Solve by hand or by using your favorite programming language.

Problem 1

Find all eigenvalues of A, and a base for the eigenspace E_λ for each eigenvalue λ:

a) $A = \begin{bmatrix} 3 & 7 \\ 7 & 3 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ c) $A = \begin{bmatrix} 2 & -4 \\ 3 & -1 \end{bmatrix}$ d) $A = \begin{bmatrix} 4 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 4 \end{bmatrix}$

e) $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ f) $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

Problem 2

Use Python to download the time series of Apple, Tesla, and Starbucks using Python:

```
import yfinance as yf

tickers = ['AAPL','TSLA','SBUX']

start = "2015-01-01"
end   = "2022-1-30"
data  = yf.download(tickers, start=start, end=end)

data.Close
```

a) Compute the daily returns of the three stocks. Report the mean, standard deviation, and covariance matrix.

b) Find the minimum-variance portfolio weights.

c) Find the mean-variance efficient portfolio.

Problem 3

$$f(x) = \frac{x^2}{x^2 + 2}.$$

a) Compute $f'(x)$ and determine where $f(x)$ is increasing/decreasing.

b) Find possible inflection points.

c) Determine the limit of $f(x)$ as $x \to \pm\infty$, and sketch the graph of $f(x)$.

Problem 4

$$f(x) = \left(e^{2x} + 4e^{-x}\right)^2.$$

a) Compute $f'(x)$ and $f''(x)$.

b) Determine where f is increasing/decreasing, and show that f is convex.

c) Find possible global extreme points for f.