Simple vs Log Returns and Normality Assumption

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Assisted by quantpy

What is the difference between simple returns and log returns?

Simple returns: The product of normally distributed variables is NOT normally distributed

Log returns: The sum of normally distributed variables DOES follow a normal distribution

In [1]:
```
# Importing dependencies

import datetime as dt
import pandas as pd
import numpy as np
import pylab
import seaborn as sns
import scipy.stats as stats
from pandas_datareader import data as pdr
import plotly.offline as pyo

pyo.init_notebook_mode(connected = True)
pd.options.plotting.backend = 'plotly'
```

In [2]:
```
# Getting the stock market data
end = dt.datetime.now()
start = dt.datetime(2018,1,1)


import yfinance as yfin
yfin.pdr_override()
df = pdr.get_data_yahoo('AAPL', start, end)
df.head()
```

```
[*********************100%***********************]  1 of 1 completed
```

Out[2]:

	Open	High	Low	Close	Adj Close	Volume
Date						

2018-01-02	42.540001	43.075001	42.314999	43.064999	40.950489	102223600
2018-01-03	43.132500	43.637501	42.990002	43.057499	40.943363	118071600
2018-01-04	43.134998	43.367500	43.020000	43.257500	41.133553	89738400
2018-01-05	43.360001	43.842499	43.262501	43.750000	41.601868	94640000
2018-01-08	43.587502	43.902500	43.482498	43.587502	41.447338	82271200

Simple Return

The formula for simple return is:

$$R_t = (P_t - P_{t-1})/P_{t-1} = P_t/P_{t-1} - 1$$

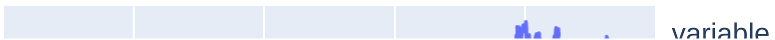
Image from https://subscription.packtpub.com/book/data/9781789618518/1/ch01lvl1sec05/converting-prices-to-returns

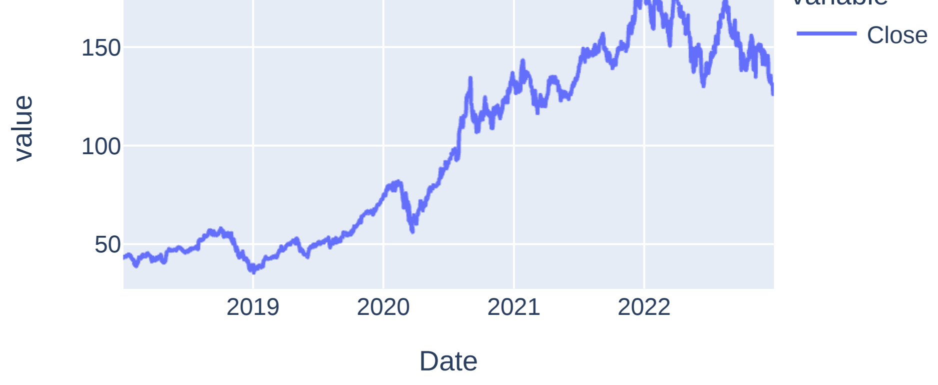
In [3]:
```
# Calculating daily simple returns

simple_returns = df.Close.pct_change().dropna()
simple_returns
```

Out[3]:
```
Date
2018-01-03   -0.000174
2018-01-04    0.004645
2018-01-05    0.011385
2018-01-08   -0.003714
2018-01-09   -0.000115
                ...
2022-12-23   -0.002798
2022-12-27   -0.013878
2022-12-28   -0.030685
2022-12-29    0.028324
2022-12-30    0.002469
Name: Close, Length: 1258, dtype: float64
```

In [4]:
```
# Plotting financial data and look at first and last share prices
df.Close.plot().update_layout(autosize = False, width = 500, height = 300)
```





In [5]:
```
print('First', df.Close[0], 'Last', df.Close[-1])
```

```
First 43.064998626708984 Last 129.92999267578125
```

In [6]:
```
# Use simple returns & attempt to compute the final price from starting price over time horizon

simple_returns.mean()
```

Out[6]:
```
0.001100790011248004
```

In [7]:
```
df.Close[0]*(1+simple_returns.mean())**len(simple_returns) # Not close as last value
```

Out[7]:
```
171.8707566899991
```

$$1 + R_t(k) = \frac{P_t}{P_{t-1}} \frac{P_{t-1}}{P_{t-2}} \cdots \frac{P_{t-k+1}}{P_{t-k}} = \frac{P_t}{P_{t-k}}$$

$$1 + R_t(k) = (1 + R_t)(1 + R_{t-1})\ldots(1 + R_{t-k+1})$$

$$1 + R_t(k) = \prod_{i=0}^{k-1}(1 + R_{t-i})$$

In [8]:
```
# Using formula above, we have that:
df.Close[0]*np.prod([(1+Rt) for Rt in simple_returns]) # Very close last value
```

Out[8]:
```
129.92999267578165
```

Simple return is not easy to compute, cannot point to the mean, doesn't desribe a lot by itself. Look into alternative methods

Log Returns:

$$r_t = \ln(1 + R_t)$$

K-period log returns:

$$r_t(k) = \ln(1 + R_t(k)) = \ln[(1 + R_t)(1 + R_{t-1})\dots(1 + R_{t-k+1})]$$

$$r_t(k) = \ln(1 + R_t(k)) = \ln(1 + R_t) + \ln(1 + R_{t-1}) + \dots + \ln(1 + R_{t-k+1})$$

$$r_t(k) = \ln(1 + R_t(k)) = r_t + r_{t-1} + \dots + r_{t-k+1} = \ln(P_t) - \ln(P_{t-k})$$

In [9]:
```
# Computing log returns:
log_returns = np.log(df.Close/df.Close.shift(1))
log_returns
```

Out[9]:
```
Date
2018-01-02          NaN
2018-01-03    -0.000174
2018-01-04     0.004634
2018-01-05     0.011321
2018-01-08    -0.003721
                 ...
2022-12-23    -0.002802
2022-12-27    -0.013976
2022-12-28    -0.031166
2022-12-29     0.027931
2022-12-30     0.002466
Name: Close, Length: 1259, dtype: float64
```

In [10]:
```
# drop nas
log_returns = np.log(df.Close/df.Close.shift(1)).dropna()
log_returns
```

Out[10]:
```
Date
2018-01-03    -0.000174
2018-01-04     0.004634
2018-01-05     0.011321
2018-01-08    -0.003721
2018-01-09    -0.000115
                 ...
2022-12-23    -0.002802
2022-12-27    -0.013976
2022-12-28    -0.031166
```

```
2022-12-29    0.027931
2022-12-30    0.002466
Name: Close, Length: 1258, dtype: float64
```

In [11]:
```
log_returns.mean()
```

Out[11]:
```
0.0008778101885308492
```

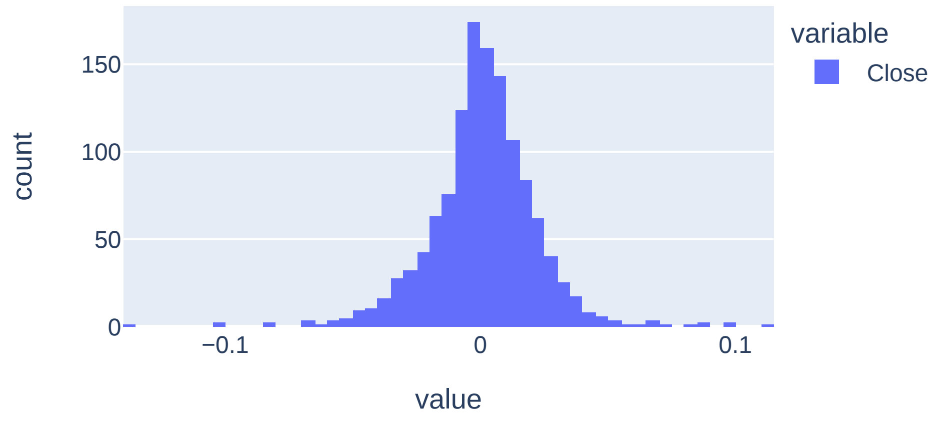
In [12]:
```
df.Close[0] * np.exp(log_returns.mean() * len(log_returns))
# Calculated end value accurately
```

Out[12]:
```
129.9299926757816
```

Look further into log returns. Note that the log distribution bounds the stock price at 0 which is consistent with reality.

In [13]:
```
# histogram of log returns

log_returns.plot(kind = 'hist').update_layout(autosize = False, width = 500, height = 300)
```



In [14]:
```
# Is this log return normally distributed
log_returns_sorted = log_returns.tolist()
log_returns_sorted.sort()
log_returns_sorted

# since this is sorted from least to greates, the following holds
worst = log_returns_sorted[0]
best = log_returns_sorted[-1]
```

```
# std of worst and best
std_worst = (worst - log_returns.mean())/log_returns.std()
std_best = (best - log_returns.mean())/log_returns.std()

print('Standard dev. worst %.2f best %.2f' %(std_worst, std_best))
# This is a very big standard deviation considering normal distribution

# Find the probability of achieving these values
print('Probability worst %.11f best %.11f' %(stats.norm(0,1).pdf(std_worst), stats.norm(0,1).pdf(std_best)))
```

```
Standard dev. worst -6.56 best 5.32
Probability worst 0.00000000018 best 0.00000028801
```

Testing for normality of financial data

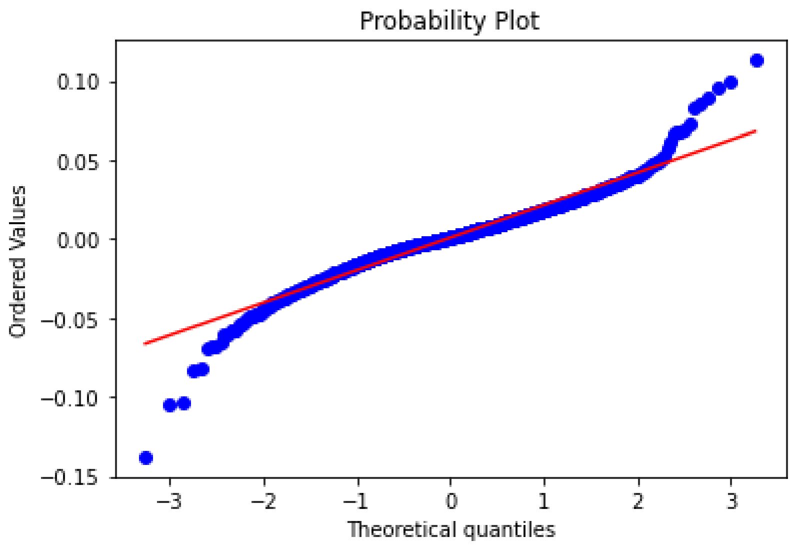
Quantile-Quantile Plots

This plots two sets of quantiles against one another i.e. theoretical quanties against the actual quantiles of the variable

In [15]:
```
stats.probplot(log_returns, dist = 'norm', plot = pylab)

print('Q-Q Plot') # Starts deviating from normally distributed line if goes away from center
```

```
Q-Q Plot
```

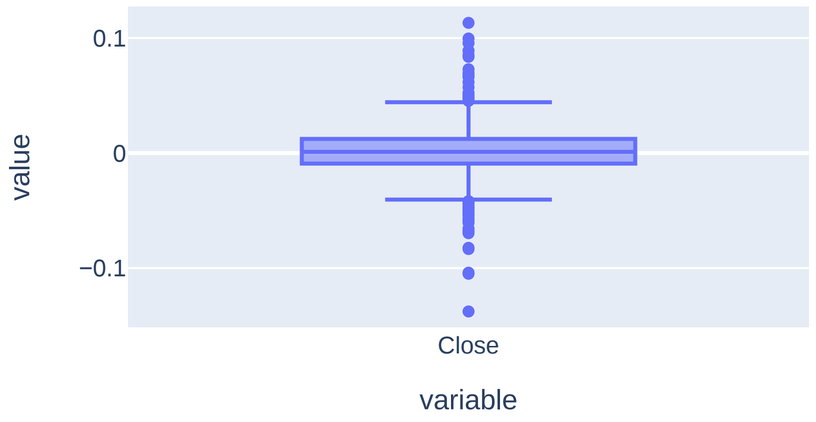


In [16]:
```
# It may not be best to use normal distribution to map full financial data
# Can be sufficient for middle data
```

Box Plots

Box plot also known as a box and whisker plot is another way to visualize the normality of a variable. it displays the distribution of data based on five-number summary i.e. minimum, first quartile (Q1), median (Q2), third quartile (Q3), and maximum

In [17]:
```
log_returns.plot(kind = 'box').update_layout(autosize = False, width = 500, height = 300)
# There is a very heavy tail, many outliers
```



From the two plots above, normal distribution might not best represent the full financial data for AAPL

Hypothesis Testing/Statistical Inference

This method can give a more objective answer

Kolmogorov Smirnov test

The Kolmogorov Smirnov test computes the distance between the empirical distances and the theoretical distribution and defines the test statistic as the suprenum of the set of those distances. The Test Statistic of the KS Test is the Kolmogorov Smirnov Statistic, which follows a Kolmogorov distribution is the null hypothesis is true. If the observed data perfectly follows a normal distribution, the value of the KS statistic will be 0. The P-Value is used to decide whether the difference is large enough to reject the null hypothesis

The advantage of this method is that the same approach can be used for comparing any distribution, not necessary just the normal distribution

In [18]:
```
ks_stat, p_value = stats.kstest(log_returns, 'norm')
print(ks_stat, p_value)
```

```
if p_value > 0.05:
    print("Probability Guassian") # normally distributed with this CI
else:
    print("Probability Not Guassian")
```

```
0.4687506939095014 5.7109740782936125e-254
Probability Not Guassian
```

Shapiro Wilk Test

The Shapiro Wilk test is the most powerful test when testing for a normal distribution. It has been developed specifically for the normal distribution and it cannot be used for testing against other distributions like for example the KS Test

In [19]:
```
sw_stat, p_value = stats.shapiro(log_returns)
print(sw_stat, p_value)


if p_value > 0.05:
    print("Probability Guassian") # normally distributed with this CI
else:
    print("Probability Not Guassian")
```

```
0.9487610459327698 1.640752811559851e-20
Probability Not Guassian
```

In [25]:
```
# For a tighter time frame
start = dt.datetime(2022,6,1)
df = pdr.get_data_yahoo('AAPL', start, end)
log_returns = np.log(df.Close/df.Close.shift(1)).dropna()
sw_stat, p_value = stats.shapiro(log_returns)
print(sw_stat, p_value)


if p_value > 0.05:
    print("Probability Guassian") # normally distributed with this CI
else:
    print("Probability Not Guassian")
```

```
[*********************100%***********************]  1 of 1 completed
0.9821842908859253 0.053514380007982254
Probability Guassian
```

We can see that when tightening the time frame, the financial data does follow a normal distribution

In conclusion, I have looked into simple return and log return and assessed

how log returns are generally more useful as they are additive. Further, I assessed the normality assumptions of this financial data to see if further analysis can be made. I have done this through visual and quantitative analysis. I have generated two plots to visually represent if the data is normally distributed and further looked into two types of hypothesis tests.