Matematikk 2 – øving 3 vår

Tekstoppgaver først, MATLAB til slutt

Tore Bergebakken

altså… det begynner på neste side.

MATLAB-koden er skrevet i Live Editor for oversiktlighetens skyld

PDFene er stitchet sammen med `pdftk *.pdf cat output submit.pdf`

Oppgave 2.7.3c)

3c) $v^2 = 4-(u-1)^2 = 4-(u^2-2u+1)$

$v^2 = -u^2+2u+3$

(2) → (1) $u^2 - 4(-u^2+2u+3) = 4$

$5u^2 - 8u - 12 - 4 = 0$

$$u = \frac{8 \pm \sqrt{64-4\cdot 5(-16)}}{2\cdot 5} = \frac{8 \pm 8\sqrt{6}}{10}$$

(a) $u = \frac{4+4\sqrt{6}}{5} \vee u = \frac{8-8\sqrt{6}}{10} < 0$

$(\,)^2 < 2\cdot(\,)$

(1) → (2) $v^2 = -\left(\frac{8(1+\sqrt{6})}{10}\right)^2 + 2\left(\frac{8(1+\sqrt{6})}{10}\right) + 3$

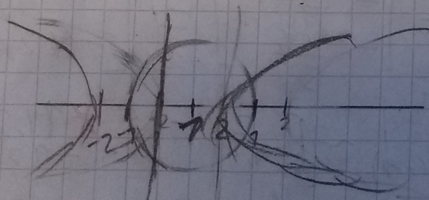
(a) $v = \pm\sqrt{-(2,7596)^2 + 2\cdot(2,7596)+3}$

$v \approx \pm 0,95069$

med (b) $\frac{8-8\sqrt{6}}{10} \approx -1,1596$ får vi $v \notin \mathbb{R}$.

$(u+1)^2 + v^2 = 4$ er en sirkel med senter i $(-1, 0)$ og radius $\sqrt{4} = 2$.

Den andre... eh, GeoGebra saves the day.



(B) løsning: $(2,7596, \pm 0,95069)$

Oppgave 2.7.4c)

4c)

$u^2 - 4v^2 = 4$ $\qquad f_1(x) = u^2 - 4v^2 - 4$

$(u-1)^2 + v^2 = 4$ $\qquad f_2(x) = (u-1)^2 + v^2 - 4$

Partiellderiverte i Jakobimatrise:

$$DF(u,v) = \begin{bmatrix} 2u & -8v \\ 2u-2 & 2v \end{bmatrix}$$

• $(u-1)^2 = u^2 - 2u + 1$
$\downarrow$ der
$2u - 2$

$$F(1,1) = \begin{bmatrix} 1-4-4 \\ 0+1-4 \end{bmatrix} = \begin{bmatrix} -7 \\ -3 \end{bmatrix}$$

$$DF(1,1) \cdot s = -F(1,1)$$

$$\begin{bmatrix} 2 & -8 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

$$2s_2 = 3 \iff s_2 = \frac{3}{2}$$

$$2s_1 = 8s_2 + 7 = 8 \cdot \left(\frac{3}{2}\right) + 7 = 12 + 7 = 19$$

$$s_1 = \frac{19}{2}$$

$$x_1 = x_0 + s = \left(1 + \frac{19}{2},\ 1 + \frac{3}{2}\right) = \left(\frac{21}{2},\ \frac{5}{2}\right)$$

feil på rekke-følge.

$$DF(x_1)\, s = -F(x_1)$$

$$\begin{bmatrix} 21 & -20 \\ 19 & 5 \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} -\frac{325}{4} \\ -\frac{185}{2} \end{bmatrix} = \begin{bmatrix} -81,25 \\ -92,5 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 21 & -20 & -81,25 \\ 19 & 5 & -92,5 \end{array}\right] \xrightarrow{(-1)} \sim \left[\begin{array}{cc|c} 2 & -25 & 11,25 \\ 19 & 5 & -92,5 \end{array}\right] \cdot \frac{1}{2}$$

$$\sim \left[\begin{array}{cc|c} 1 & -12,5 & 5,625 \\ 19 & 5 & -92,5 \end{array}\right] \cdot(-19) \sim \left[\begin{array}{cc|c} 1 & -12,5 & 5,625 \\ 0 & 242,5 & -199,375 \end{array}\right]$$

$$s_2 = \frac{-199,375}{242,5} = -\frac{319}{388} \approx -0,8221$$

$$s_1 = 5,625 - (-12,5 \cdot s_2) = 12,5 \cdot \left(-\frac{319}{388}\right) + 5,625$$

$$\approx -4,6520$$

$$x_2 = x_1 + s \approx (5,8480,\ 1,6779)$$

Oppgave 2.7.5c) FEIL A^{-1}, BRUKTE MATLAB ISTEDEN

5c) $u^2 - 4v^2 = 4$

$(u-1)^2 + v^2 = 4$

$$A_{i+1} = A_i + \frac{(\Delta_{i+1} - A_i\delta_{i+1})\delta_{i+1}^T}{\delta_{i+1}^T\delta_{i+1}}, \quad x_0 = (1,1)$$

$A_0 = I$, $x_1 = x_0 - A_0^{-1}F(x_0)$ → fra 4c

$$I^{-1} = \frac{1}{1^2-0^2}\begin{bmatrix}1 & -0\\ -0 & 1\end{bmatrix} = I \ldots$$

$$= \begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}-7\\-3\end{bmatrix} = \begin{bmatrix}1-(-7)\\1-(-3)\end{bmatrix} = \begin{bmatrix}8\\4\end{bmatrix}$$

$\delta_1 = x_1 - x_0 = (7,3)$, $\Delta_1 = F(x_1) - F(x_0)$

$$= \begin{bmatrix}8^2-4\cdot3^2-4\\ 7^2+3^2-4\end{bmatrix} - \begin{bmatrix}-7\\-3\end{bmatrix} = \begin{bmatrix}-4+7\\61+3\end{bmatrix} = \begin{bmatrix}3\\64\end{bmatrix}$$

$$A_1 = \begin{bmatrix}1&0\\0&1\end{bmatrix} + \frac{\left(\begin{bmatrix}3\\64\end{bmatrix} - \begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}7\\3\end{bmatrix}\right)[7\ 3]}{[7\ 3]\begin{bmatrix}7\\3\end{bmatrix}}$$

$$= I + \frac{\begin{bmatrix}-4\\61\end{bmatrix}[7\ 3]}{58}$$

$$= I + \frac{1}{58}\begin{bmatrix}-28 & -12\\ 427 & 183\end{bmatrix} = \frac{1}{58}\begin{bmatrix}30 & -12\\427 & 241\end{bmatrix}$$ ops... $+\begin{bmatrix}58&0\\0&58\end{bmatrix}$...

$$A_1^{-1} = \frac{1}{\frac{1}{58}(30\cdot241-(-12)\cdot427)}\begin{bmatrix}241 & -427\\-12 & 30\end{bmatrix}$$

$$A_1^{-1} = -\frac{58}{12364}\begin{bmatrix}241 & -427\\-12 & 30\end{bmatrix}$$

$$x_2 = x_1 - A_1^{-1}F(x_1)$$

$$= \begin{bmatrix}8\\4\end{bmatrix} + \frac{58}{12364}\begin{bmatrix}241 & -427\\12 & 30\end{bmatrix}\begin{bmatrix}-4\\61\end{bmatrix}$$

$$\cancel{= \frac{58}{12364}\begin{bmatrix}8+241(-4)-427(61)\\4+12(-4)+30\cdot61\end{bmatrix} \approx \begin{bmatrix}-126{,}8\\8{,}389\end{bmatrix}}$$ nope.

Med A_1 som på forrige side gir matlab $x_2 = \underline{(9{,}0892, -12{,}6703)}$

← Jeg debugger ikke mer nå.

Oppgave 2.7.6c)

6c) $x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $B_0 = I = B_0^{-1}$

$X_1 = x_0 - B_0 F(x_0) = x_0 - F(x_0)$ siden $B_0 = I$

$= \begin{bmatrix} 1 - (-7) \\ 1 - (-3) \end{bmatrix} = \boxed{\begin{bmatrix} 8 \\ 4 \end{bmatrix}}$ som i 5c.

$\delta_1 = x_1 - x_0 = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$, $\Delta_1 = \begin{bmatrix} 3 \\ 64 \end{bmatrix}$ som i 5c.

$B_1 = B_0 + \dfrac{(\delta_1 - B_0 \Delta_1)\delta_1^T B_0}{\delta_1^T B_0 \Delta_1}$

1×2 $\quad 2\times 2$

$[a\ b] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ blir 1×2.

Siden $B_0 = I$ kan vi forenkle.

$B_1 = I + \dfrac{(\delta_1 - \Delta_1)\delta_1^T}{\delta_1^T \Delta_1}$

~~Dette skal bli det samme som A_1 i 5c.~~

$(\delta_1 - \Delta_1)\delta_1^T = \begin{bmatrix} 7-3 \\ 3-64 \end{bmatrix} [7\ \ 3]$

$= \begin{bmatrix} 4 \\ -61 \end{bmatrix} [7\ \ 3] = \begin{bmatrix} 4\cdot 7 & 4\cdot 3 \\ -61\cdot 7 & -61\cdot 3 \end{bmatrix}$

$= \begin{bmatrix} 28 & 12 \\ -427 & -183 \end{bmatrix}$

$\delta_1^T \Delta_1 = [7\ \ 3] \begin{bmatrix} 3 \\ 64 \end{bmatrix} = 7\cdot 3 + 3\cdot 64 = 213$

$B_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0{,}1315 & 0{,}0563 \\ -2{,}0047 & -0{,}8592 \end{bmatrix}$ (matlab)

$x_2 = x_1 - B_1 F(x_0) = \begin{bmatrix} 8 \\ 4 \end{bmatrix} - \begin{bmatrix} 1{,}1315 & 0{,}0563 \\ -2{,}0047 & 0{,}1408 \end{bmatrix} \begin{bmatrix} -4 \\ 61 \end{bmatrix}$

$\underline{x_2 \approx (9{,}0917,\ -12{,}6076)}$ maybe

Oppgave 4.1.8b)

Kapittel 4.1

8b)

Punktene $(1,2), (3,2), (4,1), (6,3)$

Modell for linje: $y = c_1 + c_2 x$

Som matrise:

$$\begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

Skal da løse $A^T A \bar{c} = A^T y$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1+3+4+6 \\ 14 & 1+9+16+36 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 4 & 14 \\ 14 & 62 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 4+4 \\ 2+6+4+18 \end{bmatrix} = \begin{bmatrix} 8 \\ 30 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 4 & 14 & 8 \\ 14 & 62 & 30 \end{array}\right] \cdot \frac{7}{2} \; -$$

$$\sim \left[\begin{array}{cc|c} 4 & 14 & 8 \\ 0 & 13 & 2 \end{array}\right] \qquad 13 c_2 = 2 \Rightarrow c_2 = \frac{2}{13} \approx \underline{0{,}1538}$$

$$\Downarrow$$

$$4 c_1 = 8 - 14 \cdot \frac{2}{13}$$

$$c_1 = 2 - \frac{7}{13} = \frac{19}{13} \approx \underline{1{,}4615}$$

Linje

Oppgave 4.1.8b) forts., start 9b)

8b) forts.

Længda ble altså

$$\underline{\underline{y = 19/13 + 2/13 \cdot x}}$$

RMSE: $\simeq 1{,}4165 + 0{,}1538x$

$$\begin{bmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 4 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} 19/13 \\ 2/13 \end{bmatrix} = \begin{bmatrix} 21/13 \\ 25/13 \\ 27/13 \\ 31/13 \end{bmatrix} \simeq \begin{bmatrix} 1{,}615 \\ 1{,}923 \\ 2{,}077 \\ 2{,}385 \end{bmatrix}$$

$$r = y - Az = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 21 \\ 25 \\ 27 \\ 31 \end{bmatrix} \cdot \frac{1}{13} = \frac{1}{13} \begin{bmatrix} 5 \\ 1 \\ -14 \\ 8 \end{bmatrix}$$

$$RMSE = \sqrt{SE/m} = \sqrt{\frac{\frac{1}{13^2} \cdot (5^2 + 1^2 + (-14)^2 + 8^2)}{4}}$$

$$\simeq \underline{\underline{0{,}6504}}$$

9b) Parabel: $y = c_1 + c_2 x + c_3 x^2$

Samme punkter.

Oppgave 4.1.9b) forts.

9b) forts.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} \qquad a_{ij} = 1 + 3^{i+j-2} \dots$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \\ 1 & 9 & 16 & 36 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix} = \begin{bmatrix} 4 & 14 & 62 \\ 14 & 62 & 308 \\ 62 & 308 & 1634 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & 4 & 6 \\ 1 & 9 & 16 & 36 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 30 \\ 144 \end{bmatrix}$$

skal da løse

$$\left[\begin{array}{ccc|c} 4 & 14 & 62 & 8 \\ 14 & 62 & 308 & 30 \\ 62 & 308 & 1634 & 144 \end{array}\right] \left(-\tfrac{7}{2}\right) \left(-\tfrac{31}{2}\right)$$

$$\sim \left[\begin{array}{ccc|c} 4 & 14 & 62 & 8 \\ 0 & 13 & 91 & 2 \\ 0 & 91 & 673 & 20 \end{array}\right] (-7) \sim \left[\begin{array}{ccc|c} 4 & 14 & 62 & 8 \\ 0 & 13 & 91 & 2 \\ 0 & 0 & 36 & 6 \end{array}\right]$$

$$c_3 = \frac{6}{36} = \frac{1}{6} \approx 0{,}1667 \implies c_2 = \frac{2}{13} - 7 \cdot \frac{1}{6} = -\frac{79}{78} \approx -1{,}0128$$

$$c_1 = 2 - 62 \cdot \frac{1}{6} \cdot \frac{1}{4} - 14\left(-\frac{79}{78}\right) \cdot \frac{1}{4} = \frac{77}{26} \approx 2{,}961$$

Oppgave 4.1.9b) siste bit

9b) Linja blir da ca.:

$$y \approx 0{,}1667x^2 - 1{,}0128x + 2{,}967$$

ops...

$$r = b - Ac = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 9 \\ 1 & 4 & 16 \\ 1 & 6 & 36 \end{bmatrix} \begin{bmatrix} 2{,}9667 \\ -1{,}0128 \\ 0{,}1667 \end{bmatrix}$$

$$\approx \begin{bmatrix} -0{,}1154 \\ 0{,}5769 \\ -0{,}5769 \\ 0{,}1154 \end{bmatrix}$$

$$RMSE = \sqrt{\frac{SE}{m}} = \sqrt{\frac{2\cdot(0{,}1154^2 + 0{,}5769)^2}{4}}$$

$$\approx 0{,}4160$$

Oppgave 4.2.2b) – $F_3(t)$

Kapittel 4.2

2b) Data:

t	y
0	4
$1/6$	2
$1/3$	0
$1/2$	-5
$2/3$	-1
$5/6$	3

$\underline{m=6}$

Innt $F_3(t) = c_1 + c_2\cos(2\pi t) + c_3\sin(2\pi t)$

$$A = \begin{bmatrix} 1 & \cos 0 & \sin 0 \\ 1 & \cos \frac{\pi}{3} & \sin \frac{\pi}{3} \\ 1 & \cos \frac{2\pi}{3} & \sin \frac{2\pi}{3} \\ 1 & \cos \pi & \sin \pi \\ 1 & \cos \frac{4\pi}{3} & \sin \frac{4\pi}{3} \\ 1 & \cos \frac{5\pi}{3} & \sin \frac{5\pi}{3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1/2 & \sqrt{3}/2 \\ 1 & -1/2 & \sqrt{3}/2 \\ 1 & -1 & 0 \\ 1 & -1/2 & -\sqrt{3}/2 \\ 1 & 1/2 & -\sqrt{3}/2 \end{bmatrix}$$

$\sum \text{venstre} = 6 \cdot 1$

$\sum \text{midt} = 2 + 4\cdot\left(\frac{1}{2}\right)^2 = 3$

$\sum \text{høyre} = 4 \cdot \frac{3}{2^2} = \underline{3}$

$$A^T A c = A^T b$$

$\sum y = \cancel{6-6}+\underline{3}$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 12 \\ 0 \end{bmatrix}$$

$$c = \begin{bmatrix} 3/6 \\ 12/3 \\ 0/3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 4 \\ 0 \end{bmatrix}$$

Oppgave 4.2.2b) forts. $F_3(t)$, start $F_4(t)$

2b forts.

$$r = b - Ac = \begin{bmatrix} 4 \\ 2 \\ 0 \\ -5 \\ -1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1/2 & \sqrt{3}/2 \\ 1 & -1/2 & \sqrt{3}/2 \\ 1 & -1 & 0 \\ 1 & -1/2 & -\sqrt{3}/2 \\ 1 & 1/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1/2 \\ 4 \\ 0 \end{bmatrix}$$

$$= [4-4-1/2,\ 2-2-1/2,\ 0-1/2+2,\ -5-(-4)-1/2,\ -1-(-2)-1/2,\ 3-2-1/2]^T$$

$$= [-1/2,\ -1/2,\ 3/2,\ -3/2,\ 1/2,\ 1/2]^T$$

$$\|e\|_2 = \sqrt{4\left(\tfrac{1}{2}\right)^2 + 2\left(\tfrac{3}{2}\right)^2} = \frac{\sqrt{22}}{2} \approx \underline{\underline{2,3452}}$$

Til $F_4(t) = c_1 + c_2 \cos(2\pi t) + c_3 \sin(2\pi t) + c_4 \cos(4\pi t)$

$$A = \left[\begin{array}{c|c} 3\pi \text{ sam fdr} & \begin{array}{c} \cos 0 \\ \cos \frac{2\pi}{3} \\ \cos \frac{4\pi}{3} \\ \cos 2\pi \\ \cos \frac{8\pi}{3} \\ \cos \frac{10\pi}{3} \end{array} \end{array} \right] = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1/2 & \sqrt{3}/2 & -1/2 \\ 1 & -1/2 & \sqrt{3}/2 & -1/2 \\ 1 & -1 & 0 & 1 \\ 1 & -1/2 & -\sqrt{3}/2 & -1/2 \\ 1 & 1/2 & -\sqrt{3}/2 & -1/2 \end{bmatrix}$$

$\frac{10\pi}{3} \to \frac{4\pi}{3}$

$\frac{8\pi}{3} \to \frac{2\pi}{3}$

$\underline{2 + 4 \cdot \frac{1}{2^2} = 3}$

$$A^T A = \begin{bmatrix} 6 & & & \\ & 3 & & \\ & & 3 & \\ & & & 3 \end{bmatrix} \qquad A^T b = \begin{bmatrix} 3 \\ 12 \\ 0 \\ -3 \end{bmatrix}$$

$4 - 1 + 0 - 5 + 1/2 - 3/2$
$= -2 + 2/2 = -3$

Oppgave 4.2.2b) forts. $F_4(t)$

2b forts., med $F_4(t)$ ...

$$A^TAc = A^Tb$$

$$\begin{bmatrix} 6 & & & 0 \\ & 3 & & \\ & & 3 & \\ 0 & & & 3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 12 \\ 0 \\ -3 \end{bmatrix}$$

$$c = \begin{bmatrix} 3/6 \\ 12/3 \\ 0/3 \\ -3/3 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 4 \\ 0 \\ -1 \end{bmatrix}$$

fortegn, trekker fra siste kolonne i A ganget med -1.

$$\|e\|_2 = \|b - Ac\|_2, \quad e = \begin{bmatrix} -1/2 - (-1) \\ -1/2 - (1/2) \\ 3/2 - (1/2) \\ -3/2 - (-1) \\ 1/2 - 1/2 \\ 1/2 - 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1 \\ 1 \\ -1/2 \\ 0 \\ 0 \end{bmatrix}$$

$$\|e\|_2 = \sqrt{2 \cdot \left(\tfrac{1}{2}\right)^2 + 2 \cdot 1^2 + 2 \cdot 0^2}$$

$$= \sqrt{\tfrac{1}{2} + 2} = \frac{\sqrt{10}}{2} = \underline{1{,}5811}$$

Oppgave 4.2.3b) – FØRST MED FEIL y MEN RETT t OG A

3 b) Til modellen $c_1 e^{c_2 t} = y$

$$\Rightarrow \ln(y) = \ln(c_1 e^{c_2 t})$$

$$= \ln(c_1) + \ln(e^{c_2 t})$$

$$\ln y = \ln c_1 + c_2 t$$

La $k = \ln c_1$. $\boxed{\text{FEIL } y}$

Korrekt A.

$$\boxed{A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}} \qquad b = \begin{bmatrix} \ln 10 \\ \ln 5 \\ \ln 2 \\ \ln 1 \end{bmatrix} \approx \begin{bmatrix} 2{,}3026 \\ 1{,}6094 \\ 0{,}6931 \\ 0 \end{bmatrix}$$

$= \ln(10) \cdot 2 \ldots$

$$A^T b = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} \ln 10 \\ \ln 5 \\ \ln 2 \\ \ln 1 \end{bmatrix} = \begin{bmatrix} \ln(10 \cdot 5 \cdot 2 \cdot 1) \\ 0 + \ln(5 \cdot 2 \cdot 1^2) \end{bmatrix} = \begin{bmatrix} \ln(100) \\ \ln(10) \end{bmatrix}$$

$$\approx \begin{bmatrix} 4{,}6052 \\ 2{,}3026 \end{bmatrix}$$

$$\boxed{A^T A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 4 & 6 \end{bmatrix}}$$

Denne stemmer.

$$A^T A c = A^T b$$

$$\begin{bmatrix} 4 & 4 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} k \\ c_2 \end{bmatrix} = \begin{bmatrix} 2\ln(10) \\ \ln(10) \end{bmatrix} (-1) \quad \cdot \frac{1}{2} \qquad \begin{bmatrix} 2\ln(10) \\ \ln(10) \end{bmatrix}$$

men $\cdot 1/2$ isteden

$$\begin{bmatrix} 2 & 2 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} k \\ c_2 \end{bmatrix} = \begin{bmatrix} \ln 10 \\ -\ln 10 \end{bmatrix}$$

så $\begin{bmatrix} -\ln 10 \\ \ln 10 \end{bmatrix}$

$$k = \frac{\ln 10 - 2\ln 10}{2} = -\frac{1}{2}\ln 10$$

$$c_2 = -\frac{\ln 10}{2} = -\frac{1}{2}\ln 10 = \ln\frac{1}{\sqrt{10}}$$

$$k = \frac{1}{2}\left(\ln 10 - 2\left(-\frac{1}{2}\ln 10\right)\right) = \frac{1}{2} \cdot 2\ln 10 = \underline{\ln 10}$$

Oppgave 4.2.3b) – FORTSATT MED FEIL y

3b) følger altså:

Med FEIL y.

$$\begin{bmatrix} k \\ c_2 \end{bmatrix} = \begin{bmatrix} \ln 10 \\ -\frac{1}{2}\ln 10 \end{bmatrix}$$

Da er $c_1 = e^k = e^{\ln 10} = 10$

Modellen blir $y = c_1 e^{c_2 t} = \underline{10 e^{-\frac{1}{2}\ln(10) t}}$

$$= 10 \cdot (e^{\ln 10})^{-\frac{1}{2}t}$$

$$y = \underline{10 \cdot 10^{-1/2\, t}}$$

$$\|e\|_2 = \| \vec{y} - c_1 e^{c_2 \vec{t}} \|_2$$

$$= \sqrt{(1 - 10 \cdot 10^{-0})^2 + (1 - 10 \cdot 10^{-1/2})^2 + (2 - 10 \cdot 10^{-1/2})^2 + (4 - 10 \cdot 10^{-1})^2}$$

$$= \sqrt{(-9)^2 + (1 - \sqrt{10})^2 + (2 - \sqrt{10})^2 + 3^2}$$

$$\approx \underline{\underline{9{,}7477}}$$

NEI.

Oppgave 4.2.3b) – NÅ MED RETT y, SAMME t OG A

3b) for reals.

$y = [1 \;\; 1 \;\; 2 \;\; 4]^T$ NB!

Ser at jeg ender opp med (med bruk av $\ln a + \ln b = \ln(a \cdot b)$)

$$A^T b = \begin{bmatrix} \ln(1\cdot 1\cdot 2\cdot 4) \\ 0 + \ln(1\cdot 2\cdot 4^2) \end{bmatrix} = \begin{bmatrix} \ln(2^3) \\ \ln(2^5) \end{bmatrix}$$

og samme $A^T A = \begin{bmatrix} 4 & 4 \\ 4 & 6 \end{bmatrix}$. ($4^2$ siden $2 \cdot \ln 4 = \ln(4^2)$)

Løser: Bruker altså tidligere utregninger.

$$\left[\begin{array}{cc|c} 4 & 4 & 3 \ln 2 \\ 4 & 6 & 5 \ln 2 \end{array}\right] (-1)$$

$$\sim \left[\begin{array}{cc|c} 4 & 4 & 3 \ln 2 \\ 0 & 2 & 2 \ln 2 \end{array}\right]$$

$$c_2 = \frac{2}{2} \ln 2 = \ln 2$$

$$k = \frac{1}{4}(3 \ln 2 - 4 \ln 2) = -\frac{1}{4} \ln 2$$

$$c_1 = e^k = e^{-\frac{1}{4} \ln 2}$$

Modellen blir: $y = c_1 e^{c_2 t}$

$$y = e^{-\frac{1}{4}(\ln 2)} e^{(\ln 2)\cdot t} = (e^{\ln 2})^{t - \frac{1}{4}}$$

$$\underline{\underline{y = 2^{t - \frac{1}{4}}}}$$

$$\|e\|_2 = \sqrt{(1 - 2^{0 - 1/4})^2 + (1 - 2^{1 - 1/4})^2 + (2 - 2^{1 - 1/4})^2 + (4 - 2^{2 - 1/4})^2}$$

$$\approx \underline{0{,}9982}$$

Oppgave 4.5.1a)

1a) kapittel 4.5

Punktene $(0,1)$, $(1,1)$, $(0,-1)$
er sentrum i sirkler m/ $r=1$.
Får $(x_0, y_0) = (0,0)$, skal finne (x_1, y_1)

$$s_1 = \sqrt{(0-0)^2 + (0-1)^2} = \sqrt{1} = 1$$

$$s_2 = \sqrt{(0-1)^2 + (0-1)^2} = \sqrt{2}$$

$$s_3 = \sqrt{(0-0)^2 + (0-(-1))^2} = \sqrt{1} = 1$$

$$Dr(x,y) = \begin{bmatrix} \frac{x-0}{s_1} & \frac{y-1}{s_1} \\ \frac{x-1}{s_2} & \frac{y-1}{s_2} \\ \frac{x-0}{s_3} & \frac{y-(-1)}{s_3} \end{bmatrix}$$

$$Dr(0,0) = \begin{bmatrix} \frac{0}{s_1} & -\frac{1}{s_1} \\ -\frac{1}{s_2} & -\frac{1}{s_2} \\ \frac{0}{s_3} & \frac{1}{s_3} \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 1 \end{bmatrix} = \underline{\underline{A}}$$

$$(-1)^2 + \frac{1}{2} + 1^2$$

$$A^T A = \begin{bmatrix} 0 & -1/\sqrt{2} & 0 \\ -1 & -1/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ -1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{5}{2} \end{bmatrix}$$

Oppgave 4.5.1a) forts.

1a) forts.

$$r(x,y) = \begin{bmatrix} S_1(x,y) - R_1 \\ S_2(x,y) - R_2 \\ S_3(x,y) - R_3 \end{bmatrix}$$

$$r(0,0) = \begin{bmatrix} 1 - 1 \\ \sqrt{2} - 1 \\ 1 - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2} - 1 \\ 0 \end{bmatrix}$$

$$A^T r(0,0) = \begin{bmatrix} 0 & -1/\sqrt{2} & 0 \\ -1 & -1/\sqrt{2} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \sqrt{2}-1 \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{2}}{2} + \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} - 1 \\ \frac{1}{\sqrt{2}} - 1 \end{bmatrix}$$

$$A^T A v_1 = -A^T r(0,0)$$

Løser altså:

$$\left[\begin{array}{cc|c} \frac{1}{2} & \frac{1}{2} & 1 - \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{5}{2} & 1 - \frac{1}{\sqrt{2}} \end{array}\right] (-1)$$

$$\sim \left[\begin{array}{cc|c} \frac{1}{2} & \frac{1}{2} & 1 - \frac{1}{\sqrt{2}} \\ 0 & \frac{4}{2} & 0 \end{array}\right]$$

$$\text{För } v_1 = \begin{bmatrix} \left(1 - \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{0}{2}\right) \cdot 2 \\ \frac{0}{2} \end{bmatrix} = \begin{bmatrix} 2 - \frac{2}{\sqrt{2}} \\ 0 \end{bmatrix} = \begin{bmatrix} 2 - \sqrt{2} \\ 0 \end{bmatrix}$$

$$x_1 = x_0 + v_1 = \begin{bmatrix} 0 + 2 - \sqrt{2} \\ 0 + 0 \end{bmatrix} = \begin{bmatrix} 2 - \sqrt{2} \\ 0 \end{bmatrix}$$

Altså finner vi $\underline{\underline{(x_1, y_1) = (2 - \sqrt{2}, 0)}}$

2.7 - Newton's Method

CP1c

For verifying exercises 3 through 6.

The system

```
F = @(x) [x(1)^2  - 4*x(2)^2 - 4;
          (x(1)-1)^2 + x(2)^2 - 4];
DF = @(x) [2*x(1)   , -8*x(2);
           2*x(1)-2,  2*x(2)];
```

Two steps of Newton

```
newton([1; 1], F, DF, 0.5e-8, 2)
```

```
ans = 2×1
    5.8479
    1.6778
```

Decent estimate

```
newton([1; 1], F, DF, 0.5e-8, 2)
```

```
ans = 2×1
    5.8479
    1.6778
```

Turns out this system converges rather swiftly.

CP3

Two solutions for the system $u^3 - v^3 + u = 0$ and $u^2 + v^2 = 1$ - the last equation is the unit circle.

Defining the system:

```
F = @(x) [x(1)^3 - x(2)^3 + x(1);
           x(1)^2 + x(2)^2 - 1];
DF = @(x) [3*x(1)^2 + 1,  -3*x(2)^2;
           2*x(1)      ,   2*x(2) ];
```

Applying Newton's method:

```
newton([1; 1], F, DF, 0.5e-8, 10)
```

```
ans =
"Done after 6 iterations"
ans = 2×1
    0.5080
    0.8614
```

```
newton([-2; -2], F, DF, 0.5e-8, 10)
```

```
ans =
"Done after 7 iterations"
ans = 2×1
```

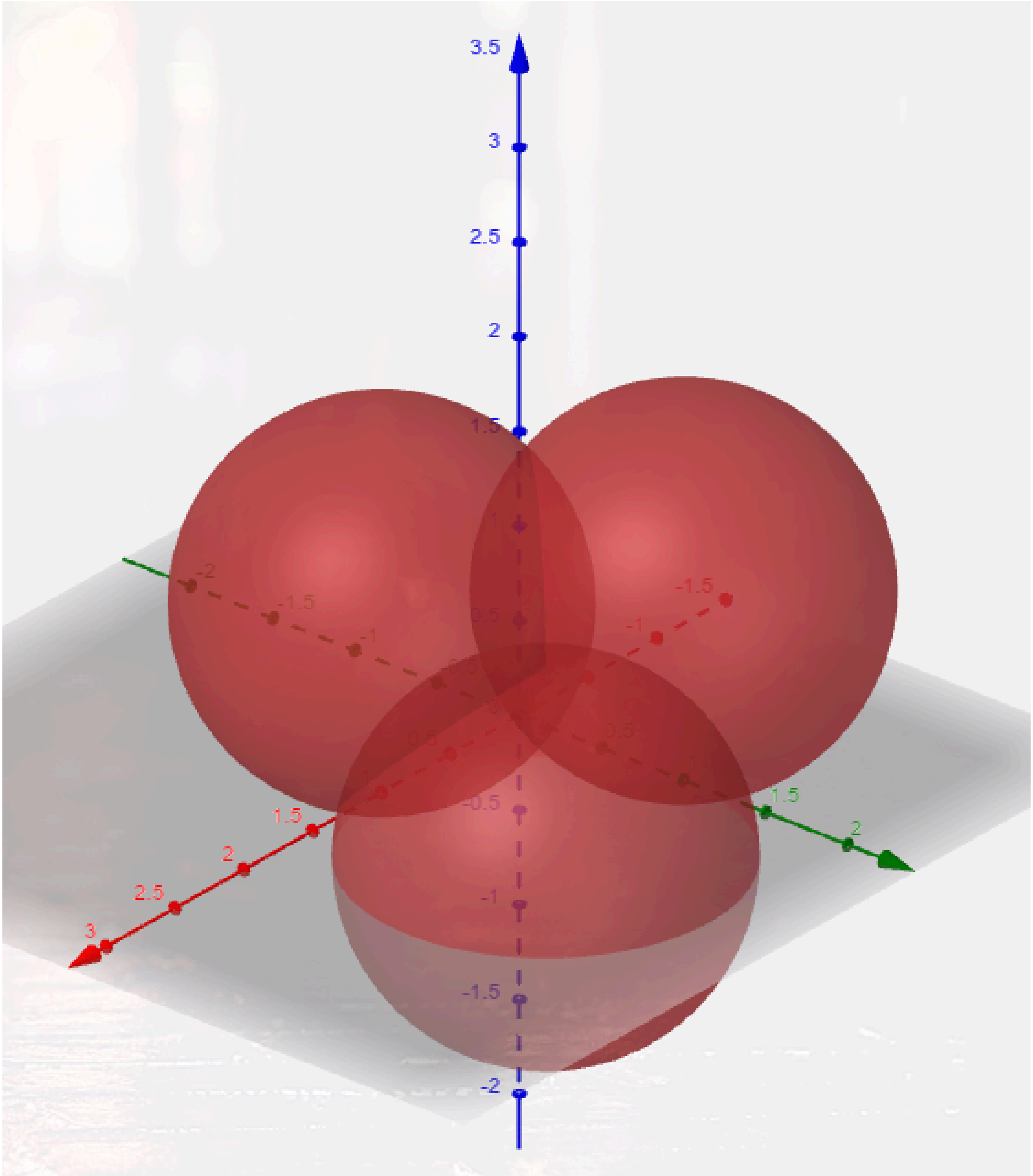
```
-0.5080
-0.8614
```

CP5a

Spheres with radius 1 and centers (1,1,0), (1,0,1), and (0,1,1).

Two points in common.

Checking them out with GeoGebra:


3.5
3
2.5
2
1.5
1
0.5
-0.5
-1
-1.5
-2
-2
-1.5
-1
1.5
2
1.5
2
2.5
3
-1.5
-1

a : Sphere((1, 1, 0), 1)

$\rightarrow \ (x - 1)^2 + (y - 1)^2 + z^2 = 1$

b : Sphere((1, 0, 1), 1)

$\rightarrow \ (x - 1)^2 + y^2 + (z - 1)^2 = 1$

c : Sphere((0, 1, 1), 1)

$\rightarrow \ x^2 + (y - 1)^2 + (z - 1)^2 = 1$

d : IntersectConic(a, b)

$\rightarrow \ X = (1,\ 0.5,\ 0.5) + (0.71\cos(t),\ 0.5\sin(t),\ 0.5\sin(t))$

e : IntersectConic(b, c)

$\rightarrow \ X = (0.5,\ 0.5,\ 1) + (-0.5\cos(t),\ -0.5\cos(t),\ 0.71\sin(t))$

Intersect(d, e)

$\rightarrow \ A = (0.33,\ 0.33,\ 0.33)$

$\rightarrow \ B = (1,\ 1,\ 1)$

Defining the matrix functions:

```
F = @(x) [(x(1) - 1)^2 + (x(2) - 1)^2 +  x(3)^2       - 1;
          (x(1) - 1)^2 +  x(2)^2       + (x(3) - 1)^2 - 1;
           x(1)^2      + (x(2) - 1)^2 + (x(3) - 1)^2 - 1];
DF = @(x) [2*(x(1) - 1) , 2*(x(2) - 1) , 2*x(3)       ;
           2*(x(1) - 1) ,  2*x(2)       , 2*(x(3) - 1);
           2*x(1)       , 2*(x(2) - 1) , 2*(x(3) - 1)];
```

Solving:

```
newton([1; 1; 1], F, DF, 0.5e-8, 10)
```

```
ans =
"Done after 1 iterations"
ans = 3×1
     1
     1
     1
```

```
newton([0; 0; 0], F, DF, 0.5e-8, 10)
```

```
ans =
"Done after 6 iterations"
ans = 3×1
    0.3333
    0.3333
    0.3333
```

Points in common: **(1, 1, 1)** and **(1/3, 1/3, 1/3)** like the book says.

CP5b

Spheres with radius 5 and centers (1,−2,0), (−2,2,−1), and (4,−2,3).

$r^2 = 5^2 = 25$

Defining the matrix functions:

```
F = @(x) [(x(1) - 1)^2 + (x(2) + 2)^2 +  x(3)^2       - 25;
          (x(1) + 2)^2 + (x(2) - 2)^2 + (x(3) + 1)^2 - 25;
          (x(1) - 4)^2 + (x(2) + 2)^2 + (x(3) - 3)^2 - 25];
DF = @(x) [2*(x(1) - 1) , 2*(x(2) + 2) , 2*x(3)       ;
           2*(x(1) + 2) , 2*(x(2) - 2) , 2*(x(3) + 1);
           2*(x(1) - 4) , 2*(x(2) + 2) , 2*(x(3) - 3)];
```

Solving:

```
newton([1; 1; 1], F, DF, 0.5e-8, 10)
```

```
ans =
"Done after 9 iterations"
ans = 3×1
    1.8889
    2.4444
    2.1111
```

```
newton([0; 0; 1], F, DF, 0.5e-8, 10)
```

```
ans =
"Done after 9 iterations"
ans = 3×1
    1.0000
    2.0000
    3.0000
```

```
newton([100; -100; 100], F, DF, 0.5e-8, 100)
```

```
ans =
"Done after 15 iterations"
```

```
ans = 3×1
    1.0000
    2.0000
    3.0000
```

Intersections in **(1.8889, 2.444, 2.1111)** and **(1,2,3)**

```
function x = newton(x, F, DF, tol, iter)
for i=1:iter
    s = linsolve(DF(x), -F(x));
    if abs(s) < tol
        sprintf("Done after %d iterations", i)
        break;
    end
    x = x + s;
end
end
```

Verifying solutions for Broyden methods

5c

```
[-4;61]*[7,3]
```

```
ans = 2×2
   -28   -12
   427   183
```

```
A=[30,-12;427,241].*(1/58)
```

```
A = 2×2
    0.5172   -0.2069
    7.3621    4.1552
```

```
x=[8;4]-inv(A)*[-4;61]
```

```
x = 2×1
    9.0892
  -12.6103
```

6c

```
I = eye(2);
[2,3]*eye(2)
```

```
ans = 1×2
     2     3
```

```
eye(2)*[2;3]
```

```
ans = 2×1
     2
     3
```

```
del=[7;3];
delta=[3;64]
```

```
delta = 2×1
     3
    64
```

```
del'*delta
```

```
ans = 213
```

```
del'*eye(2)*delta
```

```
ans = 213
```

```
(del-delta)*del'
```

```
ans = 2×2
    28    12
  -427  -183
```

```
(del-I*delta)*del'*I
```

```
ans = 2×2
    28    12
  -427  -183
```

```
B = I + ((del-delta)*del') / (del'*delta)
```

```
B = 2×2
    1.1315    0.0563
   -2.0047    0.1408
```

```
[8;4] - B*[-4;61]
```

```
ans = 2×1
    9.0892
  -12.6103
```

4.1 - Least Squares

8b

```
x=[1;3;4;6]
```

```
x = 4×1
     1
     3
     4
     6
```

```
y=[2;2;1;3]
```

```
y = 4×1
     2
     2
     1
     3
```

```
A=[1,1;1,3;1,4;1,6]
```

```
A = 4×2
     1     1
     1     3
     1     4
     1     6
```

```
b=[2;2;1;3]
```

```
b = 4×1
     2
     2
     1
     3
```

```
c=(A'*A)\(A'*b)
```

```
c = 2×1
    1.4615
    0.1538
```

```
f=@(x) c(1) + c(2)*x;
scatter(x, y)
hold on;
plot(0:0.1:10,(f(0:0.1:10)))
```

9b

```
A=[x.^0 x.^1 x.^2]
```

```
A = 4×3
     1     1     1
     1     3     9
     1     4    16
     1     6    36
```

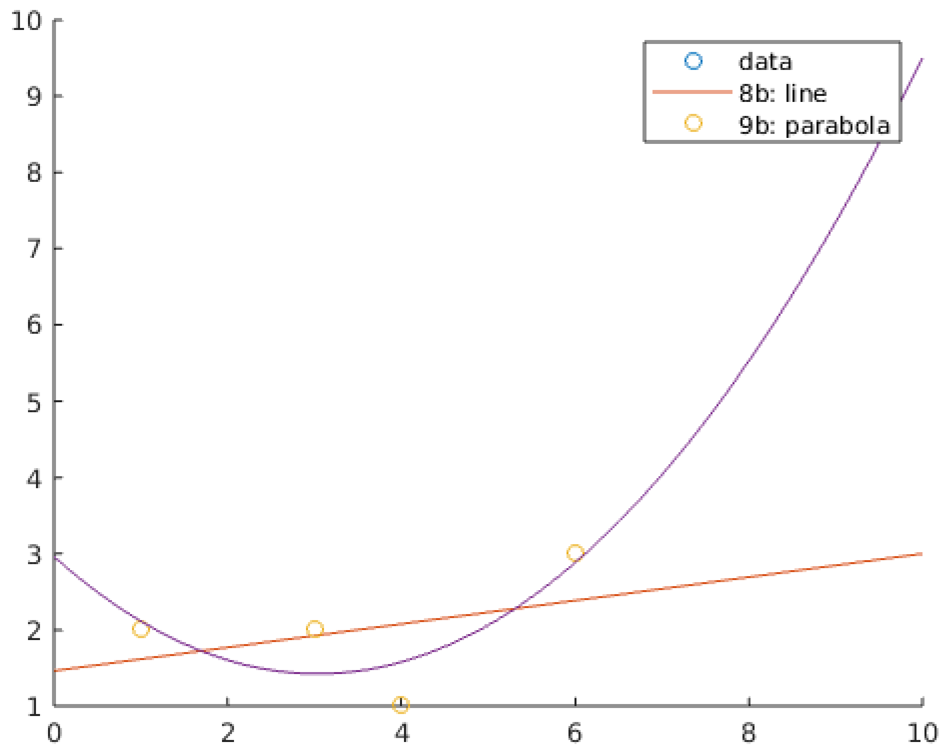
```
c=(A'*A)\(A'*b)
```

```
c = 3×1
    2.9615
   -1.0128
    0.1667
```

```
r=b-A*c
```

```
r = 4×1
   -0.1154
    0.5769
   -0.5769
    0.1154
```

```
f=@(x) c(1) + c(2)*x + c(3)*x.^2;
scatter(x, y)
plot(0:0.1:10,(f(0:0.1:10)))
legend('data', '8b: line', '9b: parabola')
hold off;
```



CP3

Data set:

```
years = [1960 1970 1990 2000]'
```

```
years = 4×1
        1960
        1970
        1990
        2000
```

```
populations = [3039585530 3707475887 5281653820 6079603571]'
```

```
populations = 4×1
10^9 ×
    3.0396
    3.7075
    5.2817
    6.0796
```

```
x = (years - 1960) / 10
```

```
x = 4×1
     0
     1
     3
     4
```

Using the method of least squares:

```
[c_line, rmse_line] = least_squares(x, populations, 2)
```

```
c_line = 2×1
10^9 ×
    2.9962
    0.7654
rmse_line = 3.6751e+07
```

```
[c_parabola, rmse_parabola] = least_squares(x, populations, 3)
```

```
c_parabola = 3×1
10^9 ×
    3.0288
    0.6787
    0.0217
rmse_parabola = 1.7130e+07
```

```
line = @(x) c_line(1) + c_line(2)*x
```

```
line = function_handle with value:
    @(x)c_line(1)+c_line(2)*x
```

```
parabola = @(x) c_parabola(1) + c_parabola(2)*x + c_parabola(3)*x.^3
```

```
parabola = function_handle with value:
    @(x)c_parabola(1)+c_parabola(2)*x+c_parabola(3)*x.^3
```

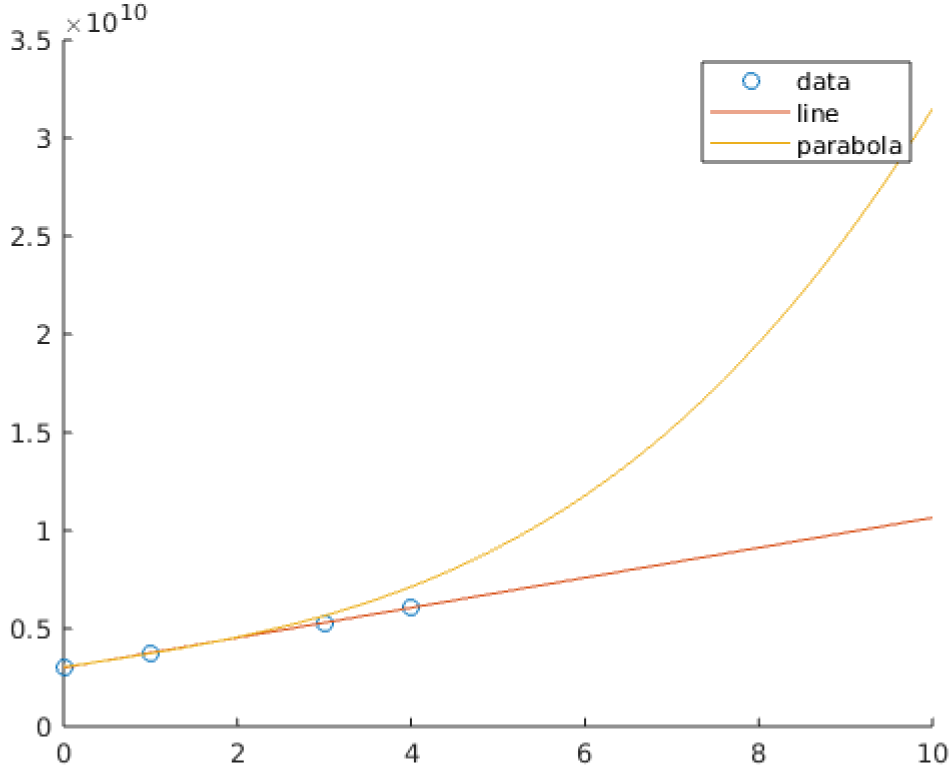
Plotting:

```
scatter(x, populations)
hold on;
axis = 0:0.1:10
```

```
axis = 1×101
         0    0.1000    0.2000    0.3000    0.4000    0.5000    0.6000    0.7000 ⋯
```

```
plot(axis, line(axis))
plot(axis, parabola(axis))
legend('data', 'line', 'parabola')
```

```
hold off;
```



Finding the root mean squared error:

```
sprintf('RMSE of line: %e', rmse_line)
```

```
ans =
'RMSE of line: 3.675109e+07'
```

```
sprintf('RMSE of parabola: %e', rmse_parabola)
```

```
ans =
'RMSE of parabola: 1.712971e+07'
```

The parabola seems like an improvement (but the graph shows the points in a fairly straight line)

```
function [c, RMSE] = least_squares(x, y, n)
[m, ~] = size(x);
% dynamic A for any n
A = x.^(0:n-1);
% the usual A'Ac=A'b
c = (A'*A)\(A'*y);
% calculate RMSE too
RMSE = sqrt(sum((y - A*c).^2) / m);
end

% could be used for plotting
function y = plottable(x, c, n)
[m, ~] = size(x);
```

```
y = zeros(m, 1);
for j=1:n
    y = y + c(j).*x.^(j-1);
end
end
```

4.2 Linearization

3b

Testing the malformed version:

```
exponentionalize([0 1 1 2]', [10 5 2 1]')
```

```
ans = 2×1
   10.0000
   -1.1513
```

Testing the corrected version:

```
c = exponentionalize([0 1 1 2]', [1 1 2 4]') % -0.25, 1->2
```

```
c = 2×1
    0.8409
    0.6931
```

```
% against what I got
format long;
log(2) - c(2)
```

```
ans =
     0
```

```
2^-0.25 - c(1)
```

```
ans =
     1.110223024625157e-16
```

```
format short;
```

CP3

Gode, gamle verdens befolking.

Data:

```
years = [1960 1970 1990 2000]'
```

```
years = 4×1
        1960
        1970
        1990
        2000
```

```
populations = [3039585530 3707475887 5281653820 6079603571]'
```

```
populations = 4×1
10^9 ×
    3.0396
    3.7075
    5.2817
    6.0796
```

Model:

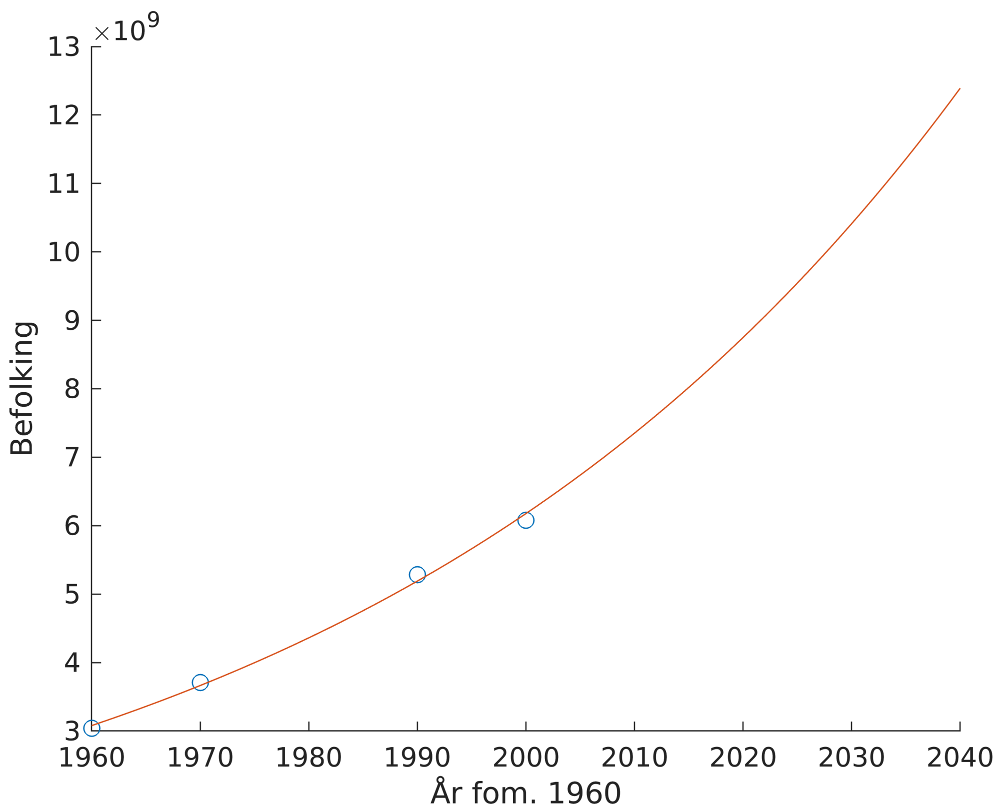
```
format long;
c = exponentionalize(years, populations)
```

```
c = 2×1
    0.000004726389209
    0.017403246298083
```

```
format short;
y = @(t) c(1).*exp(c(2).*t);
```

Plot:

```
scatter(years, populations)
hold on;
range = 1960:2040;
plot(range, y(range))
xlabel('År fom. 1960')
ylabel('Befolking')
hold off;
```



Estimate 1980:

```
sprintf('Population in 1980: %e', y(1980))
```

```
ans =
'Population in 1980: 4.361486e+09'
```

Find error of estimate:

(compared against 4,458,003,514 from worldometers.info)

```
abs(4458003514 - y(1980))
```

```
ans = 9.6518e+07
```

```
function c = exponentionalize(x, y)
y = log(y);
[n, ~] = size(x);
% let k = ln(c_1)
A = [ones(n, 1) x];
% get [k c_2]'
c = (A'*A)\(A'*y);
% then get c_1 = e^k
c(1) = exp(c(1));
end
```

4.5 Gauss-Newton!

CP1a

With 8 decimals of accuracy.

```
points = [
    0,1;
    1,1;
    0,-1;
]
```

```
points = 3×2
     0     1
     1     1
     0    -1
```

```
radii = ones(3,1);
format long;
x = gauss_newton_circles([0;0], points, radii, 0.5e-8, 100)
```

```
ans =
'Done after 25 iterations'
x = 2×1
   0.410623199054967
   0.055501396966844
```

```
format short;
```

```
function x = gauss_newton_circles(x, points, radii, tol, k)
% (matrix) functions used in the method
Si = @(x, i) sqrt((x(1) - points(i,1))^2 + (x(2) - points(i,2))^2);
r = @(x) [
    Si(x,1) - radii(1);
    Si(x,2) - radii(2);
    Si(x,3) - radii(3);
];
Dr = @(x) [
    (x(1) - points(1,1)) / Si(x,1), (x(2) - points(1,2)) / Si(x,1);
    (x(1) - points(2,1)) / Si(x,2), (x(2) - points(2,2)) / Si(x,2);
    (x(1) - points(3,1)) / Si(x,3), (x(2) - points(3,2)) / Si(x,3);
];

% Gauss-Newton algorithm
for wahtever=1:k
    A = Dr(x);
    v = (A'*A) \ (-A'*r(x));
    x = x + v;
    % test difference v against tolerance
    % no need to keep an old x lying around
    if abs(v) < tol
        sprintf('Done after %d iterations', wahtever)
        break
    end
```

```
end
end
```