Tema 1

1) $$f(x)=\begin{cases} x^2-2x & \text{si } x<0 \\ \dfrac{1}{x} & \text{si } 0\leq x<1 \\ \ln x & \text{si } x\geq 1 \end{cases}$$

Tramo 1

x	x^2-2x
∘ 0	$0^2-2.0=0$
-2	$(-2)^2-2.(-2)=8$

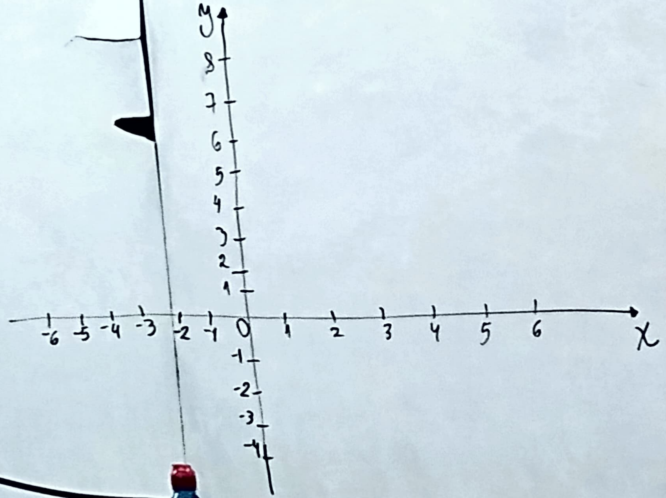
Tramo 2

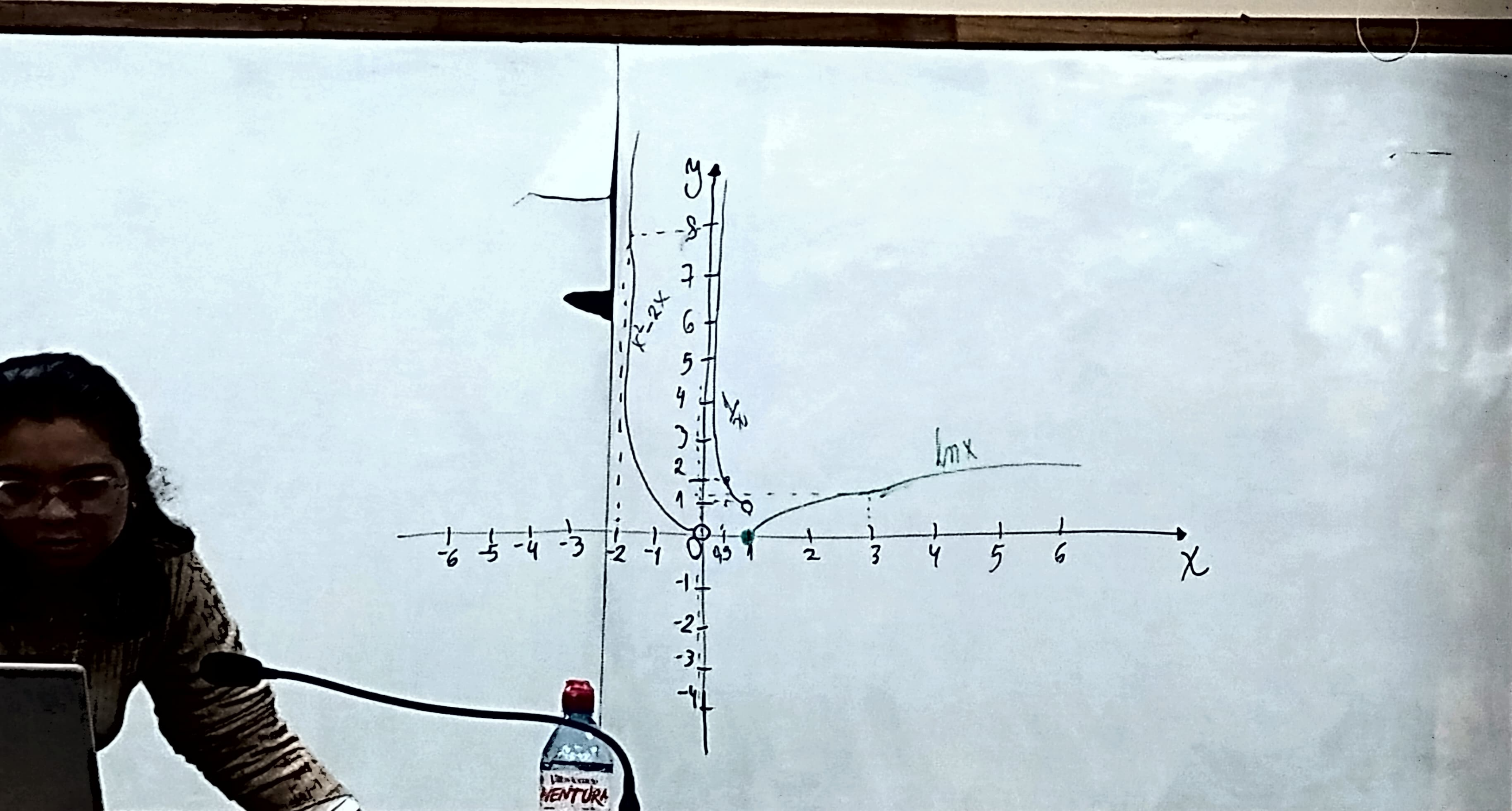
x	$\frac{1}{x}$
0,5	$\frac{1}{0,5}=2$
∘ 1	$\frac{1}{1}=1$

A.V. $\Rightarrow x=0$

Tramo 3

x	$\ln x$
• 1	$\ln(1)=0$
3	$\ln(3)=1,09$





y
x
ln x
x^2-2x
1/x

Tema 1

① $f(x) = \begin{cases} x^2-2x & \text{si } x<0 \\ \frac{1}{x} & \text{si } 0<x<1 \\ \ln x & \text{si } x\geq 1 \end{cases}$

Estudio la continuidad en $x=0$

1- $\nexists\ f(0)$

$\therefore f$ no es continua en $x=0$

$-\frac{1}{6}\quad -\frac{1}{5}\quad -\frac{1}{4}\quad -\frac{1}{3}$

Estudio de la continuidad en $x=1$

1- $\exists\ f(1)=0$

2- ¿$\exists \lim\limits_{x\to 1} f(x)$?

$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \ln x = \ln(1) = 0$

$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} \frac{1}{x} = \frac{1}{1} = 1$

Como $\lim\limits_{x\to 1^+} f(x) \neq \lim\limits_{x\to 1^-} f(x)$

$\therefore \nexists \lim\limits_{x\to 1} f(x)$

$\therefore f$ no es continua en $x=1$

Tema 1

① $f(x) = \begin{cases} x^2 - 2x & \text{si } x < 0 \\ \frac{1}{x} & \text{si } 0 \leq x < 1 \\ \ln x & \text{si } x \geq 1 \end{cases}$

c) $f'(-1)$ por definición

Derivada por definición Usamos tramo ①

Reemplazamos por $x_0 = -1$

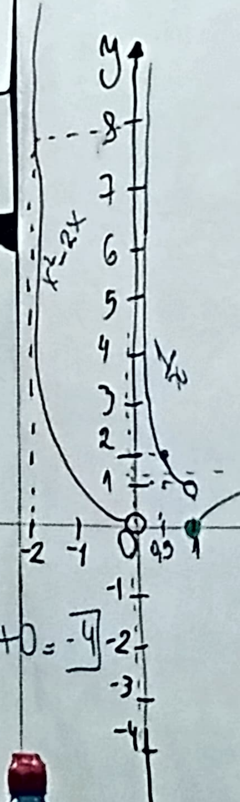
$$f(x_0 + \Delta x) = f(-1+\Delta x) = (-1+\Delta x)^2 - 2(-1+\Delta x)$$
$$= \boxed{1 - 2\Delta x + \Delta x^2 + 2 - 2\Delta x}$$

$$f(x_0) = f(-1) = (-1)^2 - 2(-1) = 1 + 2 = \boxed{3}$$

$$f'(-1) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{3} - 4\Delta x + \Delta x^2 - \cancel{3}}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{-4\Delta x + \Delta x^2}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x}(-4+\Delta x)}{\cancel{\Delta x}} = \lim_{\Delta x \to 0} -4 + \Delta x = -4 + 0 = \boxed{-4}$$

$$\therefore \boxed{f'(-1) = -4}$$



Tema 1

Límites

1)

a) $\lim\limits_{x\to\infty} \sqrt{3x^2+x} - \sqrt{3x^2+2} = \sqrt{3\infty^2+\infty} - \sqrt{3\infty^2+2} = \infty - \infty$

Resolvemos la indeterminación

$$\lim_{x\to\infty} \sqrt{3x^2+x} - \sqrt{3x^2+2} \cdot \frac{\left(\sqrt{3x^2+x} + \sqrt{3x^2+2}\right)}{\left(\sqrt{3x^2+x} + \sqrt{3x^2+2}\right)} = \lim_{x\to\infty} \frac{\left(\sqrt{3x^2+x}\right)^2 - \left(\sqrt{3x^2+2}\right)^2}{\left(\sqrt{3x^2+x} + \sqrt{3x^2+2}\right)} = \lim_{x\to\infty} \frac{\cancel{3x^2}+x-\cancel{3x^2}-2}{\sqrt{3x^2+x} + \sqrt{3x^2+2}} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{\frac{\cancel{x}}{\cancel{x}} - \frac{2}{x}}{\frac{\sqrt{3x^2+x}}{x} + \frac{\sqrt{3x^2+2}}{x}}$$

$$\lim_{x\to} \frac{1}{\quad}$$

Resolvemos la indeterminación

$$\lim_{x\to\infty} \sqrt{3x^2+x}-\sqrt{3x^2+2}\cdot\frac{(\sqrt{3x^2+x}+\sqrt{3x^2+2})}{(\sqrt{3x^2+x}+\sqrt{3x^2+2})} = \lim_{x\to\infty}\frac{(\sqrt{3x^2+x})^2-(\sqrt{3x^2+2})^2}{(\sqrt{3x^2+x}+\sqrt{3x^2+2})} = \lim_{x\to\infty}$$

$$= \lim_{x\to\infty}\frac{1-\frac{2}{x}}{\sqrt{\frac{3x^2}{x^2}+\frac{x}{x^2}}+\sqrt{\frac{3x^2}{x^2}+\frac{2}{x^2}}} = \lim_{x\to\infty}\frac{1-\underbrace{\frac{2}{x}}_{\to 0}}{\sqrt{3+\underbrace{\frac{1}{x}}_{=0}}+\sqrt{3+\underbrace{\frac{2}{x}}_{\to 0}}} = \frac{1}{\sqrt{3}+\sqrt{3}} = \boxed{\frac{1}{2\sqrt{3}}}$$

Tema 1

3) a) Determinar a y b

$f(x) = x^3 + ax^2 + bx - 7$ sabiendo que presenta un mínimo relativo en $x = 2$ y un máximo relativo $x = 4$

$f'(x) = (x^3 + ax^2 + bx - 7)'$

$f'(x) = 3x^2 + 2ax + b$

Evalúo $f'(x)$ en $x = 2$ y $x = 4$ porque son extremos deben cumplir

$f'(2) = 0 \Rightarrow 3.2^2 + 2.a.2 + b = 0$

$12 + 4a + b = 0$

$f'(4) = 0 \Rightarrow 3.4^2 + 2.a.4 + b = 0$

$48 + 8a + b = 0$

$$\begin{cases} 12 + 4a + b = 0 \\ 48 + 8a + b = 0 \end{cases}$$

1era cond

$12 + 4a + b = 0$

$\boxed{b = -4a - 12}$

$b = -4.(-9) - 12$

$\boxed{b = 24}$

2da cond

$48 + 8a + (-4a - 12) = 0$

$48 + 8a - 4a - 12 = 0$

$4a + 36 = 0$

$4a = -36$

$a = -36 : 4$

$\boxed{a = -9}$

$\therefore f(x) = x^3 - 9x^2 + 24x - 7$

Tema 1

Determinar los Puntos de inflexión

3) b) $f(x) = x^4 - 6x^2 - 3$

$f'(x) = (x^4 - 6x^2 - 3)'$

$f'(x) = 4x^3 - 12x$

$f''(x) = (4x^3 - 12x)'$

$f''(x) = 12x^2 - 12$

$f''(x) = 0$

$12x^2 - 12 = 0$

$12x^2 = 12$

$x^2 = 12 : 12$

$x = \pm\sqrt{1}$

$f'''(x) = (12x^2 - 12)'$

$f'''(x) = 24x$

$f'''(-1) = 24(-1) = -24 \neq 0$ ✓

$f'''(1) = 24 \cdot 1 = 24 \neq 0$ ✓

Los puntos de inflexión $(1, f(1))$ y $(-1, f(-1))$

$f(1) = 1^4 - 6 \cdot 1^2 - 3 = -8$; $f(-1) = (-1)^4 - 6 \cdot (-1)^2 - 3 = -8$

Tema 1

4) $f(x) = -\frac{1}{2}x^2 + 5x - 4$ determine la ecuación de la recta normal en $\overset{x_0}{x} = 2$

$f'(x) = \left(-\frac{1}{2}x^2 + 5x - 4\right)'$

$f'(x) = -\frac{1}{2} 2x + 5$

$\boxed{f'(x) = -x + 5}$

$y_0 = f(2) = -\frac{1}{2} \cdot 2^2 + 5 \cdot 2 - 4$

$y_0 = f(2) = -2 + 10 - 4$

$y_0 = f(2) = 4$ El punto es $\therefore (2, 4)$

$f'(2) = -2 + 5 = 3$

Pendiente de la recta normal

$m_N = \frac{-1}{f'(x_0)} \Rightarrow -\frac{1}{3}$

Para obtener la ecuación:

$y - y_0 = \frac{-1}{f'(x_0)} \cdot (x - x_0)$

$y - 4 = -\frac{1}{3} \cdot (x - 2)$

$y - 4 = -\frac{1}{3}x + \frac{2}{3}$

$y = -\frac{1}{3}x + \frac{2}{3} + 4$

$\boxed{y = -\frac{1}{3}x + \frac{14}{3}}$ Ecuación de la recta normal

5) a) Determine la ecuación de la curva que pasa por el punto $\left(\overset{x}{1}, \overset{f(1)}{-2}\right)$ y si la pendiente de la recta tangente a la misma está dada por $f'(x) = x^2 - x - 2$

1ero) $\int f'(x)\,dx = f(x)$

Entonces: $\int (x^2 - x - 2)\,dx = \int x^2 dx - \int x\,dx - 2\int dx$

$$f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x + C$$

$$f(1) = \frac{1^3}{3} - \frac{1^2}{2} - 2\cdot 1 + C$$

$$-2 = \frac{1}{3} - \frac{1}{2} - 2 + C$$

$$-2 = \frac{-1}{6} - 2 + C$$

$$-2 = \frac{-13}{6} + C$$

$$-2 + \frac{13}{6} = C$$

$$\boxed{\frac{1}{6} = C}$$

$$\therefore \boxed{f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{1}{6}}$$

Tema 1

5- b) Resolver: $\int \left(-3e^x + \frac{x^2\sqrt{x}}{2\sqrt[3]{x}} - \frac{2}{x^4}\right) dx$

$$\int -3e^x dx + \int \frac{x^2\sqrt{x}}{2\sqrt[3]{x}} dx - \int \frac{2}{x^4} dx$$

Por (1) $$-3\int e^x dx + \frac{1}{2}\int \frac{x^2\sqrt{x}}{\sqrt[3]{x}} dx - 2\int \frac{1}{x^4} dx$$

$$-3\int e^x dx + \frac{1}{2}\int x^{\frac{13}{6}} dx - 2\int x^{-4} dx$$

$$-3e^x + \frac{1}{2}\cdot\frac{x^{19/6}}{\frac{19}{6}} - 2\frac{x^{-3}}{-3} + C$$

$$-3e^x + \frac{1}{2}\cdot\frac{x^{19/6}}{\frac{19}{6}} + \frac{2}{3}x^{-3} + C$$

(1) $\frac{x^2\cdot\sqrt{x}}{\sqrt[3]{x}}$

$\frac{x^2\cdot x^{1/2}}{x^{1/3}} = \frac{x^{5/2}}{x^{1/3}}$

$= x^{\frac{5}{2}-\frac{1}{3}}$

$= \boxed{x^{\frac{13}{6}}}$