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Modelling and forecasting the diffusion of innovation – A 25-year review

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Abstract

The wealth of research into modelling and forecasting the diffusion of innovations is impressive and confirms its continuing importance as a research topic. The main models of innovation diffusion were established by 1970. (Although the title implies that 1980 is the starting point of the review, we allowed ourselves to relax this constraint when necessary.) Modelling developments in the period 1970 onwards have been in modifying the existing models by adding greater flexibility in various ways. The objective here is to review the research in these different directions, with an emphasis on their contribution to improving on forecasting accuracy, or adding insight to the problem of forecasting.

The main categories of these modifications are: the introduction of marketing variables in the parameterisation of the models; generalising the models to consider innovations at different stages of diffusions in different countries; and generalising the models to consider the diffusion of successive generations of technology.

We find that, in terms of practical impact, the main application areas are the introduction of consumer durables and telecommunications.

In spite of (or perhaps because of) the efforts of many authors, few research questions have been finally resolved. For example, although there is some convergence of ideas of the most appropriate way to include marketing mix-variables into the Bass model, there are several viable alternative models.

Future directions of research are likely to include forecasting new product diffusion with little or no data, forecasting with multinational models, and forecasting with multi-generation models; work in normative modelling in this area has already been published.

Keywords: Diffusion models; Technology forecasting; Telecommunications forecasting; Marketing

1. Introduction

The modelling and forecasting of the diffusion of innovations has been a topic of practical and academic interest since the 1960s when the pioneer-

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ing works of Fourt and Woodlock (1960), Mansfield (1961) Floyd (1962), Rogers (1962), Chow (1967) and Bass (1969) appeared. The interest excited by these papers can be judged by the numbers of citations for these papers on ISI Web of Science (in April 2005) which were 119, 428, 10, 988, 58 and 582 respectively. Two papers, Fourt and Woodlock, and Bass, use 'new product' rather than technology in their titles. Although the approach to modelling the diffusion of a technology or a new consumer durable is very similar, in recent years, new product applications in marketing have tended to dominate in the overall diffusion literature.

The phenomenon of innovation diffusion is shown in a stylised form in Fig. 1. Cumulative adoption and period-by-period adoptions are shown, but which of these two representations is of greater importance depends on the application. For example, in the diffusion of mobile phones, a service provider is concerned about the demand on the infrastructure and is thus concerned with cumulative adoptions; a handset supplier is concerned with meeting demand and will thus want to model and forecast period by period adoptions. In this example, the service provider will want to know the level of adoption at a particular time and the eventual number of adopters; the handset provider will want to know the rate of adoption at a given time, the timing of peak demand and the magnitude of peak demand. As a counterpoint to the smooth curves of Fig. 1, Fig. 2 shows the comparable information for the diffusion of residential telephones in the United Kingdom. The period-by-period adoptions depart fairly drastically from the bell-shaped curve. The difficulties in forecasting are also demonstrated, as in 1975, period-by-period demand appears to have peaked; decisions to expand production may have been cancelled or postponed; however, in 1979, a 43% higher peak is reached.

The main models used for innovation diffusion were established by 1970; of the eight different basic models listed in the Appendix, six had been applied in modelling the diffusion of innovations by this date. The main modelling developments in the period 1970 onwards have been in modifying the existing models by adding greater flexibility to the underlying model in various ways.

The main categories of these modifications are listed below, and in each case, the citations of a pioneering paper are quoted as a proxy for research activity in this area:

– the introduction of marketing variables in the parameterisation of the models; Robinson and Lakhani (1975)
– generalising the models to consider innovations at different stages of diffusions in different countries; Gatignon, Eliashaberg and Robertson (1989)
– generalising the models to consider the diffusion of successive generations of technology; Norton and Bass (1987).

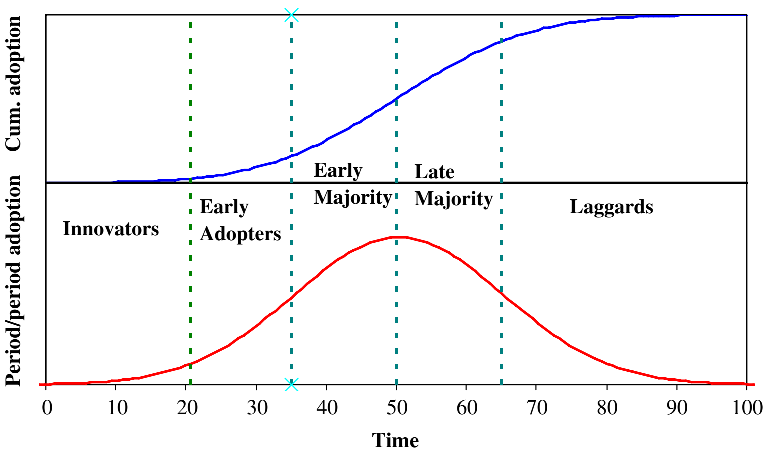

Fig. 1. Stylised diffusion curves.

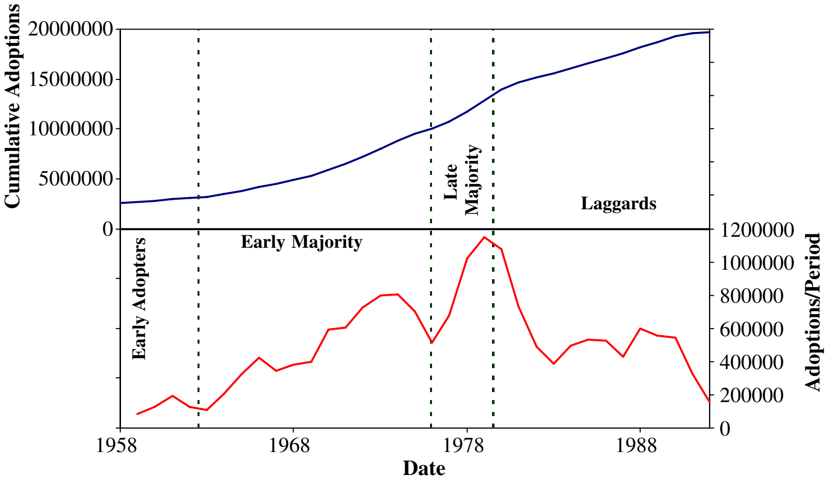

Fig. 2. Diffusion of UK residential telephones.

It is fair to say that in most of these contributions, the emphasis has been on the explanation of past behaviour rather than on forecasting future behaviour. To quantify this comment and the previous comment about the preponderance of marketing studies, the references used in this study are classified by their journals into the following categories in decreasing order in Table 1. There is little difference between the disciplines in terms of the freshness of their contributions; the average age of the marketing, forecasting and OR/management science references is 15 years, the average age of the business/economics reference is 19 years.

During the last 25 years, there have been several reviews of diffusion models. These include Meade (1984); Mahajan and Peterson (1985), Mahajan, Muller and Bass (1990, 1993), Baptista (1999), Mahajan, Muller and Wind (2000a,b) and Meade and Islam (2001). Meade emphasised several criteria for good practice in the use of growth curves for forecasting market development. These included:

– model validity: the product should be adoptable rather than consumable (i.e. there should be an obvious upper bound to the saturation level)
– statistical validity: the estimation of model parameters should be subject to significance tests
– demonstrable forecasting ability and validity: the forecast should be contextually plausible and the forecast should be accompanied by some measure of uncertainty, ideally a prediction interval.

As we will see, (a) it is still relatively easy to find applications where the model validity is dubious, (b) the application of significance tests is widespread but not ubiquitous, (c) when forecasting is included explicit discussion of uncertainty occurs in a minority of cases.

Baptista's review takes an economic viewpoint; he focuses on the diffusion of process between firms and the roles that geography and inter-firm networks play in knowledge transfer.

Mahajan, Muller and Bass offer a research agenda for the development of sounder theory for diffusion in a marketing context and more effective practice (this agenda included several topics that were currently underway). Their agenda included:

– increasing the understanding of the diffusion process at the level of the individual

Table 1
Frequency of references by journal category

Category	%
Marketing	31
Forecasting	27
OR/Management Science	13
Business/Economics	7
Other	22

– exploiting developments in hazard models as a means of incorporating marketing mix variables
– investigating the nature and effect of supply and distribution constraints
– modelling and predicting product take-off
– empirical comparisons with 'other sales forecasting' models.

Of these items, the empirical comparisons have received least attention. In this review, we shall look at modelling the diffusion of a single innovation in a single market; then the diffusion of an innovation in several (national) markets at the same time; then the diffusion of successive generations of the same innovative technology. The length of each section obviously depends on the amount of work done on the topic discussed; thus, as the latter topics are newer and less researched, the relevant sections are shorter. Within each of these topics, we shall look at issues of modelling including the introduction of explanatory variables, estimation and forecasting accuracy. The most often encountered diffusion models are described in the Appendix; we will refer to these models where necessary and keep further equations within the text to a minimum.

2. The diffusion of a single innovation in a single market

The path the cumulative adoption of an innovation takes between introduction and saturation is generally modelled by an S curve. Examination of data sets suggests that this type of model is generally appropriate. A legitimate enquiry is – *why* is cumulative diffusion S shaped? The two extreme hypotheses that explain this shape are those based on the dynamics of a (broadly homogeneous) population and those based on the heterogeneity of the population.

Taking the dynamics of the population first, Bass (1969) (see A1.01) suggests that individuals are influenced by a desire to innovate (coefficient of innovation p) and by a need to imitate others in the population (coefficient of imitation q). The probability that a potential adopter adopts at time t is driven by $(p+qF(t))$ where $F(t)$ is the proportion of adopters at time t. Relating the similarity of innovation diffusion with the spreading of an epidemic, imitation is often called a contagion effect. In a pure innovation scenario ($p>0$, $q=0$) diffusion follows a modified exponential (A1.08); in a pure imitation scenario ($p=0$, $q>0$), diffusion follows a logistic curve (A1.07). Other properties are that $(p+q)$ controls scale and (q/p) controls shape (note that the condition $(q/p)>1$ is necessary for the curve to be S-shaped).

One of the first to use a heterogeneous population argument was Rogers (1962). He suggests that populations are heterogeneous in their propensity to innovate. Rather like a military attack, the innovators (2.5% of adopters) go over the top first, followed by the early adopters (13.5%), followed by the early majority (34%), the late majority (34%) and the laggards in the rear (16%). These percentages are based on the normal distribution (e.g. innovators are 2 standard deviations or more above the mean level of innovativeness). Put another way, individuals in a system have a threshold for adoption; innovators have a very low threshold. As the innovation becomes more widely adopted the social pressure reaches more and more thresholds, since "Individual thresholds for adoption are normally distributed, thus creating the S-curve of diffusion". He reports that early adopters are better educated, more literate, have higher social status and a greater degree of upward social mobility, and are richer than later adopters.

This last property relates to income; an early mention of the income heterogeneity hypothesis was made by Duesenberry (1949). The heterogeneity of income distribution has been cited by several authors (for example, Bonus, 1973) as a driver for the S shape. The view is that the diffusion curve reflects the nature of income distribution: as the price of an innovation falls, more consumers can afford it. Provided the income distribution is bell-shaped, and the price falls monotonically, an S curve will result. In a critique of the Bass model, Russell (1980) did not like the terms innovator and imitator (a departure from the economist's world of rational agents, maximising utility under budget constraints); he preferred an individual-based model, where the individual has a threshold price. When the price of the innovation falls to this threshold, an innovation may be triggered. This argument leads via a lognormal distribution of income to an S-shaped curve. Russell refers to Bain (1963), who used the cumulative lognormal to forecast diffusion of televisions. However, Russell does allow that there may be contagion within the income strata. Liebermann and Paroush (1982) provide an economic argument that income heterogeneity, price and advertising are important drivers of the diffusion process.

Since the diffusion of an innovation is a complex process, involving large numbers of individual decisions, the diffusion of any one innovation will be due to elements of both extreme hypotheses. Van den Bulte and Stremersch (2004) performed a meta-analysis on the use of the Bass model applied to new product diffusion. The study involved 746 different Bass estimations spread over 75 consumer durables and 77 countries. The international comparison enabled them to test several sets of hypotheses, relating the diffusion to both the national culture and the nature of the product. The contagion-based hypotheses for which they found support are that (q/p) ratios are:

– negatively associated with individualism (individualism means more immunity to social contagion) or positively associated with collectivism;
– positively associated with power-distance (a measure of the hierarchical nature of the culture). The assumption here is that 'classes' tend to adopt a new product at a similar time;
– positively associated with masculinity (cultures where there is a clear distinction between gender roles).

Contrary to their expectations, they found a negative association with uncertainty avoidance (a measure of how threatened people feel when faced with a novel opportunity). A positive association is found between q/p and the Gini coefficient of income inequality, supporting the income heterogeneity hypothesis. In cases where the products concerned had competing standards e.g. VCRs (Betamax versus VHS), PCs (DOS/Windows versus Apple), they found that this technological issue dominated the social or income effects.

Rogers (1995) links other concepts to his framework (of heterogeneous innovativeness). The diffusion of an innovation will not proceed if critical mass is not reached, as may occur if there is a discontinuity in the distribution of adoption thresholds. In this context, he makes the distinction between interactive and non-interactive innovations. The first adopter of a personal computer can start writing his own programs, but the first adopter of a telephone can do nothing until the second adopter acts. This argues for a critical mass of adopters existing before diffusion really takes off. Adoption is slow before the critical mass exists (critical mass is the stage at which enough individuals have adopted that further adoption is self-sustaining) and then diffusion accelerates and a contagion effect begins to occur. Mahler and Rogers (1999) investigated reasons given by German banks for the non-adoption of twelve telecommunications innovations. The reason given in 41% of cases was the low rate of diffusion of the innovation; this reason was very highly ranked regardless of the innovative history of the institution (whether it was classified as an innovator or a laggard). They point out that in the case where there are competing standards for the innovation, each standard will need a critical mass before diffusion will accelerate.

In a study of the diffusion of imaging technology into US banks and insurance companies, Libertore and Bream (1997) found that the logistic best described this process. They found no evidence of an innovation pressure (internal influence). Through a questionnaire analysis, they found that early adoption was related to the size of the organisation, i.e. larger organisations adopted the technology earlier.

Some technologies are dependent on each other, for example, Bayus (1987) studies the relationship between the compact disc and its hardware. Rogers (1995) makes the distinction between hardware and software: software is the understanding about what the technological hardware can achieve. Rogers contends that the software diffuses faster than the hardware. Geroski (2000) suggests the use of probit models to explain the decisions of firms to adopt technology; this appears to be a way of introducing heterogeneity of firms into the adoption process.

In a study of the diffusion of twenty-five information technologies, Teng, Grover and Guttler (2002) used the Bass model to compare diffusion behaviour. They found very low coefficients of innovation (internal influence), suggesting that imitation (external influence) was the main driver for adoption in all cases. The technologies were clustered by saturation level and the coefficient of imitation, and five clusters were found. Email and fax are in one cluster with a low q and 100% saturation; spreadsheet and PCs are in a cluster with a high q and 100% saturation level (interestingly, in contradiction to Rogers, the parameter values for the hardware and software are almost identical); and imaging is in a cluster with a high q and lower saturation level.

The time at which an innovative product is introduced may have an effect on the rate of its diffusion. At present, the evidence is slightly contradictory. Kohli, Lehman and Pae (1999) consider incubation time as a factor in innovation diffusion. Incubation time is the interval between the completion of product development and the beginning of 'substantial' product sales. For example, patents on zips were taken out in 1893 and 1913 but sales did not take off until the 1930s. In a study of thirty-two products, they found a positive association between incubation time and time to peak sales and a negative association with the coefficient of innovation. They found no evidence that the incubation time was changing over time; this led them to comment that 'innate innovativeness is not increasing'. However, in a study specifically focussed on this topic, Van den Bulte (2000) examines the change in the speed of diffusion of innovations over the period 1923–1996. Diffusion speed can be measured as the slope coefficient of the logistic or the time taken to go from one level of penetration to another. He finds a significant increase in speed over the period; this increase is attributed to increased purchasing power, demographic changes and the types of product studied.

Some authors have used observed heterogeneity as a basis for qualitative forecasts of future diffusion patterns. Wareham, Levy and Shi (2004) investigate socio-economic factors underlying the diffusion of the internet and 2G mobiles in the US. Mobile adoption is positively correlated with income, occupation and living in a metropolitan area. In addition, African Americans adopted mobiles significantly faster than other ethnic groups. As African Americans and other ethnic groups were underrepresented in internet use, they suggest that a likely possible route to internet connectivity for these groups is via internet-enabled mobile devices.

On a theoretical level, some authors have examined the behaviour of populations, given specific forms of heterogeneity. Chatterjee and Eliashberg (1990) model adoption at the level of the individual, with heterogeneous perceptions of the innovation's performance. They show that the Bass and other models can be considered special cases of their micro-modelling approach. Bemmaor (1994) and Bemmaor and Lee (2002) consider a population of individuals, where each individual's probability of adoption is given by a shifted Gompertz density function. The heterogeneity is driven by the 'shift' parameter which is distributed as a gamma random variable. Bemmaor and Lee show that, under certain conditions, the observed diffusion is consistent with the Bass model. They further demonstrate that changing the parameters of the gamma distribution produces conditions under which the data are more or less skewed than the Bass. An asymmetric diffusion curve, such as the non-uniform responding logistic of Easingwood, Mahajan and Muller (1981, 1983) can be reproduced under this framework. On forecasting accuracy, Bemmaor and Lee note that the forecasting accuracy of their more flexible model is better than Bass for one-step-ahead but deteriorates for longer horizons.

The geographical location of potential adopters is another form of heterogeneity that has received some attention. In a theoretical analysis, Goldenberg, Libai, Solomon, Jan and Stauffer (2000) examine innovation diffusion via percolation theory, which describes the heterogeneity of the population in a spatial context, and precisely defines the 'micro-structure' of the population. They use simulation to demonstrate that 'social percolation' leads to a power law curve rather than exponential growth. The S-curve produced by this model has a very late point of inflexion, very close to the saturation level. In an empirical study, Baptista (2000) examined the diffusion of numerically controlled machines and microprocessors in the regions of the UK. He found that there were significant regional effects on the rate of diffusion.

2.1. The use of explanatory variables in the diffusion model

The diffusion of an innovation rarely takes place in a stable, unchanging, environment. In recognition of this, there have been many attempts to include environmental variables within the diffusion model. An early example is Tanner (1974), who used GDP/capita and the cost of car usage as additional variables in the linearised logistic (A1.17) to forecast the growth of car ownership in the UK.

Typically, this is achieved by incorporating explanatory variables into (a) market potential, (b) the probability of adoption or the hazard function or (c) both i.e. simultaneously into market potential and probability of adoption. Thus, in the first case, the environmental (or marketing mix variables) are hypothesised to determine the total number of eventual adoptions; whereas in the second case the environmental variables are hypothesised to accelerate or retard adoption. Particularly in the case when the environmental variable is the product price, one could argue that this modelling approach supports the heterogeneity argument. A falling price brings the product within reach of more potential adopters; the question is whether to represent this likely increase in adoption by increasing the market potential or by increasing the probability of adoption. We will look at these approaches in turn.

Some authors have made the saturation level (market potential) a function of price, advertising or some other measure of market activity. An early example of this was given by Mahajan and Peterson (1978) who used US housing starts to parameterise the market potential for washing machines.

The impact of price on market potential has been studied by Mahajan and Peterson (1978), Bass (1980), Bass and Bultez (1982), Kalish (1985), and Horsky (1990). One of the rationales used (see Mahajan & Peterson 1978) is that a lower price would place the product within the budgetary limitations of a greater number of buyers, thus increasing the market potential. Approaching the incorporation of price into the market potential from a different perspective, Horsky (1990) argues that the effective reservation price (e.g. price, time consumer spent to get the product, etc.) and wage rates are distributed across individuals according to an extreme value distribution. Thus the market potential (the number of consumers who will buy the product) depends on the distribution of wage and price. The market potential will increase with a reduction of average price, with increased income, or with a reduction of the dispersion of the distribution of income.

Kalish (1985) characterised the diffusion of a new product in two stages, namely awareness and adoption. He suggested that consumers will buy a product if they are aware of it and the risk adjusted price falls below their reservation level. Thus, at a particular time, the market potential is the number of individuals that find the risk-adjusted price acceptable multiplied by the percentage aware of the product.

Karshenas and Stoneman (1992) introduced explanatory variables into the market potential while modelling diffusion of colour televisions in the UK. Islam and Meade (1996) explored several formulations of the saturation level for UK business telephones using GDP-related variables. They found that the approach added insight into how the environment affected diffusion but did not lead to an increase in forecasting accuracy.

Environmental variables are introduced into the probability, or hazard rate, of adoption by a variety of routes. These include parameterising either the coefficients of innovation or imitation (or their equivalents in non-Bass models) or by introducing an extra multiplicative term. Robinson and Lakhani (1975) were the first to include price impact in the Bass model. They reformulated the Bass hazard function in this way:

$$h(t) = (\beta_0 + \beta_1 F(t))\exp(-\beta_2 P(t)) \qquad (2.1.1)$$

where $P(t)$ is a price index (with $P(0)=1$). They use this formulation as a tool to examine pricing strategies. Marginal pricing, which starts with a high price decreasing with diffusion, is shown to be a poor strategy compared to either an optimal constant price or an optimal strategy (where the price starts low, rises to a peak and then falls). Variants of (2.1.1) have been used by Dolan and Jeuland (1981) and Kalish (1983). Horsky and Simon (1983) examine the effect of advertising on the probability of adoption; they modify the hazard function thus:

$$h(t) = (\beta_0 + \beta_1 F(t) + \beta_2 \ln(A(t))) \qquad (2.1.2)$$

where $A(t)$ is advertising expenditure at time t, the coefficients β_0, β_1, and β_2 are interpreted as measuring the effects of publicity, word of mouth and advertising respectively. The model was shown to produce plausible estimates for the diffusion of a telephone banking service. Thomson and Teng (1984) proposed a model incorporating elements of (2.1.1) and (2.1.2). Generalising an approach by Thomson and Teng, Simon and Sebastian (1987) found that linking the imitation coefficient to advertising was the most effective way of using this information to model the diffusion of telephones in Western Germany. Their formulation is:

$$h(t) = \beta_0 + \beta_1 F(t) + \beta_2 \alpha(A(t))F(t) \tag{2.1.3}$$

where $\alpha(A(t))$ is a function of current and past advertising expenditure representing advertising response.

Kamakura and Balasubramanian (1988) (KB) proposed a general model, which allowed price effects in the probability of adoption and in market potential. The model is a generalisation of (A1.01)

$$f(t) = (p + qF(t))P^{\alpha}(t)\left(H(t)P^{\beta}(t) - F(t)\right) \tag{2.1.4}$$

where $H(t)$ is the proportion of households eligible to receive the innovation (for example, electrified households). This model nests simpler models, allowing for empirical testing of the nature of the price effect for particular data sets. They found that price affected adoption probability ($\alpha < 0$, $\beta = 0$) for relatively highly priced goods (refrigerators rather than blenders). Pursuing a similar theme to KB and using a similar approach to Jain and Rao (1990), Parker (1992) incorporated a price elasticity term, in a variety of models. For eleven out of twelve consumer durables, he found evidence of time-varying price elasticity.

In response to these modelling initiatives, Bass, Krishnan and Jain (1994) develop a generalised Bass model (GBM) which allows the incorporation of marketing mix variables in the modelling of new product diffusion. They achieve this by introducing a 'current marketing effort' factor, $x(t)$, into the hazard function of the Bass model.

$$h(t) = (p + qF(t))x(t) \tag{2.1.5}$$

where

$$x(t) = 1 + \beta_1 \frac{\partial P(t)}{\partial t} + \beta_2 \max\left(0, \frac{\partial A(t)}{\partial t}\right). \tag{2.1.6}$$

An attraction of this generalisation is that if the marketing effort is more or less constant, then the model simplifies to the Bass model. They demonstrate that the GBM forecasts better than the Bass model for three consumer durables over various horizons (assuming crucially that the future behaviour of the marketing variables is known). They further demonstrate empirically that their formulation is superior to making one or more of the parameters p, q, or m a function of marketing variables.

Using the GBM in a pro-active fashion in the spirit of Robinson and Lakhani, Krishnan, Bass and Jain (1999) derive optimal pricing strategies for product introduction. Using an objective function of cumulative net revenue over a planning horizon, they find that the optimal policy is to raise the price up to a specific time period and then monotonically reduce the price. The optimal timing of the peak price occurs well before peak sales. Earlier, Kalish and Lilien (1983) examine an optimal state price subsidy for accelerating the diffusion of beneficial technologies (such as alternative energy systems). In most cases, they find that the subsidy should decrease as diffusion accelerates.

In a study comparing models without and with explanatory variables, Putsis (1998) also compared different methods for estimation over several models. A central theme is the time-varying nature of the parameters of the diffusion model. These parameters may vary due to factors such as changes in the marketing mix, changes to the product or to consumer expectations. The approach offers an estimate of the current, rather than average, response to a marketing variable.

In a study using data for electrical products similar to KB, plus electronic products such as video cassette recorders (VCR) and CD players, Bottomley and Fildes (1998) used the KB modelling framework to examine the effect of price information on forecasting accuracy. They found only one case, VCRs, where the full model shown in (2.1.1) was needed (p, q, α, $\beta \neq 0$). Overall, they found little evidence of increased forecasting accuracy due to the inclusion of actual price information (let alone predicted price information).

2.2. Estimation issues in single diffusion models

Due to the nature of diffusion models, the estimation of parameters is generally a non-linear problem. Estimation of the logistic and some of its variants has been achieved by linear transformations followed by ordinary least squares (for example, see (A1.09)). A summary of these models is given by Young (1993). Lee and Lu (1987) further apply Box–Cox transformations to the linearly transformed data to forecast the diffusion of electronic switching systems in telecommunications. Meade and Islam

(1995a) used non-linear least squares to fit a range of diffusion models to telecommunications data. Another approach is adaptive estimation; this approach recognises the possibility that the parameters of the diffusion model may change over time. Meade (1985) applied the extended Kalman filter to estimating the logistic and Gompertz models for forecasting.

The most clearly documented story of the benefits of different fitting procedures involves the Bass model. This story is described below. Although the Bass model is discussed here, the lessons learnt are likely to apply to non-Bass diffusion models.

2.2.1. Estimation of the Bass model

An attraction of the Bass (1969) model, when it was introduced, was that the coefficients of innovation, p, and imitation, q, and the market potential, m, could be estimated by ordinary least squares (OLS). This exploited the property of the discrete model that the binomial expectation of new adopters at time t was

$$Y_t - Y_{t-1} = \left(p + \frac{q}{m} Y_{t-1}\right)(m - Y_{t-1}) \qquad (2.2.1)$$

given that Y_{t-1} adoptions had occurred by $t-1$. This is a discrete version of (A1.01). Empirical experience shows that the OLS approach is prone to wrong signs, implying negative probabilities, and to unstable estimates. Schmittlein and Mahajan (1982) proposed a maximum likelihood estimation (MLE) approach, using a continuous model. The likelihood function is

$$L = (1 - G(t-1))^{(M - Y_{t-1})} \prod_{i=1}^{t} (G(i) - G(i-1))^{(Y_i - Y_{i-1})}$$

where $G(t)$ is defined in (A1.03). This assumes that the adoptions are Bernoulli trials, with the probability of adoption changing between time periods. The benefits of this approach were demonstrated to be increased forecasting accuracy and more stable parameter estimates. Srinivasan and Mason (1986) (SM) argued that MLE tended to underestimate parameter standard errors and proposed a non-linear least squares (NLS) approach. They suggest the minimisation of the squared residuals $\sum u_t^2$, where

$$Y_t - Y_{t-1} = m(F(t) - F(t-1)) + u_t$$

and $F(t)$ is defined in (A1.02). They found that the fitting and forecasting performance of NLS was very similar to MLE, but both methods were superior to OLS. Their data are subsequently used by several other authors. One of the data sets used describes the adoption of mammography by a group of 209 hospitals. This data set is used here to demonstrate and contrast these two objective functions. The data are shown in Fig. 3. The adoptions peak in 1974 and forecasts are prepared 1 year ahead for the next 4 years. The relevant parameter estimates and objective functions are shown for MLE and NLS

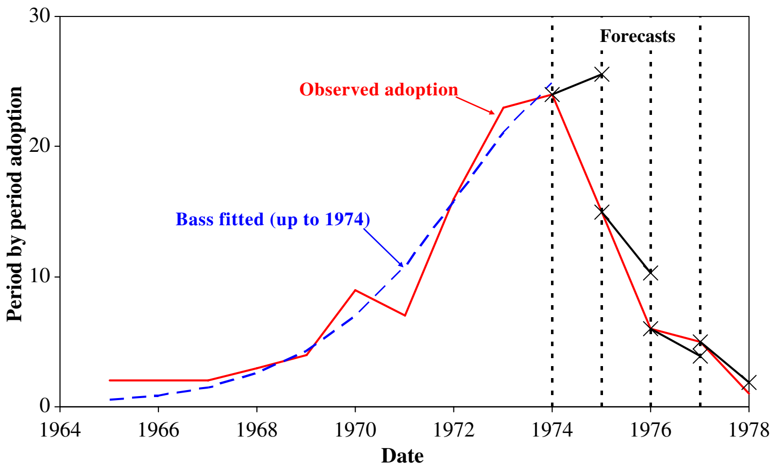

Fig. 3. Diffusion of mammography within 209 US hospitals (with NLS fit and one-period ahead forecasts).

estimation in Table 2. Note that for both procedures the estimates of market potential decrease with each new observation; the estimate of p, the coefficient of innovation tends to decrease and the estimate of q, the coefficient of imitation tends to increase with each observation. MLE favours higher estimates of market potential and the coefficient of innovation, and a lower estimate of the coefficient of imitation than NLS. The mean absolute deviation (MAD) of one-step-ahead forecasts (shown in Fig. 3) is lower for NLS. However, from 1976 onwards, the NLS estimate of market potential is less than the observed cumulative total of adoptions. In contrast, MLE estimates are plausible throughout this period.

In a different but related context, a comparison of new product trial forecasting models using consumer data sets, Hardie, Fader and Wisniewski (1998) found maximum likelihood to be noticeably better than NLS applied to period-by-period sales. However, over shorter series, they found NLS applied to cumulative sales comparable with MLE.

There are particular problems with the use of each of these objective functions. The use of MLE assumes the correct identification of the underlying random variable; Schmittlein and Mahajan assume that individual adoption is a Bernoulli trial.

Bias in the NLS estimation of the Bass model is examined by Van den Bulte and Lilien (1997). The bias includes a tendency for the saturation level to be underestimated and close to the latest observed penetration and a tendency for estimates of q to decrease as more data becomes available. In the mammography example, the former bias is very evident, but the latter is not apparent (see Table 2). They demonstrate through simulation that the biases exist, but that there is no simple solution: "expecting... a handful of noisy data points to foretell the ultimate market size and the time path of market evolution is asking too much of too little data".

The use of NLS implicitly assumes that errors have the same variance throughout the time series. To overcome this constraint, Boswijk and Franses (2005) borrow from financial econometrics to propose a new stochastic error process for the Bass model. Their approach is designed to capture heteroscedastic errors and a tendency for the data to revert to the long-term trend. They used CD diffusion data from Bewley and Griffiths (2003) for a comparison of the forecast accuracy of their approach with the SM estimation procedure. They demonstrated that the SM parameter standard errors were too low, giving over-confidence in parameter accuracy, and that the forecasts based on their formulation were more accurate for 9 out of 12 data sets.

SM discovered the location of the minimum mean square error by a gradient-based search procedure; the use of this type of search procedure implies that the objective function varies smoothly

Table 2
Parameter estimates for the adoption of mammography by 209 US hospitals using NLS and MLE

NLS estimates							
Data used up to	Cumulative sales	m	p	q	MLE	***MSE***	1-step-ahead absolute error
1974	92	188.6	0.00206	0.5453	−323.3	***2.7***	10.6
1975	107	120.3	0.00073	0.7745	−374.8	***4.0***	4.3
1976	113	110.6	0.00038	0.8703	−402.6	***4.4***	1.1
1977	118	112.0	0.00044	0.8526	−421.5	***4.2***	0.8
1978	119	111.4	0.00041	0.8607	−427.6	***3.9***	
						MAD	4.2
MLE estimates							
Data used up to	Cumulative sales	m	p	q	***MLE***	MSE	1-step-ahead absolute error
1974	92	209.0	0.00328	0.4686	***−322.3***	3.3	9.7
1975	107	160.7	0.00395	0.4998	***−368.4***	7.0	10.4
1976	113	129.4	0.00324	0.5920	***−393.4***	8.9	1.8
1977	118	126.1	0.00299	0.6125	***−413.2***	8.2	2.6
1978	119	122.4	0.00254	0.6471	***−419.8***	7.8	
						MAD	6.1

The relevant objective function values are shown in bold italics.

with the estimated parameters. Venkatesan and Kumar (2002) use a genetic algorithm as the tool for finding a minimum sum of squares (GA-NLS) rather than a sequential search-based algorithm (SSB-NLS as used by SM) to forecast the adoption of mobile telephones across seven European countries. Genetic algorithms are designed not to be trapped by local minima in a 'rough' solution surface, whereas a gradient-based method could mistakenly identify a local optimum as a global optimum. Conversely, if the surface is smooth, then a gradient-based algorithm will locate the global optimum more efficiently. Venkatesan and Kumar found that GA-NLS led to greater forecasting accuracy than was available using other implementations of NLS with the Bass model. Venkatesan, Krishnan and Kumar (2004) found that GA-NLS led to more accurate predictions of the SM data sets and removes the biases discussed by Van den Bulte and Lilien. Venkatesan, Krishnan and Kumar quote a MAD of 0.9 for one-step-ahead forecasts for the mammography data set, compared to 4.2 obtained by SSB-NLS. Unfortunately the authors do not give the corresponding in-sample mean squared error or the parameter estimates. For the GA-NLS solution to be less than the SSB-NLS solution, the objective function surface should exhibit local minima. To explore the surface, a detailed grid search with small increments was carried out over the range of likely parameter values ($0.00005 < p < 0.1$; $0.01 < q < 0.99$; $50 < m < 210$). Plots of the surface are shown around the SSB-NLS optimum for 1974 in Fig. 4, and it is clear that the surface is smooth. No evidence for the non-optimality of the SSB-NLS solution was found. To achieve the quoted out-of-sample MAD, the magnitude of the one-step-ahead error for 1975 must lie between 0 and 3.6 (since there are four errors summarised). Using the results of the grid search, the relationship between the in-sample mean squared error and the one-step-ahead forecast error was examined. This relationship is summarised in the plot in Fig. 5. The SSB-NLS minimum MSE of 2.7 gives an error of -10.6; however, in order to achieve an error magnitude of less than 3.6, the GA-NLS MSE must be greater than 3.56. This tends to suggest that, in this case, the GA-NLS solution was sub-optimal.

There is a clear consensus that using OLS to estimate the Bass model is non-optimal, but the choice between NLS and MLE is less clear. The balance of recent work has favoured NLS, but it is too early to disregard MLE. Innovative assumptions about the stochastic behaviour of adoptions can be more readily investigated by MLE. The use of heuristic computationally intensive search methods such as genetic algorithms for NLS estimation (or MLE) deserves further research.

Other authors suggest that the estimation of the parameters p, q, and m as fixed values for a given data set is misguided because parameter values vary over time. Xie, Song, Sirbu and Wang (1997) applied the augmented Kalman filter for estimating and forecasting period-by-period adoption $f(t)$. They demonstrate their method on the Bass model applied to the SM data sets, showing that it offers greater accuracy in most cases. (Mammography is one of these cases; Xie et al. quote a MAD of 3.1, less than the values given in Table 2.) They demonstrate how their framework can incorporate two or more possible models with weightings that evolve in the light of experience. Goswami and Karmeshu (2004) use simulated annealing to fit a random coefficients version of the Bass model.

2.2.2. Use of diffusion models with little or no data

In many practical situations, predictions of new product sales are desirable before sufficient data are available for model estimation. A possible means of increasing the available data is the use of higher frequency data. Putsis (1996) found that the use of seasonally adjusted quarterly data leads to greater forecasting accuracy than the use of annual data but found no further advantage with monthly data. In this situation of scarce data, forecasting by analogy is a possible approach. The Bayesian approach of Lenk and Rao (1990) is one possible approach in this context. These authors develop a hierarchical Bayes procedure for using information about prior innovations to forecast diffusion early on. The data are used in a cross-sectional manner, but the timing is based on a common time period of introduction, rather than calendar time. The advantage of this approach over a single series approach is that the experience of other innovation diffusions informs the parameter estimation for the series of interest.

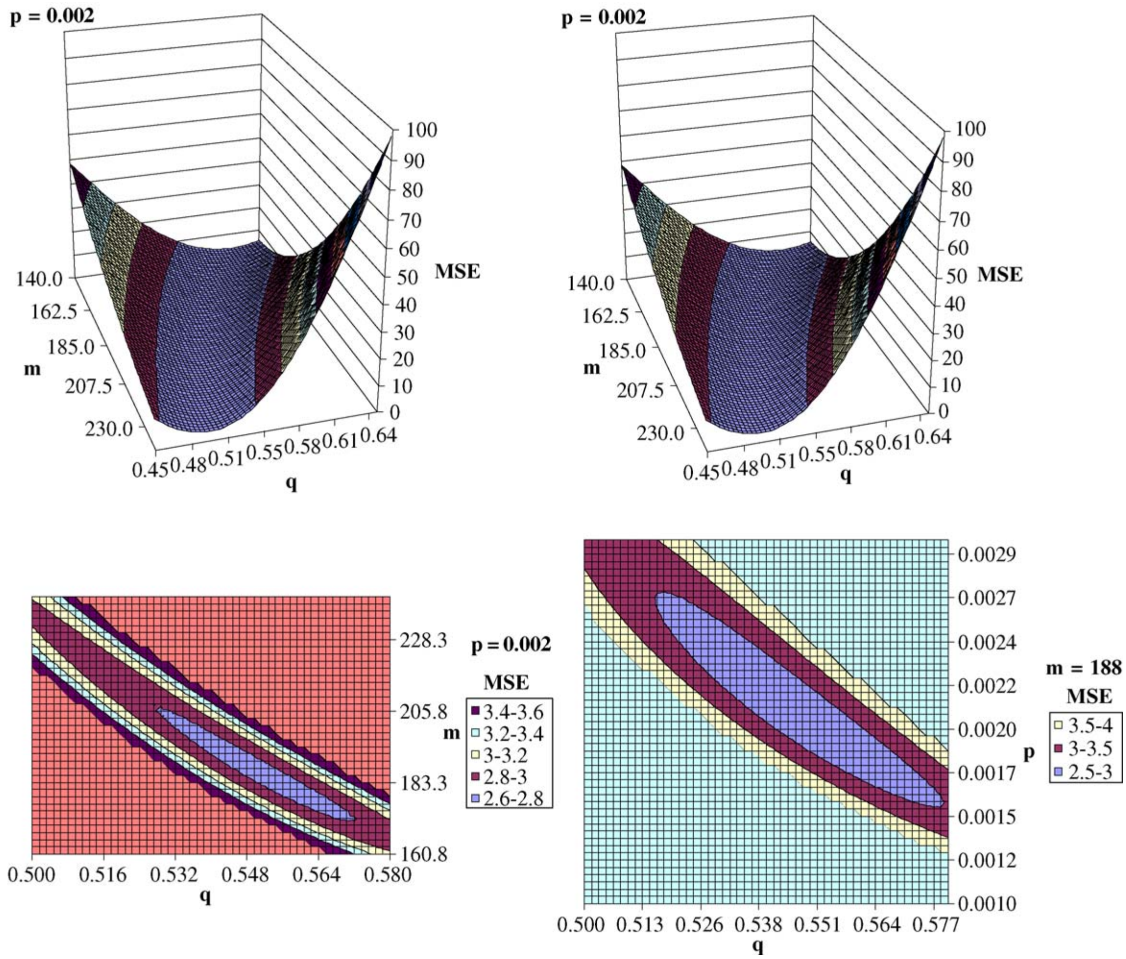

Fig. 4. The mean squared error surface for the mammography data set using observations up to and including 1974. (The first two 3D plots show MSE for wide range of values. The contour plots show the MSE surface near the optimum).

They find that their approach provides forecasts with an accuracy slightly superior to the MLE-based forecasts.

The meta-analysis of Sultan, Farley and Lehman (1990) is a useful contribution to forecasting with little data. They find that the coefficient of innovation is fairly stable across the 213 applications they examined, with an average value of 0.03. They find that the coefficient of imitation is far more variable about its average of 0.38, and argue that this finding demonstrates the coefficient's sensitivity to marketing variables, in agreement with KB's conclusions. Lee, Boatwright and Kamakura (2003) describe an application of the hierarchical Bayes procedure for forecasting sales of recorded music, pre-launch. Their model is

$$f(t) = \lambda(t)m(1 - F(t)) \tag{2.2.2}$$

where $\lambda(t)$ is a hazard function parameterised as a function of relevant exogenous variables describing the artist, the album and promotional activity. They show that the incorporation of these variables reduces pre-launch MAPEs from 69% to 52%, and that as sales data become available, the MAPE falls to around 30%.

Bass, Gordon, Ferguson and Githens (2001) prepared a pre-launch forecast of subscriptions to satellite television over a 5-year horizon. The Bass model was used with parameter values chosen by a

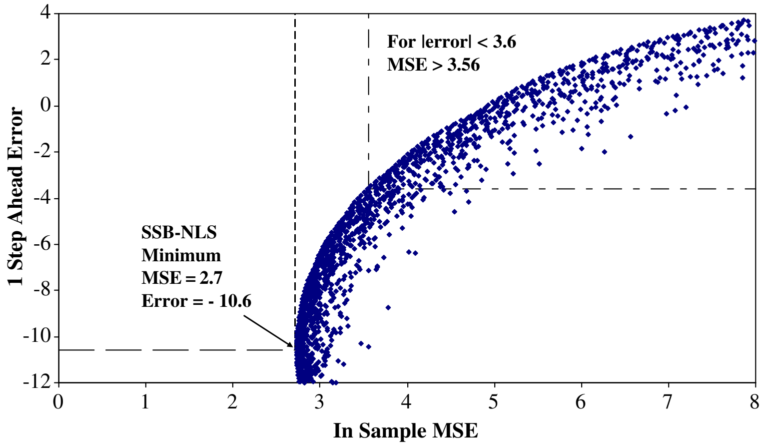

Fig. 5. The frontier of a scatter diagram showing the relationship between goodness of fit (MSE using data including 1974) and one-step-ahead forecast error.

mixture of analogy and collecting intentions data from potential consumers. They describe their forecast accuracy as 'quite good'.

2.3. Modelling constrained diffusion

In many of the historical examples used as diffusion data sets, such as telephones in the UK, US and elsewhere, supply constraints were present. In the case of fixed line telephones, the reasons relate to the slow post-war recovery and the behaviour of state monopolies. In the case of cellular telephones, supply restrictions can be a low capacity of installed base stations and the unavailability of services in some parts of the country. For example, cellular telephones in the US began operation in Chicago in 1983, in Los Angeles in 1984 and in thirty other metropolitan areas in the next phase (see Hausman, 2002). Sequential availability of interactive technology can be (and is) used as a business strategy because it allows the development of a critical mass in a particular area, before the service is launched in other parts of the country. Supply constraints continue to affect the satisfaction of demand of such desirable innovations as the iPod MP3 player. Jain, Mahajan and Muller (1991) propose an adaptation to the Bass model where there is a third category introduced of 'waiting applicant'. This model is fitted to data describing the diffusion of fixed line telephony in Israel. Islam and Fiebig (2001) extend this approach to carry out a multi-national study to estimate the saturation levels for fixed line telephony in 46 supply-restricted countries. They used pooled cross-sectional estimates to forecast supply restricted demand where little or no data is available. Ho, Savin and Terwiesch (2002) look at managing demand for new products in the presence of supply constraints. For example, this problem is faced by manufacturers of new products such as Play Stations whose introduction is eagerly awaited by enthusiasts. They have to decide how much of the product to produce before launching the product. Optimal launch timing balances inventory costs against the possible loss of customers due to impatience. The main difficulty in implementation is that optimal timing and optimal capacity depend on the coefficient of imitation, q, whose estimation is most problematic. In this case, the estimate will have to be by analogy. Kumar and Swaminathan (2003) tackle the same problem almost simultaneously. The authors differ in their findings. The problem they address is: if demand exceeds supply, should a monopolist damp down demand by building up inventory to satisfy later demand, or should he take the myopic view and satisfy demand as soon as possible? Ho et al. suggest that it is never optimal to delay filling demand, whereas Kumar and Swaminathan show that, under a particular cost structure, the strategy of building up stock can be optimal.

2.4. Modelling diffusion and replacement

In some situations, when modelling and/or forecasting sales of a new product, it is not possible to distinguish between first time purchases of the product (adoptions) and replacement purchases. When the product is becoming mature, adoptions are a small proportion of total sales and the imperative is to model total sales rather than adoptions. Olson and Choi (1985) proposed a decomposition of sales, S_t, into adoptions, y_t, and replacement sales, R_t. The Bass model was used for adoptions and R_t depends on the density function, $\lambda(t)$, the product lifetime (the Rayleigh in the case of Olson and Choi) and past sales, as shown here:

$$R_t = \sum_{i=1}^{t} \left(\int_{i-1}^{i} \lambda(x)\mathrm{d}x \right) S_{t-i} \tag{2.4.1}$$

Kamakura and Balasubramanian (1987) (KB2) used a truncated normal for $\lambda(t)$. Steffens (2001) has made the (KB2) approach dynamic by including a time-varying aggregate replacement distribution. The time to replacement is modelled as a function of price. In comparison with a static KB2 model, he found the dynamic model to be substantially more accurate than the static model over 1- to 5-year horizons. Islam and Meade (2000) compared seven different density functions for $\lambda(t)$ and a non-parametric version. They found that, over 42 data sets, there were many occasions when their maximum likelihood estimation would not converge for combinations of data and density function. Overall, their non-parametric approach dominated the parametric models in terms of forecasting accuracy.

Some authors have used cross-sectional survey data to gain further insight into the replacement decision. Fernandez (2000) shows that the time to replacement of air conditioners can be modelled in terms of demographic, environmental and cost variables. Grewal, Mehta and Kardes (2004) use attitude variables (derived from factor analysis) alongside variables describing the product and associated costs. Unfortunately, it is difficult to see how these studies can be harnessed to the time series forecasting approaches described above. Their value probably lies in a data rich environment, where an owner, who is predicted to replace soon, can be persuaded to replace sooner.

A related problem is modelling diffusion of a product with multiple purchases, for example, one household may have several televisions. This problem is addressed by Bayus, Hong and Labe (1989) and Steffens (2003).

2.5. Modelling the diffusion of multiple sub-categories

An innovative product may be available under separate sub-categories. Many products such as mobile telephones are branded; thus, in this example, the sub-categories are competing brands. Another example of sub-categorisation is where there are two or more competing standards, as with video recorders (see Section 2.2). Another categorisation that has been studied is products acquired legally or illegally.

Mahajan, Sharma and Buzzell (1993) model the effect of a new entrant to an expanding market. Their approach is a development of the Bass model as represented by (2.2.1)

$$Y_{i,t} - Y_{i,t-1} = p_i\left(m_i - Y_{i,t-1}\right) + \left(\frac{q_i}{m_i} Y_{i,t-1}\right)(M - Y_{t-1}) \tag{2.5.1}$$

where $Y_{i,t}$ represents the cumulative sales of sub-category i, m_i is the market potential of sub-category i, $Y_t = \sum_i Y_{i,t}$ and $M = \sum_i m_i$. They apply the model to competing brands of cameras offering 'instant' photographs. Krishnan, Bass and Kumar (2000) criticised this model (2.5.1) because sales of brand i were only affected by the cumulative buyers of brand i and no others (because of the term $\frac{q_i}{m_i} Y_{i,t-1}$). They proposed an alternative hazard function-based approach, where the hazard function for brand i is:

$$h_i(t) = \frac{f_i(t)}{(1 - F(t))} = p_i + q_i F(t). \tag{2.5.2}$$

Note here that the imitation process is driven by adopters of all brands rather than just brand i. An attraction of this formulation is that the sub-category models sum to a Bass model for the whole category. The model was fitted to three different markets for mobile telephones, showing that a late-arriving third brand could increase the speed of diffusion, increase the market potential or do both simultaneously.

Givon, Mahajan and Muller (1995) used a model similar to (2.5.2) for the diffusion of software where the sales of legal copies are known but the 'sales' of pirated software are not. Given an estimate for the software market potential provided by a diffusion-based model of the number of extant personal computers, using UK data for spreadsheet and word-processor sales, they demonstrate the interaction between these two sub-categories. They found that users of pirated software were a dominant influence in the imitation component of diffusion; pirate users were estimated to outnumber legal users by six to one. Givon, Mahajan and Muller (1997) extended their work to brands of the same software products and found that piracy rates differed significantly between brands.

Kim, Chang and Shocker (2000) model competition between products that fulfil the same function, rather than different brands of the same product; their work is discussed in more detail in Section 4. A range of other formulations for the interaction between the diffusions of related sub-categories of products such as prey–predator models is discussed in Shocker, Bayus and Kim (2004).

2.6. Model selection and forecasting

2.6.1. Studies of comparative forecasting accuracy

Armstrong, Brodie and McIntyre (1987) complain that little is known about the comparative performance of sales forecasting models in a given situation. In the situation of forecasting the diffusion of an innovation, one would expect a diffusion model to be more accurate than a time series model such as Holt–Winters' with a linear trend. Gottardi and Scarso (1994) compared the forecasting accuracy of ARIMA models with a selection of diffusion models and found that the non-symmetric responding logistic of Easingwood et al. (1981) (A1.12) was most accurate (lowest mean absolute percentage error). However, since many of the data sets were inappropriate as they described consumption or production, rather than diffusion, this empirical comparison is of little value.

Young (1993) used nine variations of growth curves to forecast forty-six data sets. Forecast accuracy was compared over the last three data points; Harvey's model (A1.16) was the most accurate model thirteen times, the Bass model (as implemented in (2.2.1)) was the most accurate model twelve times.

Meade and Islam (1995a) used fourteen variations of growth curve models to forecast the adoption of telephones in fifteen different countries. For a kernel of nine series and nine growth curve models, relatively free of estimation problems, they used a Friedman test to compare forecasting accuracy. The most accurate group of models contained the local logistic (A1.13), Gompertz (A1.07), logistic (A1.09) and extended logistic (equivalent to the version of the Bass model in (A1.02)); these were found to be significantly more accurate than the Bass (as implemented in (2.2.1)), the non-symmetric responding logistic (A1.12) and the flexible logistics (A1.11).

Hardie et al. (1998) (using non-diffusion, consumer goods data) found that model fit was 'largely unrelated' to forecasting performance.

Meade and Islam (1998) classified twenty-nine diffusion models into three classes according to the timing of peak diffusion in relation to introduction and saturation. This was done in order to aid model selection. Using the criteria of model fit and short-term forecast stability in conjunction with membership of each class, the prior probability that a data set belongs to each class is calculated. These prior probabilities allow the calculation of a combined forecast. In 77% of the 47 data sets examined, the combined forecast was more accurate than the best fitting model; the average improvement in root mean square error was 8%.

Bewley and Griffiths (2003) model the penetration of the compact disc (CD) in sound recording in twelve countries. They use the Bass model and versions of the flexible logistic (A1.11). They find that in ranking the relative accuracy of forecasts, the Box–Cox transformation variant of the flexible logistic substantially outperforms the Bass model.

Bass, Jain and Krishnan (2000) compared the one-step-ahead forecasting performance of four versions of the Bass model (the original, the generalised and two versions of a proportional hazard formulation of the Bass model) using three data sets. They found that one version of the proportional hazard model outperforms other models in one-step-ahead forecasts. Bemmaor and Lee (2002) compared the one to three steps ahead forecasting performance of the Bass model and the Gamma-shifted Gompertz (G-SG) model using twelve products and services. The G-SG model only out performed the Bass model for one-step-ahead forecasts. As the Bass model is a special case of the G-SG model, model parsimony becomes more important in long range forecasting.

In summary, for homogeneous data sets there is likely to be a preferred model, as shown by Meade and Islam (1995a) and Bewley and Griffiths (2003). However for heterogeneous data sets, the evidence continues to point to the non-existence of a best forecasting diffusion model, the principle asserted by Meade and Islam (2001). In this case the use of combined models suggested by Meade and Islam (1998) is a low risk approach.

2.6.2. Use of Prediction intervals

A guide to the uncertainty associated with a forecast is desirable in any circumstance. Chatfield (1993) gives the case for prediction intervals and cites reasons why they could be too narrow, such as taking parameter estimates as known values, assuming normality of errors, and assuming correct model identification. In the case of diffusion models, the case for being wary of these pitfalls is particularly strong. Several authors have suggested ways of generating prediction intervals.

Meade's (1985) use of the extended Kalman filter generated the information basis which, coupled with Monte Carlo simulation, allowed the provision of a prediction interval for an arbitrary horizon. Migon and Gamerman (1993) used a Bayesian approach to forecasting diffusion via a generalised exponential growth model. This approach allows the computation of prediction intervals and their model class includes the logistic and Gompertz curves. Meade and Islam (1995b) compare three methods for computing prediction intervals. Each method takes into account both noise and the uncertainty of the parameter estimates. The methods compared were: a Taylor series approximation of error variance; explicitly modelling the error density; and the use of boot-strapping. They found the explicit density approach the most accurate.

Bewley and Griffiths (2003) use boot-strapping to generate prediction intervals for their CD penetration forecasts. Gutiérrez, Nafidi and Gutiérrez Sánchez (2005) formulate a stochastic version of the Gompertz that allows them to provide confidence intervals for their out-of-sample forecasts.

2.7. Applications

Examples of Meade's (1984) concerns where saturation level has no bound (or meaning) are given in applications by Suslick, Harris and Allan (1995), who use the logistic to forecast US crude oil production and world consumption of copper; and by Gutiérrez et al. (2005) who forecast the consumption of natural gas in Spain.

Mahajan et al. (1990) and Lilien, Rangaswamy and van den Bulte (2000) describe generic uses of diffusion modelling in marketing. These uses include pre-launch forecasting (see Section 2.2.2); strategic decision analysis based on the product life cycle; and the determination of optimal timing of market entry.

Mahajan (1994) mentions the importance of diffusion modelling in a variety of strategic applications. These include: business valuation, where the business depends on products at various points in their life cycles; and relating capacity to demand; these issues are discussed in Section 2.3. Mahajan et al. (2000b, Table 1.1) document eight published applications: two for pre-launch and launch strategic decisions and six for post-launch strategic decisions.

In the realm of technologies studied, the area of telecommunications is particularly rich in published applications (nine of the references mention telecommunications or telephones in their titles) and unpublished conference papers by practitioners.

3. Modelling of diffusion across several countries

Modelling the diffusion of the same innovation in several countries offers a number of benefits. A practical forecasting advantage is that it helps overcome a perennial difficulty of using diffusion models for forecasting, their hunger for data. If an innovation is released in different countries at different times, it is desirable to be able to use the data from earlier adopting countries to predict the diffusion in later adopting countries. Modelling the effect of different national cultures on the diffusion process gives insight into the effect of national differences on the rate of adoption of the innovation. For example, the exercise may shed light on whether later adopting countries adopt more quickly than earlier adopters.

Addressing this last question, Takada and Jain (1991) used the Bass model for a cross-sectional analysis of the diffusion of durable goods in four Pacific Rim countries. They used the estimated coefficients to test hypotheses on country-specific effects and on lead-lag time effects on the diffusion rates. They established significant differences in the coefficients of imitation between countries with different cultures, such as US and Korea. They also found evidence that a lagged product introduction led to accelerated diffusion. The effect of lead-lag on international diffusion of innovations has been addressed more recently by Ganesh and Kumar (1996), Ganesh, Kumar and Subramanian (1997), Kumar, Ganesh and Echambadi (1998) and Kumar and Krishnan (2002). The premise is that the time lag grants additional time to potential adopters in the lagging markets to help them to understand the relative advantage of the product, better assess the technology need, and observe experience of the lead country adopters' usage of the product. Kalish, Mahajan and Muller (1995) argue that the potential adopters in the lagging countries observe the introduction and diffusion of technology in the lead country. If the product is successful in the leading countries, then the risk associated with the innovation is reduced, thus contributing to an accelerated diffusion in the lagging countries.

Gatignon et al. (1989) proposed a methodology for modelling and forecasting the multinational diffusion of innovations based on the Bass model. The market potential is estimated for each country; the coefficients of innovation and imitation are functions of national characteristics. For example, the coefficient of innovation for country i is:

$$p_i = \beta_{p,i,0} + \sum_k \beta_{p,i,k} Z_{i,k} + e_{p,i} \tag{2.6.1}$$

where $Z_{i,k}$ represents a cultural variable, $\beta_{p,i,k}$, (for i=0, 1, ...) are estimated coefficients, and $e_{p,i}$ is a disturbance term. The estimation of the model was achieved by generalised least squares. The cultural variables used to describe national cultural differences were: cosmopolitanism (communication with foreign countries by post or by travel); mobility (car ownership) and the role of women in society (proportion of women in the workforce). The model was demonstrated on six innovations ranging from lawn mowers to pocket calculators over fourteen European countries. The choice of national cultural variables, designed to capture a nation's propensity to innovate, has received much attention.

Talukdar, Sudhir and Ainslie (2002) investigated the impact of a wide range of macro-environmental variables on the parameters of the Bass model while modelling diffusion of six products across thirty-one developed and developing countries. They found that, on average, the market potential in developing countries was a third of that in developed countries; and that despite lagged introduction, the rate of adoption was slower in developing countries. They found that market potential was best explained by previous experience in the same country; in contrast, the probability of adoption was better explained by product experience in earlier adopting countries. Similar findings are reported by Desiraju, Nair and Chintagunta (2004) who modelled the diffusion of pharmaceutical drugs with the logistic model using data from 15 countries. In addition, they found that per capita expenditure on healthcare was positively related to the rate of adoption, while higher prices decreased adoption rates.

Innovativeness as a concept is discussed by Midgley and Dowling (1978), who suggested that there are two factors underlying the adoption of a new product/technology: one is the distribution of innate innovativeness in the population (echoing Rogers); the other is the communication between the population members (contagion). The realised behaviour in the adoption of an innovation depends on the interplay between these factors. Lee (1990) investigates the innovativeness of nations empirically. A cross-sectional study of 70 nations is carried out where innovativeness is represented by the proportion of the population owning a television. The significant predictor variables were GNP/capita, proportion literate, proportion of scientists and the proportion of GNP generated by the manufacturing sector. Lynn and Gelb (1996) compiled an index of national innovativeness based on ownership of a range of recently introduced products. They explain this index in terms of the national traits developed by Hofstede (1983, 1984): individualism, uncertainty avoidance and purchasing power. These variables were significant in explaining the index, but not for all individual new products. For example only purchasing power was necessary to explain ownership of video cameras. Steenkamp, Hofstede and Wedel (1999) further investigate national innovativeness alongside variables describing the individual. The dependent variable was the consumer's score on the 'Exploratory Acquisition of Products' scale developed by Baumgartner and Steenkamp (1994). On the individual level, measures of ethnocentrism, attitude towards the past (nostalgia) and education were used. On a national level, they used individualism, uncertainty avoidance and masculinity (greater emphasis on wealth and material goods in contrast to valuing people and helping others). They found that on an individual level, both ethnocentrism and attitude to the past were negatively related to innovativeness. On a national level, individualism and masculinity were positively related with innovativeness, and uncertainty avoidance was negatively related. In an exercise that provides a measure of national innovative capacity, Furman, Porter and Stern (2002) model the innovativeness of the different nations, measured by patent applications in the US in terms of variables including income (GDP), research expenditure, and levels of international trade.

Helsen, Jedidi and DeSarbo (1993) use a latent variable approach to simultaneously cluster nations into segments and estimate Bass coefficients using twenty-three variables. Their factors (groupings of variables) are mobility, health, trade, life and cosmopolitanism. They found two or three segments (country groupings) which differed depending on which innovation (colour television, VCR or CD players) was considered.

3.1. Estimation and model choice in multinational diffusion models

Islam, Fiebig and Meade (2002) compared several formulations of the parameters of the Bass and Gompertz models as functions of national variables for the diffusion of three telecommunication products. They pooled the estimation of the Gompertz growth parameter and the Bass coefficients and made the market potential for each country a function of GDP/capita as a measure of wealth and various costs associated with product adoption. They found that the pooled Gompertz model offered plausible estimates of market potential and generally produced more accurate forecasts than single national models. Kumar and Krishnan (2002) develop a multinational Bass model which incorporates both simultaneous effects and lead-lag effects. Their framework allows the first country to introduce a technology to affect subsequent countries's rates of adoption, and to allow the later countries's adoption rates to affect earlier adopting countries as well as allowing adoption in different countries to have a simultaneous effect. Talukdar et al. (2002) and Desiraju et al. (2004) used a hierarchical Bayesian framework to estimate their pooled cross-sectional models.

3.2. Applications

Gruber and Verbovan (2001) used a logistic as the basis for modelling the diffusion of telecommunications within the European Union. They interpreted the introduction of digital mobile telephones as a relaxation of the capacity constraint imposed by analogue technology. They further found that competition between suppliers increased the diffusion rate. They found a strong positive relationship between the timing of introduction (granting of licenses) and the subsequent rate of adoption, however convergence in penetration levels between early and late countries is expected to occur relatively slowly. Frank (2004) uses a similar approach to Gruber and Verboven to model the diffusion of mobile telephones in Finland. The equivalent of the imitation coefficient, q, is parameterised as a function of GDP/capita, a dummy variable identifying the introduction of GSM technology and a variable describing the proportion of fixed line telephones. However, in contrast to Gruber and Verboven, only GDP/capita was found to be significant.

Kiiski and Pohjola (2002) examine the factors influencing the cross-country diffusion of the internet. Using internet hosts per capita as the diffusion variable, with a Gompertz-based model, they identify the main determinants of diffusion as GDP per capita and access cost. They also found that a greater proportion of post 15-year-olds in tertiary education led to faster diffusion.

Kalish et al. (1995) used a diffusion model normatively to examine under which conditions waterfall (sequential entry) and sprinkler (simultaneous entry) strategies should be selected when entering international markets.

4. Modelling of diffusion across several generations of technology

Norton and Bass (1987) proposed an adaptation of the Bass model that considered different generations of a technology. Examples are the series of generations of mobile telephones and personal computers. In the Norton–Bass model, each generation of the technology attracts incremental population segments of potential adopters; in addition, later generations may attract potential adopters of earlier generations. This modelling approach effectively succeeded the models on technological substitution, where one technology replaced its predecessor. Fisher and Pry (1971), Blackman (1972) and Sharif and Kabir (1976) used variants of the logistic model for technological substitution. Examples of substitutions were diesel for steam locomotives, and steel for wood in ship hulls. Meade (1989) contrasts the dynamics of the diffusion of colour television, a retail product, with the diffusion of industrial products where the number of decision makers is small. A framework for a stochastic substitution model for modelling the properties of the different adopting populations is demonstrated. In a more recent example, the Gompertz model was used to forecast the substitution of electronic payments for cash in ten European countries by Snellman, Vesala and Humphrey (2001).

The modelling innovations of Norton and Bass were that the new (generation of) technology attracted more potential adopters and that more than two generations of technology could be considered simultaneously. Norton and Bass (1992) demonstrated their model on data from the electronics, pharmaceutical, consumer and industrial sectors. Speece and MacLachlan (1992) demonstrated that the Norton–Bass model modelled and forecast the adoption of successive generations of gallon milk containers. Mahajan and Muller (1996) extended the Norton–Bass model to allow adopters of early generations to skip generations, for example, an adopter of the first generation could replace it with third generation technology. They demonstrated their model using generations of IBM mainframe computers. Islam and Meade (1997) demonstrated that the assumption of constant coefficients of innovation and imitation (p and q) over successive generations could be relaxed. In a study of multi-national mobile telephone adoption, they demonstrated that the coefficient of imitation (q) tended to increase from generation to generation. Sohn and Ahn (2003) use the Norton–Bass model to demonstrate a cost–benefit analysis of introducing a new generation of information technology.

The case where demand data does not explicitly identify which generation of technology is purchased is discussed by Jun and Park (1999). They model consumer utility over time to allocate demand to the appropriate generation and demonstrate their model on demand data for dynamic random access memory chips.

The Norton–Bass approach is extended by Kim et al. (2000) who consider the case where several devices compete to fulfil the same function, some of which are different generations of the same device. The example they consider is mobile telephony, where the competing devices are the pager, the analogue mobile telephone, the digital mobile telephone and a device for making but not receiving calls called the CT2. Applying their approach to the mobile telephony markets of Hong Kong and Korea, the Bass model is used to represent the numbers of subscribers to the pager and the CT2; and the Norton–Bass model is used to represent the numbers of subscribers to the two generations of mobile telephones. The impact of competition between the devices is captured by making the market potential of each device a function of the subscriber base of its competitors. The out-of-sample accuracy of their approach is shown to be superior to the appropriate Bass or Norton–Bass alternative and vastly superior to the naïve alternatives of double exponential smoothing and linear regression.

An alternative modelling approach to that of Norton and Bass is demonstrated by Versluis (2002). He uses a model developed by Marchetti (1977) which divides a technology's life cycle into growth, saturation and decline. The data are the diffusion of generations of dynamic random access memory chips. The comparison with other models shows a better fit than the Norton–Bass model; however, there is no out-of-sample comparison of forecasting accuracy.

4.1. Use of explanatory variables in multi-generation models

Speece and MacLachlan (1992) found that the inclusion of price as an explanatory variable in their milk container study improved forecasting accuracy (provided the future prices were known). Padmanabhan and Bass (1993) examine optimal pricing strategies for successive generations. They find that the optimal strategy differs according to the nature of the producer (integrated monopolist or independent), the degree of cannibalisation by the newer product of the old, and the degree of foresight of the producers. Danaher, Hardie and Putsis (2001) explore the use of price as a covariate in a variety of models of generations of mobile telephones. The models they compare for adoption time are the Bass model (no price effect), generalised Bass model and a proportional hazards model which incorporates a Bass model of the baseline adoption. (This is a multi-generation version of Jain, 1992.) The proportional hazards approach provided a superior fit to the generalised Bass model, which failed to detect a price effect. They found evidence that a skimming price policy was used and was consistent with the Bass parameters (according to Kalish, 1983, various combinations of values of p and q are consistent with a skimming price policy). They found that lowering the price of the earlier generation increased take-up of that generation and led to greater take-up of the succeeding generation (because of the greater number of subscribers). They further demonstrated that lowering the price of the succeeding generation led to a greater take-up of this generation while causing a proportionately smaller decrease in take-up of the earlier generation.

Pursuing a different variation of the Bass model, Jun and Park (1999) assume customers maximise their utility in deciding when to upgrade (or adopt) a later generation of technology. The probability of adoption is a function of the utility of the consumer, which is in turn a function of the product price. Jun, Kim, Park, Park and Wilson (2002) apply and develop this approach to forecasting, firstly to the switch from analogue to digital and secondly to the adoption of two competing digital services. Both analyses use Korean telecommunications data.

5. Multi-technology models

In Section 2.2.2 we discussed the problem of forecasting the diffusion of a new product with little or no data. In this situation, using parameter estimates for a product analogous to the product of interest is a viable approach. Meade and Islam (2003) extend this idea by modelling the relationship between the times to adoption of a technology by different countries. The dependence between the times to adoption by a country of two related innovations, the fax and the cellular telephone, is modelled in two stages. For the first stage, the choice of density function for the time to adoption, a Weibull density function is used with its scale factor adapted to account for the economic and technological environments in different countries. In the second stage, describing the dependence relationship, copulas are used. The Frank and Plackett copulas, coupled with the Weibull, using eight environmental variables, are shown to provide valuable insights into the effects of environmental variables on adoption times. Once a country has adopted one technology, the model of the dependence relationship provides the conditional density of the time to adoption of the other technology.

6. Conclusions and likely further research

The wealth of research into modelling and forecasting the diffusion of innovations is impressive and confirms its continuing importance as a research topic. In terms of practical impact, the main application area is the introduction of consumer durables, particularly in telecommunications. Telecommunications is an application that lends itself to modelling the effects of all the main themes identified here: marketing mix, the multinational diffusion of services and the modelling of multi-generational diffusion.

In terms of research questions that are still open and those questions which have been resolved, the balance is strongly in favour of the former.

For example, although there is some convergence of the most appropriate way to include marketing mix-variables into the Bass model, there are several viable alternative models. This lack of closure is likely to continue simply because the processes underlying diffusion are far more complex than the models recognise and the lack of data allows only the de-selection of models which are obviously poor approximations of reality.

Future directions of research are likely to be in areas such as:

Forecasting new product diffusion with little or no data:

> there are two main reasons for this suggestion, one is that there is demand for more accuracy—any reduction of the uncertainty will be valuable—and the second is the increasing availability of cross-sectional and time series data describing consumers. The availability of this type of data will allow better estimates of model parameters. The existence of these data is likely to encourage more normative modelling to refine pricing and marketing strategies, and developments in real option valuation methods are likely to add to the value of these normative exercises. A priority among practitioners is to establish market potential as early as possible in the diffusion process, if possible before the process even begins. The identification of factors determining market potential is a fruitful area of research. Possible approaches include a new meta-analysis, specifically focussed on this agenda; or an analysis of existing meta-analyses such as those by Van den Bulte and Stremersch (2004) or Sultan et al. (1990). The development of a scale for imitation is also a profitable area of interest. This scale would complement the literature on an innovation scale (e.g. Steenkamp et al., 1999) and would further facilitate forecasting the very early stages of diffusion.

Forecasting with multinational models:

> this modelling area will continue to generate intense interest. One driver for this is the development of multinational telecommunications service providers. The launch of a new service across several companies is increasingly likely to be due to a multinational company's strategic plan, rather than the result of individual company decisions in the different countries. Again, there is scope for normative modelling for strategy evaluation.

Forecasting with multi-generation models:

> work in normative modelling in this area has already been published. The telecommunications application is likely to lead to multinational, multi-generation models for both forecasting and normative purposes.

We can be quite confident of one forecast:

> that is, our list of suggestions has omitted some major future advances in diffusion modelling and forecasting, and we look forward to discovering the nature of these omissions.

Appendix A. An annotated list of S-shaped diffusion models

Notation: X_t is the cumulative number of adopters at time t. The saturation level is usually denoted by a (except in the case of the Bass model, where the conventional notation is used). Additional parameters are denoted by b and c. In some cases, where the diffusion curve is related to a density function, μ and σ are used.

Where possible, the models are presented as equations for cumulative adoption. Those that do not fit into this category appear as non-linear trend models or non-linear autoregressive models.

A.1. Models for cumulative adoption

A.1.1. Bass model

Bass (1969) considered a population of m individuals who are both innovators (those with a constant propensity to purchase, p) and imitators (those whose propensity to purchase is influenced by the amount of previous purchasing, $q(X_{t-1}/m)$). Here we give the continuous time formulation used by Schmittlein and Mahajan (1982). The probability density function for a potential adopter making an adoption at time t is:

$$f(t) = (p + qF(t))(1 - F(t)). \tag{A1.01}$$

The corresponding cumulative density function is

$$F(t) = \frac{1 - \exp(-(p+q)t)}{1 + \exp(q/p)(-(p+q)t)}. \tag{A1.02}$$

An alternative definition is

$$G(t) = cF(t) \tag{A1.03}$$

where c is the probability of eventual adoption. The expected number of adopters at time t is $cMG(t)$, where the size of the relevant population is M.

In some cases, it will be convenient to refer to the hazard function:

$$h(t) = \frac{f(t)}{(1-F(t))}. \tag{A1.04}$$

A.1.2. Cumulative lognormal

$$X_t = a\int_0^t \frac{1}{y\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(\ln(y)-\mu)^2}{2\sigma^2}\right)\mathrm{d}y \tag{A1.05}$$

Bain (1963). Asymmetric with point of inflection before 0.5 saturation level is reached.

A.1.3. Cumulative normal

$$X_t = a\int_\infty^t \frac{1}{\sqrt{2\pi\sigma^2}}\cdot\exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right)\mathrm{d}y \tag{A1.06}$$

Rogers (1962). Its shape closely resembles logistic.

A.1.4. Gompertz

$$X_t = a\exp(-c(\exp(-bt))) \tag{A1.07}$$

Gregg, Hassel and Richardson (1964). Asymmetric about its point of inflection, which occurs before the diffusion has reached half the saturation level.

A.1.5. Log reciprocal

$$X_t = a\exp\left(\frac{1}{bt}\right) \tag{A1.08}$$

Used by McCarthy and Ryan (1976)

A.1.6. Logistic

$$X_t = \frac{a}{1+c\exp(-bt)} \tag{A1.09}$$

Gregg et al. (1964). Symmetric about its point of inflection (i.e., half the potential adopters have the product at the point of inflection). The model was used in a linearised form by Mansfield (1961), see A1.10.

There are many variations on the logistic theme, including:

$$\text{Log-logistic:}\quad X_t = \frac{a}{1+c\exp(-b\ln(t))} \tag{A1.10}$$

Tanner (1978). The replacement of time by ln(time) means that the curve is asymmetric about its point of inflection.

Flexible–logistic (FLOG) models:

$$X_t = \frac{a}{1+c\exp(-B(t))} \tag{A1.11}$$

Bewley and Fiebig (1988). A four-parameter generalization of the logistic growth curve, the FLOG model is sufficiently general to locate the point of inflection anywhere between its upper and lower bounds. By generalizing $B(t)$, the imitation effect, Bewley and Fiebig generate a range of models:

Inverse Power Transform (IPT), where

$$B(t) = b(1+kt)^{1/k} - 1$$

Exponential (ELOG), where

$$B(t) = b\frac{\exp(kt-1)}{k}$$

Box and Cox, where

$$B(t) = \left(b\frac{(1+t)^k - 1}{k}\right)$$

Non–symmetric responding logistic:

$$X_t = \frac{a}{1+c\exp(-bX_{t-1}^{\delta}t)} \tag{A1.12}$$

Easingwood et al. (1981). The underlying belief here is that the propensity to imitate, represented by b in the simple logistic model, changes in response to the number of adopters.

Local logistic: $E(X(t+L|X_t=x_t))$

$$=\frac{ax_t}{x_t+(a-x_t)\exp(-bL)} \quad \text{(A1.13)}$$

Meade (1985). Forecasts logistic growth from the last known value of diffusion.

A.1.7. Modified exponential

$$X_t=a-c\exp(-bt) \quad \text{(A1.14)}$$

Gregg et al. (1964). No point of inflection; gradient decreases monotonically to the saturation level. Essentially, this is the model used by Fourt and Woodlock (1960).

A.1.8. Weibull

$$X_t=a\left(1-\exp\left(\left(\frac{t}{c}\right)^b\right)\right) \quad \text{(A1.15)}$$

Suggested for use as a diffusion model by Sharif and Islam (1980).

A.2. Linearised trend and non-linear autoregressive models

A.2.1. Harvey

$$\ln(X_t-X_{t-1})=b+c_1t+c_2\ln(X_{t-1}) \quad \text{(A1.16)}$$

Proposed by Harvey (1984).

These remaining models assume a given saturation level and X_t represents the proportion of adopters at time t.

A.2.2. Floyd

$$\left[\frac{1}{1-X_t}\right]+\ln\left(\frac{X_t}{1-X_t}\right)=b+ct \quad \text{(A1.17)}$$

Proposed by Floyd (1962). Deleting the first term [in square brackets] in this equation gives the linearised form of the logistic proposed by Mansfield (1961).

A.2.3. Sharif and Kabir

$$\ln\left(\frac{X_t}{1-X_t}\right)+\sigma\left(\frac{1}{1-X_t}\right)=a+bt \quad \text{(A1.18)}$$

A linear combination of the Mansfield model and the Floyd model suggested by Sharif and Kabir (1976).

A.2.4. KKKI

$$\left(\frac{q-pb}{q}\right)\ln(p+qX_t)$$
$$-(b+1)\ln(1-X_t)$$
$$=c+(q+p)t \quad \text{(A1.19)}$$

Proposed by Kumar and Kumar (1992) as a technological substitution model derived from a population dynamics model by Smith (1963).

A.2.5. SBB

$$X_t=X_{t-1}\exp(b(1-X_{t-1})) \quad \text{(A1.20)}$$

Proposed by Sharma, Basu and Bhargava (1993).

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