Multi-CCD modelling of the point spread function

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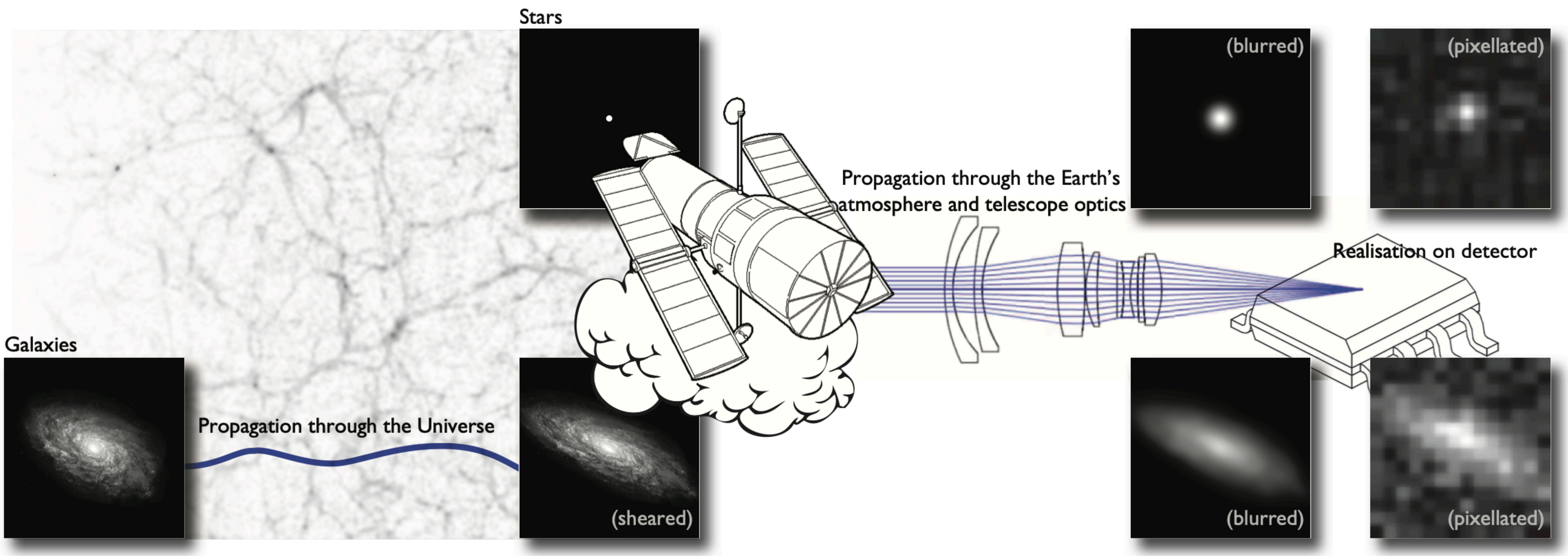
Supervisors: Jean-Luc Starck, Martin Kilbinger

Weak gravitational lensing



Credit - Kitching et al. 2011

Mandatory to correct for PSF effects → **Need the PSF at galaxy positions**

Data-driven modelling

- Use star observations to build PSF model
- Use model to recover PSF at galaxy positions

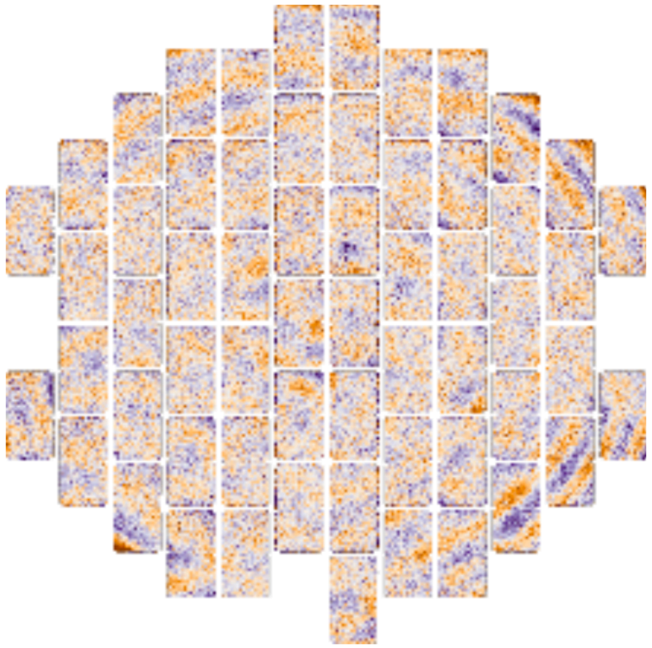
Difficulties

- Varying SNR
- Number of stars available
- Diverse field-of-view variations

State-of-the-art data-driven methods*

PSFEx [Bertin, 2011]

- Each CCD is independently modelled
- Handles polynomial variations
- Not able to capture global PSF patterns
- Issues detected with super-resolution mode
- Robust with real data
- Widely used



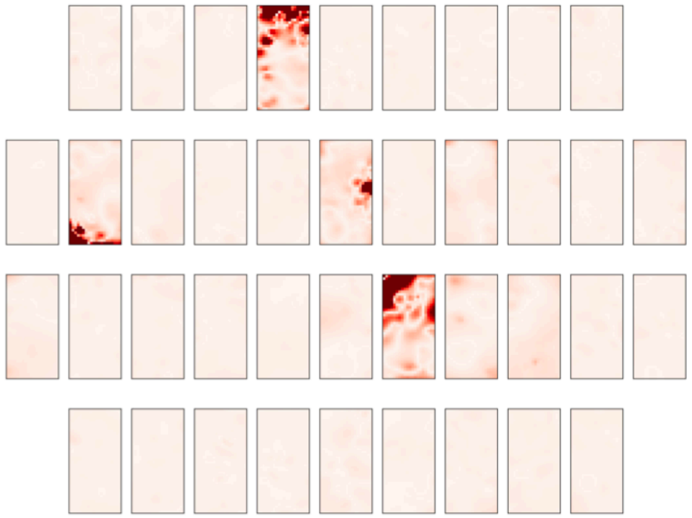
DES Y1 mean ellipticity residuals

Credit - Zuntz et al. 2018

Resolved Component Analysis (RCA)

[Ngolè et al. 2016, Schmitz et al. 2020]

- Each CCD is independently modelled
- Handles more complex PSF variations
- Better handles super-resolution w.r.t. PSFEx
- Not robust to handle real data



RCA size error

Credit - Liaudat et al. 2020

* that are publicly available

Novel PSF model

Multi-CCD PSF model

Reference: Liaudat et al. , A&A 646, A27 (2021) DOI

Joint work with: Jean-Luc Starck, Martin Kilbinger, Axel Guinot, Morgan Schmitz ..

Code available

build passing | pypi package 0.0.3 | python 3.6

https://github.com/CosmoStat/mccd

MCCD PSF Modelling

Main points

- **Extension of RCA and PSFEx**
- **Modelling the entire field-of-view simultaneously**
- **Based on a constrained matrix factorisation scheme**
- **Handles complex variations**
- **Robust to handle real data**

- **Low frequency**
- **High frequency**
- **Discontinuities**

Multi-CCD PSF model

Observation model

$$y_i^k = \mathscr{F}\left(H(u_i^k)\right) + n_i^k$$

y : **star observation thumbnail**

u : **field-of-view position**

H : **PSF field**

$\mathscr{F}$: **degradation operator** →

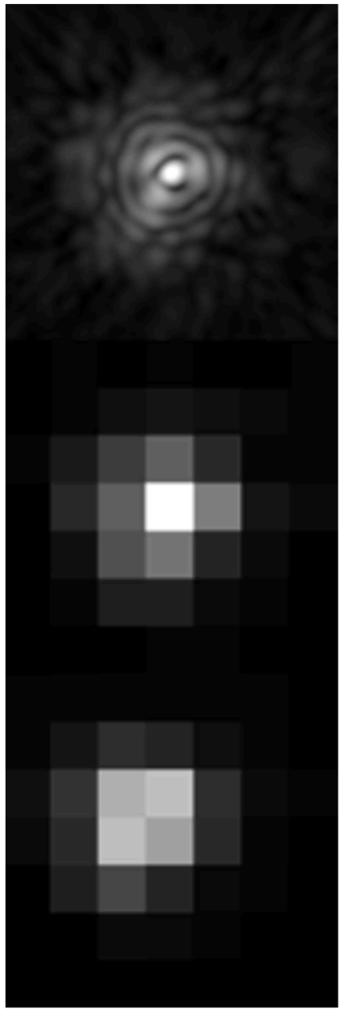
- **Intra-pixel shifts**
- **Grid sampling**
- **Possible decimation**

k : **CCD index**

i : **star index**

n : **white Gaussian noise**

Different pixel representations of the same PSF



Credit: [Krist, 2011]

Multi-CCD PSF model

Matrix factorisation scheme: learn PSF features *S*

$$\hat{H}_k = \underbrace{S_k\, A_k}_{\text{Local: } \hat{H}_k^{\text{Loc}}} + \underbrace{\tilde{S}\, \tilde{A}_k}_{\text{Global: } \hat{H}_k^{\text{Glob}}}$$

$\hat{H}_k$: **PSF reconstructions for CCD k**

$\tilde{S}$: **global PSF features**

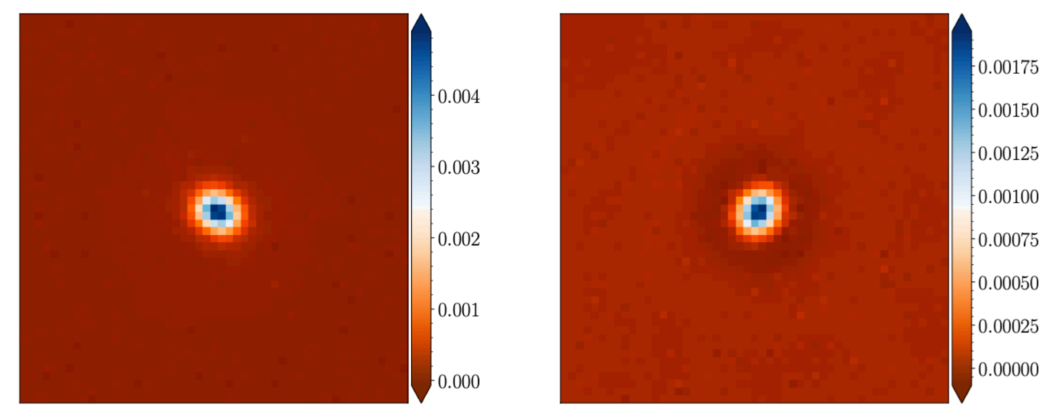
S_k : **local PSF features**

$\tilde{A}_k$: **global features weights**

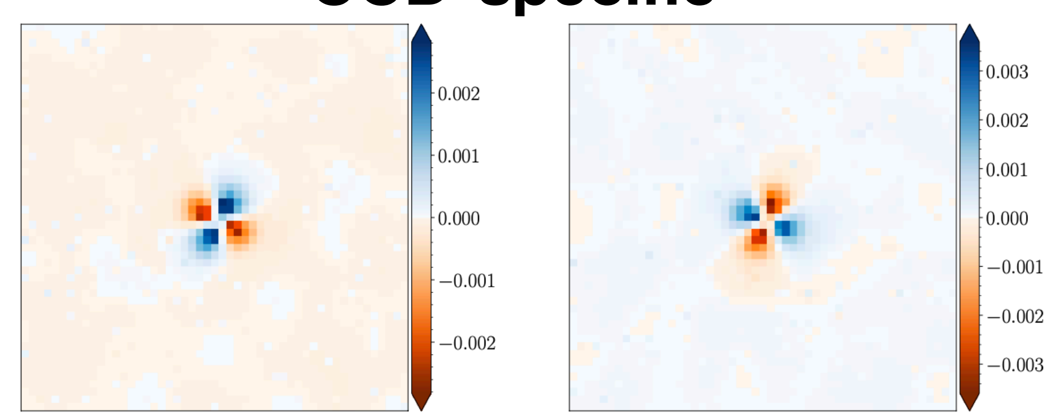
A_k : **local features weights**

PSF: linear combination of features

Common to all CCDs



CCD-specific



Multi-CCD PSF model

Enforce spatial constraints by matrix factorisation

$$\hat{H}_k = S_k \, \alpha_k \, V_k^T \; + \; \tilde{S} \, \tilde{\alpha} \, \Pi_k$$

$$A_k = \alpha_k \, V_k^T$$

$$\boxed{A_k} = \boxed{\alpha_k} \; \boxed{V_k^T}$$

Spatial variation dictionary

- Graph spatial frequencies (RCA)
  - Able to capture localised variations
  - Built with eigenvectors of the Graph's Laplacian matrix
- Polynomial spatial variations (PSFEx)

Positions of stars considered as an undirected graph

Sparse weight matrix

- Enforce a feature to specialise on a specific spatial variation

Multi-CCD PSF model

Main optimisation problem

$$
\min_{\substack{S_1,\ldots,S_N,\tilde{S}\\ \alpha_1,\ldots,\alpha_N,\tilde{\alpha}}} \Bigg\{ \sum_{k=1}^{N} \Bigg(\frac{1}{2} \| Y_k - \mathcal{F}_k (S_k \alpha_k V_k^\top + \tilde{S} \tilde{\alpha} \Pi_k) \|_F^2 + \sum_{i=1}^{r_k} \| \boldsymbol{w}_{k,i} \odot \Phi \mathbf{s}_{k,i} \|_1 + \iota_+ (S_k \alpha_k V_k^\top + \tilde{S} \tilde{\alpha} \Pi_k) + \iota_{\Omega_k} (\alpha_k) \Bigg) + \sum_{i=1}^{\tilde{r}} \| \tilde{\boldsymbol{w}}_i \odot \Phi \tilde{\mathbf{s}}_i \|_1 + \iota_{\tilde{\Omega}} (\tilde{\alpha}) \Bigg\}. \qquad (22)
$$

See publication for more details..

Multi-CCD PSF model

Recover PSFs at galaxy positions

Radial Basis Function interpolation

$$\hat{f}(u) = \sum_{i=1}^{N} \lambda_i \phi \left(\|u - u_i\| \right)$$

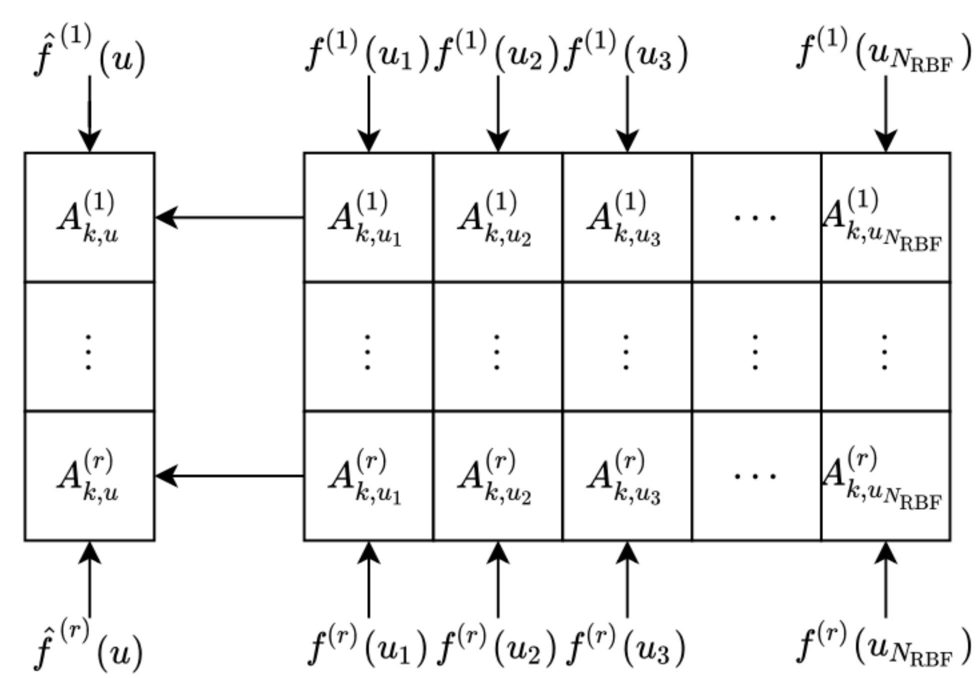
Perfect reconstruction at training position

Recover the weights for the new position *u*

$$\hat{H}_{k,u} = S_k \, \hat{A}_{k,u} + \tilde{S} \, \hat{\tilde{A}}_u$$

Interpolated from A_k and $\tilde{A}$

$$A_{k,u} \longleftarrow A_k(N_{\mathrm{RBF}})$$

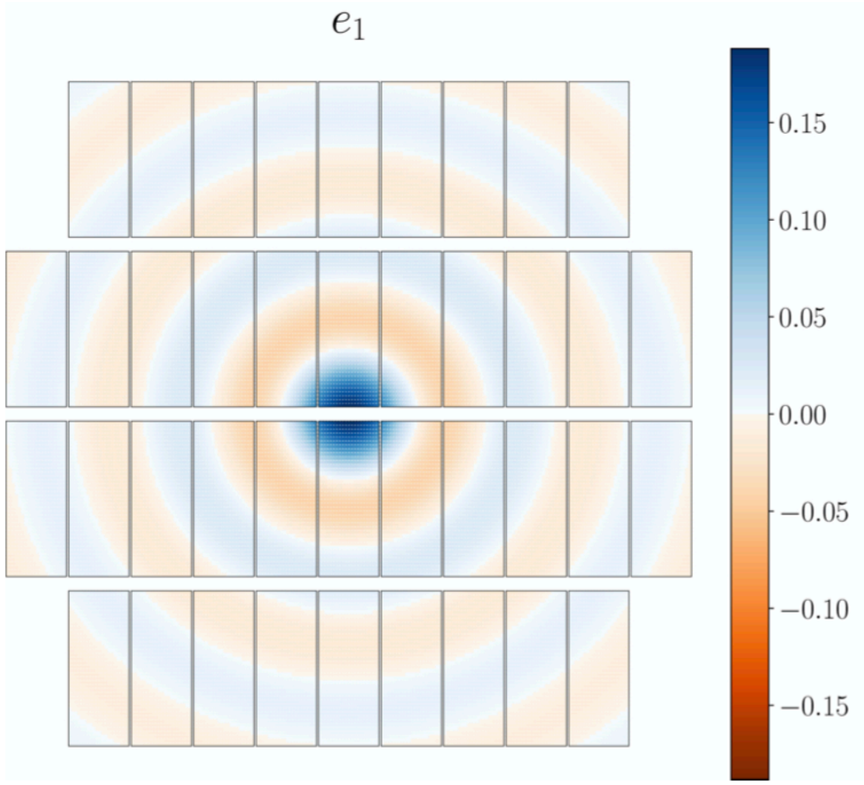


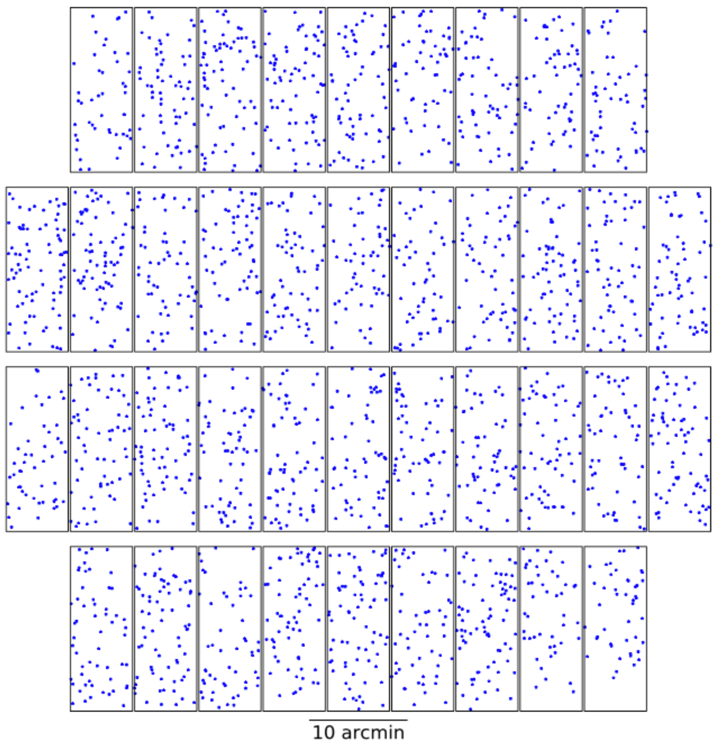
Numerical experiences: Simulations

Simulations

Description:

- **Artificially sheared images**
- **Moffat profiles**
- **Realistic positions taken from a CFIS-r exposure for training positions**
- **Testing stars in a regular grid**
- **Add Gaussian noise for desired SNR value**
- **Test for different SNR values**

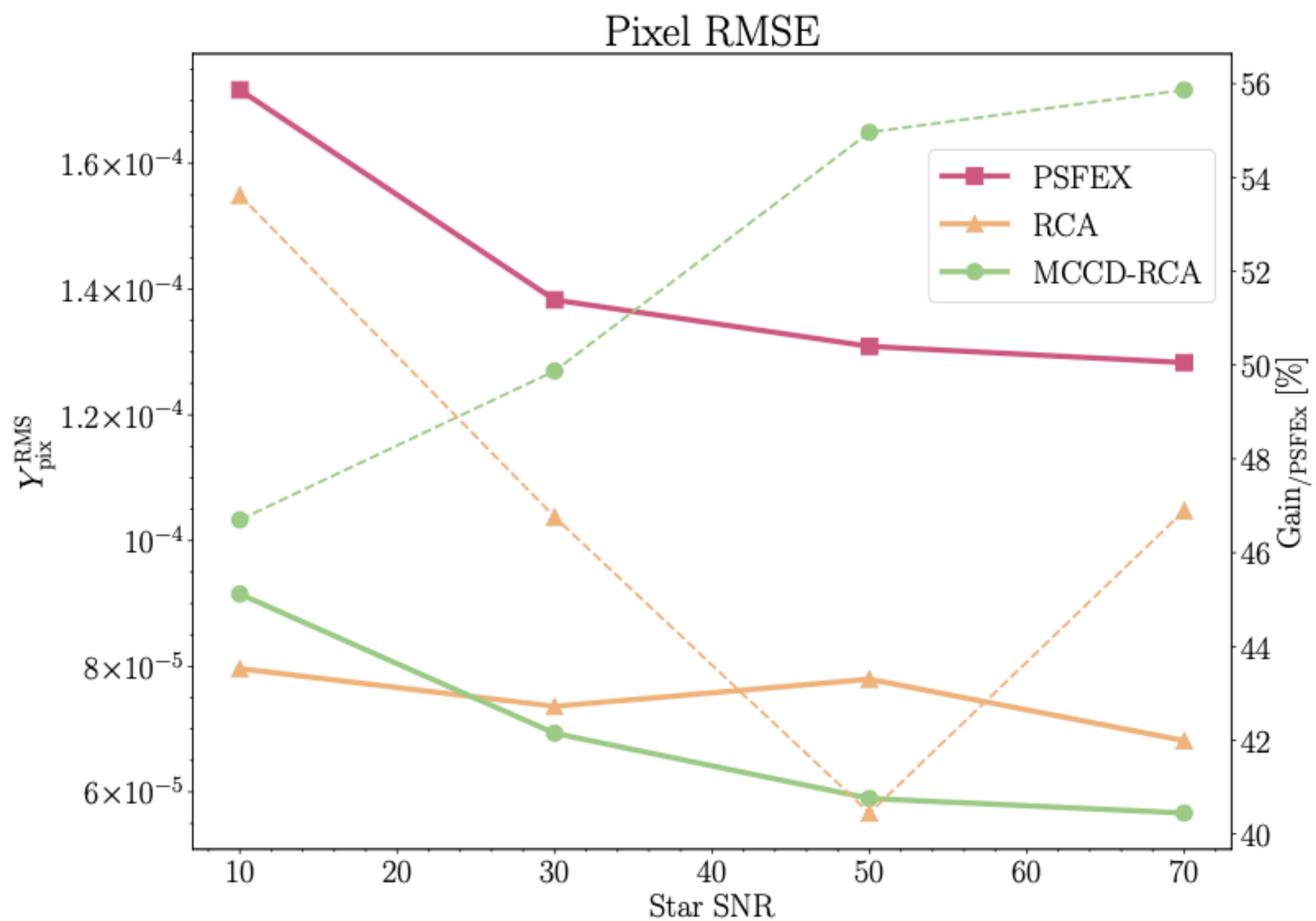




Simulation results

Pixel Root Mean Squared Error (RMSE)

- **MCCD: Average gain in pixel error of ~50% w.r.t. PSFEx**
- **Good performance of RCA**



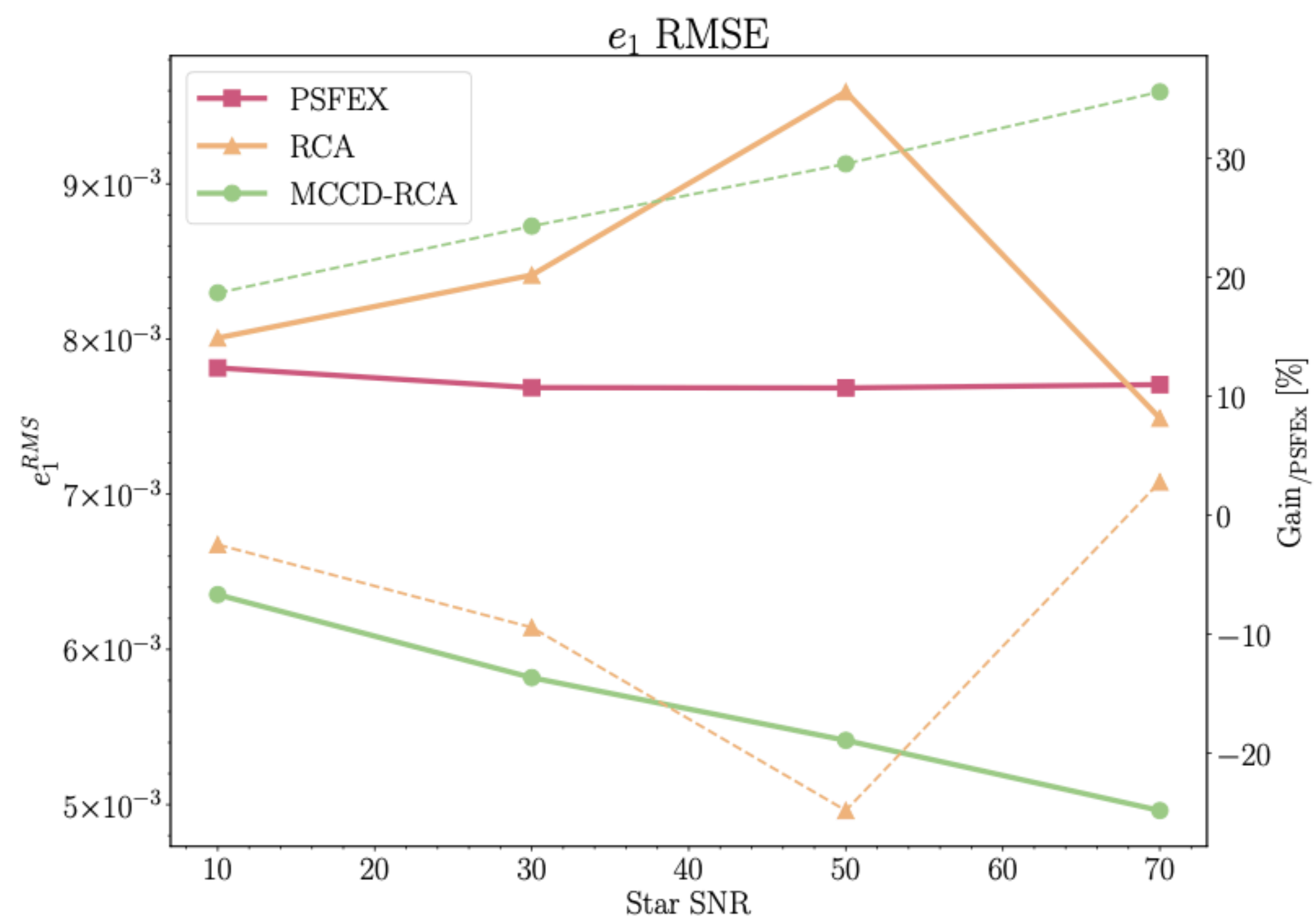
Simulation results

1st ellipticity component RMSE

- **MCCD: gain in e1 RMSE between 15% and 36% w.r.t. PSFEx**

- **Bad performance of RCA**
  - **Due to model degenerating in some CCDs**

- **Stable error of PSFEx w.r.t. SNR values**



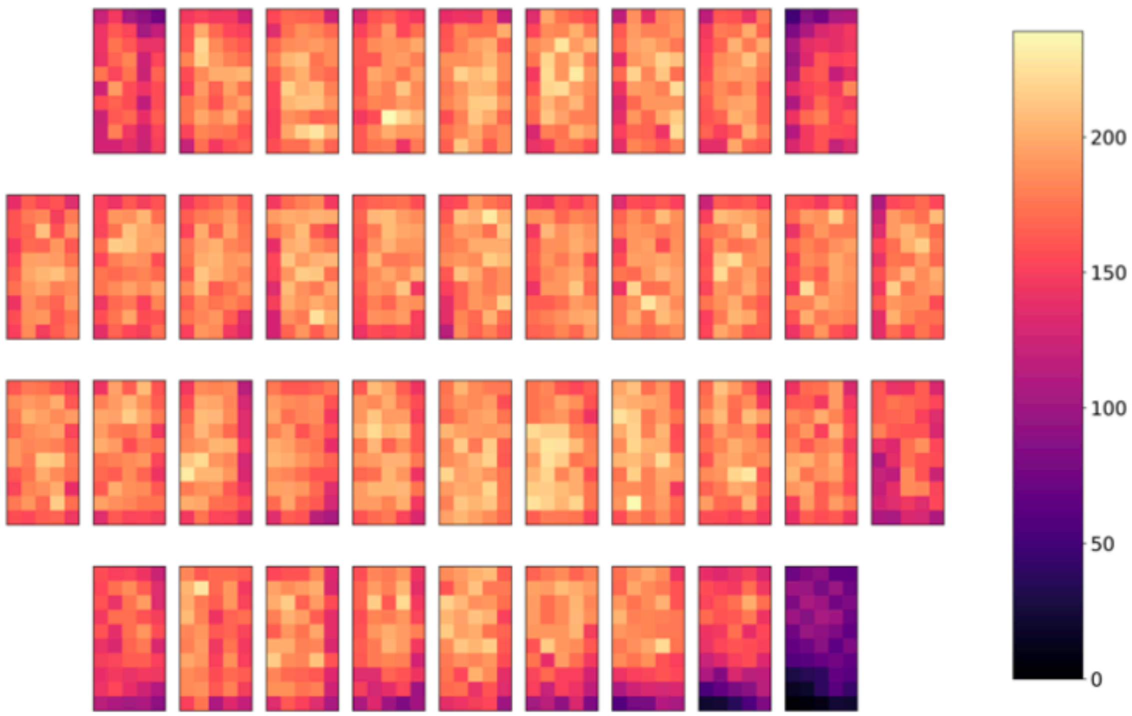
Numerical experiences: CFIS r-band images

CFIS r-band data

Description:

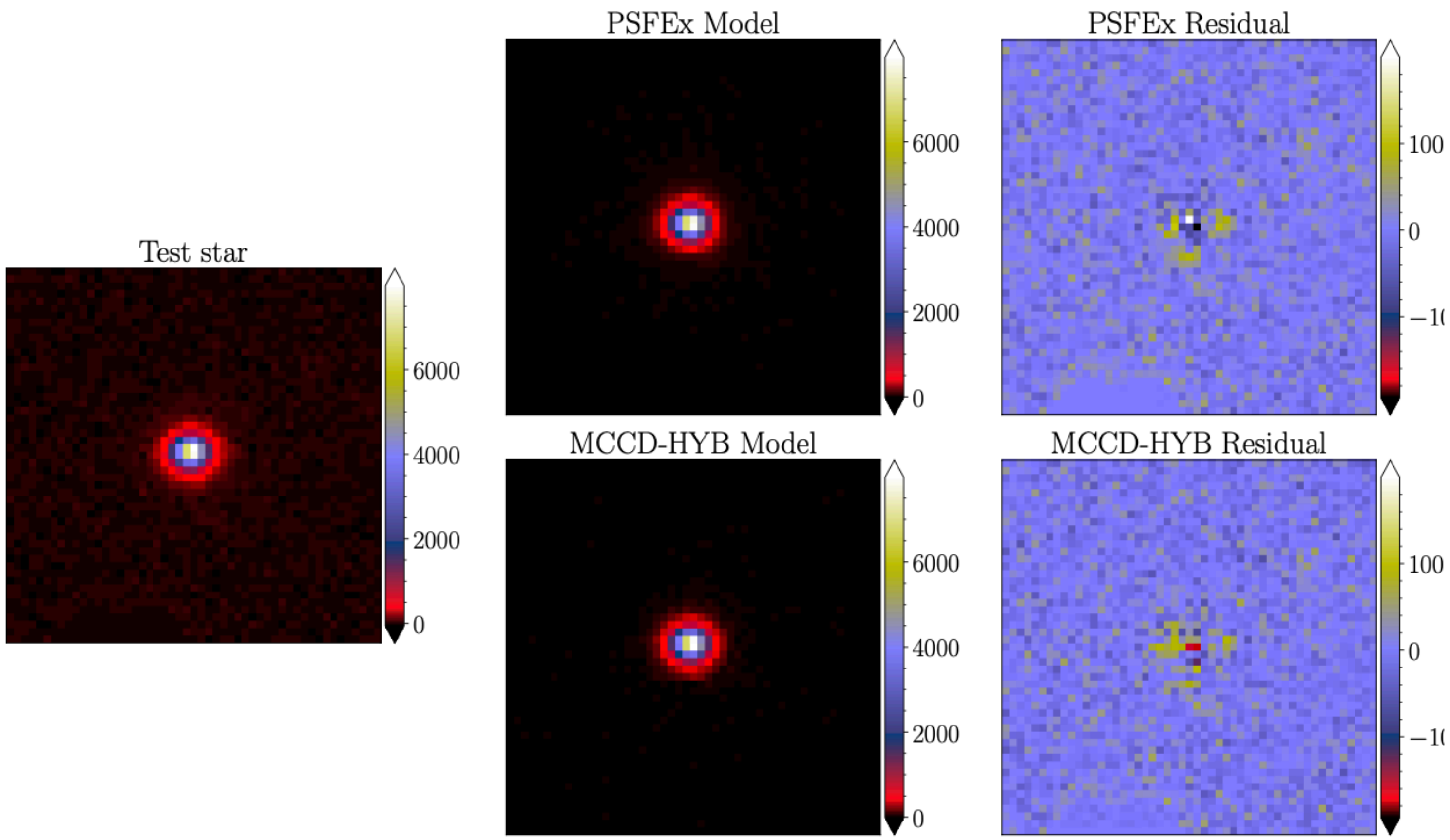
- **Total area: 50 deg^2 (W3 field)**
- **Star selection:**
  - **18 < Mag < 22**
  - **0.3 < FWHM < 1.5 [arcsec]**
- **80/20 % ratio of training/testing stars**
- **1560 training stars per exposure in average**

Field star density



CFIS r-band data

Method	Q_{p_1}	Q_{p_2}	Q_{p_3}
PSFEx	15.56	8.13	14.31
MCCD-HYB	**12.14**	**6.68**	**10.86**
$\text{Gain}_{\text{PSFEx}}$	22%	18%	24%



Model's reconstruction error

- **Qp1 : Pixel RMSE taking into consideration the noise level**

Model's noise

- **Qp2 : Mean standard deviation of model's noise**
- **Qp3 : Variation of model's noise over Qp2**

CFIS r-band data

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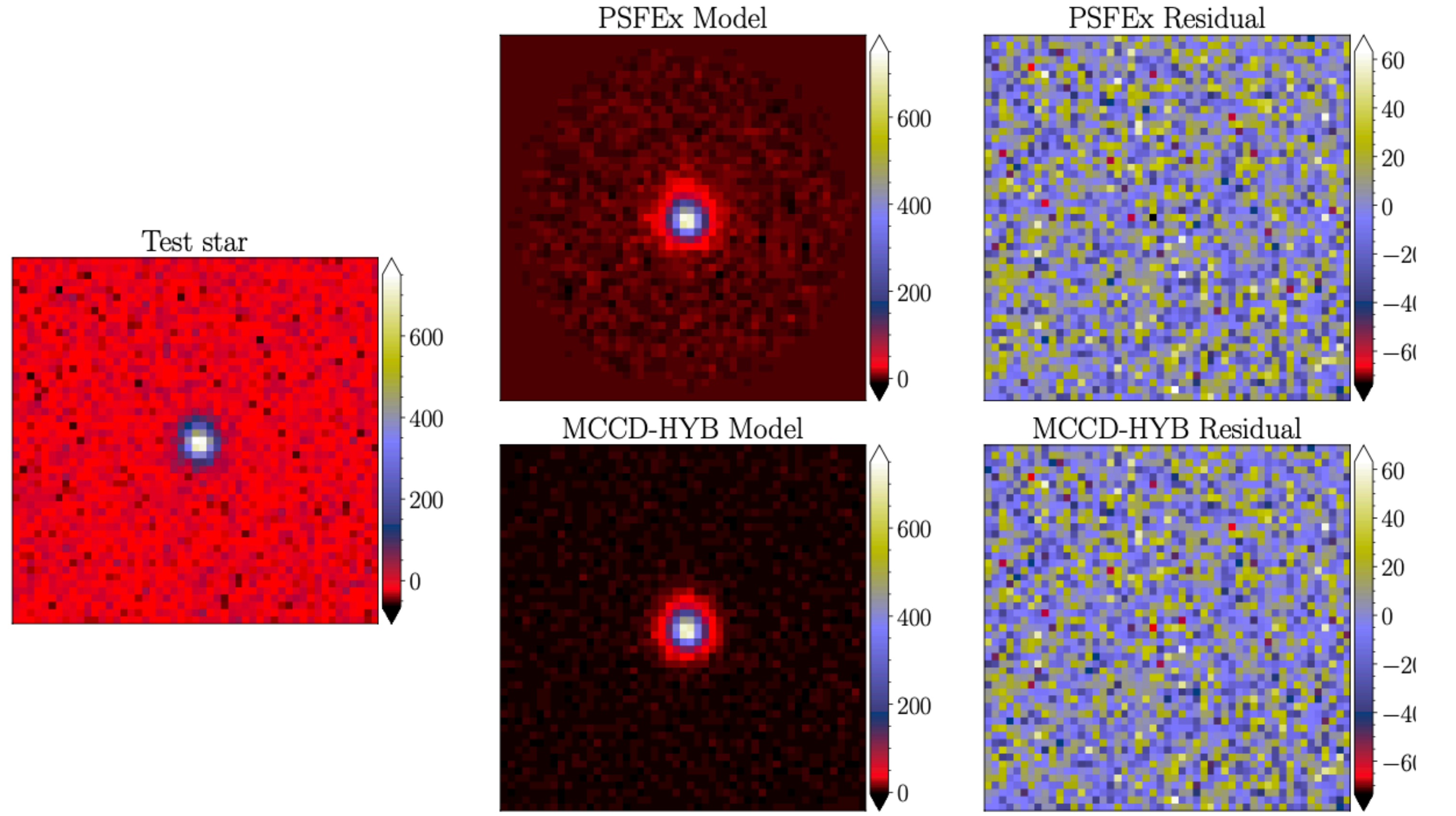

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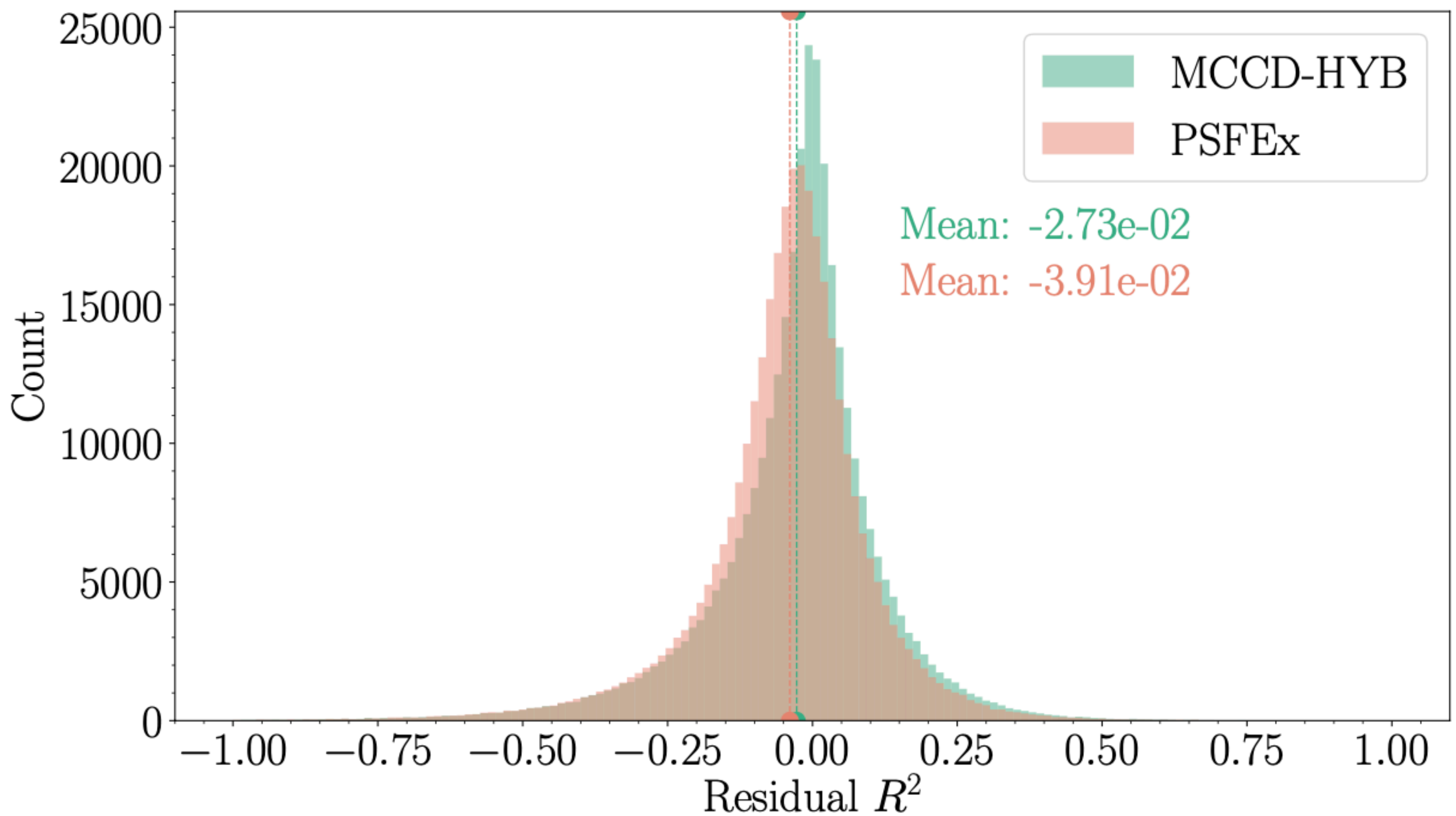
CFIS r-band data



Bigger size bias found in PSFEx: overestimation of the PSF size

MCCD has a 30% gain wrt to PSFEx in size bias

Conclusions

- New MCCD PSF model
  - Model of the entire focal plane at once
  - Handle complex field-of-view variations

- Tested with simulations
  - Better modelling of global patterns

- Tested with real data
  - Robust to handle real CFIS images
  - Better pixel error and size bias
  - Less noise in the PSF model

Perspectives

- Currently testing the MCCD PSF model in the shape measurement pipeline ShapePipe with real data.
  - Expecting results soon!

- Developing new denoising strategy based on deep learning techniques.
  - On going project with Aziz Ayed.

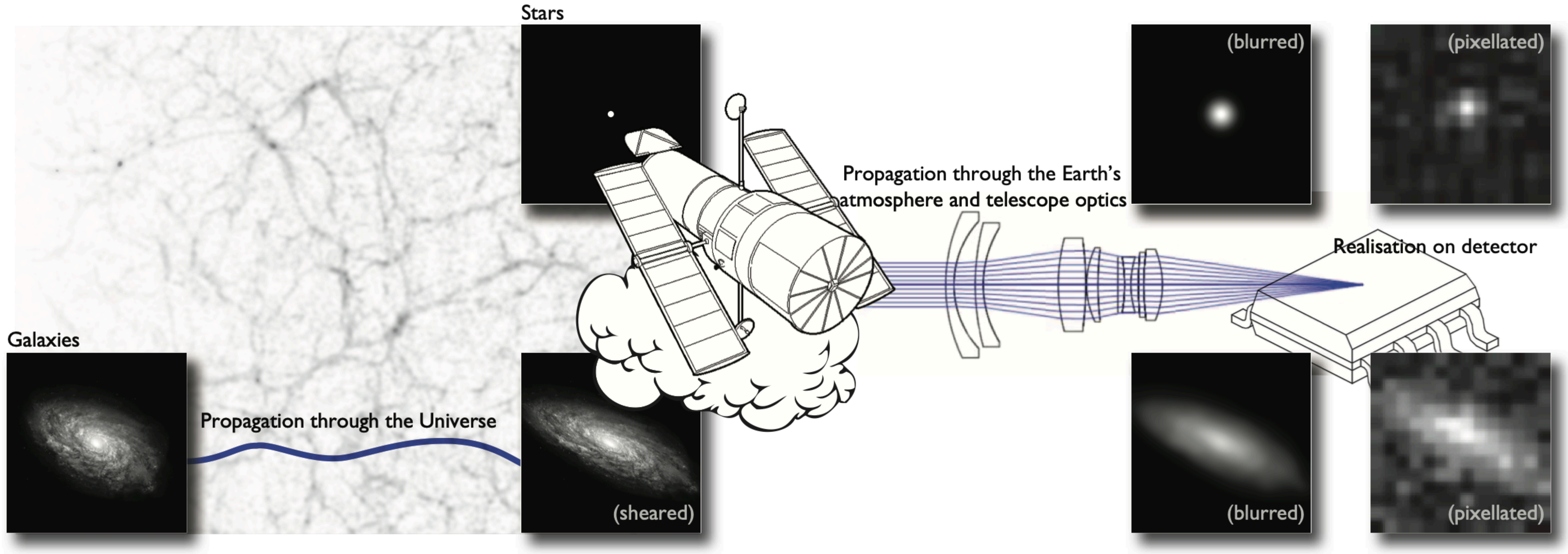
- Test the super-resolution mode of the method.

Thank you for your attention

Extra slides

Introduction

Weak gravitational lensing



Credit - Kitching et al. 2011

Mandatory to correct for PSF effects → Need the PSF at galaxy positions

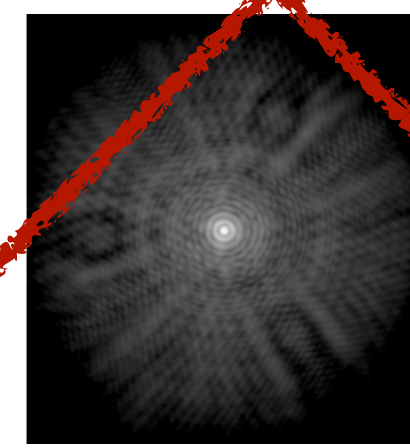
PSF modelling

Sources of the PSF →

- Optical system
- Detector effects
- Atmosphere

Parametric modelling

Characterisation of the optical system and detector i.e. Tiny Tim for HST



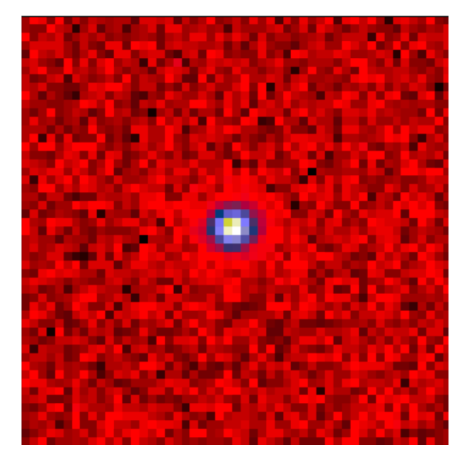
Credit: [Krist, 2011]

Data-driven modelling

- Use star observations to build PSF model
- Recover PSF at galaxy positions

Difficulties →

- Varying SNR
- Number of stars available
- Diverse field-of-view variations



CFIS r-band star. Credit: [Liaudat et al, 2020]

Method

Multi-CCD PSF model

Main optimisation problem

$$\min_{\substack{S_1,\ldots,S_N,\tilde{S}\\ \alpha_1,\ldots,\alpha_N,\tilde{\alpha}}} \Bigg\{ \sum_{k=1}^{N} \Bigg(\frac{1}{2} \| Y_k - \mathcal{F}_k(S_k \alpha_k V_k^\top + \tilde{S}\tilde{\alpha}\Pi_k) \|_F^2 + \sum_{i=1}^{r_k} \| \boldsymbol{w}_{k,i} \odot \Phi \mathbf{s}_{k,i} \|_1 + \iota_+(S_k \alpha_k V_k^\top + \tilde{S}\tilde{\alpha}\Pi_k) + \iota_{\Omega_k}(\alpha_k) \Bigg) + \sum_{i=1}^{\tilde{r}} \| \tilde{\boldsymbol{w}}_i \odot \Phi \tilde{\mathbf{s}}_i \|_1 + \iota_{\tilde{\Omega}}(\tilde{\alpha}) \Bigg\}. \qquad (22)$$

Multi-CCD PSF model

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$$

Dimensionality reduction: reduced number of PSF features in S

Multi-CCD PSF model

Main optimisation problem

$$\min_{\substack{S_1,\dots,S_N,\tilde{S}\\ \alpha_1,\dots,\alpha_N,\tilde{\alpha}}} \Bigg\{ \sum_{k=1}^{N} \Bigg(\frac{1}{2} \| Y_k - \mathcal{F}_k (S_k \alpha_k V_k^\top + \tilde{S} \tilde{\alpha} \Pi_k) \|_F^2 + \\ \sum_{i=1}^{r_k} \| \boldsymbol{w}_{k,i} \odot \Phi \mathbf{s}_{k,i} \|_1 + \underline{\iota_+ (S_k \alpha_k V_k^\top + \tilde{S} \tilde{\alpha} \Pi_k)} + \iota_{\Omega_k}(\alpha_k) \Bigg) \\ + \sum_{i=1}^{\tilde{r}} \| \tilde{\boldsymbol{w}}_i \odot \Phi \tilde{\mathbf{s}}_i \|_1 + \iota_{\tilde{\Omega}}(\tilde{\alpha}) \Bigg\}. \tag{22}$$

Positivity: PSF model should only contain positive pixels

Multi-CCD PSF model

Main optimisation problem

$$\min_{\substack{S_1,\dots,S_N,\tilde{S} \\ \alpha_1,\dots,\alpha_N,\tilde{\alpha}}} \left\{ \sum_{k=1}^{N} \left(\frac{1}{2} \| Y_k - \mathcal{F}_k(S_k \alpha_k V_k^\top + \tilde{S}\tilde{\alpha}\Pi_k) \|_F^2 + \sum_{i=1}^{r_k} \underline{\| \boldsymbol{w}_{k,i} \odot \Phi \mathbf{s}_{k,i} \|_1} + \iota_+(S_k \alpha_k V_k^\top + \tilde{S}\tilde{\alpha}\Pi_k) + \iota_{\Omega_k}(\alpha_k) \right) + \sum_{i=1}^{\tilde{r}} \underline{\| \tilde{\boldsymbol{w}}_i \odot \Phi \tilde{\mathbf{s}}_i \|_1} + \iota_{\tilde{\Omega}}(\tilde{\alpha}) \right\}. \quad (22)$$

Model denoising: PSF model should not learn noise

Sparse representation of PSF features in an wavelet decomposition

Multi-CCD PSF model

Main optimisation problem

$$\min_{\substack{S_1,\ldots,S_N,\tilde{S} \\ \alpha_1,\ldots,\alpha_N,\tilde{\alpha}}} \left\{ \sum_{k=1}^{N} \left(\frac{1}{2} \| Y_k - \mathcal{F}_k (S_k \alpha_k V_k^\top + \tilde{S} \tilde{\alpha} \Pi_k) \|_F^2 + \sum_{i=1}^{r_k} \| \boldsymbol{w}_{k,i} \odot \Phi \mathbf{s}_{k,i} \|_1 + \iota_+ (S_k \alpha_k V_k^\top + \tilde{S} \tilde{\alpha} \Pi_k) + \underline{\iota_{\Omega_k}(\alpha_k)} \right) + \sum_{i=1}^{\tilde{r}} \| \tilde{\boldsymbol{w}}_i \odot \Phi \tilde{\mathbf{s}}_i \|_1 + \underline{\iota_{\tilde{\Omega}}(\tilde{\alpha})} \right\}. \quad (22)$$

Spatial constraint: the PSF features specialise on a specific spatial variation

Sparsity enforcement of weight matrices on spatial variation dictionaries

Algorithm 1 Multi-CCD Resolved Components Analysis

Initialisation:

Preprocessing()

for $k = 1$ to K **do**

  Harmonic constraint parameters $(e_{k,i}, a_{k,i})_{1 \leq i \leq r_k} \rightarrow V_k^\top, \alpha_k^{(0,0)}$

  $0_{n_y^2 \times r_k} \rightarrow S_k^{(0,0)}$

end for

Global coordinates $\rightarrow \Pi_k, \tilde{\alpha}^{(0,0)}$ $\qquad (\tilde{\alpha}^{(0,0)} = I)$

$0_{n_y^2 \times \tilde{r}} \rightarrow \tilde{S}^{(0,0)}$

Alternate minimisation:

for $l = 0$ to l_{max} **do** — Algorithm's main iterations

  for $n = 0$ to n_G **do** — Global alternating iterations

    Noise level, $\tilde{\alpha}^{(l,n)} \rightarrow$ update $\tilde{W}^{(l,n)}$

$$\tilde{S}^{(l+1,n+1)} = \arg\min_{\tilde{S}} \{ \textstyle\sum_{k=1}^{K} \frac{1}{2} \| Y_k - \mathcal{F}_k (S_k^{(l,0)} \alpha_k^{(l,0)} V_k^\top + \tilde{S} \tilde{\alpha}^{(l,n)} \Pi_k) \|_F^2 + \sum_i \| \tilde{\boldsymbol{w}}_i^{(l,n)} \odot \Phi \tilde{\mathbf{s}}_i \|_1 \} \tag{I}$$

$$\tilde{\alpha}^{(l+1,n+1)} = \arg\min_{\tilde{\alpha}} \{ \textstyle\sum_{k=1}^{K} \frac{1}{2} \| Y_k - \mathcal{F}_k (S_k^{(l,0)} \alpha_k^{(l,0)} V_k^\top + \tilde{S}^{(l+1,n+1)} \tilde{\alpha} \Pi_k) \|_F^2 + \iota_{\tilde{\Omega}}(\tilde{\alpha}) \} \tag{II}$$

  end for

  for $n = 0$ to n_L **do** — Local alternating iterations

    for $k = 1$ to K **do** — CCD iterations

      Noise level, $\alpha_k^{(l,n)} \rightarrow$ update $W_k^{(l,n)}$

$$S_k^{(l+1,n+1)} = \arg\min_{S_k} \{ \tfrac{1}{2} \| Y_k - \mathcal{F}_k (S_k \alpha_k^{(l,n)} V_k^\top + \tilde{S}^{(l+1,n_G)} \tilde{\alpha}^{(l+1,n_G)} \Pi_k) \|_F^2 + \textstyle\sum_i \| \boldsymbol{w}_{k,i}^{(l,n)} \odot \Phi \mathbf{s}_{k,i} \|_1 + \iota_+ (S_k \alpha_k^{(l,n)} V_k^\top + \tilde{S}^{(l+1,n_G)} \tilde{\alpha}^{(l+1,n_G)} \Pi_k) \} \tag{III}$$

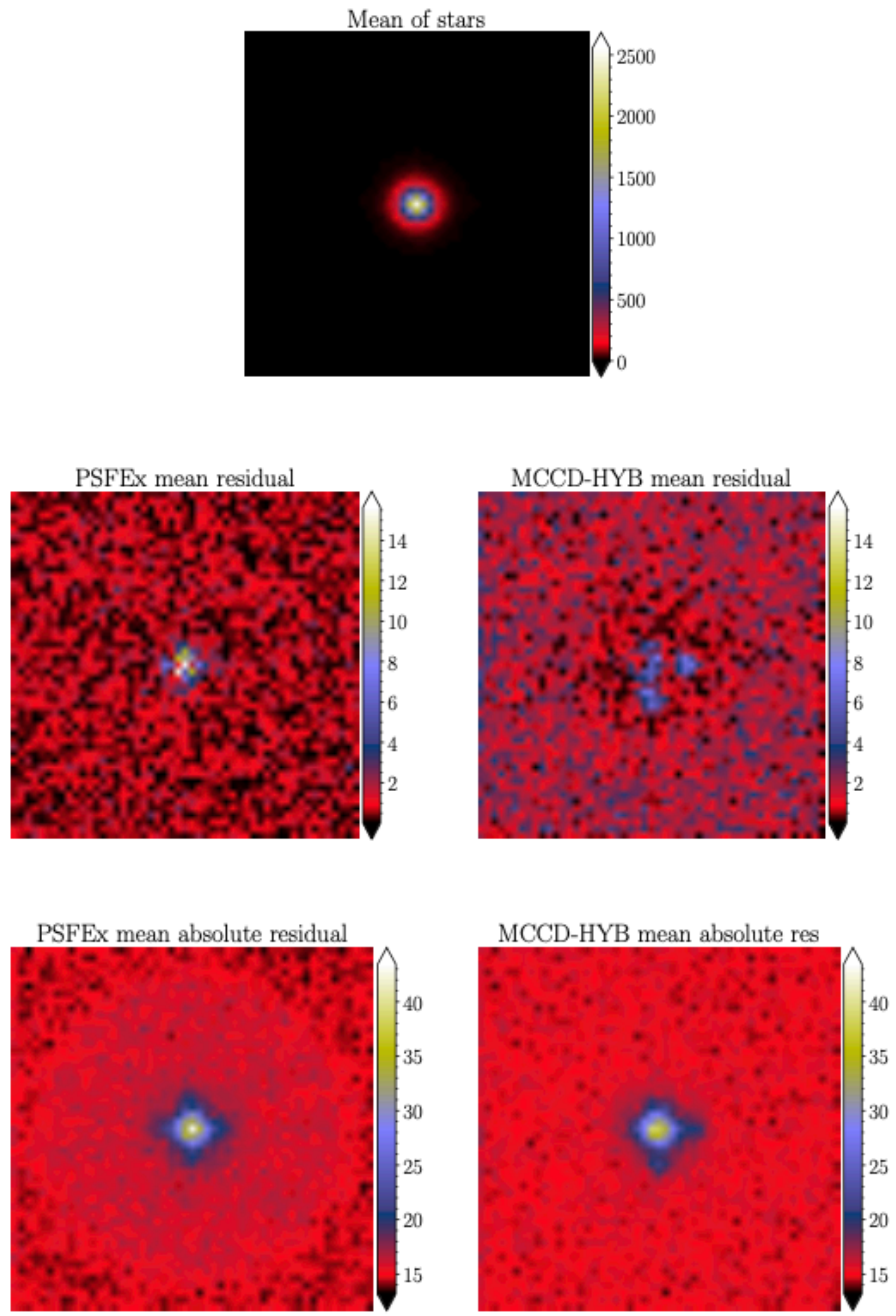
$$\alpha_k^{(l+1,n+1)} = \arg\min_{\alpha_k} \{ \tfrac{1}{2} \| Y_k - \mathcal{F}_k (S_k^{(l+1,n+1)} \alpha_k V_k^\top + \tilde{S}^{(l+1,n_G)} \tilde{\alpha}^{(l+1,n_G)} \Pi_k) \|_F^2 + \iota_{\Omega_k}(\alpha_k) \} \tag{IV}$$

    end for

  end for

end for

Results


Mean of stars
2500
2000
1500
1000
500
0
PSFEx mean residual
MCCD-HYB mean residual
14
12
10
8
6
4
2
PSFEx mean absolute residual
MCCD-HYB mean absolute res
40
35
30
25
20
15