Formulae for Oort Lore

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Contents

Abstract

Some derivation for formulas that can be useful to calculate stuff in space.

1 Conventions and Prerequisites

1.1 Notation

1.2 Variable Names

- s: Distance
- v_0: Velocity at the beginning
- v_{end}: Velocity at the end
- a: Acceleration
- a_{eff}: Effective acceleration (any gravitational has been removed: $a_{eff} = a - g$)

When we need them in vector form, they are written as $\vec{s}$ for example.

1.3 Units

We are using either km or AU instead of m.

- AU: Astronomical Unit. $1AU \approx 149 \times 10^6\,\mathrm{km}$.

1.4 Constants

- $G \approx 6.674 \times 10^{-18}\,\mathrm{N\,km^2\,kg^{-2}}$

1.5 Setup of the Solar System

Celestial Body	Distance [au]	Mass [kg]
Sun	0	
Mercury	0.4	
Venus	0.72	
Earth	1	
Mars	1.5	
Asteroid Belt	2.2-2.3	
Jupiter	5.2	
Saturn	9.5	
Uranus	19	
Neptune	30.1	
Pluto	39	
Kuiper Belt	30-50	
Scattered Disc	50-1000	
Oort Cloud	(2000-5000)-(10 000-100 000)	

2 Travel Time Without Relativity

2.1 Constant Speed

From
$$s = vt \tag{2.1}$$
follows
$$t = s/v. \tag{2.2}$$

2.2 Under Acceleration

From
$$s = 0.5at^2 \tag{2.3}$$
follows
$$t = \sqrt{2s/a} \tag{2.4}$$
and a final velocity of
$$v = at. \tag{2.5}$$

2.2.1 With Start Velocity

From
$$s = v_0 t + 0.5at^2 \tag{2.6}$$
follows
$$t = \frac{-v_0 + \sqrt{v_0^2 + 2sa}}{a} \tag{2.7}$$
and a final velocity of
$$v_1 = v_0 + at. \tag{2.8}$$
Note that the last formula can with the help of (2.7) also be written as
$$v_1 = v_0 + (-v_0 + \sqrt{v_0^2 + 2sa}) = \sqrt{v_0^2 + 2sa}, \tag{2.9}$$
from which we can get the final velocity without calculating the time first.

3 Agnus Journey Without Relativity

In our example we want to calculate the time and end velocity for the journey sections of the Agnus. We start escaping earth with four and two g at first, then take a trip to the asteroid belt with an acceleration and deceleration of one g, switching gears shortly after mars. We assume that Mars is currently located at a distance of $0.5\,\mathrm{au} = 74.8 \times 10^6\,\mathrm{km}$.

Phase one: First half of atmosphere. we want to escape earth with an effective acceleration of $a_{eff1} = a - g = 4g = 0.04\,\mathrm{km\,s^{-2}}$ until we reach a height of $s_1 = 5000\,\mathrm{km}$, which is half of the exosphere. We just assume the computer keeps the effective acceleration at a constant $4g$.

Then we reach the first destination after
$$t_1 = \sqrt{2s_1/a_{eff1}} = \sqrt{\frac{10\,000\,\mathrm{km}}{0.04\,\mathrm{km\,s^{-2}}}} = 500\,\mathrm{s} = 8.3\,\mathrm{min} \tag{3.1}$$
with a velocity of
$$v_1 = a_{eff1} t_1 = 0.04\,\mathrm{km\,s^{-2}} \cdot 500\,\mathrm{s} = 20\,\mathrm{km\,s^{-1}}, \tag{3.2}$$
which will be our start speed for the next phase.

Phase two: Second half of atmosphere. Now we want to accelerate with a burn of $a_{eff2} = 2g = 0.02\,\mathrm{km\,s^{-2}}$ until we leave the exosphere. For this, we need to pass another $s_2 = 5000\,\mathrm{km}$. Mind that we now have a non-zero start velocity of $v_1 = 20\,\mathrm{km\,s^{-1}}$.

Then we reach the second destination after

$$t_2 = \frac{-v_1 + \sqrt{v_1^2 + 2s_2 a_{eff2}}}{a_{eff2}} = \frac{-20\,\mathrm{km\,s^{-1}} + \sqrt{400\,\mathrm{km^2\,s^{-2}} + 200\,\mathrm{km^2\,s^{-2}}}}{0.02\,\mathrm{km\,s^{-2}}} \approx 224.7\,\mathrm{s} \approx 3.7\,\mathrm{min} \quad (3.3)$$

with a final velocity of

$$v_2 = v_1 + a_{eff_2} t_2 = 20\,\mathrm{km\,s^{-1}} + 0.02\,\mathrm{km\,s^{-2}} \cdot 224.7\,\mathrm{s} = 24.5\,\mathrm{km\,s^{-1}}. \quad (3.4)$$

Phase three: Mars. Now we want to go to mars and have $s_3 = 75 \times 10^6\,\mathrm{km}, v_2 = 24.5\,\mathrm{km\,s^{-1}}$ and $a_{eff3} = 0.01\,\mathrm{km\,s^{-2}}$.

Let's calculate the final velocity first, this time. It is

$$v_3 = \sqrt{v_2^2 + 2s_3 a_{eff3}} = \sqrt{600\,\mathrm{km^2\,s^{-2}} + 1.5 \times 10^6\,\mathrm{km^2\,s^{-2}}} = 1225\,\mathrm{km\,s^{-1}} \quad (3.5)$$

which is about 0.3% of the speed of light and we expect not much relativistic effects here. With this, we can also calculate the travel time and get

$$t_3 = \frac{-v_2 + v_3}{a_{eff3}} = \frac{-24.5\,\mathrm{km\,s^{-1}} + 1224\,\mathrm{km\,s^{-1}}}{0.01\,\mathrm{km\,s^{-2}}} = 120\,049\,\mathrm{s} = 33.3\,\mathrm{h}. \quad (3.6)$$

So a little more than thirty hours to get to mars this way. Nice!

Phase four: To the asteroid belt. We would like to have a velocity of $v_2 = 24.5\,\mathrm{km\,s^{-1}}$ again when we reach the asteroid belt in order to be slow down enough to dodge asteroids. To achieve this, we will further accelerate with $a_4 = 1g$ for $s_4 = 0.1\,\mathrm{au}$ and then decelerate with $a_5 = -1g$ for $s_5 = 0.6\,\mathrm{au}$.

$$v_4 = \sqrt{v_3^2 + 2s_4 a_4} = \sqrt{1\,500\,625\,\mathrm{km^2\,s^{-2}} + 300\,000\,\mathrm{km^2\,s^{-2}}} = 1342\,\mathrm{km\,s^{-1}} \quad (3.7)$$

$$t_4 = \frac{-v_3 + v_4}{a_4} = \frac{-1225\,\mathrm{km\,s^{-1}} + 1342\,\mathrm{km\,s^{-1}}}{0.01\,\mathrm{km\,s^{-2}}} = 11\,687\,\mathrm{s} = 3.2\,\mathrm{h} \quad (3.8)$$

$$v_5 = \sqrt{v_4^2 + 2s_5 a_5} = \sqrt{1\,800\,964\,\mathrm{km^2\,s^{-2}} - 1\,800\,000\,\mathrm{km^2\,s^{-2}}} = 31\,\mathrm{km\,s^{-1}} \quad (3.9)$$

$$t_5 = \frac{-v_4 + v_5}{a_5} = \frac{-1342\,\mathrm{km\,s^{-1}} + 31\,\mathrm{km\,s^{-1}}}{-0.01\,\mathrm{km\,s^{-2}}} = 131\,100\,\mathrm{s} = 36.4\,\mathrm{h} \quad (3.10)$$

It can be assumed that the difference in the final velocity origins in rounding errors.

Phase five: Through the asteroid belt. Under the assumption that we did reach the final velocity of $v_2 = 24.5\,\mathrm{km\,s^{-1}}$ - which we will maintain - and that the asteroid belt spans $s_6 = 0.1\,\mathrm{au}$, the passage takes

$$t_6 = s_6/v_2 = \frac{15 \times 10^6\,\mathrm{km}}{24.5\,\mathrm{km\,s^{-1}}} = 612 \times 10^3\,\mathrm{s} = 170\,\mathrm{h}. \quad (3.11)$$

That takes way to long, so let's add an acceleration of 1 g to that:

$$v_6 = \sqrt{v_2^2 + 2s_6 a_4} = \sqrt{600\,\mathrm{km^2\,s^{-2}} + 300\,000\,\mathrm{km^2\,s^{-2}}} = 548\,\mathrm{km\,s^{-1}} \quad (3.12)$$

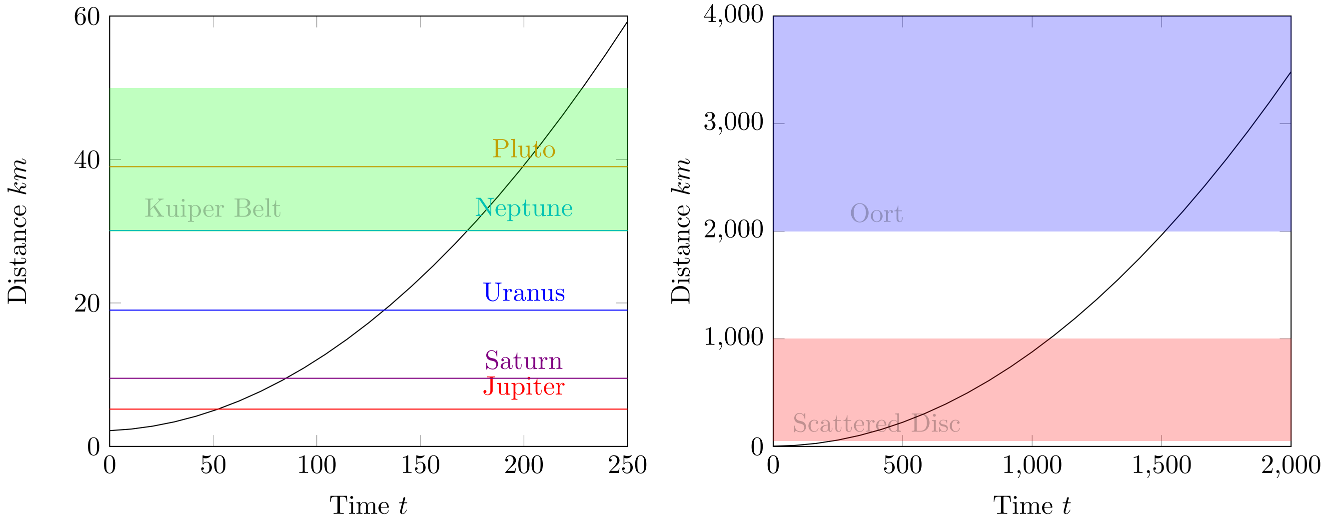
$$t_6 = \frac{-v_2 + v_6}{a_4} = \frac{-24.5\,\mathrm{km\,s^{-1}} + 548\,\mathrm{km\,s^{-1}}}{0.01\,\mathrm{km\,s^{-2}}} = 52\,350\,\mathrm{s} = 14.5\,\mathrm{h} \quad (3.13)$$

Now that looks better.

Phase six: Out to oort. Constant acceleration of $1g$ is kept. We want to have a nice chart, in which we can see when we arrive at which planet's orbit. This is delivered by the function

$$s(t) = s_0 + v_6 t + \frac{1}{2} g t^2 = 330 \times 10^6\,\mathrm{km} + 548\,\mathrm{km\,s^{-1}}\,t + 0.01\,\mathrm{km\,s^{-2}}\,t^2 = 2.2\,\mathrm{au} + 13.152 \times 10^{-3}\,\mathrm{au\,h^{-1}}\,t + 864 \times 10^{-3}\,\mathrm{au\,h^{-2}}\,t^2, \tag{3.14}$$

where we have switched the units from km and s to au and h.



4 Travel Time With Special Relativity