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Online estimation of the power coefficient in wind energy conversion systems

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Abstract— **This paper deals with the estimation of the power coefficient in a wind turbine connected to a separately excited dc generator with the aim of facilitating maximum power point tracking control design. More precisely, we bring an improvement on the observer design given in a previous work by including a possible dynamical model of the power coefficient. It is shown that the proposed observer have a better response compared to the previous observer. Additionally, it is shown that the observer is capable of handing measurement noise and that it can be easily extended to other WECS where different types of generators are employed.**

Keywords: WECS, DC generator, observer, power coefficient

I. Introduction

Wind energy is obviously one of the alternative form of energy that is clean and environment-friendly. Because of this there has been a huge interest to maximise electrical power produced by wind [1]. A wind energy conversion system (WECS) consists of a turbine, used to extract power from the wind, a transmission to gear up the rotational speed of the turbine shaft, a mechanical to electrical converter, and a controller to control the overall system behaviour. In order to maximise the power extracted from the wind, it is necessary to control the WEC at its optimum operating point. In order to achieve this, it is important to monitor the value of the power coefficient at all instant of time. This in turn implies that one should have the knowledge of the power coefficient at all time. On the other hand, such a value is not available at all time and is difficult to measure. To be more precise, the expression for the amount of power $P_m(u)$ a wind turbine is capable of producing is given by:

$$P_m(u) = \frac{1}{2} C_p(\lambda, \beta) \rho \pi R^2 u^3 \tag{1}$$

where u is the wind speed (ms^{-1}), ρ is the air density (kgm^{-3}), R is the rotor radius (m), and C_p is the power coefficient of the wind turbine. The tip-speed ratio is defined as:

$$\lambda = \frac{R\omega}{u} \tag{2}$$

where ω is the angular speed of the turbine rotor (rads^{-1}).

As mentioned above, the power coefficient C_p is the most important parameter for the WECS controller design, especially in the case of maximum power extraction. It represents the turbine efficiency, and is defined as the fraction of wind energy extracted by the turbine of the total energy that would have flowed through the area swept by the rotor blades if the turbine had not been there. In general, C_p is a non-linear function of the tip speed ratio λ and therefore changes with wind velocity. It also depends on several factors such as air density, humidity, temperature. It is calculated from the turbine design, the pitch angle β and it is sensitive to dirt on the blade surface. It is for this reason that, in practice, C_p is difficult to obtain and is different for every turbine type. In many instances, it is provided by the manufacturer documentation which are used in

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many control schemes as look-up tables to generate optimal target power references.

Various models are used to describe C_p in the literature (see e.g..[3], [4] and [5]). In [6], C_p is modelled as a third order polynomial and the problem is formulated as a search for the optimal combination of polynomial coefficients that best match the observed data. In [3], [4] and [5] an exponential term is introduced in order to improve the model given in [6]. In [7], the estimated C_p was modelled as an even order polynomial, fitted to a small portion of the measured C_p surface using a recursive least squares algorithm with a forgetting factor to account for changes in the turbine environment, such as humidity, air density, temperature, etc. In [8], a neural network was used to estimate the power output directly. However, this model is sensitive to the weather conditions, as it relies on past data to predict future events. In fact, none of these models are precise. Nonetheless, one can make use of these models in order to derive an approximate dynamical model of C_p with some uncertain terms. An observer can then be used to estimate or reduced the uncertainty and thus obtaining an appropriate estimate of the power coefficient.

This is exactly the main objective of this paper. More precisely, we propose a dynamical model of the power coefficient. We base our model based on the model given in [3]. This model is then used to propose an improvement of the C_p observer design given in [2]. A comparative work is done in order evaluate the performance of the proposed observer to that given in [2]. Finally, some conclusion are drawn.

II. System Modelling

A. DC Generator Modelling

The model of the seperately excited dc generator was derived in [2] and is recalled here. The electrical dynamics describe the field winding and the armature winding and can be represented by the following differential equations:

$$\frac{di_f}{dt} = \frac{V_f}{L_f} - \frac{R_f i_f}{L_f} \quad (3)$$

$$\frac{di_a}{dt} = \frac{K_1 i_f \omega_e}{L_a} - \frac{V_a}{L_a} - \frac{R_a i_a}{L_a} \quad (4)$$

where V_f, R_f, and L_f are the field winding voltage, resistance and inductance respectively. Similarly, V_a, R_a, and L_a are the armature winding voltage, resistance and inductance respectively, ω_e is the rotational speed of the generator (rads^{-1}), and K_1 is the induced emf constant.
We shall assume that the load is modelled by a resistance and an inductance, so that

$$V_a = R_L i_a + L_L \frac{di_a}{dt} \quad (5)$$

The mechanical dynamic equation of the dc generator is of the well-known form:

$$\frac{d\omega_e}{dt} = \frac{T_p}{J_e} - \frac{T_e}{J_e} - \frac{B_e \omega_e}{J_e} \quad (6)$$

where B_e is the coefficient of viscous friction (Nm), $T_e = K_1 i_f i_a$ is the electromagnetic torque (Nm), T_p is the drive torque (Nm), J_e is the inertia of the dc generator ($\mathrm{kgm}^2\mathrm{s}^{-2}$).
Additionally, the electromagnetic torque T_e can be written as:

$$T_e = \frac{e i_a}{\omega_e} \quad (7)$$

where e is the induced emf which can be written as:

$$e = K_1 i_f \omega_e \quad (8)$$

B. WECS Modelling

The power from the wind drives the turbine with a torque T_m and consequently the rotor of the wind turbine rotates at an angular speed ω. The transmission output torque T_p then drives the generator, which produces an electromagnetic torque T_e at a rotational speed ω_e. Note that the turbine and generator speeds are not the same due to the use of the gearbox.

The mechanical dynamics of the WECS and the dc generator can be described by the following set of equations:

$$T_m - T = J_m \dot{\omega} + B_m \omega \quad (9)$$

$$T_p - T_e = J_e \dot{\omega}_e + B_e \omega_e \quad (10)$$

$$T_p \omega_e = T\omega \quad (11)$$

where B_m and B_e are the frictional constants of the turbine and the generator respectively, T_m, T_e, T, T_p are the rotor torques at the turbine end, generator end, before and after the gear box, J_m, J_e the moment of inertia of the turbine and the generator respectively, and ω, ω_e are the rotational speed of the rotor at turbine end and generator end respectively. Note that no torsion related losses are considered here to simplify the modelling. Furthermore, the transmission is assumed ideal i.e. lossless. The transmission gear ratio is defined as:

$$\gamma = \frac{\omega_e}{\omega} \quad (12)$$

Combining the above equations, the mechanical equation of the WECS is given by:

$$J\dot{\omega} + B\omega = T_m - \gamma T_e \quad (13)$$

$$J\dot{\omega} + B\omega = \frac{P_m}{\omega} - \gamma \frac{P_e}{\omega_e} \quad (14)$$

where:

$$J = J_m + \gamma^2 J_e \tag{15}$$

$$B = B_m + \gamma^2 B_e \tag{16}$$

where P_m denotes the wind power and P_e represents the electrical power generated by dc generator. It is clear from the previous section that P_e is related to the field and armature current of the generator:

$$P_e = T_e \omega_e = K_1 i_f i_a \omega_e \tag{17}$$

Finally, the WECS model is obtained by combining the dynamics of dc generator with that of the turbine and is given by:

$$\begin{cases} \dfrac{di_f}{dt} = \dfrac{V_f}{L_f} - \dfrac{R_f i_f}{L_f} \\ \dfrac{di_a}{dt} = \dfrac{\gamma K_1 i_f \omega}{L_T} - \dfrac{R_T}{L_T} i_a \\ \dfrac{d\omega}{dt} = \dfrac{\frac{1}{2} C_p(\lambda, \beta)\rho \pi R^2 u^3}{J\omega} - \dfrac{\gamma K_1 i_f i_a}{J} - \dfrac{B\omega}{J} \end{cases} \tag{18}$$

where $R_T = R_a + R_L$ and $L_T = L_a + L_L$.

III. Observer-based C_p estimation

In [2], an observer-based estimator for the power coefficient was proposed based upon the above WECS model. For this, the power coefficient is considered as a state variable instead of a parameter and assume that the power coefficient is unknown and so that $\dfrac{dC_p}{dt} = \xi(t)$ where $\xi(t)$ is an unknown function of time. Additionally, assuming that the values of i_f, ω and V_a are measured a second order reduced model can be employed for C_p estimation:

$$\begin{cases} \dfrac{d\omega}{dt} = \dfrac{1}{2}\dfrac{C_p \rho \pi R^2 u^3}{J\omega} - \dfrac{\gamma K_1 i_f i_a}{J} - \dfrac{B\omega}{J} \\ \dfrac{dC_p}{dt} = \xi(t) \\ y = \omega \end{cases} \tag{19}$$

Consequently, the estimator is given by

$$O_1 : \begin{cases} \dfrac{d\hat{\omega}}{dt} = \dfrac{\hat{C}_p \rho \pi R^2 u^3}{2J\omega} - \dfrac{\gamma K_1 i_f \hat{i}_a}{J} - \dfrac{B\omega}{J} \\ + \dfrac{\rho \pi R^2 u^3}{2Jy} \theta l_1(\omega - \hat{\omega}) \\ \dfrac{d\hat{C}_p}{dt} = \theta^2 \dfrac{\rho \pi R^2 u^3}{2Jy} l_2(\omega - \hat{\omega}) \end{cases} \tag{20}$$

with

$$\frac{d\hat{i}_a}{dt} = \frac{\gamma K_1 i_f \omega}{L_T} - \frac{R_T}{L_T}\hat{i}_a$$

and $\theta > 0$ is a tuning parameter.

It was shown that the main advantage of this observer is that it reduces the effect of the unknown function $\xi(t)$; hence providing a good estimate of C_p. On the other hand, the main shortcomings of this observer is that it does not incorporate available information about the profile of the power coefficient. It is clearly obvious that if we include any apriori knowledge on the variation or profile of C_p, then the above observer can be considerably improved.

IV. Improved C_p Observer

In this section, we propose an improvement of the above C_p observer by including a possible dynamical model of the latter. The proposed dynmical model is given by:

$$\frac{dC_p}{dt} = \mathcal{F}_1 C_p^2 + \mathcal{F}_2 C_p + \mathcal{F}_3 + \xi(t) \tag{21}$$

where $\mathcal{F}_1 = \varsigma\chi$, $\mathcal{F}_2 = \varsigma\kappa + \chi\Gamma$ and$\mathcal{F}_3 = \kappa\Gamma$ with

$$\varsigma = \frac{c_5}{(\lambda + 0.08\beta)^2} \tag{22}$$

$$\Gamma = c_6 - \frac{c_5 c_6 \lambda + c_1 c_2 e^{-c_5 \Lambda}}{(\lambda + 0.08\beta)^2} \tag{23}$$

$$\chi = \frac{\frac{1}{2}\rho \pi R^3 u^2}{J\omega} \tag{24}$$

$$\kappa = -\frac{BR\omega + \gamma K_1 R i_f i_a}{Ju} - \frac{R\omega}{u^2}\frac{du}{dt} \tag{25}$$

$$\Lambda = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{\beta^3 + 1} \tag{26}$$

$\xi(t)$ is a bounded unknown function representing any unmodelled dynamics, λ is tip speed ratio and β is the rotor pitch angle which is assumed to be constant. The numerical values of the constants c_i are taken as

$$\begin{array}{lll} c_1 = 0.5109 & c_2 = 116 & c_3 = 0.4 \\ c_4 = 5 & c_5 = 21 & c_6 = 0.0068 \end{array}$$

Remark: The structure of the dynamics is, in fact, obtained by derivating with respect to time the model employed in [3].

Similar to the work done in [2], we derive a reduced order model of the WECS. To do this, we assume that the values of i_f, ω and V_a are measured. In practice, this is a reasonable assumption as they can be easily obtained. The voltage V_f and the wind speed u are inputs to the WECS. Since i_f is measured, its dynamics can be dropped and injected directly into the mechanical equation of the WECS. Also, since the dynamics of i_a is stable, we simply use the second equation for estimating i_a. Consequently, we shall use the following equation as an estimate for i_a:

$$\frac{d\hat{i}_a}{dt} = \frac{\gamma K_1 i_f \omega}{L_T} - \frac{R_T}{L_T}\hat{i}_a$$

This estimate will be then injected in the observer.

As a result, the following second order reduced model can be employed for C_p estimation:

$$\begin{aligned} \frac{d\omega}{dt} &= \frac{\frac{1}{2}C_p(\lambda,\beta)\rho\pi R^2 u^3}{J\omega} - \frac{\gamma K_1 i_f i_a}{J} - \frac{B\omega}{J} \\ \frac{dC_p}{dt} &= \mathcal{F}_1 C_p^2 + \mathcal{F}_2 C_p + \mathcal{F}_3 \\ y &= \omega \end{aligned} \tag{27}$$

where i_f and u are viewed as inputs to the system and ω as the output. The reduced order system in (27) can be rewritten in matrix form:

$$\dot{\mathbf{x}} = G(\mathbf{x}) + B\xi \tag{28}$$

$$y = \mathbf{C}\mathbf{x} \tag{29}$$

where

$$\mathbf{x} = \begin{pmatrix} \omega \\ C_p \end{pmatrix}$$

$$G(\mathbf{x}) = \begin{pmatrix} \frac{\frac{1}{2}x_2\rho\pi R^2 u^3}{Jy} - \frac{\gamma K_1 i_f i_a}{J} - \frac{By}{J} \\ \mathcal{F}_1 x_2^2 + \mathcal{F}_2 x_2 + \mathcal{F}_3 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \ C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

We design an observer of the form:

$$O_2 : \left\{ \dot{\hat{\mathbf{x}}} = G(\hat{\mathbf{x}}) + \mathbf{LC}\left(\mathbf{x} - \hat{\mathbf{x}}\right) \right. \tag{30}$$

where $\mathbf{L} = \begin{bmatrix} l_1 & l_2 \end{bmatrix}^T$ is the gain of the observer to be determined. To simplify, we set:

$$\Psi = \frac{\rho\pi R^2 u^3}{2Jy}$$

To analyse the stability of the model, we set $\varepsilon = \mathbf{x} - \hat{\mathbf{x}}$ and find the error dynamics to be of the form:

$$\dot{\varepsilon} = G_e(\varepsilon, y) - \mathbf{L}\varepsilon + B\xi \tag{31}$$

where:

$$G_e(\varepsilon, y) = \begin{pmatrix} \frac{\frac{1}{2}x_2\rho\pi R^2 u^3}{Jy} - \frac{\gamma K_1 i_f (i_a - \hat{i}_a)}{J} - l_1\varepsilon_1 \\ \mathcal{F}_1\left(x_2^2 - \hat{x}_2^2\right) + \mathcal{F}_2\varepsilon_2 - l_2\varepsilon_1 \end{pmatrix}$$

This turns out to be a non-linear equation in with respect to the error terms. We can obtain a first order linear approximation to the model:

$$\dot{\varepsilon} \approx J\varepsilon + B\xi \tag{32}$$

where the Jacobian matrix, J is given by:

$$J = \begin{pmatrix} -l_1 & \Psi \\ -l_2 & \mathcal{F}_2 \end{pmatrix} \tag{33}$$

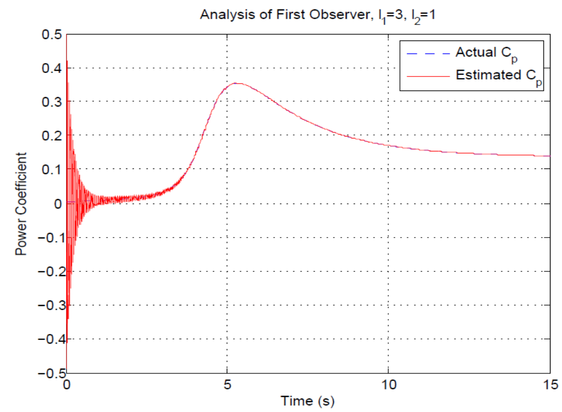

Figure 1 – Fig. 1. C_p estimation with observer O_1

We know that the system will be stable if the eigenvalues of the Jacobian about the fixed point ($\varepsilon = 0$) is negative. The eigenvalues are given by:

$$m = \frac{\tau \pm \sqrt{\tau^2 - 4\Delta}}{2} \tag{34}$$

where:

$$\tau = \mathcal{F}_2 - l_1$$

$$\Delta = \Psi l_2 - \mathcal{F}_2 l_1$$

From (34) we can get the condition for stability for the observer:

$$\frac{l_2}{l_1} > \frac{\mathcal{F}_2}{\Psi} \tag{35}$$

V. Simulation Results

Simulations were carried out to evaluate the performance of the new observer compared to O_1. Appendix 1 shows the simulation parameters used in this study. Various cases were studied: first, we examine the behaviour under constant wind speeds. Next, time-varying wind speeds with superimposed noise is introduced to simulate real conditions. Finally, we examine the case of sensor input noise to the observer, where the turbine tip speed ratio was superimposed with noise before being injected into the observer.

A. Constant Wind Speed

The model was simulated using MATLAB Simulink, with the observer gains, l_1 and l_2, set to 3 and 1 respectively. Wind speed was kept constant at $u = 3\text{ms}^{-1}$. The results of the simulation are shown in Figure 1 and Figure 2. It can be seen that the previous model produced far too much oscillation to be practical, while the new model shows much better behaviour in that respect, with acceptable convergence.

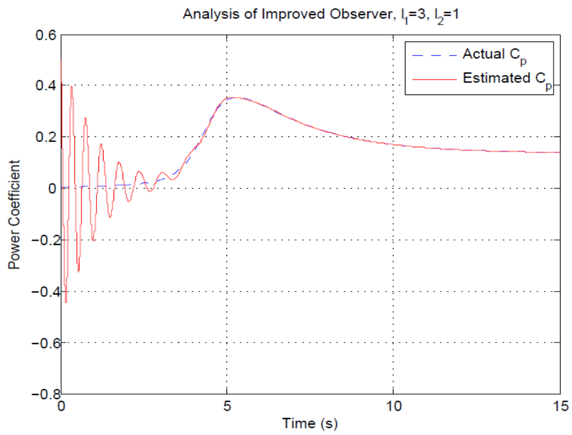

Fig 2. C_p estimation with observer O_2

B. Changing Wind Speed

The next simulation examines the behaviour of both models with the wind speed varying as an offset sinusiod with a normally distributed noise with variance of 10% (shown in Figure 3). Both models show a reasonable convergence, however with the previous model showing a large sensitivity to the noise, causing oscillations with high amplitude. The new model shows less sensitivity to the noise, showing little to no oscillation with the noisy data, a trait likely to be more desirable in practical situations. The results are shown in Figure 4 and Figure 5.

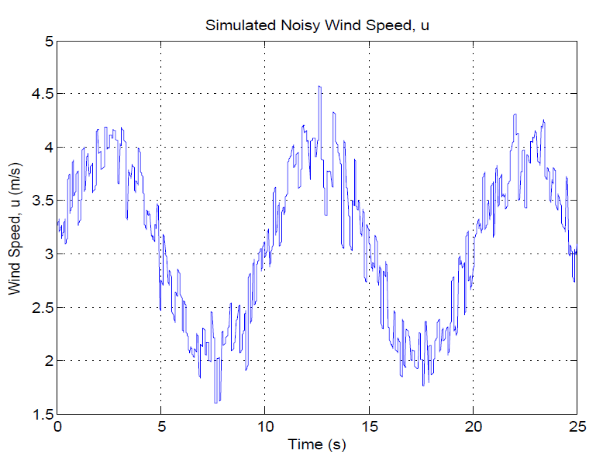

Fig. 3. Noisy and variable wind speed

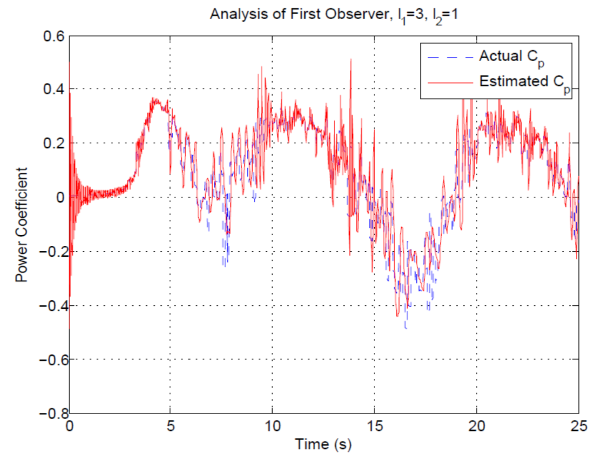

Fig. 4. C_p estimation with observer O_1 - variable and noisy wind speed

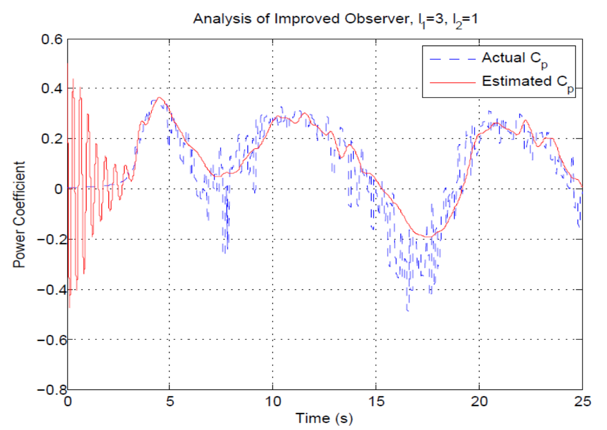

Fig. 5. C_p estimation with observer O_2 - variable and noisy wind speed

C. Measurement Noise in ω

The final simulation kept wind speed constant, but added measurement noise to the turbine angular speed measurements. A measurement noise with zero mean and a variance of 5% was superimposed onto the angular speed signal before feeding it to the observers (shown in Figure 6). The results were surprising, the previous model showed a very unstable behaviour corresponding with the noise, resulting in very large oscillations. This made the previous observer impractical in real applications. However the new model showed a greater resistance to the noise, and provided excellent convergence characteristics to the actual C_p and much smaller oscillations. The results are shown on Figure 7 and Figure 8.

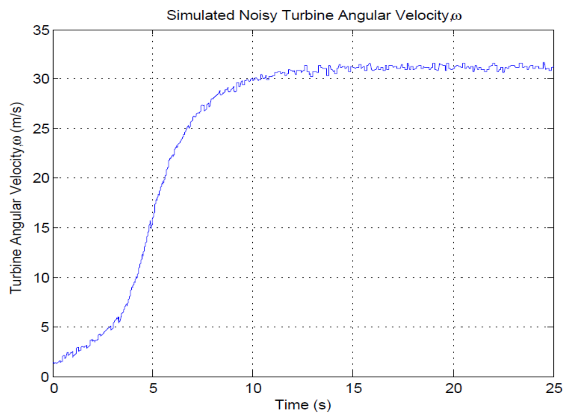

Fig. 6. Noisy wind turbine angular velocity

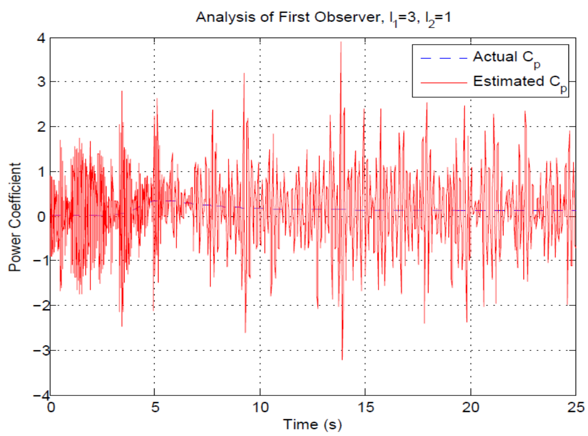

Fig. 7. C_p estimation with observer O_1 - noisy ω

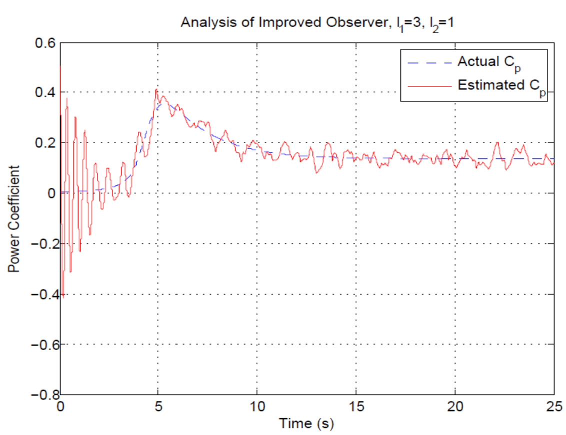

Fig. 8. C_p estimation with observer O_2 - noisy ω

VI. Conclusion

In this paper, a dynamical model the power coefficient was proposed. The model was then employed to design an improved observer design for the estimation of the power coefficient. The results have proven that the introduction of a dynmical model for C_p improved the convergence of the observer. Addtionally, the observer is robust with respect to measurement noise making it much more attractive for practical applications.

VII. Appendix 1

A. Simulation Parameters

The following numerical values are used as the model parameters in the simulation: $V_f = 240$V, $L_f = 60$H, $R_a = 1.2\Omega, K_1 = 0.353$NmA^{-2}, $J_e = 0.208$kgm^2, $R = 1.5$m, $J_m = 0.3$kgm^2, $R_L = 1.2\Omega$, $l_1 = 3$, $\theta = 3, R_f = 60\Omega$,Rated $V_a = 240$V, $L_a = 0.01$H, $B_e = 0.011$Nm, $\rho = 1.25$kgm$^{-3}, \gamma = 1 : 23.4$, $B_m = 0.0151$Nm, $L_L = 0.01$H, $l_2 = 1$ and $\beta = 5°$.

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