Lecture 2: on infinity

Hw discussion:
- Vibe check
- Questions from last lecture

Last Time:
- defined numbers (whole ones) from 0 to infinity, without defining infinity.

Question: what is infinity?
- its sometimes easier to say what it is not!

infinity is not:
- a number
- a variable

infinity is:
- an idea
- a tool
- Frustrating.

Recall:

Sets (of infinite size)

"integers": all whole numbers, Both + and - and 0.

↳ $\mathbb{Z} = \{ \ldots -3, -2, -1, 0, 1, 2, 3, \ldots \}$ ($\mathbb{Z}$, from german 'Zerlotan' or whole.)

"Natural Numbers": all whole + numbers, and 0.

$\mathbb{N} = \{0, 1, 2, 3 \ldots\}$

"Rationals": all fractions, +, -, and 0.

$\mathbb{Q} = \{ \frac{-22}{77}, -1, 0, 1, \frac{1}{2}, \frac{100}{33}, \ldots \}$ (Recall: $a = \frac{a}{1}$ $\Rightarrow \mathbb{Z} \subseteq \mathbb{Q}$)

Q for Quotient

"Reals": all numbers.

$\mathbb{R} = \{ -\pi, \frac{33}{11}, 2, 0, e, \sqrt{2}, \ldots \}$

Observe:

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$

all sets of numbers are a subset of $\mathbb{R}$.

0.25" SQUARE GRID © 2023 GeekNavy www.GeekNavy.com Product # 2D025IN8511P-NH

$\boxed{\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}}$ this is super important to remember

—X—

(recall) $|x|$ 'cardinality' or size of.

For finite sets: $a \subseteq b \rightarrow |a| \leq |b|$

subset is kinda like less than

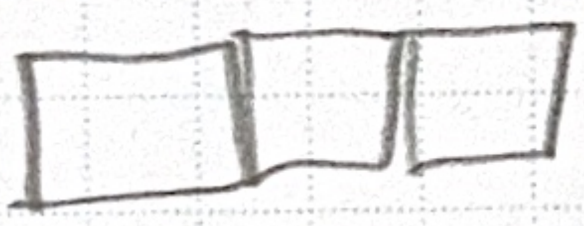
(not true for all sets)

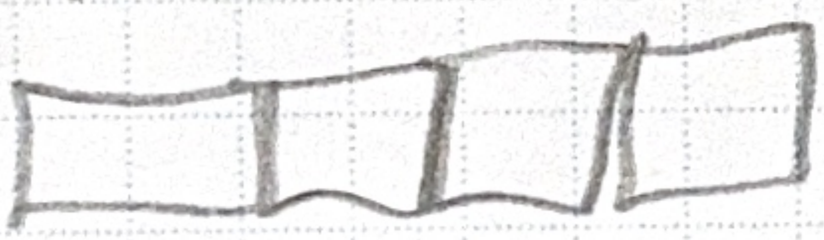
what about infinite sets?

$\boxed{\text{is } |\mathbb{N}| < |\mathbb{Z}| \text{ ?}}$

to answer that, consider 'untrustworthy Chocolate inc.'

untrustworthy chocolate inc. makes chocolate bars comprising of varying # of squares. Each square is the same size, so each bar varies in # of squares.

Bar A: 

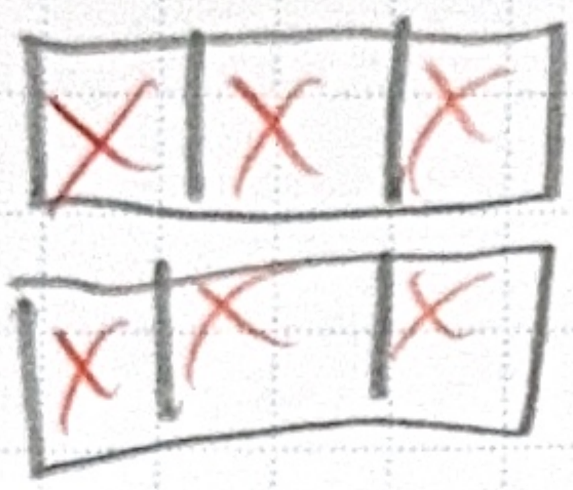
Bar B: 

if you and a friend both get an UCI bar, how can you check who got more?

Idea: You and your friend eat one square at a time. if one of you runs out, and the other friend has one or more squares left, then you recieved different sized Bars. If you Both run out at the same time, the Bars are different sizes!

Ex1:

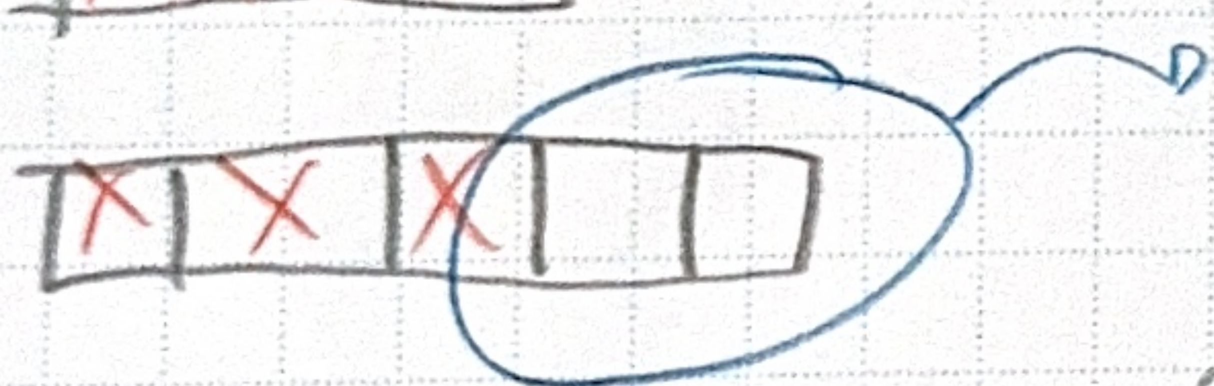
A:
B:



Both run out at same time → same size

Ex2:

A:
B:



B has leftovers → A, B got different sized Bars

Back to Sets...

the same can be done with sets!

$A = \{a, b, c\}$

$B = \{d, e, f\}$

if you can always pair one element from A with another from B, and run out at the same time, $|A| = |B|$

(elements need not equal each other, only consider pairs)

But:

$C = \{g, h\}$

$D = \{i, j, k\}$

k can't be paired with an element of C $\rightarrow |C| \neq |D|$

So if we can't always pair elements in a 1-1 manner, the sets are of different size!

So what about $\mathbb{N}$ and $\mathbb{Z}$?

Idea: Map (pair) each number in $\mathbb{N}$ to $\frac{n}{2}$ if n is even or $-\left(\frac{n+1}{2}\right)$

Simply PUT:

$\mathbb{N}$: 0, 1, 2, 3, 4, 5, 6 ....
$\mathbb{Z}$: 0, -1, 1, -2, 2, -3, 3 ....

By pairing elements in $\mathbb{N}$ and $\mathbb{Z}$ like shown, we have proven $|\mathbb{N}| = |\mathbb{Z}|$

— x —

What? This feels wrong.

Homework:

convince yourself, or conversely, try to disprove today's finding.