Our Deficit, Your Problem: Fiscal Sustainability and Exchange Rates*

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Abstract

We develop and estimate an open-economy present-value framework for the government budget constraint that embeds discount factors, exchange-rate expectations, and time-varying foreign-exchange risk premia. Using newly constructed market-value data for U.K. and U.S. public debt covering 1975-2024, we document that unexpected changes in the debt ratio are split equally between revisions to expected future surpluses and discount-rate news; the latter reflects movements in global real yields, revisions to expected real exchange rates, and UIP-premium shocks. Surplus innovations recovered from market prices is shown to materially affect the bilateral real exchange rate. We then present a tractable two-country model in which fiscal shocks in a financial hegemon propagate internationally through exchange-rate adjustments that feed back into real interest rates; the model rationalises the empirical shares and predicts "fiscal contagion" across sovereign balance sheets. A continuous-time general equilibrium framework further shows how exchange rate movements can be induced by changes to relative surpluses and debt issuance.

Keywords: Debt Sustainability, International Macroeconomics, Fiscal Deficits, Exchange Rate Determination.

JEL Classification: E63, F41, E62.

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1 Introduction

Fiscal deficits and debt sustainability have returned to the forefront of macroeconomic policy debates. Beyond the well-known link between fiscal sustainability and inflation risk, recent developments also highlight the risks that deficits pose to exchange rates. In April 2025, the U.S. dollar depreciated while Treasury yields rose — an unusual combination consistent with a lack of safe-haven inflows into U.S. government debt, hitherto regarded as a “safe asset”. This episode is part of a broader pattern: in the post-Bretton Woods era, large government deficits have repeatedly coincided with abrupt currency movements and puzzling revaluations of foreign public debt. Existing empirical work, which is often geared toward business-cycle frequencies, focuses on book-value debt, and is silent on the distinction between expected and realised deficits, struggles to explain why these international spillovers occur or how they can be reconciled with the intertemporal government budget constraint.

This paper revisits deficits and exchange rates from the perspective of the integrated government debt valuation equation and shows that cross-border adjustments in exchange rates and discount factors are required for global sovereign debt markets to clear. We start from a present-value identity that embeds the exchange rate, nominal discount rates, and time-varying foreign-exchange (FX) risk premia directly in the government budget constraint, building on present-value logic advanced by Hall and Sargent (2011), Cochrane (2019), and Jiang et al. (2024b). Using newly-constructed market-value data for U.K. (and U.S.) sovereign liabilities spanning 1975-2024, we quantify how much of local debt dynamics is driven by primary-surplus news, by global real-rate shocks, and by exchange-rate or risk-premium re-pricing. Our results establish that almost 30% of the financial aspects driving the revaluation of the market value of government debt in the United Kingdom, a representative G10 open economy, can be attributed to movements in the real exchange rate. While the domestic fiscal-surplus process remains the largest determinant of the value of U.K. government debt, accounting for real-exchange-rate-induced movements helps reconcile episodes that appear contradictory when considering only surpluses and interest-rate movements in isolation.

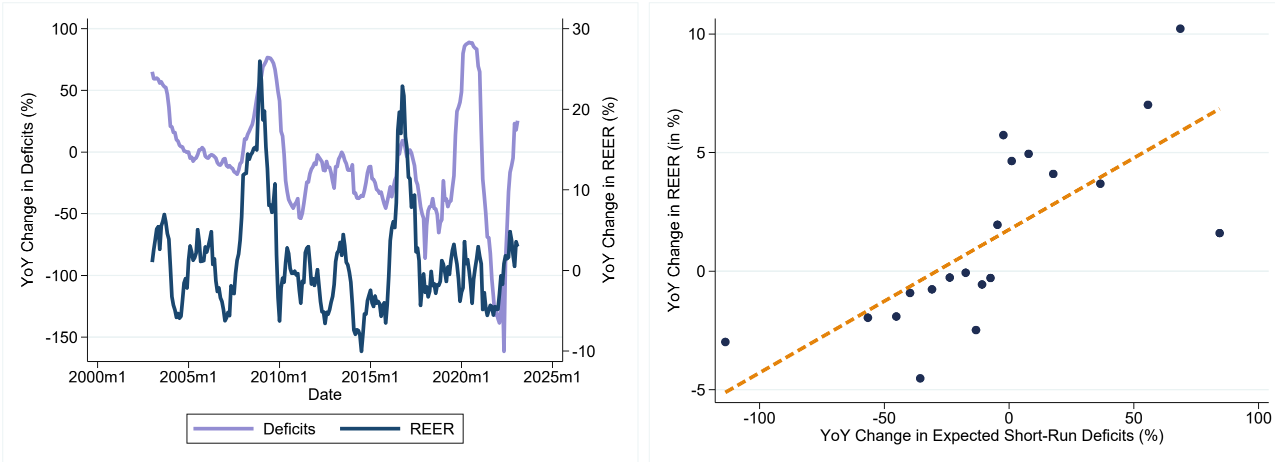
Employing Hall–Sargent market valuations, we perform a *backward* decomposition of the U.K. debt ratio and show that only about one-half of the post-1975 increase can be traced to realised primary deficits; the remainder is explained by valuation forces. Sterling depreciations and UIP-premium shocks together account for roughly a third of the upward drift, while movements in the world real yield (measured by returns on U.S. Treasuries) explain the rest. Consequently, episodes that loom large in U.K. policy debates—the late-1970s Sterling crisis, the post-1990 ERM exit, the GFC–austerity cycle, and the 2020–21 Covid shock—emerge primarily as Sterling revaluation rather than conventional flow-deficit stories.

Turning the identity *forward*, we combine the open-economy flow constraint with a VAR-based

Campbell and Shiller (1988) decomposition. Surplus news and discount-rate news remain the dominant drivers; in an international setting, the discount rate splits three ways: global real yields, expected exchange-rates, and time-varying FX risk premia. During the GFC, the collapse in world real rates offset roughly half of the deficit-induced debt surge; by contrast, the 2020 Covid spike reflects the joint force of larger expected deficits and a temporary jump in the Sterling risk premium. In short, discount rates matter as much as surpluses in understanding government balance sheets. Finally, to link unexpected deficits more directly to exchange-rate movements, we estimate how the recovered surplus processes affect the U.K. equilibrium exchange rate, finding that domestic surprise deficits are associated with depreciations, whereas foreign surprise deficits generally coincide with appreciations.

To rationalise these facts, we develop a two-country open-economy model in which fiscal shocks in a financial hegemon propagate internationally through the uncovered-interest-parity condition. Assuming a financial hegemon, we show that the threat of fiscal inflation in that country can transmit fiscal pressure to other economies by increasing the cost of servicing debt across countries. In the model, we identify a simple mechanism by which inflationary pressures at the hegemon country can spill over to other economies - a mechanism of 'international fiscal contagion', leading to fiscally-driven inflation spillovers. A deterioration of the Home country's expected surpluses tightens the Foreign intertemporal budget constraint, even when its own projected surpluses are unchanged, because currency-denominated discount factors adjust rather than current-account flows. Intuitively, fair valuation of terminal debt streams requires that a domestic devaluation of the real value of debt be matched by a comparable decline in the value of foreign debt. Thus, weaker Home surpluses necessitate either a depreciation of the Home currency or a rise in foreign prices. This mechanism operates through the interest rate charged on foreign government debt in equilibrium, with the nominal exchange rate driving much of the adjustment. We also present a continuous-time general-equilibrium model in which the market-clearing exchange rate between two economies is again heavily influenced by changes in relative surpluses.

Mechanisms linking (a) deficit changes to exchange rate movements and (b) deficit changes in one (Hegemon) country to price level changes in *another* country are gaining traction (e.g., Alberola et al. (2021), Ding and Jiang (2024), Mac Mullen (2025)). Recent empirical evidence supports a close nexus between cross-country deficits and exchange rates, but it assigns only a limited role to government-debt valuation in equilibrium. While this may be plausibly secondary in the short run, no-arbitrage in the long run provides a useful guide to the exchange-rate movements required to sustain a fair market value of government debt over time. Put differently, the literature often overlooks the consistency between short-run exchange-rate fluctuations and their long-run implications—best observed in a relatively low-frequency object such as the aggregate market value of government debt. Moreover, as laid out by Hazell and Hobler (2024) and Kawalec (2025),



(a) Changes to the REER and expected deficits over time.

(b) Binscatter of Δ expected fiscal deficits against Δ REER.

Figure 1: Dynamics of expected deficits and the REER in the U.K.

expected deficits appear to matter more for financial variables than realised deficits. Models solved via linear approximation, however, cannot sufficiently make this crucial distinction between expected and realised fiscal innovations.

Our contribution addresses these gaps. We explicitly focus on exchange rate determination from the perspective of the government debt valuation equation, recovering the link between fiscal deficits, exchange-rate movements, and the observed market value of government debt. Our main empirical exercise provides a detailed narrative of how fiscal deficit changes and exchange rate movements interact in jointly determining the equilibrium value of government debt. The long-run perspective spanning the last 50 years of macroeconomic history in the U.K. allows us to rationalize seemingly inconsistent episodes of debt valuation movements through corresponding exchange rate adjustments. This analysis is consistent with mechanisms in which changes in expected (rather than realised) deficits drive fiscal equilibrium. The accompanying theoretical frameworks further show how expected deficits in one country affect both domestic and foreign inflation by inducing exchange-rate and interest-rate movements that jointly determine the real value of government debt in equilibrium.

To further support the claim that expected deficits plausibly matter for exchange-rate movements, Figure 1 provides evidence in favour of a possible link between deficit expectations and exchange rates. The left panel plots the evolution of monthly changes to the REER (on a year-on-year basis) and of monthly changes to the expectations of fiscal deficits in the subsequent two years (again, changes are depicted on a year-on-year basis) in the United Kingdom. The right panel presents a binscatter of the two quantities against each other over the same sample period. Deficit expectations are cross-sectional averages of forecasts by private-sector institutions (banks and independent research firms) compiled by HM Treasury.

Taken together, Figure 1 suggests a plausible correlation between deficit *expectations* and real exchange rate movements: in particular, as panel (b) shows, higher deficits correlate with depreciations of the domestic currency in the context of an open economy. The figure is purely descriptive: it abstracts from realised deficits and does not identify causality. As panel (a) indicates, domestic deficit expectations are far from the only drivers of realised REER fluctuations, consistent with the broader literature.

Literature Review

The title of our contribution echoes the book of Rogoff (2025). In this paper, however, our central focus is the role borne by sovereign *deficit* dynamics for exchange rates, where we consider both domestic and foreign deficits in the way they shape exchange rate dynamics of a given country.

We primarily contribute to the literature on sovereign deficits and their ties to inflation and exchange-rate dynamics. The paper is also adjacent to work on fiscally driven price-level determination, which depends on the broader fiscal–monetary policy mix. That literature has been inaugurated by Sargent and Wallace (1981), with the subsequent work of Leeper (1991) and Woodford (1995) laying much of the modern groundwork for the models commonly in use that assign at least a share of inflationary dynamics to fiscal aggregate demand management. Literature reviews and overviews in this field are provided by Leeper and Leith (2016) and Cochrane (2023), among others. Overall support for fiscally-driven inflation has been developed both theoretically (Bassetto et al., 2024; Bassetto and Cui, 2018; Bianchi et al., 2023; Bigio et al., 2024; Campos et al., 2024; Caramp and Silva, 2023; Corsetti and Maćkowiak, 2024; Kaplan et al., 2024; Miao and Su, 2024; Smets and Wouters, 2024) and through an empirical lens (Banerjee et al., 2022; Barro and Bianchi, 2024; Chen et al., 2022a; Cochrane, 2022a; Reichlin et al., 2023).

While most of this work is set in closed-economy environments, a small but growing literature brings these insights to international models. Early explorations linking fiscal price level determination and exchange rate dynamics include Bergin (2000), Dupor (2000), Sims (1997), Sims (1999), and Woodford (1998). Initial applications of such ideas include Benigno and Missale (2004) and Corsetti and Maćkowiak (2006), who focus on the ties between public debt and currency stability especially in times of severe currency turmoil.

Alongside the renewed interest in deficit–inflation links, research has revisited the role of deficits in exchange-rate determination. An early contribution of Schmitt-Grohé and Uribe (2003) depicted the role of exchange rate adjustments vis-á-vis inflation in financing fiscal deficits within the context of emerging markets. Valchev (2020) establishes that excess returns on currency holdings can be traced to endogenous movements in bond convenience yields, which in turn depend on prevalent interactions between monetary and fiscal policy. Particularly related to our contribution are the works of Alberola et al. (2021) and Jiang (2022). Alberola et al. (2021) distinguish between Ricardian and non-Ricardian regimes, tying unexpected exchange rate movements (relative to a complete-market frictionless benchmark) to non-Ricardian fiscal regimes in which government debt lacks sufficient fiscal backing. Jiang (2022) in turn makes explicit some of the implications of applying fiscal price level determination to exchange rate dynamics, pointing out concisely that real exchange rate adjustments are linked to government surplus shocks through the cost of issuing government debt, among other factors.

Our theoretical model draws especially on the insights of Bassetto and Miller (2025), placing our modelling framework squarely within the realm of fiscally-led inflation mechanisms. One of our contributions is to add the international dimension to a tractable model of monetary–fiscal interactions, thereby rationalising a clear mechanism through which fiscal inflation in one country translates into fiscal inflation elsewhere via the interest-rate–exchange-rate nexus.

The paper also relates to the large macro–finance literature on nominal and real exchange-rate determination, which we cannot survey fully here. A partial literature review has been provided by Itskhoki (2021), including an overview about the many puzzles related to exchange rate dynamics, which standard complete-market models of exchange rate determination fail to resolve.[1] Of particular noteworthiness are the recent contributions on the exchange rate disconnect (Bacchetta and Van Wincoop, 2004; Chari et al., 2002; Corsetti et al., 2008; Duarte, 2003; Engel and West, 2005; Itskhoki and Mukhin, 2021), as our main argument links the exchange rate back to one specific macroeconomic fundamental; namely, aggregate demand managed by the fiscal authority. In that sense, our paper relates also significantly to recent contributions that push back against the narrative that exchange rates are disconnected from macroeconomic fundamentals. Particularly noteworthy in the light of our analysis are the contributions of Bodenstein et al. (2024), Kekre and Lenel (2024), and Mac Mullen (2025). Bodenstein et al. (2024) link exchange rate dynamics to trade balance shocks, which are reduced-form demand disturbances. Kekre and Lenel (2024) also attribute the majority of cross-country exchange rate variation to demand movements, albeit from the lens of resulting interest rate differentials. Finally, Mac Mullen (2025) shows that these interest rate differentials can be partially attributed to fiscal deficit dynamics, in line with the evidence and models presented in this paper.

By analysing interactions between fiscal hegemons and countries that lack the sovereign-debt privilege conferred by financial dominance, we also speak to this literature. Ellison and Scott (2020) in particular provide a long-run perspective on the valuation of U.K. national debt, which served as a cornerstone for our analysis. Our contribution is the addition of the international dimension, the elongation of their data sample, and an increased focus on inflationary factors. Jiang et al. (2024b) focus extensively on the U.S. experience in recent decades, finding evidence in favour of

[1] These include the possibility of a disconnect from macroeconomic fundamentals (Meese and Rogoff, 1983), the forward premium puzzle (Fama, 1984), the slightly negative correlation of real exchange rates and relative consumption (Backus and Smith, 1993; Corsetti et al., 2008), the Purchasing Power Parity puzzle (Rogoff, 1996), and the lower volatility of the terms of trade relative to the real exchange rate (Atkeson and Burstein, 2008), among others.

a persistent mispricing of U.S. sovereign debt. Considering the evidence presented in Chen et al. (2022b), this dominant position of U.S. sovereign debt had been inherited from the United Kingdom in the early 20th century. Ding and Jiang (2024) make explicit a number of interesting channels by which monetary and fiscal policy are intertwined in the presence of a dominant fiscal and financial hegemon.[2] Particularly noteworthy here are the recent contributions of Jiang et al. (2021a), who link the valuation of the U.S. Dollar to convenience yields enjoyed by foreign investors holding safe U.S. assets, and Jiang (2021), who argue that the U.S. fiscal cycle links currency risk premia of the Dollar relative to other currencies. Finally, Jiang et al. (2023a) and Jiang et al. (2024a) are close to our argument, showing how convenience yields on Dollar-denominated debt enter as a wedge in the Euler equations determining exchange rates.

The remainder of the paper is organised as follows. Section 2 lays out the present-value accounting framework and derives its open-economy linearisation,which we supplement with explicit links between forecasts of deficits and forecasts of exchange rates in section 3. Section 4 introduces the two-country model and shows how fiscal shocks propagate internationally through exchange-rate-discount interactions. Section 5 writes down a continuous time model to show how surpluses affect exchange rates. Section 6 concludes with policy implications for debt management and for the coordination of fiscal and exchange-rate policies.

2 Empirical Framework

We first recap the government debt valuation equation typically found in the literature. We then extend the framework to an international open economy setting.

When evaluating the framework, we make use of the data used in Cochrane (2019) for the U.S. variables, updating the macroeconomic aggregates using the data of Müller et al. (2025). For the bulk of the U.K. data analysis, we use data from Ellison and Scott (2020) on the market value of government debt, which we update using the Gilt database of Cairns and Wilkie (2023). The macroeconomic aggregates are again majorly sourced in Müller et al. (2025), with additional real exchange rate data being supplemented from the Bank of International Settlements and its Real Effective Exchange Rate Index (REER).

We define the nominal end-of-period market value of debt V_t as

$$V_t \equiv M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)}, \tag{1}$$

where M_t is non-interest bearing money, $B_t^{(t+j)}$ is the *face-value* of a zero-coupon nominal debt

[2] This literature, in turn, can be linked to insights from the literature on dominant currency paradigms, surveyed in Gopinath and Itskhoki (2022).

outstanding at the *end* of period t and due at the *beginning* of period $t+j$, $Q_t^{(t+j)}$ is the time t market price of that bond where $Q_t^{(t)} = 1$. We can divide by nominal GDP and take logs of both sides

$$v_t \equiv \log\left(\frac{V_t}{P_t Y_t}\right), \tag{2}$$

where P_t is the price level and Y_t is real GDP. We first divide by the price level to turn the market value of debt into a real variable. We then divide by GDP Y_t to turn the object into a stationary variable and hence more stable to work with in the time series.

We define the *nominal return on the portfolio of government debt*. This is defined as how the change in prices from the end of period t to the beginning of period $t+1$ affects the value of debt held between periods. The nominal return on the portfolio of government debt (holding period returns) is

$$R_{t+1}^n \equiv \frac{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}}{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}} \tag{3}$$

The denominator is the end-of-period t payout value of all government liabilities. The numerator is the market value of the debt now that each bond is one period closer to maturity. Following Hall and Sargent (2011) and Cochrane (2019), we stress the importance of using the market value of debt as opposed to the commonly reported book value of debt. Using the market value of debt ensures that discount-rate variation is priced in. Bond prices embed time-varying real yields and risk premia. Ignoring these would mechanically attribute all swings in debt-to-GDP to prospective surpluses or growth, biasing any present-value decomposition. Following commonly reported measures of the book value of government debt omits capital gains within the debt portfolio, leading to the period–by–period government budget identity not holding without ad–hoc valuation adjustments. We can take logs to get the *log nominal return* on the portfolio of government debt

$$r_{t+1}^n \equiv \log\left(R_{t+1}^n\right). \tag{4}$$

The nonlinear government flow identity is given by

$$M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)} \;=\; P_{t+1} sp_{t+1} + M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}. \tag{5}$$

Here, sp_{t+1} is the *real primary surplus* (i.e. does *not* include interest payments). At the beginning of period $t+1$, money M_t and bonds $B_t^{(t+1+j)}$ are outstanding. Following Campbell and Shiller (1988); Cochrane (2019), we provide a definition of the linearised government flow identity, with the derivation being available in Appendix B.4.

Proposition 1 (Linearised Government Debt Flow Identity, Cochrane (2019), Cochrane (2022b)) eq. (A.1)). *The linearised government debt flow identity is*

$$\rho v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - s_{t+1}, \tag{6}$$

where v_t is the log of ratio of the market value of debt to GDP at the end *of period t, r^n_{t+1} is the log nominal return on the portfolio of government bonds, π_t is inflation, g_{t+1} is the log of GDP growth, and s_{t+1} is the scaled real primary surplus to GDP ratio.*

Equation (6) is an *accounting* identity; that is, it holds in every model. We re-arrange the government flow identity to solve for surpluses[3]

$$s_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - \rho v_{t+1}.$$

In Figure 7 in the appendix, we plot the ratio of the *implied* primary surpluses to the contemporaneous market value of U.K. public debt from 1830 to 2021. Because the surplus series is expressed in "per-unit-of-debt" terms, a value of -0.2, means the government ran a primary deficit equal to 20% of the previous year's market value of debt. A description of key periods of history through this approach is provided in the corresponding appendix B.5.

We now extend equation (6) into an open economy setting. We give a full microfounded model in appendix B. Suppose we have two economies, home (H) and foreign F. Define the nominal exchange $\mathcal{E}_t$ as the number of home currencies per unit of foreign currency (i.e. an increase in $\mathcal{E}_t$ is a depreciation). Following equation (6), we can define each country's linearised government debt equation as

$$\rho_H v^H_{t+1} = v^H_t + r^{n,H}_{t+1} - \pi^H_{t+1} - g^H_{t+1} - s^H_{t+1}, \tag{7}$$

$$\rho_F v^F_{t+1} = v^F_t + r^{n,F}_{t+1} - \pi^F_{t+1} - g^F_{t+1} - s^F_{t+1}, \tag{8}$$

where $v^X_t \equiv \log\left(\frac{V^X_t}{P^X_t Y^X_t}\right)$, $r^{n,X}_{t+1}$, π^X_{t+1}, g^X_{t+1}, s^X_{t+1}, ρ_X are the analogous objects as defined above for country $X \in \{H, F\}$. Here we define the log of the exchange rate $e_t \equiv \log \mathcal{E}_t$ and therefore $\Delta e_{t+1} = e_{t+1} - e_t > 0$ is a depreciation of the home currency.

The uncovered-interest-parity (UIP) condition with a risk premium shock φ_t is

$$r^{n,H}_{t+1} - r^{n,F}_{t+1} = \Delta e_{t+1} + \varphi_{t+1}, \tag{9}$$

[3] A few comments on why we back out implied surpluses rather than use government account tables. First, it is helpful to ensure exact accounting for surpluses. The constructed surplus by definition satisfies equation (6). Present-value decompositions later in the paper therefore close exactly, with no statistical discrepancy term. The economic meaning of this approach ensures mark-to-market changes in the debt stock represent genuine gains or losses for the fiscal authority and hence the taxpayer. For example, as documented by Hall and Sargent (2011), the NIPA primary surplus in the U.S. omits these valuation effects and so understates fiscal effort when bond prices fall (higher real returns) and overstates it when prices rise. Second, we therefore place us squarely within our intuition of *expected* deficits (as the inverse of surpluses) being relevant, and our decomposition allows us to back out such expected surpluses/deficits identically.

where $\mathbb{E}_t[\varphi_{t+1}] = 0$. We can substitute the UIP condition in equation (9) into the home country's flow identity to get

$$\rho_H v^H_{t+1} = v^H_t + \left(r^{n,F}_{t+1} + \Delta e_{t+1} + \varphi_{t+1}\right) - \pi^H_{t+1} - g^H_{t+1} - s^H_{t+1}. \tag{10}$$

Equation (10) shows that *foreign* discount-rate news $r^{n,F}_{t+1}$, exchange-rate movements Δe_{t+1}, and the UIP premium φ_{t+1} now do the same fiscal work of absorbing the home country's surplus shocks as inflation and home discount-rate news did in the single-country model. For further intuition, suppose that the home country is the U.K. and foreign country is the U.S. Suppose there is a U.S. primary deficit shock. That is, suppose s^F_{t+1} falls unexpectedly ($-s^F_{t+1}\uparrow$). By equation (8), this can be offset by a corresponding increase in foreign discount rates. If this were to happen, the transmission of increased discount rates would be akin to a decrease in fiscal surpluses for the home country due to the higher cost of serving debt.

Equation (10) must still hold, so one (or a combination) of the following channels might adjust conditional on the overall value of government debt and surpluses of the home country remaining untouched:

1. *Fiscal inflation*: $\pi^H_{t+1}\uparrow$ directly erodes value of debt.
2. *Growth*: GDP growth $g^H_{t+1}\uparrow$ raises taxes and pays down deficits. This is analogous to deficits being self-financing as explored in the HANK literature (Angeletos et al., 2024).
3. *Dollar depreciates*: Δe_{t+1} moves as investors find U.S. assets less desirable.
4. *UIP-premium shock*: $\varphi_{t+1} < 0$ as investors accept lower Dollar excess return.

Therefore, any variable that moves to offset the U.S. deficit enters the U.K. constraint via $r^{n,F}_{t+1}$ or Δe_{t+1} and forces a counteracting adjustment in U.K. inflation, growth, or future surpluses.

2.1 Backward and Forward Decompositions

2.1.1 Backward Decomposition

Equipped with the decomposition of the value of government debt, we take stock of the historical arithmetics of the public balance sheet. The backward–looking decomposition asks the accounting question: *given the debt ratio we observe today, how much of it can be traced to realised primary surpluses and deficits, how much to the cumulative effect of inflation and growth, and how much to the repricing of that debt through interest-rate and exchange-rate movements?* Equation (10) allows us to examine the historical drivers of the U.K's debt.

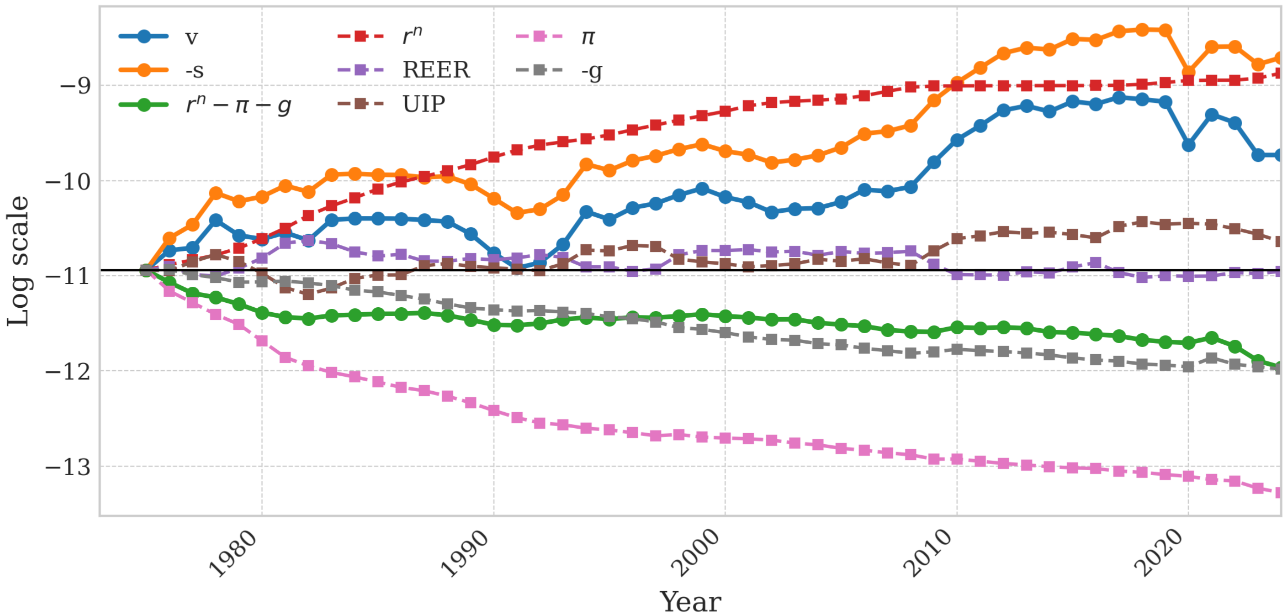


Figure 2: U.K. backward decomposition of government debt with exchange rates.

Starting from the linearised flow constraint in equation (10) for the *home* country (the U.K.)[4]

$$v^H_{t+1} = v^H_t + \left[r^{n,F}_{t+1} + \Delta e_{t+1} + \varphi_{t+1} \right] - \pi^H_{t+1} - g^H_{t+1} - s^H_{t+1}, \tag{11}$$

we iterate equation (11) from an origin date 0 to arrive at

$$v^H_t = v^H_0 + \sum_{j=1}^{t} \left[r^{n,F}_j + \Delta e_j + \varphi_j \right] - \sum_{j=1}^{t} \pi^H_j - \sum_{j=1}^{t} g^H_j - \sum_{j=1}^{t} s^H_j. \tag{12}$$

This is an accounting identity where the value of debt today is the sum of the initial value of debt, the accumulation of FX-risk-premium less inflation, growth, and surpluses. We note that this decomposition measures, and does not test, the present value relation as discussed in Cochrane (2019). We do the backward decomposition of debt in Figure 2 and summarise the components in Table 1. Figure 2 applies equation (12) as a backward accounting exercise. Each coloured path starts at zero in the base year (1975) and then *cumulates* one term of the identity forward through time. The blue line (v) is the realised market-value debt ratio; the orange line ($-s$) shows what that ratio would have been had *only* primary surpluses and deficits changed. The distance between the blue and orange series is therefore the *total valuation effect*, which we disaggregate into foreign discount-rate news ($r^{n,F}$), domestic inflation ($-\pi$), real growth ($-g$), movements in the real exchange rate (REER) and uncovered-interest-parity premia (UIP).

Primary surpluses explain only about one half of the cumulative change in U.K. debt since 1975;

[4] $\rho_H \simeq 0.97$ is the Campbell–Shiller discount factor at annual frequency. Because $\rho_H \approx 1$, we suppress it to keep notation light.

Line	Path (Start in 1975)	Interpretation
v	$v_0^H + \sum_{j \le t} \left(r_j^{n,F} + \Delta e_j + \varphi_j - \pi_j^H - g_j^H - s_j^H \right)$	Actual (market–value) debt/GDP ratio.
$-s$	$v_0^H - \sum_{j \le t} s_j^H$	Counterfactual path if *only* realised primary surpluses/deficits moved.
$r^n - \pi - g$	$v_0^H + \sum_{j \le t} \left(r_j^{n,F} - \pi_j^H - g_j^H \right)$	Debt path driven by discount-rate, inflation, and growth.
r^n	$v_0^H + \sum_{j \le t} r_j^{n,F}$	Cumulated foreign nominal return.
$-\pi$	$v_0^H - \sum_{j \le t} \pi_j^H$	Inflation erosion of real debt.
$-g$	$v_0^H - \sum_{j \le t} g_j^H$	Contribution of real growth.
$REER$	$v_0^H + \sum_{j \le t} \Delta e_j$	Cumulated Sterling depreciation.
UIP	$v_0^H + \sum_{j \le t} \varphi_j$	Cumulated FX risk-premium shocks.

Table 1: Mapping of Figure 2 series to the components of the backward decomposition in equation (12).

the remainder is valuation, split almost evenly between the global discount factor $(r^n - \pi - g)$ and exchange-rate channels (REER and UIP). Second, every regime break identified by Bordo et al. (2022) shows up as a kink in one of the valuation curves, confirming that shifts in discount factors, not just fiscal flows, drive debt dynamics. Finally, the open-economy channels matter quantitatively: a permanent 10 % Sterling depreciation raises the domestic currency value of future Dollar coupons by the same 10 %, while a 200 bp UIP shock lowers today's discount factor enough to add roughly 10% of GDP to the market value of outstanding debt—all without any change in cash surpluses.[5]

2.1.2 Forward Decomposition

The forward-looking present-value identity is an even more natural starting point: under a government debt valuation equation, the entire stream of expected future primary surpluses discounted at the rates investors require matters for today's value of debt. By writing the flow constraints in present-value form, we place the exchange rate, real discount rates, and the UIP-premium on the same footing as surpluses and growth: each enters as a discounted sum that must balance the market value of debt. This forward-looking perspective makes two things transparent. First, because the identity holds period-by-period in expectation, any news about future surpluses, inflation, discount rates, or exchange-rate movements is reflected immediately in debt valuations and currency prices. Second, following the finance literature, we are able to decompose movements in

[5] A detailed description of key macroeconomic episodes in the U.K. is provided in appendix B.5.1.

the value of debt into distinct channels of fiscal, monetary, real, and exchange rates. In short, the present-value identity turns a sequence of forward flows into one equilibrium condition we can take to the data and decompose visually into economically meaningful pieces.

We translate the government's intertemporal budget constraint into a form that can be empirically estimated with a finite-order vector autoregression (VAR), again following Cochrane (2019, 2022b). Let x_t be the state variable consistent of the variables of interest. The state variable consists of (i) the foreign nominal return that discounts both countries' bonds; (ii) real consumption growth, inflation, the primary surplus, and the market value of debt for the home country; (iii) real-exchange-rate changes Δe_t and the ex-ante UIP premium φ_t that jointly affect Dollar pricing; and (iv) short and long U.S. Treasury yields, which improve forecasts of future nominal returns and inflation. All series are annual logs or log-ratios. With the state variable, we can estimate a first-order annual VAR

$$x_{t+1} = A\,x_t + \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim \mathcal{N}(0, \Sigma),$$

on post-war U.K. data. Because all eigenvalues of A lie strictly inside the unit circle, the VAR implies finite long-run multipliers. Conditional expectations of any element of x_{t+j}, and therefore the discounted present value sum, can therefore be written in closed form:

$$E_t[x_{t+j}] = A^j x_t \implies \sum_{j=1}^{\infty} E_t[x_{t+j}] = (I - A)^{-1} A\,x_t.$$

Define selection vectors a_s, a_g, and a_r that pick out, respectively, the surplus, growth, and *nominal-return minus inflation* rows of x_t. Let $a_s, a_g, a_r, a_\pi, a_{\Delta e}, a_\varphi$ be row-selection vectors that return, respectively, $(s_t^H,\ g_t^H,\ r_t^{n,F},\ \pi_t^H,\ \Delta e_t,\ \varphi_t)$ from x_t. The infinite-horizon *expected* present-value contributions that enter the linearised budget identity are

$$PV_t^{(k)} := a_k'(I - A)^{-1} A\,x_t, \qquad k \in \{s, g, r, \pi, \Delta e, \varphi\},$$

and $PV_t^{(d)} \equiv PV_t^{(r)} - PV_t^{(\pi)}$.

Figure 3 plots the *forward* decomposition implied by the open–economy identity in equation (26). In Table 2, we describe the mapping between the formulas and Figure 3. The blue line is the realised market value of debt, while every other series is the *expectation* on the right-hand side of equation (26), taken with the information available at date t. [6]

From the perspective of this forward decomposition, real surpluses and changes to the price level are the material contributors to the equilibrium valuation of government debt. Real exchange rate movements and UIP deviations, depicted with the brown and pink lines, respectively, help to reconcile the relative under-valuation of U.K. debt vis-á-vis the surplus process until the early 2000s, as well as the over-valuation of U.K. debt implied by our VAR framework since then.

[6] Appendix B.5.2 presents a description of the emergent historical episodes.

Curve	Algebraic object	Economic meaning
v	v_t^H	Market value of debt/consumption ratio.
s	$PV_t^{(s)}$	Present value of future primary surpluses.
$-(r^n - \pi)$	$-PV_t^{(d)}$	Present value of future *real* discount rates.
r^n	$PV_t^{(r)}$	Present value of future nominal returns.
$-\pi$	$-PV_t^{(\pi)}$	Present value of future U.K. inflation.
REER	$PV_t^{(\Delta e)}$	Present value of future real-exchange-rate changes.
UIP	$PV_t^{(\varphi)}$	Present value of future UIP-premium shocks.
g	$PV_t^{(g)}$	Present value of future real growth.

Table 2: Mapping of the Figure 3 series to the analytic components in eq. (25). By construction $v_t^H = PV_t^{(s)} + PV_t^{(g)} - PV_t^{(d)}$.

Complementing this picture, appendix B.5 also provides a historical variance decomposition of the equilibrium value of U.K. sovereign debt, highlighting that while surplus news dominate in contributing to variance in the value of government debt, UIP and inflation channels nonetheless matter economically significantly for the valuation of government debt.

2.2 Linking exchange rate movements to deficit shocks

The previous exercise has shown how UIP deviations and cross-border interest rate spillovers matter significantly for the equilibrium valuation of government debt. As a final exercise, we now take the recovered surplus processes for the U.K. and the U.S. (where the U.S. surplus processes are following Cochrane (2019)) to determine how much they each contribute to medium-run movements in U.K. real exchange rates. We view this exercise as informative about the plausible contributors of secular determinants of U.K. exchange rates, showing how much of the observed variance in U.K. exchange rates can be plausibly attributed to movements in domestic surpluses or foreign surpluses, where foreign surpluses matter through foreign interest rates and associated exchange-rate and UIP-deviation channels, as explained above.

Table 3 summarizes the results by this exercise, linking the surplus innovations implied by the forward decomposition of the VAR in section 2.1 to observed year-on-year movements in U.K. real exchange rates. We weighted the U.S. surplus innovations by the relative U.S./U.K. GDP to reflect the relative size of both countries, but not weighting the surplus does not impact the qualitative results of the exercise.

Strikingly, these surplus expectation revisions relate to the observed movements in U.K. exchange rates significantly and in the direction implied by the government debt valuation equation: a positive surplus innovation in the U.K. relates to an *appreciation* of the U.K. real exchange rate, with a 1% surplus-to-GDP innovation being related to a 37 basis point relative *appreciation*.

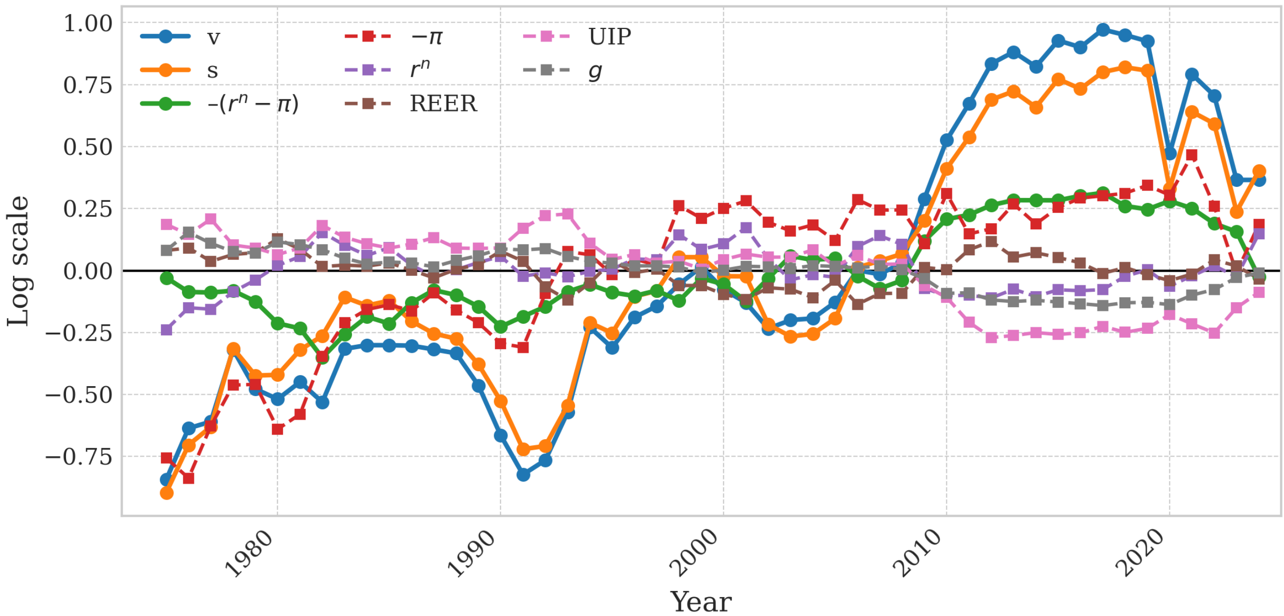


Figure 3: U.K. forward decomposition of government debt with exchange rates.

Dynamics of U.S. deficits matter an order of magnitude less, as expected and prescribed by our framework. In numbers, a 1% surplus-to-GDP innovation in the U.S. relates to a 1.8 basis point relative *depreciation* of U.K. exchange rates. The second column confirms that the *differential* between U.K. ans U.S. surpluses relates most materially to annual exchange rate movements. The third column, providing an interaction effect with local recessions, reveals that in such recessionary periods it is plausible for surplus/deficit innovations to matter through unconventional channels that are otherwise not captured.[7]

In sum, our long-run historical analysis focussing on the dynamics underpinning the valuation of government debt underscore that the domestic debt valuation equation is necessarily linked to the UIP condition and fluctuations in the real exchange rate. Additionally, we have shown how measured surplus innovations then contribute to observed movements in real U.K. exchange rates.

The link between expected surplus/deficit dynamics and exchange rates corresponds to the focal novelty of the paper. In this section, we have done so from a longer-run perspective, focussing on the mechanics of the government debt valuation equation. We support the results established herein with a *shorter-run* perspective, utilizing domestic short-run expectations of fiscal deficits to analyse their contribution to observed variation in month-on-month exchange rate revaluations.

[7] In appendix C we provide additional results based on the Engel and West (2005) reverse-regression approach, showing that exchange rates are able to predict materialized surpluses in the U.K. The link between exchange rates and surpluses follows an 'overshooting' pattern.

	Dependent variable: Δ U.K. REER		
	(1)	(2)	(3)
$\Delta\mathbb{E}(\text{Deficit})_t^{U.K.}$	0.370***	0.352***	0.358***
	(0.077)	(0.075)	(0.107)
$\Delta\mathbb{E}(\text{Deficit})_t^{U.K.} \times \mathbf{1}^{g_Y<0}$			−2.370***
			(0.615)
$\Delta\mathbb{E}(\text{Deficit})_t^{U.S.}$	−0.018***		−0.010**
	(0.005)		(0.004)
$\Delta\mathbb{E}(\text{Deficit})_t^{U.S.} \times \mathbf{1}^{g_Y<0}$			0.210***
			(0.061)
$\Delta\mathbb{E}(\text{Deficit})_t^{U.K.-U.S.}$		0.016***	
		(0.006)	
Constant	-0.006	-0.022	-0.001
	(0.010)	(0.014)	(0.009)
Observations	25	25	25
Adjusted R^2	0.469	0.320	0.523

Note: HAC-robust standard errors. *p<0.1; **p<0.05; ***p<0.01

Table 3: Long-run correlates of U.K. exchange rates

3 Expectations of forecast revisions

In this section, we present more in-depth evidence in favour of a direct link between deficits, exchange rates, and inflation, in line with the predications implied by a government debt valuation equation, and supporting the results of section 2 by using private-sector data on deficit expectations in place of the deficit innovations recovered in the VAR framework of section 2. To that goal, we now turn to a novel dataset of financial forecasters in the United Kingdom, evaluating to which degree forecasters link deficits, exchange rates, and inflation, and whether a stronger link corresponds to an improved forecast performance. A part of our contribution is the assembly of a near-30-year-long dataset of macroeconomic forecasts made by private forecasters in the United Kingdom, which has henceforth not been created to the best of our knowledge.

This exercise is broadly in the line of work of Cochrane (2018) and Barro and Bianchi (2024): we use an *innovation representation* of the government debt valuation equation in the context of international holdings of government debt to inform forecast revisions pinning down a plausible link between deficits and exchange rates. Appendix section B.6 provides a derivation of the empirical specification analyzed herein.

Data

The data that we use in this section is the universe of private sector forecasts on various macroeconomic variables, collected by the U.K. Treasury (HM Treasury, 2025). While aggregate data on average forecasts is available in machine-readable format, the individual forecasts were initially not available as such. We hand-collected all forecaster-specific data since September 1997 on a monthly basis. Further details on the data are reported in appendix E.

In the process of collecting the data, the U.K. Treasury asks a select number of banks and financial institutions (labelled 'City Forecasters') as well as a large number of independent forecasting bodies without direct vested interest (labelled 'Non-City forecasters') about their same-year and one-year-ahead predictions for a battery of macroeconomic and financial variables. Not all forecasters update their forecasts each month, but the median forecaster who is in the sample for at least three years provides updates multiple times over the year. Of particular interest to us, in line with equation (30) in the appendix, are the forecasts on sovereign deficits (officially "Public Sector Net Borrowing", or PSNB), exchange rates, real interest rates (recovered through the corresponding forecasts made on the Bank of England lending rate and CPI/RPI inflation), and of output growth. We estimate a specification aligned with equation (30). Let T be the forecast horizon, $t-1$ be the date of the previous forecast, and t be the current date. Then, we estimate generally a type of the following specification:

$$\Delta\mathbb{E}_{it}\mathcal{E}_{(T-j)\to T} = \beta_0 + \beta_1\Delta\mathbb{E}_{it}(-\hat{s}_T) + \beta_2\Delta\mathbb{E}_{it}\hat{y}_T + \beta_3\Delta\mathbb{E}_{it}\hat{r}_T + \Gamma Z_{it} + \varepsilon_{it}. \tag{13}$$

Our main specification, equation (13), deserves some explanation. $\Delta\mathbb{E}_{it}\hat{e}_{(T-12)\to T}$ on the left-hand side captures the forecast revision from $t-1$ to t ($\Delta\mathbb{E}_{it}$) of the expected change in the exchange rate from the end-of-last-year (which was j months ago) until the end of the year, which is the time period T. On the right-hand side, we have a firm-specific fixed-effect α_i.[8] Next, β_1 describes the main coefficient of interest, showing the effect of revisions in expected deficits (negative surpluses) for the year ending at T. The next coefficients β_2 and β_3 are symmetrically the coefficients on revisions of output growth and the expected real interest rate earned in the year ending at period T. Finally, ΓZ_{it} captures further control variables, and ε_{it} is the estimation error. The fact that we generally control for other forecast revisions in the estimation process of forecast revisions is thus both informed by economic theory *and* in line with evidence presented by Crump et al. (2025) that forecast revisions are occurring jointly, with important effects being missed when considering only mutual pairs of such revisions.

[8] We also control for alternative specifications with a joint firm-year fixed-effect, as per the tables below.

Results

We now present our results. First, we establish that forecasters on average link revisions indicating larger deficits to corresponding depreciations. Additionally, we provide evidence that those forecasters whose forecast revisions are closer aligned with the deficit-exchange rate-mechanism are, on average, also better in terms of their own exchange rate forecasting performance, rationalizing the link between deficits and exchange rates from the perspective of mutual innovations.

	Outcome: $\Delta\mathbb{E}(\mathcal{E}_{\text{Sterling}\mid\text{Basket}})$				
	(1)	(2)	(3)	(4)	(5)
$\Delta\mathbb{E}(\text{PSNB})$	-0.0005**	-0.0005**	-0.0005**	-0.0005**	-0.0002***
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(5.6×10^{-5})
$\Delta\mathbb{E}(g_Y)$	0.0043**	0.0053***	0.0024	0.0033	0.0047***
	(0.0018)	(0.0018)	(0.0022)	(0.0021)	(0.0011)
$\Delta\mathbb{E}(r_{t+1})$	0.0037*	0.0040**			0.0001
	(0.0018)	(0.0017)			(0.0010)
$\Delta\mathbb{E}(i_{t+1})$			0.0133***	0.0141***	
			(0.0038)	(0.0034)	
$\Delta\mathbb{E}(\pi_{RPI,t+1})$			-0.0017	-0.0020	
			(0.0015)	(0.0014)	
Observations	6,433	6,433	6,433	6,433	5,041
R^2	0.03265	0.01379	0.03565	0.01745	0.16020
Forecaster-year joint FE	✓		✓		✓
Forecaster FE		✓		✓	

Table 4: Forecast revision regressions across the universe of forecast data. *p<0.1; **p<0.05; ***p<0.01; firm-clustered standard errors in parentheses.

Table 4 relates changes to the end-of-year exchange rate forecasts to concurrent changes in other forecasts, with a set of fixed effects as specified below. The link between deficit expectation revisions and exchange rate expectation revisions is broadly robust across our specifications. In numbers, a 1 billion GBP (in 2018 GBP terms) increase in deficit expectations corresponds to a 0.05 percentage point *decrease* in end-of-year expectations of weighted exchange rates. Given that forecast revisions can amount to 100 billion GBP (as evidenced in the appendix figure 12), we can think of this mechanism as being also economically significant in addition to statistical significance.

In figure 4, we visualize the main point made by table 4: conditional on firm-specific effects and other forecast updates, we generally observe a negative relation between exchange rate forecast revisions and deficit forecast revisions. Thus, a higher deficit forecast corresponds ceteris paribus to a downwards revision in expected year-end exchange rates, in line with the prescriptions of the government debt valuation equation (30).

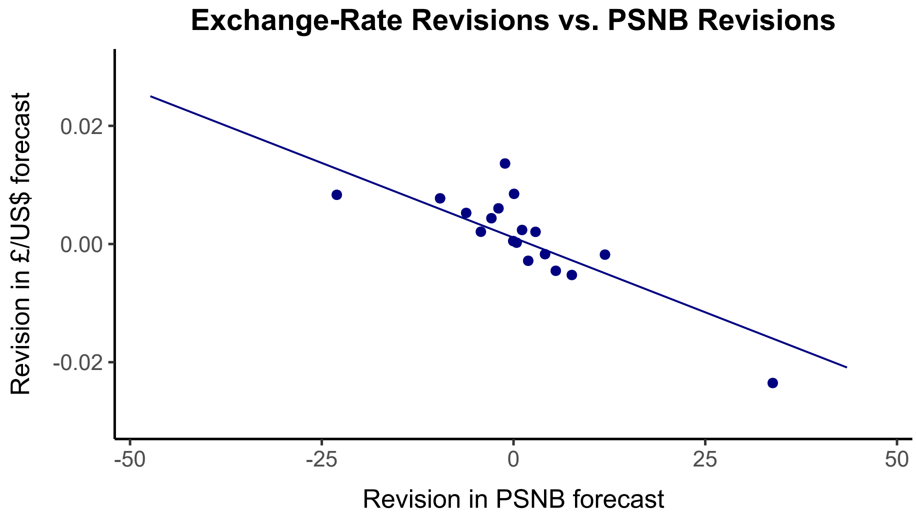


Figure 4: Binned scatter plot showing the partial correlation between deficit forecast revisions and exchange rate forecast revisions, partialling out firm fixed-effects and forecast revisions for output and real interest rates.

The results of table 4 and figure 4 partial out forecaster-specific effects. However, there is substantial heterogeneity across forecasters in how they link deficits to exchange rate revisions. This informs a further interesting question: if a forecaster presumes a tighter link between deficits and depreciations, does that correspond also to a better ability of forecasting exchange rates? We now address this question, borrowing the definition of Mincer and Zarnowitz (1969) to evaluate what makes a *good* forecast.

In detail, we proceed as follows: First, we estimate *within each firm* the following specification:

$$\mathcal{E}_T = \alpha_{i0} + \alpha_{i1}\mathbb{E}_{it}(\mathcal{E}_T) + e_i,$$

to recover the estimate α_{i1}, which captures the degree to which the forecasts made by forecaster i match the actual outcome of the exchange rate index. A perfect forecaster would obtain $\alpha_{i0} = 0$ and $\alpha_{i1} = 1$. We then rank the coefficients α_{i1} in terms of their absolute distance from 1, and check to which degree this "rank" (which is tantamount to forecasting performance) relates to the coefficient) β_{i1} recovered from a firm-specific estimation of equation (13), which therefore comes without firm-specific fixed effects. Having recovered both these elements, we estimate:

$$\hat{\beta}_{i1} = \gamma_1 \,\text{rank}(|\hat{\alpha}_{i1} - 1|) + u_i, \tag{14}$$

with the estimated coefficient $\hat{\gamma}_1$ being indicative whether better forecasters are actually characterized as those who link deficits broadly 'more' to exchange rate revaluations. We present the results of this estimation in table 5.

	Dependent variable: $\hat{\beta}_{PSNB,ER}$	
	Filtered ($n \geq 10$ obs.)	Full sample
	(1)	(2)
rank($\lvert\hat{\alpha}_{i1} - 1\rvert$)	0.838***	0.809***
	(0.157)	(0.220)
Observations	52	61
R^2	0.337	0.100
F Statistic	25.936***	6.648**
Note:	*p<0.1; **p<0.05; ***p<0.01	
	Heteroscedasticity-robust standard errors in parentheses.	

Table 5: Rank regressions depicting the link between forecast accuracy and the firm-specific regression coefficient linking forecast revisions in deficits to forecast revisions in exchange rate

The coefficients for the full sample as well as the coefficients for the restricted sample (where we estimate the firm-specific deficit-exchange-rate link only for firms appearing in at least 10 years) support the notion that forecasters whose models predicate a closer link between deficits and depreciations are also the forecasters whose exchange rate forecasting performance is, on average, better. Thus, it appears that leaning into models of exchange rate determination informed by the government debt valuation equation is a rational way of forecasting exchange rates, conditional on any given information set available at time t. While this is clearly *not* a test of the government debt valuation equation as such, it lends support to the idea of focussing on deficits and deficit expectations when trying to determine expected movements in exchange rates.

Our empirical evidence informed by forecast revisions in the U.K. across almost 30 years helped us establish two major facts: one, exchange rates contribute significantly to the revision of the equilibrium value of government debt in open economies; and second, the link between fiscal deficits and exchange rates implied by the government debt valuation equation appears to be not only present in the forecasts made by firms with a vested interest in the valuation of government debt, but those firms that are more heavily leaning into determining exchange rates with the help of the mechanisms implied by the debt valuation equation are also, on average, better forecasters. Equipped with those two pieces of evidence, we now move on to rationalize our findings theoretically: first, with a tractable model isolating our main mechanism linking between fiscal policy and exchange rates; and second, with a full-scale general equilibrium model allowing for rich international trade patterns.

4 A Simple Model of International Fiscal Inflation Spillovers

This section presents a minimal model emphasizing a fiscally-led inflation mechanism in a standard international macro context, with plausible relevance of fiscal deficits for nominal exchange rate movements and international spillovers through interest rate adjustments. What we therefore show here is a plausible mechanism how fiscal sustainability concerns in one country can spill over to the government debt valuation of another country. The interesting messages of the model are kept fully nominal, with no relevance for international goods flows, which we nullify here for expositional simplicity. One particular result will be the possibility to distinguish between *announcements* of spending at Home and their factual implementation, with particular relevance for the announcements through expectations-driven revisions of the Uncovered Interest Rate Parity (UIP) condition.

4.1 Environment

The primitives of the model rely on a simplified iteration of the framework of Bassetto and Miller (2025), but with a diminished focus on the information structure. Instead, we add an international dimension facilitating the transmission of fiscal shocks between countries. Denote the Home country by H, the Foreign country by F, and let stars (*) denote variables in the Foreign country, in line with standard notation. Heuristically, we will think of the Home country as an international hegemon determining international spillovers, and of the Foreign country as a small open economy subject to the financial influence of the hegemon, although it has a dis-taste for those spillovers, reflected in the Foreign country not enjoying holdings of Home (Hegemon) bonds. There are two periods:

1. <u>t=1</u>: there exists an initial steady-state inherited from t=0. In period one, the price levels P_1, P_1^*, as well as the nominal exchange rate $\mathcal{E}_1$, are all determined through the respective government budget constraints and cross-border debt holdings inherited from the initial period. There exists an information set about a "government spending shock". This is a shock to Home government spending in period 2, G_2, whose plausible size is revealed. Additionally, there is a prior about the policy regime in place at Home. Two policy regimes are possible: a 'fiscally-led' regime, in which taxes are a fixed quantity and the spending need will be financed through inflation; and a 'monetary-led' regime in which taxes adjust to ensure that a target price level at Home is achieved. The Foreign fiscal authority levies fixed taxes $\bar{T}^*$ in all periods, while Foreign interest rates on debt issuance i_1^* are determined in equilibrium

through the UIP condition.

2. t=2: The tax and monetary regimes from Home unfold, yielding the terminal Home price level P_2. Note that the Foreign price level P_2^* will already be perfectly revealed in period 1, being a function of equilibrium interest rates on Foreign debt from period 1 to period 2.

4.2 Home households

We begin by introducing the households, which are not fully symmetric across the two countries to simplify calculations. Relative to standard macroeconomic models, the sole key difference is that we introduce a reduced-form dis-preference term on behalf of the Foreign households for the bonds issued by Home. This may reflect a home bias in savings vehicles, or a negative perception of Home government bonds through the spillovers possible by their Hegemon status.[9]

Home households maximize the utility borne from consumption of the domestic good, adjusted for the disutility from supplying labour:[10]

$$max\, U = max \sum_{t=1}^{2} \beta^t \mathbb{E}_t \left[u(c_t) - L_t \right],$$

where c_t is consumption of the representative household, L_t is labour supplied, and the production function of the household is given by $y_t = L_t$. Disutility from labour is kept linear on purpose to simplify determination of the real equilibrium in the presence of fiscal shocks.

Each household takes as given government policy and the price level. Define B_{Ht-1} as household holdings of Home bonds at the beginning of period t, having taken them over from period $t-1$. The same applies to holdings of Foreign bonds by the Home household, defined as B_{Ft-1}. Then, the domestic household budget constraint is given by:

$$B_{Ht-1}(1+\bar{i}) + B_{Ft-1}\mathcal{E}_t(1+i^*_{t-1}) + P_t L_t = B_{Ht} + B_{Ft}\mathcal{E}_t + P_t(c_t + T_t). \tag{15}$$

Note that no debt is left outstanding in the final period for either household in equilibrium; that is, $B_{H2} = B_{F2} = B^*_{H2} = B^*_{F2} = 0$.

4.3 Home Fiscal and Monetary Authority

Next, consider the Home government. The government faces an already accrued (inherited) stock of debt B_0, held by both Home and Foreign households in some proportion. It can raise lump-sum

[9] A similar dis-taste term for the Home household for Foreign bonds can be included, but it would merely obstruct the core message through more involved algebra. Also, we could express the model through a local preference for domestic bonds, but this would not materially change the outcomes at the cost of more involved algebra again.

[10] Note that there is no goods trade in equilibrium, but only trade in financial products at this stage.

taxes T_t in each period. We follow Bassetto and Miller (2025) in assuming that $G_1 = 0$ without loss of generality. In period 1, lump-sum taxes will be set to cover interest payments, such that $T_t = \frac{B_{t-1} - \frac{B_t}{1+\bar{i}}}{P_t}$ for $t = 1$. In period $t = 2$, Home government spending and taxes are uncertain and plausibly disjoint, allowing for altered government debt dynamics.

In line with Bassetto and Miller (2025), two regimes in terms of fiscal policy are possible:

- ML (Monetary-led): in this regime, $T_2 = G_2 + \frac{B_1}{P_2^T}$, where P_2^T is an externally supplied targeted price level in period 2. Then, any spending requirement G_2 is fully soaked up by corresponding taxes, while the primary surplus $\frac{B_1}{P_2^T}$ soaks up the cost of outstanding bond holdings.
- FL (Fiscally-led): in regime FL, taxes do not respond to the spending requirement: $T_2 = \bar{T}$, where $\bar{T}$ is an exogenously supplied known constant.

Fundamentally, the assumption of introducing tax changes in period 2, but their announcement taking place in period 1, is meant to distinguish the effects of announced deficits from the effects once the deficits actually materialize. It therefore allows us to capture the effects of fiscal spending announcements on exchange rates through their forward-looking nature.

Monetary policy in the Home economy will be captured by a strict nominal interest rate peg as already in use in the Home budget constraint, $i_t = \bar{i} \, \forall t$.

4.4 Foreign block

The *Foreign* household is described by a similar problem as the domestic household, but with all variables being starred and the exchange rate adjustment in the budget constraint reflecting that the Foreign household considers the Foreign bonds as domestic, and, crucially, an additional idiosyncratic preference term capturing the dis-utility from holding Home bonds. Formally, the representative Foreign household maximizes:

$$max\ U^* = max \sum_{t=1}^{2} \beta^t \mathbb{E}_t \left[u(c_t^*) - L_t^* - \phi_F \frac{B_{Ht}^*}{P_t^*} \right].$$

The additional term at the end, $-\phi_F \frac{B_{Ht}^*}{P_t^*}$, captures the disutility from holding Home bonds. ϕ_F is an exogenous parameter capturing the weight of the disutility term, and $\frac{B_{Ht}^*}{P_t^*}$ is the real quantity of Home bonds held by Foreign households. We use a linear disutility term for simpler calculations here, but a quadratic disutility term is possible.

This maximization is performed subject to the per-period budget constraint of the Foreign household:

$$B_{Ft-1}^*(1 + i_{t-1}^*) + \frac{B_{Ht-1}^*}{\mathcal{E}_t}(1 + \bar{i}) + P_t^* L_t^* = B_{Ft}^* + \frac{B_{Ht}^*}{\mathcal{E}_t} + P_t^*(c_t^* + T_t^*). \qquad (16)$$

Now, we consider the government of Foreign. The Foreign government differs from the Home government in a number of important aspects:

- First, there is no uncertainty with respect to Foreign government spending; that is, we assume $G_t^* = 0 \; \forall t$ for simplicity.
- In terms of the policy in use, we postulate that taxes levied in the terminal period by the Foreign government are fixed; that is, $T_2^* = \bar{T}^*$. The terminal period price level in the Foreign economy will be determined by the terminal government budget constraint, with the price level adjusting to ensure that the fiscal budget constraint holds.
- Foreign interest rates are allowed to be time-varying: i_t^* is not constant. It adjusts, however, not based on a policy rule, but instead in a way that allows the UIP, adjusted for the wedge term ensuring a preference for the Foreign bonds, to hold.

Information structure: We introduce an explicit information structure that allows us to track the effect of each policy regime in conjecture with government spending on the Home and Foreign price level.

In $t = 1$, both Home and Foreign households have a (shared) prior about the policy regime and spending G_2 of Home. To keep it simple, we denote by ψ_1 the prior that the Home government will be in regime FL, and by $1 - \psi_1$ the prior that the Home government will be in regime ML. These priors will be the same in both Home and Foreign. The government spending variable can follow an arbitrary distribution $\mathcal{G}$.

Optimality Conditions: We provide detailled derivations in appendix A. The domestic household's equilibrium conditions are characterised by:

$$u'(c_t) = 1, \qquad t = 1, 2; \tag{17}$$

$$u'(c_t) = \beta(1 + \bar{i})\, \mathbb{E}_t\left[u'(c_{t+1})\frac{P_t}{P_{t+1}}\right], \qquad t = 1; \tag{18}$$

$$u'(c_t) = \beta(1 + i_t^*)\, \mathbb{E}_t\left[u'(c_{t+1})\frac{P_t}{P_{t+1}}\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right], \qquad t = 1. \tag{19}$$

For the foreign household, their equilibrium optimality conditions are characterised by:

$$u'(c_t^*) = 1, \qquad t = 1, 2; \tag{20}$$

$$u'(c_t^*) = \beta(1 + \bar{i})\, \mathbb{E}_t\left[u'(c_{t+1}^*)\frac{P_t^*}{P_{t+1}^*}\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right] - \phi_F \mathcal{E}_t, \qquad t = 1; \tag{21}$$

$$u'(c_t^*) = \beta(1 + i_t^*)\, \mathbb{E}_t\left[u'(c_{t+1}^*)\frac{P_t^*}{P_{t+1}^*}\right], \qquad t = 1. \tag{22}$$

Note that, in general, no Euler equations apply in period 2 as all bond holdings in that period are set to zero in that period by household optimality.

Definition 2 (Competitive equilibrium). *An allocation* $\{c_t, c_t^*, L_t, L_t^*, B_{Ht}, B_{Ft}, B_{Ht}^*, B_{Ft}^*, B_t, B_t^*\}_{t=1,2}$, *nominal prices* $\{P_t, P_t^*, \mathcal{E}_t\}_{t=1,2}$, *fiscal policy* $\{T_t, T_t^*\}_{t=1,2}$, *monetary policy* $\{\bar{i}, i_t^*\}_{t=1,2}$, *and government spending* $\{G_2\}$ *are called a competitive equilibrium in this economy, when:*

- *all time-t variables are t-measurable;*
- *given* $\{P_t, \bar{i}, T_t, P_t^*, i_t^*, T_t^*\}$ *and* $\{G_2\}$, *the variables* $\{c_t, L_t, B_{Ht}, B_{Ft}\}_{t=1,2}$ *solve equations (17), (18), (19), and (15); and the variables* $\{c_t^*, L_t^*, B_{Ht}^*, B_{Ft}^*\}_{t=1,2}$ *solve equations (20), (21), (22), and (16), with* $B_{H2} = B_{F2} = B_{H2}^* = B_{F2}^* = 0$.
- *The domestic government satisfies its budget constraint and policy rules:*
  - $B_t = B_{t-1}(1+\bar{i}) + P_t(G_t - T_t),\ t = 1, 2,$ *where* $B_2 = 0$.
  - $T_2 = G_2 + \frac{B_1(1+\bar{i})}{P_2^T}$ *in regime ML;* $T_2 = \bar{T}$ *in regime FL. Taxes in periods 1 and 2 adjust to ensure a constant real debt burden*
  - *Interest rates are pegged to the value* $\bar{i}$.
- *The Foreign government satisfies its budget constraint and policy rules:*
  - $B_t^* = B_{t-1}^*(1 + i_{t-1}^*) + P_t^*(G_t^* - T_t^*)$ $t = 1, 2, 3$ *and* $B_2^* = 0$.
  - *Taxes in all periods are fixed. In* $t = 1$, *taxes offset the cost of serving maturing debt, while in* $t = 2$, *taxes are simply given by some exogenous level* $\bar{T}^*$.
  - *Interest rates* i_t^* *adjust to make sure that household optimality is achieved as the sole free parameter ensuring that the UIP conditions on both households hold.*
- *Markets clear:*
  - $C_t + G_t = L_t$ *for* $t = 1, 2,$
  - $C_t^* + G_t^* = L_t^*$ *for* $t = 1, 2,$
  - $B_t = B_{Ht} + B_{Ft}$ *for* $t = 1, 2,$
  - $B_t^* = B_{Ht}^* + B_{Ft}^*$ *for* $t = 1, 2$-

In total, we have 17 endogenous variables in 17 conditions.

This model gives rise to the following two UIP conditions:

$$\frac{1}{\mathcal{E}_t} = \beta(1+\bar{i})\, \mathbb{E}_t\left[\frac{P_t^*}{P_{t+1}^*}\frac{1}{\mathcal{E}_{t+1}}\right] - \phi_F, \tag{23}$$

$$\mathcal{E}_t = \beta(1+i_t^*)\, \mathbb{E}_t\left[\frac{P_t}{P_{t+1}}\mathcal{E}_{t+1}\right]. \tag{24}$$

4.5 Solving for equilibrium

We now solve for the equilibrium of this model. We start from the terminal period ($t = 2$) and move backwards in solving the model.

To pin down the terminal exchange rate $\mathcal{E}_2$ without invoking the UIP (as there are no bonds outstanding in period 3), we start by recognizing that the model set-up requires all cross-border bond holdings to unravel at the terminal period. In an accounting sense, cross-border flows of financial goods in the terminal period must allow the Home country to repatriate the Foreign holdings of Home bonds, and vice versa. Therefore, the terminal exchange rate reflects the price level-adjusted flows that allow cross-country bond holdings to unravel. The logic is that, in equilibrium,

$$P_2(1+\bar{i})B_{H1}^* = \mathcal{E}_2 P_2^*(1+i_1^*)B_{F1} \qquad \Leftrightarrow \qquad \mathcal{E}_2 = \frac{P_2}{P_2^*}\frac{(1+\bar{i})}{(1+i_1^*)}\frac{B_{H1}^*}{B_{F1}}, \tag{25}$$

such that the non-domestic bond holdings in period 1 in combination with their respective interest rates are fully informative about the nominal exchange rate in period 2. This assumption allows us to close the outstanding stock of cross-border financial accounts in the terminal period.

In economic terms, this assumption equates the value of Foreign debt held by Home and Home debt by Foreign in the last period. Therefore, it facilitates a *positive* correlation between the value of Home and Foreign debt in the terminal period. Assume a fiscal expansion in Home, reducing the real value of Home debt in the terminal period. By this assumption, the real value of Foreign debt denoted in the Home currency must also decrease, allowing for a simple reduced-form international transmission of fiscal shocks.

This is, in a sense, an assumption about absolute hegemony of Home: since Foreign fiscal policy is assumed to be fixed, a fiscal deterioration at Home must translate to a fiscal deterioration in Foreign by depressing the market value of Foreign debt. To some extent, this rationalizes the distaste for international financial integration in Foreign, manifest in its utility function.

As alluded to previously, terminal bond holdings are zero by household optimality: $B_2 = B_2^* = B_{H2} = B_{F2} = B_{H2}^* = B_{F2}^* = 0$. Additionally, for our further analysis, we denote the measure of fiscal spending in the Home country conditional on begin in the fiscally-led regime as $\mu_1 \equiv \mathbb{E}_1(G_2|FL)$.

Conditional on the fiscal policy regime in place, we are able to pin down the price level in the terminal period at Home through the specification of the fiscal policy regime in place:

$$P_2 = \begin{cases} P_2^T & \text{if ML,} \\ \frac{B_1(1+\bar{i})}{\bar{T}-G_2} & \text{if FL.} \end{cases} \tag{26}$$

Therefore, the level of taxes levied at Home in the terminal period is given by:

$$T_2 = \begin{cases} \frac{B_1(1+\bar{i})}{P_2^T} + G_2 & \text{if ML,} \\ \bar{T} & \text{if FL.} \end{cases} \tag{27}$$

Since $G_1 = 0$ and $u'(c_t) = 1 \; \forall t$, we can simplify the Euler equation applying in period 1 in the Home household, which is:

$$u'(c_1) = \beta(1+\bar{i})\mathbb{E}_t\left[u'(c_2)\frac{P_1}{P_2}\right], \tag{28}$$

by recognizing that household optimality prescribes constant consumption across time, such that the Euler equation becomes:

$$\frac{1}{P_1} = \beta(1+\bar{i})\left[\psi_1\frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} + (1-\psi_1)\frac{1}{P_2^T}\right]. \tag{29}$$

Alternatively, we can express also the price level explicitly as:

$$P_1 = \frac{1}{\beta(1+\bar{i})}\frac{1}{\left[\psi_1\frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} + (1-\psi_1)\frac{1}{P_2^T}\right]}, \tag{30}$$

which means that we have now pinned down the domestic price level in both period 1 and period 2. It generally does not depend on the exchange rate and Foreign fiscal or monetary policy. This is because our assumptions implicitly allowed us to characterize the Home policy authorities as the 'dominant' ones, imposing their tax and interest policy on the other country through the exchange rate, with little effect vice versa. We are therefore taking here the stark assumption of an 'absolute hegemon' in terms of the monetary and fiscal policy spillovers.

Now, we focus on the Foreign country as well as the international dimension. Our main guiding point will be the two UIP conditions (23) and (24). Both conditions have to hold. However, since the Home and Foreign price levels are fully informed by the fiscal budget constraint, and the Home interest rate is fixed, equation (23) will be cleared by $\mathcal{E}_t$ only. Consequently, equation (24) will be cleared by i_t^*, the Foreign bond interest rate, as it is the only free variable ensuring that the UIP on Foreign bonds held by the domestic household holds.

To aid our discussion, we first solve for the Foreign price level in period 1. By the Foreign Euler condition on Foreign bonds,

$$\frac{1}{P_t^*} = \beta(1+i_t^*)\mathbb{E}_t\left[\frac{1}{P_{t+1}^*}\right].$$

We evaluate this at $t = 1$. Since

$$P_2^* = \frac{(1+i_1^*)B_1^*}{\bar{T}^*} \tag{31}$$

by the terminal fiscal budget constraint, we find that

$$\frac{1}{P_1^*} = \beta \frac{\bar{T}^*}{B_1^*(1+i_0^*)} \tag{32}$$

fully determines the price level. In particular, as P_2^* is fully known conditional on i_1^*, we can pin down the price level in Foreign at $t = 1$ as:

$$P_1^* = \frac{1}{\beta}\frac{B_1^*(1+i_0^*)}{\bar{T}^*}, \tag{33}$$

where i_0^* was inherited from the initial steady-state.

What is left is to determine are $\mathcal{E}_1$, $\mathcal{E}_2$, and i_1^*, which will be jointly given by our two UIP conditions and the terminal exchange rate condition. Everything else has already been accounted for.

We solve here for $(1+i_1^*)$, which will be the central object of interest capturing spillovers from expansionary fiscal policy at Home.

Proposition 3. *The equilibrium gross interest rate of Foreign in period 1 is given by:*

$$(1+i_1^*) = \phi_F \frac{(1+\bar{i})}{\bar{i}} \frac{\bar{T}^*}{B_1^*} \frac{B_{H1}^*}{B_{F1}} \frac{1}{\left[\psi_1 \frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} + (1-\psi_1)\frac{1}{P_2^T}\right]}. \tag{34}$$

This solution for the equilibrium interest rate delivers a clear message: if expectations on fiscally-led policy mixes at Home grow in period 1, the equilibrium interest rate on Foreign debt issued in period 1 *increases*. Formally:

Corollary 4 (Co-movement between Foreign interest rates and Home fiscal policy perceptions)**.** *If the terminal price level at Home is higher under the fiscally-led policy mix ($P_2|_{FL} > P_2|_{ML}$), the co-movement between Foreign interest rates in period 1 and the probability of a fiscally-led policy mix at Home are positive; that is:*

$$\frac{\partial(1+i_1^*)}{\partial\psi_1} > 0. \tag{35}$$

This result is, at first, counter-intuitive. Even though Foreign households do not enjoy Home debt in any case, fiscal policy that makes Home bonds relatively less attractive is accompanied by an increase in the cost of borrowing in Foreign. A deterioration of Home fiscal conditions correlates with a deterioration in Foreign fiscal conditions, even if Foreign fiscal policy is unchanged. This result is squarely linked to the specification of the terminal exchange rate above, which induced the linkage in fiscal sustainability between Home and Foreign. Thus, an alternative fiscal-monetary policy specification limiting the factual hegemony of Home in the determination of the exchange rate would alter our results. Our intuitions are therefore related to Ding and Jiang (2024).

At constant taxes and no changes to the aggregate bond issuance in Foreign, this higher cost of borrowing in period 1 directly translates to a higher price level in period 2 (and, thus, also higher inflation from period 1 to period 2).

Corollary 5. *The terminal price level in Foreign co-moves positively with the probability of a fiscally-led policy mix at Home:*

$$\frac{\partial P_2^*}{\partial \psi_1} > 0. \tag{36}$$

Proof. Follows immediately from equations (35) and (31). □

A higher probability of a Fiscally-led regime in Home ($\psi_1 \uparrow$) increases the domestic price level generally by equation (30). Additionally, with our specification of international financial markets, this increase in the Home price level must yield an increase in the interest rates charged on Foreign bonds for the UIP to hold. This, in turn yields an *increase* in the *Foreign terminal price level* P_2^* through the government budget constraint of Foreign, given by $P_2^* = \frac{(1+i_1^*)B_1^*}{\bar{T}^*}$. Therefore, in our simple model, we identify a minimum viable mechanism by which a deterioration in Fiscal conditions at Home spills over to higher inflation from $t = 1$ to $t = 2$ in Foreign, manifesting through a deterioration of the fiscal balance of Foreign.

Figure 5 presents these comparative statics graphically. Both the expected price level at Home, here denoted by P_2^e, as well as the materialized price level in Foreign P_2^* (which is fully determined in period one through the Foreign government budget constraint) increase in the Foreign interest rate i_1^*, which in turn increases in the probability of being in the fiscally-led regime. The sensitivity of the Foreign price level to the Home fiscal shock is of a similar order of magnitude in level terms to the Home price level sensitivity, which is a consequence of the fiscal shock being comparatively 'large' from the perspective of Foreign due to their lower bond issuance. Thus, the *relative* change of prices at Foreign is larger than at Home. This resonates the mechanism of Barro and Bianchi (2024), by which a given fiscal deterioration has larger effects on the price level when the stock of sovereign debt is *smaller*, as a relatively larger change in the price level is needed to ensure that the real deterioration of the fiscal balance is sufficiently large.

As for the exchange rate, note that there are two plausibly opposing effects, in line with equation (25): there is a possible direct effect on the terminal exchange rate if the factual policy regime is actually FL through the deterioration of Home fiscal conditions, and an opposing effect from the spillovers on the fiscal budget of Foreign. In our calibration, the latter effect dominates, which is

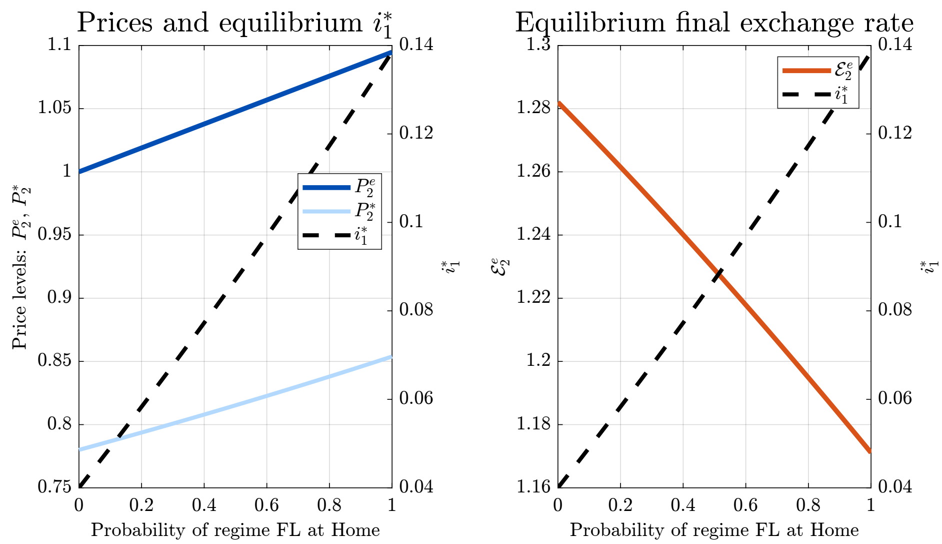


Figure 5: Comparative statics with respect to ψ_1, the probability of Home being in regime FL, for prices and equilibrium exchange rates. Calibration: $B_1 = 1$, $B_1^* = 0.15$, $B_{H1}^* = B_{F1} = 0.1$, $\bar{i} = 0.04$, $\beta = 0.9$, $\bar{T} = 1$, $\bar{T}^* = 0.2$, $\mu_1 = 0.05$, $P_2^T = 1$, $\phi_F = 0.03$.

evidenced by the relatively larger deterioration of the real value of debt at Foreign relative to Home. Here, therefore, a higher probability of the fiscally-led regime in Home leads to an *appreciation* of the Home currency in the terminal period. Effectively, this is the international extension of the mechanism presented in Barro and Bianchi (2024): because the relative adjustment of the Foreign fiscal position is larger in response to the deterioration of the Home fiscal condition, the bilateral exchange rate can appreciate from the perspective of Home.

5 Continuous Time Model

To further generalize the insights of our tractable two-period model presented in section 4, we now develop a continuous-time two country New Keynesian model of international fiscal policy spillovers. Relative to our two-period model, we now also introduce tradable goods, allowing for fiscal policy to matter furthermore through changes to aggregate demand. Both countries have unrestricted access in purchasing goods and assets from both countries. Each country has their own intermediate and final good producing firms. Nominal rigidities are present in both countries. Both monetary and fiscal policy is operating in both countries. We provide detailed derivations in Appendix D.

5.1 Domestic Household Problem

We derive the international analogue of the continuous time New Keynesian model as set out in Werning (2011).

Intratemporal Problem There is a home (H) and foreign (F) good. We define a home consumption bundle as

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

The domestic household solves the following intratemporal problem.

$$\max_{C_{Ht}, C_{Ft}} \left[(1-\gamma)^{1/\eta} C_{Ht}^{(\eta-1)/\eta} + \gamma^{1/\eta} C_{Ft}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} \tag{37}$$

$$\text{s.t.} \quad P_{Ht}C_{Ht} + P_{Ft}C_{Ft} = Z_t, \tag{38}$$

where P_{Ht} and P_{Ft} are the prices of the Home and Foreign goods in domestic currency and Z_t is total nominal expenditure. $1-\gamma$ measures the degree of home bias. The equations characterising the intratemporal problem of the domestic household is

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{39}$$

$$C_{Ht} = (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t, \tag{40}$$

$$C_{Ft} = \gamma\left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t, \tag{41}$$

$$P_t = \left[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{1/(1-\eta)}. \tag{42}$$

C_t is a CES aggregate of Home and Foreign consumption bundles C_{Ht} and C_{Ft}, with $\gamma \in (0,1)$ the expenditure weight on Foreign goods and $\eta > 0$ the elasticity of substitution. The second and third equations are the optimal demands implied by this aggregator: each bundle's quantity scales with total consumption and falls with its relative price with elasticity η. The last equation defines the dual CES price index P_t (CPI) consistent with the aggregator and these demands.
The real exchange rate S_t is defined as

$$S_t = E_t \frac{P_t^*}{P_t}, \tag{43}$$

where E_t is the nominal exchange rate. The terms of trade are given by the price of imports over the price of exports

$$\text{tot}_t = \frac{P_{Ft}}{P_{Ht}}. \tag{44}$$

Intertemporal Problem The domestic household chooses consumption C_t at price P_t, supplies labour N_t for the wage W_t, and chooses holding of domestic bonds B_t earning interest i_t and foreign bonds bonds B_t^* earning interest i_t^*. When holding foreign bonds, they need to take into account the exchange rate E_t. In order to introduce a wedge to break perfect risk-sharing, and therefore the Backus and Smith (1993) puzzle, we follow Jiang et al. (2023b, 2021b) by introducing a convenience yield for home bonds. The domestic household solves the following problem

$$\max_{\{C_t, N_t, B_t, B_t^*\}_{t\geq 0}} \int_0^\infty e^{-\rho t} \left\{ log[C_t] - \frac{N_t^{1+\phi}}{1+\phi} + \nu(B_t;\theta) \right\} dt \tag{45}$$

$$P_t C_t + \dot{B}_t + E_t \dot{B}_t^* = i_t B_t + E_t i_t^* B_t^* + W_t N_t + T_t \tag{46}$$

The necessary and sufficient conditions for domestic household optimality is given by

$$C_t N_t^\phi = \frac{W_t}{P_t} \tag{47}$$

$$\frac{\dot{C}_t}{C_t} = i_t - \pi_t + \Psi_t - \rho, \tag{48}$$

where $\Psi_t \equiv \nu_B(B_t;\theta) P_t C_t$ is the convenience yield wedge. The first equation is the intratemporal optimality (labor–leisure) condition: the marginal rate of substitution equals the real wage. The second line is the Euler equation for bonds. Because home bonds deliver utility services, the convenience yield Ψ_t raises the effective return on home bonds. The uncovered interest rate parity (UIP) condition is given by

$$i_t - i_t^* = \frac{\dot{E}_t}{E_t} - \Psi_t. \tag{49}$$

The interest rate differential is equal to the expected appreciation of the currency modified by the convenience yield term which acts as a wedge in the UIP condition relative to standard models. The transversality conditions on domestic and foreign bonds that the domestic household satisfies are

$$\lim_{T\to\infty} e^{-\rho T} \lambda_T B_T = 0 \tag{50}$$

$$\lim_{T\to\infty} e^{-\rho T} \lambda_T E_T B_T^* = 0. \tag{51}$$

5.2 Foreign Household

Intratemporal Problem The foreign household problem is symmetric to the domestic household problem. The CES demand function for the foreign household is

$$C_t^* = \left[(1-\gamma)^{1/\eta} C_{Ft}^{*(\eta-1)/\eta} + \gamma^{1/\eta} C_{Ht}^{*(\eta-1)/\eta} \right]^{\eta/(\eta-1)}. \tag{52}$$

The Foreign good gets weight $1-\gamma$; the imported U.S. good weight γ, matching the degree of home-bias symmetry. To solve the intratemporal problem, we assume that the intratemporal budget constraint is

$$P^*_{Ft}\, C^*_{Ft} + P^*_{Ht}\, C^*_{Ht} = Z^*_t,$$

where Z^*_t is the within period expenditure. The optimal consumption expenditure is symmetric to the domestic household

$$C^*_{Ft} = (1-\gamma)\left(\tfrac{P^*_{Ft}}{P^*_t}\right)^{-\eta} C^*_t, \tag{53}$$

$$C^*_{Ht} = \gamma\left(\tfrac{P^*_{Ht}}{P^*_t}\right)^{-\eta} C^*_t. \tag{54}$$

The foreign CPI is

$$P^*_t = \left[(1-\gamma) P^{*\,1-\eta}_{Ft} + \gamma\, P^{*\,1-\eta}_{Ht}\right]^{1/(1-\eta)}. \tag{55}$$

The foreign terms of trade is

$$\mathrm{tot}^*_t = \frac{P^*_{Ht}}{P^*_{Ft}}. \tag{56}$$

Intertemporal Problem The intertemporal problem for the foreign household is symmetric. They also have a convenience yield for the home country's bond. They maximise

$$\max_{\{C^*_t,\, N^*_t,\, B^*_t,\, B_t\}} \int_0^\infty e^{-\rho^* t}\left\{\log[C^*_t] - \frac{N^{*\,1+\phi^*}_t}{1+\phi^*} + \nu\left(\frac{B_t}{E_t};\theta\right)\right\} dt, \tag{57}$$

subject to the flow budget constraint

$$P^*_t C^*_t + \dot{B}^*_t + \frac{1}{E_t}\,\dot{B}_t = i^*_t B^*_t + \frac{1}{E_t}\, i_t B_t + W^*_t N^*_t + T^*_t. \tag{58}$$

The necessary and sufficient conditions for domestic household optimality are given by the Euler equation modified by the foreign's UIP wedge and a consumption-labour decision

$$\frac{\dot{C}^*_t}{C^*_t} = i^*_t - \pi^*_t + \Psi^*_t - \rho^*, \tag{59}$$

$$C^*_t\, N^{*\,\phi^*}_t = \frac{W^*_t}{P^*_t}, \tag{60}$$

where $\Psi^*_t \equiv \nu_B\left(\frac{B_t}{E_t};\theta\right) P^*_t C^*_t$ is the convenience yield wedge. The transversality conditions on domestic and foreign bonds are

$$\lim_{T\to\infty} e^{-\rho^* T}\lambda^*_T B^*_T = 0, \tag{61}$$

$$\lim_{T\to\infty} e^{-\rho^* T}\lambda^*_T \frac{B_T}{E_t} = 0. \tag{62}$$

5.3 Risk-Sharing

Following Itskhoki and Mukhin (2021), we can derive the risk-sharing condition between the home and foreign household:

$$\left[\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*}\right] - \dot{S}_t = \rho^* - \rho - \Psi_t^*. \tag{63}$$

This condition is obtained by differencing the home and foreign Euler equations and using UIP, and it links *relative* consumption growth to movements in the real exchange rate S_t. It says that home–foreign consumption growth, net of real appreciation, equals the cross-country discount-rate gap $(\rho^* - \rho)$ minus the foreign convenience-yield wedge Ψ_t^*. When $\rho = \rho^*$ and there are no convenience yields $\Psi_t^* = 0$, it collapses to the complete-markets benchmark of perfect risk-sharing $\dot{C}_t/C_t - \dot{C}_t^*/C_t^* = \dot{S}_t$. The convenience-yield wedge Ψ_t^* will resolve the Backus and Smith (1993) risk-sharing puzzle.

5.4 Domestic Final Good Producers

Final good producers are competitive and aggregate a continuum of intermediate inputs. We assume that there is a continuum of measure one of differentiated goods $Y_{Ht}(j)$ with a Dixit-Stiglitz production function:

$$Y_{Ht} = \left[\int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{64}$$

whereby $\epsilon > 1$ is the elasticity of substitution among differentiated goods. The retailers take prices of differentiated goods $p_{Ht}(j)$ as given. We can then define the retailer's problem as a cost minimisation problem:

$$\begin{aligned} \min_{Y_{Ht}(j)} \quad & \int_0^1 p_{Ht}(j) Y_{Ht}(j) dj \\ \text{s.t.} \quad & Y_{Ht} = \left[\int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}. \end{aligned} \tag{65}$$

With this problem, we can solve for the demand for each intermediate good j by the retailer. The demand for intermediate goods by the retailer is given by:

$$Y_{Ht}(j) = \left[\frac{p_{Ht}(j)}{P_{Ht}}\right]^{-\epsilon} Y_{Ht} \tag{66}$$

where we define the aggregate price index as $P_{Ht} \equiv \left[\int_0^1 p_{Ht}(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$.

5.5 Domestic Intermediate Good Producers

We first solve the firm's problem under flexible prices. This will be a useful benchmark for us when we analyse the *natural* economy.

5.5.1 Flexible Price Setting

The production function for wholesale producers is given by:

$$Y_{Ht}(j) = A_t n_t(j), \tag{67}$$

whereby A_t is a stationary productivity shock and n_t is labour hired. As wholesale producers have market power, they can choose both their goods' price level $p_{Ht}(j)$ and the amount of labour to hire $n_t(j)$ at wage rate W_t to produce output $Y_{Ht}(j)$. Wholesale producer optimally set relative prices according to a constant inverse markup of over nominal costs

$$p_{Ht}(j) = \frac{\epsilon}{\epsilon - 1}\frac{W_t}{A_t}. \tag{68}$$

5.5.2 Sticky Price Setting

We now look at the domestic intermediate firm j problem in the sticky price setting. The per-period profit is their revenue minus cost of production

$$\Pi_t(p_{Ht}(j)) = p_{Ht}(j)Y_{Ht}(j) - \frac{W_t}{A_t}Y_{Ht}(j). \tag{69}$$

To introduce nominal rigidities, firms face a Rotemberg quadratic price adjustment cost (Rotemberg, 1982)

$$\Theta_t(j) = \frac{\theta}{2}\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2 P_{Ht}Y_{Ht}. \tag{70}$$

where θ is the degree of price stickiness. The firm's optimal control problem is therefore to choose their pricing strategy in order to maximise present discounted profits subject to the adjustment cost

$$V_0(p_{H0}) = \max_{\{p_{Ht}(j)\}_{t\geq 0}} \int_0^\infty e^{-\int_0^t i(s)ds}\left\{\Pi_t(p_{Ht}) - \Theta_t(j)\right\}dt \tag{71}$$

Following Galí (2008), we restrict our attention to symmetric domestic firm pricing

$$p_{Ht}(j) = P_{Ht}. \tag{72}$$

The price setting of firms implies that the inflation rate $\pi_{Ht} \equiv \frac{\dot{P}_{Ht}}{P_{Ht}}$ is determined by

$$\left[i_t - \pi_{Ht} - \frac{\dot{Y}_{Ht}}{Y_{Ht}}\right]\pi_{Ht} = \frac{\epsilon - 1}{\theta}\left[\frac{\epsilon}{\epsilon - 1}\frac{W_t}{P_{Ht}}\frac{1}{A_t} - 1\right] + \dot{\pi}_{Ht}. \tag{73}$$

Equation (73) is the Rotemberg price–setting ordinary differential equation for producer inflation under quadratic adjustment costs. The factor in brackets on the left-hand side is the firm's effective discount rate. Inflation increases when real marginal cost $mc_t \equiv \frac{\epsilon}{\epsilon-1}\frac{W_t}{P_{H,t}A_t}$ exceeds its flexible-price target $mc_t^n = 1$, with the slope scaled by the stickiness parameter θ. As prices become perfectly flexible $\theta \to \infty$, the condition collapses to $mc_t = 1$. The foreign final and intermediate good producers face a symmetric problem.

5.6 Policy

5.6.1 Fiscal Policy

Let B_t denote the face value of outstanding nominal government debt, i_t the instantaneous nominal policy rate, P_t the CPI, and G_t real government purchases. With lump-sum taxes T_t (nominal), the consolidated nominal flow budget is

$$\dot{B}_t \;=\; i_t B_t \;+\; P_t G_t \;-\; T_t. \tag{74}$$

Define real debt $b_t \equiv B_t/P_t$ and inflation $\pi_t \equiv \dot{P}_t/P_t$. Let the (real) primary surplus be $s_t \equiv (T_t - P_t G_t)/P_t = T_t/P_t - G_t$. Dividing by the level P_t and using $\dot{b}_t = \frac{\dot{B}_t}{P_t} - \pi_t b_t$ gives the real debt law of motion

$$\dot{b}_t \;=\; (i_t - \pi_t)\, b_t \;-\; s_t. \tag{75}$$

This linear ODE has integrating factor $\mathcal{I}_t \equiv \exp\big(-\int_0^t (i_\tau - \pi_\tau)\, d\tau\big)$, so $\frac{d}{dt}(\mathcal{I}_t b_t) = -\mathcal{I}_t s_t$. Integrating from t to T and imposing the no-Ponzi/transversality condition $\lim_{T\to\infty} \mathcal{I}_T b_T = 0$ yields the government debt valuation equation where the real value of government debt is equal to the discounted stream of primary surpluses Cochrane (2023)

$$\frac{B_t}{P_t} \;=\; \int_t^{\infty} \exp\Big(-\int_t^u (i_s - \pi_s)\, ds\Big)\, s_u\, du, \tag{76}$$

5.6.2 Monetary Policy

Monetary policy follows a continuous-time Taylor rule:

$$i_t \;=\; r_t^n \;+\; \bar{\pi} \;+\; \phi_\pi\, (\pi_t - \bar{\pi}) \;+\; \phi_x\, x_t, \tag{77}$$

where $\bar{\pi}$ is the inflation target, $x_t \equiv \ln(Y_{H,t}) - \ln(Y^n_{H,t})$ is the output gap, and r^n_t is the open-economy natural real rate.

5.7 Equilibrium

We have the standard goods market clearing condition for the home and foreign good as

$$Y_{Ht} = C_{Ht} + C^*_{Ht}, \tag{78}$$

$$Y^*_{Ft} = C_{Ft} + C^*_{Ft}. \tag{79}$$

The aggregate resource constraint for the home and foreign good is given by:

$$Y_{Ht} = A_t N_t, \tag{80}$$

$$Y^*_{Ft} = A^*_t N^*_t. \tag{81}$$

We can derive the real interest rate in this setting.

Proposition 6 (Open-Economy Real Interest Rate)**.** *The domestic natural rate of interest can be decomposed into the standard Ramsey terms and three distinct open-economy adjustment channels:*

$$r^n_t = \underbrace{(\rho + g_t)}_{\text{Ramsey}} - \underbrace{\Psi_t}_{\text{Convenience Yield}} + \underbrace{\mathcal{T}_t}_{\text{Terms-of-Trade Channel}} + \underbrace{\mathcal{R}_t}_{\text{Risk-Sharing Channel}} \tag{82}$$

where $g_t \equiv \dot{A}_t / A_t$ is TFP growth and

$$\Psi_t \equiv \nu_B(B_t; \theta) P_t C_t$$

is the convenience yield on domestic bonds. The terms-of-trade channel, $\mathcal{T}_t$, reflects how changes in relative goods prices affect consumption growth, and is given by:

$$\mathcal{T}_t = \left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right) \frac{\gamma(1-\eta)\mathrm{tot}_t^{1-\eta}}{(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}} \left(\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t}\right) \tag{83}$$

The international risk-sharing channel, $\mathcal{R}_t$, captures how incomplete financial markets impact consumption growth through exchange rate dynamics and financial frictions, and is given by:

$$\mathcal{R}_t = \frac{\phi}{1+\phi} \frac{\gamma S_t^{-1} e^{-\Xi_t}}{(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}} \left(\frac{\dot{S}_t}{S_t} + \dot{\Xi}_t\right). \tag{84}$$

The natural real rate decomposes into the baseline Ramsey term plus three open-economy adjustments: a convenience yield that lowers the interest rate when home bonds deliver liquidity/safety, a terms-of-trade component capturing how movements in relative goods prices tilt intertemporal demand, and a risk-sharing component reflecting how real exchange-rate dynamics and financial wedges move relative consumption. In the closed-economy or unit-elasticity limits ($\gamma \to 0$ or $\eta \to 1$) and under perfect risk sharing ($\Xi_t = 0$), the open-economy terms vanish and r_t^n collapses to $\rho + g_t - \Psi_t$.

Proposition 7 (IS Curve)**.** *The domestic IS curve is*

$$\dot{x}_t = (i_t - \pi_t - \rho) - g_t + \Psi_t - \mathcal{T}_t - \mathcal{R}_t, \tag{85}$$

where $\dot{x}_t \equiv \frac{\dot{X}_t}{X_t}$ *where* x_t *is the log deviation of actual output from its natural level:*

$$x_t \equiv \ln(Y_{Ht}) - \ln(Y_{Ht}^n),$$

and recall that g_t *is TFP growth,* Ψ_t *is convenience yield,* $\mathcal{T}_t$ *is the terms of trade effect and* $\mathcal{R}_t$ *is the risk-sharing channel. The foreign IS curve is*

$$\dot{x}_t^* = (i_t^* - \pi_t^* - \rho^*) - g_t^* + \Psi_t^* - \mathcal{T}_t^* - \mathcal{R}_t^*. \tag{86}$$

The output gap grows when the ex-post real rate is *below* the natural real rate. TFP growth g_t and the open-economy wedges of the terms of trade $\mathcal{T}_t$ and risk-sharing $\mathcal{R}_t$ raise the natural rate (tighten demand), while a higher convenience yield Ψ_t lowers the interest rate. This is the central equation linking monetary policy (via the nominal interest rate i_t) to the real economy (via x_t).

Proposition 8 (Linearized NKPC)**.** *The home linearised New Keynesian Phillips curve is*

$$\dot{\hat{\pi}}_{Ht} = -\bar{\bar{\delta}}_H \, \hat{\pi}_{Ht} + \kappa \left[\tilde{c}_t + \phi x_t + \gamma \tau_t \right], \tag{87}$$

where $\bar{\bar{\delta}}_H \equiv \bar{i} - \overline{\dot{Y}_{Ht}/Y_{Ht}} - \bar{\pi}_H$ *is the steady-state home discount rate,* $\tilde{c}_t \equiv \hat{C}_t - \hat{C}_t^n$ *is the consumption gap,* x_t *is the output gap,* $\tau_t \equiv \ln \mathrm{tot}_t - \ln \mathrm{tot}_t^n$ *is the terms-of-trade gap.*
The foreign country New Keynesian Phillips curve is

$$\dot{\hat{\pi}}_{Ft}^* = -\bar{\bar{\delta}}_F \, \hat{\pi}_{Ft}^* + \kappa \left[\tilde{c}_t^* + \phi^* x_t^* + \gamma \tau_t^* \right], \tag{88}$$

The linearised NKPC then demand pressure into inflation dynamics: inflation decays at rate $\bar{\bar{\delta}}_H$ in the absence of cost pressures, and rises with the slope κ times the marginal–cost gap, here decomposed into the consumption gap $\tilde{c}_t$, the output gap x_t scaled by labor supply curvature ϕ, and the open-economy term $\gamma \tau_t$ capturing terms-of-trade movements.

Proposition 9 (Government Debt Valuation equation)**.** *Evaluated just before issuance at time t (so* B_{t-} *is predetermined), the CPI adjusts on impact to satisfy the government*

$$\frac{B_{t-}}{P_t} \;=\; \int_t^{\infty} \exp\Big(-\int_t^u \left[r_\tau^n + (\phi_\pi - 1)\hat{\pi}_\tau + \phi_x x_\tau \right] d\tau \Big)\, s_u \, du,$$

pinning down P_t from fiscal solvency when $\phi_\pi < 1$. For Foreign,

$$\frac{B_t^*}{P_t^*} = \int_t^\infty \exp\Big(-\int_t^u \big[r_\tau^{n*} + (\phi_\pi^* - 1)\hat{\pi}_\tau^* + \phi_x^* x_\tau^* \big] \, d\tau \Big) \, s_u^* \, du.$$

These are the governments' intertemporal budget constraints in present value under a *passive* Taylor rule. With nominal debt predetermined at issuance (B_{t-}), the *real* market value B_{t-}/P_t must equal the expected present value of future primary surpluses, discounted at the endogenous real kernel. When $\phi_\pi < 1$, the CPI P_t jumps on impact to enforce solvency. For a given stock of debt B_{t-}, a higher (lower) discounted surplus path implies a lower (higher) price level P_t today.

Formal equilibrium definition: A competitive equilibrium is then defined as a collection of processes

$$\{C_t, N_t, Y_{H,t}, P_t, P_{H,t}, \pi_t, i_t, B_t,\ C_t^*, N_t^*, Y_{F,t}^*, P_t^*, P_{F,t}^*, \pi_t^*, i_t^*, B_t^*,\ E_t, \mathrm{tot}_t, S_t\}$$

given exogenous processes $\{A_t, A_t^*,\ s_t, s_t^*,\ \Psi_t, \Psi_t^*\}$ and initial conditions (including $B_{t^-}, B_{t^-}^*$), such that households and firms are optimising and markets clear according to the equations presented throughout this section. A full definition is given in Appendix D.10.

5.8 Quantitative Exercise

We analyse the impact of a MIT shock to fiscal deficits in the home country and examine it's cross-country spillover effects. We first linearise the model around its symmetric deterministic steady state

$$\pi_t = \pi_t^* = \bar{\pi}, \quad x_t = x_t^* = 0, \quad \tau_t = 0, \quad \Psi_t = \Psi_t^* = 0, \quad g_t = g_t^* = \bar{g}.$$

Under a passive Taylor rule ($\phi_\pi < 1$), fiscal solvency pins the price level via the discount kernel $r^n + (\phi_\pi - 1)\hat{\pi} + \phi_x x$. For the shock process, we assume that surpluses follow Ornstein–Uhlenbeck processes: [11]

$$\dot{s}_t = -\varphi_s(s_t - \bar{s}) + \sigma_s \, \varepsilon_t^s, \qquad \dot{s}_t^* = -\varphi_s^*(s_t^* - \bar{s}^*) + \sigma_s^* \, \varepsilon_t^{s^*}. \tag{89}$$

Now we stack the endogenous variables in our system into z_t and shocks u_t. Then the ODE system characterizing the equilibrium conditions of the model follows

$$\dot{z}_t = A \, z_t + B \, u_t.$$

[11] This is the continuous time analogue of AR(1) processes.

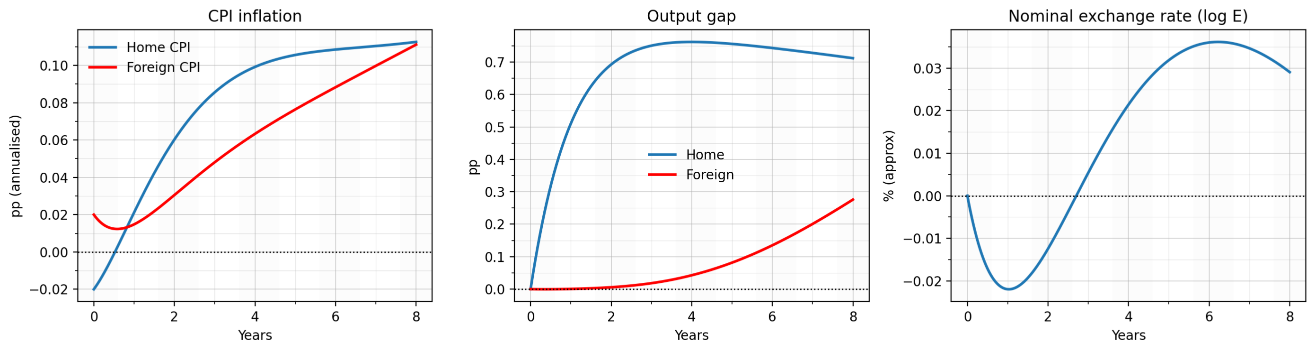


Figure 6: Impulse response function from negative home deficit shock

We can solve this system as:

$$z(t) = e^{At}z(0) + \int_0^t e^{A(t-s)}Bu_s\,ds.$$

We examine the symmetric steady state where $\bar{g}$ is constant and $\bar{\Psi} = 0$; $x = \tau = \hat{e} = 0$. We calibrate a passive Taylor rule with $\phi_\pi = 0.85$ and $\phi_x = 0$. We examine a MIT *home* negative deficit shock via s_t.

In Figure 6, we plot the impulse response function to CPI inflation and the output gap in both countries alongside the nominal exchange rate. We see that upon a negative deficit shock, the nominal exchange rate initially appreciates so as to depreciate going forward. We see that CPI inflation in the home country goes up, in line with mechanisms implied by models akin to the fiscal theory of the price level (FTPL). A crucial novelty of out model is that CPI inflation in the foreign country also increases, generating a cross-country fiscal contagion mechanism. Through the Phillips curve, output in both countries increases. Due to the Taylor rule, interest rates in both countries also rise.

6 Conclusion

This paper shows that fiscal sustainability in an open economy is jointly disciplined by expected primary surpluses and by the currency-denominated discount factor global investors apply to those surpluses, which, by extension, link fiscal surpluses to exchange rates. Using market-value debt data for the United Kingdom and the United States since 1975, we apportion roughly 50 percent of unexpected year-to-year changes in the U.K. debt ratio to revisions in surplus expectations and the remaining 50 percent to discount-rate news, with the latter being split across global yields, real-exchange-rate revisions, and time-varying UIP premia. The revisions in surplus expectations, in turn, are highly correlated to corresponding revisions in observed exchange rates. A tractable two-country model rationalises these findings: expected fiscal shocks in a financial hegemon propagate internationally through exchange-rate adjustments that feed directly into the real discount rates faced by other sovereigns, generating what we term "fiscal contagion." In the model, the core mechanism is that deficits in the financial hegemon cause a spillover to the cost of borrowing for the government of the other economy, yielding an expected future appreciation of the hegemon's currency that equates cross-country financial flows over a finite horizon. By this mechanism, the current market value of bonds in the non-hegemon economy decreases, leading additionally to an international transmission of fiscal inflation.

These results carry three policy implications. First, debt-management strategies that focus narrowly on flow deficits risk misdiagnosing solvency when large portions of debt dynamics stem from repricing of existing liabilities. Second, credible policies that stabilise the currency-denominated discount factor—through inflation control, anchoring of FX risk premia, or issuance in a diversified mix of currencies—can materially ease the fiscal burden even when deficits are large. Third, prudent coordination of fiscal and exchange-rate policies can mitigate cross-border spillovers; ignoring them risks pro-cyclical tightening just when domestic stabilisation is most needed.

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A Omitted Proofs of Simple Model

Domestic Household. The household problem can be solved very simply with a standard Lagrangian. For the domestic household,

$$\mathcal{L} = \mathbb{E}_t \sum_{t=1}^{2} \beta^t \left[u(c_t) - L_t \right.$$
$$\left. + \lambda_t \left(B_{Ht-1}(1+\bar{i}) + b_{Ft-1}(1+i^*_{t-1})\mathcal{E}_t + P_t L_t - B_{Ht} - B_{Ft}\mathcal{E}_t - P_t(c_t + T_t) \right) \right],$$

yielding the standard first-order conditions:

$$\begin{aligned}
\left[\frac{\partial \mathcal{L}}{\partial c_t}\right] &: \quad u'(c_t) - P_t \lambda_t = 0, \\
\left[\frac{\partial \mathcal{L}}{\partial L_t}\right] &: \quad -1 + P_t \lambda_t = 0, \\
\left[\frac{\partial \mathcal{L}}{\partial B_{Ht}}\right] &: \quad -\lambda_t + \beta \mathbb{E}_t \lambda_{t+1}(1+\bar{i}) = 0, \\
\left[\frac{\partial \mathcal{L}}{\partial B_{Ft}}\right] &: \quad -\lambda_t \mathcal{E}_t + \beta \mathbb{E}_t \lambda_{t+1} \mathcal{E}_{t+1}(1+i^*_t) = 0.
\end{aligned}$$

Foreign Household. For the Foreign household, the Lagrangian is given by:

$$\mathcal{L}^* = \mathbb{E}_t \sum_{t=1}^{2} \beta^t \left[u(c^*_t) - L^*_t - \phi_F \frac{B^*_{Ht}}{P^*_t} \right.$$
$$\left. + \lambda^*_t \left(\frac{B^*_{Ht-1}}{\mathcal{E}_t}(1+\bar{i}) + B^*_{Ft-1}(1+i^*_{t-1}) + P^*_t L^*_t - B^*_{Ht}\mathcal{E}_t - B^*_{Ft} - P^*_t(c^*_t + T^*_t) \right) \right],$$

yielding the first-order conditions:

$$\begin{aligned}
\left[\frac{\partial \mathcal{L}^*}{\partial c^*_t}\right] &: \quad u'(c^*_t) - P^*_t \lambda^*_t = 0, \\
\left[\frac{\partial \mathcal{L}^*}{\partial L^*_t}\right] &: \quad -1 + P^*_t \lambda^*_t = 0, \\
\left[\frac{\partial \mathcal{L}^*}{\partial B^*_{Ht}}\right] &: \quad -\phi_F \frac{1}{P^*_t} - \lambda^*_t \frac{1}{\mathcal{E}_t} + \beta \mathbb{E}_t \frac{\lambda^*_{t+1}}{\mathcal{E}_{t+1}}(1+\bar{i}) = 0, \\
\left[\frac{\partial \mathcal{L}^*}{\partial B^*_{Ft}}\right] &: \quad -\lambda^*_t + \beta \mathbb{E}_t \lambda^*_{t+1}(1+i^*_t) = 0.
\end{aligned}$$

Proof of Proposition (3)

Proof. We take equation (24) and rewrite it to:

$$(1+i_1^*) = \frac{\mathcal{E}_1}{\beta \mathbb{E}_1 \left[\frac{P_1}{P_2} \mathcal{E}_2 \right]}.$$

We now insert here equation (25) after rewriting it to $\frac{\mathcal{E}_2}{P_2} = \frac{1}{P_2^*} \frac{(1+\bar{i})}{(1+i_1^*)} \frac{B_{H1}^*}{B_{F1}}$, yielding:

$$\mathcal{E}_1 = \beta \frac{P_1}{P_2^*} (1+\bar{i}) \frac{B_{H1}^*}{B_{F1}}.$$

Use now equation (23) to replace $\mathcal{E}_1$ to obtain:

$$\frac{1}{\beta(1+\bar{i}) \mathbb{E}_1 \left[\frac{P_1^*}{P_2^*} \frac{1}{\mathcal{E}_2} \right] - \phi_F} = \beta \frac{P_1}{P_2^*} (1+\bar{i}) \frac{B_{H1}^*}{B_{F1}}.$$

Replacing $\mathcal{E}_2$ again:

$$\frac{1}{\beta(1+\bar{i}) \mathbb{E}_1 \left[\frac{P_1^*}{P_2} (1+i_1^*) \frac{B_{F1}}{B_{H1}^*} \right] - \phi_F} = \beta \frac{P_1}{P_2^*} (1+\bar{i}) \frac{B_{H1}^*}{B_{F1}}.$$

We now replace the price levels (both materialized and expected) using equations (26), (29), (30), (31), and (32) to obtain:

$$\frac{1}{\beta \frac{1}{\beta} \frac{B_1^*}{\bar{T}^*} \left[\psi_1 \frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} + (1-\psi_1) \frac{1}{P_2^T} \right] (1+i_1^*) \frac{B_{F1}}{B_{H1}^*} - \phi_F} = \frac{1}{\left[\psi_1 \frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} + (1-\psi_1) \frac{1}{P_2^T} \right]} \frac{\bar{T}}{(1+i_1^*) B_1^*} (1+\bar{i}) \frac{B_{H1}^*}{B_{F1}}. \tag{1}$$

Note here that for $\phi_F = 0$, $(1+i_1^*)$ would drop out everywhere and equilibrium would require $\bar{i} = 0$, such that we are nesting the expected results of models without such a reduced-form UIP wedge. Rewriting this expression, we obtain a closed-form solution for the foreign interest rate in equilibrium that applies for bonds issued in period 1:

$$(1+i_1^*) = \phi_F \frac{(1+\bar{i})}{\bar{i}} \frac{\bar{T}^*}{B_1^*} \frac{B_{H1}^*}{B_{F1}} \frac{1}{\left[\psi_1 \frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} + (1-\psi_1) \frac{1}{P_2^T} \right]}. \tag{2}$$

□

Proof of Corollary 4

Proof. Simply taking the first derivative of equation (34) yields:

$$\frac{\partial(1+i_1^*)}{\partial \psi_1} = -\phi_F \frac{(1+\bar{i})}{\bar{i}} \frac{\bar{T}^*}{B_1^*} \frac{B_{H1}^*}{B_{F1}} \left(\frac{1}{\left[\psi_1 \frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} + (1-\psi_1) \frac{1}{P_2^T} \right]} \right)^2 \left(\frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} - \frac{1}{P_2^T} \right).$$

Now, the assumption that $P_2|_{FL} > P_2|_{ML}$ means that $\frac{B_1(1+\bar{i})}{\bar{T}-\mu_1} > P_2^T$, which in turn implies $\frac{\bar{T}-\mu_1}{B_1(1+\bar{i})} < \frac{1}{P_2^T}$. Therefore, the last bracket is negative, negating the initial minus sign. All other elements are positive, hence this partial derivative is indeed always larger than zero. $\square$

B Appendix: Empirical Framework Microfoundation

Environment We consider a world composed of two ex-ante identical economies, labelled Home (H) and Foreign (F). A single traded good serves as the numéraire, so purchasing–power parity holds by construction. Each country is endowed with a constant flow of output, $Y_t = \bar{Y}$, which it consumes in equilibrium because there are no production or storage technologies. The representative household in either country maximises discounted expected utility where the coefficient of relative risk aversion $\gamma > 0$ and discount factor $\beta \in (0, 1)$ are common across countries.

Governments in both economies issue one-period nominal bonds denominated in their respective currencies and finance exogenous primary surpluses (or deficits) through lump-sum taxes; there are no distortionary instruments. The monetary authority sets the nominal interest rate at which the bonds are issued. Capital markets are frictionless and complete for the two assets under consideration: households in either country can freely trade both Home- and Foreign-currency bonds. Apart from these bonds, no other financial assets or contingent claims are available, and there are no costs of international portfolio adjustment.

With only one good, *purchasing-power parity (PPP)* holds every period

$$\mathcal{E}_t = \frac{P_t}{P_t^*}, \qquad \pi_t - \pi_t^* = \Delta e_t,$$

where $\mathcal{E}_t$ is the nominal exchange rate (H-currency per F-currency), P_t and P_t^* are price levels, $\pi_t = \log P_t - \log P_{t-1}$ and $e_t = \log \mathcal{E}_t$.

B.1 Household Problem

Let B_t^H denote end-of-period holdings of H-currency bonds bought at t (i.e. payoff 1 H-currency at $t+1$). Let B_t^F denote end-of-period holdings of F-currency bonds bought at t (i.e. payoff 1 F-currency at $t+1$). Finally, let $Q_t^H = 1/(1+i_t)$, $\quad Q_t^F = 1/(1+i_t^*)$ be bond prices.

The *nominal* flow budget constraint of a Home household is

$$P_t C_t + Q_t^H B_t^H + Q_t^F \mathcal{E}_t B_t^F = P_t Y_t + B_{t-1}^H + \mathcal{E}_t B_{t-1}^F + P_t T_t, \tag{3}$$

where T_t is the lump-sum transfer (tax if negative). The Foreign household has the starred analogue. Each household chooses $\{C_t, B_t^H, B_t^F\}_{t\geq 0}$ to maximise

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

subject to equation (3) and no-Ponzi conditions. Let λ_t be the nominal marginal utility. The first-order conditions are

$$\lambda_t Q_t^H = \beta \mathbb{E}_t[\lambda_{t+1}], \qquad \lambda_t Q_t^F \mathcal{E}_t = \beta \mathbb{E}_t[\lambda_{t+1} S_{t+1}]. \tag{4}$$

Additionally, the transversality condition for holdings of home and foreign bonds are

$$\lim_{T\to\infty} \mathbb{E}_t\left[\beta^T \lambda_{t+T} B_{t+T}^H\right] = 0, \qquad \lim_{T\to\infty} \mathbb{E}_t\left[\beta^T \lambda_{t+T} S_{t+T} B_{t+T}^F\right] = 0.$$

Because $\lambda_t = U'(C_t)/P_t = C_t^{-\gamma}/P_t$, equation (4) yields the Euler equation for the home bond and the foreign bond

$$1 = \beta \mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\gamma} \frac{(1+i_t)}{\Pi_{t+1}}\right], \tag{5}$$

$$1 = \beta \mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\gamma} \frac{(1+i_t^*)}{\Pi_{t+1}} \frac{S_{t+1}}{S_t}\right]. \tag{6}$$

Log-linearising around a symmetric steady state ($C_t = \bar{C}$) and dividing the two FOCs deliver the *uncovered-interest-parity (UIP)* condition

$$i_t - i_t^* = \mathbb{E}_t[\Delta e_{t+1}], \tag{7}$$

where $\Delta e_{t+1} = e_{t+1} - e_t = \log S_{t+1} - \log \mathcal{E}_t$.

Symmetric risk-sharing The economy features a single tradable good , frictionless capital markets with freely traded home and foreign bonds, and representative households in each country with identical preferences. This structure ensures that the real exchange rate is pinned to the ratio of the countries' marginal utilities of consumption. Given the single-good assumption, the real exchange rate is constant and equal to one, which in equilibrium requires the marginal utility of consumption to be equalised across both countries at all times.

With one good and identical preferences, equilibrium implies $C_t = C_t^*$, so the two Euler equations collapse to one and consumption is fixed at the endowment level. Because households share an identical utility function, the equalisation of their marginal utilities directly implies that their consumption levels must also be equal, $C_t = C_t^*$. This condition represents a state of perfect risk-sharing, where households use international asset trade to completely insure themselves against any country-specific shocks, resulting in identical consumption paths. In the context of this specific endowment economy, where output is constant and goods markets must clear , this further implies that consumption in each country is fixed at the constant endowment level. This removes all real-side volatility, thereby allowing the analysis to focus exclusively on how fiscal policy and government financing decisions drive nominal asset prices and the exchange rate.

B.2 Fiscal and Monetary Authority

The starting point is the nominal flow identity for the home government. Debt this period is equal to the principal plus interest on last period debt plus government surpluses

$$B_t = (1 + i_{t-1})B_{t-1} + P_t G_t - P_t T_t.$$

To derive the real budget constraint, we first divide the nominal budget constraint by the price level P_t:

$$\frac{B_t}{P_t} = (1 + i_{t-1})\frac{B_{t-1}}{P_t} + \frac{P_t(G_t - T_t)}{P_t}$$

Using the definitions for real debt, $b_t \equiv \frac{B_t}{P_t}$, and the real primary surplus, $s_t \equiv T_t - G_t$, the equation becomes:

$$b_t = (1 + i_{t-1})\frac{B_{t-1}}{P_t} - s_t$$

To express this in terms of lagged real debt, we rewrite $\frac{B_{t-1}}{P_t}$ as $\frac{B_{t-1}}{P_{t-1}} \cdot \frac{P_{t-1}}{P_t}$, which is equal to b_{t-1}/Π_t, where $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate.

$$b_t = (1 + i_{t-1})\frac{b_{t-1}}{\Pi_t} - s_t$$

The term $\frac{1+i_{t-1}}{\Pi_t}$ is the gross real interest rate, $1 + r_{t-1}$. This gives the exact law of motion for real government debt:

$$b_t = (1 + r_{t-1})b_{t-1} - s_t. \tag{8}$$

By symmetry, the law of motion for government debt in the foreign country is

$$b^*_t = (1 + r^*_{t-1})b^*_{t-1} - s^*_t, \tag{9}$$

where $1 + r^*_{t-1} \equiv \frac{1+i^*_{t-1}}{\Pi^*_t}$ is the gross real return rate.

The present-value government budget constraint is derived from the government's fundamental solvency condition. The principle is that the real value of outstanding debt at the start of period t, B_{t-1}/P_t, must equal the expected present value of all future primary surpluses. Future real surpluses (s_{t+j}) are discounted using the household's stochastic discount factor (SDF). With constant consumption, the nominal SDF from $t + j$ to t simplifies to $\mathcal{M}_{t,t+j} = \beta^j \frac{P_t}{P_{t+j}}$. Following asset pricing logic, the nominal value of debt, B_{t-1}, must equal the sum of expected future nominal surpluses, $P_{t+j}s_{t+j}$, discounted by the nominal SDF:

$$B_{t-1} = \mathbb{E}_t \sum_{j=0}^{\infty} \tilde{M}_{t,t+j}(P_{t+j}s_{t+j}).$$

Through further algebra and dividing by the price level, P_t we arrive at the government debt valuation equation [12]:

$$\frac{B_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{10}$$

By symmetry, the foreign country's government debt valuation equation follows

$$\frac{B^*_{t-1}}{P^*_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s^*_{t+j}. \tag{11}$$

In an *active-fiscal / passive-monetary* regime, equation (10) and (11) determines P_t, P^*_t: lower expected surpluses must be financed by higher price levels. The monetary authority follows a Taylor rule for both countries

$$1 + i_t \;=\; (1+\bar{i}) \left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi} e^{\varepsilon^m_t}, \tag{12}$$

$$1 + i^*_t \;=\; (1+\bar{i}^*) \left(\frac{\Pi^*_t}{\bar{\Pi}}\right)^{\phi^*_\pi} e^{\varepsilon^{m*}_t}, \tag{13}$$

where $\bar{\Pi}$ is the common steady-state inflation target, ϕ_π, ϕ^*_π are policy feedback coefficients and ε^m_t, ε^{m*}_t policy-shock innovations. Log-linearising around the deterministic steady state gives

$$\hat{i}_t = \phi_\pi \, \hat{\pi}_t \; + \; \varepsilon^m_t, \tag{14}$$

$$\hat{i}^*_t = \phi^*_\pi \, \hat{\pi}^*_t \; + \; \varepsilon^{m*}_t. \tag{15}$$

Market-Clearing Conditions The goods market clearing condition in both countries is

$$C_t = Y_t = \bar{Y}, \quad C^*_t = Y^*_t = \bar{Y}. \tag{16}$$

The (home) bond market clearing condition is

$$B_t = B^H_t + B^{*H}_t, \tag{17}$$

whilst the foreign bond market clearing condition is

$$B^*_t = B^F_t + B^{*F}_t. \tag{18}$$

[12] Alternatively, we could have iterated the household's budget constraint forward, impose market clearing and invoke the household's transversality condition to arrive at the government debt valuation equation.

Purchasing-power parity is

$$\mathcal{E}_t = \frac{P_t}{P_t^*}. \tag{19}$$

The nominal exchange rate is

$$\pi_t - \pi_t^* = \Delta e_t. \tag{20}$$

B.3 Equilibrium

For any variable X_t define the percentage deviation $\hat{x}_t \equiv \log\left(\frac{X_t}{\bar{X}}\right)$. At the deterministic steady state

$$\bar{C} = \bar{Y}, \quad \bar{\Pi} = \bar{\Pi}^* = 1, \quad \bar{i} = \bar{i}^* = r, \quad \bar{S} = 1, \quad \bar{s} = \bar{s}^* = 0,$$

while $(\bar{b}, \bar{b}^*)$ are free long-run debt ratios. Then equilibirum is the set of endogenous variables

$$\{\hat{\pi}_t,\ \hat{\pi}_t^*,\ \hat{i}_t,\ \hat{i}_t^*,\ \Delta\hat{e}_t,\ \hat{b}_t,\ \hat{b}_t^*\}.$$

The equilibrium conditions are:

1. Home Fisher Equation: $\hat{i}_t = \mathbb{E}_t\hat{\pi}_{t+1}$
2. Foreign Fisher Equation: $\hat{i}_t^* = \mathbb{E}_t\hat{\pi}_{t+1}^*$
3. UIP condition:$\hat{i}_t - \hat{i}_t^* = \mathbb{E}_t\Delta\hat{e}_{t+1} + \varphi_t$
4. PPP condition: $\hat{\pi}_t - \hat{\pi}_t^* = \Delta\hat{e}_t$
5. Home debt equation: $\rho\,\hat{b}_{t+1} = \hat{b}_t + \hat{i}_t - \hat{\pi}_{t+1} - \hat{s}_{t+1}$
6. Foreign debt equation: $\rho\,\hat{b}_{t+1}^* = \hat{b}_t^* + \hat{i}_t^* - \hat{\pi}_{t+1}^* - \hat{s}_{t+1}^*$
7. Home Taylor rule: $\hat{i}_t = \phi_\pi\,\hat{\pi}_t + \varepsilon_t^m$
8. Foreign Taylor rule: $\hat{i}_t^* = \phi_\pi^*\,\hat{\pi}_t^* + \varepsilon_t^{m*}$

We have the 5 shocks $(\hat{s}_{t+1}, \hat{s}_{t+1}^*, \varepsilon_t^m, \varepsilon_t^{m*}, \varphi_t)$. We have 7 endogenous variables and 8 equations. With "passive" Taylor coefficients $(0 < \phi_\pi, \phi_\pi^* < 1)$, Blanchard–Kahn conditions yield a unique stable solution.

B.4 Derivation of the linearised government debt value equation

Proof. We break this proof up into multiple steps.

Recall that the nonlinear nominal budget constraint is given by

$$M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)} \;=\; P_{t+1} sp_{t+1} + M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}.$$

Here, sp_{t+1} is the *real primary surplus* (i.e. does *not* include interest payments). At the beginning of period $t+1$, money M_t and bonds $B_t^{(t+1+j)}$ are outstanding.

Recall by definition that the nominal return on the portfolio of government debt (holding period returns) is

$$R_{t+1}^n \equiv \frac{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}}{M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)}}.$$

We can re-arrange this

$$R_{t+1}^n \left(M_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} B_t^{(t+j)} \right) = M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}.$$

Re-write the index on the left-hand side

$$R_{t+1}^n \left(M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)} \right) = M_t + \sum_{j=0}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)}$$

Use the definition of the nominal end-of-period market value of debt

$$V_t \equiv M_t + \sum_{j=0}^{\infty} Q_t^{(t+1+j)} B_t^{(t+1+j)},$$

to get

$$\boxed{M_t + \sum_{j=1}^{\infty} Q_{t+1}^{(t+j)} B_t^{(t+j)} = R_{t+1}^n V_t.}$$

Plug into equation (5) to get

$$R_{t+1}^n V_t = P_{t+1} sp_{t+1} + M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}.$$

For the right-hand side, note that we can iterate the nominal end-of-period market value of debt one period forward to get

$$V_{t+1} \equiv M_{t+1} + \sum_{j=0}^{\infty} Q_{t+1}^{(t+2+j)} B_{t+1}^{(t+2+j)}.$$

We can rewrite the index

$$V_{t+1} = M_{t+1} + \sum_{j=1}^{\infty} Q_{t+1}^{(t+1+j)} B_{t+1}^{(t+1+j)}.$$

Therefore can plug this in to conclude

$$\boxed{V_{t+1} = R^n_{t+1} V_t - P_{t+1} sp_{t+1}.} \tag{21}$$

The nominal value of government debt is increased by the nominal rate of return, and decreased by primary surpluses.

Step 2. Introduce stationarity. First, divide through by P_{t+1}

$$\begin{aligned}
\leftrightarrow \quad & V_{t+1} = V_t R^n_{t+1} - P_{t+1} sp_{t+1} \\
\leftrightarrow \quad & \frac{V_{t+1}}{P_{t+1}} = V_t \frac{R^n_{t+1}}{P_{t+1}} - sp_{t+1} \\
\leftrightarrow \quad & \frac{V_{t+1}}{P_{t+1}} = V_t \frac{R^n_{t+1}}{P_t} \frac{P_t}{P_{t+1}} - sp_{t+1} \\
\Rightarrow \quad & \frac{V_t}{P_t} R^n_{t+1} \frac{P_t}{P_{t+1}} = \frac{V_{t+1}}{P_{t+1}} + sp_{t+1}.
\end{aligned}$$

Next, divide by output at time $t+1$

$$\begin{aligned}
\leftrightarrow \quad & \frac{V_t}{P_t} R^n_{t+1} \frac{P_t}{P_{t+1}} \frac{1}{Y_{t+1}} = \frac{V_{t+1}}{Y_{t+1} P_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}} \\
\leftrightarrow \quad & \frac{V_t}{P_t} R^n_{t+1} \frac{P_t}{P_{t+1}} \frac{Y_t}{Y_{t+1}} \frac{1}{Y_t} = \frac{V_{t+1}}{Y_{t+1} P_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}} \\
\leftrightarrow \quad & \frac{V_t}{P_t Y_t} R^n_{t+1} \frac{P_t}{P_{t+1}} \frac{Y_t}{Y_{t+1}} = \frac{V_{t+1}}{Y_{t+1} P_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}} \\
\leftrightarrow \quad & \frac{V_t}{P_t Y_t} R^n_{t+1} \frac{1}{\Pi_{t+1}} \frac{1}{G_{t+1}} = \frac{V_{t+1}}{Y_{t+1} P_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}} \\
\Rightarrow \quad & \boxed{\frac{V_t}{P_t Y_t} \frac{R^n_{t+1}}{G_{t+1}} \frac{1}{\Pi_{t+1}} = \frac{V_{t+1}}{P_{t+1} Y_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}},}
\end{aligned}$$

where we defined GDP growth $G_{t+1} \equiv \frac{Y_{t+1}}{Y_t}$ and inflation rate $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$.

Step 3. Campbell-Shiller linearisation. Take logs of both sides to get

$$\begin{aligned}
& \log\left(\frac{V_t}{P_t Y_t} \frac{R^n_{t+1}}{G_{t+1}} \frac{1}{\Pi_{t+1}}\right) = \log\left(\frac{V_{t+1}}{P_{t+1} Y_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}}\right) \\
\Rightarrow \quad & \log\left(\frac{V_t}{P_t Y_t}\right) + \log\left(R^n_{t+1}\right) - \log\left(G_{t+1}\right) - \log\left(\Pi_{t+1}\right) = \log\left(\frac{V_{t+1}}{P_{t+1} Y_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}}\right).
\end{aligned}$$

We define

- $v_t \equiv \log\left(\frac{V_t}{P_t Y_t}\right)$,
- $r^n_{t+1} \equiv \log\left(R^n_{t+1}\right)$,

- $\pi_{t+1} \equiv \log(\Pi_{t+1})$,
- $g_{t+1} \equiv \log(G_{t+1})$.

So we get

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log\left(\frac{V_{t+1}}{P_{t+1}Y_{t+1}} + \frac{sp_{t+1}}{Y_{t+1}}\right).$$

Finally, define the new variable $sy_{t+1} \equiv \frac{sp_{t+1}}{Y_{t+1}}$.

$$\boxed{v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log\left(sy_{t+1} + \frac{V_{t+1}}{P_{t+1}Y_{t+1}}\right).}$$

We linearise this equation in the *level* of the surplus, not its log since the surplus may be negative. We Taylor expand the right-hand side of the above expression around the steady-state values (sy, v):

$$\begin{aligned}
\log\left\{sy_{t+1} + \frac{V_{t+1}}{P_{t+1}}\right\} &= \log\left\{sy_{t+1} + \exp\left(\log\left\{\frac{V_{t+1}}{P_{t+1}}\right\}\right)\right\} \\
&= \log\left\{sy_{t+1} + e^{v_{t+1}}\right\} \\
&\approx \log\left\{sy + e^v\right\} + \frac{1}{sy+e^v}\left(sy_{t+1} - sy\right) + \frac{e^v}{sy+e^v}\left(v_{t+1} - v\right) \\
&= \left[\log\left\{sy + e^v\right\} - \frac{1}{sy+e^v}sy - \frac{e^v}{sy+e^v}v\right] + \frac{1}{sy+e^v}sy_{t+1} + \frac{e^v}{sy+e^v}v_{t+1} \\
&= \left[\log\left\{sy + e^v\right\} - \frac{e^v}{sy+e^v}\left[v + sye^{-v}\right]\right] + \frac{1}{sy+e^v}sy_{t+1} + \frac{e^v}{sy+e^v}v_{t+1}.
\end{aligned}$$

Therefore, the linearisation is

$$\boxed{\log\left\{sy_{t+1} + \frac{V_{t+1}}{P_{t+1}}\right\} = \left[\log\left\{sy + e^v\right\} - \frac{e^v}{sy+e^v}\left[v + sye^{-v}\right]\right] + \frac{1}{sy+e^v}sy_{t+1} + \frac{e^v}{sy+e^v}v_{t+1}.}$$

We first compute the steady–state relationships. First, do the following manipulation

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log\Big\{sy_{t+1} + \exp\big(\log\{\tfrac{V_{t+1}}{P_{t+1}Y_{t+1}}\}\big)\Big\}.$$

Now evaluate at steady state

$$\begin{aligned}
&\leftrightarrow \quad v + r^n - \pi - g = \log\{sy + e^v\} \\
&\leftrightarrow \quad r^n - \pi - g = \log\{sy + e^v\} - v \\
&\leftrightarrow \quad r^n - \pi - g = \log\{sy + e^v\} - \log\{e^v\} \\
&\leftrightarrow \quad r^n - \pi - g = \log\left(\tfrac{sy+e^v}{e^v}\right) \\
&\leftrightarrow \quad \tfrac{sy+e^v}{e^v} = e^{r^n - \pi - g} \\
&\Rightarrow \quad \boxed{\tfrac{e^v}{sy+e^v} = e^{\pi + g - r^n}.}
\end{aligned}$$

Now we substitute steady-state values into the Taylor expansion. Using

$$\log\{sy + e^v\} = v + r^n - \pi - g, \qquad \frac{1}{sy + e^v} = \frac{e^{\pi+g-r^n}}{e^v}, \qquad \frac{e^v}{sy+e^v} = e^{\pi+g-r^n},$$

we can plug this into the right-hand side of the linearisation

$$\log\left\{sy_{t+1} + \frac{V_{t+1}}{P_{t+1}}\right\} = \left[\log\left\{sy + e^v\right\} - \frac{e^v}{sy+e^v}\left[v + sye^{-v}\right]\right] + \frac{1}{sy+e^v}sy_{t+1} + \frac{e^v}{sy+e^v}v_{t+1}.$$

to get the right-hand side to be

$$\left[(v + r^n - \pi - g) - e^{\pi+g-r^n}\left(v + sye^{-v}\right)\right] + \frac{e^{\pi+g-r^n}}{e^v}\, sy_{t+1} + e^{\pi+g-r^n}\, v_{t+1}.$$

We now do some algebra on this right-hand side

$$\begin{aligned}
&\leftrightarrow \quad \left[(v + r^n - \pi - g) \; - \; e^{\pi+g-r^n}\left(v + sye^{-v}\right)\right] \; + \; \frac{e^{\pi+g-r^n}}{e^v}\, sy_{t+1} \; + \; e^{\pi+g-r^n}\, v_{t+1} \\
&\leftrightarrow \quad \left[(v + r^n - \pi - g) - e^{\pi+g-r^n}\left(v + sye^{-v} + 1 - 1\right)\right] + \frac{e^{\pi+g-r^n}}{e^v}\, sy_{t+1} + e^{\pi+g-r^n}\, v_{t+1} \\
&\leftrightarrow \quad \left[(v + r^n - \pi - g) - \underbrace{e^{\pi+g-r^n}}_{=\frac{e^v}{sy+e^v}}\left(v + \tfrac{sy+e^v}{e^v} - 1\right)\right] + \frac{e^{\pi+g-r^n}}{e^v}\, sy_{t+1} + e^{\pi+g-r^n}\, v_{t+1} \\
&\leftrightarrow \quad \left[(v + r^n - \pi - g) - \frac{e^v\, v}{sy+e^v} - \underbrace{\frac{e^v}{sy+e^v}\,\frac{sy+e^v}{e^v}}_{=1} + \frac{e^v}{sy+e^v}\right] + \frac{e^{\pi+g-r^n}}{e^v}\, sy_{t+1} + e^{\pi+g-r^n}\, v_{t+1} \\
&\leftrightarrow \quad \left[(v + r^n - \pi - g) - \underbrace{\frac{e^v}{sy+e^v}}_{=e^{\pi+g-r^n}} v - 1 + \underbrace{\frac{e^v}{sy+e^v}}_{=e^{\pi+g-r^n}}\right] + \frac{e^{\pi+g-r^n}}{e^v}\, sy_{t+1} + e^{\pi+g-r^n}\, v_{t+1} \\
&\leftrightarrow \quad \left[(v + r^n - \pi - g) - e^{\pi+g-r^n} v - 1 + e^{\pi+g-r^n}\right] + \frac{e^{\pi+g-r^n}}{e^v}\, sy_{t+1} + e^{\pi+g-r^n}\, v_{t+1}.
\end{aligned}$$

Now define

$$\boxed{\rho \;\equiv\; e^{\pi+g-r^n}.}$$

Plug this in

$$\begin{aligned}
&\leftrightarrow \quad \Big[(v + r^n - \pi - g) - \rho v + \rho - 1\Big] + \frac{\rho}{e^v}\, sy_{t+1} + \rho\, v_{t+1} \\
&\leftrightarrow \quad \Big[(r^n - \pi - g) + v(1-\rho) + (\rho - 1)\Big] + \frac{\rho}{e^v}\, sy_{t+1} + \rho\, v_{t+1} \\
&\leftrightarrow \quad \Big[(r^n - \pi - g) + v(1-\rho) - (1 - \rho)\Big] + \frac{\rho}{e^v}\, sy_{t+1} + \rho\, v_{t+1} \\
&\Rightarrow \quad \boxed{(r^n - \pi - g) + (1-\rho)(v-1) + \frac{\rho}{e^v}\, sy_{t+1} + \rho\, v_{t+1}}.
\end{aligned}$$

So now go back to

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \log\left(sy_{t+1} + \frac{V_{t+1}}{P_{t+1}Y_{t+1}}\right).$$

Plug in our log-linearised expression for the right-hand side

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = (r^n - \pi - g) + (1-\rho)(v-1) + \frac{\rho}{e^v}\, sy_{t+1} + \rho\, v_{t+1}\ .$$

We suppress the small constant terms (since we treat all variables as mean-zero). deviations)

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \underbrace{\Big[(r^n - \pi - g) + (1-\rho)(v-1)\Big]}_{\text{tiny constant}} + \frac{\rho}{e^v}\, sy_{t+1} + \rho\, v_{t+1}.$$

Therefore,

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = \frac{\rho}{e^v}\, sy_{t+1} + \rho\, v_{t+1}. \tag{A.11$'$}$$

Define the scaled surplus innovation

$$s_{t+1} \;\equiv\; \frac{\rho}{e^v}\, sy_{t+1},$$

so that the equation becomes

$$v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} = s_{t+1} + \rho\, v_{t+1}.$$

Finally, re-arrange to get the linearised flow identity

$$\boxed{\rho\, v_{t+1} = v_t + r^n_{t+1} - \pi_{t+1} - g_{t+1} - s_{t+1}}$$

□

B.5 Further relegated details on the VAR framework

Here, we present the derivations underpinning the VAR model estimated in section 2. The derivations generally lean on Cochrane (2019), which we extend straightforwardly to an open economy framework.

We begin by a brief description of the historical episodes examined in figure 7, through which we are able to procure a narrative of major U.K. fiscal episodes, in line with Ellison and Scott (2020).

Great-Inflation era and the first peacetime deficits (1970–1978). From 1972 the plotted ratio turns persistently negative, bottoming near –0.15. The occured during the March 1972 "dash-for-growth" budget, which combined large tax cuts with relaxed credit controls; borrowing requirements jumped to £3.4 billion—then 5% of GDP. The demand boom collided with the 1973 oil shock, pushing CPI inflation into double digits, real wages negative, and gilt yields to post-war highs. For the first time in three centuries Britain ran substantial primary deficits outside of wartime. Market participants began to price a fiscal dominance regime: Sterling sold off, gilt investors demanded an inflation premium, and the surplus-over-debt ratio remained below zero until the 1976 IMF program.

Thatcher consolidation, the ERM Crisis, and 1990s Expansion (1979–2000). After 1979 the series climbs back into positive surpluses territory. The 1980 Medium-Term Financial Strategy imposed cash limits on departmental spending, turning a structural deficit into a modest surplus within four years. There were subsequent positive surpluses during the Thatcher administration. However, the sharp dip around 1992–93 reflects the twin blows of a deep domestic recession and "Black Wednesday". When speculative pressure forced Sterling out of the Exchange Rate Mechanism, the Treasury raised base rates to 15% in a failed defence of the parity; output contracted and tax receipts collapsed. The surplus-over-debt ratio falls accordingly. On the valuation side, the real exchanged rate dropped 15% within weeks, instantly re-inflating the domestic-currency value of foreign-currency gilts and widening the implied deficit in Figure 7. The subsequent adoption of inflation targeting stabilised expectations, and the ratio drifted back towards zero by 1995.

Between 1996 and 2000 the surplus series drifted close to balance. Robust productivity growth, a high tax base and restrained departmental spending kept the cash deficit near zero. At the same time, falling global real rates reduced the discount factor applied to future surpluses, so only tiny primary surpluses were required to keep the ratio flat. Flows and valuations move in opposite directions, leaving the debt ratio broadly unchanged.

Global Financial Crisis and its aftermath (2008–2013). Figure 7 records the deepest post-war trough during 2009–12. GDP fell 4.5% in 2009, policy aimed at stabilising the banking system pushed the headline deficit above 10% of GDP, and automatic stabilisers added another 2 percentage points to deficits. Meanwhile, Bank of England gilt purchases suppressed real yields to multi-decade lows, which the accounting identity treats as requiring even larger negative surpluses to match the observed market value of debt. Public-sector net debt jumped from 41% to 81% of GDP, the largest peace-time increase on record.

Pre-Covid recovery and Covid shock (2013–2021). From 2013 the coalition's fiscal consolidation and a stabilisation of real rates nudged the ratio back toward zero, but the path is flat rather than rising: low discount factors mean a balanced-budget stance is sufficient for debt stabilisation without the large primary surpluses seen in the 1980s. Modest surpluses in 2017-19 briefly lift the ratio into positive territory, but the Covid-19 pandemic reverses the gains. We see in the early pandemic period that implied surpluses soars to 40% of market value of debt, which seems paradoxical after the large fiscal stimulus packages announced during the pandemic. Emergency spending, furlough schemes and revenue shortfalls generate a deficit of 14.6% of GDP in 2020. Yet in Figure 7 the 2020 point *rises* to +0.42 The explanation lies in the valuation channel: gilt real yields turned sharply negative and inflation picked up, generating a large *implied* surplus in present-value terms even as the cash balance deteriorated. The post-Covid fiscal expansion was cushioned by deeply negative real rates, easing near-term debt-sustainability concerns. This is a Lucas and Stokey (1983) state-contingent financing of debt through surprise inflation. The episode encapsulates the fiscal-theory message: debt dynamics hinge on the path of discount factors and prices as much as on the arithmetic of tax minus spending.

Taken together, Figure 7 shows that shifts in the surplus-over-debt ratio align closely with the regime breaks and policy narratives documented by Bordo et al. (2022). Episodes that placed acute pressure on public finances, such as war financing, the 1970s stagflation, or the Global Financial Crisis—drives the surplus ratio deep into negative territory, whereas phases of deliberate consolidation or windfall receipts keeps surpluses persistently positive. Crucially, the accounting framework highlights that shifts in valuation terms—unexpected inflation and movements in discount rates—can do as much of the adjustment work as the cash-flow surpluses in reconciling the government budget constraint.

B.5.1 Additional details on the backward decomposition

The present-value identity in section 2 turns a sequence of forward flows into one equilibrium condition we can take to the data and decompose visually into economically meaningful pieces. We here derive how this identity can be derived.

To this end, we iterate both flow identities forward to arrive at present-value identities for the

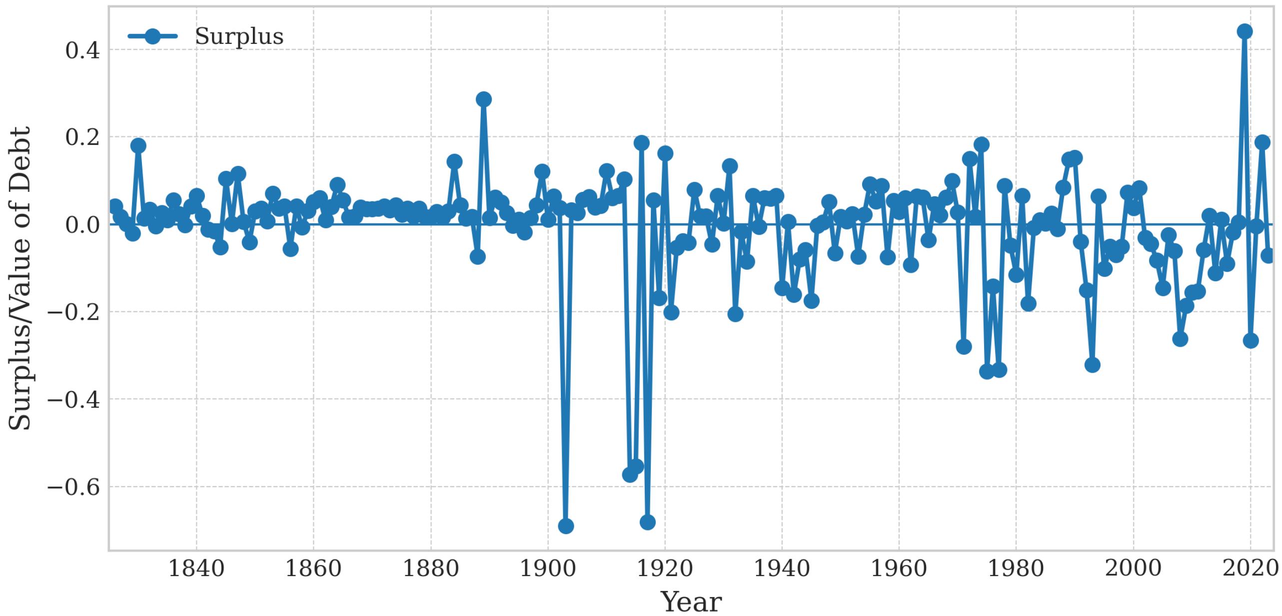

Figure 7: Implied U.K. surpluses to ensure government flow identity holds.

home and foreign debt

$$v_t^H = \sum_{j=1}^{\infty} \rho_H^{j-1} s_{t+j}^H + \sum_{j=1}^{\infty} \rho_H^{j-1} g_{t+j}^H - \sum_{j=1}^{\infty} \rho_H^{j-1} \left[r_{t+j}^{n,F} + \Delta e_{t+j} + \varphi_{t+j} - \pi_{t+j}^H \right], \tag{22}$$

$$v_t^F = \sum_{j=1}^{\infty} \rho_F^{j-1} s_{t+j}^F + \sum_{j=1}^{\infty} \rho_F^{j-1} g_{t+j}^F - \sum_{j=1}^{\infty} \rho_F^{j-1} \left[r_{t+j}^{n,F} - \pi_{t+j}^F \right]. \tag{23}$$

Investors price U.S. and U.K. bonds jointly. Persistent Dollar appreciation or a sustained fall in the UIP premium lowers the discount factor for U.S. surpluses while raising it for U.K. surpluses, binding the two budget constraints in expectation. We can rewrite the forward identity for the home country

$$\begin{aligned} v_t^H = &\sum_{j\geq 1} \rho_H^{j-1} s_{t+j}^H + \sum_{j\geq 1} \rho_H^{j-1} g_{t+j}^H - \sum_{j\geq 1} \rho_H^{j-1} r_{t+j}^{n,F} \\ &- \sum_{j\geq 1} \rho_H^{j-1} \Delta e_{t+j} - \sum_{j\geq 1} \rho_H^{j-1} \varphi_{t+j} + \sum_{j\geq 1} \rho_H^{j-1} \pi_{t+j}^H. \end{aligned} \tag{24}$$

Equation (24) also holds ex-ante. Therefore, we can take expectations based on information at time t to arrive at

$$v_t^H = \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1}\rho_H^{j-1}s_{t+j}^H\right]}_{PV_t^{(s)}} + \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1}\rho_H^{j-1}g_{t+j}^H\right]}_{PV_t^{(g)}} - \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1}\rho_H^{j-1}r_{t+j}^{n,F}\right]}_{PV_t^{(r)}}$$
$$- \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1}\rho_H^{j-1}\Delta e_{t+j}\right]}_{PV_t^{(\Delta e)}} - \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1}\rho_H^{j-1}\varphi_{t+j}\right]}_{PV_t^{(\varphi)}} + \underbrace{\mathbb{E}_t\left[\sum_{j\geq 1}\rho_H^{j-1}\pi_{t+j}^H\right]}_{PV_t^{(\pi)}}, \tag{25}$$

where $\rho_H \simeq 0.97$ is the annual Campbell–Shiller discount factor. For plotting it is convenient to gather the real discount-rate pieces as

$$PV_t^{(d)} \equiv PV_t^{(r)} - PV_t^{(\pi)},.$$

Equation (25) can therefore be written

$$v_t^H = PV_t^{(s)} + PV_t^{(g)} + PV_t^{(\pi)} - \left[PV_t^{(r)} + PV_t^{(\Delta e)} + PV_t^{(\varphi)}\right]. \tag{26}$$

Our goal is to turn the forward-looking identity in eq. (26) into *measurable* objects that can be tracked year by year.

In observing the trends in the evolution of U.K. debt (figure 2), five distinct episodes emerge.

The Sterling crisis, 1975–1981. The orange $-s$ line rises rapidly—large primary deficits—yet the blue v line increases only modestly. The gap is filled by the purple REER series and the steeply negative pink $-\pi$ series: a cumulative 25% depreciation and double-digit inflation erode the real value of gilts. Bordo et al. (2022) date this as the first peacetime episode of fiscal dominance since the 1690s; Figure 2 shows that the adjustment came predominantly through the price level and the exchange rate rather than through a fiscal retrenchment.

North-Sea revenue and disinflation, 1982–2001. During the Thatcher and early New-Labour years the orange $-s$ flattens, reflecting small primary surpluses, while the red r^n climbs as global real rates rise with the Volcker disinflation. The green $r^n - \pi - g$ bundle therefore pulls the blue v line *up* even in the face of favourable flows. Discount rates dominate: roughly half of the cumulative drift in debt over these two decades is valuation rather than cash-flow driven.

ERM exit and "stop–go" 1992–1996. The purple REER line kinks upward with Sterling's 1992 departure from the Exchange-Rate Mechanism, while the red r^n line drops as gilts rally. These offsetting valuation effects leave the blue debt path broadly unchanged, masking the cyclical widening and subsequent narrowing of primary balances.

Global Financial Crisis, 2007–2013. The financial crisis creates the largest wedge of the sample. Primary deficits widen sharply (orange), and at the same time the red r^n line *falls* as world safe real rates plunge. The two forces reinforce each other, pushing the blue debt series sharply higher. Using a similar exercise, Cochrane (2019) notes that in the U.S. decomposition about 52 % of the post-2008 rise in debt is discount-rate news; the U.K. share is similar *only* once exchange-rate channels are included, hinting at different debt dynamics for financial hegemons relative to smaller open economies.

Pandemic shock, 2020–2022. Covid-19 generates record cash deficits but also a large valuation gain: gilt real yields turn deeply negative, inflation re-accelerates (pink becomes more negative), and the brown UIP curve spikes as investors demand a wider risk premium on Sterling assets. Roughly one half of the pandemic-era debt increase is therefore absorbed by prices rather than by flows. Hence, negative real rates cushioned the fiscal expansion during this period.

B.5.2 Key macroeconomic episodes through the lens of the forward decomposition

We here present key U.K. macroeconomic historical episodes through the lens of the forward decomposition, following figure 3.

ERM crisis (1992–2001). A one-off 15 % Sterling depreciation pushes the REER path upward; the UIP premium widens, making Sterling assets riskier. Expected surpluses dip with the recession but rebound rapidly after 1997, leaving the overall debt valuation little changed.

Global Financial Crisis (2008–2012). Markets price a decade of large primary deficits (orange plummet) and a near-zero real discount rate (green surge). The two forces reinforce each other, driving v_t sharply higher.

Austerity, Brexit, and Covid (2013–2021). From 2013 the s path steepens as headline plans envisaged medium-term surpluses. In mid-2016 the REER line drops and UIP becomes negative: investors demand a higher Sterling risk premium, mechanically reducing today's debt value. At the start of the pandemic, emergency spending in 2020 raises s and v_t above 0.8 log points, yet the discount-rate term stays positive because gilt real yields plunge below –2%.

B.5.3 Variance–Decomposition

Equation (24) writes the scaled market-value of debt v_t^H as the sum of six discounted forward series. We can also construct the variance decomposition in order to ask how much of the unconditional variance of v_t^H comes from each of those six factors. Taking equation (24), abusing notation, we can take variance

$$\text{var}(v) = \text{cov}(v, \Sigma s) - \text{cov}\big[v, \Sigma(r^f + \Delta rer + uip - \pi - g)\big].$$

We then break the second covariance into its five sub-covariances. Dividing by var(v) converts each component into variance shares.

Following Cochrane (2019), we construct two alternative versions of the data series before performing the variance calculations. We construct the innovation series as follows. For each variable x_t the innovation is $\breve{x}_t = x_t - \mathbb{E}_{t-1}[x_t]$, obtained from the same VAR that underlies the forward decomposition. Cumulating the innovations isolates the portion of debt dynamics that was *unanticipated* at the time. This is an accounting of "news" rather than levels.

We also constructed a filtered series. We apply the high-pass operator

$$\theta(L) = 1 - \tfrac{1}{3}\big(L + L^2 + L^3\big)$$

to each raw series. Concretely, for debt we compute $\tilde{v}_t = v_t - \frac{1}{3}(v_{t-1} + v_{t-2} + v_{t-3})$, and analogously for $s, \pi, g, r^{n,F}, \Delta e, \varphi$. The filter removes low-frequency swings, such as wartime run-ups and long post-war declines, which dominate the unconditional variance but carry little business-cycle information. What remains is a medium-frequency measure that moves with recessions and policy cycles. Because identity (24) is algebraic, we may filter *after* forming the identity; all terms still add up exactly.

Throughout the paper we report results for all three datasets: the plain raw sums, the innovation sums that isolate true surprises, and the filtered sums that highlight cyclical movements,. The results are shown in Table 1. A positive covariance means that variable raises the variance of debt; a negative number means it moves opposite to debt, dampening variance. For compactness we report the five valuation channels with the same sign as they enter equation (24); e.g. Σr^f enters with a minus sign, so its covariance appears negative in the table. Row 2 plus Row 3 equals 1 by construction. Rows 4–8 sum to Row 3. This ensures the decomposition is internally consistent for every perspective.

Main findings. We summarise now the main variance decomposition results, presented in Table 6.

1. *Surplus news dominate.* In all three lenses the covariance between v_t and the surplus present value accounts for the largest share of variance: 83% in Plain, 94% in Innovation, and 89% in Filtered. This aligns with a FTPL-style prediction that revisions in expected primary surpluses are the primary driver of debt valuation.

	Plain	Innovation	Filtered
var(v)	1.000	1.000	1.000
cov($v, \Sigma s$)	0.832	0.940	0.890
$-$ cov[$v, \Sigma(r^f + \Delta rer + uip - \pi - g)$]	0.168	0.060	0.110
cov($v, \Sigma r^f$)	-0.452	-0.421	-0.328
cov($v, \Sigma \Delta rer$)	0.012	-0.000	0.061
cov($v, \Sigma uip$)	-0.275	-0.137	-0.230
cov($v, \Sigma \pi$)	-0.406	-0.415	-0.262
cov($v, \Sigma g$)	-0.142	-0.082	-0.125

Table 6: Variance decomposition

2. *Valuation channels offset surpluses but never overturn them.* The combined contribution of discount-rate, exchange-rate and other valuation terms is negative, reducing the explanatory share of surpluses by about 17% (Plain) to 6% (Innovation). Thus when surplus expectations improve, required returns typically rise, inflation expectations fall, or UIP premia widen—attenuating the impact on v_t, but not reversing it.

3. *Discount-rate effects are the single largest offset.* The covariance with Σr^f is -0.45 in Plain and -0.42 in Innovation: periods in which nominal discount rates are expected to be high coincide with lower debt valuations, as standard asset pricing intuition suggests.

4. *UIP and inflation channels matter materially.* Unexpected declines in the UIP premium (-0.28 Plain) or in future inflation (-0.41) both push v_t upward, but their magnitudes are about half that of the discount-rate channel.

5. *Real-exchange-rate news is small on average.* The covariance with $\Sigma \Delta rer$ is near zero in Innovation, implying that shocks to the expected path of the real exchange rate rarely move debt valuation contemporaneously. The Filtered view allocates a modest 6% share, suggesting slower-moving terms-of-trade trends do play some role.

6. *Growth news is modest and negative.* Better expected growth raises the denominator (future consumption) faster than the numerator (real surpluses), so v_t co-moves negatively with Σg (about -0.14 in Plain).

Overall, the decomposition shows that, even in an open-economy setting with currency and risk-premium channels, fiscal news remain the dominant force, while financial variables primarily modulate its impact through discounting rather than overturning it.

B.6 Open-economy forecast revaluations and the FTPL

In this part of the appendix, we derive our main specification of interest in the context of plausibly optimal forecast revisions analysed in section 3. The empirical specification that we arrive at here is closely related to the specification used in our VAR-based decomposition in section 2, but since our data does not contain forecasts on the real value of government debt, we must sidestep this one element in the very short-run.

In deriving our forecast revision framework, we assume a representative international investor, demanding a weighted fair return for a given portfolio of government assets issued by the Home country, which we here refer to as the United Kingdom for simplicity. Assume that this international investor cares with a weight of $(1-\omega)$ for the returns of 'Home' (U.K.) investors, and with a weight of ω for the returns of Foreign investors. Let $(1+i_t)$ be the pre-agreed nominal return earned by holding U.K. sovereign debt, and let $\mathcal{E}_t$ be the nominal exchange rate. Then, the overall demanded portfolio return by that representative investor is given by:

$$R_t^P = \omega(1+i_t)\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} + (1-\omega)(1+i_t) = (1+i_t)\left(1+\omega\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}-1\right)\right).$$

We postulate here that this is the factual return schedule on government debt. The government budget constraint on the total stock of sovereign debt, D_t, is then given by:

$$D_{t+1} = R_t^P(D_t - P_t s_t),$$

where R_t^P is as defined above, P_t is the price level, and s_t are real surpluses. We manipulate this equation to arrive at a standard government debt valuation equation

$$\begin{aligned}
\leftrightarrow\quad & D_t R_t^P = P_t s_t R_t^P + D_{t+1} \\
\leftrightarrow\quad & \frac{D_t}{P_t} = s_t + \frac{P_{t+1}}{P_t}\frac{1}{R_t^P}\frac{D_{t+1}}{P_{t+1}} \\
\leftrightarrow\quad & \frac{D_t}{P_t} = s_t + \frac{\Pi_{t+1}}{R_t^P}\frac{D_{t+1}}{P_{t+1}} \\
\leftrightarrow\quad & \frac{D_t}{P_t} = s_t + \frac{\Pi_{t+1}}{R_t^P}s_{t+1} + \frac{\Pi_{t+1}}{R_t^P}\frac{\Pi_{t+2}}{R_{t+1}^P}\frac{D_{t+2}}{P_{t+2}} \\
\leftrightarrow\quad & \cdots \qquad \text{(No-Ponzi condition)} \\
\Rightarrow\quad & \boxed{\frac{D_t}{P_t} = \sum_{j=0}^{\infty}\left(\prod_{l=1}^{j}\frac{\Pi_{t+l}}{R_{t+l-1}^P}\right)s_{t+j}}.
\end{aligned}$$

Now, using the Fisher equation ($1+r_t = \frac{1+i_t}{\Pi_{t+1}}$) and taking expectations, we find the targeted debt valuation equation:

$$\frac{D_t}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \frac{1}{\prod_{l=1}^{j}(1+r_{t+l})\left(1+\omega\left(\frac{\mathcal{E}_{t+l-1}}{\mathcal{E}_{t+l}}-1\right)\right)} s_{t+j}. \tag{27}$$

We take innovations: $(\mathbb{E}_t - \mathbb{E}_{t-1})$ to arrive at a model of forecast revisions on the arrival of novel information. Doing so, we obtain:

$$(\mathbb{E}_t - \mathbb{E}_{t-1})\frac{D_t}{P_t} = (\mathbb{E}_t - \mathbb{E}_{t-1})\left[\sum_{j=0}^{\infty} \frac{1}{\prod_{l=1}^{j}(1+r_{t+l})\left(1+\omega\left(\frac{\mathcal{E}_{t+l-1}}{\mathcal{E}_{t+l}}-1\right)\right)} s_{t+j}\right]. \tag{28}$$

$$\frac{D_t}{P_t} = \mathbb{E}_t\left(s_t + \frac{1}{(1+r_{t+1})\left(1+\omega\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}-1\right)\right)} s_{t+1}\right) \tag{29}$$

In the sense of such a government debt valuation equation, any innovation in surpluses must be absorbed by a corresponding innovation in the price level (that is, inflation), movements in real yields, or movements in exchange rates in proportion to their importance for the portfolio of the representative investor.

To arrive at an estimable equation mapping forecasts made by private forecasters to this debt valuation equation, we now take a first-order approximation. Additionally, and in line with the limitations of the data, we restrict ourselves to innovations in the first two years only. Put differently, we postulate that any information arriving between $t-1$ and t is uninformative for $t+j$, $j \geq 2$. Additionally, we let $D_t = D \,\forall t$, i.e., we nullify month-on-month innovations of the stock of debt due to the lack of suitable data. A first-order approximation of equation (28) around a non-stochastic steady-state restricting the right-hand side sum to the first two elements then yields:

$$-(\mathbb{E}_t - \mathbb{E}_{t-1})\frac{D}{P^2}(P_t - P) = (\mathbb{E}_t - \mathbb{E}_{t-1})\left[(s_t - s) + \frac{1}{(1+r)}(s_{t+1} - s)\right.$$
$$\left.-\frac{s}{(1+r)^2}\left[(1+r_{t+1}) - (1+r)\right] - \frac{s}{(1+r)}\frac{\omega}{\mathcal{E}}(\mathcal{E}_t - \mathcal{E}) + \frac{s}{(1+r)}\frac{\omega}{\mathcal{E}}(\mathcal{E}_{t+1} - \mathcal{E})\right].$$

Divide both sides by Y, let $\Delta\mathbb{E}_t \equiv \mathbb{E}_t - \mathbb{E}_{t-1}$, and define hatted variables as relative deviations ($\hat{x}_t \equiv \frac{X_t - X}{X}$ for any variable X). Then, we can express the previous equation as:

$$\begin{aligned}-\frac{D}{PY}\Delta\mathbb{E}_t\hat{p}_t = \frac{s}{Y}\Delta\mathbb{E}_t\hat{s}_t + \frac{1}{(1+r)}\frac{s}{Y}\Delta\mathbb{E}_t\hat{s}_{t+1} - \frac{s}{Y}\frac{1}{(1+r)}\Delta\mathbb{E}_t(\hat{R}_{t+1}) \\ -\frac{\omega}{(1+r)}\frac{s}{Y}\Delta\mathbb{E}_t\hat{e}_t + \frac{\omega}{(1+r)}\frac{s}{Y}\Delta\mathbb{E}_t\hat{e}_{t+1}.\end{aligned} \tag{30}$$

Equation (30) is informing the empirical specifications that we test in section 3.

C Additional details on the Vector Autoregression results

Complementing section 2.2, we provide additional results akin to the 'reverse regression' approach from Engel and West (2005) and Jiang (2021).

$$\frac{s^i_{t+k}}{B^i_{t+k}} = \alpha^{(k)} + \beta^{(k)}\,\Delta\log e_t^{(-i)/i} + \delta_1^{(k)}\,\frac{s^i_t}{B^i_t} + \delta_2^{(k)}\,\Delta\frac{s^i_t}{B^i_t} + \gamma_1^{(k)}\,\frac{s^{-i}_t}{B^{-i}_t} + \gamma_2^{(k)}\,\Delta\frac{s^{-i}_t}{B^{-i}_t} + \varepsilon^i_{t+k}, \tag{31}$$

that is; we test the link between the deficit/debt-ratio at time $t+k$ and the change in the real exchange rate as well as the level and the first-difference of *both* domestic and foreign deficit/debt-ratio levels. The results of this exercise are presented in table **??** and figure 8 for the U.K. and in table **??** and figure 9 for the U.S. In a nutshell, current appreciations are related to smaller local government deficits, but in the case of the U.K., the phenomenon is reversing at very long horizons.

	Dependent variable: deficit-to-debt ratio at t+k							
	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Δ REER	0.129	0.149	0.122	0.296***	0.067	−0.081	−0.086	−0.126**
	(0.104)	(0.092)	(0.080)	(0.086)	(0.051)	(0.054)	(0.065)	(0.060)
$\frac{s}{B}$-UK	0.187	−0.491**	−0.858***	−1.109***	−0.905**	−0.287	−0.142	−0.219
	(0.256)	(0.201)	(0.224)	(0.200)	(0.410)	(0.260)	(0.289)	(0.478)
$\Delta\frac{s}{B}$-UK	0.184	0.090	0.729**	0.729*	0.689	1.389**	0.604	0.706
	(0.202)	(0.276)	(0.348)	(0.405)	(0.584)	(0.626)	(0.461)	(0.608)
$\frac{s}{B}$-US	0.481**	0.814***	0.827***	0.897***	0.684	−0.005	−0.125	−0.192
	(0.194)	(0.229)	(0.256)	(0.188)	(0.421)	(0.281)	(0.291)	(0.409)
$\Delta\frac{s}{B}$-US	−0.073	−0.018	−0.647*	−0.960	−0.752	−0.824	−0.038	−0.149
	(0.246)	(0.220)	(0.386)	(0.588)	(0.673)	(0.667)	(0.458)	(0.560)
Constant	−0.010	−0.024	−0.048***	−0.058***	−0.067***	−0.095***	−0.097***	−0.109***
	(0.010)	(0.015)	(0.016)	(0.010)	(0.016)	(0.012)	(0.013)	(0.017)
Obs.	29	28	27	26	25	24	23	22
R^2	0.659	0.436	0.365	0.548	0.287	0.354	0.251	0.387
Adj. R^2	0.585	0.308	0.214	0.435	0.099	0.175	0.030	0.195

Table 7: UK: Engel and West (2005)-style regressions for the deficit-to-debt ratio. Newey-West standard errors in parentheses. *p<0.1; **p<0.05; ***p<0.01.

	Dependent variable: deficit-to-debt ratio at t+k							
	k=1	k=2	k=3	k=4	k=5	k=6	k=7	k=8
Δ REER	−0.051	0.106	0.110	0.077	0.061	0.029	0.025	−0.026
	(0.090)	(0.131)	(0.103)	(0.076)	(0.093)	(0.094)	(0.079)	(0.078)
$\frac{s}{B}$-UK	−0.183***	−0.407***	−0.581***	−0.544***	−0.500***	−0.482**	−0.455**	−0.412*
	(0.070)	(0.109)	(0.135)	(0.138)	(0.175)	(0.190)	(0.181)	(0.225)
$\Delta\frac{s}{B}$-UK	0.284***	0.405*	0.413**	0.283	0.126	0.131	−0.020	−0.169
	(0.096)	(0.215)	(0.191)	(0.220)	(0.217)	(0.250)	(0.361)	(0.432)
$\frac{s}{B}$-US	0.726***	0.453***	0.345**	0.263**	0.306**	0.350**	0.354*	0.282
	(0.108)	(0.171)	(0.170)	(0.128)	(0.121)	(0.155)	(0.193)	(0.197)
$\Delta\frac{s}{B}$-US	0.139	0.057	−0.130	−0.322*	−0.267*	−0.301	0.001	0.140
	(0.123)	(0.183)	(0.197)	(0.169)	(0.160)	(0.219)	(0.270)	(0.261)
Constant	−0.038***	−0.079***	−0.100***	−0.103***	−0.095***	−0.088***	−0.087***	−0.093***
	(0.012)	(0.018)	(0.016)	(0.015)	(0.016)	(0.016)	(0.019)	(0.026)
Obs.	52	51	50	49	48	47	46	45
R^2	0.624	0.374	0.329	0.293	0.254	0.244	0.230	0.204
Adj. R^2	0.583	0.305	0.253	0.211	0.165	0.152	0.133	0.102

Table 8: US: Engel and West (2005)-style regressions for the deficit-to-debt ratio. Newey-West standard errors in parentheses. *p<0.1; **p<0.05; ***p<0.01.

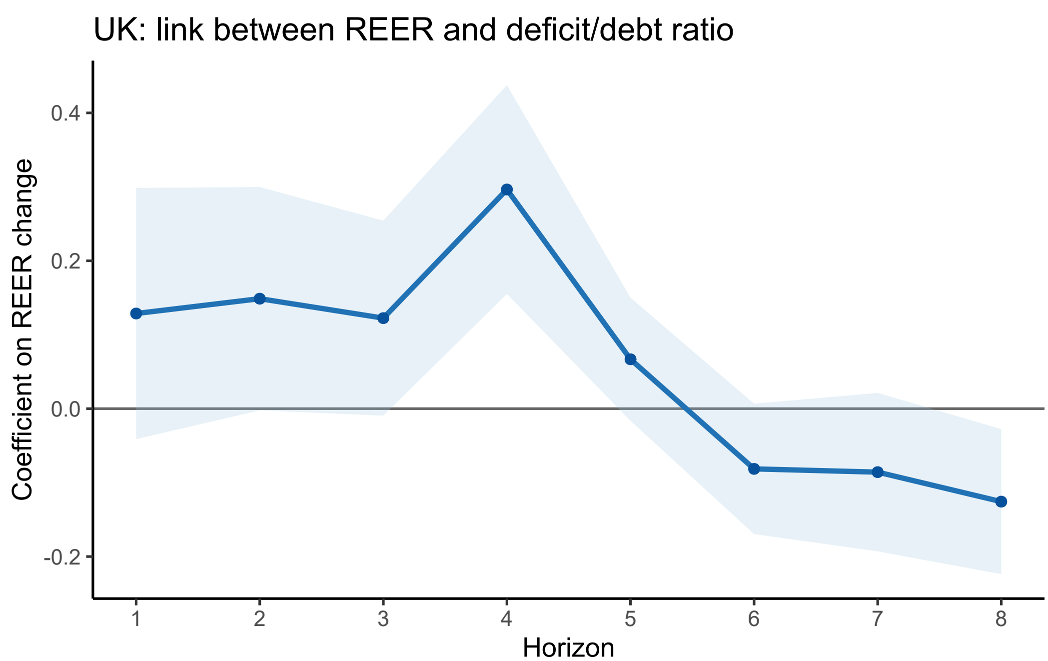

Figure 8: Engel and West (2005)-style reverse regression for the U.K.

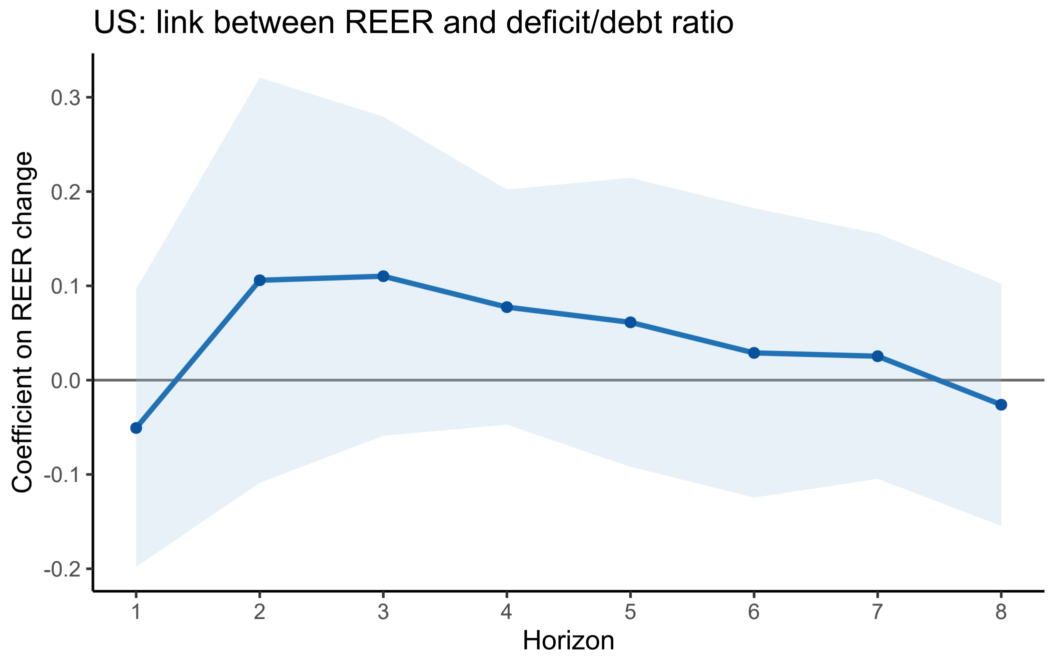

Figure 9: Engel and West (2005)-style reverse regression for the U.S.

D Derivation of International Continuous Time New Keynesian Model

D.1 Domestic Household Problem

Intratemporal Problem There is a home and foreign good. We define a home consumption bundle as

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

The domestic household solves the following intratemporal problem.

$$\max_{C_{Ht}, C_{Ft}} \left[(1-\gamma)^{1/\eta} C_{Ht}^{(\eta-1)/\eta} + \gamma^{1/\eta} C_{Ft}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} \tag{32}$$

$$\text{s.t.} \quad P_{Ht} C_{Ht} + P_{Ft} C_{Ft} = Z_t, \tag{33}$$

where P_{Ht} and P_{Ft} are the prices of the Home and Foreign goods in domestic currency and Z_t is total nominal expenditure. $1-\gamma$ measures the degree of home bias.

The equations characterising the intratemporal problem of the domestic household is

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{34}$$

$$C_{Ht} = (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t, \tag{35}$$

$$C_{Ft} = \gamma\left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t, \tag{36}$$

$$P_t = \left[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{1/(1-\eta)}. \tag{37}$$

We take first order conditions. Let ζ_t be the multiplier on the budget constraint. Form the Lagrangian

$$\mathcal{L} = \left[(1-\gamma)^{1/\eta} C_{Ht}^{(\eta-1)/\eta} + \gamma^{1/\eta} C_{Ft}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} - \zeta_t\left(P_{Ht}C_{Ht} + P_{Ft}C_{Ft} - Z_t\right).$$

Take first order conditions with respect to C_{Ht} and C_{Ft}. We have

$$\frac{\partial \mathcal{L}}{\partial C_{Ht}} = 0 \implies \left[(1-\gamma)^{1/\eta} C_{Ht}^{(\eta-1)/\eta} + \gamma^{1/\eta} C_{Ft}^{(\eta-1)/\eta}\right]^{1/(\eta-1)} (1-\gamma)^{1/\eta} C_{Ht}^{-1/\eta} = \zeta_t P_{Ht}, \tag{38}$$

$$\frac{\partial \mathcal{L}}{\partial C_{Ft}} = 0 \implies \left[(1-\gamma)^{1/\eta} C_{Ht}^{(\eta-1)/\eta} + \gamma^{1/\eta} C_{Ft}^{(\eta-1)/\eta}\right]^{1/(\eta-1)} \gamma^{1/\eta} C_{Ft}^{-1/\eta} = \zeta_t P_{Ft}. \tag{39}$$

We now simplify some algebra. Start from equation (38).

$$\begin{aligned}
&\left[\underbrace{(1-\gamma)^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}}}_{=C_t^{\frac{\eta-1}{\eta}}}\right]^{\frac{1}{\eta-1}} (1-\gamma)^{\frac{1}{\eta}} C_{Ht}^{-\frac{1}{\eta}} = \zeta P_{Ht} \\
\leftrightarrow \quad & C_t^{\frac{1}{\eta}} C_{Ht}^{-\frac{1}{\eta}} (1-\gamma)^{\frac{1}{\eta}} = \zeta P_{Ht} \\
\Rightarrow \quad & C_{Ht} = (1-\gamma)(\zeta P_{Ht})^{-\eta} C_t.
\end{aligned}$$

Do the analogous thing for equation (39) to get

$$C_{Ft} = \gamma(\zeta P_{Ft})^{-\eta} C_t.$$

Take the ratio and re-arrange:

$$\begin{aligned}
&\frac{C_{Ht}}{C_{Ft}} = \frac{(1-\gamma)(\zeta P_{Ht})^{-\eta}C_t}{\gamma(\zeta P_{Ft})^{-\eta}C_t} \\
\leftrightarrow\quad &\frac{C_{Ht}}{C_{Ft}} = \frac{(1-\gamma)(P_{Ht})^{-\eta}}{\gamma(P_{Ft})^{-\eta}} \\
\leftrightarrow\quad &\frac{C_{Ht}}{C_{Ft}} = \frac{1-\gamma}{\gamma}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{-\eta} \\
\Rightarrow\quad &C_{Ht} = C_{Ft}\frac{1-\gamma}{\gamma}\left(\frac{P_{Ft}}{P_{Ht}}\right)^{\eta}.
\end{aligned}$$

Substitute the demand ratio C_{Ht} into the CES aggregator consumption bundle:

$$\begin{aligned}
&C_t = \left[(1-\gamma)^{\frac{1}{\eta}}C_{Ht}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}}C_{Ft}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
\leftrightarrow\quad &C_t = \left[(1-\gamma)^{\frac{1}{\eta}}C_{Ft}^{\frac{\eta-1}{\eta}}\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\eta-1}{\eta}}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{1-\eta} + \gamma^{\frac{1}{\eta}}C_{Ft}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
\leftrightarrow\quad &C_t = C_{Ft}\left[(1-\gamma)^{\frac{1}{\eta}}\left(\frac{1-\gamma}{\gamma}\right)^{\frac{\eta-1}{\eta}}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{1-\eta} + \gamma^{\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
\leftrightarrow\quad &C_t = C_{Ft}\left[(1-\gamma)\left(\frac{1}{\gamma}\right)^{\frac{\eta-1}{\eta}}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{1-\eta} + \gamma^{\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
\leftrightarrow\quad &C_t = C_{Ft}\left[(1-\gamma)\,(\gamma)^{\frac{1-\eta}{\eta}}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{1-\eta} + \gamma^{\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
\leftrightarrow\quad &C_t^{\frac{\eta-1}{\eta}} = C_{Ft}^{\frac{\eta-1}{\eta}}\left[(1-\gamma)\,(\gamma)^{\frac{1-\eta}{\eta}}\left(\frac{P_{Ht}}{P_{Ft}}\right)^{1-\eta} + \gamma^{\frac{1}{\eta}}\right] \\
\leftrightarrow\quad &C_t^{\frac{\eta-1}{\eta}} = C_{Ft}^{\frac{\eta-1}{\eta}}P_{Ft}^{\eta-1}\left[(1-\gamma)(\gamma)^{\frac{1-\eta}{\eta}}P_{Ht}^{1-\eta} + \gamma^{\frac{1}{\eta}}P_{Ft}^{1-\eta}\right] \\
\leftrightarrow\quad &C_t^{\frac{\eta-1}{\eta}} = C_{Ft}^{\frac{\eta-1}{\eta}}P_{Ft}^{\eta-1}(\gamma)^{\frac{1-\eta}{\eta}}\left[\underbrace{(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}}_{=P_t^{1-\eta}}\right] \\
\leftrightarrow\quad &C_t^{\frac{\eta-1}{\eta}} = C_{Ft}^{\frac{\eta-1}{\eta}}P_{Ft}^{\eta-1}(\gamma)^{\frac{1-\eta}{\eta}}P_t^{1-\eta} \\
\leftrightarrow\quad &C_{Ft}^{\frac{\eta-1}{\eta}} = (\gamma)^{\frac{\eta-1}{\eta}}C_t^{\frac{\eta-1}{\eta}}\left(\frac{P_{Ft}}{P_t}\right)^{1-\eta} \\
\Rightarrow\quad &C_{Ft} = \gamma\left(\frac{P_{Ft}}{P_t}\right)^{-\eta}C_t
\end{aligned}$$

We can repeat the same derivations by plugging in C_{Ft} into the CES aggregator to arrive at

$$C_{Ht} = (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\eta}C_t.$$

Therefore we have

$$C_{Ht} = (1-\gamma)\left(\tfrac{P_{Ht}}{P_t}\right)^{-\eta} C_t, \tag{40}$$

$$C_{Ft} = \gamma\left(\tfrac{P_{Ft}}{P_t}\right)^{-\eta} C_t. \tag{41}$$

where P_t is defined next. We can verify that $P_t \equiv [(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}]^{\frac{1}{1-\eta}}$ is the domestic CPI. By definition, domestic CPI is defined as the price of one unit of consumption, it therefore follows by plugging in the optimal demands in equation (41) into the budget constraint:

$$\begin{aligned}
& P_{Ht}C_{Ht} + P_{Ft}C_{Ft} = Z_t \\
\leftrightarrow \quad & P_{Ht}C_{Ht} + P_{Ft}C_{Ft} = P_t C_t \\
\leftrightarrow \quad & P_{Ht}(1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t + P_{Ft}\gamma\left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t = P_t C_t \\
\leftrightarrow \quad & P_{Ht}^{1-\eta}(1-\gamma)P_t^{\eta}C_t + P_{Ft}^{1-\eta}\gamma P_t^{\eta}C_t = P_t C_t \\
\leftrightarrow \quad & P_{Ht}^{1-\eta}(1-\gamma)P_t^{\eta} + P_{Ft}^{1-\eta}\gamma P_t^{\eta} = P_t \\
\leftrightarrow \quad & P_{Ht}^{1-\eta}(1-\gamma) + P_{Ft}^{1-\eta}\gamma = P_t^{1-\eta} \\
\leftrightarrow \quad & P_t^{1-\eta} = (1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta} \\
\Rightarrow \quad & P_t = \left[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{\frac{1}{1-\eta}}.
\end{aligned}$$

Therefore, domestic CPI is defined as

$$P_t = \left[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{1/(1-\eta)}. \tag{42}$$

Finally, the real exchange rate is defined as

$$S_t = E_t \frac{P_t^*}{P_t}, \tag{43}$$

where E_t is the nominal exchange rate. The terms of trade is the price of imports over the price of exports

$$\text{tot}_t = \frac{P_{Ft}}{P_{Ht}}. \tag{44}$$

Intertemporal Problem The household chooses consumption C_t at price P_t, supplies labour N_t for the wage W_t, and chooses holding of domestic bonds B_t earning interest i_t and foreign bonds bonds B_t^* earning interest i_t^*.

$$\max_{\{C_t,N_t,B_t,B_t^*\}_{t\geq 0}} \int_0^\infty e^{-\rho t}\Bigg\{log[C_t] - \frac{N_t^{1+\phi}}{1+\phi} + \nu(B_t;\theta)\Bigg\}dt \tag{45}$$

$$P_tC_t + \dot{B}_t + E_t\dot{B}_t^* = i_tB_t + E_ti_t^*B_t^* + W_tN_t + T_t \tag{46}$$

The necessary and sufficient conditions for domestic household optimality is given by

$$C_tN_t^{\phi} = \frac{W_t}{P_t} \tag{47}$$

$$\frac{\dot{C}_t}{C_t} = i_t - \pi_t + \Psi_t - \rho, \tag{48}$$

where $\Psi_t \equiv \nu_B(B_t;\theta)P_tC_t$ is the convenience yield wedge. The UIP condition is given by

$$i_t - i_t^* = \frac{\dot{E}_t}{E_t} - \Psi_t. \tag{49}$$

The transversality conditions on domestic and foreign bonds are

$$\lim_{T\to\infty} e^{-\rho T}\lambda_T B_T = 0 \tag{50}$$

$$\lim_{T\to\infty} e^{-\rho T}\lambda_T E_T B_T^* = 0. \tag{51}$$

The current valued Hamiltonian is

$$\mathcal{H}(B_t;C_t;\lambda_t) = \Bigg[log[C_t] - \frac{N_t^{1+\phi}}{1+\phi} + \nu(B_t;\theta)\Bigg] + \lambda_t\Bigg[i_tB_t + E_ti_t^*B_t^* + W_tN_t + T_t - P_tC_t\Bigg] \tag{52}$$

The first order conditions are:

$$\begin{aligned}
[C_t]: &\quad \frac{1}{C_t} = P_t\lambda_t \\
[N_t]: &\quad N_t^{\phi} = \lambda_t W_t \\
[B_t]: &\quad \dot{\lambda}_t = \rho\lambda_t - [\lambda_t i_t] - \nu_B(B_t;\theta) \\
[B_t^*]: &\quad \dot{\lambda}_tE_t + \lambda_t\dot{E}_t = \rho\lambda_tE_t - \lambda_tE_ti_t^*.
\end{aligned}$$

Combine the first two

$$N_t^{\phi} = \frac{1}{P_tC_t}W_t$$

Therefore

$$C_tN_t^{\phi} = \frac{W_t}{P_t}$$

We want $\dot{\lambda}_t$. Take

$$\begin{aligned}
&\frac{1}{C_t} = P_t\lambda_t \\
\leftrightarrow\quad & \lambda_t = \frac{1}{P_tC_t} \\
\leftrightarrow\quad & \frac{d}{dt}\lambda_t = \frac{d}{dt}\frac{1}{P_tC_t} \\
\leftrightarrow\quad & \dot{\lambda}_t = \frac{d}{dt}\left[\frac{1}{P_tC_t}\right] \\
\leftrightarrow\quad & \dot{\lambda}_t = \frac{d}{dt}\left[P_tC_t\right]^{-1} \\
\leftrightarrow\quad & \dot{\lambda}_t = -1\left[P_tC_t\right]^{-2}\left[\frac{d}{dt}P_tC_t\right] \\
\leftrightarrow\quad & \dot{\lambda}_t = -1\left[P_tC_t\right]^{-2}\left[\dot{P}_tC_t + P_t\dot{C}_t\right] \\
\leftrightarrow\quad & \dot{\lambda}_t = -\frac{\dot{P}_tC_t}{\left[P_tC_t\right]^2} - \frac{P_t\dot{C}_t}{\left[P_tC_t\right]^2} \\
\leftrightarrow\quad & \dot{\lambda}_t = -\frac{\dot{P}_t}{P_t^2C_t} - \frac{\dot{C}_t}{P_tC_t^2}
\end{aligned}$$

Recall from the first order condition for consumption that

$$\lambda_t = \frac{1}{P_tC_t}.$$

So we can plug this in

$$\begin{aligned}
\leftrightarrow\quad & \dot{\lambda}_t = -\frac{\dot{P}_t}{P_t^2C_t} - \frac{\dot{C}_t}{P_tC_t^2} \\
\leftrightarrow\quad & \dot{\lambda}_t = -\frac{\dot{P}_t}{P_t}\lambda_t - \frac{\dot{C}_t}{C_t}\lambda_t \\
\Rightarrow\quad & \frac{\dot{\lambda}_t}{\lambda_t} = -\frac{\dot{P}_t}{P_t} - \frac{\dot{C}_t}{C_t}.
\end{aligned}$$

Define the inflation rate as

$$\pi_t = \frac{\dot{P}_t}{P_t}.$$

Plug this back in

$$\begin{aligned}
\leftrightarrow\quad & \frac{\dot{\lambda}_t}{\lambda_t} = -\frac{\dot{P}_t}{P_t} - \frac{\dot{C}_t}{C_t} \\
\Rightarrow\quad & \frac{\dot{\lambda}_t}{\lambda_t} = -\pi_t - \frac{\dot{C}_t}{C_t}.
\end{aligned}$$

Plug this into the domestic bonds first order condition

$$\begin{aligned}
\leftrightarrow \quad & \dot{\lambda}_t = \rho\lambda_t - [\lambda_t i_t] - \nu_B(B_t;\theta) \\
\leftrightarrow \quad & \frac{\dot{\lambda}_t}{\lambda_t} = \rho - i_t - \nu_B(B_t;\theta)\frac{1}{\lambda_t} \\
\leftrightarrow \quad & -\pi_t - \frac{\dot{C}_t}{C_t} = \rho - i_t - \nu_B(B_t;\theta)P_tC_t \\
\leftrightarrow \quad & \frac{\dot{C}_t}{C_t} = i_t - \pi_t + \nu_B(B_t;\theta)P_tC_t - \rho \\
\Rightarrow \quad & \frac{\dot{C}_t}{C_t} = i_t - \pi_t + \Psi_t - \rho,
\end{aligned}$$

where $\Psi_t \equiv \nu_B(B_t;\theta)P_tC_t$ is the convenience yield wedge.
Finally, go to the foreign bond FOC

$$\dot{\lambda}_t E_t + \lambda_t \dot{E}_t = \rho\lambda_t E_t - \lambda_t E_t i_t^*.$$

Divide by $\lambda_t E_t$

$$\frac{\dot{\lambda}_t}{\lambda_t} + \frac{\dot{E}_t}{E_t} = \rho - i_t^*.$$

Substitute in $\frac{\dot{\lambda}_t}{\lambda_t} = -\pi_t - \frac{\dot{C}_t}{C_t}$ to get

$$-\pi_t - \frac{\dot{C}_t}{C_t} + \frac{\dot{E}_t}{E_t} = \rho - i_t^*.$$

Therefore we have

$$\frac{\dot{C}_t}{C_t} = i_t^* - \pi_t + \frac{\dot{E}_t}{E_t} - \rho.$$

Combine this with the Euler equation charactering domestic bonds

$$\begin{aligned}
\leftrightarrow \quad & \frac{\dot{C}_t}{C_t} = i_t - \pi_t + \Psi_t - \rho \\
\leftrightarrow \quad & i_t^* - \pi_t + \frac{\dot{E}_t}{E_t} - \rho = i_t - \pi_t + \Psi_t - \rho \\
\leftrightarrow \quad & i_t^* + \frac{\dot{E}_t}{E_t} = i_t + \Psi_t \\
\Rightarrow \quad & i_t - i_t^* = \frac{\dot{E}_t}{E_t} - \Psi_t.
\end{aligned}$$

This is the UIP condition where because domestic bonds deliver utility, investors require lower financial returns on them

$$i_t - i_t^* = \frac{\dot{E}_t}{E_t} - \Psi_t.$$

Finally we have the no-transversality condition on both domestic and foreign bonds

$$\lim_{T\to\infty} e^{-\rho T}\lambda_T B_T = 0$$
$$\lim_{T\to\infty} e^{-\rho T}\lambda_T E_T B_T^* = 0.$$

D.2 Foreign Household

Intratemporal Problem The foreign household problem is symmetric to the domestic household problem. The CES demand function for the foreign household is

$$C_t^* = \left[(1-\gamma)^{1/\eta}\, C_{Ft}^{*\,(\eta-1)/\eta} + \gamma^{1/\eta}\, C_{Ht}^{*\,(\eta-1)/\eta}\right]^{\eta/(\eta-1)}. \tag{53}$$

The Foreign good gets weight $1-\gamma$; the imported U.S. good weight γ, matching the degree of home-bias symmetry. To solve the intratemporal problem, we assume that the intratemporal budget constraint is

$$P_{Ft}^*\, C_{Ft}^* + P_{Ht}^*\, C_{Ht}^* = Z_t^*,$$

where Z_t^* is the within period expenditure. The optimal consumption expenditure is symmetric to the domestic household

$$C_{Ft}^* = (1-\gamma)\left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta} C_t^*, \tag{54}$$
$$C_{Ht}^* = \gamma\left(\frac{P_{Ht}^*}{P_t^*}\right)^{-\eta} C_t^*. \tag{55}$$

The foreign CPI is

$$P_t^* = \left[(1-\gamma)P_{Ft}^{*\,1-\eta} + \gamma\, P_{Ht}^{*\,1-\eta}\right]^{1/(1-\eta)}. \tag{56}$$

The foreign terms of trade is

$$\text{tot}_t^* = \frac{P_{Ht}^*}{P_{Ft}^*}. \tag{57}$$

The equations characterising the intratemporal problem of the foreign household is

$$C_t^* = \left[(1-\gamma)^{1/\eta}\, C_{Ft}^{*\,(\eta-1)/\eta} + \gamma^{1/\eta}\, C_{Ht}^{*\,(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{58}$$
$$C_{Ft}^* = (1-\gamma)\left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta} C_t^*, \tag{59}$$
$$C_{Ht}^* = \gamma\left(\frac{P_{Ht}^*}{P_t^*}\right)^{-\eta} C_t^*, \tag{60}$$
$$P_t^* = \left[(1-\gamma)P_{Ft}^{*\,1-\eta} + \gamma\, P_{Ht}^{*\,1-\eta}\right]^{1/(1-\eta)}. \tag{61}$$

Intertemporal Problem The intertemporal problem for the foreign household is symmetric. They maximise the following

$$\max_{\{C_t^*,\, N_t^*,\, B_t^*,\, B_t\}} \int_0^\infty e^{-\rho^* t} \left\{ \log[C_t^*] - \frac{N_t^{*\,1+\phi^*}}{1+\phi^*} + \nu\left(\frac{B_t}{E_t};\theta\right) \right\} \, dt, \tag{62}$$

The flow budget constraint is

$$P_t^* C_t^* + \dot{B}_t^* + \frac{1}{E_t}\,\dot{B}_t = i_t^* B_t^* + \frac{1}{E_t}\, i_t B_t + W_t^* N_t^* + T_t^*. \tag{63}$$

The necessary and sufficient conditions for domestic household optimality is given by

$$\frac{\dot{C}_t^*}{C_t^*} = i_t^* - \pi_t^* + \Psi_t^* - \rho^*, \tag{64}$$

$$C_t^*\, N_t^{*\phi^*} = \frac{W_t^*}{P_t^*}, \tag{65}$$

where $\Psi_t^* \equiv \nu_B\left(\frac{B_t^*}{E_t};\theta\right) P_t^* C_t^*$ is the convenience yield wedge. The transversality conditions on domestic and foreign bonds are

$$\lim_{T\to\infty} e^{-\rho^* T}\lambda_T^* B_T^* = 0, \tag{66}$$

$$\lim_{T\to\infty} e^{-\rho^* T}\lambda_T^* \frac{B_T}{E_t} = 0. \tag{67}$$

The problem is overall symmetric to the domestic household problem.

D.3 Risk-Sharing

Lemma 1 (Risk-Sharing)**.** *The risk-sharing condition is given by*

$$\left[\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*}\right] - \dot{S}_t = \rho^* - \rho - \Psi_t^*. \tag{68}$$

Proof. The two Euler equations for the home and foreign household alongside the UIP condition is

$$\frac{\dot{C}_t}{C_t} = i_t - \pi_t + \Psi_t - \rho,$$

$$\frac{\dot{C}_t^*}{C_t^*} = i_t^* - \pi_t^* + \Psi_t^* - \rho^*$$

$$i_t - i_t^* = \frac{\dot{E}_t}{E_t} - \Psi_t.$$

Difference the two Euler equations

$$\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*} = i_t - i_t^* + \pi_t^* - \pi_t + \Psi_t - \Psi_t^* - \rho + \rho^*.$$

The real exchange rate was defined as

$$S_t = E_t \frac{P_t^*}{P_t}.$$

The growth of the real exchange rate is

$$\dot{S}_t = \frac{\dot{E}_t}{E_t} + \pi_t^* - \pi_t.$$

Substitute in the UIP condition

$$\dot{S}_t = i_t - i_t^* + \pi_t^* - \pi_t + \Psi_t.$$

Subtract the growth of the exchange rate with the difference of the Euler equation with

$$\begin{aligned}
& \dot{S}_t - \left[\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*}\right] = i_t - i_t^* + \pi_t^* - \pi_t + \Psi_t - [i_t - i_t^* + \pi_t^* - \pi_t + \Psi_t - \Psi_t^* - \rho + \rho^*] \\
\leftrightarrow \quad & \dot{S}_t - \left[\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*}\right] = -[-\Psi_t^* - \rho + \rho^*] \\
\leftrightarrow \quad & \dot{S}_t - \left[\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*}\right] = \Psi_t^* + \rho - \rho^* \\
\Rightarrow \quad & \left[\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*}\right] - \dot{S}_t = \rho^* - \rho - \Psi_t^*.
\end{aligned}$$

□

D.4 Domestic Final Good Producers

Final good producers are competitive producer and aggregate a continuum of intermediate inputs. We assume that there is a continuum of measure one of differentiated goods $Y_{Ht}(j)$ according to:

$$Y_{Ht} = \left[\int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{69}$$

whereby $\epsilon > 1$ is the elasticity of substitution among differentiated goods. The retailers take prices of differentiated goods $p_{Ht}(j)$ as given. We can then define the retailer's minimisation problem.

The cost minimisation problem facing the retailer is given by:

$$\begin{aligned} \min_{Y_{Ht}(j)} \quad & \int_0^1 p_{Ht}(j) Y_{Ht}(j) dj \\ \text{s.t.} \quad & Y_{Ht} = \left[\int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}. \end{aligned} \tag{70}$$

With this problem, we can solve for the demand for each intermediate good j by the retailer. The demand for intermediate goods by the retailer is given by:

$$Y_{Ht}(j) = \left[\frac{p_{Ht}(j)}{P_{Ht}} \right]^{-\epsilon} Y_{Ht} \tag{71}$$

whereby we define the aggregate price index $P_{Ht} \equiv \left[\int_0^1 p_{Ht}(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$.

The retailer's first order condition is given by:

$$p_{Ht}(j) = \lambda_t Y_{Ht}^{\frac{1}{\epsilon}} Y_{Ht}(j)^{-\frac{1}{\epsilon}}. \tag{72}$$

The **Lagrangian** is given by:

$$\mathcal{L} = \int_0^1 p_{Ht}(j) Y_{Ht}(j) dj - \lambda_t \left\{ \left[\int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} - Y_{Ht} \right\}.$$

We take a first order condition with respect to $Y_{Ht}(j)$:

$$\begin{aligned} \left[\frac{\partial \mathcal{L}}{\partial Y_{Ht}(j)} \right] : \quad & p_{Ht}(j) - \lambda_t \frac{\partial}{\partial Y_{Ht}(j)} \left\{ \left[\int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} - Y_{Ht} \right\} = 0 \\ \leftrightarrow \quad & p_{Ht}(j) - \lambda_t \left[\left\{ \frac{\epsilon}{\epsilon-1} \left[\int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}-1} \cdot \frac{\epsilon-1}{\epsilon} Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}-1} \right\} - 0 \right] = 0 \\ \leftrightarrow \quad & p_{Ht}(j) - \lambda_t \left[\left[\int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}-1} Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}-1} \right] = 0 \\ \leftrightarrow \quad & p_{Ht}(j) - \lambda_t \left[\underbrace{\left[\int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{1}{\epsilon-1}}}_{=Y_{Ht}^{\frac{1}{\epsilon}}} Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}-1} \right] = 0 \\ \leftrightarrow \quad & p_{Ht}(j) - \lambda_t \left[Y_{Ht}^{\frac{1}{\epsilon}} Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}-1} \right] = 0 \\ \Rightarrow \quad & p_{Ht}(j) - \lambda_t \left[Y_{Ht}^{\frac{1}{\epsilon}} Y_{Ht}(j)^{-\frac{1}{\epsilon}} \right] = 0. \end{aligned}$$

Now, if we go back the retailer's cost minimisation problem, we can derive the value of the Lagrange multiplier.

The Lagrange multiplier for the retailer is the aggregate price level P_{Ht}:

$$\lambda_t = P_{Ht}. \tag{73}$$

By assumption of competitive retailers, there is zero profits so then the revenue from goods sold equals the total cost:

$$\underbrace{P_{Ht}Y_{Ht}}_{\text{Total Revenue}} - \underbrace{\int_0^1 p_{Ht}(j)Y_{Ht}(j)dj}_{\text{Total Cost}} = 0$$

Now, plug in the retailer's FOC into the total cost:

$$\begin{aligned}
& P_{Ht}Y_{Ht} = \int_0^1 p_{Ht}(j)Y_{Ht}(j)dj \\
\leftrightarrow \quad & P_{Ht}Y_{Ht} = \int_0^1 \left[\lambda_t Y_{Ht}^{\frac{1}{\epsilon}} Y_{Ht}(j)^{-\frac{1}{\epsilon}}\right] Y_{Ht}(j)dj \\
\leftrightarrow \quad & P_{Ht}Y_{Ht} = \lambda_t Y_{Ht}^{\frac{1}{\epsilon}} \int_0^1 Y_{Ht}(j)^{1-\frac{1}{\epsilon}} dj \\
\leftrightarrow \quad & P_{Ht}Y_{Ht} = \lambda_t Y_{Ht}^{\frac{1}{\epsilon}} \int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj \\
\leftrightarrow \quad & P_{Ht}Y_{Ht} = \lambda_t Y_{Ht}^{\frac{1}{\epsilon}} Y_{Ht}^{\frac{\epsilon-1}{\epsilon}} \\
\leftrightarrow \quad & P_{Ht}Y_{Ht} = \lambda_t Y_{Ht} \\
\Rightarrow \quad & P_{Ht} = \lambda_t.
\end{aligned}$$

The demand for intermediate goods by the retailer is given by:

$$Y_{Ht}(j) = \left[\frac{p_{Ht}(j)}{P_{Ht}}\right]^{-\epsilon} Y_{Ht}. \tag{74}$$

Take the retailer's FOC in equation (72):

$$p_{Ht}(j) = \lambda_t Y_{Ht}^{\frac{1}{\epsilon}} Y_{Ht}(j)^{-\frac{1}{\epsilon}},$$

and then plug in the Lagrange multiplier being the aggregate price index equation (73):

$$p_{Ht}(j) = P_{Ht} Y_{Ht}^{\frac{1}{\epsilon}} Y_{Ht}(j)^{-\frac{1}{\epsilon}}.$$

Now re-arrange for the demand for intermediate good j, $Y_{Ht}(j)$:

$$
\begin{aligned}
& p_{Ht}(j) = P_{Ht} Y_{Ht}^{\frac{1}{\epsilon}} Y_{Ht}(j)^{-\frac{1}{\epsilon}} \\
\leftrightarrow \quad & \frac{p_{Ht}(j)}{P_{Ht}} = Y_{Ht}^{\frac{1}{\epsilon}} Y_{Ht}(j)^{-\frac{1}{\epsilon}} \\
\leftrightarrow \quad & \frac{p_{Ht}(j)}{P_{Ht}} = \left(\frac{Y_{Ht}}{Y_{Ht}(j)} \right)^{\frac{1}{\epsilon}} \\
\leftrightarrow \quad & \left[\frac{p_{Ht}(j)}{P_{Ht}} \right]^{\epsilon} = \frac{Y_{Ht}}{Y_{Ht}(j)} \\
\Rightarrow \quad & Y_{Ht}(j) = \left[\frac{p_{Ht}(j)}{P_{Ht}} \right]^{-\epsilon} Y_{Ht}.
\end{aligned}
$$

We are now interested in deriving what is the aggregate price level of goods P_{Ht} in this economy as a function of the individual intermediate good prices $p_{Ht}(j)$.

The aggregate price index is a non-linear combination of differentiated goods:

$$
P_{Ht} = \left[\int_0^1 p_{Ht}(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}} \tag{75}
$$

To see that result, take the Dixit-Stiglitz production function (69)

$$
Y_{Ht} = \left[\int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}.
$$

Plug in the demand for intermediate goods and re-arrange

$$
\begin{aligned}
& Y_{Ht} = \left[\int_0^1 Y_{Ht}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \\
\leftrightarrow \quad & Y_{Ht} = \left[\int_0^1 \left[\left[\frac{p_{Ht}(j)}{P_{Ht}} \right]^{-\epsilon} Y_{Ht} \right]^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} \\
\leftrightarrow \quad & Y_{Ht} = \left[\int_0^1 \left[\left[\frac{p_{Ht}(j)}{P_{Ht}} \right]^{-\epsilon} \right]^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}} Y_{Ht} \\
\leftrightarrow \quad & 1 = \left[\int_0^1 \left[\frac{p_{Ht}(j)}{P_{Ht}} \right]^{1-\epsilon} dj \right]^{\frac{\epsilon}{\epsilon-1}} \\
\leftrightarrow \quad & 1 = \left[P_{Ht}^{-(1-\epsilon)} \int_0^1 \left[p_{Ht}(j) \right]^{1-\epsilon} dj \right]^{\frac{\epsilon}{\epsilon-1}} \\
\leftrightarrow \quad & 1 = P_{Ht}^{\epsilon} \left[\int_0^1 \left[p_{Ht}(j) \right]^{1-\epsilon} dj \right]^{\frac{\epsilon}{\epsilon-1}} \\
\Rightarrow \quad & P_{Ht} = \left[\int_0^1 p_{Ht}(j)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}.
\end{aligned}
$$

Note that P_{Ht} in equation (75) is the producer price of one unit of home final good. Meanwhile, P_t

in equation (37) is the consumer price index and is the price of the household's CES consumption basket of home and foreign goods.

D.5 Domestic Intermediate Good Producers

We first solve the firm's problem under flexible prices. This will be a useful benchmark for us when we analyse the *natural* economy.

D.5.1 Flexible Price Setting

We now look at the problem for wholesale producers, these are the producers who actually produce goods. We can now state the production function for wholesale producers.

The production function for wholesale producers is given by:

$$Y_{Ht}(j) = A_t n_t(j), \tag{76}$$

whereby A_t is a stationary productivity shock. As wholesale producers have market power are monopolies, they can now chooses both the **price level** $p_{Ht}(j)$ and the amount of labour to hire $n_t(j)$ at wage rate W_t to produce output $Y_{Ht}(j)$.

The profit maximisation problem facing the wholesale producers in real terms is given by:

$$\begin{aligned} \max_{p_{Ht}(j), n_t(j)} \quad & \frac{p_{Ht}(j)}{P_{Ht}} Y_{Ht}(j) - \frac{W_t}{P_{Ht}} n_t(j) \\ \text{s.t.} \quad & Y_{Ht}(j) = A_t n_t(j) \\ & Y_{Ht}(j) = \left[\frac{p_{Ht}(j)}{P_{Ht}}\right]^{-\epsilon} Y_{Ht}. \end{aligned} \tag{77}$$

We now show what is the optimal relative price for wholesale producers. The optimal relative price for the wholesale producer is to set relative prices:

$$p_{Ht}(j) = \frac{\epsilon}{\epsilon - 1} \frac{W_t}{A_t}. \tag{78}$$

We first solve for labour in the production

$$n_t(j) = \frac{Y_{Ht}(j)}{A_t}.$$

Plug this into the firm's profit function to get new maximisation problem

$$\begin{aligned}
\max_{p_{Ht}(j)} \quad & \frac{p_{Ht}(j)}{P_{Ht}} Y_{Ht}(j) - \frac{W_t}{P_{Ht}} \frac{Y_{Ht}(j)}{A_t} \\
\text{s.t.} \quad & Y_{Ht}(j) = \left[\frac{p_{Ht}(j)}{P_{Ht}}\right]^{-\epsilon} Y_{Ht}.
\end{aligned}$$

Plug the constraint in

$$\begin{aligned}
& \frac{p_{Ht}(j)}{P_{Ht}} \left[\frac{p_{Ht}(j)}{P_{Ht}}\right]^{-\epsilon} Y_{Ht} - \frac{W_t}{P_{Ht}} \frac{1}{A_t} \left[\frac{p_{Ht}(j)}{P_{Ht}}\right]^{-\epsilon} Y_{Ht} \\
\leftrightarrow \quad & \frac{p_{Ht}(j)^{1-\epsilon}}{P_{Ht}^{1-\epsilon}} Y_{Ht} - \left[\frac{p_{Ht}(j)}{P_{Ht}}\right]^{-\epsilon} \frac{Y_{Ht} W_t}{P_{Ht} A_t}.
\end{aligned}$$

Now take first order condition with respect to $p_{Ht}(j)$:

$$\begin{aligned}
& (1-\epsilon) \frac{p_{Ht}(j)^{-\epsilon}}{P_{Ht}^{1-\epsilon}} Y_{Ht} + \epsilon p_{Ht}(j)^{-\epsilon-1} \frac{1}{P_{Ht}^{1-\epsilon}} \frac{Y_{Ht} W_t}{A_t} = 0 \\
\leftrightarrow \quad & (1-\epsilon) p_{Ht}(j)^{-\epsilon} + \epsilon p_{Ht}(j)^{-\epsilon-1} \frac{W_t}{A_t} = 0 \\
\leftrightarrow \quad & p_{Ht}(j) = -\frac{\epsilon}{(1-\epsilon)} \frac{W_t}{A_t} \\
\Rightarrow \quad & p_{Ht}(j) = \frac{\epsilon}{\epsilon - 1} \frac{W_t}{A_t}.
\end{aligned}$$

D.5.2 Sticky Price Setting

We now look at the domestic intermediate firm j problem in the sticky price setting. The per period profit is

$$\Pi_t(p_{Ht}(j)) = p_{Ht}(j) Y_{Ht}(j) - \frac{W_t}{A_t} Y_{Ht}(j). \tag{79}$$

Plug in the final good provider demand for intermediate goods

$$Y_{Ht}(j) = \left[\frac{p_{Ht}(j)}{P_{Ht}}\right]^{-\epsilon} Y_{Ht},$$

to get

$$\Pi_t(p_{Ht}(j)) = p_{Ht}(j) \left(\frac{p_{Ht}(j)}{P_t}\right)^{-\epsilon} Y_{Ht} - \frac{W_t}{A_t} \left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon} Y_{Ht}.$$

We use a Rotemberg quadratic price adjustment cost

$$\Theta_t(j) = \frac{\theta}{2} \left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2 P_{Ht} Y_{Ht}. \tag{80}$$

where θ is the degree of price stickiness. The firm's optimal control problem is

$$V_0(p_{H0}) = \max_{\{p_{Ht}(j)\}_{t\geq 0}} \int_0^{\infty} e^{-\int_0^t i(s)ds} \Big\{ \Pi_t(p_{Ht}) - \Theta_t(j) \Big\} dt \tag{81}$$

The state variable is $p_{Ht}(j)$, the control variable is $\dot{p}_{Ht}(j)$, and the co-state variable is η_t. In setting up the Hamiltonian, we note that the law of motion for p_{Ht} is trivially given by

$$\dot{p}_{Ht}(j) = \dot{p}_{Ht}(j).$$

We can form the current value Hamiltonian as

$$\mathcal{H}(p_{Ht}(j); \dot{p}_{Ht}(j); \eta_t) = \left[p_{Ht}(j) \left(\frac{p_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} - \frac{W_t}{A_t} \left(\frac{p_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} \right] - \left[\frac{\theta}{2} \left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)} \right)^2 P_{Ht} Y_{Ht} \right] + \eta_t \dot{p}_{Ht}(j). \tag{82}$$

Necessary and sufficient conditions for firm optimality conditions

$$\theta \frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)} \frac{P_{Ht}}{p_{Ht}(j)} Y_{Ht} = \eta_t \tag{83}$$

$$\dot{\eta}_t = i_t \eta_t - \left[(1-\epsilon) \left(\frac{p_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} + \epsilon \frac{W_t}{p_{Ht}(j)} \frac{1}{A_t} \left(\frac{p_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} + \theta \left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)} \right)^2 \frac{P_{Ht}}{p_{Ht}(j)} Y_{Ht} \right]. \tag{84}$$

The first order condition with respect to the control variable $\dot{p}_{Ht}(j)$ is:

$$[\dot{p}_{Ht}(j)] : \quad -\theta \frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)} \left(\frac{1}{p_{Ht}(j)} \right) P_{Ht} Y_{Ht} + \eta_t = 0.$$

The envelope condition with respect to the state variable $p_{Ht}(j)$ is given by

$$\dot{\eta}_t = i_t \eta_t - \mathcal{H}_{p_H}$$

where $\mathcal{H}_{p_H}$ is the current valued Hamiltonian differentiated with respect to state variable $p_{Ht}(j)$:

$$\begin{aligned}
\mathcal{H}_p =& \frac{\partial}{\partial p_H} \left(\left[p_{Ht}(j) \left(\frac{p_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} - \frac{W_t}{A_t} \left(\frac{p_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} \right] - \left[\frac{\theta}{2} \left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)} \right)^2 P_{Ht} Y_{Ht} \right] + \eta_t \dot{p}_{Ht}(j) \right) \\
\leftrightarrow \quad & \frac{\partial}{\partial p_H} \left(\left[(p_{Ht}(j))^{1-\epsilon} \left(\frac{1}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} - \frac{W_t}{A_t} \left(\frac{p_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} \right] - \left[\frac{\theta}{2} \left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)} \right)^2 P_{Ht} Y_{Ht} \right] + \eta_t \dot{p}_{Ht}(j) \right) \\
\leftrightarrow \quad & (1-\epsilon) \frac{p_{Ht}(j)^{-\epsilon}}{P_{Ht}^{-\epsilon}} Y_{Ht} - (-\epsilon) \frac{W_t}{A_t} \frac{p_{Ht}(j)^{-\epsilon-1}}{P_{Ht}^{-\epsilon}} Y_{Ht} - \frac{\theta}{2} (2) \left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)} \right) \left(- \frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)^2} \right) P_{Ht} Y_{Ht} \\
\leftrightarrow \quad & (1-\epsilon) \left(\frac{p_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} + \epsilon \frac{W_t}{A_t} p_{Ht}(j)^{-1} \left(\frac{p_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} + \theta \left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)} \right)^2 \left(\frac{1}{p_{Ht}(j)} \right) P_{Ht} Y_{Ht} \\
\Rightarrow \quad & (1-\epsilon) \left(\frac{p_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} + \epsilon \frac{W_t}{p_{Ht}(j)} \frac{1}{A_t} \left(\frac{p_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} Y_{Ht} + \theta \left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)} \right)^2 \frac{P_{Ht}}{p_{Ht}(j)} Y_{Ht}.
\end{aligned}$$

So plugging this back into the first order condition for the state variable, we get

$$\dot{\eta}_t = i_t\eta_t - \left[(1-\epsilon)\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon}Y_{Ht} + \epsilon\frac{W_t}{p_{Ht}(j)}\frac{1}{A_t}\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon}Y_{Ht} + \theta\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2\frac{P_{Ht}}{p_{Ht}(j)}Y_{Ht}\right].$$

We restrict our attention to symmetric domestic firm pricing.

Assumption 2 (Symmetric Equilibrium)**.** *We restrict attention to the symmetric equilibrium*

$$p_{Ht}(j) = P_{Ht}. \tag{85}$$

The symmetric equilibrium for firm pricing decision

$$\theta\pi_{Ht}Y_{Ht} = \eta_t \tag{86}$$

$$\dot{\eta}_t = i_t\eta_t - \left[(1-\epsilon)Y_{Ht} + \epsilon\frac{W_t}{P_{Ht}}\frac{1}{A_t}Y_{Ht} + \theta\pi_{Ht}^2Y_{Ht}\right]. \tag{87}$$

where we define the producer inflation rate as $\pi_{Ht} \equiv \frac{\dot{P}_{Ht}}{P_{Ht}}$.

Take firm optimality condition for control variable and impose symmetry $p_{Ht}(j) = P_{Ht}$

$$\begin{aligned}
&\theta\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\frac{P_{Ht}}{p_{Ht}(j)}Y_{Ht} = \eta_t \\
\leftrightarrow\quad &\theta\frac{\dot{P}_{Ht}}{P_{Ht}}\frac{P_{Ht}}{P_{Ht}}Y_{Ht} = \eta_t \\
\leftrightarrow\quad &\theta\underbrace{\frac{\dot{P}_{Ht}}{P_{Ht}}}_{=\pi_{Ht}}Y_{Ht} = \eta_t \\
\Rightarrow\quad &\theta\pi_{Ht}Y_{Ht} = \eta_t.
\end{aligned}$$

Take firm optimality condition for state variable

$$\begin{aligned}
\dot{\eta}_t &= i_t\eta_t - \left[(1-\epsilon)\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon}Y_{Ht} + \epsilon\frac{W_t}{p_{Ht}(j)}\frac{1}{A_t}\left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\epsilon}Y_{Ht} + \theta\left(\frac{\dot{p}_{Ht}(j)}{p_{Ht}(j)}\right)^2\frac{P_{Ht}}{p_{Ht}(j)}Y_{Ht}\right] \\
&= i_t\eta_t - \left[(1-\epsilon)\underbrace{\left(\frac{P_{Ht}}{P_{Ht}}\right)^{-\epsilon}}_{=1}Y_{Ht} + \epsilon\frac{W_t}{P_{Ht}}\frac{1}{A_t}\underbrace{\left(\frac{P_{Ht}}{P_{Ht}}\right)^{-\epsilon}}_{=1}Y_{Ht} + \theta\Bigg(\underbrace{\frac{\dot{P}_{Ht}}{P_{Ht}}}_{\equiv\pi_{Ht}}\Bigg)^2\underbrace{\frac{P_{Ht}}{P_{Ht}}}_{=1}Y_{Ht}\right] \\
&= i_t\eta_t - \left[(1-\epsilon)Y_{Ht} + \epsilon\frac{W_t}{P_{Ht}}\frac{1}{A_t}Y_{Ht} + \theta\pi_{Ht}^2Y_{Ht}\right].
\end{aligned}$$

The price setting of firms implies that the inflation rate $\pi_{Ht} \equiv \frac{\dot{P}_{Ht}}{P_{Ht}}$ is determined by

$$\left[i_t - \pi_{Ht} - \frac{\dot{Y}_{Ht}}{Y_{Ht}}\right]\pi_{Ht} = \frac{\epsilon-1}{\theta}\left[\frac{\epsilon}{\epsilon-1}\frac{W_t}{P_{Ht}}\frac{1}{A_t} - 1\right] + \dot{\pi}_{Ht}. \tag{88}$$

We want to evaluate the firm's symmetric equilibrium condition:

$$\dot{\eta}_t = i_t\eta_t - \left[(1-\epsilon)Y_{Ht} + \epsilon\frac{W_t}{P_{Ht}}\frac{1}{A_t}Y_{Ht} + \theta\pi_{Ht}^2 Y_{Ht}\right].$$

We can use the firm's symmetric equilibrium condition for the control variable and differentiate with respect to time

$$\begin{aligned}
& \eta_t = \theta\pi_{Ht}Y_{Ht} \\
\leftrightarrow \quad & \frac{d}{dt}\eta_t = \frac{d}{dt}\Big(\theta\pi_{Ht}Y_{Ht}\Big) \\
\leftrightarrow \quad & \dot{\eta}_t = \theta\frac{d}{dt}\Big(\pi_{Ht}Y_{Ht}\Big) \\
\Rightarrow \quad & \dot{\eta}_t = \theta\dot{\pi}_{Ht}Y_{Ht} + \theta\pi_{Ht}\dot{y}_t.
\end{aligned}$$

Now plug this back into the firm's symmetric equilibrium condition

$$\begin{aligned}
& \dot{\eta}_t = i_t\eta_t - \left[(1-\epsilon)Y_{Ht} + \epsilon\frac{W_t}{P_{Ht}}\frac{1}{A_t}Y_{Ht} + \theta\pi_{Ht}^2 Y_{Ht}\right] \\
\leftrightarrow \quad & \theta\dot{\pi}_{Ht}Y_{Ht} + \theta\pi_{Ht}\dot{y}_t = i_t\big[\theta\pi_{Ht}Y_{Ht}\big] - \left[(1-\epsilon)Y_{Ht} + \epsilon\frac{W_t}{P_{Ht}}\frac{1}{A_t}Y_{Ht} + \theta\pi_{Ht}^2 Y_{Ht}\right] \\
\leftrightarrow \quad & \dot{\pi}_{Ht} + \pi_{Ht}\frac{\dot{y}_t}{Y_{Ht}} = i_t\pi_{Ht} - \frac{(1-\epsilon)}{\theta} - \frac{\epsilon}{\theta}\frac{W_t}{P_{Ht}}\frac{1}{A_t} - \pi_{Ht}^2 \\
\leftrightarrow \quad & \dot{\pi}_{Ht} + \frac{(1-\epsilon)}{\theta} + \frac{\epsilon}{\theta}\frac{W_t}{P_{Ht}}\frac{1}{A_t} = i_t\pi_{Ht} - \frac{\dot{y}_t}{Y_{Ht}}\pi_{Ht} - \pi_{Ht}\pi_{Ht} \\
\leftrightarrow \quad & \dot{\pi}_{Ht} + \frac{(1-\epsilon)}{\theta} + \frac{\epsilon}{\theta}\frac{W_t}{P_{Ht}}\frac{1}{A_t} = \left[i_t - \frac{\dot{y}_t}{Y_{Ht}} - \pi_{Ht}\right]\pi_{Ht} \\
\leftrightarrow \quad & \dot{\pi}_{Ht} - \frac{(\epsilon-1)}{\theta} + \frac{\epsilon-1}{\theta}\left(\frac{\epsilon}{\epsilon-1}\frac{W_t}{P_{Ht}}\frac{1}{A_t}\right) = \left[i_t - \frac{\dot{y}_t}{Y_{Ht}} - \pi_{Ht}\right]\pi_{Ht} \\
\Rightarrow \quad & \dot{\pi}_{Ht} + \frac{\epsilon-1}{\theta}\left[\left(\frac{\epsilon}{\epsilon-1}\frac{W_t}{P_{Ht}}\frac{1}{A_t}\right) - 1\right] = \left[i_t - \frac{\dot{y}_t}{Y_{Ht}} - \pi_{Ht}\right]\pi_{Ht}
\end{aligned}$$

D.6 Foreign Final Good Producers

Final good producers are competitive and aggregate a continuum of intermediate inputs. We assume that there is a continuum of measure one of differentiated goods $Y_{Ft}^*(j)$ according to:

$$Y_{Ft}^* = \left[\int_0^1 Y_{Ft}^*(j)^{\frac{\epsilon-1}{\epsilon}}dj\right]^{\frac{\epsilon}{\epsilon-1}} \tag{89}$$

whereby $\epsilon > 1$ is the elasticity of substitution among differentiated goods.

The demand for foreign intermediate goods by the retailer is given by:

$$Y^*_{Ft}(j) = \left[\frac{p^*_{Ft}(j)}{P^*_{Ft}}\right]^{-\epsilon} Y^*_{Ft} \tag{90}$$

whereby we define the aggregate price index $P^*_{Ft} \equiv \left[\int_0^1 p^*_{Ft}(j)^{1-\epsilon} dj\right]^{\frac{1}{1-\epsilon}}$.

D.7 Foreign Intermediate Good Producers

D.7.1 Flexible Price Setting

The production function for wholesale producers is given by:

$$Y^*_{Ft}(j) = A^*_t n^*_t(j) \tag{91}$$

whereby A^*_t is a stationary productivity shock.

The optimal relative price for the wholesale producer is to set relative prices:

$$p^*_{Ft}(j) = \frac{\epsilon}{\epsilon - 1} \frac{W^*_t}{A^*_t}. \tag{92}$$

D.7.2 Sticky Price Setting

Assumption 3 (Symmetric Equilibrium)**.** *We restrict attention to the symmetric equilibrium*

$$p^*_{Ft}(j) = P^*_{Ft}. \tag{93}$$

The price setting of firms implies that the inflation rate $\pi^*_{Ft} \equiv \frac{\dot{P}^*_{Ft}}{P^*_{Ft}}$ is determined by

$$\left[i^*_t - \pi^*_{Ft} - \frac{\dot{y}^*_{Ft}}{Y^*_{Ft}}\right]\pi^*_{Ft} = \frac{\epsilon - 1}{\theta}\left[\frac{\epsilon}{\epsilon - 1}\frac{W^*_t}{P^*_{Ft}}\frac{1}{A^*_t} - 1\right] + \dot{\pi}^*_{Ft}. \tag{94}$$

D.8 Equilibrium

The aggregate home price index is

$$P_{Ht} = \frac{\epsilon}{\epsilon - 1}\frac{W_t}{A_t}. \tag{95}$$

The aggregate foreign price index is

$$P^*_{Ft} = \frac{\epsilon}{\epsilon - 1}\frac{W^*_t}{A^*_t}. \tag{96}$$

Take the home firm optimality condition for pricing

$$p_{Ht}(j) = \frac{\epsilon}{\epsilon - 1} \frac{W_t}{A_t}.$$

Impose the symmetric pricing equilibrium assumption $p_{Ht}(j) = P_{Ht}$. Do the analogous thing for the foreign firm pricing optimality condition

$$p^*_{Ft}(j) = \frac{\epsilon}{\epsilon - 1} \frac{W^*_t}{A^*_t},$$

and impose the symmetry assumption $p^*_{Ft}(j) = P^*_{Ft}$.

We have the standard goods market clearing condition. The goods market clearing conditions for the home and foreign good are

$$Y_{Ht} = C_{Ht} + C^*_{Ht}, \tag{97}$$

$$Y^*_{Ft} = C_{Ft} + C^*_{Ft}. \tag{98}$$

The aggregate resource constraint is

$$Y_{Ht} = A_t N_t, \tag{99}$$

$$Y^*_{Ft} = A^*_t N^*_t. \tag{100}$$

Take the domestic intermediate good firm production function

$$Y_{Ht}(j) = A_t n_t(j),$$

and integrate over j

$$\int_0^1 Y_{Ht}(j) dj = \int_0^1 A_t n_t(j) dj.$$

This gives

$$Y_{Ht} = A_t N_t.$$

Do this for the foreign intermediate good

$$Y_{Ft}(j)^* = A^*_t n^*_t(j),$$

and integrate as well to get

$$Y^*_{Ft} = A^*_t N^*_t.$$

Assumption 4 (Law of One Price)**.** *The law of one price holds*

$$P_{Ht} = E_t P^*_{Ht}. \tag{101}$$

$$P_{Ft} = E_t P^*_{Ft}. \tag{102}$$

$$\tag{103}$$

Remark 5. *The stronger condition of purchasing power parity holds if we assume that the home bias is identical in both countries which then implies*

$$P_t = E_t P^*_t. \tag{104}$$

Lemma 6 (Output)**.** *Output as a function of markups, terms of in both countries is given by*

$$Y_{Ht} = A_t \left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\phi}} \left[(1-\gamma) + \gamma \text{tot}_t^{1-\eta}\right]^{-\frac{1}{1+\phi}} \left\{(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}\right\}^{\frac{1}{1+\phi}}, \tag{105}$$

$$Y^*_{Ft} = A^*_t \left[\frac{\epsilon - 1}{\epsilon}\right]^{\frac{1}{1+\phi^*}} \left[(1-\gamma) + \gamma (\text{tot}^*_t)^{1-\eta}\right]^{-\frac{1}{1+\phi^*}} \left\{(1-\gamma) + \gamma S_t e^{\Xi_t}\right\}^{\frac{1}{1+\phi^*}}. \tag{106}$$

Proof. We know that the domestic price index from equation (95) is

$$P_{Ht} = \frac{\epsilon}{\epsilon - 1} \frac{W_t}{A_t}.$$

Re-arrange this

$$A_t = \frac{\epsilon}{\epsilon - 1} \frac{W_t}{P_{Ht}}.$$

Multiply and divide by P_t to get

$$\frac{W_t}{P_t} = \frac{(\epsilon - 1)}{\epsilon} \frac{P_{Ht}}{P_t} A_t.$$

We also have the household's labour supply decision

$$C_t N_t^{\phi} = \frac{W_t}{P_t}.$$

So combine them to get

$$\begin{aligned} C_t N_t^{\phi} &= \frac{W_t}{P_t} \\ \leftrightarrow \quad C_t N_t^{\phi} &= \frac{(\epsilon - 1)}{\epsilon} \frac{P_{Ht}}{P_t} A_t \end{aligned}$$

Re-arrange for A_t

$$A_t = \frac{\epsilon}{\epsilon - 1} C_t N_t^{\phi} \frac{P_t}{P_{Ht}}.$$

Next, recall the aggregate resource constraint

$$Y_{Ht} = A_t N_t \quad \Rightarrow \quad N_t = \frac{Y_{Ht}}{A_t}.$$

We plug this in

$$A_t = \frac{\epsilon}{\epsilon - 1} C_t \left[\frac{Y_{Ht}}{A_t}\right]^{\phi} \frac{P_t}{P_{Ht}}.$$

Then re-arrange for aggregate home consumption

$$C_t = \frac{\epsilon - 1}{\epsilon} \frac{P_{Ht}}{P_t} A_t^{1+\phi} Y_{Ht}^{-\phi}.$$

We want to find an expression for C_t. The domestic CPI definition is

$$P_t = \left[(1-\gamma) P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{1/(1-\eta)}.$$

Re-arrange

$$\begin{aligned}
& P_t = \left[(1-\gamma) P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{1/(1-\eta)} \\
\leftrightarrow \quad & P_t^{1-\eta} = \left[(1-\gamma) P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right] \\
\leftrightarrow \quad & \left(\frac{P_t}{P_{Ht}}\right)^{1-\eta} = (1-\gamma) + \gamma \left(\frac{P_{Ft}}{P_{Ht}}\right)^{1-\eta} \\
\leftrightarrow \quad & \left(\frac{P_t}{P_{Ht}}\right)^{1-\eta} = (1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta} \\
\leftrightarrow \quad & \left(\frac{P_t}{P_{Ht}}\right) = \left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{\frac{1}{1-\eta}} \\
\Rightarrow \quad & \frac{P_{Ht}}{P_t} = \left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{-\frac{1}{1-\eta}},
\end{aligned}$$

where recall the terms of trade is

$$\mathrm{tot}_t = \frac{P_{Ft}}{P_{Ht}}.$$

The home and foreign demand for the home good is

$$\begin{aligned}
C_{Ht} &= (1-\gamma)\left(\tfrac{P_{Ht}}{P_t}\right)^{-\eta} C_t \\
C_{Ht}^* &= \gamma \left(\tfrac{P_{Ht}^*}{P_t^*}\right)^{-\eta} C_t^*.
\end{aligned}$$

Plug this into the home goods market clearing condition

$$\begin{aligned}
& Y_{Ht} = C_{Ht} + C_{Ht}^* \\
\Rightarrow \quad & Y_{Ht} = (1-\gamma)\left(\tfrac{P_{Ht}}{P_t}\right)^{-\eta} C_t + \gamma \left(\tfrac{P_{Ht}^*}{P_t^*}\right)^{-\eta} C_t^*.
\end{aligned}$$

Since the home bias parameter is the same for both countries, purchasing power parity holds

$$\boxed{P_t = E_t P_t^*.}$$

So then use PPP and the law of one price

$$\boxed{\frac{P_{Ht}^*}{P_t^*} = \frac{P_{Ht}E_t^{-1}}{P_t E_t^{-1}} = \frac{P_{Ht}}{P_t}.}$$

Plug this back in in addition to the terms of trade

$$\begin{aligned}
Y_{Ht} &= (1-\gamma)\left(\tfrac{P_{Ht}}{P_t}\right)^{-\eta} C_t + \gamma \left(\tfrac{P_{Ht}^*}{P_t^*}\right)^{-\eta} C_t^* \\
&= (1-\gamma)\left(\tfrac{P_{Ht}}{P_t}\right)^{-\eta} C_t + \gamma \left(\tfrac{P_{Ht}}{P_t}\right)^{-\eta} C_t^* \\
&= \left(\tfrac{P_{Ht}}{P_t}\right)^{-\eta} \left[(1-\gamma)C_t + \gamma C_t^*\right] \\
&= \left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\gamma)C_t + \gamma C_t^*\right].
\end{aligned}$$

Therefore

$$Y_{Ht} = \left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\gamma)C_t + \gamma C_t^*\right].$$

We know want to use the risk-sharing condition to eliminate C_t^*. First recall the risk-sharing condition

$$\left[\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*}\right] - \dot{S}_t = \rho^* - \rho - \Psi_t^*.$$

Then recall that

$$\begin{aligned}
\frac{\dot{C}_t}{C_t} &= \frac{d}{dt}\left[\ln C_t\right], \\
\frac{\dot{C}_t^*}{C_t^*} &= \frac{d}{dt}\left[\ln C_t^*\right], \\
\dot{S}_t &= \frac{d(\ln S_t)}{dt},
\end{aligned}$$

where for $\dot{S}_t$, it holds by definition. So then the risk-sharing condition is

$$\frac{d}{dt}\left(\ln C_t - \ln C_t^* - \ln S_t\right) = \rho^* - \rho - \Psi_t^*.$$

The next step is to integrate both sides of the differential equation from an initial time $t = 0$ to a later time t. We use u as the integration variable.

$$\int_0^t \frac{d}{du}\left(\ln C_u - \ln C_u^* - \ln S_u\right) du = \int_0^t (\rho^* - \rho - \Psi_u^*) du$$

The integral on the left side, being the integral of a derivative, simplifies to the difference of the function evaluated at the endpoints:

$$[\ln C_u - \ln C_u^* - \ln S_u]_0^t = (\ln C_t - \ln C_t^* - \ln S_t) - (\ln C_0 - \ln C_0^* - \ln S_0)$$

The integral on the right side is:

$$\int_0^t (\rho^* - \rho) du - \int_0^t \Psi_u^* du = (\rho^* - \rho)t - \int_0^t \Psi_u^* du$$

Equating the two sides gives:

$$(\ln C_t - \ln C_t^* - \ln S_t) - (\ln C_0 - \ln C_0^* - \ln S_0) = (\rho^* - \rho)t - \int_0^t \Psi_u^* du$$

Define a constant, K, that collects all the initial condition terms

$$K \equiv \ln C_0 - \ln C_0^* - \ln S_0$$

Moving this to the right side of the integrated equation gives:

$$\ln C_t - \ln C_t^* - \ln S_t = (\rho^* - \rho)t - \int_0^t \Psi_u^* du + K$$

Now introduce Ξ_t where the constant K is absorbed into Ξ_t

$$\Xi_t \equiv (\rho^* - \rho)t - \int_0^t \Psi_u^* du + K$$

Using this definition, the integrated equation is written compactly as:

$$\ln C_t - \ln C_t^* - \ln S_t = \Xi_t$$

The final step is to solve for C_t^* from the logarithmic equation. We first isolate the $\ln C_t^*$ term:

$$\ln C_t^* = \ln C_t - \ln S_t - \Xi_t$$

Now, we exponentiate both sides to remove the logarithms:

$$e^{\ln C_t^*} = e^{(\ln C_t - \ln S_t - \Xi_t)}$$

Using the property that $e^{a+b} = e^a e^b$ and $e^{\ln x} = x$:

$$C_t^* = e^{\ln C_t} \cdot e^{-\ln S_t} \cdot e^{-\Xi_t}$$
$$C_t^* = C_t \cdot (e^{\ln S_t})^{-1} \cdot e^{-\Xi_t}$$

This gives the final expression for foreign consumption C_t^* in level form:

$$C_t^* = C_t S_t^{-1} e^{-\Xi_t}.$$

So if we go back to

$$Y_{Ht} = \left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\gamma)C_t + \gamma\, C_t^*\right].$$

We have

$$Y_{Ht} = \left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\gamma)C_t + \gamma C_t S_t^{-1} e^{-\Xi_t}\right].$$

Which simplifies

$$Y_{Ht} = C_t \left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}\right].$$

So now go back and use

$$C_t = \frac{\epsilon - 1}{\epsilon} \frac{P_{Ht}}{P_t} A_t^{1+\phi} Y_{Ht}^{-\phi}.$$

We therefore have

$$\begin{aligned}
& Y_{Ht} = C_t \left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}\right] \\
\leftrightarrow \quad & Y_{Ht} = \frac{\epsilon - 1}{\epsilon} \frac{P_{Ht}}{P_t} A_t^{1+\phi} Y_{Ht}^{-\phi} \left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}\right] \\
\leftrightarrow \quad & Y_{Ht}^{1+\phi} = \frac{\epsilon - 1}{\epsilon} \frac{P_{Ht}}{P_t} A_t^{1+\phi} \left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}\right] \\
\leftrightarrow \quad & Y_{Ht} = A_t \left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\phi}} \left[\frac{P_{Ht}}{P_t}\right]^{\frac{1}{1+\phi}} \left\{\left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}\right]\right\}^{\frac{1}{1+\phi}} \\
\leftrightarrow \quad & Y_{Ht} = A_t \left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\phi}} \left[\left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{-\frac{1}{1-\eta}}\right]^{\frac{1}{1+\phi}} \left\{\left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}\right]\right\}^{\frac{1}{1+\phi}} \\
\Rightarrow \quad & Y_{Ht} = A_t \left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\phi}} \left[(1-\gamma) + \gamma \mathrm{tot}_t^{1-\eta}\right]^{-\frac{1}{1+\phi}} \left\{(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}\right\}^{\frac{1}{1+\phi}}.
\end{aligned}$$

where in the last line, we used in the exponents

$$\frac{-1}{(1-\eta)(1+\phi)} + \frac{\eta}{(1-\eta)(1+\phi)} = -\frac{1}{1+\phi}.$$

The result for Y_{Ft} holds by symmetry where we use the foreign CPI is

$$P_t^* = \left[(1-\gamma) P_{Ft}^{*\,1-\eta} + \gamma\, P_{Ht}^{*\,1-\eta}\right]^{1/(1-\eta)}.$$

The foreign terms of trade is

$$\mathrm{tot}_t^* = \frac{P_{Ht}^*}{P_{Ft}^*}.$$

$\square$

Proposition 7 (Open-Economy Real Interest Rate)**.** *The domestic natural rate of interest can be decomposed into the standard Ramsey terms and three distinct open-economy adjustment channels:*

$$r_t^n = (\rho + g_t) - \underbrace{\Psi_t}_{\text{Convenience Yield}} + \underbrace{\mathcal{T}_t}_{\text{Terms-of-Trade Channel}} + \underbrace{\mathcal{R}_t}_{\text{Risk-Sharing Channel}} \tag{107}$$

where $g_t \equiv \dot{A}_t/A_t$ is TFP growth and Ψ_t is the convenience yield on domestic bonds. The terms-of-trade channel, $\mathcal{T}_t$, reflects how changes in relative goods prices affect consumption growth, and is given by:

$$\mathcal{T}_t = \left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right) \frac{\gamma(1-\eta)\text{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\text{tot}_t^{1-\eta}} \left(\frac{\dot{\text{tot}}_t}{\text{tot}_t}\right) \tag{108}$$

The international risk-sharing channel, $\mathcal{R}_t$, captures how incomplete financial markets impact consumption growth through exchange rate dynamics and financial frictions, and is given by:

$$\mathcal{R}_t = \frac{\phi}{1+\phi} \frac{\gamma S_t^{-1} e^{-\Xi_t}}{(1-\gamma)+\gamma S_t^{-1} e^{-\Xi_t}} \left(\frac{\dot{S}_t}{S_t} + \dot{\Xi}_t\right). \tag{109}$$

Proof. Take the domestic Euler equation

$$\frac{\dot{C}_t}{C_t} = i_t - \pi_t + \Psi_t - \rho.$$

Re-arrange for the real rate $r_t \equiv i_t - \pi_t$

$$r_t = \rho + \frac{\dot{C}_t}{C_t} - \psi_t.$$

Take the labour supply condition

$$C_t N_t^{\phi} = \frac{W_t}{P_t}.$$

Plug in the resource constraint

$$Y_{Ht} = A_t N_t,$$

into the labour supply condition to get

$$\frac{W_t}{P_t} = C_t \left[\frac{Y_{Ht}}{A_t}\right]^{\phi}.$$

Then plug into the Dixit-stiglitz pricing

$$P_{Ht} = \frac{\epsilon}{\epsilon - 1}\frac{W_t}{A_t},$$

to get

$$P_{Ht} = \frac{\epsilon}{\epsilon - 1}\frac{P_t C_t}{A_t}\left[\frac{Y_{Ht}}{A_t}\right]^{\phi}.$$

Re-arrange for C_t

$$C_t = \frac{\epsilon - 1}{\epsilon}\frac{P_{Ht}}{P_t}A_t^{1+\phi}Y_{Ht}^{-\phi}.$$

We previously showed

$$\frac{P_{Ht}}{P_t} = G_t^{-\frac{1}{1-\eta}}(\text{tot}_t) \equiv \left[(1-\gamma) + \gamma \text{tot}_t^{1-\eta}\right]^{-\frac{1}{1-\eta}},$$

and

$$Y_{Ht} = A_t\left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\phi}}\left[(1-\gamma) + \gamma \text{tot}_t^{1-\eta}\right]^{-\frac{1}{1+\phi}}\left\{(1-\gamma) + \gamma S_t^{-1}e^{-\Xi_t}\right\}^{\frac{1}{1+\phi}}.$$

So therefore

$$haC_t = \frac{\epsilon - 1}{\epsilon}G_t^{-\frac{1}{1-\eta}}(\text{tot}_t)A_t^{1+\phi}\left[A_t\left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\phi}}\left[(1-\gamma) + \gamma \text{tot}_t^{1-\eta}\right]^{-\frac{1}{1+\phi}}\left\{(1-\gamma) + \gamma S_t^{-1}e^{-\Xi_t}\right\}^{\frac{1}{1+\phi}}\right]^{-\phi}.$$

This simplifies to

$$C_t = \left[\frac{\epsilon - 1}{\epsilon}\right]^{\frac{1}{1+\phi}}G_t^{-\frac{1}{1-\eta}}(\text{tot}_t)A_t\left[G_t^{-\frac{1}{1+\phi}}(\text{tot}_t)\left\{(1-\gamma) + \gamma S_t^{-1}e^{-\Xi_t}\right\}^{\frac{1}{1+\phi}}\right]^{-\phi},$$

which becomes

$$C_t = \left[\frac{\epsilon - 1}{\epsilon}\right]^{\frac{1}{1+\phi}}G_t^{\frac{\phi}{1+\phi}-\frac{1}{1-\eta}}(\text{tot}_t)A_t\left\{(1-\gamma) + \gamma S_t^{-1}e^{-\Xi_t}\right\}^{\frac{-\phi}{1+\phi}}.$$

Take logs of both sides

$$\log(C_t) = \frac{1}{1+\phi}\log\left[\frac{\epsilon - 1}{\epsilon}\right] + \left[\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right]\log\left[G_t\right] + \log\left[A_t\right] - \frac{\phi}{1+\phi}\log\left[(1-\gamma) + \gamma S_t^{-1}e^{-\Xi_t}\right]$$

Differentiate with respect to t and use $\frac{\dot{X}_t}{X_t} = \frac{d}{dt}\log(X_t)$. So then we see that the first term on the right-hand side is 0. The third term clearly evaluates to

$$\frac{d}{dt}\log[A_t] = \frac{\dot{A}_t}{A_t}.$$

For the second term, the log expression is:

$$\left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right)\ln\left[G_t\right]$$

To differentiate this, we keep the constant coefficient and differentiate $\ln(G_t)$ using the chain rule:

$$\frac{d}{dt}\left\{\left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right)\ln\left[G_t\right]\right\} = \left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right)\frac{\dot{G}_t}{G_t}$$

To find $\frac{\dot{G}_t}{G_t}$, we first differentiate $G_t = (1-\gamma) + \gamma \text{tot}_t^{1-\eta}$ with respect to time:

$$\dot{G}_t = \frac{dG_t}{dt} = 0 + \gamma(1-\eta)\text{tot}_t^{-\eta} \cdot \dot{\text{tot}}_t$$

Therefore, the growth rate is:

$$\begin{aligned}
\frac{\dot{G}_t}{G_t} &= \frac{\gamma(1-\eta)\text{tot}_t^{-\eta}\dot{\text{tot}}_t}{(1-\gamma)+\gamma\text{tot}_t^{1-\eta}} \\
&= \frac{\gamma(1-\eta)\text{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\text{tot}_t^{1-\eta}} \left(\frac{\dot{\text{tot}}_t}{\text{tot}_t}\right)
\end{aligned}$$

Hence

$$\frac{\dot{G}_t}{G_t} = \frac{\gamma(1-\eta)\text{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\text{tot}_t^{1-\eta}} \left(\frac{\dot{\text{tot}}_t}{\text{tot}_t}\right).$$

For the fourth term, we have

$$-\frac{\phi}{1+\phi} \ln\left[(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}\right]$$

Let's define $Z_t \equiv (1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}$ for clarity. The term is $-\frac{\phi}{1+\phi}\ln(Z_t)$. The derivative is

$$-\frac{\phi}{1+\phi}\frac{\dot{Z}_t}{Z_t}.$$

To find $\dot{Z}_t$, we differentiate Z_t with respect to time, which requires the product rule on $S_t^{-1}e^{-\Xi_t}$:

$$\begin{aligned}
\dot{Z}_t = \frac{dZ_t}{dt} &= \gamma\left[\left(\frac{d}{dt}S_t^{-1}\right)e^{-\Xi_t} + S_t^{-1}\left(\frac{d}{dt}e^{-\Xi_t}\right)\right] \\
&= \gamma\left[(-S_t^{-2}\dot{S}_t)e^{-\Xi_t} + S_t^{-1}(e^{-\Xi_t}(-\dot{\Xi}_t))\right]
\end{aligned}$$

Factoring out common terms gives:

$$\dot{Z}_t = -\gamma S_t^{-1} e^{-\Xi_t}\left(\frac{\dot{S}_t}{S_t} + \dot{\Xi}_t\right)$$

Therefore, the growth rate of Z_t is:

$$\frac{\dot{Z}_t}{Z_t} = \frac{-\gamma S_t^{-1} e^{-\Xi_t}}{(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}}\left(\frac{\dot{S}_t}{S_t} + \dot{\Xi}_t\right)$$

Combine the results to arrive at consumption growth rate being

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{A}_t}{A_t} + \left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right)\frac{\dot{G}_t}{G_t} - \frac{\phi}{1+\phi}\frac{\dot{Z}_t}{Z_t}.$$

And plug in our expressions to get

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{A}_t}{A_t} + \left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right) \frac{\gamma(1-\eta)\text{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\text{tot}_t^{1-\eta}} \left(\frac{\dot{\text{tot}}_t}{\text{tot}_t}\right) + \frac{\phi}{1+\phi} \frac{\gamma S_t^{-1} e^{-\Xi_t}}{(1-\gamma)+\gamma S_t^{-1} e^{-\Xi_t}} \left(\frac{\dot{S}_t}{S_t} + \dot{\Xi}_t\right).$$

So then go back to the Euler equation

$$r_t = \rho + \frac{\dot{C}_t}{C_t} - \psi_t.$$

And plug in our expression for consumption growth

$$r_t = \rho + \frac{\dot{A}_t}{A_t} + \left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right) \frac{\dot{G}_t}{G_t} - \frac{\phi}{1+\phi} \frac{\dot{Z}_t}{Z_t} - \psi_t.$$

Finally for notation, define the terms-of-trade channel $\mathcal{T}_t \equiv \frac{\dot{G}_t}{G_t}$ and the risk-sharing channel $\mathcal{R}_t \equiv \frac{\dot{Z}_t}{Z_t}$. Also this is the real rate under the flexible setting so it is actually the natural rate

$$r_t^n = \rho + \frac{\dot{A}_t}{A_t} + \left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right) \frac{\dot{G}_t}{G_t} - \frac{\phi}{1+\phi} \frac{\dot{Z}_t}{Z_t} - \psi_t.$$

□

Remark 8. *Note that as $\gamma \to 0$ where we have full home-bias, this formula collapses back into the closed economy setting $r_t = \rho - \Psi_t + \dot{A}_t/A_t$. Additionally, if we have unit elasticity $\eta \to 1$, we remove the terms-of-trade dynamics.*

Theorem 9 (Integral Representation of the Non-Linear Phillips Curves)**.** *Home producer price inflation, π_{Ht}, is the expected present discounted value of future real marginal costs:*

$$\pi_{Ht} = \int_t^\infty e^{-\int_t^u \left(i_s - \frac{\dot{y}_{Hs}}{y_{Hs}} - \pi_{Hs}\right) ds} \frac{\epsilon - 1}{\theta} (mc_u - 1) du, \tag{110}$$

where the real marginal cost, mc_t, is given by:

$$mc_t \equiv \frac{\epsilon}{\epsilon - 1} \frac{W_t}{P_{Ht} A_t}. \tag{111}$$

Foreign producer price inflation, $\pi^*_{F,t}$, is the expected present discounted value of future foreign real marginal costs:

$$\pi^*_{F,t} = \int_t^\infty e^{-\int_t^u \left(i^*_s - \frac{\dot{y}^*_{F,s}}{y^*_{Fs}} - \pi^*_{Fs}\right) ds} \frac{\epsilon - 1}{\theta} (mc^*_u - 1) du, \tag{112}$$

where the foreign real marginal cost, mc^*_t, is given by:

$$mc^*_t \equiv \frac{\epsilon}{\epsilon - 1} \frac{W^*_t}{P^*_{F,t} A^*_t}. \tag{113}$$

Proof. We start from the price-setting equation derived from the sticky-price producer firm's problem:

$$\dot{\pi}_{Ht} + \frac{\epsilon-1}{\theta}\left[\frac{\epsilon}{\epsilon-1}\frac{W_t}{P_{Ht}}\frac{1}{A_t} - 1\right] = \left[i_t - \frac{\dot{Y}_{Ht}}{Y_{Ht}} - \pi_{Ht}\right]\pi_{Ht}$$

This is a non-linear Riccati equation . To solve it without linearization, we can use an integrating factor as it is a first-order ODE that is quadratic in inflation π_{Ht}. Move all terms involving π_{Ht} to the left side and define the firm's markup-adjusted real marginal cost as $mc_t \equiv \frac{\epsilon}{\epsilon-1}\frac{W_t}{P_{Ht}A_t}$.

$$\dot{\pi}_{Ht} - \left[i_t - \frac{\dot{Y}_{Ht}}{Y_{Ht}} - \pi_{Ht}\right]\pi_{Ht} = -\frac{\epsilon-1}{\theta}\left(mc_t - 1\right)$$

We define the firm's full, effective discount rate as δ_t. This rate is endogenous because it depends on inflation itself:

$$\delta_t \equiv i_t - \frac{\dot{Y}_{Ht}}{Y_{Ht}} - \pi_{Ht}$$

Substituting this definition allows us to write the equation in a form suitable for an integrating factor:

$$\dot{\pi}_{Ht} - \delta_t\pi_{Ht} = -\frac{\epsilon-1}{\theta}\left(mc_t - 1\right)$$

The integrating factor, I_t, is defined based on the coefficient of the π_{Ht} term:

$$I_t = e^{\int_0^t -\delta_s ds} = e^{-\int_0^t\left(i_s - \frac{\dot{y}_{Hs}}{y_{Hs}} - \pi_{Hs}\right)ds}$$

Multiplying the equation by the integrating factor I_t allows the left side to be expressed as a single derivative. We then integrate from t to ∞ to find the stable forward-looking solution.

$$\begin{aligned}
&\dot{\pi}_{Ht} - \delta_t\pi_{Ht} = -\frac{\epsilon-1}{\theta}\left(mc_t - 1\right)\\
\leftrightarrow\quad & e^{\int_0^t -\delta_s ds}\dot{\pi}_{Ht} - e^{\int_0^t -\delta_s ds}\delta_t\pi_{Ht} = -e^{\int_0^t -\delta_s ds}\frac{\epsilon-1}{\theta}\left(mc_t - 1\right)\\
\leftrightarrow\quad & \frac{d}{dt}\left(e^{-\int_0^t \delta_s ds}\pi_{Ht}\right) = -e^{-\int_0^t \delta_s ds}\frac{\epsilon-1}{\theta}(mc_t - 1)\\
\leftrightarrow\quad & \int_t^\infty \frac{d}{du}\left(e^{-\int_0^u \delta_s ds}\pi_{Hu}(u)\right)du = -\int_t^\infty e^{-\int_0^u \delta_s ds}\frac{\epsilon-1}{\theta}(mc_u - 1)du\\
\leftrightarrow\quad & \left[e^{-\int_0^u \delta_s ds}\pi_{Hu}(u)\right]_t^\infty = -\int_t^\infty e^{-\int_0^u \delta_s ds}\frac{\epsilon-1}{\theta}(mc_u - 1)du
\end{aligned}$$

Applying the no-bubble (transversality) condition, $\lim_{u\to\infty} e^{-\int_0^u \delta_s ds}\pi_{Hu}(u) = 0$, gives:

$$-e^{-\int_0^t \delta_s ds}\pi_{Ht} = -\int_t^\infty e^{-\int_0^u \delta_s ds}\frac{\epsilon-1}{\theta}(mc_u - 1)du$$

$$\pi_{Ht} = e^{\int_0^t \delta_s ds}\int_t^\infty e^{-\int_0^u \delta_s ds}\frac{\epsilon-1}{\theta}(mc_u - 1)du$$

Combining the exponential terms gives the final integral equation for inflation:

$$\pi_{Ht} = \int_t^\infty e^{-\int_t^u \delta_s ds} \frac{\epsilon - 1}{\theta}(mc_u - 1)du$$

Substituting the definition of δ_s back in yields the complete, implicit solution:

$$\pi_{Ht} = \int_t^\infty e^{-\int_t^u \left(i_s - \frac{\dot{y}_{Hs}}{y_{Hs}} - \pi_{Hs}\right)ds} \frac{\epsilon - 1}{\theta}(mc_u - 1)du.$$

□

Proposition 10 (Output Growth)**.** *Output growth is given by IS term, terms-of-trade and risk-sharing channels*

$$\frac{\dot{Y}_{Ht}}{Y_{Ht}} = (i_t - \pi_t + \Psi_t - \rho) - \frac{(1+\phi)}{1+\phi\eta}\mathcal{T}_t - \frac{1+\phi}{\phi}\mathcal{R}_t. \tag{114}$$

Foreign output growth is given by the foreign IS term, and the corresponding terms-of-trade and risk-sharing channels:

$$\frac{\dot{Y}^*_{F,t}}{Y^*_{F,t}} = (i^*_t - \pi^*_t + \Psi^*_t - \rho^*) - \frac{1+\phi^*}{1+\phi^*\eta}\mathcal{T}^*_t - \frac{1+\phi^*}{\phi^*}\mathcal{R}^*_t. \tag{115}$$

Proof. The market-clearing condition for home-produced goods is that they must be purchased by either domestic or foreign households:

$$Y_{Ht} = C_{Ht} + C^*_{Ht}$$

The derivation starts with the equation for total home output (Y_{Ht}), which is determined by the sum of home and foreign demand. This is expressed in terms of each country's total consumption basket (C_t and C^*_t) and the terms of trade (tot_t).

$$Y_{Ht} = \left[(1-\gamma) + \gamma\,\text{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \left[(1-\gamma)C_t + \gamma\, C^*_t\right]$$

Use the risk-sharing condition to express foreign consumption (C^*_t) in terms of domestic consumption (C_t), the real exchange rate (S_t), and the risk-sharing wedge (Ξ_t)

$$C^*_t = C_t S_t^{-1} e^{-\Xi_t}$$

Combine the risk-sharing condition and the goods market clearing condition. We then get

$$Y_{Ht} = C_t \cdot \left[(1-\gamma) + \gamma\,\text{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}} \cdot \left[(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}\right]$$

To get from levels to growth rates, we take the natural logarithm of the equation and then differentiate with respect to time. Using the rule that for any variable X, $d(\ln(X))/dt = \dot{X}/X$, we

get:
$$\frac{\dot{Y}_{Ht}}{Y_{Ht}} = \frac{\dot{C}_t}{C_t} + \frac{d}{dt}\left(\ln\left(\left[(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}}\right)\right) + \frac{d}{dt}\left(\ln\left((1-\gamma)+\gamma S_t^{-1}e^{-\Xi_t}\right)\right)$$
This equation shows that the growth rate of domestic output is equal to the growth rate of domestic consumption plus two adjustment terms that capture open-economy effects. Then recall the domestic household's Euler equation

$$\frac{\dot{C}_t}{C_t} = i_t - \pi_t + \Psi_t - \rho.$$

Now, we can replace the consumption growth rate term, $\frac{\dot{C}_t}{C_t}$, with the domestic household's Euler equation:

$$\begin{aligned}\frac{\dot{Y}_{Ht}}{Y_{Ht}} = \underbrace{(i_t - \pi_t + \Psi_t - \rho)}_{\text{Standard IS Channel (from Consumption)}} &+ \underbrace{\frac{d}{dt}\left(\ln\left(\left[(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}}\right)\right)}_{\text{Terms-of-Trade Channel}} \\ &+ \underbrace{\frac{d}{dt}\left(\ln\left((1-\gamma)+\gamma S_t^{-1}e^{-\Xi_t}\right)\right)}_{\text{Expenditure-Switching Channel}}\end{aligned}$$

For the terms-of-trade channel, we need to differentiate

$$\ln\left(\left[(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}}\right) = \frac{\eta}{1-\eta}\ln\left(\left[(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}\right]\right) = \frac{\eta}{1-\eta}\ln G_t.$$

We previously showed

$$\frac{d}{dt}\ln G_t = \frac{\dot{G}_t}{G_t} = \frac{\gamma(1-\eta)\mathrm{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\mathrm{tot}_t^{1-\eta}}\left(\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t}\right).$$

So then
$$\frac{d}{dt}\frac{\eta}{1-\eta}\ln G_t = \frac{\eta\gamma\mathrm{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\mathrm{tot}_t^{1-\eta}}\left(\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t}\right).$$

We previously defined

$$\mathcal{T}_t = \left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right)\frac{\gamma(1-\eta)\mathrm{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\mathrm{tot}_t^{1-\eta}}\left(\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t}\right),$$

so then we can write

$$\frac{\eta\gamma\mathrm{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\mathrm{tot}_t^{1-\eta}}\left(\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t}\right) = \left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\right)^{-1}\frac{1}{(1-\eta)}\mathcal{T}_t.$$

Combine the terms inside the parentheses by finding a common denominator, which is $(1+\phi)(1-\eta)$:

$$\begin{aligned}\frac{\phi}{1+\phi}-\frac{1}{1-\eta} &= \frac{\phi(1-\eta)-1(1+\phi)}{(1+\phi)(1-\eta)}\\ &= \frac{\phi-\phi\eta-1-\phi}{(1+\phi)(1-\eta)}\\ &= \frac{-1-\phi\eta}{(1+\phi)(1-\eta)}\end{aligned}$$

Invert the expression:

$$\left(\frac{-1-\phi\eta}{(1+\phi)(1-\eta)}\right)^{-1} = \frac{(1+\phi)(1-\eta)}{-1-\phi\eta} = -\frac{(1+\phi)(1-\eta)}{1+\phi\eta}$$

Multiply by the remaining term and cancel out the $(1-\eta)$ term:

$$\begin{aligned}&\left(-\frac{(1+\phi)(1-\eta)}{1+\phi\eta}\right)\frac{1}{(1-\eta)}\mathcal{T}_t\\ &= -\frac{(1+\phi)}{1+\phi\eta}\mathcal{T}_t.\end{aligned}$$

Therefore we have

$$\ln\left(\left[(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}\right]^{\frac{\eta}{1-\eta}}\right) = -\frac{(1+\phi)}{1+\phi\eta}\mathcal{T}_t.$$

For the expenditure switching channel

$$\frac{d}{dt}\left(\ln\left((1-\gamma)+\gamma S_t^{-1}e^{-\Xi_t}\right)\right) = \frac{d}{dt}\ln Z_t = \frac{\dot{Z}_t}{Z_t} = \frac{-\gamma S_t^{-1}e^{-\Xi_t}}{(1-\gamma)+\gamma S_t^{-1}e^{-\Xi_t}}\left(\frac{\dot{S}_t}{S_t}+\dot{\Xi}_t\right).$$

We previously defined risk-sharing channel

$$\mathcal{R}_t = \frac{\phi}{1+\phi}\frac{\gamma S_t^{-1}e^{-\Xi_t}}{(1-\gamma)+\gamma S_t^{-1}e^{-\Xi_t}}\left(\frac{\dot{S}_t}{S_t}+\dot{\Xi}_t\right).$$

So then

$$\frac{-\gamma S_t^{-1}e^{-\Xi_t}}{(1-\gamma)+\gamma S_t^{-1}e^{-\Xi_t}}\left(\frac{\dot{S}_t}{S_t}+\dot{\Xi}_t\right) = -\frac{1+\phi}{\phi}\mathcal{R}_t.$$

Recall that the last two terms is the terms-of-trade channel $\mathcal{T}_t$ and the risk-sharing channel $\mathcal{R}_t$. We therefore get

$$\frac{\dot{Y}_{Ht}}{Y_{Ht}} = (i_t-\pi_t+\Psi_t-\rho)-\frac{(1+\phi)}{1+\phi\eta}\mathcal{T}_t-\frac{1+\phi}{\phi}\mathcal{R}_t.$$

□

Theorem 11 (IS Curve)**.** *The domestic IS curve is*

$$\dot{x}_t = (i_t-\pi_t-\rho)-g_t+\Psi_t-\mathcal{T}_t-\mathcal{R}_t, \tag{116}$$

where $\dot{x}_t \equiv \frac{\dot{X}_t}{X_t}$ where x_t is the log deviation of actual output from its natural level:

$$x_t \equiv \ln(Y_{Ht}) - \ln(Y_{Ht}^n),$$

and recall that g_t is TFP growth, Ψ_t is convenience yield, $\mathcal{T}_t$ is the terms of trade effect and $\mathcal{R}_t$ is the risk-sharing channel. The foreign IS curve is

$$\dot{x}_t^* = (i_t^* - \pi_t^* - \rho^*) - g_t^* + \Psi_t^* - \mathcal{T}_t^* - \mathcal{R}_t^*. \tag{117}$$

Proof. The natural level of home output under flexible prices is

$$Y_{Ht}^n = A_t \left(\frac{\epsilon - 1}{\epsilon}\right)^{\frac{1}{1+\phi}} \left[(1-\gamma) + \gamma \text{tot}_t^{1-\eta}\right]^{-\frac{1}{1+\phi}} \left\{(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}\right\}^{\frac{1}{1+\phi}}.$$

Then the evolution of the natural rate of output is

$$\frac{\dot{Y}_{Ht}^n}{Y_{Ht}^n} = (r_t^n + \Psi_t - \rho) - \frac{(1+\phi)}{1+\phi\eta}\mathcal{T}_t - \frac{1+\phi}{\phi}\mathcal{R}_t.$$

Actual output growth is given by

$$\frac{\dot{Y}_{Ht}}{Y_{Ht}} = (i_t - \pi_t + \Psi_t - \rho) - \frac{1+\phi}{1+\phi\eta}\mathcal{T}_t - \frac{1+\phi}{\phi}\mathcal{R}_t.$$

Now define the output gap x_t as the log deviation of actual output from its natural level:

$$x_t \equiv \ln(Y_{Ht}) - \ln(Y_{Ht}^n).$$

The evolution of the output gap ($\dot{x}_t$) is the difference between the growth rate of actual output and the growth rate of natural output:

$$\dot{x}_t = \frac{\dot{Y}_{Ht}}{Y_{Ht}} - \frac{\dot{Y}_{Ht}^n}{Y_{Ht}^n}$$

Substituting these into the equation for $\dot{x}_t$:

$$\begin{aligned} \dot{x}_t = &\left[(i_t - \pi_t + \Psi_t - \rho) - \frac{1+\phi}{1+\phi\eta}\mathcal{T}_t - \frac{1+\phi}{\phi}\mathcal{R}_t\right] \\ &- \left[(r_{n,t} + \Psi_t - \rho) - \frac{1+\phi}{1+\phi\eta}\mathcal{T}_t - \frac{1+\phi}{\phi}\mathcal{R}_t\right] \end{aligned}$$

The terms cancel out to leave

$$\dot{x}_t = (i_t - \pi_t) - r_t^n.$$

Now recall we previously showed that the domestic natural rate of interest can be decomposed into the standard Ramsey terms and three distinct open-economy adjustment channels:

$$r_t^n = (\rho + g_t) - \underbrace{\Psi_t}_{\text{Convenience Yield}} + \underbrace{\mathcal{T}_t}_{\text{Terms-of-Trade Channel}} + \underbrace{\mathcal{R}_t}_{\text{Risk-Sharing Channel}}$$

where $g_t \equiv \dot{A}_t / A_t$ is TFP growth and Ψ_t is the convenience yield on domestic bonds. The terms-of-trade channel, $\mathcal{T}_t$, reflects how changes in relative goods prices affect consumption growth, and is given by:

$$\mathcal{T}_t = \left(\frac{\phi}{1+\phi} - \frac{1}{1-\eta} \right) \frac{\gamma(1-\eta)\text{tot}_t^{1-\eta}}{(1-\gamma) + \gamma \text{tot}_t^{1-\eta}} \left(\frac{\dot{\text{tot}}_t}{\text{tot}_t} \right)$$

The international risk-sharing channel, $\mathcal{R}_t$, captures how incomplete financial markets impact consumption growth through exchange rate dynamics and financial frictions, and is given by:

$$\mathcal{R}_t = \frac{\phi}{1+\phi} \frac{\gamma S_t^{-1} e^{-\Xi_t}}{(1-\gamma) + \gamma S_t^{-1} e^{-\Xi_t}} \left(\frac{\dot{S}_t}{S_t} + \dot{\Xi}_t \right).$$

So plug this into the IS curve to get

$$\dot{x}_t = (i_t - \pi_t - \rho) - g_t + \Psi_t - \mathcal{T}_t - \mathcal{R}_t.$$

□

The output gap expands ($\dot{x}_t > 0$) whenever the actual real interest rate is below the natural real interest rate, and contracts when it is above. This is the central equation linking monetary policy (via nominal interest rate i_t) to the real economy (via x_t).

Theorem 12 (Linearized NKPC)**.** *The home linearised New Keynesian Phillips curve is*

$$\dot{\hat{\pi}}_{Ht} = -\bar{\delta}_H \, \hat{\pi}_{Ht} + \kappa \left[\tilde{c}_t + \phi x_t + \gamma \tau_t \right], \tag{118}$$

where $\bar{\delta}_H \equiv \bar{i} - \overline{\dot{Y}_{Ht}/Y_{Ht}} - \bar{\pi}_H$, $\tilde{c}_t \equiv \hat{C}_t - \hat{C}_t^n$ *is the consumption gap,* x_t *is the output gap,* $\tau_t \equiv \ln \text{tot}_t - \ln \text{tot}_t^n$ *is the terms-of-trade gap.*
The foreign country New Keynesian Phillips curve is

$$\dot{\hat{\pi}}^*_{Ft} = -\bar{\delta}_F \, \hat{\pi}^*_{Ft} + \kappa \left[\tilde{c}^*_t + \phi^* x^*_t + \gamma \tau^*_t \right], \tag{119}$$

Proof. First start from the Rotemberg price-setting ODE

$$\dot{\pi}_{Ht} + \left(i_t - \tfrac{\dot{Y}_{Ht}}{Y_{Ht}} - \pi_{Ht} \right) \pi_{Ht} = \frac{\epsilon - 1}{\theta} \left(\frac{\epsilon}{\epsilon - 1} \frac{W_t}{P_{Ht} A_t} - 1 \right).$$

To simplify notation, define

$$\delta_t = i_t - \tfrac{\dot{Y}_{Ht}}{Y_{Ht}} - \pi_{Ht},$$
$$\mathrm{mc}_t = \frac{\epsilon}{\epsilon - 1}\frac{W_t}{P_{Ht}A_t},$$
$$\kappa = \frac{\epsilon - 1}{\theta}.$$

Therefore we have

$$\dot{\pi}_{Ht} = -\delta_t \pi_{Ht} + \kappa\big(\mathrm{mc}_t - 1\big).$$

To linearize around steady state, we first write each variable as a steady-state deviation:

$$\pi_{Ht} = \bar{\pi}_H + \hat{\pi}_{Ht}, \qquad \delta_t = \bar{\delta}_H + \hat{\delta}_t, \qquad \mathrm{mc}_t = 1 + \widehat{\mathrm{mc}}_t,$$

with bars denoting steady state values and hats denoting first-order deviations. Recall that linearisation is invariant to an affine relabelling of variables. Recall that the steady state marginal cost is $\overline{\mathrm{mc}} = 1$. Since $\bar{\pi}_H$ is constant,

$$\dot{\pi}_{Ht} = \dot{\hat{\pi}}_{Ht}.$$

Substitute the expressions in:

$$\dot{\hat{\pi}}_{Ht} = -(\bar{\delta}_H + \hat{\delta}_t)(\bar{\pi}_H + \hat{\pi}_{Ht}) + \kappa\big((1 + \widehat{\mathrm{mc}}_t) - 1\big).$$

Expand the product:

$$\dot{\hat{\pi}}_{Ht} = -\bar{\delta}_H\bar{\pi}_H - \bar{\delta}_H\hat{\pi}_{Ht} - \bar{\pi}_H\hat{\delta}_t - \hat{\delta}_t\,\hat{\pi}_{Ht} + \kappa\,\widehat{\mathrm{mc}}_t.$$

By linearisation, we drop the second-order term $-\hat{\delta}_t\,\hat{\pi}_{Ht}$. Now impose *zero steady-state inflation*, $\bar{\pi}_H = 0$. Then

$$\boxed{\dot{\hat{\pi}}_{Ht} = -\bar{\delta}_H\,\hat{\pi}_{Ht} + \kappa\,\widehat{\mathrm{mc}}_t.}$$

Also, we know that under flexible prices, the natural (flexible price) marginal cost is

$$\mathrm{mc}_t^n = 1.$$

Also recall that marginal cost at steady state $\overline{\mathrm{mc}} = 1$. So then the natural marginal cost deviation is 0

$$\widehat{\mathrm{mc}_t^n} = \ln\left(\mathrm{mc}_t^n\right) - \ln(\overline{\mathrm{mc}^n}) = \ln(1) - \ln(1) = 0.$$

So then the marginal cost gap is equal to the marginal cost deviation from steady state

$$\begin{aligned}
\widehat{\mathrm{mc}}_t^{\mathrm{gap}} &\equiv \ln(\mathrm{mc}_t) - \ln(\mathrm{mc}_t^n) \\
&= \ln(\mathrm{mc}_t) - 0 \\
&= \ln(\mathrm{mc}_t) \\
&= \ln(\mathrm{mc}_t) - \underbrace{\ln(\overline{\mathrm{mc}})}_{=0} \\
&= \widehat{\mathrm{mc}}_t.
\end{aligned}$$

Therefore

$$\boxed{\dot{\hat{\pi}}_{Ht} = -\bar{\delta}_H \, \hat{\pi}_{Ht} + \kappa \, \widehat{\mathrm{mc}}_t^{\mathrm{gap}}.}$$

This is the desired first-order linearization around a zero-inflation steady state.

Next, linearize real marginal cost. Using intratemporal optimality $W_t/P_t = C_t N_t^{\phi}$ and rewrite as

$$\frac{W_t}{P_{Ht}} = C_t N_t^{\phi} \frac{P_t}{P_{Ht}}.$$

Linearise to get

$$\Rightarrow \quad \widehat{W}_t - \widehat{P}_{Ht} = \widehat{C}_t + \phi \, \widehat{N}_t + (\widehat{P_t/P_{Ht}}).$$

Take the marginal cost

$$\mathrm{mc}_t = \frac{\epsilon}{\epsilon - 1} \frac{W_t}{P_{Ht} A_t}.$$

Linearise this and substitute in the household optimality condition

$$\widehat{\mathrm{mc}}_t = (\widehat{W}_t - \widehat{P}_{Ht}) - \widehat{A}_t = \widehat{C}_t + \phi \, \widehat{N}_t + (\widehat{P_t/P_{Ht}}) - \widehat{A}_t.$$

Since $Y_{Ht} = A_t N_t$, then $\widehat{N}_t = \widehat{Y}_{Ht} - \widehat{A}_t$, giving

$$\widehat{\mathrm{mc}}_t = \widehat{C}_t + \phi \, \widehat{Y}_{Ht} + (\widehat{P_t/P_{Ht}}) - (1 + \phi) \widehat{A}_t.$$

Analogously, define the natural (flexible price) marginal cost as

$$\widehat{\mathrm{mc}}_t^n = \widehat{C}_t^n + \phi \, \widehat{Y}_{Ht}^n + \widehat{\left(\frac{P_t^n}{P_{Ht}^n} \right)} - (1 + \phi) \widehat{A}_t^n.$$

And noting that $\widehat{A}_t = \widehat{A}_t^n$, we have that the marginal cost gap

$$\widehat{\mathrm{mc}}_t^{\mathrm{gap}} \equiv \widehat{\mathrm{mc}}_t - \widehat{\mathrm{mc}}_t^n = \left(\widehat{C}_t - \widehat{C}_t^n \right) + \left(\phi \, \widehat{Y}_{Ht} - \phi \, \widehat{Y}_{Ht}^n \right) + \left(\widehat{\frac{P_t}{P_{Ht}}} - \widehat{\left(\frac{P_t^n}{P_{Ht}^n} \right)} \right).$$

We now want to log-linearise the CPI–PPI wedge P_t/P_{Ht} and map it into the terms of trade. With

$$P_t = \left[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}, \qquad \mathrm{tot}_t \equiv \frac{P_{F,t}}{P_{Ht}},$$

we have

$$\ln\frac{P_t}{P_{Ht}} = \frac{1}{1-\eta}\ln\left[(1-\gamma) + \gamma\,\mathrm{tot}_t^{1-\eta}\right].$$

Therefore taking the partial derivative

$$\frac{\partial \ln(P_t/P_{Ht})}{\partial \ln \mathrm{tot}_t} = \frac{\gamma\,\mathrm{tot}_t^{1-\eta}}{(1-\gamma) + \gamma\,\mathrm{tot}_t^{1-\eta}},$$

and, evaluating at a symmetric steady state where $\overline{P}_H = \overline{P}_F$, this implies $\overline{\mathrm{tot}} = 1$, and therefore steady state

$$\left.\frac{\partial \ln(P_t/P_{Ht})}{\partial \ln \mathrm{tot}_t}\right|_{\overline{\mathrm{tot}}=1} = \frac{\gamma\,(1)^{1-\eta}}{(1-\gamma) + \gamma\,(1)^{1-\eta}} = \gamma.$$

So then a first-order Taylor expansion of $\ln\left(\frac{P_t}{P_{Ht}}\right)$ is

$$\ln\left(\frac{P_t}{P_{Ht}}\right) \approx \ln\left(\frac{\overline{P}}{\overline{P}_H}\right) + \gamma\left(\ln \mathrm{tot}_t - \ln \mathrm{tot}\right)$$

In the natural (flexible-price) economy, this is

$$\ln\left(\frac{P_t^n}{P_{Ht}^n}\right) \approx \ln\left(\frac{\overline{P^n}}{\overline{P_H^n}}\right) + \gamma\left(\ln \mathrm{tot}_t^n - \ln \mathrm{tot}^n\right).$$

Then if we take the difference

$$\ln\left(\frac{P_t}{P_{Ht}}\right) - \ln\left(\frac{P_t^n}{P_{Ht}^n}\right) = \gamma\left[(\ln \mathrm{tot}_t - \ln \mathrm{tot}) - (\ln \mathrm{tot}_t^n - \ln \mathrm{tot}^n)\right] = \gamma\left[\ln \mathrm{tot}_t - \ln \mathrm{tot}_t^n\right].$$

So therefore

$$\underbrace{\ln\left(\frac{P_t}{P_{Ht}}\right) - \ln\left(\frac{P_t^n}{P_{Ht}^n}\right)}_{\equiv\left(\frac{P_t}{P_{Ht}}\right)^{\mathrm{gap}}} = \gamma\left[\underbrace{\ln \mathrm{tot}_t - \ln \mathrm{tot}_t^n}_{\equiv \tau_t}\right],$$

where we define the terms-of-trade gap as

$$\tau_t \equiv \ln \mathrm{tot}_t - \ln \mathrm{tot}_t^n,$$

and the CPI-PPI wedge gap

$$\left(\frac{P_t}{P_{Ht}}\right)^{\mathrm{gap}} \equiv \ln\left(\frac{P_t}{P_{Ht}}\right) - \ln\left(\frac{P_t^n}{P_{Ht}^n}\right).$$

So therefore the CPI-PPI wedge gap is

$$\left(\frac{P_t}{P_{Ht}}\right)^{\text{gap}} = \gamma\tau_t.$$

Then use

$$\left(\widehat{\frac{P_t}{P_{Ht}}} - \widehat{\left(\frac{P_t^n}{P_{Ht}^n}\right)}\right) = \left(\frac{P_t}{P_{Ht}}\right)^{\text{gap}}.$$

So then the marginal cost gap becomes

$$\widehat{\text{mc}}_t^{\text{gap}} = \left(\widehat{C}_t - \widehat{C}_t^n\right) + \phi\left(\widehat{Y}_{Ht} - \widehat{Y}_{Ht}^n\right) + \gamma\tau_t.$$

Then recall that the output gap

$$x_t \equiv \ln(Y_{Ht}) - \ln(Y_{Ht}^n) = \widehat{Y}_{Ht} - \widehat{Y}_{Ht}^n.$$

So marginal cost gap is

$$\widehat{\text{mc}}_t^{\text{gap}} = \left(\widehat{C}_t - \widehat{C}_t^n\right) + \phi x_t + \gamma\tau_t.$$

So plug this into the linearised pricing ODE

$$\begin{aligned}
\dot{\hat{\pi}}_{Ht} &= -\bar{\delta}_H\,\hat{\pi}_{Ht} + \kappa\,\widehat{\text{mc}}_t^{\text{gap}} \\
&= -\bar{\delta}_H\,\hat{\pi}_{Ht} + \kappa\left[\left(\underbrace{\widehat{C}_t - \widehat{C}_t^n}_{\equiv\tilde{c}_t}\right) + \phi x_t + \gamma\tau_t\right].
\end{aligned}$$

So therefore

$$\dot{\hat{\pi}}_{Ht} = -\bar{\delta}_H\,\hat{\pi}_{Ht} + \kappa\left[\tilde{c}_t + \phi x_t + \gamma\tau_t\right].$$

□

D.9 Policy

Fiscal Policy Let B_t denote the face value of outstanding nominal government debt, i_t the instantaneous nominal policy rate, P_t the CPI, and G_t real government purchases. With lump-sum taxes T_t (nominal), the consolidated nominal flow budget is

$$\dot{B}_t \;=\; i_t B_t \;+\; P_t G_t \;-\; T_t. \tag{120}$$

Define real debt $b_t \equiv B_t/P_t$ and inflation $\pi_t \equiv \dot{P}_t/P_t$. Let the (real) primary surplus be $s_t \equiv (T_t - P_t G_t)/P_t = T_t/P_t - G_t$. Dividing (120) by P_t and using $\dot{b}_t = \frac{\dot{B}_t}{P_t} - \pi_t b_t$ gives the real debt law of motion

$$\dot{b}_t \;=\; (i_t - \pi_t)\, b_t \;-\; s_t. \tag{121}$$

This linear ODE has integrating factor $\mathcal{I}_t \equiv \exp\big(-\int_0^t (i_\tau - \pi_\tau)\, d\tau\big)$, so $\frac{d}{dt}(\mathcal{I}_t b_t) = -\mathcal{I}_t s_t$. Integrating from t to T and imposing the no-Ponzi/transversality condition $\lim_{T\to\infty} \mathcal{I}_T b_T = 0$ yields the forward solution (government debt valuation)

$$b_t \;=\; \int_t^\infty \exp\Big(-\int_t^u (i_s - \pi_s)\, ds\Big)\, s_u\, du, \qquad \frac{B_t}{P_t} \;=\; \int_t^\infty e^{-\int_t^u i_s ds}\, \frac{T_u - P_u G_u}{P_u}\, du. \tag{122}$$

Monetary Policy Monetary policy follows a continuous-time Taylor rule with an inflation coefficient strictly below one:

$$i_t \;=\; r_t^n \;+\; \bar{\pi} \;+\; \phi_\pi\,(\pi_t - \bar{\pi}) \;+\; \phi_x\, x_t, \qquad 0 \le \phi_\pi < 1, \quad \phi_x \ge 0, \tag{123}$$

where $\bar{\pi}$ is the inflation target, $x_t \equiv \ln(Y_{H,t}) - \ln(Y_{H,t}^n)$ is the output gap, and r_t^n is the open-economy natural real rate. The ex-post real rate entering the discount kernel is

$$i_t - \pi_t \;=\; r_t^n \;+\; (\phi_\pi - 1)\,\hat{\pi}_t \;+\; \phi_x\, x_t, \qquad \hat{\pi}_t \equiv \pi_t - \bar{\pi}. \tag{124}$$

Debt Valuation Substituting (124) into (76) gives

$$b_t \;=\; \int_t^\infty \exp\Big(-\int_t^u \big[r_\tau^n + (\phi_\pi - 1)\hat{\pi}_\tau + \phi_x x_\tau\big]\, d\tau\Big)\, s_u\, du. \tag{125}$$

Evaluated just before issuance at time t (so B_{t-} is predetermined), the CPI adjusts on impact to satisfy

$$\frac{B_{t-}}{P_t} \;=\; \int_t^\infty \exp\Big(-\int_t^u \big[r_\tau^n + (\phi_\pi - 1)\hat{\pi}_\tau + \phi_x x_\tau\big]\, d\tau\Big)\, s_u\, du,$$

pinning down P_t from fiscal solvency when $\phi_\pi < 1$. For Foreign,

$$\frac{B_t^*}{P_t^*} \;=\; \int_t^\infty \exp\Big(-\int_t^u \big[r_\tau^{n*} + (\phi_\pi^* - 1)\hat{\pi}_\tau^* + \phi_x^* x_\tau^*\big]\, d\tau\Big)\, s_u^*\, du.$$

D.10 Equilibrium

Given exogenous processes $\{A_t, A_t^*, s_t, s_t^*, \Psi_t, \Psi_t^*\}$ and initial conditions (including B_{t-}, B_{t-}^*), an equilibrium is a collection of processes

$$\{C_t, N_t, Y_{Ht}, P_t, P_{Ht}, \pi_t, i_t, B_t,\; C_t^*, N_t^*, Y_{Ft}^*, P_t^*, P_{Ft}^*, \pi_t^*, i_t^*, B_t^*,\; E_t,\; \mathrm{tot}_t,\; S_t\}$$

that satisfy the following conditions:

1. **Households (Home and Foreign).**

Intratemporal optimality (labor supply):

$$C_t N_t^{\phi} = \frac{W_t}{P_t}, \qquad C_t^*(N_t^*)^{\phi^*} = \frac{W_t^*}{P_t^*}.$$

Euler equations with convenience yield:

$$\frac{\dot{C}_t}{C_t} = i_t - \pi_t + \Psi_t - \rho, \qquad \frac{\dot{C}_t^*}{C_t^*} = i_t^* - \pi_t^* + \Psi_t^* - \rho^*.$$

UIP and risk sharing:

$$i_t - i_t^* = \frac{\dot{E}_t}{E_t} - \Psi_t, \qquad \left(\frac{\dot{C}_t}{C_t} - \frac{\dot{C}_t^*}{C_t^*}\right) - \frac{\dot{S}_t}{S_t} = \rho^* - \rho - \Psi_t^*,$$

where $S_t \equiv E_t P_t^*/P_t$ is the real exchange rate.

CES demands and CPI (Home):

$$C_{Ht} = (1-\gamma)\left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t, \qquad C_{Ft} = \gamma\left(\frac{P_{Ft}}{P_t}\right)^{-\eta} C_t,$$

$$P_t = \left[(1-\gamma)P_{Ht}^{1-\eta} + \gamma P_{Ft}^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

CES demands and CPI (Foreign, symmetric):

$$C_{Ft}^* = (1-\gamma)\left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta} C_t^*, \qquad C_{Ht}^* = \gamma\left(\frac{P_{Ht}^*}{P_t^*}\right)^{-\eta} C_t^*,$$

$$P_t^* = \left[(1-\gamma)(P_{Ft}^*)^{1-\eta} + \gamma(P_{Ht}^*)^{1-\eta}\right]^{\frac{1}{1-\eta}}.$$

2. **Firms and price setting.**

Final-good aggregation and PPI (Home):

$$P_{Ht} = \left(\int_0^1 p_{Ht}(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}}, \qquad Y_{Ht}(j) = \left(\frac{p_{Ht}(j)}{P_{Ht}}\right)^{-\varepsilon} Y_{Ht}.$$

Foreign (symmetric):

$$P_{Ft}^* = \left(\int_0^1 p_{Ft}^*(j)^{1-\varepsilon}\, dj\right)^{\frac{1}{1-\varepsilon}}, \qquad Y_{Ft}^*(j) = \left(\frac{p_{Ft}^*(j)}{P_{Ft}^*}\right)^{-\varepsilon} Y_{Ft}^*.$$

Technology and feasibility:

$$Y_{Ht} = A_t N_t, \qquad Y_{Ft}^* = A_t^* N_t^*.$$

Real marginal costs:

$$mc_t \equiv \frac{\varepsilon}{\varepsilon-1}\frac{W_t}{P_{Ht}A_t}, \qquad mc_t^* \equiv \frac{\varepsilon}{\varepsilon-1}\frac{W_t^*}{P_{Ft}^*A_t^*}.$$

Rotemberg price setting (exact PPI inflation ODEs):

$$\Big(i_t - \pi_{Ht} - \frac{\dot{Y}_{Ht}}{Y_{Ht}}\Big)\pi_{Ht} = \frac{\varepsilon-1}{\theta}\Big(mc_t - 1\Big) - \dot{\pi}_{Ht},$$

$$\Big(i_t^* - \pi_{Ft}^* - \frac{\dot{Y}_{Ft}^*}{Y_{Ft}^*}\Big)\pi_{Ft}^* = \frac{\varepsilon-1}{\theta}\Big(mc_t^* - 1\Big) - \dot{\pi}_{Ft}^*.$$

Integral (PV) representation of the nonlinear NKPCs:

$$\pi_{Ht} = \int_t^\infty \exp\Big\{-\int_t^u \big[i_s - \tfrac{\dot{Y}_{Hs}}{Y_{Hs}} - \pi_{Hs}\big]\,ds\Big\}\,\frac{\varepsilon-1}{\theta}\,\big(mc_u - 1\big)\,du,$$

$$\pi_{Ft}^* = \int_t^\infty \exp\Big\{-\int_t^u \big[i_s^* - \tfrac{\dot{Y}_{Fs}^*}{Y_{Fs}^*} - \pi_{Fs}^*\big]\,ds\Big\}\,\frac{\varepsilon-1}{\theta}\,\big(mc_u^* - 1\big)\,du.$$

3. **Technology, feasibility, and market clearing.**

   Goods market clearing (each good):

$$Y_{Ht} = C_{Ht} + C_{Ht}^*, \qquad Y_{Ft}^* = C_{Ft} + C_{Ft}^*.$$

   Law of One Price (good level):

$$P_{Ht} = E_t P_{Ht}^*, \qquad P_{Ft} = E_t P_{Ft}^*.$$

   Terms of trade and identity:

$$\mathrm{tot}_t \equiv \frac{P_{Ft}}{P_{Ht}}, \qquad \tau_t \equiv \ln \mathrm{tot}_t = \ln P_{Ft} - \ln P_{Ht}, \qquad \dot{\tau}_t = \pi_{Ft} - \pi_{Ht}.$$

4. **Natural real rate and IS (open economy).**

   Natural real rate decomposition (flex-price allocation):

$$r_t^n = (\rho + g_t) - \Psi_t + T_t + R_t, \qquad g_t \equiv \frac{\dot{A}_t}{A_t},$$

$$T_t = \Big(\frac{\phi}{1+\phi} - \frac{1}{1-\eta}\Big)\,\frac{\gamma(1-\eta)\,\mathrm{tot}_t^{1-\eta}}{(1-\gamma)+\gamma\,\mathrm{tot}_t^{1-\eta}}\,\frac{\dot{\mathrm{tot}}_t}{\mathrm{tot}_t},$$

$$R_t = \frac{\phi}{1+\phi}\,\frac{\gamma\,S_t^{-1}e^{-\Xi_t}}{(1-\gamma)+\gamma\,S_t^{-1}e^{-\Xi_t}}\,\Big(\frac{\dot{S}_t}{S_t} + \dot{\Xi}_t\Big),$$

with $S_t \equiv E_t P_t^*/P_t$. Foreign analogs r_t^{n*}, T_t^*, R_t^* are defined symmetrically.

$$r_t^{n*} = (\rho^* + g_t^*) - \Psi_t^* + T_t^* + R_t^*, \qquad g_t^* \equiv \frac{\dot{A}_t^*}{A_t^*}.$$

$$T_t^* = \Big(\frac{\phi^*}{1+\phi^*} - \frac{1}{1-\eta}\Big)\,\frac{\gamma(1-\eta)\,(\mathrm{tot}_t^*)^{\,1-\eta}}{(1-\gamma)+\gamma(\mathrm{tot}_t^*)^{\,1-\eta}}\,\frac{\dot{\mathrm{tot}}_t^*}{\mathrm{tot}_t^*}, \qquad \mathrm{tot}_t^* \equiv \frac{P_{Ht}^*}{P_{Ft}^*},$$

$$R_t^* = \frac{\phi^*}{1+\phi^*}\,\frac{\gamma\,S_t e^{\Xi_t}}{(1-\gamma)+\gamma S_t e^{\Xi_t}}\Big(\frac{\dot{S}_t}{S_t} + \dot{\Xi}_t\Big), \qquad S_t \equiv \frac{E_t P_t^*}{P_t}.$$

IS curve for the output gap $x_t \equiv \ln Y_{Ht} - \ln Y_{Ht}^n$:

$$\dot{x}_t = (i_t - \pi_t) - r_t^n, \qquad \text{equivalently} \qquad \dot{x}_t = (i_t - \pi_t - \rho) - g_t + \Psi_t - T_t - R_t.$$

IS curve for the foreign output gap $x_t^* \equiv \ln Y_{Ft}^* - \ln(Y_{Ft}^{n,*})$:

$$\dot{x}_t^* = (i_t^* - \pi_t^*) - r_t^{n*}, \qquad \text{equivalently} \qquad \dot{x}_t^* = (i_t^* - \pi_t^* - \rho^*) - g_t^* + \Psi_t^* - T_t^* - R_t^*.$$

5. **Policy blocks.**

Monetary policy (passive Taylor rule):

$$i_t = r_t^n + \bar{\pi} + \phi_\pi(\pi_t - \bar{\pi}) + \phi_x x_t, \quad 0 \le \phi_\pi < 1,\ \phi_x \ge 0,$$

$$i_t - \pi_t = r_t^n + (\phi_\pi - 1)\hat{\pi}_t + \phi_x x_t, \qquad \hat{\pi}_t \equiv \pi_t - \bar{\pi},$$

Monetary policy (passive Taylor rule, foreign):

$$i_t^* = r_t^{n*} + \bar{\pi}^* + \phi_\pi^*(\pi_t^* - \bar{\pi}^*) + \phi_x^* x_t^*, \quad 0 \le \phi_\pi^* < 1,\ \phi_x^* \ge 0,$$

$$i_t^* - \pi_t^* = r_t^{n*} + (\phi_\pi^* - 1)\hat{\pi}_t^* + \phi_x^* x_t^*, \qquad \hat{\pi}_t^* \equiv \pi_t^* - \bar{\pi}^*.$$

Fiscal block (real GBC and valuation):

$$\dot{b}_t = (i_t - \pi_t) b_t - s_t, \qquad b_t \equiv \frac{B_t}{P_t},$$

$$b_t = \int_t^\infty \exp\Big\{-\int_t^u \big[r_\tau^n + (\phi_\pi - 1)\hat{\pi}_\tau + \phi_x x_\tau\big]\,d\tau\Big\}\,s_u\,du,$$

Fiscal block (real GBC and valuation, foreign):

$$\dot{b}_t^* = (i_t^* - \pi_t^*)\,b_t^* - s_t^*, \qquad b_t^* \equiv \frac{B_t^*}{P_t^*},$$

$$b_t^* = \int_t^\infty \exp\Big\{-\int_t^u \big[r_\tau^{n,*} + (\phi_\pi^* - 1)\hat{\pi}_\tau^* + \phi_x^* x_\tau^*\big]\,d\tau\Big\}\,s_u^*\,du.$$

6. **External asset pricing and exchange rate.**

Exchange-rate level as a ratio of fiscal present values (under passive rules):

$$\mathcal{S}_t \equiv \int_t^{\infty} \exp\Big\{ -\int_t^u \big[r_\tau^n + (\phi_\pi - 1)\hat{\pi}_\tau + \phi_x x_\tau\big]\, d\tau \Big\} s_u \, du,$$

$$\mathcal{S}_t^* \equiv \int_t^{\infty} \exp\Big\{ -\int_t^u \big[r_\tau^{n,*} + (\phi_\pi^* - 1)\hat{\pi}_\tau^* + \phi_x^* x_\tau^*\big]\, d\tau \Big\} s_u^* \, du,$$

$$\boxed{E_t = \frac{B_{t-}}{B_{t-}^*} \cdot \frac{\mathcal{S}_t^*}{\mathcal{S}_t}} \quad \text{(under PPP)}.$$

Exchange-rate growth decomposition:

$$\frac{\dot{E}_t}{E_t} = (r_t^{n*} - r_t^n) + (\phi_\pi^* - 1)\hat{\pi}_t^* - (\phi_\pi - 1)\hat{\pi}_t + \phi_x^* x_t^* - \phi_x x_t + \left(\frac{s_t}{\mathcal{S}_t} - \frac{s_t^*}{\mathcal{S}_t^*} \right).$$

D.11 Final set of linearised system

Home block

$$\dot{x}_t = (\phi_\pi - 1)\hat{\pi}_t + \phi_x x_t - \hat{r}_t^n, \tag{126}$$

$$\dot{\hat{\pi}}_{H,t} = -\bar{\delta}_H \hat{\pi}_{H,t} + \kappa\big(\tilde{c}_t + \phi x_t + \gamma \tau_t\big), \tag{127}$$

$$\hat{r}_t^n = (g_t - \bar{g}) - \Psi_t + T_t + R_t. \tag{128}$$

Foreign block (symmetric)

$$\dot{x}_t^* = (\phi_\pi^* - 1)\hat{\pi}_t^* + \phi_x^* x_t^* - \hat{r}_t^{n*}, \tag{129}$$

$$\dot{\hat{\pi}}_{F,t}^* = -\bar{\delta}_F^* \hat{\pi}_{F,t}^* + \kappa^*\big(\tilde{c}_t^* + \phi^* x_t^* + \gamma^* \tau_t\big), \tag{130}$$

$$\hat{r}_t^{n*} = (g_t^* - \bar{g}^*) - \Psi_t^* + T_t^* + R_t^*. \tag{131}$$

Open-economy wedges and identities

$$\tau_t = \ln P_{Ft} - \ln P_{Ht}, \qquad \dot{\tau}_t = \hat{\pi}_{F,t} - \hat{\pi}_{H,t}, \tag{132}$$

$$i_t - i_t^* = \dot{\hat{e}}_t - \Psi_t, \qquad \text{under PPP: } \dot{\hat{e}}_t = \hat{\pi}_t - \hat{\pi}_t^*. \tag{133}$$

Policy (linear form)

$$\hat{i}_t = \hat{r}_t^n + \phi_\pi \hat{\pi}_t + \phi_x x_t, \qquad 0 \le \phi_\pi < 1, \tag{134}$$

$$\hat{i}_t^* = \hat{r}_t^{n*} + \phi_\pi^* \hat{\pi}_t^* + \phi_x^* x_t^*, \qquad 0 \le \phi_\pi^* < 1. \tag{135}$$

Fiscal

$$\dot{\hat{b}}_t = \bar{b}\big[(\phi_\pi - 1)\hat{\pi}_t + \phi_x x_t + \hat{r}^n_t\big] - \hat{s}_t, \qquad \bar{b} \equiv \bar{B}/\bar{P},$$

$$\dot{\hat{b}}^*_t = \bar{b}^*\Big[(\phi^*_\pi - 1)\,\hat{\pi}^*_t + \phi^*_x\, x^*_t + \hat{r}^{n*}_t\Big] - \hat{s}^*_t, \qquad \bar{b}^* \equiv \frac{\bar{B}^*}{\bar{P}^*}$$

Shock processes (OU)

$$\dot{s}_t = -\varphi_s(s_t - \bar{s}) + \sigma_s\,\varepsilon^s_t, \qquad \dot{s}^*_t = -\varphi^*_s(s^*_t - \bar{s}^*) + \sigma^*_s\,\varepsilon^{s^*}_t, \tag{136}$$

$$\dot{g}_t = -\varphi_g(g_t - \bar{g}) + \sigma_g\,\varepsilon^g_t, \qquad \dot{g}^*_t = -\varphi^*_g(g^*_t - \bar{g}^*) + \sigma^*_g\,\varepsilon^{g^*}_t, \tag{137}$$

$$\dot{\Psi}_t = -\varphi_\Psi \Psi_t + \sigma_\Psi\,\varepsilon^\Psi_t, \qquad \dot{\Psi}^*_t = -\varphi^*_\Psi \Psi^*_t + \sigma^*_\Psi\,\varepsilon^{\Psi^*}_t. \tag{138}$$

Compact state space. Stack $z_t \equiv [x_t, \hat{\pi}_{H,t}, x^*_t, \hat{\pi}^*_{F,t}, \tau_t, \hat{e}_t, \hat{b}_t, \hat{b}^*_t]'$, shocks $u_t \equiv [g_t - \bar{g}, g^*_t - \bar{g}^*, \Psi_t, \Psi^*_t, s_t - \bar{s}, s^*_t - \bar{s}^*]'$. Then

$$\dot{z}_t = A\,z_t + B\,u_t, \qquad z(t) = e^{At}z(0) + \int_0^t e^{A(t-s)}Bu_s\,ds.$$

Specialisation used to generate the IRFs

- **Steady state.** Symmetric countries; $\bar{\pi}$ (often 0 in continuous time); $\bar{g}$ constant; $\bar{\Psi} = 0$; $x = \tau = \hat{e} = 0$.
- **Policy.** Passive Taylor rules with $\phi_\pi = 0.85$ (foreign analogous) and $\phi_x \geq 0$.
- **Shock.** Temporary *home* surplus (deficit) shock via s_t (OU with short half-life).
- **Objects plotted.** $\{\hat{\pi}_{H,t},\ \hat{\pi}^*_{F,t},\ \hat{e}_t,\ x_t,\ x^*_t,\ i_t,\ i^*_t\}$.

E Further details on the forecast data in section 3

To make more sense of the novel forecast dataset provided in section 3, we add some more descriptive statistics related to the forecast dataset in this part of the appendix. Overall, we have 88 unique forecasters in our data-sample, spanning almost 28 complete years (Sep-1997 until Jun-2025) yielding a total of 15,089 unique forecaster-month observations that contain at least some novel forecast being made.[13]

Figure 10 provides information on the number of unique forecasters appearing in the data and providing novel forecasts at least once a year. We observe a steady downwards trend in the number of forecasters who provide sufficient information to be considered in out analysis. With the exception of the two most recent years, however, there are continually at least 30 individual forecasters available in each year that provide at least one sufficient set of forecasts to be considered in the analysis.

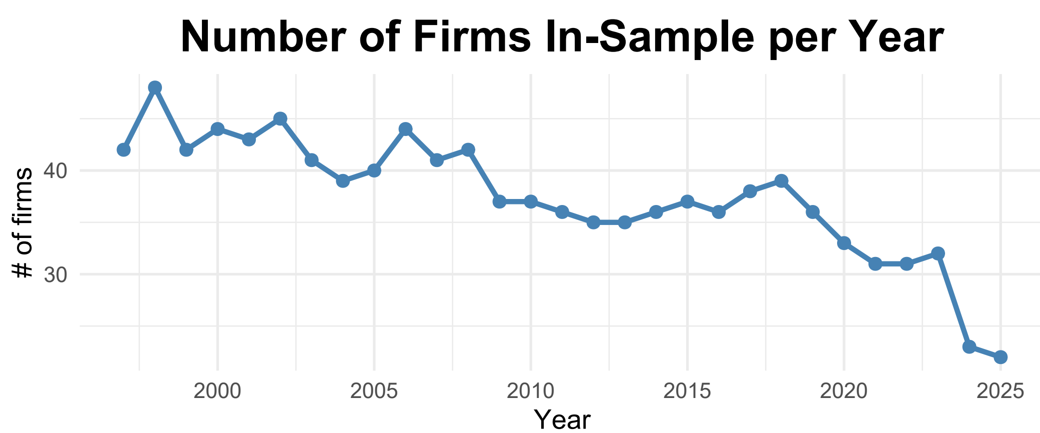

Figure 10: Number of unique forecasters appearing in the data in each year.

Of particular interest, of course, is the frequency of forecast revisions. Forecasts are made in rolling two-year windows, and so the number of forecast changes *within* a given year is informative for the number of valid observations that we can obtain. Figure 11 provides this information, showing how often firms in the sample on average revise their forecasts for a given forecasting window. Conditioning on the forecasters who provide sufficiently complete observations of forecasts, we observe a relatively high number of forecast revisions, averaging 6-8 per year per forecaster. These within-year forecast revisions are the centre of the analysis in section 3.

In table 9 we provide information on the average forecast revisions in the sample of forecasters. Generally, the median revision of each individual variable is centred around zero. Since the mean

[13]In the data, we generally drop the forecasts made by the European Commission, the OECD, and the IMF, since their forecasts reflect general model estimates that are relatively non-reflective of novel information coming in at monthly frequencies.

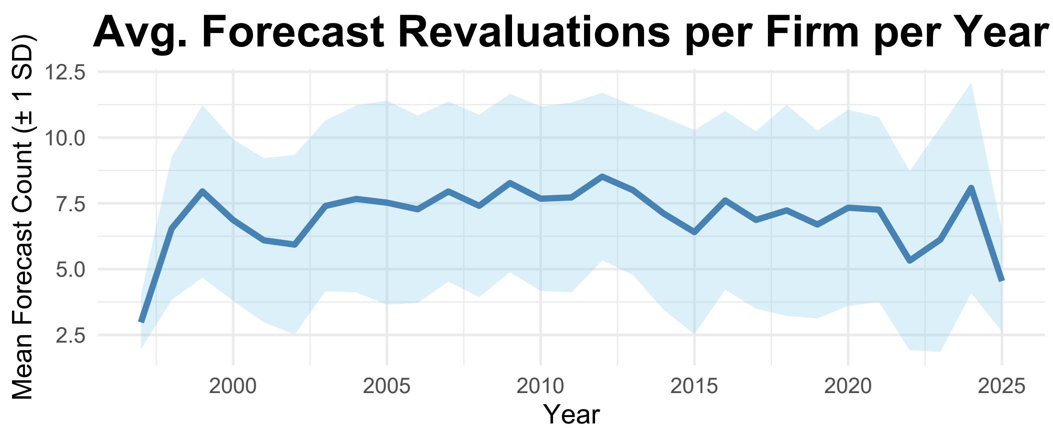

Figure 11: Number of forecast revisions per firm per year, on average. Shaded bands indicate one standard deviation around the mean of per-firm forecast revisions

of the PSNB (deficit) forecast revisions is larger than the median, this indicates a possible slight right-skew in the distribution of forecast revisions. Overall, forecasters appear to stick to their original forecasts (made at the beginning of a given forecast year), unless they are receiving novel information that forces them to amend their forecasts in accordance with their needs.

Variable	Mean	Median	SD	Q1	Q3
$\Delta\mathbb{E}_t$(PSNB) (in GBP Billion)	1.111	0.000	13.188	-0.995	1.848
$\Delta\mathbb{E}_t(g_Y)$ (in %)	-0.052	0.000	0.616	-0.100	0.100
$\Delta\mathbb{E}_t(\pi_{RPI})$ (in p.p.)	0.027	0.000	0.580	-0.100	0.200
$\Delta\mathbb{E}_t(\mathcal{E})$ (in %)	0.000	0.000	0.057	-0.003	0.008

Table 9: Summary Statistics for forecast revisions across the entire sample.

Given our particular interest in the forecasts of deficits (PSNB) and of exchange rates, figure 12 depicts the distribution of forecast revisions in both variables across time. As is to be expected, the average forecast revision is generally not significantly different from zero, but there is substantial disagreement in terms of the actual revisions. In terms of the forecasts for the exchange rate (right-hand panel), the Brexit episode stands out as one of consensus expectations of depreciation. As for deficit (PSNB) forecasts, the Covid-episode and the subsequent stimulus packages were generally underestimated when forecasts were made, followed by a gradual upwards revision of deficit expectations alongside each individual year.

To complement the depiction of the forecasts, we also consider the average of the *actual* forecasts made by the forecasters. Table 10 summarizes the average forecasts of our four main variables of interest. Since 1997, average annual deficits were predicted to hover around 60 billion GBP in real (2018) terms, while real output growth has been predicted to remain on average at slightly below 2%. In contrast, retail price inflation expectations were closer to 3 percentage points, while the average exchange rate index reflects broad expectations of depreciations relative to the

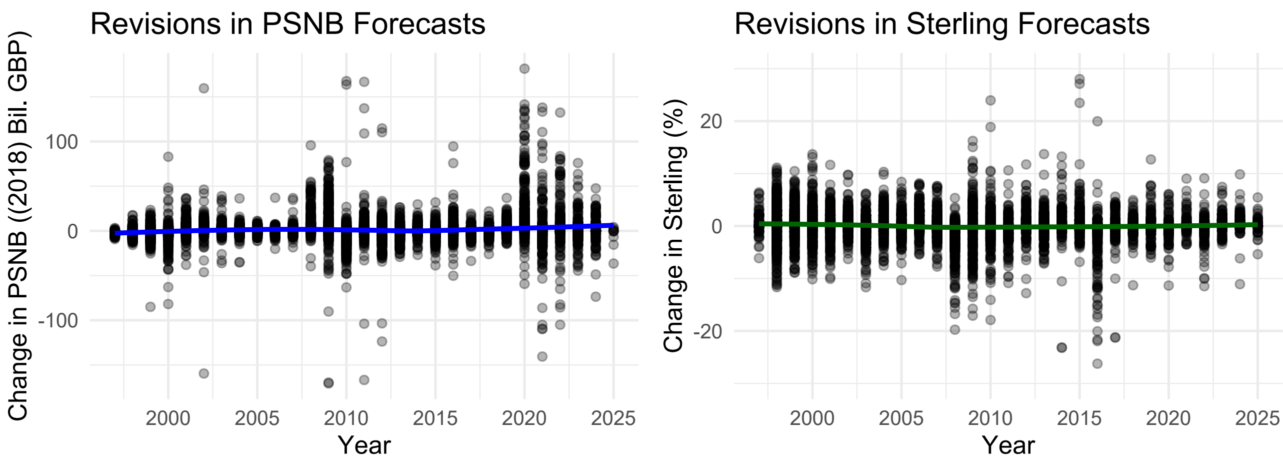

Figure 12: Distribution of forecast revisions in exchange rates and deficits over time.

benchmark period, which here is January 2005.

Variable	Mean	Median	SD	Q1	Q3
PSNB (in GBP Billion)	60.241	43.575	54.801	26.318	90.074
g_Y (in %)	1.749	1.900	2.003	1.200	2.600
π_{RPI} (in p.p.)	2.881	2.800	1.446	2.300	3.400
$\mathcal{E}_{\text{Sterling}\mid\text{Basket}}$ (in %)	90.889	92.600	9.446	82.100	98.931

Table 10: Summary statistics of actual forecasts made by forecasters