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Project cost risk analysis: A Bayesian networks approach for modeling dependencies between cost items

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Abstract

Uncertainty of cost items is an important aspect of complex projects. Cost uncertainty analysis aims to help decision makers to understand and model different factors affecting funding exposure and ultimately estimate the cost of project. The common practice in cost uncertainty analysis includes breaking the project into cost items and probabilistically capturing the uncertainty of each item. Dependencies between these items are important and if not considered properly may influence the accuracy of cost estimation. However these dependencies are seldom examined and there are theoretical and practical obstacles in modeling them.

This paper proposes a quantitative assessment framework integrating the inference process of Bayesian networks (BN) to the traditional probabilistic risk analysis. BNs provide a framework for presenting causal relationships and enable probabilistic inference among a set of variables. The new approach explicitly quantifies uncertainty in project cost and also provides an appropriate method for modeling complex relationships in a project, such as common causal factors, formal use of experts' judgments, and learning from data to update previous beliefs and probabilities. The capabilities of the proposed approach are explained by a simple example.

Keywords: Project cost analysis; Bayesian networks; Common cause; Dependency

1. Introduction

To make cost estimates, project managers use cost analysis; a discipline that attempts to forecast the ultimate cost of a project. The difficulty about this analysis, especially for complex projects, is that there are a lot of uncertainties about cost items such as technology, productivity of human resources, economic conditions, market conditions, prices, inflation and other future risks and events. In general uncertainty occurs for a number of reasons:

- Uniqueness (no similar experience)
- Variability (trade-off between performance measure like time, cost, and quality)
- Ambiguity (lack of clarity, data, structure, and bias in estimates)

Cost uncertainty analysis is an important aspect of cost estimation that helps decision makers to understand not only the potential funding exposure but also the nature of risks for a particular project or program and possible responses to them. Without considering the uncertainty involved, there is a high risk that the actual cost of a project exceeds what it was originally anticipated, which in turn causes several other risks such as delays and performance problems (Elkjaer, 2000; Lai et al., 2008).

Several techniques including Regression modeling, Artificial Neural Networks (ANNs), feature-based method (FBM) and case-based reasoning (CBR) are proposed for modeling risk and uncertainty in project cost analysis. Section 2 briefly reviews a number of notable techniques. However none of

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these techniques capture the dependencies between project cost items consistent to real word conditions.

Simulation-based techniques are the state-of-the-art approach that are adopted by many project management software tools and are arguably the best practice available (Arena et al., 2006; Chapman and Ward, 2011; Chou et al., 2009; Cooper et al., 2005). Monte Carlo simulations (MCSs) treat cost uncertainty with the input distributions derived from elicitation sessions with technical experts or, occasionally, from historical data. In particular, the ultimate goal is to estimate a cumulative distribution function (equivalently, a probability distribution function) for the final cost, which in principle contains all of the uncertainty information and allows the computation and comparison of different project riskiness.

Steadily improving software and the wide availability of powerful desktop computers make this approach straightforward to be implemented in practice. It is also attractive to analysts because of its conceptual simplicity and the widespread use of simulation technology elsewhere in the business world (Cooper et al., 2005).

However, a debate continues over the inclusion of explicit correlation in simulated quantities. The cost items may not be independent—that is, the high cost of one element may affect the cost of another because of shared technology or manufacturing resources. Such correlation—if it exists—must be captured, or the final distribution will not accurately represent the uncertainty in the final cost (Book and Young, 1997; Vose, 2000). The difficulty of doing so is an obstacle to applying quantitative risk methods. However, modeling this correlation is known to be much more difficult for human subjects to do without using various conditioning arrangements (Kadane and Wolfson, 1998). This problem is compounded by the fact that correlations between random variables, which can only take positive values (as duration and cost can), are not unconstrained. Unlike normal distributions, where the correlations between unlimited sets of normal random variables can be arbitrarily specified, subject to an overall condition of a positive definite correlation matrix, the constraints on correlated positive random variables are stronger and far less intuitive.

As is the common practice of combining probability distribution, MCS employs a correlation coefficient matrix to model dependencies between distributions of cost items (Chapman and Ward, 2011). Usually, the same project participants that estimated the cost ranges will estimate the correlation. It ranges between $+1$ and -1. A correlation coefficient of $+1$ signifies a perfect positive relationship, while -1 shows a perfect negative one. Practically, it is difficult to estimate these correlation coefficients because the estimator has to specify all the joint distributions among cost items, which is highly impractical when the number of items is large (Yang, 2011). Moreover, matrix theory implies that a correlation matrix will not have any negative determinants in real life. Yet, with several people sitting around in a room, the correlation matrix that results may not be feasible. Adjusting the coefficients allows the user to ensure that the correlation matrix is at least not demonstrably impossible. This is a minimal test and does not ensure us that the correlation coefficients are “right” in any sense.

The dependencies between project cost items have seldom been examined (Chou et al., 2009). In order to consider the dependencies and correlations between the cost items, this paper proposes a quantitative assessment framework integrating the inference process of a Bayesian network to the traditional probabilistic risk analysis. Bayesian networks (BNs) are a framework that presents probabilistic relationships and enables probabilistic inference among a set of variables (Heckerman et al., 1995). BNs provide an intuitively compelling approach for handling causal relationships and external influences. The application of BNs as a tool for mapping causal dependencies between frequencies and severities of risk events is demonstrated for modeling financial institutions’ operational risk (Mittnik and Starobinskaya, 2010).

The proposed model of this paper handles the correlation problem by modeling the uncertainty of common characteristics and performance indicators which affect cost items. The motivation behind this paper is derived from the fact that despite a causal relationship between uncertainty sources and cost items; this causality is not modeled in current state-of-the-art project cost risk analysis techniques (such as simulation techniques).

The purpose of the proposed approach is to improve the assessment of likelihood distribution of project cost. The new approach explicitly quantifies uncertainty in project cost and also provides an appropriate method for modeling complex relationships and factors in project, such as: causal relation between variables, common causal factors, formal use of expert’s judgments, and learning from data to update previous beliefs and probabilities.

The remainder of this paper is organized as follows. In Section 2, we briefly review previous approaches for project cost estimating. Section 3 describes the concept of correlation and dependency between cost items. An overview of BN methodology and its application to modeling dependency, and developing the proposed model is covered in Section 4. For clarification purposes, a simple example is studied in Section 5. The paper ends with the conclusion.

2. Advanced techniques for project cost estimating: a review

Several techniques and models have been developed to support better project cost estimating. Cost estimates can be provided as probabilistic or deterministic values. As each cost item is a random variable representing an unknown future cost, a deterministic value should be applied only when detailed or specific cost estimates are available from a reliable source. Deterministic values can be achieved through definitive formulation, linear programming, and optimization approaches. On the other hand, probabilistic cost estimating should be utilized during the early project development stages, especially when the reliability of information is questionable. Probabilistic models treat the future final cost of a project as a random variable and use formal probability methods to quantify its uncertainty.

Regression modeling, MCS, and Artificial Neural Networks (ANNs) are commonly used to estimate pre-conceptual project cost. Regression modeling and ANNs normally derive deterministic values, whereas MCS normally derives a probabilistic range.

Zhang and Fuh (1998) studied the applicability of a feature-based cost estimation using a back-propagation neural network for estimating the cost of packaging products. Gunaydin and Doghan (2004) investigated the utility of neural network methodology to overcome cost estimation problems in the early phases of building design processes. Magnussen and Olsson (2006) provided a comparative analysis of cost estimates of major public investment projects. Peeters and Madauss (2008) described the different techniques, memo-technically called the 5C approach, and developed the use of a computerized tool, PRICE, to support these techniques against cost overruns in the space sector. Chou (2009, 2011) introduced streamlining Monte Carlo simulation processes and input probability distribution selection via hypothesis, and specification of correlation between simulated variables to create an early-stage cost distribution for budget allocation. Roy et al. (2011) presented the various data and information requirements for detailed cost estimating in the automotive industry. Their research project has identified the common cost estimation process model within the identified industry sector. The study identified the types of data and information requirements for cost estimating. Schexnayder et al. (2003) reported that 31 departments of transportation (DOTs) used lane-mile average values of previous projects to evaluate project cost during the pre-conceptual stage.

Analytical technique (bottom-up estimate) allows the estimation of the cost of a project from a decomposition of the work into manageable tasks, operations, or activities with easy calculation (PMI, 2013). The generic technique for cost estimation is the activity based costing (ABC) system that focuses on calculating the costs incurred on performing the activities to manufacture a product (Niazi et al., 2006). Ben-Arieh and Qian (2003) presented a methodology of applying ABC to evaluate the cost of the design and development activity for machined parts being produced in a controlled manufacturing facility. Tornberg et al. (2002) studied the possibilities of ABC and the modeling of design, purchasing and manufacturing processes in providing helpful cost information for product designers.

Parametric estimating technique, also known as feature-based method (FBM) or multiple regression analysis (MRA), estimates costs from product features, functions, and characteristics. It is a technique that applies validated relationships between a project's known technical and cost characteristics given a subset of factual data. Usually it is a necessity to get the detailed design to know the sketch of equipment to use feature-based method. Comprehensive studies have been carried out to describe the non-linear relationships between cost and project characteristics and generated higher-level predictability depending on the quality of the underlying data source and the sophisticated statistical techniques employed to develop the model (Emsley et al., 2002; Kwak and Watson, 2005; Lowe et al., 2006; Saito et al., 1991).

Artificial intelligence (AI) plays an innovative role in the cost estimating process though a considerable effort is needed to realize the effectiveness by validating the computing outcomes. For cost estimation, AI has been extensively utilized in the disciplines of construction industry (Dogan et al., 2006; Emsley et al., 2002) as well as manufacturing sectors, computer & software engineering, process industry, concurrent engineering, R&D management, and product development. A typical representative AI technique, ANNs, is a feasible alternative for early cost prediction owing to its ability to model complex systems given a minimal amount of data input. Numerous studies have proved its prediction power and excellent accuracy at the early stages of building and highway construction (Emsley et al., 2002).

Another prevalent AI technique is the case-based reasoning (CBR) technology using the nearest neighbor algorithm that is based on the rationale of characterizing projects with some essential attributes. These attributes are then used with weighting values to match similar cases. CBR requires a series of systematic procedures to extract the relevant knowledge from the experience, integrate a case into an established knowledge structure, index the case for later matching, adapt a new case with similar cases, and save to knowledge base. Many different studies have identified CBR as particularly appropriate for application to the conceptual stage of construction projects (An et al., 2007; Chou, 2009; Chua et al., 2001).

Simulation technique is commonly adopted to evaluate the probability of project total cost (Chapman and Ward, 2011; Yang, 2011). Monte Carlo simulation can simulate a random selection process multiple times to create multiple realizations. The primary simulation processes cover data collection, random-number generation, model formulation, analysis, and visual presentation. The steps in MCS are as follows: (a) identify the major and uncertain work components; (b) generate reproducible random numbers and random variates that reflect the true nature of the modeled uncertain item (using defined statistical distributions and a correlation matrix to model variability of and dependency between each uncertain item); (c) apply the model to calculate the desired output parameters with a predetermined number of iterations for the desired confidence and simulation error; (d) repeat steps (a)–(c) for a large number of realizations; and, (e) collect statistics on means, standard deviations, and each iteration's output to generate conclusions and draw graphs.

Finally MCS calculates the probability distribution function (PDF) and cumulative distribution function (CDF) of total project cost by generating random-numbers and modeling formulation. The mean value (μ) and standard deviation (σ) are calculated on the basis of the frequency distribution of the total project cost:

$$\mu = \frac{1}{\mathrm{n}} \sum\nolimits_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Y_i} \tag{1}$$

$$\sigma = \frac{1}{\mathrm{n}} \left(\sum\nolimits_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Y_i^2} - \mathrm{n}(\mu)^2 \right). \tag{2}$$

The mean μ is calculated by adding all the values for the total project cost ($\mathrm{Y_i}$) and dividing the sum by the number of iterations for the total cost (n). The standard deviation σ is a measure of the spread of the distribution.

There are some alternative methodologies and probabilistic approaches attempting to address the cost risk issue. Methods such as fault trees (Diekemann et al., 1996), propagation of errors (Morgan et al., 1990), and method of moments (Arena et al., 2006) are among the proposed approaches for handling uncertainty in project cost analysis.

This section discussed about advanced techniques and methodologies in project cost risk analysis. However, most of these techniques don't either consider the dependencies (assuming that cost items are independent) or use stochastic dependencies (correlation coefficient matrix) which are not sufficient to adequately assess project cost uncertainty. Therefore, more realistic models are needed to capture causal relationships/dependencies between project cost items, consistent to real word conditions.

3. Determination of dependency

Once distributions are determined, an estimator needs to specify the correlation and/or dependency between items accurately. Correlations between cost components/distributions are generally one of four types (Cooper et al., 2005): (a) common caused correlation: two distributions may be correlated because there is a common cause or driver affecting each distribution; for instance, the need to expand a right-of-way may affect an excavated area and increase the cost for both cost items, and vice versa. Such a correlation is very common in construction projects; (b) cause and effect or cascade correlation: Two distributions may be correlated when a problem in one distribution leads to a problem in another; for instance, an equipment breakdown in an earlier activity may cause delays, and replacing or repairing the equipment may not be as efficient as using the original one and, thus, may have ripple effects on the other activities; (c) compound consequence correlations: If two risks emerge, their combined effects may be greater than their individual effect and may have an exponential effect on overall risks. In this case, the correlation may no longer be linear; (d) other statistical dependence correlations: This refers to dependencies that are generally poorly defined and understood. A number of studies (Chau, 1995; Clark, 2001; Ranasinghe, 2000; Ranasinghe and Russell, 1992; Touran, 1993; Touran and Suphot, 1997; Wall, 1997; Yang, 2005) indicated that overlooking a correlation between cost components significantly influences the degree of risk associated with variances. For instance, if Variance (A + B) = Variance (A) + Variance (B) + 2 R · SQRT [Variance (A) · Variance (B)], the variance is the square of the standard deviation and R is the correlation coefficient between A and B. In this case, the positive correlation can lead to an underestimated variance, whereas a negative correlation can lead to an overestimation of variance.

When variable correlations are integrated into a model, inaccurate correlations can occur due to differences among distributions of cost items. Ranked correlations can be used to eliminate such a problem as such correlations make few assumptions about data distributions. Wall (1997) applied a pragmatic approach that involves a mixture of objective and subjective determinations to deal with correlations between variables. Wall concluded that this approach would lead to huge errors in estimates when the models do not consider correlations. It is more important to input correlations among construction elements than to choose the best-fitted distribution to represent cost elements (i.e., Beta or lognormal distributions). The next section presents a BN framework for modeling dependencies by capturing the common project causes which affect several cost items.

4. Proposed approach

This section explains a new methodology using the BN method to model the dependencies between various cost items. First a brief introduction of BNs and its application to modeling dependency is presented. Then the framework of the proposed method is described.

4.1. Bayesian networks (BNs): An overview

A Bayesian network (BN), also called a causal network or Bayesian belief network, is a powerful tool for knowledge representation and reasoning under conditions of uncertainty (Cheng et al., 2002), and visually presents the probabilistic relationships among a set of variables (Heckerman, 1997). It is frequently applied in real-world problems such as diagnosis, forecasting, automated vision, sensor fusion, and manufacturing control (Heckerman et al., 1995). It has been extended to other applications including software risk management (Fan and Yu, 2004), transportation (Ulegine et al., 2007), project scheduling (Khodakarami et al., 2007), ecosystem and environmental management (Uusitalo, 2007) and assessing new product development project (Chin et al., 2009). A Bayesian network has many advantages such as suitability for small and incomplete data sets, structural learning possibility, combination of different sources of knowledge, explicit treatment of uncertainty and support for decision analysis, and fast responses (Uusitalo, 2007).

A Bayesian belief network consists of qualitative and quantitative parts (van der Gaag, 1996). The qualitative part of a BN, the so-called structural learning is the graphical representation of independence holding among variables and has the form of a directed acyclic graph (DAG) that is popular in the statistics, the machine learning, and the artificial intelligence societies. The quantitative part of a BN, the so-called parameter learning, finds dependence relations as joint conditional probability distributions among variables using cause and consequence relationships from the qualitative part and data of variables. The network is commonly represented as a graph, which is a set of nodes and arrows. The nodes represent the probabilistic variables and the arrows represent the causal relationships between these variables. The nodes, which are the starting ones and do not have an inward arrow, are called the parent nodes. The other nodes, which have inward arrows connected to them, are the child nodes. In order to run the calculations it is necessary to define the states and probabilities for each node.

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One of the main advantages of BNs is their ability to model causal relationship among variables. This can be done (using Bayes rule) from cause to effect as well as from effect to cause. Bayes rule can be expressed as follows:

$$P\left({}^{A}/_{B}\right) = \frac{P\left({}^{B}/_{A}\right)P(A)}{P(B)}. \tag{3}$$

In a cost estimation context, this can be interpreted as follows. Fig. 1 shows a naïve Bayes BN model. Variables B_1, B_2 …, B_n represent n cost items, and variable A represents a 'common cause' to B, such as shared 'staff quality'. Let's start with a prior probability P(A), representing our belief about A before observing any relevant evidence. P(B|A) represents the likelihood for cost elements based on observing the level of staff quality. Suppose that B_i is observed. In Eq. (3), our revised belief for the probability of staff quality, the posterior probability $P(A|B_i)$, is obtained by multiplying the prior probability of staff quality P(A) by the likelihood $P(B_i|A)$ and then normalizing the results by dividing by the constant $P(B_i)$. Now, this revised belief for staff quality can be entered to predicting B_j (which are not observed yet) by using Eq. (4).

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}. \tag{4}$$

By common causes we are trying to capture:

- Organizational issues, which their variability can lead to cost uncertainty; such as people quality. Expert judgment can be used to provide estimates on such a complex or otherwise poorly understood phenomena.
- Variables such as available technology, materials, economic conditions, etc. which their problem can lead to project cost overrun. In order to support the identification of these factors, the most important cost overrun causes, extracted from reviewed articles, are summarized and presented in Appendix A.

This structure provides an appropriate mechanism for modeling dependencies in project. Furthermore BNs have following principal advantages (Luu et al., 2009): (1) Bayesian networks provide great flexibility in their capacities for accepting input and providing output; (2) Bayesian networks have the ability to allow the value of a variable to be entered as a known input or to evaluate the likelihood of a variable as an output of the system; (3) BNs can readily calculate the probability of events before and after the introduction of evidence and update its diagnosis or prediction; (4) Bayesian networks may be developed using expert opinion instead of requiring historical data; (5) Bayesian networks also allow variables to be added or removed without significantly affecting the remainder of the network because modifications to the network may be isolated; and (6) BNs gain insight into relationships among variables of the process due to its graphical display and (7) after a BN is constructed, sensitivity analysis is capable of analyzing how much a specific node is influenced by other nodes (sensitivity is represented by entropy: a larger entropy between nodes produces a bigger influence).

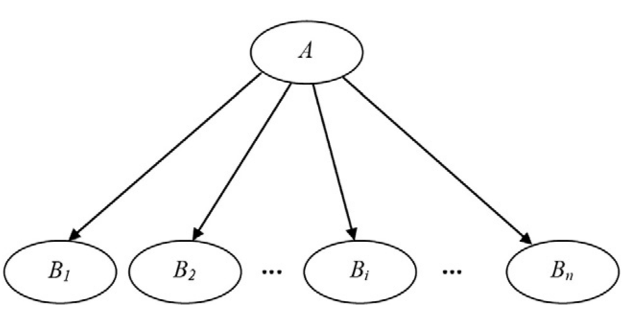

Fig. 1. A naïve BN model.

There are numerous tools that enable users to build BN models and run the propagation calculations. With such tools, it is possible to perform fast propagation in large BNs (with hundreds of variables) and display the probability graphs for under study variables.[1]

4.2. Model framework

The model employs a BN technology to conduct a causal analysis on important variables influencing project cost and provides probabilistic results which can improve our decisions. The schematic framework of the proposed model is portrayed in Fig. 2.

4.2.1. Inputs

The BN model has two main inputs:

- Cost items, $CI_i(i = 1, 2,\ldots,n)$. This is essentially a qualitative analysis that can start from WBS.
- All possible common causes to cost items, CC_j ($j = 1, 2,\ldots, m$). The common causes can be identified from published literature, brainstorming sessions and conducting interviews with an expert group. In fact, these factors may be varied in different types of project. In order to support identification of these factors, we summarized the most important causes of cost overrun by reviewing published articles (see Appendix A).

4.2.2. BN

The network should be constructed as indicated in Fig. 3. In this step, the prior probability of common causes (P(CC)), and probability distribution of each cost item conditional on different states of its relevant common causes (P(CI|CC)) must

[1] We used AgenaRisk toolset (www.agenarisk.co.uk) to build the presented models in this paper.

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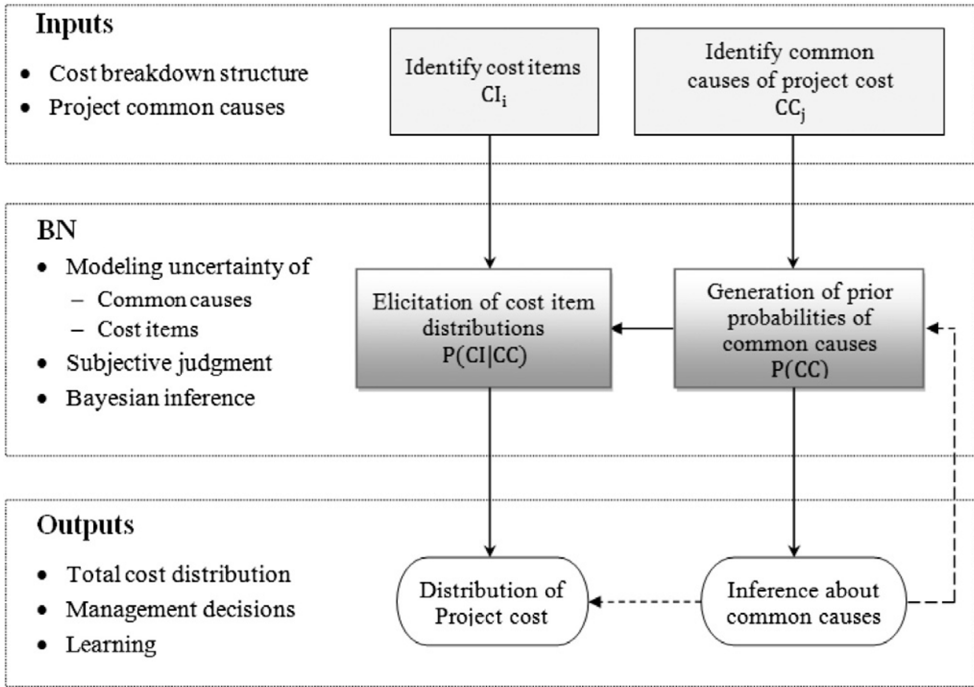

Fig. 2. A framework for using BN for project cost risk analysis.

be specified. The node "Total cost" sums up the cost items and shows project total cost.

4.2.2.1. Generation of prior probabilities of common causes. The common cause is considered as a 'ranked' variable with three levels: low, average, and high. In this study, the new systematic approach in determining the probabilities of a Bayesian network, proposed by Chin et al. (2009) is used. Suppose there are n states S_1, S_2, …, S_n of a node N which has no parent, and the probability of each state S_i, i.e., $P(S_i)$ need to be specified. Traditionally, $P(S_i)$ is specified directly by experts, using their knowledge and experiences. When the number of states is small, such a method may be feasible. With the increase in states of a variable, estimating probabilities directly to all states at one time may inevitably involve biases and inaccuracies.

An alternative way is to perform pair-wise comparisons between states for generating their probabilities. Since there are only two instead of n states considered at one time in a pair-wise comparison, it should be much easier to provide judgments by pair wise comparisons than the direct estimation of probabilities. In the new approach, the prior probability of each state of a node can be determined by the following pair-wise comparison matrix:

	S_1	S_2	…	S_n	ω
S_1	a_{11}	a_{12}	…	a_{1n}	ω_1
S_2	a_{21}	a_{22}	…	a_{2n}	ω_2
…	…	…	…	…	…
S_n	a_{n1}	a_{n2}	…	a_{nn}	ω_n
λ_{max}=		CI=		CR=	

In the above matrix, a_{ij}(i = 1, 2,…,n;j = 1, 2,…,n) can be specified by questions like "comparing the state S_i with S_j,

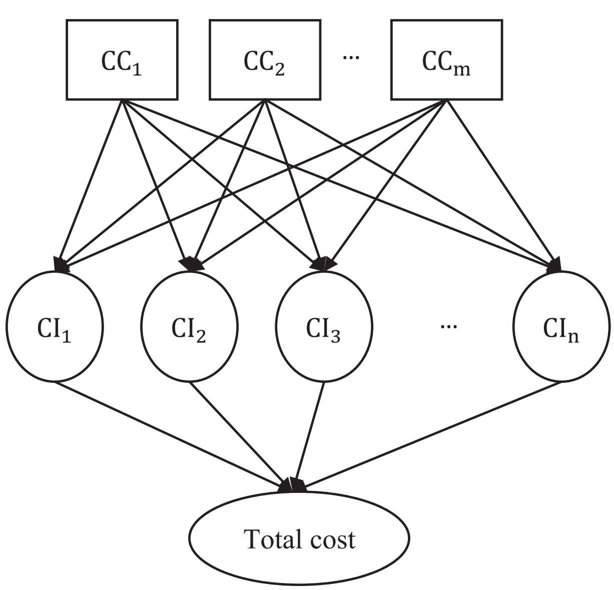

Fig. 3. Schematic of proposed BN model.

Table 1
Random consistency index.

n	1	2	3	4	5	6	7	8	9
RI	0	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45

which one is more likely to occur and how much more likely?" and the value of a_{ij} represents the multiple of the likelihood of the presence of S_i over that of S_j. Note that from the meaning of a_{ij}, we can find that $a_{ji} = 1/a_{ij}$ and $a_{ii} = 1$, so there are $n(n-1)$ different comparisons in the above pair-wise comparison matrix. However, it is sufficient to provide $n(n-1)$ inter-related comparisons rather than all the $n(n-1)$ different comparisons, although it is useful to have more comparisons for checking consistency.

Similar to Saaty's AHP, the relative priorities of S_i can be generated from the maximum eigenvector $\omega = (\omega_1,\ldots,\omega_n)^T$ of the matrix S_i and the consistency of the pair-wise comparison matrix can be checked by the consistency ratio CR = CI/RI, where CI is the consistency index, which is defined by $(\lambda_{max} - n)/(n-1)$ (λ_{max} is the maximum eigenvalue corresponding to ω), and RI is a random index related to n as shown in Table 1. A pair-wise comparison matrix with CR less than 0.10 is considered acceptable.

Since the sum of all the elements in ω is 1 and its *i*th element ω_i represents the relative importance of the state S_i among all the states, it is natural to interpret ω_i as the prior probability of stat S_i. In other words, we have

$$P(S_i) = \omega_i. \quad (5)$$

4.2.2.2. Generation of conditional distributions. To date, there are still controversies regarding which type of distribution provides a better fit to historical data in construction. Whereas beta distributions were commonly used to fit the durations of construction operations (AbouRizk and Halpin, 1992; Farid and

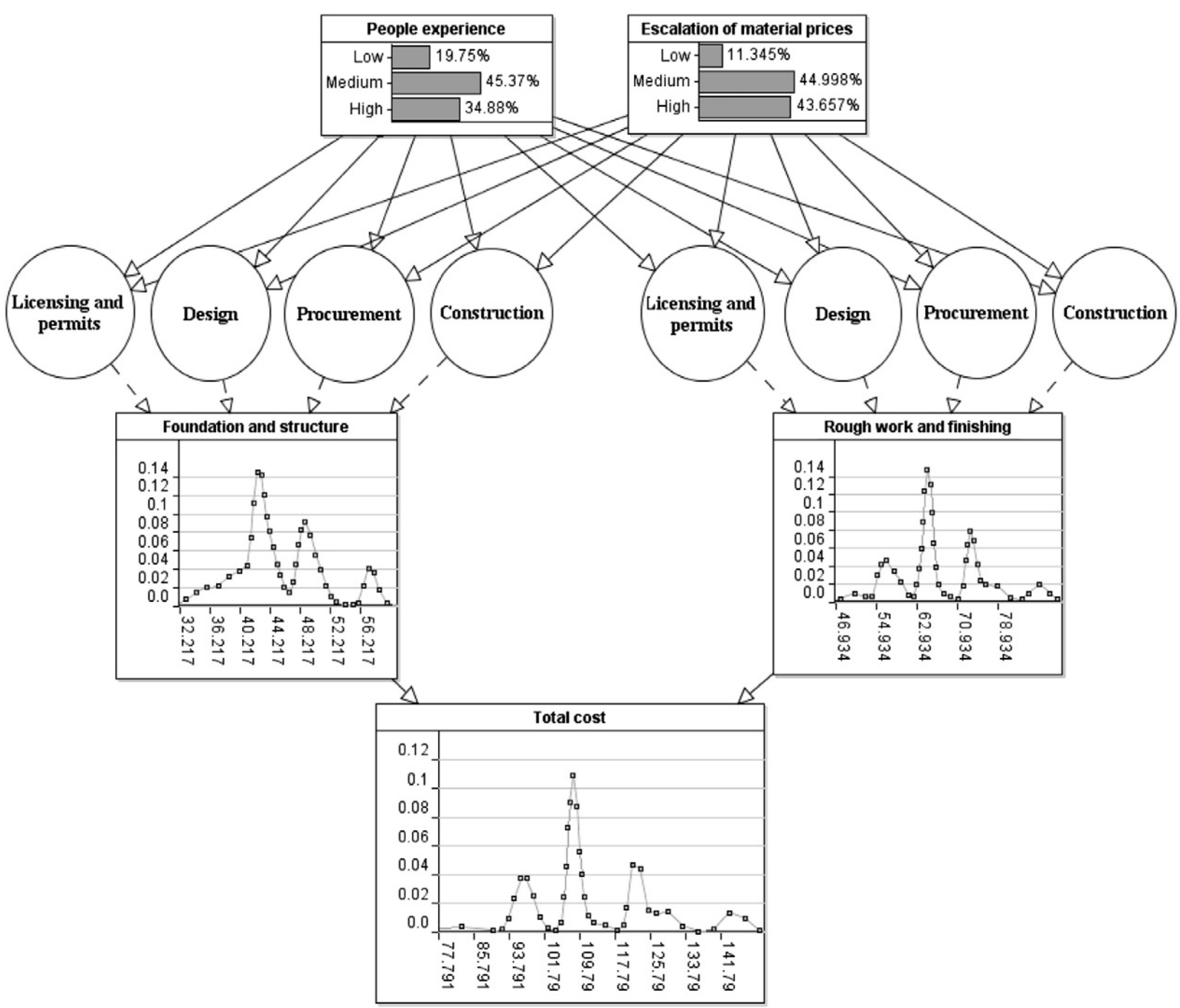

Fig. 4. BN model for example project cost analysis.

Koning, 1994), others showed that lognormal distributions are also good in describing concrete delivery and placement process (Graham et al., 2005). As for construction cost, lognormal distributions were found to provide a good fit for building projects (Touran and Wiser, 1992; Wall, 1997) and highway bridges (Chou et al., 2009), but Weibull distributions performed better in modeling cost data of pavement (Salem et al., 2003). Touran and Suphot (1997) suggested that cost items of office buildings may have different distributions, e.g., gamma distributions for masonry and Erlang distributions for electrical work.

In the view of the controversies in the choice of distribution and estimation process, the proposed approach should be distribution-free and non-parametric. In regard to this requirement, the proposed model is designed to be flexible enough to accommodate any type of distribution.

In fact one of advantages of BNs is their ability to combine different types of data from various sources. Even though we may have little or no historical data, there is often an abundance of *expert* (but subjective) judgment, as well as diverse information and data on indirectly related risks. These are the types of situation that can be successfully addressed using BNs. Unlike classical statistical analysis methods that strongly depend on the distribution of variables and require formal procedures for eliciting these distribution, a BN is "agnostic" about the type of data in any variable and about the way that the NPTs are defined.

For the sake of simplicity here, we use the triangle distribution, with the parameters "upper", "lower", and "most-likely", for all of the cost items under consideration. The method of three point estimates is familiar to most estimators and has been included as standard practice in PMBOK (PMI, 2013). The underlying distribution is customarily assumed to be a continuous triangle because it can model asymmetrical situations and because of its simplicity (AbouRizk and Halpin, 1992; Arena et al., 2006; Goodpasture, 2003; Morgan et al., 1990; Wang, 2002). Expert should be asked to provide the upper, lower, and most-likely values for cost of the Work Breakdown Structure (WBS) element under consideration (or for the technical characteristic that drives cost). During the elicitation, the expert should be pushed to think of reasons that the range could be larger (especially in the upper direction), and to explain the reasoning behind the answers. This extension will counteract the tendencies to overoptimistically narrow distributions and will give the expert and the person conducting the elicitation insight into issues that might be useful in further elicitation or analysis.

4.2.3. Outputs

Having entered the probabilities we can now use Bayesian probability to do various types of analysis. Bayesian probability

Table 2
Prior probability of "People experience" (PE) and "Escalation of material prices" (EMP).

P(PE = low) = 0.1975	P(EMP = low) = 0.1134
P(PE = medium) = 0.4037	P(EMP = medium) = 0.4498
P(PE = low) = 0.3988	P(EMP = low) = 0.4364

Table 3
The parameters of triangle distribution for cost items ($M).

	People experience	Low			Medium			High		
	Escalation of material prices	Low	Medium	High	Low	Medium	High	Low	Medium	High
Foundation and structure	Licensing and permits	(2.82, 3.1, 3.41)	(3.01, 3.3, 3.92)	(3.26, 3.5 ,4.31)	(2.08, 2.9, 3.57)	(2.88, 3.1, 3.77)	(3.09, 3.3, 4.02)	(1.96, 2.7, 3.73)	(2.94, 3, 4.11)	(2.74, 3.2, 4.29)
	Design	(3.61, 4.1, 5.33)	(4.37, 4.9, 5.56)	(5.21, 5.4, 6.39)	(3.18, 3.8, 4.77)	(4.01, 4.3, 4.69)	(4.11, 5.1, 6.04)	(2.64, 3.5, 4.78)	(3.55, 4.1, 5.26)	(4.05, 4.9, 6.13)
	Procurement	(21.14, 22, 23.64)	(23.82, 24.5, 25.78)	(26.42, 27, 28.26)	(17.8, 18.5, 19.91)	(19.07, 20.9, 21.12)	(22.13, 23, 24.61)	(16.33, 17, 18.64)	(17.06, 18, 19.37)	(20.31, 21, 22.16)
	Construction	(15.48, 16, 17.66)	(16.86, 18.6, 19.99)	(20.54, 21, 22.39)	(11.01, 12, 13.61)	(14.66, 15.2, 16.13)	(16.25, 17, 18.38)	(10.56, 11, 12.28)	(12.3, 13.1, 14.73)	(14.13, 15, 16.31)
Rough work and finishing	Licensing and permits	(3.72, 4, 5.36)	(4.68, 5.5, 6.75)	(6.24, 7, 8.18)	(3.44, 3.8, 4.53)	(4.21, 4.5, 4.94)	(5.86, 6, 6.82)	(2.71, 3.5, 4.68)	(3.39, 4.2, 4.54)	(4.83, 5.5, 6.77)
	Design	(6.17, 7, 8.41)	(7.69, 8.4, 9.37)	(9.23, 10, 11.80)	(4.76, 5.5, 6.97)	(6.02, 6.5, 7.38)	(7.27, 8, 9.55)	(3.71, 4, 5.49)	(4.43, 5.3, 6.55)	(5.98, 7, 8.22)
	Procurement	(30.57, 31, 32.61)	(35.72, 36.3, 38.11)	(39.41, 40, 42.6)	(28.26, 29, 30.17)	(30.85, 31.4, 32.06)	(33.4, 34.5, 36.56)	(24.84, 25, 27.47)	(27.9, 28.5, 31.58)	(31.7, 32.5, 35.19)
	Construction	(24.15, 25, 28.17)	(25.46, 27, 31.15)	(27.1, 29.5, 31.61)	(19.87, 21, 22.75)	(21.94, 23, 24.53)	(24.4, 25.5, 26.85)	(16.56, 17, 18.91)	(17.23, 18, 20.19)	(18.97, 19.5, 21.43)

is all about revising probabilities in the light of actual observations of events. The model takes advantages of the predicting as well as diagnostic capabilities of BN. This can yield powerful analyses including predicting the marginal probability distribution of the total project cost, various scenario analysis, and updating our beliefs about common causes in light of new observations in cost items (i.e. learning). All of these provide insightful information for decision makers and hence better decisions regarding project costs.

4.3. Example

In this section we bring a small example which demonstrates the applicability of the proposed model. This example is drawn from a recent case experienced by one of the authors and refers to a hospital building project.

A team of experts including 10 experienced members of different project's stakeholders (i.e. sponsor, contractor and consultant) participated in the study. Cost items and common causes were identified through brainstorming sessions and conducting interviews with the expert team. For example there were two items in level 1 of the WBS; 'Foundation and structure' and 'Rough work and finishing'. Each of these items consisted of four work packages, as the cost items; 'Licensing and permits', 'Design', 'Procurement', and 'Construction'. The identified common causes are "People experience" and "Escalation of material prices". A node is created in the BN for each cost item and common cause as shown in Fig. 4.

The prior probability of common cause nodes were elicited from the expert team via questionnaires and applying the methods described in Section 4.2.2.1. The experts were asked to perform pair-wise comparisons between states of each common cause node for generating their probabilities (see Appendix B-1). For instance, the prior probability of the states of the node "People experience" and "Escalation of material prices" are estimated as in Table 2.

The conditional probability of cost items were elicited from the expert team via questionnaires and applying the methods described in Section 4.2.2.2. The experts were asked to provide the upper, lower, and most-likely values for each cost item under consideration conditional to the states of the relevant common cause (see Appendix B-2). A statistical software package was employed to examine the goodness-of-fit of data and eliminate subjectivity and select the best-fitted model. For instance, the parameters of triangle distribution elicited for conditional distribution of cost items are shown in Table 3.

After the structure of the BN is completed and probabilities are determined, the inference can be performed to see project cost uncertainty. The dependency and correlation among cost items are captured in nodes "People experience" and "Escalation of material prices". Fig. 4 illustrates the whole BN and the PDF of project cost in node "Total cost". This output provides a range of all possible costs AND an understanding of which outcomes are more probable. The mean cost is \$113.3 million and there is a 95% probability that the total cost of the project will not exceed \$144.56 million. In addition, through sensitivity analysis, the model results can provide an understanding of which cost items or common causes should be focused on by management to reduce the project cost risks; "Construction" of work package, Rough work and finishing, and "People experience" are the most important cost item and common cause respectively.

5. Discussion

The purpose of risk modeling is to get insight into the system performance, represent or express the uncertainties,

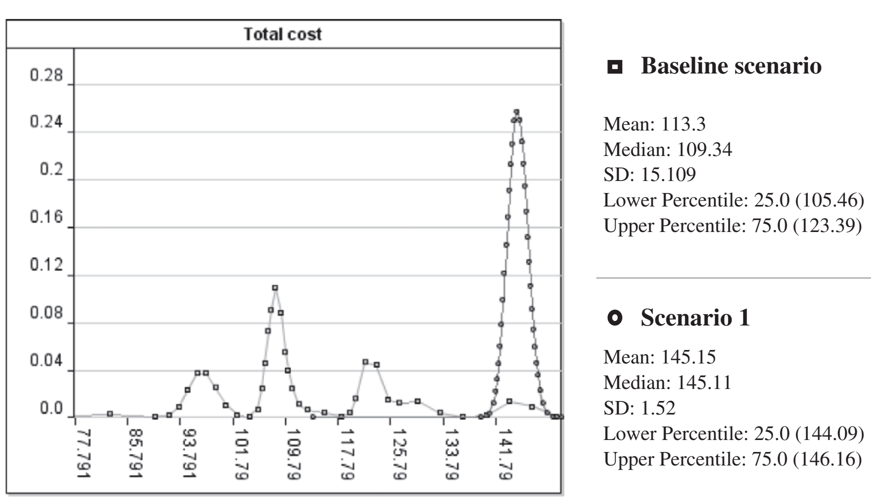

Fig. 5. Comparison of project total cost (\$ million) when the level of People experience and Escalation of material prices changes.

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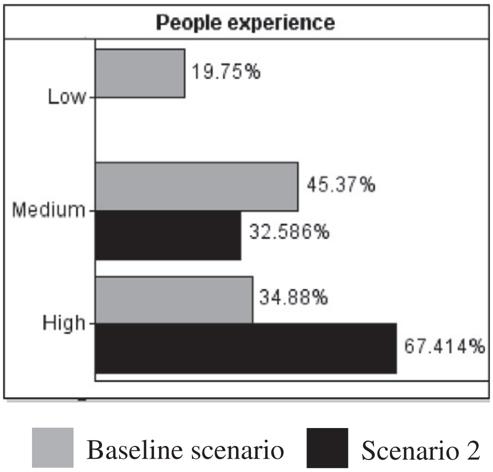

Fig. 6. Level of required People experience when there is a budget constraint (Scenario 2).

identify the risk contributors and see the effect of changes (Aven, 2012). The risk analysis provides adequate decision support and this cannot be done without incorporating expert judgment. Validation of risk assessment tools is not straightforward as principally an objective risk description or measurement does not exist. However, it is essential to discuss the goodness or appropriateness of the model to be used in a risk analysis. The BN presented in this paper is a model for assessing uncertainty/risk in project cost. The model aims to capture and measure uncertainty in the background knowledge (namely common causes) as a rationale for modeling the dependency between project cost items. Including the background knowledge component in the risk description is essential for providing an informative risk picture supporting the decision making.

This section explores the capabilities of the proposed model in assessing the uncertainty of cost quantities based on observations made on other related variables. It is possible to enter observations anywhere in the model to perform not only predictions but also many types of trade-off and explanatory analysis. Here we can analyze the project total cost from different aspects and perform different 'what if?' type analyses.

For example, Fig. 5 shows how the distribution of the project total cost changes when we know that the "People experience" is "low" and the probability of "Escalation of material prices" is "high" (Scenario 1). It shows the predictions for 'Total Cost' in the case where we enter an extreme of the common causes. While there is a significant uncertainty about project total cost in the Baseline scenario, the resulting distribution under Scenario 1 shows a meaningful reduction of standard deviation (from 15.1 to 1.52) and increase of mean (from 109 to 145.11 $M) for project total cost.

Another possible analysis in this model is the trade-off analysis between project cost and people experience when there is a budget constraint. In other words we are interested to know about the level of 'People experience' required if we want the 'Total cost' not to exceed from a given value. For example, consider there is a budget constraint of $95 million on project cost (Scenario 2). Fig. 6 shows how the probability distribution of required People experience to meet this budget constraint is skewed toward high.

One important advantage of BNs is their capability for parameter learning which is updating our belief about one variable when new information arrives. This is shown in the next scenario. Suppose the activity 'Design' of item 'Foundation and structure' actually finishes in $5.6 million and the 'Escalation of material prices' is observed as 'medium'. Because this work package has taken more cost than was expected, the level of People experience has probably not been sufficient. By entering this observation in the completed work

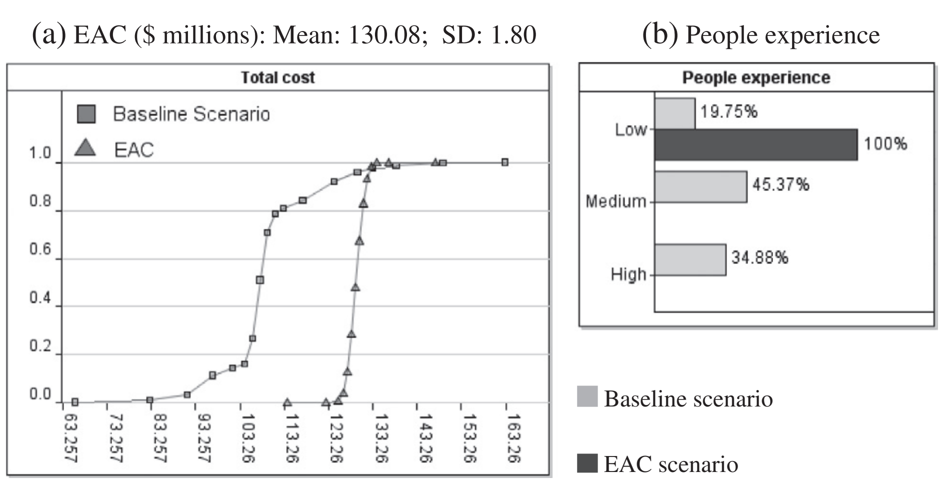

Fig. 7. New evidence in the cost items and escalation of prices, updates estimate at completion and belief about the actual level of People experience.

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package (Design), the model automatically updates the cost of other work packages which are not started yet (Rough work and uncertainty), and consequently updates the project 'Total cost' (Fig. 7a). This result is equivalent to the concept of estimate at completion (EAC) in the earned value management (EVM) technique. EVM has been widely used to estimate the total cost of project at completion (Cheng et al., 2010; Pajares and López-Paredes, 2011), based on actual performance up to any given point in the project. Estimates at completion may differ, based on the assumptions made about future performance. The assumption generally associated with EVM is that past performance is a good predictor of future performance, and (in) efficiencies, observed to date will prevail to completion. This is an interesting feature of the model which incorporates uncertainty in earned value concept. After the actual cost of work performed is entered, the model updates the distribution of cost estimation at completion, based upon the uncertainty of the remaining cost items and variability of project performance (which can be captured in common causes). This can help the project manager to take appropriate control action. Also this can update the analyst's belief about the actual level of People experience as is shown in (Fig. 7b).

6. Conclusion

Project cost analysis is one of the main challenges of project managers. In recent decades, through advances in computer technology and availability of simulation software, different methods are developed and used for cost risk analysis. However simulation based methods have theoretical and practical limitations and are not able to model complex cause and effect relationships among variables. More advanced techniques are required to capture different aspects of uncertainty in project cost estimation. This paper has proposed a new approach that makes it possible to incorporate uncertainty and causality in project cost estimating. The approach brings the full weight and power of BN analysis to bear on the problem of project cost risk analysis. This makes it possible to:

- Express uncertainty about the final cost for each cost item and the whole project with full probability distributions
- Model the dependency among various cost items using the causal relationships and conditional dependency among them
- Perform complex "what-if?" analysis
- Learn from data so that the predictions become more relevant and accurate.

The application of the approach was explained by the use of a simple example. The aim of this paper is to demonstrate how advanced artificial intelligence tools such as BN can be employed to capture complex issues such as uncertainty in project cost analysis. The model proposed here is not the ultimate model. In fact, one of the advantages of BN modeling is that the network can be easily (although it needs some knowledge of probability) tailored and modified to fit to the problem. For example the same method can be used to model schedule uncertainty. Also when required (e.g. for critical phases of project) the model can be expanded by adding more variables to quantify decision making scenarios such as management reactions to cost overrun.

The framework proposed in this paper can help us move to a new generation of cost risk assessment tools that are better informed by available knowledge and data and hence, more valid and useful. However, the presented case study is not intended to be comprehensive. More empirical research is required in order to justify the applicability of the model in real size projects (Pitchforth and Mengersen, 2013).

When studying several hundred cost items and numerous common causes, the BN becomes too complex in term of efficiency of inference process as well as constructing the model. For our future work, the so-called Object Oriented Bayesian network (OOBN) approach (Koller and Pfeffer, 1997) would be sought. By using OOBNs, complex models can be constructed easily using inter-related objects. This not only facilitates the process of building the model but also speeds up the inference process.

Appendix A. Major cost overruns in reviewed articles

Author	Findings
Mansfield et al. (1994)	1) Poor contract management 2) Financing and payment of completed works 3) Changes in site conditions 4) Shortages of materials 5) Imported materials and plant items 6) Design changes 7) Subcontractors and nominated suppliers
Arditi et al. (1985)	1) Problems with materials 2) Financial problems of both owner and contractors 3) Organization deficiencies in both owners' and contractors' company 4) Lack of qualified/technical workers 5) Extra works
Frimpong et al. (2003)	1) Monthly payment difficulties from agencies 2) Poor contractor management 3) Material procurement 4) Poor technical performances 5) Escalation of material prices
Kaliba et al. (2009)	1) Bad or inclement weather due to heavy rains and floods 2) Scope changes 3) Environmental protection and mitigation costs 4) Schedule delay 5) Strikes 6) Technical challenges 7) Inflation and local government pressures

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Appendix B. Sample questionnaires

1. *Sample questionnaire for elicitation of prior probabilities of common causes*

Dear Expert,

Please provide your opinion about pair-wise comparison of any two states. For each cell of the matrix first judge which member of the pair is weaker. Then assign a relative weight to the other member.

	Low	Medium	High
Low	1		
Medium		1	
High			1

2. *Sample questionnaire for elicitation of conditional probability of cost items*

Dear Expert,

Please provide your opinion about the minimum, maximum, and most likely values ($) of the cost item (Procurement) conditional on its common causes' states.

People experience		Low			Medium			High		
Escalation of material prices		Low	Medium	High	Low	Medium	High	Low	Medium	High
Procurement (Foundation and structure)	min									
	max									
	most likely									

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