Formally Verifying the KZG Polynomial Commitment Scheme

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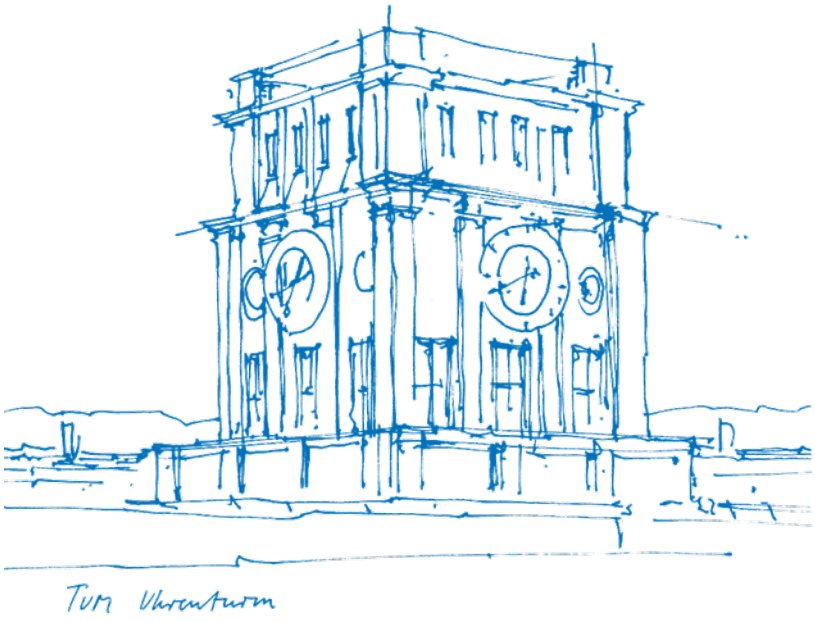
Chair for Logic and Verification
28. Mai 2024



Outline

- Motivation
- Polynomial Commitment Schemes
- KZG
- Formal Verification

Motivation

What is formal verification?

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- providing a formal proof

What is formal verification?

- providing a formal proof
  $\Rightarrow$ providing a machine-checkable proof

What is formal verification?

- providing a formal proof
  $\Rightarrow$ providing a machine-checkable proof
- undecidable

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- providing a formal proof
  $\Rightarrow$ providing a machine-checkable proof
- undecidable
  $\Rightarrow$ *Pain in the A***

Why formally verify the KZG?

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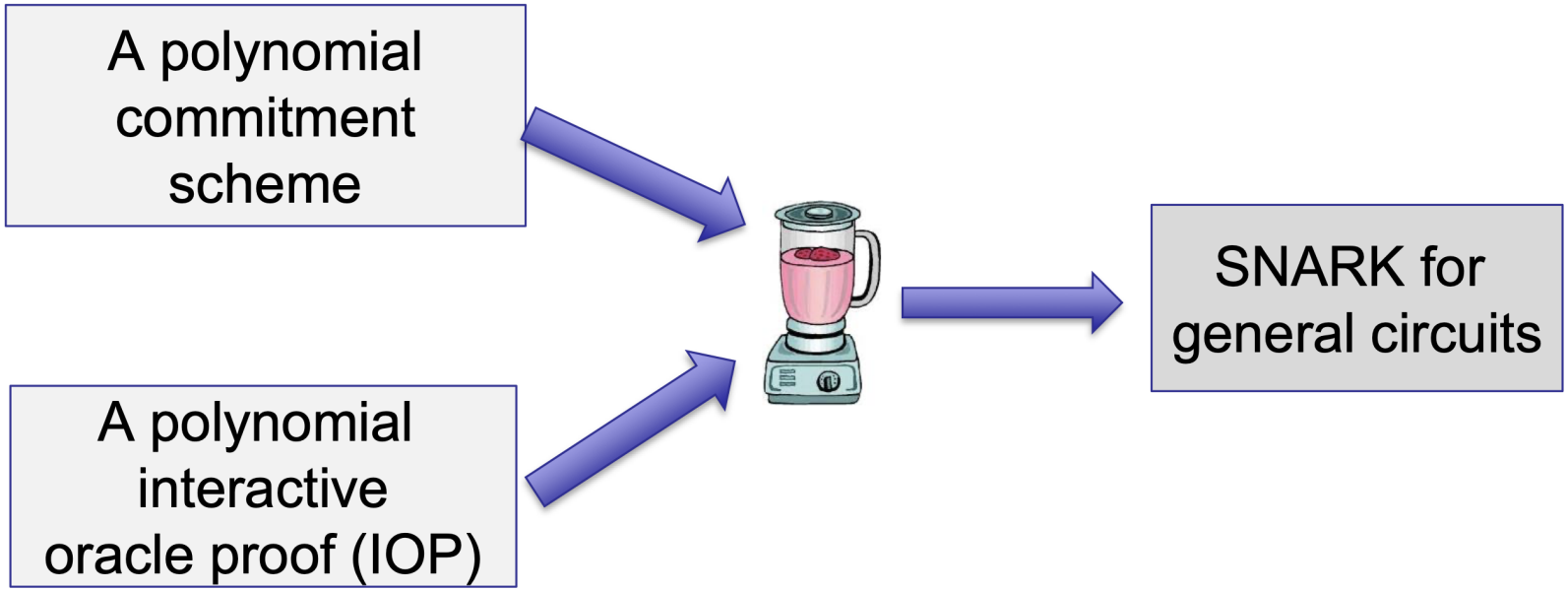

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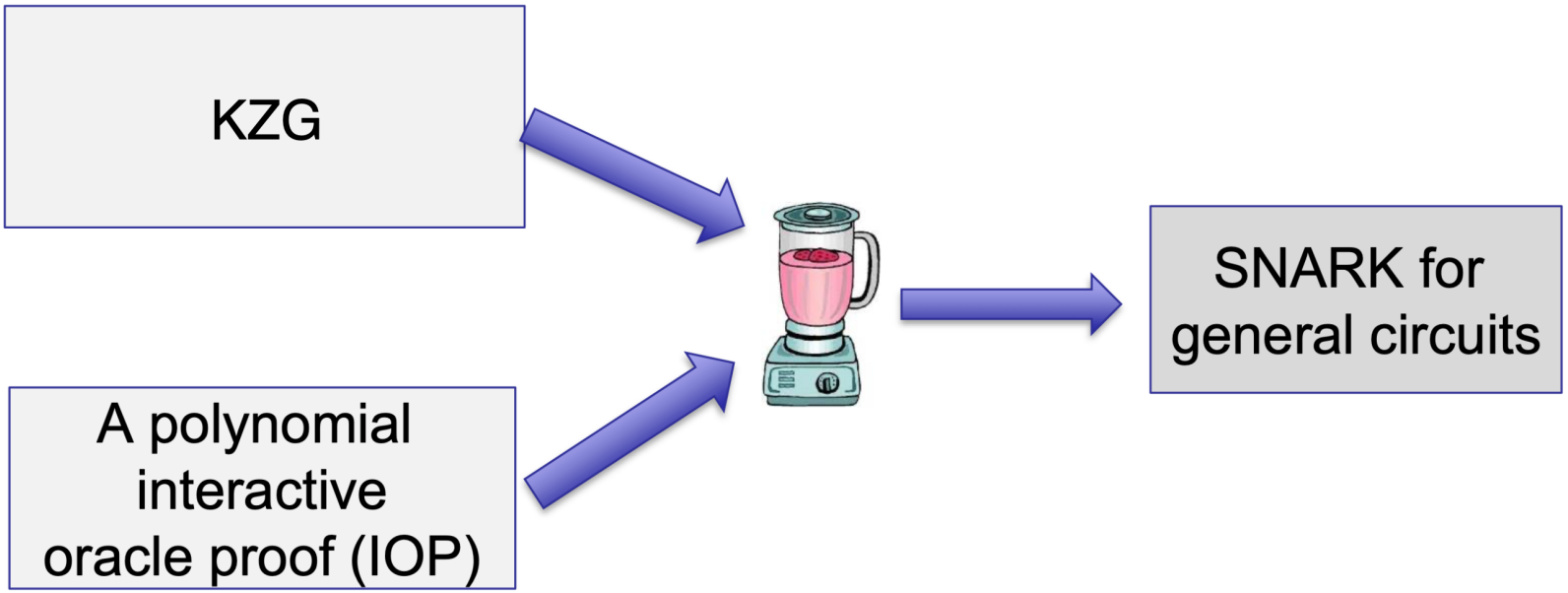

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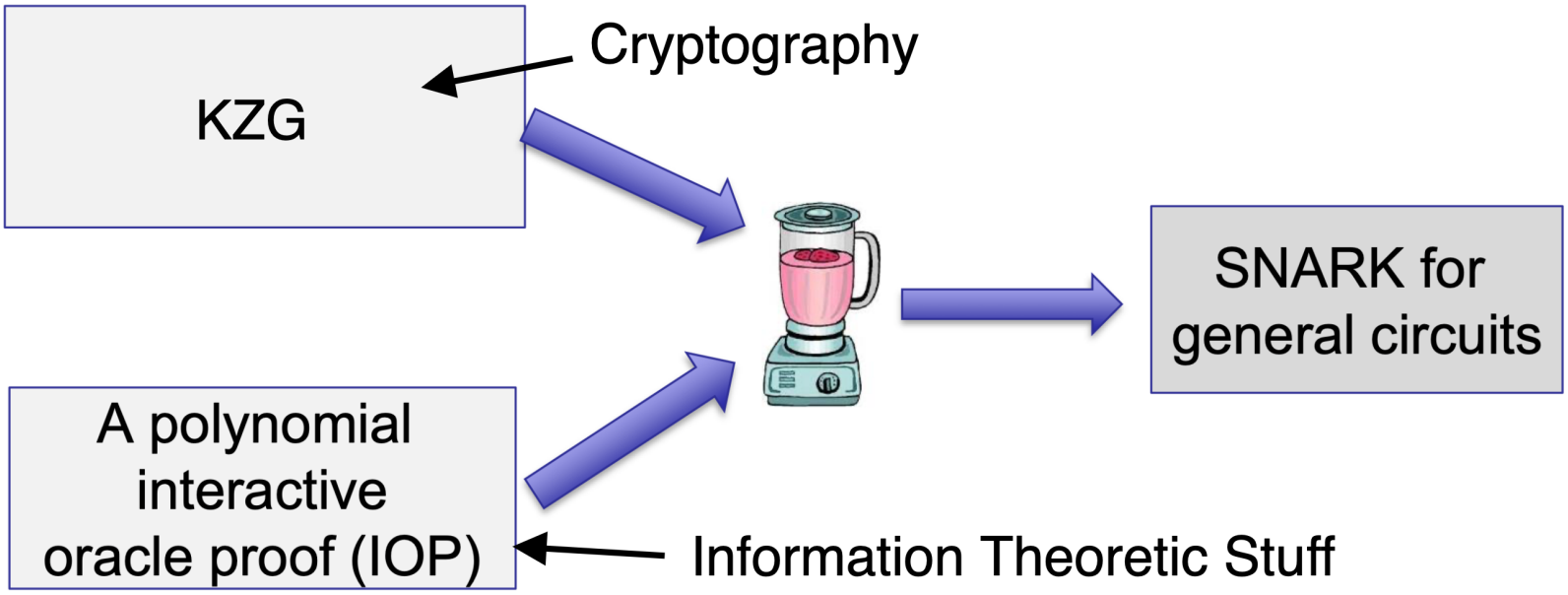
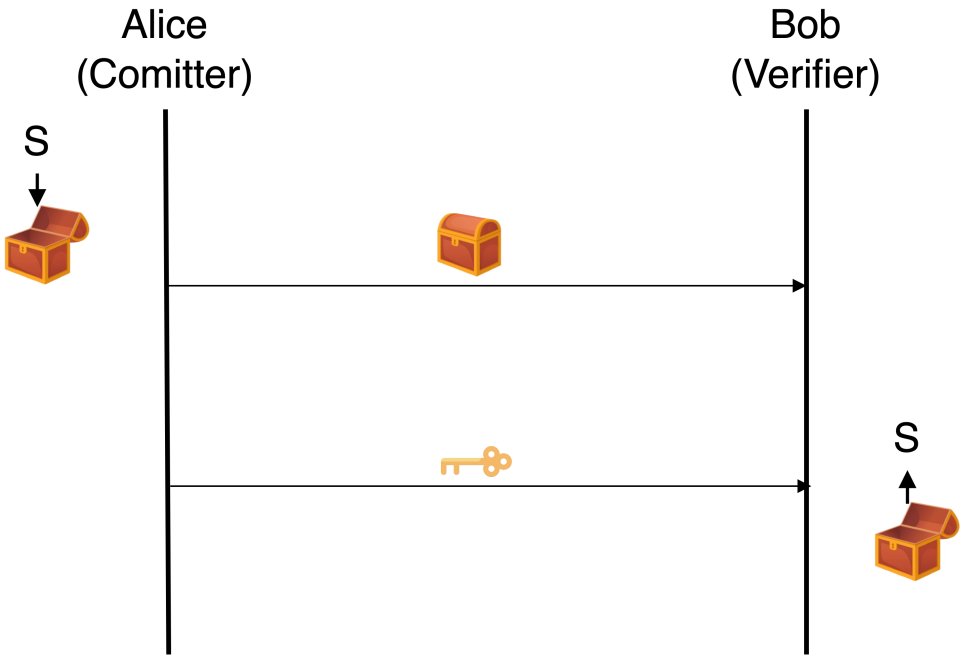


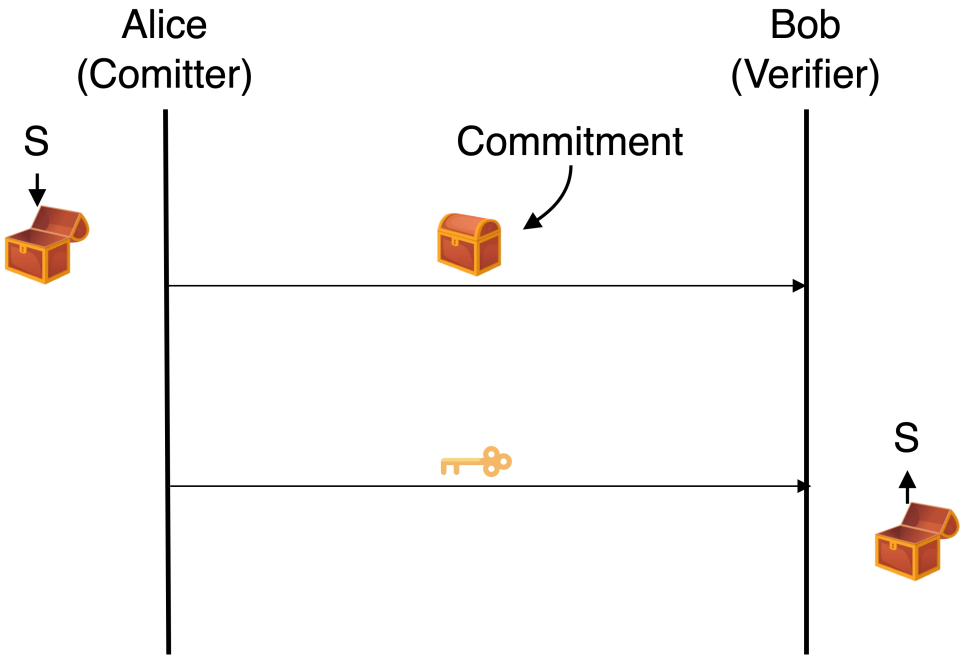
Polynomial Commitment Scheme

Commitment Scheme

Commitment Scheme



Commitment Scheme



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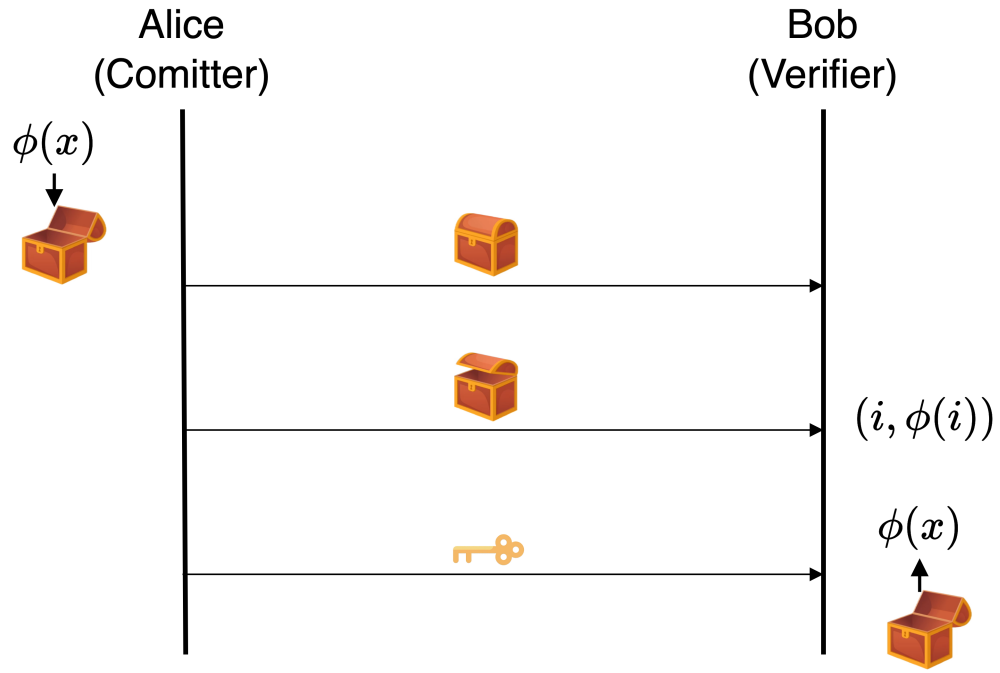
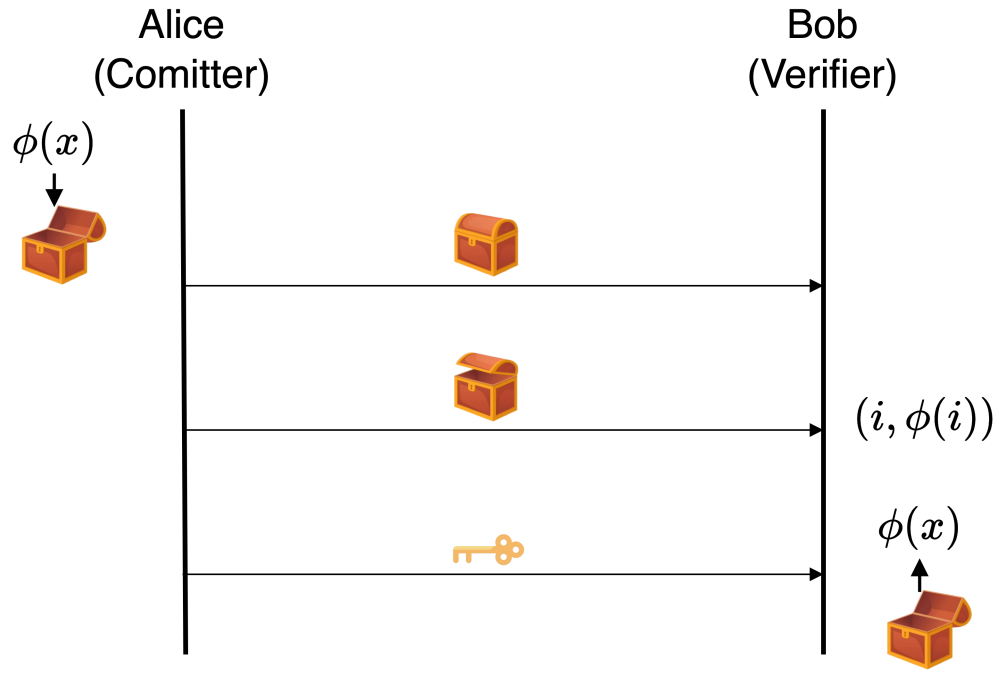


The KZG

KZG - Polynomial Commitment Scheme



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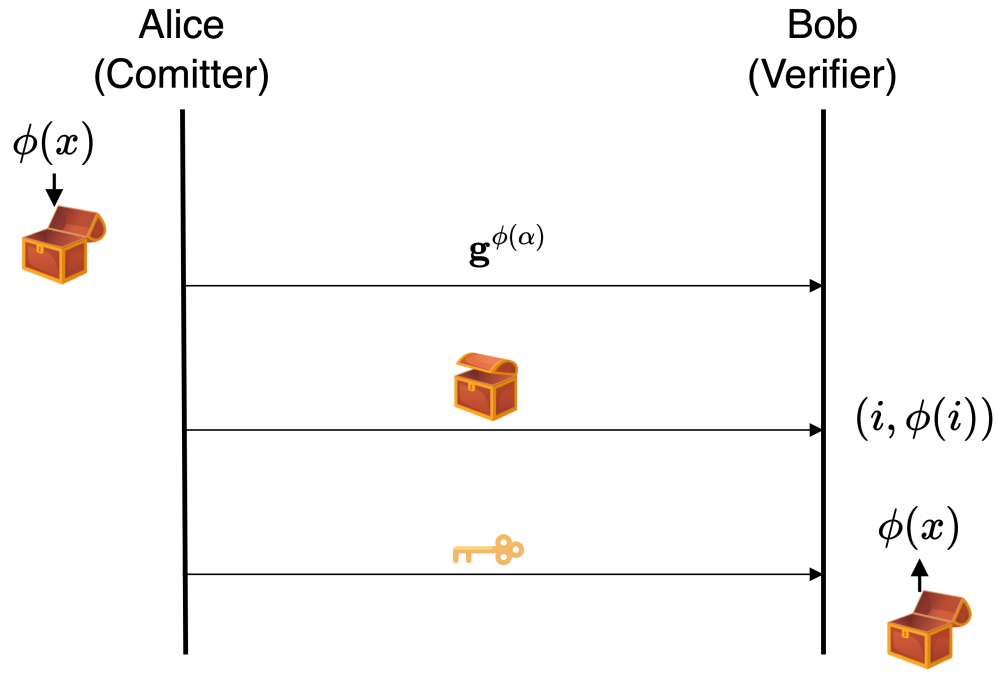

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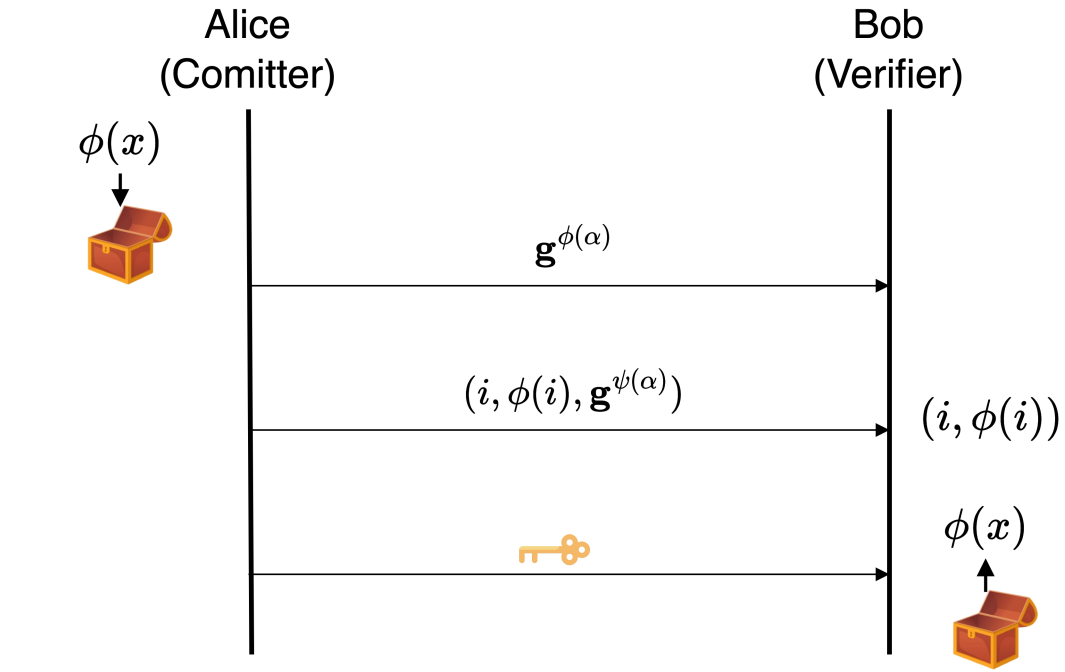

KZG - how to prove a peek? (Intuition)

Commitment - $\mathbf{g}^{\phi(\alpha)}$

Peek - $(i, \phi(i), \mathbf{g}^{\psi(\alpha)})$

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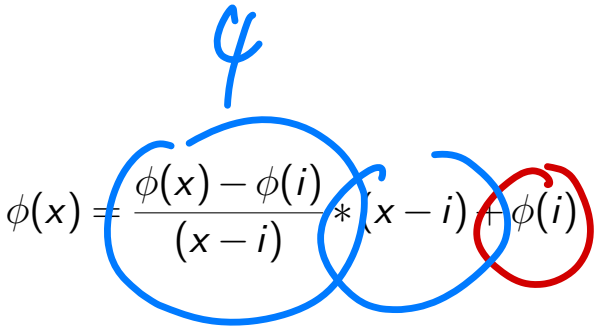

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$$\phi(x) = \psi(x) * (x - i) + \phi(i)$$

KZG - how to prove a peek? (Intuition)

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$x = i$

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$\phi(i) = \phi(i)$

KZG - how to prove a peek? (Intuition)

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g^{α}

$$\phi(x) = \psi(x) * (x - i) + \phi(i)$$

$G \times G \to G_T$

$e(g^a, g^b) = g^{a \cdot b}$

$\Downarrow$ Schwarz-Zippel

$$\phi(\alpha) = \psi(\alpha) * (\alpha - i) + \phi(i)$$

$g^{\phi(\alpha)^{\alpha}}$

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$$\Downarrow \text{ Schwarz-Zippel}$$

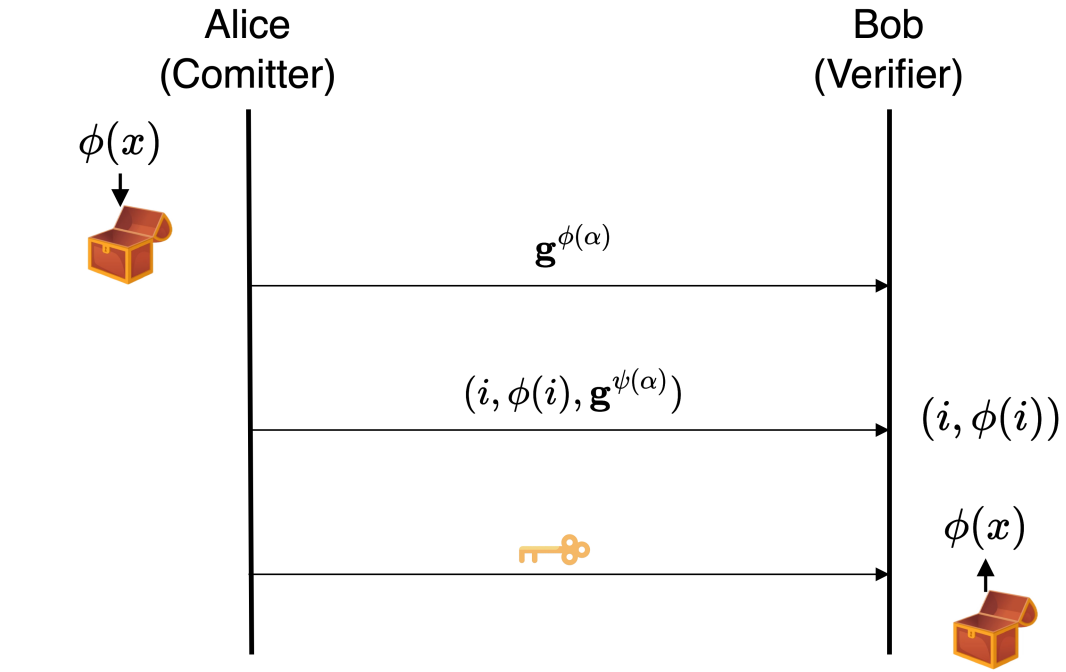
$$\phi(\alpha) = \psi(\alpha) * (\alpha - i) + \phi(i)$$

$$\Downarrow \text{ Pairing}$$

$$e\left(\mathbf{g}^{\phi(\alpha)}, \mathbf{g}\right) = e\left(\mathbf{g}^{\psi(\alpha)}, \frac{\mathbf{g}^{\alpha}}{\mathbf{g}^{i}}\right) \cdot e\left(\mathbf{g}^{\phi(i)}, \mathbf{g}\right)$$

Disclaimer: This was soley intuition.
Not security proofs.

KZG - Polynomial Commitment Scheme



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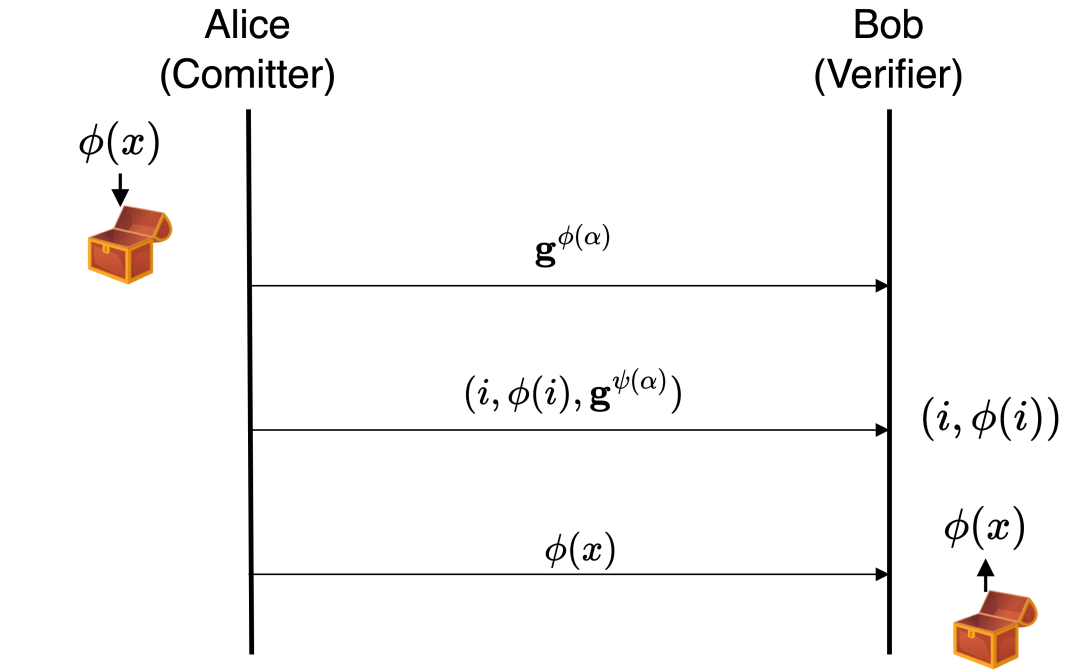

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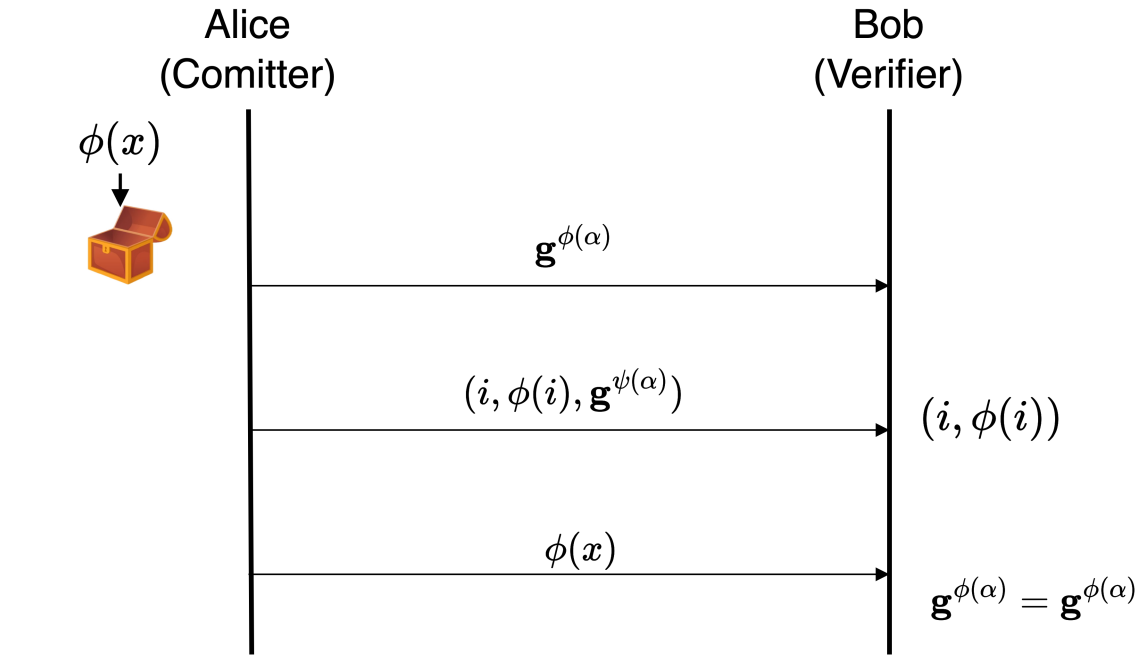

Formal Verification

What do we formally verify? (Highlevel)

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Security proofs for the KZG:

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- polynomial binding

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What do we formally verify? (Highlevel)

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- polynomial binding $\rightarrow$ Alice cannot change $\phi(x)$ in 📦
- evaluation binding $\rightarrow$ Alice cannot reveal 📦$:=(i, \phi(i))$ and 📦$' := (i, \phi(i)')$ with 📦$\neq$ 📦$' \equiv \phi(i) \neq \phi(i)'$

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Standard PCS Security Properties

What do we formally verify? (Highlevel)

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Standard PCS Security Properties SNARK precondition

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Games against adversary Eve

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$\rightarrow$ games as probabilistic algorithms

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$$Pr(\text{Eve wins game}) \approx 0$$

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Games against adversary Eve

→ games as probabilistic algorithms

→ Eve modeled as arbitrary probabilistic algorithm

→ Goal: prove

$$Pr(\text{Eve wins game}) \approx 0$$

Example Hiding Game:

$$\left(\begin{array}{c} C \leftarrow \text{📦}, \\ wtnss \leftarrow \text{📦📦📦📦📦📦}\ldots, \\ \phi' \leftarrow Eve(C, wtnss), \\ : \ \phi = \phi' \end{array}\right)$$

How do we prove this?

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$\Rightarrow$ Reduction from Hard Problem

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Example Discrete Log:

$$\left(\begin{array}{c} a \xleftarrow{\$} \mathbb{Z}_p \\ a' \leftarrow Eve(\mathbf{g}^a) \\ : a = a' \end{array}\right)$$

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$$\left(\begin{array}{c} C \leftarrow \text{📦} := \mathbf{g}^{\phi(\alpha)}, \\ \mathit{wtnss} \leftarrow \text{📦📦📦📦📦📦}\ldots, \\ \phi' \leftarrow \mathit{Eve}(C, \mathit{wtnss}), \\ : \ \phi = \phi' \end{array}\right)$$

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Reduction:

let $I = [(y, \mathbf{g}^b), \ldots]$

How do we prove this?

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Reduction:

let $I = [(y, \mathbf{g}^b), \ldots]$

$$\begin{pmatrix} \mathbf{g}^{\phi(\alpha)} \leftarrow \text{interpolate}((x, \mathbf{g}^a) \# I), \\ \mathit{wtnss} \leftarrow \text{📦📦📦📦📦📦}\ldots, \\ \phi' \leftarrow \mathit{Eve}(\mathbf{g}^{\phi(\alpha)}, \mathit{wtnss}), \\ : \ \phi = \phi', \\ \phi(x) \simeq \alpha \end{pmatrix}$$

How do we prove this?

$\Rightarrow$ Reduction from Hard Problem

Example Discrete Log:

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$\Rightarrow$ Eve efficient $\Longrightarrow$ DL broken

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$\Rightarrow$ Eve efficient $\implies$ DL broken
$\Rightarrow$ Is this reduction cryptographically sound?

How do we prove this?

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NO!

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$\Rightarrow$ Is this reduction cryptographically sound?
NO!
$\Rightarrow$ "no adversary can break hiding for arbitrary polynomials"
vs "for arbitrary polynomials no adversary can break hiding"

A Better Way - Sequences of Games [Shoup04]

Transformation via Game-hops

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Transformation via Game-hops $\Rightarrow$ show security game $=$ hardness game for some adversary

$$\text{Security Game} = \left(\quad \dots \quad \right) = \left(\quad \dots \quad \right) \leq \left(\quad \dots \quad \right) = \cdots = \left(\quad \dots \quad \right) = \text{Hardness Assumption Game}$$

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2. Failure-event:

$$\begin{pmatrix} a \xleftarrow{\$} \mathbb{Z} \\ a' \leftarrow \mathit{Eve}(\mathbf{g}^a) \\ : a = a' \end{pmatrix} \approx \begin{pmatrix} a \xleftarrow{\$} \mathbb{Z} \\ a' \leftarrow \mathit{Eve}(\mathbf{g}^{\frac{1}{a}}) \\ : \frac{1}{a} = a' \end{pmatrix}$$

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2. Failure-event:

$$\begin{pmatrix} a \xleftarrow{\$} \mathbb{Z} \\ a' \leftarrow \mathit{Eve}(\mathbf{g}^a) \\ : a = a' \end{pmatrix} \approx \begin{pmatrix} a \xleftarrow{\$} \mathbb{Z} \\ a' \leftarrow \mathit{Eve}(\mathbf{g}^{\frac{1}{a}}) \\ : \frac{1}{a} = a' \end{pmatrix}$$

3. Over-estimation:

A Better Way - Sequences of Games [Shoup04]

Transformation via Game-hops $\Rightarrow$ show security game $=$ hardness game for some adversary

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$\Rightarrow$ prove game-hop correctness in Higher Order Logic (HOL) (machine checked)

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- first step to formaliy verify KZG-based SNARKs

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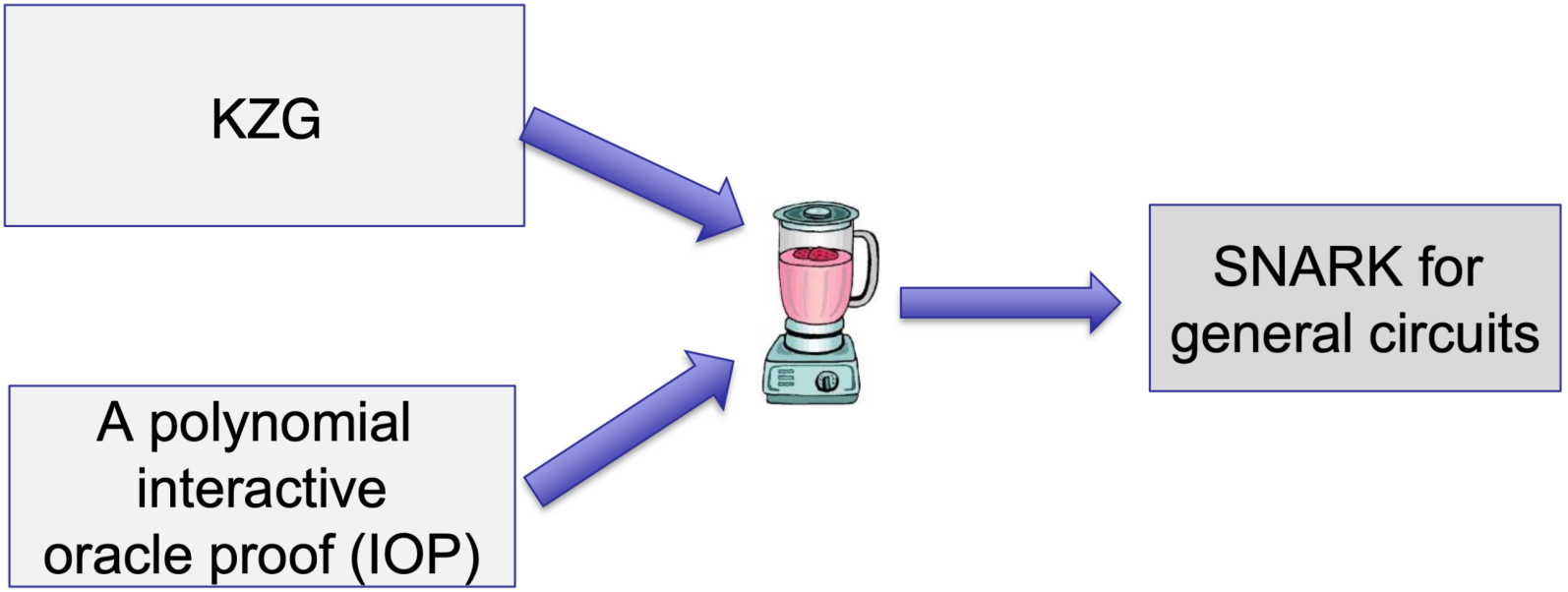

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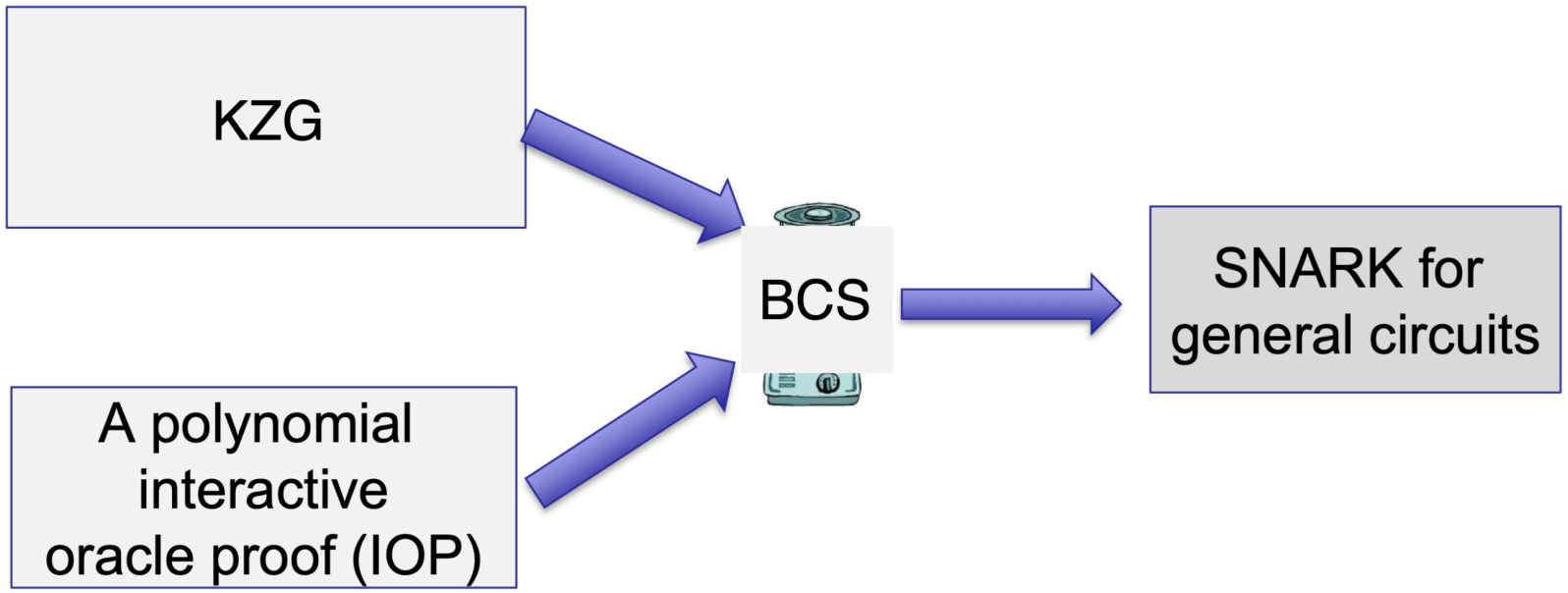

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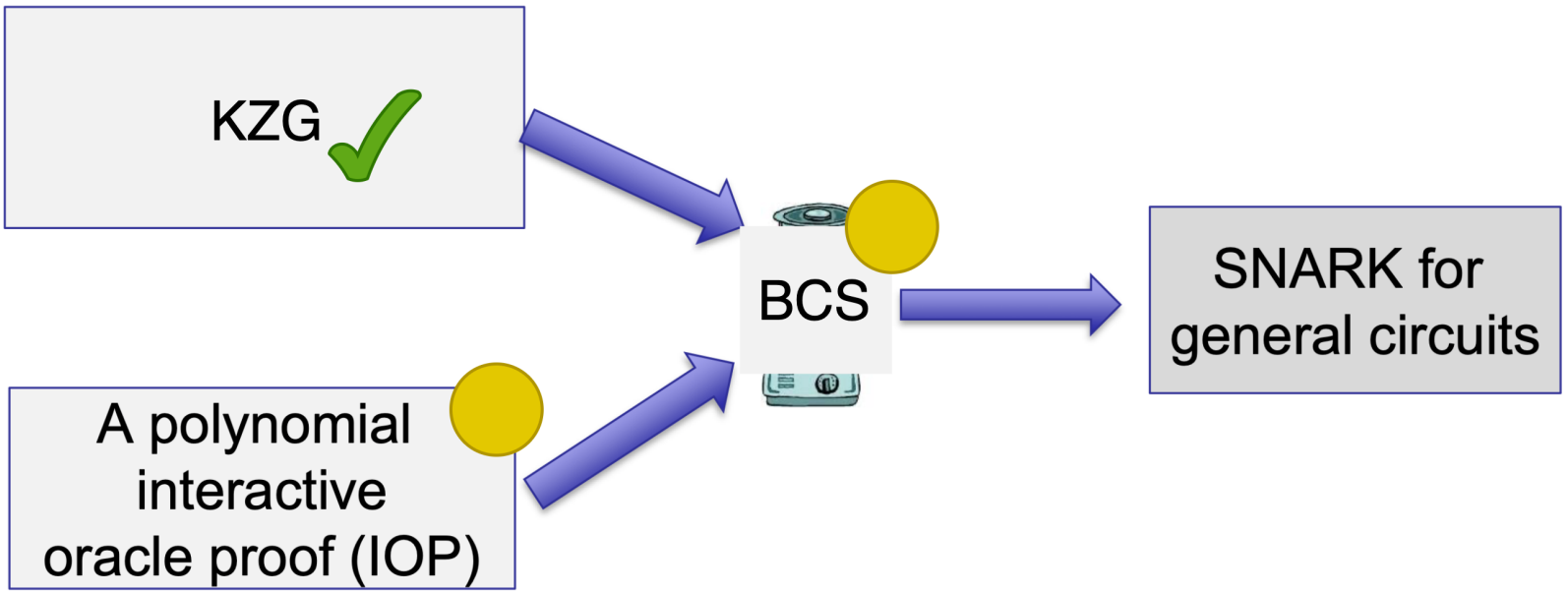

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Formally Verifying the KZG Polynomial Commitment Scheme

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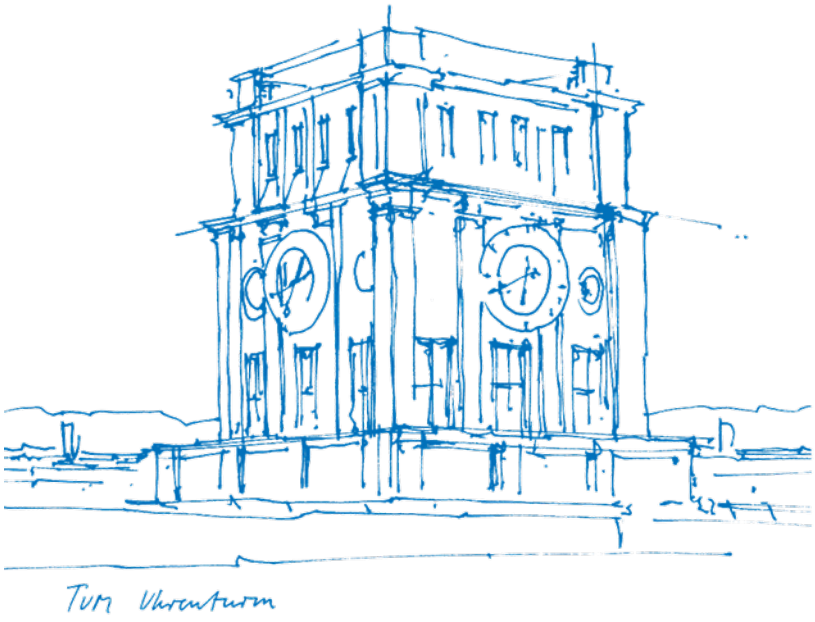
Chair for Logic and Verification
28. Mai 2024



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