Solve by hand or by using your favorite programming language.

Problem 1

Solve the following linear systems using Gaussian elimination:

a) $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 11 \\ 2 & -1 & 3 & 3 \\ 3 & 2 & 5 & 12 \end{array}\right]$ b) $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 11 \\ 2 & -1 & 3 & 3 \\ 3 & 2 & 5 & 12 \end{array}\right]$ c) $\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 8 \\ 1 & 3 & 1 & 5 & 28 \\ 2 & 4 & 2 & 9 & 48 \end{array}\right]$

Problem 2

Determine the number for solutions that the following linear system has:

$$\begin{array}{ccccccc} x & + & y & + & 2z & = 6 \\ x & + & 2y & + & 4z & = 13 \\ x & + & 3y & + & 9z & = 24 \end{array}$$

Problem 3

We consider the homogeneous linear system with coefficient matrix

$$A = \begin{bmatrix} 1 & 1 & 4 & -1 \\ 5 & 5 & -1 & 4 \\ 7 & 6 & 3 & 3 \end{bmatrix}$$

Describe the set of solutions geometrically. How many degrees of freedom are there? Does this change if we change the red coefficient in the second row?

In problems 4-5, we consider the vectors given by

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 4 \\ -1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}, \quad v_4 = \begin{bmatrix} -1 \\ 3 \\ -7 \end{bmatrix}, \quad v_5 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}.$$

Problem 4

Determine if the vectors are linearly independent:

a) $\{v_1, v_2\}$ b) $\{v_1, v_2, v_3\}$ c) $\{v_1, v_2, v_5\}$ d) $\{v_2, v_3, v_4\}$ e) $\{v_1, v_2, v_3, v_4\}$

Problem 5

Compute the dimension of V, and find a base $\mathcal{B}$ of V:

a) $V = \text{span}(v_1, v_2)$ b) $V = \text{span}(v_1, v_2, v_3)$ c) $V = \text{span}(v_1, v_2, v_5)$

d) $V = \text{span}(v_1, v_2, v_3, v_4)$