# Debt Indexation, Determinacy, and Inflation\*

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#### **Abstract**

Contrary to popular belief, inflation-indexed government debt can boost inflationary outcomes in response to deficit shocks, conditional on a sufficient lack of future fiscal backing. I formalize this insight through a state-of-the-art calibrated HANK model with multiple asset types, showing that the inflationary effect of a 1% deficit-to-GDP shock increases by 0.21 percentage points when the share of inflation-indexed debt moves from zero to levels observed in the U.K. The main mechanism is that the price level becomes partially backward-looking through the presence of inflation-indexed debt. Empirical evidence from high-powered fiscal deficit shocks supports this finding, which has additional implications for the distinction between 'fiscally-led' mechanisms and 'HANK-type' mechanisms surpassing Ricardian equivalence.

**Keywords:** Debt Indexation, Government Debt, Fiscal-Monetary Interactions.

**JEL Codes:** E52, E63, H63

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#### 1 Introduction

Rivaling theories contribute to our understanding of the 2021-2023 inflationary episode, ranging from labor market tightness, through supply chain disruptions, to fiscal deficits and the associated monetary response. This paper adds a novel, yet overlooked aspect to this ongoing debate: the *composition* of government debt, with a particular distinction between purely nominal debt and *inflation-indexed debt*, which differs from the commonly modeled government obligations as its face value changes with the gross rate of inflation. As the government budget balance plays a crucial role for aggregate demand, and thereby co-determines the price level, an interesting feedback loop arises when the face value of a part of government debt itself changes with gross inflation. In this paper, I show the qualitative and quantitative importance of this mechanism, conditional on fiscal deficits being not completely backed by future surpluses.

Indexed debt is not a mere theoretical curiosity. Figure 1 shows the share of inflation-indexed debt as part of the overall sovereign debt stock over time in a number of countries. While there is considerable heterogeneity across countries, indexed bonds are present across the board, and have been so for the past three decades. This paper mostly focuses on the United Kingdom (U.K.) and the United States (U.S.), since these two indexed debt markets are the largest ones in both absolute and relative terms.<sup>1</sup>

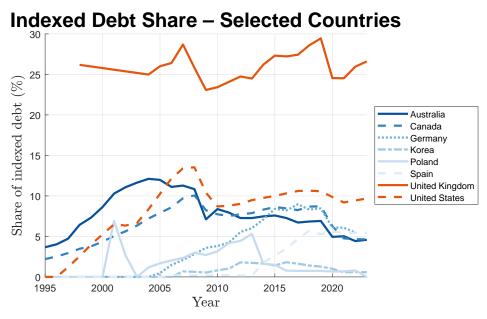


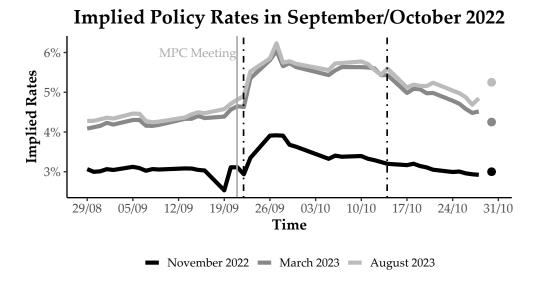
Figure 1: The share of inflation-indexed debt in the total sovereign debt portfolio over time. Data source: BIS (2024).

Analyzing the role of the government debt structure for inflation requires a delicate treatment of the interactions between fiscal and monetary policy. While a *fiscally-led policy mix*; that is, a

<sup>&</sup>lt;sup>1</sup>Ex-post realized real yields differ between the two types of bonds, for instance due to inflation risk premia and liquidity risk premia (Gürkaynak et al., 2010). Looking at market yields at constant maturity for the U.S., historical data provided by the Fed Board of Governors (2024) confirm a permanently positive differential in market yields on 10-year constant-maturity TIPS relative to standard Treasuries on every day since 2003.

fiscal authority committing to policy *not* adhering to its intertemporal budget constraint, is not a predicament for an analysis of the role of deficits for inflation, it enhances the role of fiscal policy as drivers of inflationary dynamics in macroeconomic general equilibrium models (Leeper, 1991; Sims, 2011; Ascari et al., 2023; Rachel and Ravn, 2025). I therefore motivate this paper further with the help of a specific fiscal measure that can be seen as a real-world example of fiscal policy forcing the hand of the monetary policy authority (and thus informing a possibly fiscally-led policy mix): the 'U.K. mini-budget' in September 2022.<sup>2</sup>

Figure 2 plots the market-implied monetary policy rates in the window around the mini-budget announcement and its subsequent cancellation. The first solid line depicts the date of a Bank of England Monetary Policy Committee (MPC) meeting, which occurred ahead of the announcement of the 'mini-budget' fiscal policy measure, with the MPC minutes being released on the 22nd of September 2022, one day ahead of the fiscal policy announcement. This timing is useful for the argument insofar as the meeting likely communicated the Bank of England's stance on future rate changes clearly, taking all available information up to that point into account (Braun et al., 2024). Nonetheless, implied policy rates rose sharply in the subsequent days, just after the announcement of the mini budget (denoted by the first dotted line), with the shift amounting to a 120bps increase in expected policy rates for the upcoming twelve months ahead. After the scraping of the mini-budget (second dotted line), expected policy rates swiftly returned to their 'pre-shock' levels.<sup>3</sup>



**Figure 2:** Expectations of nominal interest rates in the United Kingdom for three scheduled MPC meetings around the 'mini-budget' announcement in September 2022, defined as the break-even rates under which Overnight Indexed Swaps are fairly priced at the prevailing forward interest rate curve (Bloomberg data series GB0BFR). The dots at the end reflect the policy rates after each meeting has taken place.

<sup>&</sup>lt;sup>2</sup>For a more detailed argument related to this fiscal shock, see Leeper (2023), NIESR (2022), and section 2.1.

<sup>&</sup>lt;sup>3</sup>The expected monetary policy response was partially driven by a concurrent funding mismatch in liability-driven investment strategies of defined-benefit pension funds that were closely tied to movements in yields of sovereign bonds. See Pinter (2023) for a detailed exposition of this point.

This event therefore resonates well with the possible idea of (at least partially) fiscally-led policy mixes: financial market participants clearly expected changes to the monetary policy stance beyond the very short term, plausibly in response to an announced fiscal policy measure, as also explored in section 2.1, although risk or liquidity premiums likely also contributed to the observed movements.

I motivate the relevance of indexed debt for inflation further in the paper, using data to narratively pin down the shock induced by the 'U.K. mini-budget', tracking it through market expectations on sovereign deficits and government bond price revaluations. With that data, I estimate a sizable *fiscal inflation multiplier*, which grows with the share of inflation-indexed debt. My second empirical finding, based on an exercise with local projections using exogenously identified shocks, shows that inflation-indexed debt boosts observed consumer price inflation in response to deficit shocks.

I next move on to introducing inflation-indexed debt in a one-equation model to pin down the main mechanism by which indexed debt matters for the price level through the government debt valuation equation: the price level becomes a state variable without further ado, such that previous price levels matter for the determination of the current price level, even without further sources of stickiness in the economy. For such an economy with inflation-indexed debt, I prove the uniqueness of the corresponding dynamic stationary general equilibrium.

Thereafter, I analyze the combined effects of household heterogeneity and the presence of inflationindexed debt to fiscal deficit shocks in a full heterogeneous-agent New Keynesian model á la Kaplan et al. (2018), making use of the methods pioneered in Auclert et al. (2021) to solve heterogeneousagent models up to first-order in aggregate variables, while preserving heterogeneity with respect to the individual agents in this economy. I additionally pay attention to the different insurance properties of inflation-indexed debt in models with incomplete markets, accounting for the possible windfall gains that can be earned by governments as in Brunnermeier et al. (2024). Inflationindexed debt matters quantitatively by increasing the volatility of inflation by roughly 2.6% per each percentage point increase of the share of indexed debt in the overall debt portfolio. In terms of the level impact, for an economy with a close to 30% share of inflation-indexed debt in the sovereign debt portfolio (as observed in the U.K.), the resulting inflation rate increases by 0.2 percentage points in response to a 1% deficit-to-GDP shock relative to a baseline case without inflation-indexed debt, which is quantitatively highly relevant. I furthermore establish that the classic notions of 'active/passive monetary/fiscal policy', as derived by Leeper (1991), do not directly translate into the world with inflation-indexed debt, even though similarities in the determination of saddle pathstable equilibria prevail.

As a final corollary, I prove that inflation-indexed debt allows to discriminate to some degree between various mechanisms overcoming Ricardian Equivalence. Once inflation-indexed debt is present, there is no complete equivalence between mechanisms overcoming Ricardian equivalence driven by incomplete markets and those arising through fiscally-led policy mixes.

#### Literature Review

This paper contributes to the burgeoning literature on fiscal-monetary interactions, pioneered in Sargent and Wallace (1981) and formalized through Leeper (1991). Initial contributions focusing on the possibility of a fiscally-led policy mix include Sims (1994) and Woodford (1995), who also coined the terminology behind the 'Fiscal Theory of the Price Level' (FTPL), whose mechanisms apply in the presented framework as well.<sup>4</sup> More succinct summaries of the literature are provided by Leeper and Leith (2016) and Cochrane (2023). Bassetto and Cui (2018), Miao and Su (2021), Liemen and Posch (2022), Ascari et al. (2023), Bianchi et al. (2023), and Smets and Wouters (2024) provide advances in analyzing fiscally-led policy mixes in OLG and New Keynesian models, while empirical support for the possibility of fiscally-driven inflation has been developed in Cochrane (2022a), Cochrane (2022b), Chen et al. (2022), Barro and Bianchi (2023), Cloyne et al. (2023), and Ascari et al. (2024).<sup>5</sup> A narrative example of a specific fiscal shock informing inflation rates is provided by Hazell and Hobler (2024), who focus on the 2021 Georgia Senate election runoff.

The focus of models emphasizing the link between fiscal-monetary policy interactions and the price level recently shifted towards models with explicitly *non-Ricardian* fiscal policy, thereby making real interest rates endogenous to fiscal frameworks. This endogeneity is important insofar as fiscal-monetary interactions ultimately constitute criteria that constrain the *transversality condition* on government debt to hold for only *one* candidate price level, but the transversality condition itself depends on the real interest rate.<sup>6</sup>

Many such models fall under the category of heterogeneous-agent New Keynesian (HANK) frameworks, or try to overcome Ricardian equivalence through other mechanisms causing market incompleteness. Brunnermeier et al. (2022), Kaplan et al. (2023), Campos et al. (2024), and Kwicklis (2025) have all applied fiscal price level determination to rich heterogeneous-agent frameworks. A second type of such models considers New Keynesian models with mortality frictions, exemplified by Angeletos et al. (2024), Dupraz and Picco (2025), Nakamura et al. (2025), and Rachel and Ravn (2025). Angeletos et al. (2024) negate the need to consider fiscally-led policy mixes when determining the price level, finding quantitatively identical responses of inflation to expansionary fiscal shocks in such NK-OLG models under monetary-led policy mixes. An overview over the state-of-the-art is provided by Kaplan (2025b), and a complementing succinct mathematical description of the link between non-Ricardian properties of households and fiscal-monetary interactions can be

<sup>&</sup>lt;sup>4</sup>I do not explicitly define the FTPL in this paper. Instead, I follow Brunnermeier et al. (2022), who define 'core aspects', or themes, commonplace in the FTPL literature. By that definition, FTPL-type thinking is applicable when (a) governments issue some liabilities in their own currency, (b) the government faces a debt valuation equation over which it has some agency in place of an 'intertemporal budget constraint', (c) the price level plays a key role in ensuring debt sustainability in equilibrium, (d) additional assumptions, e.g., on fiscal-monetary policy mixes, complement equilibrium uniqueness, and (e) the framework could deliver price level uniqueness even without nominal frictions.

<sup>&</sup>lt;sup>5</sup>A complementary point is that the effects of monetary policy, in turn, are also constrained and informed by the underlying fiscal reaction function, as shown by Caramp and Silva (2023, 2025).

<sup>&</sup>lt;sup>6</sup>Brunnermeier et al. (2022) and Kaplan et al. (2023) provide conditions under which such models admit (unique) forward-looking equilibria expressed through the price level. Their notions of uniqueness are challenged by Hagedorn (2021, 2024), who argues that the endogeneity of the real interest rate in incomplete-markets models 'breaks' determinacy and allows a continuum of initial price levels to exist.

#### found in Kaplan (2025a).

My contribution is to introduce a second type of assets (inflation-indexed debt) with a feedback loop between asset holdings and the price level, quantifying the importance that such indexed debt has for inflation dynamics in a calibrated state-of-the-art macroeconomic model. In doing so, I specifically pay attention to the different insurance properties borne by the two types of debt, thereby mitigating concerns related to misspecifications of aggregate transversality conditions (Brunnermeier et al., 2024). I furthermore show that the non-Ricardian effects of fiscal policy arising through a fiscally-led policy mix and non-Ricardian effects arising through market incompleteness as in HANK are not necessarily equal once inflation-indexed debt is present in the model.

I also contribute explicitly to the literature on inflation-linked government bonds. Such bonds were introduced in economic and financial research long ago, especially in relation to the introduction of TIPS in the U.S. in 1997. One of the earliest contributions in this field is Fischer (1975), who derives household demand for such assets in a multi-asset framework. The special insurance properties of such inflation-linked debt are extensively discussed in Campbell and Shiller (1996), Barr and Campbell (1997), Garcia and van Rixtel (2007), Gürkaynak et al. (2010) and Andreasen et al. (2021), while Barro (2003) characterize the optimal issuance strategy for indexed debt under a fiscal authority operating under a tax-smoothing objective. Notably, Sims (2013) briefly mentions the possible detrimental consequences of indexed debt in fiscally-led policy frameworks. This paper builds on his remarks, providing a rigorous framework nesting his intuitions. Schmid et al. (2024) provide a systematic analysis of inflation-indexed debt as a policy tool, emphasizing its role as an ex-ante commitment device against inflation. In the contribution, I leverage the unique properties of inflation-indexed debt, which express themselves mostly through the induction of a backward-looking component in the government budget equilibrium condition and through the insurance premia they bear. This paper's focus thus effectively rests on the 'ex-post' effects that inflation-indexed debt can have in the face of expansionary government spending shocks.

In the later sections of the paper, I rely on modern computational methods to efficiently solve and estimate heterogeneous-agent methods, as in Kaplan et al. (2018) and Bayer and Luetticke (2020). In particular, I leverage the efficient computation algorithms pioneered in Auclert et al. (2021) and some of the refinements of Auclert et al. (2024b) to solve a model with heterogeneous households, two types of assets, and fiscal-monetary interactions.<sup>7</sup>

The rest of the paper is structured as follows. Section 2 exposes the relevance of indexed debt for materialized and expected inflation in the face of fiscal shocks, after which I introduce inflation-indexed debt in a one-equation framework in section 3. I introduce the main quantitative model in section 4. Section 5 discusses the calibration and the estimation method, and I present the quantitative findings in section 6. Section 7 isolates the main mechanism from the quantitative model in a tractable manner. Finally, section 8 concludes.

<sup>&</sup>lt;sup>7</sup>To motivate the relevance of household heterogeneity applied to holdings of sovereign debt, figure A.1 in the appendix provides evidence on the skew of household holdings of such debt, sorted by their respective income decile. This skew is even more pronounced for inflation-indexed debt.

## 2 Empirical evidence: indexed debt and inflationary dynamics

To motivate the relevance of indexed debt as a possible driver of the net present value of government debt and, therefore, of price level dynamics through the government debt valuation equation, I provide two pieces of evidence. First, I build on Hazell and Hobler (2024) and provide a narrative analysis of a powerful fiscal shock, but in an environment with high levels of indexed debt, finding decisively larger inflation multipliers in response to deficit shocks compared to theirs. Second, I employ a long-running series of exogenous fiscal policy shocks in a local projection applied to pin down the effects that inflation-indexed debt has on inflation itself when unexpected deficit-increasing measures occur.

# 2.1 Narrative evidence on the effect of deficits under indexed debt: the 2022 U.K. 'mini-budget'

I now provide narrative evidence on the effects of indexed debt through a clearly identified fiscal policy shock: the September 2022 U.K. fiscal policy announcement, commonly dubbed the 'minibudget'. I focus on this specific shock for two reasons: first, the event was largely unexpected in terms of its magnitude, allowing a clear identification of the effects of fiscal shortfalls on inflation. Second, this exercise is a complement to Hazell and Hobler (2024), who exploit probabilistic variation on Democrat Senate control around the 2021 Georgia Senate run-off election to infer the expected effects of expansionary fiscal policy on the price level. This paper provides a similar exercise in an environment with high levels of inflation-indexed debt, complementing their estimates.

The institutional setup of U.K. fiscal policy serves as an excellent device for identifying the 2022 'mini-budget' episode as a clear fiscal shock. Fiscal policy in the U.K. is shaped by regular fiscal announcements, which set up the broad guidelines for expected sovereign income and spending in a given fiscal year. From 1980 until 2016, the larger 'budget announcement' would usually occur in early spring (coinciding with the beginning of a new fiscal year), supplemented by shorter budget statements in the fall of the same year. Between 2017 and 2019, the regular budget announcement was moved to fall, with the spring season being used usually for supplementary statements. Beginning in 2020, the main budget announcement started ti take place in early spring again.

In spring 2022, then-Chancellor Rishi Sunak provided a budget statement, scheduled to be followed up by a full budget announcement in November 2022. In-between, and therefore outside of the usual bi-annual statement/announcement cycle, then-Chancellor Kwasi Kwarteng (who had since been appointed) presented a Ministerial Statement dubbed "The Growth Plan", with fiscal policy measures amounting to 150 Billion GBP, or approximately 5% of the GDP of the United Kingdom (NIESR, 2022). This statement did *not* constitute a budget announcement in the usual sense, being placed outside of the bi-annual budget statement cycle. The release of all budget statements made by the British government is usually supplemented with a concurrently released report by the Office for Budget Responsibility (OBR), an independent auditor supervising budgetary questions in the United Kingdom. In the case of the 'mini-budget', no such independent forecast of the

budgetary consequences of the statement was publicly released, as the ruling government denied the release of the forecast created by the OBR. Since then, the forecast that made by the OBR at that time has been released, although it is only of limited relevance with respect to the eventual policy measures announced as the report was made 18 days ahead of the budget announcement, thus not capturing the full extent of the fiscal policy proposals.<sup>8</sup>

The episode of early fall 2022 is characterized by this fiscal policy announcement and its expected effects. In particular, the effects of the fiscal policy announcement are (in the very short-term) plausibly shielded from unrelated monetary policy news (both in terms of the interest rate level and in terms of the signaling of the state of the economy), since the preceding Bank of England Monetary Policy Committee decision was released one day *before* the announcement of the fiscal policy measure, on September 22, 2022.

#### 2.1.1 The size of the shock

The most important question begets the *size* of the fiscal policy shock, which is *not* equal to the overall size of the fiscal package, as the policy announcement had been expected ahead of the budget statement. Ignoring this would bias the estimated effects of the policy announcement. Additionally, the probability of the fiscal policy measures being implemented upon announcement need not equal 100%, which might cause a downwards bias of the estimates.

To address these points, I follow the lead of Hazell and Hobler (2024), albeit with some limitations caused by impeded data availability. First, I establish the actual size of the expected debt-financing of the announced fiscal measures through their impact on the budget balance. This serves as the factual upper bound of the size of the shock component. Here, I utilize two estimates:

- The first is based on a direct reading of the corresponding budget statement. A reading of the implied policy measures yields an increase of the Debt Management Office's Net Financing Requirement from GBP 161.7 billion to GBP 234.1 billion in 2022-23, such that the corresponding upper bound of the shock (through the increase in borrowing requirements) would be GBP 72.4 billion.
- The second is an analysis by the *Institute of Fiscal Studies*, which predicted a GBP  $\sim$ 60 billion funding shortfall.<sup>10</sup>

The *shock* impacting the expected path of debt on the day of the announcement is not equivalent to the sum of additional deficits, since the policy package had been expected, but its full extent was simply not known. To isolate the shock component, I exploit private sector forecasts on *Public Sector Net Borrowing*, which are aggregated on a monthly basis and released by the U.K. Treasury.<sup>11</sup>

<sup>&</sup>lt;sup>8</sup>The forecast can be found under https://obr.uk/docs/dlm\_uploads/FOI-Information-on-preparatory-work-for-the-mini-budget.pdf.

<sup>&</sup>lt;sup>9</sup>The statement is available under https://www.gov.uk/government/publications/the-growth-plan-2022-documents.

<sup>&</sup>lt;sup>10</sup>The report is available under: https://ifs.org.uk/articles/mini-budget-response.

<sup>&</sup>lt;sup>11</sup>The forecast summaries are available under https://www.gov.uk/government/collections/data-forecasts.

These are forecasts of the factual borrowing requirement of the U.K. government in each fiscal year, provided both by financial market participants as well as other independent forecasters. I collect data on the forecasts provided in the period between September 1, 2022, and September 22, 2022 (i.e., until the day before the shock) and compare these forecasts with the ones collected between October 1, 2022, and October 10, 2022. Unfortunately, data is not collected at narrower time intervals around the 'mini-budget' announcement. This limitation causes - if anything - a downwards bias of the estimated deficit-inflation multiplier. 12

For October 2022 (i.e., after the announcement of the fiscal package), forecasts were provided by Barclays Capital, Goldman Sachs, JP Morgan, Beacon Economic Forecasting, CEBR, Heteronomics, ICAEW, Kern Consulting, and Oxford Economics. The total mean forecast revision of Public Sector Net Borrowing for the 2022-23 and 2023-24 Fiscal Years lies at GBP 47.4 billion, vastly exceeding all other non-crisis forecast revisions. This confirms the initial intuition that the 'mini-budget' shock was indeed economically significant and to a large degree unexpected. Given that this forecast revision is also below the upper bound of the shock size, the following analysis works with this estimate of a GBP 47.4 billion funding shortfall, equivalent to 1.27% of annual GDP in 2022 (GBP 47.4 billion / GBP 3.732 trillion). Relative to Hazell and Hobler (2024), the 'mini-budget' shock component equals 60% of the size of their shock relative to local nominal GDP.

#### 2.1.2 Linking the deficit shock to expected inflation

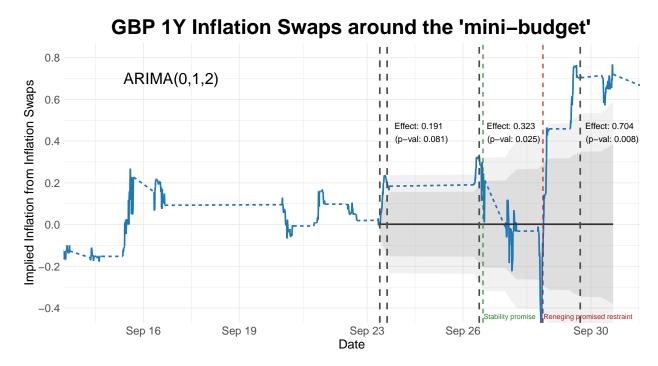
I now introduce data capturing expected inflation through a high-frequency identification strategy that measures the expected inflationary consequence of the underlying stimulus. The analysis thus follows Hazell and Hobler (2024), postulating that around the 'mini-budget announcement' the dynamics of asset prices  $y_t$  can be summarized by the process:

$$y_t = \begin{cases} \varepsilon_t & \text{if } t < T, \\ \varepsilon_t + \alpha_t & \text{if } t \ge T, \end{cases}$$
 (1)

where T denotes the time period at which fiscal stimulus occurred. I set the shock period T to September 23, 2022, 09.30am, coinciding with the beginning of the budget statement in parliament. Denoting by j a counter of periods after the event, denote by  $\hat{\alpha}_{T+j} = y_{T+j} - \mathbb{E}_T \left[ y_{T+j} | \alpha_{T+j} = 0 \right]$  the estimate of the causal effect of the shock in the narrow window around the announcement.

<sup>&</sup>lt;sup>12</sup>The deficit-inflation multiplier, measuring the effect of a change in sovereign borrowing on expected inflation, is larger for a given change in expected inflation when the borrowing shock is *smaller*. Here, it is easier to over-estimate the size of the shock, since the October data has been collected two weeks after the fiscal announcement, and forecasters might have by then already expected the package to be unwinding. See appendix A.3 for further details on the narrative around the 'mini-budget'.

<sup>&</sup>lt;sup>13</sup>The only periods with larger absolute adjustments in the expected two-year budget deficit forecast were April 2020 (GBP 147.4 billion), May 2020 (GBP 114.9 billion), and May 2009 (GBP 50 billion). Outside of the GFC and Covid periods, the largest absolute month-on-month average forecast revision was GBP 20.8 billion in October 2019, less than half of the forecast change in October 2022.



**Figure 3:** Implied inflation expectations from one-year GBP Inflation swaps in the period around the 'mini-budget' shock, with data normalized to 0 for September 23, 2022, 09:30am. The gray fan-chart depicts 68% and 95% confidence intervals for implied inflation based on a forecast of the swap price from the moment of the shock onward, with the model being chosen optimally in accordance with the Bayesian Information Criterion.

The main quantity of interest is one-year ahead expected inflation, as implied through GBP-indexed inflation swaps traded at the London Stock Exchange. Figure 3 summarizes the movements of expected one-year ahead inflation, derived through inflation swaps, around the 'mini-budget' shock on September 23, 2022.

The gray dashed vertical lines are points at which meaningful estimates of changes to one-year ahead inflation expectations can be recovered. The first vertical line depicts the beginning of the shock, as implied by the beginning of the budget speech announcing the 'mini-budget' measures in detail. The second line measures one-year ahead inflation expectations on the same day at 3:00pm, 5.5 hours after the budget speech commenced. Even though markets can credibly be expected to take a couple of days to incorporate fiscal news into forming inflation expectations (Bahaj et al., 2023), there is a significant response of implied inflation on the day of the 'mini-budget' announcement. Looking at the effect measured on the next trading day, a further magnification of the effect occurs, yielding an implied year-on-year inflationary response of 0.277% to the narratively identified shock.

Between the third and the fourth vertical line implied inflation drops sharply. This drop is consistent with the expectation that the fiscal spending announcement might end up being unraveled, indicated by the green dashed line labeled 'Stability Promise'. I provide a narrative description of

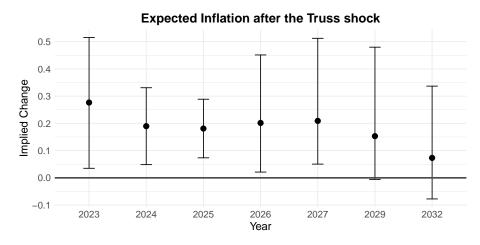
<sup>&</sup>lt;sup>14</sup>Since inflation swaps operate with a two-month indexation lag in the context of the U.K., I adjust the prices of the swaps to reflect this lag, again in line with Hazell and Hobler (2024).

the events in this period in appendix A.3, including a brief description of the role played by the troubles on LDI markets.

On September 28 & 29, there is again a significant up-shoot in the expected inflation measure. While by then other events might contaminate the evolution of inflation swap prices, the observed sharp appreciations perfectly coincide with statements of the Treasury that *despite* the market turmoil, the proposed fiscal package will be followed through, superseding previous statements of a release of a stabilizing medium-term fiscal plan. This event is depicted by the red dashed line. The elevated levels of expected inflation then continued to persist well into October, when an eventual unraveling occurred in parallel to an overhaul of the ruling government that enacted the fiscal package in the first place.

To remain conservative in terms of the implied size of the expected inflation adjustment, yet consistent with the literature, I postulate that the response of inflation swaps until September 26 can be considered the baseline change in one-year ahead inflation expectations.

The Inflation Multiplier: the resulting baseline estimate of the one-year ahead inflation multiplier, which captures the response of year-on-year inflation to a 1% deficit-to-GDP shock, is therefore  $0.277/1.27 \approx 0.22\%$ . This estimate exceeds the *two-year* inflation multiplier found by Hazell and Hobler (2024) of 0.19% by 15%, despite the assumptions ensuring that the inflation multiplier estimate errs on the conservative side. Taking the point estimate of 0.603 (which aligns closest with the forecast change to the budget deficit introduced in the last subsection), the inflation multiplier would amount to  $0.603/1.27 \approx 0.475\%$ , more than double the estimate of Hazell and Hobler (2024) and vastly above existing estimates for other countries. The effects of the deficit shock were expected to be persistent, as implied by inflation swaps for longer horizons depicted in figure 4.



**Figure 4:** Implied change in inflation expectations for various forecast horizons, as implied by GBP inflation swaps on September 26, 2022, 12:00pm relative to the pre-shock period. 95% confidence bands are shown through bins.

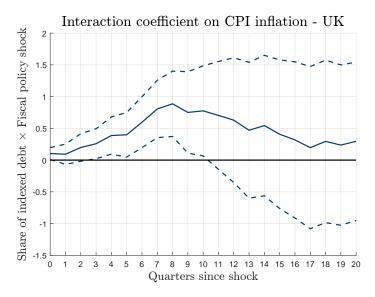
Appendix section A.3 provides further evidence on the nature of the 'mini-budget shock episode', which confirms an element of surprise in relation to the size of the unveiled fiscal package that contributed to the turmoil on financial markets reflected in the pricing of inflation swaps.

#### 2.2 Evidence on the ex-post inflationary effect of inflation-indexed debt

A shortcoming of the previous narrative analysis is that the inflation measure is one of *expected* inflation recovered through the pricing of inflation swaps. I now address this issue, providing evidence on the effect of inflation-indexed debt on *realized* inflation. To do so, I leverage the time series of narratively identified exogenous fiscal policy surprises  $\varepsilon_t^F$  provided by Mierzwa (2024), and combine it with a novel time series of the market value of inflation-indexed debt as a share of the overall market value of U.K. sovereign debt. I take this to be the main indicator for the prevalence of inflation-indexed debt. Equipped with these time series, I estimate the following local projection to measure the dynamic impact of inflation-indexed debt on the price level:

$$\log P_{t+h} - \log P_{t-1} = \alpha_h + \beta_h \Delta \omega_t \varepsilon_t^F + \delta_{1h} \Delta \omega_t + \delta_{2h} \varepsilon_t^F + \Gamma_h Z_{t-1} + e_{t+h}, \tag{2}$$

where  $h \geq 0$  indexes the forecast horizon considered and  $Z_{t-1}$  is a vector of control variables specified below. Of particular interest is the coefficient  $\beta_h$ , which captures the cross-effect of the identified fiscal shock  $\varepsilon_t^F$  and the growth in the share of inflation-indexed debt  $\Delta\omega_t$  present in the economy at time t.<sup>15</sup>



**Figure 5:** IRF implied by the local projection (2) through the coefficients  $β_h$ . The control vector Z contains the first four lags of the real GDP growth rate, the Bank of England Official Bank Rate, the change in the weighted real exchange rate, a same-period recession indicator, year-fixed effects, and the first lag of the price level difference. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction). Confidence intervals are provided at the 90% level. Sample length: 1970 Q1 - 2019 Q2.

Figure 5 depicts the impulse-responses estimated through the local projection (2). The crucial

<sup>&</sup>lt;sup>15</sup>I work with the growth rate of the share of inflation-indexed debt in the total debt portfolio as the share of inflation-indexed debt  $\omega_t$  is trending, as shown in appendix figure A.3. Using the level of inflation-indexed debt can then induce a spurious state-dependence. More generally, since  $\omega_t$  is endogenous to fiscal policy decisions, using the change in the share of indexed debt  $\Delta\omega_t$  can reduce the magnitude of the bias when the level of  $\omega_t$  is strongly correlated with other low-frequency components of fiscal policy that affect the outcome variable (Gonçalves et al., 2024). Effectively, using the change  $\Delta\omega_t$  here captures the local derivative of inflation with respect to  $\varepsilon_t^F$  when the policy variable  $\omega_t$  itself also changes contemporaneously. Using instead the past level of indexed debt,  $\omega_{t-1}$ , does not significantly alter the results.

observation is that the interaction effect between the share of inflation-indexed debt and the fiscal policy shock is significantly positive in the ten quarters after the shock. In economic terms, the coefficients imply that a 1% increase in the combined measure of the change of the share of inflation-indexed debt and the narratively identified fiscal shock (measured as a percentage of GDP) itself leads to an increase of the price level of almost 1% in the two years after the shock. This evidence clearly links the share of inflation-indexed debt to inflation observed in response to expansionary fiscal shocks. Further details related to this analysis as well as an application to U.S. data are provided in appendix A.

## 3 Intuition from a one-equation price level-determination model

This section explicitly introduces inflation-indexed debt in the government budget constraint and the resulting debt valuation equation. The analysis herein can be thought of as a partial equilibrium analysis that isolates the effect of indexed debt through the government budget constraint, sidelining general equilibrium effects. I derive the novel result that the price level itself becomes a *state variable* in the intertemporal government budget equilibrium, i.e., today's price level becomes a function of the past price level. This observation holds true despite the lack of other inertia, and it gives rise to a double role of the price level as a state variable and a market-clearing jump variable.

The per-period government budget constraint in a world with indexed debt is given by

$$B_{t-1} + \frac{P_t}{P_{t-1}} b_{t-1} = P_t s_t + Q_t B_t + q_t b_t,$$

where  $B_t$  denotes the face value of non-indexed government debt issued at time t at price  $Q_t$ ,  $b_t$  denotes the issuance value of indexed-government debt issued at time t at price  $q_t$ , lowercase letters correspond to the values for inflation-indexed debt,  $s_t$  are primary real surpluses raised (i.e., the inverse of deficits), and  $P_t$  denotes the price level. The cost of maturing inflation-indexed debt  $b_{t-1}$  is scaled by the gross inflation rate,  $P_t/P_{t-1}$ .  $^{16}$ 

To close this model as simply as possible, let  $Q_t = \frac{1}{1+i_t}$  and  $q_t = \frac{1}{1+r_t}$ , i.e., the price of bonds equals the inverse of the respective relevant gross interest rate. By letting the price of indexed debt be equal to the inverse of the real interest rate, expectations of the face value payment adjustment of indexed debt through inflation are taken care of. Iterating this equation forwards, dividing both sides by  $P_t$ , and making use of the Fisher equation gives rise to the following relationship:

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \sum_{i=0}^{\infty} \prod_{l=1}^{j} \frac{1}{1 + r_{t+l}} s_{t+j}.$$
 (3)

<sup>&</sup>lt;sup>16</sup>See Hall and Sargent (2011) for a verification that this is the correct specification for indexed debt that is in line with how its face value adjusts empirically, absent indexation lags that I sideline here for simplicity.

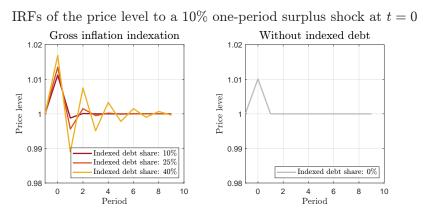
<sup>&</sup>lt;sup>17</sup>This equals the fair price of either type of debt for a household maximizing its cumulative discounted utility with an exogenous endowment and perfect foresight.

This equation is the baseline government debt valuation equation without accounting for the differences in the insurance properties borne by the two types of debt, which made it possible to use the simplified bond pricing kernels  $Q_t$  and  $q_t$  as defined in the last paragraph. Clearly, the price level itself becomes a state variable: the real value of maturing inflation-indexed bonds depends on the past price level, not on today's price level. Intuitively, the real value of inflation-indexed bonds depends on the past price level, because the face value payment of that bond is unity at yesterday's prices. The term in orange is the novel addition relative to canonical models of fiscal inflation and is the centerpiece of this paper.

#### Sample IRFs in partial equilibrium in finite horizon

I now explore the properties of the government debt valuation equation using impulse-responses to the price level under various levels of inflation-indexed debt. The goal is to explore how indexed debt changes the mechanisms inducing fiscal inflation in the clearest possible way.

The model is set up (in terms of outstanding bonds and expected surpluses), such that  $P_{-1}=1$ . The initial state is set up such that the present discounted value (PDV) of surpluses is equal to the real value of the stock of debt in each period. The economy has a finite horizon of 11 periods  $t \in \{-1,0,1,...,9\}$ , such that all debt has to be repaid by the government in period 9 by appropriate surpluses. All these assumptions jointly ensure a price level of  $P_t=1 \ \forall t$  in the absence of shocks. The impulse to the system is a one-period decrease of surpluses by 10% in period 0, announced at the same time. After period 0, the PDV of surpluses returns to its pre-shock value.



**Figure 6:** IRFs to a 10% decrease in the surplus in t = 0 for various levels of indexed debt.

Figure 6 shows the reaction of the price level in response to a one-period decrease in surpluses (which is reversed in t=1), announced in the period 0. The right-side panel illustrates the "standard" response induced by the government debt valuation equation in a world without inflation-indexed debt. In period 0, the decrease in real surpluses induces a temporary upwards adjustment of the price level proportional to the decrease in surpluses, which returns back to its initial state subsequently, since the PDV of surpluses is equal to the pre-shock value after period 1.

However, when the share of inflation-indexed debt is strictly positive, the impact response is ex-

acerbated: given that the initial price level  $P_{-1}$  is fixed in the moment of the shock at time 0, it is not possible to devalue the stock of inflation-indexed debt when the shock occurs. Therefore, the devaluation of the remaining (non-indexed) stock of bonds must be *larger* relative to the case without inflation-indexed debt: the price level must go up by a larger amount in the shock period because of the existence of inflation-indexed debt.

The periods following the shock yield further novel dynamics relative to a baseline model without indexed debt. Instead of returning to the pre-shock value once the shock vanishes, the price level oscillates when indexed debt is present. How can this be? Since the PDV of surpluses returns to its pre-shock level in t=1, the stock of debt is suddenly worth *too little*: inflation-indexed debt is not worth much due to the high price level at t=0, which is the correct factor to adjust such debt to 'real' terms in period 1. But since the funding shortfall is now gone, the real value of non-indexed debt ( $B_1/P_1$ ) must actually *increase* to make up the 'under-valuation' of indexed debt: therefore,  $P_1$  must *decrease* (increasing the real value of non-indexed debt) to let the debt valuation equation hold. In the subsequent period, the price level from the previous period is now *too low*, increasing the value of indexed debt and pushing down the real value of non-indexed debt through a higher price level. This mechanism repeats itself until convergence to the initial equilibrium.<sup>18</sup>

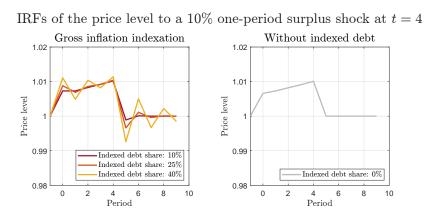


Figure 7: IRFs to a 10% decrease in one-period surpluses in t = 4 conditional on the share of indexed debt.

Figure 7 repeats the exercise for a similar decrease of surpluses at a later time (in period 4), announced in period 0. Due to the early announcement, the PDV of surpluses already decreases in period 0, remaining below its initial value until period 4. The oscillations induced by inflation-indexed debt decrease in size until period 4, and subsequently pick up from period 4 onwards in line with the mechanism described above. The fact that the oscillations are decreasing in magnitude leading up to the shock is caused by the PDV of surpluses *not* being constant between periods 0 and 4 in this example: the closer period 4 becomes, the more the PDV of surpluses actually decreases as the shock period gets discounted by less. This buffers the price level oscillations on the way to the period of the shock.

<sup>&</sup>lt;sup>18</sup>Cochrane (2001) explores a similar result in figure 4 of his paper, driven by a non-geometric maturity structure and the presence of long-term debt.

## 4 A Quantitative General Equilibrium Model

Having studied the relevance of indexed debt in a simplified framework that isolates its effect on the debt valuation equation, I now introduce inflation-indexed debt in a rich state-of-the-art macroeconomic model. Given that inflation-indexed debt delivers desirable insurance features to households by providing an income smoothing source that yields certain real returns, the chosen model must necessarily bear relevance to imperfect consumption smoothing, borrowing constraints, and market imperfections precluding perfect risk-sharing across households. Otherwise, households would be indifferent between the two types of debt up to first-order. Here, I work with a heterogeneous-agent model in the spirit of Kaplan et al. (2018), utilizing the efficient algorithms for solving such models provided by Auclert et al. (2021) and paying close attention to the peculiarities of determinacy in incomplete-market models exposed by Brunnermeier et al. (2024), which are intimately related to the government debt valuation equation.

**Households:** Heterogeneous households are indexed by i. Such households choose consumption,  $c_{it}$ , labor supply,  $N_{it}$  (adjusted by a shifter to the relative disutility of labor supply  $\zeta$ ), and asset holdings  $B_{it}$  and  $b_{it}$  to maximize their cumulative discounted utility

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( u(c_{it}) - \zeta v(N_{it}) \right) \right],$$

subject to two budget constraints - one for the aggregate household budget, and one for the semantically separate evolution of indexed debt in the household savings portfolio:

$$\begin{split} P_t c_{it} + Q_t B_{it} &= \frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta} di} (1-\tau_{it}) W_t N_{it} + B_{i,t-1} - d_{it} \mathbb{1}_{\{adj_{it}=1\}}, \\ q_t b_{it} &= \Pi_t b_{i,t-1} + d_{it} \mathbb{1}_{\{adj_{it}=1\}}, \end{split}$$

where  $Q_t$  and  $q_t$  are the nominal prices for non-indexed debt  $B_{it}$  and indexed debt  $b_{it}$ , respectively.  $W_{it} \equiv w_{it}P_t$  denotes the nominal wage level, adjusted by hours worked  $N_{it}$  and scaled by the idiosyncratic productivity disturbance  $\frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta}di}$  and the distortionary income tax rate  $\tau_{it}$ . Through the two separate budget constraints, I posit that consumption is only possible from the non-indexed savings portfolio. Effectively, I postulate that indexed debt cannot be transformed to consumption as easily as non-indexed debt. This assumption reflects the significantly smaller liquidity of inflation-indexed bond markets, even relative to their market size (Andreasen et al., 2021; Fleming and Krishnan, 2012) and is necessary to ensure the existence of an ex-ante real yield differential. Without any adjustment friction, expected yields would equalize and there would be no incentive to hold both types of debt through a no-arbitrage argument. <sup>19</sup>

 $d_{it}$  captures household-specific transfers from the 'main' budget constraint to the relatively less

<sup>&</sup>lt;sup>19</sup>Complementary evidence on the use of inflation-indexed government bonds by households for inflation hedging within the context of the U.S. is provided by Nagel and Yan (2022).

accessible portfolio of indexed bond holdings, which are only allowed to happen when the exogenous portfolio rebalancing variable  $adj_{it}$  is equal to 1, which happens with probability  $\nu$ . This is a Calvo-type friction, applied to portfolio holdings of the household.<sup>20</sup> Finally, households are subject to borrowing constraints  $B_{it} \geq -\underline{B}$ ,  $b_{it} \geq -\underline{b}$ .

In the following, let  $\varepsilon_i \equiv \frac{e_i^{1-\theta}}{\int e_i^{1-\theta} di}$  be a simplified descriptor of the Markov chain pinning down idiosyncratic productivity. To solve the household block, it is necessary to distinguish whether a household is able to adjust its holdings of indexed debt in a given period  $(adj_{it}=1)$  or not  $(adj_{it}=0)$ . I now define corresponding value functions for households, noting that the state variables are therefore the household-specific past asset holdings  $(B_-,b_-)$ , the Markov chain realization  $\varepsilon_i$ , and the adjustment state  $adj_i$ . The subscript i is dropped in the following for notational simplicity, yielding the following value functions:

• adjuster, adj = 1:

$$V_t(1,\varepsilon;B_-,b_-) = \max_{c,B,b,N} u(c) - \zeta v(N) + \beta \mathbb{E} \left[ V_{t+1}(adj',\varepsilon',B,b) | \varepsilon \right]$$
 (4)

subject to a unified budget constraint (created by replacing  $d_{it}$  in the former budget constraint) and the borrowing constraints:

$$Pc + QB + qb = \varepsilon(1 - \tau)WN + B_{-} + \Pi b_{r}$$
  
 $B \ge -B; \quad b \ge -b,$ 

where adj' is i.i.d., with probability  $\mathbb{P}(adj'=1)=\nu$ .

• non-adjuster, adj = 0: Here, b does not enter the decision set and is taken to be an unchangeable state operating in the background, with the next-period income arising through holdings of non-indexed debt being a function of the indexed debt holdings with which the household entered the current period.

$$V_{t}(0,\varepsilon,B_{-},b_{-}) = \max_{c,B,N} u(c) - \zeta v(N) + \beta \mathbb{E} \left[ V_{t+1} \left( adj',\varepsilon',B,\frac{\Pi}{q} b_{-} | \varepsilon \right) \right]$$
 (5)

subject to the budget and borrowing constraints:

$$Pc + QB = \varepsilon(1 - \tau)WN + B_{-},$$
 
$$B \ge -\underline{B}.$$

The goal is to recover policy functions  $c(\cdot)$ ,  $B(\cdot)$ ,  $b(\cdot)$ , and  $N(\cdot)$  that solve the household problem in both instances. The above problem generally yields first-order conditions that depend on the adjustment possibilities that an agent enjoys in a given period. Denote by  $\lambda_{it}$ ,  $\mu_{it}^B$ , and  $\mu_{it}^b$  the re-

<sup>&</sup>lt;sup>20</sup>Such Calvo-type sticky portfolio arrangements have been present in macroeconomic models since at least Graham and Wright (2007) and have prominently been used in heterogeneous-agent models by Auclert et al. (2024b) and Bayer et al. (2024).

spective state-dependent constraint multipliers. For the adjusters,  $adj_{it} = 1$ , the relevant first-order conditions from that household problem are given by

$$\begin{aligned} \{c\}: & u'(c) = P\lambda_{it} \\ \{N\}: & \zeta v'(N) = \lambda_{it} \varepsilon (1 - \tau) w P \\ \{B\}: & Q\lambda_{it} = \beta \mathbb{E} \left[ V_{B,i,t+1} \right] + \mu_{it}^{B} \\ \{b\}: & q\lambda_{it} = \beta \mathbb{E} \left[ V_{b,i,t+1} \right] + \mu_{it}^{B}, \end{aligned}$$

while the envelope conditions, using  $\lambda_{it} = \frac{u'(c)}{P}$  from the first-order condition on c, are given by:

$$\begin{split} V_{B,i,t} &= \frac{u'(c)}{P}, \\ V_{b,i,t} &= \begin{cases} \frac{u'(c)}{P} \Pi = \frac{u'(c)}{P_-} & \text{if } adj_{it} = 1 \\ \beta \frac{\Pi}{q} \mathbb{E} \left[ V_{b,i,t+1} \right] & \text{if } adj_{it} = 0. \end{cases} \end{split}$$

The conditions for equilibrium jointly imply the following Euler equations:

$$\begin{split} \frac{Q}{P}u'(c) &\geq \beta \mathbb{E}\left[V_{B,i,t+1}\right],\\ \frac{q}{P}u'(c) &\geq \beta \mathbb{E}\left[V_{b,i,t+1}\right],\\ \zeta v'(N) &= u'(c)\varepsilon(1-\tau)w, \end{split}$$

where the inequalities are strict if the respective asset holdings are at their lower bound.

This household block defines pricing kernels for the bonds that are on offer by the government, conditional on the households pricing the bonds being unconstrained. For non-indexed debt, the first-order conditions for households on the Euler equation imply that

$$Q_t = \beta \mathbb{E}_t \left[ \frac{u'(c_{i,t+1})}{u'(c_{it})} \frac{P_t}{P_{t+1}} \right] := \mathbb{E}_t \left[ \mathcal{M}_{i,t,t+1} \right], \tag{6}$$

where  $\mathcal{M}$  denotes the household-specific stochastic discount factor (SDF). For indexed bonds, applying the definition of the SDF,

$$q_t = \beta \mathbb{E}_t \left[ \frac{u'(c_{i,t+1})}{u'(c_{it})} \right] := \mathbb{E}_t \left[ \mathcal{M}_{i,t,t+1} \Pi_{t+1} \right]. \tag{7}$$

**Firms and production:** To focus on the effects of indexed debt and its interaction with households facing uninsurable idiosyncratic income risk, I model the production block in a parsimonious way, following Auclert et al. (2024). In particular, the model requires that the aggregate effects of idiosyncratic productivity risk are 'small' for the production firms relative to the aggregate effects of

aggregate risks.<sup>21</sup>

Hence, in this model there exists a continuum of monopolistically competitive firms k that produce goods of variety k, with each being produced in accordance with a linear production function  $Y_{kt} = A_{kt}N_{kt}$ .  $A_{kt}$  evolves according to an AR(1) process in logs,

$$\log A_{kt} = \rho_a \log A_{k,t-1} + \sigma_{\epsilon} \epsilon_{kt},$$

where that  $0 \le \rho_a \le 1$ . The firm profit function is defined as

$$D_{kt} = \frac{P_{kt}}{P_t} Y_{kt} - \frac{W_t}{P_t} N_{kt} = \left(\frac{P_{kt}}{P_t} - \frac{W_t}{P_t} \frac{1}{A_{kt}}\right) A_{kt}^{1-\zeta} \left(\frac{P_{kt}}{P_t}\right)^{-\zeta} Y_t.$$

Following Auclert et al. (2024), a log-linearized approximation to the solution of the profit-maximization problem of monopolistically competitive firms yields a Phillips Curve of the form:

$$\hat{\pi}_t = (\varphi + \sigma)\kappa \sum_{l=0}^{\infty} \beta^l \hat{y}_{t+l}$$
 (8)

where  $(\varphi + \sigma)$  is the sum of the Frisch elasticity of labor supply and the inverse of the elasticity of intertemporal substitution, as in standard New Keynesian models, and  $\kappa = \frac{\lambda(1-\beta(1-\lambda))}{1-\lambda}$  is the slope of the Phillips curve, where  $\lambda$  is the probability with which monopolistically competitive firms can adjust their prices.

**Fiscal policy:** Fiscal policy is characterized by two elements. The first one is the government debt valuation equation. Characterizing this equation is only possible through invoking a suitable transversality condition on government debt.

As pointed out by Brunnermeier et al. (2024), individual transversality conditions on household asset holdings do *not* imply directly that a similar transversality condition holds for aggregate debt stocks under incomplete markets. With incomplete markets and endogenous real interest rates, the government debt valuation equation may ultimately fail to deliver a unique price level based off a simple aggregate 'transversality condition' on government debt, since there is no guarantee that such a condition holds when markets are incomplete. Appendix section B.1 illustrates this point.

Instead, I here follow the approach undertaken in Brunnermeier et al. (2024), which is dubbed the *dynamic trading perspective*. By that approach, I start from household unit-level budget constraints. Aggregating them up under suitable bond pricing kernels, I obtain a dynamic aggregate constraint on sovereign debt, which constitutes the government debt valuation equation. By doing so, I account for the benefits of the two debt products in partially overcoming the effects of market incompleteness borne by households, which allows me to leverage household-level transversality conditions. This final expression of the government debt valuation equation still equates of the

<sup>&</sup>lt;sup>21</sup>See proposition 4 of Auclert et al. (2024) for a detailed exposition of this point.

real value of today's debt holdings to a suitably discounted fiscal surplus term:

**Proposition 1** *In a model with both non-indexed and inflation-indexed debt and incomplete markets, the government debt valuation equation can be expressed as:* 

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \tilde{\mathcal{M}}_{t,t+k} \bar{A}_{t+k} \right], \tag{9}$$

where  $\tilde{M}_{t,t+k} = \sum_i M_{i,t,t+k} \Pi_{t,t+k+1} \frac{A_{i,t+k}}{\bar{A}_{t+k}}$  is the weighted average SDF across all households i, adjusted for inflation, with weights being proportionate to  $A_{i,t+k}$ .  $\bar{A}_t = \frac{1}{N_i} \sum_i A_{it}$  is the average of the term  $A_{it}$ , which captures the surpluses raised by the government from each household i and the utility-weighted windfall gain that households enjoy from holding inflation-indexed debt:

$$A_{it} \equiv c_{it} - \varepsilon_{it}(1 - \tau_{it})w_t N_t + \left[Cov_t \left( \mathcal{M}_{i,t,t+1}, \Pi_{t+1} \right) + \mathcal{M}_{i,t,t+1} \left( \mathbb{E}_t \Pi_{t+1} - \Pi_{t+1} \right) \right] \frac{b_{it}}{P_t}.$$

#### **Proof.** See appendix B.1. ■

 $A_{it}$  therefore expresses the full portfolio return earned by household i for each additional unit of net worth, consisting of the net utility gain from saving, the insurance premium on indexed debt (captured through the covariance term), and the possible windfall gain/loss from surprise inflation through indexed debt (captured through the last term). Depending on the precise nature of market incompleteness and the prevailing fiscal and monetary rules, equation (9) (co-)determines the price level at time t, given some previous price level  $P_{t-1}$ .

I close the government block by assuming the taxation rule

$$\frac{\tau_t}{\tau} = \left(\frac{s_{B,t-1}}{s_B}\right)^{\gamma_B} \left(\frac{s_{b,t-1}}{s_b}\right)^{\gamma_b} e^{\zeta_t},\tag{10}$$

where  $\tau_t \equiv \frac{T_t}{Y_t}$  are surpluses raised by the government as a fraction of output, and  $s_{B,t} \equiv \frac{Q_t B_t}{P_t Y_t}$ ,  $s_{b,t} \equiv \frac{q_t b_t}{P_t Y_t}$  are the real market values of the two existing types of debt.  $\zeta_t$  is an AR(1) shock to the quantity of lump-sum taxes raised, and the policy reaction coefficients to deviations of the market values of both types of debt from their steady-state values are given by  $\gamma_B$  and  $\gamma_b$ . Steady-state values are denoted without time subscripts. In log-linearized terms, this relationship becomes:

$$\hat{\tau}_t = \gamma_B \hat{s}_{B,t-1} + \gamma_b \hat{s}_{b,t-1} + \zeta_t. \tag{11}$$

**Monetary policy**: Monetary policy follows an inertial Taylor rule with weights on both inflation and output deviations from steady-state:

$$\left(\frac{R_t^n}{R^n}\right) = \left(\frac{R_{t-1}^n}{R^n}\right)^{\rho_M} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{1-\rho_M} e^{\nu_t} \tag{12}$$

where  $v_t$  is an AR(1) shock to the conduct of monetary policy. In exact log-linearized terms,

$$\hat{r}_{t}^{n} = \rho_{M} \hat{r}_{t-1}^{n} + (1 - \rho_{M}) \left[ \phi_{\pi} \hat{\pi}_{t} + \phi_{y} \hat{y}_{t} \right] + \nu_{t}.$$
(13)

**Market clearing:** Market clearing on the three markets in this economy is defined as follows:

 Goods market: on the goods market, aggregate consumption and production are equalized, taking into account the loss from price adjustment costs on the producer's behalf:

$$C_t + G_t + \frac{\mu/(\mu - 1)}{2\kappa} \left(\log(1 + \pi_t)\right)^2 Y_t = Y_t.$$
 (14)

• Labor market: labor supply and demand must be equal:

$$\sum_{i} N_{it} = \sum_{k} N_{kt}. \tag{15}$$

Asset market: for each class of assets, the supply by the government must be equal to cumulative household demand:

$$B_t = \sum_i B_{it} \tag{16a}$$

$$b_t = \sum_i b_{it}. (16b)$$

Equilibrium: I now characterize a competitive equilibrium in this economy:

**Definition 1 (Competitive Equilibrium)** A competitive equilibrium of the heterogeneous-agent economy is an allocation  $\{C_t, N_t, Y_t, B_t, b_t, Y_{it}, N_{it}, d_t, \tau_t\}_{t=0}^{\infty}$ , together with prices  $\{P_t, P_{it}, w_t, \pi_t, Q_t, q_t, R_t^n\}_{t=0}^{\infty}$  and exogenous variables  $\{v_t, Z_t, G_t\}_{t=0}^{\infty}$ , such that:

- all agents maximize their utility with suitable policy functions on  $c(\cdot)$ ,  $N(\cdot)$ ,  $B(\cdot)$ , and  $b(\dot)$ , solving the type-dependent value functions (4) or (5),
- all firms maximize their PDV of profits,
- the government does not violate its per-period budget constraint, levies taxes in accordance with its fiscal rule, and the price level is determined through equation (9),
- the central bank follows its policy rule (12),
- all markets clear ((15), (16a), (16b), equation (14) follows from Walras' Law), and
- the distribution of household wealth and productivity  $\Gamma_t(B,b,z)$  evolves by its law of motion and is determined in the long-run by the fixed point of its evolution:

$$\Gamma_{t+1}(\mathcal{B}, \delta, z') = \int_{\{(B, b, z): g_t(B, b, z) \in (\mathcal{B}, \delta)\}} Pr(z'|z) d\Gamma_t(\mathcal{B}, \delta, z).$$

I close the model by defining the utility function of consumption for each household i as  $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$ , and the disutility function of labor supply as  $v(N) = \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\alpha}}$ .

**Steady-state**: in the following I will consider a log-linearized approximation around the deterministic steady-state with respect to aggregate variables. That steady-state is characterized by a zero inflation rate,  $\Pi=1$ , such that bond prices are equal to the household discount rate,  $Q=\beta$  and  $q=\beta$  in the absence of uncertainty. I furthermore normalize steady-state output to 1.

#### Steady-state determinacy under simplified real interest rate determination

Denote variables in steady-state with the subscript 'ss'. A necessary precondition for the analysis of the the dynamic implications of indexed debt is the existence of a *unique* steady-state. This is not trivial under heterogeneous agents and non-Ricardian fiscal policy, since the real interest rate is an endogenous object and multiple combinations for the initial price level might exist in dependence on the real interest rate (Hagedorn, 2021). Here, I propose that this economy with inflation-indexed debt can yield a unique price level in a stationary equilibrium if the real interest rate is determined *outside* the government budget equation (taking off the 'double burden' of determining both the initial price level  $P_0$  and the steady-state real interest rate  $r_{\rm ss}$  that this condition would alternatively be subject to), although some additional restrictions must be made. This statement is formalized in the following:

**Proposition 2 (Stationary equilibrium determinacy)** *Under incomplete markets, with non-negative steady-state inflation, and abstracting from aggregate uncertainty, the intertemporal government budget equation can determine a unique initial price level in stationary equilibrium even in the presence of inflation-indexed debt for non-negative steady-state inflation rates if*  $\frac{b_{ss}}{b_{ss}+B_{ss}} < 1$ ,  $r_{ss} > 0$ , and if a steady-state asset demand function  $\delta(r_{SS})$  exists and is invertible.

#### **Proof.** See appendix B.2. ■

Therefore, the model yields a unique initial price level with positive steady-state levels inflation-indexed debt, provided that the real interest rate is pinned down outside of the government debt valuation equation. This result is reflective of Hagedorn (2021, 2024) under a suitable application to a setting with inflation-indexed debt. With this result at hand, the model operates with a unique steady-state, sidelining worries related to stationary equilibrium multiplicity.

The intuition behind the proof is the following: the intertemporal government budget equilibrium without inflation-indexed debt relates the price level to the real interest rate, which is determined on the asset market. With inflation-indexed debt, steady-state inflation itself becomes another element of the intertemporal government budget equilibrium. That inflation rate, which is posited to be pinned down by fiscal policy in the stationary equilibrium, is directly related to the real interest rate through the Fisher equation. Then, with the real interest rate (and, thus, implicitly inflation) being pinned down by asset market equilibrium, there is only one plausible real interest rate that manages to uniquely pin down the price level from the government budget constraint.

## 5 Calibration and computational approach

Parameter	Description	Value	Source/Target	
Firms				
Υ	Steady-state output	1	Normalization	
ε	Elasticity of substitution between product varieties	9	Firm mark-up of 11% (Auclert et al., 2024a)	
κ	Slope of price Phillips curve	0.055	Hazell et al. (2022), Gagliardone et al. (2023), Benigno and Eggertsson (2023)	
Households				
$\sigma$	Inverse of intertemporal elasticity of substitution	1	Simplification for simulation	
$\varphi$	Inverse of Frisch elasticity of labor supply	1	Simplification for simulation	
<u>B</u>	Lower bound of non-indexed debt holdings	0		
<u>b</u>	Lower bound of indexed debt holdings	0		
$ ho_z$	Persistence of $AR(1)$ shocks to household productivity	0.966	Auclert et al. (2021)	
$\sigma_z$	Standard deviation of AR(1) shocks to household productivity	0.92	Auclert et al. (2021)	
Government				
T/G	Steady-state surplus, measured by the tax-to-government spending ratio	1.03	See explanation below	
r*	Natural rate of interest	0.015	Benigno et al. (2024)	
$ ho_M$	Inertia in Taylor-type interest rate rule	0	Simplification	
$\phi_{\pi}$	Monetary policy reaction to inflation deviations from steady-state	{0.5, 1.5}	For fiscally-led/monetary-led policy mix (Bianchi et al., 2023)	
$\phi_y$	Monetary policy reaction to output deviations from steady-state	0	Bianchi et al. (2023)	
$\gamma_B$	Fiscal policy reaction to deviations of market value of non-indexed debt from steady-state	{0.5, 1.5}	For fiscally-led/monetary-led policy mix (Bianchi et al., 2023)	
$\gamma_b$	Fiscal policy reaction to deviations of market value of indexed debt from steady-state	0.5		
Computation	al parameters			
$n_z$	Number of points in asset grid for household productivity shock	11		
$n_b$	Number of points in asset grid for indexed debt	50		
$n_B$	Number of points in asset grid for non-indexed debt	50		
$\bar{B}$	Maximum holdings of non-indexed debt in asset grid	5000		
$\bar{b}$	Maximum holdings of indexed debt in asset grid	5000	Approximation to Auclert et al. (2024)	
T	Number of periods used in simulations of Jacobians	300	Auclert et al. (2021)	

Table 1: Baseline parametrization for the quantitative estimation

The parametrization of the economy is summarized by table 1, while the endogenously calibrated parameters are given by table 2. I follow overall the approach of Auclert et al. (2021), as I apply a conceptually similar algorithm. In the preferred calibration, I vary government spending G and the household discount factor  $\beta$  to ensure that the goods and labor markets debt clear. Finally, the market for inflation-indexed debt is targeted with the help of  $\nu$ , the probability of being able to access the portfolio of indexed debt actively. The market for non-indexed debt is not targeted, but clears with a tolerance of  $10^{-5}$ , while targeted market clearing conditions clear with close to machine precision ( $10^{-15}$ ). To compare various policy combinations, I here consider base-

line active/passive policy coefficients (determining whether a given policy mix is fiscally-led or monetary-led) as given by Bianchi et al. (2023). The policy coefficients  $\{\phi_{\pi}, \phi_{y}, \gamma_{B}, \gamma_{b}\}$  should thus be taken as indicative and related to suitable active/passive policy combinations in the sense of Leeper (1991), but not as calibrated feedback rules.<sup>22</sup> When deviating from the baseline parameterizations summarized in tables 1 and 2, I will explicitly introduce novel parameters as suitable.

Debt/GDP shares	HH discount factor	$\mathbb{P}(\text{adjustment})$	Govt. spending		
	Main calibration: U.K. debt portfolio				
B: 0.8176, b: 0.3024	$\beta = 0.9569$	$\nu = 0.2293$	G = 0.4987		
	Counterfactual: U.S. debt shares				
B: 1.0171, b: 0.1029	$\beta = 0.9569$	$\nu = 0.1385$	G = 0.4986		
	Counterfactual: no indexed debt				
B: 1.12, b: 0.	$\beta = 0.9570$	$\nu = 0.0052$	G = 0.4986		

Table 2: Calibrated parameters across different debt scenarios

The calibration delivers overall reasonable estimates of the endogenous parameters that are in line with the parametrization of Auclert et al. (2021). The level of government spending is not targeted to its empirical counterpart, yet the estimated government spending share of GDP is only slightly above the share of government spending in GDP in 2024 (44.4%).

Finally, to pin down both the price level and the tax rate in steady-state, I exogenously fix the tax rate to be 3% higher than government spending in GDP, such that surpluses are equal to one percent of the government spending-to-GDP ratio. This assumption runs counter to currently observed budget deficits, but solving the model under steady-state deficits is infeasible under positive steady-state real interest rates.<sup>23</sup> However, the assumption of positive surpluses in steady-state remains qualitatively and quantitatively in line with recent long-run forecasts of the current budget deficit for the United Kingdom, provided by the OBR (2024) in their historical official forecasts database (table CB).<sup>24</sup>

By imposing zero steady-state inflation, I nullify the possibility of distortionary inflation that could induce a wedge to the long-run natural rate. In terms of economic aggregates, the steady-state is thus well-described by the above calibration. Through the normalization of output to unity and the calibrated share of government spending of  $\sim 0.4985$ , consumption in steady-state is implied to be equal to 0.5015 by market clearing, while taxation is equal to 0.5134.

In terms of government debt, I mainly compare three different steady-state calibrations: one which follows the observed modal split of sovereign debt into non-indexed and indexed debt in the United Kingdom (which is the G7 country with the highest share of indexed debt, such that B = 0.8176

<sup>&</sup>lt;sup>22</sup>Macroeconomists tended to focus on calibrations in which  $\phi_{\pi}$ , the parameter capturing the reaction of monetary policy to deviations of the inflation rate from steady-state, is larger than one, commonly known as the 'Taylor Principle'. As Nakamura et al. (2025) show, this notion is rejected in the data. Instead, the Taylor principle can be reinterpreted as *prescriptive*, that is, as a policy prescription according to which the central bank ought to act, but not as *descriptive*. This supports the idea of calibrating models with  $\phi_{\pi} < 1$ .

<sup>&</sup>lt;sup>23</sup>Kaplan et al. (2023) solve a model with negative surpluses and a negative steady-state real interest rate.

 $<sup>^{24}</sup>$ Conditional on a  $\sim 40\%$  share of government spending in GDP, the projected 1% budget surplus in the long-run as a share of GDP is equivalent to a ratio of sovereign income to spending of 1.025.

and b=0.3024), and two counterfactual calibrations with either a split between indexed and non-indexed debt in accordance to the U.S. sovereign debt portfolio (i.e., B=1.0171 and b=0.1029), or the complete absence of any indexed debt (i.e., B=1.12 and b=0). I therefore exogenously postulate the same steady-state gross bond supply across the calibrations, given that bond supply as a share of GDP is a relatively low-frequency variable, but just vary the shares of the two types of bonds. Many of the exercises will resolve around the differences between these calibrations, as I will mainly focus on the effect that indexed government debt has on economic aggregates.

Even though government debt aggregates are exogenously supplied in steady-state for all calibrations, the distribution of debt across households cannot be deduced immediately from the calibration itself, as it is generally dependent on the properties of the idiosyncratic process to income in a way that is not fully captured by the calibration itself. Figure 8 plots the distribution of debt holdings across households in two cases - once for the standard calibration to the U.K., and once for the counterfactual calibration where steady-state issuance of indexed government debt is set to 0. For the calibration without indexed debt supply, plotted in panel (b), no households hold indexed debt, while indexed bonds are held by 24% of all households in the main calibration.

Even though the distribution of debt is not a targeted moment, the model replicates the empirically observed significant skew in the distribution of debt holdings across households. Appendix A.1 shows that the simulated distributions of either type of debt are close to empirical data.

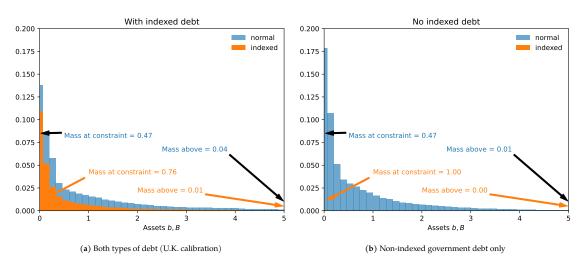


Figure 8: Steady-state distributions of asset holdings across households in the calibrated steady-state

#### Computational details - using the Sequence-Space Jacobian

The simulation of the model is linear in aggregates, but non-linear in idiosyncratic shocks. It is derived by using the Sequence-Space Jacobian method developed in Auclert et al. (2021), which itself constitutes an evolution of the methods pioneered by Reiter (2009). The computational method I employ generates perfect-foresight solutions in aggregates in response to time-zero perturbations of exogenous disturbances, but it maintains the non-linearity underlying the responses of heterogeneous households.

First, the heterogeneous household block is solved, taking aggregate prices as given, for both the policy functions (through backwards iteration) and the distribution of asset holdings (through forwards iteration). Both solve with a numerical tolerance of up to  $10^{-14}$ , and are subsequently used to inform other blocks of the model (such as firm optimality, government policies, and market clearing) and to generate updates of aggregates where necessary. The two components (heterogeneous households and aggregate) interact and iterate until convergence, which is reached with a numerical threshold of  $10^{-9}$ , which is reasonable given the high degree of complexity underlying household behavior in the presence of two types of assets.

## 6 Quantitative insights: the effects of indexed debt and policy rules

With the computational algorithm at hand, I solve and estimate the model's aggregate impulseresponses for a number of shocks, using the parametrization from table 1, but varying the calibration of the debt shares in line with table 2. Here, I will mostly focus on the effects of unanticipated disturbances to *government spending*  $G_t$ , which directly influence the surpluses raised by the government in any given period.<sup>25</sup>

Before considering the dynamics implied by this model in detail, I first look at the role that inflation-indexed debt plays for the amplification of shocks as evidenced through simulated moments, in line with the principal focus of the paper. To get a detailed grasp of the effect of inflation-indexed debt for aggregate variables, I compare the simulated volatility of a number of macroeconomic aggregates across all calibrations (U.K. calibration, counterfactual U.S. distribution of debt across the two types, and issuance of non-indexed debt only) and across both 'common' policy combinations (passive monetary/active fiscal (fiscally-led) and active monetary/passive fiscal (monetary-led)). The results of this exercise are presented in table 3.

	Normalized standard deviations across policy scenarios								
	PM/AF-U.K.	PM/AF-U.S. split	PM/AF-NoIndex	AM/PF-U.K.	AM/PF-U.S. split	AM/PF-NoIndex			
G	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000			
Y	0.994010	1.024624	0.909531	0.845227	0.893975	0.835870			
С	0.125033	0.162543	0.103052	0.249651	0.245962	0.288624			
$\pi$	0.170823	0.154582	0.094898	0.116095	0.113646	0.099419			
r	0.124260	0.132743	0.090154	0.155643	0.144316	0.146941			
N	0.994010	1.024624	0.909531	0.845227	0.893975	0.835870			

**Table 3:** Normalized standard deviations of aggregate variables in response to fiscal shocks with  $\rho_G = 0.5$ 

<sup>&</sup>lt;sup>25</sup>Appendix C provides an overview of the dynamic responses to expansionary monetary policy shocks.

The three left-hand columns can be summarized in one line and yield one of the major quantitative insights of the paper: the volatility of economic aggregates increases in the presence of inflation-indexed debt, conditional on being in the fiscally-led policy case. Of particular interest in that regard is the fourth row of table 3, which captures the volatility of inflation in response to government spending shocks. Therefore, conditional on being in the fiscally-led policy scenario, the unweighted volatility of inflation is around 80% higher in the calibration to British indexed/non-indexed debt shares relative to the counterfactual without any inflation-indexed debt being present. With a calibrated share of indexed sovereign debt of about 30%, on average, a one percentage point increase in the share of inflation-indexed debt more or less corresponds to an approximately 2.6% increase in the volatility of inflation in response to uncovered government spending shocks.

This effect is far from linear, as evidenced by the second column which shows that the U.S. calibration attains elevated levels of volatility, too, despite the share of indexed debt being less than half of the share of indexed debt in the calibration to U.K. debt shares. To the best of my knowledge, this paper is among the first to quantitatively evaluate the impact that inflation-indexed debt can have on the volatility of inflation, and how such changes in volatility are directly related to the monetary-fiscal policy nexus, as the inflation volatility increase is evidently much smaller under the monetary-led policy scenario, amounting to a difference of only 17%.

#### Impulse-response functions to expansionary fiscal spending shocks

I will now look in more detail at the impulse responses to government spending shocks and the role borne by inflation-indexed debt when an unexpected government spending increase occurs. As the persistence of any shock is relevant, the model will be simulated under different possible autocorrelations of the fiscal shock to highlight the role of persistence and the forward-looking nature of the intertemporal government budget constraint. The initial focus rests on the case of a fiscally-led policy mix in line with the first parametrization introduced in table 2, i.e., the baseline calibration to the U.K. economy. Figure 9 plots IRFs of aggregate variables in response to a 100bp expansionary fiscal shock that increases the need for fiscal spending when the shock is highly persistent, i.e.,  $\rho_G = 0.8$ .

A number of observations is worth highlighting: the responses of consumption and tax rates are in line with canonical macroeconomic models and the expected reactions in response to the fiscal expansion: in response to the fiscal expansion, there is an instantaneous increase in output that persists alongside the expenditure increase. In general, there is little amplification in output from the spending increase. For the main calibration (blue line), private consumption is partially crowded out (leading to an impact change of output of less than one), while for lower levels of inflation-indexed debt (orange and green lines), there is a slight multiplying effect that is in the ballpark of modern estimates of the impact of expansionary fiscal policy (Auerbach and Gorodnichenko, 2012; Ramey and Zubairy, 2018; Ramey, 2019).

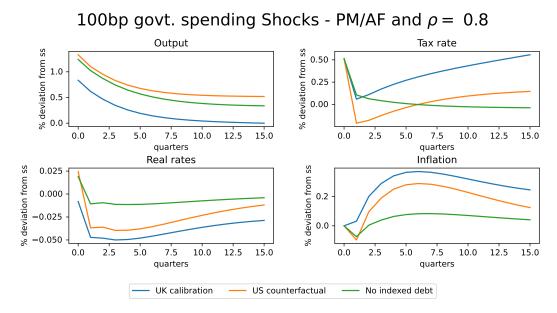


Figure 9: IRFs to the government spending shock under a fiscally-led policy mix.

Whether or not the output effect of government spending is more or less than 1 is intricately linked to the tax rate changes in response to the government spending change. Recall that this analysis concerns a *fiscally-led policy mix*, which signifies that the tax adjustment does not cover the full government spending increase. Indeed, this is the case here. The dynamics of the tax rate, however, are clearly linked to the share of inflation-indexed debt: without such debt being present (green line), the tax rate monotonically decays back to its steady-state. Once inflation-indexed debt is present, however, the equilibrium tax rule admits a 'V-shape': after an initial increase in the tax rate on impact, the tax rate briefly decreases on the expectation of the shock being only temporary.<sup>26</sup> Over time, however, the tax rate subsequently increases to cover the additional expenses arising from the cost of servicing indexed debt.

The evolution of the real interest rate in response to the fiscal impulse is again tightly linked to the share of inflation-indexed debt in each calibration. In the calibration without indexed debt (green line), the real interest rate briefly appreciates on impact of the shock due to expected deflationary pressure before returning immediately to the vicinity of its steady-state level. With positive levels of inflation-indexed debt, the impact change of the real interest rate is ambiguous and depends on the exact level of inflation-indexed debt. After the impact period, real interest rates depreciate sharply by up to 50 basis points in the view of expected inflationary pressure coming from the cost of serving inflation-indexed debt. This cost is increasing in the share of such debt in the economy.

Finally, the panel on the bottom right quantifies the main interest of this paper - the annualized rate of inflation in response to the fiscal expansion. In the present model, the price pressure aris-

<sup>&</sup>lt;sup>26</sup>Negative deviations of the tax rate from equilibrium, as temporarily observed in the calibration to U.S. debt shares, are possible as the real value of government bonds decreases below steady-state (which is related to the large negative real return shock that lowers the prices of bonds).

ing from a fiscal expansion is only minimal without inflation-indexed debt, peaking at about 0.08% six quarters after the shock. Inflation-indexed debt, however, proves to magnify inflationary pressure quite significantly: with positive levels of inflation-indexed debt, annualized rates of inflation peak at 0.38% in the U.K. calibration and 0.29% in the counterfactual with debt shares as in the U.S., respectively. This implied debt-inflation multiplier therefore aligns well with the empirical evidence presented in section 2, being placed between the first and the third tercile of the range of admissible estimates arising from the high-powered 'mini-budget shock'.

The multiplier for the U.S. calibration furthermore fits well with the evidence presented in Hazell and Hobler (2024), who find a debt-inflation multiplier in the U.S. of 0.19%. The model therefore attributes a significant share of the differences in the debt-inflation dynamics between the U.S. and the U.K. to the differences in the share of inflation-indexed debt, confirming the intuition laid out by the model.

Confirming the intuition behind the mechanisms underpinning inflation-indexed debt, the 'break' observed in the inflationary response in period 1 can be directly attributed to inflation-indexed debt as well: since inflation-indexed debt feeds back to the price level only with a one-period lag (as derived in the simple model in section 3), the inflationary pressure is not increasing monotonically, but only with this one period lag as seen in figure 9.

Summarizing, I therefore find that turning off the debt indexation channel of government debt (i.e., setting inflation-indexed debt to zero) nullifies all dynamics beyond the first-order dynamics of the spending shock, nesting the expected reaction to a fiscal expansion under a fiscally-led policy mix with non-Ricardian households: output and inflation co-move in general, but no higher-order dynamics are observed. Once inflation-indexed debt is present, however, inflationary pressure becomes more pronounced and persistent, accompanied by tax changes that reflect the need of the fiscal authority to cover the additional expenses arising from serving the cost of maturing inflation-indexed debt.<sup>27</sup>

An important factor in the analysis is the persistence of the government spending shock,  $\rho_G$ . As the persistence of the shock underlying figure 9 is relatively high, the observed dynamics are tightly connected to intertemporal substitution motives for the household. To highlight the 'barebones' reaction of the economy to a one-off government spending shock (and to highlight the corresponding relevance of inflation-indexed debt in this world), I consider a non-persistent fiscal shock next.

<sup>&</sup>lt;sup>27</sup>Complementary impulse-response functions of bond prices and quantities, as well as of the price level itself, are provided in appendix C.

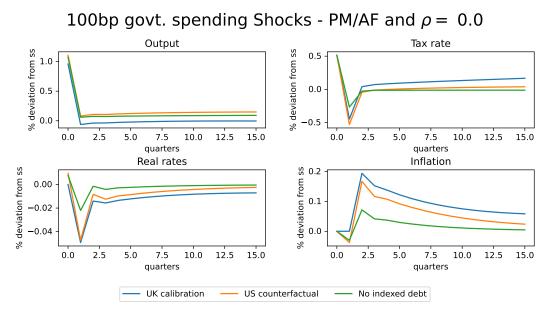


Figure 10: IRFs to the government spending shock under a fiscally-led policy mix - counterfactual without indexed debt.

Figure 10 summarizes the aggregate response of the economy to a fiscal spending shock when the government spending shock is not persistent at all,  $\rho_G = 0$ . Unsurprisingly, the persistence of output is virtually zero, too. As the government spending shock is short-lived, intertemporal substitution motives matter less, leading to less of a crowding out of consumption on impact. Between the three indexed-debt-share calibrations, there is a minimal difference in in output even after the shock dies out, which will be linked explicitly to the behavior of the tax-inflation nexus.

Focusing next on the top-right panel shows that the fiscal authority again covers the shock to a limited extent (nominally 50%, in line with the calibration). Once the shock dies out, there is a brief small deviation of the tax rate below zero in the subsequent period across all calibrations, which is mostly linked to a temporary crowding out through changes in the equilibrium value of both types of debt and related wealth effects on the households. In the case without indexed debt (green line), the tax rate is subsequently flat at zero, while it remains slightly positive in the cases with inflation-indexed debt (orange and blue lines).

The fact that tax rates remain elevated can be directly linked to materialized inflation rates, depicted in the bottom right panel. Compared to figure 9, inflation rates in figure 10 are generally less than half as large, but there is still a pronounced increase in inflation rates from two quarters after the initial shock onwards. This is again related to a devaluation of the surplus-backing of the stock of government debt, which must be counterfinanced by a devaluation of the stock of debt. While this reduction in the shock impact is also the case without indexed debt (in the green line), the presence of such debt magnifies the inflationary pressure from the fiscal expansion by a factor of two. The differences between the calibrations to U.K. and U.S. debt shares are marginal, since most of the differences are covered by differences in the tax rate in the medium-run.

Real rates correspondingly turn negative at the initial shock, followed by a gradual increase back to their respective steady-state value. Inflation rates therefore also mirror the pattern of real rates in line with the Fisher equation (as the monetary authority remains passive), such that there is an uptick in inflation shortly on impact followed by a gradual unwinding relative to the observed steady-state of inflation.

Finally, I highlight what changes in the simulations when a monetary-led policy mix is considered instead, corresponding to fiscal policy turning 'passive' in the language of Leeper (1991). The calibration of the policy parameters in this case follows from table 1. Figure 11 summarizes the results from this exercise for highly persistent fiscal shocks,  $\rho_G = 0.8$ .

The response of output turns out to be qualitatively similar *on impact* relative to the fiscally-led policy mix for the calibration without indexed debt, but for both calibrations with positive indexed debt levels, output is generally more sensitive to the government spending shock - there is less crowding out both on impact and in the long-run. For the case without indexed debt, however, observe that the output reaction to the government spending impulse turns negative in the medium run (after nine quarters), reflecting that under monetary-led policy mixes fiscal policy generates smaller wealth effects on behalf of the households, such that the increased spending will be financed in part by a later reduction in available resources.

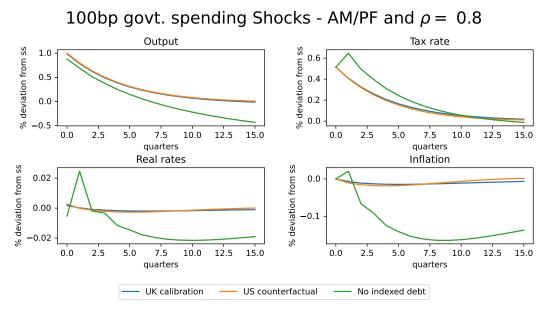


Figure 11: IRFs to the government spending shock under a monetary-led policy mix.

Across the board, there is little quantitative difference between the U.K. and the U.S. debt share calibrations. Since this specification follows a conventional monetary-led policy mix, fiscal policy, as exemplified through the tax rate, passively adjusts to ensure that the government budget constraint holds. It does so by increasing tax rates by a consistently higher margin relative to the

fiscally-led policy case. Because the tax rule shifts correspondingly, the real value of government debt remains unchanged, which is reflected in the absence of large movements of real rates and, correspondingly, of materialized inflation rates.

The model without inflation-indexed debt behaves vastly differently in the monetary-led policy mix, generally featuring a smaller response of output coincident with a sharper increase in tax rates and a temporary increase in real rates above their equilibrium level, followed by a depreciation of real rates in the medium-run. Inflation rates, after being positive on impact, similarly exhibit deflationary pressure. The reason for that lies in the absence of risk-sharing among households and the imperfect insurance properties borne by normal (non-indexed) bonds: in the light of such incomplete markets, the government spending measure exhibits a greater degree of Ricardian disequivalence, impacting households through a negative wealth effect. This negative wealth effect contributes to a reduction in household demand and a slight deflationary pressure, as materialized in figure 11.

Appendix C presents further omitted simulation results, in particular related to the IRFs of bond prices and interest rates, household policy functions and monetary policy shocks. In particular the revaluation of the bonds as expressed through their prices are of interest, as they confirm the above arguments that the revaluation of the intertemporal government budget equation belongs to the main determinants of the inflationary response.<sup>28</sup>

## Decomposing the price level response in the monetary-led policy mix

The government debt valuation equation (9) allows the decomposition of the drivers of inflation in the model by postulating that this equation is informing the rate of inflation under the fiscally-led policy mix (following Bianchi et al. (2023) and Kaplan et al. (2023)) and decomposing the various drivers of inflation in general equilibrium across the U.K. and the U.S. debt share calibrations. Consider the government debt valuation equation, depicted suppressing household heterogeneity for expositional simplicity:

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \tilde{\mathcal{M}}_{t,t+k} \left[ c_{t+k} - \varepsilon_{t+k} (1 - \tau_{t+k}) w_{t+k} N_{t+k} \right] + \left[ Cov_t \left( \tilde{\mathcal{M}}_{t+k,t+k+1}, \Pi_{t+k+1} \right) + \tilde{\mathcal{M}}_{t+k,t+k+1} \left( \mathbb{E}_{t+k} \Pi_{t+k+1} - \Pi_{t+k+1} \right) \right] \frac{b_{i,t+k}}{P_{t+k}} \right].$$
(17)

Through a linear approximation, I recover the terms in blue, yellow, green, and red explicitly, claiming that all further terms (depicted in purple) are higher-order terms captured by the remaining household heterogeneity.<sup>29</sup>

<sup>&</sup>lt;sup>28</sup>In further work, I aim to quantify the effect of household heterogeneity more fully with the help of non-linear impulse-response functions that might showcase significant amplification of observed responses in labor supplied, output, and inflation for the case with inflation-indexed debt.

<sup>&</sup>lt;sup>29</sup>In terms of the measurement of the purple terms, I simply attain the residual difference between all other terms and the gross rate of inflation to the purple terms, mirroring the fact that the solution algorithm delivers a solution that

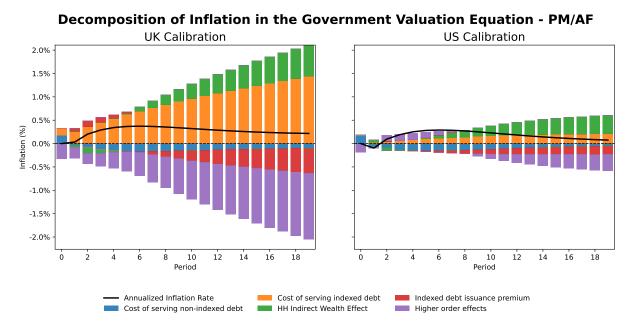


Figure 12: Decomposition of inflation in response to a 1% government spending shock under a fiscally-led policy mix.

Figure 12 shows the results of this decomposition of inflation in the two leading calibrations (to debt shares as observed in the U.K. and the U.S.) under a conventionally fiscally-driven policy mix, as defined in table 1.

Across both calibrations, the cost of serving maturing inflation-indexed debt indeed contributes significantly to observed inflation rates, as do the wealth effects on the households which are net-positive (except for the first few periods, in which the tax rate change outweighs the benefits borne from holding sovereign debt). As the inflation rate increases, there is a premium that the government can exert from issuing indexed debt, creating a medium-run deflationary pressure (red bars). A relatively important part of the inflationary dynamics are higher-order effects, which are generally deflationary in the medium-run across both calibrations, arising from the movements in the cross-section of the stochastic discount factor through the path of taxation that yield depressed consumption levels and, therefore, deflationary pressure on aggregate.

Even though the absolute difference in observed inflation levels is relatively small, the decomposition reveals that the volatility of the individual aspects increases sharply in the share of inflation-indexed debt. The magnitude of the individual contributors to inflationary/deflationary pressure are almost an order of magnitude larger in the calibration to U.K. debt shares, but the effects are canceling each other out to a significant extent.

#### The interaction between policy rules and the share of indexed debt

Having considered the general response of a battery of macroeconomic variables in response to surprise government spending shocks, I now zoom into the role borne jointly by inflation-indexed

is linear in aggregates, but plausibly non-linear in idiosyncratic elements.

debt and fiscal-monetary policy interactions, as exemplified through their respective policy rules (10) and (12) for inflation on impact of such shocks. To that goal, I fix the monetary policy coefficients at the levels summarized by table 1 and vary the share of inflation-indexed debt in the government debt portfolio,  $\omega_t = \frac{b_t}{B_t + b_t}$ , between  $[0,1)^{30}$ , while also varying the strength of the fiscal policy reaction coefficients,  $\{\gamma_B, \gamma_b\}$ , between [0,1], which are the coefficients under which fiscal policy is conventionally considered "active". This analysis is restricted to cases in which  $\gamma_B = \gamma_b$  here, such that no variation of impact inflation is induced by a change of the *relative* prevalence of the two types of debt, as the share of inflation-indexed debt is kept at its respective calibrated steady-state value.

The first case considered is the one of conventionally passive monetary policy ( $\phi_{\pi}=0.5$ ). The reaction of the price level in the first year after the impulse across the policy combinations and various shares of inflation-indexed debt in the sovereign debt portfolio is depicted in figure 13.

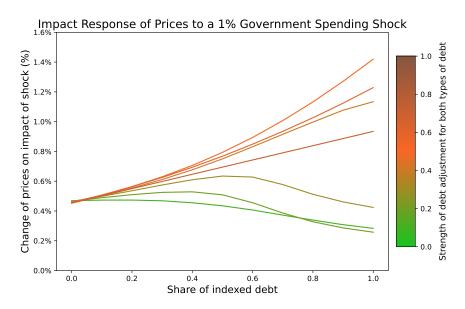


Figure 13: Cumulative one-year reaction of prices in response to fiscal spending shocks under a fiscally-led policy mix.

On the x-axis, I vary the share of indexed debt in the total debt portfolio (while maintaining a constant overall relation between the gross stock of debt and GDP), while the colors indicate the chosen fiscal reaction coefficients  $\gamma_B = \gamma_b$ . Thus, orange and especially brown colors reflect 'less active' fiscal policy in the conventional sense (as more of the shock is covered by corresponding tax raises), while greener colors reflect 'more active' fiscal policy.

I first focus on the leftmost point at which the share of inflation-indexed debt is exactly zero. The observed response is a direct mapping of well-known results in the literature on fiscal-monetary interactions, which are therefore nested by the model. The 'more active' fiscal policy is (i.e., as  $\gamma_b, \gamma_B \rightarrow 0$ ), the stronger is the immediate impact on the price level when the shock occurs (the green lines are the highest for a zero share of inflation-indexed debt). This result is fully in line

<sup>&</sup>lt;sup>30</sup>Recall that it is impossible for all debt to be inflation-indexed in line with proposition 2.

with existing evidence from fiscal-monetary interaction models, and reflects that the necessary devaluation of a fully nominal debt stock is higher when income taxation does not react at all to the expansionary government spending shock. The difference across the various policy scenarios, however, is relatively small in the impact period.

The more interesting dynamics occur when moving to the right in the above picture, i.e., as the share of inflation-indexed debt in the government debt portfolio is gradually increased. The interaction between the adjustment coefficients and the share of indexed debt into two categories *based* off the value of the adjustment coefficients of fiscal policy can be broadly categorized as follows:

- $\gamma_B = \gamma_b \gtrsim 0.3$  (orange-brown lines): Here, fiscal policy adjusts by covering comparatively more of the government spending shock through a corresponding increase in taxation. The higher the share of inflation-indexed debt, the greater is the change of the price level on impact, as the debt stock that can be devalued once the shock manifests itself is comparatively smaller, leading to a larger needed depreciation of that (smaller) stock of non-indexed debt.
- $\gamma_B = \gamma_b \lesssim 0.3$  (green lines): in these cases, the taxation schedule of the government covers only very little of the additional expense coming from the government spending shock. For realistic levels of inflation-indexed debt (below 40%), there is a general increase in the response of prices on impact, in line with standard predictions. The more indexed debt is present in the economy, the higher the devaluation of the outstanding debt stock must be, as in the case above. As the share of inflation-indexed debt becomes very large, however, a surprising effect comes into play: the impact change of the price level actually starts to become smaller. This reduction in the size of the impact change is directly related to the *real* side of the economy, as evidenced in figure 9: as the share of inflation-indexed debt increases, the real expansion of the economy becomes less pronounced, and the net worth of the wealthier households that hold large quantities of inflation-indexed debt decreases (as shown in figure C.6 in the appendix). This holds particularly true when the taxation reaction of the government is rather small. Through the Phillips Curve, then, downwards pressure on prices is exerted, overcoming some of the inflationary pressure induced through the government budget equilibrium.

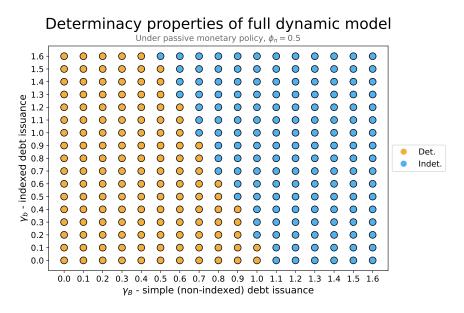
#### Fiscal-monetary policy combinations and determinacy

As a final exercise, I consider explicitly for which values of the fiscal and monetary policy parameters the linearized system implies a unique, saddle path-stable equilibrium. In doing so, I exploit the 'winding number criterion' developed in Auclert et al. (2023), which is suitable given the choice of the sequence-space Jacobian as the primary device to solve the full dynamic model.<sup>31</sup>

<sup>&</sup>lt;sup>31</sup>A detailed exposition of the 'winding number criterion' can be found in Auclert et al. (2023). Intuitively, it is related to the Blanchard and Kahn (1980)-conditions, which are cast in state-space. The winding number criterion provides a generalizable mapping of the Blanchard-Kahn conditions onto the sequence-space, i.e., allowing for infinitely many 'quasi-roots' of the linearized system. The prerequisites to apply the winding number criterion, such as the quasi-Toeplitz property of the generalized Jacobian, are not violated (the corresponding results are available upon request).

Figure 14 summarizes the determinacy properties of the model on an equispaced grid of the fiscal policy reaction parameters  $\gamma_B$  and  $\gamma_b$ , setting  $\phi_\pi = 0.5$ . The model thus effectively says that monetary policy does *not* raise interest rates more than one-for-one with inflation, which is conventionally dubbed 'passive monetary policy' (or, equivalently, a fiscally-led policy mix). Interestingly, the results relate to canonical determinacy principles in line with Leeper (1991), even if they do not fully overlap. In particular, uniqueness of the equilibrium path is reached for conventional values of the non-indexed debt reaction parameter  $\gamma_B$  when inflation-indexed debt issuance in response to fiscal shocks is zero (which is evidenced by the last row of figure 14).

Once the taxation schedule is directly related to inflation-indexed debt deviations from equilibrium as well ( $\gamma_b > 0$ ), two interesting phenomena arise. First, a trade-off in the government debt rule arises, by which an increased reactiveness of the taxation schedule to deviations of the value of indexed debt  $b_t$  from steady-state must be paid off with a smaller reactiveness with respect to the market value of non-indexed debt  $B_t$ . Second, this trade-off is non-linear: in particular, despite monetary policy being conventionally 'passive', it is possible for governments to react *more than one-for-one* ( $\gamma_b > 1$ ) with their taxation schedule in relation to deviations of inflation-indexed debt from steady-state, provided that the adjustment with respect to non-indexed debt  $B_t$  is small enough.



**Figure 14:** Determinacy of the generalized Jacobian in relation to choices for the fiscal and monetary policy reaction coefficients when monetary policy can be considered conventionally 'passive'.

## 7 The effects of indexed debt in a tractable General Equilibrium model

Section 6 explored in a rich quantitative model how inflation-indexed debt and policy rules interact to give rise to inflationary pressure in response to fiscal deficit shocks. In that model, however, characterizing the effect of the joint role borne by indexed debt and the prevalent policy rules for inflation is not possible analytically.

I now overcome this limitation by introducing indexed debt in a tractable framework with possibly non-Ricardian fiscal policy, drawing on Angeletos et al. (2024) and Nakamura et al. (2025), which is a New Keynesian model with mortality risk (NK-OLG model). The major goal of this exercise is to show how indexed debt can matter for inflation in a tractable general equilibrium framework. As the model framework used for this analytical exercise mainly relies on Angeletos et al. (2024), with the only changes being done to (i) the aggregation on the aggregate demand side; and (ii) to the fiscal and monetary policy rules that reflect the presence of indexed debt, I relegate the full derivations to appendix D.

The main result emphasizing the *relevance* of inflation-indexed debt for inflation in this model is that it qualifies the previously claimed equivalence between fiscally-led policy mixes and HANK-type mechanisms inducing fiscal inflation. This idea can be summarized in the following proposition:

**Proposition 3** Let  $\pi_{\varepsilon,0}^{FD,RANK}$  denote period 0 inflation in a New Keynesian (RANK) framework with inflation-indexed debt and under a fiscally-led policy mix; and let  $\pi_{\varepsilon,0}^{MD,HANK}$  denote period 0 inflation in a NK-OLG ( $\approx$  HANK) framework with inflation-indexed debt, strictly positive mortality risk, a simplified aggregate demand equation, and under a monetary-led policy mix. Let  $\kappa$  be the slope of the Phillips curve,  $\theta$  the share of inflation-indexed debt,  $\tau_y$  the share of output taxed (in deviations from steady-state),  $\beta$  the discount rate, and let  $1-\omega$  be the mortality risk of a household. If

$$\tau_{y} > \beta \frac{D^{SS}}{Y^{SS}} \left[ \frac{\kappa \theta}{1 - \beta} - (1 - \omega) \left( \frac{1}{\sigma} - \frac{\kappa}{1 - \beta} \right) \right], \tag{18}$$

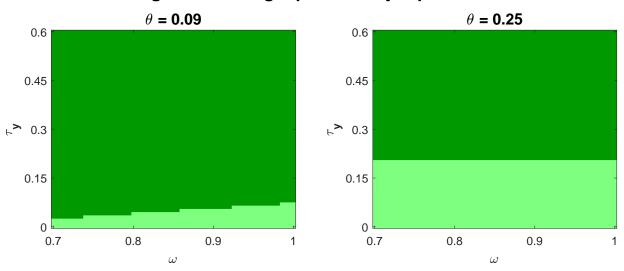
then impact inflation in response to an expansionary fiscal shock is higher in the policy limit point for the fiscally-led RANK economy relative to the monetary-led HANK economy, i.e.,  $|\pi_{\varepsilon,0}^{FD,RANK}| > |\pi_{\varepsilon,0}^{MD,HANK}|$ , conditional on

$$\kappa < \frac{1 - \beta}{\sigma}.\tag{19}$$

#### **Proof.** See appendix D. ■

Why does this matter? By this result, I qualify recent insightful evidence that the effects of deficit shocks for inflation in RANK economies with fiscally-led policy mixes can be replicated up to first-order by models with monetary-led policy mixes, which are the more common calibrations, conditional on the existence of mortality risk (e.g., Angeletos et al. (2024)). The underlying idea is that both models induce a form of Ricardian dis-equivalence which can be parametrized in a way by which the two classes of models mirrors each other, refuting the need to analyze models with fiscally-led policy mixes. The major insight coming from inflation-indexed debt for this debate is that this point does not hold exactly once the model contains multiple types of assets issued by the government with some real return differential.

# Regions with larger price level jumps in RANK



**Figure 15:** The parametrizations under which either of the inequalities described by proposition 4 is fulfilled, indicating larger deviations of the price level from steady-state in response to deficit shocks in the RANK model. The dark green area denotes places where the magnitude of inflation is larger in the RANK case, the light green area denotes places where *de*inflationary pressure is instead amplified. Calibration:  $D^{SS} = 1$ ,  $Y^{SS} = 1$ ,  $\kappa = 0.025$ ,  $\beta = 0.97$ ,  $\sigma = 1$ ,  $\tau_d = 0$ ,  $\phi = \frac{1-\omega+\varepsilon}{\sigma}$ .

Figure 15 summarizes the parameter space in terms of the tax-base channel  $\tau_y$  and the household mortality risk  $\omega$  for which the inequalities stated in proposition 4 hold. For conventional values of  $\tau_y$  and  $\omega$ , we can observe that inflation-indexed debt usually raises materialized inflation rates. If the tax base channel  $\tau_y$  were very small (and the share of inflation-indexed debt would simultaneously be large), inflation-indexed debt could enhance deflationary pressure (light green color), while for common calibrations of tax base channels, inflation-indexed debt amplifies the existing positive inflation on impact. Therefore, when the share of inflation-indexed debt increases, this simple model generally exhibits a higher propensity to inflationary 'catastrophes' on the RANK side of the equilibrium when indexed debt is present.

### 8 Conclusion

This paper introduced inflation-indexed debt into non-Ricardian general equilibrium models. I first provided empirical evidence on the role of inflation-indexed debt as a major determinant of inflationary dynamics with the help of local projections applied to the U.K. and the U.S., as well as with a specific high-powered fiscal shock in the U.K. in September 2022. Next, I established in a simplified model that such debt itself suffices to make the price level a backward-looking state variable: the previous price level therefore matters directly for the determination of today's price level. I then introduced inflation-indexed debt in a state-of-the-art macroeconomic model with imperfect markets and household heterogeneity, ensuring the existence of a unique steady-state

<sup>&</sup>lt;sup>32</sup>This result has been achieved by nullifying the intertemporal substitution channel of windfall gains arising from indexed debt holdings, which was done by an appropriate adjustment of the monetary policy rule. Forgoing this channel generally increases the area under which indexed debt amplifies price level deviations from steady-state.

before providing model-driven evidence that inflation-indexed debt can indeed exacerbate the inflationary response to government spending shocks, in particular when fiscal policy is considered conventionally 'active' in the sense of Leeper (1991). Finally, I contrasted fiscally-led policy mixes and other mechanisms inducing non-Ricardian household behavior in a tractable model to complement the quantitative analysis, establishing that inflation-indexed debt operates under mechanisms that are not present in models that abstain from non-Ricardian dynamics induced through the fiscal block.

Both the empirical and theoretical results derived in this paper thus tarnish the classic notion that inflation-indexed bonds always limit inflation in a given country by offering governments a commitment device to 'not inflate the debt away', as argued by Campbell and Shiller (1996). While this notion can remain true absent government deficit shocks, the results point out that once the government budget is ex-post (after debt issuance) in disarray, the inflationary consequences of funding shortfalls can increase in the share of inflation-indexed debt. Issuance of indexed debt can therefore backfire despite its great ability to serve as an ex ante commitment device following Schmid et al. (2024).

Despite these conclusions, more can be done to emphasize the interaction between inflation-indexed debt and materialized inflation rates. A complete estimation of the model based off long-running samples of U.K. and U.S. data with a particular focus to the precise coefficients of the fiscal and monetary rules would further strengthen te conclusions of this paper, e.g., through a historical decomposition. Another refinement can be the inclusion of long-term government debt: as Cochrane (2001) and Barro and Bianchi (2023) show, the maturity structure of government debt matters for the trade-off between front-loaded and delayed inflation responses to deficit shocks.

Finally, inflation-indexed debt can inform recent policy debates on the possible regressivity or progressivity of inflation as implicit taxation. As evidenced by figure A.1 in the appendix, inflation-indexed debt, which serves as an insurance device against unexpected inflation, seems to be particularly skewed in household portfolios towards the highest decile of the income distribution. A more thorough analysis of the welfare effects of unexpected inflation to households at varying income levels should therefore be considered as a further policy-relevant application in due course.

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# **Appendix**

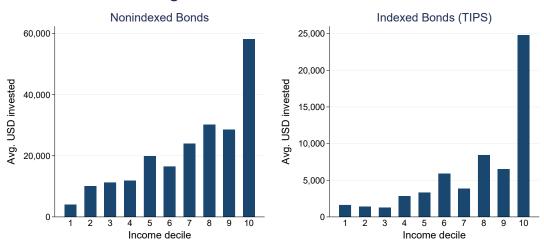
# A Additional empirical evidence

### A.1 The distribution of government debt holdings across households

Publicly available microdata reinforces the idea of an unequal distribution of indexed debt in household portfolios. This brief section focuses on the U.S. due to the superior availability of household-level data on asset holdings.

Figure A.1 plots the real (2017) Dollar value of nonindexed and indexed government debt holdings of households questioned in the U.S. Survey of Consumer Finances (SCF), separated by income deciles.<sup>33</sup> The left-hand panel of figure A.1 reflects the well-known left-skew of bond holdings of households in the income distribution, by which households at the upper end of the income distribution hold a significantly larger share of sovereign bonds. The right-hand panel of figure A.1 reflects a less well-known observation: this left skew is *vastly* more pronounced for indexed sovereign bonds, with the top income decile holding almost 40% of such bonds in the sample.

# Bond holdings in USD in the income distribution



**Figure A.1:** Distribution of indexed and non-indexed debt holdings across household income deciles, denoted in real (2017) USD. Data source: Survey of Consumer Finances (U.S.); sample period: 2014-2019.

Figure A.2 provides further evidence in favor of the distribution implied by the model, which has not been a targeted moment. Just as in the model (figure 8), the density of the asset distribution of both indexed and non-indexed bonds exhibits a significant skew, which is more pronounced overall for inflation-indexed debt. In particular, the size of the bins, even if not exactly matched, broadly reflects the distribution of the model very well.

<sup>&</sup>lt;sup>33</sup>I chose income deciles due to their clear definition in the survey with a single question. Constructing individual wealth variables is possible with the survey data, albeit this process is subject to particular choices about what to consider as household wealth. For most definitions of wealth, the results continue to hold qualitatively.

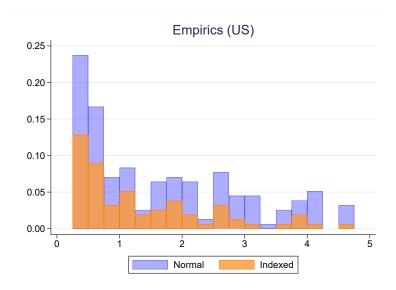


Figure A.2: Density of indexed and non-indexed debt holdings in the U.S. Survey of Consumer Finances; snapshot from October 2019.

### A.2 Further details on the Local Projection exercise

To shed further light on the evidence presented in section 2.2, I here provide the additional information related to the local projection exercise on U.K. data presented in figure 5 and introduce additional evidence using U.S. data in a similar exercise.

Figure A.3 plots the time series of the level and the first-difference of indexed debt in the U.K., showing the secular increase in the share of indexed debt in the sovereign bond portfolio since 1980.

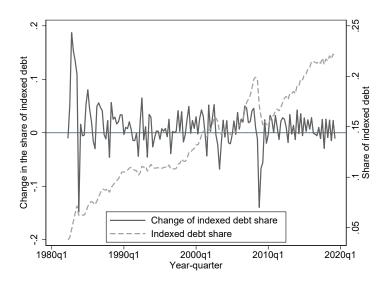


Figure A.3: Time series of the level of the share of inflation-indexed debt in the U.K. as well as the first-difference thereof.

Table A.1 gives the concrete numbers corresponding to figure 5, specifying the exact coefficients

of the interaction effect of  $\Delta\omega_t\,\varepsilon_t$  and the individual effects of the change in the indexed debt share  $\Delta\omega_t$  and the exogenous fiscal shock  $\varepsilon_t$  on the cumulative price level change from the pre-shock period -1 until the period specified.

Dependent variable: log(Cumulative Inflation)											
Forecast period:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)			
Fiscal Shock	-0.01**	-0.01	-0.01	-0.01	-0.02	-0.02	-0.02	-0.02			
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)			
Index Share	0.02***	0.02**	0.03**	0.02	0.02	0.02	0.03	0.03			
	(0.00)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)			
Fiscal Shock × Index Share	0.10*	0.09	0.20	0.26*	0.39**	0.40*	0.60**	0.81***			
	(0.06)	(0.10)	(0.13)	(0.14)	(0.18)	(0.21)	(0.24)	(0.27)			
Additional Controls	Y	Y	Y	Y	Y	Y	Y	Y			
Observations	155	154	153	152	151	150	149	148			
$R^2$	0.412	0.518	0.559	0.630	0.592	0.599	0.575	0.602			

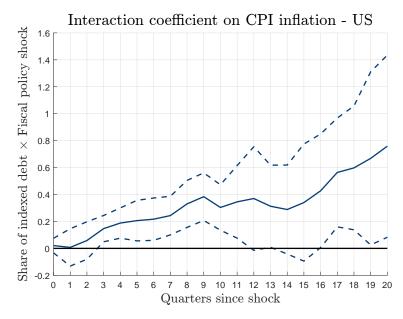
**Table A.1:** Local Projection results for the U.K. The controls include the first four lags of the real GDP growth rate, the Bank of England Bank Rate, the change in the weighted real exchange rate, a same-period recession indicator, year-fixed effects, and the first lag of the price level difference. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction).

While the share of indexed debt itself does not impact medium-term inflation, the interaction effect of the share of indexed debt with the identified fiscal shock follows the pattern given in figure 5.

To ensure that this mechanism is not idiosyncratic to the U.K., I provide now the results of a similar exercise applied to the U.S. Here, I utilize the U.S. fiscal shock series provided by Mierzwa (2024). I leverage the identified fiscal shocks and estimate the same local projection specification (equation (2)) to estimate the role played by inflation-indexed debt in exacerbating the effects of fiscal spending shocks in the U.S. Table A.2 and figure A.4 summarize the results of this exercise using data since 1980, which is the earliest period for which identified fiscal shocks are available.

Dependent variable: log(Cumulative Inflation)										
Lag periods:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)		
Fiscal Shock	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00		
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)		
Index Share	0.00	-0.01	-0.01	-0.01	-0.00	-0.00	-0.01	-0.01		
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)		
Fiscal Shock × Index Share	0.02	0.01	0.06	0.15**	0.19***	0.21**	0.22**	0.24***		
	(0.03)	(0.08)	(0.08)	(0.06)	(0.07)	(0.09)	(0.10)	(0.09)		
Additional Controls	Y	Y	Y	Y	Y	Y	Y	Y		
Observations	161	160	159	158	157	156	155	154		
$R^2$	0.324	0.371	0.474	0.531	0.543	0.542	0.559	0.554		

**Table A.2:** Local Projection results for the U.S. The controls include the first four lags of the real GDP growth rate, the Federal Funds Rate, the change in the weighted real exchange rate, a same-period recession indicator, year-fixed effects, and the first lag of the price level difference. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction).



**Figure A.4:** IRFs implied by a local projection in the style of equation (2). The controls include the first four lags of the real GDP growth rate, the Federal Funds Rate, the change in the weighted real exchange rate, a same-period recession indicator, year-fixed effects, and the first lag of the price level difference. Confidence intervals are provided at the 90% level. Sample length: 1980 Q1 - 2019 Q4.

The results here paint a supporting picture, as the interaction effect between the change in the share of inflation-indexed debt and the identified fiscal shock appears to be statistically significant in the medium-term again, though the level of the effect is smaller than in the U.K..

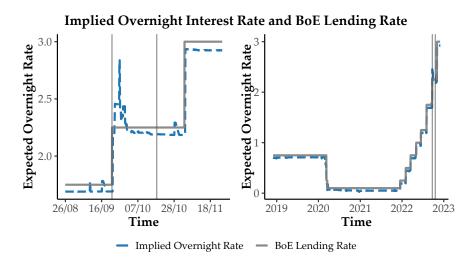
#### A.3 Additional evidence on the September 2022 'mini-budget' shock

In this part of the appendix, I provide additional details on the fiscal policy measure dubbed "The Growth Plan" in September 2022 in the United Kingdom, commonly known as the 'mini-budget', and the reaction on financial markets to its announcement. To do so, I utilize ticker-frequency data on market prices (and thus yields), debt quantities, and risk perceptions. The data comes from the Bank of England (BoE), the U.K. Treasury, and Bloomberg Financial Services.

I begin by examining the degree to which the policy announcement can be informative about the propensity of a type of 'fiscally-led policy mix' in a wider sense, i.e., whether the policy measures around this particular fiscal shock can be placed in a context at which monetary policy passively adjusts to the fiscal policy measure, taking the fiscal announcement as given.<sup>34</sup> A possible measure that is plausibly related to debt sustainability concerns introduced by the budget announcement as well as to the prospective monetary reaction are expected overnight interest rates. These are the interest rates used for overnight bank lending activities on financial markets, instrumented using

<sup>&</sup>lt;sup>34</sup>Determining uniquely whether a given policy announcement, or a given time period, clearly relates to a monetary-led or a fiscally-led policy mix in a narrow sense, i.e., in relation to the respective policy rules and how they inform the stability of the underlying economic system, is generally not possible purely based off time-series data. Simply put, the "Taylor Principle" cannot be tested as its impact on the uniqueness properties surrounding macroeconomic models depends on off-equilibrium threats that cannot be observed under the condition of the Taylor Principle itself holding (Cochrane, 2011; Neumeyer and Nicolini, 2025).

swaps on overnight lending between the day at question and the day of the next monetary policy meeting. Normally, these swaps follow the prevailing nominal interest rate closely (with a spread of a couple of basis points), as any other rate would induce arbitrage by the possibility of a risk-free hedge using the current overnight nominal interest rate. As figure A.5 shows, however, the turmoil introduced by the 'mini-budget' caused a remarkable wedge between the two rates:



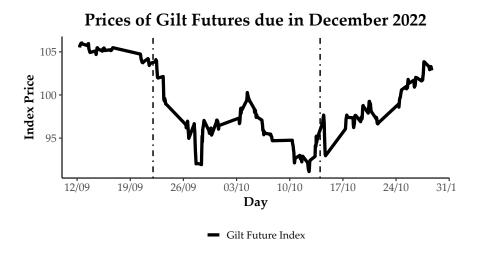
**Figure A.5:** The prevailing Bank of England Lending Rate and the Implied Overnight Interest Rate, derived by instrumenting overnight interest swaps from today to the expected next meeting of the BoE Monetary Policy Committee.

As can be inferred from the right-hand side panel for a period of five years, and on the left-hand side in more detail for the period of interest, the implied overnight interest rate follows the BoE lending rate closely, exhibiting jumps around meeting dates of the BoE Monetary Policy Committee in alignment with monetary policy decisions.

The period of the mini-budget, which commenced one day after a Monetary Policy Committee (MPC) meeting (September 23 and September 22, respectively), induced movements in the expected overnight rates that were not observed at any other point in time - despite no MPC meeting in near sight. Expected overnight rates shot up far beyond the then-prevailing BoE bank lending rate by up to 50 basis points. Such movements can be caused by an array of different possibilities: it could be either that fiscal policy caused a shift in market expectations of monetary policy in the short-term, thus implying that monetary policy was considered to be 'reactive' to the fiscal policy announcement, or that the mini-budget was expected to have such detrimental consequences on inflation that the BoE was required to react immediately, or it might be reflective of liquidity issues in the swap market in the same period.

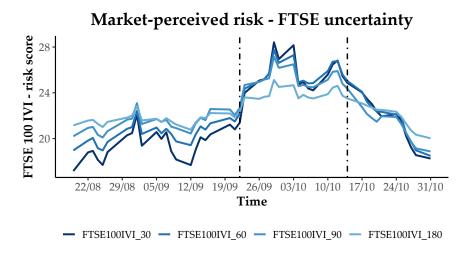
Correspondingly, and as can be inferred from figure A.6, prices of Gilt futures dropped sharply during the period in which the U.K. mini-budget was expected to be put in place.

<sup>&</sup>lt;sup>35</sup>On September 27, then-BoE chief economist Huw Pill stated that the proposed U.K. government budget might require a "significant monetary response", indicating readiness on behalf of the BoE to adjust the monetary stance, but no concrete emergency meeting date had been proposed at that point.



**Figure A.6:** Evolution of the weighted Gilt Future Index for futures due in December 2022, weighting Gilt prices based on a normalized face value of 100, after adjusting for expected inflation. Data source: Bloomberg.

An important caveat is that reducing the observed dynamics to expected revaluations of bonds and prospective interest rate movements does not capture all aspects related to this fiscal policy announcement. Uncertainty surrounding the proposed policy measures might have also been an important contributor to market reactions. Figure A.7 plots the FTSE 100 IVI Index that can plausibly serve as a proxy for uncertainty.

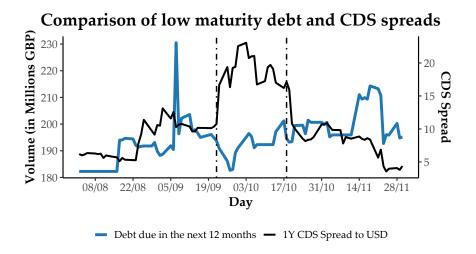


**Figure A.7:** The uncertainty index over equity of the largest publicly traded British companies. The lines measure implied uncertainty in 30-day, 60-day, 90-day, and 180-day forward-looking windows.

The mini-budget episode coincided with an inversion of the market-perceived risk relative to the forecast horizon: whereas in the periods before and after the mini-budget announcement market risk was perceived to be higher in the medium-term (180 days) than in the short-term (30 or 60 days), the opposite has been the case during this short-lived fiscal episode.

A final and related aspect is the possibility of elevated default risk, which has been omitted from the

model presented in this paper. In figure A.8, I present the Credit Default Swap spread alongside the quantity of Gilts maturing in the near-term around September 2022, which plausibly indicate near-term fiscal refinancing pressures.



**Figure A.8:** Government debt maturing over the course of the next 12 months, plotted against the 1-year spread of Credit Default Swaps on U.K. Gilts over USD futures. Data source: Bloomberg.

During the period of the 'mini-budget' announcement, the amount of maturing government debt was at rather low levels, i.e., there was no inherent pressure to refinance a large quantity of maturing obligations around the time of the 'mini-budget' announcement. Yet, the black line, which plots short-term Credit Default Swaps based off U.K. Gilts, was elevated during the 'mini-budget' episode, confirming a disconnect of the total amount of maturing debt from perceived market risk. While the two measures have a correlation of 0.49 in the period leading up to the announcement of the mini budget, that correlation drops to -0.51 between the middle of September and the middle of October 2022.

All these data points reinforce the idea that the 'mini-budget' announced by the U.K. Treasury in September 2022 was indeed an unexpected fiscal measure that significantly subverted perceived fiscal sustainability, with wide-spread ramifications for expected real returns on government bonds, expected inflation and interest rates, and elevated risk levels.

#### FAQ: the 'mini-budget shock' episode

In addition to the evidence derived using market data, I here present narrative viewpoints complementing the understanding of the 'mini-budget' episode.

Why did markets possibly reverse the uptick in inflation expectations initially?

- On September 26, around 4.00pm, Kwasi Kwarteng announced to publish a 'medium-term fiscal plan', which possibly indicated greater restraint in fiscal policy: (Bloomberg).
- On September 28, a further plausible shock to perceptions of fiscal sustainability occurred: Moody's explicitly deemed the mini-budget to put U.K. debt sustainability in danger, fol-

lowed by a same-day upshoot of inflation expectations: (Reuters).

• Likewise, on September 28, in a reversal of expectations caused by the September 26 statement, the Treasury explicitly rejected for the first time since the initial announcement any idea of reneging on the additional budget shortfall, thereby re-affirming expectations about the fiscal policy measure actually being pushed through. See: (BBC). On the very same day, the Bank of England intervened in bond markets by re-starting long-dated government bond purchases, which, again, plausibly re-affirmed the idea that the Treasury will not back down. This was announced at around 11.20am - see (X). Swap breakeven rates shot up by 60bps in the three hours after the statement made by the Bank of England.

Why were inflation swaps priced much higher in August 2022 compared to the dynamics occurring in September and October 2022?

- On August 17, 2022, the ONS released a report of CPI inflation being 10.1%, breaking the 10% barrier for the first time in 40 years, also beating the private sector forecasts decisively (Bloomberg). This occurred alongside a significant depreciation of the British Pound (FT). Likewise, implied interest rate raises corresponding to expectations of vastly more aggressive monetary tightening from that period onward alongside a yield curve inversion appeared around August 15 (FT).
- Implied one-year ahead inflation expectations peaked at around 8% in August. This is still *vastly* below the forecasts released in August 2022 by major financial market actors: the Goldman Sachs forecast of one-year ahead inflation amounted to 14.8%, with a 'negative' scenario of 22.1% annual inflation for the U.K. implied in their August 2022 briefings (FT). Relative to that forecast, the change in inflation swaps implied in that policy uncertainty episode was relatively benign. This period of increased inflation expectations also coincided with record prices on natural gas spot markets in the U.K..

### B Derivations and proofs from the main text

#### **B.1** Derivations from section 4

#### On the transversality condition with uninsurable idiosyncratic risk

I first show why it is not possible to directly arrive at a government debt valuation equation starting from aggregate debt quantities, following the logic of Brunnermeier et al. (2024).

To illustrate this point, start from the government budget constraint.

$$B_{t-1} + \Pi_t b_{t-1} = P_t s_t + Q_t B_t + q_t b_t.$$

This is the government budget constraint given some (real) surplus schedule  $s_t$  and bond pricing kernels  $Q_t$ ,  $q_t$ . All elements are multiplied by the unweighted average household SDF  $\mathcal{M}_{t,t+1}$  and

<sup>&</sup>lt;sup>36</sup>All debt in this model is single-period. I briefly expose the effects of long-term debt in appendix E.

divided by the current price level  $P_t$  to obtain

$$\mathcal{M}_{t,t+1} \frac{B_{t-1}}{P_t} + \mathcal{M}_{t,t+1} \frac{b_{t-1}}{P_{t-1}} = \mathcal{M}_{t,t+1} s_t + Q_t \mathcal{M}_{t,t+1} \Pi_{t+1} \frac{B_t}{P_{t+1}} + q_t \mathcal{M}_{t,t+1} \frac{b_t}{P_t}.$$

Adding and subtracting elements suitably on the right-hand side, I can express this equation as:

$$\begin{split} \mathcal{M}_{t,t+1} \frac{B_{t-1}}{P_t} + \mathcal{M}_{t,t+1} \frac{b_{t-1}}{P_{t-1}} &= \mathcal{M}_{t,t+1} s_t + \left( Q_t \mathcal{M}_{t,t+1} \Pi_{t+1} - \mathcal{M}_{t+1,t+2} \right) \frac{B_t}{P_{t+1}} \\ &+ \left( q_t \mathcal{M}_{t,t+1} - \mathcal{M}_{t+1,t+2} \right) \frac{b_t}{P_t} + \mathcal{M}_{t+1,t+2} \left( \frac{B_t}{P_{t+1}} + \frac{b_t}{P_t} \right). \end{split}$$

Iterating on this expression until T, dividing the resulting expression by the SDF, and taking limits  $T \to \infty$  ultimately gives:

$$\begin{split} \frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} &= \mathbb{E}_t \left[ \sum_{l=0}^{\infty} \frac{\mathcal{M}_{t+l,t+l+1}}{\mathcal{M}_{t,t+1}} s_{t+l} + \frac{Q_{t+l} \mathcal{M}_{t+l,t+l+1} \Pi_{t+l+1} - \mathcal{M}_{t+l+1,t+l+2}}{\mathcal{M}_{t,t+1}} \frac{B_{t+l}}{P_{t+l+1}} \right. \\ &+ \frac{q_{t+l} \mathcal{M}_{t+l,t+l+1} - \mathcal{M}_{t+l+1,t+l+2}}{\mathcal{M}_{t,t+1}} \frac{b_{t+l}}{P_{t+l}} \right] + \lim_{T \to \infty} \frac{\mathcal{M}_{T+1,T+2}}{\mathcal{M}_{t,t+1}} \left( \frac{B_T}{P_{T+1}} + \frac{b_T}{P_T} \right). \end{split} \tag{B.1}$$

This expression nests the usual debt valuation equation under complete markets, making use of  $Q_t \mathcal{M}_{t,t+1} \Pi_{t+1} = \mathcal{M}_{t+1,t+2}$  and  $q_t \mathcal{M}_{t,t+1} = \mathcal{M}_{t+1,t+2}$ .

It would be a mistaken belief that the current price level is determined by this integrated government budget constraint. This logic would require the last limiting term to vanish and go to zero. However, this is *not* necessarily the case: even though the transversality condition holds on the household level as a consequence of household optimality and a no-Ponzi condition, it *cannot* be aggregated to derive the aggregate transversality condition directly off-the-shelf. The reason is that the unweighted average SDF  $\mathcal{M}_{t,t+1}$  is discarding the heterogeneity of underlying consumption (which led to the rise of household-specific discount factors), and thus ignores the possibility of the government possibly earning an excess return on its debt issuance. This can be considered a 'safe asset premium' (Brunnermeier et al., 2024) and is reflective of the inherent value that such debt bears to households in partially overcoming market incompleteness, possibly yielding different 'fundamental' valuations of government debt by the household vis-à-vis the government.

However, assuming additionally perfect insurance of idiosyncratic risk, it would be possible to define a simple average SDF that is consistent across household and government valuations,

$$\bar{\mathcal{M}}_{t,t+1} \equiv \sum_i \frac{B_{it} + b_{it}}{B_t + b_t} \mathcal{M}_{i,t,t+1},$$

under which the final limiting term and all wedges would vanish due to a fair bond pricing valuation, creating a 'standard formulation' of the government debt valuation equation:

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[ \sum_{l=0}^{\infty} \frac{\bar{\mathcal{M}}_{t+l,t+l+1}}{\bar{\mathcal{M}}_{t,t+1}} s_{t+l} \right].$$

#### Derivation of equation (9) (proof of proposition 1)

This section presents the derivations underlying a *dynamic trading perspective* for asset valuation laid out in Brunnermeier et al. (2024), which avoids fallacies related to a possibly nonexistent aggregate transversality condition by defining the differences in government debt valuation between households and the government, which are rooted in the insurance properties that government bonds bear for households. This allows the leveraging of household-level transversality conditions to derive an aggregate government debt valuation equation that only holds for one initial candidate price level.

The starting point for the valuation equation of government debt is the household budget constraint, which is given by

$$P_{t}c_{it} + Q_{t}B_{it} + q_{t}b_{it} = \varepsilon_{it}(1 - \tau_{it})P_{t}w_{t}N_{t} + B_{i,t-1} + \Pi_{t}b_{i,t-1}$$

for each household *i*. Following the results derived in the household block, let households price bonds in accordance with their *SDF*:

$$B_{i,t-1} + \Pi_t b_{i,t-1} = \mathbb{E}_t \left( \mathcal{M}_{i,t,t+1} \right) B_{it} + \mathbb{E}_t \left( \Pi_{t+1} \mathcal{M}_{i,t,t+1} \right) b_{it} + P_t (c_{it} - \varepsilon_{it} w_t N_t (1 - \tau_{it})).$$

Splitting up the second expectation term yields:

$$B_{i,t-1} + \Pi_t b_{i,t-1} = \mathbb{E}_t \left( \mathcal{M}_{i,t,t+1} \right) B_{it} + \mathbb{E}_t \left( \mathcal{M}_{i,t,t+1} \right) \mathbb{E}_t \left( \Pi_{t+1} \right) b_{it} + b_{it} Cov_t \left( \mathcal{M}_{i,t,t+1}, \Pi_{t+1} \right) + P_t (c_{it} - \varepsilon_{it} w_t N_t (1 - \tau_{it})).$$

Dividing all elements by  $P_t$  and adding/subtracting relevant terms on the right-hand side allows to iterate on the resulting expression:

$$\begin{split} \frac{B_{i,t-1} + \Pi_t b_{i,t-1}}{P_t} &= \mathbb{E}_t \left( \mathcal{M}_{i,t,t+1} \right) \Pi_{t+1} \left[ \frac{B_{it} + \Pi_{t+1} b_t}{P_{t+1}} \right] + \left( c_{it} - \varepsilon_{it} (1 - \tau_{it}) w_t N_t \right) \\ &+ Cov_t \left( \mathcal{M}_{i,t,t+1}, \Pi_{t+1} \right) \frac{b_{it}}{P_t} + \mathbb{E}_t \left( \mathcal{M}_{i,t,t+1} \right) \frac{b_{it}}{P_t} \left( \mathbb{E}_t \Pi_{t+1} - \Pi_{t+1} \right). \end{split}$$

Now, start iterating on this expression. The first iteration yields:

$$\begin{split} \frac{B_{i,t-1} + \Pi_t b_{i,t-1}}{P_t} &= \mathbb{E}_t \left( \mathcal{M}_{i,t,t+1} \right) \Pi_{t+1} \left[ \mathbb{E}_{t+1} \left( \mathcal{M}_{i,t+1,t+2} \right) \Pi_{t+2} \left[ \frac{B_{i,t+1} + \Pi_{t+2} b_{i,t+1}}{P_{t+2}} \right] \right. \\ & \left. \left( c_{i,t+1} - \varepsilon_{i,t+1} (1 - \tau_{i,t+1}) w_{t+1} N_{t+1} \right) + Cov_{t+1} \left( \mathcal{M}_{i,t+1,t+2}, \Pi_{t+2} \right) \frac{b_{i,t+1}}{P_{t+1}} \right. \\ & \left. + \mathbb{E}_{t+1} \left( \mathcal{M}_{i,t+1,t+2} \right) \frac{b_{i,t+1}}{P_{t+1}} \left( \mathbb{E}_{t+1} \Pi_{t+2} - \Pi_{t+2} \right) \right] \\ & \left. + \left( c_{it} - \varepsilon_{it} (1 - \tau_{it}) w_t N_t \right) + Cov_t \left( \mathcal{M}_{i,t,t+1}, \Pi_{t+1} \right) \frac{b_{it}}{P_t} + \mathbb{E}_t \left( \mathcal{M}_{i,t,t+1} \right) \frac{b_{it}}{P_t} \left( \mathbb{E}_t \Pi_{t+1} - \Pi_{t+1} \right). \end{split}$$

Continuing rolling over, applying the LIE, and simplifying SDFs by making use of the identity  $\mathcal{M}_{i,t,t+k}\mathcal{M}_{i,t+k,t+l} = \mathcal{M}_{i,t,t+l} \ \forall t,k,l$  obtains:

$$\begin{split} \frac{B_{i,t-1} + \Pi_{t}b_{i,t-1}}{P_{t}} &= \mathbb{E}_{t} \left[ \sum_{k=0}^{\infty} \mathcal{M}_{i,t,t+k} \Pi_{t+1,t+k+1} \left\{ \left( c_{i,t+k} - \varepsilon_{i,t+k} (1 - \tau_{i,t+k}) w_{t+k} N_{t+k} \right) \right. \\ &+ \left[ Cov_{t+k} \left( \mathcal{M}_{i,t+k,t+k+1}, \Pi_{t+k+1} \right) + \mathcal{M}_{t+k,t+k+1} \left( \mathbb{E}_{t+k} \Pi_{t+k+1} - \Pi_{t+k+1} \right) \right] \frac{b_{i,t+k}}{P_{t+k}} \right\} \right] \\ &+ \lim_{T \to \infty} \left\{ \mathbb{E}_{t} \left[ \mathcal{M}_{i,t,t+T} \left( \frac{B_{i,t+T} + \Pi_{t+T+1} b_{i,t+T}}{P_{t+T}} \right) \right] \right\}, \end{split}$$
 (B.2)

where I use the notation  $\Pi_{t+1,t+k+1}$  to define gross inflation from period t+1 to period t+k+1. This expression is the integrated household budget constraint at optimality, from which the integrated *government* budget constraint is derived.

Crucially, household optimality implies  $\lim_{T \to \infty} \frac{B_{i,T} + \Pi_{T+1} b_{i,T}}{P_T} \leq 0$ , while a no-Ponzi condition on household debt holdings ensures that  $\lim_{T \to \infty} \frac{B_{i,T} + \Pi_{T+1} b_{i,T}}{P_T} \geq 0$ . Furthermore, by the definition of the SDF and the properties of the CRRA utility function,  $\lim_{T \to \infty} M_{i,t,T} \neq \pm \infty$ . Therefore, the final limit converges to 0 and must not be considered.

The formulation of equation (B.2) is intuitive: the real value of household bond holdings is equal to its expected discounted consumption benefits from today to infinity (as future net consumption earnings are suitably discounted with the SDF, which is a mirror image of the price of the two bonds), adjusted suitably for additional surprise earnings enjoyed from holdings of *indexed* sovereign debt: these are decreased by surprise inflation through its (negative) covariance with the SDF (as higher *future* inflation pushes the SDF down), and increased by surprise inflation through a level effect (since such inflation yields a windfall gain relative to what was paid for the indexed bond in the previous period).

Aggregating the household-level constraints up to an integrated government budget constraint and making use of the asset market clearing conditions  $B_t = \sum_i B_{it}$  and  $b_t = \sum_i b_{it}$ , and of the idea that the household TVCs hold individually, yields the following expression:

$$\begin{split} &\frac{B_{t-1} + \Pi_{t}b_{t-1}}{P_{t}} = \sum_{i} \left\{ \mathbb{E}_{t} \left[ \sum_{k=0}^{\infty} \mathcal{M}_{i,t,t+k} \Pi_{t+1,t+k+1} \left\{ \left( c_{i,t+k} - \varepsilon_{i,t+k} (1 - \tau_{i,t+k}) w_{t+k} N_{t+k} \right) \right. \right. \\ & + \left. \left[ Cov_{t+k} \left( \mathcal{M}_{i,t+k,t+k+1}, \Pi_{t+k+1} \right) + \mathcal{M}_{t+k,t+k+1} \left( \mathbb{E}_{t+k} \Pi_{t+k+1} - \Pi_{t+k+1} \right) \right] \frac{b_{i,t+k}}{P_{t+k}} \right\} \right] \right\}. \end{split}$$
 (B.3)

I simplify this equation by taking the summation into the expectation and switching the sums. To further simplify the integrated government debt valuation equation, create the variable  $A_{it}$  which captures the surpluses raised by the government from each household i:

$$A_{it} \equiv c_{it} - \varepsilon_{it}(1 - \tau_{it})w_t N_t + \left[Cov_t \left( \mathcal{M}_{i,t,t+1}, \Pi_{t+1} \right) + \mathcal{M}_{i,t,t+1} \left( \mathbb{E}_t \Pi_{t+1} - \Pi_{t+1} \right) \right] \frac{b_{it}}{P_t},$$

which is the full portfolio return of household i of holding an additional unit of net worth. This expression describes what the government factually can raise as surpluses from each household i. Define  $\bar{A}_t = \sum_i A_{it}$  as the sum of all individual-level surpluses. Then, rewrite the implied intertemporal government budget constraint (B.3) to:

$$\frac{B_t}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \left( \sum_{i} M_{i,t,t+k} \Pi_{t,t+k+1} \frac{A_{i,t+k}}{\bar{A}_{t+k}} \right) \bar{A}_{t+k} \right].$$

Defining the household value-weighted SDF  $\tilde{\mathcal{M}}_{t,t+k} = \sum_{i} \mathcal{M}_{i,t,t+k} \Pi_{t,t+k+1} \frac{A_{i,t+k}}{\bar{A}_{t+k}}$  gives the final government debt valuation equation:

$$\frac{B_t}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[ \sum_{k=0}^{\infty} \tilde{M}_{t,t+k} \bar{A}_{t+k} \right], \tag{B.4}$$

where  $\tilde{M}_{t,t+k}$  is now the weighted average SDF across all households i, adjusted for inflation, with weights being proportionate to  $A_{i,t+k}$ , consisting of the net utility gain from saving, the insurance premium on indexed debt (captured through the covariance term), and the possible windfall gain/loss from surprise inflation (captured through the last term in the definition of  $A_{i,t+k}$ ). Equation (B.4) is the government debt valuation equation, at times called 'the FTPL equation', that informs the price level at time t, given some previous price level  $P_{t-1}$ .

#### B.2 Proof of proposition 2

I first show that a unique steady-state can be attained even with endogenous real interest rates when indexed debt is present. Complementing this proof, I show how indexed debt translates into a model where taxation is assumed to cover all interest expenses over time on a stationary equilibrium path, following Hagedorn (2021). I therefore maintain a 'true Balanced Growth Path' (BGP) with a constant real value of the debt portfolio thanks to an appropriate taxation schedule.

To apply the ideas of Hagedorn (2021), I start of with his steady-state taxation function, but rewrite it to account for possible non-zero steady-state inflation and some positive level of indexed debt, since the presence of both changes the nominal value of taxation over time. The aim of this step is to find an asset demand function that depends only on model primitives and allows the derivation of the asset market equilibrium.<sup>37</sup> Doing so requires pinning down steady-state asset demand under incomplete markets in closed-form, for which I leverage the results of Acemoglu and Jensen (2015).

To find the steady-state level of taxation consistent with the bond issuance schedule that keeps the real value of bonds constant, I begin with an arbitrary per-period government budget constraint (setting  $G_t = 0$ , such that real surpluses are  $s_t = t_t$ , or, in nominal terms,  $P_t s_t = P_t t_t =: T_t$ ):

$$B_{t-1} + \frac{P_t}{P_{t-1}} b_{t-1} = T_t + Q_t B_t + q_t b_t.$$

 $Q_t$  and  $q_t$  must be equal to some constant values in steady-state. Without aggregate uncertainty, the bond prices arising through asset demand must solely depend on the offered interest rates, since cross-sectional risks average out. Thus, *in steady-state*:

$$\begin{split} B_{ss} + \Pi_{ss}b_{ss} &= T_{ss} + Q_{ss}B_{ss} + q_{ss}b_{ss} \\ \Leftrightarrow B_{ss} + \Pi_{ss}b_{ss} &= T_{ss} + \frac{1}{1+i_{ss}}B_{ss} + \frac{1}{1+r_{ss}}b_{ss} \\ \Leftrightarrow T_{ss} &= \left(1 - \frac{1}{1+i_{ss}}\right)B_{ss} + \left(\Pi_{ss} - \frac{1}{1+r_{ss}}\right)b_{ss}. \end{split}$$

Using the Fisher equation,  $\Pi_{ss}-\frac{1}{1+r_{ss}}=\frac{1+i_{ss}}{1+r_{ss}}-\frac{1}{1+r_{ss}}=\frac{i_{ss}}{1+r_{ss}}$ , and therefore:

$$T_{ss} = \frac{i_{ss}}{1 + i_{ss}} B_{ss} + \frac{i_{ss}}{1 + r_{ss}} b_{ss},$$

which can be expressed in real terms (as the household cares about real taxation) as

$$t_{ss} = \frac{r_{ss}}{1 + i_{ss}} B_{ss} + \frac{r_{ss}}{1 + r_{ss}} b_{ss}.$$

Define by  $S_t\left(\Omega_t,\{1+r_l,t_l\}_l^\infty\right)$  the cumulative asset demand function under incomplete markets, which depends on the household distribution of wealth  $\Omega_t$ , real interest rates  $1+r_t$ , and tax levels  $t_t$ , and is well-defined under standard regularity conditions (Acemoglu and Jensen, 2015). To relate steady-state taxation more clearly to gross asset demand, fix the shares of  $B_{ss}$  and  $b_{ss}$  of gross asset demand  $S_{ss}$  in steady-state. Denoting by  $\theta$  the share of indexed debt  $b_{ss}$  in the steady-state asset portfolio, the taxation term in steady-state finally becomes

 $<sup>^{37}</sup>$  For the sake of completeness, I here specify the approach Hagedorn (2021) takes to determine steady-state taxation. He specifies the per-period government budget constraint as  $B_{t+1}=(1+i_t)B_t-T_t \Leftrightarrow T_t=(1+i_t)B_t-B_{t+1}$  to arrive in steady-state at  $T_{ss}=i_{ss}S_{ss}$ , where  $S_{ss}$  is steady-state asset demand. in real terms,  $t_{ss}=:\frac{T_{ss}}{P_{ss}}=r_{ss}S_{ss}$ .

$$t_{ss} = \left[ (1 - \theta) \frac{r_{ss}}{1 + i_{ss}} + \theta \frac{r_{ss}}{1 + r_{ss}} \right] S_{ss}.$$

Under such steady-state taxes, the gross asset demand function arising from heterogeneous household demand  $(S_{t+1} = \delta(\Omega_t; 1 + r_t, 1 + r_{t+1}, 1 + r_{t+2}, ...; t_t, t_{t+1}, ...))$  simplifies to the following mapping in steady-state:

$$S_{ss} = \mathcal{S}\left(\Omega_{ss}; 1 + r_{ss}, 1 + r_{ss}, 1 + r_{ss}, \ldots; \left\lceil (1 - \theta) \frac{r_{ss}}{1 + i_{ss}} + \theta \frac{r_{ss}}{1 + r_{ss}} \right\rceil S_{ss}, \left\lceil (1 - \theta) \frac{r_{ss}}{1 + i_{ss}} + \theta \frac{r_{ss}}{1 + r_{ss}} \right\rceil S_{ss}, \ldots \right).$$

With  $i_{ss}$  being equal to some constant set by the monetary policymaker in steady-state and the taxation function just derived, asset demand is derived by finding the fixed point of the above equation, which yields asset demand as a function of the real interest rate  $r_{ss}$ , following Acemoglu and Jensen (2015):

#### Asset demand: S(r).

By the previous derivations, I now directly leverage asset supply in real terms as the left-hand side of the derivations of the asset market equation evaluated in steady-state, such that the stationary asset market equilibrium must be pinned down by

$$S(r) = \frac{B}{\tilde{P}} + \frac{b}{\tilde{P}(1 + \pi_{ss})},$$

or, making use of the Fisher equation,

$$S(r) = \frac{B}{\tilde{p}} + \frac{b}{\tilde{p}} \frac{(1+r_{ss})}{(1+i_{ss})}.$$

Solving the asset market equilibrium requires taking a stance on the source of  $\pi_{ss}$ , the (possibly) non-zero steady-state inflation rate in this economy. Following Hagedorn (2021), I postulate that the only possible non-zero steady-state inflation rate is the one consistent with a corresponding increase in taxation over time alongside this inflationary path:

$$1+\pi_{ss}=\frac{T'-T}{T},$$

where variables with a prime denote next period values. Since *T* represents nominal taxes, the above statement is equivalent to the claim that *real* taxes remain constant.

Given the bond portfolio on offer, express the above condition as follows:

$$1 + \pi_{ss} = (1 - \theta) \frac{B' - B}{B} + \theta \frac{b' - b}{b} \cdot (1 + \pi_{ss})$$

$$\Leftrightarrow 1 + \pi_{ss} = \frac{(1 - \theta) \frac{B' - B}{B}}{1 - \theta \frac{b' - b}{b}},$$

where the inflation-adjustment on the right-hand side in the first line follows from the adjustment of the face value of inflation-indexed debt. This bond issuance schedule therefore pins down steady-state inflation.

*Using the debt valuation equation to determine the price level*: I now invoke the above derivations within the debt valuation equation to pin down the price level uniquely, provided that the real interest rate is recovered through the asset market equilibrium.

The steady-state real interest rate can be recovered from the asset market through household demand, provided that this demand function is invertible, as

$$r_{ss} = S^{-1} \left( \frac{B}{\tilde{P}} + \frac{b}{\tilde{P}(1 + \pi_{ss})} \right),$$

which can be inserted in the government budget equilibrium  $(\frac{B}{\tilde{p}} + \frac{B}{\tilde{P}(1+\pi_{ss})} = \sum_{j=0}^{\infty} \left(\frac{1}{1+r_{ss}}\right)^{j} \bar{s})$  with  $r_{ss} > 0$  (such that the right-hand side can be rewritten as a geometric sum,  $\sum_{j=0}^{\infty} \left(\frac{1}{1+r_{ss}}\right)^{j} = \frac{1+r_{ss}}{r_{ss}}$ ) to get the following condition:

$$\frac{B_{ss}+b_{ss}(1+\pi_{ss})}{\tilde{p}}=\bar{s}\frac{1+r_{ss}}{r_{ss}}.$$

The fixed point of this equation pins down the price level uniquely, given asset market optimality. Using the previously derived definition of the surplus process, i.e.,  $\bar{s} = t_{ss} = \frac{r_{ss}}{1+i_{ss}} B_{ss} + \frac{r_{ss}}{1+r_{ss}} b_{ss}$ :

$$\frac{B_{ss} + b_{ss}(1 + \pi_{ss})}{\tilde{p}} = \left[ \frac{r_{ss}}{1 + i_{ss}} B_{ss} + \frac{r_{ss}}{1 + r_{ss}} b_{ss} \right] \frac{1 + r_{ss}}{r_{ss}}.$$

Using the Fisher equation  $((1+i_{ss})=(1+r_{ss})(1+\pi_{ss}))$ , this equilibrium relation is simplified to:

$$\frac{B_{ss}}{\tilde{P}} + \frac{b}{\tilde{P}(1+\pi_{ss})} = (1+\pi_{ss})B + b,$$

which eventually pins down the price level as

$$\tilde{P} = \frac{B_{ss} + b_{ss}(1 + \pi_{ss})}{(1 + \pi_{ss})B_{ss} + b_{ss}}.$$

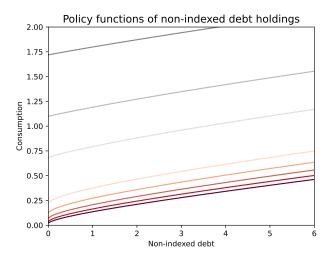
From the taxation schedule, I recover the steady-state inflation rate. I simplify this schedule by utilizing the steady-state growth rates  $\frac{B'-B}{B}$  =:  $g_B$  and  $\frac{b'-b}{b}$  =:  $g_b$ , such that steady-state inflation

becomes  $1+\pi_{ss}=\frac{(1-\theta)\frac{B'-B}{B}}{1-\theta\frac{b'-b}{b}}=\frac{(1-\theta)g_B}{1-\theta g_b}$ . Then, the initial price level is given by:

$$\tilde{P} = \frac{B_{ss} + b_{ss} \frac{(1-\theta)g_B}{1-\theta g_b}}{B_{ss} \frac{(1-\theta)g_B}{1-\theta g_b} + b_{ss}},$$

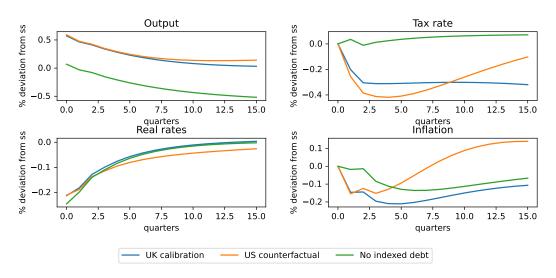
with the bond growth rates themselves being fiscal choice variables in the stationary equilibrium.

### C Further simulation results



**Figure C.1:** Household policy functions for demand of non-indexed debt in the calibrated HANK model for unconstrained households. The policy functions for low values of idiosyncratic productivity start to become positive only for strictly positive levels of non-indexed debt due to the possibility to purchase inflation-indexed debt stock.

# 25 bp monetary policy shocks - PM/AF and $\rho = 0.8$



**Figure C.2:** IRFs to a 25bps expansionary monetary shock - under a fiscally-led policy mix and  $\rho = 0.8$ .

# 25 bp monetary policy shocks - PM/AF and $\rho = 0.0$

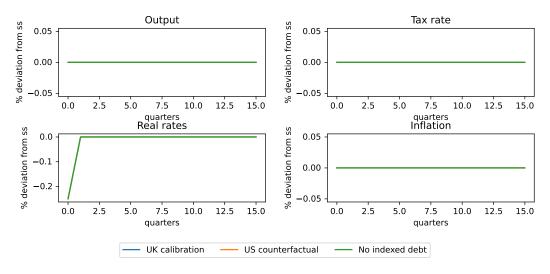


Figure C.3: IRFs to a 25bps expansionary monetary shock - under a fiscally-led policy mix and  $\rho = 0$ .

# 25 bp monetary policy shocks - AM/PF and $\rho = 0.8$

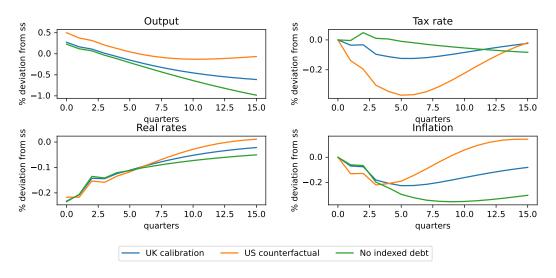


Figure C.4: IRFs to a 25bps expansionary monetary shock - with under a monetary-led policy mix and  $\rho=0.8$ .

# 25 bp monetary policy shocks - AM/PF and $\rho = 0.0$

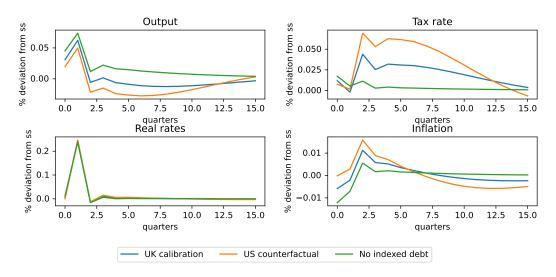


Figure C.5: IRFs to a 25bps expansionary monetary shock - with under a monetary-led policy mix and  $\rho=0$ .

For the main policy scenario (the 'fiscally-led policy mix'), I furthermore provide additional evidence on changes of quantities directly informing the intertemporal government budget constraint (9).

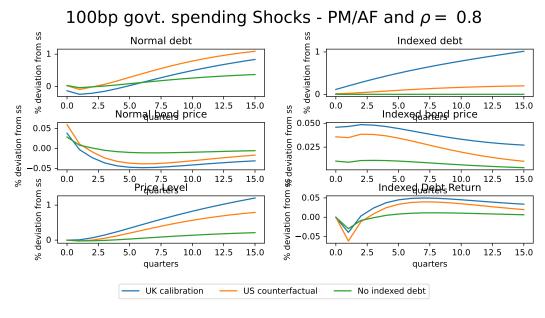
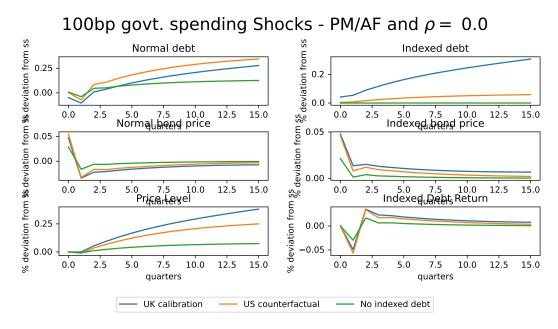


Figure C.6: Further IRFs to a 100bps expansionary fiscal spending shock - under a fiscally-led policy mix and  $\rho = 0.8$ .



**Figure C.7:** Further IRFs to a 100bps expansionary fiscal spending shock - under a fiscally-led policy mix and  $\rho = 0$ .

### D A tractable exposition of the effects of inflation-indexed debt

In this section, I develop a simplified iteration of Angeletos et al. (2024), adding inflation-indexed debt but at the same time reducing spillovers from mortality risk to aggregate demand. This is done to facilitate a simpler characterization of equilibrium inflation rates.<sup>38</sup> In that sense, the model presented here is also related to Woodford (2019) and Nakamura et al. (2025). The model is fundamentally a representative-agent New Keynesian-Overlapping Generations (RANK-OLG) model in the spirit of Blanchard (1985). The mortality friction can be considered a proxy for liquidity risk commonplace in canonical HANK models while maintaining superior tractability properties.

I use this framework to compare two different sources of potentially non-Ricardian fiscal policy: one is the idea of a fiscally-led policy mix (i.e., the commitment to not repay current deficits in equivalent real terms) under RANK, the other one is the household mortality friction (RANK-OLG, or 'quasi-HANK'), but under a monetary-led policy mix. Instead of laying out the cases of the fiscally-led policy mix under RANK and the monetary-led policy mix under HANK separately, I analyze both cases jointly as a dynamic system, keeping the parametrization of fiscal policy, monetary policy, and the mortality friction opaque for as long as possible.

For this model, uppercase variables define the level values of variables, while lowercase variables are log-deviations from steady-state. The steady-state will be log-linearized around zero inflation ( $\Pi^{SS}=1$ ), and the fiscal variables debt ( $d_t$ ), taxes ( $t_t$ ), and assets ( $a_t$ ) will all be measured in absolute deviations from steady-state (not log deviations) to ensure that zero-debt steady-states are not excluded.

<sup>&</sup>lt;sup>38</sup>A similar exercise with a *full* characterization of the effects of indexed debt on aggregate demand can be found in Ellison and Kawalec (2025)

#### D.1 Household block

The probability of surviving from one period to another is captured by  $\omega \in (0,1]$ . Households are replaced by new ones whenever they die. They maximize expected utility, given by

$$\mathbb{E}_{t} \left[ \sum_{k=0}^{\infty} (\beta \omega)^{k} \left[ \frac{C_{t+k}^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} - \nu \frac{L_{t+k}^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} \right] \right]. \tag{D.1}$$

The household budget constraint: households can trade in a risk-free annuity as in Angeletos et al. (2024), earning a specified nominal rate of return  $R_t^P$ . That annuity consists of a representative share of the government debt portfolio, which consists of regular debt  $B_t^R$  earning a gross return  $I_t$ , where  $I_t$  is the gross nominal interest rate, and *inflation-indexed debt*  $B_t^I$ , which earns a gross rate of return  $I_t \frac{P_{t+1}}{P_t}$ , reflecting the face value adjustment of such debt in line with materialized inflation. The gross portfolio return of the household consists of a weighted average of the returns earned by the two individual asset classes, where I specify the constant share of inflation-indexed debt in the government bond portfolio as  $\theta$ :

$$R_t^P = \theta I_t \frac{P_{t+1}}{P_t} + (1 - \theta)I_t = I_t \left( 1 + \theta \left( \frac{P_{t+1}}{P_t} - 1 \right) \right), \tag{D.2}$$

which captures the pre-death probability rate of return on the portfolio of government debt owned as the only savings asset by each household. The remainder of the budget constraint follows Angeletos et al. (2024) closely: all households receive labor income and dividends  $W_tL_{it} + Q_{it}$ , are taxed in accordance with a taxation rule, and all old households make a contribution  $S_{it}$  to a social fund whose proceeds are distributed to newborn households, eliminating wealth effects from mortality risk.<sup>39</sup> The household-specific budget constraint is then given by:

$$P_{t+1}A_{i,t+1} = \frac{R_t^P}{\omega} P_t \left( A_{it} + \underbrace{Y_{it}}_{\equiv W_t L_{it} + Q_{it}} - C_{it} - T_{it} + S_{it} \right).$$
 (D.3)

I otherwise retain all other household-side assumptions from Angeletos et al. (2024): dividends are identical across households i, labor supply is intermediated by unions to obtain  $L_{it} = L_t$ , and income and taxes faced by households are equalized. Taking expectations and subsequently making use of the Fisher equation yields the following expression of the budget constraint in real terms:

$$A_{i,t+1} = R_t \left( 1 + \theta \mathbb{E}_t \Pi_{t+1} \right) \frac{1}{\omega} (A_{it} + Y_{it} - C_{it} - T_{it} + S_{it}), \tag{D.4}$$

where  $R_t$  is the ex-ante real interest rate.

The transfers are specified as  $S_{it} = S^{new} = D^{SS} \ge 0$  and  $S_{it} = S^{old} = -\frac{1-\omega}{\omega}D^{SS} \le 0$ , such that  $(1-\omega)S^{new} + \omega S^{old} = 0$ .

This household problem yields the following set of first-order conditions:

$$\{C_{it}\}: C_{it}^{-\frac{1}{\sigma}} - \lambda_{it} \frac{R_t}{\omega} (1 + \theta \mathbb{E}_t (\Pi_{t+1} - 1)) = 0,$$
 (D.5a)

$$\{L_{it}\}: \qquad L_{it}^{\frac{1}{\varphi}} + \lambda_{it} \frac{R_t}{\omega} (1 + \theta \mathbb{E}_t (\Pi_{t+1} - 1)) \frac{\partial Y_{it}}{\partial L_{it}} = 0, \tag{D.5b}$$

$$\{A_{i,t+1}\}: -\lambda_{it} + \beta \mathbb{E}_t \left[ R_{t+1} (1 + \theta(\Pi_{t+2} - 1)) \lambda_{i,t+1} \right] = 0,$$
 (D.5c)

The first-order conditions jointly yield the Euler equation for consumption:

$$C_{it}^{-\frac{1}{\sigma}} \left[ R_t (1 + \theta \mathbb{E}_t \Pi_{t+1}) \right]^{-1} = \beta \mathbb{E}_t \left[ C_{i,t+1}^{-\frac{1}{\sigma}} \right]. \tag{D.6}$$

I linearize this expression through a first-order approximation to obtain a linearized form of the Euler equation.

$$c_{it} = -\sigma(r_t + \theta \mathbb{E}_t \pi_{t+1}) + \mathbb{E}_t c_{i,t+1}, \tag{D.7}$$

where the novel term induced by the presence of indexed debt is  $\theta \mathbb{E}_t \pi_{t+1}$ .

At this point, it is possible to characterize the intertemporal *aggregate* household budget constraint with the help of this household-level Euler equation, which could then be used to derive an aggregate demand equation. Under this expression, characterizing equilibrium inflation rates analytically remains possible, but demand changes induced by households that leave/enter the Euler equation through the presence of mortality risk obstruct the core message of how indexed debt can matter through the intertemporal government budget constraint. In the following, I therefore align the model closer to Woodford (2019) and Nakamura et al. (2025), postulating instead that the effect from mortality risk on aggregate demand can be captured by discounting future variables adequately on the aggregate Euler equation in line with mortality risk. Then, the aggregate Euler equation can be expressed as

$$c_t = \omega \mathbb{E}_t c_{t+1} - \sigma (i_t - \omega \mathbb{E}_t \pi_{t+1} + \theta \mathbb{E}_t \pi_{t+1}).$$

Using market clearing  $c_t = y_t$  and expressing the right-hand side in terms of the real interest rate  $r_t$ , this (simplified) aggregate demand equation becomes:

$$y_t = \omega \mathbb{E}_t y_{t+1} - \sigma(r_t + (1 - \omega + \theta) \mathbb{E}_t \pi_{t+1}). \tag{D.8}$$

The presence of indexed debt can offset the discounting of the interest rate channel that is induced by the presence of mortality risk in the effect of expected inflation on today's aggregate demand.

<sup>&</sup>lt;sup>40</sup>This intertemporal aggregate household budget constraint features a similar indexed-debt adjustment term and is given by:  $c_t = (1 - \beta \omega) \left( a_t + \mathbb{E}_t \sum_{s=0}^{\infty} (\beta \omega)^s (y_{t+s} - t_{t+s}) \right) - \beta \left( \sigma \omega - (1 - \beta \omega) \frac{A^{SS}}{\gamma^{SS}} \right) \mathbb{E}_t \left[ \sum_{s=0}^{\infty} (\beta \omega)^s (r_{t+s} + \theta \pi_{t+1+s}) \right].$ 

#### D.2 Supply side

The supply side of the model follows the specification common to canonical New Keynesian models. The New Keynesian Phillips Curve arises as a consequence of Calvo pricing frictions for a price optimization problem of monopolistically competitive wholesalers:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t[\pi_{t+1}],\tag{D.9}$$

which can be iterated forward to express inflation as a function of current and future output gaps:

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t[y_{t+k}]. \tag{D.10}$$

# D.3 Fiscal and monetary policy

I begin the discussion of the government block by deriving the government budget constraint, which differs somewhat relative to Angeletos et al. (2024). The simple per-period budget constraint of the government is defined as:

$$D_{t+1} = R_t^P (D_t - P_t T_t),$$

where  $R_t^P \equiv I_t(1+\theta(\Pi_{t+1}-1))$  captures the portfolio return that the government has to pay households. I therefore postulate that the government portfolio has a *fixed* share of inflation-indexed debt  $\theta$ , in line with the characterization of the household portfolio. Linearizing this constraint to express the evolution of the total debt portfolio  $d_{t+1}$  in deviations from steady-state gives rise to the following log-linearized budget constraint:

$$d_{t+1} = \frac{1}{\beta} (d_t - t_t) + \frac{D^{SS}}{Y^{SS}} r_t - \frac{D^{SS}}{Y^{SS}} ((1 - \theta) \pi_{t+1} - \mathbb{E}_t \pi_{t+1}).$$
 (D.11)

The crucial novelty here is the adjustment of future inflation by  $(1-\theta)$ . Intuitively, this adjustment captures the idea that inflation-indexed debt cannot be devalued through surprise inflation, as the face value of that part of the debt stock remains unchanged in present real terms irrespective of the rate of inflation. Therefore, the ability of governments to inflate away debt in real terms is constrained.

The analysis furthermore retains the no-Ponzi condition of Angeletos et al. (2024), i.e.,  $\mathbb{E}_t \left[ \lim_{T \to \infty} \beta^T d_{t+T} \right] = 0$ . Starting off the steady-state where  $x_{-1} = 0 \ \forall x \in \{d, t, r, y, \pi\}$ , equation (D.11) pins down the initial change in the debt stock as a function of surprise inflation:

$$d_0 = -\frac{D^{SS}}{\gamma^{SS}}(1-\theta)\pi_0.$$

To close the model, appropriate fiscal and monetary policy rules must be specified. The monetary policy rule deserves a special treatment as it is a point of departure from Angeletos et al. (2024). Following equation (D.8), inflation-indexed debt can induce an intertemporal substitution effect through the Euler equation due to the possibility of windfall gains in the presence of surprise inflation. As I intend to eliminate this effect and solely focus on the relevance of inflation-indexed debt through wealth effects induced by taxation, I postulate a monetary policy rule that absorbs the effect of inflation-indexed debt and of the mortality risk on the inflation adjustment in the aggregate demand equation:

$$r_t = \phi y_t - (1 - \omega + \theta) \mathbb{E}_t \pi_{t+1}. \tag{D.12}$$

This policy rule ensures that there is no distortion on intertemporal demand induced by windfall gains or losses from surprise inflation. Heuristically, central banks *care about the real interest rate* that is relevant to the aggregate of surviving households. Denoting this policy-relevant interest rate by  $\tilde{r}_t \equiv r_t + (1 - \omega + \theta)\mathbb{E}_t \pi_{t+1}$ , the monetary policy rule can likewise be expressed as  $\tilde{r}_t = \phi y_t$ , nesting the specification of Angeletos et al. (2024).

Given that the monetary rule also absorbs the effect of inflation-indexed debt on the government budget constraint in the case of surprise inflation, I also introduce a dependence of the tax rule on the share of inflation-indexed debt, reflecting that the tax schedule must ensure that the quantity of taxes raised accounts for the possible cost incurred by the higher service cost of inflation-indexed debt in the presence of higher inflation. The fiscal rule otherwise remains the same as in Angeletos et al. (2024), such that I define for  $\tau_d$ ,  $\tau_y \in [0,1)$ :

$$t_t = -\varepsilon_t + \tau_d(d_t + \varepsilon_t) + \tau_y y_t + \beta \frac{D^{SS}}{\gamma_{SS}} \theta \mathbb{E}_t \pi_{t+1}, \tag{D.13}$$

where the last term reflects the novel adjustment of taxes to the expected costs incurred by inflation. Heuristically, governments know that surprise inflation can erode their budget balance (through higher face value payments on indexed debt), and they therefore adjust their taxation schedule to cover these expenses. Defining the quantity of taxes raised net of face value outlays for indexed debt as  $\tilde{t}_t \equiv t_t - \beta \frac{D^{SS}}{Y^{SS}} \theta \mathbb{E}_t \pi_{t+1}$  again nests the model of Angeletos et al. (2024). The quantity  $\tilde{t}_t$  reflects the discretionary tax revenue, i.e., the tax revenue available for the government once immediate obligations have been taken care of.

#### D.4 Equilibrium and general model properties

The definition of the *competitive equilibrium* is standard and kept brief on purpose.

**Definition 2** A competitive equilibrium is a path  $\{c_t, y_t, \pi_t, a_t, d_t, t_t, r_t\}_{t=0}^{\infty}$  that satisfies the aggregate demand function (D.8), the NKPC (D.9), market clearing  $(c_t = y_t \text{ and } a_t = d_t)$ , the government's flow budget constraint (D.11), as well as the monetary and fiscal policy rules (D.12) and (D.13).

Equations (D.8), (D.9), and (D.11) (jointly with the monetary and fiscal rules (D.12) and (D.13)) yield a first-order difference system, which will be the centerpiece of the analysis in this section:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -(1-\omega+\theta)\frac{D^{SS}}{\gamma^{SS}} & 1 \end{bmatrix} \mathbb{E}_{t} \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ d_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1+\sigma\phi}{\omega} & 0 & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{\gamma^{SS}}\phi - \frac{\tau_{y}}{\beta} & 0 & \frac{1}{\beta}(1-\tau_{d}) \end{bmatrix} \begin{bmatrix} y_{t} \\ \pi_{t} \\ d_{t} + \varepsilon_{t} \end{bmatrix}, \tag{D.14}$$

which can be rewritten to:

$$\mathbb{E}_{t} \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ d_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1+\sigma\phi}{\omega} & 0 & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{\gamma^{SS}}\phi - \frac{\tau_{y}}{\beta} - \frac{D^{SS}}{\gamma^{SS}} \frac{\kappa(1-\omega+\theta)}{\beta} & \frac{D^{SS}}{\gamma^{SS}} \frac{(1-\omega+\theta)}{\beta} & \frac{1}{\beta}(1-\tau_{d}) \end{bmatrix} \begin{bmatrix} y_{t} \\ \pi_{t} \\ d_{t} + \varepsilon_{t} \end{bmatrix}.$$
(D.15)

The properties of the model depend on the eigenvalues of the previous matrix. Since the matrix is lower triangular, its eigenvalues are trivially given by the elements of its diagonal:

$$\lambda_1 = \frac{1 + \sigma \phi}{\omega}; \qquad \lambda_2 = \frac{1}{\beta}: \qquad \lambda_3 = \frac{1}{\beta}(1 - \tau_d).$$
 (D.16)

To satisfy the necessary conditions for a unique saddle-path equilibrium, two eigenvalues must lie outside the unit circle, and one inside, since the system contains exactly one state variable.

### D.5 Solving the model in the limit case

Defining a 'limit point' between policy mixes that are prospectively "more/less fiscally/monetary-led" is possible by considering at which values of the core policy parameters  $\phi$  and  $\tau_d$  the associated eigenvalues are exactly one. Doing so for the first and last eigenvalues (since  $\lambda_2$  is trivially > 1), I establish the following parameter combination as the 'policy limit point':

$$\phi = -\frac{1-\omega}{\sigma}, \qquad \tau_d = 1-\beta.$$

To consider the dynamic properties of the system, I now focus on the aforementioned *limit point* between the fiscally-led and the monetary-led policy mix, but with two slight tweaks. First, I perturb the monetary policy parameter by a small value  $\epsilon/\sigma>0$  to ensure that the eigenvalue associated with the aggregate demand relation of the model matrix in equation (D.15) lies strictly inside the unit circle, such that  $\phi=-\frac{1-\omega+\epsilon}{\sigma}$ . Thereby, I ensure that the analysis retains the focus on the 'equivalence result' in terms of impact inflation between HANK models and the fiscally-led policy mix, following Angeletos et al. (2024). Second, to retain the comparability of my results to Angeletos et al. (2024), I consider values of  $\tau_d$  slightly below the limit point, letting  $\tau_d \to 0^+$ .

I denote the eigenvector associated with the stable eigenvalue as  $(\chi_1, \chi_2, 1)'$ , such that the element pertaining to the state variable itself is normalized to 1. The evolution of all three endogenous

variables can then be expressed in terms of the stable eigenvalue and its associated eigenvector:

$$y_t = \chi_1(d_t + \varepsilon_t); \qquad \pi_t = \chi_2(d_t + \varepsilon_t); \qquad \mathbb{E}_t d_{t+1} = \rho_d(d_t + \varepsilon_t).$$
 (D.17)

The three coefficients are given by the solution to the system  $(A - \lambda_2 I)\chi = 0$ , with  $\chi_3 = 1$ . That system is specified as:

$$\begin{bmatrix} 1 - \frac{\epsilon}{\omega} & 0 & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ -\left(\frac{D^{SS}}{\gamma^{SS}} \frac{(1-\omega+\epsilon)}{\sigma} + \frac{\tau_{y}}{\beta} + \frac{D^{SS}}{\gamma^{SS}} \frac{\kappa(1-\omega+\theta)}{\beta}\right) & \frac{D^{SS}}{\gamma^{SS}} \frac{(1-\omega+\theta)}{\beta} & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} \chi_{1} \\ \chi_{2} \\ 1 \end{bmatrix} = \rho_{d} \begin{bmatrix} \chi_{1} \\ \chi_{2} \\ 1 \end{bmatrix}.$$
 (D.18)

The first equation implied by the system pins down  $\rho_d$ , the persistence of the state variable:

$$\rho_d = 1 - \frac{\epsilon}{\omega} < 1.$$

The remaining two equations are given by:

$$\begin{split} -\chi_1\frac{\kappa}{\beta} + \chi_2\frac{1}{\beta} &= \left(1 - \frac{\epsilon}{\omega}\right)\chi_2 \\ -\left[\frac{D^{SS}}{Y^{SS}}\frac{(1 - \omega + \epsilon)}{\sigma} + \frac{\tau_y}{\beta} + \frac{D^{SS}}{Y^{SS}}\frac{\kappa(1 - \omega + \theta)}{\beta}\right]\chi_1 + \frac{D^{SS}}{Y^{SS}}\frac{(1 - \omega + \theta)}{\beta}\chi_2 + \frac{1}{\beta} &= 1 - \frac{\epsilon}{\omega}. \end{split}$$

Thanks to the lower triangular structure of the matrix, the resulting system of equations can be solved easily, pinning down  $\chi_1$  and  $\chi_2$  uniquely and yielding the sensitivity of inflation on impact in response to the fiscal shock  $\varepsilon_0$ . This process gives

$$\pi_0^{\varepsilon} \equiv \chi_2 \varepsilon_0 = \frac{\kappa \left(\frac{1}{\beta} - 1 + \frac{\epsilon}{\omega}\right)}{\left[\frac{D^{SS}}{Y^{SS}} \frac{(1 - \omega + \epsilon)}{\sigma} + \frac{\tau_y}{\beta}\right] \left[1 - \beta \left(1 - \frac{\epsilon}{\omega}\right)\right] - \frac{D^{SS}}{Y^{SS}} \kappa \left(1 - \frac{\epsilon}{\omega}\right) (1 - \omega + \theta)} \varepsilon_0. \tag{D.19}$$

Without inflation-indexed debt ( $\theta=0$ ), this expression would be trivially positive as  $\omega\to 1$ , nesting the standard RANK case as exposed in Angeletos et al. (2024). Additionally, for the 'proper' limit point between fiscally-led and monetary-led policy mixes (i.e., for  $\varepsilon\to 0$ ), mortality risk *only* matters in direct relation to the existence of inflation-indexed debt; that is, the effects of both are closely intertwined.

**Proposition 4** *If impact inflation is positive in the policy limit point; that is, if* 

$$\tau_{y} > \beta \frac{D^{SS}}{\gamma^{SS}} \left[ \frac{\kappa \theta}{1 - \beta} - (1 - \omega) \left( \frac{1}{\sigma} - \frac{\kappa}{1 - \beta} \right) \right],^{41} \tag{D.20}$$

 $<sup>^{41}</sup>$ This proposition therefore implies that as  $\theta$  increases, impact inflation in response to deficit shocks might eventually

then impact inflation in response to an expansionary fiscal shock is higher in the policy limit point for the fiscally-led RANK economy relative to the monetary-led HANK economy if

$$\kappa < \frac{1 - \beta}{\sigma}.\tag{D.21}$$

If impact inflation is negative in the policy limit point; that is, if

$$\tau_{y} < \beta \frac{D^{SS}}{Y^{SS}} \left[ \frac{\kappa \theta}{1 - \beta} - (1 - \omega) \left( \frac{1}{\sigma} - \frac{\kappa}{1 - \beta} \right) \right], \tag{D.22}$$

then impact deflation in response to an expansionary fiscal shock is smaller in the policy limit point for the fiscally-led RANK economy relative to the monetary-led HANK economy under the same condition.

**Proof.** Evaluating the inflation expression (D.19) for both FD-RANK and MD-HANK economies against each other gives:

$$\begin{split} \pi_0^{\varepsilon,FD,RANK} > \pi_0^{\varepsilon,MD,HANK} \\ \Leftrightarrow \frac{\kappa \left(\frac{1}{\beta} - 1\right)}{\frac{\tau_y}{\beta} (1 - \beta) - \frac{D^{SS}}{Y^{SS}} \kappa \theta} > \frac{\kappa \left(\frac{1}{\beta} - 1\right)}{\left[\frac{D^{SS}}{Y^{SS}} \frac{(1 - \omega)}{\sigma} + \frac{\tau_y}{\beta}\right] (1 - \beta) - \frac{D^{SS}}{Y^{SS}} \kappa (1 - \omega + \theta)} \\ \Leftrightarrow 0 < \frac{D^{SS}}{Y^{SS}} (1 - \omega) \left(\frac{1 - \beta}{\sigma} - \kappa\right) \\ \Leftrightarrow \kappa < \frac{1 - \beta}{\sigma}, \end{split}$$

which is the definition stated in the proposition. A similar derivation applies for the case in which the inflationary impact of both models is negative. ■

This result overturns the possible irrelevance of the interaction between mortality risk and the fiscal-monetary policy mix for inflation, as argued for in a related model by Angeletos et al. (2024). The presence of inflation-indexed debt and the way it partially overcomes market incompleteness can cause a direct link between mortality risk, the fiscal-monetary policy mix, and inflation.

For most common calibrations, the conditions stated in the proposition are fulfilled. Labeling  $\tau_y$  as the tax base channel; that is, as the proportion of income taxed, the inequality (D.20) is fulfilled even for elevated levels of inflation-indexed debt, conditional on a relatively flat slope of the Phillips curve as found by Hazell et al. (2022).

Figure D.1 provides a comparison of impact inflation as a function of the inverse of mortality risk  $\omega$  in the cases of fiscally-led policy mixes and monetary-led policy mixes. The size of the shock is normalized to reflect a 1% deficit-to-GDP shock. Additionally, I vary the level of inflation-indexed debt as shares of the total debt stock in the plot between 0 and 0.2, with the latter being at the higher

end of observed levels around the world.<sup>42</sup>

#### Cumulative two-year inflation following a 1% deficit-to-GDP shock Fiscally-led policy mix Monetary-led policy mix 0.25 0.25 $\theta = 0.08$ $\theta = 0.08$ 0.2 0.2 $\theta = 0.2$ $\theta = 0.2$ Change of prices in %% Change of prices in %% 0.15 0.1 0.05 0.05 0 -0.05 -0.05 0.8 0.85 0.9 0.95 0.8 0.85 0.9 0.95

**Figure D.1:** The role of household (quasi-)heterogeneity and indexed debt across policy regimes. The fiscally-led policy mix is defined by the parameters  $\tau_d = 0$  and  $\phi = -0.2$ , while under the monetary-led policy mix  $\tau_d = 0.4$  and  $\phi = 0.2$ . The remaining calibration is:  $D^{SS} = 1$ ,  $Y^{SS} = 1$ ,  $\kappa = 0.025$ ,  $\beta = 0.97$ ,  $\sigma = 1$ ,  $\tau_d = 0$ ,  $\phi = \frac{1 - \omega + \epsilon}{\sigma}$ .

The right-hand side, which focuses on the monetary-led policy mix, features zero inflation across the board. This result does not come as a surprise due to the block-exogeneity of the debt equation and the fact that its associated eigenvalue must be the stable one under a monetary-led policy mix: here, inflation must necessarily be equal to zero on any saddle-path stable equilibrium, as inflation would otherwise be unbounded.<sup>43</sup>

On the left-hand side, the dynamics of inflation in response to a deficit are more interesting. First, the price level change under a fiscally-led policy mix is generally increasing in the mortality risk  $1-\omega$ : as  $\omega$  falls (and households are more likely to die), prices react more to pressure coming from fiscal deficits in the fiscally-led policy mix. Inflation-indexed debt, however, does matter for the exact inflation level and for the interactions between mortality risk and the fiscally-led policy mix: under realistic calibrations, inflation-indexed debt generally increases the change of the price level on impact, with the effects being particularly pronounced when the economy admits realistic levels of mortality risk.

Since the shock is equivalent to a 1% deficit-to-GDP shock, it is possible to calculate the 'fiscal inflation multiplier' in the spirit of Hazell and Hobler (2024), which measures the percent change in the rate of inflation following a 1% change in deficits relative to GDP. For realistic parametrizations of  $\omega \approx 0.8$  (Angeletos et al., 2024), changing the share of inflation-indexed debt from 0 to 20% boosts the fiscal inflation multiplier by 0.04 percentage points. The observed results are on the

<sup>&</sup>lt;sup>42</sup>The only OECD member country with higher shares of inflation-indexed debt in the total debt stock is the United Kingdom with approximately 28% of the total market value of debt being indexed to inflation.

 $<sup>^{43}</sup>$ Without the block-exogeneity of the debt equation, which was introduced here for analytical convenience, the parametric combination assumed under the monetary-led policy mix could feature non-zero inflation rates for  $\omega < 1$ . More generally, a continuous determinate policy space would exist, making a clear-cut distinction between fiscally-led and monetary-led policy mixes difficult (Rachel and Ravn, 2025).

lower end relative to the evidence on the effects of inflation-indexed debt in the quantitative model of section 6, but fit qualitatively in the same story.

I generalize this discussion to say something about the degree to which the effects of indexed debt are increasing in heterogeneity:

**Proposition 5** The effects of inflation-indexed debt are increasing in the degree of quasi-heterogeneity, i.e.,

$$\frac{\left(\partial \pi_0^{\varepsilon}\right)^2}{\partial \omega \partial \theta} > 0,\tag{D.23}$$

if and only if the tax base channel of debt is sufficiently large; that is, if:

$$\tau_y > \beta \frac{D^{SS}}{Y^{SS}} \left( \frac{\kappa \theta}{1 - \beta} - \frac{1 - \omega}{\sigma} \right).$$
(D.24)

The probability of this being the case therefore decreases in the share of inflation-indexed debt,  $\theta$ .

**Proof.** Start with equation (D.19). Taking the partial derivative w.r.t.  $\theta$  yields:

$$\frac{\partial \pi_0^{\varepsilon}}{\partial \theta} = \frac{\kappa^2 \left(\frac{1}{\beta} - 1\right) \frac{D^{SS}}{Y^{SS}}}{\left\{ \left[\frac{D^{SS}}{Y^{SS}} \frac{(1 - \omega)}{\sigma} + \frac{\tau_y}{\beta}\right] (1 - \beta) - \frac{D^{SS}}{Y^{SS}} \kappa (1 - \omega + \theta) \right\}^2} \varepsilon_0.$$

Differentiating the previous expression with respect to  $\omega$  gives:

$$\frac{\left(\partial \pi_0^{\varepsilon}\right)^2}{\partial \omega \partial \theta} = \frac{\left(\frac{D^{SS}}{Y^{SS}}\kappa\right)^2 \left(\frac{1}{\beta} - 1\right) \left[\frac{1 - \beta}{\sigma} - \kappa\right]}{\left\{\left[\frac{D^{SS}}{Y^{SS}} \frac{(1 - \omega)}{\sigma} + \frac{\tau_y}{\beta}\right] (1 - \beta) - \frac{D^{SS}}{Y^{SS}}\kappa (1 - \omega + \theta)\right\}^3}.$$

If the last expression is larger than zero, indexed debt  $\theta$  indeed expands the measured effects of quasi-heterogeneity on price level adjustments in the initial period. That expression is positive if and only if its numerator and denominator have the same signs. The numerator of the previous expression is trivially positive under  $\kappa < \frac{1-\beta}{\sigma}$ . Now, the denominator in turn is only positive if the tax base channel is sufficiently strong, which is the case if:

$$\tau_y > \beta \frac{D^{SS}}{Y^{SS}} \left( \frac{\kappa \theta}{1 - \beta} - \frac{1 - \omega}{\sigma} \right).$$
(D.25)

Therefore, irrespective of inflation on impact being positive or negative in response to a fiscal innovation, the size of the overall price level shift can be curbed by inflation-indexed debt when the tax base channel is sufficiently weak.

#### D.6 Moving beyond the limit point

The previous analysis was restricted to the 'quasi-limit' point where  $\phi = -\frac{1-\omega+\epsilon}{\sigma}$ ,  $\tau_d \to 0^+$ . This section now generalizes prior insights to a wider feasible set of monetary and fiscal policy combinations.

The full first-order system in this framework with inflation-indexed debt is given by:

$$\mathbb{E}_{t} \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ d_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1+\phi\sigma}{\omega} & 0 & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_{y}}{\beta} - \frac{D^{SS}}{Y^{SS}} \frac{\kappa(1-\omega+\theta)}{\beta} & \frac{D^{SS}}{Y^{SS}} \frac{(1-\omega+\theta)}{\beta} & \frac{1}{\beta}(1-\tau_{d}) \end{bmatrix} \begin{bmatrix} y_{t} \\ \pi_{t} \\ d_{t} + \varepsilon_{t} \end{bmatrix}, \tag{D.26}$$

such that the general eigenvalue system associated with the stable eigenvalue  $\rho_d$  is now defined as:

$$\begin{bmatrix} \frac{1+\phi\sigma}{\omega} & 0 & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta} - \frac{D^{SS}}{Y^{SS}}\frac{\kappa(1-\omega+\theta)}{\beta} & \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\beta} & \frac{1}{\beta}(1-\tau_d) \end{bmatrix} \begin{bmatrix} \chi_1 \\ \chi_2 \\ 1 \end{bmatrix} = \rho_d \begin{bmatrix} \chi_1 \\ \chi_2 \\ 1 \end{bmatrix}.$$
 (D.27)

This process, in turn, yields the following system of three equations in the three unknowns  $\rho_d$ ,  $\chi_1$  and  $\chi_2$ :

The first equation gives  $\rho_d = \frac{1+\phi\sigma}{\omega}$ , which is the analytical expression of the corresponding eigenvalue. The second of the three equations in turn yields:

$$\chi_1 = \frac{1 - \beta \rho_d}{\kappa} \chi_2,$$

which can be inserted for  $\chi_1$  in the third condition to obtain:

$$\left\{\left[\frac{D^{SS}}{Y^{SS}}\left(\phi-\frac{\kappa(1-\omega+\theta)}{\beta}\right)-\frac{\tau_y}{\beta}\right]\left(\frac{1-\beta\rho_d}{\kappa}\right)+\frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\beta}\right\}\chi_2=\rho_d-\frac{1}{\beta}(1-\tau_d).$$

I rewrite this condition to:

$$\chi_2 = \frac{\rho_d - \frac{1}{\beta}(1 - \tau_d)}{\left(\frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta}\right)\left(\frac{1 - \beta\rho_d}{\kappa}\right) + \frac{D^{SS}}{Y^{SS}}(1 - \omega + \theta)\rho_d}.$$

Finally, insert for  $\rho_d$  to obtain the expression pinning down impact inflation in the general case:

$$\pi_0^{\varepsilon} \equiv \chi_2 \varepsilon_0 = \frac{\frac{1}{\omega} (1 + \phi \sigma) - \frac{1}{\beta} (1 - \tau_d)}{\left(\frac{D^{SS}}{Y^{SS}} \phi - \frac{\tau_y}{\beta}\right) \left(\frac{1 - \frac{\beta}{\omega} (1 + \phi \sigma)}{\kappa}\right) + \frac{D^{SS}}{Y^{SS}} \frac{(1 - \omega + \theta)}{\omega} (1 + \phi \sigma)} \varepsilon_0.$$
 (D.28)

This general expression can be used for a more in-depth analysis of the impact of inflation-indexed debt on impact inflation in a more general setting with fiscal and monetary policy being outside the limit point.

**Proposition 6** For a sufficiently fiscally-led policy mix; that is, for  $\tau_d < 1 - \frac{\beta}{\omega}(1 + \phi\sigma)$ , inflation-indexed debt always boosts inflation in response to a fiscal transfer shock, conditional on the equilibrium being existent and the eigenvalue associated with the aggregate demand equation being the stable one, which restricts monetary policy to

$$\begin{split} \phi \in & \left( -\frac{1}{\sigma}, \min \left\{ -\frac{1-\omega}{\sigma}, \right. \right. \\ & \left. \frac{\frac{D^{SS}}{Y^{SS}} \left( 1 + \frac{\kappa \sigma (1-\omega + \theta)}{\omega} - \frac{\beta}{\omega} \right) - \sqrt{\left[ \frac{D^{SS}}{Y^{SS}} \left( 1 + \frac{\kappa \sigma (1-\omega + \theta)}{\omega} - \frac{\beta}{\omega} \right) \right]^2 - 4 \frac{D^{SS}}{Y^{SS}} \frac{\beta}{\omega} \sigma \left[ \frac{\tau_y}{\omega} - \frac{\tau_y}{\beta} + \frac{D^{SS}}{Y^{SS}} \frac{\kappa (1-\omega + \theta)}{\omega} \right]}{2 \frac{D^{SS}}{Y^{SS}} \frac{\beta}{\omega} \sigma} \right] \\ & \left. 2 \frac{D^{SS}}{Y^{SS}} \frac{\beta}{\omega} \sigma \right] \end{split}$$

The magnitude of the effect of inflation-indexed debt on impact inflation is decreasing in the strength of tax adjustment to debt issuance  $\tau_d$ ; that is,  $\frac{(\partial \pi_0^\varepsilon)^2}{\partial \theta \partial \tau_d} < 0$ , always.

**Proof.** Taking the first partial derivative of impact inflation with respect to the share of inflation-indexed debt gives:

$$\frac{\partial \pi_0^{\varepsilon}}{\partial \theta} = \frac{\left[\frac{1}{\beta}(1 - \tau_d) - \frac{1}{\omega}(1 + \phi\sigma)\right] \frac{D^{SS}}{Y^{SS}} \frac{1}{\omega}(1 + \phi\sigma)}{\left[\left(\frac{\tau_y}{\beta} - \phi \frac{D^{SS}}{Y^{SS}}\right) \left(\frac{1 - \frac{\beta}{\omega}(1 + \phi\sigma)}{\kappa}\right) - \frac{D^{SS}}{Y^{SS}} \frac{(1 - \omega + \theta)}{\omega}(1 + \phi\sigma)\right]^2} \varepsilon_0.$$
(D.29)

This fraction is positive if and only if its numerator is positive, which is the case when both bracketed elements are positive, i.e., when either of the following sets of inequalities holds:

$$\begin{split} \phi &> -\frac{1}{\sigma}; \qquad \tau_d < 1 - \frac{\beta}{\omega} (1 + \phi \sigma). \\ \phi &< -\frac{1}{\sigma}; \qquad \tau_d > 1 - \frac{\beta}{\omega} (1 + \phi \sigma). \end{split}$$

The second case, however, is ruled out by the previous existence restriction on  $\phi$ . Therefore, for sufficiently active fiscal policy (reflected by small, or even negative values of  $\tau_d$ ), indexed debt can boost inflation if an equilibrium exists, which must be supported by monetary policy not reacting

positively to inflationary pressure as before.<sup>44</sup>

To establish the second result, it suffices to take the partial derivative of equation (D.29) with respect to  $\tau_d$  and the result follows, since

$$\frac{(\partial \pi_0^{\varepsilon})^2}{\partial \theta \partial \tau_d} = \frac{-\frac{1}{\beta} \frac{D^{SS}}{Y^{SS}} \frac{1}{\omega} (1 + \phi \sigma)}{\left[ \left( \frac{\tau_y}{\beta} - \phi \frac{D^{SS}}{Y^{SS}} \right) \left( \frac{1 - \frac{\beta}{\omega} (1 + \phi \sigma)}{\kappa} \right) - \frac{D^{SS}}{Y^{SS}} \frac{(1 - \omega + \theta)}{\omega} (1 + \phi \sigma) \right]^2} \varepsilon_0 < 0.$$
(D.30)

Relative to the results established in the limit point between the fiscally-led and the monetary-led policy mixes, I therefore confirm here that inflation-indexed debt is generally amplifying inflationary pressure, but especially under fiscally-led policy mixes. In particular, as the tax adjustment to debt issuance becomes minimal; that is, as  $\tau_d$  decreases, the corresponding inflationary pressure arising from fiscal shocks becomes even more amplified.

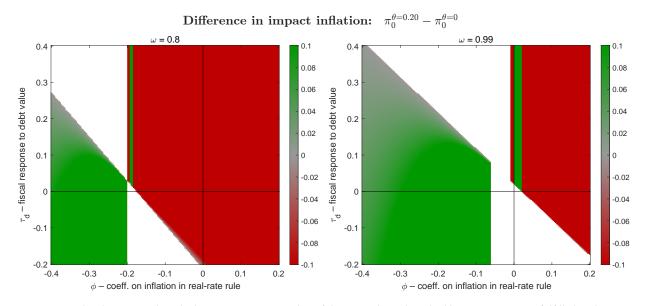


Figure D.2: Plot showing under which parametrizations either of the inequalities described by proposition 3 is fulfilled, indicating larger impact inflation when the share of inflation-indexed debt is higher. Green areas indicate places where either of the sets of inequalities hold, red areas indicate the opposite. White areas are the parts of the policy space where no saddle-path stable equilibrium exists. Calibration:  $D^{SS} = 1.2, Y^{SS} = 1, \kappa = 0.05, \beta = 0.99, \sigma = 1, \tau_y = 0.25$ .

Figure D.2 visualizes what was spelled out in proposition 6 for two instances - one with 'moderate quasi-heterogeneity' (left panel) and once in the *almost*-quasi-representative agent case. The bottom-left part of each panel is the area that would be conventionally considered 'fiscally dominant' in the sense that the model features a 'passive' monetary authority in the sense of Leeper (1991). Conditional on the existence of equilibrium, the presence of inflation-indexed debt increases inflationary pressure on impact across the board in the areas conventionally associated

<sup>&</sup>lt;sup>44</sup>The possibility that both elements in the numerator are *negative* is ruled out, since  $\phi$  cannot be smaller than  $-\frac{\omega}{\sigma}$  in any saddle-path equilibrium (Angeletos et al., 2024).

with a fiscally-led policy mix. For the area under which monetary policy is conventionally considered 'active' (the top-right area), the opposite is generally the case, except for a small region determined generally by  $\phi$ . Overall, however, proposition 6 shows the possibility of increased inflationary pressures under higher levels of inflation-indexed debt whenever fiscal policy is considered conventionally active, and at times even when the opposite is the case.

### E Long-term debt and debt indexation

In this part of the appendix, I briefly derive the debt valuation equation under complete markets with long-term debt.

Due to the assumption of complete markets and following the exposition in the main body of the text, bond pricing kernels for long-term assets maturing at time (t + j) evaluated at time t are given by:

$$Q_t^{(t+j)} = \mathbb{E}_t \left( \beta^j \frac{P_t}{P_{t+j}} \right), \qquad q_t^{(t+j)} = \beta^j,$$

reflecting that inflation-indexed debt always has the same price, as its face value accounts for changes to the price level between issuance and redemption. That being said, indexed debt is *not* fully equivalent to a real claim in the sense that its payout value is not scaled by the prevailing price level.

In this context, the government flow budget condition is given by:

$$B_{t-1}^{(t)} + \Pi_t b_{t-1}^{(t)} = P_t s_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} \left( B_t^{(t+j)} - B_{t-1}^{(t+j)} \right) + \sum_{j=1}^{\infty} q_t^{(t+j)} \left( b_t^{(t+j)} - \Pi_t b_{t-1}^{(t+j)} \right).$$

This condition states that in each period t, the payout of maturing debt (left-hand side) must be equal to the nominal surpluses raised plus the possible income from issuing additional debt maturing as a later point in the future (relative to what had already been issued before). Governments can also redeem more bonds than they issue, in which case either of the sums on the right-hand side can also be negative.

That flow condition keeps track of mounting payments on inflation-indexed debt by adjusting the prospective cost of serving indexed debt in each period by the accumulated face value payments, given by  $\left(b_t^{(t+j)} - \Pi_t b_{t-1}^{(t+j)}\right)$ . In sum, surpluses on the right-hand side of the previous equation get diminished when inflation  $\Pi_t$  from the last period has been high, as that inflation is reflected in the obligations that the government will have as that long-term inflation-indexed debt matures.

Grouping terms in the previous equation yields:

$$\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)} + \sum_{j=0}^{\infty} \Pi_t b_{t-1}^{(t+j)} = P_t s_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} + B_t^{(t+j)} + \sum_{j=1}^{\infty} q_t^{(t+j)} b_t^{(t+j)}.$$
 (E.1)

Let the real value of debt now be defined as:

$$V_t := \sum_{j=0}^{\infty} Q_t^{(t+j)} \frac{B_{t-1}^{(t+j)}}{P_t} + \sum_{j=0}^{\infty} \frac{b_{t-1}^{(t+j)}}{P_{t-1}}.$$

Focus on the right-hand side of equation (E.1). I now rewrite the two summative terms again to obtain  $V_{t+1}$ . Dividing those terms by  $P_t$ :

$$\sum_{j=1}^{\infty} Q_t^{(t+j)} \frac{B_t^{(t+j)}}{P_t} + \sum_{j=1}^{\infty} q_t^{(t+j)} \frac{b_t^{(t+j)}}{P_t}.$$

Shifting the index from j = 1 to j = 0 gives:

$$\sum_{j=0}^{\infty} Q_t^{(t+j+1)} \frac{B_t^{(t+j+1)}}{P_t} + \sum_{j=0}^{\infty} q_t^{(t+j+1)} \frac{b_t^{(t+j+1)}}{P_t}.$$

 $q_t^{(t+j+1)} = \beta q_{t+1}^{(t+j+1)}$  by the bond pricing kernels defined previously. Thus, the previous expression becomes

$$\beta \left[ \sum_{j=0}^{\infty} Q_{t+1}^{(t+j+1)} \frac{B_t^{(t+j+1)}}{P_{t+1}} + \sum_{j=0}^{\infty} q_{t+1}^{(t+j+1)} \frac{b_t^{(t+j+1)}}{P_t} \right] = \beta V_{t+1}$$

by the previous definition of  $V_t$ . Now, applying a transversality condition of the form

$$\lim_{T \to \infty} \beta^T \left[ \sum_{j=0}^{\infty} Q_{t+T}^{(t+j+T)} \frac{B_{t+T}^{(t+j+T)}}{P_{t+T}} + \sum_{j=0}^{\infty} q_{t+T}^{(t+j+T)} \frac{b_{t+T}^{(t+j+T)}}{P_{t+T}} \right] = 0,$$

obtains the debt valuation equation with inflation-indexed debt:

$$\sum_{t=0}^{\infty} Q_t^{(t+j)} \frac{B_{t-1}^{(t+j)}}{P_t} + \sum_{j=0}^{\infty} q_t^{(t+j)} \frac{b_{t-1}^{(t+j)}}{P_{t-1}} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j}, \tag{E.2}$$

which is a straightforward generalization of the government debt valuation equation exposed, for instance, in Cochrane (2001).