PHYS 509 PS 3

Instructions: show your work. Justify all steps.

1. File *exp1000.txt* contains a simulated list of decay times from an experiment.
   (a) Perform a least squares fit to this distribution for the lifetime τ, to find the best estimate and also its uncertainty. Plot your fit results on your histogram of the data.
   (b) Plot a scan of the χ^2 about the minimum over a range that shows at least the $\pm 3\sigma$ region with lines indicating the 1, 2 and 3 σ levels on the χ^2 plot. Make a table with the symmetric and asymmetric confidence intervals for 1, 2 and 3 σ limits.

2. Use the same file as above, but perform a maximum likelihood fit for τ.
   (a) Perform a likelihood fit to this distribution for the lifetime τ, to find the best estimate and also its uncertainty. Plot your fit result on a histogram of the data.
   (b) Plot a scan of the likelihood L about the minimum over a range that shows at least the $\pm 3\sigma$ region with lines indicating the 1, 2 and 3 σ levels on the plot. Make a table with the symmetric and asymmetric confidence intervals for 1, 2 and 3 σ limits.

3. Do the same fits as above, but with the file *exp30.txt*, which only has 30 entries.
   (a) Do the least squares fit on this file, with the same plots as above. Explain your histogram binning, how you determined it, and any other special treatment you had to make.
   (b) Do the ML fit, again with the same plots as above.

4. Derive an expression for the *convolution* of an exponential decay with lifetime τ, with a Gaussian resolution function. Assume the Gaussian is centered at 0, with variance σ^2. You should be able to express this function using an exponential and an erfc function.

5. Download the file *exp_smeared.txt*, which contains a set of simulated data from an experiment measuring the lifetime of a particle, but with Gaussian smearing due to detector resolution.

   Perform a ML fit to the data to extract the lifetime τ and detector resolution σ, along with the full covariance matrix of these estimators. Use the Kolmogorov-Smirnov test to determine the goodness of fit for this model.

   Plot your fit result on top of a histogram of your data.

6. Download the file *exp_smear_bckgnd.txt*, which contains data from a signal which is a smeared exponential similar to the one above, but also with a background of random events with uniform probability over the experiment's region of operation, which is from $t = -3$ to $t = 10$ seconds. The detector's time resolution is determined elsewhere to be a Gaussian $N(0, \sigma^2)$ with $\sigma = 0.2$s.

   Perform a ML fit to the data to extract the lifetime τ, fraction of the sample that is signal, along with the full covariance matrix of these estimators.

   Plot your fit result on a histogram of the data, and also on the same plot show the fitted background and (smeared) signal fits.

7. File *gauss_bckgnd.txt* contains the counts from an experiment with a Gaussian signal of unknown mean and sigma, along with a flat background. There are 50 bins, with edges listed in the file.
   (a) Do a least squares fit on the data for the amplitude, mean, and sigma of the signal (S, μ_0, σ_0), and the background level μ_B. The *significance* of your signal is defined as S/σ_S, where σ_S^2 is the variance of S returned by your fit.
   (b) Plot the histogram of the data along with the total fit result, the signal component and the background.
   (c) Choose the bins that you consider contain the bulk of the signal based upon the fit. How many background events B do you estimate are in this region, and what would you estimate is the statistical uncertainty σ_B on this number. Explain your reasoning.
   (d) Given the number of signal events S you obtained from your fit, what would the uncertainty σ_S on this be if the data were background free? Compare this to what you actually obtained in the fit.
   (e) Considering all the bins you chose in your signal region, what is your estimate of the uncertainty on the size of statistical fluctuations in this region, expressed in terms of S and B? What is your expected signal significance, then, and how does it compare with your fitted value?

8. In an undergraduate lab, students perform an experiment to measure g, by dropping a long clear plastic plate with a dark line marked every 10 cm over a 1 m length. The lines are sensed by a photosensor, and the time t that each passes the sensor is recorded. The clock of the timer starts when the first line is sensed, so there are 11 measurements, starting with $t = 0$ for the line at 0 cm. The plate is held by hand some point above the photosensor before releasing. The file *gmeasure.txt* contains the times for the 11 markings, each with an uncertainty of 5 ms.
   (a) What is the equation for the position of the plate $z(t)$, where z = vertical position of the 0 cm line marking wrt the photosensor position? (For uniformity, let's take z increasing the downward direction.)
   (b) Since your experimentally measured variable is t, invert this so t is the dependent variable and then fit for g. Plot the data, with errorbars, along with your fit result.
   (c) Instead, treat z as the dependent variable, and fit for g using the original equation you had. You will need to convert the uncertainty on t to an approximate uncertainty on z at each point that will change as your fit parameters change. Plot the data, with errorbars, along with your fit result. Compare with the previous results.