ecture 1

- syllabus
- ethos of the course
- the importance of engagement for success
- there are No rongers or wrongers in todays lecture.

—X—

What even is Math? → Q

- Numbers?
- scary symbols?
- Shapes?
- ???

There are no Right ars wers. Only perspectives

- Geometers say shapes, lines, and angles
- algabraists say numbers, symbols, and structures.
- ANAlysts say the properties of numbers and Functions.

All of them are Right, However each perspective is unique to their Fields.

to Me?

Math, all of it, is the study of Objects - Numbers, Shapes, symbols, and Manipulations (Morphisms) on the Objects

- The 'categorical' view of Math

Example:

Algebra is the study of families of numbers and Manipulations on them (Functions)

This is just My perspective. to Begin answering somewhat satisfyingly our Questions, lets ask another Question:

What are numbers? How do you define them? Q

↳ Quantities (Cardinals, aka 'amount of things' numbers)
- there are Theones most people use, most of the time

↳ Position (Relative) → Ordinals (1st, 2nd, 3rd, etc).
- denote relative order of objects.
  - order

You can have 5 apples, But you can't come in 5 Place in a race,

You can come in 3rd in a race, But you can't have 35th dollars in your Bank.

2 seperate systems that use the same symbols

Giuseppe Peano (1858-1932), notable for some of the first Rigorous definitions of numbers. (wrong)

14
Analysis (real)
Complex Analysis

order to understand numbers, we need to understand a simpler structure

Set: an unordered collection of things.

- ↳ $\mathbb{Z}$ (German - Zeitoten (whole))
- ↳ set of people in a class
- ↳ set of all even numbers that I happen, like

Sets can have anything in them

$S = \{a, b, c \ldots, y, z\} = \{b, a, c, d, \ldots y, z\}$

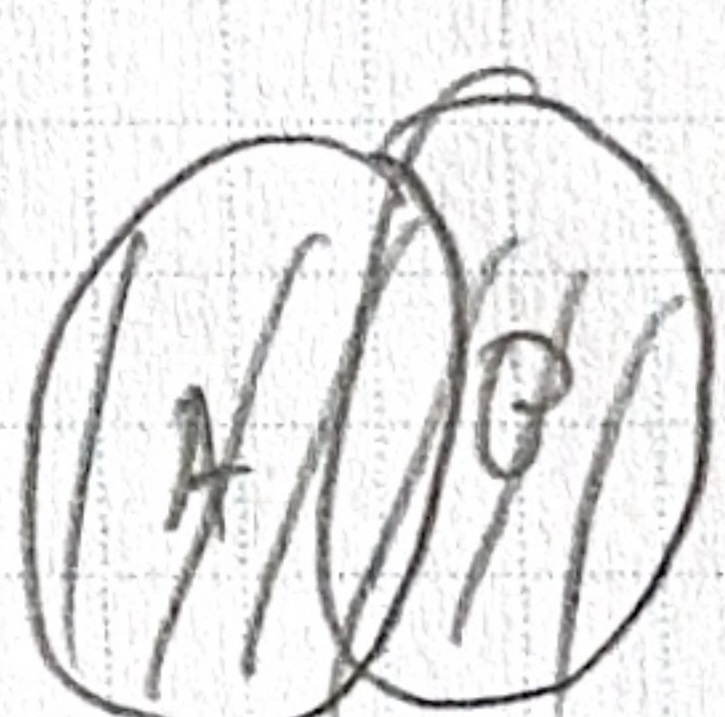
operations on sets

→ Union (adding) ∃ the combination of all elements in Both sets

Intersect (subtracting) ∃ collection of all things 2 sets share.

intersection: $A \cap B$

Union:



$A \cup B$ union

Special sets:

$\emptyset$ = null, or empty set, No numbers

$\mathcal{U}$ = "universe" "top set" all numbers $\mathcal{U}$

set equality: Sets A and B are equal if they share the same exact elements

Ex

$A = \{a, b, c\}$, $B = \{b, c, a\}$ $A = B$

$C = \{B, C\}$, $A \neq C$

$A = B$ if $A \cap B = A$ And $A \cap B = B$

sets'. A subset is a set, usually smaller, of elements shared by both sets.

$A \subseteq B$ 'A is a subset of B'

even numbers $\subseteq$ all numbers

Students in this class $\subseteq$ all students

All sets $\subseteq \mathcal{U}$ and No set is a subset of $\emptyset$ ($\forall A (A \subseteq \emptyset)$)

($\forall x$ ...)

Peano axioms for addition

↳ What is an axiom? Ω

Axiom is a statement or rule Assumed true, that cannot be proved.

" toddler: why gone "

what is a number? → via Peano

$\Omega = 0$

For all n, define a "successor" function
↳ next number

$S(n) = n \cup \{n\}$

$\rightarrow 1 = S(0) = \{\emptyset\}$

$2 = S(1) = S(S(0)) = \{0, 1\} = \{\emptyset, \{\emptyset\}\} = \{0, \{0\}\}$

this way, all whole numbers are defined, between 0 and infty

Wait... what is infinity? → next class.

Other properties from Peano

$0+a=a$: "0 is ~~additive~~ idempotent under addition"

$a+S(b)=S(a+b)$ "linearity"

Conclusion:

often what we assume to be true in math has some core beliefs behind them.

There will be several, often frustrating, analysis of seemingly fundamental questions in math, such as the one we had today

→ Zermelo-Frankel, Gödel, Russell, all great mathematicians wrestled with this topic.

Questions?

Homework:

Using the definition of the successor above, try to define normal multiplication ($\cdot$) beginning with the following facts:

$0 \cdot a = 0$ and $a \cdot S(b) = a + (a \cdot b)$

end lecture 1