Introduction to diversification

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Motivation

"There is no such thing as a free lunch"

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- Diversification

Diversification

- Central idea in finance
- Risk-Reward (no free lunch)
- You can achieve the same expected return with lower overall risk (!!)

Example

Assume that you have \$200 and that you would like to invest one of two stocks, A and B, over the next two periods, 1 and 2. The share price is \$100 for both stocks today.

Period	1	2	Value of position after period 2
Stock A	+100%	-50%	\$100
Stock B	-50%	+100%	\$100

Example

The return after period two is zero!

But what if we invested in both stocks?

Example

Now you invest $100 in each stock and rebalance every period to keep the ratio 50/50 (assume you can buy fractions of a share).

Period	1	2	Value of position after period 2
Stock A	+100%	-50%	$100
Stock B	-50%	+100%	$100
A+B	$200 + 50	62.5 +250	$312.5

Portfolio of correlated stocks

- Stocks in a portfolio usually co-move in one way or another
- Risk (Portfolio variance?)
- Systematic and idiosyncratic risk

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Portfolio of correlated stocks

- Variance of a single stock: $\sigma_i^2 = T^{-1}\sum_{t=1}^{T}(r_i - \bar{r})^2$

- Variance of a portfolio of n assets: $\sigma_p^2 = \omega'\Sigma\omega$,

where $\omega' = [\omega_1, ..., \omega_n]$ is a vector of weights and Σ is an n$\times$n covariance matrix.

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$$\sigma_p^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j Cov(s_i, s_j) = \sum_{i=1}^{n} \omega_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i \omega_j \sigma_{i,j}.$$

Portfolio of correlated stocks

Suppose the portfolio is equal-weighted. We can write

1. $\overline{\sigma}^2 = n^{-1} \sum_{i=1}^{n} \sigma_i^2$
2. $\overline{\text{Cov}} = (n(n-1))^{-1} \sum_{i=1}^{n} \sum_{j \neq i}^{n} \sigma_{i,j}$

which we can insert in the original equation:

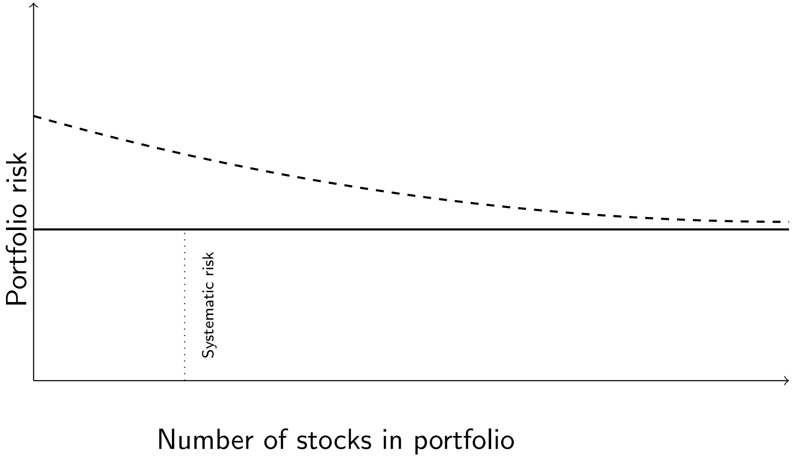
$$\sigma_p^2 = \frac{1}{n^2}\overline{\sigma}^2 + \frac{n-1}{n}\overline{\text{Cov}}$$

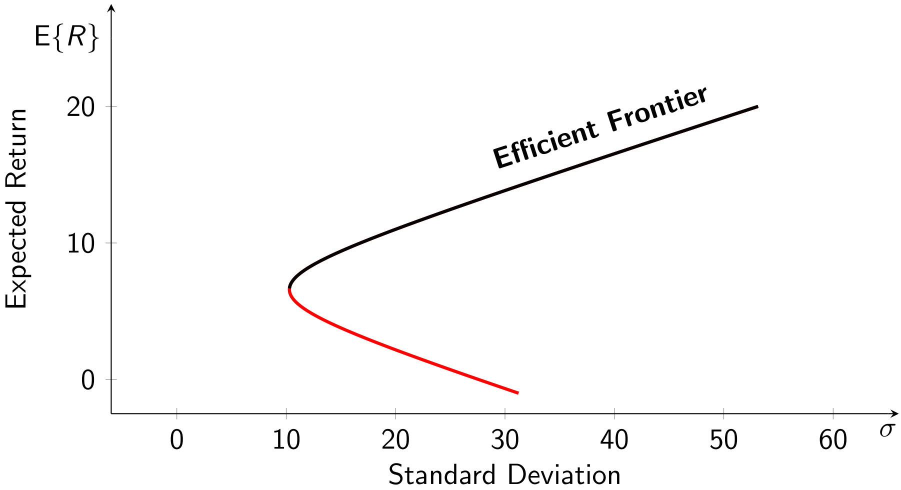
Portfolio of correlated stocks

Result:

$$\lim_{n \to \infty} \sigma_p^2 = \overline{Cov} \quad \Rightarrow \text{ only covariances matter!}$$

Diversification





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