 Question 7.34

A binary mixture containing 40 mol percent of the more volatile component (mvc) is fed to a continuous distillation column to produce a top product containing 90 mol percent and a bottom product containing 10 mol percent of the mvc. If the operating reflux ratio is 1.5 times the minimum, estimate the number of theoretical stages required when the feed is a saturated vapour. The relative volatility of the mvc is 8.5.
Solution:

3 stages.

[Question end]

Q.7.35 Question 7.35

A methanol/isopropanol (i.e., propan-2-ol) feed is continuously fractionated in a plate column to produce a top product containing 95 mol% methanol and a bottom product containing 10 mol% methanol. The feed, which has a composition 40 mol% methanol, is introduced as liquid at its bubble point temperature and the column operates with a reflux ratio of 4. A blank *xy* diagram is available in Fig. 20 and VLE data is available in Table 7. The proposed column design utilises a thermosyphon reboiler, which is a total reboiler (i.e., not an equilibrium stage). What is the overall column efficiency if the Murphree stage efficiencies are 80% in the rectifying section and 60% in the stripping section?   **[20 marks]**

Table 7: VLE data for methanol-isopropanol mixtures. The most volatile compound is methanol.

x_A	0	0.05	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
y_A	0	0.085	0.167	0.317	0.45	0.57	0.67	0.75	0.83	0.895	0.955	1.00

Note: A blank x-y diagram is available in Fig. 20.
Solution:
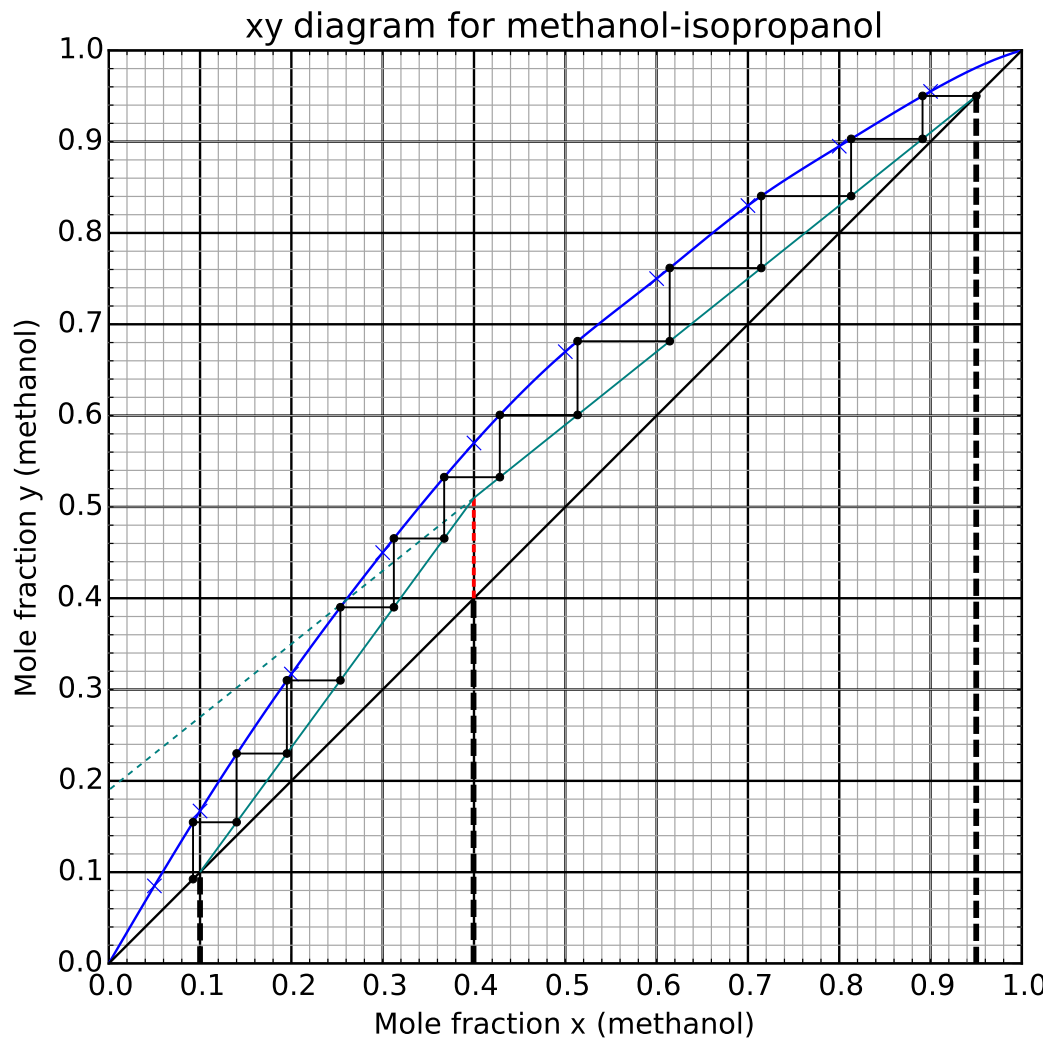
Figure 36: Ideal solution for the methanol-isopropanol system.

Intercept is $x_D/(R + 1) = 0.95/5 = 0.19$.

q-line is vertical as the feed is at its bubble point.

Just under 12 stages required for 100% efficiency.
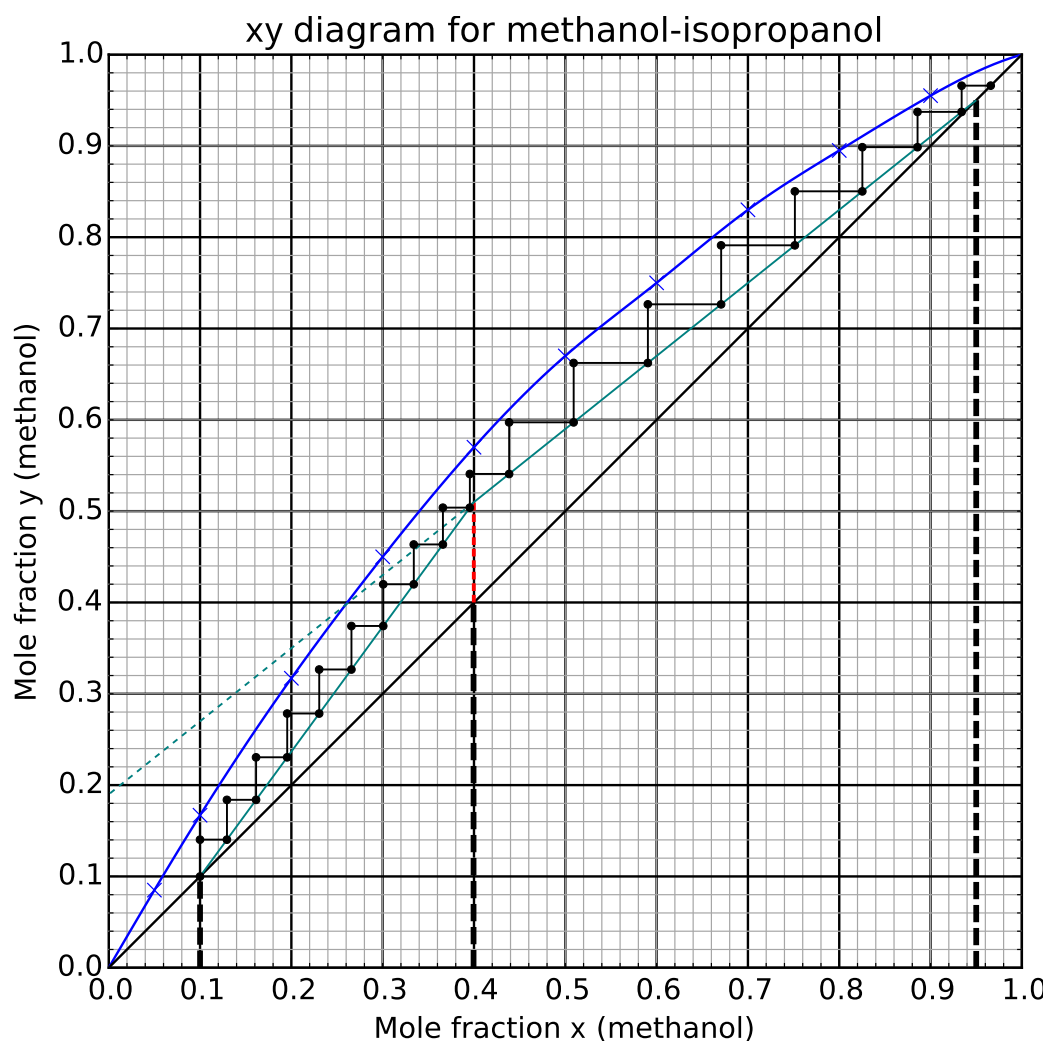
Figure 37: Murphree solution for the methanol-isopropanol system.

Note, whole stages can only be reported, and as the reboiler is a total reboiler it is NOT a stage for distillation (so efficiency is applied to all stages).

18 stages when Murphree efficiency is taken into account, so overall efficiency is 2/3 or 66.66%.

[Question total: 20 marks]

8   Week 8: X-Y adsorption and multi-stage batch distillation

Q.8.36   Question 8.36   **Recommended**

It is desired to absorb 90% of the acetone in a gas containing 1.0 mol% acetone in air in a counter-current stage tower. The total inlet gas flow to the tower is 30.0 kg mol h^{-1}, and the total inlet pure water flow to be used to absorb the acetone is 90 kg mol H$_2$O h^{-1}. The process is to operate isothermally at 300 K and a total pressure of 101.3 kPa. The equilibrium relation for the acetone (A) in the gas-liquid is $y_A = 2.53\,x_A$. Determine the number of theoretical stages required for this separation using a XY-plot (do not assume the change in flow-rates are negligible). **[20 marks]**

Solution:
We start by solving the mass balances to find the inert flow rates, and the concentrations at one end of the absorber. We need to remind ourselves of the numbering convention for the streams, so here is the standard drawing.
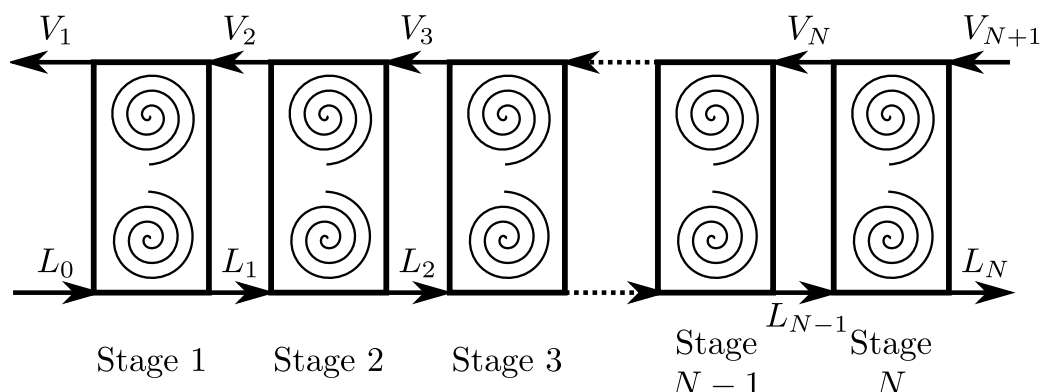
So, converting the text to math, we have

$$V_{N+1} = 30 \text{ kg mol h}^{-1} \qquad\qquad y_{A,N+1} = 0.01$$
$$L_0 = 90 \text{ kg mol h}^{-1} \qquad\qquad x_{A,0} = 0$$

The inert gas is the air, so using the conditions at the gas inlet we have $V' = V_{N+1}(1 - y_{A,N+1}) = 30(1 - 0.01) = 29.7$ kg mol h^{-1}.

The water entering the process is pure, so we have $L' = L_0 = 90$ kg mol h^{-1}. Now we need all of the concentrations at both ends of the absorber so we can plot the operating line equation. So we need the concentration of the gas phase V_1. The problem states that 90% of the acetone is absorbed into the liquid. Entering the absorber in V_{N+1}, we have $0.01 \times 30 = 0.3$ kg mol h^{-1} of Acetone. Only 10% of this is in the exit gas stream, or $0.1 \times 0.3 = 0.03$ kg mol h^{-1} of Acetone in V_1. The concentration of V_1 is then $y_{A,1} = 0.03/(0.03 + 29.7) \approx 0.001$. We can work out the liquid concentrations at the other end of the absorber in a similar manner. 90% of the Acetone entering in the inlet gas is absorbed into the liquid, or $0.9 \times 0.3 = 0.27$ kg mol h^{-1}. The total flow rate is then $L_N = L' + 0.9 \times 0.3 = 90.27$ kg mol h^{-1}, and the concentration is $x_{A,N} = 0.27/90.27 \approx 0.003$. A summary of all of this information is now given

$$V_{N+1} = 30 \text{ kg mol h}^{-1} \qquad\qquad y_{A,N+1} = 0.01$$
$$L_N = 90.27 \qquad\qquad x_{A,N} = 0.003$$
$$V_1 = 29.73 \text{ kg mol h}^{-1} \qquad\qquad y_{A,1} = 0.001$$
$$L_0 = 90 \text{ kg mol h}^{-1} \qquad\qquad x_{A,0} = 0$$

We have everything we need to draw the operating line, which is given by the equation

$$\frac{y_{A,n+1}}{1 - y_{A,n+1}} = \frac{L'}{V'}\frac{x_{A,n}}{1 - x_{A,n}} + \frac{y_{A,1}}{1 - y_{A,1}} - \frac{L'}{V'}\frac{x_{A,0}}{1 - x_{A,0}}$$

This is where the solution deviates from previous solutions, as we want to plot the graph as X versus Y to obtain a straight operating line. Changing in variables to $X = x/(1 - x)$ and $Y = y/(1 - y)$, the operating line becomes

$$Y_{A,n+1} = \frac{L'}{V'}X_{A,n} + Y_{A,1} - \frac{L'}{V'}X_{A,0}$$

And substituting in the known values, we have

$$Y_{A,n+1} = \frac{90}{29.7} X_{A,n} + \frac{0.001}{1-0.001} - \frac{90}{29.7} X_{A,0}^{0}$$
$$= 3.03\, X_{A,n} + 0.001$$

The operating line will go from $X_{A,0} = 0/(1-0) = 0$ to $X_{A,N} = 0.003/(1-0.003) \approx 0.003$. We can use the operating line equation to find the corresponding Y values, or just directly use the y'values we've already determined. To plot the operating line, we draw a straight line between the points $(X_{A,0} = 0, Y_{A,1} = 0.001)$ and $(X_{A,N} = 0.003, Y_{A,N+1} = 0.01)$.

We also need to know the VLE line in these coordinates. The original VLE data equation is

$$y_A = 2.53\, x_A$$

We need it in terms of Y_A, which gives

$$Y_A = \frac{y_A}{1-y_A} = \frac{2.53\, x_A}{1 - 2.53\, x_A}$$

To plot this, we need to generate a table of data from $x_A = 0$ until we have $Y_A > 0.01$.

x_A	X_A	Y_A
0	0	0
0.0003	0.00030009	0.0007595765
0.0006	0.0006003602	0.0015203078
0.0009	0.0009008107	0.0022821966
0.0012	0.0012014417	0.0030452454
0.0015	0.0015022534	0.0038094569
0.0018	0.0018032458	0.0045748338
0.0021	0.0021044193	0.0053413787
0.0024	0.0024057739	0.0061090944
0.0027	0.0027073097	0.0068779835
0.003	0.0030090271	0.0076480487
0.0033	0.0033109261	0.0084192927
0.0036	0.0036130068	0.0091917182
0.0039	0.0039152696	0.0099653279
0.0042	0.0042177144	0.0107401246

Now this table and data is a pain to generate and plot, but you only need to plot it once for each system (acetone-water, or CO_2-water, etc.). So if you are given XY VLE data from the start, it is actually easier to perform the absorber design using XY when compared to xy. Plotting this data and the two points of the operating line, gives the following graph
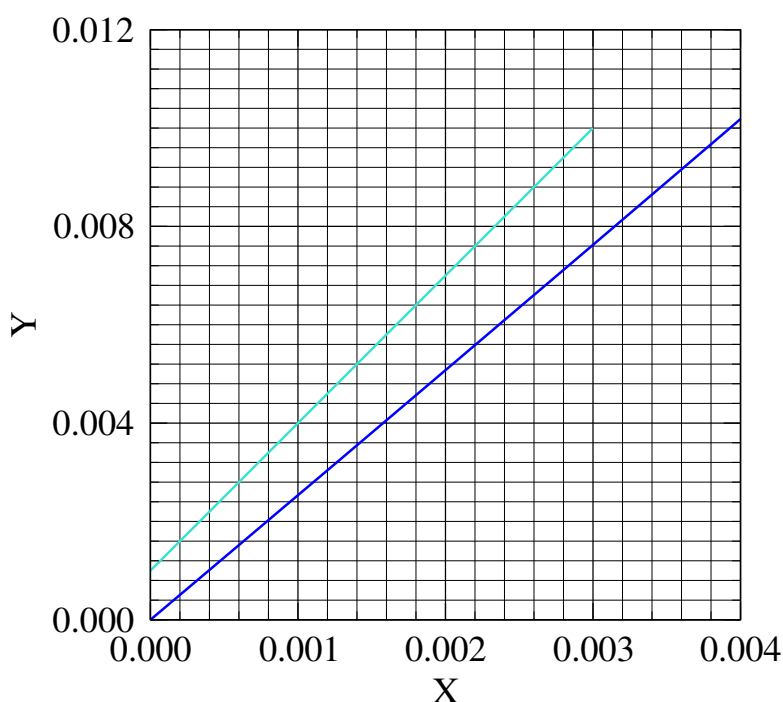
which we can then step to obtain the solution of ≈ 5.1 theoretical stages.
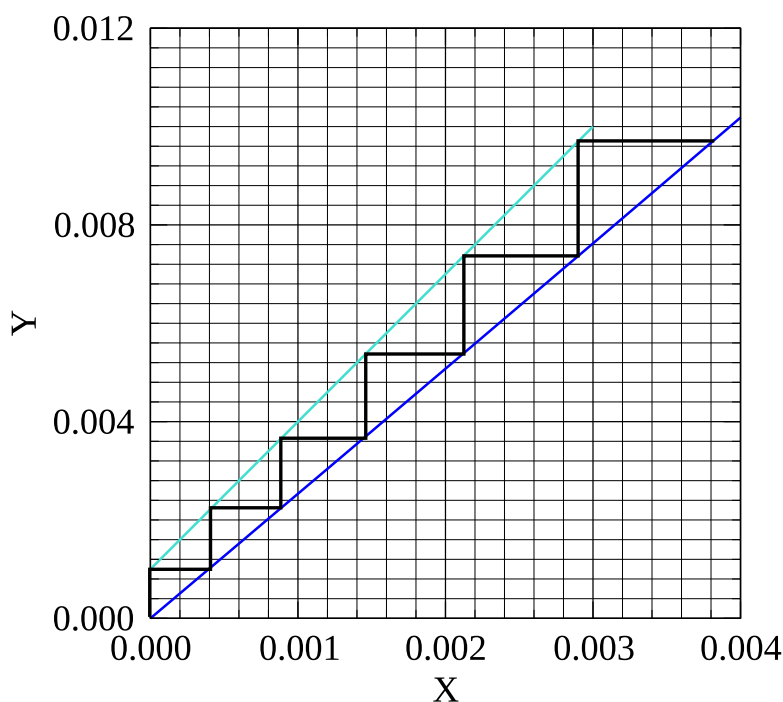
[Question total: 20 marks]

Q.8.37 Question 8.37   **Recommended**

When 10 000 kg of a mixture of heptane (mol. mass 100 g) and n-octane (mol. mass 114 g) is boiled and undergoes batch rectification in a column comprising three theoretical stages (including the still) with a reflux ratio of 1.2, the initial distillate contains 85 mol% n-heptane. **Note:** VLE data is provided in Fig. 38.

a) Determine the initial composition of the liquid in the still.
   Solution:
   If the distillate composition is $x_D = 0.85$, the intercept with the axis is $y(x = 0) = x_D/(R + 1) \approx 0.386$. Performing the design, we have the following diagram
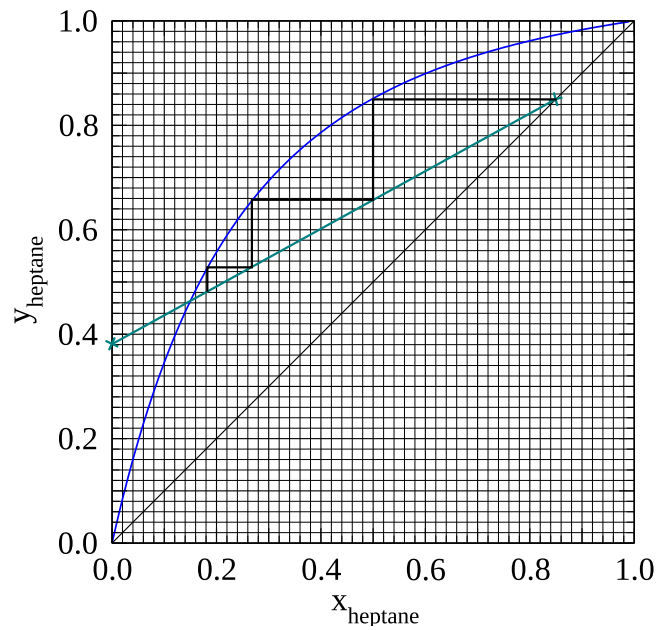
Therefore the initial still concentration is roughly 0.18.

b) If the process is continued at the same reflux ratio until the still composition has been reduced to half its initial value, calculate the fraction of the initial charge vapourised and the average composition of the distillate collected.

Solution:

Here we will need several distillation values to perform the integration AND to find the final distillate composition. We're looking for the distillation concentration where the bottoms concentration is **0.09**.
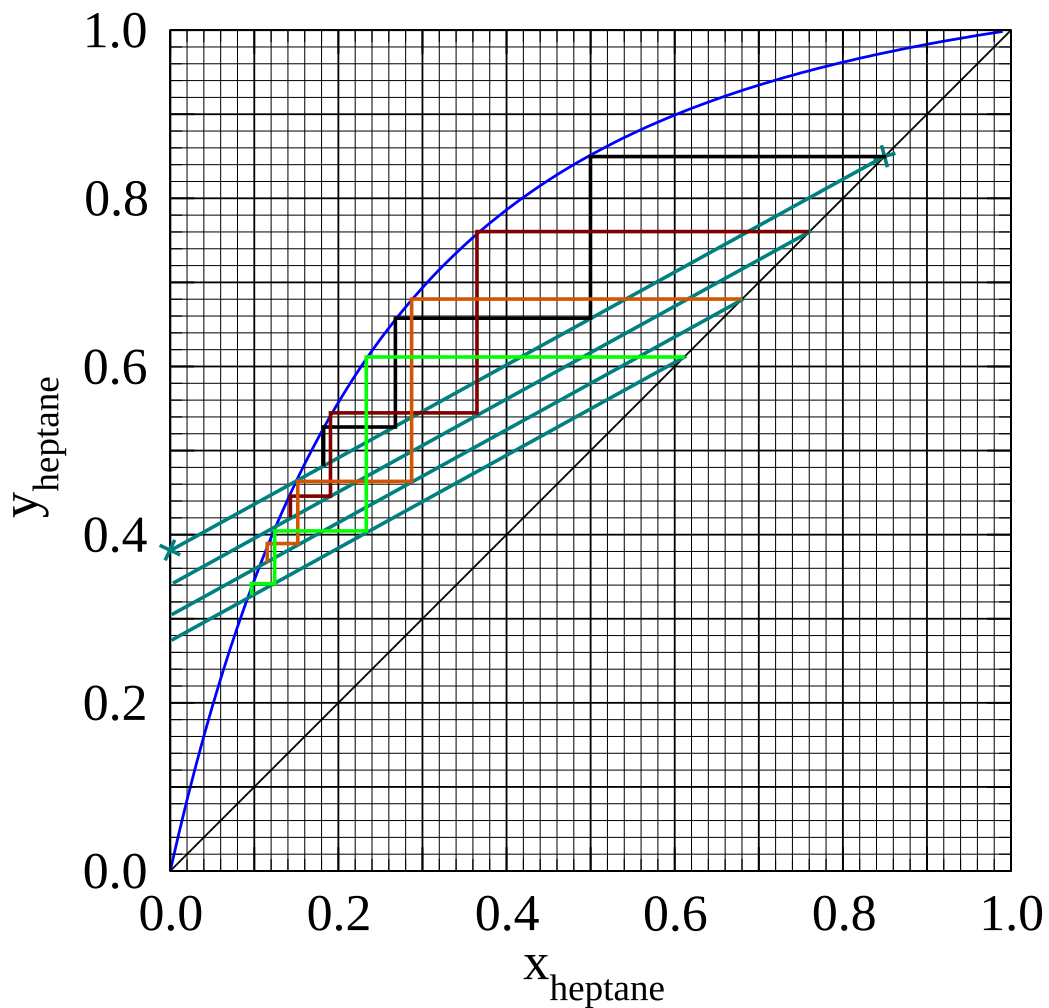
Performing the distillations, we have the following results

x_D	x_S	$(x_D - x_S)^{-1}$
0.85	0.18	1.49
0.76	0.14	1.61
0.68	0.11	1.75
0.61	0.09	1.92

The value of the kernel $((x_D - x_S)^{-1})$ doesn't change an awful lot over the integration area, so this is probably enough points to carry out the integration.

We need to integrate $(x_D - x_S)^{-1}$ with respect to x_S, for the Rayleigh equation (modified for multi-stage distillation)

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_{x_{initial}}^{x_{final}} \frac{dx_S}{x_D - x_S}$$

Using the trapezium rule, we can calculate the area under the first section of the curve

$$\int_{0.18}^{0.14} \frac{dx_S}{x_D - x_S} \approx 0.5\,(1.49 + 1.61)\,(0.14 - 0.18) \approx -0.062$$

Adding up each trapezium, we have

$$\int_{0.18}^{0.09} \frac{dx_S}{x_D - x_S} \approx -0.062 - 0.050 - 0.037 \approx -0.149$$

Using the full rayleigh equation, we have

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_{0.18}^{0.09} \frac{dx_S}{x_D - x_S} = -0.149$$
$$L_{final}/L_{initial} = 0.8616$$

We cannot use the 10000 kg in Rayleigh's equation here as we have used mole fractions in the balance thus L must have units of moles. Mass units could be directly used for L if x and y were also mass fractions. At this point all we can state that 86.16% of the initial charge in moles remains in the still.

To determine the average concentration of the distillate we need to perform a mass balance:

$$x_{initial}\, L_{initial} - x_{final}\, L_{final} = x_{avg.}\,(L_{initial} - L_{final})$$

Dividing by $L_{initial}$ and rearranging for $x_{avg.}$:

$$x_{avg.} = \frac{0.18 - 0.09 \times 0.8616}{1 - 0.8616} \approx 0.74$$

c) If the boil-up rate in the still is constant and equal to 10 kmol h^{-1}, how long will the process take?
Solution:
A trap here is that the units of the boil up are in kmol, but the initial charge was specified in kg. The average molecular mass of the liquid in the still at the start ($x_{initial} = 0.18$) is

$$0.18 \times 100 + 0.82 \times 114 = 111.5 \text{ g mol}^{-1}$$

This means that there are $10000/111.5 \approx 89.7$ kmol in the still at the start. There are $89.7\exp(-0.149) = 77.3$ kmol remaining in the still. The total distillation time is then $(89.7 - 77.3)/10 \approx 1.24$ h.

d) Why could the above separation not be achieved using a simple Rayleigh distillation?
Solution:
The additional stages in the column allow a higher recovery rate of the volatile component. The use of a single stage would greatly reduce the overall distillate composition collected for the same fraction of vapourised feed.

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

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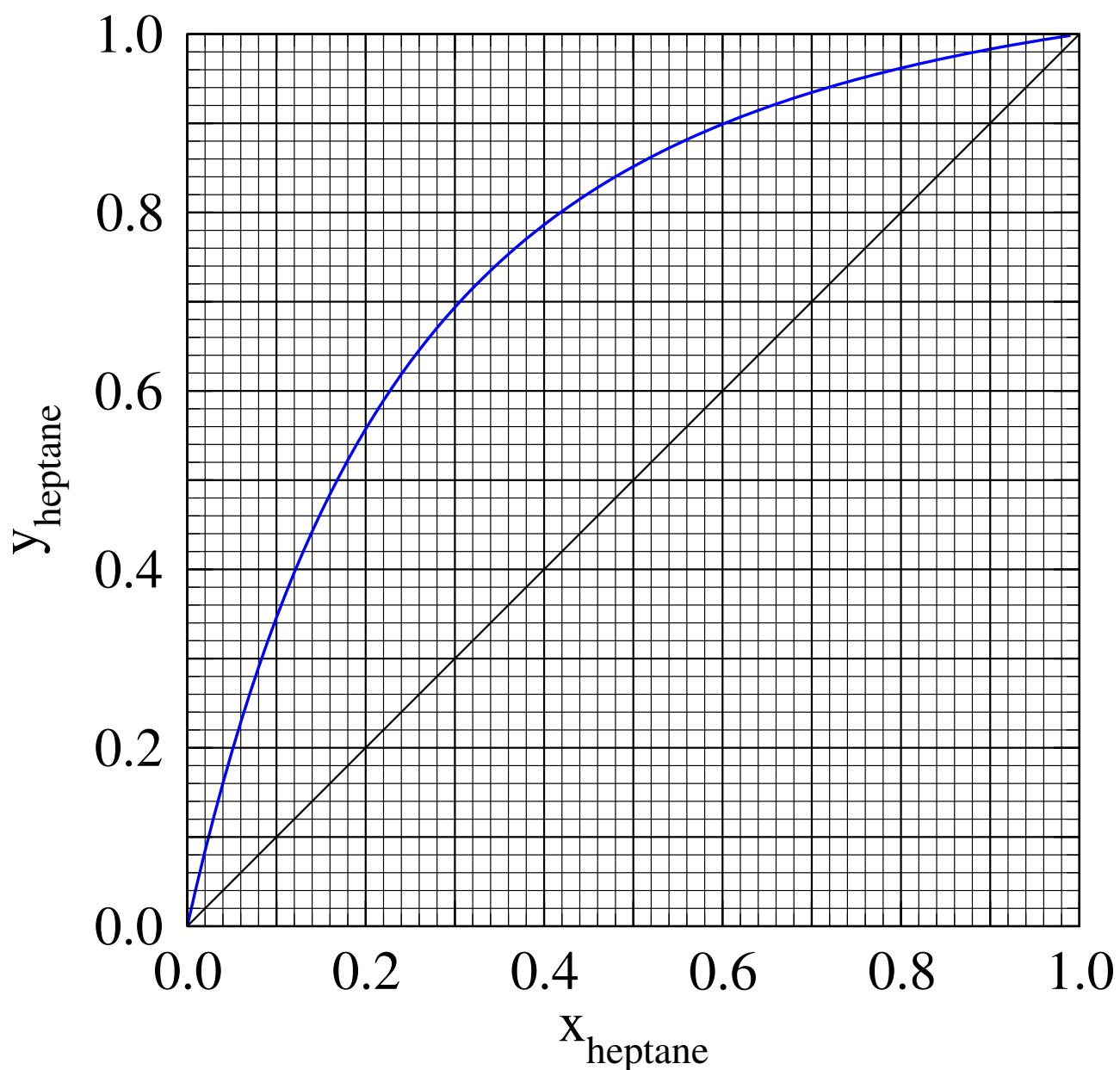
Figure 38: VLE data for the heptane-octane system at 1 atm. For use in Q. 8.37.

[Question end]

9   Week 9: Ponchon-Savarit Distillation

Q.9.38   **Question 9.38**   **Recommended**

In the production of high-purity ethanol, grape alcohol is distilled from a feedstock concentration of 3.3 mol% to 74 mol%. Do not assume constant molar overflow in this case. VLE data are provided in Fig. 26 and enthalpy data are available in Fig. 43. Assuming a multi-stage batch *rectification* column is used, calculate:

a) Demonstrate the mole fractions of 3.3 mol% and 74 mol% ethanol are equivalent to 10% ABV (alcohol by volume) and 90% ABV (you may assume that water and ethanol have constant densities of 998 kg/m³ and 789 kg m⁻³).
Solution:
The conversion from 10% ABV to mole basis is as follows. We first work out the molar concentrations

$$0.1\,(\mathrm{m^3/m^3}) \times 789\,(\mathrm{kg\ m^{-3}})/46\,(\mathrm{kg\ kmol^{-1}}) = 1.71\,(\mathrm{kmol\ ethanol\ m^{-3}})$$
$$0.9\,(\mathrm{m^3/m^3}) \times 998\,(\mathrm{kg\ m^{-3}})/18\,(\mathrm{kg\ kmol^{-1}}) = 49.9\,(\mathrm{kmol\ water\ m^{-3}})$$

The molar fraction is then $1.71/(49.9 + 1.71) = 3.3$ mol% ethanol. Performing the same calculation for 90%, we have 15.4 (kmol ethanol m⁻³) and 5.54 (kmol water m⁻³) giving 74 mol%.

b) What is the *initial* minimum number of trays required at the feedstock concentration to achieve the required product concentration?   **[5 marks]**
Solution:
Using the Ponchon-Savarit construction (see Fig. 39) for infinite reflux ratio, we have a minimum of just over 4 theoretical stages.

c) What is the *initial* minimum reflux ratio required to perform this separation?   **[5 marks]**
Solution:
Here, we take equilibrium lines of the system and determine the height they intercept with $x = x_D$ (see Fig. 40). The highest intercept is the pinch point in the system. Here, as in many systems, it is at the feed tie-line and yields a minimum reflux ratio of,

$$R = \frac{h'_d - h_{v,d}}{h_{v,d} - h_{l,d}} = \frac{132500 - 40333}{40333 - 533} \approx 2.32.$$

d) Assuming an *initial* reflux ratio of $R = 4$, determine the theoretical number of stages required to perform the *initial* separation.   **[5 marks]**
Solution:
The actual pole point location is backed out of the same equation for the reflux ratio calculation,

$$R = \frac{h'_d - h_{v,d}}{h_{v,d} - h_{l,d}}$$
$$h'_d = R\,(h_{v,d} - h_{l,d}) + h_{v,d}$$
$$= 4\,(40333 - 533) + 40333$$
$$= 199500 \approx 200000$$

The design is performed in Fig. 41, giving **5** theoretical stages.

Figure 39: The minimum number of plates required to perform the distillation (see Q. 9.38b).
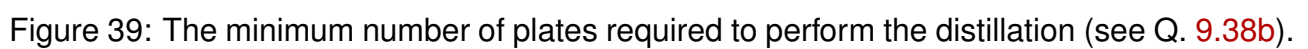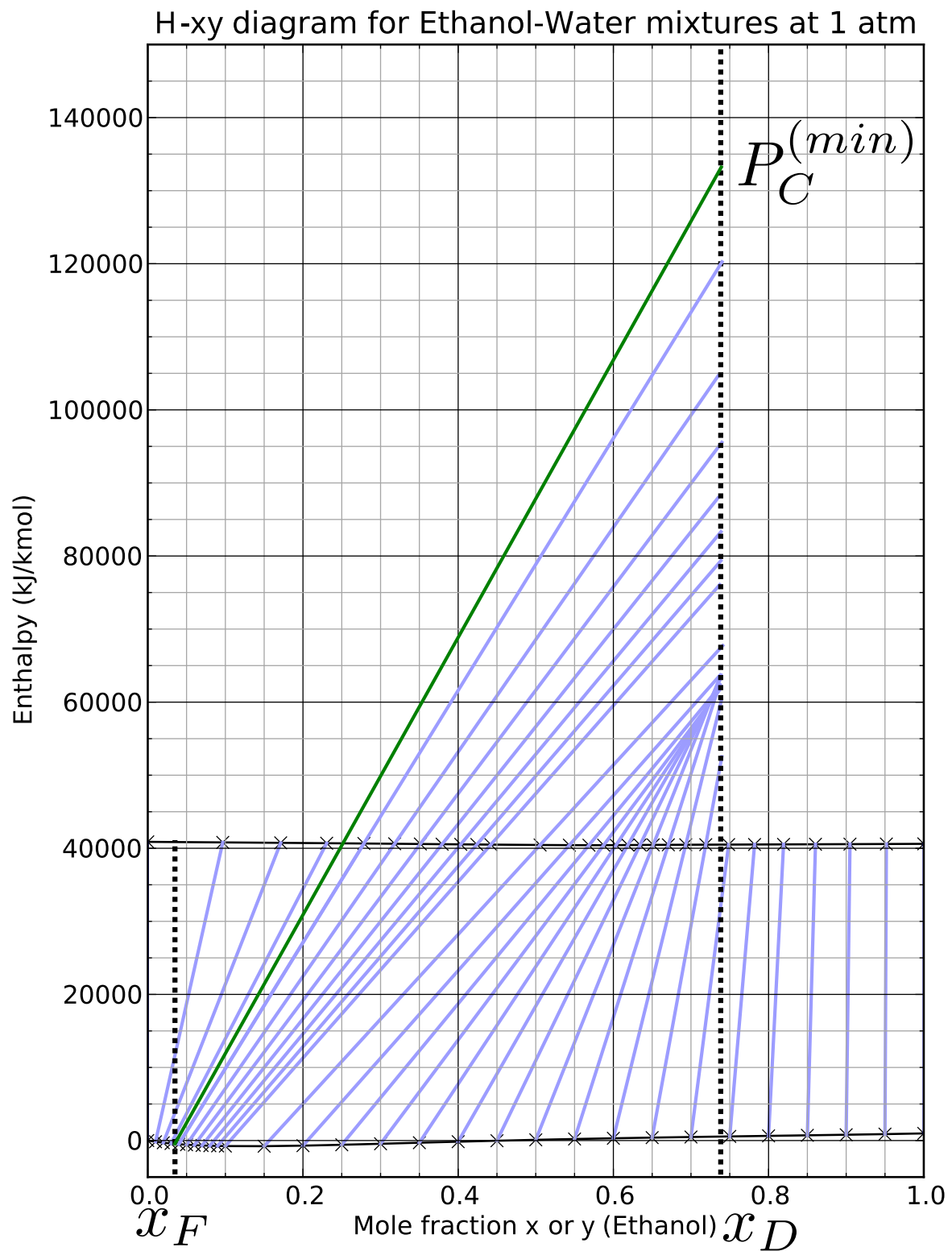
Figure 40: The minimum reflux ratio to perform the separation (solution to Q. 9.38c).
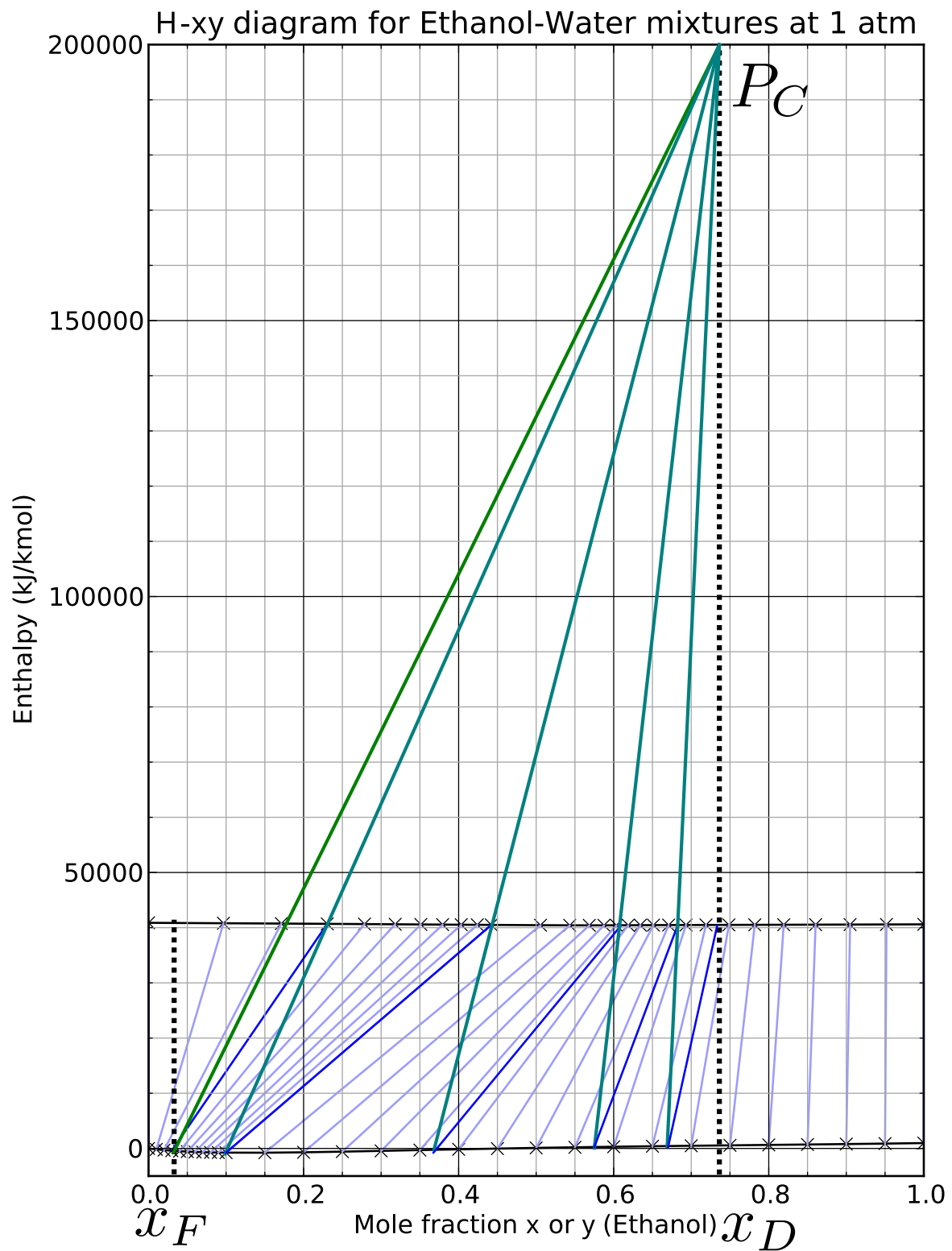
Figure 41: The solution to Q. 9.38d).

e) Assuming the reflux ratio is varied to keep the top product concentration constant over the batch distillation, what is the minimum liquid concentration which can be reached in the still? **[5 marks]**

Solution:

This is equivalent to asking what liquid concentration would yield the top product given an infinite reflux ratio and **5** theoretical stages. We can simply extend our infinite reflux ratio calculation (see Fig. 42). This yields a minimum bottom concentration of 0.08%.
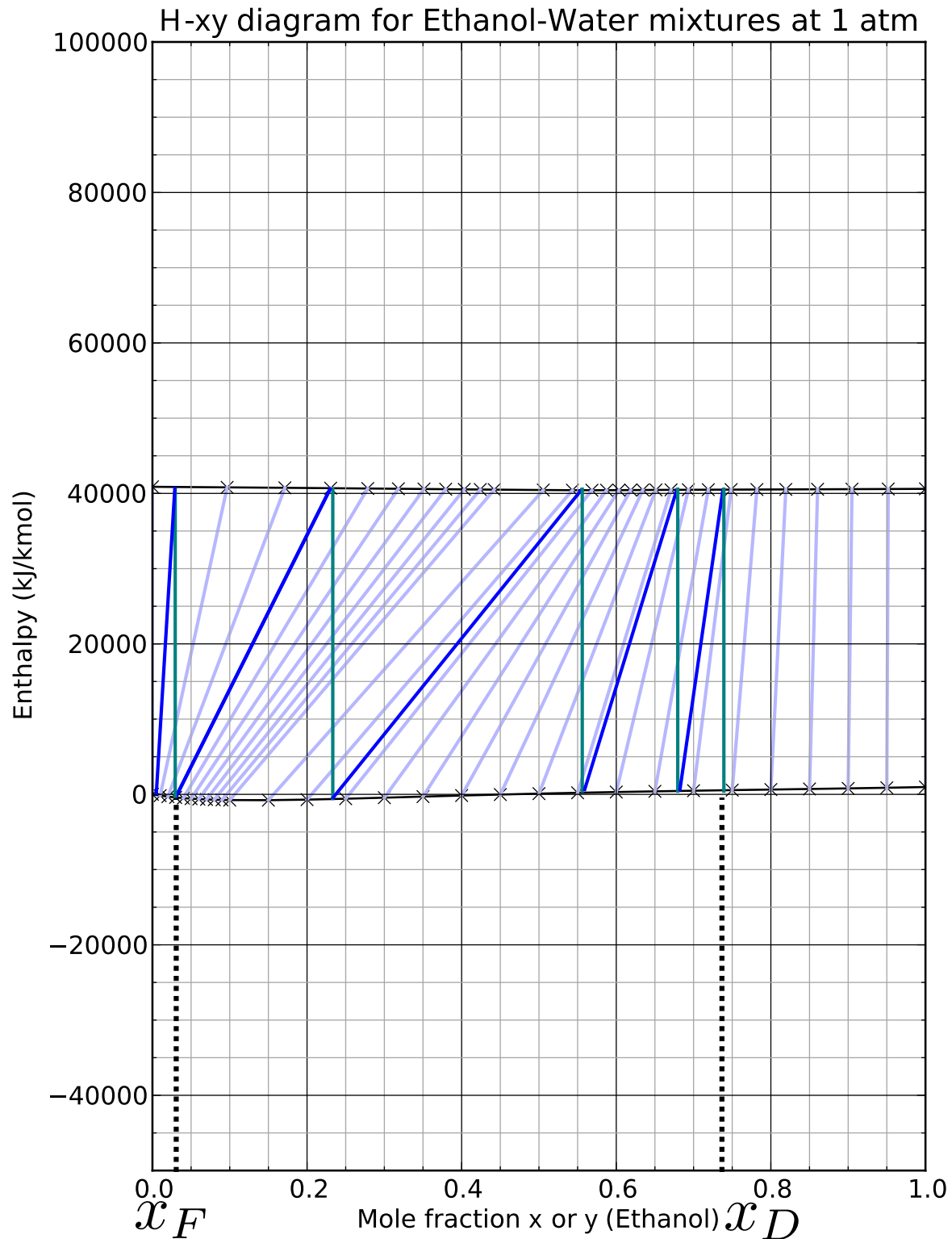
Figure 42: Five plates at infinite reflux ratio. The solution to Q. 9.38e).

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

Student ID:
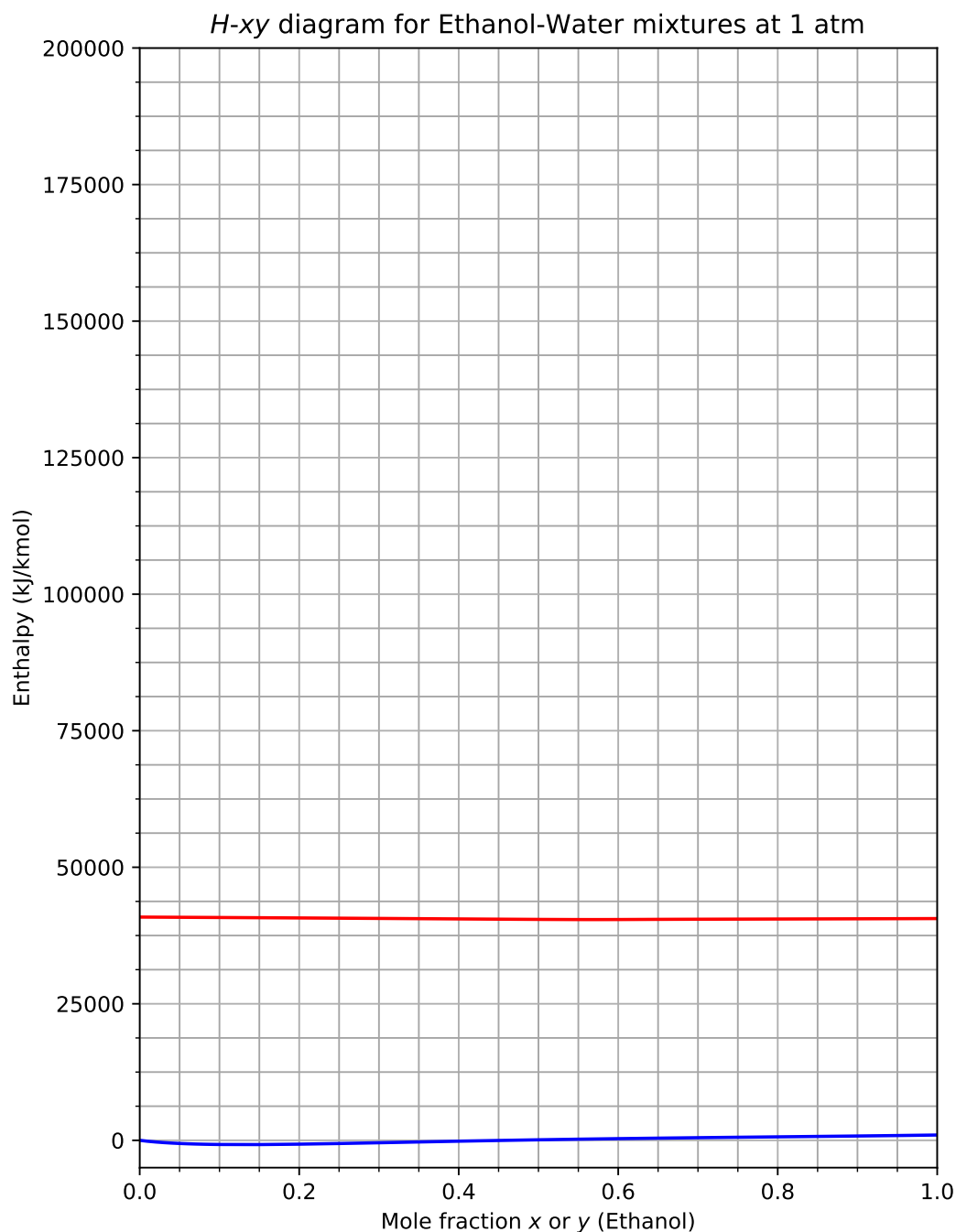
Figure 43: The enthalpy-concentration diagram for a ethanol-water mixture at atmospheric pressure. Data was calculated using the NASA Glenn thermodynamic database, and latent heats of mixing estimated using the equation given in doi:10.1016/0021-9614(75)90261-X (it should be noted that the latent heats of mixing have an almost negligble effect for this system). For use in Q. 9.38.

[Question total: 20 marks]

Q.9.39   Question 9.39

a) Under what circumstances does the Ponchon-Savarit method provide a better estimate of the number of theoretical stages required for a given distillation application, when compared to the McCabe-Thiele method? **[4 marks]**
Solution:
The Ponchon-Savarit method (or enthalpy-concentration method) is more suitable than the McCabe-Thiele method whenever the assumption of Constant Molar Overflow (CMO) is violated to a significant degree. This may occurr if sensible heat changes are significant, there are strong concentration effects (heats of mixing/solution) on the enthlapy, or latent heats change dramatically.

If the McCabe-Thiele method was modified to include these effects, the operating lines would appear curved. The Ponchon-Savarit method bypasses the CMO assumption by explicitly including an enthalpy balance (hence the need for a *H-xy* diagram).

b) Using overall balances, component balances, and heat balances demonstrate that the condenser pole point (P_c), the reboiler pole point (P_R) and the feed point (P_F) must all lie on the same straight line on the enthalpy-composition diagram. **[6 marks]**
Solution:

Taking an overall balance we have:

$$F = W + D \qquad\qquad\qquad x_F\, F = x_W\, W + x_D\, D$$

And eliminating F gives:

$$\frac{W}{D} = \frac{x_D - x_F}{x_F - x_W}$$

We can also perform an enthalpy balance:

$$Q_B + h_F\, F = h_{l,W}\, W + h_{l,D}\, D + Q_c$$

Introducing $h'_w = h_{l,w} - \frac{Q_B}{W}$ and $h'_D = h_{l,D} + Q_c/D$ into the energy balance we have

$$h_F\, F = h'_W\, W + h'_D\, D + Q_c$$
$$\frac{W}{D} = \frac{h'_D - h_F}{h_F - h'_W}$$

Combining these two gives:

$$\frac{h'_D - h_F}{h_F - h'_W} = \frac{x_D - x_F}{x_F - x_W}$$

If h_F and x_F are our free parameters, then it is clear to see that this equation has a solution at $(x_F, h_F) = (x_D, h'_D)$ and $(x_F, h_F) = (x_W, h'_W)$. Therefore, the feed point $P_F = (x_F, h_F)$, condenser pole point $P_C = (x_D, h'_D)$, and reboiler point $P_W = (x_W, h'_W)$, all lie on the same (straight) line on a *H-x* diagram.

[Question total: 10 marks]

Q.9.40 Question 9.40

a) A mixture of *n*-hexane and *n*-octane is to be separated by distillation to produce a bottom product containing no more than 3 mol% *n*-hexane and a top product of at least 95% purity of *n*-hexane. An enthalpy-composition chart is given in Fig. 44 which you should use and include in your solution booklet.

  a) Determine the minimum number of theoretical stages required to achieve the desired separation. **[6 marks]**

Solution:

[1/6]   The minimum number of trays corresponds to total reflux$_1^{\checkmark}$. In this case, the operating

[1/6]   lines are all straight (the point P_C is at $h'_C \to \infty$).$_1^{\checkmark}$

Performing the stepping using circled stage numbers (marks for quality and accuracy of

[3/6]   graphical construction$_3^{\checkmark}$), we have 3.87 theoretical stages required$_1^{\checkmark}$.

[1/6]
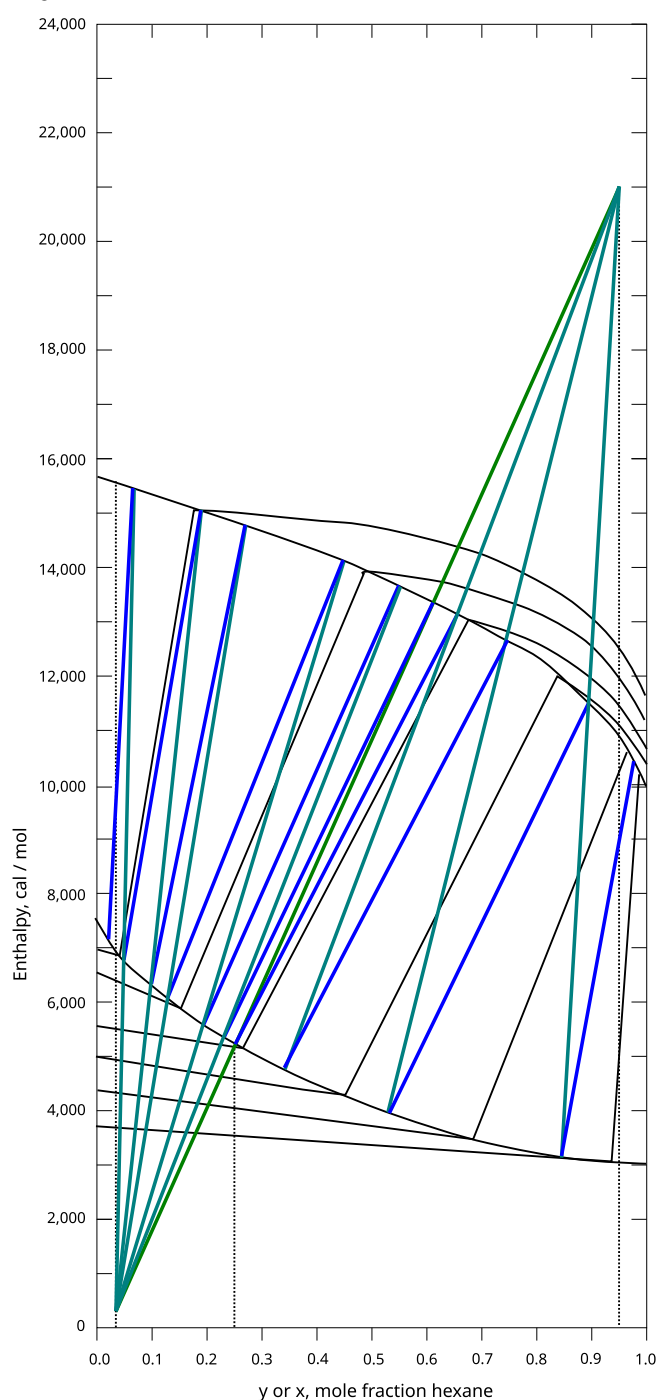
b) Calculate the actual number of theoretical stages required if the reflux ratio is 1.25 and the feed, having a composition of 0.25 mole fraction *n*-hexane, is supplied as liquid at its boiling point. **[11 marks]**
   Solution:
   If the feed is a liquid at its boiling point, we can instantly determine the feed point as

[1/11] $P_F = (x_F, h_L(x_F))_1^{\checkmark}$. We only need one pole to determine the line which the other pole lies on. The most convenient pole is the condenser due to the specification of the reflux ratio.

With a reflux ratio of 1.25, the condenser condenses 2.25 moles for each mole of distillate produced. Therefore we have

$$\frac{Q_C}{D} = 2.25\,(h_{V,D} - h_{L,D}) = 2.25 \times (11000 - 3000) = 18000 \text{ cal mol}^{-1} \qquad (6)$$

[2/11]   This implies that $h_D' = h_{L,D} + Q_C/D = 21000$ cal mol^{-1}.$_2^{\checkmark}$

Plotting $P_C = (x_D, h_D')$ and $P_F = (x_F, h_L(x_F))$ (by passing a line through the feed point P_F and P_C), we then look for where the line reaches x_W to determine the reboiler point P_R. Then we can begin stepping from the feed stage using stage numbers in squares
[7/11] (marks for graphical construction $\frac{\checkmark}{7}$) to find that almost exactly 9 theoretical stages are
[1/11] needed$_1^{\checkmark}$.

c) Determine the number of trays in the column if the overall efficiency is 70% and a partial reboiler is used. If you do not have an answer, use an estimate of 9 ideal stages for the previous question. **[3 marks]**
   Solution:
   The previous question gave approximately 9 ideal stages. One stage is accounted for by
[1/3] the partial reboiler$_1^{\checkmark}$. The total number of plates is then 8/0.7=11.43$_1^{\checkmark}$. As these are real
[1/3] trays a whole number must be used thus there are 12 trays in the column$_1^{\checkmark}$.

[1/3]

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

Student ID:

Figure 44: Enthalpy concentration diagram for hexane-octane mixtures at 760 mmHg. For use in Q. 9.40.

[Question total: 20 marks]

Q.9.41 Question 9.41

It is required to separate 1 kg s^{-1} of a solution of ammonia in water, containing 30% by mass of ammonia, to give a top product of 99.5% purity and a weak solution containing 10% by mass of ammonia. You may assume that the feed and output streams are liquids at their boiling points. Constant molar overflow does not apply in this system.

T-xy data is available in Fig. 45 and enthalpy-concentration data in Fig. 46.

a) Determine the number of theoretical stages required to perform the separation assuming reflux is 20% in excess of the minimum.   **[13 marks]**

Solution:

Solving the overall mass balance,

$$D + W = 1 \qquad\qquad 0.995\, D + 0.1\, W = 1 \times 0.3$$
$$D = 0.22 \text{ kg s}^{-1} \qquad\qquad W = 0.78 \text{ kg s}^{-1}$$

[2/13] The feed flashes to $y(x = 0.3) = 0.91$, extending this line to the $x = 0.995$ line gives an
[1/13] intercept at ≈ 2000 kJ kg^{-1}.

[1/13] The latent heat of vaporization at $x = 0.995$ is $\approx 1540 - 310 = 1230$ kJ kg^{-1}.

The minimum reflux ratio is then,

$$R_{min} = \frac{2000 - 1540}{1230} = 0.374$$

[1/13]

The actual reflux rate is $1.2\, R_{min} = 0.449$, meaning the condenser pole point is at $1.449 *$
[1/13] $1230 + 310 = 2092$ kJ kg^{-1}.
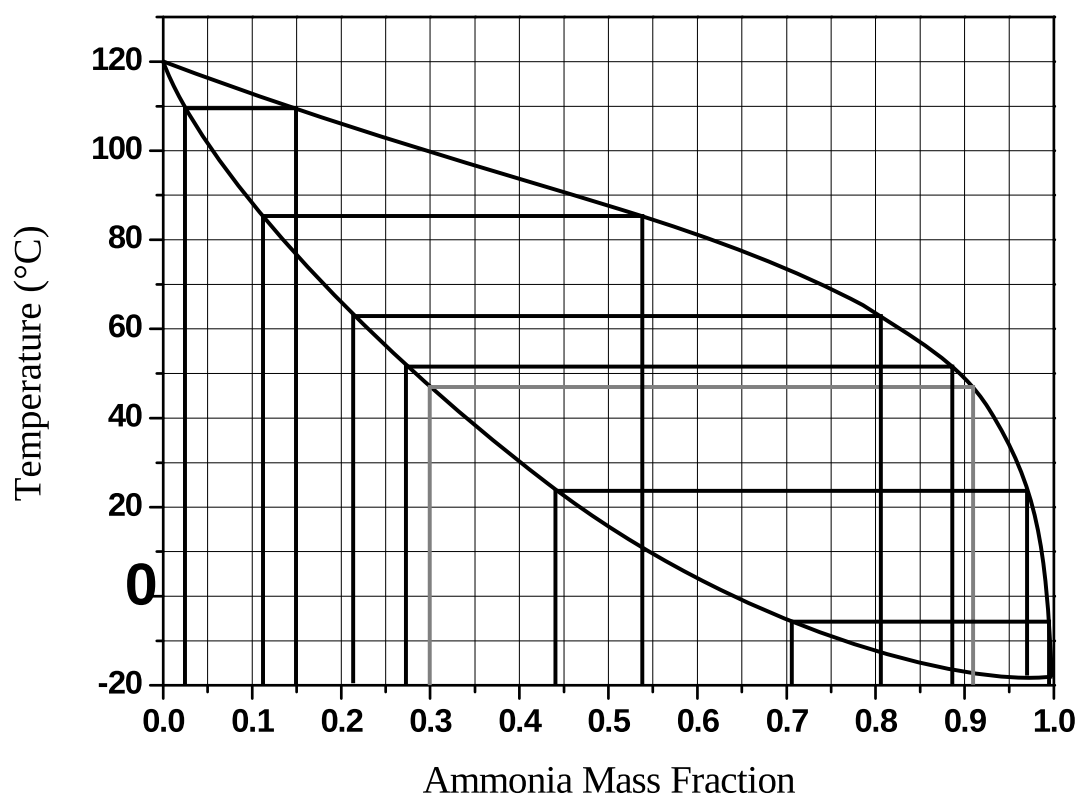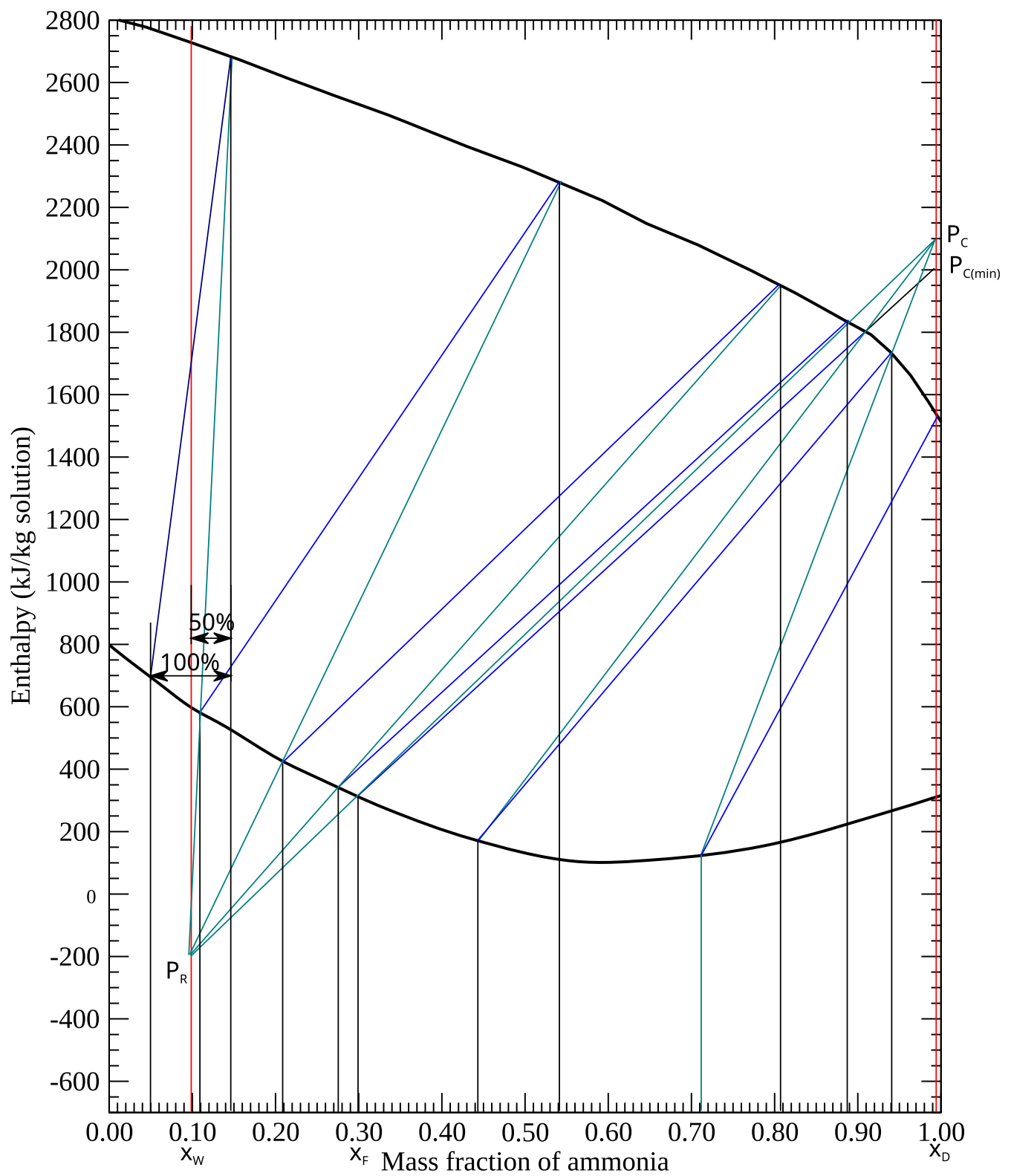
[5/13]   Care must be taken in the construction of the graph.✓₅

[2/13]   The solution is 6.5 theoretical stages. ✓₂

b) Calculate the heat to be rejected in the condenser.   **[4 marks]**
   Solution:
   The heat rejected per kg of distillate is the height of the condenser pole point from the **[1/4]** saturated liquid line,$\checkmark_1$ $2092 - 310 = 1782$ kJ kg^{-1}.$\checkmark_1$ The total duty is then $0.22 * 1782 =$ **[1/4]** 392 kW$\checkmark_2$.

[2/4] c) A partial reboiler and a tray column with a overall overall ideal efficiency of 60% is proposed to be used. Calculate the number of trays needed.   **[3 marks]**
   Solution:

[1/3] Partial reboilers approach ideal operation$\checkmark_1$, thus the number of ideal stages in the column **[1/3]** is 5.5.$\checkmark_1$. The total number of real trays is then $5.5/0.6 = 9.167$ which is rounded up to 10 **[1/3]** whole real trays.$\checkmark_1$

[Question total: 20 marks]

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

Student ID:

Figure 45: The *T-xy* diagram for a ammonia-water mixture at atmospheric pressure. For use in Q. 9.41.

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

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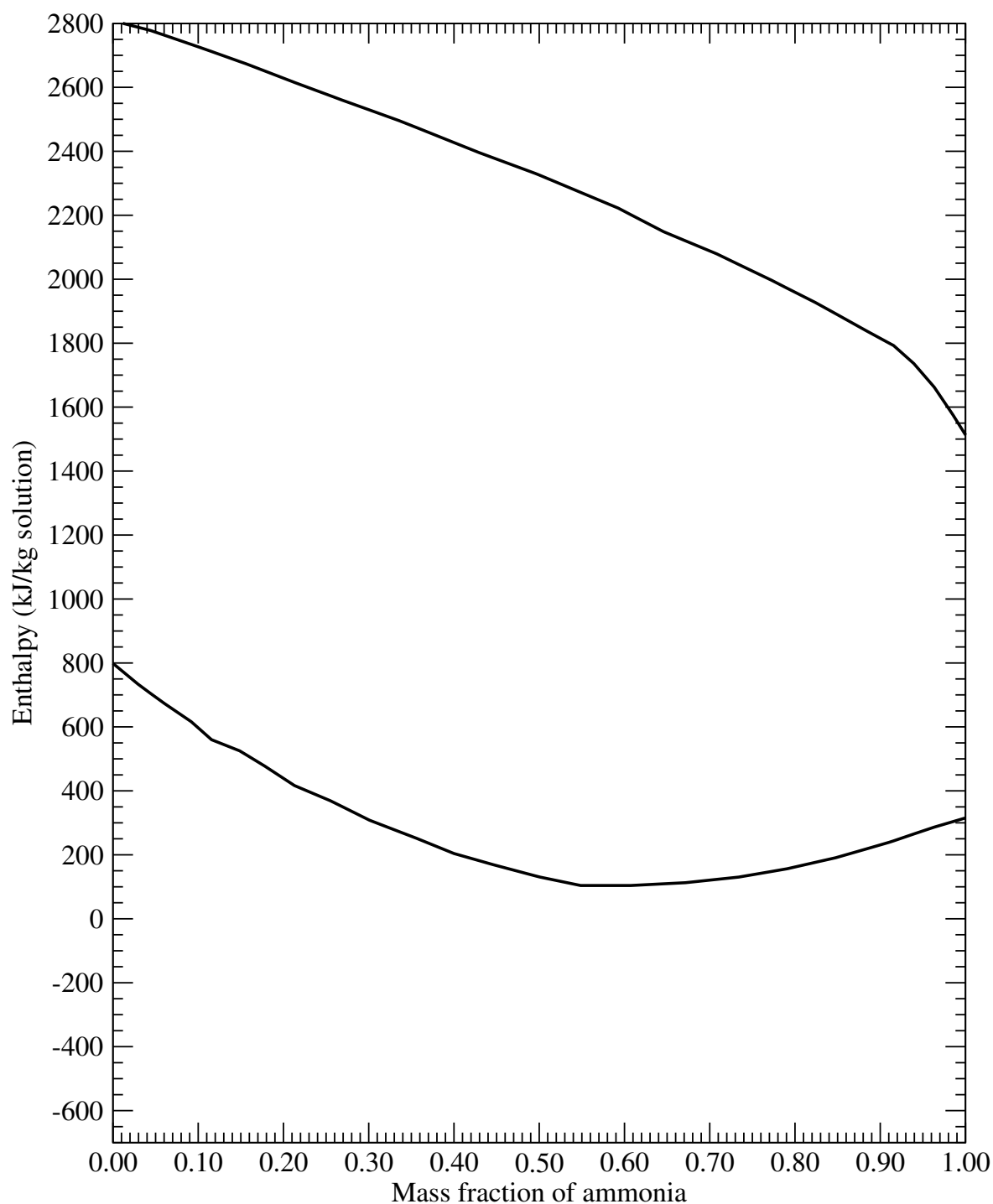
Figure 46: The enthalpy-concentration diagram for a ammonia-water mixture at atmospheric pressure. For use in Q. 9.41.

Q.9.42   Question 9.42

Using the enthalpy-composition diagram for hexane-octane in Fig. 47, construct a temperature-composition (T-x-y) diagram and a x-y diagram at the same pressure of 760 mmHg. The boiling temperatures of the pure components are 68.5 and 125.8°C.

Solution:

Use tie lines to give x and y for each temperature, plot these as T-x-y and x-y points.

[Question end]

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

Student ID:
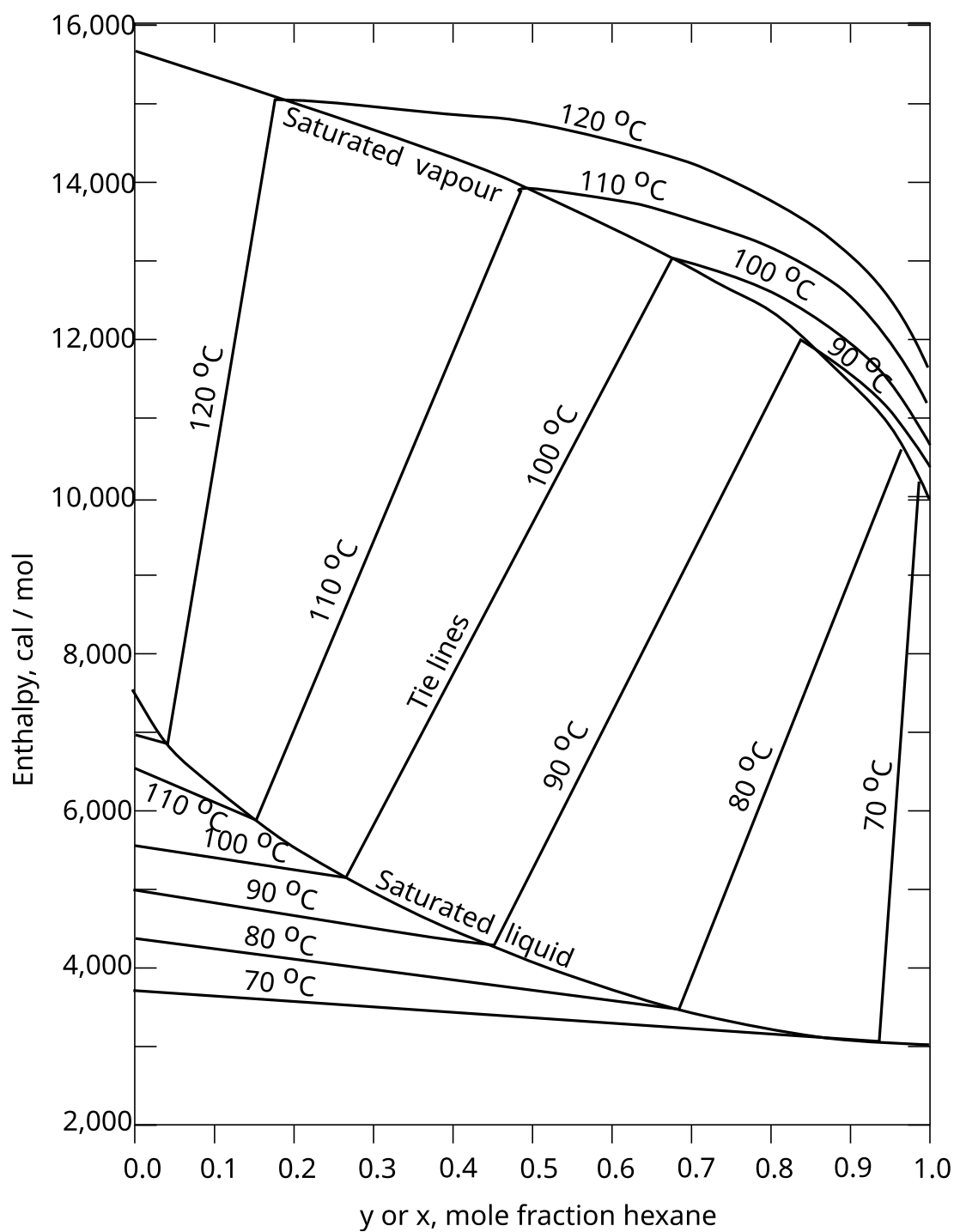
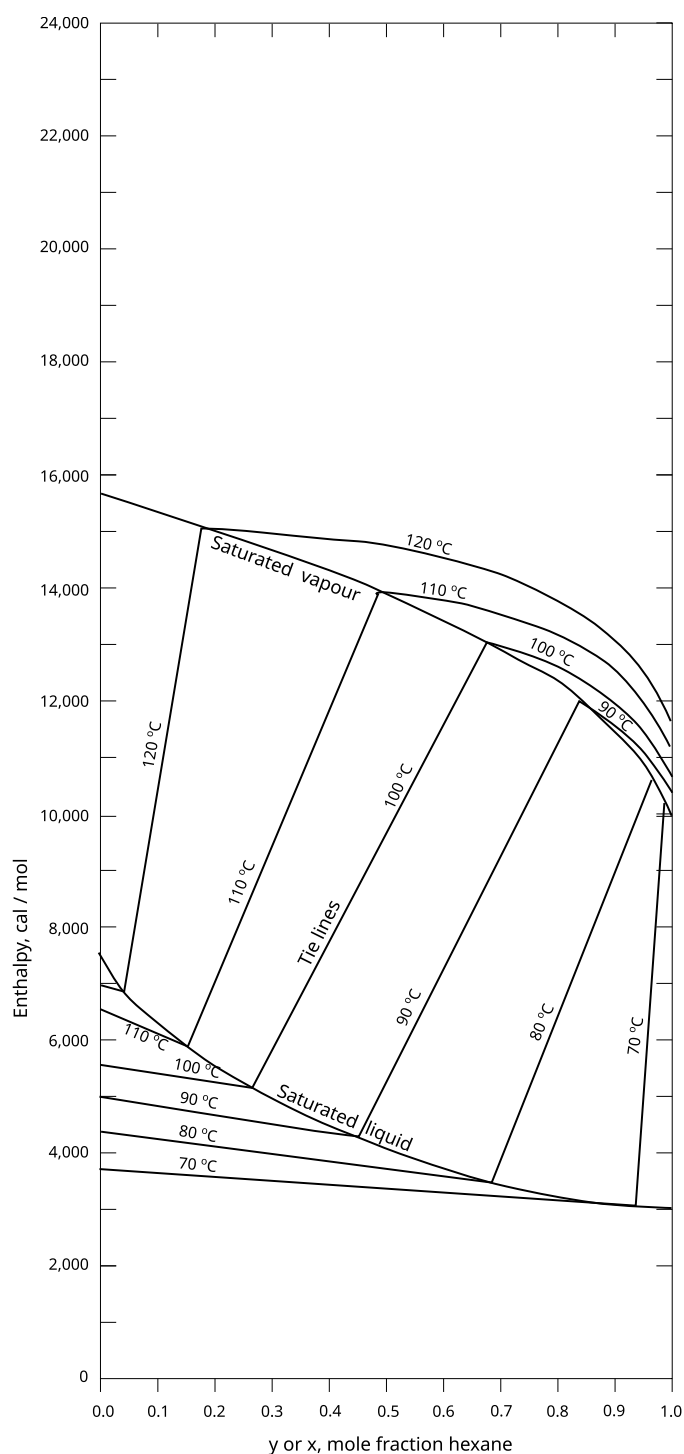Figure 47: Enthalpy concentration diagram for hexane-octane mixtures at 760 mmHg. For use in Q. 9.42, and 9.43.

Q.9.43 Question 9.43

An equimolar mixture of hexane and octane is throttled from a moderately high pressure into a flash vessel maintained at a pressure of 760 mmHg. Determine the temperature at which the vessel must be maintained for each of the following degrees of vaporisation (V/F): 0.00, 0.25, 0.50, 0.75, 1.00. You will need the enthalpy-composition diagram for hexane-octane in Fig. **??**.

Solution:

T = 87.5, 93, 99, 104.5, 109.5

[Question end]

Q.9.44 Question 9.44

An aqueous solution of acetic acid is to be enriched by simple (differential) distillation. The flask is initially charged with 0.025 kmol of solution of composition 0.600 mol fraction water.

x	0.300	0.350	0.400	0.450	0.500	0.550	0.600
y	0.420	0.483	0.537	0.592	0.638	0.685	0.728

a) Distillation is continued until the composition of the liquid in the flask has fallen to 0.300 mol fraction water. Assuming that the vapour is completely condensed and collected in a cooled reciever, calculate the quantity and composition of the distillate.
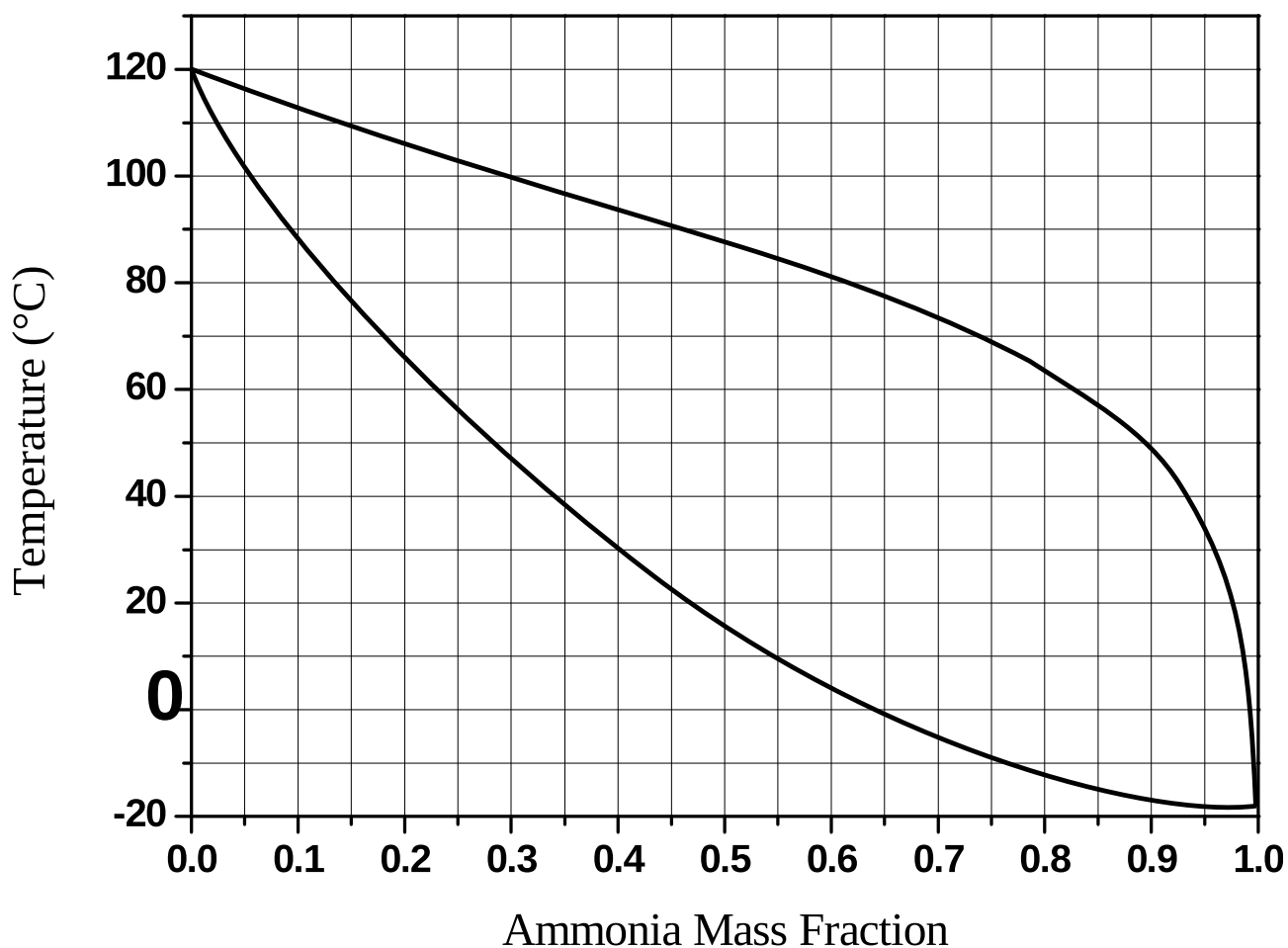
Solution:

Distillate collected $= 0.0223$ kmol, x_D = 0.637.

b) Repeat the calculation for the case in which distillation is stopped when the composition of the liquid in the still has reached 0.500 mol fraction water.

Solution:

Distillate collected $= 0.0131$ kmol, x_D = 0.69.

c) Explain why the separation is so poor, particularly in the case studied in Q. a.

Solution:

Separation is poor as there is only one stage, and the relative volatility is quite small. After a time in batch distillation, the composition of the distillate product is weaker in the mvc than the starting material in the still, so the further the operation goes on, the less overall separation occurs.

[Question end]

Q.9.45 Question 9.45

Two miscible liquids, A and B, having equal molar heats of vaporisation, are being separated by batch rectification. At a particular point of the distillation, the composition of the vapour leaving the still is 44% A and that of the overhead product being produced at that time is 95% A.

a) If the reflux ratio is set at 3:1, estimate:

    i) The number of equilibrium stages in the column.

    Solution:

    4 including the still.

    ii) The temperature of the liquid leaving each stage.

    Solution:

Stage	0 (still)	1	2	3
T (°C)	134.5	128	116.5	107.5

iii) The temperature and composition of the liquir in the still.
Solution:
x_{still} = 0.15 and T = 134.5 °C

T (°C)	138.0	131.5	126.2	121.5	117.2	113.5	110.0	107.0	104.0
x_A	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900
y_A	0.330	0.530	0.660	0.750	0.820	0.875	0.920	0.955	0.980

The boiling temperatures of the pure components are 102.0°C and 145.0°C.

b) Using the equilibrium data and the equation of the operating line, make a stage to stage calculation (non-graphical) of the stream compositions.
Solution:
From the top y_3 = x_D = 0.95, x_3 = 0.79, y_2 = 0.83, x_2 = 0.515, y_1 = 0.624, x_1 = 0.275, y_{still} = 0.444, x_{still} = 0.15

[Question end]

Q.9.46 Question 9.46
A mixture of the same two liquids, A & B, as in Q. 9.45, is to be distilled in a batch recti-fication column, containing 3 equilibrium stages, to produce a distillate rich in A. When the column is operated with a reflux ratio of 2.5 the initial product composition is 89% A. Distil-lation continues at a constant reflux ratio until the composition of the liquor in the still has fallen to 10.5% A.
Compare the above with operation at a constant product composition (to produce the same amount of distillate of the same composition). Which is the more economical mode of oper-ation for this separation?
Solution:
 The reflux directly represents the economic cost of the process, as energy is required to boil and condense it. We are then trying to find out how much reflux is used to produce exactly the same distillate.
For the constant R process, plot the VLE curve and operating line, then plot 3 steps down from the initial distillate point (x_D, x_D) to get the initial still composition (about 0.185) remembering that the still is one of the 3 equilibrium stages. For several decreasing values of x_D, repeat the above to calculate x_{still} until x_{still} reaches the final composition of 10.5%. Then, using the Rayleigh equation you can determine $x_{D,average}$ for the whole process ($x_{D,average} \approx 0.82$), and how much product is actually collected. You can then multiply this by R, to determine how much reflux was processed (about 21) which is the cost of this mode of operation.
For the constant reflux ratio process, set x_D to a constant value x_D = $x_{D,average}$ = 0.82. Use repeated McCabe-Thiele as above to determine the values of D for each R. Collect values of R vs D over the range of operation required to produce the same amount of distillate, and integrate this to detemine the total quantity refluxed (about 22).
Thus, constant reflux ratio is cheaper to run.

[Question end]

Q.9.47 Question 9.47
Determine the values of "q" and the gradient of the "q-line" for a feed having each of the following states:

a) The molar enthalpies of saturated liquid and saturated vapour are 20 MJ kmol^{-1} and 50 MJ kmol^{-1} respectively; the molar enthalpy of the feed is:

  i) 18 MJ kmol^{-1}.
     Solution:
     q = 1.067, gradient=15.925

  ii) 19 MJ kmol^{-1}.
     Solution:
     q = 1.033, gradient=31.303

  iii) 20 MJ kmol^{-1}.
     Solution:
     q = 1.0, gradient=∞

  iv) 21 MJ kmol^{-1}.
     Solution:
     q = 0.967, gradient=-29.303

  v) 30 MJ kmol^{-1}.
     Solution:
     q = 0.667, gradient=2.003

  vi) 52 MJ kmol^{-1}.
     Solution:
     q = -0.067, gradient=0.0628

b) The molar enthalpies of saturated liquid and saturated vapour are 9.0 MJ kmol^{-1} and 48.6 MJ kmol^{-1} respectively. The bubble point and dew point temperatures are 93 °C and 99.3 °C, and the molar heat capacities of the liquid and vapour are 75 KJ kmol^{-1} K^{-1} and 36 KJ kmol^{-1} K^{-1} respectively. The feed has each of the following temperatures:

  i) 80.0 °C.
     Solution:
     q = 1.025, gradient=41

  ii) 90.0 °C.
     Solution:
     q = 1.006, gradient=167.667

  iii) 99.5 °C.
     Solution:
     q = -1.8×10^{-4}, gradient=1.79×10^{-4}

  iv) 120.0 °C.
     Solution:
     q = -0.0188, gradient=0.0185

[Question end]

Q.9.48 Question 9.48

An equimolar mixture of carbon tetrachloride and toluene is to be fractionated at atmospheric pressure to produce a top product containing 4 mol% toluene and a bottom product containing 4 mol% carbon tetrachloride. The feed is liquid at its bubble point and it may be assumed that the molar enthalpies of the liquid and of the vapour are linear funtions of composition. Enthalpic and VLE data is available in Table 8 & 9 and a blank chart is provided in Fig. 48. Using the Ponchon-Savarit method, calculate:

a) The minimum reflux ratio.   **[5 marks]**
    Solution:

    In this question, the student must plot the enthalpic data, and add the tie-lines themselves before performing the graphical constructions listed below.

    Table 8 only includes the enthalpic data for pure carbon tetrachloride and pure toluene. These are the points at $x = 0$ and $x = 1$, but no data is available for the intermediate region. If we assume ideal mixtures, then we can just draw straight lines between these points (as performed below). This only applies if the enthalpy of a mixture is just the sum of the pure component enthalpies (i.e., there is no heat of solution).
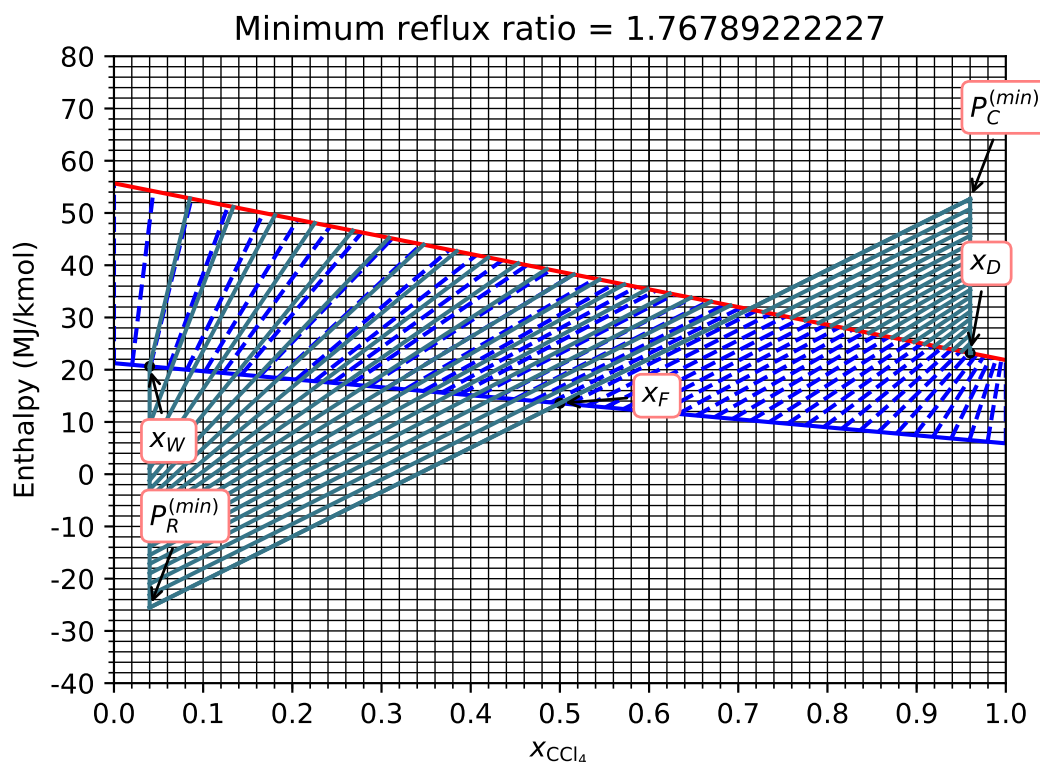
b) The minimum number of equilibrium stages.   **[7 marks]**
    Solution:

Minimum trays=7
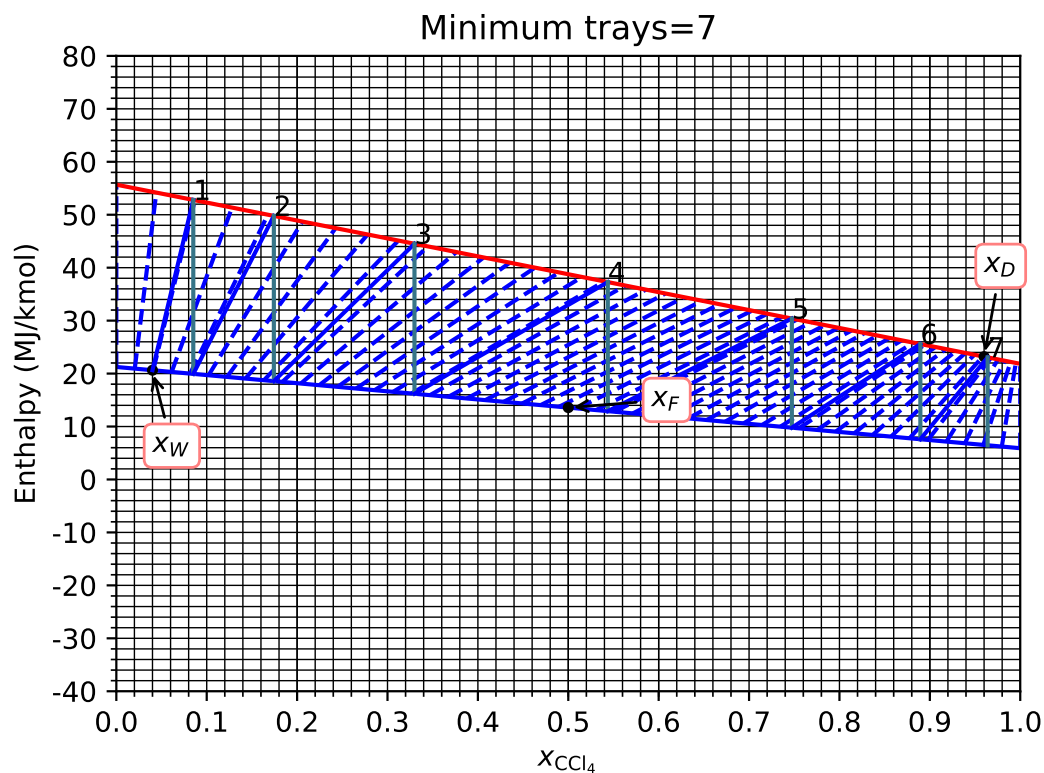
c) The number of equilibrium stages when the reflux ratio from the condenser is 2.5. **[8 marks]**

Solution:
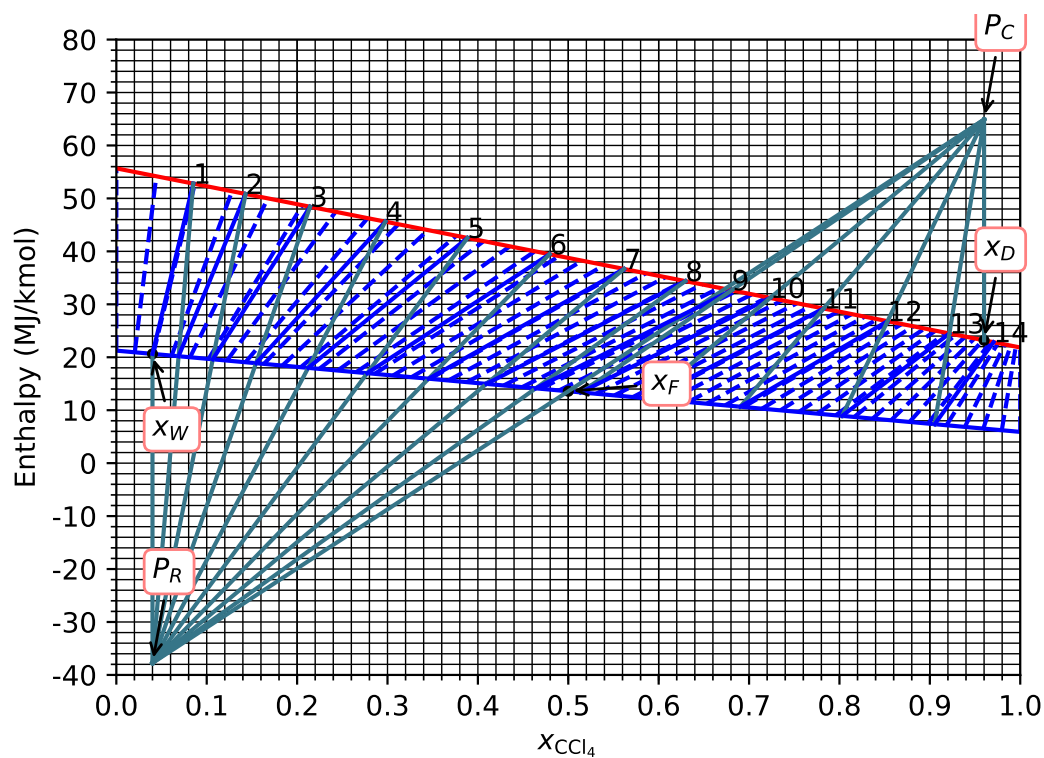
	Boiling Point (°C)	h_l (kJ kmol^{-1})	h_{fg} (kJ kmol^{-1})
CCl$_4$	76.4	5904 at 76.4 °C	15940
toluene	110.4	21250 at 110.4 °C	34430

Table 8: Enthalpic data for carbon tetrachloride (CCl$_4$) and toluene. This is required for Q. 9.48.

y_{CCl_4}	0.0	0.37	0.62	0.79	0.92	1.0
x_{CCl_4}	0.0	0.20	0.40	0.60	0.80	1.0

Table 9: Vapour-liquid equilibrium for carbon tetrachloride and toluene. This is required for Q. 9.48.

Data sheet handout. If submitting answers to the question, ensure you submit this graph as part of your solution.

Student ID:
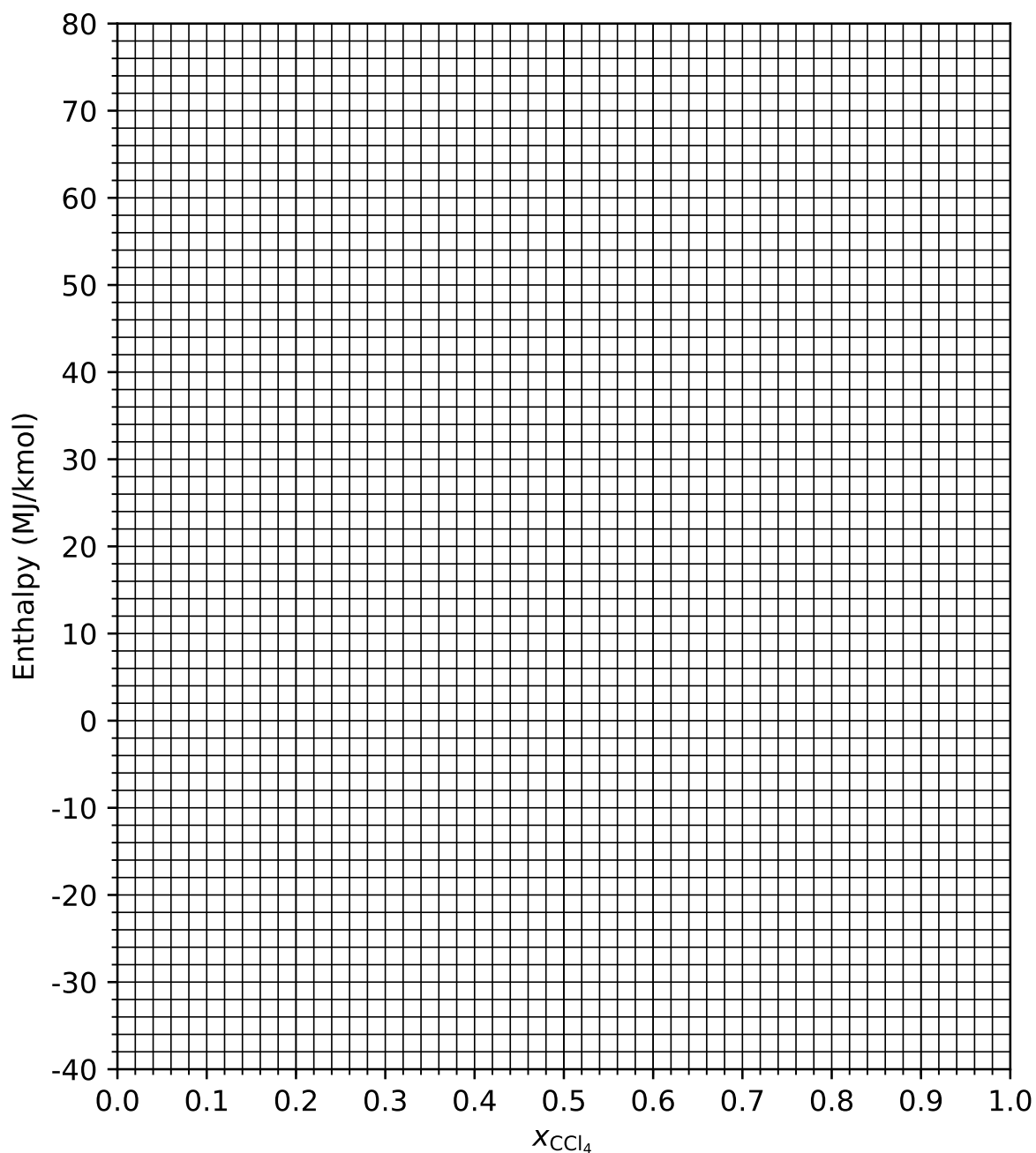
Figure 48: A blank H-x-y diagram for Q. 9.48

[Question total: 20 marks]

Part II

Datasheet from the exam

DATASHEET

Conversion from Celsius to Fahrenheit:

$$^\circ F = {^\circ C} \times 1.8 + 32$$

Operating lines:

$$y_n = x_{n+1}\frac{R}{R+1} + \frac{x_D}{R+1} \qquad \text{Enrichment line} \qquad (7)$$

$$y_m = x_{m+1}\frac{L_m}{V_m} - x_W\frac{W}{V_m} \qquad \text{Stripping line} \qquad (8)$$

$$y = x\frac{q}{q-1} - \frac{x_F}{q-1} \qquad \text{q-line} \qquad (9)$$

$$\frac{y_{A,n+1}}{1 - y_{A,n+1}} = \frac{L'}{V'}\frac{x_{A,n}}{1 - x_{A,n}} + \frac{y_{A,1}}{1 - y_{A,1}} - \frac{L'}{V'}\frac{x_{A,0}}{1 - x_{A,0}} \qquad \text{Absorption} \qquad (10)$$

Relative volatility

$$y_A = \frac{\alpha\, x_A}{1 + (\alpha - 1)x_A} \qquad (11)$$

Rayleigh's equation

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = \int_{x_{initial}}^{x_{final}} \frac{\mathrm{d}x}{y - x} \qquad (12)$$

If the relative volatility is constant:

$$\ln\left(\frac{L_{final}}{L_{initial}}\right) = (\alpha - 1)^{-1}\ln\left(\frac{x_{final}(1 - x_{initial})}{x_{initial}(1 - x_{final})}\right) + \ln\left(\frac{1 - x_{initial}}{1 - x_{final}}\right) \qquad (13)$$

Quadratic equation:

$$a\,x^2 + b\,x + c = 0 \qquad\qquad x = \frac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a} \qquad (14)$$

Ponchon-Savarit equations:

$$P_C = (R + 1)(h_V(x = x_D) - h_L(x = x_D)) + h_L(x = x_D) \qquad (15)$$

Table 10: Thermodynamic properties of saturated steam by temperature, calculated using the NASA CEA database and the vapour pressure data of Wexler or Wagner and Pruss (1990). The reference state is the triple point of saturated liquid water.

T (°C)	P (bar)	$C_{p,l}$ (kJ kg^{-1} K^{-1})	$C_{p,v}$ (kJ kg^{-1} K^{-1})	h_l (kJ kg^{-1})	h_{lv} (kJ kg^{-1})	h_v (kJ kg^{-1})	s_l (kJ kg^{-1} K^{-1})	s_v (kJ kg^{-1} K^{-1})
0.01	0.00612	4.22	1.888	0.000611	2501.0	2501.0	-6.161e-08	9.155
1	0.00657	4.216	1.889	4.177	2499.0	2503.0	0.01526	9.129
2	0.00706	4.213	1.89	8.392	2496.0	2505.0	0.03061	9.103
3	0.00758	4.21	1.89	12.6	2494.0	2506.0	0.04589	9.076
4	0.00814	4.208	1.891	16.81	2491.0	2508.0	0.0611	9.051
5	0.00873	4.205	1.892	21.02	2489.0	2510.0	0.07625	9.025
6	0.00935	4.203	1.892	25.22	2487.0	2512.0	0.09134	8.999
7	0.01	4.201	1.893	29.43	2484.0	2514.0	0.1064	8.974
8	0.0107	4.199	1.894	33.63	2482.0	2516.0	0.1213	8.949
9	0.0115	4.197	1.895	37.82	2480.0	2517.0	0.1362	8.924
10	0.0123	4.196	1.896	42.02	2477.0	2519.0	0.1511	8.9
12	0.014	4.193	1.898	50.41	2472.0	2523.0	0.1806	8.851
14	0.016	4.191	1.899	58.79	2468.0	2527.0	0.2099	8.804
16	0.0182	4.188	1.901	67.17	2463.0	2530.0	0.239	8.757
18	0.0206	4.187	1.904	75.55	2458.0	2534.0	0.2678	8.711
20	0.0234	4.185	1.906	83.92	2454.0	2537.0	0.2965	8.666
25	0.0317	4.182	1.912	104.8	2442.0	2547.0	0.3673	8.557
30	0.0425	4.18	1.918	125.7	2430.0	2556.0	0.4368	8.452
35	0.0563	4.179	1.925	146.6	2418.0	2565.0	0.5052	8.352
40	0.0738	4.179	1.932	167.5	2406.0	2574.0	0.5724	8.256
45	0.0959	4.179	1.94	188.4	2394.0	2582.0	0.6386	8.163
50	0.124	4.18	1.948	209.3	2382.0	2591.0	0.7038	8.075
55	0.158	4.181	1.957	230.2	2370.0	2600.0	0.768	7.99
60	0.199	4.183	1.966	251.2	2358.0	2609.0	0.8312	7.908
65	0.25	4.185	1.976	272.1	2345.0	2618.0	0.8935	7.83
70	0.312	4.188	1.987	293.0	2333.0	2626.0	0.955	7.754
75	0.386	4.192	1.999	314.0	2321.0	2635.0	1.016	7.681
80	0.474	4.196	2.012	334.9	2308.0	2643.0	1.075	7.611
85	0.579	4.2	2.026	355.9	2295.0	2651.0	1.134	7.543
90	0.702	4.205	2.042	377.0	2283.0	2660.0	1.193	7.478
95	0.846	4.211	2.059	398.0	2270.0	2668.0	1.25	7.415
100	1.01	4.217	2.077	419.1	2256.0	2676.0	1.307	7.354
110	1.43	4.23	2.121	461.4	2230.0	2691.0	1.419	7.238
120	1.99	4.246	2.174	503.8	2202.0	2706.0	1.528	7.129
130	2.7	4.265	2.237	546.4	2174.0	2720.0	1.635	7.026
140	3.62	4.286	2.311	589.2	2144.0	2733.0	1.739	6.929
150	4.76	4.31	2.396	632.3	2114.0	2746.0	1.842	6.837
160	6.18	4.338	2.492	675.6	2082.0	2757.0	1.943	6.749
170	7.92	4.369	2.599	719.2	2049.0	2768.0	2.042	6.665
180	10.0	4.406	2.716	763.2	2014.0	2777.0	2.14	6.584

Table 11 continued: Thermodynamic properties of saturated steam by temperature.

T (°C)	P (bar)	$C_{p,l}$ (kJ kg^{-1} K^{-1})	$C_{p,v}$ (kJ kg^{-1} K^{-1})	h_l (kJ kg^{-1})	h_{lv} (kJ kg^{-1})	h_v (kJ kg^{-1})	s_l (kJ kg^{-1} K^{-1})	s_v (kJ kg^{-1} K^{-1})
190	12.6	4.447	2.846	807.6	1978.0	2785.0	2.236	6.506
200	15.5	4.494	2.99	852.4	1940.0	2792.0	2.331	6.43
250	39.8	4.865	4.012	1086.0	1715.0	2801.0	2.793	6.072
300	85.9	5.752	6.223	1345.0	1405.0	2750.0	3.255	5.706

Table 11: Thermodynamic properties of saturated steam by pressure, calculated using the NASA CEA database and the vapour pressure data of Wexler or Wagner and Pruss (1990). The reference state is the triple point of saturated liquid water.

P (bar)	T (°C)	$C_{p,l}$ (kJ kg^{-1} K^{-1})	$C_{p,v}$ (kJ kg^{-1} K^{-1})	h_l (kJ kg^{-1})	h_{lv} (kJ kg^{-1})	h_v (kJ kg^{-1})	s_l (kJ kg^{-1} K^{-1})	s_v (kJ kg^{-1} K^{-1})
0.01	6.97	4.201	1.893	29.3	2484.0	2514.0	0.1059	8.975
0.015	13.0	4.192	1.898	54.69	2470.0	2525.0	0.1956	8.827
0.02	17.5	4.187	1.903	73.43	2459.0	2533.0	0.2606	8.723
0.025	21.1	4.184	1.907	88.43	2451.0	2539.0	0.3119	8.642
0.03	24.1	4.183	1.91	101.0	2444.0	2545.0	0.3543	8.577
0.035	26.7	4.181	1.914	111.8	2438.0	2550.0	0.3907	8.521
0.04	29.0	4.181	1.917	121.4	2432.0	2554.0	0.4224	8.473
0.045	31.0	4.18	1.919	130.0	2427.0	2557.0	0.4507	8.431
0.05	32.9	4.18	1.922	137.8	2423.0	2561.0	0.4763	8.394
0.055	34.6	4.179	1.924	144.9	2419.0	2564.0	0.4995	8.36
0.06	36.2	4.179	1.927	151.5	2415.0	2567.0	0.5209	8.329
0.065	37.6	4.179	1.929	157.6	2412.0	2569.0	0.5407	8.301
0.07	39.0	4.179	1.931	163.4	2408.0	2572.0	0.5591	8.275
0.075	40.3	4.179	1.933	168.8	2405.0	2574.0	0.5763	8.25
0.08	41.5	4.179	1.935	173.9	2402.0	2576.0	0.5925	8.227
0.085	42.7	4.179	1.936	178.7	2400.0	2578.0	0.6078	8.206
0.09	43.8	4.179	1.938	183.3	2397.0	2580.0	0.6223	8.186
0.095	44.8	4.179	1.94	187.6	2394.0	2582.0	0.6361	8.167
0.12	49.4	4.18	1.947	206.9	2383.0	2590.0	0.6963	8.085
0.14	52.5	4.18	1.953	220.0	2376.0	2596.0	0.7366	8.031
0.16	55.3	4.181	1.958	231.6	2369.0	2601.0	0.772	7.985
0.18	57.8	4.182	1.962	241.9	2363.0	2605.0	0.8035	7.944
0.2	60.1	4.183	1.966	251.4	2358.0	2609.0	0.832	7.907
0.22	62.1	4.184	1.971	260.1	2352.0	2613.0	0.8579	7.874
0.24	64.1	4.185	1.974	268.1	2348.0	2616.0	0.8818	7.844
0.26	65.8	4.186	1.978	275.6	2343.0	2619.0	0.904	7.817
0.28	67.5	4.187	1.982	282.6	2339.0	2622.0	0.9246	7.791
0.3	69.1	4.188	1.985	289.2	2335.0	2625.0	0.9439	7.767
0.32	70.6	4.189	1.989	295.5	2332.0	2627.0	0.9621	7.745
0.34	72.0	4.19	1.992	301.4	2328.0	2630.0	0.9793	7.725
0.36	73.3	4.19	1.995	307.0	2325.0	2632.0	0.9956	7.705

Table 11 continued: Thermodynamic properties of saturated steam by pressure.

P (bar)	T (°C)	$C_{p,l}$ (kJ kg^{-1} K^{-1})	$C_{p,v}$ (kJ kg^{-1} K^{-1})	h_l (kJ kg^{-1})	h_{lv} (kJ kg^{-1})	h_v (kJ kg^{-1})	s_l (kJ kg^{-1} K^{-1})	s_v (kJ kg^{-1} K^{-1})
0.38	74.6	4.191	1.998	312.4	2322.0	2634.0	1.011	7.686
0.4	75.9	4.192	2.001	317.6	2318.0	2636.0	1.026	7.669
0.42	77.0	4.193	2.004	322.5	2316.0	2638.0	1.04	7.652
0.44	78.2	4.194	2.007	327.2	2313.0	2640.0	1.054	7.636
0.46	79.3	4.195	2.01	331.8	2310.0	2642.0	1.067	7.621
0.48	80.3	4.196	2.013	336.2	2307.0	2644.0	1.079	7.607
0.5	81.3	4.197	2.016	340.5	2305.0	2645.0	1.091	7.593
0.55	83.7	4.199	2.022	350.5	2299.0	2649.0	1.119	7.561
0.6	85.9	4.201	2.029	359.8	2293.0	2653.0	1.145	7.531
0.65	88.0	4.203	2.035	368.5	2288.0	2656.0	1.169	7.504
0.7	89.9	4.205	2.041	376.7	2283.0	2659.0	1.192	7.479
0.75	91.8	4.207	2.047	384.4	2278.0	2662.0	1.213	7.456
0.8	93.5	4.209	2.053	391.6	2274.0	2665.0	1.233	7.434
0.85	95.1	4.211	2.059	398.5	2269.0	2668.0	1.252	7.413
0.9	96.7	4.213	2.065	405.1	2265.0	2670.0	1.269	7.394
0.95	98.2	4.214	2.07	411.4	2261.0	2673.0	1.286	7.376
1	99.6	4.216	2.076	417.4	2258.0	2675.0	1.303	7.359
1.1	102.0	4.22	2.087	428.8	2250.0	2679.0	1.333	7.327
1.2	105.0	4.223	2.097	439.3	2244.0	2683.0	1.361	7.298
1.3	107.0	4.226	2.108	449.1	2238.0	2687.0	1.387	7.271
1.4	109.0	4.229	2.118	458.4	2232.0	2690.0	1.411	7.246
1.5	111.0	4.232	2.128	467.1	2226.0	2693.0	1.434	7.223
1.6	113.0	4.235	2.138	475.3	2221.0	2696.0	1.455	7.201
1.7	115.0	4.238	2.147	483.2	2216.0	2699.0	1.475	7.181
1.8	117.0	4.241	2.157	490.7	2211.0	2701.0	1.494	7.162
1.9	119.0	4.244	2.166	497.8	2206.0	2704.0	1.513	7.144
2	120.0	4.247	2.175	504.7	2202.0	2706.0	1.53	7.127
2.5	127.0	4.26	2.22	535.4	2181.0	2717.0	1.607	7.052
3	134.0	4.272	2.262	561.5	2163.0	2725.0	1.672	6.992
3.5	139.0	4.283	2.302	584.3	2148.0	2732.0	1.727	6.94
4	144.0	4.294	2.34	604.7	2133.0	2738.0	1.777	6.895
4.5	148.0	4.305	2.377	623.2	2120.0	2743.0	1.821	6.856
5	152.0	4.315	2.413	640.2	2108.0	2748.0	1.861	6.821
6	159.0	4.335	2.48	670.5	2086.0	2756.0	1.931	6.759
7	165.0	4.353	2.543	697.1	2066.0	2763.0	1.992	6.707
8	170.0	4.371	2.603	721.0	2047.0	2768.0	2.046	6.662
9	175.0	4.388	2.66	742.7	2030.0	2773.0	2.094	6.621
10	180.0	4.405	2.715	762.7	2014.0	2777.0	2.138	6.585
15	198.0	4.485	2.964	844.7	1946.0	2791.0	2.315	6.443
20	212.0	4.562	3.19	908.6	1890.0	2798.0	2.447	6.339
25	224.0	4.638	3.404	962.0	1840.0	2802.0	2.554	6.256
30	234.0	4.714	3.612	1008.0	1795.0	2803.0	2.646	6.186
35	243.0	4.791	3.817	1050.0	1753.0	2803.0	2.725	6.125
40	250.0	4.869	4.022	1087.0	1713.0	2801.0	2.797	6.07

Table 11 continued: Thermodynamic properties of saturated steam by pressure.

P (bar)	T ($^\circ$C)	$C_{p,l}$ (kJ kg^{-1} K^{-1})	$C_{p,v}$ (kJ kg^{-1} K^{-1})	h_l (kJ kg^{-1})	h_{lv} (kJ kg^{-1})	h_v (kJ kg^{-1})	s_l (kJ kg^{-1} K^{-1})	s_v (kJ kg^{-1} K^{-1})
45	257.0	4.949	4.228	1122.0	1676.0	2798.0	2.861	6.02
50	264.0	5.032	4.438	1155.0	1640.0	2794.0	2.921	5.974
60	276.0	5.208	4.877	1214.0	1571.0	2785.0	3.027	5.89
70	286.0	5.4	5.354	1267.0	1505.0	2773.0	3.122	5.815
80	295.0	5.614	5.883	1317.0	1442.0	2759.0	3.208	5.745
90	303.0	5.854	6.476	1364.0	1379.0	2743.0	3.287	5.679
100	311.0	6.127	7.147	1408.0	1318.0	2725.0	3.36	5.616
120	325.0	6.813	8.819	1491.0	1194.0	2686.0	3.496	5.494