1) $A = \{x \in \mathbb{R} \,/ (x+2)\cdot(x-1) < 0\}$

Intervalos, cotas, extremos y gráfico.

2) $\begin{cases} x, & x \leq 0 \\ |x-2| - 2, & 0 < x < 4 \\ 1, & x \geq 4 \end{cases}$

Gráfico, dominio, imagen y continuidad en $x = 0$ y $x = 4$

3) $\lim_{x \to -1} \left(\frac{1}{x+1} - \frac{4}{x^3+x+2} \right)$ $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

4) Determinar analíticamente el valor de a y b de $f(x) = x^3 + ax^2 + bx - 7$ para que sea un mínimo relativo en $x = 4$ y máximo relativo en $x = 2$.

Graficar si es posible.

5) $\int \frac{\ln x^2}{x^{-3}} dx$ $\int_{-1}^{1} \int_{0}^{1} \left(x^2 \cdot y - y \right) dx dy$

6) Calcular el área delimitada entre las funciones $y = 2x + 3$, $y = x + 2$ y la recta que pasa por los puntos (3, 5) y (2, 7)

7) Determinar el valor de verdad de siguiente proposición $3 \cdot \left(x \cdot z_x + y \cdot z_y \right) = 6$ siendo $z = \ln\left(x^2 + xy + y^2 \right)$

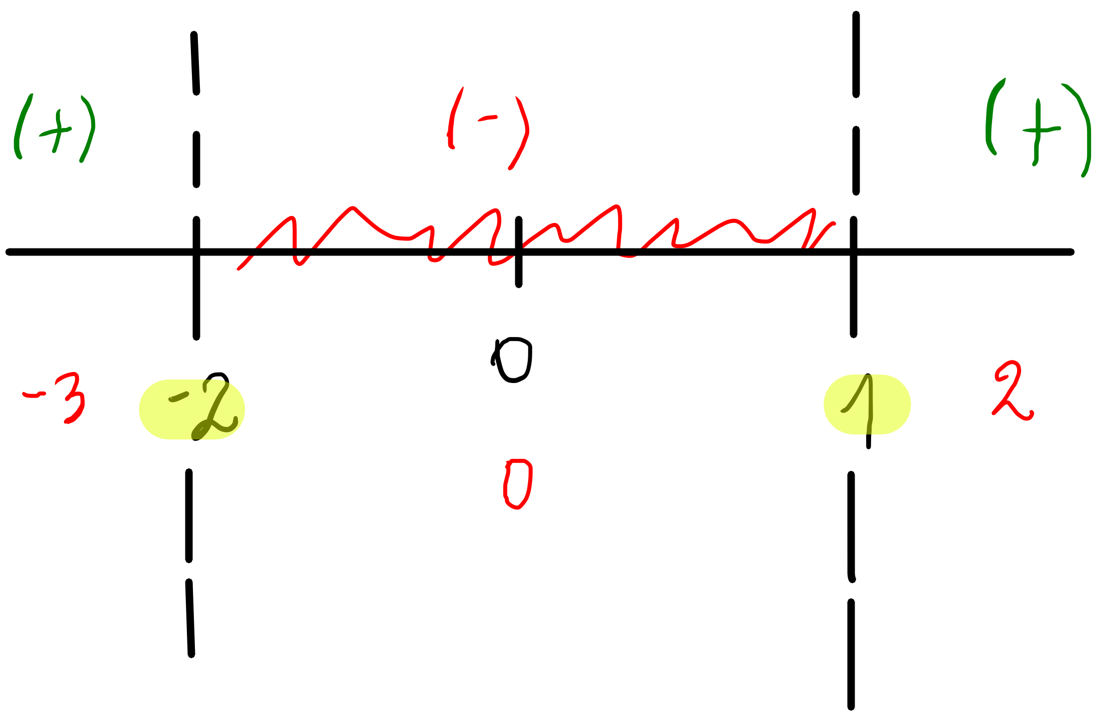
8) Hallar los iterados o sucesivos de $f(x,y) = \frac{3x^2 + y^2}{x^4 + 2x^2 \cdot y + y^2}$ en P(-1, 1). ¿Existe límite doble? Justificar

9) Hallar extremos y puntos de ensilladura si existen de $f(x,y) = x^2 + y^3 - 3xy$

1) $A = \{x \in \mathbb{R} \,/\, (x+2)(x-1) < 0\}$

$x_1 = -2$ $\quad x_2 = 1$

$(x+2)(x-1)$



$\Rightarrow \therefore -2 < x < 1$

$C.S = \{k \in \mathbb{R} \mid k \geq 1\}$

$C.I = \{j \in \mathbb{R} \mid j \leq -3\}$

Amplitud: $1-(-2) = 3$

Supremo: 1 $\quad$ Maximo: -

Infimo: -3 $\quad$ Minimo: -

Centro: $(a+b)/2$

$(-2+1)/2 = -\frac{1}{2} = -0{,}5$

Radio: Amplitud / 2

$3/2 = 1{,}5$

$E(-0{,}5;\ 1{,}5)$

2) Continuidad

$x = 0$

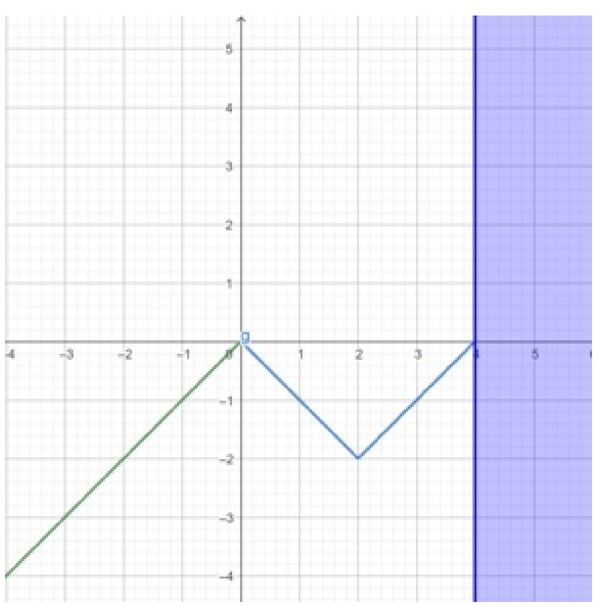
$\exists f(0) = 0$; $\exists \lim_{x \to 0^-} x = 0$; $\lim_{x \to 0^+} |x-2|-2 = 0$; $\exists \lim_{x \to 0} f(x)$

$\exists f(0) = \lim_{x \to 0} f(x)$

$\therefore$ $x=0$ es continuo en $f(x)$

$x = 4$

$\exists f(4) = 1$; $\exists \lim_{x \to 4^-} |x-2|-2 = 0$; $\lim_{x \to 4^+} 1 = 1$; $\nexists \lim_{x \to 4} f(x)$



$\therefore$ $x=4$ es discontinua esencial en $f(x)$

3) $\lim\limits_{x\to -1}\left(\frac{1}{x+1}-\frac{4}{x^3+x+2}\right) \Rightarrow \lim\limits_{x\to -1}\left(\frac{1}{x+1}-\frac{4}{(x+1)(x^2-x+2)}\right)$

$$\begin{array}{c|cccc} & 1 & 0 & 1 & 2 \\ -1 & & -1 & 1 & -2 \\ \hline & 1 & -1 & 2 & \boxed{0} \end{array}$$

$\Rightarrow \lim\limits_{x\to -1}\left(\frac{x^2-x+2}{(x+1)(x^2-x+2)}-\frac{4}{(x+1)(x^2-x+2)}\right)$

$x^2-x+2 \Rightarrow x \notin \mathbb{R}$

$\Rightarrow \lim\limits_{x\to -1}\left(\frac{x^2-x-2}{(x+1)(x^2-x+2)}\right)$

$x^2-x-2=(x+1)(x-2)$

$\frac{1\pm\sqrt{(-1)^2-4\cdot 1\cdot(-2)}}{2}$

$\Rightarrow \lim\limits_{x\to -1}\left(\frac{\cancel{(x+1)}(x-2)}{\cancel{(x+1)}(x^2-x+2)}\right)$

$\frac{1\pm 3}{2} = \begin{cases} x_1 = 2 \\ x_2 = -1 \end{cases}$

$\Rightarrow \lim\limits_{x\to -1}\frac{x-2}{x^2-x+2} = \frac{-1-2}{(-1)^2+1+2} = \frac{-3}{4}$

$\therefore \frac{-3}{4}$

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{e^x-1}\right)=\left(\frac{e^x-1}{x(e^x-1)}-\frac{x}{x(e^x-1)}\right)=\lim_{x\to 0}\frac{e^x-1-x}{x(e^x-1)}$$

$$\Rightarrow \lim_{x\to 0}\frac{e^x-1-x}{e^x\cdot x-x}=\frac{0}{0}\Rightarrow \text{l'hopital}$$

$$\Rightarrow \lim_{x\to 0}\frac{e^x-1}{x\cdot e^x+e^x-1}=\frac{0}{0}\Rightarrow \text{l'hopital}$$

$$\Rightarrow \lim_{x\to 0}\frac{e^x}{x\cdot e^x+2\cdot e^x}=\frac{1}{2}$$

$$\therefore \frac{1}{2}$$

4) $f(x) = x^3 + ax^2 + bx - 7 \Rightarrow x = 2$

$f'(x) = 3x^2 + 2xa + b$

$f'(2) = 3 \cdot (2)^2 + 2 \cdot 2 \cdot a + b$

$= 12 + 4a + b$

$\Rightarrow x = 4$

$f'(4) = 3 \cdot 4^2 + 2 \cdot 4a + b$

$= 48 + 8a + b$

$$\begin{cases} (1) & 12 + 4a + b \\ (2) & 48 + 8a + b \end{cases}$$

(1) $12 + 4a + b = 0$

$12 + 4a = -b$

$4a = -b - 12$

$a = \frac{-b}{4} - \frac{12}{4}$

$a = \frac{-b}{4} - 3$

(2) $48 + 8 \cdot \left(\frac{-b}{4} - 3\right) + b = 0$

$48 - 2b - 24 + b = 0$

$24 - 2b + b = 0$

$24 - b = 0$

$b = 24$

(1) $12 + 4a + 24 = 0$

$36 + 4a = 0$

$4a = -36$

$a = -\frac{36}{4}$

$a = -9$

$$\boxed{f(x) = x^3 - 9x^2 + 24x - 7 = 0}$$

$f'(x) = 3x^2 - 18x + 24$

$f''(x) = 6x - 18$

$x=2 \Rightarrow f''(2) = 6\cdot 2 - 18 = -6 < 0 \therefore$ Es max relativo

$x=4 \Rightarrow f''(4) = 6\cdot 4 - 18 = 6 > 0 \therefore$ Es mín relativo

5) $\int \frac{\ln(x^2)}{x^{-3}} = \int \ln(x^2) \cdot \frac{1}{x^{-3}} \Rightarrow \ln(x^2) \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{2x}{x^2}$

$$\boxed{\begin{array}{ll} u = \ln(x^2) & dv = x^3 \\ du = \frac{1}{x^2} \cdot 2x & v = \frac{x^4}{4} \end{array}}$$

$\Rightarrow \ln(x^2) \cdot \frac{x^4}{4} - \frac{1}{4} \cdot 2 \int \frac{\cancel{x^5}}{\cancel{x^2}}$

$\Rightarrow \ln(x^2) \cdot \frac{x^4}{4} - \frac{1}{2} \int x^3\,dx$

$\ln(x^2) \cdot \frac{x^4}{4} - \frac{1}{2} \cdot \frac{x^4}{4} \Rightarrow \boxed{\frac{\ln(x^2) \cdot x^4}{4} - \frac{x^4}{8} + C}$

$\int_{-1}^{1} \int_{0}^{1} (x^2 \cdot y - y)\,dx\,dy \Rightarrow \int_{-1}^{1} \left\{ 2x \cdot y - y \right\} \Big|_0^1 dy \Rightarrow \int_{-1}^{1} \Big[(2 \cdot 1 \cdot y -) - (2 \cdot 0 \cdot y - y) \Big] dy \Rightarrow \int_{-1}^{1} \big[(2y - y) - (-y) \big] dy \Rightarrow \int_{-1}^{1} (2y - y + y)\,dy \Rightarrow$

$$\int_{-1}^{1} 2y \cdot dy = \left[\frac{\cancel{2}y^2}{\cancel{2}}\right]\Bigg|_{-1}^{1} \Rightarrow y^2 \cdot dy \Rightarrow \left[(1)^2 - (-1)^2\right] = 0$$

6) $y = 2x + 3$, $y = x + 2$, $(3, 5)$ y $(2, 7)$

$$\frac{x-3}{2-3} = \frac{y-5}{7-5} \Rightarrow \frac{x-3}{-1} = \frac{y-5}{2} \Rightarrow (x-3) \cdot 2 = -1 \cdot (y-5) \Rightarrow$$

$$2x - 6 = -y + 5 \Rightarrow$$

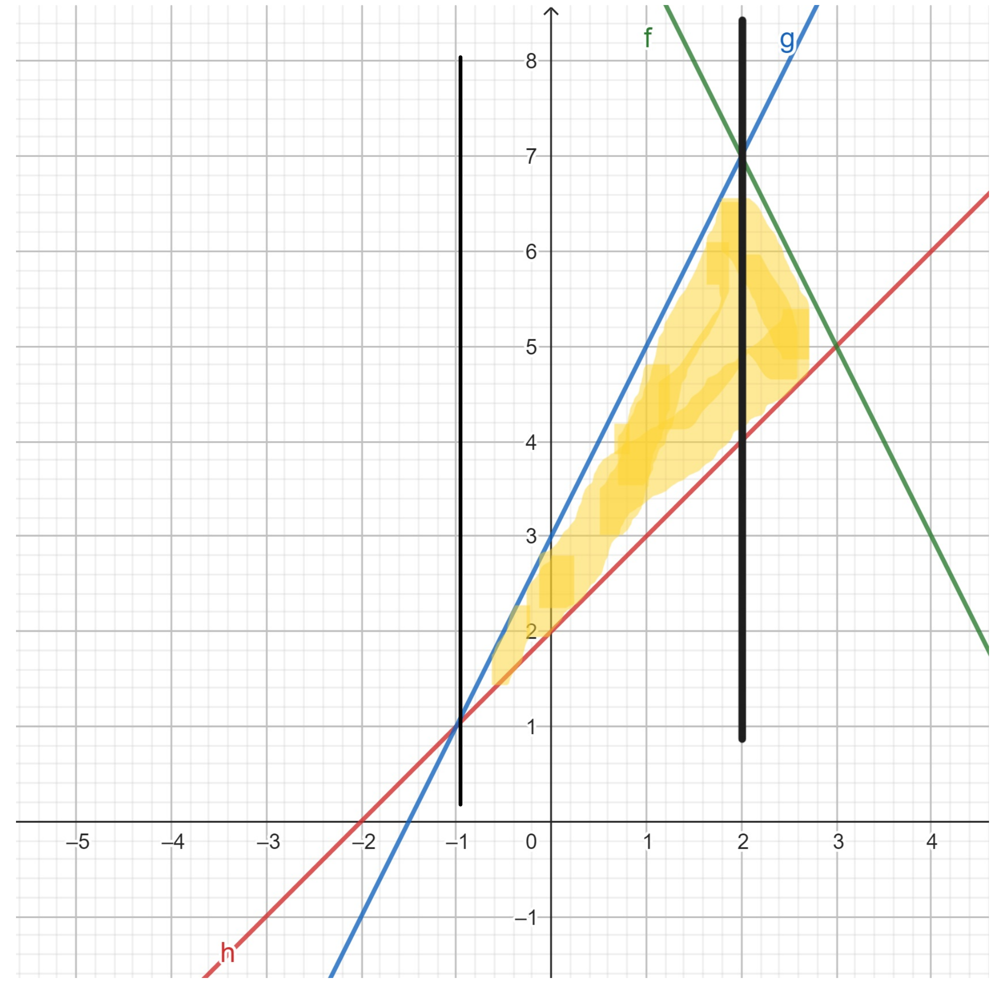
$$2x - 11 = -y \Rightarrow y = -2x + 11$$

$$\left.\begin{aligned} y &= -2x + 11 \\ y &= 2x + 3 \end{aligned}\right\} \quad \begin{aligned} -2x + 11 &= 2x + 3 \\ -2x - 2x &= 3 - 11 \\ -4x &= -8 \\ x &= \frac{-8}{-4} \\ x &= 2 \end{aligned}$$

$$\left.\begin{aligned} y &= -2x + 11 \\ y &= x + 2 \end{aligned}\right\} \quad \begin{aligned} -2x + 11 &= x + 2 \\ -2x - x &= 2 - 11 \\ -3x &= -9 \\ x &= \frac{-9}{-3} \\ x &= 3 \end{aligned}$$

$$\begin{cases} y = 2x+3 \\ y = x+2 \end{cases} \qquad \begin{aligned} 2x+3 &= x+2 \\ 2x - x &= 2-3 \end{aligned}$$

$$\boxed{x = -1}$$



$$A_1 = \int_{-1}^{2} \left[(2x+3) - (x+2) \right] dx$$

$$= \int_{-1}^{2} (x+1)\, dx \Rightarrow \left[\frac{x^2}{2} + x \right] \Bigg|_{-1}^{2} =$$

$$\left[\left(\frac{4}{2} + 2 \right) - \left(\frac{1}{2} - 1 \right) \right] = \left[4 + \frac{1}{2} \right] = \frac{9}{2}$$

$$\boxed{A_1 = \frac{9}{2}}$$

$$A_2 = \int_{2}^{3} \left[(-2x+11) - (x+2) \right] dx = \int_{2}^{3} (-3x+9)\, dx \Rightarrow$$

$$\left[-3\frac{x^2}{2} + 9x \right] \Bigg|_{2}^{3} = \left[\left(-\frac{27}{2} + 27 \right) - \left(-6 + 18 \right) \right] = \left[\left(\frac{27}{2} \right) - 12 \right] = \frac{3}{2}$$

$$\boxed{A_2 = \frac{3}{2}}$$

$$A = A_1 + A_2 \Rightarrow \frac{9}{2} + \frac{3}{2} = 6 \qquad \therefore \boxed{A = 6}$$

7) $Z = \ln(x^2 + xy + y^2)$

$$Z'_x = \frac{2x+y}{x^2+xy+y^2} \qquad Z'_y = \frac{2y+x}{x^2+xy+y^2}$$

$$3 \cdot \left[x \cdot \frac{2x+y}{x^2+xy+y^2} + y \cdot \frac{2y+x}{x^2+xy+y^2} \right] \Rightarrow 3 \cdot \left[\frac{(2x^2+xy) + (2y^2+xy)}{x^2+xy+y^2} \right]$$

$$\Rightarrow 3 \cdot \left[\frac{2x^2+2xy+2y^2}{x^2+xy+y^2} \right] \Rightarrow 3 \cdot \left[2 \cdot \frac{\cancel{(x^2+xy+y^2)}}{\cancel{x^2+xy+y^2}} \right]$$

$\Rightarrow 3 \cdot 2 = 6$ $\therefore$ **La afirmación es verdadera**

8) $$\lim_{(x,y)\to(-1,1)} \frac{3x^2+y^2}{x^4+2x^2y+y^2} = \frac{3 \cdot (-1)^2 + 1^2}{(-1)^4 + 2 \cdot (-1)^2 \cdot 1 + 1^2} = \cancel{\frac{4}{4}} = 1$$

$\therefore$ **Existe límite doble**

9) $f(x,y) = x^2 + y^3 - 3xy$

$f'_x = 2x - 3y$

$f''_{xx} = 2$

$f''_{xy} = -3$

$f'_y = 3y^2 - 3x$

$f''_{yy} = 6y$

$f''_{yx} = -3$

$$\Rightarrow \begin{cases} 2x - 3y = 0 & (1) \\ 3y^2 - 3x = 0 & (2) \end{cases}$$

(1) $2x = 3y$

$x = \frac{3y}{2}$

(2) $3y^2 - 3\cdot\left(\frac{3y}{2}\right) = 0$

$3y^2 - \frac{9y}{2} = 0$

$y_1 = \frac{3}{2}$ $\quad y_2 = 0$

(1) $2x - 3\left(\frac{3}{2}\right) = 0$

$2x - \frac{9}{2} = 0$

$x_1 = \frac{9}{4}$

$2x - 3\cdot(0) = 0$

$2x = 0$

$x_2 = 0$

$P1: \left(\frac{9}{4}, \frac{3}{2}\right)$ $\quad P2: (0,0)$

$$H(x,y) = \begin{vmatrix} 2 & -3 \\ -3 & 6y \end{vmatrix} = 12y - 9$$

$P1: H\left(\frac{9}{4}, \frac{3}{2}\right) = 9 > 0 \Rightarrow f''_{xx}\left(\frac{9}{4}, \frac{3}{2}\right) = 2 > 0 \therefore$ **P1 es un mínimo relativo**

$P2: H(0,0) = -9 < 0 \therefore$ **P2 es un punto de ensilladura**