Synthetic graphene in real bilayers and synthetic bilayers of real graphene

Tobias Grass

L4G at ICFO, 27. 4. 2017

UNIVERSITY OF MARYLAND 1856

JOINT QUANTUM INSTITUTE

First part: Artificial graphene / real bilayer

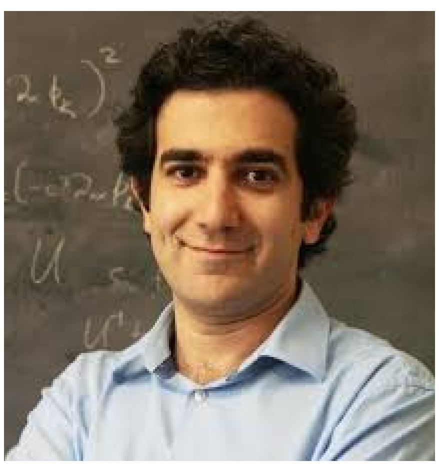
Work published in:
2D Mater. 4 (2017) 015039

Ravindra Chhajlany   Leticia Tarruell   Maciej Lewenstein   Vittorio Pellegrini

(U Poznan)   *(ICFO)*   *(IIT Genova)*

Second part: Real graphene / artificial bilayer

Work in progress!



Mohammad Hafezi   Michael Gullans   Areg Ghazaryan   Pouyan Ghaemi

(JQI / NIST)   *(City College New York)*

First part:

Artificial graphene / real bilayer

Why to build a bilayer?

- Coulomb drag
  [cf. B. N. Narozhny and A. Levchenko, Rev. Mod. Phys. 88, 025003 (2016)]

  with two layers of graphene
  [e.g. Gorbachev, R. V. *et al. (Manchester)* Strong Coulomb drag and broken symmetry in double-layer graphene. Nat. Phys. **8**, 896 (2012)]

  or two layers of non-relativistic 2DEG

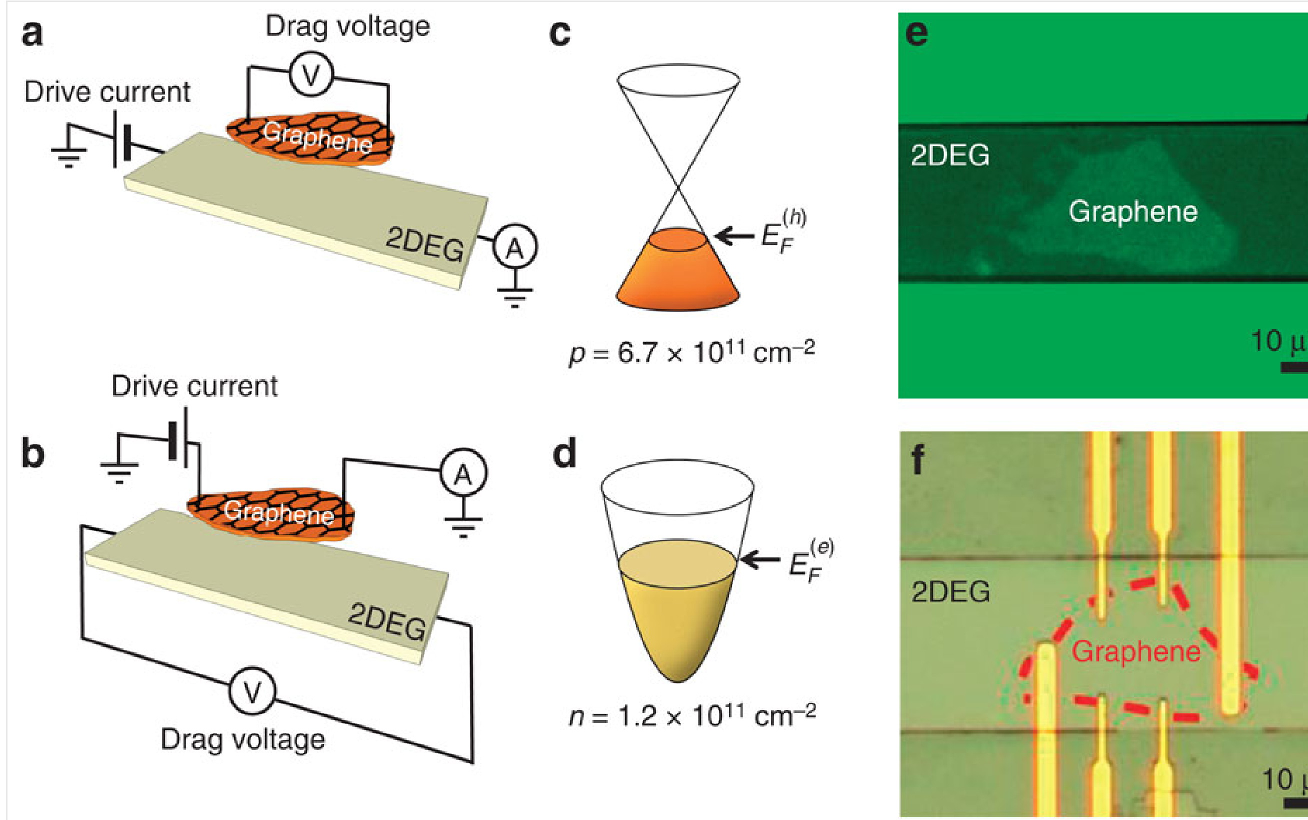
  or heterostructures (graphene + 2DEG)
  [see figure]

- Single-particle effects when combining graphene and hexagonal boron nitride

  [Yankowitz, M. *et al. (Tucson)* Emergence of superlattice Dirac points in graphene on hexagonal boron nitride. Nat. Phys. **8**, 382 (2012)]

Let's see what we can do with artificial graphene...

From
Anomalous low-temperature Coulomb drag in graphene-GaAs heterostructures
A. Gamucci, D. Spirito, M. Carrega, B. Karmakar, A. Lombardo, M. Bruna, L. N. Pfeiffer, K. W. West, A. C. Ferrari, M. Polini & V. Pellegrini
Nature Communications **5**, Article number: 5824 | doi:10.1038/ncomms6824

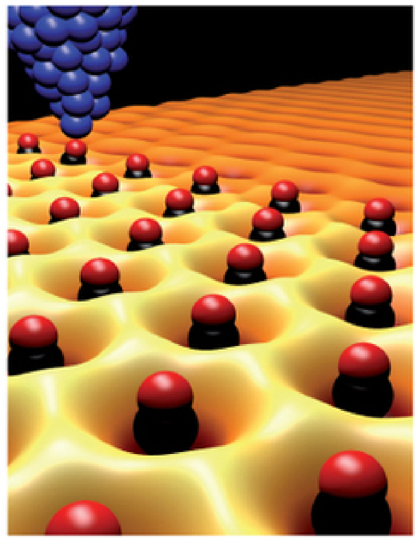


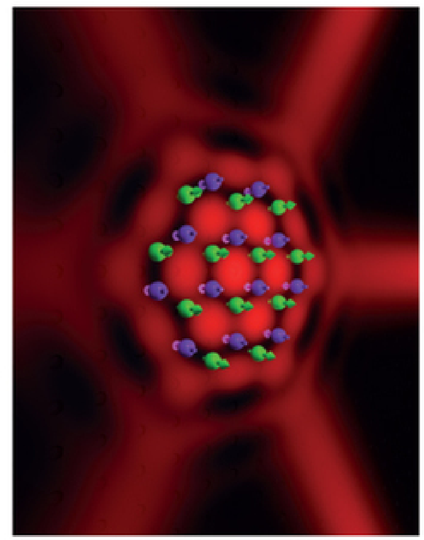
(**a**,**b**) Configurations for the Coulomb drag measurements. In **a**, a voltage drop V_{drag} appears in graphene, in response to a drive current I_{drive} flowing in the 2DEG. In **b**, the opposite occurs. The drag voltage is measured with a low-noise voltage amplifier coupled to a voltmeter as a function of the applied bias. The drive current is also monitored. (**c**) Conical massless Dirac fermion band structure of low-energy carriers in SLG. The SLG in this work is hole-doped. (**d**) Parabolic band structure of ordinary Schrödinger electrons in the 2DEG. (**e**) Optical micrograph of the device before the deposition of Ohmic contacts. The SL flake becomes visible in green light after the sample is coated with a polymer (PMMA)[31]. The scale bar is 10 μm long. (**f**) Optical microscopy image of the contacted SLG on the etched 2DEG GaAs channel. The red dashed line denotes the SLG boundaries. The scale bar is 10 μm long.

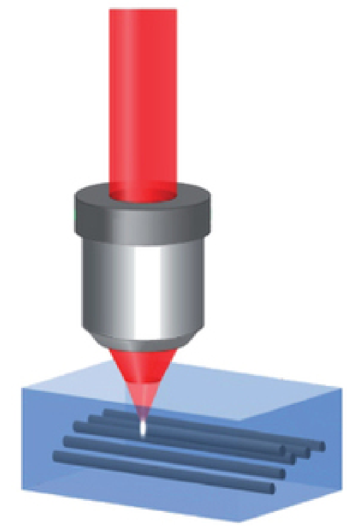
Artificial graphene

Subject particles to hexagonal lattice potential!

Semiconductors	Molecules	Cold atoms	Photonic crystals
a	**b**	**c**	**d**



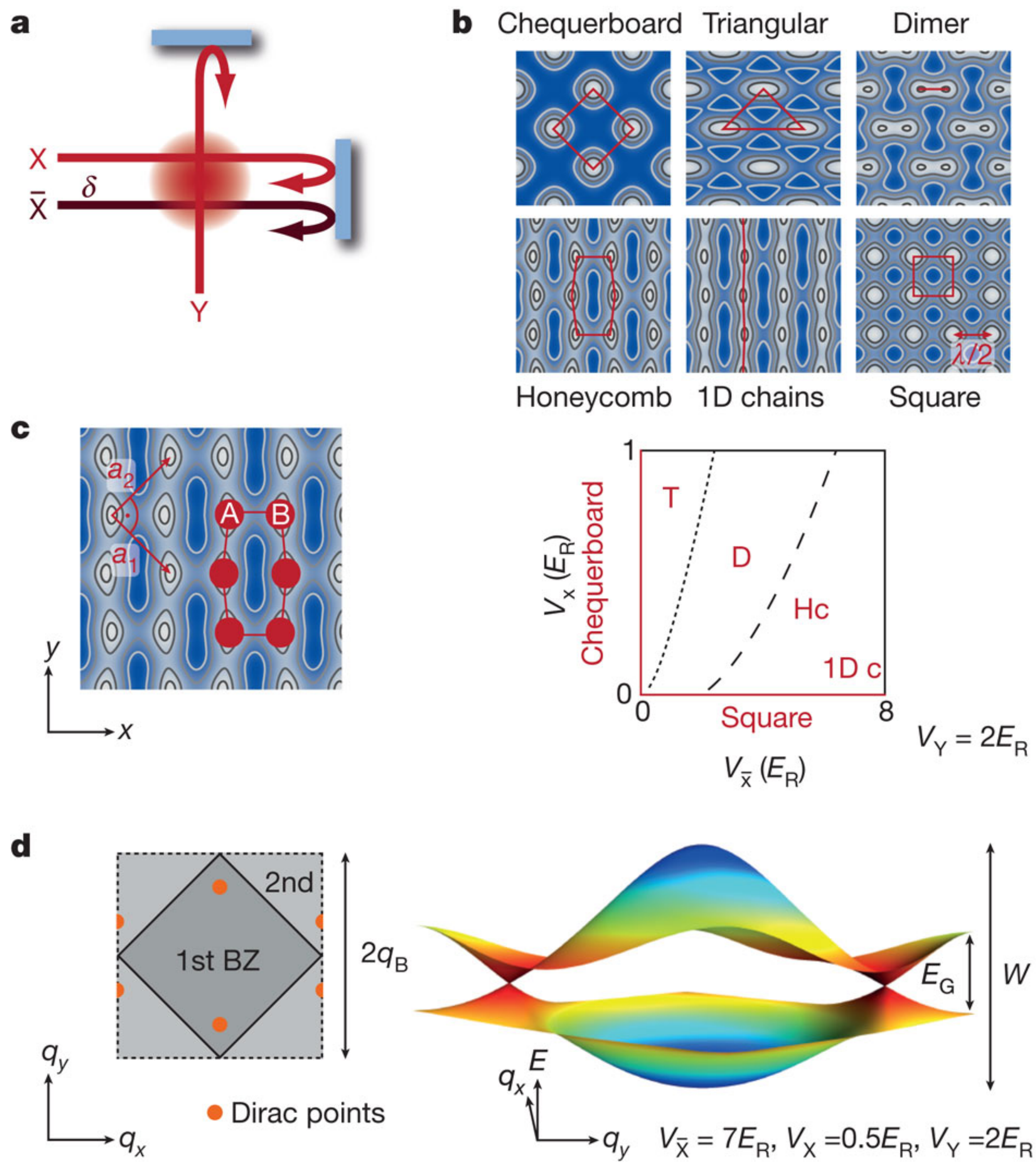




Semiconductors	Molecules	Cold atoms	Photonic crystals
$V_0 \approx 10$ K	$V_0 \approx 10^3$ K	$V_0 \approx 10$ mK	$\Delta n \approx 10^{-3}$
$d \approx 20$–100 nm	$d \approx 1$–3 nm	$d \approx 500$ nm	$d \approx 10$ µm–10 mm
$N \approx 10$–10^7	$N \approx 10^2$–10^3	$N \approx 10^5$	$N \approx 10$–10^3
$U \approx 10$ K	$U \approx 600$ K	$U \approx 100$ nK	n/a
$V \approx 1$ K	$V << U$	$V << U$	n/a
$t \approx 1$–10 K	$t \approx 10^3$ K	$t \approx 0.1$–10^3 nK	$c_0 \approx$ cm^{-1}
$T_F \approx 0.1$–100 K	$T_F \approx 5$–500 K	$T_F \approx 100$ nK	n/a

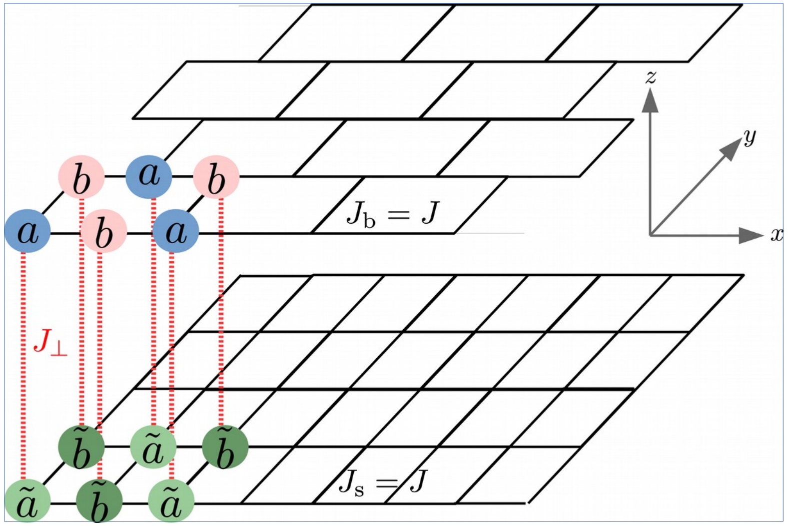
From review article: Marco Polini *et al.* , Artificial honeycomb lattices for electrons, atoms and photons, Nat. Nano 8, 625 (2013)

Cold atom artificial graphene



[Leticia Tarruell *et al.* Creating, moving and merging Dirac points with a Fermi gas in a tunable honeycomb lattice. Nature 483, 302 (2012)]

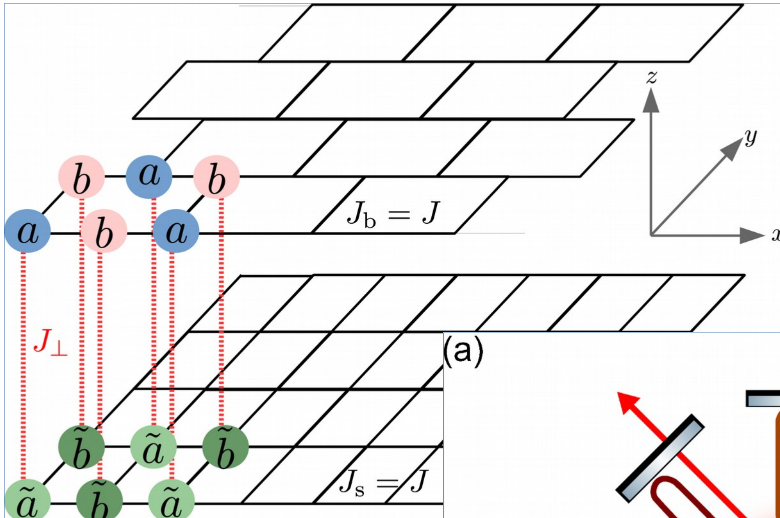
Bilayer setup

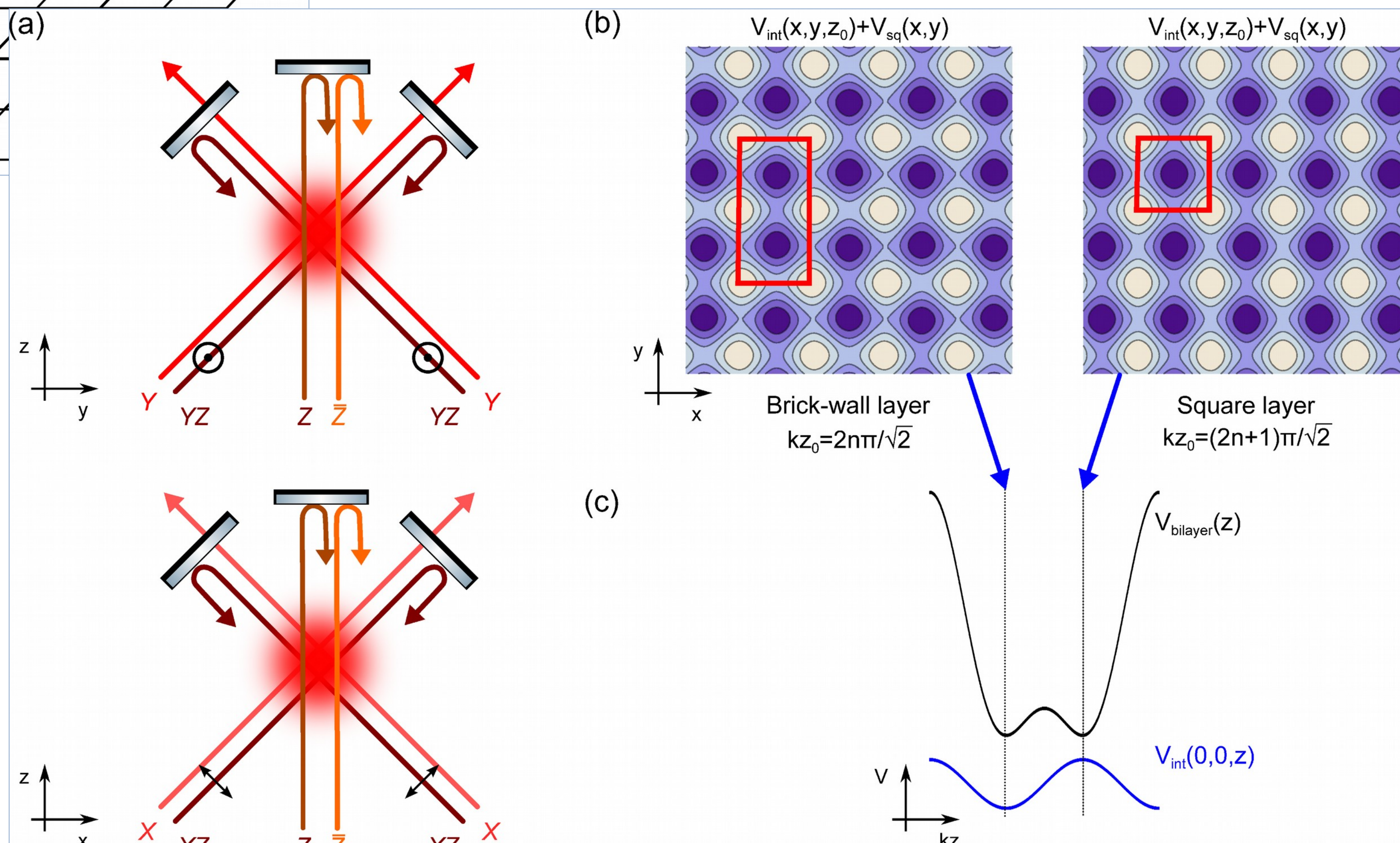


Graphene layer on top of "normal" layer:
Combine brick-wall lattice and square lattice!

Bilayer setup

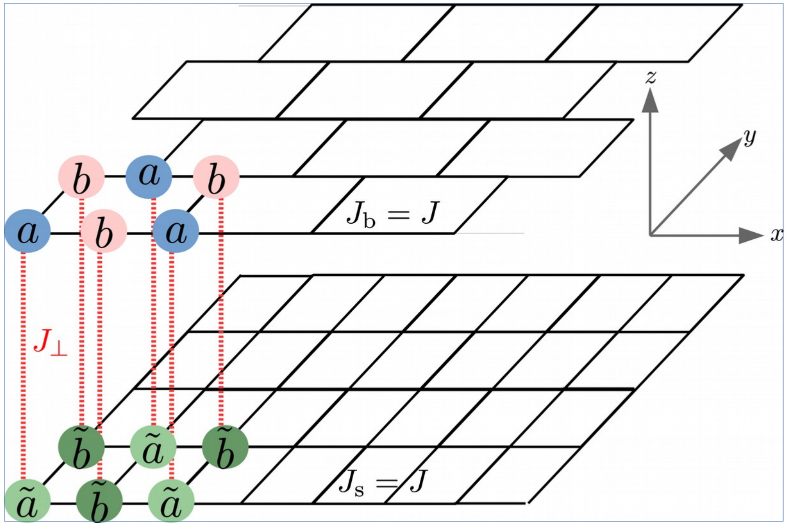
Graphene layer on top of “normal” layer:
Combine brick-wall lattice and square lattice!





Possible with depicted arrangement of ten laser beams.

Bandstructure in the bilayer



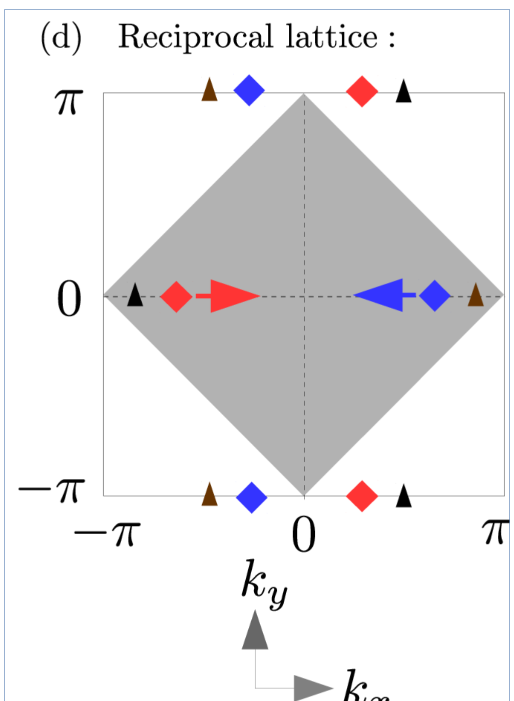
Tight-binding Hamiltonian:

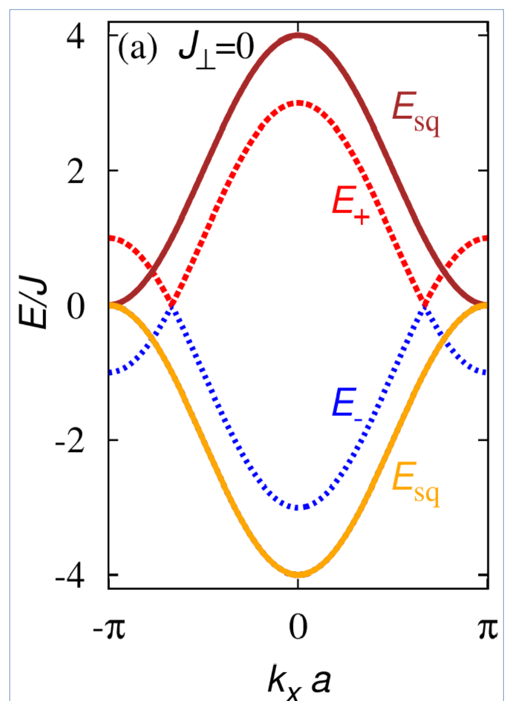
$$H_{\rm tb} = -\,J_{\rm b}\sum_{\mathbf{i}\in A}\left(b^\dagger_{\mathbf{i}+\hat{x}}a_{\mathbf{i}} + b^\dagger_{\mathbf{i}-\hat{x}}a_{\mathbf{i}} + b^\dagger_{\mathbf{i}+\hat{y}}a_{\mathbf{i}}\right) - J_{\rm s}\sum_{\langle\mathbf{i},\mathbf{j}\rangle}\tilde{b}^\dagger_{\mathbf{j}}\tilde{a}_{\mathbf{i}} - J_\perp\sum_{\mathbf{i}}\left(a^\dagger_{\mathbf{i}}\tilde{a}_{\mathbf{i}} + b^\dagger_{\mathbf{i}}\tilde{b}_{\mathbf{i}}\right) + \text{H.c.},$$

Uncoupled bands:

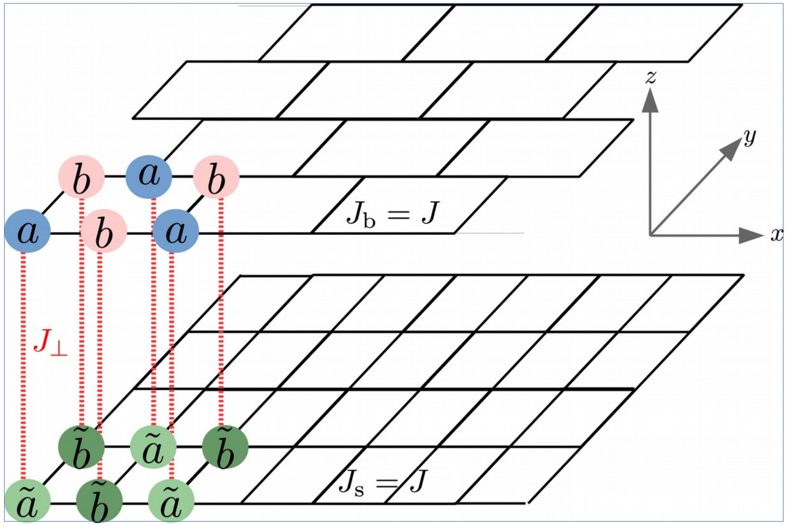
$$E_\pm(\mathbf{k}) = \pm\, E_{\rm br}(\mathbf{k}) = \pm J\sqrt{3 + 2\cos(2k_x a) + 2\cos[(k_x+k_y)a] + 2\cos[(k_x-k_y)a]},$$

$$E_{\rm sq}(\mathbf{k}) = -\,2J\left[\cos(k_x a) + \cos(k_y a)\right],$$





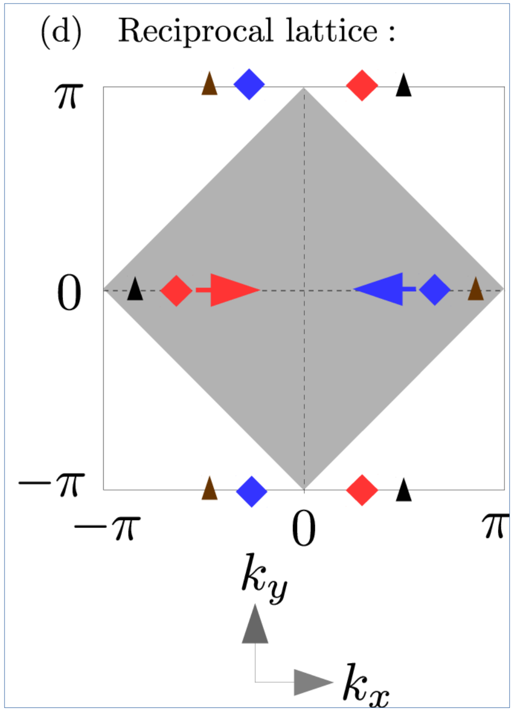
Bandstructure in the bilayer

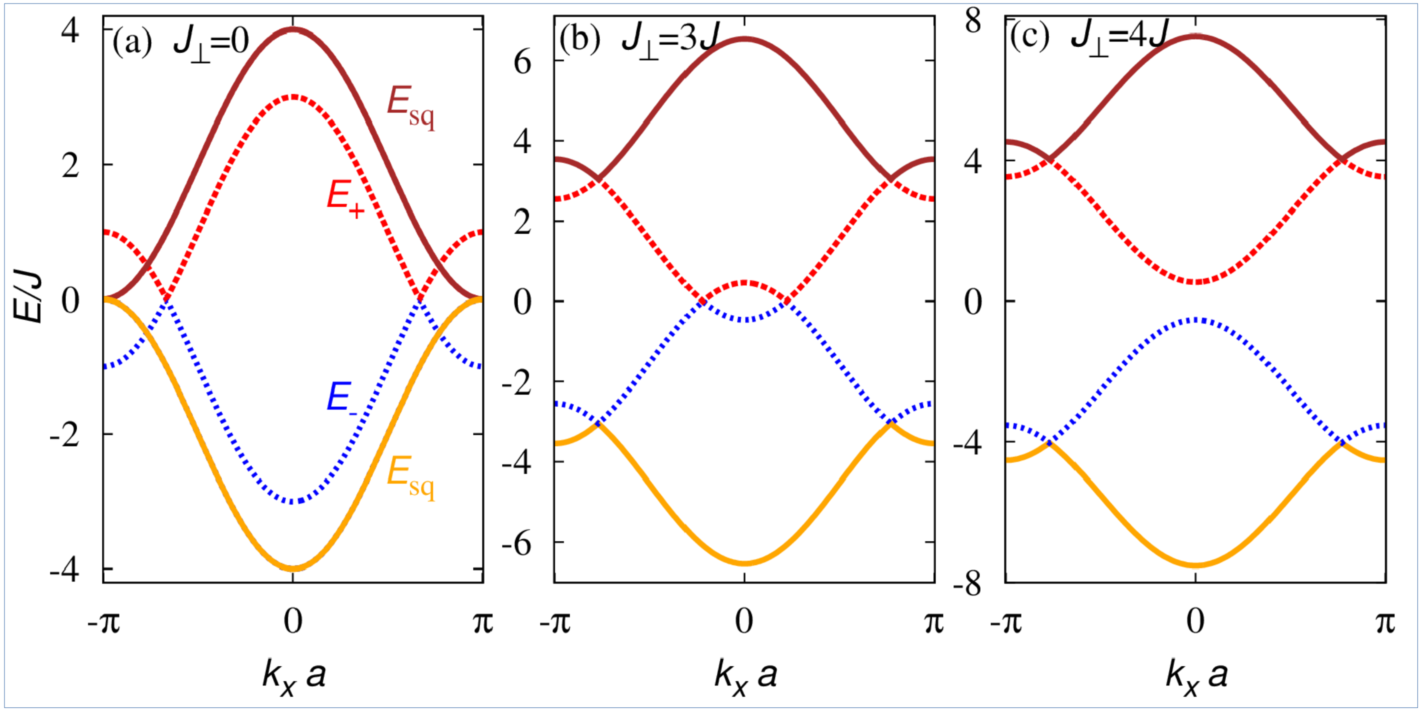


Tight-binding Hamiltonian:

$$H_{\rm tb} = -J_{\rm b}\sum_{\mathbf{i}\in A}\left(b^\dagger_{\mathbf{i}+\hat{x}}a_{\mathbf{i}} + b^\dagger_{\mathbf{i}-\hat{x}}a_{\mathbf{i}} + b^\dagger_{\mathbf{i}+\hat{y}}a_{\mathbf{i}}\right) - J_{\rm s}\sum_{\langle\mathbf{i},\mathbf{j}\rangle}\tilde{b}^\dagger_{\mathbf{j}}\tilde{a}_{\mathbf{i}} - J_\perp\sum_{\mathbf{i}}\left(a^\dagger_{\mathbf{i}}\tilde{a}_{\mathbf{i}} + b^\dagger_{\mathbf{i}}\tilde{b}_{\mathbf{i}}\right) + \text{H.c.}$$

- Interlayer coupling generates new Dirac points at intersections of square-layer band and brick-wall layer band.
- Interlayer coupling shifts graphene Dirac points towards the BZ center where they finally merge.



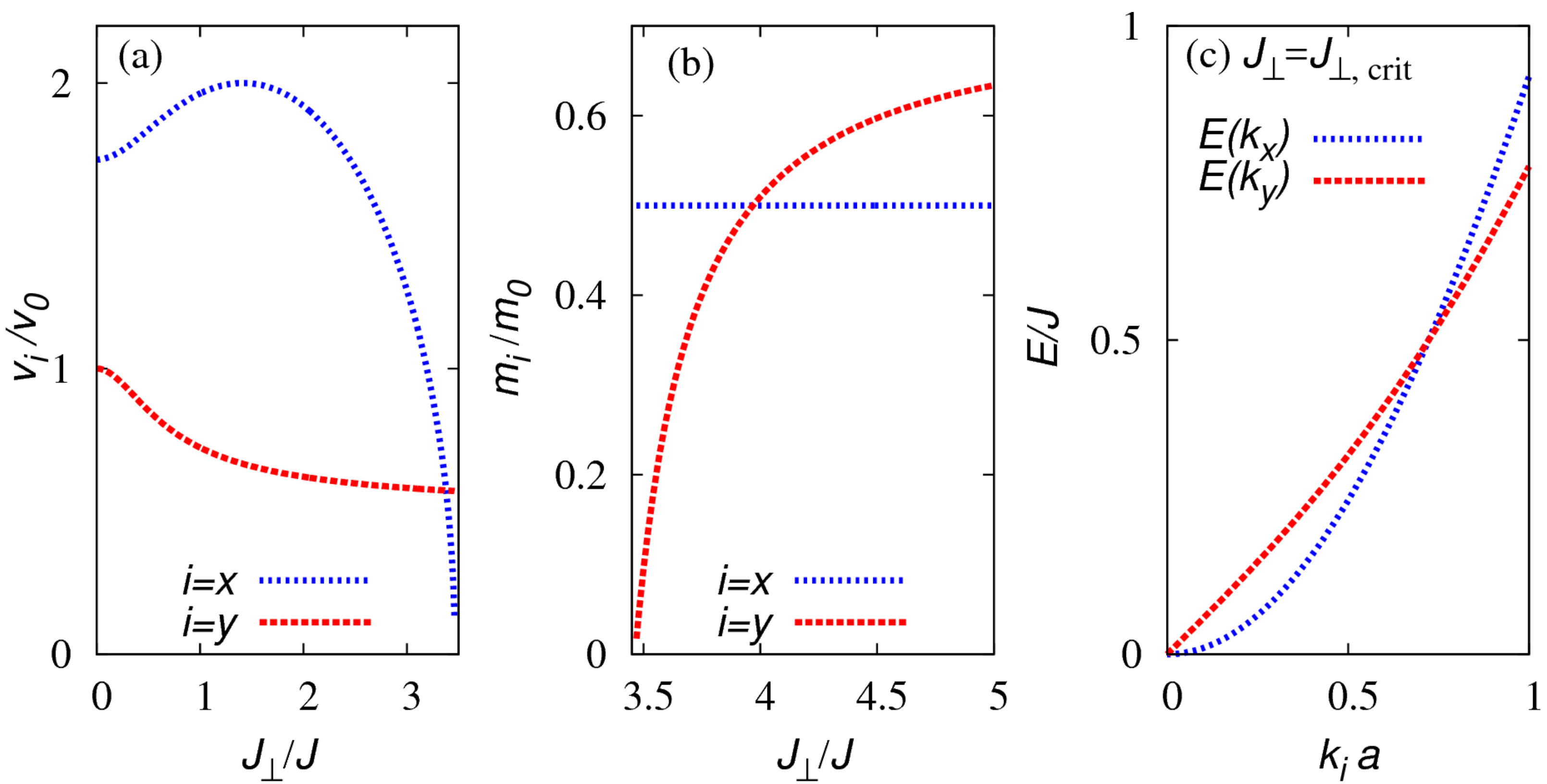


A curiosity: Bandstructure at merging point

Excitations characterized by gap, velocity, effective mass:

$$E(k) \sim \Delta + \hbar v k + \frac{\hbar^2 k^2}{2m} + \dots$$

Along the two in-plane directions, excitations look very differently when Dirac points merge:
Coexistence of massless and massive excitations!



See also:
G. Montambaux, F. Piéchon, J.-N. Fuchs, and M. O. Goerbig, Phys. Rev. B 80, 153412 (2009)
P. Dietl, F. Piéchon, and G. Montambaux, Phys. Rev. Lett. 100, 236405 (2008)

Mean-field for attractive interactions

Fill bilayer with spin-1/2 fermions → interactions on doubly occupied sites:

$$H_{\rm int} = U \sum_{\mathbf{i}} (a^\dagger_{\mathbf{i}\uparrow} a^\dagger_{\mathbf{i}\downarrow} a_{\mathbf{i}\downarrow} a_{\mathbf{i}\uparrow} + b^\dagger_{\mathbf{i}\uparrow} b^\dagger_{\mathbf{i}\downarrow} b_{\mathbf{i}\downarrow} b_{\mathbf{i}\uparrow} + \tilde{a}^\dagger_{\mathbf{i}\uparrow} \tilde{a}^\dagger_{\mathbf{i}\downarrow} \tilde{a}_{\mathbf{i}\downarrow} \tilde{a}_{\mathbf{i}\uparrow} + \tilde{b}^\dagger_{\mathbf{i}\uparrow} \tilde{b}^\dagger_{\mathbf{i}\downarrow} \tilde{b}_{\mathbf{i}\downarrow} \tilde{b}_{\mathbf{i}\uparrow}),$$

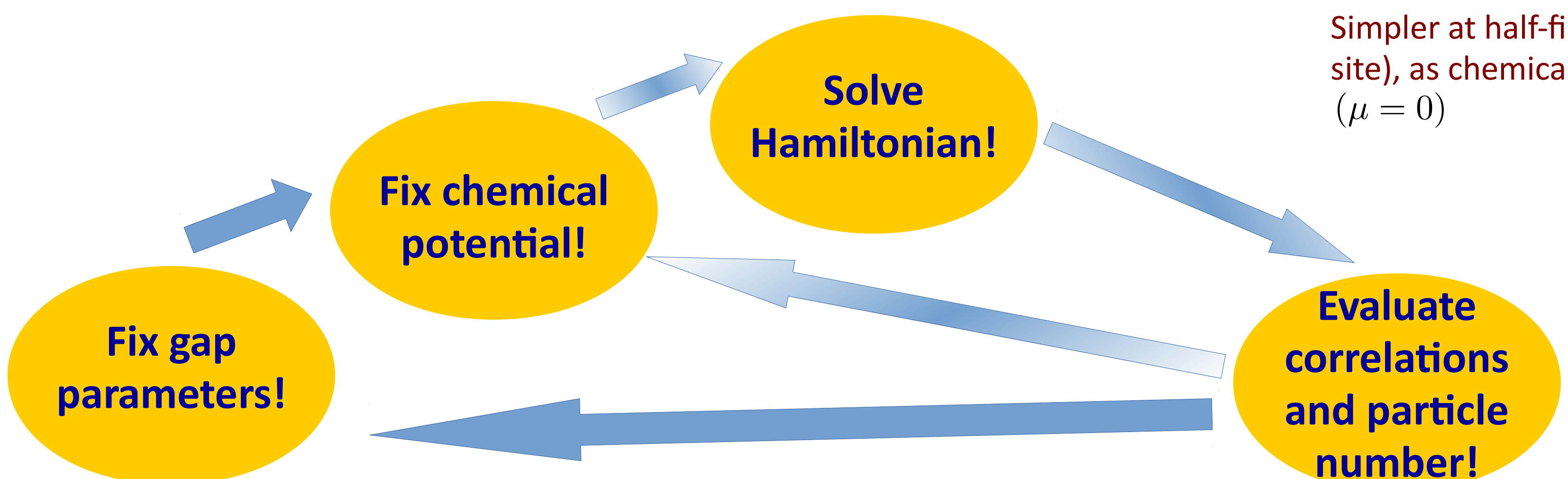
Mean-field decoupling for attractive interactions (*U=-u<0*):

$$\Delta_{\rm br} \equiv (4u/N) \sum_{\mathbf{k}} \langle a_{-\mathbf{k}\downarrow} a_{\mathbf{k}\uparrow} \rangle = (4u/N) \sum_{\mathbf{k}} \langle b_{-\mathbf{k}\downarrow} b_{\mathbf{k}\uparrow} \rangle, \qquad \Delta_{\rm sq} \equiv (4u/N) \sum_{\mathbf{k}} \langle \tilde{a}_{-\mathbf{k}\downarrow} \tilde{a}_{\mathbf{k}\uparrow} \rangle = (4u/N) \sum_{\mathbf{k}} \langle \tilde{b}_{-\mathbf{k}\downarrow} \tilde{b}_{\mathbf{k}\uparrow} \rangle.$$

Quadratic Hamiltonian:

$$H_{\rm BCS} = \sum_{\sigma \neq \sigma'} \sum_{\mathbf{k}} \left(a^\dagger_{\mathbf{k}\sigma}, a_{-\mathbf{k}\sigma'}, b^\dagger_{\mathbf{k}\sigma}, b_{-\mathbf{k}\sigma'}, \tilde{a}^\dagger_{\mathbf{k}\sigma}, \tilde{a}_{-\mathbf{k}\sigma'}, \tilde{b}^\dagger_{\mathbf{k}\sigma}, \tilde{b}_{-\mathbf{k}\sigma'} \right) \cdot \begin{pmatrix} -\mu & \Delta_{\rm br} & -J^{\rm br}_{\mathbf{k}} & 0 & -J_\perp & 0 & 0 & 0 \\ \Delta^*_{\rm br} & \mu & 0 & J^{\rm br}_{\mathbf{k}} & 0 & J_\perp & 0 & 0 \\ -J^{\rm br}_{-\mathbf{k}} & 0 & -\mu & \Delta_{\rm br} & -0 & 0 & -J_\perp & 0 \\ 0 & J^{\rm br}_{-\mathbf{k}} & \Delta^*_{\rm br} & \mu & 0 & 0 & 0 & J_\perp \\ -J_\perp & 0 & 0 & 0 & -\mu & \Delta_{\rm sq} & -J^{\rm sq}_{\mathbf{k}} & 0 \\ 0 & J_\perp & 0 & 0 & \Delta^*_{\rm sq} & \mu & 0 & J^{\rm sq}_{\mathbf{k}} \\ 0 & 0 & -J_\perp & 0 & -J^{\rm sq}_{\mathbf{k}} & 0 & -\mu & \Delta_{\rm sq} \\ 0 & 0 & 0 & J_\perp & 0 & J^{\rm sq}_{\mathbf{k}} & \Delta^*_{\rm sq} & \mu \end{pmatrix} \cdot \begin{pmatrix} a_{\mathbf{k}\sigma} \\ a^\dagger_{-\mathbf{k}\sigma'} \\ b_{\mathbf{k}\sigma} \\ b^\dagger_{-\mathbf{k}\sigma'} \\ \tilde{a}_{\mathbf{k}\sigma} \\ \tilde{a}^\dagger_{-\mathbf{k}\sigma'} \\ \tilde{b}_{\mathbf{k}\sigma} \\ \tilde{b}^\dagger_{-\mathbf{k}\sigma'} \end{pmatrix}$$

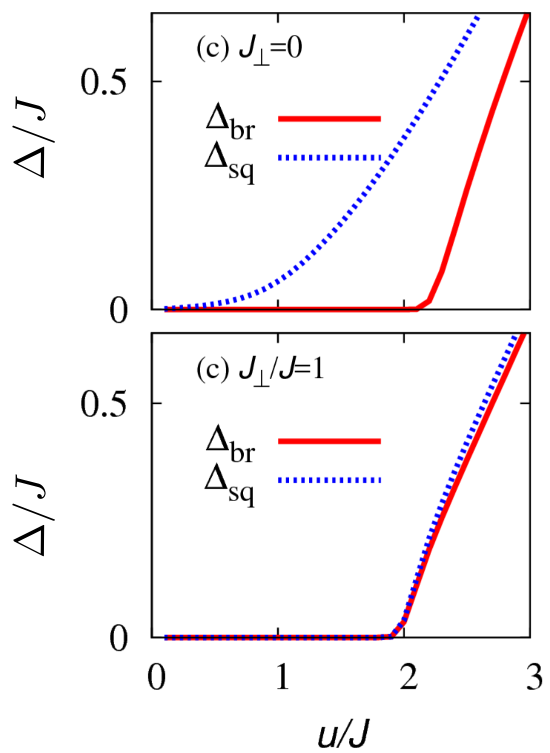
Self-consistent solution:



Simpler at half-filling (one atom per site), as chemical potential is fixed: $(\mu = 0)$

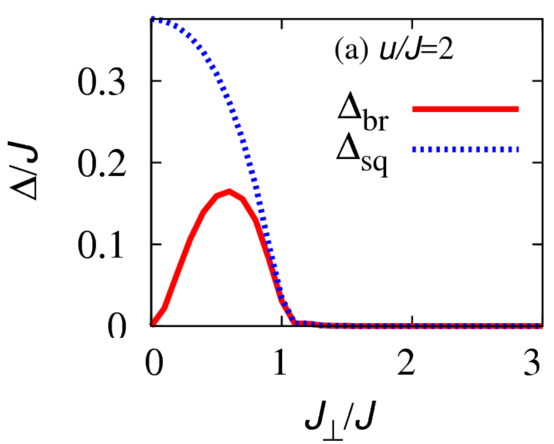
Superfluid vs. semimetal

Half-filling (1 atom per site):



Uncoupled brick-wall layer exhibits SM-SF transition.

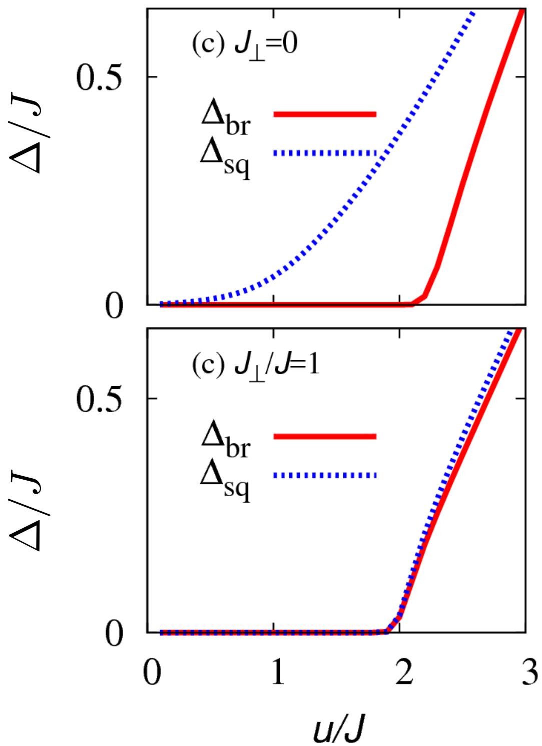
The whole system exhibits SM behavior for finite coupling.



Strong coupling suppresses, weak coupling enhances SF behavior.

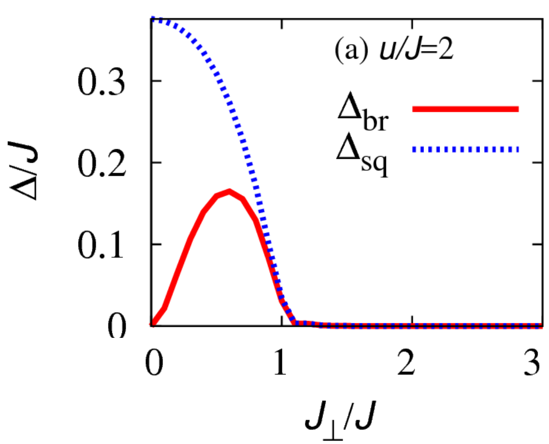
Superfluid vs. semimetal

Half-filling (1 atom per site):



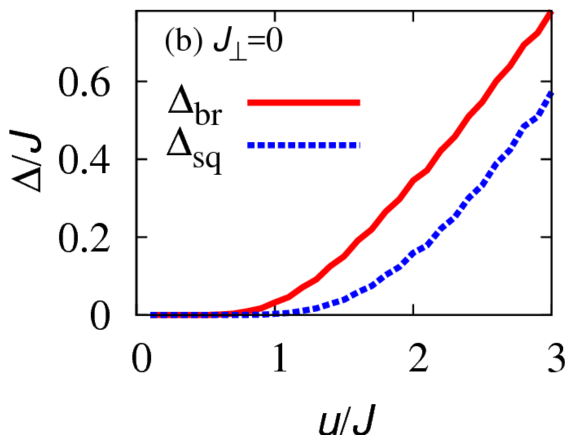
Uncoupled brick-wall layer exhibits SM-SF transition.

The whole system exhibits SM behavior for finite coupling.

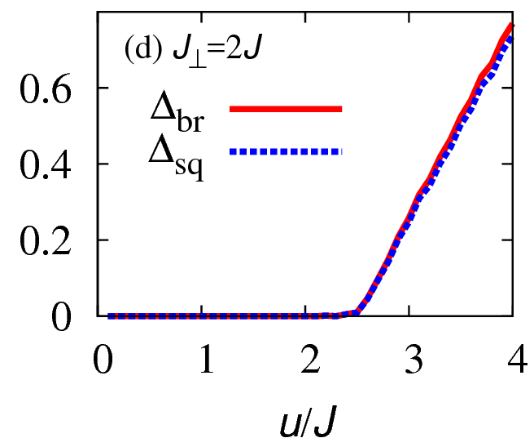


Strong coupling suppresses, weak coupling enhances SF behavior.

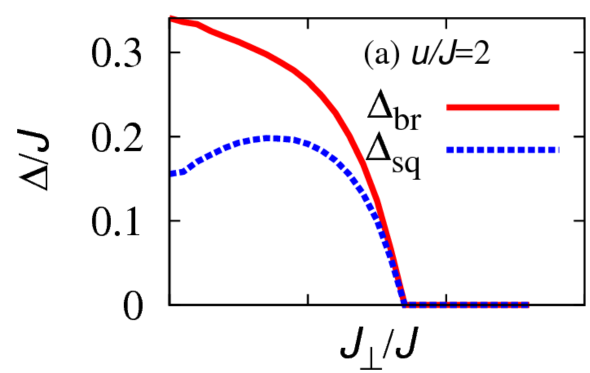
1/4-filling (half atom per site):



No SM phase in uncoupled system



SM-SF transition at finite coupling



Strong coupling suppresses SF phase

Quantum magnetism

Effective Hamiltonian for strongly repulsive system: $H_{\text{eff}} = \sum_{ij} J_{ij}^{\text{ex}} \mathbf{S}_i \cdot \mathbf{S}_j$ with $J_{ij}^{\text{ex}} = J_{ij}^2/U$

Neel-to-dimer transition upon increasing interlayer coupling:

The staggered magnetization vanishes in both layers simultaneously for $g_{\text{crit}} = \frac{J_\perp}{J} \approx 2.2$

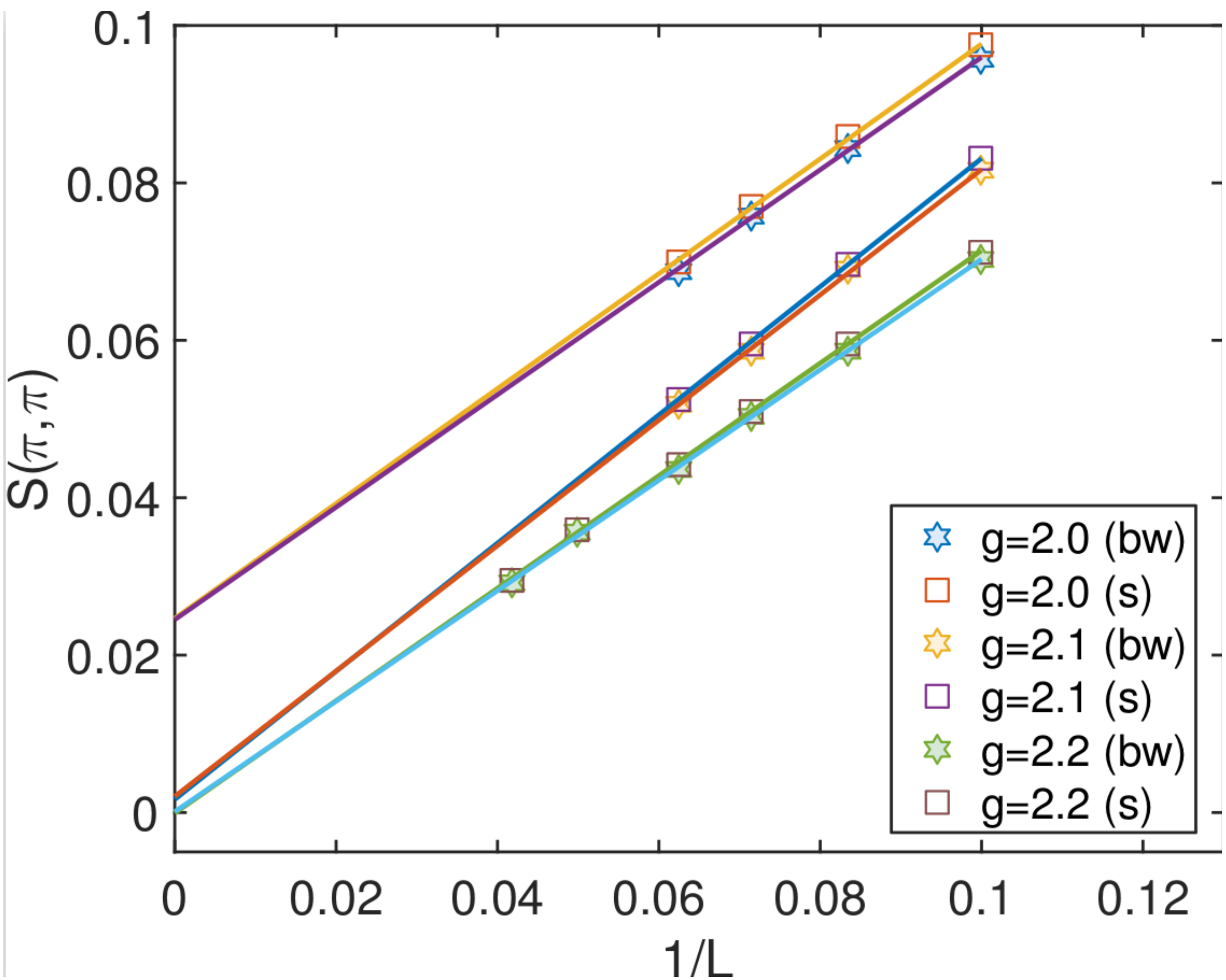
(QMC data for up to 1152 sites)

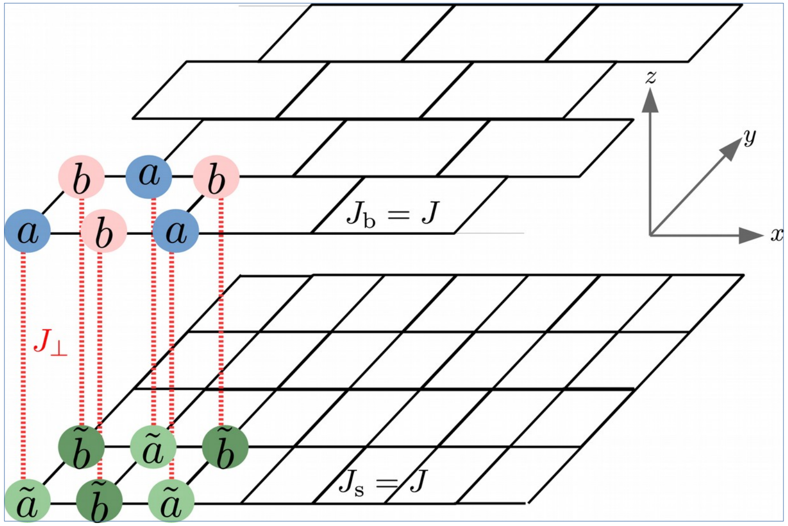
$$S(\pi,\pi) = \langle m_{\text{s}}^2 \rangle$$

$$m_{\text{s}} = \frac{1}{N_{\text{sites}}} \left(\sum_{i \in A} \mathbf{S}_i - \sum_{i \in B} \mathbf{S}_i \right)$$

Two square layers: $g_{\text{crit}} \approx 2.5$

Two hexagonal layers: $g_{\text{crit}} \approx 1.6$





Summary ...

- Shift and merge Dirac points by layer coupling!

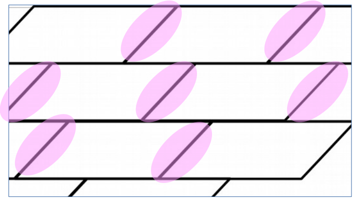
- New Dirac points!
- Give rise to SM-SF transition at filling one-fourth.
- Interlayer coupling can enhance or suppress SF phase.

- Magnetic transition in Mott phase:
  long-range Neel to dimer.

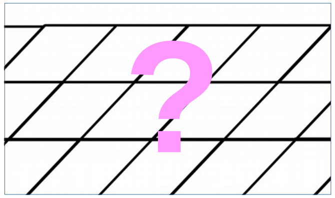
... and Outlook

- What happens when one layer becomes topological?
  Proximity-induced topological order?
  → Haldane model
  → shaken gauge field

- Frustrated quantum magnetism,
  e.g. anisotropic coupling strengths:



Unique dimerization in brick-wall



Degenerate configurations in square layer

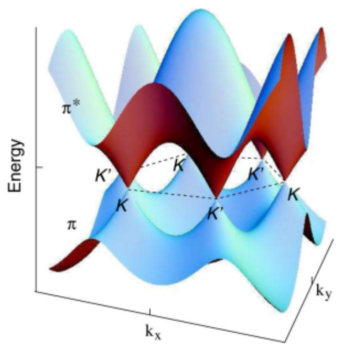
Second part:

Real graphene / artificial bilayer

More precisely:

Light-controlled Fractional Quantum Hall effect in graphene

Graphene in magnetic field: Landau levels



Effective Hamiltonian around Dirac point:

$$H_\xi = \xi v_{\mathrm{F}}(p_x\sigma_x + p_y\sigma_y)$$

$$\xi = \pm \text{ for } K, K'$$

Pauli matrices represent sublattice structure!

In magnetic field:

$$p_i \to \Pi_i = p_i + eA_i$$

$$\Pi_x = \frac{\hbar}{\sqrt{2}l_{\mathrm{B}}}(a^\dagger + a) \quad \text{and} \quad \Pi_y = \frac{\hbar}{i\sqrt{2}l_{\mathrm{B}}}(a^\dagger - a)$$

$$H_\xi = \xi\sqrt{2}\frac{\hbar v_{\mathrm{F}}}{l_{\mathrm{B}}}\begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}$$

“Standard” Landau level wave functions:

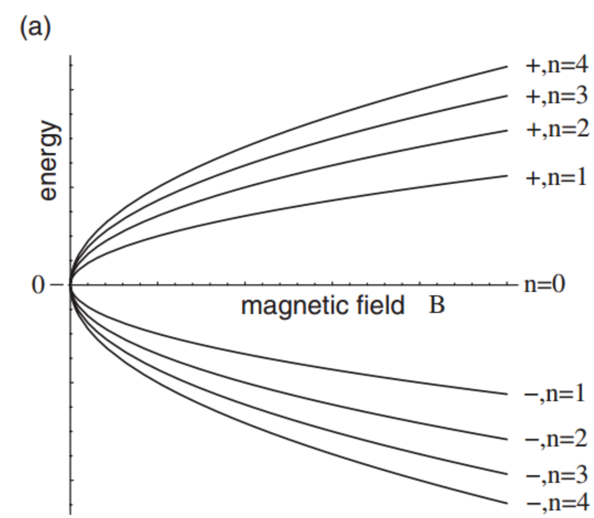
$$a^\dagger\varphi_{n,m} = \varphi_{n+1,m}$$

Graphene Landau level wave functions:

$$\Psi_{n=0} = \begin{pmatrix} 0 \\ \varphi_{0,m} \end{pmatrix} \quad \text{and} \quad \Psi_{n>0} = \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi_{n-1,m} \\ \xi\lambda\varphi_{n,m} \end{pmatrix}$$

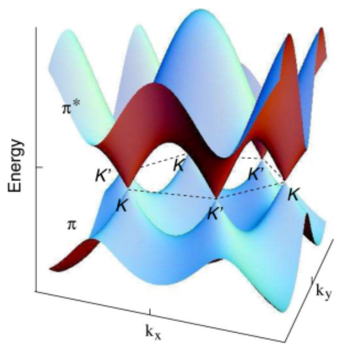
$$\lambda = \pm$$

At energies $\epsilon_{\lambda n} = \lambda\frac{\hbar v_{\mathrm{F}}}{l_{\mathrm{B}}}\sqrt{2n}$



See also review article: M. Goerbig, *Electronic properties of graphene in a strong magnetic field*, Rev. Mod. Phys. **83** 4 (2011)

Graphene in magnetic field: Landau levels



Effective Hamiltonian around Dirac point:

$$H_\xi = \xi v_{\rm F}(p_x\sigma_x + p_y\sigma_y)$$

$$\xi = \pm \text{ for } K, K'$$

Pauli matrices represent sublattice structure!

In magnetic field:

$$p_i \to \Pi_i = p_i + eA_i$$

$$\Pi_x = \frac{\hbar}{\sqrt{2}l_{\rm B}}(a^\dagger + a) \quad \text{and} \quad \Pi_y = \frac{\hbar}{i\sqrt{2}l_{\rm B}}(a^\dagger - a)$$

$$H_\xi = \xi\sqrt{2}\frac{\hbar v_{\rm F}}{l_{\rm B}}\begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}$$

"Standard" Landau level wave functions:

$$a^\dagger \varphi_{n,m} = \varphi_{n+1,m}$$

Graphene Landau level wave functions:

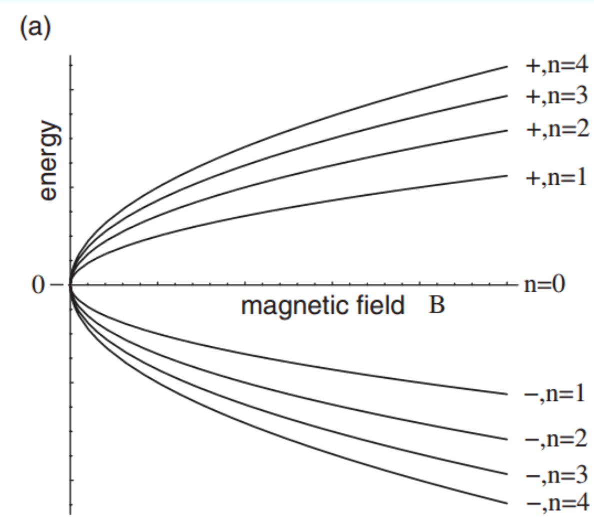
$$\Psi_{n=0} = \begin{pmatrix} 0 \\ \varphi_{0,m} \end{pmatrix} \quad \text{and} \quad \Psi_{n>0} = \frac{1}{\sqrt{2}}\begin{pmatrix} \varphi_{n-1,m} \\ \xi\lambda\varphi_{n,m} \end{pmatrix}$$

$$\lambda = \pm$$

At energies $\epsilon_{\lambda n} = \lambda\frac{\hbar v_{\rm F}}{l_{\rm B}}\sqrt{2n}$

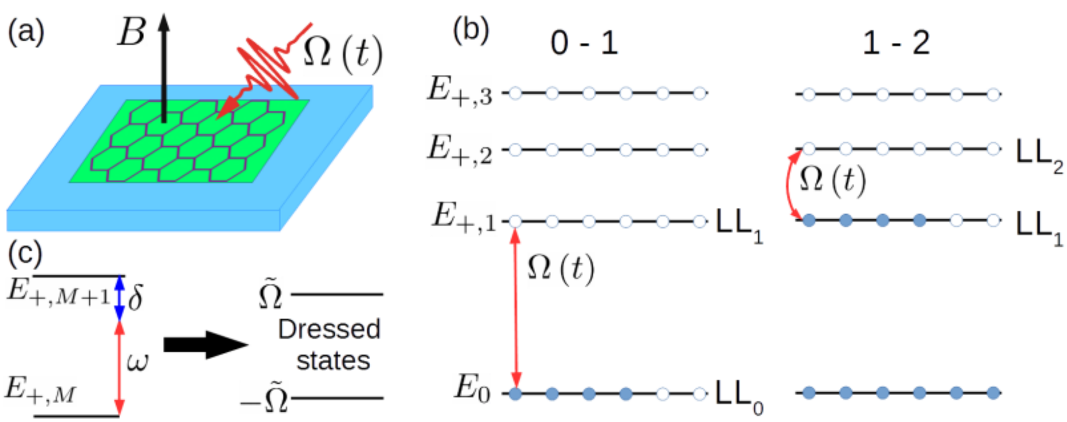
Relativistic Landau levels:

- Spinor wave function (modifies interactions in terms of Haldane pseudopotentials)
- Spin and valley degeneracy: 4 bands per energy level (IQH plateaux at Δν=4)
- Particle-hole symmetry (first IQH plateaux at ν=2)
- Non-equidistant energy levels!



See also review article: M. Goerbig, *Electronic properties of graphene in a strong magnetic field*, Rev. Mod. Phys. **83** 4 (2011)

Optical coupling of graphene Landau levels



What's the quantum number *m*?

Symmetric gauge → angular momentum component:

$$l = \hbar(m - n)$$

Coupling without angular momentum transfer:

$$\{n+1, m+1\} \leftrightarrow \{n, m\} \quad \text{e.g.} \quad \mu = 1$$

Landau gauge → Momentum component:

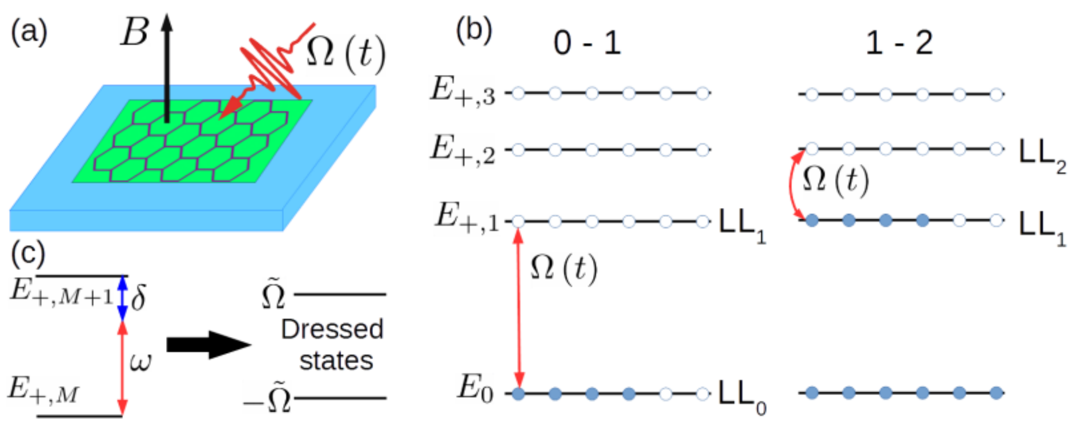
$$k_x = 2\pi m / L_x$$

Coupling without momentum transfer: $\mu = 0$

$$H_0(t) = \sum_m \left[\frac{\Delta E}{2} \left(c^\dagger_{n+1,m} c_{n+1,m} - c^\dagger_{n,m} c_{n,m} \right) + \hbar\Omega \left(c^\dagger_{n+1,m+\mu} c_{n,m} \mathrm{e}^{-i(\Delta E - \delta)t} + \text{h.c.} \right) \right.$$

Areg Ghazaryan, Michael J. Gullans, Pouyan Ghaemi, Mohammad Hafezi, *Light-induced fractional quantum Hall phases in graphene* (arXiv:1612.08748)

Optical coupling of graphene Landau levels



What's the quantum number *m*?

Symmetric gauge → angular momentum component:

$$l = \hbar(m - n)$$

Coupling without angular momentum transfer:

$$\{n+1, m+1\} \leftrightarrow \{n, m\} \quad \text{e.g.} \quad \mu = 1$$

Landau gauge → Momentum component:

$$k_x = 2\pi m / L_x$$

Coupling without momentum transfer: $\mu = 0$

$$H_0(t) = \sum_m \left[\frac{\Delta E}{2} \left(c^\dagger_{n+1,m} c_{n+1,m} - c^\dagger_{n,m} c_{n,m} \right) + \hbar\Omega \left(c^\dagger_{n+1,m+\mu} c_{n,m} \mathrm{e}^{-i(\Delta E - \delta)t} + \text{h.c.} \right) \right.$$

In rotating frame after RWA:

$$H_0 = \sum_m \left(\hbar\Omega c^\dagger_{n+1,m+\mu} c_{n,m} + \text{h.c.} + \hbar\delta c^\dagger_{n+1,m} c_{n+1,m} \right)$$

Interactions after RWA:

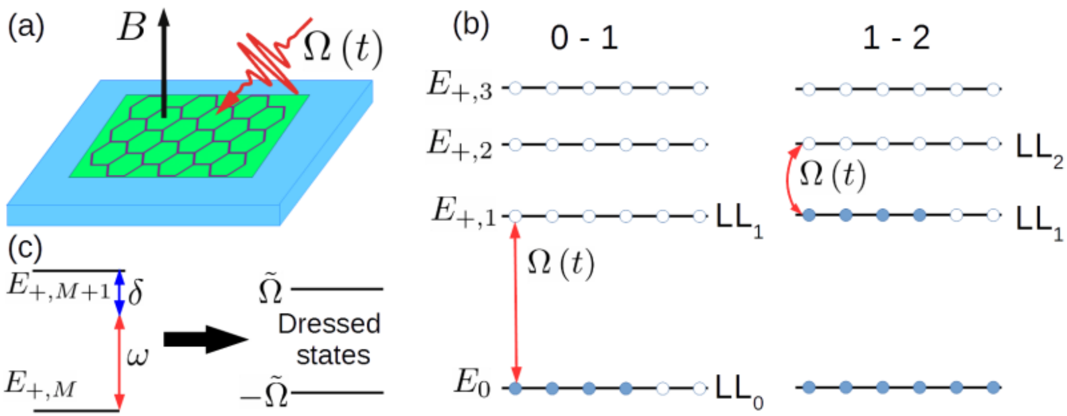
$$V^{n_1,n_2,n_3,n_4 (\mathrm{RWA})}_{m_1,m_2,m_3,m_4} = \delta_{n_1+n_2-n_3-n_4} V^{n_1,n_2,n_3,n_4}_{m_1,m_2,m_3,m_4}$$

Tunable Fractional Quantum Hall Hamiltonian
(assuming spin and valley polarization):

$$H = H_0 + V^{(\mathrm{RWA})}$$

Areg Ghazaryan, Michael J. Gullans, Pouyan Ghaemi, Mohammad Hafezi, *Light-induced fractional quantum Hall phases in graphene* (arXiv:1612.08748)

Optical coupling of graphene Landau levels



Three scenarios (all work in progress):

Strong coupling

Low-energy manifold: lower dressed Landau level.

How does dressing modify interactions?

Weak coupling

Both Landau levels can be occupied.

Bilayer quantum Hall phases?

Pulsed coupling

System is prepared in the lower level.

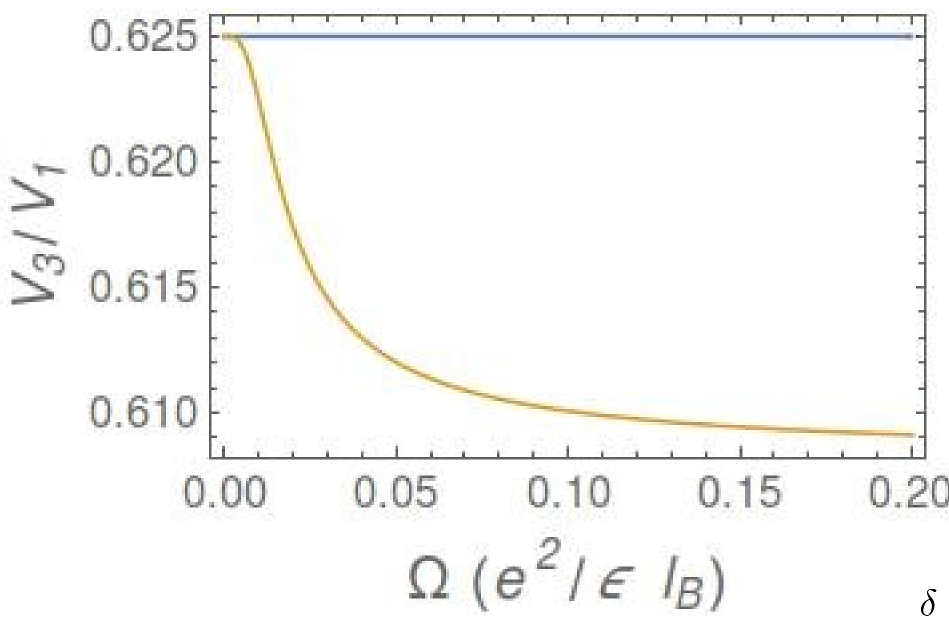
Transfer between Landau levels?

Strong coupling: Haldane pseudopotentials

$$V^{n_1 n_2, n_3 n_4}_{m_1 m_2, m_3 m_4} = \sum_{m,M} V^{n_1 n_2, n_3 n_4}_{m} P^{m_1 m_2}_{mM} P^{m_3 m_4}_{mM}$$

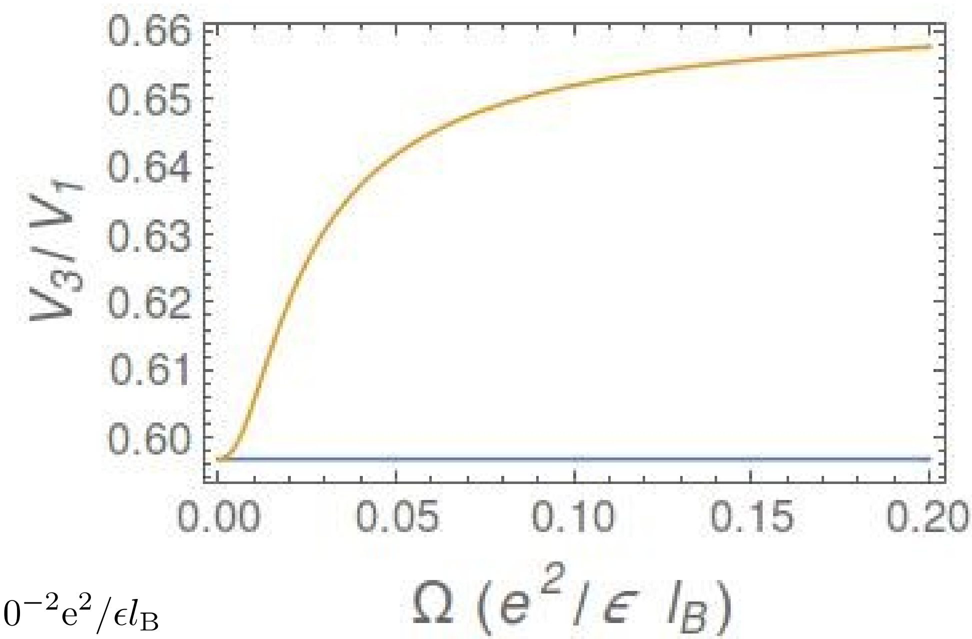
Dressed Landau level: $|s, m\rangle = \cos[\theta(\delta, \Omega)]|n+1, m+1\rangle + \sin[\theta(\delta, \Omega)]|n, m\rangle$

LL0 ↔ LL1



$\delta = 10^{-2} \mathrm{e}^2/\epsilon l_{\mathrm{B}}$

LL1 ↔ LL2



Modification of pseudopotentials are rather small.

We checked at filling ν=2/3 that strongly coupled system forms a PH-conjugated Laughlin phase, just as the uncoupled system does.

Effects at other filling factors?

Weak coupling: bilayer quantum Hall phases

Exact diagonalization on torus, at filling ν=2/3:

- For weak coupling between n=0 and n=1, system polarizes in n=0, forming a PH Laughlin phase.
- For weak coupling between n=1 and n=2, or n=-1 and n=1, system is completely depolarized between both levels, forming a singlet state.

Known quantum Hall states at ν=2/3:

	Layer polarization	*Quasiparticles*	*Torus degeneracy*
PH Laughlin	polarized	Abelian	3
$\mathbb{Z}_4$ (Read-Rezayi)	polarized	Non-Abelian	15
330-Halperin	singlet	Abelian	9
112-Halperin	singlet	Abelian	3
CF (Jain)	singlet	Abelian	3
Interlayer Pfaffian	singlet	Non-Abelian	9
Intralayer Pfaffian	singlet	Non-Abelian	27
Fibonacci	singlet	Non-Abelian	6

PHYSICAL REVIEW B
covering condensed matter and materials physics

Highlights Recent Accepted Authors Referees Search Press About

Non-Abelian two-component fractional quantum Hall states

Maissam Barkeshli and Xiao-Gang Wen
Phys. Rev. B **82**, 233301 – Published 2 December 2010

PHYSICAL REVIEW LETTERS

Highlights Recent Accepted Collections Authors Referees Search Press Ab

Fibonacci Anyons From Abelian Bilayer Quantum Hall States

Abolhassan Vaezi and Maissam Barkeshli
Phys. Rev. Lett. **113**, 236804 – Published 3 December 2014

PHYSICAL REVIEW B
covering condensed matter and materials physics

Highlights Recent Accepted Authors Referees Search P

Rapid Communication

Non-Abelian phases in two-component $\nu = 2/3$ fractional quantum Hall states: Emergence of Fibonacci anyons

Zhao Liu, Abolhassan Vaezi, Kyungmin Lee, and Eun-Ah Kim
Phys. Rev. B **92**, 081102(R) – Published 5 August 2015

PHYSICAL REVIEW B
covering condensed matter and materials physics

Highlights Recent Accepted Authors Referees Search Press About

Abelian and non-Abelian states in $\nu = 2/3$ bilayer fractional quantum Hall systems

Michael R. Peterson, Yang-Le Wu, Meng Cheng, Maissam Barkeshli, Zhenghan Wang, and Sankar Das Sarma
Phys. Rev. B **92**, 035103 – Published 2 July 2015

PHYSICAL REVIEW B
covering condensed matter and materials physics

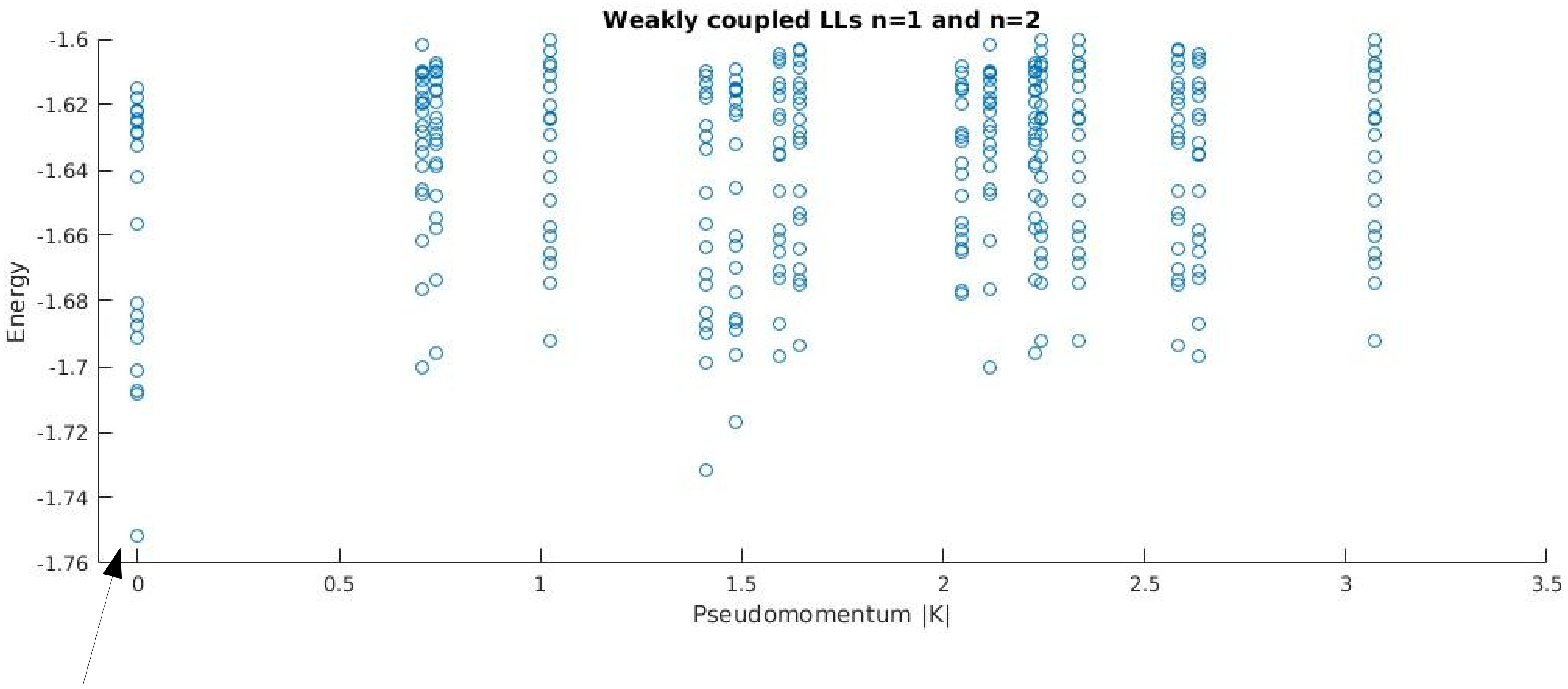
Highlights Recent Accepted Authors Referees Searc

Competing Abelian and non-Abelian topological orders in $\nu = 1/3 + 1/3$ quantum Hall bilayers

Scott Geraedts, Michael P. Zaletel, Zlatko Papić, and Roger S. K. Mong
Phys. Rev. B **91**, 205139 – Published 27 May 2015

Weak coupling LL1→LL2 at ν=2/3

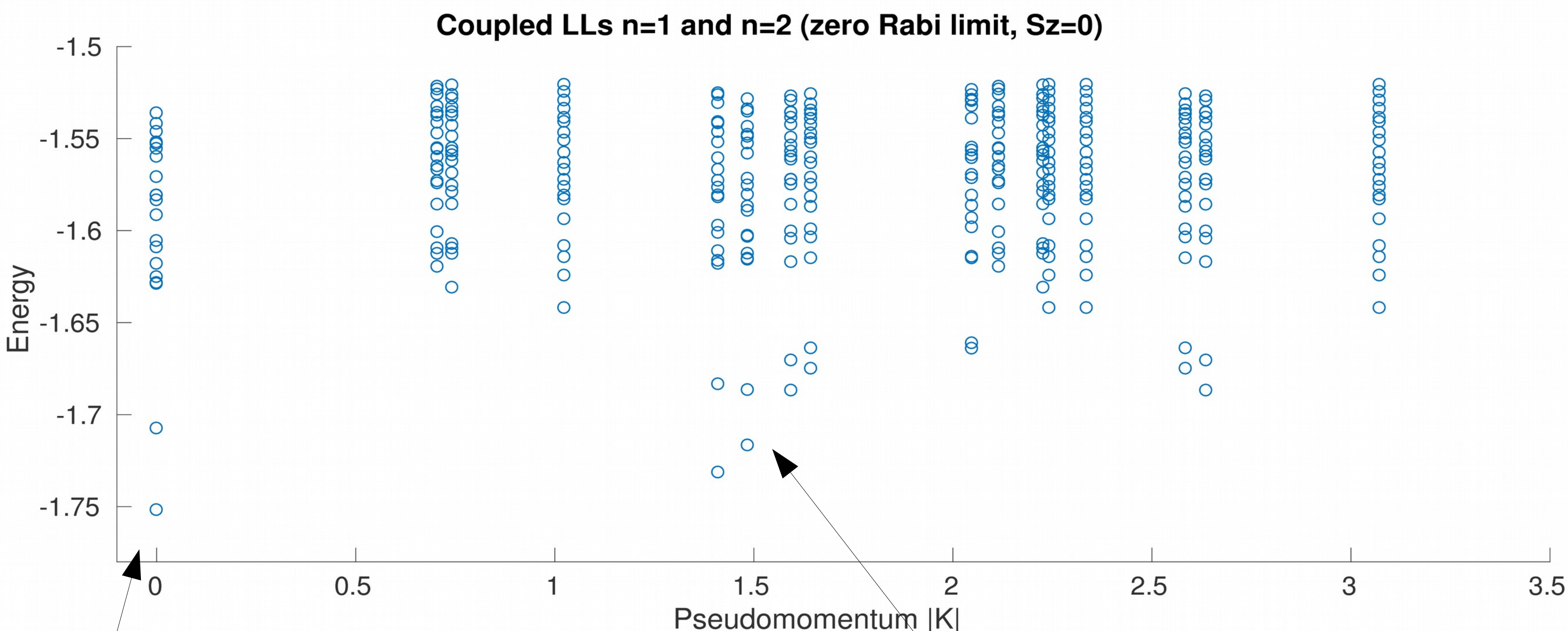
$$\nu = 2/3, N = 8, \Omega = 0.001, \delta = 0.02$$



- 3-fold degenerate singlet
- but no good overlap with Jain state or 112-Halperin state
- Overlap with 330-state: 0.44

Weak coupling LL1→LL2 at ν=2/3

$$\nu = 2/3, N = 8, \Omega \to 0, \delta = 0.02$$

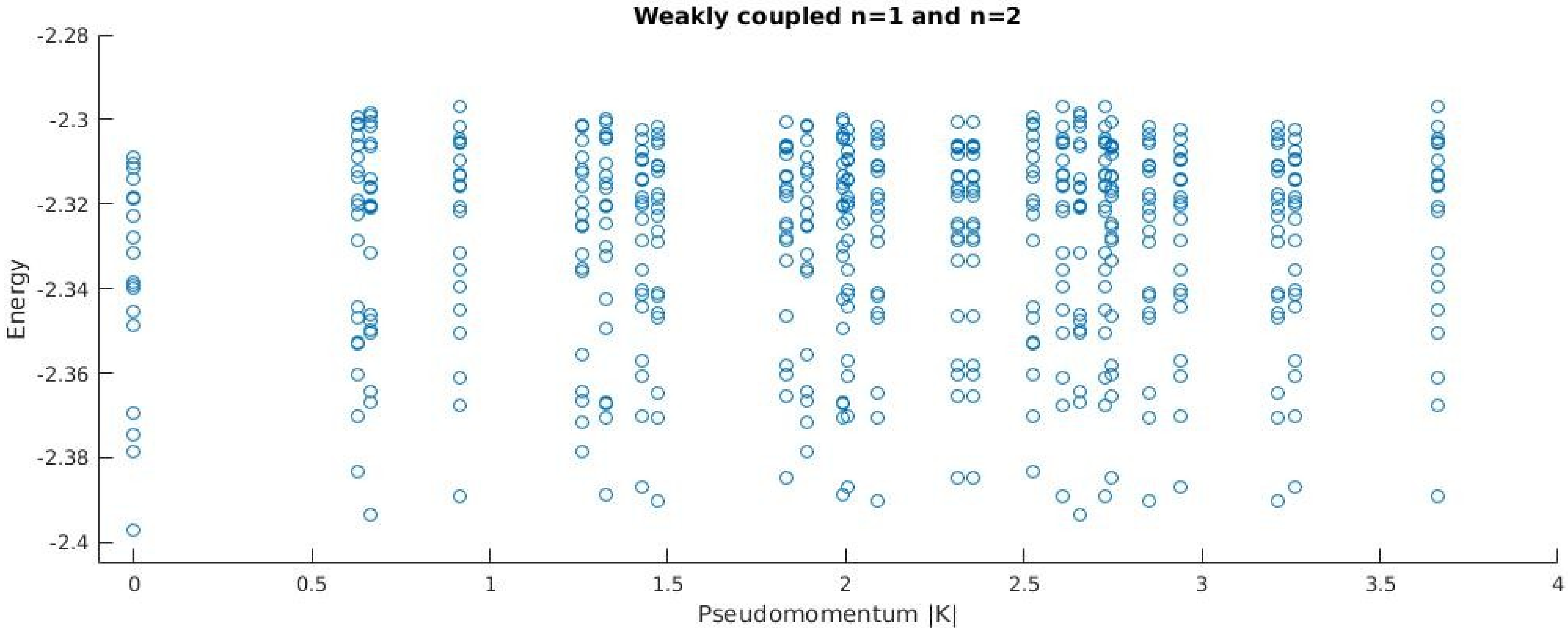


Six-fold quasidegeneracy at *K*=0 ?
Fibonacci phase?

Roton
instability?

Weak coupling LL1→LL2 at *ν*=2/3

$\nu = 2/3, N = 10, \Omega = 0.001, \delta = 0.02$



- No gap for *N*=10
- Even/odd effect?

Pulse: LL0→LL1

On the single-particle level, a π-pulse coupling "flips" the LL index:

$$\varphi_{n,m} \to \varphi_{n+1,m+1} = a^\dagger b^\dagger \varphi_{n,m}$$

Does this also work on the many-body level?

$$\Psi \to \prod_{i=1}^{N} a_i^\dagger b_i^\dagger \Psi$$

If so, this could be used to produce quasiholes:

$$\Psi = \Psi_{\mathrm{L}} \sim \prod_{i<j} (z_i - z_j)^3$$

Start with Laughlin state in LLL!

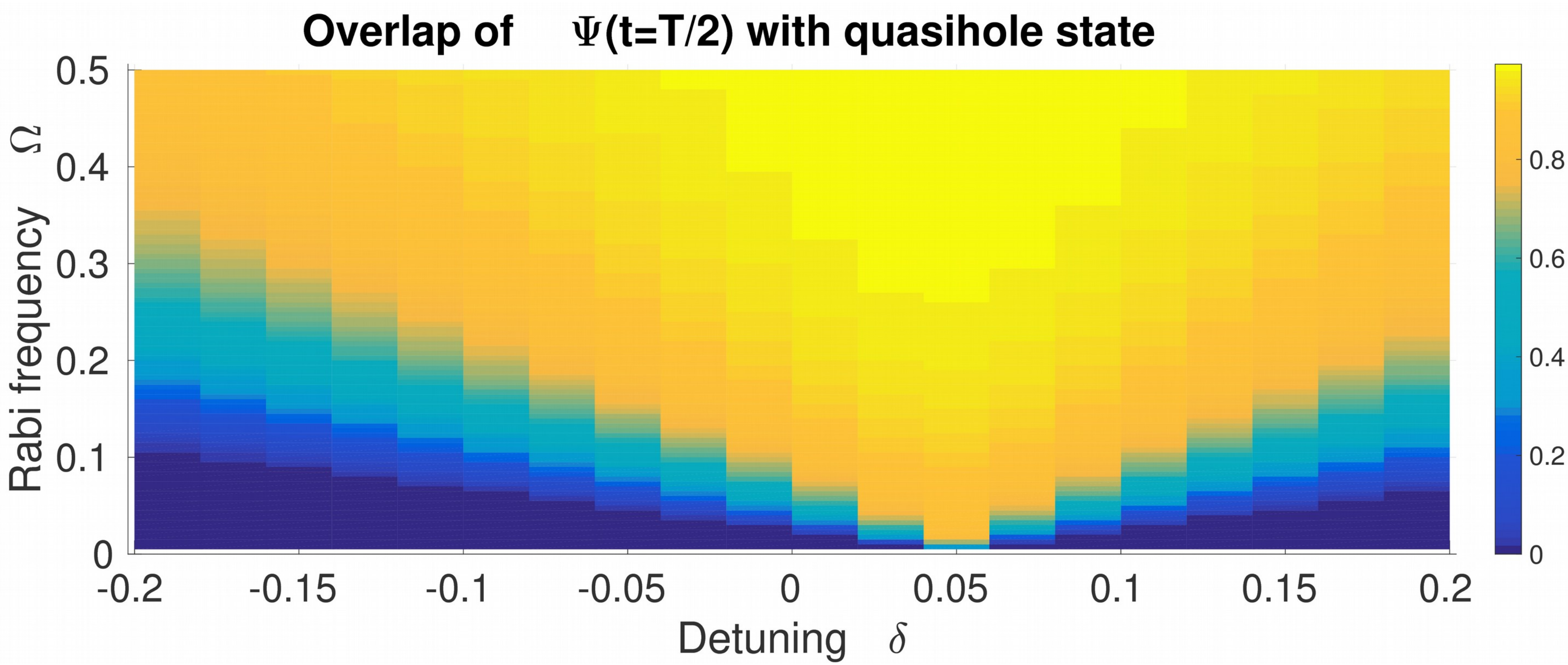
$$\left(\prod_i b_i^\dagger\right) \Psi \sim \left(\prod_i z_i\right) \Psi_{\mathrm{L}} \sim \Psi_{\mathrm{qh}}$$

Shift in *m*-quantum numbers produces quasihole!

$$\left(\prod_i a_i^\dagger\right) \Psi_{\mathrm{qh}} \sim \Psi'_{\mathrm{qh}}$$

Shift in *n*-quantum numbers translates state into higher Landau level!

Pulse: LL0→LL1



$$\Psi = \Psi_{\rm L} \sim \prod_{i<j} (z_i - z_j)^3$$

Start with Laughlin state in LLL!

$$\left(\prod_i b_i^\dagger\right) \Psi \sim \left(\prod_i z_i\right) \Psi_{\rm L} \sim \Psi_{\rm qh}$$

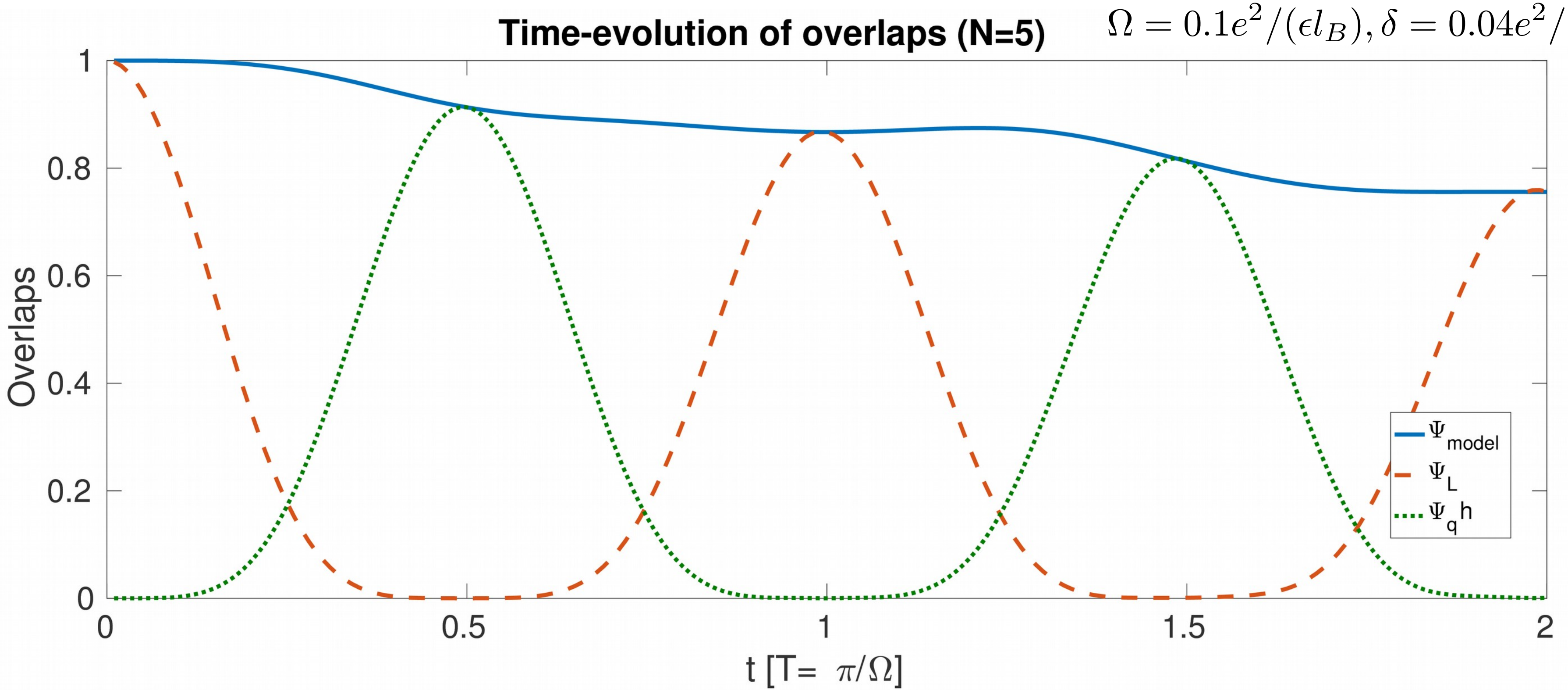
Shift in *m*-quantum numbers produces quasihole!

$$\left(\prod_i a_i^\dagger\right) \Psi_{\rm qh} \sim \Psi'_{\rm qh}$$

Shift in *n*-quantum numbers translates state into higher Landau level!

Pulse: LL0→LL1

$\Omega = 0.1e^2/(\epsilon l_B), \delta = 0.04e^2/(\epsilon l_B)$



We model the wave function by superposition of initial state, quasihole state, and edge-like excitations:

$$\Psi_{\mathrm{model}}(t) = \sum_{s=0}^{N} \sqrt{\binom{N}{s}} \cos(\Omega t)^{N-s} \sin(\Omega t)^s \Psi^{(s)},$$

$$\Psi^{(s)} = \sum_{\{k_1,\ldots,k_s\}} (-1)^{\sum_{j=1}^{s} k_j} (-i)^{\mathrm{mod}(s,2)} \frac{1}{\sqrt{\binom{N}{s}}} \prod_{j=1}^{s} a^{\dagger}_{k_j} b^{\dagger}_{k_j} \Psi_{\mathrm{L}}.$$

Measure fractional charge/statistics by interference of Laughlin and quasihole state?

But there is never a superposition of (only) these two states.

Summary & Outlook

- Graphene Landau levels can be coupled using light.
- Tool to control quantum Hall phases:

Engineer Haldane pseudopotentials!

Create new degree of freedom (for bilayer phases)!

Create (and braid) quasiholes generated by light!

Thank you!