Connectivity of Undirected Graphs

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1 Introduction

A graph is a set of vertices connected by edges. When an edge connects two vertices together, those two vertices are considered adjacent. In this project, we will be examining undirected graphs and their connectivity given different minimum distances required to be connected the nearest adjacent node.

Using a Euclidean graph generated in a 20x20 space, we can test how this threshold distance affects the number of edges present in a random distribution of vertices and we can then use these edges to determine the probability of the graph being connected for a given threshold distance. A graph is considered connected if for every pair of vertices v and w the distance between v and w is finite and contained along a path.

For our results, it is expected that we get sigmoidal regressions for both the relationship between threshold distance and number of edges as well as the relationship between threshold distance and the probability of connectivity for the graph.

2 Procedures

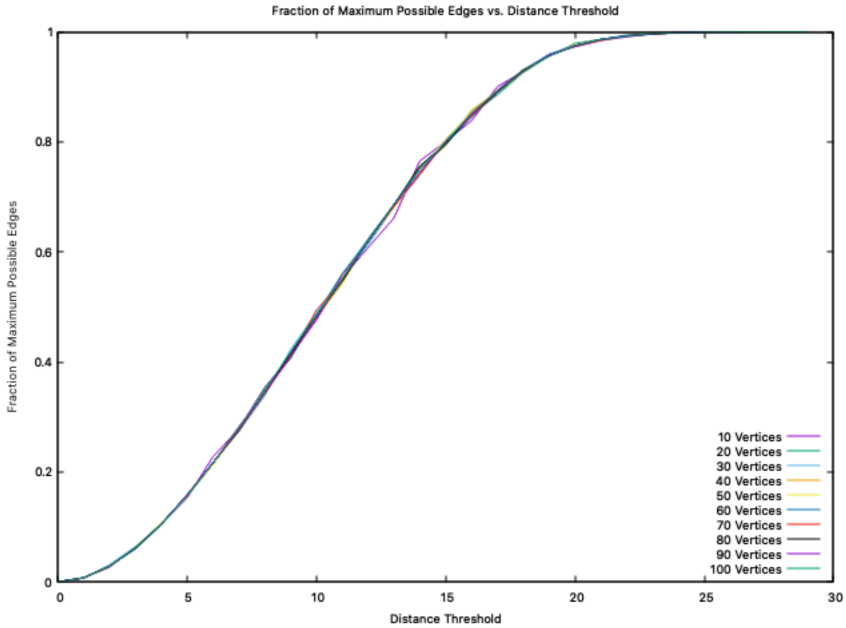
The following data structures were used in the graph class

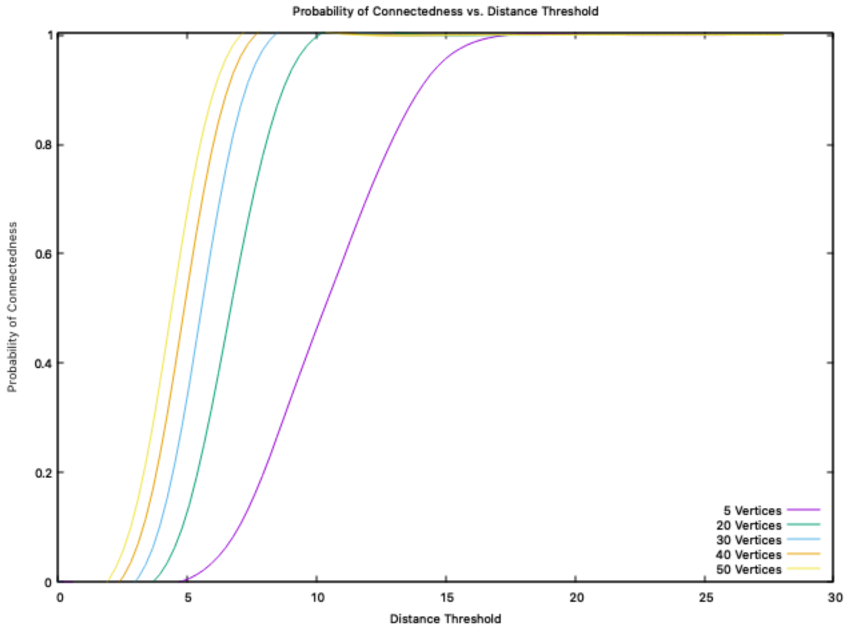
- A vector¡point¿ structure to store vertices
- A vector¡vector¡int¿¿ to create an adjacency graph
- A helper vector¡bool¿ to keep track of connectivity during searches
- Variables to track the threshold distance of the graph, the number of vertices, and the number of edges

Using these data structures, we built a method which would iterate through graph for each pair of points and check the number of points that existed for a given distance threshold and another method which checked the connectivity of the graph for a given distance threshold by using a recursive depth-first search algorithm.

For each graph, multiple trials were run in order to obtain more accurate average values for each given distance threshold, and for the graph of the number of edges, the edges were divided by the maximum amount of possible edges for a given number of vertices so that values would be presented as a fraction between 0 and 1. The data obtained was stored in text files and plotted using the CLI tool gnuplot.

3 Data


Fraction of Maximum Possible Edges vs. Distance Threshold
Fraction of Maximum Possible Edges
Distance Threshold
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
30
10 Vertices
20 Vertices
30 Vertices
40 Vertices
50 Vertices
60 Vertices
70 Vertices
80 Vertices
90 Vertices
100 Vertices


Probability of Connectedness vs. Distance Threshold
Probability of Connectedness
Distance Threshold
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
30
5 Vertices
20 Vertices
30 Vertices
40 Vertices
50 Vertices

4 Conclusion

As expected, the regression for both graphs was sigmoidal. The point at which half of the possible edges occurred was when the threshold distance was approximately 1/3 of the maximum distance at 10 or 11. The connectivity of the graph seemed to increase much more rapidly with respect to the distance threshold when there were more points, which makes sense intuitively because with a random distribution this simply means there is a higher chance of points being with the distance threshold.

The graphs are asymptotic and for both graphs, as the highest possible value for the threshold is reached, the fraction of edges formed as well as the probability of connectivity approach 1. The change observed for varying amounts of vertices in the graph of probability is only because the amount of space is not proportional to the number of points, and as the amount of points increase it is more likely in a set space that the graph will be connected.