Physics 3102 - Quantum Transport

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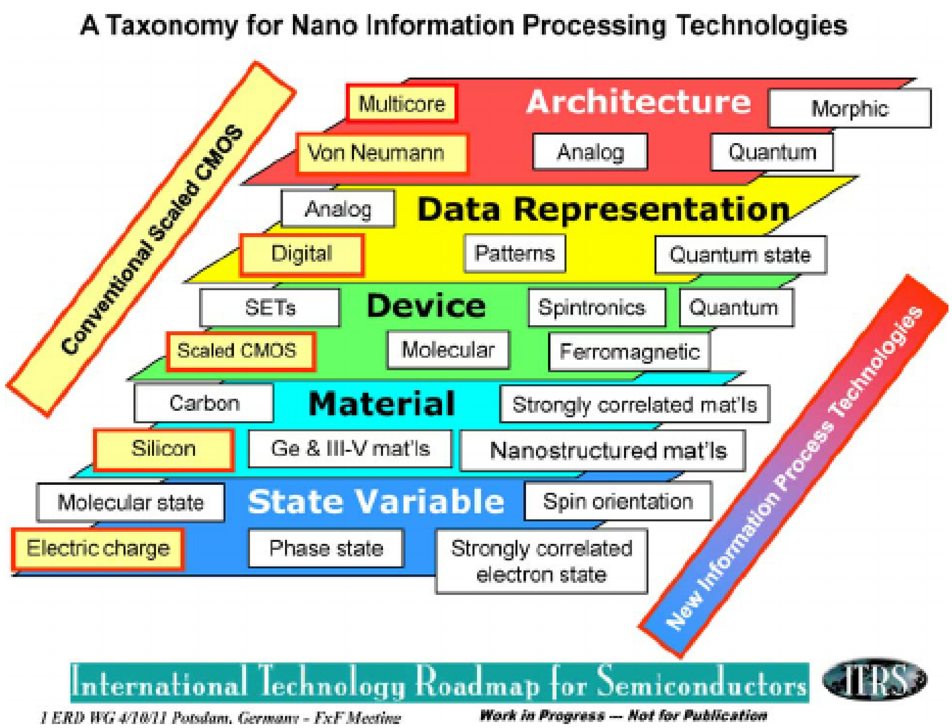
Introduction:
Notes written in fall 2025 following Sergey Frolov's University of Pittsburgh, Spring 2013 youtube lectures.

Contents

1 Introduction

- Quantum transport is the study of how charged particles (like electrons) move through materials and devices.
- To get our transport to behave quantum mechanically we can:
  - shrink the size of our device to be comparable to the electron's wavelength (nanoscale devices)
  - cool the device to very low temperatures to reduce thermal vibrations (would want energy scale of wavefunction to be larger than the thermal energy)
- metal–oxide–semiconductor field-effect transistor (MOSFET) cannot be made arbitrarily small because of quantum tunneling through the gate oxide layer (CMOS is the manufacturing process)



- Key phenomena in quantum transport include:
  - Conductance quantization (ballistic transport in low-dimensional systems)
  - Quantum interference
  - Tunneling and Coulomb blockade
  - Mesoscopic superconductivity
  - Quantum bits (qubits)
  - Topological quantum phases
- Examples of new device concepts:
  - quantum computing (superposition states used for qm algos with speedup over classical algos)
  - spintronics (intrinsic spin used for new logic devices)
  - optoelectronics (device to generate or detect radiation on single photon level)

- Quantum of Charge: $e = 1.6 \times 10^{-19} C$
- Quantum of Conductance: $G_0 = \frac{e^2}{h} = 3.87 \times 10^{-5} \Omega^{-1}$

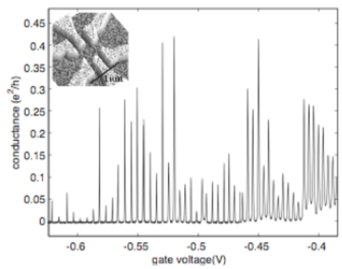


Figure 1.1: test

- Quantum of Resistance: $R_0 = \frac{h}{e^2} = 25.8 k\Omega$

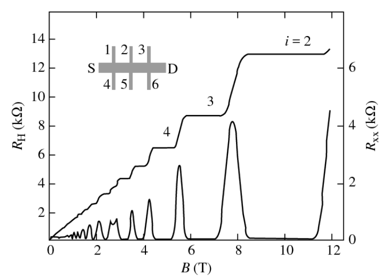


Figure 1.2: Quantum Hall Effect: In a 2D electron gas subjected to a strong perpendicular magnetic field, the Hall resistance becomes quantized in integer multiples of R_0, leading to precisely defined resistance plateaus.

- Quantum of Magnetic Flux: $\Phi_0 = \frac{e}{h} = 4.14 \times 10^{-15}$ webers
- Basic electrons: mass m_e, charge e, spin 1/2
  - PIAB
  - Waves in waveguides
  - Coulomb blockade
  - Zeeman effect
- Electrons+interactions+bandstructure effects:
  - Band structure effects $\rightarrow$ effect mass like 0.044 m_e in GaAs (great that it is lower so that it has higher mobility and easier to get quantum effects) and 0 in graphene
  - electron-electron interaction $\rightarrow$ fractional quantum Hall effect, kondo effect (electron with spin on island, conduction electrons in leads screen the spin)
  - Hyperfine coupling $\rightarrow$ interaction with lattice nuclear spin "bath"
  - Spin-orbit coupling $\rightarrow$ (also a band structure effect) interaction with electric fields in a lattice, motion of electron through electric field is seen as magnetic field in electron's rest frame, couples to spin of electron (i.e. control of movement of electron can control its spin)

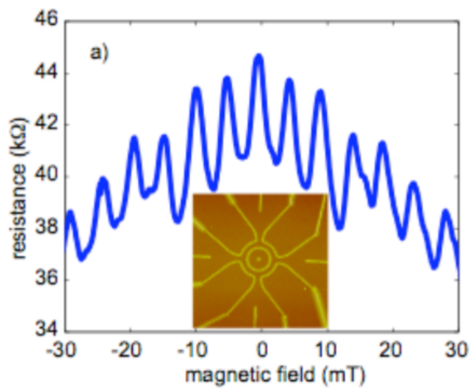


Figure 1.3: The Aharonov-Bohm effect: electrons traveling in a region with zero magnetic field can still be affected by a magnetic flux enclosed by their path, leading to observable phase shifts in their wavefunctions.

  - superconductivity $\rightarrow$ pairing of two electrons into (spin singlets called cooper pairs) a boson flow of supercurrent without resistance

- The quantum hall effect (2D): Si-MOSFET (Klaus von Klitzing 1980) [Online]
- Quantum point contacts: gates over 2DEG forms 1D channel in semiconductor heterostructure, apply source and drain bias and you measure quantized conductance (B.J. van Wees 1988) [Online]
- Single spin, single shot readout: quantum dot in GaAs (Elzerman 2004) [Online]
- Single spin control with magnetic field (Koppens 2006) [Online]
- Coupling between two single spins (Petta 2005) [Online]
- Superconducting Flux Qubit: Mooij 1999 [Online]
  - SQUID does the measurement of the flux qubit.

2 Energy and Length Scales

Charging energy (associated with one electron) $E_C = \frac{e^2}{2C}$

- Capacitance of a sphere of radius R: $C = 4\pi\epsilon_0 R$.
- What is the energy needed to charge the sphere with one electron?

Radius R	Capacitance C	Charging Energy E/k_B
$10\mu m$	10^{-15} F	0.84 K
$1\mu m$	10^{-16} F	8.4 K
$0.1\mu m$	10^{-17} F	84 K
$0.01\mu m$	10^{-18} F	840 K

Overview energy scales:

- $k_B T$: thermal energy scale at room temp is 26 meV
- eV: energy an electron gains when crossing a potential difference V
- μ chemical potential the energy required to add one particle to the system
- $\mu_B B$: Zeeman energy of an electron in a magnetic field B
- E_F: Fermi energy (it is the energy of the highest occupied electron state); for metals it is on the order of a few eV. For semiconductors it can be much smaller.
- ΔE: level spacing, energy difference between single-particle states calculated from the Schrödinger equation
- E_C: charging energy (more on this when we cover coulomb blockade)
- E_T: Thouless energy the energy scale of coherence effects

Length scales (quantum regime):

- for mesescopic and nanoscale structures: device dimensions are comparable to the fundamental size of electron. This causes their properties to be influenced by quantum mechanical effects.
- size of electron is essentially given by fermi wavelength λ_F
  - in most metals, electron density is very large (10^{21}cm^{-3}), and fermi is thus order of a few nanometers.
  - in semiconductors, lower carrier density ($\ll 10^{21}$cm^{-3}) and thus larger fermi wavelength of several tens of nanometers.
- Bloch's theorem says that electrons in a perfect crystal lattice will oscillate without scattering.
- scattering happens in real materials. electrons scatter from any disorder, such as defects and impurities but also from other electrons and phonons.
- Golden rule in Quantum mechanics says scattering from static potential does not change the energy of the electrons, so scattering off fixed impurity is elastic, and scattering off phonons or other electrons is inelastic.
- elastic and inelastic scattering length scales are material and temperature dependent.

- mean free path l is the average distance an electron travels between scattering events (correlation between initial and final momentum is lost – randomization).

$$l = v_F \tau$$

  when τ is the mean free time between scattering events.

- Mobility: $\mu = \frac{e\tau}{m}$ where τ is the mean free time between scattering events.
- Conductivity: $\sigma = \frac{ne^2\tau}{m} = ne\mu$
- 2D diffusion constant $D = \frac{v_F^2\tau}{2}$
- 1D diffusion constant $D = v_F^2\tau$
- There exists an elastic mean free path, and an inelastic mean free path.
- Coherent vs incoherent transport:
  - disruption of interference effects from electron phase breaking time τ_φ (often dont distinguish from inelastic scattering time τ_i but they are not the same because inelastic scattering does not always cause phase breaking).
  - phase coherence length $L_\varphi = \sqrt{D\tau_\varphi}$, is avg distance electrons diffuse in the material before their phase is disrupted through scattering.
  - To observe interference effects, this length must be comparable to device size which requires experiments be performed at low temperatures (to reduce phonon scattering).
- phase coherence for a small loop of wire at low temperatures with an applied magnetic field can lead to Aharonov-Bohm oscillations in the conductance of the loop as a function of magnetic field. If you make the loop bigger then 100 microns you will not see the oscillations because the phase coherence length is too small.
- transport regimes: since submicron structure can be fabricated on length sclaes smaller than the average impurity spacing in semiconductors, we can study different transport regimes:
  - Diffusive transport: electrons scatter multiple times off impurities, phonons, other electrons, etc. Mean free path l much smaller than device length L ($l \ll L$)
  - Quasi-ballistic transport: electrons scatter a few times, $l \sim L$
  - Ballistic transport: electrons travel through the device without scattering, $l \gg L$
- When is transport diffusive/ballistic and classical/quantum?
  - diffusive and classical: $\lambda_F, l_i, l_e \ll L$
  - diffusive and quantum: $\lambda_F, l_e \ll L, l_i$
  - ballistic and classical: $\lambda_F \ll L < l_e, l_i$
  - ballistic and quantum: $\lambda_F, L < l_e < l_i$
- Current-voltage characteristics: Superconductor-Insulator-Superconductor (SIS) junction (josephson junction): different than ohms law!
  - Current biased: as we increase I, voltage V across junction remains zero until I reaches critical current I_c at which point V jumps to a finite value and then increases linearly with I.
  - Voltage biased: as we increase V, current I across junction remains zero until V reaches critical voltage V_c at which point I jumps to a finite value and then increases linearly with V.

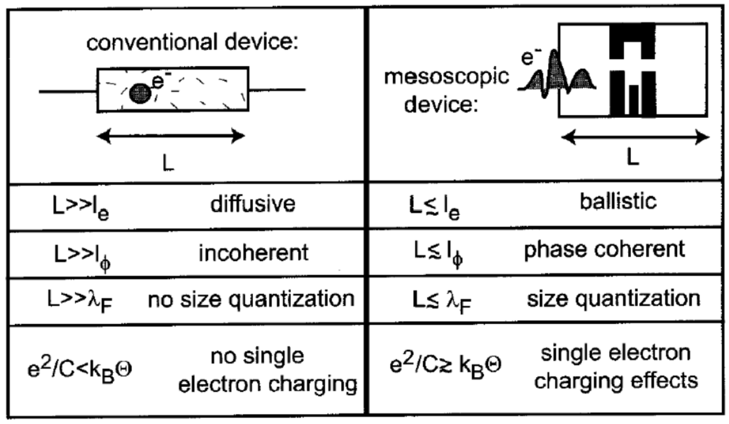


Figure 2.1: Different length scales relevant for electron transport in a conductor. The Fermi wavelength λ_F characterizes the quantum nature of electrons, while the mean free path l indicates the average distance an electron travels between scattering events. The phase coherence length L_φ represents the distance over which an electron maintains its quantum phase coherence, crucial for observing quantum interference effects.

		GaAs(100)	Si (100)	UNITS
Effective Mass	m	0.067	0.19	$m_e = 9.1 \times 10^{-28}$ g
Spin Degeneracy	g_s	2	2	
Valley Degeneracy	g_v	1	2	
Dielectric Constant	ε	13.1	11.9	$\varepsilon_0 = 8.9 \times 10^{-12}$ F m^{-1}
Density of States	$\rho(E) = g_s g_v (m/2\pi\hbar^2)$	0.28	1.59	10^{11} cm^{-2} meV^{-1}
Electronic Sheet Density[a]	n_s	4	1-10	10^{11} cm^{-2}
Fermi Wave Vector	$k_F = (4\pi n_s/g_s g_v)^{1/2}$	1.58	0.56–1.77	10^6 cm^{-1}
Fermi Velocity	$v_F = \hbar k_F/m$	2.7	0.34–1.1	10^7 cm/s
Fermi Energy	$E_F = (\hbar k_F)^2/2m$	14	0.63–6.3	meV
Electron Mobility[a]	μ_e	10^4–10^6	10^4	cm^2/V·s
Scattering Time	$\tau = m\mu_e/e$	0.38–38	1.1	ps
Diffusion Constant	$D = v_F^2\tau/2$	140–14000	6.4–64	cm^2/s
Resistivity	$\rho = (n_s e \mu_e)^{-1}$	1.6–0.016	6.3–0.63	kΩ
Fermi Wavelength	$\lambda_F = 2\pi/k_F$	40	112–35	nm
Mean Free Path	$l = v_F\tau$	10^2–10^4	37–118	nm
Phase Coherence Length[b]	$l_\phi = (D\tau_\phi)^{1/2}$	200–···	40–400	nm$(T/\mathrm{K})^{-1/2}$
Thermal Length	$l_T = (\hbar D/k_B T)^{1/2}$	330–3300	70–220	nm$(T/\mathrm{K})^{-1/2}$
Cyclotron Radius	$l_{cycl} = \hbar k_F/eB$	100	37–116	nm$(B/\mathrm{T})^{-1}$
Magnetic Length	$l_m = (\hbar/eB)^{1/2}$	26	26	nm$(B/\mathrm{T})^{-1/2}$

- Cronnenwett et al. 2002 ([Online]) Quantum point contact ”waterfall plot” $\rightarrow$ conductance quantization.
- Ono et al. 2002 ([Online]) Spin blockade in a quantum dot. non-symmetric current-voltage characteristics due to pauli exclusion principle.

3 Materials for Quantum Transport

- 2D Electron Gas (2DEG):
  - Came from industry that was needing faster/smaller devices (note they have large mean free paths i.e. small number of impurities, and high electron mobility).
  - Often in semiconductor heterostructures, formed at interface.
  - Also in Metal-oxide-semiconductor (MOS) structures.
  - Materials are very well matched at interface so there are few defects.
  - To fill 2DEG you can either apply gate voltage or use doping.
  - Electron mobility goes up as temperature goes down (less phonon scattering).

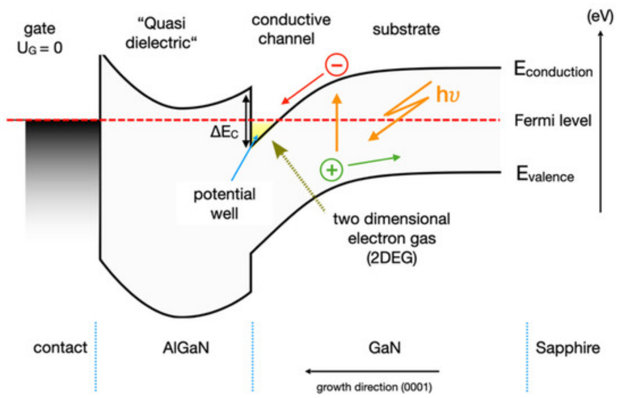


Figure 3.1: 2DEG in a semiconductor heterostructure.

- Place metal on surface and then heat it up (anneal) so that it diffuses through the semiconductor and creates a contact to the 2DEG.
- To make quantum dots we have metals on surface that we apply (very negative) voltage to (gates) to locally deplete the 2DEG.

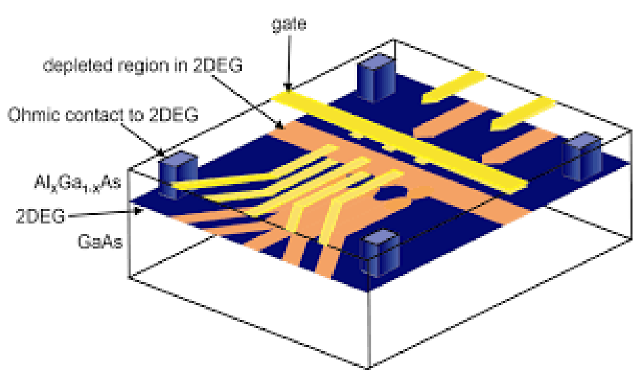


Figure 3.2: Example quantum transport device using a 2DEG.

- Metallic or semiconductor nanowires (1D systems):

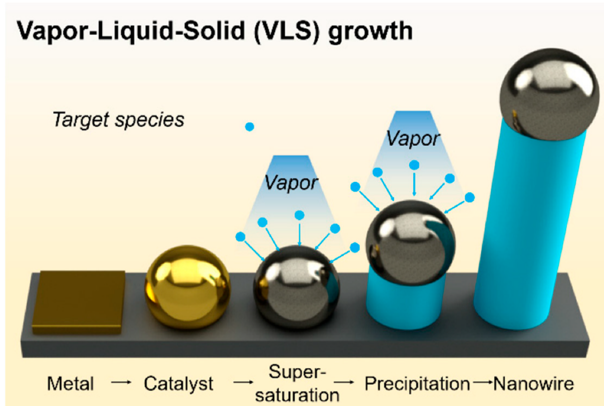


Figure 3.3: VLS grown nanowire process.

- Semiconducting quantum dots (0D systems):
  - electron motion strongly confined in all 3 dimensions.
  - quantization so that these are like artificial atoms.
  - can be made in 2DEG by applying voltages to gates.
  - dominant transport is single electron tunneling.
  - can also be by pillars etched out of 2deg and also self-assembled (strained induced) InAs dots.
- Metallic clusters & semiconductor dots (0D): made by chemists, synthesized in solution and can make different sizes that can emit different colors.
- Single molecules and chains of atoms: two electrodes placed with tiny gap and then deposit molecules and some will bridge the gap.
- Carbon base material-systems: fullerenes (1985, 0D ”buckyballs”), carbon nanotubes (1991, 1D), graphene (2004, 2D).
- Free electrons in three dimensions:
  - Schrodinger equation: $-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi$.
  - The eigenstates are: $\psi_{\vec{k}}(\vec{r}) = \frac{1}{(2\pi)^3}e^{i\vec{k}\cdot\vec{r}}$ with $\vec{k} = (k_x, k_y, k_z)$.
  - The energy eigenvalues are: $E_{\vec{k}} = \frac{\hbar^2\vec{k}^2}{2m} = \frac{\hbar^2}{2m}(k_x^2, k_y^2, k_z^2)$.
  - The unitary volume of the state in k-space is $(2\pi)^3$.
  -
  - The number of states in a 3D k-space volume $d\vec{k} = dk_x dk_y dk_z$ is given by:

    $$g(\vec{k})d\vec{k} = \frac{2}{(2\pi)^3}d\vec{k}$$

    where the factor of 2 is for spin degeneracy.

- $d\vec{k}$ can be written as:

$$d\vec{k} = \frac{4}{3}\pi((k+dk)^3 - k^3) = 4\pi k^2 dk$$

  leading to:

$$g(\vec{k})d\vec{k} = \frac{1}{\pi^2}k^2 dk$$

- By calculating k^2 and dk/dE:

$$k^2 = \frac{2mE}{\hbar^2} \quad , \quad \frac{dk}{dE} = \sqrt{\frac{2m}{E\hbar^2}}\frac{1}{2}$$

$$\frac{1}{\pi^2}k^2 dk = \frac{1}{\pi^2}\left(\frac{2mE}{\hbar^2}\right)\frac{dk}{dE}dE = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}dE$$

- Which is the density of states in 3D for free electrons:

$$\boxed{g(E) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}}$$

- Can also do this for 2D and 1D and 0D systems:

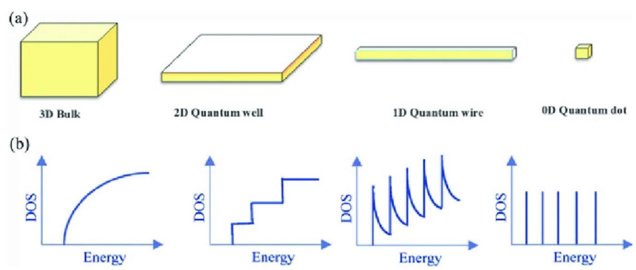


Figure 3.4: Density of states in different dimensions.

- Band strucutre and symmetry points
- Direct vs indirect band gaps
  - Wider bands are heavier effective mass
  - Narrower bands are lighter effective mass
- Bands vs sub-bands: sub bands based on excited states in quantum wells (not in 3D bulk material)
- Newish quantum transport materials: Bi2Se3 topological insulator
- MoS2 (transition metal dichalcogenides)
- LAO/STO (lanthanum aluminate/strontium titanate)

4 Technology

- Crystal growth (normally done out of house, so that we ensure it is high quality) → nanofabrication (normally done in house because no one wants to do it for you)→ Low-T electrical measurements (science!)
- Molecular Beam Epitaxy (MBE):
  - MBE is performed in an ultra-high vacuum (UHV) chamber.
  - A heat source evaporates pure elements, which then condense on a heated substrate.
  - Atoms self-assemble into crystalline layers.
  - The process involves ballistic (of atoms) deposition (molecular flow).
  - Temperature is a critical parameter.
  - Effusion cells are used to carry very pure elements (like Gallium, Arsenic, Aluminum, Indium).
  - A RHEED (reflection high energy electron diffraction) gun reflects electrons off the surface and reads the pattern with a sensor to monitor growth in real time. Signal oscillates, intensity determined by surface roughness.
- Metal Organic Vapor Phase Epitaxy (MOVPE) (or MOCVD):
  - Temperature usually 600-1200C
  - Uses metal-organic precursors (like trimethylgallium, triethylgallium, arsine, phosphine) that decompose on the hot substrate to deposit the desired elements (i.e. organics leave and leave semiconductor elements behind).
  - Used to make nanowires
- Pulsed Laser Deposition (PLD):
  - UHV chamber with window
  - laser shoots at target material, ablates material which deposits on substrate
  - used to make complex oxides (LAO), HTSCs (YBCO, BSCO etc.), and also semiconductors
- nanofabrication: pattern definition (photolithography, Electron beam lithography, and nano imprint lithography), Thin film deposition (evaporation, sputtering using plasma), Etching (chemical etching – wet and dry, Ion milling)

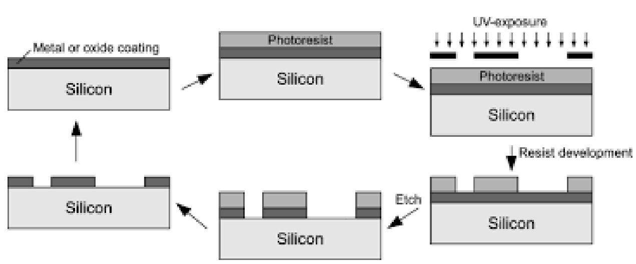


Figure 4.1: photolithography process

- Excimer Laser Stepper: resolution is around 65nm, can make smaller features possible, fast, very expensive.
- Stamping nanostructures: PDSM stamp, nanoimprint, step and flash imprint lithography (around 22nm resolution accross large area, fast, need master, overlay is difficult).

- Electron Beam Lithography (EBL): a pattern generator to SEM (beam is scanned across surface), very high resolution (sub 10nm), slow, serial process, need conductive substrate or coating. Several million dollars.
- Focused Ion Beam (FIB): use atoms like Ga, to mill away material. Pretty similar properties to EBL.
- Nanofabrication journey:
  - you dont get unlimited good semiconductor wafers
  - find defects and figure out if you can avoid them in your device design.
  - practice with just GaAs at first without the 2DEG
- Low-T technology:
  - Liquid helium 4 at 4.2K (bath cryostat).
  - Pumped helium 4 – 1.5K (note that superfluid transition is at 2.17K).
  - Helium 3 – 0.3K (expensive, rare, hard to handle)
  - Dilution Refrigerator (mix of helium 3 and helium 4) – 0.01K
  - Adiabatic demagnetization refrigerator – 0.001K, use solids and apply magnetic fields to cool down (entropy increases, temperature goes down), no helium 3 required.
- Cooling electrons down: electron temperature is not the same thing as the lattice temperature (phonons) i.e. the temp of the fridge.
  - Let electrons have time to cool down by wrapping wires around stages of fridge. Electron-phonon coupling. Add resistor to cool down electrons.
  - Need to filter out high frequency noise (RF) from room temperature electronics. Use RC filters, copper powder filters, pi-filters, etc.

5 Ballistic Transport

- Diffusive transport: electrons scatter multiple times off impurities, phonons, other electrons, etc. Mean free path l much smaller than device length L ($l \ll L$)
- Quasi-ballistic transport: electrons scatter a few times, $l \sim L$
- The Drude model describes diffusive transport, based on the free electron gas picture. Electrical resistance comes from scattering (information is lost when scattering).
- This gives us equation for velocity of electron in an electric field and mobility of electron, and conductivity.

$$\vec{v}_d = -\frac{e\vec{E}\tau}{m}$$

$$\mu_d = \frac{e\tau}{m}$$

$$\sigma = \frac{ne^2\tau}{m} = ne\mu_d$$

- Einstein relation: electrons near E_F diffuse due to density gradient.
- Ohms law and drude model are in diffusive regime
- In the ballistic regime, electrons travel through the device without scattering. The resistance comes from the contacts (interfaces) to the device, not from within the device itself.
- With non-local measurement (voltage is not measured along the current path to electrical ground) you can measure electron focussing in a magnetic field (famous non-local measurement is quantum hall effect).
- Split gate hetero structure in 2DEG GaAs/AlGaAs, apply negative voltage to deplete 2DEG under gates and create a narrow constriction (quantum point contact, QPC). When you make the constriction narrow enough you get quantized conductance in units of $G_0 = \frac{2e^2}{h}$.
- Constricts to 1D transport, so electrons can only move forward or backward. The conductance is given by the Landauer-Buttiker formula.
- QPC are not perfect, if you increase temp the plateaus get washed away. For finite temp we must consider a finite occupation of the energy levels.
- The width of the derivative of the fermi function is $\sim 3.5k_BT$. If $3.5k_BT$ is comparable to the sub-band spacing then the plateaus get washed out.
- Transmission resonance can also wash out the plateaus.
- Reflection can be from impurities or Backscattering from abrupt constriction at QPC. Tapered/smooth and short potential constriction is better for transmission closer to $T = 1$ and $R = 0$.
- Can measure in nanowires also but you need very high magnetic fields to see quantized conductance.
- Perpendicular magnetic field also makes plateaus more stretched out in gate voltage.
- Parallel magnetic field splits spin degeneracy and you get $1e^2/h$ plateaus.
-

6 Quantum Point Contacts II

- GaAs/AlGaAs heterostructure has electrons that are confined to a 2D layer in the interface between the two materials. The electrons can move freely in the 2D plane but are confined in the third dimension. These electroncs have very high mobility! with mean free path around 25 microns.
- Use gates to apply electric fields to deplete the 2DEG under the gates and create a narrow constriction (quantum point contact, QPC). When you make the constriction narrow enough you get quantized conductance.
- These are modes in the solution to the Schrodinger equation in the constriction. Each mode contributes $2e^2/h$ to the conductance (factor of 2 from spin degeneracy).
- First two papers on this topic are from van Wees et al. and Wharam et al. in 1988.
- See also Sara Cronenwett PhD thesis for more interesting plots like the waterfall plot.
- It shows conductance plateaus which become spin resolved if you apply a magnetic field.
- At 0 B you can also get 1/2 plateaus with high bias (but this is not spin-resolved).
- 0.7 or 0.8 plateaus have been debated over for several decades. Likely from interaction effects. When you apply large field it goes to 0.5 plateau.
- Adiabatic Quantum Transport: Many modes, some closed some open. Smoothly varying potential. No scattering between modes. This is called the Landauer Buttiker formulism of Quantum Transport.
- Transmission and Reflection of modes.
- Incoming amplitudes $\vec{a}$ and outgoing amplitudes $\vec{b}$.

$$\vec{b} = \hat{s}\vec{a}$$

$$\boxed{\begin{bmatrix} \vec{b}_L \\ \vec{b}_R \end{bmatrix} = \begin{bmatrix} \hat{r} & \hat{t}' \\ \hat{t} & \hat{r}' \end{bmatrix} \begin{bmatrix} \vec{a}_L \\ \vec{a}_R \end{bmatrix}}$$

- Unitarity of S-matrix: $\hat{s}^\dagger \hat{s} = \hat{I}$. Express conservation of the number of particles.
- Fundamental relation is also the Onsager Relation, for symmetry with respect to the time reversal:

$$s_{\alpha n;\beta m}(B) = s_{\beta m;\alpha n}(-B)$$

- You can inject a 2DEG by adding electrons to surface of liquid helium. Lower density electrons, less screening, much stronger Coulomb interactions. If you lower the temperature enough you can get a Wigner crystal (held together in crystal lattice form by Coulomb repulsion). Harder to create quantum confinment because these are real electrons (not effective) here have larger mass of 1 (compared to GaAs 2DEG electrons of 0.067) and thus much smaller wavelength.
- Groups have tried to make QPCs in this system but it is very challenging.
- Spin effects of QPCs, apply magnetic field in plane (no orbital effects), adds Zeeman energy to electrons in the 2DEG. Extra plateaus show in conductance at 0.5, 1.5, 2.5, etc. $2e^2/h$.
- The factor of 2 normally comes from spin degeneracy. The magnetic field breaks this degeneracy and splits the modes.

- Gate voltage at odd integer multiples of e^2/h only allows electrons with spins oriented along the field to go through the QPC. This is a spin filter (or injector)!
- Non local spin valve: inject spins at one QPC and detect them at another QPC.

7 Coulomb Blockade

- Charging effects
- Occur in bulk metal.
- charge quantization effects only observable in small volume well separated from its environment.
- The total capacitance of the island (to the ground, leads, etc):

$$C_\Sigma = C + C_{\text{self}} + C_{\text{res}}$$

- With isolated island: charge is quantized, $Q = ne$.
- Energy to add an electron to the island:

$$E = E_c n^2$$
$$E_c = \frac{e^2}{2C_\Sigma}$$

- If charging energy is not available from external voltage sources, T, etc, then transport is blocked.
- Capacitances: isolated sphere $C = \epsilon_0 \epsilon_r 2\pi d$, isolated disk $C = \epsilon_0 \epsilon_r 4d$, parallel plate $C = \epsilon_0 \epsilon_r A/d$, nanotube with diameter, r, above a ground place at distance h $C = \frac{2\pi\epsilon_0\epsilon_r L}{\ln(2h/r)}$.
- Quick estimate: for capacitance per unit length $C' = \epsilon_0 \epsilon_r = \epsilon_r 10\text{aF/um}$
- Mesoscopic systems, typically C in 1-100 fF range. Ec in 0.5-50meV range.
- What is the energy needed to charge the sphere with one electron? Energy of charged capacitor: $E = qV/2 = q^2/2C$

R	C	E/k_B	E
10 μm	1.1×10^{-15} F	0.84 K (^{3}He)	70 μeV
1 μm	1.1×10^{-16} F	8.4 K (LHe)	0.7 meV
0.1 μm	1.1×10^{-17} F	84 K (LN_2)	7 meV
0.01 μm	1.1×10^{-18} F	840 K (spg)	70 meV

- Need $E_c > kT$ in order to observe charging effects.
- Conditions for observing Coulomb blockade:
  - $\Delta t = RC$ is typical time to charge the island, then Heisenberg uncertainty relation: $\Delta E \Delta t = (e^2/C)RC > \hbar$
  - $R \gg \frac{\hbar}{e^2} \approx 25.8k\Omega$
  - Requirement on temperature/voltage: $kT, eV < E_c$
  - Typical experimental setup: quantum dot connected to source and drain via tunnel barriers, capacitively coupled to gate.
- Tunnel Junction: two conductors separated by thin insulator. Electrons can tunnel through the insulator. Ohms law does not apply.
- Island: single electron box, constant interaction model. no current flows. island potential is given by charge on island plus induced charge from gate.

- We can add electroncs one by one to a single electron box by changing the gate voltage.
- Double barrier circuit: single electron transistor (SET).
- Chemical potential is the energy an electrons needs to have in order to enter the system. $\mu(N) = U(N) - U(N-1) = (N-1/2)e^2/C - eV_{\text{ext}}$.
- Current flows if a dot chemical potential lies inside the bias window (window between source and drain chemical potentials).

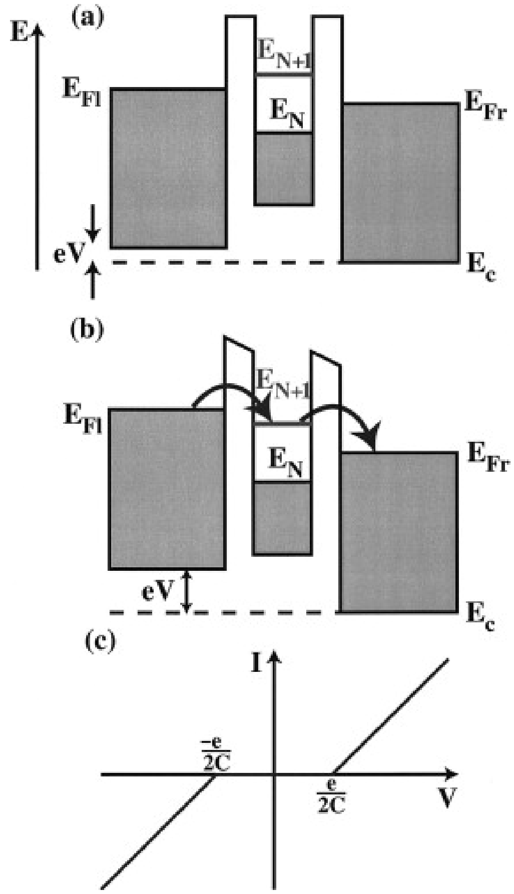


Figure 7.1: Coulomb blockade.

- First measurements of the Coulomb Effect: I. Giaever and H.R. Zeller, Phys. Rev. Lett. 20, 1504 (1968).
- 20 years later: Observation of Single-Electron Charging Effects in Small Junction by T.A. Fulton and G.J. Dolan, Phys. Rev. Lett. 59, 109 (1987).
- Aside: Shadow Evaporation: (last lecture 2DEG and apply gates to apply potential barriers). But in metals, we can use shadow evaporation to create small tunnel junctions.
- This technique is used for SETs, SQUIDs, Quantum Dots with tunnel barriers, other devices with small tunnel junctions.
- Steps:

– Evaporate metal onto insulator.
– Apply resist mask with undercut.
– Evaporate metal at angle 1.
– Oxidize to form tunnel barrier.
– Evaporate metal at angle 2.
– Lift off resist.
– Result: two metal layers separated by thin oxide layer.

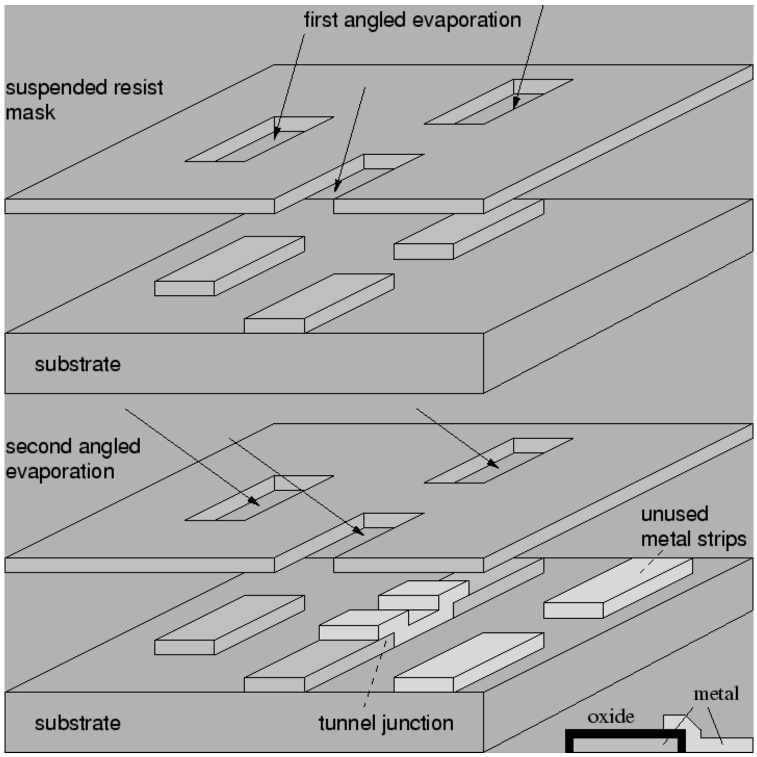


Figure 7.2: Shadow evaporation.

- Coulomb diamonds: the lines define a region in which there is no current. This region is called the coulomb diamond. At zero bias, current flows at the blue degeneract points.
- Gates are even used now to even shift the latter of the energy levels in the dot on top of also the source and drain gates. So that is how we get the Source/Drain gate voltage vs Gate voltage (on the dot) Coulomb diamond plot.
- At the degeneracy points, the energy cost for N and N+1 electrons is the same.
- On the lines, either the source or drain chemical potential is aligned with a dot chemical potential.
- Inside diamond, no current flows
- Outside diamond, current flows.
- Slope of lines tells you about the coupling capacitancebetween the source/drain/gate and the island.

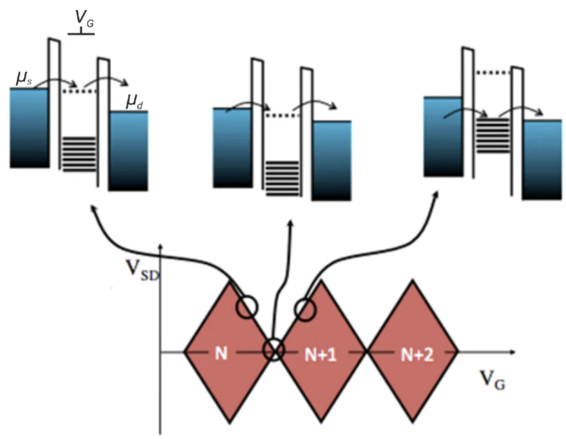


Figure 7.3: Charge stability diagram also known as Coulomb diamonds.

- You can also figure out the charging energy of the capacitor by looking how stretched from a square diamond it is (compare width and height from centre of diamond). Conversion factor to figure out how much gate voltage needs to be applied to overcome the charging energy.
- Gate traces and stability diagrams are used to characterize quantum dots. Inside the coulomb diamonds, the number of electrons on the island is fixed and no current flows. Outside the Coulomg islands, the number of electrons fluctuates and current flows.
- A low-bias gate trace shows peaks in conductance each time the number of electrons on the dot changes by one.
- Asymmetric coupling: Coulomb staircase (Matsumoto et al. Appl. Phys. Lett. 68, 34, 1996).
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