Debt Indexation, Determinacy, and Inflation*

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Abstract

In this paper, I analyze the importance of *inflation-indexation* of a part of the stock of government debt. I establish empirically using both Local Projections and a narratively identified powerful fiscal event that sovereign deficit shocks are more inflationary when the share of government-issued inflation-indexed debt is higher. I leverage this finding to introduce inflation-indexed debt in macroeconomic models focusing on interactions of fiscal and monetary policy through policy rules, showing that: (i) even absent further frictions, inflation-indexed debt makes the price level backward-looking (i.e., it becomes a jump-state variable), (ii) it alters bounds that pin down 'active fiscal policy', (iii) it allows us to discriminate to some degree between 'fiscally-led' mechanisms and 'HANK-type' mechanisms surpassing Ricardian equivalence, and (iv) in a calibrated HANK model, changing the share of inflation-indexed government debt from a baseline case without indexed debt to levels observed in the US increases the inflationary effect of a 1% deficit-to-GDP shock by 0.21 percentage points under a fiscally-led policy mix, while there is no sizable effect under a monetary-led policy mix.

Keywords: Debt Indexation, Government Debt, Fiscal-Monetary Interactions.
JEL Codes: E52, E63, H63

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1 Introduction

This paper aims to shed light on the role that inflation-indexed debt plays in magnifying inflation in response to changes in fiscal deficits. It should be immediate that if the government budget balance plays a crucial role in (co-)determining the price level, then an interesting feedback loop may arise if the face value of a part of government debt itself changes either with the price level or its rate of change. I motivate this idea by using data on a specific narrative fiscal shock tracked through market expectations on sovereign deficits and government bond price revaluations, showing a sizable fiscal inflation multiplier related to the share of inflation-indexed debt. My second empirical finding, based on an exercise with local projections using exogenously identified shocks that increase fiscal deficits, reflects that inflation-indexed debt indeed appears to boost ex-post inflation outcomes in response to such shocks.

I next move on to introducing inflation-indexed debt in a one-equation model of government debt-driven price level evaluation. In that model, I establish that the price level becomes a state variable without further ado: previous price levels matter for the determination of the current price level when inflation-indexed debt is present, even without further sources of stickiness in the economy. For such an economy with inflation-indexed debt, I prove uniqueness of the stationary equilibrium in a corresponding dynamic general equilibrium economy. As a corollary, I prove that inflation-indexed debt allows to discriminate to some degree between various mechanisms overcoming Ricardian Equivalence. Once inflation-indexed debt is present, there is no equivalence between mechanisms overcoming Ricardian equivalence driven by incomplete markets and those arising through fiscally-led policy mixes.

Finally, I analyze the combined effects of household heterogeneity and the presence of inflation-indexed debt to monetary and fiscal spending shocks in a full heterogeneous-agent New Keynesian (HANK) model á la Kaplan et al. (2018), making use of the methods pioneered in Auclert et al. (2021) to solve heterogeneous-agent models up to first-order in aggregate variables, while preserving heterogeneity with respect to the individual agents in this economy.[1] I additionally pay attention to the different insurance properties of inflation-indexed debt in models with incomplete markets, following the lead of Brunnermeier et al. (2024). I find that inflation-indexed debt matters quantitatively by increasing the volatility of inflation by roughly 2.6% per each percentage point increase of the share of indexed debt in the overall debt portfolio. In terms of the level impact of inflation-indexed debt, I find that an economy with a 30% share of such debt in the sovereign debt portfolio (as is the case in the UK, for instance), the resulting inflation rate increases by 0.2 percentage points in response to a 1% deficit-to-GDP shock relative to a baseline case without inflation-indexed debt, which is quantitatively highly relevant. I furthermore establish that the classic notions of 'active/passive monetary/fiscal policy', as derived by Leeper (1991), do not directly translate into the world with inflation-indexed debt, even though similarities in the

[1] Crucially, this preserves non-linear risk aversion motives for holding indexed debt, as indexed debt will be the principal insurance device of households against inflationary shocks.

determination of saddle-path stable equilibria prevail.

To establish that indexed debt is not a mere theoretical curiosity, figure 1 shows the share of inflation-indexed debt as part of the overall sovereign debt stock over time in a number of countries. While there is considerable heterogeneity across countries, indexed bonds are present across the board, and have been so for the past three decades. This paper mostly focuses on the UK and the US, since these two indexed debt markets are the largest ones in both absolute and relative terms.[2]

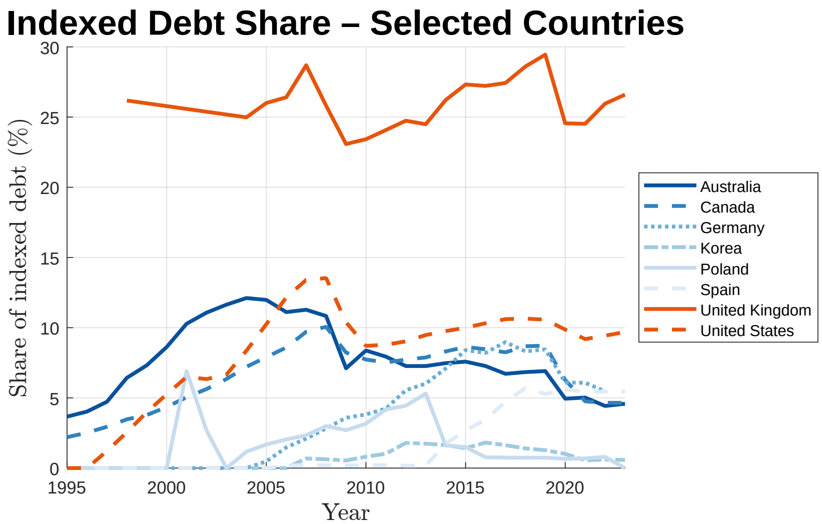


Figure 1: The share of inflation-indexed debt in the total sovereign debt portfolio in selected countries over time. Data source: BIS (2024).

Analyzing the role of the government debt structure for materialized inflation requires a delicate treatment of the interactions between fiscal and monetary policy. While a fiscally-led policy mix in the sense of Leeper (1991) is not a predicament for an analysis of the role of government deficits for inflation, it enhances the role of fiscal policy as drivers of inflationary dynamics in macroeconomic general equilibrium models (Leeper, 1991; Sims, 2011; Ascari et al., 2023; Bianchi et al., 2023). Instead of providing a full picture supporting a possibly fiscally-led policy mix,[3] I motivate this paper by considering a specific policy example: the 'UK mini-budget' in September 2022, which can be considered an exogenous fiscal policy disturbance.[4]

To indicate that this might be an example of a fiscal shock driving implications for monetary policy adjustment (and thus informing a possible fiscally-led policy mix), figure 2 plots the market-implied policy rates in the window around the mini-budget announcement and its cancellation. The first solid line depicts the date of a Bank of England MPC meeting, which occurred just ahead

[2]Ex-post realized real yields differ between the two types of bonds, for instance due to inflation risk premia and liquidity risk premia (Gürkaynak et al., 2010). Looking at market yields at constant maturity, historical data provided by the Fed Board of Governors (2024) confirms a permanently positive differential in market yields on 10-year constant-maturity TIPS relative to standard US treasuries on every day since 2003.

[3]Relevant evidence for the UK, the US, and the Euro Area is provided by Ascari et al. (2024); Barro and Bianchi (2023); Bianchi et al. (2023); Chen et al. (2022); Cochrane (2022b); Leeper (2023); Smets and Wouters (2024).

[4]For a more detailed argument related to this fiscal shock, see Leeper (2023), NIESR (2022), and section 2.2.

of the detailed policy announcement of the 'mini-budget' fiscal policy measure, with the MPC minutes being released on the 22nd of September 2022, one day ahead of the fiscal policy announcement. This is useful for the argument insofar as the meeting likely communicated the Bank of England's stance on future rate changes clearly, taking all available information up to that point into account (Braun et al., 2024). Nonetheless, implied policy rates rose sharply a couple of days after the meeting of the Bank of England's MPC, just after the announcement of the mini budget (denoted by the first dotted line), with the shift amounting to a 120bps increase in expected policy rates one year ahead. After the scraping of the mini-budget (second dotted line), expected policy rates swiftly returned to their 'pre-shock' levels.[5]

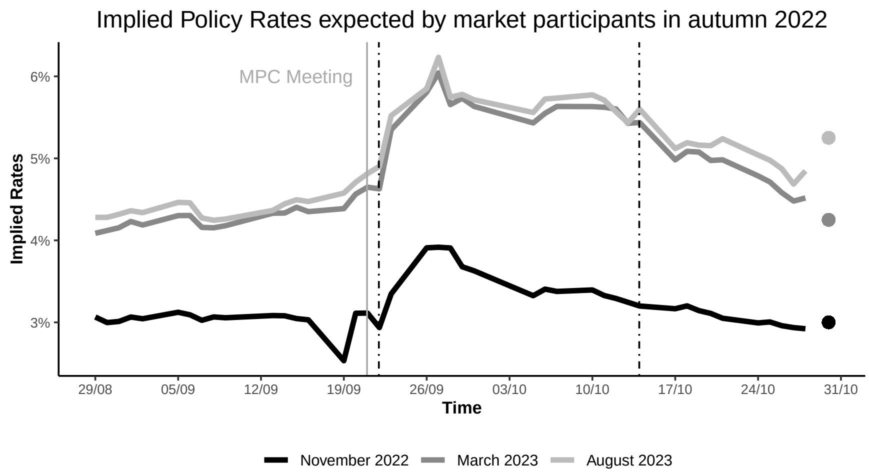


Figure 2: Expectations of nominal interest rates in the United Kingdom for the three MPC meetings after the 'mini-budget' announced in September 2022. The dots at the end reflect the factual values of nominal policy rates after each meeting has taken place.

This event therefore resonates well with the possible idea of (at least partially) fiscally-led policy mixes: financial market participants clearly expected changes to the monetary policy stance beyond the very short term in response to an announced fiscal policy measure.

Literature Review

ADD: Nakamura/Steinsson new paper, Barro 2003 (Optimal Management of Indexed Debt), more of Nico Caramp's stuff, other paper with Martin, everything from Greg Kaplan, TIm Willems Game-Theory-FTPL, Dupraz-Picco recent paper on fiscal requirements with non-Ricardian households, Sims-2016-JacksonHole, whatever else is on the slides but not here...

This paper contributes to the burgeoning literature on fiscal-monetary interactions, pioneered in Sargent and Wallace (1981) and formalized through Leeper (1991). Initial contributions focusing on the possibility of a fiscally-led policy mix include Sims (1994) and Woodford (1995), who

[5]Note that the expected monetary policy response was partially driven by a concurrent funding mismatch in liability-driven investment strategies of defined-benefit pension funds that were closely tied to movements in yields of sovereign bonds. See Pinter (2023) for a detailed exposition of this point.

also coined the terminology behind the 'Fiscal Theory of the Price Level' (FTPL), whose mechanisms apply in the presented framework as well.[6] More succinct summaries of the literature are provided by Leeper and Leith (2016) and Cochrane (2023). Bassetto and Cui (2018), Liemen and Posch (2022), Ascari et al. (2023), and Bianchi et al. (2023) provide advances in analyzing fiscally-led policy mixes in standard OLG and New-Keynesian models, while empirical support for the possibility of fiscally-driven inflation has been developed in Barro and Bianchi (2023), Cochrane (2022a), Chen et al. (2022), and Cloyne et al. (2023), especially in light of the recent inflationary episode. A narrative example of a recent fiscal shock informing inflation rates is provided by Hazell and Hobler (2024), who focus on the 2021 Georgia Senate election runoff.

Applications of fiscally-driven price level determination in recent papers shifted the focus towards models with an endogenous real interest rate. This is important insofar as such models fundamentally constitute criteria that constrain the *transversality condition* on government debt to hold for only *one* candidate price level, but that transversality condition itself depends on the real interest rate. Brunnermeier et al. (2020), Miao and Su (2021), and Kaplan et al. (2023) each provide conditions under which such models nonetheless admit (unique) forward-looking equilibria expressed through the price level, however, their notions of uniqueness are challenged by Hagedorn (2021, 2024), who argues that the endogeneity of the real interest rate in incomplete-markets models 'breaks' determinacy and allows a continuum of initial price levels to exist. I contribute to this literature by explaining and partially overcoming this seeming discrepancy, qualifying the criteria under which one can obtain unique price levels even in incomplete-market settings with inflation-indexed debt.

In a recent contribution closely related to the importance of transversality conditions (which lie at the heart of price level and inflation determination), Brunnermeier et al. (2024) lay out how differences in the valuation of 'safe' assets can induce an aggregate transversality condition to fail, even if individual transversality conditions hold. I contribute to this idea by laying out the properties of this idea in a model of the fiscal price level determination with indexed debt, paying attention to the different insurance properties borne by both types of debt.

I also contribute to the literature on inflation-linked government bonds. Such bonds were introduced in economic and financial research long ago, especially in relation to the introduction of TIPS in the US in 1997. One of the earliest contributions in this field is Fischer (1975), who derives household demand for such assets in a multi-asset framework. The special insurance properties of such inflation-linked debt are extensively discussed in Campbell and Shiller (1996), Barr and Campbell (1997), Garcia and van Rixtel (2007), Gürkaynak et al. (2010) and Andreasen et al. (2021). Notably, Sims (2013) briefly mentions the possible detrimental consequences of indexed debt in fiscally-led

[6] Note that I do not explicitly define the 'Fiscal Theory of the Price Level' in this paper. A possible definition, as for instance in practical use in Brunnermeier et al. (2020), relates to the uniqueness of the price level under which the transversality condition on government debt holds. In general, therefore, 'the FTPL' is not fully equivalent to the analysis of dynamic equilibrium economies under fiscally-led policy mixes. Under certain conditions, the 'FTPL equation' can be seen as the equation (co-)determining the price level under fiscally-led policy mixes, but the underlying government debt valuation equation is generally present in all macroeconomic models that feature a fiscal authority.

policy frameworks. This paper builds on his remarks, providing a rigorous framework nesting his intuitions. Schmid et al. (2024) provide a systematic analysis of inflation-indexed debt as a policy tool, emphasizing its role as an ex-ante commitment device against inflation. In the contribution, I leverage the unique properties of inflation-indexed debt, which express themselves mostly through the induction of a backward-looking component in the government budget equilibrium condition and through the insurance premia they bear. This paper's focus thus effectively rests on the 'ex-post' effects that inflation-indexed debt can have in the face of expansionary government spending shocks.

In the later sections of the paper, I majorly rely on modern computational methods to efficiently solve and estimate heterogeneous-agent methods, as in Kaplan et al. (2018), Bayer and Luetticke (2020), and Achdou et al. (2022). In particular, I leverage the efficient computation algorithms pioneered in Auclert et al. (2021) and some of the refinements of Auclert et al. (2024b) to solve a model with heterogeneous households, two types of assets, and fiscal-monetary interactions.[7]

Finally, this paper is not the first to link fiscal-monetary interactions to heterogeneous-agent frameworks. Brunnermeier et al. (2020), Kaplan et al. (2023), and in particular Kwicklis (2024), who links a fiscally-led policy mix to the canonical HANK framework of Kaplan et al. (2018), have all applied fiscal price level determination to heterogeneous-agent frameworks. Angeletos et al. (2024), on the contrary, negate the need for FTPL-type dynamics in analyzing changes to the price level, finding quantitatively identical responses of inflation to expansionary fiscal shocks in HANK models. My contribution is to introduce a second type of assets (inflation-indexed debt) with a feedback loop between asset holdings and the price level, quantifying the importance that such indexed debt has for inflation dynamics in a calibrated state-of-the-art macroeconomic model. I furthermore show that non-Ricardian effects of fiscal policy arising through a fiscally-led policy mix and non-Ricardian effects arising through market incompleteness as in HANK are generally *not* equal once inflation-indexed debt is present in the model.

The rest of the paper is structured as follows. Section 2 briefly exposes the relevance of indexed debt form materialized and expected inflation in the face of fiscal shocks, after which I introduce inflation-indexed debt in simplified economic frameworks in sections 3 and 4. I introduce the main quantitative model in section 5. Section 6 discusses the calibration and estimation methods, and I present the quantitative findings in section 7. Finally, section 8 concludes.

2 Empirical evidence: indexed debt and inflationary dynamics

To motivate the relevance of indexed debt as a possible driver of the net present value of government debt and, therefore, of price level dynamics through its debt valuation equation, I provide two pieces of evidence: first, I employ a long-running series of exogenously supplied fiscal policy

[7] To motivate the relevance of household heterogeneity applied to holdings of sovereign debt, figure E.1 in the appendix provides evidence on the skew of household holdings of such debt, sorted by their respective income decile. It furthermore establishes that this skew is *even more pronounced* for inflation-indexed debt.

shocks in a local projection applied to both the UK and the US to pin down the effects that inflation-indexed debt has on inflation itself when fiscal spending disturbances affect the economy. Second, I follow up on Hazell and Hobler (2024) and provide a narrative analysis of a specific fiscal shock in a high-indexed-debt environment, finding decisively larger inflation multipliers in response to deficit shocks compared to theirs.

To complement this picture, appendix F provides a measure of 'net fiscal shocks' in the UK with two types of sovereign debt under the assumption of *complete markets*, showing that the *unexplained* component of revaluations of sovereign debt, induced (among other things) by the complete markets assumption, is closely linked to the share of inflation-indexed debt in the government bond portfolio, which solidifies the need to consider interactions between incomplete-markets and the two types of debt.[8]

2.1 Evidence on the ex-post inflationary effect of inflation-indexed debt

This section procides direct evidence on the effect that inflation-indexed debt can have on inflation, making use of the series of narratively identified tax shocks provided by Mierzwa (2024).

I leverage his time series of exogenous fiscal policy surprises, and combine it with a novel long-running series of inflation-indexed debt, taking the share of inflation-indexed debt in the overall sovereign debt portfolio as the main indicator for the intensity of the prevalence of inflation-indexed debt. Equipped with these time series, I estimate the following local projection (Jordà, 2005) to measure the dynamic impact of inflation-indexed debt on changes in the price level:

$$\log P_{t+h} - \log P_{t-1} = \alpha_h + \beta_h \Delta\omega_t \varepsilon_t^F + \delta_{1h}\Delta\omega_t + \delta_{2h}\varepsilon_t^F + \Gamma_h Z_{t-1} + e_{t+h}, \tag{1}$$

where $h \geq 0$ indexes the forecast horizon considered and Z_{t-1} is a vector of control variables specified below. Of particular interest is the coefficient β_h, which captures the cross-effect of the identified fiscal shock ε_t^F and the growth in the share of inflation-indexed debt $\Delta\omega_t$ present in the economy at time t.[9]

Figure 3 depicts the impulse-responses from the preferred local projection specification. The crucial observation is a positive interaction effect between the share of inflation-indexed debt present in the economy and the fiscal policy shock, directly after the fiscal shock occurs. In economic terms, the coefficients imply that a 1% increase in the combined measure of the change of the share of inflation-indexed debt and the narratively identified fiscal shock (measured as a percentage of

[8]The analysis is limited to one country and one time period only in that appendix, focusing majorly on the importance of indexed debt while minimizing the need to account for cross-sectional heterogeneity. For a cross-country exercise *without* indexed debt that focuses directly on the empirical link between fiscal surprises, corresponding bond revaluations, and inflation, see Barro and Bianchi (2023).

[9]I work with the growth rate of the share of inflation-indexed debt in the total debt portfolio to capture the effect of the joint variation in the indexed debt share and the fiscal spending behavior, postulating that previous levels of inflation-indexed debt are already accounted for in the government debt valuation equation prior to the shock occurring. Econometrically, this specification follows Cloyne et al. (2023).

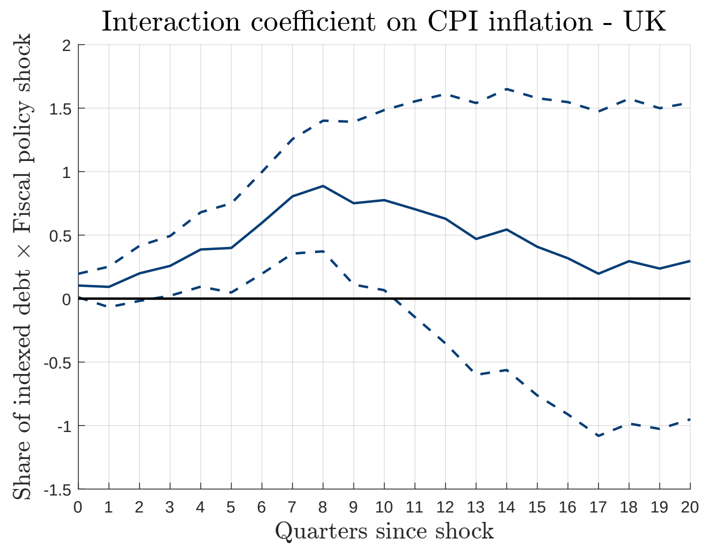


Figure 3: IRF implied by the local projection (1) through the coefficients β_h. The control vector Z contains the first four lags of the real GDP growth rate, the short-run UK bank rate, the change in the weighted real exchange rate, a same-period recession indicator, year-fixed effects, and the first lag of the price level difference. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction). Confidence intervals are provided at the 90% level. Sample length: 1970 Q1 - 2019 Q2.

GDP) itself leads to an increase of the price level of almost 1% in the two years after the shock.[10] To the degree to which one can plausibly attain exogeneity to the narrative fiscal innovations at use here, this brief exercise therefore allows to claim a clear link between the share of inflation-indexed debt and the inflation incurred in response to expansionary fiscal shocks.

2.2 Narrative evidence on shocks in high-indexed-debt environments: the 2022 UK 'mini-budget'

I now provide a piece of narrative evidence directly related to a clearly identified fiscal policy shock: the September 2022 UK fiscal policy announcement, commonly dubbed the 'mini-budget'. Here, I focus on this specific shock episode of the UK for two reasons: first, as will be argued now, the event was largely unexpected in terms of its magnitude, allowing a clear identification of the effects of fiscal shortfalls on inflation. Second, this exercise is a complement to Hazell and Hobler (2024), who exploit probabilistic variation on Democrat Senate control around the 2021 Georgia Senate run-off election to infer the expected effects of expansionary fiscal policy on the price level. This paper provides a similar exercise in an environment with high levels of inflation-indexed debt, complementing their existing estimates.

The institutional set-up of UK fiscal policy serves as an excellent device for identifying the 2022 'mini-budget' episode as a clear fiscal shock. Fiscal policy in the UK is shaped by regular fiscal announcements, which set up the broad guidelines for expected sovereign income and spending in a given fiscal year. From 1980 until 2016, the larger 'budget announcement' would usually occur in early spring (coinciding with the beginning of a new fiscal year), supplemented by shorter budget

[10]Further details related to this analysis as well as an application to US data are provided in appendix E.

statements in the fall of the same year. Between 2017 and 2019, the regular budget announcement was moved to fall, with the spring season being used usually for supplementary announcements. From 2020 onwards, the main budget statement was again placed in the early spring season.

In spring 2022, then-Chancellor Rishi Sunak provided a budget statement, which was followed up by a full budget announcement in November 2022. In-between, and therefore outside of the usual bi-annual statement/announcement cycle, then-Chancellor Kwasi Kwarteng (who had since been appointed) presented a Ministerial Statement dubbed "The Growth Plan", with fiscal policy measures amounting to 150 Billion GBP, or approximately 5% of the GDP of the United Kingdom (NIESR, 2022). This statement did *not* constitute a budget announcement in the usual sense, being placed outside of the bi-annual budget statement cycle. The release of all budget statements made by the British government is usually supplemented with a concurrently released report by the Office for Budget Responsibility (OBR), an independent auditor supervising budgetary questions in the United Kingdom. In the particular case of the 'mini-budget', no such independent forecast of the budgetary consequences of the statement was publicly released, as the ruling government denied the release of the concurrent forecast provided by the OBR. Since then, the forecast that was provided by the OBR at that time has been released, although it is only of limited relevance with respect to the eventual policy measures announced as the report was made 18 days ahead of the budget announcement, thus not capturing the full extent of the fiscal policy proposals.[11]

The episode of early fall 2022 is characterized by this fiscal policy announcement and its expected effects. In particular, the effects of the fiscal policy announcement are plausibly shielded from monetary policy effects (both in terms of the policy level and in terms of the signaling of the state of the economy), since the preceding Bank of England Monetary Policy Committee decision was released one day *before* the announcement of the fiscal policy measure, on September 22, 2022.

2.2.1 The size of the shock

The most important question begets the *size* of the fiscal policy shock, which is *not* equal to the overall size of the fiscal announcement, as the fiscal policy announcement had been expected ahead of the final budget statement. Ignoring this would contribute to an upwards bias of the estimates of the effects of the policy announcement. Additionally, the probability of the fiscal policy measures being implemented upon announcement need not equal 100%, which might cause a downwards bias of the estimates.

To address these important questions, I follow the lead of Hazell and Hobler (2024), albeit with some limitations caused by impeded data availability. First, I establish the actual *full* size of the fiscal policy package through its direct impact on the budget. This serves as the factual upper bound of the size of the possible shock. Here, I utilize two estimates:

[11] The forecast can be found under https://obr.uk/docs/dlm_uploads/FOI-Information-on-preparatory-work-for-the-mini-budget.pdf.

- The first is based on a direct reading of the corresponding budget statement.[12] Algorithms summarizing the implied policy measures robustly predict that the "Debt Management Office's Net Financing Requirement increasing from GBP 161.7 billion to GBP 234.1 billion in 2022-23", such that the corresponding upper bound of the shock would be GBP 72.4 billion.
- The second is an analysis by the *Institute of Fiscal Studies*, which predicts "a GBP 60 billion hole in the budget".[13]

As alluded to above, the shock element impacting the expected path of debt and inflation on the day of the announcement was not equivalent to the full size of the announced measures. Instead, the announcement of the fiscal policy package was expected, but its full extent was simply not known. To arrive at estimates of the shock component to the expected fiscal funding shortfall, I exploit forecasts on *Public Sector Net Borrowing*, which are aggregated on a monthly basis and released by the UK Treasury.[14] These are forecasts about the factual borrowing requirement of the government, provided both by financial market participants as well as other independent forecasters. I collect data on the forecasts provided in the period between September 1, 2022, and September 22, 2022 (i.e., until the day before the shock) and compare these forecasts with the ones collected between October 1, 2022, and October 10, 2022. Unfortunately, data is not collected at narrower time intervals, but this would - if anything - cause a downwards bias of the estimated debt-price multiplier.[15]

For September 2022, the forecasts were provided by Barclays Capital, Netwest Markets, the British Chambers of Commerce, Beacon Economic Forecasting, CEBR, Liverpool Macro Research, and Oxford Economics. The total mean forecast revision of Public Sector Net Borrowing for the 2022-23 and 2023-24 Fiscal Years lies at GBP 47.4 billion, vastly exceeding all other non-crisis period forecast revisions.[16] This confirms the initial intuition that the 'mini-budget' shock was indeed economically significant and to a large degree unexpected. Given that this forecast revision is also below the upper bound of the shock size, the following analysis works with this estimate of a GBP 47.4 billion funding shortfall, equivalent to 1.27% of annual GDP in 2022 (GBP 47.4 billion / GBP 3.732 trillion). Relative to the fiscal shock analyzed in Hazell and Hobler (2024), the shock equals 60% of the size of their shock relative to local nominal annualized GDP. Unlike Hazell and Hobler (2024), however, I do not provide a discussion about the nature of the stimulus as this paper mostly concerns the degree to which the stimulus coincided with changes to expected inflation.

[12] The statement is available under https://www.gov.uk/government/publications/the-growth-plan-2022-documents.

[13] The report is available under: https://ifs.org.uk/articles/mini-budget-response.

[14] The forecast summaries are available under https://www.gov.uk/government/collections/data-forecasts.

[15] The multiplier in terms of the influence of a change in sovereign borrowing on expected inflation is larger for a given change in expected inflation when the borrowing shock is *smaller*. Here, it is easier to over-estimate the size of the 'shock' aspect, since I include data that is two weeks away from the fiscal announcement, by when financial markets might have already priced in more of the fiscal response.

[16] The only periods with larger absolute adjustments in the two-year budget deficit forecast were April 2020 (GBP 147.4 billion), May 2020 (GBP 114.9 billion), and May 2009 (GBP 50 billion). Outside of the GFC and Covid periods, the largest absolute month-on-month average forecast revision was GBP 20.8 billion in October 2019, less than half of the forecast change in October 2022.

2.2.2 Linking the deficit shock to expected inflation

I now introduce data capturing expected inflation from a standard high-frequency identification strategy that measures the expected inflationary consequence of the underlying stimulus. In this, the analysis follows Hazell and Hobler (2024), postulating that around the 'mini-budget announcement' the dynamics of asset prices y_t can be summarized by the process:

$$y_t = \begin{cases} \varepsilon_t & \text{if } t < T, \\ \varepsilon_t + \alpha_t & \text{if } t \geq T, \end{cases} \tag{2}$$

where T denotes the time period at which fiscal stimulus occurred. I set the shock period T to September 23, 2022, 09.30am, coinciding with the beginning of the budget statement in parliament. Denoting by j a counter of periods after the event, denote by $\hat{\alpha}_{T+j} = y_{T+j} - \mathbb{E}_T \left[y_{T+j} | \alpha_{T+j} = 0 \right]$ the estimate of the causal effect of the shock in the narrow window around the announcement.

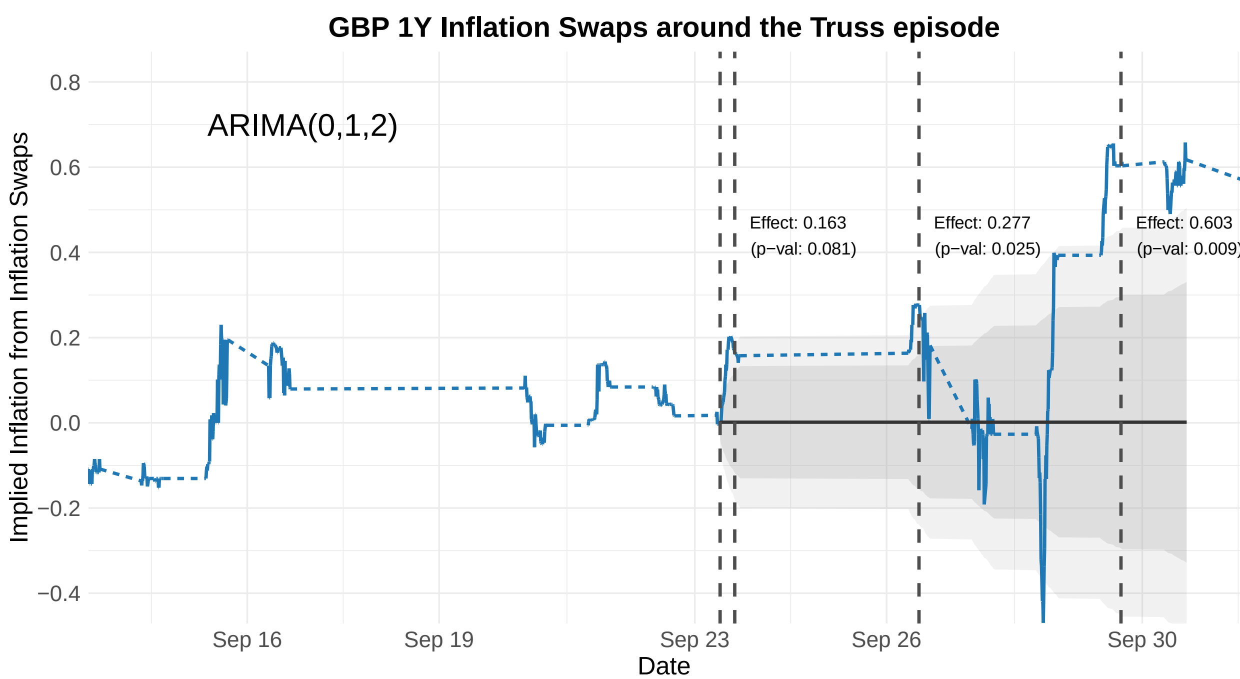


Figure 4: Implied inflation expectations from one-year GBP Inflation swaps in the period around the 'mini-budget' shock, with data normalized to 0 for September 23, 2022, 09:30am. The gray fan-chart depicts 68% and 95% confidence intervals for implied inflation based on a forecast of the swap price from the moment of the shock onward, with the model being chosen optimally in accordance with the Bayesian Information Criterion.

The main quantity of focus is one-year ahead expected inflation, as implied through GBP-indexed inflation swaps traded at the London Stock Exchange.[17] Figure 4 summarizes the movements of expected one-year ahead inflation around the 'mini-budget shock' on September 23, 2022, as implied by one-year ahead inflation swaps.

[17]Since inflation swaps operate with a two-month indexation lag in the context of the UK, I adjust the prices of the swaps to reflect this lag, again in line with Hazell and Hobler (2024).

With the dashed vertical lines, I plot points at which one can expect to recover meaningful estimates about the response of one-year ahead inflation implied by financial markets. The first vertical line depicts the beginning of the shock, as implied by the beginning of the budget speech announcing the 'mini-budget' measures in detail. The second line measures one-year ahead inflation expectations on the same day at 3:00pm, 5.5 hours after the budget speech commenced. Even though markets can credibly be expected to take a couple of days to incorporate news into forming inflation expectations (Bahaj et al., 2023), there is a significant response of implied inflation on the same-day. Looking at the effect measured on the next trading day, a further magnification of the effect occurs, yieldingan implied year-on-year inflationary response of 0.277% to the narratively identified shock.

Between the third and the fourth vertical line exists observe a sharp drop in implied inflation. This is consistent with a growing expectation that the fiscal spending announcement might end up being unraveled. I provide a narrative description of the events in this period in appendix E.3, including a brief description of the role played by the troubles on LDI markets.

On September 28 & 29, there is again a significant up-shoot in the expected inflation measure. While by then other events might contaminate the evolution of inflation swap prices, the observed sharp appreciations perfectly coincide with statements of the Treasury that *despite* the market turmoil, the proposed fiscal package will be followed through, superseding previous statements of a release of a stabilizing medium-term fiscal plan. The elevated levels of expected inflation then continued to persist well into October, when an eventual unraveling occurred in parallel to an overhaul of the ruling government that enacted the fiscal package in the first place.

To remain conservative in terms of the implied size of the expected inflation adjustment, yet consistent with the literature, I postulate that the response of inflation swaps until September 26 can be considered the baseline change in one-year ahead inflation expectations.

The Inflation Multiplier: the resulting baseline estimate of the one-year ahead inflation multiplier, which captures the response of year-on-year inflation to a 1% deficit-to-GDP shock, is therefore $0.277/1.27 \approx 0.22\%$. This exceeds the *two-year* inflation multiplier found by Hazell and Hobler (2024) of 0.19% by 15%. This is despite the assumptions ensuring that the inflation multiplier estimate errs on the conservative side. Taking the point estimate of 0.603 (which aligns closest with the forecast change to the budget deficit introduced in the last subsection), the inflation multiplier would amount to $0.603/1.27 \approx 0.475\%$, more than double the estimate of Hazell and Hobler (2024) and vastly above existing estimates for other countries.

Finally, the effects of the shock were expected to be relatively persistent, as implied by inflation swaps for longer horizons depicted in figure 5.

In appendix section E.3 I provide some further evidence on the nature of the 'mini-budget shock episode', which confirms an element of surprise in relation to the size of the unveiled fiscal package that contributed to the turmoil on financial markets reflected in the pricing of inflation swaps.

Equipped with this evidence on the effects of inflation-indexed debt on both the revaluation of sovereign debt and inflation, I now introduce inflation-indexed debt in simplified economic frameworks to lay out the mechanisms under which such debt operates in canonical models.

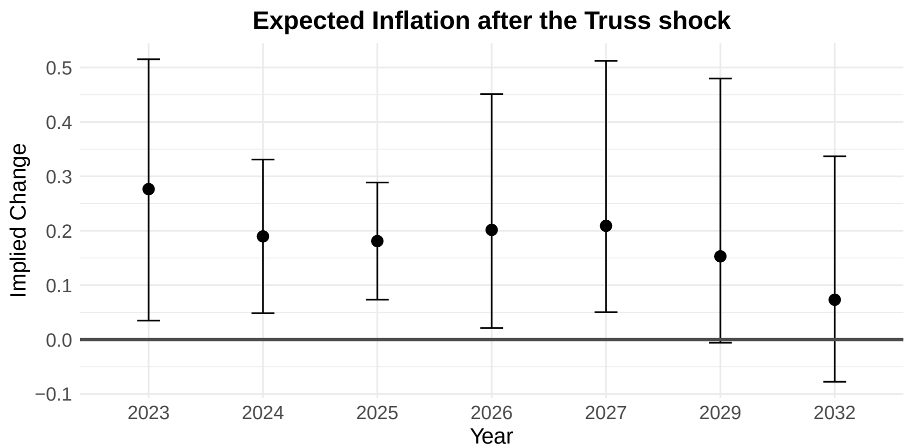


Figure 5: Implied inflation expectations for various forecast horizons, as implied by GBP inflation swaps on September 26, 2022, 12:00pm. 95% confidence bands are indicated.

3 Intuition from a one-equation price level-determination model

This section explicitly introduces inflation-indexed debt in the government budget constraint and the resulting debt valuation equation, which therefore effectively constitutes a partial equilibrium analysis focusing on the importance of the government budget for price level dynamics. I derive the novel result that the price level itself becomes a *state variable* in the intertemporal government budget equilibrium, i.e., today's price level becomes a function of the past price level. This is despite the lack of other inertia, and it gives rise to a double role of the price level as a state variable and a market-clearing jump variable.

I begin by deriving the intertemporal government budget equilibrium with indexed debt, starting off with the case of 'fair' bond pricing, i.e., abstracting from insurance premia on either type of debt. The per-period government budget constraint in a world with indexed debt is given by

$$B_{t-1} + \frac{P_t}{P_{t-1}} b_{t-1} = P_t s_t + Q_t B_t + q_t b_t,$$

where notation follows the previous section, i.e., B_t denotes the face value of non-indexed government debt issued at time t at price Q_t, b_t denotes the issuance value of indexed-government debt issued at time t at price q_t, lowercase letters correspond to the values for inflation-indexed debt, s_t are net real surpluses raised (inverse of deficits), and P_t denotes the price level. The cost of maturing inflation-indexed debt b_{t-1} is scaled by the gross inflation rate, P_t/P_{t-1}.[18]

[18]See Hall and Sargent (2011) for a verification that this is the correct specification for indexed debt in line with how its face value adjusts empirically.

To close this model as simply as possible, let $Q_t = \frac{1}{1+i_t}$ and $q_t = \frac{1}{1+r_t}$, i.e., the price of bonds equals the inverse of their respective relevant gross interest rate. Using the real interest rate to determine the price of inflation-indexed deb factually takes into account expectations on the payment being indexed to the ex-post inflation rate.[19] Iterating this equation forwards after dividing both sides by P_t and making use of the Fisher equation gives the following relationship:

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \sum_{j=0}^{\infty} \prod_{l=1}^{j} \frac{1}{1+r_{t+l}} s_{t+j}. \tag{3}$$

This is the simple intertemporal budget equilibrium with indexed debt, but without accounting for the differences in the insurance properties borne by the two types of debt, which made it possible to use the simplified bond pricing kernels Q_t and q_t as defined in the last paragraph.[20] Indeed, *the price level itself becomes a state variable*: the real value of maturing inflation-indexed bonds depends on the past price level, not on today's price level. Intuitively, the real value of inflation-indexed bonds depends on the past price level, because the face value payment of that bond is unity *at yesterday's prices*. The term in orange is the novel addition relative to canonical models of fiscal inflation and is the centerpiece of this paper.

Sample IRFs in partial equilibrium in finite horizon

I now briefly explore the properties of this intertemporal budget equilibrium relationship using impulse-responses to the price level under various levels of inflation-indexed debt (as derived above in equation (3)). The goal is to explore how indexed debt changes the mechanisms inducing fiscal inflation in relation to surplus shortages in the clearest possible way.

The model is set up (in terms of outstanding bonds and expected surpluses), such that $P_{-1} = 1$. The initial state is therefore the one in which the PDV of surpluses is equal to the real value of the stock of debt in each period. The economy has a finite horizon of 11 periods $t \in \{-1, 0, 1, ..., 9\}$, such that all debt has to be repaid by the government in period 9 by appropriate surpluses. This setup ensures a price level of $P_t = 1 \ \forall t$ in the absence of any shocks. The impulse to the system is a one-period decrease of surpluses by 10% in period 0, announced at the same time. After the shock period, the PDV of surpluses will therefore return to its pre-shock value.

Figure 6 highlights the reaction of the price level in response to a decrease in surpluses in period 0, announced in the same period. The right-side panel illustrates the "standard" response induced by the government debt valuation equation in a world without inflation-indexed debt. In period 0, the decrease in real surpluses induces a temporary upwards adjustment of the price level proportional to the decrease in surpluses, which returns back to its initial state subsequently, since the PDV of

[19] Possible insurance premia on inflation-indexed debt will be introduced later in general equilibrium.

[20] Effectively, due to the Fisher equation, the above bond pricing kernels impose the absence of any insurance premia or other valuation wedges, allowing both types of bonds to yield exactly the same realized returns. Section 5 discusses in more detail bond pricing kernels *without* this simplification.

surpluses is equal to the pre-shocked value from period 1 onwards.

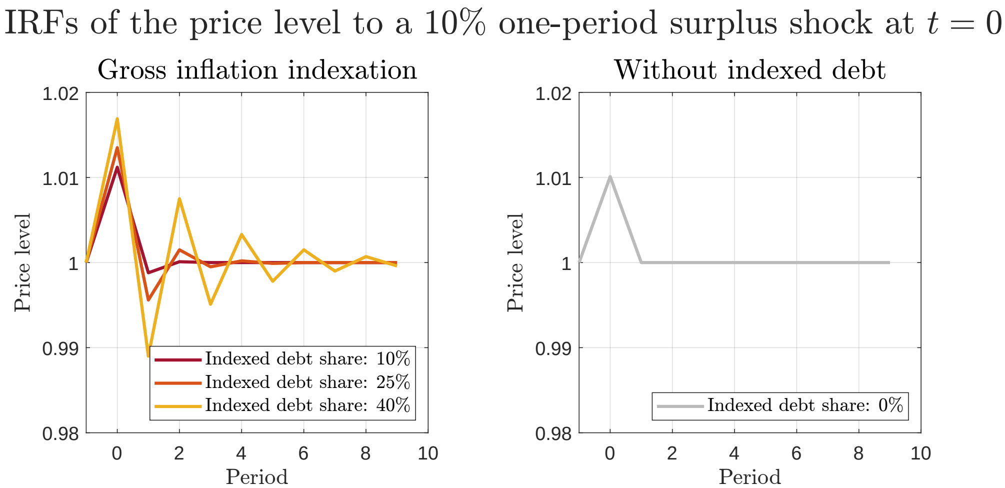


Figure 6: IRFs to a 10% decrease in one-period surpluses in $t = 0$ conditional on the share of indexed debt.

However, when the share of inflation-indexed debt is strictly positive, the impact response is exacerbated: given that the initial price level P_{-1} is fixed in the moment of the shock at time 0, it is not possible to devalue the stock of inflation-indexed debt when the shock occurs. Therefore, the devaluation of the remaining (non-indexed) stock of bonds must be *larger* relative to the case without inflation-indexed debt: the price level must go up by a larger amount in the shock period when inflation-indexed debt is present.

The periods following the shock yield further dynamics that are not observed under a standard model of price level determination through the government debt valuation equation (as depicted in the right panel). Instead of returning to the pre-shock value once the shock vanishes, the price level oscillates when indexed debt is present in the economy. Since from $t = 1$ onwards the PDV of surpluses returns to its pre-shock level, in $t = 1$ the stock of debt is suddenly worth *too little*: inflation-indexed debt is not worth much due to the high price level at $t = 0$, which is the correct factor to adjust such debt to 'real' terms in period 1. But since the funding shortfall is now gone, this implies that the real value of non-indexed debt (B_1/P_1) must actually *increase* to make up the 'under-valuation' of indexed debt: therefore, P_1 must *decrease* (increasing the real value of non-indexed debt) to let the government budget equilibrium hold. In the subsequent period, the price level from the previous period is now *too low*, increasing the value of indexed debt and pushing down the real value of non-indexed debt through a higher price level. This mechanism repeats itself until convergence to the initial equilibrium.[21]

[21] Cochrane (2001) explores a similar result in figure 4 of his paper, driven by a non-geometric maturity structure and the presence of long-term debt.

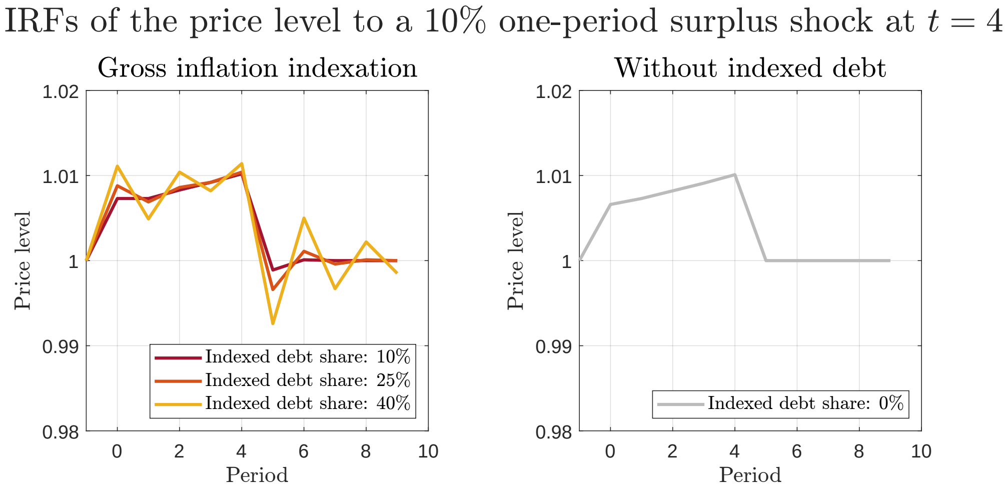


Figure 7: IRFs to a 10% decrease in one-period surpluses in $t = 4$ conditional on the share of indexed debt.

Figure 7 repeats the exercise for a similar decrease of surpluses at a later time (in period 4), announced in period 0. Due to the early announcement, the PDV of surpluses already decreases in period 0, remaining below its initial value until period 4, inclusive. The oscillations induced by inflation-indexed debt decrease in size until period 4 (after being larger immediately following the announcement in period 0), and subsequently pick up from period 4 onwards in line with the mechanism described above. The fact that the oscillations are decreasing in magnitude leading up to the shock is caused by the PDV of surpluses *not* being constant between periods 0 and 4 in this example: the closer one gets to period 4, the more the PDV of surpluses actually decreases as the 'future' shock gets discounted by less. This buffers the price level oscillations on the way to the period of the shock.

This concludes the introduction of inflation-indexed debt in the simplest possible model. Before moving towards the full quantitative evaluation of the importance of inflation-indexed debt, I first focus on the importance of considering *both* inflation-indexed debt and market incompleteness in a simpler general equilibrium model.

4 The role of indexed debt in overcoming Ricardian Equivalence

Angeletos et al. (2024) recently have proven through an insightful 'quasi-HANK' framework (with market incompleteness through mortality risk) that the effects of fiscally-led policy mixes can be replicated, up to first order, under monetary-led policy mixes in models with market incompleteness, as both models induce a form of Ricardian dis-equivalence which can be parametrized in a way by which both classes of models mirrors each other. In a sense, this refutes the necessity for considering monetary-fiscal interactions in ways moving beyond monetary policy rules adhering to the Taylor Principle and, thereby, the need to invoke the government debt valuation equation as a limiting condition pinning down sustainable paths of government debt when analyzing the dynamics of the price level.

This matters for the presented contribution, as the simultaneous consideration of market imperfections commonplace in HANK models that induce Ricardian dis-equivalence might suffice to generate the mechanisms intended to capture. I briefly leverage the modeling framework presented by Angeletos et al. (2024) to qualify this point, providing evidence that under the presence of inflation-indexed debt the mere consideration of non-Ricardian elements inherent to heterogeneous-agent models masks effects that matter *only* in cases of fiscally-led policy mixes. This, in turn, lays the analytical groundwork for the later claim that the monetary-fiscal policy mix determines the extent to which government deficit shocks are becoming more inflationary in the share of inflation-indexed debt present in the economy.

As the model framework used for this analytical exercise mainly relies on the contribution of Angeletos et al. (2024) with the only changes being done to (1) the aggregation on the aggregate demand side; and (2) to the fiscal and monetary policy rules that reflect that policymakers care about interest rates faced and taxes paid by *households*, I relegate the concrete derivations to Appendix D. This section leverages their HANK model with mortality risk (in the spirit of Blanchard (1985)) in a reduced-form setting with a simplified annuity scheme while adding inflation-indexed debt to the portfolio of savings products available to each household, nullifying through amended policy rules any effect of indexed debt beyond the government debt valuation equation. The main finding about the *relevance* of inflation-indexed debt in breaking the equivalence between fiscally-led policy mixes and HANK-type mechanisms breaking Ricardian Equivalence can be summarized in the following proposition:

Proposition 1 *Let $\pi_{\varepsilon,0}^{FD,RANK}$ denote impact inflation in a framework following Angeletos et al. (2024) with inflation-indexed debt, no quasi-heterogeneity, and under a fiscally-led policy mix; and let $\pi_{\varepsilon,0}^{MD,HANK}$ denote impact inflation with inflation-indexed debt, quasi-heterogeneity, and under a monetary-led policy mix. Let κ be the slope of the Phillips curve, θ the share of inflation-indexed debt, τ_y the fiscal policy rule reaction parameter to output deviations from steady-state, β the discount rate, and ω the mortality risk of a given household discriminating whether an economy is considered to be of type 'RANK' ($\omega = 1$) or 'HANK' ($\omega < 1$).*

If either of the following conditions holds:

$$\tau_y > \frac{\beta}{1-\beta}\frac{D^{SS}}{Y^{SS}}\kappa\theta, \tag{4}$$

$$\tau_y < \frac{\beta\omega}{1-\beta\omega}\frac{D^{SS}}{Y^{SS}}\kappa\theta, \tag{5}$$

then impact price level changes in response to an expansionary fiscal shock are larger in absolute terms in the policy limit point where fiscally-led and monetary-led policy mixes collapse ($\tau_d \to 0, \phi \to 0$) for the RANK economy with a fiscally-led policy mix relative to the a HANK economy with a monetary-led policy mix, i.e., $|\pi_{\varepsilon,0}^{FD,RANK}| > |\pi_{\varepsilon,0}^{MD,HANK}|$.

Proof. See appendix D. ■

For most common calibrations (including the ones preferred by Angeletos et al. (2024)), this condition is fulfilled, which are plotted in figure D.1. In particular, the first of the two inequalities in the proposition is likely to be fulfilled as τ_y/κ is a large number for common calibrations of the fiscal rule and the Phillips Curve parameter, including Angeletos et al. (2024), especially once τ_y is interpreted as the tax base channel capturing the elasticity of tax income with respect to output. Only for very small values of τ_y and relatively low shares of inflation-indexed debt θ, it is possible for inflation-indexed debt to mitigate the volatility induced by inflation-indexed debt in the light of expansionary fiscal shocks.

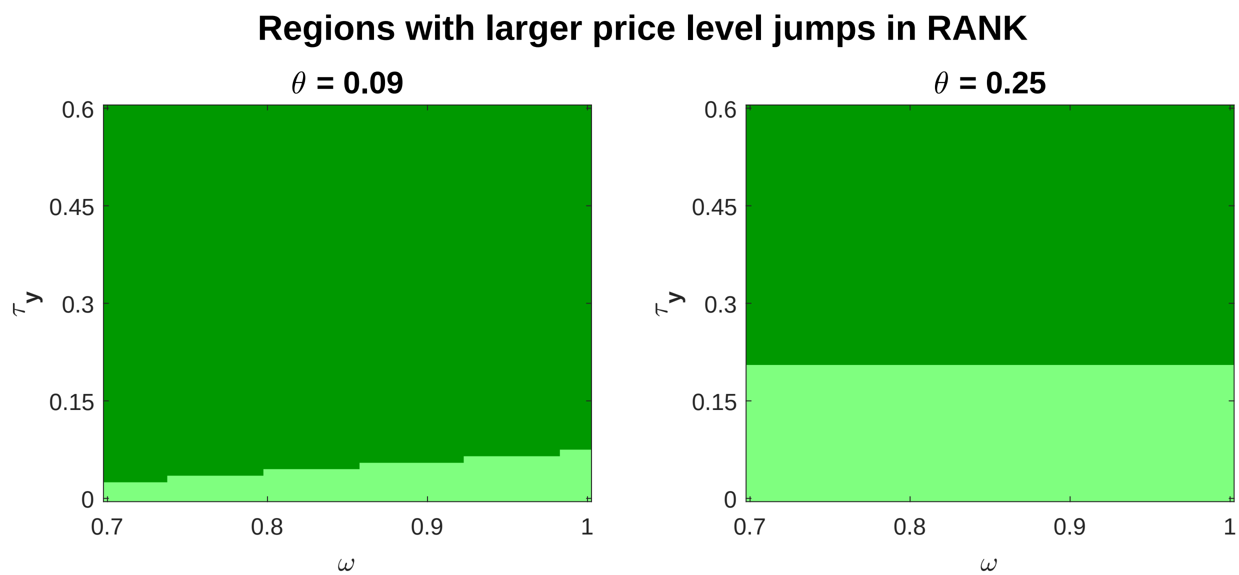


Figure 8: Plot showing under which parametrizations either of the inequalities described by proposition 4 is fulfilled, indicating larger deviations of the price level from steady-state in response to deficit shocks. The dark green area denotes places where the first the magnitude of inflation is larger approaching from the HANK limit (with $\pi_0^\varepsilon > 0$), the light green area denotes places where *dis*inflationary pressure is instead amplified. Calibration: $D^{SS} = 1$, $Y^{SS} = 1$, $\kappa = 0.025$, $\beta = 0.97$, $\sigma = 1$, $\tau_d = 0$, $\phi = \frac{1-\omega+\epsilon}{\sigma}$.

Figure D.1 summarizes the parameter space in terms of the tax-base channel τ_y and the household mortality risk (which can be interpreted as a proxy for liquidity effects in HANK models) ω for which the inequalities stated in proposition 4 hold. For conventional values of τ_y and ω, we can observe that inflation-indexed debt usually raises materialized inflation rates. If the tax base channel τ_y were very small (and the share of inflation-indexed debt would simultaneously be large), inflation-indexed debt could enhance deflationary pressure (light green color), while for common calibrations of tax base channels, inflation-indexed debt amplifies the existing positive inflation on impact.[22] Therefore, when the share of inflation-indexed debt increases, this simple model generally exhibits a higher propensity of inflationary 'catastrophes' when approaching the limit point between fiscally-led and monetary-led policy mixes.

[22]Recall that this result has been achieved by nullifying the intertemporal substitution channel of windfall gains arising from indexed debt holdings, which was done by an appropriate adjustment of the monetary policy rule. Forgoing this channel generally increases the area under which indexed debt amplifies price level deviations from steady-state.

5 A Quantitative General Equilibrium Model

Having studied the relevance of indexed debt in simplified models, I now introduce inflation-indexed debt in a state-of-the-art macroeconomic model. Given that inflation-indexed debt delivers desirable insurance features to households by providing an income smoothing source that yields a constant value in real terms, the chosen model must necessarily bear relevance to imperfect consumption smoothing, borrowing constraints, and market imperfections precluding perfect risk-sharing across households. Here, I work with a heterogeneous-agent model in the spirit of Kaplan et al. (2018), utilizing the efficient algorithms for solving the model provided by Auclert et al. (2021) and paying close attention to limitations of determinacy in incomplete-market models as exposed by Brunnermeier et al. (2024), which are intimately related to the determinacy-establishing properties that arise from analyzing the intertemporal government budget constraint.[23]

Households: Heterogeneous households are indexed by i. Such households choose consumption, c_{it}, labor supply, N_{it}, and asset holdings B_{it} and b_{it} to maximize their cumulative discounted utility

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(u(c_{it}) - v(N_{it}) \right) \right]$$

subject to two budget constraints - one for the aggregate household budget, and one for the semantically separate evolution of indexed debt:

$$\begin{aligned} P_t c_{it} + Q_t B_{it} &= \frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta} di} (1-\tau_{it}) W_t N_{it} + B_{i,t-1} - d_{it} \mathbb{1}_{\{adj_{it}=1\}}, \\ q_t b_{it} &= \Pi_t b_{i,t-1} + d_{it} \mathbb{1}_{\{adj_{it}=1\}}, \end{aligned}$$

where Q_t and q_t are the nominal prices for non-indexed and indexed debt, respectively, whose holdings are denoted by B_{it} and b_{it}. $W_{it} \equiv w_{it} P_t$ denotes the nominal wage level, adjusted by hours worked N_{it} and scaled by the idiosyncratic productivity disturbance $\frac{e_{it}^{1-\theta}}{\int e_{it}^{1-\theta} di}$ and taxes τ_{it}. d_{it} captures idiosyncratic transfers from non-indexed bond holdings to indexed bond holdings, which are only allowed to happen when the exogenous portfolio rebalancing variable adj_{it} is equal to 1, which happens with probability ν.[24] Finally, households are also subject to standard borrowing constraints

$$B_{it} \geq -\underline{B}, \qquad b_{it} \geq -\underline{b}.$$

Effectively, I posit that consumption is only possible directly from the non-indexed savings port-

[23] Auclert et al. (2024b) furthermore provide analytical conditions for determinacy in economies with many bond types and bonds-in-the-utility function. Such an approach is sidestepped for now while recognizing its importance for future research.

[24] Such Calvo-type sticky debt arrangements have been present in macroeconomic models for a long time, see, e.g., Graham and Wright (2007), and have prominently been used in heterogeneous-agent models by Auclert et al. (2024b) and Bayer et al. (2024).

folio, i.e., I postulate that indexed debt cannot be transformed to consumption as easily as non-indexed debt. This assumption reflects the significantly smaller liquidity of inflation-indexed bond markets, even relative to their market size (Andreasen et al., 2021; Fleming and Krishnan, 2012) and is required for the ex-ante expected yields of both types of debt to be different. Without any adjustment friction, expected yields would equalize and there would be no incentive to hold both types of debt through a no-arbitrage argument.[25]

To solve the household block, the crucial determinant is whether a household is able to adjust its holdings of indexed debt in a given period ($adj_{it} = 1$) or not ($adj_{it} = 0$). In the following, let $\varepsilon_i \equiv \frac{e_i^{1-\theta}}{\int e_i^{1-\theta} di}$ be a simplified descriptor of the Markov chain pinning down idiosyncratic productivity. I now define corresponding value functions for households, noting that the state variables are therefore the household-specific past asset holdings (B_-, b_-), the Markov chain realization ε_i, and the adjustment state adj_i. The subscript i is dropped in the following for notational simplicity, yielding the following value functions:

- adjuster, $adj = 1$:

$$V_t(1, \varepsilon; B_-, b_-) = \max_{c,B,b,N} u(c) - v(N) + \beta \mathbb{E}\left[V_{t+1}(adj', \varepsilon', B, b)|\varepsilon\right] \tag{6}$$

  subject to the budget constraint and the borrowing constraints:

$$\begin{aligned} Pc + QB + qb = \varepsilon(1-\tau)WN + B_- + \Pi b_, \\ B \geq -\underline{B}; \quad b \geq -\underline{b}, \end{aligned}$$

  where adj' is i.i.d., with probability $\mathbb{P}(adj' = 1) = \nu$.

- non-adjuster, $adj = 0$: Here, b does not enter the decision set and is taken to be a state operating in the background, with the next-period income from non-indexed debt being automatically adjusted based off previously held indexed debt.

$$V_t(0, \varepsilon, B_-, b_-) = \max_{c,B,N} u(c) - v(N) + \beta \mathbb{E}\left[V_{t+1}\left(adj', \varepsilon', B, \frac{\Pi}{q} b_- | \varepsilon\right)\right] \tag{7}$$

  subject to the budget and borrowing constraints:

$$\begin{aligned} Pc + QB = \varepsilon(1-\tau)WN + B_-, \\ B \geq -\underline{B}. \end{aligned}$$

The goal is to recover policy functions $c(\cdot), B(\cdot), b(\cdot)$, and $N(\cdot)$ that solve the household problem in both instances. The above problem generally yields first-order conditions that depend on the adjustment possibilities that an agent enjoys in a given period. Denote by λ_{it}, μ_{it}^B, and μ_{it}^b the re-

[25] Evidence on the use of inflation-indexed government bonds by households for inflation hedging within the context of the US is provided by Nagel and Yan (2022).

spective state-dependent constraint multipliers. For the adjusters, $adj_{it} = 1$, the relevant first-order conditions from that household problem are given by

$$\begin{aligned}
\{c\}: && u'(c) &= P\lambda_{it} \\
\{N\}: && v'(N) &= \lambda_{it}\varepsilon(1-\tau)wP \\
\{B\}: && Q\lambda_{it} &= \beta\mathbb{E}\left[V_{B,i,t+1}\right] + \mu^B_{it} \\
\{b\}: && q\lambda_{it} &= \beta\mathbb{E}\left[V_{b,i,t+1}\right] + \mu^b_{it},
\end{aligned}$$

while the envelope conditions, using $\lambda_{it} = \frac{u'(c)}{P}$ from the first-order condition on c, are given by:

$$\begin{aligned}
V_{B,i,t} &= \frac{u'(c)}{P}, \\
V_{b,i,t} &= \begin{cases} \frac{u'(c)}{P}\Pi = \frac{u'(c)}{P_-} & \text{if } adj_{it} = 1 \\ \beta\frac{\Pi}{q}\mathbb{E}\left[V_{b,i,t+1}\right] & \text{if } adj_{it} = 0. \end{cases}
\end{aligned}$$

The conditions for equilibrium jointly imply the following Euler equations:

$$\begin{aligned}
\frac{Q}{P}u'(c) &\geq \beta\mathbb{E}\left[V_{B,i,t+1}\right], \\
\frac{q}{P}u'(c) &\geq \beta\mathbb{E}\left[V_{b,i,t+1}\right], \\
v'(N) &= u'(c)\varepsilon(1-\tau)w,
\end{aligned}$$

where the inequalities are strict if the respective asset holdings are at their respective lower bound.

This household block defines pricing kernels for the bonds that are on offer by the government, conditional on the households pricing the bonds being unconstrained. For non-indexed debt, the first-order conditions for households on the Euler equation imply that

$$Q_t = \beta\mathbb{E}_t\left[\frac{u'(c_{i,t+1})}{u'(c_{it})}\frac{P_t}{P_{t+1}}\right] := \mathbb{E}_t\left[\mathcal{M}_{i,t,t+1}\right], \tag{8}$$

where $\mathcal{M}$ denotes the household-specific stochastic discount factor (SDF). For indexed bonds, applying the definition of the SDF,

$$q_t = \beta\mathbb{E}_t\left[\frac{u'(c_{i,t+1})}{u'(c_{it})}\right] := \mathbb{E}_t\left[\mathcal{M}_{i,t,t+1}\Pi_{t+1}\right]. \tag{9}$$

—

Firms and production

To focus on the effects of indexed debt and its interaction with households facing uninsurable idiosyncratic income risk, I model the production block in a parsimonious yet tractable way, following Auclert et al. (2024). In particular, the model requires that the aggregate effects of idiosyncratic productivity risk are 'small' for the production firms relative to the aggregate effects of aggregate risks.[26]

There exists a continuum of monopolistically competitive firms k that produce goods of variety k, which make each use of a linear production function $Y_{kt} = A_{kt}N_{kt}$. A_{kt} evolves according to an AR(1) process in logs,

$$\log A_{kt} = \rho_a \log A_{k,t-1} + \sigma_\epsilon \epsilon_{kt},$$

where that $0 \leq \rho_a \leq 1$. The firm profit function is standard and defined as

$$D_{kt} = \frac{P_{kt}}{P_t} Y_{kt} - \frac{W_t}{P_t} N_{kt} = \left(\frac{P_{kt}}{P_t} - \frac{W_t}{P_t} \frac{1}{A_{kt}} \right) A_{kt}^{1-\zeta} \left(\frac{P_{kt}}{P_t} \right)^{-\zeta} Y_t.$$

Following Auclert et al. (2024), a log-linearized approximation to the solution of the profit-maximization problem of monopolistically competitive firm yields a Phillips Curve of the form:

$$\hat{\pi}_t = (\varphi + \sigma)\kappa \sum_{l=0}^{\infty} \beta^l \hat{y}_{t+l} \tag{10}$$

where $(\varphi + \sigma)$ is the sum of the Frisch elasticity of labour supply and the inverse of the elasticity of intertemporal substitution, as in standard New Keynesian models.

—

Fiscal policy: I next move on to deriving the intertemporal government budget equilibrium in this economy. This condition is fundamentally a criterion related to nominal determinacy through the transversality condition on government debt. This is the extent to which 'FTPL'-type mechanisms are present in this model (as is the case in any macroeconomic model with a fiscal sector, even if it is not the major determinant of price level dynamics).

As pointed out by Brunnermeier et al. (2024), individual transversality conditions on household asset holdings do *not* imply that a similar transversality condition holds necessarily for aggregate debt stocks under incomplete markets. Therefore, with incomplete markets and endogenous real interest rates, the government debt valuation equation may ultimately fail to deliver a unique price level based off a simple aggregate 'transversality condition' on government debt, since there is no guarantee that such a condition holds in aggregate when markets are incomplete.

To illustrate this point, start naively from the government budget constraint, aiming to derive an integrated version of it in the hopes of finding a unique debt valuation equation.

$$B_{t-1} + \Pi_t b_{t-1} = P_t s_t + Q_t B_t + q_t b_t$$

[26] See proposition 4 of Auclert et al. (2024) for a detailed exposition of this point.

is the standard government budget constraint, given some surplus schedule s_t and bond pricing kernels Q_t, q_t.[27] All elements are multiplied by the unweighted average household SDF $\mathcal{M}_{t,t+1}$ and divided by the current price level P_t to obtain

$$\mathcal{M}_{t,t+1}\frac{B_{t-1}}{P_t} + \mathcal{M}_{t,t+1}\frac{b_{t-1}}{P_{t-1}} = \mathcal{M}_{t,t+1}s_t + Q_t\mathcal{M}_{t,t+1}\Pi_{t+1}\frac{B_t}{P_{t+1}} + q_t\mathcal{M}_{t,t+1}\frac{b_t}{P_t}.$$

Adding and subtracting elements suitably on the right-hand side, re-express this equation as:

$$\begin{aligned}\mathcal{M}_{t,t+1}\frac{B_{t-1}}{P_t} + \mathcal{M}_{t,t+1}\frac{b_{t-1}}{P_{t-1}} &= \mathcal{M}_{t,t+1}s_t + \left(Q_t\mathcal{M}_{t,t+1}\Pi_{t+1} - \mathcal{M}_{t+1,t+2}\right)\frac{B_t}{P_{t+1}} \\ &+ \left(q_t\mathcal{M}_{t,t+1} - \mathcal{M}_{t+1,t+2}\right)\frac{b_t}{P_t} + \mathcal{M}_{t+1,t+2}\left(\frac{B_t}{P_{t+1}} + \frac{b_t}{P_t}\right).\end{aligned}$$

Iterating on this expression until T, dividing the resulting expression by the SDF, and taking limits $T \to \infty$ ultimately gives:

$$\begin{aligned}\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t\Bigg[\sum_{l=0}^{\infty}\frac{\mathcal{M}_{t+l,t+l+1}}{\mathcal{M}_{t,t+1}}s_{t+l} + \frac{Q_{t+l}\mathcal{M}_{t+l,t+l+1}\Pi_{t+l+1} - \mathcal{M}_{t+l+1,t+l+2}}{\mathcal{M}_{t,t+1}}\frac{B_{t+l}}{P_{t+l+1}} \\ + \frac{q_{t+l}\mathcal{M}_{t+l,t+l+1} - \mathcal{M}_{t+l+1,t+l+2}}{\mathcal{M}_{t,t+1}}\frac{b_{t+l}}{P_{t+l}}\Bigg] + \lim_{T\to\infty}\frac{\mathcal{M}_{T+1,T+2}}{\mathcal{M}_{t,t+1}}\left(\frac{B_T}{P_{T+1}} + \frac{b_T}{P_T}\right).\end{aligned} \tag{11}$$

Note that this expression nests the standard fiscal-theoretic case with complete markets, since in this case $Q_t\mathcal{M}_{t,t+1}\Pi_{t+1} = \mathcal{M}_{t+1,t+2}$ and $q_t\mathcal{M}_{t,t+1} = \mathcal{M}_{t+1,t+2}$.

Seeing this integrated government budget constraint, one could mistakenly believe that the current price level is determined by this equation, conditional on the previous price level P_{t-1}. This logic requires the last limiting term to vanish and go to zero. However, this is *not* necessarily the case: even though the transversality condition holds on the household level as a consequence of household optimality and a no-Ponzi condition, it *cannot* be aggregated to derive a concurrent aggregate transversality condition directly off-the-shelf: the reason for that is that the unweighted average SDF $\mathcal{M}_{t,t+1}$ is discarding the heterogeneity of underlying consumption (which led to the rise of household-specific discount factors), and thus ignores the possibility of the government possibly earning an excess return on its debt issuance. This can be considered a 'safe asset premium' (Brunnermeier et al., 2024) and is reflective of the inherent value that such debt bears to households in partially overcoming the market incompleteness, possibly yielding different 'fundamental' valuations of government debt by the household vis-à-vis the government.

Instead, one can follow the approach undertaken in Brunnermeier et al. (2024), which is dubbed the *dynamic trading perspective*, and aggregate household unit-level budget constraints to obtain a dynamic aggregate constraint on sovereign debt, which factually is a mirror image of the usual

[27] All debt in this model is single-period. I briefly expose the workings of the government budget constraint with long-term indexed and non-indexed debt in appendix C.

'FTPL equation'. Accounting for the benefits of the two debt products in partially overcoming market incompleteness borne by households, and thereby being able to leverage household-level transversality conditions, I find an expression of the intertemporal budget equilibrium in terms of the real value of today's debt holdings and a suitably-discounted surplus term:

Proposition 2 *In a model with both non-indexed and inflation-indexed debt and incomplete markets, the integrated government budget constraint ('the FTPL equation') can be expressed as:*

$$\frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \tilde{\mathcal{M}}_{t,t+k} \bar{A}_{t+k} \right], \tag{12}$$

where $\tilde{\mathcal{M}}_{t,t+k} = \sum_i \mathcal{M}_{i,t,t+k} \Pi_{t,t+k+1} \frac{A_{i,t+k}}{\bar{A}_{t+k}}$ is the weighted average SDF across all households i, adjusted for inflation, with weights being proportionate to $A_{i,t+k}$. $\bar{A}_t = \frac{1}{N_i} \sum_i A_{it}$ is the average of the term A_{it}, which captures the surpluses raised by the government from each household i and the utility-weighted windfall gain that households enjoy when holding inflation-indexed debt:

$$A_{it} \equiv c_{it} - \varepsilon_{it}(1 - \tau_{it}) w_t N_t + \left[Cov_t \left(\mathcal{M}_{i,t,t+1}, \Pi_{t+1} \right) + \mathcal{M}_{i,t,t+1} \left(\mathbb{E}_t \Pi_{t+1} - \Pi_{t+1} \right) \right] \frac{b_{it}}{P_t}.$$

Proof. See appendix A.1. ■

A_{it} therefore expresses the full portfolio return of household i of holding an additional unit of net worth, consisting of the net utility gain from saving, the insurance premium on indexed debt (captured through the covariance term), and the possible windfall gain/loss from surprise inflation through indexed debt (captured through the last term). Equation (12) is 'the FTPL equation' that is used to pin down the price level at time t, given some previous price level P_{t-1}.

Maintaining this equation as determining the price level, I close the government block by assuming a simple taxation rule as in standard Fisherian models,

$$\frac{\tau_t}{\tau} = \left(\frac{s_{B,t-1}}{s_B} \right)^{\gamma_B} \left(\frac{s_{b,t-1}}{s_b} \right)^{\gamma_b} e^{\zeta_t}, \tag{13}$$

where $\tau_t \equiv \frac{T_t}{Y_t}$ are surpluses raised by the government as a fraction of output, and $s_{B,t} \equiv \frac{Q_t B_t}{P_t Y_t}, s_{b,t} \equiv \frac{q_t b_t}{P_t Y_t}$ are the real market values of the two existing types of debt. ζ_t is a standard AR(1) shock to the quantity of lump-sum taxes raised, and the policy reaction coefficients to deviations of the market values of both types of debt from their steady-state values are given by γ_B and γ_b. Steady-state values are denoted without time subscripts. In log-linearized terms, this relationship becomes:

$$\hat{\tau}_t = \gamma_B \hat{s}_{B,t-1} + \gamma_b \hat{s}_{b,t-1} + \zeta_t. \tag{14}$$

Monetary policy: Monetary policy follows an inertial Taylor rule with positive weights on both inflation and output deviations from steady-state:

$$\left(\frac{R_t^n}{R^n}\right) = \left(\frac{R_{t-1}^n}{R^n}\right)^{\rho_M} \left[\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y}\right]^{1-\rho_M} e^{\nu_t} \tag{15}$$

where ν_t is an AR(1) shock to the conduct of monetary policy. In exact log-linearized terms,

$$\hat{r}_t^n = \rho_M \hat{r}_{t-1}^n + (1-\rho_M)\left[\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t\right] + \nu_t. \tag{16}$$

Market clearing: Market clearing on the three markets of relevance in this economy is defined as follows:

- Goods market: on the goods market, aggregate consumption and production are equalized, taking into account the loss from price adjustment costs on the producer's behalf:

$$C_t + G_t + \frac{\mu/(\mu-1)}{2\kappa}\left(log(1+\pi_t)\right)^2 Y_t = Y_t. \tag{17}$$

- Labor market: labor supply and demand must be equalized:

$$\sum_i N_{it} = \sum_k N_{kt}. \tag{18}$$

- Asset market: for each class of assets, the supply by the government must be equal to cumulative household demand:

$$B_t = \sum_i B_{it} \tag{19a}$$

$$b_t = \sum_i b_{it}. \tag{19b}$$

Equilibrium: I now characterize a competitive equilibrium in this economy:

Definition 1 (Competitive Equilibrium) *A* competitive equilibrium *of the heterogeneous-agent economy is an allocation* $\{C_t, N_t, Y_t, B_t, b_t, Y_{it}, N_{it}, d_t, \tau_t\}_{t=0}^{\infty}$*, together with prices* $\{P_t, P_{it}, w_t, \pi_t, Q_t, q_t, R_t^n\}_{t=0}^{\infty}$ *and exogenous variables* $\{\nu_t, Z_t, G_t,\}_{t=0}^{\infty}$*, such that:*

- *all agents maximize their utility with suitable policy functions on* $c(\cdot), N(\cdot), B(\cdot)$*, and* $b(\dot{)}$*, solving the type-dependent value functions (6) or (7),*
- *all firms maximize their PDV of profits,*
- *the government does not violate its per-period budget constraint, levies taxes in accordance with its fiscal rule, and the price level is determined through equation (12),*
- *the central bank follows its policy rule (15),*
- *all markets clear ((18), (19a), (19b), equation (17) follows from Walras' Law), and*

- *the distribution of household wealth and productivity $\Gamma_t(B, b, z)$ evolves by its law of motion and is determined in the long-run by the fixed point of its evolution:*

$$\Gamma_{t+1}(\mathcal{B}, \mathcal{b}, z') = \int_{\{(B,b,z):g_t(B,b,z)\in(\mathcal{B},\mathcal{b})\}} Pr(z'|z)d\Gamma_t(\mathcal{B}, \mathcal{b}, z).$$

I close the model by defining the utility function of consumption for each household i as $u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}$, and the disutility function of labor supply as $v(N) = \frac{N^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$.

Steady-state: in the following I will consider a log-linearized approximation around the deterministic steady-state with respect to aggregate variables. That steady-state is characterized by a zero inflation rate, $\Pi = 1$, such that bond prices are equal to the household discount rate, $Q = \beta$ and $q = \beta$ in the absence of uncertainty. I furthermore normalize steady-state output to 1. The remainder of the steady-state is characterized explicitly in line with the calibration introduced in section 6.

Steady-state determinacy with a simplified real interest rate determination

To provide a brief characterization supporting the possible uniqueness of the steady-state despite the high complexity of the model, I briefly invoke the framework of Hagedorn (2021) with an appropriate adjustment to include inflation-indexed debt, featuring the determination of the real interest rate with the help of asset market clearing.

I provide a general treatment of a possible equilibrium of the non-Ricardian economy with inflation-indexed debt and possibly heterogeneous agents, taking into account the ramifications that bond revaluations can have on asset markets in general equilibrium. I propose that inflation-indexed debt can yield price level uniqueness in a stationary equilibrium if the real interest rate is determined *outside* the government budget equation (taking off the 'double burden' of determining both the initial price level P_0 and the real interest rate r_{ss} that this condition would alternatively be subject to), although some additional restrictions must be made. This statement is formalized in the following:

Proposition 3 (Stationary equilibrium determinacy) *Under incomplete markets, with non-negative steady-state inflation, and abstracting from aggregate uncertainty, the intertemporal government budget equation can determine a unique initial price level in stationary equilibrium even in the presence of inflation-indexed debt for non-negative steady-state inflation rates if $\frac{b}{b+B} < 1$, $r_{ss} > 0$, and if a steady-state asset demand function $\mathcal{S}(r_{SS})$ exists and is invertible.*

Proof. See appendix A.2. ■

Therefore, the model framework yields a unique initial price level in the setting with inflation-indexed debt, provided that the real interest rate is pinned down outside of the government debt valuation equation. The present result is, in a sense, a qualification of the results of Hagedorn (2021, 2024), applied suitably to a setting with inflation-indexed debt. This is done to ensure that one can operate with a clear, unique steady-state and analyze shocks to the economy without worrying about stationary equilibrium multiplicity.

The intuition behind the proof is the following: the intertemporal government budget equilibrium without inflation-indexed debt relates the price level to the real interest rate, which is determined on the asset market. With inflation-indexed debt, steady-state inflation itself becomes another element of the intertemporal government budget equilibrium. That inflation rate, which is posited to be pinned down by fiscal policy in the stationary equilibrium, is directly related to the real interest rate through the Fisher equation. Then, with the real interest rate (and implicitly inflation as well) being pinned down by asset market equilibrium, there is only one plausible real interest rate that manages to uniquely pin down the price level from the government budget constraint.

Equipped with the results on steady-state uniqueness in abstract general equilibrium models (that feature endogenous real interest rates, but do not take a stance how they arise), I now present the computational approach for the simulations with the help of the full-fledged general equilibrium model.

6 Calibration and computational approach

The parametrization of the economy used in the dynamic simulations is summarized by table 1. I follow overall the approach of Auclert et al. (2021), as I apply a conceptually similar algorithm. In the preferred calibration, I vary government spending G and the household discount factor β to ensure that the goods market and the asset market for non-indexed clear. Finally, the market for inflation-indexed debt is targeted with the help of ν, the probability of being able to access the portfolio of indexed debt actively. These endogenous parameters are summarized by table 2. The market for non-indexed debt is not targeted, but clears with a tolerance of $1e-5$, while targeted market clearing conditions clear with close to machine precision ($1e-15$). To compare various policy combinations, I here consider baseline active/passive policy coefficients (determining whether a given policy mix is fiscally-led or monetary-led) as given by Bianchi et al. (2023) within their NK-DSGE model. The parameters related to policy coefficients in the table below, $\{\phi_\pi, \phi_y, \gamma_B, \gamma_b\}$, should be taken as indicative and related to suitable active/passive policy combinations in the sense of Leeper (1991). When deviating from the baseline parameterizations mentioned in the table, I will explicitly introduce novel policy coefficients as suitable.

Parameter	Description	Value	Source/Target
Firms			
Y	Steady-state output	1	Normalization
ε	Elasticity of substitution between product varieties	9	Firm mark-up of 11% (Auclert et al., 2024a)
κ	Slope of price Phillips curve	0.055	Hazell et al. (2022), Gagliardone et al. (2023), Benigno and Eggertsson (2023)
Households			
σ	Inverted intertemporal elasticity of substitution	1	Simplification for simulation

φ	Inverse Frisch elasticity of labor supply	1	Simplification for simulation
$\underline{B}$	Lower bound for non-indexed debt holdings	0	
$\underline{b}$	Lower bound for indexed debt holdings	0	
ρ_z	Persistence of AR(1) shocks to household productivity	0.966	Auclert et al. (2021)
σ_z	Standard deviation of AR(1) shocks to household productivity	0.92	Auclert et al. (2021)
Government			
T/G	Steady-state surplus, measured by the tax-to-government spending ratio	1.03	See explanation below
r^*	Natural rate of interest	0.015	Benigno et al. (2024)
ρ_M	Inertia in Taylor-type interest rate rule	0	Simplification
ϕ_π	Monetary policy reaction to inflation deviations from steady-state	$\{0.5, 1.5\}$	For fiscally-led/monetary-led policy mix (Bianchi et al., 2023)
ϕ_y	Monetary policy reaction to output deviations from steady-state	0	Bianchi et al. (2023)
γ_B	Fiscal policy reaction to non-indexed debt	$\{0.5, -0.5\}$	For fiscally-led/monetary-led policy mix (Bianchi et al., 2023)
γ_b	Fiscal policy reaction to indexed debt	0.5	Observed low volatility in changes to indexed debt issuance
Simulation			
n_z	Number of points in asset grid for household productivity shock	11	
n_b	Number of points in asset grid for indexed debt	50	
n_B	Number of points in asset grid for non-indexed debt	50	
$\bar{B}$	Maximum holdings of non-indexed debt in asset grid	5000	
$\bar{b}$	Maximum holdings of indexed debt in asset grid	5000	Approximation to Auclert et al. (2024)
T	Number of periods used in simulations of Jacobians	300	Auclert et al. (2021)

Table 1: Baseline parametrization for the quantitative estimation

Debt/GDP shares	HH discount factor	$\mathbb{P}$(adjustment)	Govt. spending
	Main calibration: UK debt portfolio		
B: 0.8176, b: 0.3024	$\beta = 0.9569$	$\nu = 0.2293$	$G = 0.4987$
	Counterfactual: US debt shares		
B: 1.0171, b: 0.1029	$\beta = 0.9569$	$\nu = 0.1385$	$G = 0.4986$
	Counterfactual: no indexed debt		
B: 1.12, b: 0.	$\beta = 0.9570$	$\nu = 0.0052$	$G = 0.4986$

Table 2: Endogenous parameters across different debt calibration scenarios

The calibration delivers overall reasonable estimates of the endogenous parameters that are in line with the parametrization of Auclert et al. (2021). The level of government spending is not targeted to its empirical counterpart, yet the estimated government spending share of GDP is pretty much equal to government spending in the UK in 2020 (49.96%) and only slightly above the share of government spending in GDP in 2024 (44.4%).

Finally, note that to pin down both the price level and the tax rate in steady-state, I exogenously fix the tax rate to be 3% higher than government spending in GDP, such that surpluses are equal to one percent of the government spending-to-GDP ratio. Note that this assumption runs counter to currently observed budget surpluses in the UK and in the US, which are decidedly negative. The proposed model, however, has issues in solving for perpetual deficits, conditional on the long-run real interest rate being positive.[28] However, the assumption of positive surpluses in steady-state remains qualitatively and quantitatively in line with recent long-run forecasts of the current budget deficit for the United Kingdom, provided by the OBR (2024) in their historical official forecasts database (table CB).[29]

The entire steady-state is derived under the assumption of zero steady-state inflation, rendering limited relevance to the role of distorting inflation or interest rates different from the long-run natural rate. In terms of economic aggregates, the steady-state is thus well-described by the above calibration. Thanks to the normalization of output to unity and the calibrated share of government spending of ~ 0.4985, one can deduce that consumption in steady-state is equal to 0.5015 by market clearing, while taxation is equal to 0.5134.

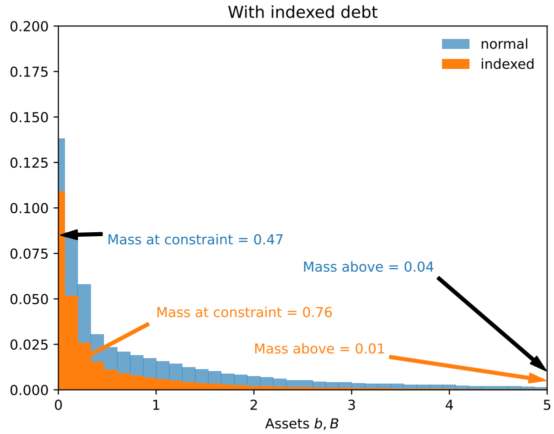
In terms of government debt, the model will operate under and compare three different steady-state calibrations: one which follows the observed modal split of sovereign debt into non-indexed and indexed debt (such that $B = 0.8176$ and $b = 0.3024$) in the United Kingdom (which is the G7 country with the highest share of indexed debt), and two counterfactual calibrations with either a split between indexed and non-indexed debt similar to the US (i.e., $B = 1.0171$ and $b = 0.1029$), or the complete absence of any indexed debt (i.e., $B = 1.12$ and $b = 0$). I therefore exogenously postulate the same steady-state bond supply across the calibrations, given that bond supply as a share of GDP is a relatively low-frequency variable, and given that the distribution of assets is an equilibrium outcome that depends on this supplied quantity. Many of the exercises will resolve around the differences between these calibrations, as I will mainly focus on the effect that indexed government debt has on economic aggregates.

Even though government debt aggregates are exogenously supplied in steady-state for all calibrations, the distribution of debt across households cannot be deduced immediately from the calibration itself, as it is generally dependent on the properties of the idiosyncratic process to income in a way that is not fully captured by the calibration itself. Figure 9 plots the distribution of debt holdings across households in two cases - once for the standard calibration to the UK, and one for the counterfactual calibration where steady-state issuance of indexed government debt is set to 0. For the calibration without indexed debt supply, plotted in panel (b), no households hold indexed debt in steady-state, while indexed bonds are held by 24% of all households in the main calibration.

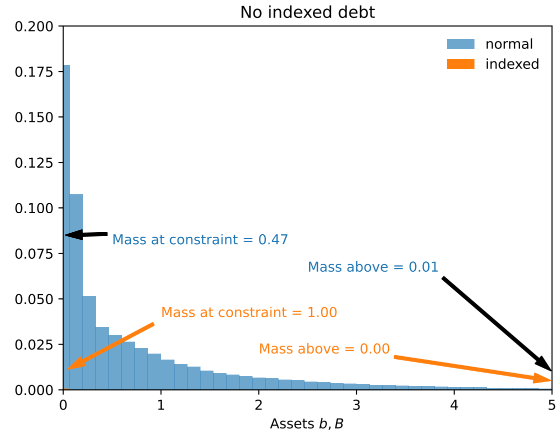
Furthermore, even though the distribution of debt is by all means not a targeted moment, the model provides (reassuringly) a significant skew in the distribution of debt holdings across simulated households. Given the larger presence of non-indexed sovereign debt, the holdings thereof are of course larger across the entire distribution, reflecting the evidence for the US provided in section E.1 in the appendix.

[28] Kaplan et al. (2023) solve a model with negative surpluses and a negative steady-state real interest rate, but this is computationally difficult to implement for the chosen algorithm.

[29] Conditional on a $\sim 40\%$ share of government spending in GDP, the projected 1% budget surplus in the long-run as a share of GDP is equivalent to a ratio of sovereign income to spending of 1.025.



(a) Both types of debt (UK calibration)



(b) Non-indexed government debt only

Figure 9: Steady-state distributions of asset holdings across households in the calibrated steady-state

Computational details - using the Sequence-Space Jacobian:

The solution to the model that is linear in aggregates, but non-linear in idiosyncratic shocks, is derived by using the Sequence-Space Jacobian method developed in Auclert et al. (2021), which itself constitutes an evolution of the methods pioneered by Reiter (2009). The computational method I employ therefore generates perfect-foresight solutions in aggregates in response to time-zero perturbations of exogenous disturbances, but it maintains the non-linearity underlying the responses of heterogeneous households.

First solve the heterogeneous household block is solved, taking aggregate prices as given, for both the steady-state policy functions (through backwards iteration) and the steady-state distribution of asset holdings (through forwards iteration). Both solve with a numerical tolerance of up to $1e-14$, and are subsequently used to inform other blocks of the model (such as firm optimality, government policies, and market clearing) and to generate updates of aggregates where necessary. The two components (heterogeneous-agent and aggregate) interact and iterate until convergence, which is reached with a numerical threshold of $1e-9$ in the solution that is linear in aggregates, which is reasonable given the high degree of complexity underlying household behavior in the presence of two types of assets. The discretization of exogenous disturbances and the asset grid remain in line with the calibration of Auclert et al. (2021), who prove that the numerical error induced by such a discretization indeed remains almost negligible.

7 Quantitative insights in HANK with indexed debt and rich policy interactions

With the computational algorithm at hand, I solve and estimate the model's aggregate impulse-responses for a number of shocks, using the parametrization from table 1, but varying the calibration of the debt shares in line with table 2. Here, I will mostly focus on the effects of unanticipated disturbances to *government spending* G_t, which directly influence the surpluses raised by the government in any given period.[30]

Before considering the dynamics implied by this rich model in detail, I first look at the role that inflation-indexed debt plays for the amplification of shocks as evidenced through simulated moments, in line with the principal focus of the paper. To get a more detailed grasp behind that role borne by the presence of inflation-indexed debt for aggregates, I compare the simulated volatility of a number of macroeconomic aggregates across all calibrations (UK calibration, counterfactual US distribution of debt across the two types, and issuance of non-indexed debt only) and across both 'standard' policy combinations (passive monetary/active fiscal (fiscally-led) and active monetary/passive fiscal (monetary-led)). The results of this exercise are presented in table 3.

	Normalized standard deviations across policy scenarios					
	PM/AF-UK	PM/AF-US split	PM/AF-NoIndex	AM/PF-UK	AM/PF-US split	AM/PF-NoIndex
G	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
Y	0.994010	1.024624	0.909531	0.845227	0.893975	0.835870
C	0.125033	0.162543	0.103052	0.249651	0.245962	0.288624
π	0.170823	0.154582	0.094898	0.116095	0.113646	0.099419
r	0.124260	0.132743	0.090154	0.155643	0.144316	0.146941
N	0.994010	1.024624	0.909531	0.845227	0.893975	0.835870

Table 3: Normalized standard deviations of aggregate variables in response to fiscal shocks with $\rho_G = 0.5$

The three left-hand columns can be summarized in one line and yield one of the major quantitative insights of the paper: the volatility of economic aggregates increases in the presence of inflation-indexed debt, conditional on being in the fiscally-led policy case. Of particular interest in that regard is the fourth row of table 3, which captures the volatility of inflation in response to government spending shocks. Here, one can see that, conditional on being in the fiscally-led policy scenario, the unweighted volatility of inflation is around 80% higher in the calibration to British indexed/non-indexed debt shares relative to the counterfactual without any inflation-indexed debt being present. With a calibrated share of indexed sovereign debt of about 30%, on average, a one percentage point increase in the share of inflation-indexed debt more or less corresponds to an approximately 2.6% increase in the volatility of inflation in response to uncovered government spending shocks.

This effect is far from linear, as evidenced by the second column which shows that the US calibration attains elevated levels of volatility, too, despite the share of indexed debt being less than half

[30] Appendix B provides an overview of the dynamic responses to expansionary monetary policy shocks.

of the share of indexed debt in the calibration to UK debt shares. To the best of my knowledge, this paper is among the first to quantitatively evaluate the impact that inflation-indexed debt can have on the volatility of inflation, and how such changes in volatility are directly related to the monetary-fiscal policy nexus, as the inflation volatility increase is evidently much smaller under the monetary-led policy scenario, amounting to a difference of only 17%.

Impulse-response functions implied by the model

I will now look in more detail at the impulse responses to government spending shocks and the role borne by inflation-indexed debt when an unexpected government spending increase occurs. As the persistence of any shock is relevant, the model will be simulated under different possible autocorrelations of the fiscal shock to highlight the role of persistence and the forward-looking nature of the intertemporal government budget constraint. The initial focus rests on the case of a *fiscally-led policy mix* in line with the first parametrization introduced in table 2, i.e., the baseline calibration to the UK economy. Figure 10 plots IRFs of aggregate variables in response to a 100bp expansionary fiscal shock that increases the need for fiscal spending when the shock is highly persistent, i.e., $\rho_G = 0.8$.

A number of observations is worth highlighting: the responses of consumption and tax rates are in line with canonical macroeconomic models and the expected reactions in response to the fiscal expansion: in response to the fiscal expansion, there is an instantaneous increase in output that persists alongside the expenditure increase. In general, there is little amplification in output from the spending increase. For the main calibration (blue line), private consumption is partially crowded out (leading to an impact change of output of less than one), while for lower levels of inflation-indexed debt (orange and green lines), there is a slight multiplying effect that is in the ballpark of modern estimates of the impact of expansionary fiscal policy (Auerbach and Gorodnichenko, 2012; Ramey and Zubairy, 2018; Ramey, 2019).

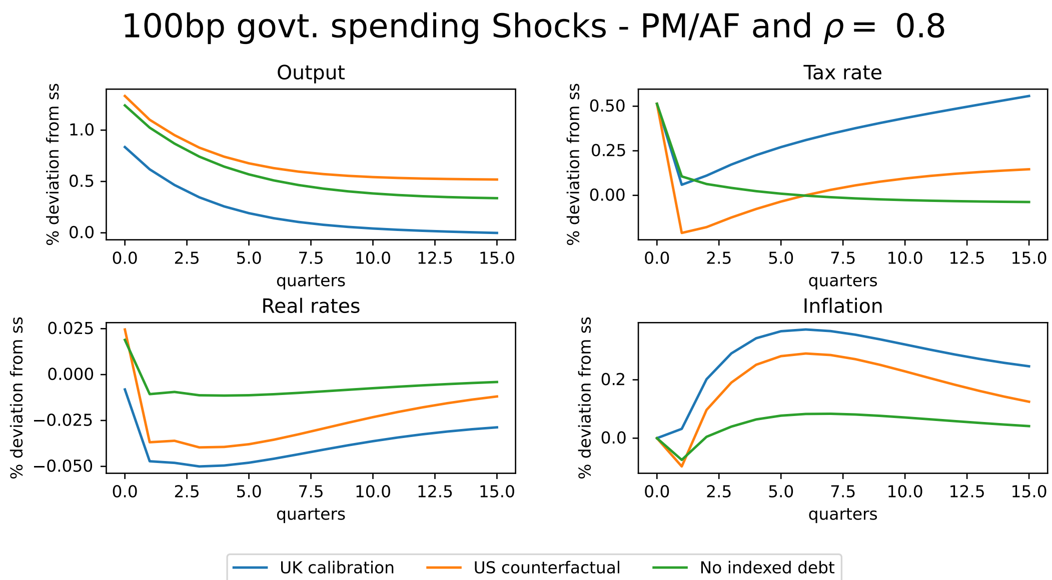


Figure 10: IRFs to the government spending shock under a fiscally-led policy mix.

Whether or not the output effect of government spending is more or less than 1 is intricately linked to the tax rate changes in response to the government spending change. Recall that this analysis concerns a *fiscally-led policy mix*, which signifies that the tax adjustment does not cover the full government spending increase. Indeed, this is the case here. The dynamics of the tax rate, however, are clearly linked to the share of inflation-indexed debt: without such debt being present (green line), the tax rate monotonically decays back to its steady-state. Once inflation-indexed debt is present, however, the equilibrium tax rule admits a 'V-shape': after an initial increase in the tax rate on impact, the tax rate briefly decreases on the expectation of the shock being only temporary.[31] Over time, however, the tax rate subsequently increases to cover the additional expenses arising from the cost of servicing indexed debt.

The evolution of the real interest rate in response to the fiscal impulse is again tightly linked to the share of inflation-indexed debt in each calibration. In the calibration without indexed debt (green line), the real interest rate briefly appreciates on impact of the shock due to expected deflationary pressure before returning immediately to the vicinity of its steady-state level. With positive levels of inflation-indexed debt, the impact change of the real interest rate is ambiguous and depends on the exact level of inflation-indexed debt. After the impact period, real interest rates depreciate sharply by up to 50 basis points in the view of expected inflationary pressure coming from the cost of serving inflation-indexed debt. This cost is increasing in the share of such debt in the economy.

Finally, the panel on the bottom right quantifies the main interest of this paper - the annualized rate of inflation in response to the fiscal expansion. In the present model, the price pressure aris-

[31] *Negative* deviations of the tax rate from equilibrium, as temporarily observed in the calibration to US debt shares, are possible as the real value of government bonds decreases below steady-state (which is related to the large negative real return shock that lowers the prices of bonds).

ing from a fiscal expansion is only minimal without inflation-indexed debt, peaking at about 0.08% six quarters after the shock materializes. Inflation-indexed debt, however, proves to magnify inflationary pressure quite significantly: with positive levels of inflation-indexed debt, annualized rates of inflation peak at 0.38% in the UK calibration and 0.29% in the counterfactual with debt shares as in the US, respectively. This implied debt-inflation multiplier therefore aligns well with the empirical evidence presented in section 2, being placed between the first and the third tercile of the range of admissible estimates arising from the high-powered 'mini-budget shock'.

The multiplier for the US calibration furthermore fits well with the evidence presented in Hazell and Hobler (2024), who find a debt-inflation multiplier in the US of 0.19%. The model therefore attributes a significant share of the differences in the debt-inflation dynamics between the US and the UK to the differences in the share of inflation-indexed debt, confirming the intuition laid out by the model.

Confirming the intuition behind the mechanisms underpinning inflation-indexed debt, the 'break' observed in the inflationary response in period 1 can be directly attributed to inflation-indexed debt as well: since inflation-indexed debt feeds back to the price level only with a one-period lag (as derived in the simple model in section 3), the inflationary pressure is not increasing monotonically, but only with this one period lag as seen in figure 10.

Summarizing, I therefore find that turning off the debt indexation channel of government debt (i.e., setting inflation-indexed debt to zero) nullifies all dynamics beyond the first-order dynamics of the spending shock, nesting the standard expected reaction to a fiscal expansion under a fiscally-led policy mix with non-Ricardian households: output and inflation co-move in general, but no higher-order dynamics are observed. Once inflation-indexed debt is present, however, inflationary pressure becomes more pronounced and persistent, accompanied by tax changes that reflect the need of the fiscal authority to cover the additional expenses arising from serving the cost of maturing inflation-indexed debt.[32]

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An important factor in the analysis is the persistence of the government spending shock, ρ_G. As the persistence of the shock underlying figure 10 is relatively high, the observed dynamics are tightly connected to intertemporal substitution motives for the household. To highlight the 'barebones' reaction of the economy to a one-off government spending shock (and to highlight the corresponding relevance of inflation-indexed debt in this world), I consider a non-persistent fiscal shock next.

[32]Complementary impulse-response functions of bond prices and quantities, as well as of the price level itself, are provided in appendix B.

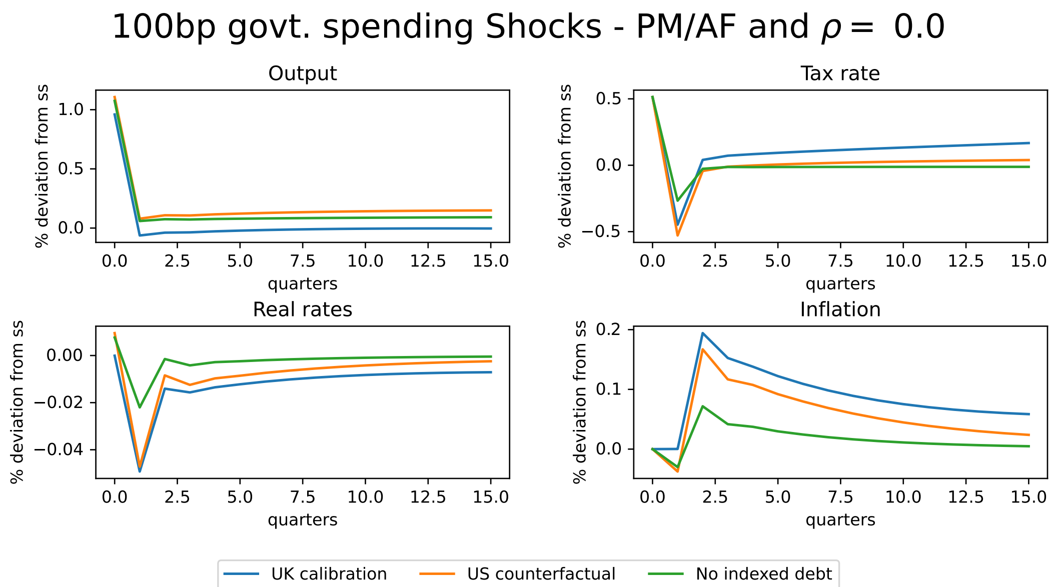

Figure 11: IRFs to the government spending shock under a fiscally-led policy mix - counterfactual without indexed debt.

Figure 11 summarizes the aggregate response of the economy to a fiscal spending shock when the government spending shock is not persistent at all, $\rho_G = 0$. Unsurprisingly, the persistence of output is virtually zero, too. As the government spending shock is short-lived, intertemporal substitution motives matter less, leading to less of a crowding out of consumption on impact. Between the three indexed-debt-share calibrations, there is a minimal difference in in output even after the shock dies out, which will be linked explicitly to the behavior of the tax-inflation nexus.

Focusing next on the top-right panel shows that the fiscal authority again covers the shock to a limited extent (nominally 50%, in line with the calibration). Once the shock dies out, there is a brief small deviation of the tax rate below zero in the subsequent period across all calibrations, which is mostly linked to a temporary crowding out through changes in the equilibrium value of both types of debt and related wealth effects on the households. In the case without indexed debt (green line), the tax rate is subsequently flat at zero, while it remains slightly positive in the cases with inflation-indexed debt (orange and blue lines).

The fact that tax rates remain elevated can be directly linked to materialized inflation rates, depicted in the bottom right panel. Compared to figure 10, materialized inflation rates in figure 11 are generally less than half as large, but there is still a pronounced increase in inflation rates from two quarters after the initial shock onwards. This is again related to a devaluation of the surplus-backing of the stock of government debt, which must be counterfinanced by a devaluation of the stock of debt. While this is also the case without indexed debt (in the green line), the presence of such debt magnifies the inflationary pressure from the fiscal expansion by a factor of two. The differences between the US-debt-shares and UK-debt-shares calibrations are marginal, since most of the differences are covered by differences in the tax rate in the medium-run.

Real rates correspondingly turn negative at the initial shock, followed by a gradual increase back to their respective steady-state value. Inflation rates therefore also mirror the pattern of real rates in line with the Fisher equation (as the monetary authority remains passive), such that there is an uptick in inflation shortly on impact followed by a gradual unwinding relative to the observed steady-state of inflation.

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Finally, I highlight what changes in the simulations when a *monetary-led policy mix* is considered instead, corresponding to fiscal policy turning 'passive' in the language of Leeper (1991). The calibration of the policy parameters in this case follows from table 1. Figure 12 summarizes the results from this exercise for highly persistent fiscal shocks, $\rho_G = 0.8$.

The response of output turns out to be qualitatively similar *on impact* relative to the fiscally-led policy mix for the calibration without indexed debt, but for both calibrations with positive indexed debt levels, output is generally more sensitive to the government spending shock - there is less crowding out both on impact and in the long-run. For the case without indexed debt, however, observe that the output reaction to the government spending impulse turns negative in the medium run (after nine quarters), reflecting that under monetary-led policy mixes fiscal policy generates smaller wealth effects on behalf of the households, such that the increased spending will be financed in part by a later reduction in available resources.

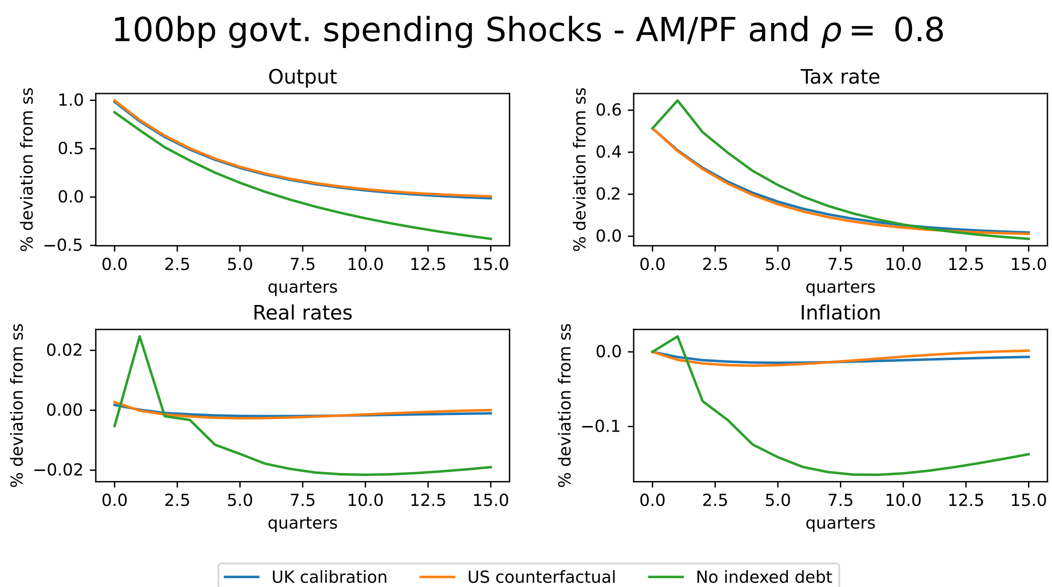

Figure 12: IRFs to the government spending shock under a monetary-led policy mix.

Across the board, there is little quantitative difference between the UK and the US debt share calibrations. Since this specification follows a conventional monetary-led policy mix, fiscal policy, as exemplified through the tax rate, passively adjusts to ensure that the government budget constraint holds. It does so by increasing tax rates by a consistently higher margin relative to the fiscally-led policy case. Because the tax rule shifts correspondingly, the real value of government debt remains unchanged, which is reflected in the absence of large movements of real rates and, correspondingly, of materialized inflation rates.

The model without inflation-indexed debt behaves vastly differently in the monetary-led policy mix, generally featuring a smaller response of output coincident with a sharper increase in tax rates and a temporary increase in real rates above their equilibrium level, followed by a depreciation of real rates in the medium-run. Inflation rates, after being positive on impact, similarly exhibit deflationary pressure. The reason for that lies in the absence of risk-sharing among households and the imperfect insurance properties borne by normal (non-indexed) bonds: in the light of such incomplete markets, the government spending measure exhibits a greater degree of Ricardian disequivalence, impacting households through a negative wealth effect. This negative wealth effect contributes to a reduction in household demand and a slight deflationary pressure, as materialized in figure 12.

Appendix B presents further omitted simulation results, in particular related to the IRFs of bond prices and interest rates, household policy functions and monetary policy shocks. In particular the revaluation of the bonds as expressed through their prices are of interest, as they confirm the above arguments that the revaluation of the intertemporal government budget equation belongs to the main determinants of the inflationary response.[33]

Decomposing the price level response in the monetary-led policy mix

The government debt valuation equation (12) allows the decomposition of the drivers of inflation in the model by postulating that this equation is informing the rate of inflation under the fiscally-led policy mix (following Bianchi et al. (2023) and Kaplan et al. (2023)) and decomposing the various drivers of inflation in general equilibrium across the UK and the US debt share calibrations. Consider the government debt valuation equation, depicted suppressing household heterogeneity for expositional simplicity:

$$\begin{aligned} \frac{B_{t-1}}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \Bigg[\sum_{k=0}^{\infty} \tilde{M}_{t,t+k} \big[c_{t+k} - \varepsilon_{t+k}(1-\tau_{t+k}) w_{t+k} N_{t+k} \\ + \Big[Cov_t \left(\tilde{M}_{t+k,t+k+1}, \Pi_{t+k+1} \right) + \tilde{M}_{t+k,t+k+1} \left(\mathbb{E}_{t+k} \Pi_{t+k+1} - \Pi_{t+k+1} \right) \Big] \frac{b_{i,t+k}}{P_{t+k}} \Big] \Bigg]. \end{aligned} \tag{20}$$

Through a linear approximation, I recover the terms in blue, yellow, green, and red explicitly, claiming that all further terms (depicted in purple) are higher-order terms captured by the remaining household heterogeneity.[34]

[33]In further work, I aim to quantify the effect of household heterogeneity more fully with the help of non-linear impulse-response functions that might showcase significant amplification of observed responses in labor supplied, output, and inflation for the case with inflation-indexed debt.

[34]In terms of the measurement of the purple terms, I simply attain the residual difference between all other terms and the gross rate of inflation to the purple terms, mirroring the fact that the solution algorithm delivers a solution that

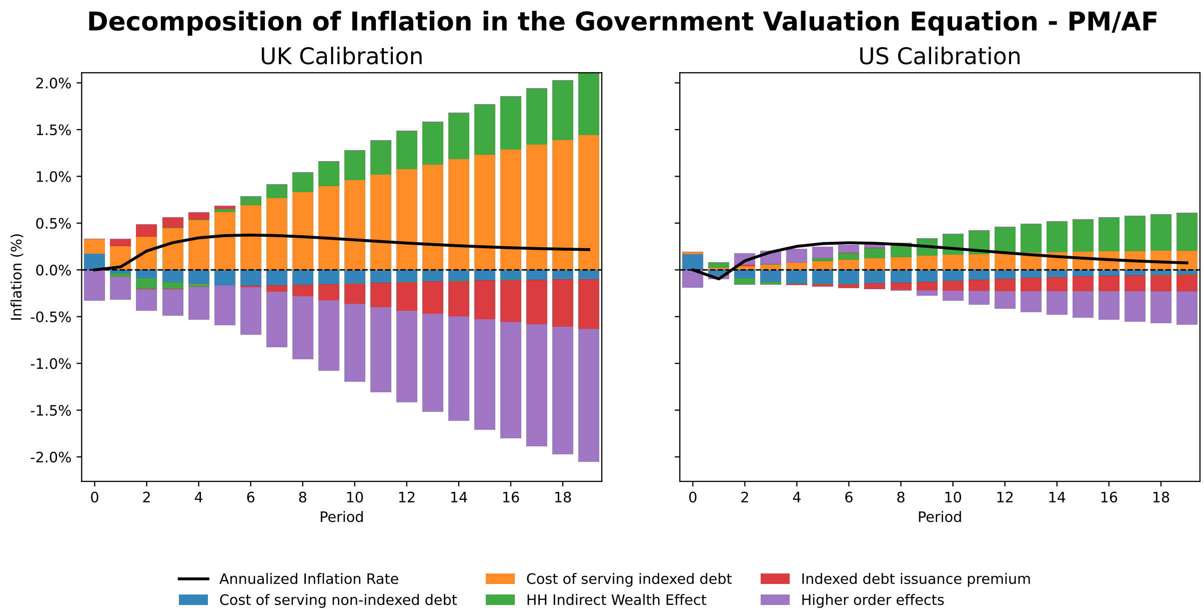

Figure 13: Decomposition of inflation in response to a 1% government spending shock under a fiscally-led policy mix.

Figure 13 shows the results of this decomposition of inflation in the two leading calibrations (to debt shares as observed in the UK and the US) under a conventionally fiscally-driven policy mix, as defined in table 1.

Across both calibrations, one can observe that the cost of serving maturing inflation-indexed debt indeed contributes significantly to observed inflation rates, as do the wealth effects on the households which are net-positive (except for the first few periods, in which the tax rate change outweighs the benefits borne from holding sovereign debt). As the inflation rate increases, there is a premium that the government can exert from issuing indexed debt, creating a medium-run deflationary pressure (red bars). A relatively important part of the inflationary dynamics are higher-order effects, which are generally deflationary in the medium-run across both calibrations, arising from the movements in the cross-section of the stochastic discount factor through the path of taxation that yield depressed consumption levels and, therefore, deflationary pressure on aggregate.

Even though the absolute difference in observed inflation levels is relatively small, the decomposition reveals that the volatility of the individual aspects increases sharply in the share of inflation-indexed debt. The magnitude of the individual contributors to inflationary/deflationary pressure are almost an order of magnitude larger in the calibration to UK debt shares, but the effects are canceling each other out to a significant extent.

is linear in aggregates, but plausibly non-linear in idiosyncratic elements.

First-year responses of the price level depending on the share of inflation-indexed debt and the fiscal policy response

Having considered the general response of a battery of macroeconomic variables in response to surprise government spending shocks, I now zoom into the role borne jointly by inflation-indexed debt and fiscal-monetary policy interactions, as exemplified through their respective policy rules (13) and (15) for inflation on impact of such shocks. To that goal, I fix the monetary policy coefficients at the levels summarized by table 1 and vary the share of inflation-indexed debt in the government debt portfolio, $\omega_t = \frac{b_t}{B_t + b_t}$, between $[0, 1)$[35], while also varying the strength of the fiscal policy reaction coefficients, $\{\gamma_B, \gamma_b\}$, between $[0, 1]$, which are the coefficients under which fiscal policy is conventionally considered "active". Note that this analysis is restricted to cases in which $\gamma_B = \gamma_b$ here, such that no variation of impact inflation is induced by a change of the *relative* prevalence of the two types of debt, as the share of inflation-indexed debt is kept at its respective calibrated steady-state value.

The first case considered is the one of conventionally passive monetary policy ($\phi_\pi = 0.5$). The reaction of the price level in the first year after the impulse across the policy combinations and various shares of inflation-indexed debt in the sovereign debt portfolio is depicted in figure 14.

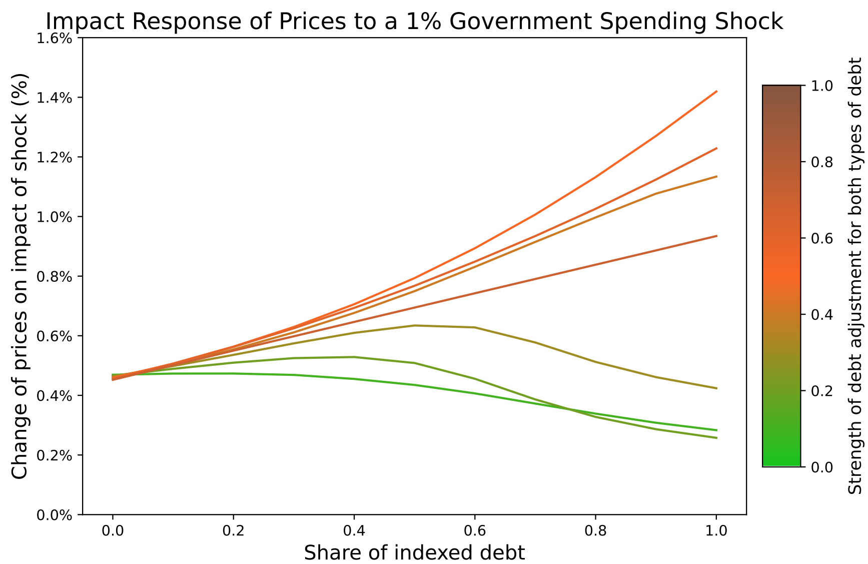

Figure 14: Cumulative one-year reaction of prices in response to fiscal spending shocks under a fiscally-led policy mix.

On the x-axis, I vary the share of indexed debt in the total debt portfolio (while maintaining a constant overall relation between the gross stock of debt and GDP), while the colors indicate the chosen fiscal reaction coefficients $\gamma_B = \gamma_b$. Thus, orange and especially brown colors reflect 'less active' fiscal policy in the conventional sense (as more of the shock is covered by corresponding tax raises), while greener colors reflect 'more active' fiscal policy.

[35]Recall that it is impossible for all debt to be inflation-indexed in line with proposition 3.

I first focus on the leftmost point at which the share of inflation-indexed debt is exactly zero. One can observe the conventional response that is expected in such models of fiscal-monetary interactions. The 'more active' fiscal policy is (i.e., as $\gamma_b, \gamma_B \to 0$), the stronger is the immediate impact on the price level when the shock occurs (the green lines are the highest for a zero share of inflation-indexed debt). This is fully in line with existing evidence from fiscal-monetary interaction models, and reflects that the necessary devaluation of a fully nominal debt stock is higher when income taxation does not react at all to the expansionary government spending shock. The difference across the various policy scenarios, however, is relatively small in this impact period.

The more interesting dynamics occur as one moves to the right in the above picture, i.e., as the share of inflation-indexed debt in the government debt portfolio is gradually increased. The interaction between the adjustment coefficients and the share of indexed debt into two categories *based off the value of the adjustment coefficients of fiscal policy* can be broadly categorized as follows:

- $\gamma_B = \gamma_b \gtrsim 0.3$ (orange-brown lines): Here, fiscal policy adjusts by covering comparatively more of the government spending shock through a corresponding increase in taxation. The higher the share of inflation-indexed debt, the greater is the change of the price level on impact, as the debt stock that can be devalued once the shock manifests itself is comparatively smaller, leading to a larger needed depreciation of that (smaller) stock of non-indexed debt.
- $\gamma_B = \gamma_b \lesssim 0.3$ (green lines): in these cases, the taxation schedule of the government covers only very little of the additional expense coming from the government spending shock. For realistic levels of inflation-indexed debt (below 40%), there is a general increase in the response of prices on impact, in line with standard predictions. The more indexed debt is present in the economy, the higher the devaluation of the outstanding debt stock must be, as in the case above. As the share of inflation-indexed debt becomes very large, however, a surprising effect comes into play: the impact change of the price level actually starts to become smaller. This is directly related to the *real* side of the economy, as evidenced in figure 10: as the share of inflation-indexed debt increases, the real expansion of the economy becomes less pronounced, and the net worth of the wealthier households that hold large quantities of inflation-indexed debt decreases (as shown in figure B.6 in the appendix). This holds particularly true when the taxation reaction of the government is rather small. Through the Phillips Curve, then, downwards pressure on prices is exerted, overcoming some of the inflationary pressure induced through the government budget equilibrium.

Fiscal-monetary policy combinations and determinacy

As a final exercise, I consider explicitly for which values of the fiscal and monetary policy parameters one can establish determinacy of the linearized system. In doing so, I exploit the 'winding number criterion' developed in Auclert et al. (2023), which is suitable given the choice of the sequence-space Jacobian as the primary device to solve the full dynamic model.[36]

Figure 15 summarizes the determinacy properties of the model on an equispaced grid of the fiscal policy reaction parameters γ_B and γ_b, setting $\phi_\pi = 0.5$. The model thus effectively says that monetary policy does *not* raise interest rates more than one-for-one with inflation, which is conventionally dubbed 'passive monetary policy' (or, equivalently, a fiscally-led policy mix). Interestingly, the results relate to canonical determinacy principles in line with Leeper (1991), even if they do not fully overlap. In particular, uniqueness of the equilibrium path is reached for conventional values of the non-indexed debt reaction parameter γ_B when inflation-indexed debt issuance in response to fiscal shocks is zero (which is evidenced by the last row of figure 15).

Once the taxation schedule is directly related to inflation-indexed debt deviations from equilibrium as well ($\gamma_b > 0$), two interesting phenomena arise. First, a trade-off in the government debt rule arises, by which an increased reactiveness of the taxation schedule to deviations of the value of indexed debt b_t from steady-state must be paid off with a smaller reactiveness with respect to the market value of non-indexed debt B_t. Second, this trade-off is non-linear: in particular, despite monetary policy being conventionally 'passive', it is possible for governments to react *more than one-for-one* ($\gamma_b > 1$) with their taxation schedule in relation to deviations of inflation-indexed debt from steady-state, provided that the adjustment with respect to non-indexed debt B_t is small enough.

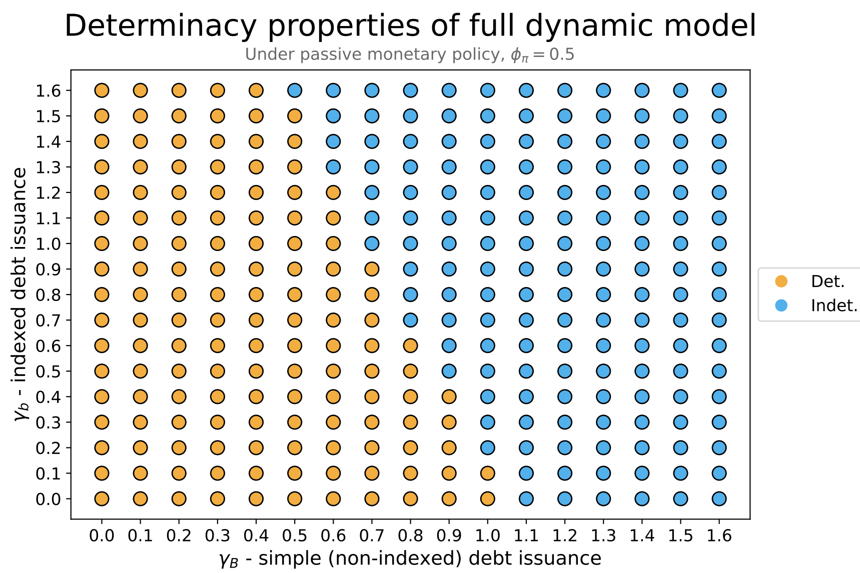

Figure 15: Determinacy of the generalized Jacobian in relation to choices for the fiscal and monetary policy reaction coefficients when monetary policy can be considered conventionally 'passive'.

[36] A more detailed exposition of the 'winding number criterion' can be found in Auclert et al. (2023). Intuitively, one can relate this criterion to the Blanchard and Kahn (1980)-condition, which is cast in state-space. The winding number criterion provides a generalizable 'mapping' of the Blanchard-Kahn conditions for the sequence-space, i.e., allowing infinitely many quasi-'roots' of the linearized system. Note that the prerequisites to apply the winding number criterion, such as the quasi-Toeplitz property of the generalized Jacobian, are not violated (the corresponding results are available upon request).

8 Discussion, summary, and next steps

This paper introduced inflation-indexed debt into canonical non-Ricardian general equilibrium models. I first provided support for the role of inflation-indexed debt as a major determinant of inflationary dynamics with the help of local projections applied to the UK and the US, as well as with a specific high-powered fiscal shock in the UK in September 2022. Next, I established in a simplified model that such debt itself suffices to make the price level a backward-looking state variable: the previous price level therefore matters directly for the determination of today's price level. This was followed up by a brief discussion of existing comparisons between fiscally-led policy mixes and other mechanisms inducing non-Ricardian household behavior, establishing that inflation-indexed debt operates under mechanisms that are otherwise not present in models with non-Ricardian mechanisms that abstain from an explicit fiscal block. Finally, I introduced inflation-indexed debt in a state-of-the-art macroeconomic model with imperfect markets and household heterogeneity, ensuring the existence of a unique steady-state before providing model-driven evidence that inflation-indexed debt can indeed exacerbate the inflationary response to government spending shocks, in particular when fiscal policy is considered conventionally 'active' in the sense of Leeper (1991).

Both the empirical and theoretical results derived in this paper thus tarnish the classic notion that inflation-indexed bonds always limit inflation in a given country by offering governments a commitment device to 'not inflate the debt away', as exposed in Campbell and Shiller (1996). While this notion can remain true absent government deficit shocks, the results point out that once the government budget is ex-post (after debt issuance) in disarray, the inflationary consequences of funding shortfalls can increase in the share of inflation-indexed debt. Issuance of indexed debt can therefore backfire despite its great ability to serve as an ex ante commitment device following Schmid et al. (2024).

Despite these conclusions, more can be done to emphasize the interaction between inflation-indexed debt and materialized inflation rates. In the near future, I intend to provide a complete estimation of the model based off long-running samples of UK and US data, which allowing to pin down the drivers of inflation across time through a historical decomposition. Another refinement should see the inclusion of long-term government debt: as Cochrane (2001) and Barro and Bianchi (2023) show, the maturity structure of government debt hugely matters for the trade-off between a front-loaded and a delayed inflation response to sovereign deficit shocks.

Finally, inflation-indexed debt can enhance the understanding in an important sphere of recent policy debates on the possible regressivity or progressivity of inflation as implicit taxation. As evidenced by figure E.1 in the appendix, inflation-indexed debt, which serves as a principal insurance device against unexpected inflation, seems to be particularly skewed in household portfolios towards the highest decile of the income distribution. A more thorough analysis of the welfare effects of unexpected inflation to households at varying income deciles should therefore be considered as a further policy-relevant application in due course.

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Appendix

A Derivations and proofs from the main text

A.1 Derivations from section 5

Derivation of equation (12) (proof of proposition 2)

This section presents the derivations underlying a *dynamic trading perspective* for asset valuation laid out in Brunnermeier et al. (2024), which avoids fallacies related to a possibly nonexistent aggregate transversality condition by clearly defining the valuation differences of government debt between households and the government based off the insurance properties that government bonds bear for households. This allows to leverage household-level transversality conditions to derive an aggregate FTPL-type condition that only holds for one initial candidate price level.

The starting point for this valuation equation of government debt is the household budget constraint, which was given by

$$P_t c_{it} + Q_t B_{it} + q_t b_{it} = \varepsilon_{it}(1-\tau_{it})P_t w_t N_t + B_{i,t-1} + \Pi_t b_{i,t-1}$$

for each household i. Following the results derived in the household block, let households price bonds in accordance with their *SDF*:

$$B_{i,t-1} + \Pi_t b_{i,t-1} = \mathbb{E}_t\left(M_{i,t,t+1}\right)B_{it} + \mathbb{E}_t\left(\Pi_{t+1}M_{i,t,t+1}\right)b_{it} + P_t(c_{it} - \varepsilon_{it} w_t N_t(1-\tau_{it})).$$

Splitting up the second expectation term yields:

$$B_{i,t-1}+\Pi_t b_{i,t-1} = \mathbb{E}_t\left(M_{i,t,t+1}\right)B_{it}+\mathbb{E}_t\left(M_{i,t,t+1}\right)\mathbb{E}_t\left(\Pi_{t+1}\right)b_{it}+b_{it}Cov_t\left(M_{i,t,t+1},\Pi_{t+1}\right)+P_t(c_{it}-\varepsilon_{it}w_tN_t(1-\tau_{it})).$$

Divide all elements by P_t and add/subtract relevant terms on the right-hand side to ensure that one can iterate on the resulting expression:

$$\begin{aligned}\frac{B_{i,t-1}+\Pi_t b_{i,t-1}}{P_t} &= \mathbb{E}_t\left(M_{i,t,t+1}\right)\Pi_{t+1}\left[\frac{B_{it}+\Pi_{t+1}b_t}{P_{t+1}}\right] + (c_{it} - \varepsilon_{it}(1-\tau_{it})w_t N_t) \\ &+Cov_t\left(M_{i,t,t+1},\Pi_{t+1}\right)\frac{b_{it}}{P_t} + \mathbb{E}_t\left(M_{i,t,t+1}\right)\frac{b_{it}}{P_t}\left(\mathbb{E}_t\Pi_{t+1} - \Pi_{t+1}\right).\end{aligned}$$

Now, start iterating on this expression. The first iteration yields:

$$\frac{B_{i,t-1} + \Pi_t b_{i,t-1}}{P_t} = \mathbb{E}_t \left(\mathcal{M}_{i,t,t+1} \right) \Pi_{t+1} \Bigg[\mathbb{E}_{t+1} \left(\mathcal{M}_{i,t+1,t+2} \right) \Pi_{t+2} \left[\frac{B_{i,t+1} + \Pi_{t+2} b_{i,t+1}}{P_{t+2}} \right]$$
$$(c_{i,t+1} - \varepsilon_{i,t+1}(1 - \tau_{i,t+1}) w_{t+1} N_{t+1}) + Cov_{t+1} \left(\mathcal{M}_{i,t+1,t+2}, \Pi_{t+2} \right) \frac{b_{i,t+1}}{P_{t+1}}$$
$$+ \mathbb{E}_{t+1} \left(\mathcal{M}_{i,t+1,t+2} \right) \frac{b_{i,t+1}}{P_{t+1}} \left(\mathbb{E}_{t+1} \Pi_{t+2} - \Pi_{t+2} \right) \Bigg]$$
$$+ (c_{it} - \varepsilon_{it}(1 - \tau_{it}) w_t N_t) + Cov_t \left(\mathcal{M}_{i,t,t+1}, \Pi_{t+1} \right) \frac{b_{it}}{P_t} + \mathbb{E}_t \left(\mathcal{M}_{i,t,t+1} \right) \frac{b_{it}}{P_t} \left(\mathbb{E}_t \Pi_{t+1} - \Pi_{t+1} \right).$$

Continuing rolling over, applying the LIE, and simplifying SDFs by making use of the identity $\mathcal{M}_{i,t,t+k} \mathcal{M}_{i,t+k,t+l} = \mathcal{M}_{i,t,t+l}\ \forall t, k, l$ obtains:

$$\begin{aligned} \frac{B_{i,t-1} + \Pi_t b_{i,t-1}}{P_t} = \mathbb{E}_t \Bigg[\sum_{k=0}^{\infty} \mathcal{M}_{i,t,t+k} \Pi_{t+1,t+k+1} \Big\{ &(c_{i,t+k} - \varepsilon_{i,t+k}(1 - \tau_{i,t+k}) w_{t+k} N_{t+k}) \\ + \left[Cov_{t+k} \left(\mathcal{M}_{i,t+k,t+k+1}, \Pi_{t+k+1} \right) + \mathcal{M}_{t+k,t+k+1} \left(\mathbb{E}_{t+k} \Pi_{t+k+1} - \Pi_{t+k+1} \right) \right] &\frac{b_{i,t+k}}{P_{t+k}} \Big\} \Bigg] \\ + \lim_{T \to \infty} \left\{ \mathbb{E}_t \left[\mathcal{M}_{i,t,t+T} \left(\frac{B_{i,t+T} + \Pi_{t+T+1} b_{i,t+T}}{P_{t+T}} \right) \right] \right\}&, \end{aligned} \tag{A.1}$$

where I use the notation $\Pi_{t+1,t+k+1}$ to define gross inflation from period $t+1$ to period $t+k+1$. This is the integrated household budget constraint at optimality, from which the integrated *government* budget constraint is derived.

Crucially, note that household optimality implies $\lim_{T \to \infty} \frac{B_{i,T} + \Pi_{T+1} b_{i,T}}{P_T} \leq 0$, while a no-Ponzi condition on household debt holdings ensures that $\lim_{T \to \infty} \frac{B_{i,T} + \Pi_{T+1} b_{i,T}}{P_T} \geq 0$. Furthermore, by the definition of the SDF and the properties of a standard CRRA utility function, $\lim_{T \to \infty} \mathcal{M}_{i,t,T} \neq \pm\infty$. Therefore, the final limit converges to 0 and must not be considered.[37]

The formulation of equation (A.1) is intuitive: the real value of household bond holdings is equal to its expected discounted consumption benefits from today to infinity (as future net consumption earnings are suitably discounted with the SDF, which is a mirror image of the price of the two bonds), adjusted suitably for additional surprise earnings enjoyed from holdings of *indexed* sovereign debt: these are decreased by surprise inflation through its (negative) covariance with the SDF (as higher *future* inflation pushes the SDF down), and increased by surprise inflation through a level effect (since such inflation yields a windfall gain relative to what was paid for the indexed bond in the previous period).

Aggregating these individual household bond constraints up to an integrated government budget constraint and making use of the asset market clearing conditions $B_t = \sum_i B_{it}$ and $b_t = \sum_i b_{it}$, and

[37] Even though this idea resembles the core idea behind Brunnermeier et al. (2020) and Brunnermeier et al. (2024), this approach also overcomes the issues raised by Hagedorn (2024) by taking into account the dynamic trading (flow) benefits of government debt across time. This ensures the transversality conditions to hold for only one initial price level.

of the idea that the household TVCs hold individually yields the following expression:

$$\begin{aligned}\frac{B_{t-1}+\Pi_t b_{t-1}}{P_t} &= \sum_i \Bigg\{ \mathbb{E}_t \Bigg[\sum_{k=0}^{\infty} M_{i,t,t+k}\Pi_{t+1,t+k+1} \big\{ \big(c_{i,t+k} - \varepsilon_{i,t+k}(1-\tau_{i,t+k}) w_{t+k} N_{t+k} \big) \\ &+ \big[Cov_{t+k}\left(M_{i,t+k,t+k+1}, \Pi_{t+k+1} \right) + M_{t+k,t+k+1}\left(\mathbb{E}_{t+k}\Pi_{t+k+1} - \Pi_{t+k+1} \right) \big] \frac{b_{i,t+k}}{P_{t+k}} \Big\} \Bigg] \Bigg\}. \end{aligned} \tag{A.2}$$

I simplify this equation by noting that one can take the summation into the expectation and switch around the order of summation. To further simplify the integrated government debt valuation equation, create the variable A_{it} which captures the surpluses raised by the government from each household i:

$$A_{it} \equiv c_{it} - \varepsilon_{it}(1-\tau_{it}) w_t N_t + \big[Cov_t\left(M_{i,t,t+1}, \Pi_{t+1} \right) + M_{i,t,t+1}\left(\mathbb{E}_t \Pi_{t+1} - \Pi_{t+1} \right) \big] \frac{b_{it}}{P_t},$$

which is the full portfolio return of household i of holding an additional unit of net worth. Alternatively, one can view this as what the government factually can raise as surpluses from each household i.

Define $\bar{A}_t = \sum_i A_{it}$ as the sum of all individual-level surpluses. Then, rewrite the implied intertemporal government budget constraint (A.2) to:

$$\frac{B_t}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \left(\sum_i M_{i,t,t+k} \Pi_{t,t+k+1} \frac{A_{i,t+k}}{\bar{A}_{t+k}} \right) \bar{A}_{t+k} \right],$$

or, defining the *household value-weighted SDF* $\tilde{M}_{t,t+k} = \sum_i M_{i,t,t+k} \Pi_{t,t+k+1} \frac{A_{i,t+k}}{\bar{A}_{t+k}}$, one finally arrives at:

$$\frac{B_t}{P_t} + \frac{b_{t-1}}{P_{t-1}} = \mathbb{E}_t \left[\sum_{k=0}^{\infty} \tilde{M}_{t,t+k} \bar{A}_{t+k} \right], \tag{A.3}$$

where $\tilde{M}_{t,t+k}$ is now the weighted average SDF across all households i, adjusted for inflation, with weights being proportionate to $A_{i,t+k}$, consisting of the net utility gain from saving, the insurance premium on indexed debt (captured through the covariance term), and the possible windfall gain/loss from surprise inflation (captured through the last term in the definition of $A_{i,t+k}$). Equation (A.3) is 'the FTPL equation' that is used to pin down the price level at time t, given some previous price level P_{t-1}.

A.2 Proof of proposition 3

I first show that determinacy can indeed be achieved under an FTPL-type mechanism when indexed debt is present, provided a suitable theory of the real interest rate, before showing how indexed debt translates into a model where taxation is assumed to cover all interest expenses over time on the stationary equilibrium path, following Hagedorn (2021). I therefore maintain a 'true Balanced Growth Path' (BGP) with a constant real value of the debt portfolio thanks to an appropriate taxation schedule.

To apply the framework of Hagedorn (2021), rewrite the steady-state taxation function to account for possible non-zero steady-state inflation and some positive level of indexed debt, since the presence of both changes the nominal value of taxation over time. The aim is to find an asset demand function depending only on model primitives.[38] To do so, one must pin down steady-state asset demand under incomplete markets in a closed-form solution, for which I leverage the results of Acemoglu and Jensen (2015).

To find the steady-state level of taxation consistent with the bond issuance schedule that keeps the real value of bonds constant (provided that inflation devalues the non-indexed bonds), begin with an arbitrary per-period government budget constraint (setting $G_t = 0$, such that real surpluses are $s_t = \tau_t$, or, in nominal terms, $P_t s_t = P_t \tau_t =: T_t$):

$$B_{t-1} + \frac{P_t}{P_{t-1}} b_{t-1} = T_t + Q_t B_t + q_t b_t.$$

Q_t and q_t must be equal to some constant values in steady-state. Without aggregate uncertainty, the bond prices arising through asset demand must solely depend on the offered interest rates, since cross-sectional risks average out. Thus, *in steady-state*:

$$\begin{aligned} B_{ss} + \Pi_{ss} b_{ss} &= T_{ss} + Q_{ss} B_{ss} + q_{ss} b_{ss} \\ \Leftrightarrow B_{ss} + \Pi_{ss} b_{ss} &= T_{ss} + \frac{1}{1+i_{ss}} B_{ss} + \frac{1}{1+r_{ss}} b_{ss} \\ \Leftrightarrow T_{ss} &= \left(1 - \frac{1}{1+i_{ss}}\right) B_{ss} + \left(\Pi_{ss} - \frac{1}{1+r_{ss}}\right) b_{ss}. \end{aligned}$$

Using the Fisher equation, $\Pi_{ss} - \frac{1}{1+r_{ss}} = \frac{1+i_{ss}}{1+r_{ss}} - \frac{1}{1+r_{ss}} = \frac{i_{ss}}{1+r_{ss}}$, and therefore:

$$T_{ss} = \frac{i_{ss}}{1+i_{ss}} B_{ss} + \frac{i_{ss}}{1+r_{ss}} b_{ss},$$

which can be expressed in real terms (as the household cares about real taxation) as

[38] For the sake of completeness, I here specify the approach Hagedorn (2021) takes to determine steady-state taxation. He specifies the per-period government budget constraint as $B_{t+1} = (1+i_t)B_t - T_t \;\Leftrightarrow\; T_t = (1+i_t)B_t - B_{t+1}$ to arrive in steady-state at $T_{ss} = i_{ss} S_{ss}$, where S_{ss} is steady-state asset demand. in real terms, $\tau_{ss} =: \frac{T_{ss}}{P_{ss}} = r_{ss} S_{ss}$.

$$\tau_{ss} = \frac{r_{ss}}{1+i_{ss}} B_{ss} + \frac{r_{ss}}{1+r_{ss}} b_{ss}.$$

Define by $S_t\left(\Omega_t, \{1+r_l, \tau_l\}_l^{\infty}\right)$ the cumulative asset demand function under incomplete markets, which depends on the household distribution of wealth Ω_t, real interest rates $1+r_t$, and tax rates τ_t, and is well-defined under standard regularity conditions (Acemoglu and Jensen, 2015). To relate steady-state taxation more clearly to gross asset demand, fix the shares of B_{ss} and b_{ss} of gross asset demand S_{ss} in steady-state. Denoting by ω the share of indexed debt b_{ss} in the steady-state asset portfolio, the taxation term in steady-state finally becomes

$$\tau_{ss} = \left[(1-\omega)\frac{r_{ss}}{1+i_{ss}} + \omega \frac{r_{ss}}{1+r_{ss}}\right] S_{ss}.$$

Under such steady-state taxes, the gross asset demand function arising from heterogeneous household demand ($S_{t+1} = \mathcal{S}(\Omega_t; 1+r_t, 1+r_{t+1}, 1+r_{t+2}, ...; \tau_t, \tau_{t+1}, ...)$) simplifies to the following mapping in steady-state:

$$S_{ss} = \mathcal{S}\left(\Omega_{ss}; 1+r_{ss}, 1+r_{ss}, 1+r_{ss}, ...; \left[(1-\omega)\frac{r_{ss}}{1+i_{ss}} + \omega\frac{r_{ss}}{1+r_{ss}}\right] S_{ss}, \left[(1-\omega)\frac{r_{ss}}{1+i_{ss}} + \omega\frac{r_{ss}}{1+r_{ss}}\right] S_{ss}, ...\right).$$

With i_{ss} being equal to some constant set by the monetary policymaker in steady-state and the taxation function just derived, asset demand can again be derived by finding the fixed point of the above equation, which yields asset demand as a function of the real interest rate r_{ss}, following Acemoglu and Jensen (2015):

$$\underline{\text{Asset demand:}} \qquad S(r).$$

From the previous derivations, one can directly leverage asset supply in real terms as the left-hand side of the derivations of the fiscal theory equation evaluated in steady-state, such that the stationary asset market equilibrium must be pinned down by

$$S(r) = \frac{B}{\tilde{P}} + \frac{b}{\tilde{P}(1+\pi_{ss})},$$

or, making use of the Fisher equation,

$$S(r) = \frac{B}{\tilde{P}} + \frac{b}{\tilde{P}}\frac{(1+r_{ss})}{(1+i_{ss})}.$$

An important question relates to the source of π_{ss}, the posited non-zero steady-state inflation rate in this economy. Following the contribution of Hagedorn (2021), posit that the only possible non-zero steady-state inflation rate is the one consistent with a corresponding increase in taxation over time alongside this inflationary path:

$$1 + \pi_{ss} = \frac{T' - T}{T},$$

where variables with a prime denote next period values. Since T represents nominal taxes, the above statement is equivalent to the claim that *real* taxes remain constant.

Given the bond portfolio on offer, express the above condition as follows:

$$\begin{aligned} 1 + \pi_{ss} &= (1-\omega)\frac{B' - B}{B} + \omega\frac{b' - b}{b} \cdot (1 + \pi_{ss}) \\ \Leftrightarrow 1 + \pi_{ss} &= \frac{(1-\omega)\frac{B'-B}{B}}{1 - \omega\frac{b'-b}{b}}, \end{aligned}$$

where the inflation-adjustment on the right-hand side in the first line follows from the adjustment of the face value of inflation-indexed debt. This bond issuance schedule therefore can be considered to pin down steady-state inflation.

Using the FTPL-style equation to determine the price ***level***: one can now invoke the above derivations within the FTPL-style equation to pin down the price level uniquely, provided that the real interest rate is recovered from the asset market.

Following the above reasoning, that steady-state real interest rate can indeed be recovered from the asset market through household demand, provided that this demand function is invertible, as

$$r_{ss} = S^{-1}\left(\frac{B}{\tilde{P}} + \frac{b}{\tilde{P}(1 + \pi_{ss})}\right),$$

which can be inserted in the stationary intertemporal FTPL equilibrium ($\frac{B}{\tilde{P}} + \frac{B}{\tilde{P}(1+\pi_{ss})} = \sum_{j=0}^{\infty}\left(\frac{1}{1+r_{ss}}\right)^j \bar{s}$) with $r_{ss} > 0$ (such that the right-hand side can be rewritten as a geometric sum, $\sum_{j=0}^{\infty}\left(\frac{1}{1+r_{ss}}\right)^j = \frac{1+r_{ss}}{r_{ss}}$) to get the following condition:

$$\frac{B_{ss} + b_{ss}(1 + \pi_{ss})}{\tilde{P}} = \bar{s}\frac{1 + r_{ss}}{r_{ss}},$$

and the fixed point of this equation pins down the price level uniquely, given asset market optimality. To be precise, given the earlier definition of the surplus process, i.e., $\bar{s} = \tau_{ss} = \frac{r_{ss}}{1+i_{ss}}B_{ss} + \frac{r_{ss}}{1+r_{ss}}b_{ss}$:

$$\frac{B_{ss} + b_{ss}(1 + \pi_{ss})}{\tilde{P}} = \left[\frac{r_{ss}}{1 + i_{ss}}B_{ss} + \frac{r_{ss}}{1 + r_{ss}}b_{ss}\right]\frac{1 + r_{ss}}{r_{ss}}.$$

Using the Fisher equation ($(1 + i_{ss}) = (1 + r_{ss})(1 + \pi_{ss})$), this equilibrium relation is simplified to:

$$\frac{B_{ss}}{\tilde{P}} + \frac{b}{\tilde{P}(1+\pi_{ss})} = (1+\pi_{ss})B + b,$$

which eventually pins down the price level as

$$\tilde{P} = \frac{B_{ss} + b_{ss}(1+\pi_{ss})}{(1+\pi_{ss})B_{ss} + b_{ss}}.$$

From the taxation schedule (which is a fiscal variable itself, actively managed by fiscal policy), one can recover the steady-state inflation rate. Simplify this by utilizing the steady-state growth rates $\frac{B'-B}{B} =: g_B$ and $\frac{b'-b}{b} =: g_b$, such that steady-state inflation becomes $1+\pi_{ss} = \frac{(1-\omega)\frac{B'-B}{B}}{1-\omega\frac{b'-b}{b}} = \frac{(1-\omega)g_B}{1-\omega g_b}$. Thus, the initial price level in this steady-state is given by:

$$\tilde{P} = \frac{B_{ss} + b_{ss}\frac{(1-\omega)g_B}{1-\omega g_b}}{B_{ss}\frac{(1-\omega)g_B}{1-\omega g_b} + b_{ss}},$$

with the bond growth rates themselves being fiscal choice variables in the stationary equilibrium.

B Further simulation results

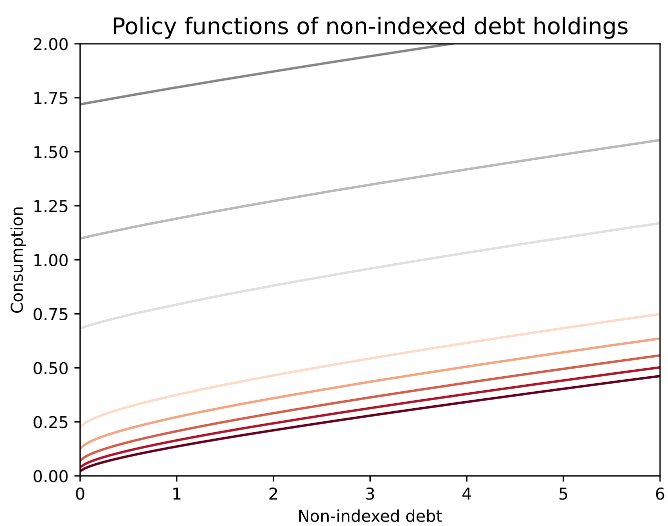

Figure B.1: Household policy functions for demand of non-indexed debt in the calibrated HANK model for unconstrained households. Note that the policy functions for low values of idiosyncratic productivity start to become positive only for strictly positive levels of non-indexed debt due to the possibility to purchase inflation-indexed debt stock.

Next, this appendix showcases dynamic impulse-responses of aggregate variables in response to a 25bps expansionary monetary policy shock, as well as the results of a full non-linear estimation of the model, which encountered significant numerical instabilities.

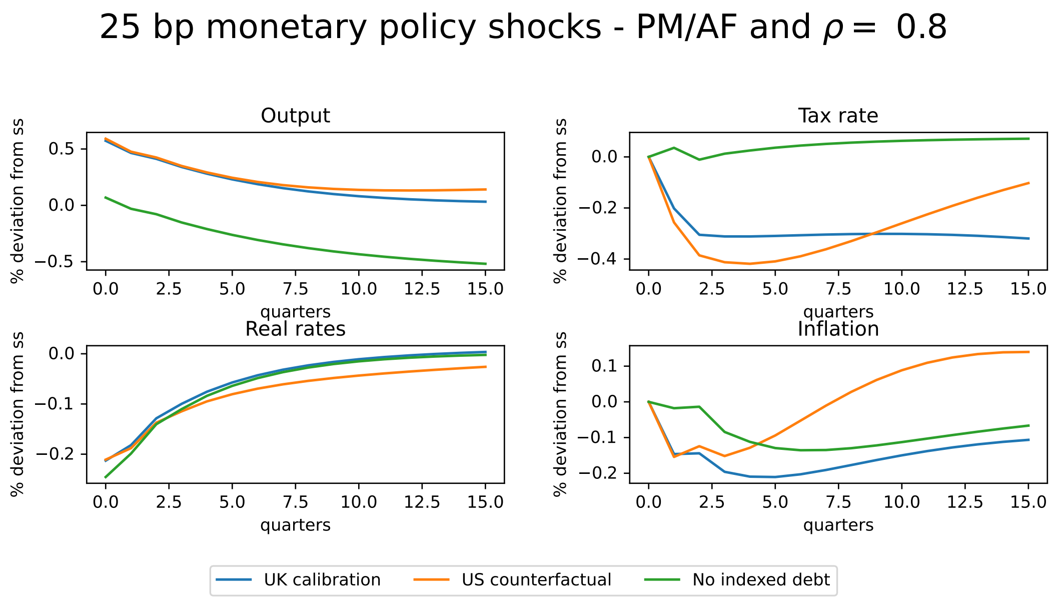

Figure B.2: IRFs to a 25bps expansionary monetary shock - under a fiscally-led policy mix and $\rho = 0.8$.

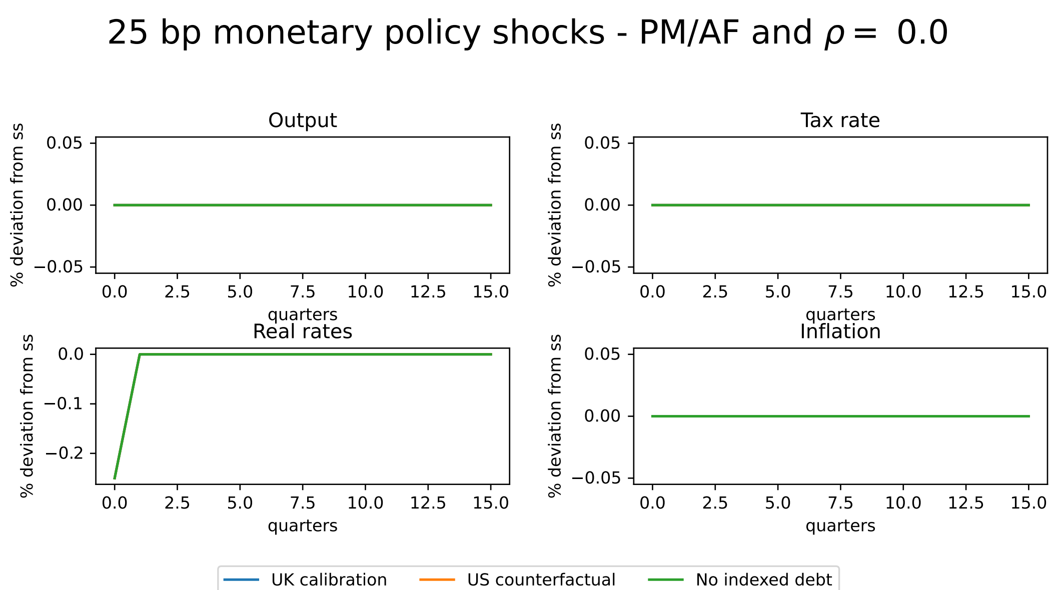

Figure B.3: IRFs to a 25bps expansionary monetary shock - under a fiscally-led policy mix and $\rho = 0$.

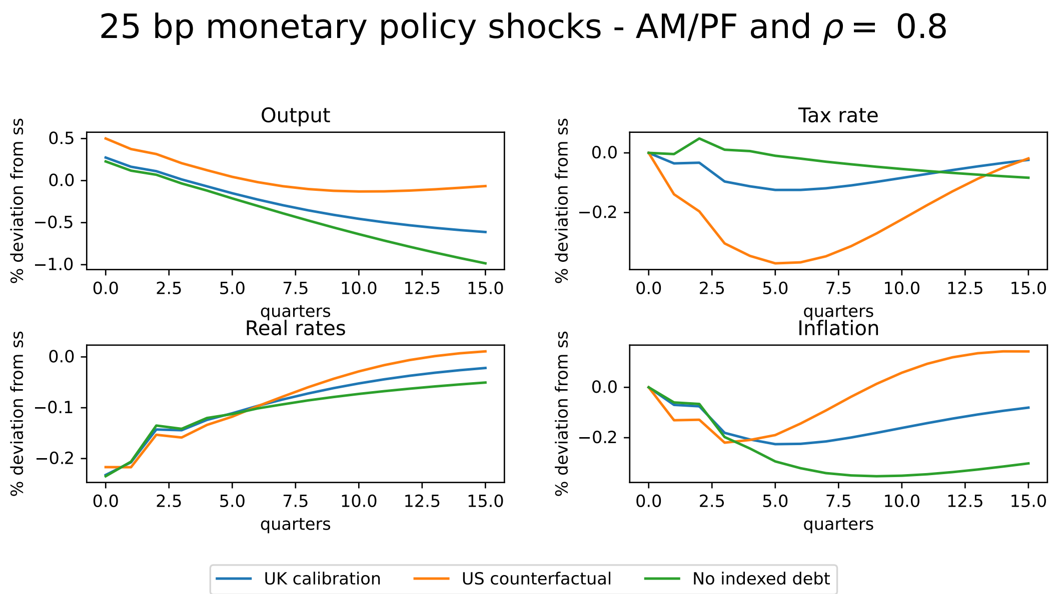

Figure B.4: IRFs to a 25bps expansionary monetary shock - with under a monetary-led policy mix and $\rho = 0.8$.

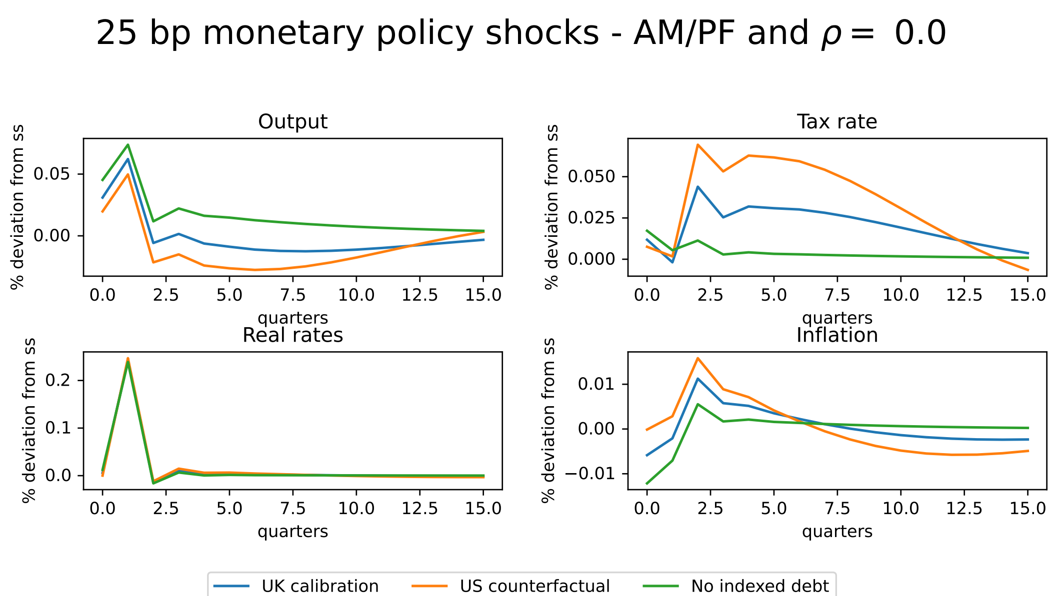

Figure B.5: IRFs to a 25bps expansionary monetary shock - with under a monetary-led policy mix and $\rho = 0$.

For the main policy scenario (the 'fiscally-led policy mix'), I furthermore provide additional evidence on changes of quantities directly informing the intertemporal government budget constraint (12).

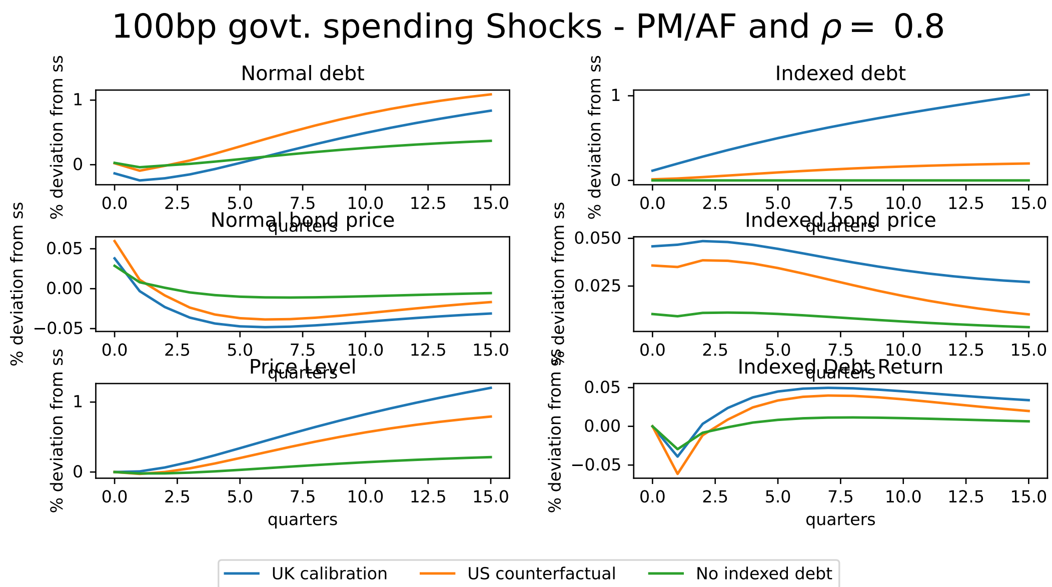

Figure B.6: Further IRFs to a 100bps expansionary fiscal spending shock - under a fiscally-led policy mix and $\rho = 0.8$.

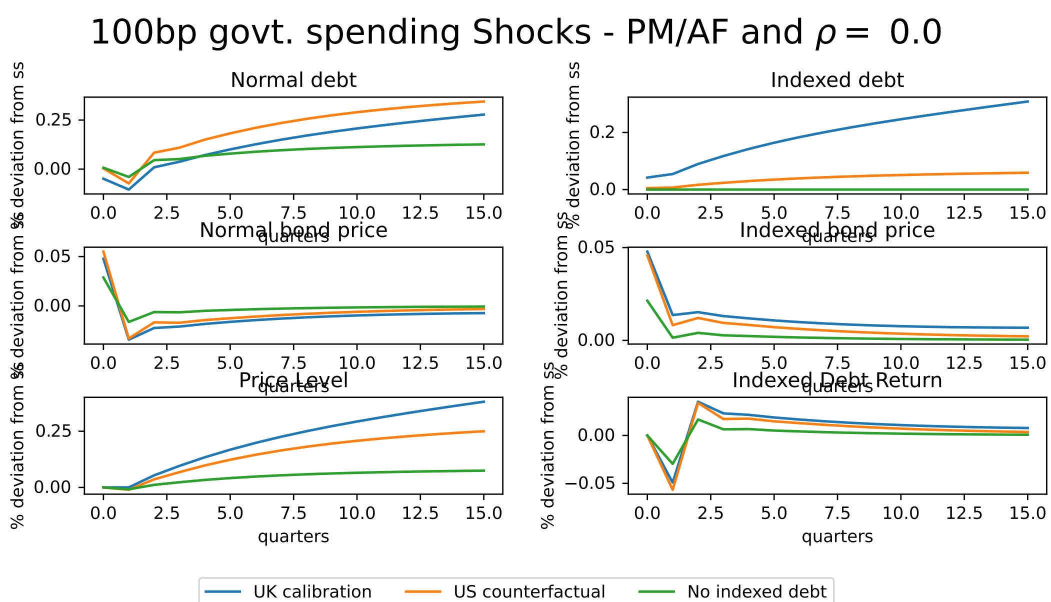

Figure B.7: Further IRFs to a 100bps expansionary fiscal spending shock - under a fiscally-led policy mix and $\rho = 0$.

C Long-term debt and debt indexation

In this part of the appendix, I briefly derive the debt valuation equation under complete markets with long-term debt.

Following the assumption of complete markets and the exposition in the main body of the text, standard bond pricing kernels for long-term assets maturing at time $(t + j)$ evaluated at time t are given by:

$$Q_t^{(t+j)} = \mathbb{E}_t \left(\beta^j \frac{P_t}{P_{t+j}} \right), \tag{C.4}$$

$$q_t^{(t+j)} = \beta^j, \tag{C.5}$$

reflecting that inflation-indexed debt always has the same price, as its face value accounts for changes to the price level between issuance and redemption, but is not fully equivalent to a real claim.

In this context, the government flow budget condition is given by:

$$B_{t-1}^{(t)} + \Pi_t b_{t-1}^{(t)} = P_t s_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} \left(B_t^{(t+j)} - B_{t-1}^{(t+j)} \right) + \sum_{j=1}^{\infty} q_t^{(t+j)} \left(b_t^{(t+j)} - \Pi_t b_{t-1}^{(t+j)} \right). \tag{C.6}$$

This condition states that in each period t, the payout of maturing debt (left-hand side) must be equal to the nominal surpluses raised *plus* the possible income from issuing additional debt maturing as a later point in the future (relative to what had already been issued before). Of course, governments can also redeem more bonds than they issue, in which case either of the sums on the right-hand side can also be negative.

That flow condition keeps track of mounting payments on inflation-indexed debt by adjusting the prospective cost of serving indexed debt in each period by the accumulated face value payments, given by $\left(b_t^{(t+j)} - \Pi_t b_{t-1}^{(t+j)} \right)$. In sum, surpluses on the right-hand side of the previous equation get diminished when inflation Π_t from the last period has been high, as that inflation is reflected in the obligations that the government will have as that long-term inflation-indexed debt matures.

Grouping terms of the previous equation yields:

$$\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)} + \sum_{j=0}^{\infty} \Pi_t b_{t-1}^{(t+j)} = P_t s_t + \sum_{j=1}^{\infty} Q_t^{(t+j)} + B_t^{(t+j)} + \sum_{j=1}^{\infty} q_t^{(t+j)} b_t^{(t+j)}. \tag{C.7}$$

Let the real value of debt now be defined as:

$$V_t := \sum_{j=0}^{\infty} Q_t^{(t+j)} \frac{B_{t-1}^{(t+j)}}{P_t} + \sum_{j=0}^{\infty} \frac{b_{t-1}^{(t+j)}}{P_{t-1}}. \tag{C.8}$$

Focus on the right-hand side of equation (C.7). I now rewrite the two summative terms again to obtain V_{t+1}. Dividing those terms by P_t:

$$\sum_{j=1}^{\infty} Q_t^{(t+j)} \frac{B_t^{(t+j)}}{P_t} + \sum_{j=1}^{\infty} q_t^{(t+j)} \frac{b_t^{(t+j)}}{P_t}. \tag{C.9}$$

Shifting the index from $j = 1$ to $j = 0$ gives:

$$\sum_{j=0}^{\infty} Q_t^{(t+j+1)} \frac{B_t^{(t+j+1)}}{P_t} + \sum_{j=0}^{\infty} q_t^{(t+j+1)} \frac{b_t^{(t+j+1)}}{P_t}. \tag{C.10}$$

Now, note that $q_t^{(t+j+1)} = \beta q_{t+1}^{(t+j+1)}$ by the bond pricing kernels defined previously. Thus, the previous expression becomes

$$\beta \left[\sum_{j=0}^{\infty} Q_{t+1}^{(t+j+1)} \frac{B_t^{(t+j+1)}}{P_{t+1}} + \sum_{j=0}^{\infty} q_{t+1}^{(t+j+1)} \frac{b_t^{(t+j+1)}}{P_t} \right] = \beta V_{t+1} \tag{C.11}$$

by the previous definition of V_t. Now, applying a transversality condition of the form

$$\underset{T\to\infty}{lim} \beta^T \left[\sum_{j=0}^{\infty} Q_{t+T}^{(t+j+T)} \frac{B_{t+T}^{(t+j+T)}}{P_{t+T}} + \sum_{j=0}^{\infty} q_{t+T}^{(t+j+T)} \frac{b_{t+T}^{(t+j+T)}}{P_{t+T}} \right] = 0, \tag{C.12}$$

obtains the debt valuation equation with inflation-indexed debt:

$$\sum_{j=0}^{\infty} Q_t^{(t+j)} \frac{B_{t-1}^{(t+j)}}{P_t} + \sum_{j=0}^{\infty} q_t^{(t+j)} \frac{b_{t-1}^{(t+j)}}{P_{t-1}} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j}, \tag{C.13}$$

which is a straightforward generalization of the government debt valuation equation exposed, for instance, in Cochrane (2001).

D A tractable exposition of the effects of inflation-indexed debt

In this section, I briefly develop a simplified iteration of Angeletos et al. (2024), adding inflation-indexed debt but at the same time reducing spillovers from mortality risk to aggregate demand. This is done to facilitate a simpler characterization of equilibrium inflation rates.[39] In that sense, the model is also closely related to Woodford (2019) and Nakamura et al. (2025). The model is fundamentally an OLG-NK model in the spirit of Blanchard (1985). The idea is that the mortality friction can be considered a proxy for liquidity risk commonplace in canonical HANK models while maintaining superior tractability properties. Instead of laying out the cases of the fiscally-led policy mix under RANK and the monetary-led policy mix under HANK separately, I analyze both cases jointly as a dynamic system, keeping the parametrization of fiscal policy, monetary policy, and the mortality friction opaque for as long as possible. This is done to minimize the length of the analysis.

For this model, uppercase variables define the level values of variables, while lowercase variables are log-deviations from steady-state. The steady-state will be log-linearized around zero inflation ($\Pi^{SS} = 1$), and the fiscal variables debt (d_t), taxes (t_t), and assets (a_t) will all be measured in

[39] A similar exercise with a *full* characterization of the effects of indexed debt on aggregate demand can be found in Ellison and Kawalec (2025)

absolute deviations from steady-state (not log deviations) to ensure that zero-debt steady-states are not excluded.

D.1 Household block

The probability of surviving from one period to another is captured by $\omega \in (0,1]$. Households are replaced by new ones whenever they die. They maximize expected utility, given by

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\omega)^k \left[\frac{C_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} - \nu \frac{L_t^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}} \right] \right]. \tag{D.14}$$

The household budget constraint: households can trade in a risk-free annuity as in Angeletos et al. (2024), earning a specified nominal rate of return R_t^P. That annuity consists of a representative share of the government debt portfolio, which consists of regular debt B_t^R earning a gross return I_t, where I_t is the gross nominal interest rate, and *inflation-indexed debt* B_t^I, which earns a gross rate of return $I_t \frac{P_{t+1}}{P_t}$, reflecting the face value adjustment of such debt in line with materialized inflation. The gross portfolio return of the household consists of a weighted average of the returns earned by the two individual asset classes, where I specify the constant share of inflation-indexed debt in the government bond portfolio as θ:

$$R_t^P = \theta I_t \frac{P_{t+1}}{P_t} + (1-\theta) I_t = I_t \left(1 + \theta \left(\frac{P_{t+1}}{P_t} - 1 \right) \right), \tag{D.15}$$

which captures the pre-death probability rate of return on the portfolio of government debt owned as the only savings asset by each household. The remainder of the budget constraint follows Angeletos et al. (2024) closely: all households receive labor income and dividends $W_t L_{it} + Q_{it}$, are taxed in accordance with a taxation rule, and all old households make a contribution S_{it} to a social fund whose proceeds are distributed to newborn households, eliminating wealth effects from mortality risk.[40] The household-specific budget constraint is then given by:

$$P_{t+1} A_{i,t+1} = \frac{R_t^P}{\omega} P_t \left(A_{it} + \underbrace{Y_{it}}_{\equiv W_t L_{it} + Q_{it}} - C_{it} - T_{it} + S_{it} \right). \tag{D.16}$$

I otherwise retain all other household-side assumptions from Angeletos et al. (2024): dividends are identical across households i, labor supply is intermediated by unions to obtain $L_{it} = L_t$, and income and taxes faced by households are equalized. Taking expectations and subsequently making use of the Fisher equation yields the following expression of the budget constraint in real terms:

$$A_{i,t+1} = R_t \left(1 + \theta \mathbb{E}_t \Pi_{t+1} \right) \frac{1}{\omega} (A_{it} + Y_{it} - C_{it} - T_{it} + S_{it}), \tag{D.17}$$

[40] The transfers are specified as $S_{it} = S^{new} = D^{SS} \geq 0$ and $S_{it} = S^{old} = -\frac{1-\omega}{\omega} D^{SS} \leq 0$, such that $(1-\omega)S^{new} + \omega S^{old} = 0$.

where R_t is the ex-ante real interest rate.

This household problem yields a standard set of first-order conditions:

$$\{C_{it}\}: \qquad C_{it}^{-\frac{1}{\sigma}} - \lambda_{it}\frac{R_t}{\omega}(1+\theta\mathbb{E}_t(\Pi_{t+1}-1)) = 0, \tag{D.18a}$$

$$\{L_{it}\}: \qquad L_{it}^{\frac{1}{\varphi}} + \lambda_{it}\frac{R_t}{\omega}(1+\theta\mathbb{E}_t(\Pi_{t+1}-1))\frac{\partial Y_{it}}{\partial L_{it}} = 0, \tag{D.18b}$$

$$\{A_{i,t+1}\}: \qquad -\lambda_{it} + \beta\mathbb{E}_t\left[\frac{R_{t+1}}{\omega}(1+\theta(\Pi_{t+2}-1))\lambda_{i,t+1}\right] = 0, \tag{D.18c}$$

The first-order conditions jointly yield a standard Euler equation for consumption:

$$C_{it}^{-\frac{1}{\sigma}}\left[\frac{R_t}{\omega}(1+\theta\mathbb{E}_t\Pi_{t+1})\right]^{-1} = \beta\mathbb{E}_t\left[C_{i,t+1}^{-\frac{1}{\sigma}}\right]. \tag{D.19}$$

I linearize this expression through a standard first-order approximation to obtain a linearized form of the Euler equation.

$$c_{it} = -\sigma(r_t + \theta\mathbb{E}_t\pi_{t+1}) + \mathbb{E}_t c_{i,t+1}, \tag{D.20}$$

where the novel term induced by the presence of indexed debt is $\theta\mathbb{E}_t\pi_{t+1}$.

The usual way forward at this point would be to characterize the intertemporal *aggregate* household budget constraint with the help of this household-level Euler equation, which could then be used to derive an aggregate demand equation.[41] Under this expression, characterizing equilibrium inflation rates analytically remains possible, but demand changes induced by households that leave/enter the Euler equation through the presence of mortality risk obstruct the core message of how indexed debt can matter through the intertemporal government budget constraint. In the following, I therefore align the model closer to Woodford (2019) and Nakamura et al. (2025), postulating instead that the effect from mortality risk on aggregate demand can be captured by discounting future variables adequately on the aggregate Euler equation in line with mortality risk. Then, the aggregate Euler equation can be expressed as

$$c_t = \omega\mathbb{E}_t c_{t+1} - \sigma(i_t - \omega\mathbb{E}_t\pi_{t+1} + \theta\mathbb{E}_t\pi_{t+1}).$$

Using market clearing $c_t = y_t$ and expressing the right-hand side in terms of the real interest rate r_t, we can express this (simplified) aggregate demand equation as

$$y_t = \omega\mathbb{E}_t y_{t+1} - \sigma(r_t + (1-\omega+\theta)\mathbb{E}_t\pi_{t+1}). \tag{D.21}$$

[41] This intertemporal aggregate household budget constraint features a similar indexed debt adjustment term and is given by: $c_t = (1-\beta\omega)\left(a_t + \mathbb{E}_t\sum_{s=0}^{\infty}(\beta\omega)^s(y_{t+s}-t_{t+s})\right) - \beta\left(\sigma\omega - (1-\beta\omega)\frac{A^{SS}}{Y^{SS}}\right)\mathbb{E}_t\left[\sum_{s=0}^{\infty}(\beta\omega)^s(r_{t+s}+\theta\pi_{t+1+s})\right]$.

The presence of indexed debt can offset the discounting of the interest rate channel that is induced by the presence of mortality risk in the effect of expected inflation on today's aggregate demand.

D.2 Supply side

The supply side of the model is fully standard and follows the standard specification commonplace in canonical New Keynesian models. In particular, the New Keynesian Phillips Curve arises as a consequence of standard Calvo pricing frictions included in a standard firm problem:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t[\pi_{t+1}], \tag{D.22}$$

which can be iterated forward to express inflation as a function of current and future output gaps:

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t[y_{t+k}]. \tag{D.23}$$

D.3 The Government Block

I begin the discussion of the government block by a brief derivation of the government budget constraint, which differs somewhat relative to Angeletos et al. (2024). The simple per-period budget constraint of the government is defined as:

$$D_{t+1} = R_t^P (D_t - P_t T_t),$$

where $R_t^P \equiv I_t(1+\theta(\Pi_{t+1}-1))$ captures the portfolio return that the government has to pay households. I therefore postulate that the government portfolio has a *fixed* share of inflation-indexed debt θ, in line with the characterization of the household portfolio. Linearizing this constraint to express the evolution of the total debt portfolio d_{t+1} in deviations from steady-state gives rise to the following log-linearized budget constraint:

$$d_{t+1} = \frac{1}{\beta}(d_t - t_t) + \frac{D^{SS}}{Y^{SS}} r_t - \frac{D^{SS}}{Y^{SS}}\left((1-\theta)\pi_{t+1} - \mathbb{E}_t \pi_{t+1}\right). \tag{D.24}$$

The crucial novelty here is the adjustment of future inflation by $(1-\theta)$. Intuitively, this captures the idea that inflation-indexed debt cannot be devalued through surprise inflation, as the face value of that part of the debt stock remains unchanged in present real terms irrespective of the rate of inflation. Therefore, the ability of governments to inflate away debt in real terms is constrained.

The analysis furthermore retains the no-Ponzi condition of Angeletos et al. (2024), i.e., $\mathbb{E}_t\left[\lim_{T\to\infty} \beta^T d_{t+T}\right] = 0$. Starting off the steady-state where $x_{-1} = 0 \; \forall x \in \{d, t, r, y, \pi\}$, equation (D.24) pins down the initial change in the debt stock as a function of surprise inflation:

$$d_0 = -\frac{D^{SS}}{Y^{SS}}(1-\theta)\pi_0.$$

To close the model one must specify appropriate fiscal and monetary policy rules.

The monetary policy rule deserves a special treatment as it is a point of departure from Angeletos et al. (2024). Following equation (D.21), inflation-indexed debt can induce an intertemporal substitution effect through the Euler equation due to the possibility of windfall gains in the presence of surprise inflation. As I intend to eliminate this effect and solely focus on the relevance of inflation-indexed debt through wealth effects induced by taxation, I postulate a monetary policy rule that absorbs the effect of inflation-indexed debt and of the mortality risk on the inflation adjustment in the aggregate demand equation:

$$r_t = \phi y_t - (1 - \omega + \theta)\mathbb{E}_t \pi_{t+1}. \tag{D.25}$$

This policy rule ensures that there is no distortion on intertemporal demand induced by windfall gains or losses from surprise inflation. Heuristically, central banks *care about the real interest rate that is relevant to the aggregate of surviving households*. Denoting this policy-relevant interest rate by $\tilde{r}_t \equiv r_t + (1-\omega+\theta)\mathbb{E}_t\pi_{t+1}$, the monetary policy rule can likewise be expressed as $\tilde{r}_t = \phi y_t$, nesting the specification of Angeletos et al. (2024).

Given that the monetary rule also absorbs the effect of inflation-indexed debt on the government budget constraint in the case of surprise inflation, I also introduce a dependence of the tax rule on the share of inflation-indexed debt, reflecting that the tax schedule must ensure that the quantity of taxes raised accounts for the possible cost incurred by the higher service cost of inflation-indexed debt in the presence of higher inflation. The fiscal rule otherwise remains the same as in Angeletos et al. (2024), such that I define for $\tau_d, \tau_y \in [0, 1)$:

$$t_t = -\varepsilon_t + \tau_d(d_t + \varepsilon_t) + \tau_y y_t + \beta\frac{D^{SS}}{Y^{SS}}\theta\mathbb{E}_t\pi_{t+1}, \tag{D.26}$$

where the last term reflects the novel adjustment of taxes to the expected costs incurred by inflation. Heuristically, governments *know* that surprise inflation can erode their budget balance (through higher face value payments on indexed debt), and they therefore adjust their taxation schedule to cover these expenses. Defining the quantity of taxes raised *net of* face value outlays for indexed debt as $\tilde{t}_t \equiv t_t - \beta\frac{D^{SS}}{Y^{SS}}\theta\mathbb{E}_t\pi_{t+1}$ again nests the model of Angeletos et al. (2024). The quantity $\tilde{t}_t$ reflects the discretionary tax revenue, i.e., the tax revenue available for the government once immediate obligations have been taken care of.

D.4 Equilibrium and general model properties

The definition of the *competitive equilibrium* is standard and kept brief on purpose.

Definition 2 *A competitive equilibrium is a stochastic path $\{c_t, y_t, \pi_t, a_t, d_t, t_t, r_t\}_{t=0}^{\infty}$ that satisfies the aggregate demand function, the NKPC, market clearing ($c_t = y_t$ and $a_t = d_t$), the government's flow budget constraint and its no-Ponzi condition, as well as the fiscal and monetary policy rules.*

Equations (D.21), (D.22), and (D.24) (jointly with the monetary and fiscal rules (D.25) and (D.26)) yield a first-order difference system, which will be the centerpiece of the analysis in this section:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -(1-\omega+\theta)\frac{D^{SS}}{Y^{SS}} & 1 \end{bmatrix} \mathbb{E}_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ d_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1+\sigma\phi}{\omega} & 0 & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta} & 0 & \frac{1}{\beta}(1-\tau_d) \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ d_t + \varepsilon_t \end{bmatrix}, \tag{D.27}$$

which can be rewritten to:

$$\mathbb{E}_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ d_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1+\sigma\phi}{\omega} & 0 & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta} - \frac{D^{SS}}{Y^{SS}}\frac{\kappa(1-\omega+\theta)}{\beta} & \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\beta} & \frac{1}{\beta}(1-\tau_d) \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ d_t + \varepsilon_t \end{bmatrix}. \tag{D.28}$$

The properties of the model depend on the eigenvalues of the previous matrix. Since the matrix is lower triangular, its eigenvalues are trivially given by the elements of its diagonal, such that the eigenvalues are given by:

$$\lambda_1 = \frac{1+\sigma\phi}{\omega}; \qquad \lambda_2 = \frac{1}{\beta}: \qquad \lambda_3 = \frac{1}{\beta}(1-\tau_d). \tag{D.29}$$

To satisfy the necessary conditions for a unique saddle-path equilibrium, two eigenvalues must lie outside the unit circle, and one inside, since the system contains exactly one state variable.

D.5 Solving the model in the limit case

Defining a 'limit point' between policy mixes that are prospectively "more/less fiscally/monetary-led" is possible by considering at which values of the core policy parameters ϕ and τ_d the associated eigenvalues are exactly one. Doing so for the first and last eigenvalues (since λ_2 is trivially > 1), we can establish the following parameter combination as the 'policy limit point':

$$\phi = -\frac{1-\omega}{\sigma}, \qquad \tau_d = 1-\beta.$$

To consider the dynamic properties of the system, consider first precisely the aforementioned *limit point between the fiscally-led and the monetary-led policy mix*, but with two slight tweaks. First, amend the monetary policy parameter by a small value $\epsilon > 0$ to ensure that the eigenvalue associated with the aggregate demand relation of the model matrix in equation (D.28) lies strictly inside the unit circle. Thereby, I ensure that the analysis retains the focus on the 'equivalence result' in terms of impact inflation between HANK models and the fiscally-led policy mix, following Angeletos et al.

(2024). Therefore, we let $\phi = -\frac{1-\omega+\epsilon}{\sigma}$. Second, to follow the analysis of Angeletos et al. (2024) yet again, we consider values of τ_d slightly below the limit point, such that $\tau_d \to 0^+$.

Now, let the eigenvector associated with the stable eigenvalue be denoted as $(\chi_1, \chi_2, 1)'$, such that the element pertaining to the state variable itself is normalized to 1. The evolution of all three endogenous variables can then be expressed in terms of the stable eigenvalue and its associated eigenvector:

$$y_t = \chi_1(d_t + \varepsilon_t); \qquad \pi_t = \chi_2(d_t + \varepsilon_t); \qquad \mathbb{E}_t d_{t+1} = \rho_d(d_t + \varepsilon_t). \tag{D.30}$$

The three coefficients are given by the solution to the system $(A - \lambda_2 I)\chi = 0$, with $\chi_3 = 1$. That system is specified as:

$$\begin{bmatrix} 1 - \frac{\epsilon}{\omega} & 0 & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ -\left(\frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\epsilon)}{\sigma} + \frac{\tau_y}{\beta} + \frac{D^{SS}}{Y^{SS}}\frac{\kappa(1-\omega+\theta)}{\beta}\right) & \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\beta} & \frac{1}{\beta} \end{bmatrix} \begin{bmatrix} \chi_1 \\ \chi_2 \\ 1 \end{bmatrix} = \rho_d \begin{bmatrix} \chi_1 \\ \chi_2 \\ 1 \end{bmatrix}. \tag{D.31}$$

From the first equation implied by the system, we can directly infer ρ_d, the persistence of the state variable:

$$\rho_d = 1 - \frac{\epsilon}{\omega} < 1.$$

This remaining two equations are given by:

$$-\chi_1 \frac{\kappa}{\beta} + \chi_2 \frac{1}{\beta} = \left(1 - \frac{\epsilon}{\omega}\right)\chi_2$$

$$-\left[\frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\epsilon)}{\sigma} + \frac{\tau_y}{\beta} + \frac{D^{SS}}{Y^{SS}}\frac{\kappa(1-\omega+\theta)}{\beta}\right]\chi_1 + \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\beta}\chi_2 + \frac{1}{\beta} = 1 - \frac{\epsilon}{\omega}.$$

Thanks to the lower triangular structure of the matrix, the resulting system of equations can be solved easily, pinning down χ_1 and χ_2 uniquely and yielding the sensitivity of inflation on impact in response to the fiscal shock ε_0. This process gives

$$\pi_0^{\varepsilon} \equiv \chi_2 \varepsilon_0 = \frac{\kappa\left(\frac{1}{\beta} - 1 + \frac{\epsilon}{\omega}\right)}{\left[\frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\epsilon)}{\sigma} + \frac{\tau_y}{\beta}\right]\left[1 - \beta\left(1 - \frac{\epsilon}{\omega}\right)\right] - \frac{D^{SS}}{Y^{SS}}\kappa\left(1 - \frac{\epsilon}{\omega}\right)(1-\omega+\theta)}\varepsilon_0. \tag{D.32}$$

Note that without inflation-indexed debt ($\theta = 0$), this expression would be trivially positive as $\omega \to 1$, nesting the standard RANK case as exposed in Angeletos et al. (2024). Additionally, for the 'proper' limit point between fiscally-led and monetary-led policy mixes (i.e., for $\epsilon \to 0$), we observe that mortality risk *only* matters in direct relation to the existence of inflation-indexed debt; that is, the effects of both are closely intertwined.

Proposition 4 *If impact inflation is positive in the policy limit point; that is, if*

$$\tau_y > \beta \frac{D^{SS}}{Y^{SS}} \left[\frac{\kappa\theta}{1-\beta} - (1-\omega)\left(\frac{1}{\sigma} - \frac{\kappa}{1-\beta}\right)\right], \text{[42]} \tag{D.33}$$

then impact inflation in response to an expansionary fiscal shock is higher in the policy limit point for the fiscally-led RANK economy relative to the monetary-led HANK economy if

$$\kappa < \frac{1-\beta}{\sigma}. \tag{D.34}$$

If impact inflation is negative in the policy limit point; that is, if

$$\tau_y < \beta \frac{D^{SS}}{Y^{SS}} \left[\frac{\kappa\theta}{1-\beta} - (1-\omega)\left(\frac{1}{\sigma} - \frac{\kappa}{1-\beta}\right)\right], \tag{D.35}$$

then impact deflation in response to an expansionary fiscal shock is smaller in the policy limit point for the fiscally-led RANK economy relative to the monetary-led HANK economy under the same condition.

Proof. Evaluating the inflation expression (D.32) for both FD-RANK and MD-HANK economies against each other gives:

$$\begin{aligned} \pi_0^{\varepsilon,FD,RANK} &> \pi_0^{\varepsilon,MD,HANK} \\ \Leftrightarrow \frac{\kappa\left(\frac{1}{\beta}-1\right)}{\frac{\tau_y}{\beta}(1-\beta) - \frac{D^{SS}}{Y^{SS}}\kappa\theta} &> \frac{\kappa\left(\frac{1}{\beta}-1\right)}{\left[\frac{D^{SS}}{Y^{SS}}\frac{(1-\omega)}{\sigma} + \frac{\tau_y}{\beta}\right](1-\beta) - \frac{D^{SS}}{Y^{SS}}\kappa(1-\omega+\theta)} \\ \Leftrightarrow 0 &< \frac{D^{SS}}{Y^{SS}}(1-\omega)\left(\frac{1-\beta}{\sigma} - \kappa\right) \\ \Leftrightarrow \kappa &< \frac{1-\beta}{\sigma}, \end{aligned}$$

which is the definition stated in the proposition. A similar derivation applies for the case in which the inflationary impact of both models is negative. ■

This is an interesting result as it overturns a possible irrelevance of the interaction between mortality risk and the fiscal-monetary policy mix for materialized inflation rates, as argued for in a related model by Angeletos et al. (2024). The presence of inflation-indexed debt and the way it partially overcomes market incompleteness can cause a direct link between mortality risk, the fiscal-monetary policy mix, and the materialized rates of inflation.

For most common calibrations, the conditions stated in the proposition are fulfilled. Labeling τ_y as the tax base channel; that is, as the proportion of income taxed, we can observe that even for

[42]Note that this proposition therefore implies that as θ increases, impact inflation in response to deficit shocks might eventually not be positive.

elevated levels of inflation-indexed debt the inequality (D.33) is fulfilled, conditional on a relatively flat slope of the Phillips curve as found in Hazell et al. (2022).

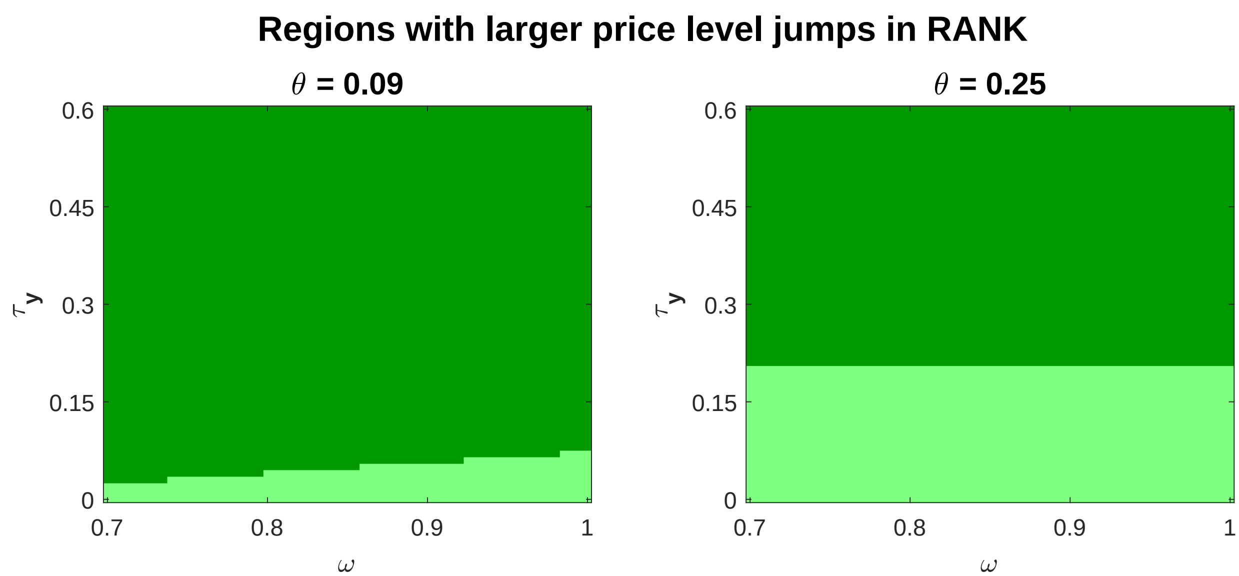

Figure D.1: Plot showing under which parametrizations either of the inequalities described by proposition 4 is fulfilled, indicating larger deviations of the price level from steady-state in response to deficit shocks. The dark green area denotes places where the first the magnitude of inflation is larger approaching from the HANK limit (with $\pi_0^\epsilon > 0$), the light green area denotes places where *dis*inflationary pressure is instead amplified. Calibration: $D^{SS} = 1$, $Y^{SS} = 1$, $\kappa = 0.025$, $\beta = 0.97$, $\sigma = 1$, $\tau_d = 0$, $\phi = \frac{1-\omega+\epsilon}{\sigma}$.

Figure D.1 summarizes the parameter space in terms of the tax-base channel τ_y and the household mortality risk (which can be interpreted as a proxy for liquidity effects in HANK models) ω for which the inequalities stated in proposition 4 hold. For conventional values of τ_y and ω, inflation-indexed debt usually raises materialized inflation rates. If the tax base channel τ_y were very small (and the share of inflation-indexed debt would simultaneously be large), inflation-indexed debt could enhance deflationary pressure (light green color), while for common calibrations of tax base channels, inflation-indexed debt amplifies the existing positive inflation on impact.[43] Therefore, when the share of inflation-indexed debt increases, this simple model generally exhibits a higher propensity of inflation volatility first and foremost when approaching the limit point between fiscally-led and monetary-led policy mixes.

[43]Recall that this result has been achieved by nullifying the intertemporal substitution channel of windfall gains arising from indexed debt holdings, which was done by an appropriate adjustment of the monetary policy rule. Forgoing this channel generally increases the area under which indexed debt amplifies price level deviations from steady-state.

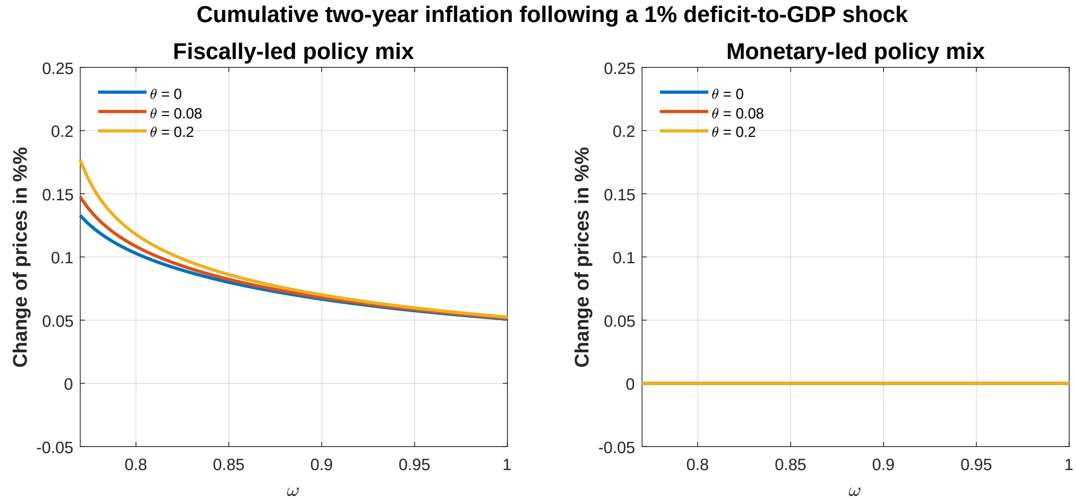

Figure D.2: The role of household (quasi-)heterogeneity and indexed debt across policy regimes. In the fiscally-led policy mix, we set $\tau_d = 0$ and $\phi = -0.2$, while in the monetary-led policy mix, we set $\tau_d = 0.4$ and $\phi = 0.2$. The remaining calibration is unchanged relative to previous plots.

Figure D.2 provides a comparison of impact inflation as a function of the inverse of mortality risk ω in the cases of fiscally-led policy mixes and monetary-led policy mixes. The size of the shock is normalized to reflect a 1% deficit-to-GDP shock. Additionally, we provide three distinct levels of inflation-indexed debt as shares of the total debt stock, varying between 0 and 0.2, which is at the higher end of observed levels around the world.[44]

The right-hand side, which focuses on the monetary-led policy mix, features zero inflation across the board. This does not come as a surprise due to the block-exogeneity of the debt equation and the fact that its associated eigenvalue must be the stable one under a monetary-led policy mix: here, inflation must necessarily be equal to zero on any saddle-path stable equilibrium, as inflation would otherwise be unbounded.[45]

On the left-hand side, the dynamics of inflation in response to a deficit are much more interesting. First, the price level change under a fiscally-led policy mix is generally increasing in the mortality risk $1-\omega$: as ω falls (and households are more likely to die), prices react more to pressure coming from fiscal deficits in the fiscally-led policy mix. Inflation-indexed debt, however, does matter for the exact inflation level and for the interactions between mortality risk and the fiscally-led policy mix: under realistic calibrations, inflation-indexed debt generally increases the change of the price level on impact, with the effects being particularly pronounced when the economy admits realistic levels of mortality risk.

Since the shock is equivalent to a 1% deficit-to-GDP shock, it is possible to calculate the 'fiscal

[44]The only OECD member country with higher shares of inflation-indexed debt in the total debt stock is the United Kingdom with approx. 28% of the total market value of debt.

[45]Note that if one were to overcome the block-exogeneity of the debt equation, which was introduced here for analytical convenience, then the parametric combination assumed under the monetary-led policy mix could feature non-zero inflation rates for $\omega < 1$. More generally, a continuous determinate policy space would exist, making a clear-cut distinction between fiscally-led and monetary-led policy mixes difficult (Rachel and Ravn, 2025).

inflation multiplier' in the spirit of Hazell and Hobler (2024), which measures the percent change in the rate of inflation following a 1% change in deficits relative to GDP. For realistic parametrizations of $\omega \approx 0.8$ (Angeletos et al., 2024), changing the share of inflation-indexed debt from 0 to 20% boosts the fiscal inflation multiplier by 0.04 percentage points. The observed results are on the lower end relative to the evidence on the effects of inflation-indexed debt in the quantitative model of section 7, but fit qualitatively in the same story.

We can generalize this discussion to say something about the degree to which the effects of indexed debt are increasing in heterogeneity:

Proposition 5 *The effects of inflation-indexed debt are increasing in the degree of quasi-heterogeneity, i.e.,*

$$\frac{\left(\partial \pi_0^{\varepsilon}\right)^2}{\partial \omega \partial \theta} > 0, \tag{D.36}$$

if and only if the tax base channel of debt is sufficiently large; that is, if:

$$\tau_y > \beta \frac{D^{SS}}{Y^{SS}} \left(\frac{\kappa \theta}{1-\beta} - \frac{1-\omega}{\sigma} \right). \tag{D.37}$$

The probability of this being the case therefore decreases in the share of inflation-indexed debt, θ.

Proof. We take equation (D.32) and take the partial derivative w.r.t θ:

$$\frac{\partial \pi_0^{\varepsilon}}{\partial \theta} = \frac{\kappa^2 \left(\frac{1}{\beta} - 1\right) \frac{D^{SS}}{Y^{SS}}}{\left\{ \left[\frac{D^{SS}}{Y^{SS}} \frac{(1-\omega)}{\sigma} + \frac{\tau_y}{\beta} \right] (1-\beta) - \frac{D^{SS}}{Y^{SS}} \kappa (1 - \omega + \theta) \right\}^2} \varepsilon_0.$$

Differentiating the previous expression with respect to ω, we then obtain:

$$\frac{\left(\partial \pi_0^{\varepsilon}\right)^2}{\partial \omega \partial \theta} = \frac{\left(\frac{D^{SS}}{Y^{SS}} \kappa\right)^2 \left(\frac{1}{\beta} - 1\right) \left[\frac{1-\beta}{\sigma} - \kappa \right]}{\left\{ \left[\frac{D^{SS}}{Y^{SS}} \frac{(1-\omega)}{\sigma} + \frac{\tau_y}{\beta} \right] (1-\beta) - \frac{D^{SS}}{Y^{SS}} \kappa (1 - \omega + \theta) \right\}^3}.$$

If the last expression is larger than zero, indexed debt θ indeed expands the measured effects of quasi-heterogeneity on price level adjustments in the initial period. That expression is positive if and only if its numerator and denominator have the same signs. The numerator of the previous expression is trivially positive under $\kappa < \frac{1-\beta}{\sigma}$. Now, the denominator in turn is only positive if the tax base channel is sufficiently strong, which is the case if:

$$\tau_y > \beta \frac{D^{SS}}{Y^{SS}} \left(\frac{\kappa \theta}{1-\beta} - \frac{1-\omega}{\sigma} \right). \tag{D.38}$$

■

Therefore, irrespective of inflation on impact being positive or negative in response to a fiscal innovation, the size of the overall price level shift can be curbed by inflation-indexed debt when the tax base channel is sufficiently weak.

D.6 Moving beyond the limit point

The previous analysis was restricted to the 'quasi-limit' point where $\phi = -\frac{1-\omega+\epsilon}{\sigma}, \tau_d \to 0^+$. This section now generalizes prior insights to a wider feasible set of monetary and fiscal policy combinations.

The full first-order system in this framework with inflation-indexed debt is given by:

$$\mathbb{E}_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ d_{t+1} \end{bmatrix} = \begin{bmatrix} \frac{1+\phi\sigma}{\omega} & 0 & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta} - \frac{D^{SS}}{Y^{SS}}\frac{\kappa(1-\omega+\theta)}{\beta} & \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\beta} & \frac{1}{\beta}(1-\tau_d) \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ d_t + \varepsilon_t \end{bmatrix}, \tag{D.39}$$

such that the general eigenvalue system associated with the stable eigenvalue ρ_d is now defined as:

$$\begin{bmatrix} \frac{1+\phi\sigma}{\omega} & 0 & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta} - \frac{D^{SS}}{Y^{SS}}\frac{\kappa(1-\omega+\theta)}{\beta} & \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\beta} & \frac{1}{\beta}(1-\tau_d) \end{bmatrix} \begin{bmatrix} \chi_1 \\ \chi_2 \\ 1 \end{bmatrix} = \rho_d \begin{bmatrix} \chi_1 \\ \chi_2 \\ 1 \end{bmatrix}. \tag{D.40}$$

This, in turn, yields the following system of three equations in the three unknowns ρ_d, χ_1 and χ_2:

$$\begin{aligned} \left(\frac{1+\phi\sigma}{\omega}\right)\chi_1 &= \rho_d\chi_1, \\ -\frac{\kappa}{\beta}\chi_1 + \frac{1}{\beta}\chi_2 &= \rho_d\chi_2, \\ \left(\frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta} - \frac{D^{SS}}{Y^{SS}}\frac{\kappa(1-\omega+\theta)}{\beta}\right)\chi_1 + \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\beta}\chi_2 + \frac{1}{\beta}(1-\tau_d) &= \rho_d. \end{aligned}$$

We can immediately deduce that $\rho_d = \frac{1+\phi\sigma}{\omega}$, which is the analytical expression of the corresponding eigenvalue. From the second of the three conditions, we can deduce that

$$\chi_1 = \frac{1-\beta\rho_d}{\kappa}\chi_2,$$

which we can insert for χ_1 in the third condition to obtain:

$$\left\{\left[\frac{D^{SS}}{Y^{SS}}\left(\phi - \frac{\kappa(1-\omega+\theta)}{\beta}\right) - \frac{\tau_y}{\beta}\right]\left(\frac{1-\beta\rho_d}{\kappa}\right) + \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\beta}\right\}\chi_2 = \rho_d - \frac{1}{\beta}(1-\tau_d),$$

which we can rewrite to:

$$\chi_2 = \frac{\rho_d - \frac{1}{\beta}(1-\tau_d)}{\left(\frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta}\right)\left(\frac{1-\beta\rho_d}{\kappa}\right) + \frac{D^{SS}}{Y^{SS}}(1-\omega+\theta)\rho_d}.$$

We can now insert for ρ_d to obtain the expression pinning down impact inflation in the general case:

$$\pi_0^\varepsilon \equiv \chi_2\varepsilon_0 = \frac{\frac{1}{\omega}(1+\phi\sigma) - \frac{1}{\beta}(1-\tau_d)}{\left(\frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta}\right)\left(\frac{1-\frac{\beta}{\omega}(1+\phi\sigma)}{\kappa}\right) + \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\omega}(1+\phi\sigma)}\varepsilon_0. \tag{D.41}$$

This general expression can be used for a more in-depth analysis of the impact of inflation-indexed debt on impact inflation in a more general setting with fiscal and monetary policy being outside the limit point.

Proposition 6 *For a sufficiently fiscally-led policy mix; that is, for $\tau_d < 1 - \frac{\beta}{\omega}(1+\phi\sigma)$, inflation-indexed debt always boosts inflation in response to a fiscal transfer shock, conditional on the equilibrium being existent and the eigenvalue associated with the aggregate demand equation being the stable one, which restricts monetary policy to*

$$\phi \in \left(-\frac{1}{\sigma}, \min\left\{-\frac{1-\omega}{\sigma},\right.\right.$$
$$\left.\left.\frac{\frac{D^{SS}}{Y^{SS}}\left(1+\frac{\kappa\sigma(1-\omega+\theta)}{\omega}-\frac{\beta}{\omega}\right) - \sqrt{\left[\frac{D^{SS}}{Y^{SS}}\left(1+\frac{\kappa\sigma(1-\omega+\theta)}{\omega}-\frac{\beta}{\omega}\right)\right]^2 - 4\frac{D^{SS}}{Y^{SS}}\frac{\beta}{\omega}\sigma\left[\frac{\tau_y}{\omega}-\frac{\tau_y}{\beta}+\frac{D^{SS}}{Y^{SS}}\frac{\kappa(1-\omega+\theta)}{\omega}\right]}}{2\frac{D^{SS}}{Y^{SS}}\frac{\beta}{\omega}\sigma}\right\}\right)$$

The magnitude of the effect of inflation-indexed debt on impact inflation is decreasing in the strength of tax adjustment to debt issuance τ_d; that is, $\frac{(\partial\pi_0^\varepsilon)^2}{\partial\theta\partial\tau_d} < 0$, always.

Proof. Taking the first partial derivative of impact inflation with respect to the share of inflation-indexed debt, we obtain:

$$\frac{\partial\pi_0^\varepsilon}{\partial\theta} = \frac{\left[\frac{1}{\beta}(1-\tau_d) - \frac{1}{\omega}(1+\phi\sigma)\right]\frac{D^{SS}}{Y^{SS}}\frac{1}{\omega}(1+\phi\sigma)}{\left[\left(\frac{\tau_y}{\beta} - \phi\frac{D^{SS}}{Y^{SS}}\right)\left(\frac{1-\frac{\beta}{\omega}(1+\phi\sigma)}{\kappa}\right) - \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\omega}(1+\phi\sigma)\right]^2}\varepsilon_0. \tag{D.42}$$

This fraction is positive if and only if its numerator is positive. This is the case when both bracketed elements are positive, i.e., when either of the following sets of inequalities holds:

$$\phi > -\frac{1}{\sigma}; \qquad \tau_d < 1 - \frac{\beta}{\omega}(1+\phi\sigma).$$

$$\phi < -\frac{1}{\sigma}; \qquad \tau_d > 1 - \frac{\beta}{\omega}(1+\phi\sigma).$$

The second case, however, is ruled out by the previous existence restriction on ϕ. Therefore, for sufficiently active fiscal policy (reflected by small, or even negative values of τ_d), indexed debt can boost inflation if an equilibrium exists, which must be supported by monetary policy not reacting positively to inflationary pressure as before.[46]

To establish the second result, it suffices to take the partial derivative of equation (D.42) with respect to τ_d and the result follows, since

$$\frac{(\partial\pi_0^{\varepsilon})^2}{\partial\theta\partial\tau_d} = \frac{-\frac{1}{\beta}\frac{D^{SS}}{Y^{SS}}\frac{1}{\omega}(1+\phi\sigma)}{\left[\left(\frac{\tau_y}{\beta} - \phi\frac{D^{SS}}{Y^{SS}}\right)\left(\frac{1-\frac{\beta}{\omega}(1+\phi\sigma)}{\kappa}\right) - \frac{D^{SS}}{Y^{SS}}\frac{(1-\omega+\theta)}{\omega}(1+\phi\sigma)\right]^2}\varepsilon_0 < 0. \tag{D.43}$$

■

Relative to the results established in the limit point between the fiscally-led and the monetary-led policy mixes, we therefore confirm that inflation-indexed debt is generally proving to amplify inflationary pressure especially under fiscally-led policy mixes. In particular, as the tax adjustment to debt issuance becomes minimal; that is, as τ_d decreases, the corresponding inflationary pressure arising from fiscal shocks becomes even more amplified.

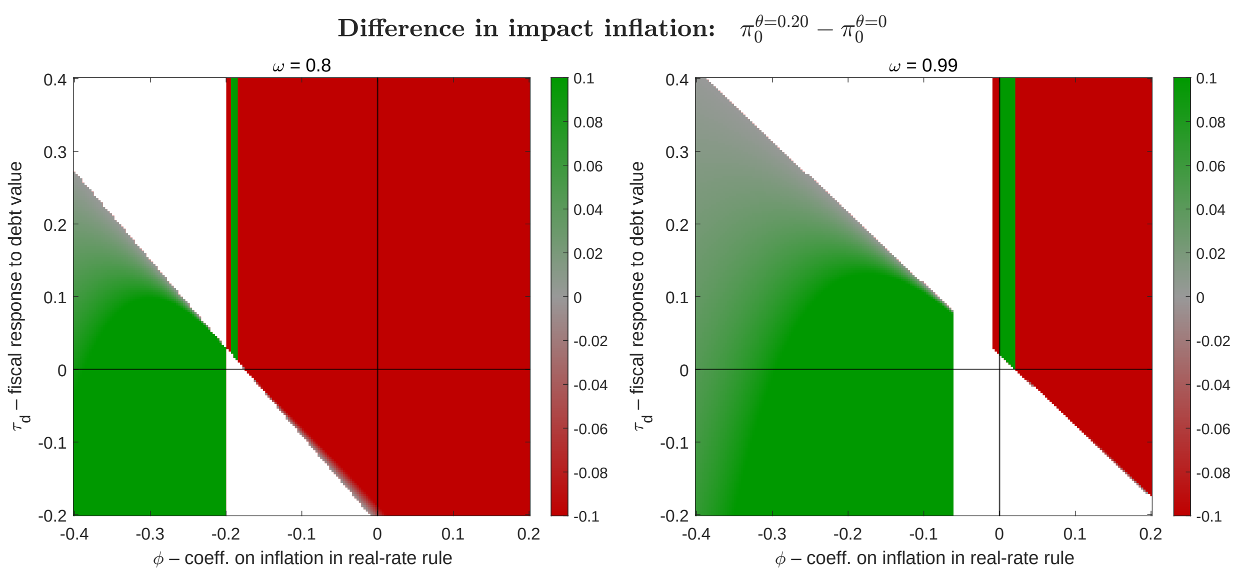

Figure D.3: Plot showing under which parametrizations either of the inequalities described by proposition 3 is fulfilled, indicating larger impact inflation when the share of inflation-indexed debt is higher. Green areas indicate places where either of the sets of inequalities hold, red areas indicate the opposite. White areas are the parts of the policy space where no saddle-path stable equilibrium exists. Calibration: $D^{SS} = 1.2, Y^{SS} = 1, \kappa = 0.05, \beta = 0.99, \sigma = 1, \tau_y = 0.25$.

Figure D.3 visualizes what was spelled out in proposition 6 for two instances - one with 'moder-

[46] The possibility that both elements in the numerator are *negative* is ruled out, since ϕ cannot be smaller than $-\frac{\omega}{\sigma}$ in any saddle-path equilibrium (Angeletos et al., 2024).

ate quasi-heterogeneity' (left panel) and once in the *almost*-quasi-representative agent case. The bottom-left part of each panel is the area that would be conventionally-considered 'fiscally dominant' in the sense that there we have a 'passive' monetary authority in the sense of Leeper (1991). Conditional on the existence of equilibrium, the presence of inflation-indexed debt increases inflationary pressure on impact across the board in the areas conventionally associated with a fiscally-led policy mix. For the area under which we would conventionally consider monetary policy 'active' (the top-right area), the opposite is generally the case, except for a small region determined generally by ϕ. Overall, however, proposition 6 makes the case for increased inflationary pressures under higher levels of inflation-indexed debt whenever fiscal policy is considered conventionally active, and at times even when the opposite is the case.

E Additional empirical evidence

E.1 The distribution of government debt holdings across households

Publicly available microdata reinforces the case for the relevance of the distribution of indexed debt in the household portfolio. This brief section focuses on the US due to the superior availability of household-level data on asset holdings.

Figure E.1 plots the real (2017) Dollar value of nonindexed and indexed government debt holdings of households questioned in the US Survey of Consumer Finances (SCF), separated by income deciles.[47,48] The left-hand panel of figure E.1 reflects the well-known left-skew of bond holdings of households in the income distribution, by which households at the upper end of the income distribution hold a significantly larger share of sovereign bonds. The right-hand panel of figure E.1 reflects a less well-known observation: this left skew is *vastly* more pronounced for indexed sovereign bonds (TIPS), with the top income decile holding almost 40% of all outstanding TIPS in the sample.

[47] I chose income deciles due to their clear definition in the survey with a single question. Constructing individual wealth variables is possible with the survey data, albeit this is subject to individual choices about what to consider as household wealth. For most definitions of wealth, the results continue to hold qualitatively.

[48] Using household survey responses to develop a profile of sovereign bond ownership is not perfect, since most sovereign bonds owned by households are only held indirectly through insurance companies and pension funds, but it can be considered a first-pass proxy for the ultimate holdings of such savings vehicles.

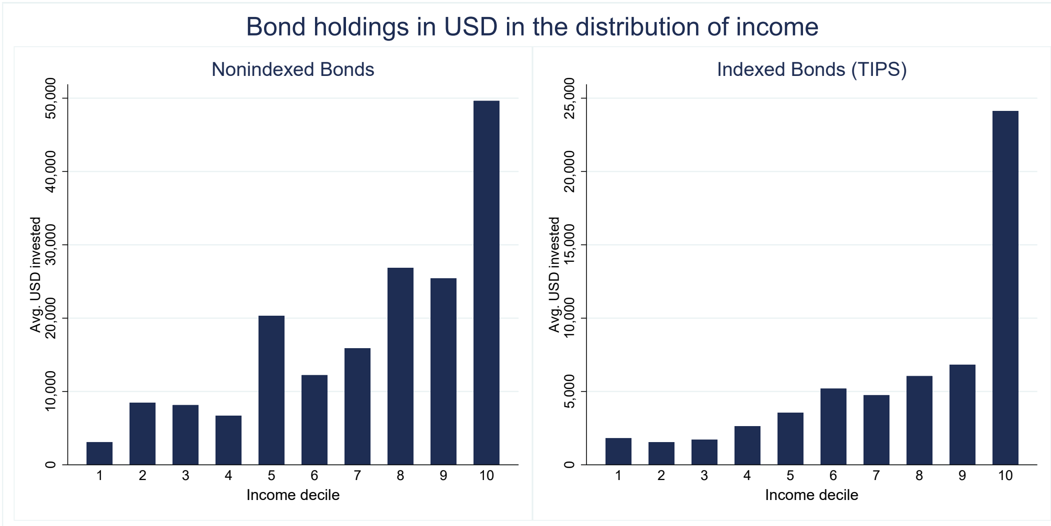

Figure E.1: Distribution of indexed and non-indexed debt holdings across household income deciles, denoted in real (2017) USD. Data source: Survey of Consumer Finances (US); sample period: 2014-2019.

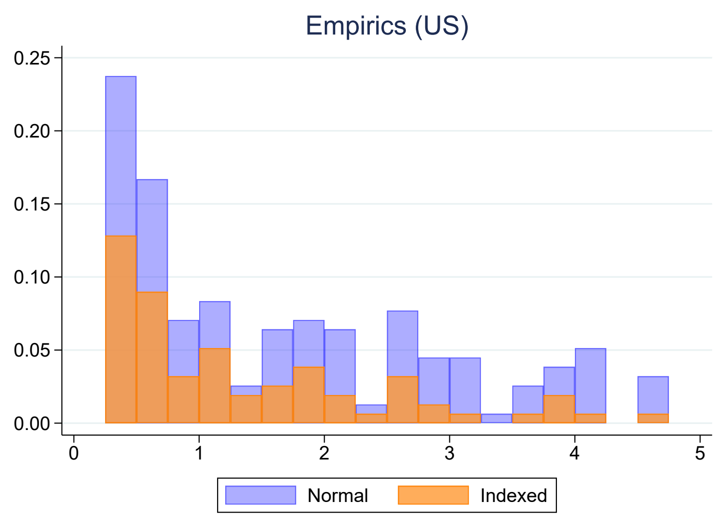

Figure E.2: Density of indexed and non-indexed debt holdings in the US Survey of Consumer Finances; snapshot from October 2019.

Figure E.2 provides further evidence in favor of the model calibration, which has not been a targeted moment. Just as in the model (figure 9), the distribution of both indexed and non-indexed bonds exhibits a significant skew, which is more pronounced overall for inflation-indexed debt. In particular, the size of the bins, even if not exactly matched, broadly reflects the distribution of the model very well.

As is standard in models forgoing extreme-value distributions of exogenous innovations, the model struggles with matching the tails of the distribution. In the context of the macroeconomic implications of the model, this is not of substantial concern.

E.2 Further details on the Local Projections

To shed further light on the evidence presented in section 2.1, I here provide the result tables from the local projection on UK data presented in figure 3 and introduce additional evidence using US data in a similar exercise.

First, table E.1 summarizes the results given in figure 3, specifying the exact coefficients on the interaction effect of $\Delta\omega_t \varepsilon_t$ and the individual effects of the change in the indexed debt share $\Delta\omega_t$ and the identified fiscal shock ε_t on cumulative price level change from the pre-shock period -1 until the period specified above all columns.

	Dependent variable: log(Cumulative Inflation)							
Lag periods:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Fiscal Shock	-0.01**	-0.01	-0.01	-0.01	-0.02	-0.02	-0.02	-0.02
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)
Index Share	0.02***	0.02**	0.03**	0.02	0.02	0.02	0.03	0.03
	(0.00)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Fiscal Shock × Index Share	0.10*	0.09	0.20	0.26*	0.39**	0.40*	0.60**	0.81***
	(0.06)	(0.10)	(0.13)	(0.14)	(0.18)	(0.21)	(0.24)	(0.27)
Additional Controls	Y	Y	Y	Y	Y	Y	Y	Y
Observations	155	154	153	152	151	150	149	148
R^2	0.412	0.518	0.559	0.630	0.592	0.599	0.575	0.602

Table E.1: Local Projection results for the UK. Additional controls include past four-quarter lags of GDP growth, the Bank Rate, real exchange rate growth, and year and recession dummies. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction).

While the share of indexed debt itself does not seem to impact medium-term inflation significantly, the interaction effect of the share of indexed debt with the identified fiscal shock follows the pattern given in figure 3.

To ensure that this mechanism is not idiosyncratic to the UK, I utilize again the data provided by Mierzwa (2024) within an econometric specification adjacent to Cloyne et al. (2023), but in relation to the series of US fiscal shocks identified therein. I leverage the number of identified fiscal shocks and estimate the same local projection specification (equation (1)) to estimate the role played by inflation-indexed debt in exacerbating the effects of fiscal spending shocks. Table E.2 and figure E.3 summarize the results of this exercise using data since 1980, which is the earliest period for which identified fiscal shocks are available.

	Dependent variable: log(Cumulative Inflation)							
Lag periods:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Fiscal Shock	0.00	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)
Index Share	0.00	-0.01	-0.01	-0.01	-0.00	-0.00	-0.01	-0.01
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Fiscal Shock × Index Share	0.02	0.01	0.06	0.15**	0.19***	0.21**	0.22**	0.24***
	(0.03)	(0.08)	(0.08)	(0.06)	(0.07)	(0.09)	(0.10)	(0.09)
Additional Controls	Y	Y	Y	Y	Y	Y	Y	Y
Observations	161	160	159	158	157	156	155	154
R^2	0.324	0.371	0.474	0.531	0.543	0.542	0.559	0.554

Table E.2: Local Projection results for the US. Additional controls include past four-quarter lags of GDP growth, the Federal Funds Rate, real exchange rate growth, and year and recession dummies. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction).

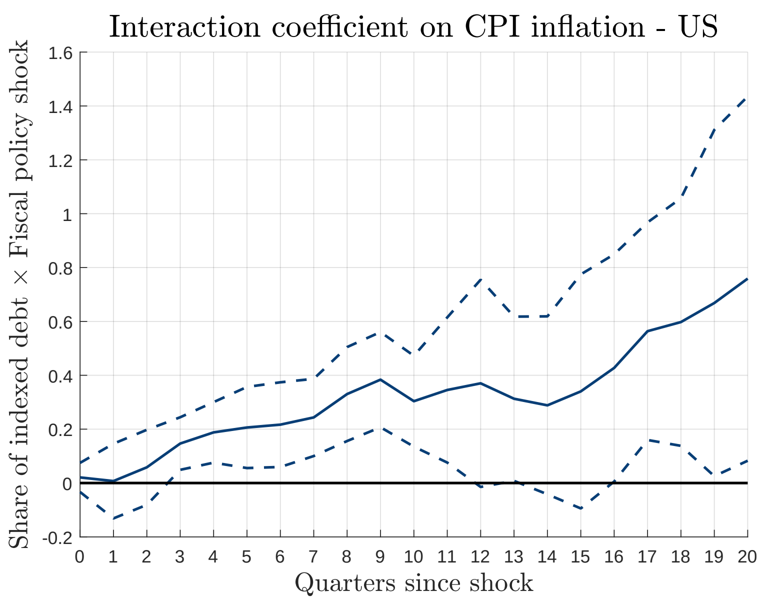

Figure E.3: IRFs implied by a local projection in the style of equation (1). The control vector Z consists of the first four lags of the real GDP growth rate, the short-run nominal interest rate, the change in the weighted real exchange rate, and a same-period recession indicator. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction). Confidence intervals are provided at the 90% level. Sample length: 1980 Q1 - 2019 Q4.

The results here paint a supporting picture, as the interaction effect between the change in the share of inflation-indexed debt and the identified fiscal shock appears to be statistically significant in the medium-term again, even though the level of the effect is not as pronounced as in the UK.

Given the institutional arrangements in the US and the main estimation exercise in the paper, the US example is used to provide another piece of evidence by focusing *only* on fiscal policy surprises occurring in periods that can be considered as being supported by *active fiscal policy* (a fiscally-led policy mix) in the sense of Leeper (1991). To be precise, I leverage the Bayesian DSGE estimation of Chen et al. (2022), assigning the label of 'active fiscal policy' to periods in which the posterior probability of a fiscally-led policy regime exceeds 0.8. By this process, 44% (33/75) of the original shock observations in the period 1980-2019 remain as happening in fiscally-dominant periods. The resulting exercise with the reduced shock sample yields the results presented in table E.3 and in figure E.4.

	Dependent variable: log(Cumulative Inflation)							
Lag periods:	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Fiscal Shock (Active Periods)	-0.00	-0.00	-0.00	-0.00	-0.01	-0.01	-0.01	-0.01
	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Index Share	0.00	-0.00	-0.00	-0.00	0.00	0.00	-0.00	-0.00
	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)	(0.02)	(0.02)	(0.02)
Fiscal Shock × Index Share	0.01	-0.08	-0.02	0.18	0.18	0.14	0.12	0.28
	(0.07)	(0.13)	(0.20)	(0.21)	(0.22)	(0.25)	(0.27)	(0.29)
Additional Controls	Y	Y	Y	Y	Y	Y	Y	Y
Observations	161	160	159	158	157	156	155	154
R^2	0.329	0.381	0.480	0.534	0.545	0.546	0.563	0.555

Table E.3: Local Projection results for the US under a fiscally-led policy mix shocks, following Chen et al. (2022). Additional controls include past four-quarter lags of GDP growth, the Federal Funds Rate, real exchange rate growth, and year and recession dummies. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction).

Restricting the focus on the sample periods for which one can assign a relatively high probability of a fiscally-led policy regime, there is at first no significant response of cumulative inflation to the fiscal shock, followed by a gradually significant and positive response in the interaction effect of the indexed debt share and the identified fiscal shock in the medium-term, broadly in line with the previous results. The magnitude of the interaction effect more or less doubles relative to the previous analysis without the restriction on periods of active fiscal policy only.

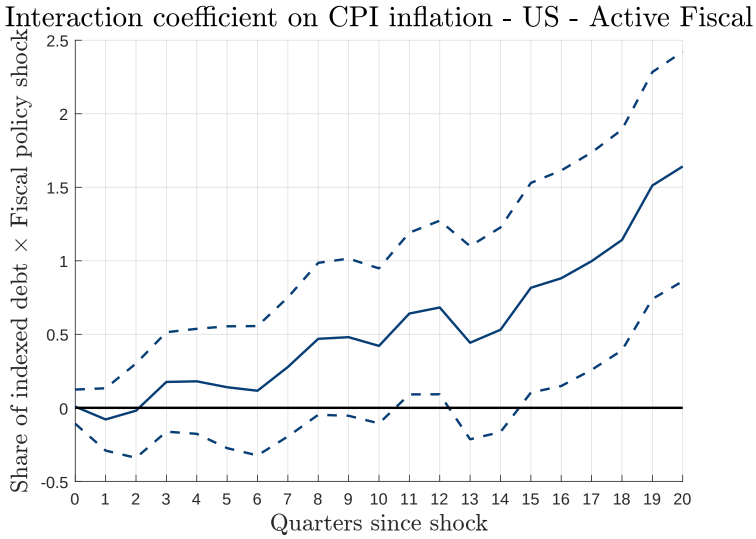

Figure E.4: IRFs implied by a local projection in the style of equation (1). The control vector Z consists of the first four lags of the real GDP growth rate, the short-run nominal interest rate, the change in the weighted real exchange rate, and a same-period recession indicator. Standard errors are robust to heteroskedasticity and autocorrelation (Newey-West correction). Confidence intervals are provided at the 90% level. Sample length: 1980 Q1 - 2019 Q4.

E.3 Additional evidence on the nature of the September 2022 'Truss shock'

In this appendix, I provide additional evidence on the fiscal policy shock dubbed "The Growth Plan" in September 2022 in the United Kingdom, commonly dubbed the 'mini-budget'. This fiscal policy announcement is placed in the context of general trends that were observed in the data at the same period, i.e., mainly in September and October 2022. For all analysis carried out in the following, ticker-frequency data on market prices has been used. The data comes from the Bank of England (BoE), the UK Treasury, and Bloomberg Financial Services.

I begin by examining the degree to which the policy announcement can be informative about the propensity of a type of 'fiscally-led policy mix' in a wider sense, i.e., whether the policy measures around this particular fiscal shock can be placed in a context at which monetary policy passively adjusts to the fiscal policy measure, taking the fiscal announcement as given.[49] The preferred measure that is plausibly related to both sustainability concerns introduced by the budget announcement and the prospective monetary reaction comes from expected overnight interest rates. These are interest rates for overnight bank lending activities on financial markets, instrumented using swaps on overnight lending between the day at question and the day of the next monetary policy meeting. Normally, and as is the case for vast periods of the time, those swaps follow the prevailing nominal market rate closely, as any other rate would induce arbitrage by the possibility of a risk-free hedge using the current nominal interest rate. As figure E.5 shows, however, the turmoil

[49] Determining uniquely whether a given policy announcement, or a given time period, clearly relates to a monetary-led or a fiscally-led policy mix in a narrow sense, i.e., in relation to the respective policy rules and how they inform the stability of the underlying economic system, is generally not possible purely based off economic data. Simply put, the "Taylor Principle" cannot be tested as its impact on the uniqueness properties surrounding macroeconomic models relies on off-equilibrium threats that cannot be observed under the condition of the Taylor Principle itself holding (Cochrane, 2011; Neumeyer and Nicolini, 2025).

introduced by the UK mini budget caused a remarkable wedge between the two rates:

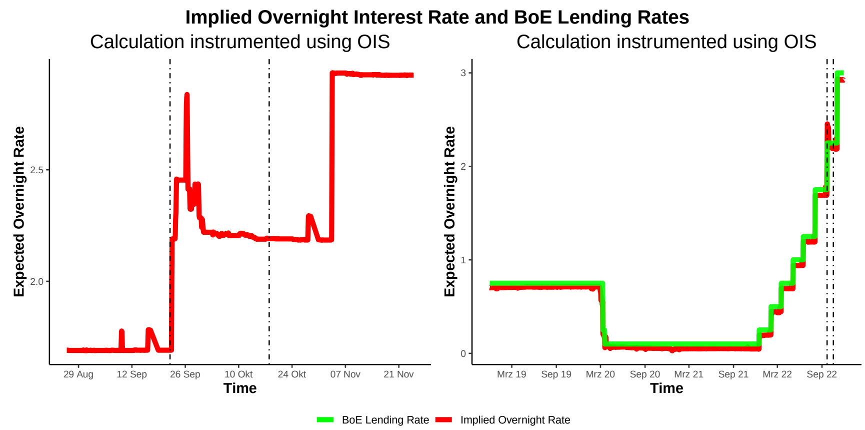

Figure E.5: The prevailing Bank of England Lending Rate and the Implied Overnight Interest Rate, derived by instrumenting overnight interest swaps from today to the expected next meeting of the BoE monetary policy committee. Data source: Bloomberg.

As can be inferred from the right-hand side panel for a period of almost three years, and on the left-hand side in more detail for the period of interest, the imputed overnight interest rate follows the BoE lending rate closely, exhibiting jumps around meeting dates of the BoE Monetary Policy Committee in alignment with monetary policy decisions.

The period of the mini-budget, which commenced one day after a Monetary Policy Committee (MPC) meeting (September 23 and September 22, respectively), induced movements in the expected overnight rates that were not observed at any other point in time - despite no MPC meeting in near sight,[50] the expected overnight rates shot up far beyond the then-prevailing BoE bank lending rate by up to 50 basis points. Such movements can be caused by an array of different possibilities: it could be either that fiscal policy caused a shift in market expectations of monetary policy in the short-term, thus implying that monetary policy was considered to be 'reactive' to the fiscal policy announcement, or that the mini-budget was expected to have such detrimental consequences on inflation that the BoE was required to react immediately, or it might be reflective of liquidity issues the swap market in the same period.

Correspondingly, and as can be inferred from figure E.6, prices of Gilt futures dropped sharply during the period in which the UK mini-budget was expected to be put in place.

[50]Only on the September 27, then-BoE chief economist Huw Pill stated that the proposed UK government budget might require a "significant monetary response", indicating readiness on behalf of the BoE to adjust the monetary stance, but no concrete emergency meeting date had been proposed at that point.

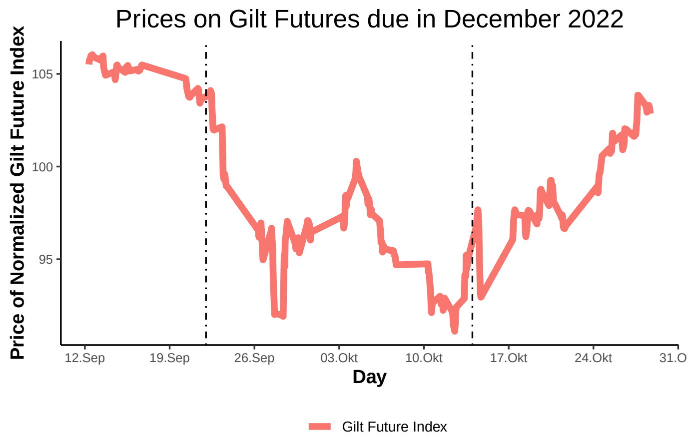

Figure E.6: Evolution of the weighted Gilt Future Index for futures due in Decemebr 2022, weighting Gilt prices based on a normalized face value of 100, after adjusting for expected inflation. Data source: Bloomberg.

Since the expectations were that the mini-budget was supposed to be financed through a further issuance of Gilts, the sharp decrease in the weighted index of Gilt future prices can be considered a mechanical movement, caused by the expected sharp increase in the supply of Gilts.

The depreciation of the British Pound can be similarly useful for the case of a fiscally-led policy innovation occurring in September 2022. Here, I provide evidence on the *relative* depreciation of the GBP over the Sterling Overnight Index Average (SONIA) by using the spread between swaps on GBP futures and SONIA futures, which concern unsecured transactions in the Sterling market on future dates. Figure E.7 shows that the spread between GBP future and SONIA swaps shot up following the announcement of the mini-budget across all maturity dates. However, the yield curve of the spreads increased in *curvature* in its 'inverse-U' shape following the announcement of the mini budget, as is evidenced by the right-hand panel.

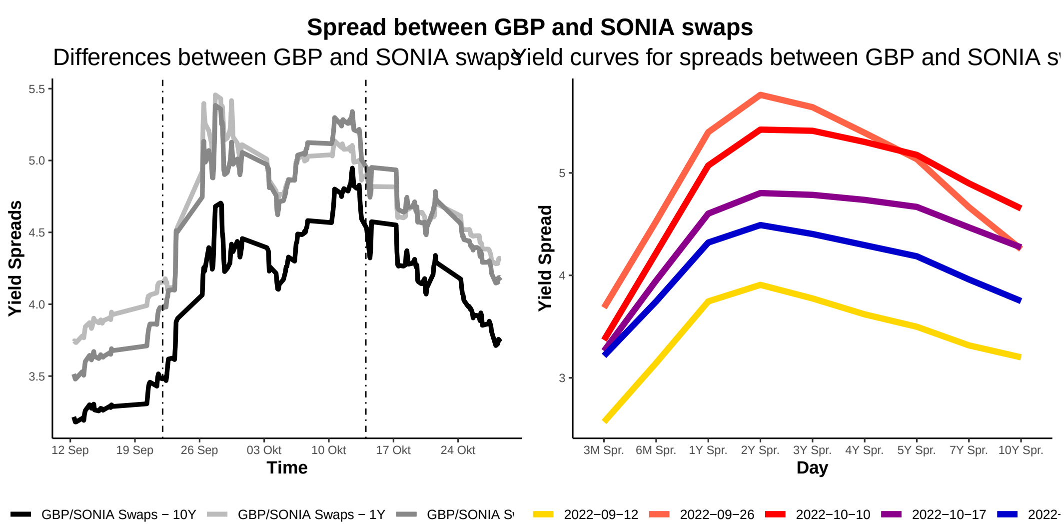

Figure E.7: Yield Spreads between GBP future swaps and SONIA swaps on the overnight Sterling market. The left-hand panel plots spreads over time, while the right-hand panel plots yield curves on selected dates. Data source: Bloomberg.

An important caveat to these findings is that reducing all dynamics to expected revaluations of bonds is unlikely to capture the the full richness of macroeconomic dynamics in response to this deficit shock itself. As evidence of that, consider figure E.8, which plots uncertainty around the period of the mini-budget (measured through the FTSE 100 IVI Index), showing that overall market uncertainty beyond sovereign debt markets had been elevated at the same time as well.

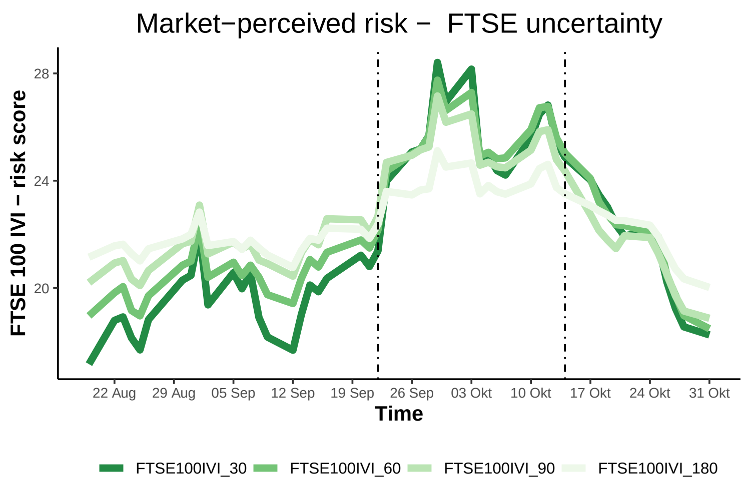

Figure E.8: The uncertainty index over equity of the largest publicly traded British companies. The lines measure implied uncertainty in 30-day, 60-day, 90-day, and 180-day forward-looking windows.

The mini-budget episode coincided with an inversion of the market-perceived risk relative to the forecast horizon: whereas in the periods before and after the mini-budget announcement market risk was perceived to be higher in the short-medium-term (180 days) than in the short-term (30 or 60 days), the opposite is the case during this short-lived fiscal episode.

Finally, financial market data allows also the direct consideration of default risk, an aspect that has been omitted from the model presented in this paper. In figure E.9, I present the Credit Default Swap spread as well as the number of Gilts maturing in the near-term around September 2022.

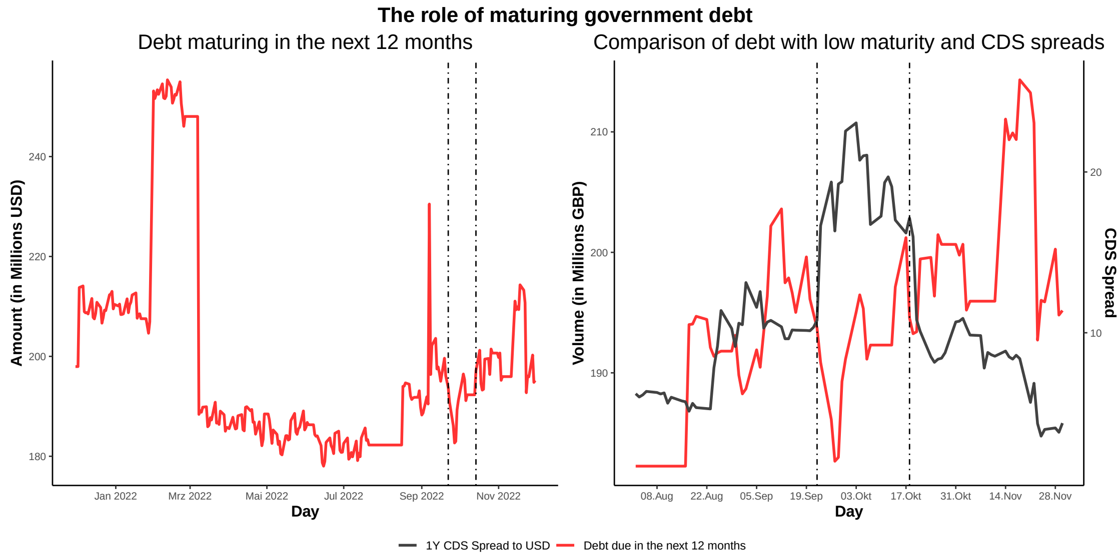

Figure E.9: Government debt maturing over the course of the next 12 months, plotted against the 1-year spread of Credit Default Swaps on UK Gilts over USD futures. Data source: Bloomberg.

Crucially, the nominal amount of maturing government debt was at rather low levels, in particular shortly following the announcement of the mini-budget. The black line, which plots short-term corresponding Credit Default Swaps based off UK Gilts, confirms that the total amount of debt due in the near future 'disconnected' from the perceived market risk at the time of the mini-budget: while the two measures have a correlation of 0.49 in the period leading up to the announcement of the mini budget, that correlation drops to -0.51 for the period between the middle of September and the middle of October 2022.

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All these data points reinforce the idea that the 'mini-budget' announced by the UK Treasury in September 2022 was indeed an unexpected fiscal measure that significantly subverted perceived fiscal sustainability, with wide-spread ramifications for expected real returns on government bonds, expected inflation and interest rates, and elevated risk levels.

FAQ: the 'mini-budget shock' episode

In addition to the evidence depicted above derived using the market impact of the mini-budget episode, I here present narrative viewpoints complementing the understanding of the mini-budget episode.

Why did markets reverse the uptick in inflation expectations initially?

- On September 26, around 4.00pm, Kwasi Kwarteng announced to publish a 'medium-term fiscal plan', which possibly indicated greater restraint in fiscal policy: (Bloomberg).
- On September 28, a second shock to fiscal sustainability occurred: Moody's explicitly deemed the mini-budget to put UK debt sustainability in danger, followed by a same-day upshoot of inflation expectations: (Reuters).

- Likewise, on September 28, in a reversal of expectations caused by the September 26 statement, the Treasury explicitly rejected for the first time since the initial announcement any idea of reneging on their budget shortfall, thereby re-affirming expectations about the fiscal policy measure actually being pushed through. See: (BBC). On the very same day, the Bank of England intervened in bond markets by re-starting long-dated government bond purchases, which, again, re-affirmed the idea that the Treasury will not back down as the Bank of England decided it must act despite no policy having been enacted at that point. This was announced at around 11.20am - see (X). Swap breakeven rates shot up by 60bps in the three hours after the statement made by the Bank of England.

Why were inflation swaps priced much higher in August 2022 compared to the dynamics occurring in September and October 2022?

- On August 17, 2022, the ONS released a report of CPI inflation being 10.1%, breaking the 10% barrier for the first time in 40 years, also beating the city forecast of 9.8% decisively (Bloomberg). This occurred alongside a significant depreciation of the Pound (FT). Likewise, implied interest rate raises corresponding to expectations of vastly more aggressive monetary tightening from that period onward alongside a yield curve inversion appeared around August 15 (FT).
- Implied one-year ahead inflation expectations peaked at around 8% in August. Note that this is still *vastly* below the forecasts released in August 2022 by major financial market actors: the Goldman Sachs forecast of one-year ahead inflation amounted to 14.8%, with a 'negative' scenario of 22.1% annual inflation for the UK implied in their August 2022 briefings (FT). Relative to that forecast, the change in inflation swaps implied in that policy uncertainty episode was relatively benign. This period of increased inflation expectations coincided also with record prices on natural gas spot markets in the UK.

F Bond market revaluations in response to narrative fiscal shocks in the UK

FInally, I leverage a novel dataset capturing the entire universe of UK sovereign debt (comprising of normal Gilts, inflation-indexed Gilts, treasury bonds, treasury strips, etc.) in the period from 2000 until 2010 to analyze bond revaluations in response to unanticipated fiscal spending shocks. The starting point of this analysis are the derivations from Cochrane (2023), and most notably the equilibrium valuation of government debt under complete markets:

$$\frac{Q_t B_{t-1}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j},$$

where Q_t refers to the market price of government debt, B_{t-1} is the quantity of outstanding (marketable) sovereign debt, P_t is the price level, and s_{t+j} denotes the real government surplus at time $t + j$ expected at time t.

Accounting for the existence of long-term and inflation-indexed debt, this relationship changes to:

$$\frac{\sum_{j=0}^{\infty} q_t^{(t+j)} b_{t-1}^{(t+j)}}{P_{t-1}} + \frac{\sum_{j=0}^{\infty} Q_t^{(t+j)} B_{t-1}^{(t+j)}}{P_t} = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j s_{t+j}.$$

where superscripts denote the time of maturity of the bond at question and lowercase variables denote quantities and prices of inflation-indexed debt.[51]

Moving this equation one period forward ($t \mapsto t+1$) and multiplying/dividing the latter element on the left-hand side by P_t gives:

$$\sum_{j=0}^{\infty} \frac{q_{t+1}^{(t+1+j)} b_t^{(t+1+j)}}{P_t} + \frac{\sum_{j=0}^{\infty} Q_{t+1}^{(t+1+j)} B_t^{(t+1+j)}}{P_t} \frac{P_t}{P_{t+1}} = \mathbb{E}_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}.$$

Now, take first-differences of the expected values of the above relationship to capture surprise revaluations: $\Delta\mathbb{E}_{t+1} \equiv \mathbb{E}_{t+1} - \mathbb{E}_t$, based on objects that are not deterministic in t. This yields the following core equation:

$$\frac{\sum_{t=0}^{\infty} b_t^{(t+j)} \Delta\mathbb{E}_{t+1}\left(q_{t+1}^{(t+j)}\right) + \sum_{j=0}^{\infty} B_t^{(t+1+j)} \Delta\mathbb{E}_{t+1}\left(Q_{t+1}^{(t+1+j)} \frac{P_t}{P_{t+1}}\right)}{P_t} = \Delta\mathbb{E}_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}, \tag{F.44}$$

which will be the main relationship pinning down the 'net fiscal shock measure'.

What are the terms including 'differenced expectations', and can they be recovered empirically? Consider the first term in the numerator of the fraction:

$$\Delta\mathbb{E}_{t+1}\left(q_{t+1}^{(t+j)}\right) = \underbrace{q_{t+1}^{(t+1+j)}}_{\text{Spot price after innovation}} - \underbrace{\mathbb{E}_t q_{t+1}^{(t+1+j)}}_{t \text{ to } t+1 \text{ forward price before innovation}} := \textit{Forecast Error},$$

i.e., the surprise revaluation can be captured by means of differences between spot and forward prices. Note, however, that forward prices of bonds are hardly observable: while some forward contracts exist on bond ETFs, individual forwards can hardly be found, in particular for inflation-linked bonds that are comparatively illiquid.

The other innovation term that must be considered proves similarly cumbersome:

$$\Delta\mathbb{E}_{t+1}\left(Q_{t+1}^{(t+1+j)} \frac{P_t}{P_{t+1}}\right) = Q_{t+1}^{(t+j)} \frac{P_t}{P_{t+1}} - \mathbb{E}_t\left(Q_{t+1}^{(t+1+j)} \frac{P_t}{P_{t+1}}\right).$$

The first part is the ex post price of nominal debt adjusted for inflation, which is an easily retrievable object. The second part of the term equals the ex ante joint expectation over the nominal bond

[51] Derivations of similar relationships under the same complete-markets assumption are given in section 3.

return and the inverse of inflation.[52]

In this brief exercise, I aim to simplify the second term by considering sufficiently tight windows around fiscal announcement dates: 2 days before and after a shock, such that $\Delta t = 4$ days. In a span of four days, the price level is approximately unchanged, $\frac{P_t}{P_{t+1}} \approx 1$, thereby restricting the attention to the forecast errors in nominal and real bond prices.

The right-hand side stochastic term, $\Delta \mathbb{E}_{t+1} \sum_{j=0}^{\infty} \beta^j s_{t+1+j}$, is said to be the 'shock' in the narrow timeframe: this is posited to be some fiscal announcement, making the implicit assumption that any surprise to fiscal surpluses is fully self-contained and does not have further effects beyond the announced fiscal measure. The fiscal shocks considered here are the narrative shocks provided by Cloyne (2013), which limits the current sample size to the years 2000-2010. The construction of the shock series is modeled after the seminal paper of Romer and Romer (2010), and makes use of official documents released by UK legislators to ensure that only truly 'exogenous' discretionary tax changes are considered.

Data on bond prices and quantities are obtained from the Bank of England, the Treasury, Thompson Reuters, and the UK Government Securities Database maintained by Cairns and Wilkie (2023). The data were cross-checked across sources and corrected whenever errors were encountered, thus retaining the full information on traded government bonds, as it is necessary to paint the *complete* picture of the universe of British sovereign debt.

The final net shock measure consists then of the difference between the fiscal policy announcement given by Cloyne (2013) and the bond market innovation described above:

$$NetShock_{t+1} = \Delta \mathbb{E}_{t+1} \left[\sum_{j=0}^{\infty} \beta^j s_{t+1+j} - \sum_{t=0}^{\infty} b_t^{(t+j)} \left(q_{t+1}^{(t+j)} \right) - \frac{\sum_{j=0}^{\infty} B_t^{(t+1+j)} \left(Q_{t+1}^{(t+1+j)} \frac{P_t}{P_{t+1}} \right)}{P_t} \right]. \tag{F.45}$$

In words, this 'net shock measure' is a measure of the 'action' of the discretionary tax change (first term), adjusted for the 'reaction' of sovereign bond markets, i.e., the revaluation of outstanding debt (second term). Should the assumptions behind the derivation of this net shock measure be perfectly correct, one could expect the measure to be 0 at all times: if surpluses change, bond markets react by adjusting the real value of outstanding debt. If, instead, this measure has some variation, and it co-varies with the share of indexed debt outstanding, one may consider this evidence that inflation-indexed debt can be among the principal drivers informing fiscal theory when moving beyond the posited simplifying assumptions in deriving equation (F.45).[53]

[52]By the Fisher equation, this is the ex ante expected real return on holding nominal debt conditional on the risk premium.

[53]A more in-depth analysis of sovereign income and spending, including a detailed analysis of corresponding bond revaluations and yields borne on sovereign debt, is provided by Chen et al. (2022).

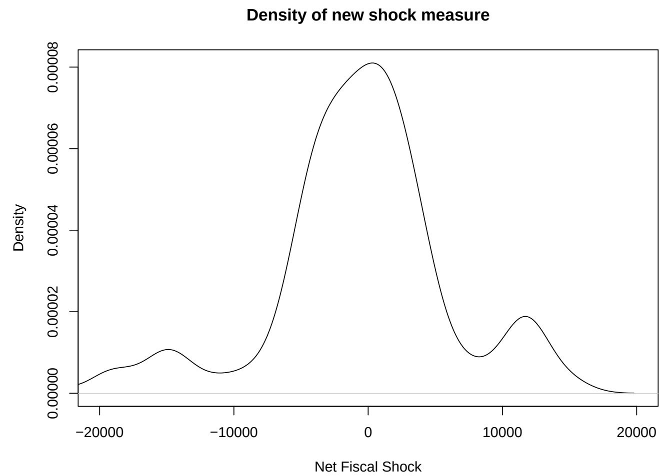


Figure F.1: Estimated Kernel density of the net shock measure described by equation (F.45) for the United Kingdom, 2000-2010.

Figure F.1 plots the density of this shock measure, showing that it is quite centered around zero, but not exactly symmetric. This may either be related to an issue in the definition of the shock series, problems with the assumption of the expectations hypothesis being a reasonable approximation to bond market behavior in such narrow time frames,[54] or it may be related to market incompleteness and possible differences in insurance premia between the two types of bonds.

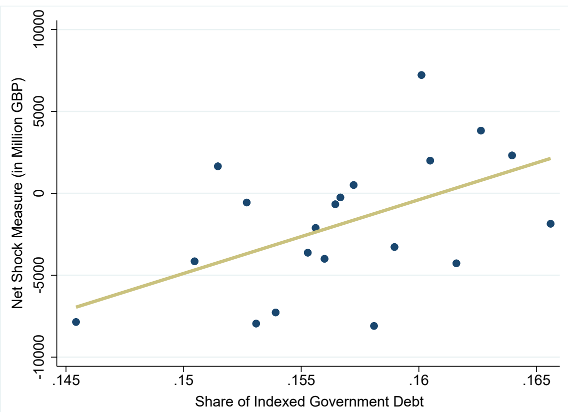
I will now argue that the last case might bear some relevance with the help of the following simple estimation exercise:

$$NetShock_t = \alpha + \beta \frac{\sum_{j=0}^{\infty} B_t^{*(t+j)}}{\sum_{j=0}^{\infty} b_t^{(t+j)} + B_t^{(t+j)}} + \Gamma_t X_t + \varepsilon_t, \tag{F.46}$$

i.e., I project the net shock measure on the share of inflation-indexed debt and a set of controls, which can include a year fixed effect to account for long-run changes in spending behavior as well as a recession indicator. The results of this exercise are given by the following figure F.2 and the corresponding regression table.

Despite the low variation in the share of indexed government debt in the United Kingdom from 2000 to 2010, the results showcase a possible correlation between the share of indexed debt and the unexplained variation in UK bond market valuations of government debt. As mentioned above, those results must be taken with caution, but this provides a stepping stone informing the analysis of the relevance of inflation-indexed debt in relation to surprise fiscal shocks, and how the two jointly influence real and nominal economic outcomes.

[54] The expectations hypothesis was used to derive the otherwise unobservable forward prices of all traded bonds.



	Dependent variable: Shock Measure	
	(1)	(2)
Share of indexed debt	450.134***	445.734***
	(154.966)	(153.894)
Recession indicator	.	1.588
	.	(3.729)
Constant	−61.745***	−61.88***
	(20.662)	(20.796)
Year-FE	Yes	Yes
Observations	88	88
R^2	0.2907	0.2928
Residual Std. Error	7.936	7.979
Note:		*p<0.1; **p<0.05; ***p<0.01

Figure F.2 & Table F.1: OLS results for the relationship between the share of indexed debt and the new net shock measure in the United Kingdom, 2000-2010. The figure plots the results for the preferred specification (2). Standard errors are robust to heteroskedasticity.