Debt Indexation and the Fiscal Theory of the Price Level*

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Abstract

In this paper, we analyze the importance of *inflation-indexation* of a part of the stock of government debt. We first establish that the degree to which uncovered sovereign spending shocks are inflationary is increasing in the share of inflation-indexed debt in the overall government debt portfolio. We leverage this finding to introduce inflation-indexed debt in a model of the Fiscal Theory of the Price Level (FTPL), where we show that: (i) even absent further frictions, inflation-indexed debt makes the price level backward-looking (i.e., it becomes a state variable), (ii) it tightens bounds that pin down 'active fiscal policy', and (iii) in a calibrated HANK model, a one percentage point increase in the share of inflation-indexed debt in overall government debt increases the volatility of the response of inflation to government spending by up to 4% relative to a no-indexed debt baseline case in a world of fiscal dominance.

Keywords: Debt Indexation, Fiscal Theory of the Price Level, Fiscal-Monetary Interactions.
JEL Codes: E32, E63, G12

1 Introduction

This paper aims to shed light on the role that indexed debt plays in a model of the Fiscal Theory of the Price Level (FTPL). Such indexed debt has yet not been introduced in models of the FTPL, despite the possible importance of such debt, whose face value changes with the gross rate of inflation. It should be immediate that if the government budget balance indeed plays a crucial role in determining the price level (as the FTPL claims to be the case), then an interesting feedback loop may arise if the face value of a part of that debt *itself* changes either with the price level or its rate of change. We motivate this idea by using data on narrative fiscal shocks through government bond price revaluations, showing that an unexplained component in sovereign debt revaluations (relative to a simple complete-markets FTPL model) is correlated with the share of marketable indexed debt outstanding. Our second empirical finding, based on an exercise with local projections using again exogenously identified fiscal shocks, reflects that inflation-indexed debt appears to boost ex-post inflation outcomes in response to narratively identified expansionary fiscal shocks.

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We next move on to modeling the FTPL with inflation-indexed debt. In a simple one-equation model, we establish that the price level becomes a state variable without further ado: previous price levels matter for the determination of the current price level in a model of the FTPL with inflation-indexed debt, even without further sources of stickiness being present in the economy. For such an economy with inflation-indexed debt, we are able to prove uniqueness of the stationary equilibrium in a corresponding dynamic economy.

This paper therefore also adds to recent debates about determinacy properties of FTPL models in non-Ricardian economies. Farmer and Zabczyk (2019) and Hagedorn (2021, 2024) particularly stand out, arguing that the FTPL cannot yield determinacy in models where the real interest rate is itself an equilibrium object. Here, we mostly restrict ourselves to the contribution of Hagedorn (2021), establishing further conditions under which an incomplete-markets model with the FTPL can determine the initial price level uniquely when inflation-indexed debt is present.

Finally, we analyze the combined effects of household heterogeneity and the presence of inflation-indexed debt to monetary and fiscal spending shocks in a full heterogeneous-agent New Keynesian (HANK) model á la Kaplan et al. (2018), making use of the methods pioneered in Auclert et al. (2021) to solve heterogeneous-agent models up to first-order in aggregate variables, while preserving heterogeneity with respect to the individual agents in this economy.[1] Here, inflation-indexed debt matters quantitatively by increasing the volatility of inflation on average by up to 4% for each one percentage point increase of the share of indexed debt in the overall debt portfolio when the economy is calibrated to the UK in the post-Covid period. We furthermore establish that the classic notions of 'active/passive monetary/fiscal policy', as derived by Leeper (1991), do not directly translate into the world with inflation-indexed debt, even though similarities in the determination of unique equilibria prevail.

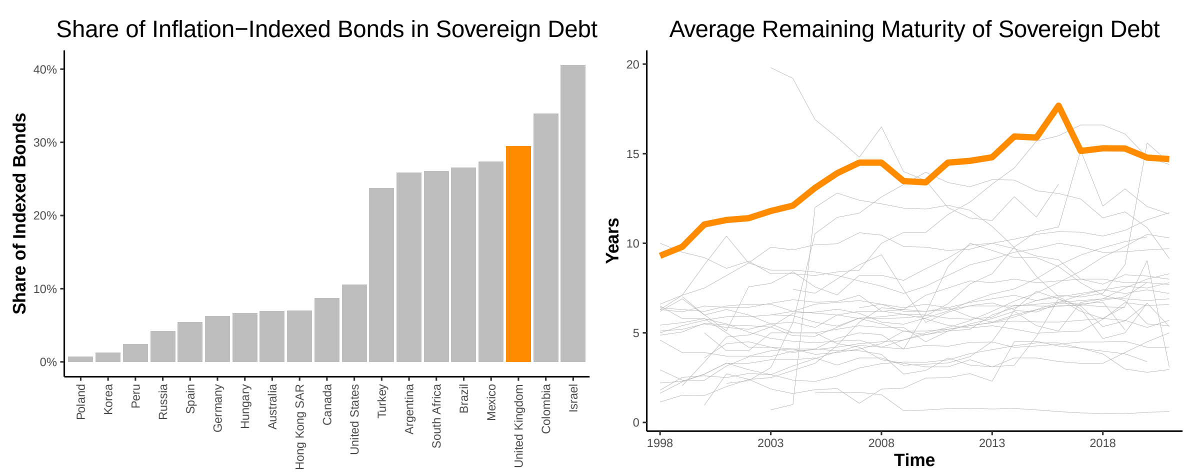


Figure 1: The share of inflation-indexed debt in the total sovereign debt portfolio and its average weighted maturity over time. The UK is marked orange, while grey lines and bars indicate other BIS member countries. Data source: BIS (2024).

[1]Crucially, this allows us to preserve non-linear risk aversion motives for holding indexed debt, as indexed debt will be the principal insurance device of households against inflationary shocks.

To establish the relevance of our idea behind introducing indexed debt, figure 1 shows both the share of inflation-indexed debt as part of the overall debt stock and its maturity over time. While there is considerable heterogeneity across countries, indexed bonds are present across the board. We will mostly focus on the UK both in the empirical motivation and in the theoretical specification due to the significant presence of inflation-indexed debt, as evidenced by figure 1.[2]

While 'fiscal dominance' in the sense of Leeper (1991) is not a predicament for the FTPL, it usually enhances the role of fiscal policy as drivers of inflationary dynamics in macroeconomic general equilibrium models (Leeper, 1991; Sims, 2011; Bianchi et al., 2023). Instead of providing a full picture supporting possible fiscal dominance,[3] we motivate this paper by considering a specific policy example: the UK 'mini-budget' considered in September 2022, which can be considered an exogenous fiscal policy disturbance.[4]

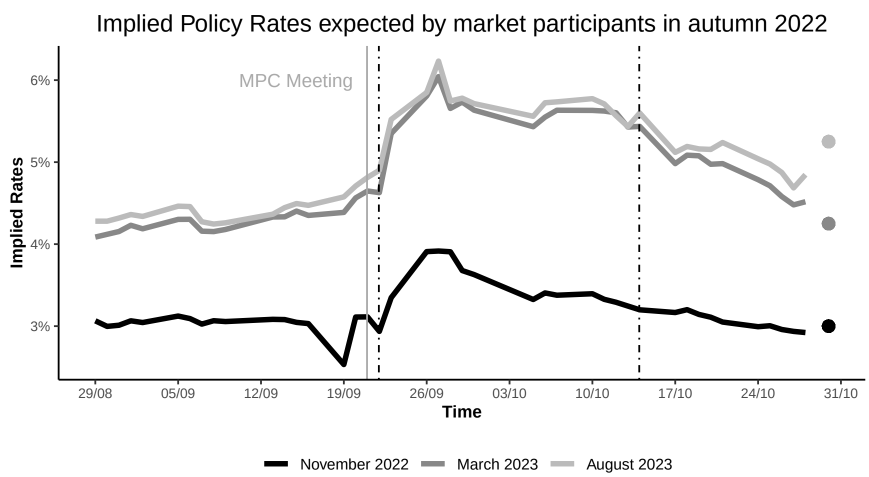


Figure 2: Expectations of nominal interest rates in the United Kingdom for the three MPC meetings after the 'mini-budget' announced in September 2022. The dots at the end reflect the factual values of nominal policy rates after each meeting has taken place.

To indicate that this might be an example of a fiscal shock driving implications for monetary policy adjustment (and thus informing the fiscal-monetary mix through 'fiscal dominance'), figure 2 plots the market-implied policy rates in the window around the mini-budget announcement and its cancellation. The first solid line depicts the date of a Bank of England MPC meeting, which occurred just ahead of the detailed policy announcement of the 'mini-budget' fiscal policy measure, with the MPC minutes being released on the 22nd of September 2022, one day ahead of the fiscal policy announcement. This is useful for our argument insofar as the meeting likely communicated the Bank of England's stance on future rate changes clearly, taking all available information up to

[2] Ex-post realized real yields differ between the two types of bonds as well, for instance due to inflation risk premia and liquidity risk premia (Gürkaynak et al., 2010). Looking at market yields at constant maturity, historical data provided by the Fed Board of Governors (2024) confirms a permanently positive differential in market yields on 10-year constant-maturity TIPS relative to standard US treasuries on every day since 2003.

[3] Relevant evidence for both the UK and the US is provided by Barro and Bianchi (2023); Bianchi et al. (2023); Chen et al. (2022); Cochrane (2022b); Leeper (2023); Smets and Wouters (2024).

[4] For a more detailed argument related to this specific fiscal shock, see Leeper (2023) and NIESR (2022).

that point into account (Braun et al., 2024). Nonetheless, implied policy rates rose sharply a couple of days after the meeting of the Bank of England's MPC, just after the announcement of the mini budget (denoted by the first dotted line), with the shift amounting to a 120bps increase in expected policy rates one year ahead. After the scraping of the mini-budget (second dotted line), expected policy rates swiftly returned to their 'pre-shock' levels.[5]

This event resonates well with the possible idea of (at least partial) fiscal dominance: financial market participants clearly expected changes to the monetary policy stance beyond the very short term in response to an announced fiscal policy measure.

Literature Review

This paper contributes to the burgeoning literature on fiscal-monetary interactions, pioneered in Sargent and Wallace (1981) and formalized through Leeper (1991). Initial contributions focusing on the possibility of fiscal dominance include Sims (1994) and Woodford (1995), who coined the terminology behind the 'Fiscal Theory'. More succinct summaries of the literature and of the state-of-the-art of the FTPL are provided by Leeper and Leith (2016) and Cochrane (2023). Bassetto and Cui (2018), Liemen and Posch (2022), and Bianchi et al. (2023) provide advances of the FTPL in standard OLG and New-Keynesian models, while empirical support for the possibility of fiscally-driven inflation has been developed in Barro and Bianchi (2023), Cochrane (2022a), Chen et al. (2022), and Cloyne et al. (2023), mirroring the interest in possible fiscal drivers in the recent inflationary episode. A specific example of a recent fiscal shock informing inflation rates is provided by Hazell and Hobler (2024), who focus on the 2021 Georgia Senate election runoff.

Applications of the FTPL in recent papers shifted the focus towards models with an endogenous real interest rate. This is important insofar as the FTPL is fundamentally a criterion that constrains the transversality condition on government debt to hold for only *one* candidate price level, but that transversality condition itself fundamentally depends on the real interest rate. Brunnermeier et al. (2020), Miao and Su (2021), and Kaplan et al. (2023) each provide conditions under which the FTPL nonetheless admits (unique) solutions expressed through the price level, however, their notions of uniqueness are challenged by Hagedorn (2021, 2024), who argues that the endogeneity of the real interest rate in incomplete-markets models 'breaks' the FTPL and allows a continuum of initial price levels to exist. We contribute to this literature by explaining and partially overcoming this seeming discrepancy, qualifying the criteria under which the FTPL can admit unique price levels even in incomplete-market settings with inflation-indexed debt.

In a recent contribution closely related to the importance of transversality conditions (which, as we just mentioned, lie at the heart of the FTPL), Brunnermeier et al. (2024) lays out how differences in the valuations of 'safe' assets can induce an aggregate transversality condition to fail, even if individual transversality conditions hold. We contribute to this idea by laying out the properties

[5] Note that the expected monetary policy response was partially driven by a concurrent funding mismatch in liability-driven investment strategies of defined-benefit pension funds that were closely tied to movements in yields of sovereign bonds. See Pinter (2023) for a detailed exposition of this point.

of this idea in a model of the FTPL with indexed debt, paying attention to the different insurance properties borne by both types of debt.

We also contribute to the literature on inflation-linked government bonds. Such bonds were introduced in economic and financial research long ago, especially in relation to the introduction of TIPS in the US in 1997. One of the earliest contributions in this field is Fischer (1975), who derives household demand for such assets in a partial-equilibrium multi-asset framework. The special insurance properties of such inflation-linked debt are extensively discussed in Barr and Campbell (1997), Garcia and van Rixtel (2007), Gürkaynak et al. (2010) and Andreasen et al. (2021). Schmid et al. (2024) provide a systematic analysis of inflation-indexed debt as a policy tool for governments, emphasizing its role as an ex-ante commitment device against inflation. In our contribution, we leverage the unique properties of inflation-indexed debt within frameworks of the FTPL, which express themselves mostly through the induction of a backward-looking component in the FTPL 'core' equation and through the insurance premia they bear. Our focus thus effectively rests on the 'ex-post' effects that inflation-indexed debt can have in the face of expansionary government spending shocks.

In the later sections of the paper, we majorly rely on modern computational methods to efficiently solve and estimate heterogeneous-agent methods, as in Kaplan et al. (2018), Bayer and Luetticke (2020), and Achdou et al. (2022). In particular, we leverage the efficient computation algorithms pioneered in Auclert et al. (2021) and some of the refinements of Auclert et al. (2024) to solve a model with heterogeneous households, two types of assets, and the FTPL in a matter of minutes.[6]

Finally, we are not the first to link fiscal dominance to heterogeneous-agent frameworks. Brunnermeier et al. (2020), Kaplan et al. (2023), and in particular Kwicklis (2024), who links fiscal dominance to the canonical HANK framework of Kaplan et al. (2018), have all applied the FTPL to heterogeneous-agent frameworks. Similarly worth mentioning is the paper by Angeletos et al. (2024), who negate the need for FTPL models, finding quantitatively identical responses of inflation to expansionary fiscal shocks in HANK models. Our contribution here is to introduce a second type of assets (inflation-indexed debt) with a feedback loop between asset holdings and the price level, quantifying the importance that such indexed debt can have for inflation dynamics in a calibrated model applied to a country with high levels of inflation-indexed debt. Such effects are unlikely to arise up to first-order in simple HANK models without the FTPL, which we will explore in further research.

The rest of the paper is structured as follows. Section 2 briefly exposes the relevance of indexed debt for bond revaluations and ex-post inflation in the face of fiscal shocks, after which we introduce inflation-indexed debt in simplified economic frameworks in section 3. We introduce the main quantitative model in section 4. Section 5 discusses the calibration and estimation methods, and we present our quantitative findings in section 6. Finally, section 7 concludes.

[6] To motivate the relevance of household heterogeneity applied to holdings of sovereign debt, figure D.1 in the appendix provides evidence on the skew of household holdings of such debt, sorted by their respective income decile. It furthermore establishes that this skew is *even more pronounced* for inflation-indexed debt.

2 The importance of indexed debt in sovereign bond revaluations and for inflationary dynamics

To motivate the relevance of indexed debt as a possible driver of the net present value of government debt and, therefore, of price level dynamics under the FTPL, we proceed in two steps: first, we derive a measure of 'net fiscal shocks' in the UK with two types of sovereign debt under the assumption of *complete markets*, showing that the *unexplained* component of revaluations of sovereign debt, induced (among other things) by the complete markets assumption, is closely linked to the share of inflation-indexed debt in the government bond portfolio, which solidifies the need to consider interactions between incomplete-markets and the two types of debt.[7] Second, we employ a long-running series of exogenously supplied UK fiscal policy shocks in a local projection to solidify the effects that inflation-indexed debt has on inflation itself when fiscal spending disturbances affect the economy and provide some robustness checks coming from a similar exercise on US data.

2.1 Determining unexplained sovereign bond revaluations

We leverage a novel dataset capturing the entire universe of UK sovereign debt (comprising of normal gilts, inflation-indexed gilts, treasury bonds, treasury strips, etc.) in the period from 2000 until 2010 to analyze bond revaluations in response to unanticipated fiscal spending shocks. The starting point of this analysis are the canonical FTPL derivations from Cochrane (2023), and most notably the equilibrium valuation of government debt:

$$\frac{Q_t B_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j},$$

where Q_t refers to the price of government debt, B_{t-1} is the quantity of outstanding (marketable) sovereign debt, P_t is the price level, and s_{t+j} denotes the real government surplus at time $t+j$.

Accounting for the existence of long-term and inflation-indexed debt, this relationship changes to:

$$\frac{\sum_{j=0}^{\infty} q_t^{(t+j)} b_{t-1}^{(t+j)}}{P_{t-1}} + \frac{\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

where superscripts denote the time of maturity of the bond at question and lowercase variables denote quantities and prices of inflation-indexed debt.[8] Note that we therefore define 'indexed debt' as a type of debt instrument whose principal payment is multiplied by the gross inflation rate between bond issuance and redemption, thereby mirroring the factual payment adjustment process commonplace in sovereign bond markets.

[7] We limit ourselves to one country and one time period only, focusing majorly on the importance of indexed debt while minimizing the need to account for cross-sectional heterogeneity. For a cross-country exercise *without* indexed debt that focuses directly on the empirical link between fiscal surprises and inflation, see Barro and Bianchi (2023).

[8] Derivations of similar relationships under the same complete-markets assumption are given in section 3.

We now move this equation one period forward ($t \mapsto t+1$) and multiply/divide the latter element on the left-hand side by P_t:

$$\sum_{j=0}^{\infty} \frac{q_{t+1}^{(t+1+j)} b_t^{(t+1+j)}}{P_t} + \frac{\sum_{j=0}^{\infty} Q_{t+1}^{(t+1+j)} B_t^{(t+1+j)}}{P_t} \frac{P_t}{P_{t+1}} = \mathbb{E}_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}.$$

We now take first-differences of the expected values of the above relationship to capture surprise revaluations: $\Delta\mathbb{E}_{t+1} \equiv \mathbb{E}_{t+1} - \mathbb{E}_t$, based on objects that are not deterministic in t. This yields the following core equation:

$$\frac{\sum_{t=0}^{\infty} b_t^{(t+j)} \Delta\mathbb{E}_{t+1}\left(q_{t+1}^{(t+j)}\right) + \sum_{j=0}^{\infty} B_t^{(t+1+j)} \Delta\mathbb{E}_{t+1}\left(Q_{t+1}^{(t+1+j)} \frac{P_t}{P_{t+1}}\right)}{P_t} = \Delta\mathbb{E}_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}, \qquad (1)$$

which will be the main relationship pinning down our 'net fiscal shock measure'.

What are the terms including 'differenced expectations', and can we recover them empirically? Consider the first term in the numerator of the fraction:

$$\Delta\mathbb{E}_{t+1}\left(q_{t+1}^{(t+j)}\right) = \underbrace{q_{t+1}^{(t+1+j)}}_{\text{Spot price after innovation}} - \underbrace{\mathbb{E}_t q_{t+1}^{(t+1+j)}}_{t \text{ to } t+1 \text{ forward price before innovation}} := \textit{Forecast Error},$$

i.e., the surprise revaluation can be captured by means of differences between spot and forward prices. Note, however, that forward prices of bonds are hardly observable: while some forward contracts exist on bond ETFs, individual forwards can hardly be found, in particular for inflation-linked bonds with lower market liquidity.

The other innovation term that we must consider proves similarly cumbersome:

$$\Delta\mathbb{E}_{t+1}\left(Q_{t+1}^{(t+1+j)} \frac{P_t}{P_{t+1}}\right) = Q_{t+1}^{(t+j)} \frac{P_t}{P_{t+1}} - \mathbb{E}_t\left(Q_{t+1}^{(t+1+j)} \frac{P_t}{P_{t+1}}\right).$$

The first part is the ex post price of nominal debt adjusted for inflation, which is an easily retrievable object. The second part of the term equals the ex ante joint expectation over the nominal bond return and the inverse of inflation.[9]

In our brief exercise, we aim to simplify the second term by considering sufficiently tight windows around fiscal announcement dates: 2 days before and after a shock, such that $\Delta t = 4$ days. In a span of four days, the price level is approximately unchanged, $\frac{P_t}{P_{t+1}} \approx 1$, thereby restricting our attention to the forecast errors in nominal and real bond prices.

The right-hand side stochastic term, $\Delta\mathbb{E}_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}$, is said to be the 'shock' in the narrow timeframe: we posit this to be some fiscal announcement, making the implicit assumption that

[9] By the Fisher equation, this is the ex ante expected real return on holding nominal debt conditional on the risk premium.

any surprise to fiscal surpluses is fully self-contained and does not have further effects beyond the announced fiscal measure. The fiscal shocks considered here are the narrative shocks provided by Cloyne (2013), which limits the current sample size to the years 2000-2010. The construction of the shock series is modelled after the seminal paper of Romer and Romer (2010), and makes use of official documents released by UK legislators to ensure that only truly 'exogenous' discretionary tax changes are considered.

Data on bond prices and quantities are obtained from the Bank of England, the Treasury, Thompson Reuters, and the UK Government Securities Database maintained by Cairns and Wilkie (2023). The data were cross-checked across sources and corrected whenever errors were encountered, allowing us to avoid losing information on traded government bonds, as we need to paint the full picture of the universe of British sovereign debt.

Our final net shock measure consists then of the difference between the fiscal policy announcement given by Cloyne (2013) and the bond market innovation described above:

$$NetShock_{t+1} = \Delta\mathbb{E}_{t+1}\left[\sum_{j=0}^{\infty}\beta^j s_{t+1+j} - \sum_{t=0}^{\infty} b_t^{(t+j)}\left(q_{t+1}^{(t+j)}\right) - \frac{\sum_{j=0}^{\infty} B_t^{(t+1+j)}\left(Q_{t+1}^{(t+1+j)}\frac{P_t}{P_{t+1}}\right)}{P_t}\right]. \tag{2}$$

In words, this 'net shock measure' is a measure of the 'action' of the discretionary tax change (first term), adjusted for the 'reaction' of sovereign bond markets, i.e., the revaluation of outstanding debt (second term). Should our assumptions behind the derivation of this net shock measure be perfectly correct, we could expect the measure to be 0 at all times: if surpluses change, bond markets react by adjusting the real value of outstanding debt. If, instead, this measure has some variation, and it co-varies with the share of indexed debt outstanding, we may consider this evidence that inflation-indexed debt can be among the principal drivers informing fiscal theory when moving beyond our posited simplifying assumptions in deriving equation (2).[10]

Figure E.1 in the appendix plots the density of this shock measure, showing that it is quite centered around zero, but not exactly symmetric. This may either be related to an issue in the definition of the shock series, problems with the assumption of the expectations hypothesis being a reasonable approximation to bond market behaviour in such narrow timeframes,[11] or it may be related to market incompleteness and possible differences in insurance premia between the two types of bonds.

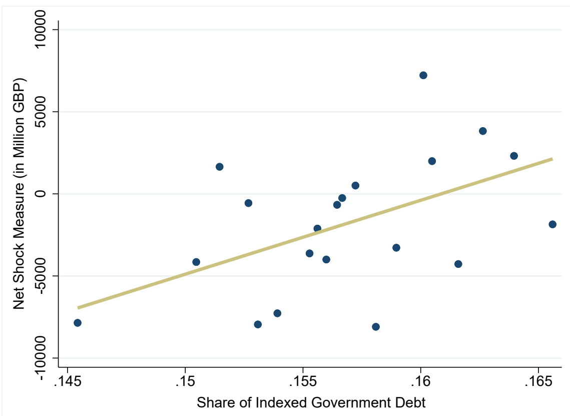
We will now argue that the last case might bear some relevance with the help of the following simple estimation exercise:

[10] A more in-depth analysis of sovereign income and spending, including a detailed analysis of corresponding bond revaluations and yields borne on sovereign debt, is provided by Chen et al. (2022).

[11] The expectations hypothesis was used to derive the otherwise unobservable forward prices of all traded bonds.

$$NetShock_t^s = \alpha + \beta \frac{\sum_{j=0}^{\infty} B_t^{*(t+j)}}{\sum_{j=0}^{\infty} b_t^{(t+j)} + B_t^{(t+j)}} + \Gamma_s X_s + \varepsilon_t^s, \tag{3}$$

i.e., we project the net shock measure on the share of inflation-indexed debt and a set of controls, which can include a year fixed effect to account for long-run changes in spending behaviour as well as a recession indicator. The results of this exercise are given by the following figure 3 and the corresponding regression table.



	Dependent variable: Shock Measure	
	(1)	(2)
Share of indexed debt	450.134***	445.734***
	(154.966)	(153.894)
Recession indicator	.	1.588
	.	(3.729)
Constant	−61.745***	−61.88***
	(20.662)	(20.796)
Year-FE	Yes	Yes
Observations	88	88
R^2	0.2907	0.2928
Residual Std. Error	7.936	7.979
Note:		*p<0.1; **p<0.05; ***p<0.01

Figure 3 & Table 1: OLS results for the relationship between the share of indexed debt and the new net shock measure in the United Kingdom, 2000-2010. The figure plots the results for our preferred specification (2). Standard errors are robust to heteroskedasticity.

Despite the low variation in the share of indexed government debt in the United Kingdom from 2000 to 2010, the results showcase a possible correlation between the share of indexed debt and the unexplained variation in UK bond market valuations of government debt. As mentioned above, those results must be taken with caution, but this provides a stepping stone informing our analysis of the relevance of inflation-indexed debt in relation to surprise fiscal shocks, and how the two jointly influence real and nominal economic outcomes.

2.2 Evidence on the inflationary effect of inflation-indexed debt

Having established that fiscal spending surprises alter the composition of the government budget constraint beyond what is implied by said constraint itself in a frictionless world, we are now providing direct evidence on the effect that inflation-indexed debt can have on inflation, making use of the series of narratively identified tax shocks in the UK provided by Mierzwa (2024).

We leverage his time series of exogenous fiscal policy surprises, and combine it with our novel long-running series of inflation-indexed debt, taking the share of inflation-indexed debt of the overall sovereign debt portfolio as our main indicator for the intensity of the prevalence of inflation-indexed debt. Equipped with these time series, we estimate the following local projection (Jordà, 2005) to measure the dynamic impact of inflation-indexed debt on changes in the rate of inflation:

$$\log P_{t+h} - \log P_{t-1} = \alpha_h + \beta_h \Delta\omega_t \varepsilon_t^F + \delta_{1h}\Delta\omega_t + \delta_{2h}\varepsilon_t^F + \Gamma_h Z_{t-1} + e_{t+h}, \tag{4}$$

where $h \geq 0$ indexes the forecast horizon considered and Z_{t-1} is a vector of control variables specified below. Of particular interest to us is the coefficient β_h, which captures the cross-effect of the identified fiscal shock ε_t^F and the growth in the share of inflation-indexed debt $\Delta\omega_t$ present in the economy at time t.[12] Of particular interest to us is the coefficient β_h, which captures the cross-effect of the identified fiscal shock ε_t^F and the share of inflation-indexed debt ω_t present in the economy at time t. In our estimation, the utilize the entire sample provided by Mierzwa (2024), i.e., from 1970 Q1 until 2019 Q2.

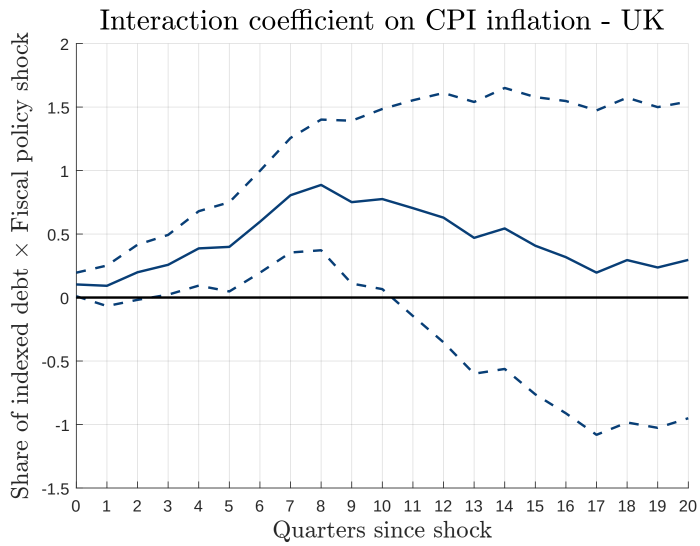


Figure 4: IRF implied by the local projection (4) through the coefficients β_h. The control vector Z contains the first four lags of the real GDP growth rate, the short-run UK bank rate, the change in the weighted real exchange rate, a same-period recession indicator, year-fixed effects, and the first lag of the price level difference. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction). Confidence intervals are provided at the 90% level. Sample length: 1970 Q1 - 2019 Q2.

Figure 4 depicts the impulse-responses from our preferred local projection specification. Crucially, we can observe a positive interaction effect between the share of inflation-indexed debt present in the economy and the fiscal policy shock, directly after the fiscal shock occurs. In economic terms, the coefficients imply that a 1% increase in the combined measure of the change of the share of inflation-indexed debt and the narratively identified fiscal shock (measured as a percentage of GDP) itself leads to an increase of the price level of almost 1% in the two years after the shock.[13]

Equipped with this evidence on the effects of inflation-indexed debt on both the revaluation of sovereign debt and inflation, we now introduce inflation-indexed debt in simplified economic frameworks to lay out the mechanisms under which such debt operates in the FTPL.

[12] We work with the growth rate of the share of inflation-indexed debt in the total debt portfolio to capture the effect of the joint variation in the indexed debt share and the fiscal spending behavior, postulating that previous levels of inflation-indexed debt are already accounted for in the government budget valuation equation prior to the shock occurring. Econometrically, we therefore simply follow Cloyne et al. (2023).

[13] Further details related to this analysis as well as an application to US data are provided in appendix D.

3 Intuition from a simplified model

Introducing inflation-indexed debt in the FTPL

We first explicitly introduce inflation-indexed debt in a barebones version of the FTPL, which will be effectively in partial equilibrium. We derive the novel result that the price level itself becomes a *state variable* in the intertemporal government budget equilibrium, i.e., today's price level becomes a function of the past price level. This is despite the lack of other inertia, and it gives rise to a double role of the price level as a state variable and a market-clearing variable.

We begin by deriving the intertemporal government budget equilibrium with indexed debt, starting off with the case of 'fair' bond pricing, i.e., abstracting from insurance premia on either type of debt. The per-period government budget constraint in a world with indexed debt is given by

$$B_{t-1} + \frac{P_t}{P_{t-1}} b_{t-1} = P_t s_t + Q_t B_t + q_t b_t,$$

where notation follows the previous section, i.e., B_t denotes the terminal value of non-indexed government debt issued at time t at a price Q_t, lowercase letters correspond to the values for inflation-indexed debt, s_t are net real surpluses raised, and P_t denotes the price level. The cost of maturing inflation-indexed debt b_{t-1} is scaled by the gross inflation rate, P_t/P_{t-1}.[14]

To remain closely aligned with canonical simplified models of the FTPL (Hagedorn, 2021; Cochrane, 2023), we let $Q_t = \frac{1}{1+i_t}$ and $q_t = \frac{1}{1+r_t}$, i.e., the price of bonds equals the inverse of their respective relevant gross interest rate. By using the real interest rate to determine the price of inflation-indexed debt, we factually take into account expectations on the payment being indexed to the ex-post inflation rate.[15] Iterating this equation forwards after dividing both sides by P_t and making use of the Fisher equation, we find the following relationship:

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \sum_{j=0}^{\infty} \prod_{l=1}^{j} \frac{1}{1+r_{t+l}} s_{t+j}. \tag{5}$$

[14] The last point about scaling the cost (or, equivalently, the payment) of maturing indexed debt by the gross inflation rate, P_t/P_{t-1}, deserves special highlighting, as this deviates from the structure used, e.g., in Cochrane (2023), chapter 8. This partial equilibrium model as well as all further models in the paper consider the cashless limit of the economy. Such a setup bears the question as to what the price level measures. This, in turn, is a question of the unit of account of the cashless limit of the economy. Since nominal bonds are paying out some value independent of the price level at all times, we can consider these nominal bonds to be the unit of account. The price level thus measures the price of the production good in units of non-indexed bonds. What an indexed bond therefore might be can be split up into two lines of thinking. The first way, not employed here (but in a battery of other papers), is that an indexed bond's payment must be such that the unit of account does not matter, i.e., the current price level must be tied to the current payout of the bond to get rid of this unit of account. This would lead to an indexed bond payment of $P_{t+1} b_t$. Instead, we posit that if P_t is truly the price of output in terms of the non-indexed bonds, then all that an indexed bond does is to ensure that by investing into such a bond today, its payoff tomorrow will allow its holder to purchase the number of consumption goods 'as if' she were to pay today's price for consumption goods (despite being in the next period). This is achieved by adjusting this bond payout by the ratio of the price levels. Thus, the payout would be $(P_{t+1}/P_t) b_t$, or, equivalently, $\Pi_t b_t$. See Schmid et al. (2024) for a recent example using a similar definition of non-indexed and indexed debt.

[15] Possible insurance premia on inflation-indexed debt will be introduced later in General Equilibrium.

This is the simple intertemporal budget equilibrium with indexed debt, but without accounting for the differences in the insurance properties borne by the two types of debt, which allowed us to make use of the simplified bond pricing kernels Q_t and q_t as defined in the last paragraph.[16] Indeed, *the price level itself becomes a state variable*: the real value of maturing inflation-indexed bonds depends on the past price level, not on today's price level. Intuitively, the real value of inflation-indexed bonds depends on the past price level, because the face value payment of that bond is unity at *at yesterday's prices*. The term in orange is the novel addition relative to canonical models of the FTPL and will be the centerpiece of this paper.

Sample IRFs in partial equilibrium with finite horizon

We now briefly explore the properties of this intertemporal budget equilibrium relationship using impulse-responses to the price level under various levels of indexed debt under inflation indexation (as derived above in equation (5)). The goal is to explore how indexed debt changes the implications of the FTPL in relation to surplus shortages in the clearest possible way.

We set up the model (in terms of outstanding bonds and expected surpluses), such that $P_{-1} = 1$. The initial state is therefore the one in which the PDV of surpluses is equal to the real value of the stock of debt in each period. The economy has a finite horizon of 11 periods $t \in \{-1, 0, 1, ..., 9\}$, such that all debt has to be repaid by the government in period 9 by appropriate surpluses. This setup ensures a price level of $P_t = 1 \; \forall t$ in the absence of any shocks. The impulse to the system is a one-period decrease of surpluses by 10% in period 0, announced at the same time. After the shock period, the PDV of surpluses will therefore return to its pre-shock value.

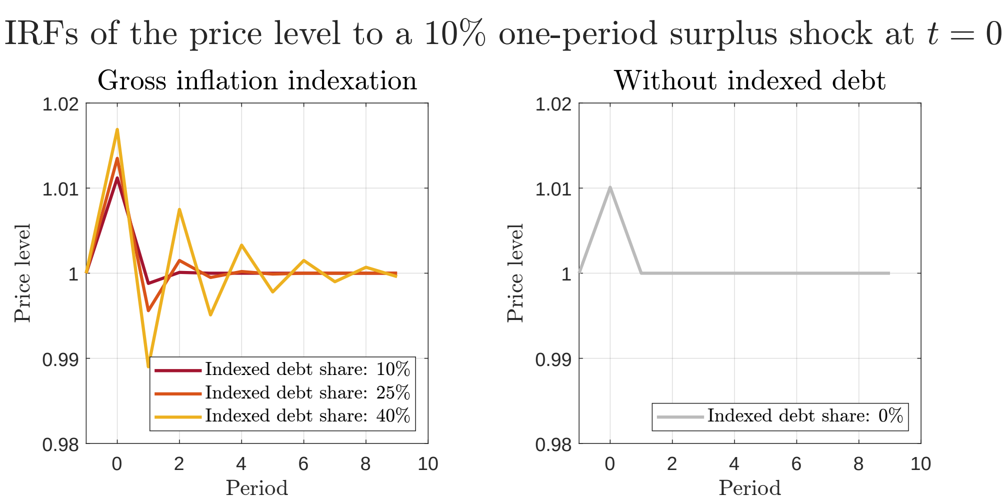


Figure 5: IRFs to a 10% decrease in one-period surpluses in $t = 0$ conditional on the share of indexed debt.

Figure 5 highlights the reaction of the price level in response to a decrease in surpluses in period

[16]Effectively, due to the Fisher equation, the above bond pricing kernels impose the absence of any insurance premia or other valuation wedges, allowing both types of bonds to yield exactly the same realized returns. Section 4 discusses in more detail bond pricing kernels *without* this simplification.

0, announced in the same period. The right-side panel illustrates the standard FTPL response in a world without inflation-indexed debt. In period 0, the decrease in real surpluses induces a temporary upwards adjustment of the price level proportional to the decrease in surpluses, which returns back to its initial state subsequently, since the PDV of surpluses is equal to the pre-shocked value from period 1 onwards.

However, when we have a strictly positive share of inflation-indexed debt, the impact response is already exacerbated: given that the initial price level P_{-1} is fixed in the moment of the policy announcement at time 0, it is not possible to devalue the stock of inflation-indexed debt when the shock occurs. Therefore, the devaluation of the remaining (non-indexed) stock of bonds must be *larger* relative to the case without inflation-indexed debt: the price level must go up by a larger amount in the shock period when we have inflation-indexed debt.

The periods following the shock yield further exciting dynamics that are not observed under a 'standard' FTPL model as in the right panel. Instead of returning to the pre-shock value once the shock vanishes, we can observe observe oscillating behavior of the price level when indexed debt is present in the economy. Since from $t = 1$ onwards the PDV of surpluses returns to its pre-shock level, we are in a situation in $t = 1$ in which the stock of debt is suddenly worth *too little*: inflation-indexed debt is not worth much due to the high price level at $t = 0$, which is the correct factor to adjust such debt to 'real' terms in period 1. But since the funding shortfall is now gone, this implies that the real value of non-indexed debt (B_1/P_1) must actually *increase* to make up the 'under-valuation' of indexed debt: therefore, P_1 must *decrease* (increasing the real value of non-indexed debt) to let the government budget equilibrium hold. In the subsequent period, the price level from the previous period is now *too low*, increasing the value of indexed debt and pushing down the real value of non-indexed debt through a higher price level. This mechanism repeats itself until convergence to the initial equilibrium.[17]

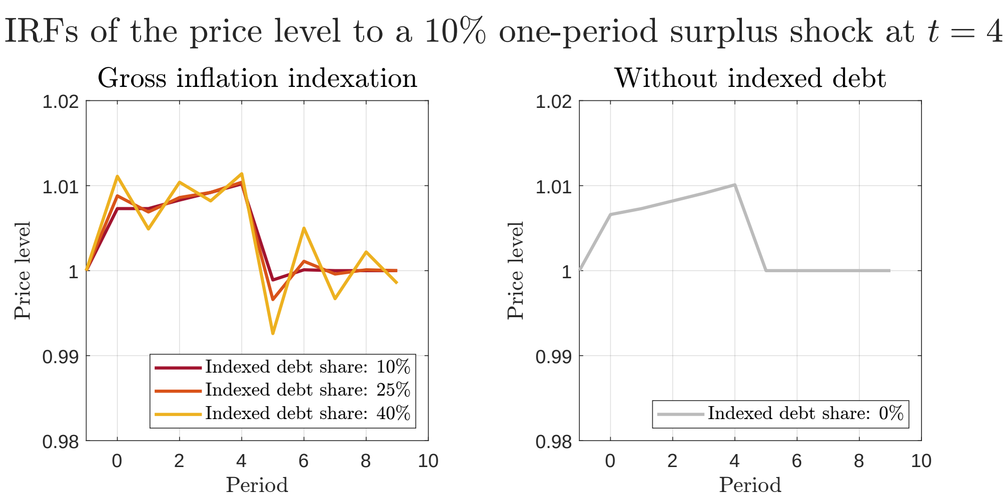


Figure 6: IRFs to a 10% decrease in one-period surpluses in $t = 4$ conditional on the share of indexed debt.

[17] Cochrane (2001) explores a similar result in figure 4 of his paper, driven by a non-geometric maturity structure and the presence of long-term debt.

Figure 6 repeats the above exercise for a similar decrease of surpluses at a later time (in period 4), announced in period 0. Due to the early announcement, the PDV of surpluses already decreases in period 0, remaining below its initial value until period 4, inclusive. The oscillations induced by inflation-indexed debt decrease in size until period 4 (after being larger immediately following the announcement in period 0), and subsequently pick up from period 4 onwards in line with the mechanism described above. The fact that the oscillations are decreasing in magnitude leading up to the shock is caused by the PDV of surpluses *not* being constant between periods 0 and 4 in this example: the closer we get to period 4, the more the PDV of surpluses actually decreases, because we get closer to the period with the smaller surpluses and thereby discount that period less and less. This buffers the price level oscillations on our way to the period of the shock.

A further exposition of the importance of inflation-indexed debt within the context of a simple macroeconomic model (akin to Leeper (1991)), including a brief discussion of the determinacy properties, can be found in appendix C.

4 The FTPL with heterogeneous households and indexed debt

Having studied the relevance of indexed debt in simplified models, we now introduce inflation-indexed debt together with the FTPL in a state-of-the-art macroeconomic model. Given that inflation-indexed debt delivers desirable insurance features to households by providing an income smoothing source that yields a constant value in real terms, the chosen model must necessarily bear relevance to imperfect consumption smoothing, borrowing constraints, and market imperfections precluding perfect risk-sharing across households. Consequently, we choose to work with a heterogeneous-agent model in the spirit of Kaplan et al. (2018), utilizing the efficient algorithms for solving the model provided by Auclert et al. (2021) and paying close attention to limitations of determinacy in incomplete-market models as exposed by Brunnermeier et al. (2024).[18]

Households: We index heterogeneous households by i. Such households choose consumption, c_{it}, labor supply, N_{it}, and asset holdings B_{it} and b_{it} to maximize their cumulative discounted utility

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left(u(c_{it})-v(N_{it})\right)\right]$$

subject to two budget constraints - one for the aggregate household budget, and one for the semantically separate evolution of indexed debt:

$$P_t c_{it} + Q_t B_{it} = \frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta} di}(1-\tau_{it})W_t N_{it} + B_{i,t-1} - d_{it}\mathbb{1}_{\{adj_{it}=1\}},$$

$$q_t b_{it} = \Pi_t b_{i,t-1} + d_{it}\mathbb{1}_{\{adj_{it}=1\}},$$

[18] Auclert et al. (2024) furthermore provide analytical conditions for determinacy in economies with many bond types and bonds-in-the-utility function. We sidestep such an approach for now, recognizing its importance for future research.

where Q_t and q_t are the nominal prices for non-indexed and indexed debt, respectively, whose holdings are denoted by B_{it} and b_{it}. $W_{it} \equiv w_{it} P_t$ denotes the nominal wage level, adjusted by hours worked N_{it} and scaled by the idiosyncratic productivity disturbance $\frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta} di}$ and taxes τ_{it}. d_{it} captures idiosyncratic transfers from non-indexed bond holdings to indexed bond holdings, which are only allowed to happen when the exogenous portfolio rebalancing variable adj_{it} is equal to 1.[19] Finally, households are also subject to standard borrowing constraints

$$B_{it} \geq -\underline{B}, \qquad b_{it} \geq -\underline{b}.$$

Effectively, we posit that consumption is only possible directly from the non-indexed savings portfolio, i.e., we postulate that indexed debt cannot be transformed to consumption as easily as non-indexed debt. This assumption reflects the significantly smaller liquidity of inflation-indexed bond markets, even relative to their market size (Andreasen et al., 2021; Fleming and Krishnan, 2012) and is required for the ex-ante expected yields of both types of debt to be different. Without any adjustment friction, expected yields would equalize and there would be no incentive to hold both types of debt through a no-arbitrage argument.

To solve the household block, the crucial determinant will be whether a household will be able to adjust its holdings of indexed debt in a given period ($adj_{it} = 1$) or not ($adj_{it} = 0$). In the following, let $\varepsilon_i \equiv \frac{e_i^{1-\theta}}{\int e_i^{1-\theta} di}$ be a simplified descriptor of the Markov chain pinning down idiosyncratic productivity. We now define corresponding value functions for households, noting that the state variables are therefore the household-specific past asset holdings (B_-, b_-), the Markov chain realization ε_i, and the adjustment state adj_i. We drop the subscript i in the following for notational simplicity. We then find the following value functions:

- adjuster, $adj = 1$:

$$V_t(1, \varepsilon; B_-, b_-) = \max_{c,B,b,N} u(c) - v(N) + \beta \mathbb{E}\left[V_{t+1}(adj', \varepsilon', B, b) | \varepsilon\right] \tag{6}$$

  subject to the budget constraint and the borrowing constraints:

$$\begin{aligned} Pc + QB + qb = \varepsilon(1-\tau)WN + B_- + \Pi b_, \\ B \geq -\underline{B}; \quad b \geq -\underline{b}, \end{aligned}$$

  where adj' is i.i.d., with probability $\mathbb{P}(adj' = 1) = \nu$.

- non-adjuster, $adj = 0$: Here, b does not enter the decision set and is taken to be a state operating in the background, with the next-period income from non-indexed debt being automat-

[19]Such Calvo-type sticky debt arrangements have been present in macroeconomic models for a long time, see, e.g., Graham and Wright (2007), and have prominently been used in heterogeneous-agent models by Auclert et al. (2024) and Bayer et al. (2024).

ically adjusted based off previously held indexed debt.

$$V_t(0, \varepsilon, B_-, b_-) = \max_{c,B,N} u(c) - v(N) + \beta \mathbb{E}\left[V_{t+1}\left(adj', \varepsilon', B, \frac{\Pi}{q} b_- | \varepsilon \right) \right] \tag{7}$$

subject to the budget and borrowing constraints:

$$\begin{aligned} Pc + QB &= \varepsilon(1-\tau)WN + B_-, \\ B &\geq -\underline{B}. \end{aligned}$$

The goal is to recover policy functions $c(\cdot), B(\cdot), b(\cdot)$, and $N(\cdot)$ that solve the household problem in both instances. The above problem generally yields Bellman-Lagrange functions depending on the adjustment type that an agent enjoys in a given period. Denote by λ_{it}, μ_{it}^B, and μ_{it}^b the respective state-dependent constraint multipliers. For the adjusters, $adj_{it} = 1$, the relevant first-order conditions from that household problem are given by

$$\begin{aligned} \{c\} :& \qquad u'(c) = P\lambda_{it} \\ \{N\} :& \qquad v'(N) = \lambda_{it}\varepsilon(1-\tau)wP \\ \{B\} :& \qquad Q\lambda_{it} = \beta\mathbb{E}\left[V_{B,i,t+1}\right] + \mu_{it}^B \\ \{b\} :& \qquad q\lambda_{it} = \beta\mathbb{E}\left[V_{b,i,t+1}\right] + \mu_{it}^b, \end{aligned}$$

while the envelope conditions, using $\lambda_{it} = \frac{u'(c)}{P}$ from the first-order condition on c, are given by:

$$\begin{aligned} V_{B,i,t} &= \frac{u'(c)}{P}, \\ V_{b,i,t} &= \begin{cases} \frac{u'(c)}{P}\Pi = \frac{u'(c)}{P_-} & \text{if } adj_{it} = 1 \\ \beta\frac{\Pi}{q}\mathbb{E}\left[V_{b,i,t+1}\right] & \text{if } adj_{it} = 0. \end{cases} \end{aligned}$$

The conditions for equilibrium jointly imply the following Euler equations:

$$\begin{aligned} \frac{Q}{P}u'(c) &\geq \beta\mathbb{E}\left[V_{B,i,t+1}\right], \\ \frac{q}{P}u'(c) &\geq \beta\mathbb{E}\left[V_{b,i,t+1}\right], \\ v'(N) &= u'(c)\varepsilon(1-\tau)w, \end{aligned}$$

where the inequalities are strict if the respective asset holdings are at their respective lower bound.

This household block also clearly defines pricing kernels for the bonds that are on offer by the government, conditional on the households pricing the bonds being unconstrained. For non-indexed debt, the first-order conditions for households on the Euler equation imply that

$$Q_t = \beta \mathbb{E}_t \left[\frac{u'(c_{i,t+1})}{u'(c_{it})} \frac{P_t}{P_{t+1}} \right] := \mathbb{E}_t \left[\mathcal{M}_{i,t,t+1} \right], \tag{8}$$

where $\mathcal{M}$ denotes the household-specific stochastic discount factor (SDF). For indexed bonds, applying the definition of the SDF,

$$q_t = \beta \mathbb{E}_t \left[\frac{u'(c_{i,t+1})}{u'(c_{it})} \right] = \mathbb{E}_t \left[\mathcal{M}_{i,t,t+1} \Pi_{t+1} \right]. \tag{9}$$

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Firms and production

To focus on the effects of indexed debt and its interaction with households facing uninsurable idiosyncratic income risk, we model the production block in a parsimonious yet tractable way, following Auclert et al. (2024). In particular, we require in the following that the aggregate effects of idiosyncratic productivity risk are 'small' for the production firms relative to the aggregate effects of aggregate risks.[20]

There exists a continuum of monopolistically competitive firms k that produce goods of variety k, which make each use of a linear production function $Y_{kt} = A_{kt} N_{kt}$. A_{kt} evolves according to an AR(1) process in logs,

$$\log A_{kt} = \rho_a \log A_{k,t-1} + \sigma_\epsilon \epsilon_{kt},$$

where we note that $0 \leq \rho_a \leq 1$. The firm profit function is standard and defined as

$$D_{kt} = \frac{P_{kt}}{P_t} Y_{kt} - \frac{W_t}{P_t} N_{kt} = \left(\frac{P_{kt}}{P_t} - \frac{W_t}{P_t} \frac{1}{A_{kt}} \right) A_{kt}^{1-\zeta} \left(\frac{P_{kt}}{P_t} \right)^{-\zeta} Y_t.$$

Following Auclert et al. (2024), a log-linearized approximation to the solution of the profit-maximization problem of monopolistically competitive firm yields a Phillips Curve of the form:

$$\hat{\pi}_t = (\varphi + \sigma) \kappa \sum_{l=0}^{\infty} \beta^l \hat{y}_{t+l} \tag{10}$$

where $(\varphi + \sigma)$ is the sum of the Frisch elasticity of labour supply and the inverse of the elasticity of intertemporal substitution, as in standard New Keynesian models.

—

Fiscal policy: We next move on to deriving the intertemporal government budget equilibrium in this economy. Recall from the introduction that the FTPL is fundamentally a criterion related to the transversality condition on government debt, since the FTPL equation merely implies a relation-

[20]See proposition 4 of Auclert et al. (2024) for a detailed exposition of this point.

ship on government debt under which the transversality condition on government debt can only hold for one price level in a given period (see Cochrane (2023) and Hagedorn (2024) for a more detailed discussion of this point). However, as pointed out by Brunnermeier et al. (2024), individual transversality conditions on household asset holdings do *not* imply that a similar transversality condition holds necessarily for aggregate debt stocks under incomplete markets. Therefore, with incomplete markets and endogenous real interest rates, the FTPL may ultimately fail to deliver a unique price level based off the aggregate 'transversality condition' on government debt, since there is no guarantee that such a condition holds in aggregate when markets are incomplete.

To illustrate this point, we first naively start from the government budget constraint, aiming to derive an integrated version of it in the hopes of finding a unique debt valuation equation.

$$B_{t-1} + \Pi_t b_{t-1} = P_t s_t + Q_t B_t + q_t b_t$$

is the standard government budget constraint, given some surplus schedule s_t and bond pricing kernels Q_t, q_t. We multiply all elements by the unweighted average household SDF $\mathcal{M}_{t,t+1}$ and divide all elements by the current price level P_t to obtain

$$\mathcal{M}_{t,t+1}\frac{B_{t-1}}{P_t} + \mathcal{M}_{t,t+1}\frac{b_{t-1}}{P_{t-1}} = \mathcal{M}_{t,t+1}s_t + Q_t\mathcal{M}_{t,t+1}\Pi_{t+1}\frac{B_t}{P_{t+1}} + q_t\mathcal{M}_{t,t+1}\frac{b_t}{P_t}.$$

Adding and subtracting elements suitably on the right-hand side, we re-express this equation as:

$$\begin{aligned}\mathcal{M}_{t,t+1}\frac{B_{t-1}}{P_t} + \mathcal{M}_{t,t+1}\frac{b_{t-1}}{P_{t-1}} &= \mathcal{M}_{t,t+1}s_t + \left(Q_t\mathcal{M}_{t,t+1}\Pi_{t+1} - \mathcal{M}_{t+1,t+2}\right)\frac{B_t}{P_{t+1}} \\ &\quad + \left(q_t\mathcal{M}_{t,t+1} - \mathcal{M}_{t+1,t+2}\right)\frac{b_t}{P_t} + \mathcal{M}_{t+1,t+2}\left(\frac{B_t}{P_{t+1}} + \frac{b_t}{P_t}\right).\end{aligned}$$

Iterating on this expression until T, dividing the resulting expression by the SDF, and taking limits $T \to \infty$, we end up finding:

$$\begin{aligned}\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t\Bigg[\sum_{l=0}^{\infty}\frac{\mathcal{M}_{t+l,t+l+1}}{\mathcal{M}_{t,t+1}}s_{t+l} + \frac{Q_{t+l}\mathcal{M}_{t+l,t+l+1}\Pi_{t+l+1} - \mathcal{M}_{t+l+1,t+l+2}}{\mathcal{M}_{t,t+1}}\frac{B_{t+l}}{P_{t+l+1}} \\ + \frac{q_{t+l}\mathcal{M}_{t+l,t+l+1} - \mathcal{M}_{t+l+1,t+l+2}}{\mathcal{M}_{t,t+1}}\frac{b_{t+l}}{P_{t+l}}\Bigg] + \lim_{T\to\infty}\frac{\mathcal{M}_{T+1,T+2}}{\mathcal{M}_{t,t+1}}\left(\frac{B_T}{P_{T+1}} + \frac{b_T}{P_T}\right).\end{aligned} \tag{11}$$

Note that this expression nests the standard FTPL case under complete markets, since in this case $Q_t\mathcal{M}_{t,t+1}\Pi_{t+1} = \mathcal{M}_{t+1,t+2}$ and $q_t\mathcal{M}_{t,t+1} = \mathcal{M}_{t+1,t+2}$.

Seeing this integrated government budget constraint, one could mistakenly believe that the current price level is determined by this equation, conditional on the previous price level P_{t-1}. This logic requires the last limiting term to vanish and go to zero. However, this is *not* necessarily the case: even though the transversality condition holds on the household level as a consequence of household optimality and a no-Ponzi condition, it *cannot* be aggregated to derive a concurrent aggregate transversality condition directly off-the-shelf: the reason for that is that the unweighted average SDF $\mathcal{M}_{t,t+1}$ is discarding the heterogeneity of underlying consumption (which led to the rise of household-specific discount factors), and thus ignores the possibility of the government possibly earning an excess return on its debt issuance. This can be considered a 'safe asset premium' (Brunnermeier et al., 2024) and is reflective of the inherent value that such debt bears to households in partially overcoming the market incompleteness, possibly yielding different 'fundamental' valuations of government debt by the household vis-à-vis the government.

Instead, we can follow the approach undertaken in Brunnermeier et al. (2024), which is dubbed the *dynamic trading perspective*, and aggregate household unit-level budget constraints to obtain a dynamic aggregate constraint on sovereign debt, which factually is a mirror image of the usual FTPL equation. Accounting for the benefits of the two debt products in partially overcoming market incompleteness borne by households, and thereby being able to leverage household-level transversality conditions, we find that we can still express the intertemporal budget equilibrium in terms of the real value of today's debt holdings and a suitably-discounted surplus term:

Proposition 1 *In a model with both non-indexed and inflation-indexed debt and incomplete markets, the integrated government budget constraint ('the FTPL equation') can be expressed as:*

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t\left[\sum_{k=0}^{\infty} \tilde{\mathcal{M}}_{t,t+k}\bar{A}_{t+k}\right], \tag{12}$$

where $\tilde{\mathcal{M}}_{t,t+k} = \sum_i \mathcal{M}_{i,t,t+k}\Pi_{t,t+k+1}\frac{A_{i,t+k}}{\bar{A}_{t+k}}$ *is the weighted average SDF across all households* i*, adjusted for inflation, with weights being proportionate to* $A_{i,t+k}$*.* $\bar{A}_t = \frac{1}{N_i}\sum_i A_{it}$ *is the average of the term* A_{it}*, which captures the surpluses raised by the government from each household i and the utility-weighted windfall gain that households enjoy when holding inflation-indexed debt:*

$$A_{it} \equiv c_{it} - \varepsilon_{it}(1-\tau_{it})w_t N_t + \left[Cov_t\left(\mathcal{M}_{i,t,t+1}, \Pi_{t+1}\right) + \mathcal{M}_{i,t,t+1}\left(\mathbb{E}_t\Pi_{t+1} - \Pi_{t+1}\right)\right]\frac{b_{it}}{P_t}.$$

Proof. See appendix A.1. ■

A_{it} therefore also expresses the full portfolio return of household i of holding an additional unit of net worth, consisting of the net utility gain from saving, the insurance premium on indexed debt (captured through the covariance term), and the possible windfall gain/loss from surprise inflation through indexed debt (captured through the last term). Equation (12) is 'the FTPL equation' that is used to pin down the price level at time t, given some previous price level P_{t-1}.

Maintaining this equation as determining the price level, we close the government block by assuming a simple taxation rule as in standard Fisherian models (see, e.g., section C in the appendix),

$$\frac{\tau_t}{\tau} = \left(\frac{s_{B,t-1}}{s_B}\right)^{\gamma_B} \left(\frac{s_{b,t-1}}{s_b}\right)^{\gamma_b} e^{\zeta_t}, \tag{13}$$

where $\tau_t \equiv \frac{T_t}{Y_t}$ are surpluses raised by the government as a fraction of output, and $s_{B,t} \equiv \frac{Q_t B_t}{P_t Y_t}$, $s_{b,t} \equiv \frac{q_t b_t}{P_t Y_t}$ are the real market values of the two existing types of debt. ζ_t is a standard AR(1) shock to the quantity of lump-sum taxes raised, and the policy reaction coefficients to deviations of the market values of both types of debt from their steady-state values are given by γ_B and γ_b. Steady-state values are denoted without time subscripts. In log-linearized terms, this relationship is given by:

$$\hat{\tau}_t = \gamma_B \hat{s}_{B,t-1} + \gamma_b \hat{s}_{b,t-1} + \zeta_t. \tag{14}$$

Monetary policy: We allow monetary policy to follow an inertial Taylor rule with positive weights on both inflation and output deviations from steady-state:

$$\left(\frac{R_t^n}{R^n}\right) = \left(\frac{R_{t-1}^n}{R^n}\right)^{\rho_M} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{1-\rho_M} e^{\nu_t} \tag{15}$$

where ν_t is an AR(1) shock to the conduct of monetary policy. In exact log-linearized terms,

$$\hat{r}_t^n = \rho_M \hat{r}_{t-1}^n + (1-\rho_M)\left[\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t\right] + \nu_t. \tag{16}$$

Market clearing: Finally we define market clearing on the three markets of relevance in this economy as follows:

- Goods market: on the goods market, aggregate consumption and production are equalized, taking into account the loss from price adjustment costs on the producer's behalf:

$$C_t + G_t + \frac{\mu/(\mu-1)}{2\kappa}\left(log(1+\pi_t)\right)^2 Y_t = Y_t. \tag{17}$$

- Labor market: labor supply and demand must be equalized:

$$\sum_i N_{it} = \sum_k N_{kt}. \tag{18}$$

- Asset market: for each class of assets, the supply by the government must be equal to cumulative household demand:

$$B_t = \sum_i B_{it} \tag{19a}$$

$$b_t = \sum_i b_{it}. \tag{19b}$$

Equilibrium: We characterize a competitive equilibrium in this economy as follows:

Definition 1 (Competitive Equilibrium) *A* competitive equilibrium *of the heterogeneous-agent economy is an allocation* $\{C_t, N_t, Y_t, B_t, b_t, Y_{it}, N_{it}, d_t, \tau_t\}_{t=0}^{\infty}$*, together with prices* $\{P_t, P_{it}, w_t, \pi_t, Q_t, q_t, R_t^n\}_{t=0}^{\infty}$ *and exogenous variables* $\{\nu_t, Z_t, G_t, \}_{t=0}^{\infty}$*, such that:*

- *all agents maximize their utility with suitable policy functions on* $c(\cdot), N(\cdot), B(\cdot)$*, and* $b(\dot{)}$*, solving the type-dependent value functions (6) or (7),*
- *all firms maximize their PDV of profits,*
- *the government does not violate its per-period budget constraint, levies taxes in accordance with its fiscal rule, and the price level is determined through equation (12),*
- *the central bank follows its policy rule (15),*
- *all markets clear ((18), (19a), (19b), equation (17) follows from Walras' Law), and*
- *the distribution of household wealth and productivity* $\Gamma_t(B, b, z)$ *evolves by its law of motion and is determined in the long-run by the fixed point of its evolution:*

$$\Gamma_{t+1}(\mathcal{B}, \mathscr{b}, z') = \int_{\{(B,b,z):g_t(B,b,z)\in(\mathcal{B},\mathscr{b})\}} Pr(z'|z)d\Gamma_t(\mathcal{B}, \mathscr{b}, z).$$

We close the model by defining the utility function of consumption for each household i as $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, and the disutility function of labor supply as $v(N) = \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$.

Steady-state: in the following, we will consider a log-linearized approximation around the deterministic steady-state with respect to aggregate variables. That steady-state will be characterized by a zero inflation rate, $\Pi = 1$, such that bond prices are equal to the household discount rate, $Q = \beta$ and $q = \beta$ in the absence of uncertainty. We furthermore normalize steady-state output to 1. The remainder of the steady-state will be characterized explicitly in line with the calibration introduced in section 5.

Steady-state determinacy with a simplified real interest rate determination

To provide a brief characterization supporting the possible uniqueness of the steady-state despite the high complexity of the model, we briefly invoke the framework of Hagedorn (2021) with an appropriate adjustment to include inflation-indexed debt, featuring the determination of the real interest rate with the help of asset market clearing.

We provide a general treatment of a possible equilibrium of the non-Ricardian economy with inflation-indexed debt and possibly heterogeneous agents, taking into account the ramifications that bond revaluations can have on asset markets in general equilibrium. Our proposal is that inflation-indexed debt can yield price level uniqueness in a stationary equilibrium if the real interest rate is determined *outside* the FTPL equation (taking off the 'double burden' of determining both the initial price level P_0 and the real interest rate r_{ss} that the FTPL would alternatively be subject to), although some additional restrictions must be made. This statement is formalized in the following:

Proposition 2 (Stationary equilibrium determinacy in the sense of Hagedorn (2021)) *Under incomplete markets, with non-negative steady-state inflation, and abstracting from aggregate uncertainty, the FTPL can determine a unique initial price level in stationary equilibrium even in the presence of inflation-indexed debt and a positive inflation rate if* $\frac{b}{b+B} < 1$*,* $r_{ss} > 0$*, and a steady-state asset demand function* $\mathcal{S}(r_{SS})$ *exists and is invertible.*

Proof. See appendix A.2. ■

Therefore, the FTPL yields a unique initial price level in our setting with inflation-indexed debt, provided that the real interest rate is pinned down outside of the FTPL equation. The present result is, in a sense, a qualification of the results of Hagedorn (2021, 2024), applied suitably to a setting with inflation-indexed debt. This is done to ensure that we can operate with a clear, unique steady-state and analyze shocks to the economy without worrying about stationary equilibrium multiplicity.

The intuition behind the proof is the following: the intertemporal government budget equilibrium without inflation-indexed debt relates the price level to the real interest rate, which is determined on the asset market. With inflation-indexed debt, steady-state inflation itself becomes another element of the intertemporal government budget equilibrium. That inflation rate, which we posit to be pinned down by fiscal policy in the stationary equilibrium, is directly related to the real interest rate through the Fisher equation. Then, with the real interest rate (and implicitly inflation as well) being pinned down by asset market equilibrium, we only receive one plausible real interest rate that allows us to uniquely pin down the price level from the government budget constraint.

Equipped with our results on steady-state uniqueness in abstract general equilibrium models (that feature endogenous real interest rates, but do not take a stance how they arise), we are now moving on to describing the computational approach for our simulations with the help of the full-fledged general equilibrium model.

5 Calibration and computational approach

The calibration of the economy used in the dynamic simulations is summarized by table 2. We follow overall the approach of Auclert et al. (2021), as we apply a conceptionally similar algorithm. In our preferred calibration, we vary government spending G and the household discount factor β to ensure that the goods market and the asset market for non-indexed clear. Finally, the market for inflation-indexed debt is targeted with the help of ν, the probability of being able to access the portfolio of indexed debt actively. These endogenous parameters are summarized by table 3. The market for non-indexed debt is not targeted, but clears with a tolerance of $1e-5$, while targeted market clearing conditions clear with close to machine precision ($1e-15$). To compare various policy combinations, we mostly restrict ourselves to baseline active/passive policy coefficients as given by Bianchi et al. (2023) within their NK-DSGE model. The parameters related to policy coefficients in the table below, $\{\phi_\pi, \phi_y, \gamma_B, \gamma_b\}$, should be taken as indicative and related to suitable active/passive policy combinations in the sense of Leeper (1991). When deviating from the baseline parameterizations mentioned in the table, we will explicitly introduce novel policy coefficients as suitable.

Parameter.	Description	Value	Source/Target
Firms			
Y	Steady-state output	1	Normalization
ε	Elasticity of substitution between product varieties	3	Firm mark-up of 50%
κ	Slope of price Phillips curve	0.05	Conventional estimate
Households			
σ	Inverted intertemporal elasticity of substitution	1	Simplification for simulation
φ	Inverse Frisch elasticity of labor supply	1	Simplification for simulation
$\underline{B}$	Lower bound for non-indexed debt holdings	0	
$\underline{b}$	Lower bound for indexed debt holdings	0	
ρ_z	Persistence of AR(1) shocks to household productivity	0.966	Auclert et al. (2021)
σ_z	Standard deviation of AR(1) shocks to household productivity	0.92	Auclert et al. (2021)
Government			
T/G	Steady-state surplus, measured by the tax-to-government spending ratio	1.025	See explanation below
r^*	Natural rate of interest	0.015	Benigno et al. (2024)
ρ_M	Inertia in Taylor-type interest rate rule	0	Simplification
ϕ_π	Monetary policy reaction to inflation deviations from steady-state	$\{0.5, 1.5\}$	Bianchi et al. (2023)
ϕ_y	Monetary policy reaction to output deviations from steady-state	0	Bianchi et al. (2023)
γ_B	Fiscal policy reaction through non-indexed debt	$\{0.5, 1.5\}$	
γ_b	Fiscal policy reaction through indexed debt	$\{0.5, 1.5\}$	
B	Steady-state holdings of non-indexed debt	0.9	Non-indexed British sovereign debt relative to GDP
b	Steady-state holdings of non-indexed debt	0.22	Indexed British sovereign debt relative to GDP
Simulation			
n_z	Number of points in asset grid for household productivity shock	11	
n_b	Number of points in asset grid for indexed debt	50	
n_B	Number of points in asset grid for non-indexed debt	50	
$\bar{B}$	Maximum holdings of non-indexed debt in asset grid	5000	
$\bar{b}$	Maximum holdings of indexed debt in asset grid	5000	Approximation to Auclert et al. (2024)
T	Number of periods used in simulations of Jacobians	300	Auclert et al. (2021)

Table 2: Baseline parametrization for the quantitative estimation

Debt/GDP shares	HH discount factor	$\mathbb{P}$(adjustment)	Govt. spending
	Main calibration: UK debt portfolio		
B: 0.9, b: 0.22	$\beta = 0.9546$	$\nu = 0.1448$	$G = 0.3705$
	Counterfactual: US debt shares		
B: 1.0171, b: 0.1029	$\beta = 0.9547$	$\nu = 0.1034$	$G = 0.3704$
	Counterfactual: no indexed debt		

B: 1.12, b: 0.	$\beta = 0.9549$	$\nu = 0.0447$	$G = 0.3703$

Table 3: Endogenous parameters across different debt calibration scenarios

The calibration delivers overall reasonable estimates of the endogenous parameters that are in line with the parametrization of Auclert et al. (2021). The level of government spending is not targeted to its empirical counterpart, yet the estimated government spending share is only slightly below the level of government spending in the UK prior to the pandemic (OBR, 2024b).

Finally, note that to pin down both the price level and the tax rate in steady-state, we exogenously fix the tax rate to be 2.5% higher than government spending in GDP, such that surpluses are equal to one percent of the government spending-to-GDP ratio. Note that this assumption runs counter to currently observed budget surpluses in the UK, which are decidedly negative. The proposed model, however, has issues in solving for perpetual deficits, conditional on the long-run real interest rate being positive.[21] However, the assumption of positive surpluses in steady-state remains qualitatively and quantitatively in line with some of the recent long-run forecasts of the current budget deficit provided by OBR (2024a) in their historical official forecasts database (table CB).[22]

The entire steady-state is derived under the assumption of zero steady-state inflation, rendering limited relevance to the role of distorting inflation or interest rates different from the long-run natural rate. In terms of economic aggregates, the steady-state is thus well-described by the above calibration. Thanks to the normalization of output to unity and the calibrated share of government spending of 0.3705, we can deduce that consumption in steady-state will be equal to 0.6295 by market clearing, while taxation will be equal to 0.3815.

In terms of government debt, we will operate with and compare two different steady-state calibrations: one which follows the observed modal split of sovereign debt into non-indexed and indexed debt (such that $B = 0.9$ and $b = 0.22$), and two counterfactual calibrations where we either postulate a split between indexed and non-indexed debt as in the US (i.e., $B = 1.02$ and $b = 0.1$), or the complete absence of any indexed debt (i.e., $B = 1.12$ and $b = 0$), but maintaining in both cases the overall cumulative debt level observed in the UK. We therefore exogenously postulate the same steady-state bond supply across our calibrations, given that bond supply as a share of GDP is a relatively low-frequency variable, and given that the distribution of assets is an equilibrium outcome that depends on this supplied quantity. Many of our exercises will resolve around the differences between these calibrations, as we will mainly focus on the effect that indexed government debt has on economic aggregates.

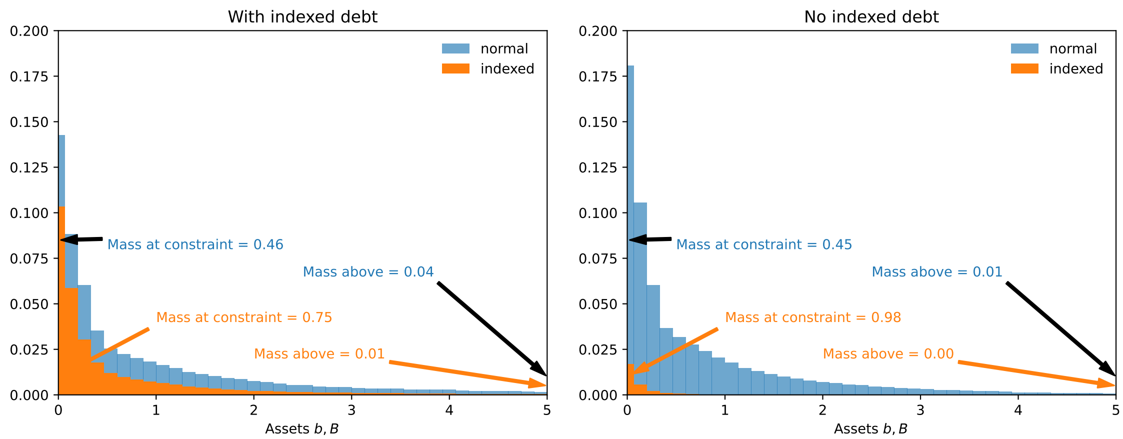
Even though government debt aggregates are exogenously supplied in steady-state for all calibrations, the distribution of debt across households cannot be deduced immediately from the calibration itself, as it is generally dependent on the properties of the idiosyncratic process to income

[21] Kaplan et al. (2023) solve a model with negative surpluses and a negative steady-state real interest rate, but this is computationally difficult to implement for our chosen algorithm.

[22] Conditional on a 40% share of government spending in GDP, the projected 1% budget surplus in the long-run as a share of GDP is equivalent to a ratio of sovereign income to spending of 1.025.

in a way that is not fully captured by the calibration itself. Figure 7 plots the distribution of debt holdings across households in two cases - once for the standard calibration to the UK, and one for the counterfactual calibration where steady-state issuance of indexed government debt is set to 0.[23]

Note that the distribution of debt is by all means not a targeted moment, yet we can find, reassuringly, a significant skew in the distribution of debt holdings across simulated households. Given the larger presence of non-indexed sovereign debt, the holdings thereof are of course larger across the entire distribution, reflecting the evidence for the US provided in figure D.1 in the appendix.



(**a**) Both types of debt (UK calibration)

(**b**) Non-indexed government debt only

Figure 7: Steady-state distributions of asset holdings across households in the calibrated steady-state

Computational details - using the Sequence-Space Jacobian:

The solution to the model that is linear in aggregates, but non-linear in idiosyncratic shocks, is derived by using the Sequence-Space Jacobian method developed in Auclert et al. (2021), which itself constitutes an evolution of the methods pioneered by Reiter (2009). The computational method we employ therefore generates perfect-foresight solutions in aggregates in response to time-zero perturbations of exogenous disturbances, but it maintains the non-linearity underlying the responses of heterogeneous households.

We first solve the heterogeneous household block, taking aggregate prices as given, for both the steady-state policy functions (through backwards iteration) and the steady-state distribution of asset holdings (through forwards iteration). Both solve with a numerical tolerance of up to $1e{-}7$, and are subsequently used to inform other blocks of the model (such as firm optimality, government policies, and market clearing) and to generate updates of aggregates where necessary. The two components (heterogeneous-agent and aggregate) interact and iterate until convergence, which is reached with a numerical threshold of $1e - 6$ in the solution that is linear in aggregates, which is reasonable given the high degree of complexity underlying household behavior in the presence of

[23]Note that the plot for the case without indexed debt shows some marginally non-zero holdings of indexed debt for a very small share of households. These are numerical inconsistencies, but even this counterfactual calibration still solves with a tolerance of $1e - 3$.

two types of assets. The discretization of exogenous disturbances and the asset grid remain in line with the calibration of Auclert et al. (2021).

6 Quantitative insights from the HANK-FTPL model

With the computational algorithm at hand, we solve and estimate the model's aggregate impulse-responses for a number of shocks, using the parametrization from table 2, but varying the calibration of the debt shares in line with table 3. Here, we will mostly focus on the effects of unanticipated disturbances to *government spending* G_t, which directly influence the surpluses raised by the government in any given period.[24] Note that the notions of 'active' and 'passive' fiscal policy in this section are akin to the definitions underlying such policy combinations in Leeper (1991).

Before considering impulse-response functions in detail, we first look at the role that inflation-indexed debt plays for the amplification of shocks when the FTPL is at work as evidenced through simulated moments, in line with the principal focus of the paper. To get a more detailed grasp behind that role borne by the presence of inflation-indexed debt for aggregates, we compare the simulated volatility of a number of macroeconomic aggregates across all calibrations (UK calibration, counterfactual US distribution of debt across the two types, and issuance of non-indexed debt only) and across both 'standard' policy combinations (passive monetary/active fiscal and active monetary/passive fiscal). The results of this exercise are presented in table 4.

	Normalized standard deviations across policy scenarios					
	PM/AF-UK	PM/AF-US split	PM/AF-NoIndex	AM/PF-UK	AM/PF-US split	AM/PF-NoIndex
G	1.527525	1.527525	1.527525	1.527525	1.527525	1.527525
Y	1.51838	1.565195	1.388928	1.291344	1.365381	1.277081
C	0.190974	0.248259	0.157339	0.381226	0.375627	0.440722
π	0.260869	0.236133	0.144924	0.177253	0.17356	0.151822
r	0.189807	0.202811	0.137661	0.237697	0.220391	0.22438
N	1.51838	1.565195	1.388928	1.291344	1.365381	1.277081

Table 4: Normalized standard deviations of aggregate variables in response to fiscal shocks with $\rho_G = 0.5$

The three left-hand columns can be summarized in one line and yield the main quantitative insight of the paper: the volatility of economic aggregates increases in the presence of inflation-indexed debt, conditional on being in the active fiscal policy case. Of particular interest in that regard is the fourth row of table 4, which captures the volatility of inflation in response to government

[24] Appendix B provides an overview of the dynamic responses to expansionary monetary policy shocks.

spending shocks. Here, we can see that, conditional on being in the active fiscal policy scenario, the unweighted volatility of inflation is around 80% higher in the British sovereign debt case relative to the counterfactual without any inflation-indexed debt being present in response to government spending shocks. With a calibrated share of indexed sovereign debt of about 20%, we can therefore say that, on average, a one percentage point increase in the share of inflation-indexed debt more or less corresponds to an approximately 4% increase in the volatility of inflation in response to uncovered government spending shocks. Of course, this effect is far from linear, as evidenced by the second column which shows that the US calibration attains elevated levels of volatility, too, despite the share of indexed debt being only half of the UK share of indexed debt. To the best of our knowledge, we are among the first to quantitatively evaluate the impact that inflation-indexed debt can have on the volatility of inflation, and how such changes in volatility are directly related to the monetary-fiscal policy nexus, as the inflation volatility increase is evidently much smaller under the active monetary/passive fiscal scenario, amounting to a difference of only 17%.

—

We are now ready to look in more detail at the impulse responses to government spending shocks and the role borne by inflation-indexed debt when an unexpected government spending increase occurs. In general, we will allow for different possible autocorrelations of the fiscal shock to highlight the role of persistence and the forward-looking nature of the FTPL as well. We begin by focusing on the case of *active fiscal policy* in line with the first parametrization introduced in table 3, i.e., the baseline calibration to the UK economy. Figure 8 plots IRFs of aggregate variables in response to a 100bp expansionary fiscal shock that increases the need for fiscal spending when the shock is highly persistent, i.e., $\rho_G = 0.8$.

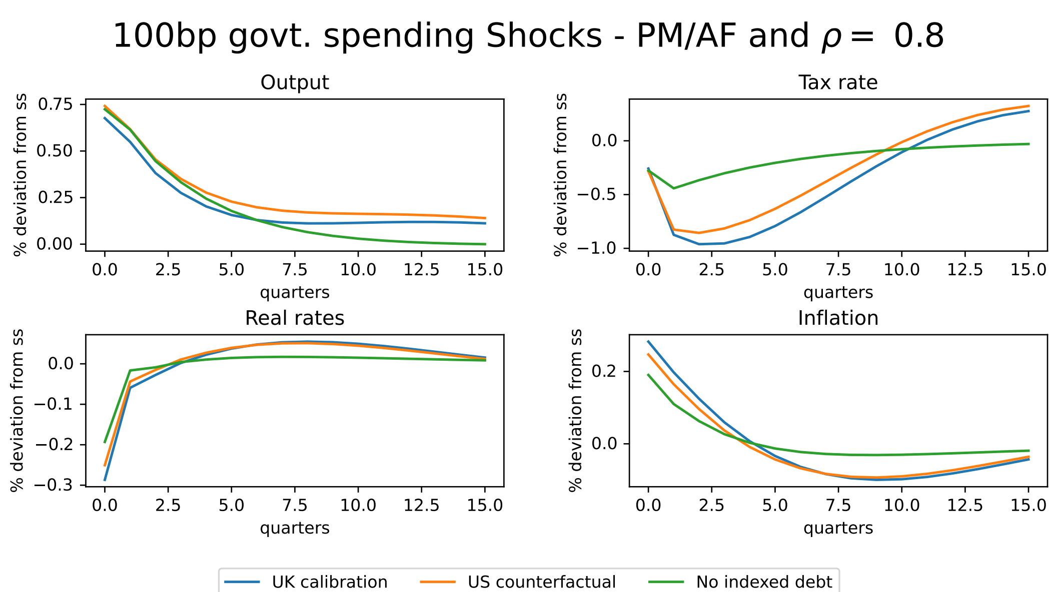


Figure 8: IRFs to the government spending shock with active fiscal policy.

A number of observations is worth highlighting: the responses of consumption and tax rates are in line with canonical macroeconomic models and the expected reactions in response to the fiscal expansion: there is a corresponding increase in output (which partially crowds out consumption as the fiscal shock itself is quite persistent) as the tax adjustment does *not* cover the additional expenses arising from the shock, in line with our specification of active fiscal policy. Notably, the tax rate deviation from steady-state is at first *negative*, highlighting that the real value of government bonds decreases below steady-state (which is related to the large negative real return shock that lowers the prices of bonds).

The response of aggregate inflation (and correspondingly of ex-post real interest rates) in the cases with positive levels of inflation-indexed debt confirms our analysis from the simplified model: while the fiscal expansion is related to inflationary pressure on impact, it translates to deflation in the medium-run before showcasing inflationary patterns in the longer-term. That medium-run slight deflationary pressure is relatively persistent. Across the specifications with positive levels of inflation-indexed debt, we therefore note that observed inflation rates exhibit the 'wave' pattern that we expected to arise through the presence of inflation-indexed debt and the dependence of the current price level on the future price level. Relative to the toy model discussed in section 3, the oscillations are more spread out here due to the relevance of production and household sectors in general equilibrium. In particular, the medium-term lack of price pressure is a direct consequence of the crowding out of household consumption due to the increase in government spending, which translates into temporarily lower inflation as households act under perfect foresight with respect to aggregate variables. Afterwards, as the government spending shock begins to die out, households push their consumption levels up, thereby leading to an increase in inflation rates that is accommodated by active fiscal policy that allows the devaluation of debt across time, with the persistence of this devaluation mechanism increasing in the share of inflation-indexed debt.

Turning off the debt indexation channel of government debt (i.e., setting inflation-indexed debt to zero) nullifies all dynamics beyond the first-order dynamics of the spending shock, allowing us to nest the standard expected reaction to a fiscal expansion ín a world of active fiscal policy with non-Ricardian households: output and inflation comove, increasing above their respective steady-state values. Since we remain in a scenario of active fiscal policy, the government spending increase is not fully covered by a corresponding movement in taxation in that case either. The response of inflation is overall devoid of any persistence beyond the initial inflationary pressure from the shortfall of government income, and the response of the real interest rate is factually not persistent at all, since the real rate is perfectly forecastable after impact and is therefore taken into account by households. The magnitude of the response of tax rates is approximately a third of the response in the case with inflation-indexed debt, reflecting the decrease in the burden on taxation due to the absence of (costly) inflation-indexed debt.

An important factor in our above analysis is the persistence of the government spending shock, ρ_G. As the persistence of the shock underlying figure 8 is relatively high, the observed dynamics are tightly connected to intertemporal substitution motives for the household. To highlight the 'barebones' reaction of our economy to a one-off government spending shock (and to highlight the corresponding relevance of inflation-indexed debt in this world), we consider a non-persistent fiscal shock next.

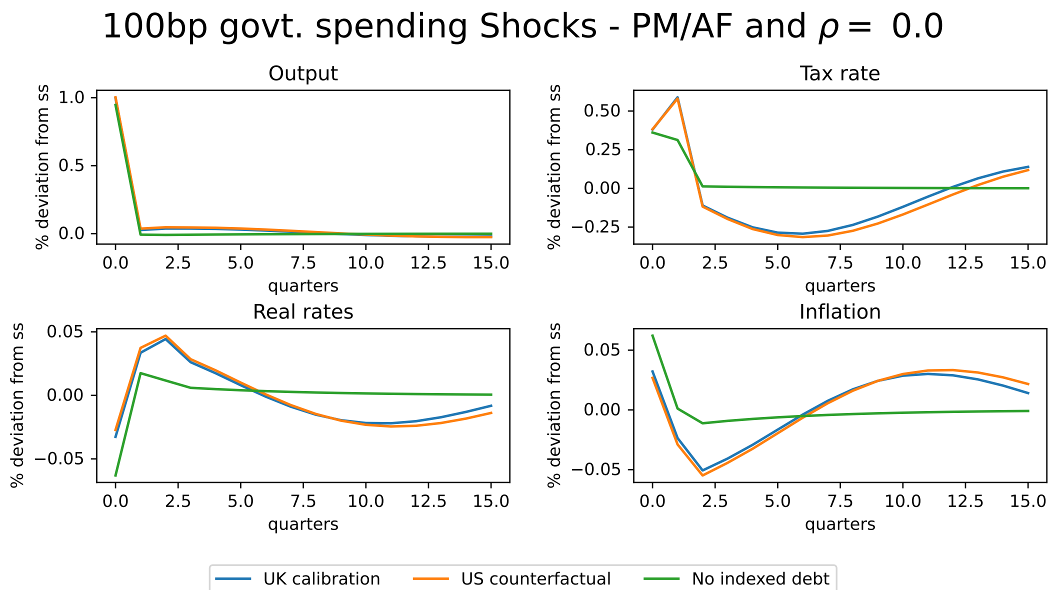


Figure 9: IRFs to the government spending shock with active fiscal policy - counterfactual without indexed debt.

Figure 9 summarizes the aggregate response of the economy to a fiscal spending shock when the government spending shock is not persistent at all, $\rho_G = 0$. Unsurprisingly, the persistence of output is virtually zero, too. As the government spending shock is short-lived, intertemporal substitution motives matter less, leading to less of a crowding out of consumption. Nonetheless, the fiscal authority of course continues to adjust the tax rate, where interesting differences between the cases of no indexed debt and strictly positive levels of indexed debt arise: while the tax rate returns to steady-state after two periods in the case without inflation-indexed debt (as should be expected), the tax rate shoots up more initially for the two scenarios with indexed debt, which is followed by a decrease of the tax rate below its steady-state, in line with the previously mentioned 'overshooting' of the devaluation of inflation-indexed debt. Interestingly, the differences between the UK calibration and the counterfactual with US debt shares are minuscule, reflecting the importance that the persistence of the shock has for the propagation of fiscal funding shortages.

Real rates correspondingly turn negative at the initial shock, followed by a gradual increase and a temporal positive deviation of real rates from their steady-state value. Inflation rates mirror this pattern in line with the Fisher equation (as the monetary authority remains passive), such that there is an uptick in inflation shortly on impact followed by a gradual unwinding relative to the observed steady-state of inflation. The volatility of inflation is generally one order of magnitude smaller relative to the case of persistent spending shocks, as expected.

Finally, we want to highlight what in our simulations changes when monetary policy turns *active*, while fiscal policy turns passive, in line with the calibration from table 2. Figure 10 summarizes the results from this exercise for highly persistent fiscal shocks, $\rho_G = 0.8$.

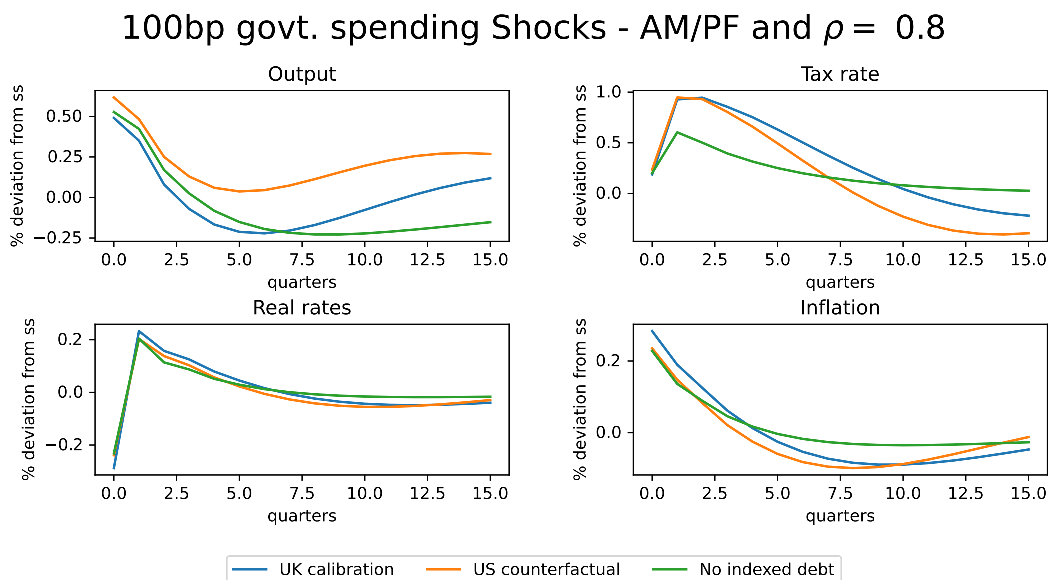


Figure 10: IRFs to the government spending shock with active monetary policy.

The responses of output and inflation turn out to be qualitatively similar *on impact* relative to the passive monetary policy case discussed above, while the reaction of tax rates differs markedly, as tax increases now cover the spending shock. This similarity in output and inflation across the calibrations will inform the discussion of the determinacy properties below. The change in output is larger on impact for the calibration to US debt shares as there is less crowding out of consumption relative to the UK calibration, while the inflationary impact is largest for the UK calibration.

After the initial impact from the unexpected shock, the response of inflation remains qualitatively similar across all three calibrations, i.e., irrespective of the share of debt that is indexed to inflation. This development is mirrored by qualitatively similar response of real interest rates present in the economy. This solidifies that inflation-indexed debt plays a role in particular when we have 'active' fiscal policy management in the sense of Leeper (1991), since in these situations the fiscal authority remains the prevailing driving force behind aggregate price changes.

The responses of output and tax rates show that these results are indeed driven mainly by the change in the policy mix, since the different calibrations induce varying observed impulse-responses after the initial period. Note that all this is true despite the economy being simulated in perfect-foresight after the initial shock. The fact that the fiscal expansion is met again with slight deflationary pressure further down the road in the cases with strictly positive levels of inflation-indexed debt reflects the interest rate-driven revaluation of government bonds that feeds back to changes in the price level through the FTPL relationship.

Appendix B presents further omitted simulation results, in particular related to the IRFs of bond prices and interest rates, household policy functions and monetary policy shocks. In particular the revaluation of the bonds as expressed through their prices are of interest, as they confirm the above arguments that the revaluation of the FTPL equation belongs to the main determinants of the inflationary response.[25]

Impact responses of the price level depending on the share of inflation-indexed debt and the fiscal policy response

Having considered the general response of a battery of macroeconomic variables in response to surprise government spending shocks, we now zoom into the role borne jointly by inflation-indexed debt and the debt-related policy coefficients for inflation on impact of such shocks. To that goal, we fix the monetary policy coefficients at the levels summarized by table 2 and vary the share of inflation-indexed debt in the government debt portfolio, $\omega_t = \frac{b_t}{B_t+b_t}$, between $[0,1)$[26], while also varying the strength of the fiscal policy reaction coefficients, $\{\gamma_B, \gamma_b\}$, between $[0,1]$, which are the coefficients under which fiscal policy is conventionally considered "active". Note that we restrict ourselves to cases in which $\gamma_B = \gamma_b$ here, such that no variation of impact inflation is induced by a change of the *relative* prevalence of the two types of debt, as the share of inflation-indexed debt is kept at its respective calibrated steady-state value.[27]

The first case that we consider is the one of conventionally passive monetary policy ($\phi_\pi = 0.5$). The impact reaction of the price level in this case is summarized by figure 11.

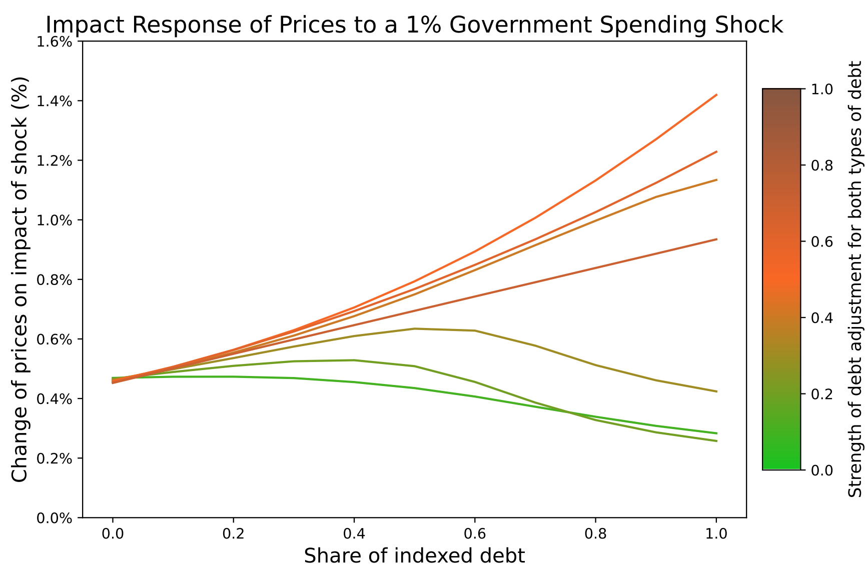


Figure 11: Impact reaction of prices in response to fiscal spending shocks under conventionally passive monetary policy.

[25]In further work, we aim to quantify the effect of household heterogeneity more fully with the help of non-linear impulse-response functions that might showcase significant amplification of observed responses in labor supplied, output, and inflation for the case with inflation-indexed debt.

[26]Recall that we cannot postulate all debt to be inflation-indexed in line with proposition 2.

[27]Appendix figure B.8 provides evidence on the effects of variation of the fiscal policy reaction coefficients γ_b and γ_B for the impact reaction on the price level, keeping the overall stock of debt fixed at the 'UK Calibration' values.

On the x-axis, we vary the share of indexed debt in the total debt portfolio (while maintaining a constant overall relation between the gross stock of debt and GDP), while the colors indicate the chosen fiscal reaction coefficients $\gamma_B = \gamma_b$. Thus, orange and especially brown colors reflect 'less active' fiscal policy in the conventional sense (as more of the shock is covered by corresponding tax raises), while greener colors reflect 'more active' fiscal policy.

We first focus on the leftmost point at which the share of inflation-indexed debt is exactly zero. There, we observe the conventional response that is expected in such models of fiscal-monetary interactions. The 'more active' fiscal policy is (i.e., as $\gamma_b, \gamma_B \to 0$), the stronger is the immediate impact on the price level when the shock occurs (the green lines are the highest for a zero share of inflation-indexed debt). This is fully in line with existing evidence from fiscal-monetary interaction models, and simply reflects that the necessary devaluation of a fully nominal debt stock is higher when income taxation does not react at all to the expansionary government spending shock. The difference across the various policy scenarios, however, is relatively small in this impact period.

The more interesting dynamics occur as we move to the right in the above picture, i.e., as we gradually increase the share of inflation-indexed debt in the government debt portfolio. We can broadly categorize the interaction between the adjustment coefficients and the share of indexed debt into two categories *based off the value of the adjustment coefficients of fiscal policy*:

- $\gamma_B = \gamma_b \gtrsim 0.3$ (orange-brown lines): Here, fiscal policy adjusts by covering comparatively more of the government spending shock through a corresponding increase in taxation. We observe uniformly that the higher the share of inflation-indexed debt, the greater is the change of the price level on impact, as the debt stock that can be devalued once the shock manifests itself is comparatively smaller, leading to a larger needed depreciation of that (smaller) stock of non-indexed debt.

- $\gamma_B = \gamma_b \lesssim 0.3$ (green lines): in these cases, the taxation schedule of the government covers only very little of the additional expense coming from the government spending shock. For realistic levels of inflation-indexed debt (below 40%), we can observe a general increase in the response of prices on impact, in line with standard predictions. The more indexed debt we have, the higher the devaluation of the outstanding debt stock must be, as in the case above. As the share of inflation-indexed debt becomes very large, however, we observe a surprising effect: the impact change of the price level actually starts to become smaller. This is directly related to the *real* side of the economy, as evidenced in figure 8: as the share of inflation-indexed debt increases, the real expansion of the economy becomes less pronounced, and the net worth of the wealthier households that hold large quantities of inflation-indexed debt decreases (as shown in figure B.6 in the appendix). This holds particularly true when the taxation reaction of the government is rather small. Through the Phillips Curve, then, we can observe downwards pressure on prices, overcoming some of the inflationary pressure induced through the government budget equilibrium.

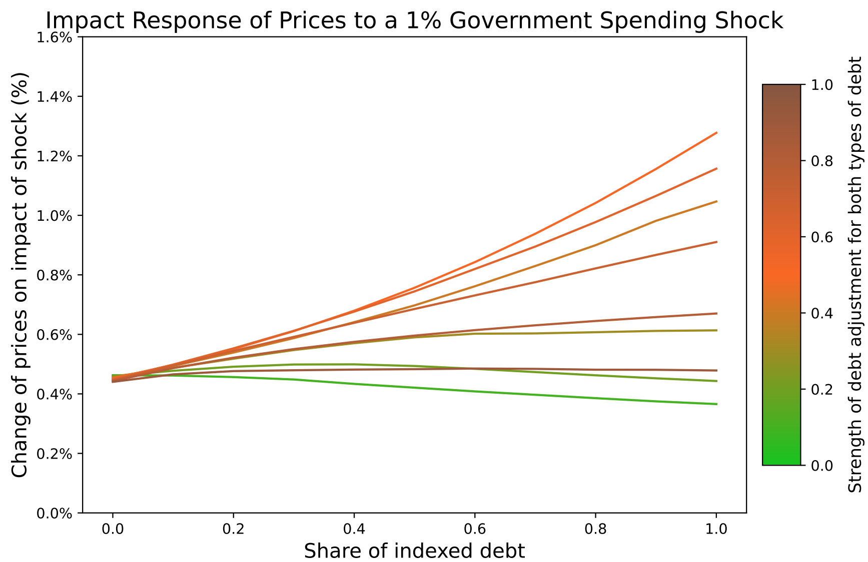


Figure 12: Impact reaction of prices in response to fiscal spending shocks under conventionally active monetary policy.

Figure 12 repeats the above exercise, but for conventionally 'active' monetary policy with a reaction coefficient of $\phi_\pi = 1.5$. The principal results from the previous exercise under $\phi_\pi = 0.5$ translate to this case: while at zero inflation-indexed debt, the impact response of prices is larger under 'more active' fiscal policy scenarios, we again observe a widening of the responses as we increase the share of inflation-indexed debt and, except for the case of near-zero reaction coefficients, the degree to which the sovereign spending shock is inflationary is again increasing in the share of indexed debt. This effect remains particularly pronounced for reaction coefficients of around 0.5, i.e., when only half of the corresponding bond revaluation induced by the shock is covered by an increase in taxation, although the overall magnitudes are generally smaller relative to the case when monetary policy reacts less in terms of a possible interest rate adjustment.

Finally, I want to emphasize again that we are only considering the *impact* response of prices, not the cumulative response. This is an artifact of the idea that we are considering deviations from steady-state here, and with finite spending shocks, the model naturally will converge back to its steady-state at the old price level, as long as there is no permanent change in debt issuance. Further exercises with this model will therefore center on *permanently* increasing the issuance of either type of debt.

Fiscal-monetary policy combinations and determinacy

As a final exercise, we consider explicitly for which values of the fiscal and monetary policy parameters we can establish determinacy of the linearized system. In doing so, we exploit the 'winding number criterion' developed in Auclert et al. (2023), which is suitable given our choice of the sequence-space Jacobian as the primary device to solve the full dynamic model.[28]

[28] A more detailed exposition of the 'winding number criterion' can be found in Auclert et al. (2023). Intuitively, one can relate this criterion to the Blanchard and Kahn (1980)-condition, which is cast in state-space. The winding number

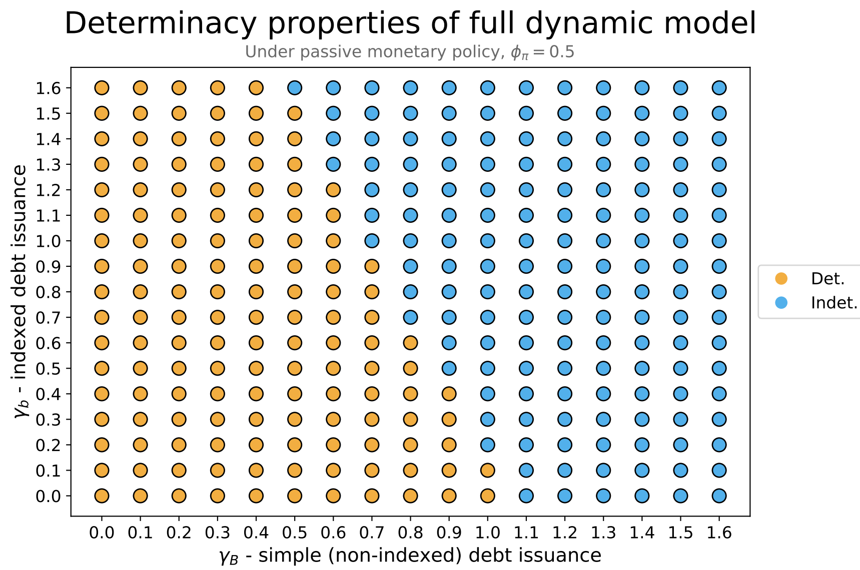


Figure 13: Determinacy of the generalized Jacobian in relation to choices for the fiscal and monetary policy reaction coefficients when monetary policy can be considered conventionally 'passive'.

Figure 13 summarizes the determinacy properties of the model on an equispaced grid of the fiscal policy reaction parameters γ_B and γ_b, setting $\phi_\pi = 0.5$. We thus effectively say that monetary policy does *not* raise interest rates more than one-for-one with inflation, which is conventionally dubbed 'passive monetary policy'. Interestingly, the results relate to canonical determinacy principles in line with Leeper (1991), even if they do not fully overlap. In particular, uniqueness of the equilibrium path is reached for conventional values of the non-indexed debt reaction parameter γ_B when inflation-indexed debt issuance in response to fiscal shocks is zero (which is evidenced by the last row of figure 13).

Once we allow the taxation schedule to be directly related to inflation-indexed debt deviations from equilibrium as well ($\gamma_b > 0$), two interesting phenomena arise. First, a trade-off in the government debt rule arises, by which an increased reactiveness of the taxation schedule to deviations of the value of indexed debt b_t from steady-state must be paid off with a smaller reactiveness with respect to the market value of non-indexed debt B_t. Second, this trade-off is non-linear: in particular, despite monetary policy being conventionally 'passive', it is possible for governments to react *more than one-for-one* ($\gamma_b > 1$) with their taxation schedule in relation to deviations of inflation-indexed debt from steady-state, provided that the adjustment with respect to non-indexed debt B_t is small enough. This is partially in line with our results from the analysis of a simplified Fisherian model in appendix C, to which we refer the interested reader for a closed-form analysis of determinacy properties with inflation-indexed debt in a simpler macroeconomic model.

criterion provides a generalizable 'mapping' of the Blanchard-Kahn conditions for the sequence-space, i.e., allowing infinitely many quasi-'roots' of the linearized system. Note that the prerequisites to apply the winding number criterion, such as the quasi-Toeplitz property of the generalized Jacobian, are not violated (the corresponding results are available upon request).

7 Discussion, summary, and next steps

This paper introduced inflation-indexed debt into models of the Fiscal Theory of the Price Level. We first provided support for the role of inflation-indexed debt as a major determinant of inflationary dynamics with the help of local projections applied to the UK and the US. Next, we established in a simplified model that such debt itself suffices to make the price level a backward-looking state variable: the previous price level therefore matters directly for the determination of today's price level. Finally, we introduced inflation-indexed debt in a state-of-the-art macroeconomic model with imperfect markets and household heterogeneity, ensuring the existence of a unique steady-state before providing model-driven evidence that inflation-indexed debt can indeed exacerbate the inflationary response to government spending shocks under the FTPL, in particular when fiscal policy is considered conventionally 'active' in the sense of Leeper (1991).

Both the empirical and theoretical results derived in this paper thus tarnish the classic notion that inflation-indexed bonds always limit inflation in a given country by offering governments a commitment device to 'not inflate the debt away', as exposed in Campbell and Shiller (1996). While this notion can remain true absent government spending shocks, our results point out that once the government budget is ex-post (after debt issuance) in disarray, the inflationary consequences of funding shortfalls can increase in the share of inflation-indexed debt. Issuance of indexed debt can therefore backfire despite its great ability to serve as an ex ante commitment device following Schmid et al. (2024).

Despite these conclusions, this project remains far from finished. In particular, the external validity of the results could be further strengthened by resorting to a more rigorous calibration to the UK economy. A further interesting avenue relates to the recent contribution of Angeletos et al. (2024), who find that models of the FTPL are not yielding novel insights relative to simple HANK models in terms of how the two models break Ricardian Equivalence to induce nominal and real effects of expansionary government spending shocks. While the proofs in their paper have a clear and unrefutable implication for simple models of the FTPL, we theorize that the quantitative equivalence between HANK and FTPL models will be difficult to replicate once inflation-indexed debt is accounted for, as such debt has limited first-order effects in simple HANK models, but sizable effects under the FTPL, as detailed above. This point is foremost on our agenda for further research. Another improvement to the paper could see a more thorough analysis of empirical drivers of net fiscal shocks (as developed in section 2) that overcomes the current limitations on the sample size.

Finally, inflation-indexed debt can enhance our understanding in an important sphere of recent policy debates on the possible regressivity or progressivity of inflation as implicit taxation. As evidenced by figure D.1 in the appendix, inflation-indexed debt, which serves as a principal insurance device against unexpected inflation, seems to be particularly skewed in household portfolios towards the highest decile of the income distribution. A more thorough analysis of the welfare effects of unexpected inflation to households at varying income deciles should therefore be considered as a further policy-relevant application in due course.

References

Acemoglu, D. and M. K. Jensen (2015). Robust Comparative Statics in Large Dynamic Economies. *Journal of Political Economy 123*(3), 587–640.

Achdou, Y., J. Han, J.-M. Lasry, P.-L. Lions, and B. Moll (2022). Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach. *The Review of Economic Studies 89*(1), 45–86.

Andreasen, M. M., J. H. Christensen, and S. Riddell (2021). The TIPS Liquidity Premium. *Review of Finance 25*(6), 1639–1675.

Angeletos, G.-M., C. Lian, and C. Wolf (2024). Deficits and Inflation: HANK meets FTPL. *NBER Working Paper* (w33102).

Auclert, A., B. Bardóczy, M. Rognlie, and L. Straub (2021). Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models. *Econometrica 89*(5), 2375–2408.

Auclert, A., R. Rigato, M. Rognlie, and L. Straub (2024). New Pricing Models, Same Old Phillips Curves? *The Quarterly Journal of Economics 139*(1), 121–186.

Auclert, A., M. Rognlie, and L. Straub (2023). Determinacy and Existence in the Sequence Space. *Manuscript*.

Auclert, A., M. Rognlie, and L. Straub (2024). The Intertemporal Keynesian Cross. *Journal of Political Economy 132*(12), 000–000.

Barr, D. G. and J. Y. Campbell (1997). Inflation, Real Interest Rates, and the Bond Market: A Study of UK Nominal and Index-Linked Government Bond Prices. *Journal of Monetary Economics 39*(3), 361–383.

Barro, R. J. and F. Bianchi (2023). Fiscal Influences on Inflation in OECD Countries, 2020-2022. Technical report, National Bureau of Economic Research.

Bassetto, M. and W. Cui (2018). The Fiscal Theory of the Price Level in a World of Low Interest Rates. *Journal of Economic Dynamics and Control 89*, 5–22.

Bayer, C., B. Born, and R. Luetticke (2024). Shocks, Frictions, and Inequality in US Business Cycles. *American Economic Review 114*(5), 1211–1247.

Bayer, C. and R. Luetticke (2020). Solving Discrete Time Heterogeneous Agent Models with Aggregate Risk and Many Idiosyncratic States by Perturbation. *Quantitative Economics 11*(4), 1253–1288.

Benigno, G., B. Hofmann, G. N. Barrau, and D. Sandri (2024). Quo vadis, r*? The Natural Rate of Interest After the Pandemic. *BIS Quarterly Review 4*.

Bianchi, F., R. Faccini, and L. Melosi (2023). A Fiscal Theory of Persistent Inflation. *The Quarterly Journal of Economics 138*(4), 2127–2179.

BIS (2024). Bank for International Settlements Debt Securities Statistics. Data retrieved from `https://data.bis.org/topics/DSS/tables-and-dashboards`.

Blanchard, O. J. and C. M. Kahn (1980). The Solution of Linear Difference Models under Rational Expectations. *Econometrica*, 1305–1311.

Braun, R., S. Miranda-Agrippino, and T. Saha (2024). Measuring Monetary Policy in the UK: The UK Monetary Policy Event-Study Database. *Journal of Monetary Economics*, 103645.

Brunnermeier, M., S. Merkel, and Y. Sannikov (2024). Safe Assets. *Journal of Political Economy forth-*

coming.

Brunnermeier, M. K., S. A. Merkel, and Y. Sannikov (2020). The Fiscal Theory of Price Level with a Bubble. Technical report, National Bureau of Economic Research.

Cairns, A. and D. Wilkie (2023). British Government Securities Database. Data retrieved from the Heriot-Watt University Institute and Faculty of Actuaries, https://www.macs.hw.ac.uk/~andrewc/gilts/.

Campbell, J. Y. and R. J. Shiller (1996). A Scorecard for Indexed Government Debt. *NBER Macroeconomics Annual 11*, 155–197.

Chen, X., E. M. Leeper, and C. Leith (2022). Strategic Interactions in US Monetary and Fiscal Policies. *Quantitative Economics 13*(2), 593–628.

Chen, Z., Z. Jiang, H. Lustig, S. Van Nieuwerburgh, and M. Z. Xiaolan (2022). Exorbitant Privilege Gained and Lost: Fiscal Implications. *National Bureau of Economic Research WP no. w30059*.

Cloyne, J. (2013). Discretionary Tax Changes and the Macroeconomy: New Narrative Evidence from the United Kingdom. *American Economic Review 103*(4), 1507–1528.

Cloyne, J., O. Jorda, and A. M. Taylor (2023). State-Dependent Local Projections: Understanding Impulse Response Heterogeneity. *NBER Working Paper* (w30971).

Cloyne, J., J. Martinez, H. Mumtaz, and P. Surico (2023). Do Tax Increases Tame Inflation? In *AEA Papers and Proceedings*, Volume 113, pp. 377–381. AEA.

Cochrane, J. (2023). *The Fiscal Theory of the Price Level*. Princeton University Press.

Cochrane, J. H. (2001). Long-Term Debt and Optimal Policy in the Fiscal Theory of the Price Level. *Econometrica 69*(1), 69–116.

Cochrane, J. H. (2022a). Fiscal Histories. *Journal of Economic Perspectives 36*(4), 125–46.

Cochrane, J. H. (2022b). The Fiscal Roots of Inflation. *Review of Economic Dynamics 45*, 22–40.

Farmer, R. and P. Zabczyk (2019). *A Requiem for the Fiscal Theory of the Price Level*. International Monetary Fund.

Fed Board of Governors (2024). Selected Interest Rates (Daily) - H.15. Data retrieved from the website of the Board of Governors of the Federal Reserve Systems, https://www.federalreserve.gov/releases/h15/.

Fischer, S. (1975). The Demand for Index Bonds. *Journal of Political Economy 83*(3), 509–534.

Fleming, M. J. and N. Krishnan (2012). The Microstructure of the TIPS Market. *Economic Policy Review*.

Garcia, J. A. and A. A. van Rixtel (2007). Inflation-Linked Bonds from a Central Bank Perspective. *ECB Occasional Paper* (62).

Graham, L. and S. Wright (2007). Nominal Debt Dynamics, Credit Constraints and Monetary Policy. *The BE Journal of Macroeconomics 7*(1).

Gürkaynak, R. S., B. Sack, and J. H. Wright (2010). The TIPS Yield Curve and Inflation Compensation. *American Economic Journal: Macroeconomics 2*(1), 70–92.

Hagedorn, M. (2021). A Demand Theory of the Price Level. CEPR discussion paper no. DP11364.

Hagedorn, M. (2024). The Failed Theory of the Price Level. CEPR discussion paper no. DP17816.

Hazell, J. and S. Hobler (2024). Do Deficits Cause Inflation? A High Frequency Narrative Approach. *mimeo*.

Jordà, Ò. (2005). Estimation and Inference of Impulse Responses by Local Projections. *American Economic Review 95*(1), 161–182.

Kaplan, G., B. Moll, and G. L. Violante (2018, March). Monetary Policy According to HANK. *American Economic Review 108*(3), 697–743.

Kaplan, G., G. Nikolakoudis, and G. L. Violante (2023). Price Level and Inflation Dynamics in Heterogeneous Agent Economies. *National Bureau of Economic Research WP no. w31433*.

Kwicklis, N. (2024). Output and Inflation in an Active-Fiscal, Passive-Monetary HANK. mimeo.

Leeper, E. M. (1991). Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies. *Journal of Monetary Economics 27*(1), 129–147.

Leeper, E. M. (2023). Fiscal Dominance: How Worried Should We Be? *Mercatus Policy Brief Series*.

Leeper, E. M. and C. Leith (2016). Understanding Inflation as a Joint Monetary–Fiscal Phenomenon. In *Handbook of Macroeconomics*, Volume 2, pp. 2305–2415. Elsevier.

Liemen, M. O. and O. Posch (2022). FTPL and the Maturity Structure of Government Debt in the New-Keynesian Model. *Available at SSRN 4157228*.

Miao, J. and D. Su (2021). Fiscal and Monetary Policy Interactions in a Model with Low Interest Rates. Unpublished Working Paper. Retrieved from: https://people.bu.edu/miaoj/MiaoSuN08.pdf.

Mierzwa, S. (2024). Spillovers from Tax Shocks to the Euro Area. *Oxford Economic Papers*, gpae024.

NIESR (2022). An Independent Assessment of the Mini-Budget - 23 September 2022. available online at https://niesr.ac.uk/wp-content/uploads/2022/09/September-2022-Mini-Budget-Response.pdf, retrieved on 25 May 2024.

OBR (2024a). Office of Budget Responsibility Historical official forecasts database. Data retrieved from https://obr.uk/data/.

OBR (2024b). Office of Budget Responsibility Public Finances Databank. Data retrieved from https://obr.uk/public-finances-databank-2023-24/.

Pinter, G. (2023). An Anatomy of the 2022 Gilt Market Crisis. Bank of England Working Paper No. 1019.

Reiter, M. (2009). Solving Heterogeneous-Agent Models by Projection and Perturbation. *Journal of Economic Dynamics and Control 33*(3), 649–665.

Romer, C. D. and D. H. Romer (2010). The Macroeconomic Effects of Tax Changes: Estimates Based on a New Measure of Fiscal Shocks. *American Economic Review 100*(3), 763–801.

Sargent, T. J. and N. Wallace (1981). Some Unpleasant Monetarist Arithmetic. *Federal Reserve Bank of Minneapolis Quarterly Review 5*(3), 1–17.

Schmid, L., V. Valaitis, and A. T. Villa (2024). Government Debt Management and Inflation with Real and Nominal Bonds. Technical report, Centre for Macroeconomics (CFM).

Sims, C. A. (1994). A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy. *Economic Theory 4*, 381–399.

Sims, C. A. (2011). Stepping on a Rake: The Role of Fiscal Policy in the Inflation of the 1970s.

European Economic Review 55(1), 48–56.

Smets, F. and R. Wouters (2024). Fiscal Backing, Inflation and US Business Cycles.

Woodford, M. (1995). Price-Level Determinacy Without Control of a Monetary Aggregate. In *Carnegie-Rochester Conference Series on Public Policy*, Volume 43, pp. 1–46. Elsevier.

Appendix

A Derivations and proofs from the main text

A.1 Derivations from section 4

Derivation of equation (12) (proof of proposition 1)

We here present the derivations underlying a *dynamic trading perspective* for asset valuation laid out in Brunnermeier et al. (2024), which avoids fallacies related to a possibly nonexistent aggregate transversality condition by clearly defining the valuation differences of government debt between households and the government based off the insurance properties that government bonds bear for households. This allows us to leverage household-level transversality conditions to derive an aggregate FTPL-type condition that only holds for one initial candidate price level.

The starting point for this valuation equation of government debt is the household budget constraint, which we recall was given by

$$P_t c_{it} + Q_t B_{it} + q_t b_{it} = \varepsilon_{it}(1 - \tau_{it}) P_t w_t N_t + B_{i,t-1} + \Pi_t b_{i,t-1}$$

for each household i. Following our results derived in the household block, we let households price bonds in accordance with their *SDF*:

$$B_{i,t-1} + \Pi_t b_{i,t-1} = \mathbb{E}_t\left(M_{i,t,t+1}\right) B_{it} + \mathbb{E}_t\left(\Pi_{t+1} M_{i,t,t+1}\right) b_{it} + P_t(c_{it} - \varepsilon_{it} w_t N_t (1 - \tau_{it})).$$

Splitting up the second expectation term, we get

$$B_{i,t-1} + \Pi_t b_{i,t-1} = \mathbb{E}_t\left(M_{i,t,t+1}\right) B_{it} + \mathbb{E}_t\left(M_{i,t,t+1}\right) \mathbb{E}_t\left(\Pi_{t+1}\right) b_{it} + b_{it} Cov_t\left(M_{i,t,t+1}, \Pi_{t+1}\right) + P_t(c_{it} - \varepsilon_{it} w_t N_t (1 - \tau_{it})).$$

We divide all elements by P_t and add/subtract relevant terms on the right-hand side to ensure that we can iterate on the resulting expression:

$$\begin{aligned}\frac{B_{i,t-1} + \Pi_t b_{i,t-1}}{P_t} &= \mathbb{E}_t\left(M_{i,t,t+1}\right) \Pi_{t+1} \left[\frac{B_{it} + \Pi_{t+1} b_t}{P_{t+1}}\right] + (c_{it} - \varepsilon_{it}(1 - \tau_{it}) w_t N_t) \\ &\quad + Cov_t\left(M_{i,t,t+1}, \Pi_{t+1}\right) \frac{b_{it}}{P_t} + \mathbb{E}_t\left(M_{i,t,t+1}\right) \frac{b_{it}}{P_t} \left(\mathbb{E}_t \Pi_{t+1} - \Pi_{t+1}\right).\end{aligned}$$

We can now start iterating on this expression. The first iteration yields:

$$\frac{B_{i,t-1}+\Pi_t b_{i,t-1}}{P_t} = \mathbb{E}_t\left(\mathcal{M}_{i,t,t+1}\right)\Pi_{t+1}\left[\mathbb{E}_{t+1}\left(\mathcal{M}_{i,t+1,t+2}\right)\Pi_{t+2}\left[\frac{B_{i,t+1}+\Pi_{t+2}b_{i,t+1}}{P_{t+2}}\right]\right.$$
$$(c_{i,t+1}-\varepsilon_{i,t+1}(1-\tau_{i,t+1})w_{t+1}N_{t+1}) + Cov_{t+1}\left(\mathcal{M}_{i,t+1,t+2},\Pi_{t+2}\right)\frac{b_{i,t+1}}{P_{t+1}}$$
$$\left. + \mathbb{E}_{t+1}\left(\mathcal{M}_{i,t+1,t+2}\right)\frac{b_{i,t+1}}{P_{t+1}}\left(\mathbb{E}_{t+1}\Pi_{t+2}-\Pi_{t+2}\right)\right]$$
$$+(c_{it}-\varepsilon_{it}(1-\tau_{it})w_t N_t) + Cov_t\left(\mathcal{M}_{i,t,t+1},\Pi_{t+1}\right)\frac{b_{it}}{P_t} + \mathbb{E}_t\left(\mathcal{M}_{i,t,t+1}\right)\frac{b_{it}}{P_t}\left(\mathbb{E}_t\Pi_{t+1}-\Pi_{t+1}\right).$$

Continuing rolling over, applying the LIE, and simplifying SDFs by making use of the identity $\mathcal{M}_{i,t,t+k}\mathcal{M}_{i,t+k,t+l} = \mathcal{M}_{i,t,t+l}\ \forall t,k,l$, we eventually end up with:

$$\frac{B_{i,t-1}+\Pi_t b_{i,t-1}}{P_t} = \mathbb{E}_t\left[\sum_{k=0}^{\infty}\mathcal{M}_{i,t,t+k}\Pi_{t+1,t+k+1}\left\{(c_{i,t+k}-\varepsilon_{i,t+k}(1-\tau_{i,t+k})w_{t+k}N_{t+k})\right.\right.$$
$$\left.\left. + \left[Cov_{t+k}\left(\mathcal{M}_{i,t+k,t+k+1},\Pi_{t+k+1}\right) + \mathcal{M}_{t+k,t+k+1}\left(\mathbb{E}_{t+k}\Pi_{t+k+1}-\Pi_{t+k+1}\right)\right]\frac{b_{i,t+k}}{P_{t+k}}\right\}\right] \tag{A.1}$$
$$+\lim_{T\to\infty}\left\{\mathbb{E}_t\left[\mathcal{M}_{i,t,t+T}\left(\frac{B_{i,t+T}+\Pi_{t+T+1}b_{i,t+T}}{P_{t+T}}\right)\right]\right\},$$

where we use the notation $\Pi_{t+1,t+k+1}$ to define gross inflation from period $t+1$ to period $t+k+1$. This is the integrated household budget constraint at optimality, from which we hope to derive the integrated government budget constraint.

Crucially, we note that household optimality implies $\lim_{T\to\infty}\frac{B_{i,T}+\Pi_{T+1}b_{i,T}}{P_T} \leq 0$, while a no-Ponzi condition on household debt holdings ensures that $\lim_{T\to\infty}\frac{B_{i,T}+\Pi_{T+1}b_{i,T}}{P_T} \geq 0$. Furthermore, by the definition of the SDF and the properties of a standard CRRA utility function, $\lim_{T\to\infty}\mathcal{M}_{i,t,T} \neq \pm\infty$. Therefore, the final limit converges to 0 and must not be considered.[29]

The formulation of equation (A.1) is intuitive: the real value of household bond holdings is equal to its expected discounted consumption benefits from today to infinity (as future net consumption earnings are suitably discounted with the SDF, which is a mirror image of the price of the two bonds), adjusted suitably for additional surprise earnings enjoyed from holdings of *indexed* sovereign debt: these are decreased by surprise inflation through its (negative) covariance with the SDF (as higher *future* inflation pushes the SDF down), and increased by surprise inflation through a level effect (since such inflation yields a windfall gain relative to what was paid for the indexed bond in the previous period).

We now aggregate these individual household bond constraints up to an integrated government budget constraint. We make use of the asset market clearing conditions $B_t = \sum_i B_{it}$ and $b_t = \sum_i b_{it}$ and of the idea that the household TVCs hold individually to get the following expression:

[29] Even though this idea resembles the core idea behind Brunnermeier et al. (2020) and Brunnermeier et al. (2024), we are also overcoming the issues raised by Hagedorn (2024) by taking into account the dynamic trading (flow) benefits of government debt across time. This ensures the transversality conditions to hold for only one initial price level.

$$
\begin{aligned}
\frac{B_{t-1}+\Pi_t b_{t-1}}{P_t} &= \sum_i \Bigg\{ \mathbb{E}_t \Bigg[\sum_{k=0}^{\infty} M_{i,t,t+k}\Pi_{t+1,t+k+1} \big\{ \left(c_{i,t+k} - \varepsilon_{i,t+k}(1-\tau_{i,t+k}) w_{t+k} N_{t+k}\right) \\
&+ \left[Cov_{t+k}\left(M_{i,t+k,t+k+1}, \Pi_{t+k+1}\right) + M_{t+k,t+k+1}\left(\mathbb{E}_{t+k}\Pi_{t+k+1} - \Pi_{t+k+1}\right)\right] \frac{b_{i,t+k}}{P_{t+k}} \big\} \Bigg] \Bigg\}.
\end{aligned} \tag{A.2}
$$

We simplify this equation by noting that we can take the summation into the expectation and switch around the order of summation. To further simplify the integrated government budget valuation equation, we create the variable A_{it} which captures the surpluses raised by the government from each household i:

$$
A_{it} \equiv c_{it} - \varepsilon_{it}(1-\tau_{it}) w_t N_t + \left[Cov_t\left(M_{i,t,t+1}, \Pi_{t+1}\right) + M_{i,t,t+1}\left(\mathbb{E}_t \Pi_{t+1} - \Pi_{t+1}\right)\right] \frac{b_{it}}{P_t},
$$

which is the full portfolio return of household i of holding an additional unit of net worth. Alternatively, one can view this as what the government factually can raise as surpluses from each household i.

We additionally define $\bar{A}_t = \sum_i A_{it}$ as the sum of all individual-level surpluses. We can then rewrite the implied intertemporal government budget constraint (A.2) to:

$$
\frac{B_t}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \left(\sum_i M_{i,t,t+k} \Pi_{t,t+k+1} \frac{A_{i,t+k}}{\bar{A}_{t+k}} \right) \bar{A}_{t+k} \right],
$$

or, defining the *household value-weighted SDF* $\tilde{M}_{t,t+k} = \sum_i M_{i,t,t+k}\Pi_{t,t+k+1}\frac{A_{i,t+k}}{\bar{A}_{t+k}}$, we finally arrive at:

$$
\frac{B_t}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \tilde{M}_{t,t+k} \bar{A}_{t+k} \right], \tag{A.3}
$$

where $\tilde{M}_{t,t+k}$ is now the weighted average SDF across all households i, adjusted for inflation, with weights being proportionate to $A_{i,t+k}$, consisting of the net utility gain from saving, the insurance premium on indexed debt (captured through the covariance term), and the possible windfall gain/loss from surprise inflation (captured through the last term in the definition of $A_{i,t+k}$). Equation (A.3) is 'the FTPL equation' that is used to pin down the price level at time t, given some previous price level P_{t-1}.

A.2 Proof of proposition 2

We first show that determinacy can indeed be achieved with the FTPL when indexed debt is present, provided that we include a suitable theory of the real interest rate, before showing how indexed debt translates into a model where taxation is assumed to cover all interest expenses over time on the stationary equilibrium path, following Hagedorn (2021). We therefore maintain a 'true Balanced Growth Path' (BGP) with a constant real value of the debt portfolio thanks to an appropriate taxation schedule.

To apply the framework of Hagedorn (2021), we have to rewrite the steady-state taxation function to account for possible non-zero steady-state inflation and some positive level of indexed debt, since the presence of both changes the nominal value of taxation over time. We still aim to find an asset demand function depending only on model primitives.[30] To do so, we must pin down steady-state asset demand under incomplete markets in a closed-form solution, for which we will leverage the results of Acemoglu and Jensen (2015).

To find the steady-state level of taxation consistent with the bond issuance schedule that keeps the real value of bonds constant (provided that inflation devalues the non-indexed bonds), we begin with an arbitrary per-period government budget constraint (setting $G_t = 0$, such that real surpluses are $s_t = \tau_t$, or, in nominal terms, $P_t s_t = P_t \tau_t =: T_t$):

$$B_{t-1} + \frac{P_t}{P_{t-1}} b_{t-1} = T_t + Q_t B_t + q_t b_t.$$

Q_t and q_t must be equal to some constant values in steady-state. Without aggregate uncertainty, the bond prices arising through asset demand must solely depend on the offered interest rates, since cross-sectional risks average out. Thus, *in steady-state*, we have that:

$$\begin{aligned} B_{ss} + \Pi_{ss} b_{ss} &= T_{ss} + Q_{ss} B_{ss} + q_{ss} b_{ss} \\ \Leftrightarrow B_{ss} + \Pi_{ss} b_{ss} &= T_{ss} + \frac{1}{1+i_{ss}} B_{ss} + \frac{1}{1+r_{ss}} b_{ss} \\ \Leftrightarrow T_{ss} &= \left(1 - \frac{1}{1+i_{ss}}\right) B_{ss} + \left(\Pi_{ss} - \frac{1}{1+r_{ss}}\right) b_{ss}. \end{aligned}$$

Using the Fisher equation, we can see that $\Pi_{ss} - \frac{1}{1+r_{ss}} = \frac{1+i_{ss}}{1+r_{ss}} - \frac{1}{1+r_{ss}} = \frac{i_{ss}}{1+r_{ss}}$, and therefore:

$$T_{ss} = \frac{i_{ss}}{1+i_{ss}} B_{ss} + \frac{i_{ss}}{1+r_{ss}} b_{ss},$$

which can be expressed in real terms (as the household cares about real taxation) as

$$\tau_{ss} = \frac{r_{ss}}{1+i_{ss}} B_{ss} + \frac{r_{ss}}{1+r_{ss}} b_{ss}.$$

Define by $S_t\left(\Omega_t, \{1+r_l, \tau_l\}_l^\infty\right)$ the cumulative asset demand function under incomplete markets, which depends on the household distribution of wealth Ω_t, real interest rates $1+r_t$, and tax rates τ_t, and is well-defined under standard regularity conditions (Acemoglu and Jensen, 2015). To relate

[30] For the sake of completeness, we want to specify the approach Hagedorn (2021) takes to determine steady-state taxation. He specifies the per-period government budget constraint as $B_{t+1} = (1+i_t)B_t - T_t \Leftrightarrow T_t = (1+i_t)B_t - B_{t+1}$ to arrive in steady-state at $T_{ss} = i_{ss} S_{ss}$, where S_{ss} is steady-state asset demand. in real terms, $\tau_{ss} =: \frac{T_{ss}}{P_{ss}} = r_{ss} S_{ss}$.

steady-state taxation more clearly to gross asset demand, we fix the shares of B_{ss} and b_{ss} of gross asset demand S_{ss} in steady-state. Denoting by ω the share of indexed debt b_{ss} in the steady-state asset portfolio, the taxation term in steady-state finally becomes

$$\tau_{ss} = \left[(1-\omega)\frac{r_{ss}}{1+i_{ss}} + \omega\frac{r_{ss}}{1+r_{ss}}\right] S_{ss}.$$

Under such steady-state taxes, the gross asset demand function arising from heterogeneous household demand ($S_{t+1} = \mathcal{S}(\Omega_t; 1+r_t, 1+r_{t+1}, 1+r_{t+2}, ...; \tau_t, \tau_{t+1}, ...)$) simplifies to the following mapping in steady-state:

$$S_{ss} = \mathcal{S}\left(\Omega_{ss}; 1+r_{ss}, 1+r_{ss}, 1+r_{ss}, ...; \left[(1-\omega)\frac{r_{ss}}{1+i_{ss}} + \omega\frac{r_{ss}}{1+r_{ss}}\right] S_{ss}, \left[(1-\omega)\frac{r_{ss}}{1+i_{ss}} + \omega\frac{r_{ss}}{1+r_{ss}}\right] S_{ss}, ...\right).$$

With i_{ss} being equal to some constant set by the monetary policymaker in steady-state and the taxation function that we just derived, asset demand can again be derived by finding the fixed point of the above equation, which would yield asset demand as a function of the real interest rate r_{ss}, following Acemoglu and Jensen (2015):

$$\underline{\text{Asset demand:}} \qquad S(r).$$

From our previous derivations, we directly leverage asset supply in real terms as the left-hand side of our derivations of the fiscal theory equation evaluated in steady-state, such that the stationary asset market equilibrium must be pinned down by

$$S(r) = \frac{B}{\tilde{P}} + \frac{b}{\tilde{P}(1+\pi_{ss})},$$

or, making use of the Fisher equation,

$$S(r) = \frac{B}{\tilde{P}} + \frac{b}{\tilde{P}}\frac{(1+r_{ss})}{(1+i_{ss})}.$$

An important question relates to the source of π_{ss}, the posited non-zero steady-state inflation rate in this economy. Following the contribution of Hagedorn (2021), we posit that the only possible non-zero steady-state inflation rate is the one consistent with a corresponding increase in taxation over time alongside this inflationary path:

$$1+\pi_{ss} = \frac{T'-T}{T},$$

where variables with a prime denote next period values. Since T represents nominal taxes, the above statement is equivalent to the claim that *real* taxes remain constant.

Given the bond portfolio on offer, we can express the above condition as follows:

$$\begin{aligned} 1+\pi_{ss} &= (1-\omega)\frac{B'-B}{B}+\omega\frac{b'-b}{b}\cdot(1+\pi_{ss}) \\ \Leftrightarrow 1+\pi_{ss} &= \frac{(1-\omega)\frac{B'-B}{B}}{1-\omega\frac{b'-b}{b}}, \end{aligned}$$

where the inflation-adjustment on the right-hand side in the first line follows from the adjustment of the face value of inflation-indexed debt. This bond issuance schedule therefore can be considered to pin down steady-state inflation.

Using the FTPL to determine the price ***level***: We can now invoke the above derivations within the FTPL to pin down the price level uniquely, provided that we can recover the real interest rate from the asset market.

Following our above reasoning, that steady-state real interest rate can indeed be recovered from the asset market through household demand, provided that this demand function is invertible, as

$$r_{ss} = S^{-1}\left(\frac{B}{\tilde{P}}+\frac{b}{\tilde{P}(1+\pi_{ss})}\right),$$

which we can insert in the stationary intertemporal FTPL equilibrium ($\frac{B}{\tilde{P}}+\frac{B}{\tilde{P}(1+\pi_{ss})} = \sum_{j=0}^{\infty}\left(\frac{1}{1+r_{ss}}\right)^j\bar{s}$) with $r_{ss}>0$ (such that the right-hand side can be rewritten as a geometric sum, $\sum_{j=0}^{\infty}\left(\frac{1}{1+r_{ss}}\right)^j = \frac{1+r_{ss}}{r_{ss}}$) to get the following condition:

$$\frac{B_{ss}+b_{ss}(1+\pi_{ss})}{\tilde{P}} = \bar{s}\frac{1+r_{ss}}{r_{ss}},$$

and the fixed point of this equation pins down the price level uniquely, given asset market optimality. To be precise, given our earlier definition of the surplus process, i.e., $\bar{s}=\tau_{ss}=\frac{r_{ss}}{1+i_{ss}}B_{ss}+\frac{r_{ss}}{1+r_{ss}}b_{ss}$, we have

$$\frac{B_{ss}+b_{ss}(1+\pi_{ss})}{\tilde{P}} = \left[\frac{r_{ss}}{1+i_{ss}}B_{ss}+\frac{r_{ss}}{1+r_{ss}}b_{ss}\right]\frac{1+r_{ss}}{r_{ss}}.$$

Using the Fisher equation ($(1+i_{ss})=(1+r_{ss})(1+\pi_{ss})$), we can simplify this equilibrium relation to:

$$\frac{B_{ss}}{\tilde{P}}+\frac{b}{\tilde{P}(1+\pi_{ss})} = (1+\pi_{ss})B+b,$$

which eventually pins down the price level as

$$\tilde{P} = \frac{B_{ss} + b_{ss}(1 + \pi_{ss})}{(1 + \pi_{ss})B_{ss} + b_{ss}}.$$

From the taxation schedule (which is a fiscal variable itself, actively managed by fiscal policy), we can recover the steady-state inflation rate. We simplify this by utilizing the steady-state growth rates $\frac{B'-B}{B} =: g_B$ and $\frac{b'-b}{b} =: g_b$, such that steady-state inflation becomes $1 + \pi_{ss} = \frac{(1-\omega)\frac{B'-B}{B}}{1-\omega\frac{b'-b}{b}} = \frac{(1-\omega)g_B}{1-\omega g_b}$. Thus, the initial price level in this steady-state is given by:

$$\tilde{P} = \frac{B_{ss} + b_{ss}\frac{(1-\omega)g_B}{1-\omega g_b}}{B_{ss}\frac{(1-\omega)g_B}{1-\omega g_b} + b_{ss}},$$

with the bond growth rates themselves being fiscal choice variables in the stationary equilibrium.

B Further simulation results

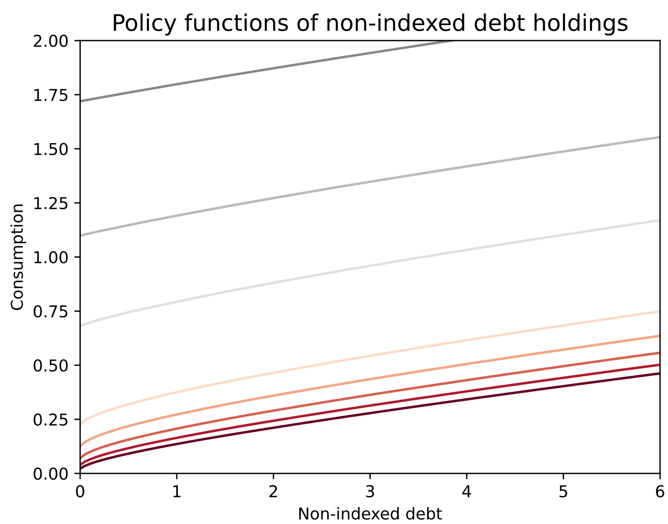


Figure B.1: Household policy functions for demand of non-indexed debt in the calibrated HANK model for unconstrained households. Note that the policy functions for low values of idiosyncratic productivity start to become positive only for strictly positive levels of non-indexed debt due to the possibility to purchase inflation-indexed debt stock.

Next, this appendix showcases dynamic impulse-responses of aggregate variables in response to a 25bps expansionary monetary policy shock, as well as the results of a full non-linear estimation of the model, which encountered significant numerical instabilities.

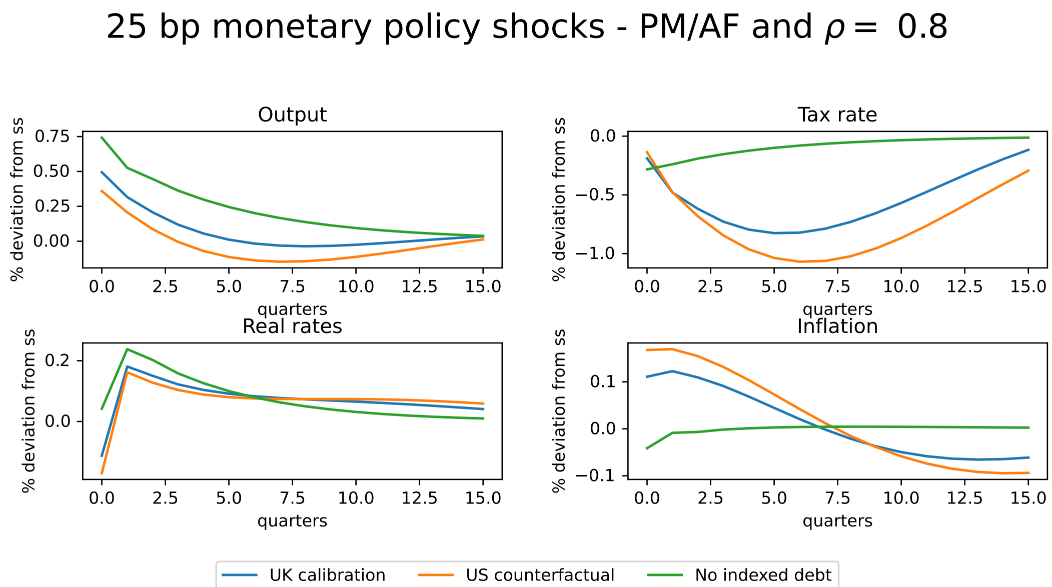


Figure B.2: IRFs to a 25bps expansionary monetary shock - with active fiscal policy and $\rho = 0.8$.

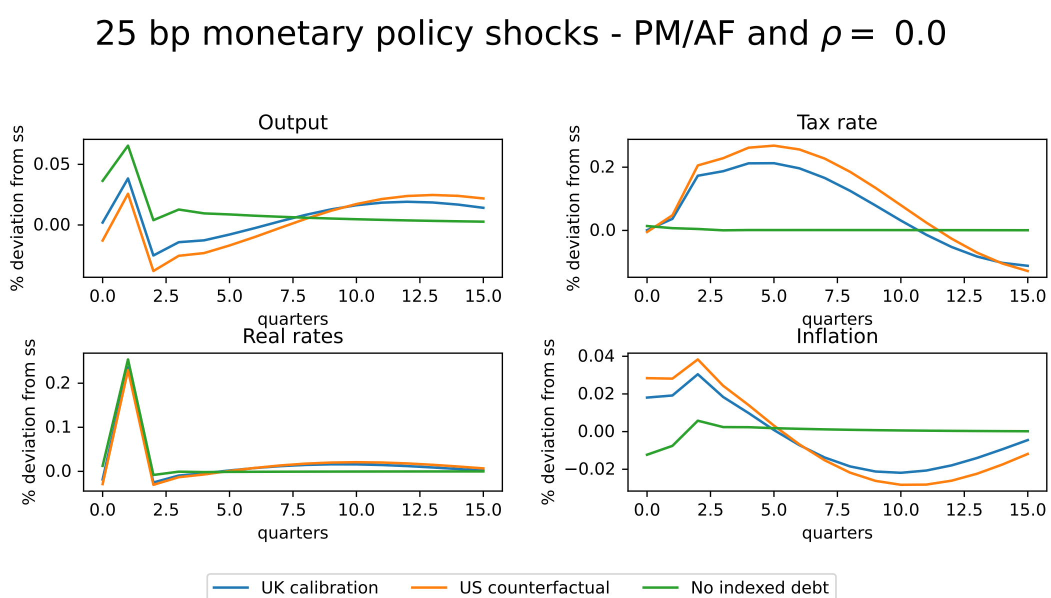


Figure B.3: IRFs to a 25bps expansionary monetary shock - with active fiscal policy and $\rho = 0$.

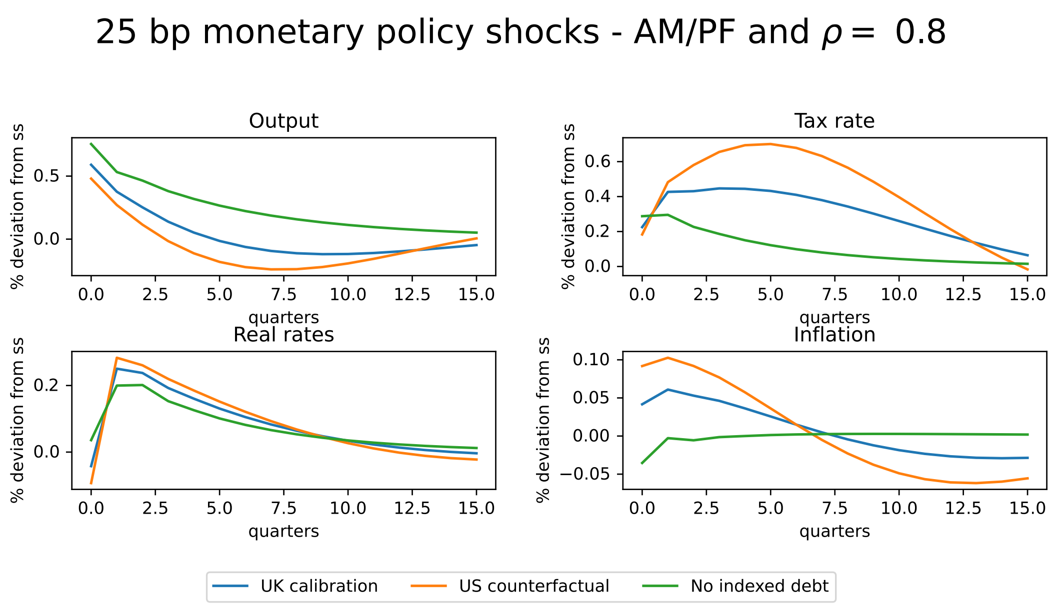


Figure B.4: IRFs to a 25bps expansionary monetary shock - with active monetary policy and $\rho = 0.8$.

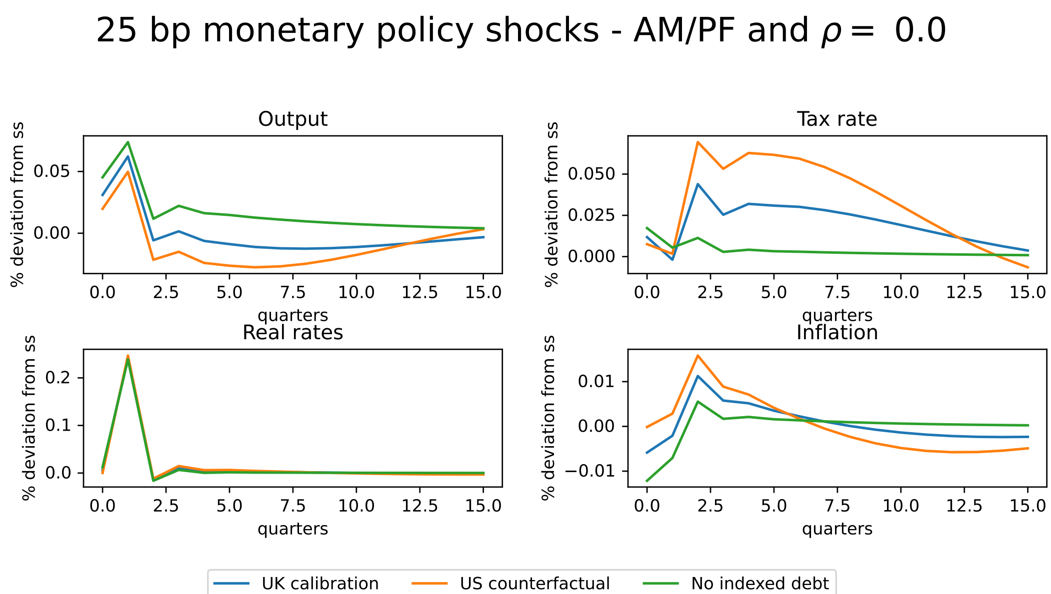


Figure B.5: IRFs to a 25bps expansionary monetary shock - with active monetary policy and $\rho = 0$.

For our main policy scenario ('passive monetary policy' and 'active fiscal policy'), we furthermore provide additional evidence on changes of quantities directly informing the core FTPL equation (12).

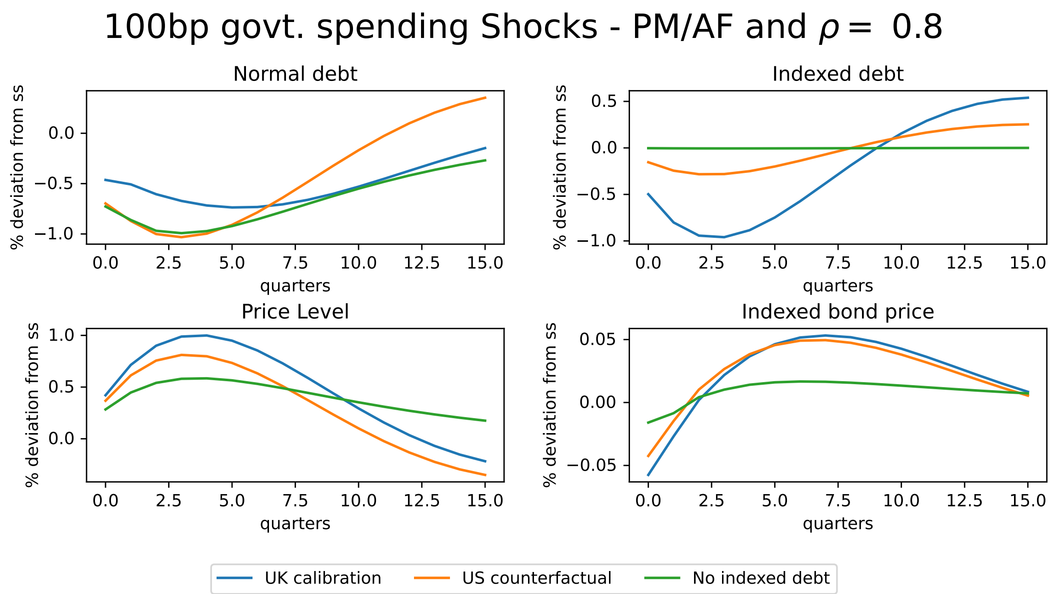


Figure B.6: Further IRFs to a 100bps expansionary fiscal spending shock - with passive monetary/active fiscal policy and $\rho = 0.8$.

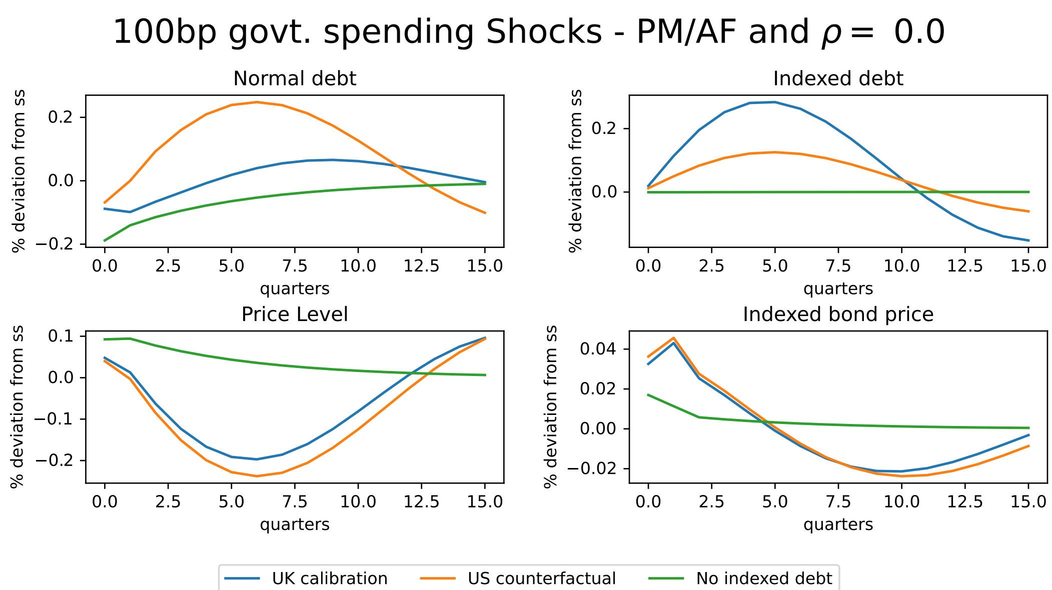


Figure B.7: Further IRFs to a 100bps expansionary fiscal spending shock - with passive monetary/active fiscal policy and $\rho = 0$.

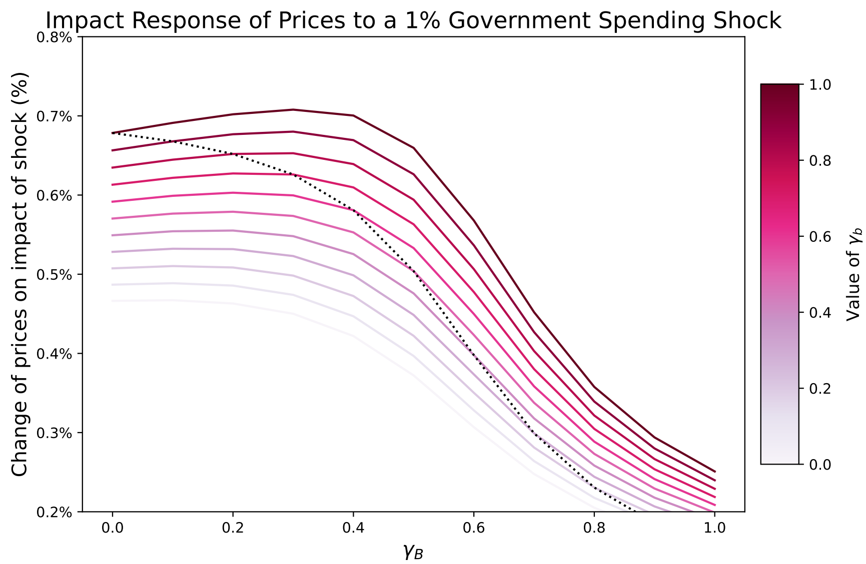


Figure B.8: Impact reaction of prices in response to fiscal spending shocks under conventionally passive monetary policy in the UK calibration, depending on the reaction coefficients of fiscal policy γ_b and γ_B. The black dotted line summarizes impact responses of prices when $\gamma_b + \gamma_B = 1$.

C Fiscal rules and inflation-indexed debt: an example from a Fisherian model

To illustrate simply how indexed debt enters a canonical macroeconomic model in general equilibrium on top of the FTPL, we consider here a Fisherian economy with a representative household saving in the two types of government bonds available. We do so to analyze how the presence of indexed debt can influence equilibrium properties in dynamic economies, as this will be an important consideration in the final dynamic model. In the toy model that we lay out here, a representative household receives a constant stream of goods Y, solving

$$\max_{\{c_t, B_t, b_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where B_t and b_t denote nominal quantities of non-indexed and inflation-indexed debt, respectively, subject to the flow budget constraint

$$P_t c_t + Q_t B_t + q_t b_t = P_t (Y - T_t) + B_{t-1} + \Pi_t b_{t-1},$$

such that the face value payment of indexed debt b_{t-1} is scaled by the level of gross inflation Π_t. T_t here denotes gross lump-sum taxes raised by the government on the income stream enjoyed by the household. The household optimality conditions are given by

$$
\begin{aligned}
\{c_t\}: &\qquad u'(c_t) = \lambda_t P_t \\
\{B_t\}: &\qquad \lambda_t Q_t = \beta \mathbb{E}_t \lambda_{t+1} \\
\{b_t\}: &\qquad \lambda_t q_t = \beta \mathbb{E}_t \lambda_{t+1} \Pi_{t+1}.
\end{aligned}
$$

Using market clearing $c_t = Y \ \forall t$, we obtain standard bond pricing kernels:

$$Q_t = \beta \mathbb{E}_t \left(\frac{P_t}{P_{t+1}} \right); \qquad q_t = \beta,$$

reflecting that inflation-indexed bonds are completely risk-free in a world of representative households and perfect credit markets.

Government: The simple flow budget constraint of the government is given by

$$B_{t-1} + \Pi_t b_{t-1} = P_t T_t + Q_t B_t + q_t b_t,$$

with the bond pricing kernels being given by the household first-order conditions. For simplicity, we assume here that $s_t = T_t$, i.e., we abstain from government spending without loss of generality.

Under this simple setting, a standard fiscal policy rule in a Fisherian model sees a government react to deviations of both types of debt in *real* terms from their respective steady-state levels:

$$\frac{\tau_t}{\tau} = \left(\frac{s_{B,t-1}}{s_B} \right)^{\gamma_B} \left(\frac{s_{b,t-1}}{s_b} \right)^{\gamma_b} e^{\zeta_t},$$

where $\tau_t \equiv \frac{T_t}{Y}$ are surpluses raised by the government as a fraction of output, and $s_{B,t} \equiv \frac{Q_t B_t}{P_t Y}, s_{b,t} \equiv \frac{q_t b_t}{P_t Y}$ are the real market values of the two existing types of debt. ζ_t is a standard AR(1) shock to the quantity of lump-sum taxes raised, and the policy reaction coefficients are given by γ_B and γ_b.

The central bank follows a simplified monetary rule for maximum analytical tractability:

$$\frac{R_{n,t}}{R_n} = \left(\frac{\Pi_t}{\Pi} \right)^{\phi},$$

where $R_{n,t} = 1 + i_t$ is the gross nominal interest rate. Note also that under the present setting $Q_t = \frac{1}{R_{n,t}}$, i.e., the price of the nominal bond must be the inverse of the gross nominal interest rate.

Linearizing the simple Fisherian model: we denote variables in their log-deviations from steady-state with hats. A simple (log)-linearization around the zero-inflation steady-state gives us the following system of equations:

- Nominal bond prices: using $Q_t = \frac{1}{R_{n,t}}$ and $Q_t = \beta \mathbb{E}_t \left(\frac{1}{\Pi_{t+1}} \right)$, we get $\frac{1}{R_{n,t}} = \beta \mathbb{E}_t \left(\frac{1}{\Pi_{t+1}} \right)$. To a first-order approximation,

$$\hat{r}_{n,t} = \mathbb{E}_t \hat{\pi}_{t+1}. \tag{C.4}$$

- Monetary rule: taking simply logs of the monetary rule, we get

$$\hat{r}_{n,t} = \phi \hat{\pi}_t. \tag{C.5}$$

- Law of motion of debt: we take the law of motion of debt, divide it by $(P_t Y)$, and apply the debt share definitions from above:

$$\frac{Q_t B_t}{P_t Y} + \frac{q_t b_t}{P_t Y} + \frac{T_t}{Y} = \frac{Q_{t-1} B_{t-1}}{P_{t-1} Y} \frac{1}{Q_{t-1}} \frac{P_{t-1}}{P_t} + \frac{q_{t-1} b_{t-1}}{P_{t-1} Y} \frac{1}{q_{t-1}} \Pi_t \frac{1}{\Pi_t}$$
$$\Leftrightarrow s_{B,t} + s_{b,t} + \tau_t = s_{B,t-1} R_{n,t-1} \frac{1}{\Pi_t} + s_{b,t-1} \frac{1}{\beta}.$$

This expression cannot be log-linearized exactly, but we can obtain a first-order Taylor approximation:

$$s_B + s_b + \tau + (s_{B,t} - s_B) + (s_{b,t} - s_b) + (\tau_t - \tau)$$
$$= s_B R_n \frac{1}{\Pi} + s_b \frac{1}{\beta} + \frac{R_n}{\Pi}(s_{B,t-1} - s_B) + \frac{s_B}{\Pi}(R_{n,t-1} - R_n) - \frac{s_B R_n}{\Pi^2}(\Pi_t - \Pi) + \frac{1}{\beta}(s_{b,t-1} - s_b).$$

Getting rid of the steady-state, using $\Pi = 1$ and $R_n = \frac{1}{\beta}$, and dividing all elements by s_B, we obtain:

$$\hat{s}_{B,t} + \frac{s_b}{s_B}\hat{s}_{b,t} + \frac{\tau}{s_B}\hat{\tau}_t = \frac{1}{\beta}\left[\hat{s}_{B,t-1} + \hat{r}_{n,t-1} - \hat{\pi}_t + \frac{s_b}{s_B}\hat{s}_{b,t-1}\right].$$

Now, in steady-state, $\bar{Q} = \bar{q} = \beta$. Thus, $\frac{s_b}{s_B} = \frac{b}{B}$. Also, from the flow budget constraint evaluated at steady-state, we know that $PT = (1-\beta)(B+b)$, such that the steady-state taxation term can be rewritten as $\frac{\tau}{s_B} = \frac{\tau PY}{QB} = \frac{1}{\beta}\frac{TP}{B} = \frac{1-\beta}{\beta}\left(\frac{B+b}{B}\right)$. Combining these elements, we finally obtain the linearized budget constraint:

$$\hat{s}_{B,t} + \frac{b}{B}\hat{s}_{b,t} = \frac{1}{\beta}\left[\hat{s}_{B,t-1} + \hat{r}_{n,t-1} - \hat{\pi}_t + \frac{b}{B}\hat{s}_{b,t-1} - (1-\beta)\frac{B+b}{B}\hat{\tau}_t\right]. \tag{C.6}$$

- Fiscal rule: the fiscal rule can be log-linearized exactly:

$$\hat{\tau}_t = \gamma_B \hat{s}_{B,t-1} + \gamma_b \hat{s}_{b,t-1} + \zeta_t. \tag{C.7}$$

We can combine equations (C.4)+(C.5) and (C.6)+(C.7), respectively, to attain the following system of difference equations:

$$\mathbb{E}_t \hat{\pi}_{t+1} = \phi \hat{\pi}_t, \tag{C.8}$$

$$\hat{s}_{B,t} + \frac{b}{B}\hat{s}_{b,t} = \frac{1}{\beta}\left[\hat{s}_{B,t-1} + \frac{b}{B}\hat{s}_{b,t-1}\right] + \frac{1}{\beta}\left[\hat{r}_{n,t-1} - \hat{\pi}_t - (1-\beta)\frac{B+b}{B}\zeta_t\right] - \frac{1-\beta}{\beta}\frac{B+b}{B}\left[\gamma_B \hat{s}_{B,t-1} + \gamma_b \hat{s}_{b,t-1}\right]. \tag{C.9}$$

The presence of indexed debt introduces a third first-differenced variable into this policy-side system of equations, $\hat{s}_b$. We therefore need to close this system with some further condition.[31]

—

The first idea is a straightforward *simplification* of the problem by imposing the following relationship in terms of the debt issuance between the two types of debt offered: $\gamma_b = \frac{b}{B}\gamma_B$, such that the issuance of the two types of debt is inversely proportional to their relative share of overall outstanding debt in the steady-state. The resulting log-linearized 'fiscal' equation can then be expressed as:

$$\hat{s}_{B,t} + \frac{b}{B}\hat{s}_{b,t} = \frac{1}{\beta}\left[1 - (1-\beta)\frac{B+b}{B}\gamma_B\right]\left[\hat{s}_{B,t-1} + \frac{b}{B}\hat{s}_{b,t-1}\right] + \frac{1}{\beta}\left[\hat{r}_{n,t-1} - \hat{\pi}_t - (1-\beta)\frac{B+b}{B}\zeta_t\right].$$

Under this set-up, we can define the novel state variable $\hat{s}_{D,t}$, where D stands for the "total debt stock", such that $\hat{s}_{D,t} \equiv \hat{s}_{B,t} + \frac{b}{B}\hat{s}_{b,t}$.

$$\hat{s}_{D,t} = \frac{1}{\beta}\left[1 - (1-\beta)\frac{B+b}{B}\gamma_B\right]\hat{s}_{D,t-1} + \frac{1}{\beta}\left[\hat{r}_{n,t-1} - \hat{\pi}_t - (1-\beta)\frac{B+b}{B}\zeta_t\right]. \tag{C.10}$$

Together with equation (C.8), we get the following system of equations:

$$\underbrace{\begin{bmatrix} 1 & 0 \\ \frac{1}{\beta} & 1 \end{bmatrix}}_{A_0} \mathbb{E}_t \begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{s}_{D,t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \phi & 0 \\ 0 & \frac{1}{\beta}\left[1 - (1-\beta)\frac{B+b}{B}\gamma_B\right] \end{bmatrix}}_{A_1} \begin{bmatrix} \hat{\pi}_t \\ \hat{s}_{D,t} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\beta}\left[\hat{r}_{n,t-1} - (1-\beta)\frac{B+b}{B}\zeta_t\right]. \end{bmatrix}$$

Note that $A_0^{-1} = \begin{bmatrix} 1 & -\frac{1}{\beta} \\ 0 & 1 \end{bmatrix}$. We are interested in the determinacy properties of the matrix

$$Z \equiv A_0^{-1}A_1 = \begin{bmatrix} 1 & -\frac{1}{\beta} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \phi & 0 \\ 0 & \frac{1}{\beta}\left[1 - (1-\beta)\frac{B+b}{B}\gamma_B\right] \end{bmatrix} = \begin{bmatrix} \phi & 0 \\ -\frac{1}{\beta}\frac{1}{\beta}\left[1 - (1-\beta)\frac{B+b}{B}\gamma_B\right] & \frac{1}{\beta}\left[1 - (1-\beta)\frac{B+b}{B}\gamma_B\right] \end{bmatrix}.$$

The eigenvalues of Z are given by

$$\left\{\phi, \quad \frac{1}{\beta}\left[1 - (1-\beta)\frac{B+b}{B}\gamma_B\right]\right\}.$$

[31] Note that we restrict all policy reaction parameters to be *non-negative* in the following discussion for expositional simplicity: $\phi \geq 0, \gamma_B \geq 0, \gamma_b \geq 0$.

Following Blanchard and Kahn (1980), saddle-path stability of this system requires one eigenvalue inside and one outside the unit circle. We thus have saddle-path stability under the following policy combinations:

- "Active monetary/passive fiscal": $\phi > 1$ & $\gamma_B > \frac{B}{B+b}$,
- "Passive monetary/active fiscal": $\phi < 1$ & $\gamma_B < \frac{B}{B+b}$,

such that the restriction to pursue active fiscal policy is *tighter* relative to the result established in Leeper (1991). Intuitively, 'fiscal activism' now requires the fiscal reaction coefficient to be sufficiently far away from unity, since 'almost passive' fiscal reactions might actually yield passive fiscal authorities due to the debt stock not being sufficiently devalued for a stable path of the price level, as the change in the real value of non-indexed debt might not be enough to cover the required devaluation of the debt stock in the face of a fiscal shock.

—

The other possible approach to close the economy is the 'shadow economy' trick used by Bianchi et al. (2023). Following Bianchi et al. (2023), we construct a 'shadow economy' that has the same monetary block, but a simplified fiscal block with *only* non-indexed debt: $b_t = 0\ \forall t$. The underlying assumption behind this 'shadow economy' is tantamount to postulating that the fiscal authority *only* reacts with non-indexed debt in response to fiscal disturbances: when a spending shortage or surplus occurs, the fiscal authority only reacts by adjusting the stock of non-indexed debt.[32] This simplified fiscal block is summarized by the log-linearized equation

$$\hat{s}_{B,t} = \frac{1}{\beta}\left[1-(1-\beta)\gamma_B\right]\hat{s}_{B,t-1} + \frac{1}{\beta}\left[\hat{r}_{n,t-1} - \hat{\pi}_t - (1-\beta)\zeta_t\right], \tag{C.11}$$

which is the standard Fisherian model with non-indexed debt only. Combining equations (C.8), (C.9), and (C.11), we obtain the following system of linear difference equations:

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{\beta} & \frac{b}{B} & 1 \\ \frac{1}{\beta} & 0 & 1 \end{bmatrix}}_{=A_0} \mathbb{E}_t \begin{bmatrix} \hat{\pi}_{t+1} \\ \hat{s}_{b,t+1} \\ \hat{s}_{B,t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \phi & 0 & 0 \\ 0 & \frac{1}{\beta}\left[\frac{b}{B} - (1-\beta)\frac{B+b}{B}\gamma_b\right] & \frac{1}{\beta}\left[1-(1-\beta)\frac{B+b}{B}\gamma_B\right] \\ 0 & 0 & \frac{1}{\beta}\left[1-(1-\beta)\gamma_B\right] \end{bmatrix}}_{=A_1} \begin{bmatrix} \hat{\pi}_t \\ \hat{s}_{b,t} \\ \hat{s}_{B,t} \end{bmatrix} + C \begin{bmatrix} \hat{r}_{n,t} \\ \zeta_t. \end{bmatrix}$$

The determinacy properties of this system again depend on $Z \equiv A_0^{-1}A_1$. This matrix Z is given by:

$$Z = \begin{bmatrix} \phi & 0 & 0 \\ 0 & \frac{1}{\beta}\left[1-(1-\beta)\frac{B+b}{b}\gamma_b\right] & -\frac{1-\beta}{\beta}\gamma_B \\ 0 & 0 & \frac{1}{\beta}\left[1-(1-\beta)\gamma_B\right], \end{bmatrix}$$

[32]Following Bianchi et al. (2023), an otherwise more common example related to standard monetary economics are 'shadow economies' used to close models with price rigidities, in which the 'shadow economy' is one with flexible prices and no inefficient shocks.

with corresponding eigenvalues

$$\left\{\phi, \quad \frac{1}{\beta}\left[1-(1-\beta)\gamma_B\right], \quad \frac{1}{\beta}\left[1-(1-\beta)\frac{B+b}{b}\gamma_b\right]\right\},$$

and since the system consists of one forward-looking and two backward-looking variables, we now need one eigenvalue outside the unit circle in modulus and two inside to ensure determinacy. Relevant policy mixes inducing saddle-path stability are thus given by:[33]

- *AM/PF*: $\phi > 1, \gamma_B > 1, \gamma_b > \frac{b}{B+b}$;
- *PM/AF$_B$/PF$_b$*: $\phi < 1, \gamma_B < 1, \gamma_b > \frac{b}{B+b}$;
- *PM/AF$_B$/PF$_b$*: $\phi < 1, \gamma_B > 1, \gamma_b < \frac{b}{B+b}$,

such that active fiscal policy can choose what type of debt to actively take on in response to fiscal shocks. For better intuition, it is instructive to consider two specific types of debt policies:

- $\gamma_b = \gamma_B \frac{b}{B+b}$: under such a debt issuance rule, there are only two distinct eigenvalues, similar to the case above due to the induced co-movement of both types of debt in response to deviations from steady-state.
- $\gamma_b = \gamma_B$: such a fiscal rule would indicate a fiscal authority reacting with similarly-sized deviations of both types of debt from their respective steady-state values in response to shocks to the revenue generated by taxation. In that case, we recover the eigenvalues

$$\left\{\phi, \frac{1}{\beta}\left[1-(1-\beta)\gamma_B\right], \frac{1}{\beta}\left[1-(1-\beta)\frac{B+b}{b}\gamma_B\right]\right\}, \tag{C.12}$$

  depending on one fiscal policy reaction parameter and one monetary policy reaction parameter only. Policy combinations supporting saddle-path stability are then given by:

  – *AM/PF*: $\phi > 1, \gamma_B > 1, \gamma_B > \frac{b}{B+b}$;
  – *PM/AF-1*: $\phi < 1, \gamma_B > 1, \gamma_B < \frac{b}{B+b}$;
  – *PM/AF-2*: $\phi < 1, \gamma_B < 1, \gamma_B > \frac{b}{B+b}$.

  Note that case PM/AF-1 is not possible, so the only viable active fiscal policy combination here is PM/AF-2. Clearly, relative to a standard Leeper (1991)-model, it implies tighter bounds, ruling out 'fully active' fiscal policies of the type $\gamma_B = 0$ so long as $b > 0$: fiscal policy therefore cannot be 'fully active' in the traditional sense, as such behavior would mean an unbounded devaluation of the debt stock as not enough surpluses are raised to service the spiraling costs of indexed debt.

This Fisherian model therefore established that indexed debt can greatly influence the conduct of 'active' fiscal policy in the sense of Leeper (1991): in particular, the conditions for active saddle-

[33] In the following list, "A" refers to *active*, "P" refers to *passive* (in the sense of Leeper (1991)), "M" refers to *monetary*, and "F" refers to *fiscal* policies. Additionally, fiscal policies are subscripted with non-indexed ("B") and indexed ("b") debt.

path stable fiscal policy are unequivocally tightened across all our examples for specific policy rule combinations. While these insights rationalize the importance of indexed debt for determinacy properties of dynamic economic models, the implications of this simple model need not directly translate to state-of-the-art dynamic macroeconomic models.

D Additional evidence

The distribution of government debt holdings across households

Looking at publicly available microdata, a case for the relevance of the distribution of indexed debt in the household portfolio can be made. Please note that we leverage data on US households here, not on UK households, since we could not find publicly available microdata on UK households that explicitly capture holdings of inflation-indexed sovereign debt.

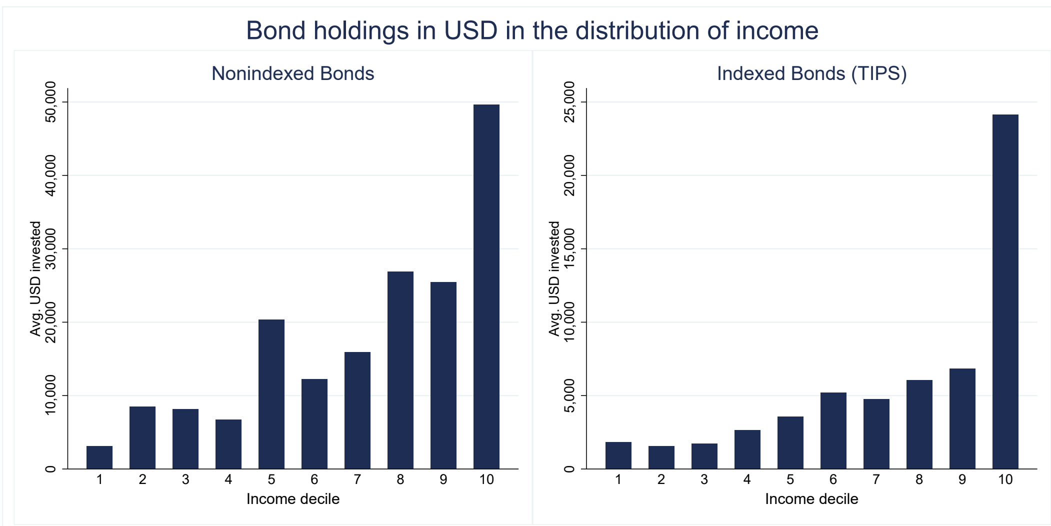


Figure D.1: Distribution of indexed and non-indexed debt holdings across household income deciles, denoted in real (2017) USD. Data source: Survey of Consumer Finances (US); sample period: 2014-2019.

Figure D.1 plots the real (2017) Dollar value of nonindexed and indexed government debt holdings of households questioned in the US Survey of Consumer Finances (SCF), separated by income deciles.[34,35] The left-hand panel of figure D.1 reflects the well-known left-skew of bond holdings of households in the income distribution, by which households at the upper end of the income distribution hold a significantly larger piece of sovereign bonds. The right-hand panel of figure D.1 reflects a less well-known observation: this left skew is *vastly* more pronounced for indexed sovereign bonds (TIPS), with the top income decile holding almost 40% of all outstanding TIPS in

[34] We chose income deciles due to their clear definition in the survey with a single question. Constructing individual wealth variables is possible with the survey data, albeit this is subject to individual choices about what to consider as household wealth. For most definitions of wealth, the results continue to hold qualitatively.

[35] Admittedly, using household survey responses to develop a profile of sovereign bond ownership is very far from perfect, since most sovereign bonds owned by households are only held indirectly through insurance companies and pension funds (and a vast share of sovereign bonds are held by Monetary Financial Institutions). Thus, finding the distribution of bonds held through such investment vehicles is of primary importance.

the sample. This relative difference in the distribution of indexed bond holdings relative to nonindexed bond holdings is likely related to severe differences in the pricing of indexed and nonindexed bonds relative to the benchmark model above, which in turn would have significant influence on the determination of the price level.[36]

Further details on the Local Projections

To shed further light on the evidence presented in section 2.2, we here provide the result tables from the local projection on UK data presented in figure 4 and introduce additional evidence using US data with a similar exercise.

First, table D.1 summarizes the results given in figure 4, specifying the exact coefficients on the interaction effect of $\Delta\omega_t\varepsilon_t$ and the individual effects of the change in the indexed debt share $\Delta\omega_t$ and the identified fiscal shock ε_t on cumulative price level change from the pre-shock period -1 until the period specified above all columns.

	Dependent variable: log(Cumulative Inflation)							
Lag periods:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Fiscal Shock	-0.01**	-0.01	-0.01	-0.01	-0.02	-0.02	-0.02	-0.02
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)
Index Share	0.02***	0.02**	0.03**	0.02	0.02	0.02	0.03	0.03
	(0.00)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Fiscal Shock × Index Share	0.10*	0.09	0.20	0.26*	0.39**	0.40*	0.60**	0.81***
	(0.06)	(0.10)	(0.13)	(0.14)	(0.18)	(0.21)	(0.24)	(0.27)
Additional Controls	Y	Y	Y	Y	Y	Y	Y	Y
Observations	155	154	153	152	151	150	149	148
R^2	0.412	0.518	0.559	0.630	0.592	0.599	0.575	0.602

Table D.1: Local Projection results for the UK. Additional controls include past four-quarter lags of GDP growth, the Bank Rate, real exchange rate growth, and year and recession dummies. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction).

While the share of indexed debt itself does not seem to impact medium-term inflation significantly, the interaction effect of the share of indexed debt with the identified fiscal shock follows the pattern given in figure 4.

To ensure that we are not solely picking up variation idiosyncratic to the UK, we utilize again the data provided by Mierzwa (2024) within an econometric specification adjacent to Cloyne et al. (2023), but here in relation to the series of US fiscal shocks identified therein. We leverage the number of identified fiscal shocks and estimate the same local projection specification (equation (4)) to estimate the role played by inflation-indexed debt in exacerbating the effects of fiscal spending shocks. Table D.2 and figure D.2 summarize this exercise for the entirety of available data since

[36]Reaching the levels of skewness observed in the cross-sectional distribution of TIPS holdings is far from an easy feat. Thinking about this issue will be of utmost importance once the more urgent issues around the basic definition of the model are cleared.

1980, which is the beginning of the sample period for which we have fiscal shocks.[37]

	Dependent variable: log(Cumulative Inflation)							
Lag periods:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Fiscal Shock	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)
Index Share	0.00	-0.01	-0.01	-0.01	-0.00	-0.00	-0.01	-0.01
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Fiscal Shock × Index Share	0.02	0.01	0.06	0.15**	0.19***	0.21**	0.22**	0.24***
	(0.03)	(0.08)	(0.08)	(0.06)	(0.07)	(0.09)	(0.10)	(0.09)
Additional Controls	Y	Y	Y	Y	Y	Y	Y	Y
Observations	161	160	159	158	157	156	155	154
R^2	0.324	0.371	0.474	0.531	0.543	0.542	0.559	0.554

Table D.2: Local Projection results for the US. Additional controls include past four-quarter lags of GDP growth, the Federal Funds Rate, real exchange rate growth, and year and recession dummies. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction).

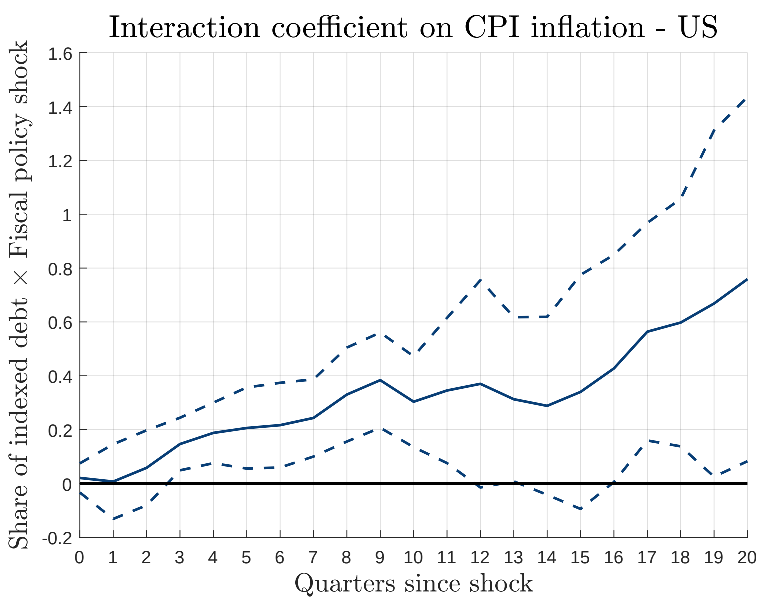


Figure D.2: IRFs implied by a local projection in the style of equation (4). The control vector Z consists of the first four lags of the real GDP growth rate, the short-run nominal interest rate, the change in the weighted real exchange rate, and a same-period recession indicator. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction). Confidence intervals are provided at the 90% level. Sample length: 1980 Q1 - 2019 Q4.

The results here paint a supporting picture, as the interaction effect between the change in the share of inflation-indexed debt and the identified fiscal shock appears to be statistically significant in the medium-term again, even though the level of the effect is not as pronounced as in the UK.

Given the institutional arrangements in the US and the main estimation exercise in the paper, we use the US example to provide another piece of evidence by focusing *only* on fiscal policy surprises

[37] We thereby utilize comparable time periods in our analysis of both the US and the UK.

occurring in periods that can be considered as being supported by an *active fiscal policy* in the sense of Leeper (1991). To be precise, we leverage the Bayesian DSGE estimation of Chen et al. (2022), assigning the label of 'active fiscal policy' to periods in which the posterior probability of a fiscally dominant regime exceeds 0.8. This leaves us with 44% (33/75) of the original shock observations in the period 1980-2019. The resulting exercise with the reduced shock sample yields the results presented in table D.3 and in figure D.3.

Dependent variable: log(Cumulative Inflation)								
Lag periods:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Fiscal Shock (Active Periods)	-0.00	-0.00	-0.00	-0.00	-0.01	-0.01	-0.01	-0.01
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Index Share	0.00	-0.00	-0.00	-0.00	0.00	0.00	-0.00	-0.00
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Fiscal Shock × Index Share	0.01	-0.08	-0.02	0.18	0.18	0.14	0.12	0.28
	(0.07)	(0.13)	(0.20)	(0.21)	(0.22)	(0.25)	(0.27)	(0.29)
Additional Controls	Y	Y	Y	Y	Y	Y	Y	Y
Observations	161	160	159	158	157	156	155	154
R^2	0.329	0.381	0.480	0.534	0.545	0.546	0.563	0.555

Table D.3: Local Projection results for the US with active fiscal policy shocks, following Chen et al. (2022). Additional controls include past four-quarter lags of GDP growth, the Federal Funds Rate, real exchange rate growth, and year and recession dummies. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction).

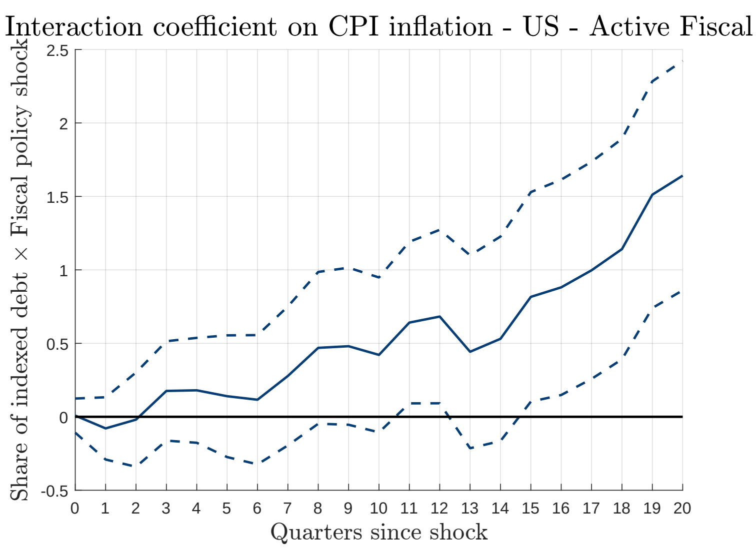


Figure D.3: IRFs implied by a local projection in the style of equation (4). The control vector Z consists of the first four lags of the real GDP growth rate, the short-run nominal interest rate, the change in the weighted real exchange rate, and a same-period recession indicator. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction). Confidence intervals are provided at the 90% level. Sample length: 1980 Q1 - 2019 Q4.

Restricting our focus only on the sample periods to which we can assign a relatively high probability of a fiscally dominant regime, we can observe at first no significant response of cumulative inflation to the fiscal shock, followed by a gradually significant and positive response in the interaction effect of the indexed debt share and the identified fiscal shock in the medium-term, broadly in line with our previous results. The magnitude of the interaction effect more or less doubles relative to our previous analysis without the restriction on periods of active fiscal policy only.

E Omitted plots

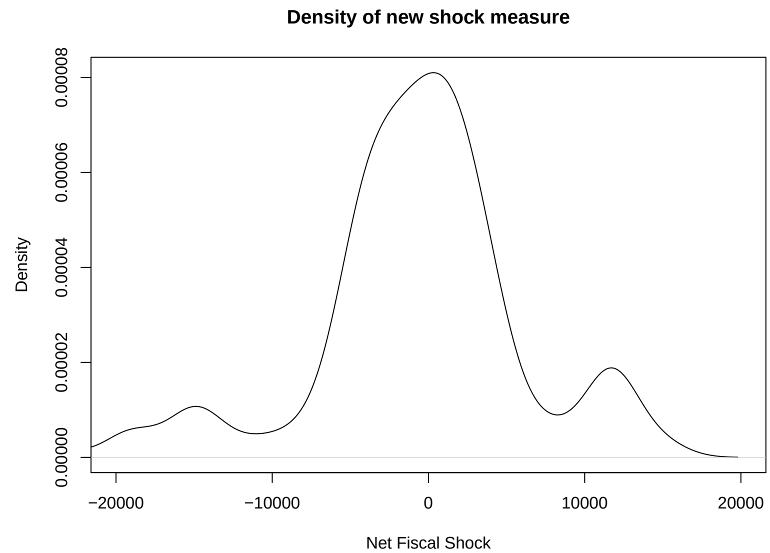


Figure E.1: Estimated Kernel density of the net shock measure described by equation (2) for the United Kingdom, 2000-2010.