On the paradox of pesticides

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ABSTRACT. The paradox of pesticides was observed experimentally, which says that pesticides may dramatically increase the population of a pest when the pest has a natural predator. Here we use a mathematical model to study the paradox. We find that the timing for the application of pesticides is crucial for the resurgence or non-resurgence of the pests. In particular, regularly applying pesticides is not a good idea as also observed in experiments [7] [3]. In fact, the best time to apply pesticides is when the pest population is reasonably high.

1. Introduction

The paradox of pesticides says that pesticides may dramatically increase the population of a pest when the pest has a natural predator. Right after the application of the pesticide, of course the pest population shall decrease. But the pest may resurge later on resulting in a population well beyond the crop's economic threshold.

In the experiment of [9], pesticides (pyrethroids) are applied in apple orchards for the main goal of killing flying insects (lepidopteran and dipteran pests). The side effect of this is that later on the populations of other pests (phytophagous mites: panonychus ulmi and tetranychus urticae) feeding on the apple tree's leaves resurge above the crop's economic thresholds, while the populations of these pests on the leaves not sprayed with the pesticides remain low (below the crop's economic thresholds). These phytophagous mites are the preys of some predator mites (phytoseiid predator typhlodromus caudiglans and stigmaeid predator zetzellia mali). The pesticides kill both the leaf eating mites and the predator mites. One of the predator mites (typhlodromus caudiglans) is almost wiped out by the pesticides, and resurges very slowly. The population of the other predator mite (zetzellia mali) is only slightly affected by the pesticides, but it alone cannot control the resurgence of the leaf eating mites. The pesticide damage to the populations of the leaf eating mites is not as severe as that to the population of the predator mite (typhlodromus caudiglans). After the spraying of the pesticides, the dynamics described above

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lasts 11 weeks. The phenomena just described have been well documented in many experiments [14] [5] [6] [2] [16] [4] [12] [8] [1].

In the experiment of [7] [3], pesticide effects on arthropods in the rice field are studied. There are four groups of arthropods: herbivores, detritivores, predators, parasitoids. Herbivores eat rice plants, detritivores eat detritus in the rice field, and predators and parasitoids eat herbivores and detritivores. Pesticides can kill every group of arthropods. Since they move around faster and more often, predators contact the pesticides more often, and therefore are killed more. This leads to more abundance of herbivores. A new scheme for spraying pesticides was introduced in [7], that is, applying pesticides only when the pest densities are high enough, rather than applying regularly. In contrast to regularly applying pesticides every week, the new scheme dramatically reduces the density of herbivores among all arthropods, which are the main destroyers of the rice field; enhances the densities of predators and detritivores; and slightly reduces the density of parasitoids.

Mathematical modeling on pest resurgence has also been studied, e.g. in [11], a time-discrete model generalizing the Nicholson-Bailey model [13], is studied and concludes that the paradox of pesticides could be caused essentially by the interspecific relationship and the intraspecific density effect.

To avoid the paradox of pesticides, farmers can make use of Integrated Pest Management [17] [18]. Mathematical models on integrated pest management are also studied [15].

In this paper, we study a time-continuous model on pest resurgence, based upon the classical Lotka-Volterra model. We agree with the experimental work [7] on that the timing of applying the pesticides is crucial for the paradox of pesticides. We believe that when the timing is right, even though the pesticides only kill the pests rather than their predators, the pests may still resurge in abundance. We also believe that the amount of pesticides applied is also important. We think that the issue of the amount of pesticides is more delicate than the theory of hormesis [10] [12] [8] (that is, small amounts of pesticides enhance pest population, whereas large amounts of pesticides reduce pest population).

2. The mathematical model

To model the effect of pesticides on pest resurgence, we study the Lotka-Volterra system with forcing,

(2.1)
$$\frac{dH}{dt} = H(a - bP) - \alpha \Delta(t - T),$$

$$\frac{dP}{dt} = P(bcH - d) - \beta \Delta(t - T),$$

(2.2)
$$\frac{dP}{dt} = P(bcH - d) - \beta \Delta(t - T),$$

 β) are positive constants, $\Delta(t)$ is an approximation of the delta function, and the $\Delta(t-T)$ terms represent the effects of pesticides. Specifically, we choose $\Delta(t-T)$ to be

$$\Delta(t-T) = 1/\epsilon$$
, when $t \in [T, T+\epsilon]$; = 0, otherwise.

Without the pesticide terms, the system (2.1)-(2.2) reduces to the classical Lotka-Volterra system,

$$\frac{dH}{dt} = H(a - bP).$$

(2.3)
$$\frac{dH}{dt} = H(a - bP),$$

$$\frac{dP}{dt} = P(bcH - d),$$

The nontrivial steady state of this system (2.3)-(2.4),

$$(2.5) H = \frac{d}{bc}, P = \frac{a}{b},$$

is neutrally stable. The phase plane diagram of (2.3)-(2.4) is shown in Figure 1.

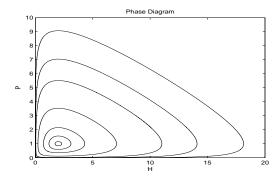


FIGURE 1. The phase plane diagram of (2.3)-(2.4), a = b = d =1, c = 0.5, with different starting points of (H, P): (1.8, 0.9), (1.4, 0.7), (1.0, 0.5), (0.5, 0.25), (0.2, 0.1), (0.1, 0.05), (0.04, 0.02).

The key points on the resolution of the paradox of pesticides are as follows:

(1) The timing of applying the pesticides is crucial. If the pesticides are applied when the populations of both the pests and the predators are relatively large, then a decrease in both populations can be achieved,

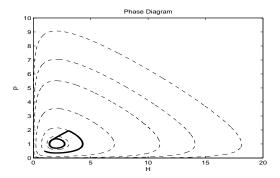


FIGURE 2. A pesticide forced orbit of (2.1)-(2.2), a=b=d=1, $c=0.5,\, T=3.5,\, \epsilon=0.07,\, \alpha=1.4,\, \beta=0.7.$ In this case, the pest population deceases and maintains at a lower amplitude oscillation.

see Figure 2. On the other hand, if the pesticides are applied when either the pest's population or the predator's population is relatively small, then a dramatic increase in the resurgent pest's population occurs, leading to pest's population well beyond the crop's economic threshold, see Figures 3 and 4. From these figures, it is clear that even though the pesticides only kill the pests rather than their predators (that is, right after the application of the pesticides, the pest's population decreases, while the predator's population maintains the same), the pests still resurge in abundance beyond the crop's economic threshold. This is because that when the population of the pests decreases, the predator's population will decreases too, since the predators feed on the pests. It is the relatively minimal values of both the pest's population and the predator's population that decide how large the cycle which they are going to sit on in the phase plane.

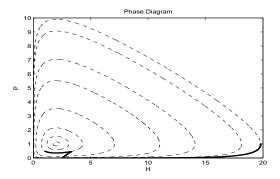


FIGURE 3. A pesticide forced orbit of (2.1)-(2.2), a = b = d = 1, $c = 0.5, T = 2, \epsilon = 0.047, \alpha = 0.94, \beta = 0.47$. In this case, the pest population resurges with dramatic increase beyond the crop's economic threshold.

(2) The amount of pesticides applied is also important. In the case of Figure 2, if the amount of pesticides is large enough, a dramatic increase in the resurgent pest's population can still occur as shown in Figure 5.

3. Spatial Effect

To study the spatial effect, we introduce the following specific model,

(3.1)
$$\frac{\partial H}{\partial t} - \frac{\partial P}{\partial x} \frac{\partial H}{\partial x} = \frac{\partial^2 H}{\partial x^2} + H(1 - P) - \alpha \Delta (t - T, x),$$
(3.2)
$$\frac{\partial P}{\partial t} + \frac{\partial H}{\partial x} \frac{\partial P}{\partial x} = \frac{\partial^2 P}{\partial x^2} + P(0.5H - 1) - \beta \Delta (t - T, x),$$

(3.2)
$$\frac{\partial P}{\partial t} + \frac{\partial H}{\partial x} \frac{\partial P}{\partial x} = \frac{\partial^2 P}{\partial x^2} + P(0.5H - 1) - \beta \Delta (t - T, x),$$

where the second term on the left hand side represents 'convection' due to predator chasing and prey escaping, the first term on the right hand side represents population 'diffusion', and the last term on the right hand side represents pesticide effect now depending upon both space and time. Temporally the pesticide term is still near Delta function. Two type of boundary conditions are studied:

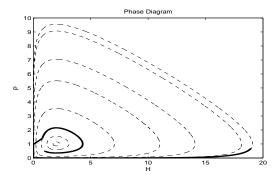


FIGURE 4. A pesticide forced orbit of (2.1)-(2.2), a=b=d=1, $c=0.5,\ T=5,\ \epsilon=0.039,\ \alpha=0.78,\ \beta=0.39.$ In this case, the pest population resurges with dramatic increase beyond the crop's economic threshold.

• Neumann boundary condition,

$$\partial_x H = \partial_x P = 0, \quad x = 0, L;$$

• Dirichlet boundary condition,

$$H = P = 0, \quad x = 0, L.$$

Without the pesticide terms ($\alpha = \beta = 0$), for a variety of initial conditions, we observe that the asymptotic state under the Neumann boundary condition is spatially uniform, while the asymptotic state under the Dirichlet boundary condition is the same steady state which appears to be an attractor; see Figures 6 and 7.

For the paradox of pesticides, the asymptotic states are not important. One has to look at the entire temporal dynamics, in fact, the intial stage dynamics is much more important. Imagine that at the initial temporal stage, the population of the pest reaches beyond the crop's economic threshold, the damage to the crops is done, then the later dynamics is meaningless. In Figure 8, we plot the 'phase plane diagram' of the initial temporal stage dynamics of the spatial mid-points of

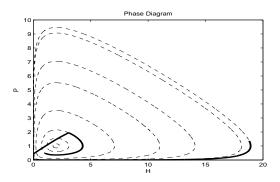


FIGURE 5. A pesticide forced orbit of (2.1)-(2.2), a=b=d=1, $c=0.5,\,T=3.5,\,\epsilon=0.149,\,\alpha=2.98,\,\beta=1.49.$ In contrast to Figure 2, here large amount of pesticides is applied, and results in that the pest population resurges with dramatic increase beyond the crop's economic threshold.

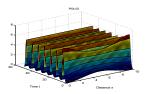
H(t,x) and P(t,x) for the intial condition

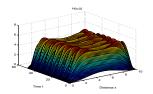
$$H = \epsilon [1 - \cos(2\pi x/L)], \ P = 1 - \cos(2\pi x/L);$$

with various values of ϵ . In Figure 8, we only show the most important initial 'one-loop' for each orbit. This initial 'one-loop' is the most relevant for the crop application. We view Figure 8 as the counterpart of Figure 1, and study the paradox of pesticide for the system (3.1)-(3.2). In Figure 9, the spatial-temporal profiles of H and P corresponding to the ($\epsilon=0.03125$) one-loop in 8 are shown. The counterparts for the Dirichlet boundary condition are shown in Figures 8 and 11.

4. Conclusion

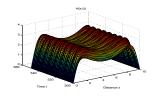
Via a mathematical model, we studied the experimentally observed paradox of pesticides. Our conclusion is that the timing for the application of pesticides is fundamental for the control of pest population. Regularly applying pesticides is

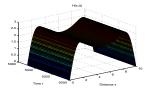




(a) Neumann b.c.

(b) Dirichlet initial





- (c) Dirichlet transient
- (d) Dirichlet asymptotic

FIGURE 6. The asymptotic dynamics of the system (3.1)-(3.2) without the pesticide terms ($\alpha = \beta = 0$). The specific initial condition used here is $H = P = \sin^2(\pi x/L)$. (a) is for the Neumann boundary condition. (b)-(d) are for the Dirichlet boundary condition; (b) shows the dynamics of the initial temporal stage, (c) shows the transient stage dynamics, and (d) shows the asymptotic dynamics.

not a good control as also observed in experiments [7] [3]. In fact, the best time to apply pesticides is when the pest population is reasonably high. We also studied a generalized partial differential equation model, we arrive at the same conclusion.

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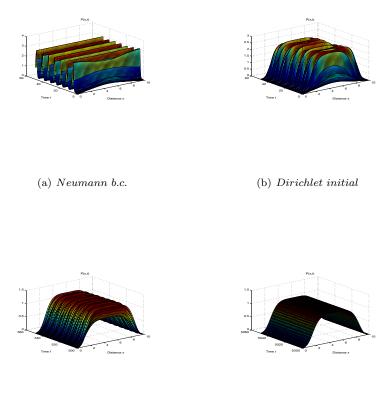


FIGURE 7. The same with Figure 6 except for variable P.

(d) Dirichlet asymptotic

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(c) Dirichlet transient

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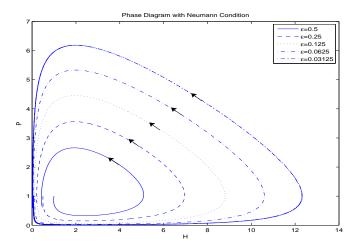
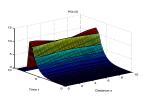
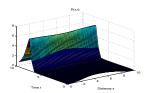


FIGURE 8. The phase plane diagram of the spatial mid-points of H(t,x) and P(t,x) under the system (3.1)-(3.2) without the pesticide terms ($\alpha = \beta = 0$); and for the Neumann boundary condition with the intial condition $H = \epsilon [1 - \cos(2\pi x/L)]$, $P = 1 - \cos(2\pi x/L)$ with various values of ϵ .

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(a) H profile

(b) P profile

FIGURE 9. The the spatial-temporal profiles of H and P corresponding to the ($\epsilon = 0.03125$) one-loop in Figure 8.

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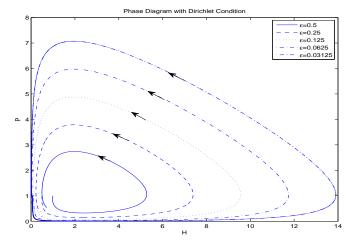


FIGURE 10. The phase plane diagram of the spatial mid-points of H(t,x) and P(t,x) under the system (3.1)-(3.2) without the pesticide terms ($\alpha=\beta=0$); and for the Dirichlet boundary condition with the intial condition $H=\epsilon[1-\cos(2\pi x/L)],$ $P=1-\cos(2\pi x/L)$ with various values of ϵ .

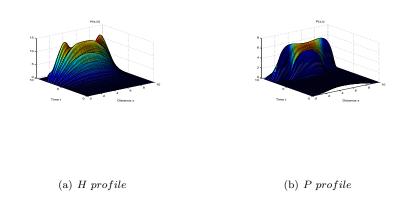


FIGURE 11. The the spatial-temporal profiles of H and P corresponding to the ($\epsilon=0.03125$) one-loop in Figure 10.