Inflation-Indexed Debt and the Risks of Fiscal Dominance*

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Abstract

The origins of recent U.S. inflation are the subject of much debate. One argument has its roots in an unfunded expansion of debt-driven government spending, in what has been labelled *fiscal dominance* (Leeper, 1991) or a *fiscally-led policy mix* (Bianchi et al., 2023). We show that the risks of such fiscal dominance depend on the degree to which government debt is indexed to inflation. Inflation-indexation has a nonlinear effect on the existence of saddlepath equilibria, and amplifies the inflationary effects of deficit shocks when policy is fiscally led. Empirical evidence links inflation-indexed debt to low central bank independence, a high probability of suspending fiscal rules, and a larger reaction of inflation to fiscal shocks.

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1 Introduction

Recent high inflation has renewed interest in the interactions between fiscal and monetary policy. The inflation experienced in the U.S. in 2021-23 coincided with a large debt-financed fiscal expansion that, if not accompanied by corresponding promises to repay the debt in equivalent real terms in the future, must lead to a depreciation in the real value of government debt through an inflation-driven debt devaluation process. The possibility of fiscal policy dominating in this way has contributed to concerns about the sustainability of government finances, especially given record absolute debt levels and debt-to-GDP ratios only previously seen in the aftermath of wars.[1]

This paper shows that the risks associated with fiscal dominance are heightened when the face value of at least some of a government's outstanding debt is automatically uprated by the gross rate of inflation, as with Treasury Inflation-Protected Securities (TIPS) in the U.S. and Index-Linked Gilts in the U.K. We find that inflation-indexed debt has a nonlinear impact on the existence of saddle path-stable equilibria, and that it can induce more volatility in inflation if there are deficit-driven government transfer shocks and policy is fiscally-led. In short, fiscal dominance is more consequential when a higher proportion of government debt is indexed to inflation.

That inflation-indexed debt affects the risks of fiscal dominance has so far not been recognised. Although a number of theoretical (Angeletos et al., 2024; Ascari et al., 2023; Bianchi et al., 2023; Cochrane, 2022b) and empirical (Barro and Bianchi, 2023; Hazell and Hobler, 2024; Hilscher et al., 2022) papers explore the recent inflationary episode using models that emphasise fiscal-monetary interactions and the extent to which debt-driven fiscal expansions can fuel inflation, they all abstract from inflation-indexed debt and so overlook the role it plays in determining the risks and consequences of fiscal dominance and a fiscally-led policy mix.[2]

We begin with a simple Fisherian environment where households save using government debt that is partially indexed to inflation. Following the analytical approach in Leeper (1991), we show that inflation-indexed debt restricts the admissible space of monetary policies that supports a fiscally-led policy mix. This occurs because inflation-indexed debt imposes an additional constraint on fiscal policy, namely that it must fund any uprating of face values required in response to inflation. We then progress to a non-Ricardian general equilibrium model in the spirit of Blanchard (1985) and Angeletos et al. (2024), where we find that inflation-indexed debt tends to amplify the reaction of inflation to debt-financed transfer shocks. The relationship is nonlinear since it flattens out and even inverts at high levels of inflation-indexed debt.

It is challenging to use empirical data to establish whether the policy mix in a particular period is monetary or fiscally-led (Cochrane, 2011; Neumeyer and Nicolini, 2025). We offer three distinct pieces of evidence to suggest that inflation-indexed debt plays a role in inflation dynamics through

[1] The increased burden of servicing debt is one area of concern. For example, interest payments on federal debt in the U.S. are expected to reach 7% of GDP by 2050.

[2] Complementary explanations for the 2021-2023 inflationary episode include broad supply shocks and bounded rationality (Benigno, 2022; Beaudry et al., 2024), shocks to commodity prices with sticky labor (Bernanke and Blanchard, 2023), and self-fulfilling inflation expectations (Acharya and Benhabib, 2024).

the risks of fiscal dominance. Firstly, using a long sample of global data, we see that the share of inflation-indexed debt correlates positively with inflation and negatively with the Central Bank Independence Index of Romelli (2024). Secondly, we use cross-country data from Davoodi et al. (2022) to show that fiscal rules are more likely to be suspended in countries that have a high proportion of government debt indexed to inflation. Thirdly, we combine the narratively-identified U.S. tax shocks of Mierzwa (2024) with the classification of policy in Chen et al. (2022) to demonstrate that amplification of fiscal shocks is only when policy is fiscally-led.

The paper is organised as follows. Section 2 introduces inflation-indexed debt in a simple Fisherian model, deriving determinacy conditions and characterising the behaviour of inflation when the policy mix is fiscally led. Section 3 deepens the analysis of inflation-indexed debt in a general equilibrium setting. The empirical evidence is in Section 5, before Section 6 concludes.

Literature Review

This paper contributes to the burgeoning literature on fiscal-monetary interactions, pioneered in Sargent and Wallace (1981) and formalized through Leeper (1991). Initial contributions focusing on the possibility of a fiscally-led policy mix include Sims (1994) and Woodford (1995). More succinct summaries of the literature are provided by Leeper and Leith (2016) and Cochrane (2023). Empirical and theoretical support for the possibility of fiscally-driven inflation has been developed in Barro and Bianchi (2023), Caramp and Silva (2021), Cochrane (2022a), Chen et al. (2022), and Cloyne et al. (2023), mirroring the interest in possible fiscal drivers in the recent inflationary episode. In particular, Bianchi et al. (2023) and Smets and Wouters (2024) extensively analyse the importance of the interactions between fiscal and monetary policy for fiscally-driven inflation in canonical general equilibrium models. Additional evidence utilizing novel model dimensions, such as monetary unions, interest rate boundaries, and game-theoretic interactions between fiscal and monetary policymakers are provided by Reichlin et al. (2023); Miao and Su (2024), and Corsetti and Maćkowiak (2024).

Closest related to us in terms of our modelling framework are Angeletos et al. (2024) and Rachel and Ravn (2025), who themselves work with a model building on the seminal contribution of Blanchard (1985), facilitating a tractable way of introducing non-Ricardian fiscal policies using simple transfer shocks, which do not directly alter the resource constraint present in the economy.[3] Angeletos et al. (2024) and Rachel and Ravn (2025) prove the conditions for existence and uniqueness of dynamic competitive equilibria in a NK-OLG model, delivering boundary conditions under which equilibria can be characterized. Rachel and Ravn (2025) deliver two particularly insightful results related to our work; first, indeterminacy becomes a smaller issue once households are non-Ricardian, i.e., adherence to the Taylor principle is not strictly necessary to deliver unique saddle-path stable equilibria even when fiscal policy is conventionally 'passive'; and second, the classic

[3]Working with government spending shocks, for instance, would alter the resource constraint in the economy. Without further assumptions on the household utility borne by such government spending, however, any such government spending would be 'wasteful'.

distinction between 'active' and 'passive' policies should not be taken literally once households are non-Ricardian. Relative to this body of work, our additional consideration of inflation-indexed debt innovates on a number of dimensions. First, inflation-indexed debt overcomes the singularity result of Angeletos et al. (2024) that RANK-FTPL and HANK models converge to the same limit point; second, inflation-indexed debt generally increases the response of inflation in response to transfer shocks; and finally, inflation-indexed debt alters one of the conclusions of Rachel and Ravn (2025), namely that the explosive region becomes more problematic under non-Ricardianness, providing an additional anchor favouring determinate equilibria.

Banerjee et al. (2022) evaluates the prevalence of fiscally-driven policy mixes empirically with a focus on a wider set of advanced economies. A narrative example of a recent fiscal shock informing inflation rates is provided by Hazell and Hobler (2024), who focus on the 2021 Georgia Senate election run-off, with similar supplementary evidence on a specific policy shock in the UK being provided by Kawalec (2025).

Albeit indirectly, we also contribute to the literature on inflation-linked government bonds. The plausibly earliest contribution in this field is Fischer (1975), who derives household demand for indexed bonds in a multi-asset optimal allocation framework. The special insurance properties of such inflation-linked debt are extensively discussed in Barr and Campbell (1997), Garcia and van Rixtel (2007), Gürkaynak et al. (2010) and Andreasen et al. (2021). Notably, Sims (2013) briefly mentions the possible detrimental consequences of indexed debt in frameworks that extensively feature fiscally-driven policy mixes. This paper expands on his remarks using simple, analytical modelling frameworks. Schmid et al. (2024) provide a systematic analysis of inflation-indexed debt as a policy tool, emphasizing its role as an ex-ante commitment device against inflation. With our contribution, we do not invalidate the conclusions of Schmid et al. (2024) about the desire for indexed bonds as plausibly overcoming a certain degree of market incompleteness, but we instead add to their results by qualifying the possibility of risks associated with the issuance of inflation-indexed debt that have henceforth been hidden from the literature.

2 Example from a Fisherian model

We illustrate how inflation-indexed debt matters in a canonical Fisherian model with a representative household saving in a government bond that is partially indexed to inflation. We do so to analyse how the presence of indexed debt can influence the equilibrium properties in dynamic economies, and provide a brief characterization of possible price level dynamics.

A representative household receives a constant stream of goods Y, maximizing its expected present value of utility from consumption, discounted at the time-invariant rate β:

$$\max_{\{c_t, b_t\}_{t=0}^{\infty}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

subject to the flow budget constraint

$$P_t c_t + q_t b_t = P_t (Y - T_t) + \pi_t^{\theta} b_{t-1}, \tag{1}$$

where b_t is an asset that is a *partially*-indexed government-issued security. The asset is indexed to the gross rate of inflation by θ, in a manner that is reflective of how the face value of indexed debt is formed empirically (Hall and Sargent, 2011). The government-issued asset is bought at the market price q_t. T_t denotes real lump-sum taxes raised by the government.

The household optimality conditions are standard:

$$\begin{aligned} \{c_t\}: &\qquad u'(c_t) = \lambda_t P_t \\ \{b_t\}: &\qquad \lambda_t q_t = \beta \, \mathrm{E}_t\big[\lambda_{t+1} \, \pi_{t+1}^{\theta}\big] \end{aligned}$$

The asset pricing equation is:

$$q_t = \beta \, \mathrm{E}_t\left[\frac{u'(c_{t+1})}{u'(c_t)} \, \pi_{t+1}^{\theta-1}\right]. \tag{2}$$

The flow budget constraint of the government is:

$$\pi_t^{\theta} b_{t-1} = P_t T_t + q_t b_t, \tag{3}$$

with the bond pricing kernel being given by the household first-order conditions and T_t denoting real surpluses raised by the government, which are equivalent to the lump-sum taxes raised.

The fiscal policy rule reacts to the debt-to-GDP ratio by adjusting tax rate τ_t:

$$\frac{\tau_t}{\tau} = \left(\frac{s_{t-1}}{s}\right)^{\gamma} e^{\varphi_t}; \qquad s_t \equiv \frac{q_t b_t}{P_t Y}, \quad \tau_t \equiv \frac{T_t}{Y},$$

where $\tau_t \equiv \frac{T_t}{Y}$ are surpluses raised by the government as a fraction of output, and $s_t \equiv \frac{q_t b_t}{P_t Y}$ is the market value of the government-issued partially-indexed bond relative to GDP. φ_t is a standard AR(1) shock to the present lump-sum tax rate (the exogenous fiscal policy disturbance in the model), and the fiscal policy reaction coefficient is γ.

The monetary policy rule targets the return $R_t = 1/q_t$ on the partially-indexed bond:

$$R_t = \frac{1}{\beta} \pi_t^{\phi}. \tag{4}$$

For completeness, we explore the model properties under a standard Taylor Rule in appendix A.

Equilibrium

Combining the household budget constraint (1) with the government budget constraint (3), consumption must be equal to the endowment:

$$c_t = Y \ \forall t,$$

and the government bond pricing kernel simplifies to:

$$q_t = \beta \, \mathrm{E}_t\big[\pi_{t+1}^{\theta-1}\big],$$

reflecting the partial indexation of the government-issued asset and the resulting dependence of its price on expected inflation. We now linearize the equilibrium conditions around a deterministic zero-inflation steady-state ($\pi = 1$), consistent with household optimality and the fiscal and monetary rules. We denote variables in their log-deviations from steady-state with hats. Log-linearization around the zero-inflation steady-state gives the following system of equations:

1. Nominal bond prices:

$$-\hat{R}_t = (\theta - 1)\, \mathrm{E}_t \hat{\pi}_{t+1}. \tag{5}$$

2. Monetary rule:

$$\hat{R}_t = \phi \hat{\pi}_t. \tag{6}$$

3. Law of motion of debt:

$$\frac{\pi_t^{\theta} b_{t-1}}{P_t Y} = \frac{P_t T_t}{P_t Y} + \frac{q_t b_t}{P_t Y} \quad \Longrightarrow \quad \pi_t^{\theta-1} s_{t-1} R_{t-1} \; = \; \tau_t + s_t.$$

Log-linearized:

$$(\theta - 1)\, \hat{\pi}_t + \hat{s}_{t-1} + \hat{R}_{t-1} = \frac{\tau}{\tau + s}\, \hat{\tau}_t + \frac{s}{\tau + s}\, \hat{s}_t.$$

$$\tau + s = sR\pi^{\theta-1}, \qquad \frac{1}{R} = \beta\pi^{\theta-1} \quad \Longrightarrow \quad \tau + s = \frac{s}{\beta}, \qquad \frac{\tau}{s} = \frac{1-\beta}{\beta}.$$

$$(\theta - 1)\, \hat{\pi}_t + \hat{s}_{t-1} + \hat{R}_{t-1} = (1 - \beta)\, \hat{\tau}_t + \beta\, \hat{s}_t. \tag{7}$$

4. Fiscal rule:

$$\hat{\tau}_t = \gamma\, \hat{s}_{t-1} + \varphi_t. \tag{8}$$

Combining (5) and (6):

$$\phi\,\hat{\pi}_t = (1-\theta)\,\mathrm{E}_t\hat{\pi}_{t+1}, \tag{9}$$

Forward (8) one period, take expectations at t and use $-\hat{R}_t = (\theta-1)\mathrm{E}_t\hat{\pi}_{t+1}$:

$$\hat{s}_t = (1-\beta)\,\mathrm{E}_t\hat{\tau}_{t+1} + \mathrm{E}_t\beta\hat{s}_{t+1}, \tag{10}$$

in which inflation does not directly appear. Substitute in for $\hat{\tau}_{t+1} = \gamma\hat{s}_t + \varphi_{t+1}$ to obtain:

$$\hat{s}_t = (1-\beta)\gamma\hat{s}_t + (1-\beta)\,\mathrm{E}_t\varphi_{t+1} + \mathrm{E}_t\beta\hat{s}_{t+1}. \tag{11}$$

The system can be written as:

$$\underbrace{\begin{pmatrix} 1-\theta & 0 \\ 0 & \beta \end{pmatrix}}_{\equiv A_0} \mathrm{E}_t \begin{pmatrix} \hat{\pi}_{t+1} \\ \hat{s}_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} \phi & 0 \\ 0 & 1-\gamma(1-\beta) \end{pmatrix}}_{\equiv A_1} \begin{pmatrix} \hat{\pi}_t \\ \hat{s}_t \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ -(1-\beta) \end{pmatrix}}_{\equiv C} \mathrm{E}_t\varphi_{t+1}. \tag{12}$$

The matrix relevant for the determinacy properties is

$$Z = A_0^{-1}A_1 = \begin{pmatrix} \dfrac{\phi}{1-\theta} & 0 \\ 0 & \dfrac{1-\gamma(1-\beta)}{\beta} \end{pmatrix}. \tag{13}$$

Its eigenvalues are $\dfrac{\phi}{1-\theta}$ and $\dfrac{1-\gamma(1-\beta)}{\beta}$. A *monetary-led* equilibrium requires $\phi > 1-\theta$ and $\gamma > 1$. A *fiscally-led* equilibrium requires $\phi < 1-\theta$ and $\gamma < 1$. The introduction of inflation-indexed debt hence matters only for the monetary, not fiscal, side of the model.

Figure 1 visualizes the determinacy properties of the model. In a nutshell, the presence of inflation-indexed debt shifts how *monetary* policy impacts the existence of a unique, saddle path-stable equilibrium. Once the share of inflation-indexed debt turns positive (right panel), the space under which a unique monetary-led equilibrium exists increases; that is, central banks can follow "more passive" monetary policies without running into the indeterminate (yellow) space. At the same time, a fiscal authority wishing to engage in activist fiscal policy without negative repercussions on the stability properties of the system must be aware that this requires an even more 'passive' monetary authority relative to conventional wisdom.

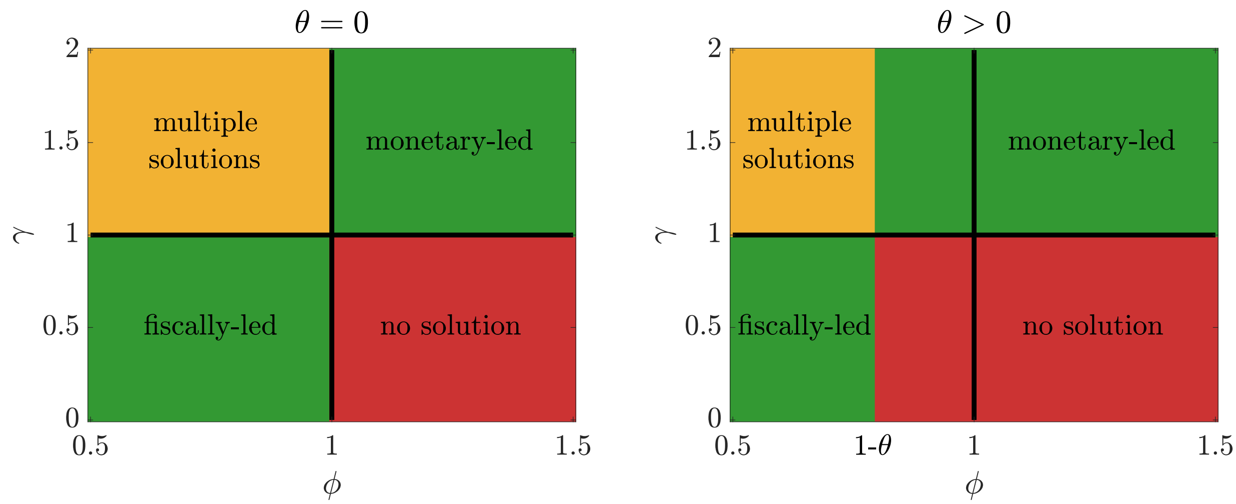

Figure 1: Determinacy properties of the Fisherian model.

The intuition behind this phenomenon is that indexed debt works as an 'automatic stabilizer': as the central bank targets the return rate of the partially-indexed bond, any change it induces on its net return rate has *more* stabilizing effects relative to a canonical Taylor Rule, as a presumed increase in the net policy rate directly decreases the expected future inflationary pressure, thereby stabilizing the system beyond the simple first-order effect of the nominal policy rate adjustment.

To derive debt-inflation dynamics under the fiscally-led policy mix, the block-recursivity of the model means we need to solve the debt equation forward.[4] The debt equation is:

$$(1-(1-\beta)\gamma)\,\hat{s}_t = (1-\beta)\,\mathrm{E}_t\varphi_{t+1} + \mathrm{E}_t\beta\,\hat{s}_{t+1}.$$

so solving forward implies:

$$\hat{s}_t = \sum_{i=0}^{\infty}\left(\frac{\beta}{1-(1-\beta)\gamma}\right)^{i+1}\left(\frac{1-\beta}{1-(1-\beta)\gamma}\right)\mathrm{E}_t\varphi_{t+i+1}.$$

When φ_t is an AR(1) process with persistence ρ then $\mathrm{E}_t\varphi_{t+i+1} = \rho^{i+1}\varphi_t)$ and:

[4]The solution for the monetary-led policy mix trivially features $\pi_t = 0\,\forall t$, as otherwise inflation would be explosive. The value of debt under a monetary-led policy mix evolves as $\hat{s}_t = \frac{1-(1-\beta)\gamma}{\beta}\hat{s}_{t-1} - \varphi_t$.

$$
\begin{aligned}
\hat{s}_t &= \sum_{i=0}^{\infty}\left(\frac{\beta\rho}{1-(1-\beta)\gamma}\right)^{i+1}\left(\frac{1-\beta}{1-(1-\beta)\gamma}\right)\varphi_t \\
&= \left(\frac{\beta\rho}{1-(1-\beta)\gamma}\right)\frac{1}{1-\dfrac{\beta\rho}{1-(1-\beta)\gamma}}\left(\frac{1-\beta}{1-(1-\beta)\gamma}\right)\varphi_t \\
&= \left(\frac{\beta\rho}{1-(1-\beta)\gamma-\beta\rho}\right)\left(\frac{1-\beta}{1-(1-\beta)\gamma}\right)\varphi_t,
\end{aligned}
$$

which is independent of θ.

We solve for inflation dynamics in equilibrium by taking the linearized budget constraint and substituting in for the monetary policy rule to obtain:

$$
\hat{\pi}_t = \frac{\phi}{1-\theta}\hat{\pi}_{t-1} - \frac{\beta}{1-\theta}\hat{s}_t + \frac{1-(1-\beta)\gamma}{1-\theta}\hat{s}_{t-1} - \frac{1-\beta}{1-\theta}\varphi_t. \tag{14}
$$

Inflation-indexed debt here has the expected effect of scaling *surprise* inflation: When $\mathrm{E}_{t-1}\hat{\pi}_t \neq \hat{\pi}_t$, the fact that the equilibrium price for indexed debt had been *too low* in period $t-1$ induces additional surprise inflation in period t.

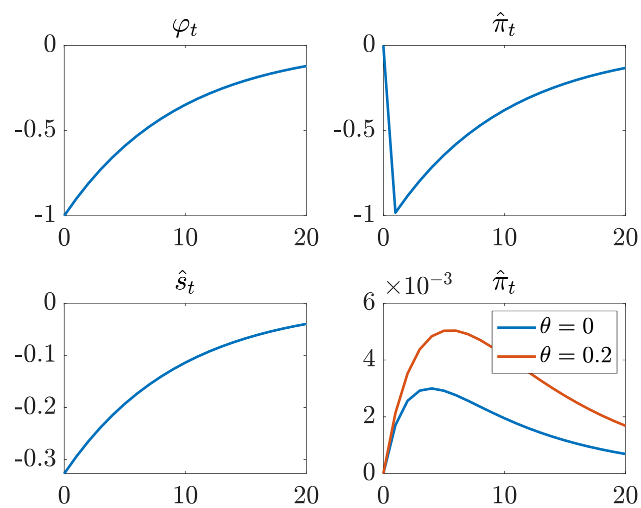

Figure 2: Dynamics of fiscally-led equilibrium in Ricardian model.

Figure 2 plots the equilibrium paths of the tax rate, the debt-to-GDP ratio, and the rate of inflation in response to a deficit shock φ_t for two levels of inflation-indexed debt: $\theta = 0$ and $\theta = 0.2$. As expected, the reaction of the tax rate and the debt-to-GDP ratio are independent of the share of inflation-indexed debt. The key difference lies in the observed rate of inflation: inflation-indexed debt causes additional loading on inflation under a fiscally-led policy mix. Higher equilibrium inflation is needed to ensure sufficient devaluation of the government debt stock, since only the *non-indexed* share of the debt stock can be devalued. The inflation that follows a deficit shock ias increasing in the share of inflation-indexed debt when the fiscal authority does *not* promise a complete repayment of borrowing.

In sum, inflation-indexed debt in the simple model makes the policy space qualifying as a 'fiscally-led policy mix' smaller, but the consequences of being within that space become more detrimental: the inflationary pressure arising from an expansionary deficit shock is increasing in the share of outstanding inflation-indexed debt.

3 A non-Ricardian General Equilibrium Model with Indexed Debt

We proceed with a richer, albeit tractable, general equilibrium framework based on Angeletos et al. (2024), who develop an OLG-NK model in the spirit of Blanchard (1985).[5] They introduce a mortality friction that breaks Ricardian equivalence and can be considered as a general proxy for the liquidity risk in canonical heterogeneous-agent models. It allows for the analysis of purely distributive *transfer* shocks with relevant real and nominal effects, facilitating a clean analysis of the effects of inflation-indexed debt.

3.1 Model framework

As in the previous section, government bonds D_t are partially inflation-indexed, and can heuristically be decomposed into constant proportions of non-indexed and indexed debt:

$$D_t \equiv (1-\theta)B_t + \theta\tilde{B}_t, \tag{15}$$

where θ is the time-invariant share of inflation-indexed debt. Inflation is zero in steady-state, with debt d_t, taxes t_t and household assets a_t measured in absolute deviations to allow for the possibility of zero-debt steady-states.

3.1.1 Households

The probability of the household surviving (i.e., the inverse of the mortality risk) from one period to another is $\omega \in (0,1]$. Households are replaced whenever they die, keeping overall population levels constant. Household i maximizes the present discounted value of its utility:

$$\mathrm{E}_t \sum_{t=0}^{\infty} (\beta\omega)^k \left(\frac{C_{i,t+k}^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} - \nu \frac{L_{i,t+k}^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}} \right), \tag{16}$$

where β is a common discount factor, ν scales the disutility of labour supply L_{it}, σ is the elasticity of intertemporal substitution and φ is the Frisch elasticity of labour supply.

[5] Rachel and Ravn (2025) work in a similar framework.

Households trade the partially-indexed government bond, D_t, earning a nominal rate of return R_t^D/ω if the household survives. In effect, there is a risk-free return of R_t^n/ω on the fraction $1-\theta$ of debt that is not indexed and a return of $\frac{R_t^n}{\omega}\left(\frac{P_{t+1}}{P_t}\right)$ on the fraction θ that is indexed:

$$R_t^D = (1-\theta)R_t^n + \theta R_t^n\left(\frac{P_{t+1}}{P_t}\right) = R_t^n\left(1+\theta\left(\frac{P_{t+1}}{P_t}-1\right)\right). \tag{17}$$

The budget constraint follows Angeletos et al. (2024), in that all households receive labour income $W_t L_{it}$ and dividends X_{it}, and pay lump-sum taxes T_{it}. Existing households make a contribution S_{it} to a social fund whose proceeds are distributed to newborn households, eliminating wealth effects from mortality risk.[6] The household-specific budget constraint is:

$$P_{t+1}A_{i,t+1} = \frac{R_t^D}{\omega}P_t\left(A_{i,t} + \underbrace{Y_{i,t}}_{\equiv W_t L_{i,t}+X_{i,t}} - C_{i,t} - T_{i,t} + S_{i,t}\right). \tag{18}$$

We retain all other household-side assumptions from Angeletos et al. (2024). Dividends are identical across households i, labour supply is intermediated by unions to obtain $L_{it} = L_t$, and income and taxes faced by households are equalized. Taking expectations and defining the *ex ante* real interest rate through the Fisher equation as $R_t = R_t^n \mathrm{E}_t P_t/P_{t+1}$, the first-order conditions are:

$$\{C_t\}: \qquad C_t^{-\frac{1}{\sigma}} - \lambda_t\frac{R_t}{\omega}(1+\theta\mathrm{E}_t(\Pi_{t+1}-1)) = 0, \tag{19a}$$

$$\{L_t\}: \qquad -\nu L_t^{\frac{1}{\varphi}} + \lambda_t\frac{R_t}{\omega}(1+\mathrm{E}_t\theta(\Pi_{t+1}-1))\frac{\partial Y_t}{\partial L_t} = 0, \tag{19b}$$

$$\{A_{t+1}\}: \qquad -\lambda_t + \beta\omega\mathrm{E}_t\left(\lambda_{t+1}\frac{R_{t+1}}{\omega}(1+\theta\mathrm{E}_{t+1}(\Pi_{t+2}-1))\right) = 0, \tag{19c}$$

which implies the Euler equation for consumption:

$$\frac{C_t^{-\frac{1}{\sigma}}}{R_t(1+\mathrm{E}_t\theta(\Pi_{t+1}-1)} = \beta\mathrm{E}_t C_{t+1}^{-\frac{1}{\sigma}}. \tag{20}$$

The log-linearized Euler equation is:

$$c_t = -\sigma(r_t + \theta\mathrm{E}_t\pi_{t+1}) + \mathrm{E}_t c_{t+1}. \tag{21}$$

The present value budget constraint is obtained from the Euler equation and per-period budget

[6] The transfers are $S_{it} = S^{new} = D^{SS} \geq 0$ and $S_{it} = S^{old} = -\frac{1-\omega}{\omega}D^{SS} \leq 0$, such that $(1-\omega)S^{new} + \omega S^{old} = 0$.

constraint. As in Angeletos et al. (2024), it is a generalization of the Permanent Income Hypothesis:

$$
\begin{aligned}
c_t =&(1-\beta\omega)\left(a_t + \mathrm{E}_t \sum_{s=0}^{\infty}(\beta\omega)^s(y_{t+s}-t_{t+s})\right) \\
&-\beta\left(\sigma\omega-(1-\beta\omega)\frac{A^{SS}}{Y^{SS}}\right)\mathrm{E}_t\left[\sum_{s=0}^{\infty}(\beta\omega)^s(r_{t+s}+\theta\pi_{t+1+s})\right].
\end{aligned} \tag{22}
$$

a_t is net asset holdings of households and there is an additional inflation adjustment in the last term. This is the novelty relative to Angeletos et al. (2024), capturing income from inflation for holders of inflation-indexed debt.

3.1.2 Aggregate supply

The supply side of the model is standard and follows the canonical New Keynesian structure. The New Keynesian Phillips Curve is due to pricing frictions in the firm's problem:

$$\pi_t = \kappa y_t + \beta \mathrm{E}_t \pi_{t+1}, \tag{23}$$

which iterated forward gives inflation as a function of current and future output gaps:

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \mathrm{E}_t y_{t+k}. \tag{24}$$

3.1.3 Government

The government budget constraint is:

$$D_{t+1} = R_t^D(D_t - P_t T_t),$$

where D_t is the stock of government debt and $R_t^D = R_t^n(1+\theta(\Pi_{t+1}-1))$ is the *ex post* return the government has to pay. The return is defined in terms of the *realised* rate of inflation, from t to $t+1$. Dividing both sides by P_t and denoting the real value of government debt by $D_t^R = D_t/P_t$,[7] a linear approximation around a zero-inflation steady-state yields:

$$(D_{t+1}^R - D^{R,SS}) + D^{R,SS}(\Pi_{t+1}-1) = R^{D,SS}[(D_t^{R,SS} - D^{R,SS}) - (T_t - T^{SS})] + (D^{R,SS} - T^{SS})(R_t^D - R^{D,SS}).$$

Dividing both sides by steady-state output Y^{SS} and defining $d_t = (D_t^R - D^{R,SS})/Y^{SS}$ for debt and $t_t = (T_t^R - T^{SS})/Y^{SS}$ for taxes, the linearized budget constraint becomes:

$$d_{t+1} + \frac{D^{SS}}{Y^{SS}}\pi_{t+1} = \frac{1}{\beta}(d_t - t_t) + \frac{D^{SS}}{Y^{SS}} r_t^D,$$

[7] The resulting government budget constraint is $D_{t+1}^R \Pi_{t+1} = R_t^D(D_t^R - T_t)$.

where we simplify using the steady-state nominal return $R^{D,SS} = 1/\beta$ and the steady-state government budget constraint $D^{SS} = (D^{SS} - T^{SS})/\beta$.

The *ex post* return r_t^D is related to the risk-free nominal return r_t^n and *realised* inflation π_{t+1} through the inflation-indexation of the government bond, with the risk-free nominal return r_t^n itself related to the *ex ante* real return r_t and *expected* inflation $\mathrm{E}_t\pi_{t+1}$ through the Fisher equation. As approximate log-deviations from steady state, the returns satisfy:

$$\begin{aligned} r_t^D &= r_t^n + \theta\pi_{t+1}, \\ r_t &= r_t^n - \mathrm{E}_t\pi_{t+1}, \\ r_t^D &= r_t + \mathrm{E}_t\pi_{t+1} + \theta\pi_{t+1}. \end{aligned}$$

The final expression for the government budget constraint is:

$$d_{t+1} = \frac{1}{\beta}(d_t - t_t) + \frac{D^{SS}}{Y^{SS}} r_t - \frac{D^{SS}}{Y^{SS}}\left((1-\theta)\pi_{t+1} - \mathbb{E}_t\pi_{t+1}\right). \tag{25}$$

The novelty here is the multiplication of *realized* future inflation by $(1-\theta)$, which is the share of non-indexed debt. Intuitively, inflation-indexed debt cannot be devalued through surprise inflation. The ability of governments to devalue their debt in real terms is therefore constrained by indexed debt, with implications for the evolution of the debt stock in the presence of inflation.[8]

The steady-state of the model requires $x_{-1} = 0 \ \forall x \in \{d, t, r, y, \pi\}$. Starting from the steady state, equation (25) defines the initial change in the debt stock as a function of surprise inflation:

$$d_0 = -\frac{D^{SS}}{Y^{SS}}(1-\theta)\pi_0,$$

and the presence of inflation-indexed debt generally reduces the initial reaction of the value of the debt stock to inflationary pressure.

We close the model with the monetary and fiscal and policy rules in Angeletos et al. (2024):

$$\begin{aligned} r_t &= \phi y_t, & (26)\\ t_t &= -\varepsilon_t + \tau_d(d_t + \varepsilon_t) + \tau_y y_t. & (27) \end{aligned}$$

3.2 Aggregate demand

Aggregate demand is derived by imposing market clearing conditions $a_t = d_t$ and $y_t = c_t$ and substituting the monetary policy rule (26) into the intertemporal budget constraint (22):

[8] We furthermore impose the no-Ponzi condition of Angeletos et al. (2024), i.e., $\mathbb{E}_t \lim_{T\to\infty} \beta^T d_{t+T} = 0$, which arises from rational optimizing behaviour on behalf of the households.

$$y_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\omega)^s \left[\left(1 - \frac{\Gamma\phi}{1-\beta\omega}\right) y_{t+s} - t_{t+s} - \frac{\Gamma\theta}{1-\beta\omega}\pi_{t+1+s}\right]\right), \tag{28}$$

where $\Gamma \equiv \beta\left(\sigma\omega - (1-\beta\omega)\frac{D^{SS}}{Y^{SS}}\right)$.

In recursive form, the aggregate demand relationship simplifies to:

$$\mathrm{E}_t y_{t+1} - \frac{\Gamma\theta}{\beta\omega}\mathrm{E}_t\pi_{t+1} - (1-\beta\omega)\mathrm{E}_t d_{t+1} = \frac{\beta\omega(1-\tau_y) + \Gamma\phi + \tau_y}{\beta\omega} y_t - \frac{(1-\beta\omega)(1-\tau_d)}{\beta\omega}(d_t + \varepsilon_t), \tag{29}$$

where taxes have been substituted out using fiscal policy rule (27).

Equation (29) generalizes the aggregate demand equation to account for inflation-indexed debt. Through θ, inflation indexation weakens the relationship between current output and future inflation. Together with the New Keynesian Phillips Curve (23) and the government debt valuation equation (25), it gives a full characterization of the model's dynamics.

3.3 Equilibrium

Definition 1 *A competitive equilibrium is a series $\{c_t, y_t, \pi_t, a_t, d_t, t_t, r_t\}_{t=0}^{\infty}$ that satisfies the aggregate demand function, the New Keynesian Phillips Curve, market clearing, the government's flow budget constraint, the no-Ponzi condition, and the monetary and fiscal policy rules.*

3.4 Without mortality risk

We begin by abstaining from mortality risk and setting $\omega = 1$. This representative agent New Keynesian (RANK) environment retains the fundamental block recursivity of the system, whereby the dynamics of output and inflation are independent of the dynamics of debt. To see this, substitute the government budget constraint (25) into the aggregate demand equation (29) under $\omega = 1$:

$$\mathrm{E}_t y_{t+1} - \theta\sigma\frac{D^{SS}}{Y^{SS}}\mathrm{E}_t\pi_{t+1} = (1+\sigma\phi)y_t, \tag{30}$$

The output-inflation block in the absence of mortality risk is therefore independent of the evolution of government debt, even with indexed debt (Kaplan, 2025; Rachel and Ravn, 2025). This mirrors the canonical formulations of RANK models, except for the adjustment for indexed debt. The above equation combines with the New Keynesian Phillips Curve (23) and the government budget constraint (25) to define the first-order system:

$$\mathbb{E}_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ d_{t+1} \end{bmatrix} = \begin{bmatrix} 1 + \phi\sigma - \frac{D^{SS}}{Y^{SS}}\frac{\kappa\sigma\theta}{\beta} & \frac{D^{SS}}{Y^{SS}}\frac{\sigma\theta}{\beta} & 0 \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta} - \frac{D^{SS}}{Y^{SS}}\frac{\kappa\theta}{\beta} & \frac{D^{SS}}{Y^{SS}}\frac{\theta}{\beta} & \frac{1}{\beta}(1-\tau_d) \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ d_t + \varepsilon_t \end{bmatrix}. \tag{31}$$

The system is block recursive for all $\theta \geq 0$, so inflation indexation of debt in itself does not overcome the result that government debt is block-exogenous with respect to output and inflation in RANK models (Leeper, 1991; Angeletos et al., 2024). The policy space that supports a unique saddle-path stable equilibrium will therefore still split into areas conventionally labelled 'active" and 'passive' with respect to the actions of each policymaker.

The system has one predetermined variable d_t and two forward looking variables (y_t, π_t), so determinancy requires one eigenvalue inside and two eigenvalues outside the unit circle. In the fiscally-led equilibrium, the eigenvalue $\beta^{-1}(1-\tau_d)$ associated with the government budget constraint must be outside the unit circle, which requires $\tau_d < 1-\beta$. The conditions under which the other two eigenvalues straddle the unit circles in the fiscally-led equilibrium are as follows:

Proposition 1 *To ensure a unique saddlepath equilibrium when $\omega = 1$ under a fiscally-led policy mix $\tau_d < 1-\beta$, the restriction on ϕ, the monetary policy parameter, tightens with the degree of inflation-indexed debt. The formal restriction for a fiscally-led equilibrium in the absence of mortality risk is:*

$$\max\left\{-\frac{1}{\sigma},\ -\frac{1}{\sigma}\left[2-\frac{\frac{D^{SS}}{Y^{SS}}\kappa\sigma\theta}{1+\beta}\right]\right\} < \phi < -\frac{\frac{D^{SS}}{Y^{SS}}\kappa\theta}{1-\beta}. \tag{32}$$

Proof. See appendix B. ■

The conditions on the monetary policy parameter ϕ are tighter for determinacy in the fiscally-led regime with a positive share of inflation-indexed debt. A larger share θ of indexed debt, steeper Phillips curve (higher κ), or a higher steady-state debt-to-GDP ratio $\frac{D^{SS}}{Y^{SS}}$, all push the right-hand bound in (32) leftward, so increased monetary passivity is needed to sustain fiscal dominance. Otherwise, there are multiple equilibria with self-fulfilling fluctuations. The bounds on ϕ for monetary dominance can also tighten when the share of indexed debt is sufficiently high.[9] Figure 3 depicts Proposition 1, without indexed debt in the left panel and with indexed debt ($\theta = 0.2$) in the right panel. The affected combinations for a unique monetary-led equilibrium are not restricted by the presence of inflation-indexed debt (the top-right green area stays the same in the left and right panels), but any fiscally-led equilibrium can only be sustained by increased degrees of monetary passivity (the bottom-left green area gets smaller).

[9] See Appendix C for technical details.

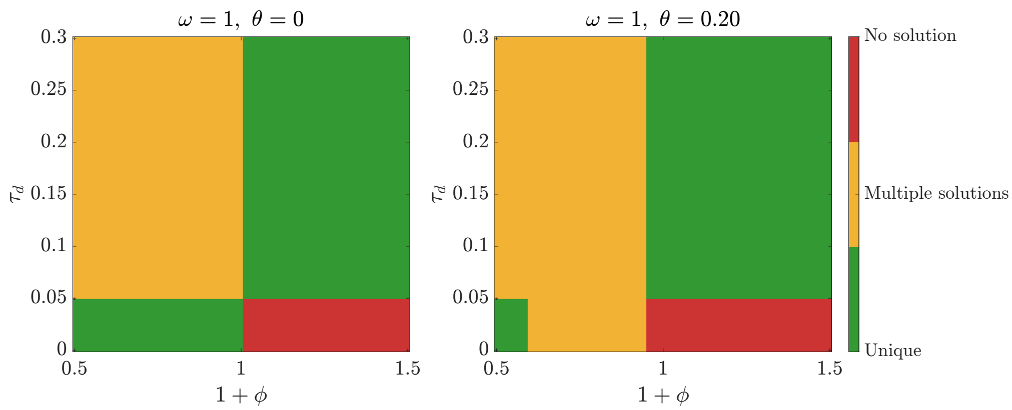

Figure 3: The determinacy properties of the RANK model.

3.5 With mortality risk

The first-order system for the joint dynamics of $\{y_t, \pi_t, d_t\}$ with $\omega \neq 1$ is:

$$\begin{pmatrix} 1 & -\frac{\Gamma\theta}{\beta\omega} & -(1-\beta\omega) \\ 0 & 1 & 0 \\ 0 & -\theta\frac{D^{SS}}{Y^{SS}} & 1 \end{pmatrix} \mathrm{E}_t \begin{pmatrix} y_{t+1} \\ \pi_{t+1} \\ d_{t+1} \end{pmatrix} = \begin{pmatrix} \frac{\beta\omega(1-\tau_y)+\Gamma\phi+\tau_y}{\beta\omega} & 0 & -\frac{(1-\beta\omega)(1-\tau_d)}{\beta\omega} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta} & 0 & \frac{1}{\beta}(1-\tau_d) \end{pmatrix} \begin{pmatrix} y_t \\ \pi_t \\ d_t + \varepsilon_t \end{pmatrix}. \tag{33}$$

Multiplying by the inverse of the left-hand matrix defines a system in the usual form:

$$\mathrm{E}_t \begin{pmatrix} y_{t+1} \\ \pi_{t+1} \\ d_{t+1} \end{pmatrix} = A \begin{pmatrix} y_t \\ \pi_t \\ d_t + \varepsilon_t \end{pmatrix}. \tag{34}$$

This characteristic polynomial of A is relatively complex, precluding an *exact* parametric characterization of the conditions for saddlepath stable equilibria. Nonetheless, it is possible to provide an approximate characterization of the policy space. The details are in Appendix D.

Proposition 2 *The feasible region for a unique saddle path-stable equilibrium is constrained by a band $\phi^-(\tau_d;\theta) < \phi < \phi^+(\tau_d;\theta)$ when $\tau_d^0 > \tau_d > \tau_d^*$. The band shifts with θ, with higher levels of θ making it less likely that a unique saddlepath equilibrium exists.*

Proof. See appendix D. ■

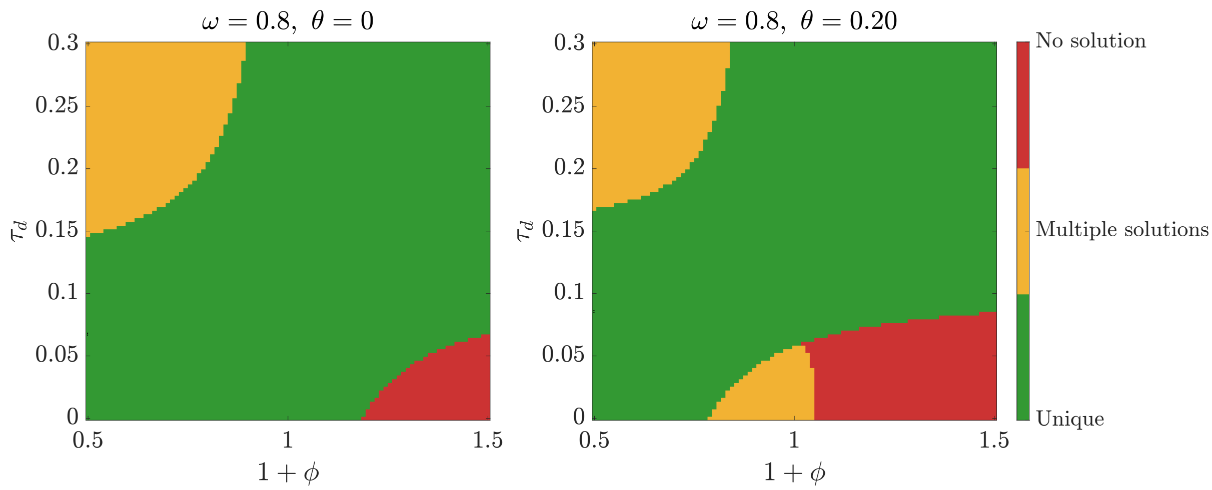

Figure 4: Visualization of Proposition 2.

Figure 4 visualizes Proposition 2, showing parameterizations of fiscal policy τ_d and monetary policy $1 + \phi$ for which the first-order system with mortality risk has a unique saddlepath solution. The left panel depicts the policy space without inflation-indexed debt, replicating the result in Angeletos et al. (2024) and Rachel and Ravn (2025) that mortality risk supports a unique equilibrium for a continuous policy space with no clear distinction between fiscally and monetary led equilibria. The right panel of Figure 4 depicts the same system when inflation-indexed debt is 20% of the overall government debt stock.[10] In line with Proposition 2, the green area shrinks at low values of τ_d and a lower monetary policy parameter $1+\phi$ is needed to support a unique saddlepath equilibrium. In contrast, the green area expands slightly for high values of τ_d and the range of monetary policy parameters that supports equilibrium is larger with inflation-indexed debt.

What is the intuition behind this result? Inflation-indexed debt can be thought of as a type of *automatic stabilizer* inherent to the conduct of fiscal policy. For a given expansionary, deficit-financed fiscal measure, for instance, the presence of inflation-indexed debt should stabilize the value of that part of the government debt stock, as such debt retains its real value from the perspective of the household. However, when a fiscal policy authority commits to being *very* expansionary (i.e., committing to $\tau_d \searrow 0$), we run into a scenario where self-fulfilling fluctuations are possible if the *monetary* authority in turn engages in interest rate policies by which real interest rates remain approximately stable.

To see this, start from considering the fiscal policy authority. When it reacts insufficiently by appropriate taxation to changes of the real value of debt from equilibrium, it is possible that the raised additional taxes are *insufficient* to serve the additional face value of maturing inflation-indexed debt in response to an expansionary fiscal shock. When the monetary authority intends to keep real rates at least approximately stable, this problem compounds for the fiscal authority as the overall debt service cost also does not decrease. Through the combination of the presence of indexed

[10] This is above the share of TIPS in the stock of US Federal Debt, which is about 7.5%, but below the share of inflation-linked Gilts in the stock of UK Government debt, which is around 25%.

debt, very expansionary fiscal policy, and a monetary authority that intends to 'fight back' against inflationary pressure, it is then possible for self-fulfilling fluctuations to materialize.

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Under such a rich system, characterizing inflation analytically becomes difficult. We are nonetheless interested in the effect of inflation-indexed debt, and therefore perform the following experiment: for the entirety of the (relevant) policy space, we calculate the one-year change in the price level in response to a 1% deficit-to-GDP shock under two scenarios: once setting the share of inflation-indexed debt to 20% ($\theta = 0.2$), and once without any inflation-indexed debt ($\theta = 0$). Then, we take the difference of the two observed inflation rates, allowing us to characterize the impact of inflation-indexed debt on price level dynamics in the model. Figure 5 shows the results of this exercise, varying the core fiscal reaction parameter τ_d on the y-axis and the monetary policy parameter ϕ on the x-axis.

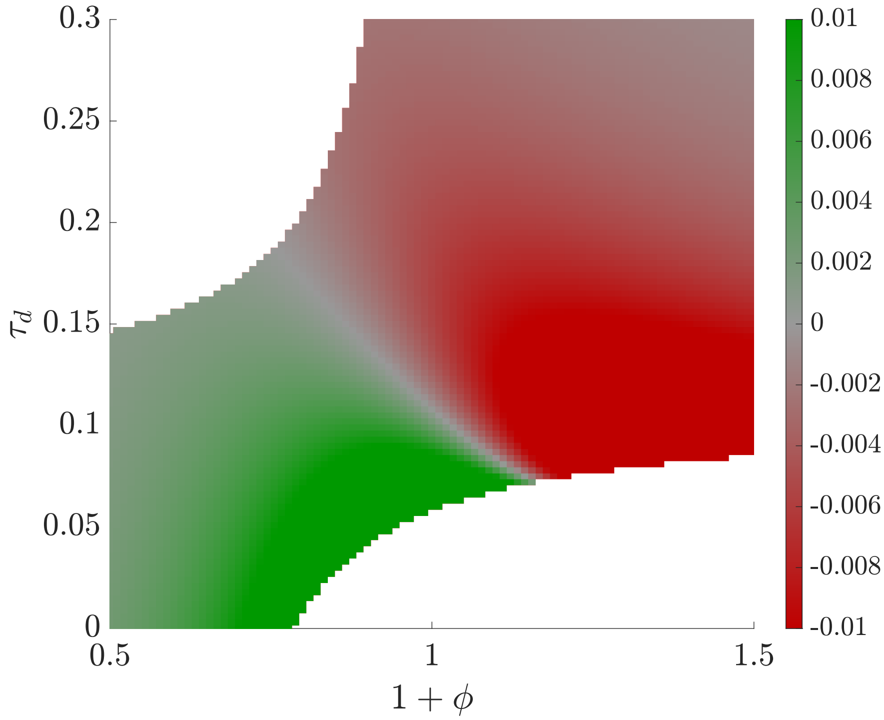


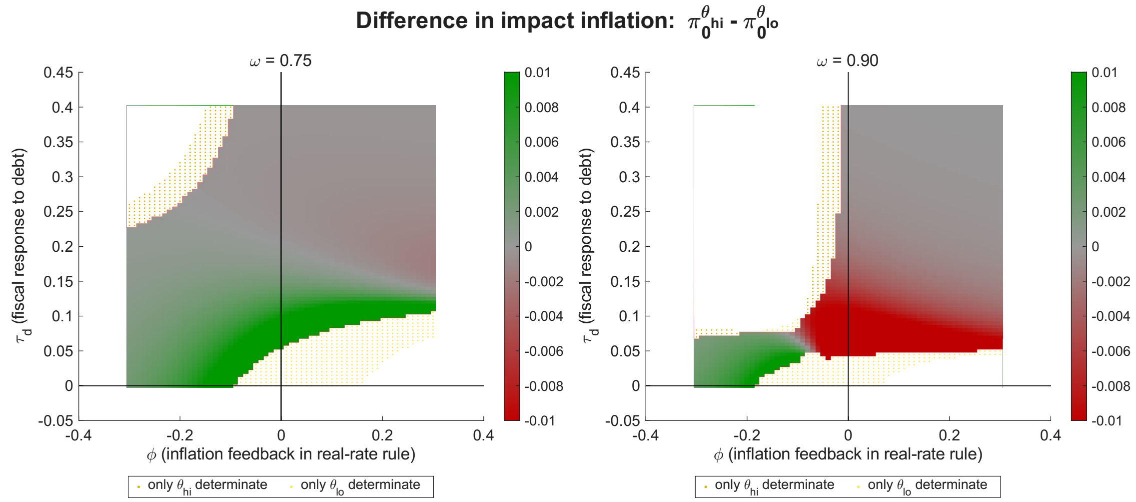
Figure 5: One-year inflation rates in response to a 1% deficit-to-GDP shock in dependence on the policy parameters.

While the clear-cut distinction between fiscally-led and monetary-led policy mixes has become seemingly difficult under the presence of mortality risk, as clear from figure 4 and the interpretation in Rachel and Ravn (2025), figure 5 shows a very interesting novel result that arises due to the presence of inflation-indexed debt: we have one clear area in which inflation-indexed debt has a net *positive* impact on inflation, and one clear area in which it has a net *negative* impact on inflation, i.e., an area where inflation-indexed debt can be *dis*inflationary.[11] Low values of τ_d and ϕ are increasing

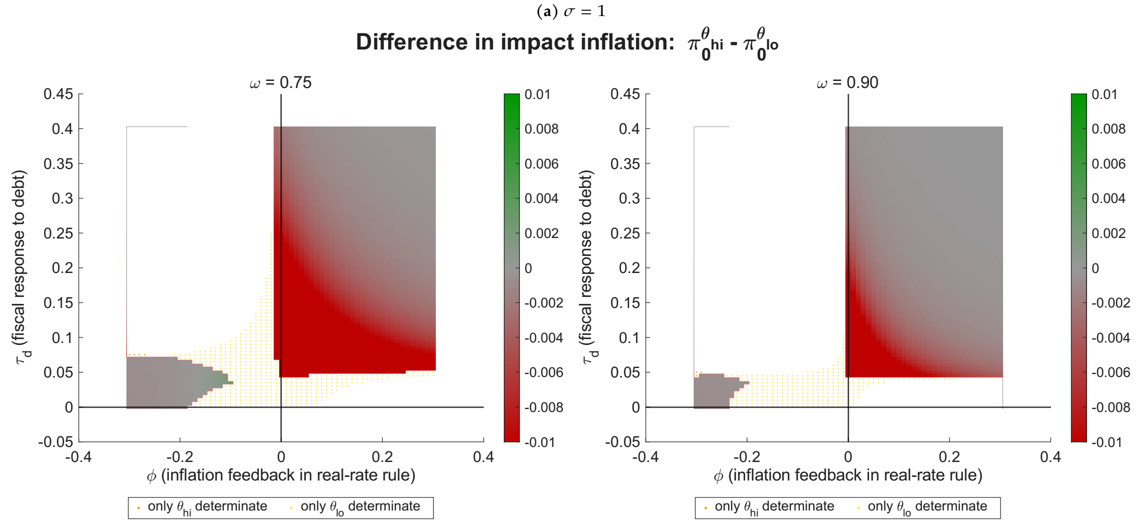
[11] This was not the case in the RANK model, where inflation-indexed debt was either inflationary (under a fiscally-led policy mix), or had no impact on the unique solution $\pi_t = 0$ (in the monetary-led policy mix).

the propensity of indexed debt to be inflationary, which can therefore be seen as a tool in helping measure approximate fiscal dominance, even when no clearly distinct policy areas persist.

How is it possible for inflation-indexed debt to induce slight *dis*inflationary pressure in response to a deficit shock? This holds particularly true for high values of ϕ, i.e., for monetary reaction parameters that induce a real rate increase in equilibrium. In such circumstances, the real value of inflation-indexed debt is *increasing* once a deficit shock materializes through the promise of the central bank to crush additional inflationary pressure arising from the deficit shock, yieding a surprise *positive* revaluation of the outstanding indexed debt stock. Then, the overall value of the stock of government debt is increasing through the presence of inflation-indexed debt, yielding some disinflationary pressure in equilibrium. The presented response of equilibrium inflation in dependence on the share of inflation-indexed debt also rationalizes the result in figure 4 to some extent, as the overall volatility of the price level is clearly most pronounced through the presence of indexed debt for low values of τ_d, irrespective of the monetary reaction parameter, which mostly influences the *direction* of the observed price level volatility.



(a) $\sigma = 1$



(b) $\sigma = 6$

Figure 6: The effect of risk aversion on the model, contrasting the cases of $\sigma = 1$ and $\sigma = 6$.

Relative to proposition 2, figure 6 sheds additional light on the effect of risk aversion. In particular, the relatively even-sided effect on determinacy properties caused by the presence of indexed debt (shifting the determinate space generally upwards) becomes increasingly *one-sided* as risk aversion goes up: for higher levels of risk aversion, inflation-indexed debt still reduces the feasible policy space for activist fiscal policy (restricting the number of monetary reaction parameters for which any given value of τ_d yields a unique saddle path-stable equilibrium); however, it does *not* increase the policy space under relatively passive fiscal policy (in the plot, the ratio of orange dots is virtually zero). Therefore, for high levels of risk aversion, inflation-indexed debt is generally only reducing the feasible policy space by eliminating some fiscal-monetary policy combinations.

4 Alternative policy rules

In the previous section, we restricted ourselves to relatively simple policy rules that were broadly in line with Angeletos et al. (2024) and Rachel and Ravn (2025). However, it is logical for the presence of inflation-indexed debt to alter the conduct of policymakers: both monetary and fiscal policy might anticipate some of the effects of indexed debt explored in section 2, internalizing them in their own policymaking. We here explore such alternative policy rules, showing how the shift in the determinate space can be unwound, or even inverted, with appropriate policy rules.

In section 2, we specified the monetary rule as equation (26) and the fiscal rule as equation (27). However, the presence of inflation-indexed debt can plausibly be accounted for by either policy authority, especially in the light of the possibly detrimental consequences in terms of equilibrium (in)existence or (dis)inflationary pressure.

It is possible to create a myriad of such policy specifications. Here, we focus on one particular instance, comprised of a monetary authority that aims to mitigate the impact of inflation-indexed debt on the aggregate demand equation, paired with a fiscal authority that is aware of the additional cost of serving such indexed debt upon the realization of a positive inflationary shock.

The alternative monetary rule: following equation (22), inflation-indexed debt manifests in the household policy function by inducing a direct wedge on equilibrium real interest rates. A monetary authority tasked with providing broad economic stabilization might then prefer to nullify this effect to the degree possible within its mandate. In this case, the central bank can simply follow a real rate rule that internalizes the additional pressure on the portfolio-adjusted real rate earned by the household; formally:

$$r_t = \phi y_t - \theta \mathbb{E}_t \pi_{t+1} \tag{35}$$

after log-linearization. The novel term is given by $-\theta \mathbb{E}_t \pi_{t+1}$. Under such a monetary rule, the policy rate set is effectively a *portfolio-adjusted real rate*, which acts as a stabilizing tool on the aggregate demand side in the presence of unexpected inflationary shocks.

Fiscal policy, in turn, can try to internalize anticipated expenditures arising from the presence of such debt. In that case, fiscal policy can simply be specified to follow:

$$t_t = -\varepsilon_t + \tau_d(d_t + \varepsilon_t) + \tau_y y_t + \beta \frac{D^{SS}}{Y^{SS}} \theta \tau_\pi \mathbb{E}_t \pi_{t+1} \tag{36}$$

after log-linearization. Here, the novel term in the rule is $\beta \frac{D^{SS}}{Y^{SS}} \theta \tau_\pi \mathbb{E}_t \pi_{t+1}$, with the novel policy parameter τ_π. The terms $\beta \frac{D^{SS}}{Y^{SS}} \theta$ weight this novel reaction parameter in line with intertemporal discounting and the overall outstanding indexed debt stock. Factually, such a fiscal rule ensures that a fiscal policy authority expecting increased outlays for inflation-indexed debt attempts to raise additional funds through taxation to cover such expenses. Conditional on the return rates for either assets being equivalent in real terms, *expected* inflationary pressure would then even yield a windfall gain on behalf of the fiscal authority.

With these two rules, the aggregate demand equation (29) changes to the following specification:

$$\mathbb{E}_t y_{t+1} - \tau_\pi \frac{D^{SS}}{Y^{SS}} \theta \mathbb{E}_t \pi_{t+1} - (1-\beta\omega)\mathbb{E}_t d_{t+1} = \frac{\beta\omega(1-\tau_y) + \Gamma\phi + \tau_y}{\beta\omega} y_t - \frac{(1-\beta\omega)(1-\tau_d)}{\beta\omega}(d_t + \varepsilon_t). \tag{37}$$

Future inflation continues to matter for household aggregate demand, but now only through the distortionary taxation induced by the fiscal authority in proportion to its anticipated additional cost of serving maturing indexed debt. The initial 'base' effect through a wealth channel induced by a mismatch between the real interest rate set by the central bank and the factual portfolio return has been nullified.

The third equilibrium condition, the intertemporal debt valuation equation (initially given by equation (25)), similarly changes to the following expression after taking expectations and inserting our novel tax rule:

$$\mathbb{E}_t d_{t+1} + \tau_\pi \theta \frac{D^{SS}}{Y^{SS}} \mathbb{E}_t \pi_{t+1} = \left(\frac{D^{SS}}{Y^{SS}} \phi - \frac{\tau_y}{\beta} \right) y_t + \frac{1-\tau_d}{\beta}(d_t + \varepsilon_t). \tag{38}$$

The wedge term arising from the additional cost of serving indexed debt has vanished due to the adjustment of the monetary rule. Additionally, expected inflationary pressure can have a net positive impact on fiscal balance sheet through the taxation adjustment.

Summarizing equations (37) and (38), together with the NKPC (23), gives us the following 3×3 first-difference system:

$$\mathbb{E}_t \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ d_{t+1} \end{bmatrix} = \begin{bmatrix} a_{11} & \frac{D^{SS}}{Y^{SS}} \frac{\tau_\pi \, \theta \, (\beta\omega^2 - \omega + 1)}{\beta\omega} & \frac{1-\beta\omega}{\beta}(1-\tau_d)\left(1 - \frac{1}{\omega}\right) \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ \frac{D^{SS}}{Y^{SS}}\left(\phi + \frac{\kappa \, \tau_\pi \, \theta}{\beta}\right) - \frac{\tau_y}{\beta} & -\frac{D^{SS}}{Y^{SS}} \frac{\tau_\pi \, \theta}{\beta} & \frac{1}{\beta}(1-\tau_d) \end{bmatrix} \begin{bmatrix} y_t \\ \pi_t \\ d_t + \varepsilon_t \end{bmatrix}, \tag{39}$$

where $a_{11} \equiv 1 + \phi\sigma - \left(1-\beta\omega\right)\left(1-\frac{1}{\omega}\right)\left(\frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta}\right) - \frac{D^{SS}}{Y^{SS}} \frac{\kappa \, \tau_\pi \, \theta}{\beta\omega}\left(\beta\omega^2 - \omega + 1\right)$.

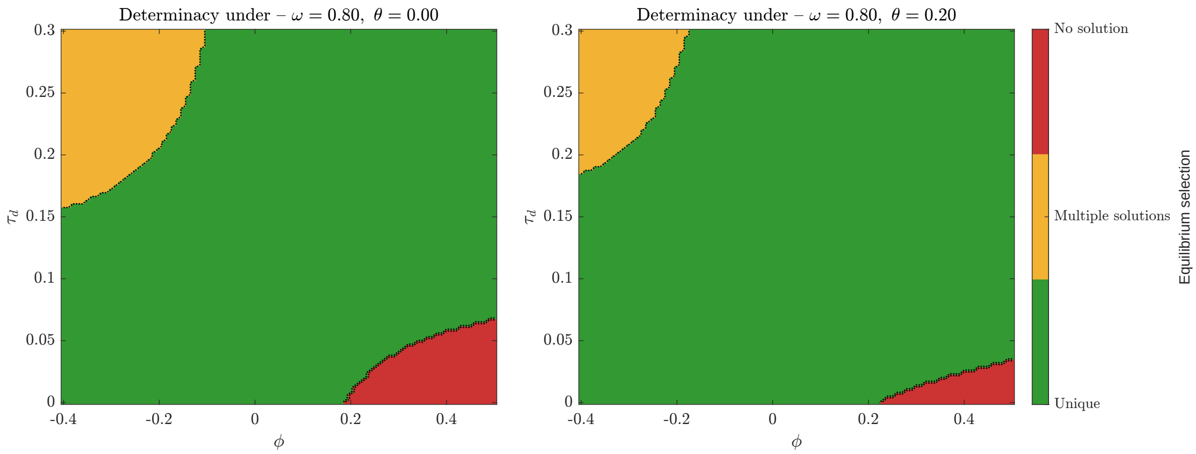


Figure 7: The determinacy properties of the model with mortality risk in dependence on the presence of indexed debt for the novel monetary and fiscal rules (35) and (36). The novel policy parameter τ_π has been set to 1.0.

Figure 7 depicts the impact of these novel rules on the determinacy properties of the system. In short, changing the fiscal and monetary rules to reflect a partial internalization of the anticipated effects of indexed debt *widens* the space under which saddle path-stable equilibria are possible unilaterally. In particular, even for relatively low values of τ_d, which indicate fiscal authorities committing to very expansionary policy measures, determinate equilibria are possible even for central bank policies that are tightening the economy in the presence of high inflation rates. In this scenario, the presence of indexed debt allows a foray into the area conventionally labelled as "active/active", which has thus far not been possible in other models, despite its possible real-world relevance in the light of the post-Covid inflationary episode.

Intuitively, the presence of the novel term in the fiscal rule facilitates more 'active' fiscal rules, since the presence of indexed debt does not weigh negatively on the fiscal budget constraint, since additional outlays for inflation-indexed debt are covered by the novel 'inflation stabilizer' in the fiscal rule (36).

What is the impact of the novel rules on inflationary pressure in the face of deficit shocks? Figure 8 sheds some light on this answer.

When the monetary rule absorbs the household-level windfall gain arising from their holdings of indexed debt in the presence of surprise inflation, the presence of indexed debt does *not* induce any additional inflationary pressure; by that same logic, the government's intertemporal budget constraint does not deteriorate through the presence of additional indexed debt. If anything, it is possible for inflation-indexed debt to depress inflationary pressure in response to deficit shocks when the government receives an additional windfall gain through its taxation in the wake of expected inflation. Note that, of course, the inflationary pressure in response to the deficit shock remains positive and particularly elevated for low levels of τ_d, but the change in the fiscal and monetary rules simply allows for a reduction in the magnitude of that pressure when indexed debt is present. Simply put, the *additional* inflationary pressure coming through indexed debt can be nullified, but nothing more.

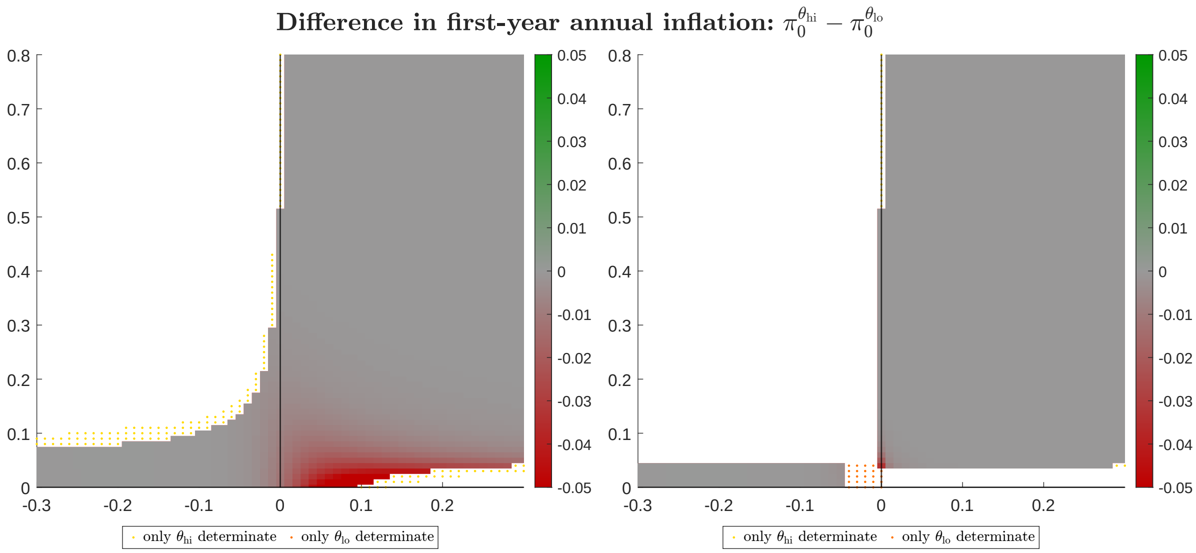


Figure 8: The difference in one-year inflation in the model with mortality risk between the cases $\theta = 0.2$ and $\theta = 0$ with fiscal rules (35) and (36). The novel policy parameter τ_π has been set to 1.0.

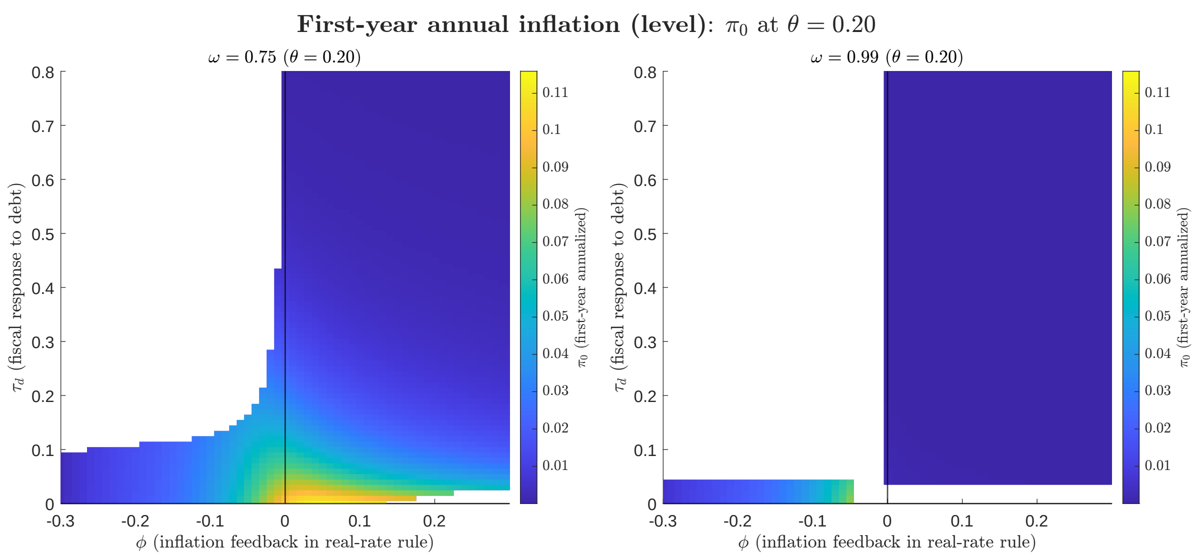


Figure 9: The level of one-year inflation rates with inflation-indexed debt ($\theta = 0.2$) under the policy rules (35) and (36). The novel policy parameter τ_π has been set to 1.0.

5 Fiscally-led Policy Mixes and Materialized Inflation empirically

We now supplement the theoretical derivations from sections 2 and 3 with an empirical analysis evaluating the possibility of fiscally-led policy regimes. Given that empirically distinguishing between two such clear-cut regimes is a futile task (Neumeyer and Nicolini, 2025), we mostly provide correlational evidence that hints at the relevance of our indexed debt mechanism in periods that are more likely to be associated with fiscally-led policy mixes. At the end of this section, we utilize exogenously identified fiscal shocks and an exogenously identified regime classification to provide quasi-causal evidence in favor of our mechanism for the specific context of the US.

5.1 Correlations and non-causal regressions: the coincidence of probable fiscally-led policy mixes and inflation-indexed debt

At first, we intend to provide certain aspects motivating our analysis of inflation-indexed debt as a potential driver of inflationary episodes, as well as of the broad relevance of our indexed debt mechanism.

To do so, we first examine in a simple descriptive statistic whether inflation-indexed debt levels are descriptively related to materialized inflation rates. This task is conducted in figure 10, which plots the share of inflation-indexed debt against observed gross annual CPI inflation rates in the sample of BIS Member Countries issuing inflation-indexed debt since 1990.

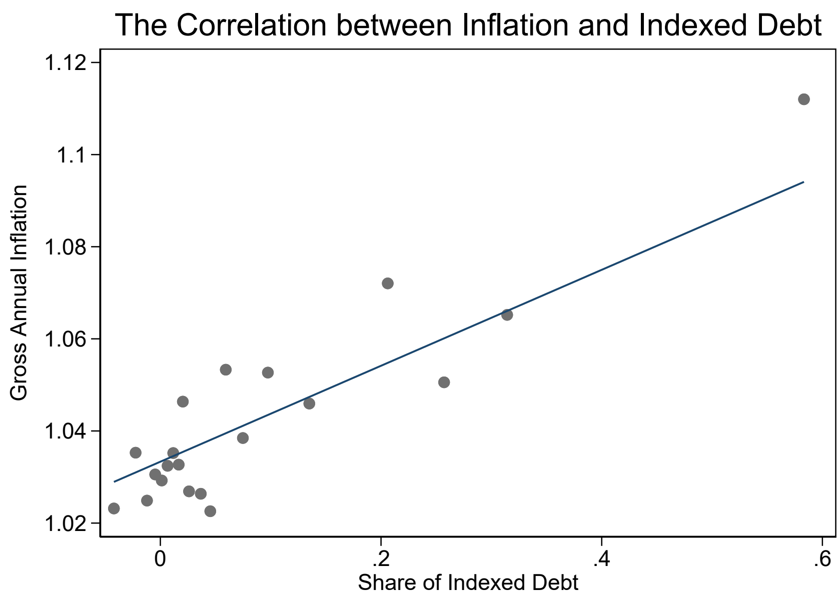
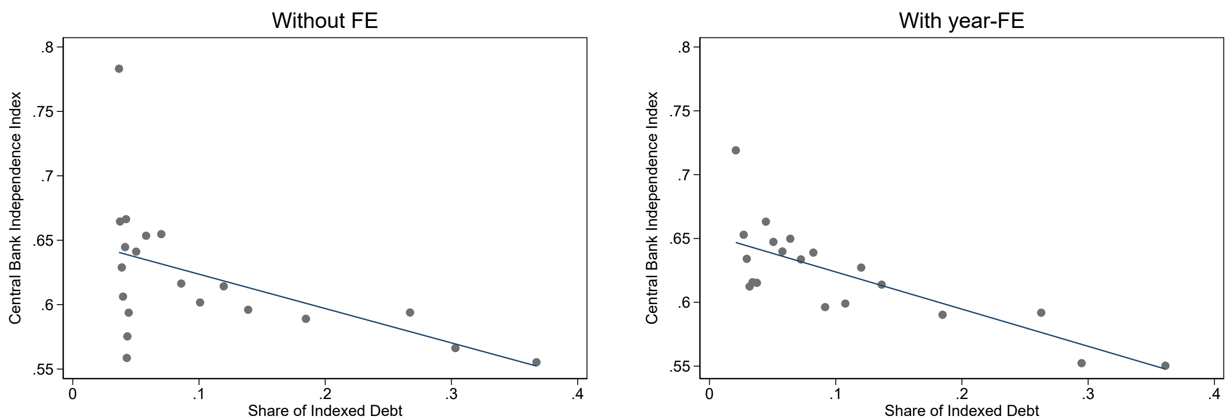


Figure 10: Binscatter of inflation-indexed debt against materialized inflation rates, controlling for the presence of fiscal rules, the degree of central bank independence, and a time-fixed effect in an annual sample of all BIS Member Countries issuing inflation-indexed debt.

While only correlational, the evidence presented here hinders the traditional message of inflation-indexed debt serving as a device ensuring that issuing governments will successfully curb inflationary pressure (Campbell and Shiller, 1996): there seems to be a clear positive correlation between observed shares of indexed debt and annual inflation rates in our long-running sample.

Inflation-indexed debt therefore does not seem to eliminate inflationary pressures altogether. But how much of this is informed through the link between inflation-indexed debt and the monetary-fiscal policy mix? As alluded to before, and as established rigorously by Cochrane (2011) and Neumeyer and Nicolini (2025), estimating policy rules with empirical data in the hopes of distinguishing between fiscally-led and monetary-led policy mixes is a futile task. We therefore explicitly do not do so, and now mostly present non-causal evidence that suggestively hints at an effect of inflation-indexed debt on measures that can, to a certain extent, be proxies for the possibilities of monetary-led and fiscally-led policy mixes.

A natural proxy evaluating the monetary side is the degree of central bank independence, here measured through the Central Bank Independence Index of Romelli (2024). Intuitively, the index captures the degree to which a central bank is not constrained through other factors (e.g., partisan considerations) in its decision-making. A less independent central bank might hint at a reduced probability of a monetary-led policy mix. Figure 11 relates this independence index to the share of inflation-indexed debt present at a given country and at a given point in time. In this logic, we therefore follow Banerjee et al. (2022).



(**a**) Estimates without fixed effects

(**b**) Estimates with year fixed effects

Figure 11: Binscatter of the relationship between the share of inflation-indexed debt and the measured degree of central bank independence following Romelli (2024).

While again at best correlational, the evidence is again striking: this time, we observe a clearly negative correlational relationship between the share of inflation-indexed debt in the overall debt stock and the measured degree of central bank independence. While not making any causal statements, the observed data hints at a diminished probability of monetary-led policy mixes occurring when large swaths of the government debt stock are indexed to gross inflation.

The following table 1 provides the numbers corresponding to figure 11, providing evidence on the correlation between measured central bank independence and the share of inflation-indexed debt in the overall government debt stock.[12]

[12]In appendix E, we provide some further robustness exercises.

Dep. var.	Central Bank Independence Index (Romelli, 2024)							
CB board index	0.687***	0.680***	0.505***	0.483***	0.686***	0.615***	0.500***	0.485***
	(0.0194)	(0.0201)	(0.0152)	(0.0169)	(0.0192)	(0.0187)	(0.0149)	(0.0164)
Fiscal Rule Intensity	0.0102***	0.0130***	0.0118***	0.00798***	0.0114***	0.0447***	0.00994***	0.00621***
	(0.00361)	(0.00397)	(0.00183)	(0.00183)	(0.00370)	(0.00445)	(0.00191)	(0.00181)
Indexed debt share	-0.182***	-0.198***	0.258***	0.105***	-0.313***	0.410***	0.253***	0.0666**
	(0.0413)	(0.0424)	(0.0336)	(0.0287)	(0.0490)	(0.0763)	(0.0414)	(0.0322)
FisRules × IndexDebt			-0.0893***			-0.490***	-0.0955***	
			(0.0153)			(0.0411)	(0.0161)	
Inflation					-0.00206*	-0.00125	-0.00155***	-0.00222***
					(0.00113)	(0.00108)	(0.000376)	(0.000377)
IndexDebt × Inflation					0.0169***	0.000288	0.00147	0.00382***
					(0.00387)	(0.00383)	(0.00132)	(0.00123)
Constant	0.251***	0.252***	0.326***	0.344***	0.260***	0.253***	0.337***	0.365***
	(0.0115)	(0.0120)	(0.00885)	(0.0112)	(0.0121)	(0.0112)	(0.00927)	(0.0115)
Obs.	605	605	605	605	605	605	605	605
R^2	0.707	0.713	0.978	0.980	0.717	0.778	0.979	0.981
F	482.5	451.3	296.8	41.32	304.1	317.9	210.2	42.46
R^2_{adj}	0.705	0.697	0.977	0.978	0.715	0.765	0.978	0.979
RMSE	0.106	0.107	0.030	0.029	0.104	0.095	0.029	0.028
Year-FE	No	Yes	No	Yes	No	Yes	No	Yes
Country-FE	No	No	Yes	Yes	No	No	Yes	Yes

Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$

Table 1: Regressions on possible predictors of central bank independence following Romelli (2024)

While establishing a proxy for the monetary side of the policy mix has already been ambitious, guidance for the fiscal side is even more difficult to come by due to the clearly simultaneous nature of fiscal policy setting with overall macroeconomic dynamics (Ramey and Zubairy, 2018). To nonetheless get a semblance of the idea, we exploit data from Davoodi et al. (2022) on the implementation of *fiscal rules* around the globe. While the nature of such rules and their implementation are relatively arbitrary, we zoom into *periods of rule suspension*; that is, periods in which previously established fiscal rules were suspended. In doing so, we focus on the question whether any rule had been suspended, as well as whether specific fiscal rules (expenditure rules, budget balancing rules, and deficit rules) were suspended. While again an imperfect proxy, we interpret here periods of fiscal rule suspension as periods of governments being willing to run fiscal policies that are normally considered unsustainable, therefore contributing to a possible fiscally-led policy mix.

The results of the logit regression predicting factors influencing the probability of a given fiscal rule suspension (for each of the four previously mentioned sets of rule suspensions) are presented in table 2.[13] We generally observe a clear positive correlational relationship between the share of inflation-indexed debt being present and the probability of rule suspension, except for periods of deficit rule suspension.[14] Taking the fifth column as an example (for our most comprehensive

[13]Appendix E.1 shows additional results in relation to past levels of inflation-indexed debt.

[14]The cross-country variation on the suspension of debt rules is quite low, however, impeding the statistical power

estimation), we can observe that each percentage point increase in the share of inflation-indexed debt is correlated with a 0.634% increase in the probability of the suspension of any fiscal rule that had previously been put in place.

Dep. var.	AnyR	ER	BR	DR	AnyR	ER	BR	DR
CB independence	-2.2267	-3.5126	-2.9157	0.4831	0.6227	13.5366	0.0185	0.9860
	(1.8226)	(3.9415)	(2.0313)	(4.9218)	(2.1084)	(10.9178)	(1.9724)	(4.8700)
CB board index	1.6759	4.8314	1.7574	-0.2362	-1.2981	-6.5182	-1.5719	-1.2533
	(1.5110)	(3.7678)	(1.7029)	(3.0294)	(1.4926)	(6.1250)	(1.4332)	(2.7346)
Indexed debt share	3.0572***	4.7854***	3.1249***	0.3678	6.3852***	13.6669***	6.4125***	2.0403
	(0.6578)	(1.0479)	(0.6751)	(0.9102)	(1.1965)	(4.0720)	(1.2509)	(1.3204)
Inflation	-0.0162	-0.0255	-0.0767*	-0.1554*	-0.0204	-0.1393**	-0.1335**	-0.2160*
	(0.0154)	(0.0219)	(0.0379)	(0.0674)	(0.0775)	(0.0432)	(0.0458)	(0.0958)
Constant	-3.1932***	-5.6121***	-2.7544***	-4.1307**	-1.1790	-8.8706*	-0.4267	-1.0932
	(0.5008)	(0.5327)	(0.4616)	(1.5229)	(0.9511)	(4.1165)	(0.8817)	(1.8025)
Obs.	652	652	652	652	285	261	285	95
ll	-103.8148	-51.7716	-91.2659	-42.2145	-59.0467	-18.4910	-56.0453	-24.6104
χ^2	27.9986	66.0330	27.5217	16.0084	70.0490	69.0798	70.0949	8.8445
p	0.0000	0.0000	0.0000	0.0030	0.0000	0.0000	0.0000	0.2640
R^2	0.1014	0.2750	0.1122	0.0218	0.3550	0.6779	0.3194	0.1034
Year-FE	No	No	No	No	Yes	Yes	Yes	Yes

Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table 2: Rule Suspension Regression. The outcome variables are defined as: *AnyR* = "Any Fiscal Rule Suspended", *ER* = "Expenditure Rule Suspended", *BR* = "Budget Balancing Rule Suspended", *DR* = "Deficit Rule Suspended".

To visualize the results of table 2, figure 12 predicts the probability of the suspension of any fiscal rule as a function of the share of inflation-indexed debt. To maximize insights, figure 12 performs this analysis at both higher inflation levels of 5% annualized CPI inflation, as well as for lower levels of 1% annualized CPI inflation.

Irrespective of the level of inflation, figure 12 shows a general clear positive link between the share of inflation-indexed debt and the propensity of the suspension of any fiscal rule. That being said, the effect seems to be quite limited for most advanced economies, which boast inflation-indexed debt shares of below 30%.[15] There is, however, a significant difference between low-inflationary and high-inflationary environments in the propensities of a rule suspension, with the probability being higher in *low* inflation environments. While this might seem surprising, it can easily be rationalized as high-inflation environments are usually characterized by a lack of *surprise* inflationary pressure, as higher inflation levels usually imply that corresponding market expectations (and therefore market prices of inflation-indexed debt) reflect expectations of pronounced inflationary

of the estimation.

[15]Note, however that up to 80% of debt being indexed to factors other than the price level is possible. For instance, many emerging market economies issue large amounts of debt denominated to foreign currencies, which work tantamount to inflation-indexed debt in simple models of the Fiscal Theory of the Price Level (Cochrane, 2023).

episodes. But inflation-indexed debt becomes especially costly during surprise inflation episodes, which must arise from an initial point of relative price stability.

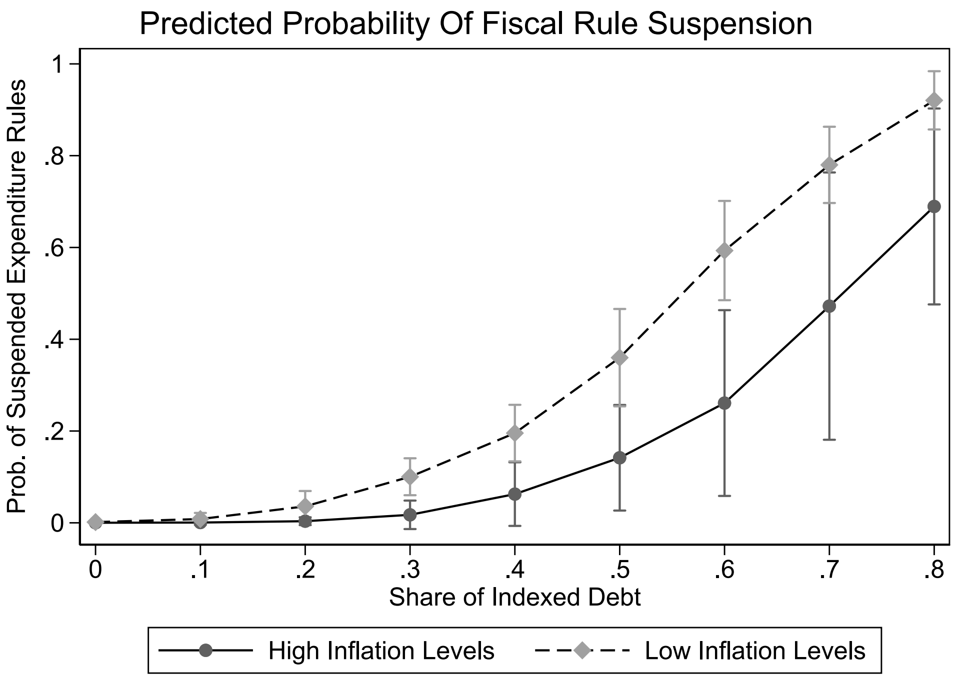


Figure 12: The relationship between the share of inflation-indexed debt and the predicted probability of a suspension of fiscal rules in high- and low-inflationary environments, following the estimation in table 2.

Even though the non-causal evidence presented here is useful to set the ideas, we now intend to move on to the degree possible and provide a semblance of causal evidence on the role that inflation-indexed debt might play in shaping inflationary dynamics across policy regimes. While identifying policy regimes is generally hardly possible (meaning that evaluating the effect of indexed debt on the propensity of either policy regime in a causal sense is prohibitively difficult), we can attempt to find evidence on the inflationary consequence of inflation-indexed debt in either policy regime, conditional on supplying the prevalence of each regime exogenously.

5.2 Causal evidence on the link between fiscally-led policy mixes and materialized inflation in the light of high shares of indexed debt

To round up our empirical exercise, this subsection presents direct evidence on the effect that inflation-indexed debt can have on inflation, making use of the series of narratively identified tax shocks in the US provided by Mierzwa (2024).

We leverage his time series of exogenous fiscal policy surprises, and combine it with a corresponding series of inflation-indexed debt, taking the share of inflation-indexed debt in the overall sovereign debt portfolio as our main indicator for the intensity of the prevalence of inflation-indexed debt. Equipped with these time series, we estimate the following local projection (Jordà, 2005) to measure the dynamic impact of inflation-indexed debt on changes in the rate of inflation:

$$\log P_{t+h} - \log P_{t-1} = \alpha_h + \beta_h \omega_{t-1} \varepsilon_t^F + \delta_{1h} \omega_{t-1} + \delta_{2h} \varepsilon_t^F + \Gamma_h Z_{t-1} + e_{t+h}, \tag{40}$$

where $h \geq 0$ indexes the forecast horizon considered and Z_{t-1} is a vector of control variables specified below. Of particular interest to us is the coefficient β_h, which captures the cross-effect of the identified fiscal shock ε_t^F and the past level of the share of inflation-indexed debt ω_{t-1}.[16] We estimate this equation separately in for two different periods, creating effectively a *state-dependent local projection*, as described by Jordà and Taylor (2025). We estimate the specification (40) separately for periods of *fiscally-led policy mixes* and *monetary-led policy mixes*, as identified by Chen et al. (2022) for the United States.[17] We label the regimes here 'active fiscal policy' and 'passive fiscal policy' in line with Chen et al. (2022).

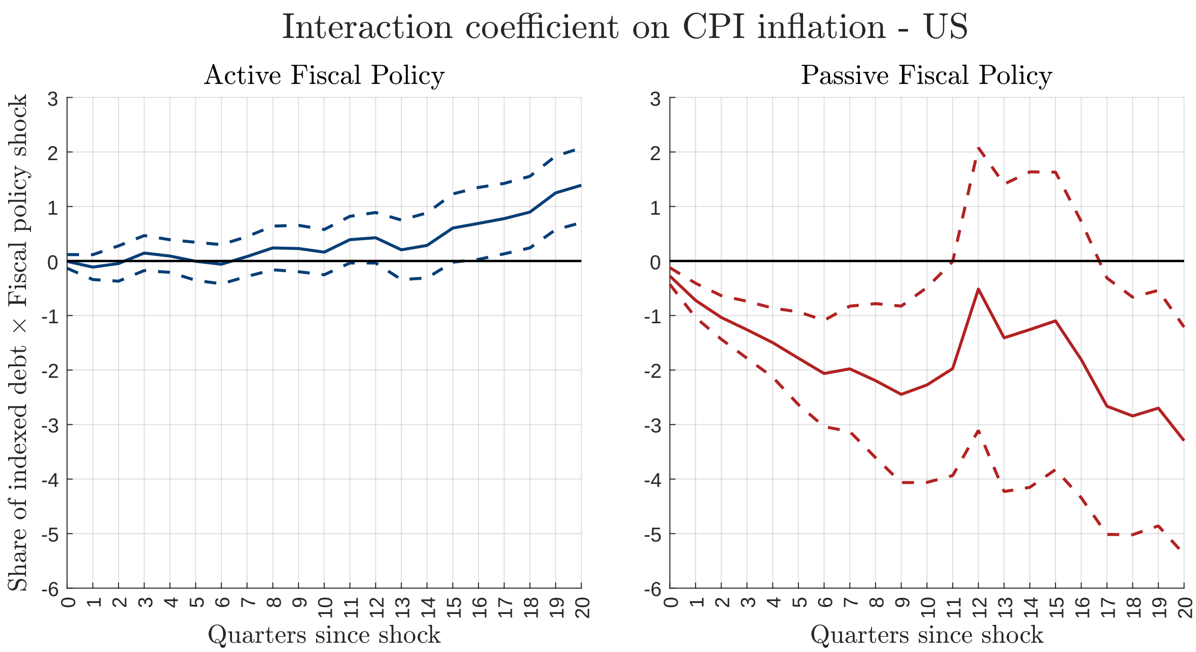


Figure 13: IRF implied by the local projection (40) through the coefficients β_h. The control vector Z contains the first four lags of the real GDP growth rate, the short-run UK bank rate, the change in the weighted real exchange rate, a same-period recession indicator, year-fixed effects, and the first lag of the price level difference. Standard errors are robust to heteroscedasticity and autocorrelation (Newey-West correction). Confidence intervals are provided at the 90% level. Sample length: 1970 Q1 - 2019 Q2.

Figure 13 summarizes this exercise, with the left-hand panel showcasing the fiscally-led policy mix and the right-hand panel showing the monetary-led policy mix. Under the monetary-led policy mix, the response of the price level on the interaction between the share of inflation-indexed debt and the identified fiscal policy shock is relatively imprecisely estimated, but - if anything - there

[16] We use last period's level of inflation-indexed debt to avoid running into a simultaneity bias.

[17] While the results in their paper only extend for the period until the Great Financial Crisis, we utilize their characterization to further pin down the prevalent policy mix until 2020, inclusive. Our results are qualitatively robust to characterizing the prevalent fiscal-monetary regime as Bianchi and Melosi (2022) do.

is at least a temporary negative effect on the price level when more debt is indexed. In the case of a fiscally-led policy mix, however, medium-term price pressures arising from the identified fiscal policy disturbance are significantly increasing in the share of inflation-indexed debt. In the period 4-5 years after the initial shock, a 1% deficit-to-GDP shock accustomed by a corresponding change of 1% in the share of inflation-indexed debt boosts observed changes to the price level by up to 1.5% relative to a case without a corresponding change in the share of inflation-indexed debt. This is an economically meaningful effect that would remain hidden without consideration of inflation-indexed debt for inflationary dynamics.

6 Conclusion

This paper has looked at fiscal-monetary interactions through the lens of a policy instrument insufficiently considered in dynamic macroeconomic models thus far: inflation-indexed sovereign debt, issued by a multitude of sovereigns.

Inflation-indexed debt highlights two aspects associated with the risks of what is conventionally known as fiscally-led policy mixes. First, recent results emphasizing that the clear distinction between fiscally- and monetary-led policy mixes are overcome once market imperfections such as liquidity risk are added appear to be qualified by the presence of indexed debt: in particular, fiscal policy conventionally considered active is strikingly related to additional inflationary pressure through the presence of inflation-indexed debt, even under such market imperfections. Intuitively, since cost pressures translate into higher debt service burdens for governments when they have issued more indexed debt, monetary and fiscal policies must react sufficiently to cover this additional cost, leading to the emergence of 'quasi fiscally-led' policy mechanisms. Second, it increases the risk of inflationary disasters when we actually are within such fiscally-led policy mixes - that is, when a fiscal deficit shock materializes, it is more inflationary when we actually have higher shares of inflation-indexed debt *and* we are simultaneously operating under policy rules considered to be conventionally fiscally-led.

Other than further robustifying our theoretical results by providing analytic characterizations of medium-run inflationary pressures in the simplified models provided that feature inflation-indexed debt, our top priority is the introduction of a calibrated medium-scale model that provides an exact quantitative evaluation of the effects of inflation-indexed debt. As the tax base channel seems to matter for the effects of such debt, we hope to exploit cross-country variation in this tax base channel to derive robust predictions on the effects of inflation-indexed debt in such medium-scale models.

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Appendix

A The Fisherian model under a standard Taylor Rule

In place of the bond return rate-targeting monetary rule in section 2, we here explore the dynamic properties of the Fisherian framework under a standard policy rule defined on the simple nominal interest rate.

The central bank would then follow a simplified monetary rule on the gross nominal interest rate:

$$R_t^n = \frac{1}{\beta}\Pi_t^{\phi}, \tag{A.1}$$

where $R_{n,t} \equiv 1 + i_t$ is the gross nominal interest rate, and ϕ is the monetary policy parameter of interest that will indicate whether the economy operates under a monetary-led policy mix or not. This monetary rule implies a zero-inflation steady-state consistent with household optimality (captured by the bond pricing equations), under which $R^n = \frac{1}{\beta}$. Thus, under the present setting, $Q_t = \frac{1}{R_t^n}$, i.e., the price of the nominal bond must be the inverse of the gross nominal interest rate as all bonds are short-term and the relevant interest rate is under control of the monetary authority.

*Equilibrium: The equilibrium is derived exactly as in the main body of the paper, but making use of the monetary rule (A.1). Expressing the system in log-deviations from a deterministic zero-inflation steady-state, we obtain the following system of equations:

- Nominal bond prices: using $q_t = \frac{1}{R_t}$ and $q_t = \beta\, \mathbb{E}_t\big[\Pi_{t+1}^{\theta-1}\big]$, we get, to a first-order approximation,

$$-\hat{R}_t = (\theta - 1)\, \mathbb{E}_t \hat{\pi}_{t+1}. \tag{A.2}$$

- Monetary rule: taking logs of the monetary rule, we get

$$\hat{R}_t^n = \phi \hat{\pi}_t. \tag{A.3}$$

- Law of motion of debt: as in the main text,

$$(\theta - 1)\, \hat{\pi}_t + \hat{s}_{t-1} + \hat{R}_{t-1} = (1-\beta)\, \hat{\tau}_t + \beta\, \hat{s}_t. \tag{A.4}$$

- Fiscal rule: the fiscal rule can be log-linearized exactly:

$$\hat{\tau}_t = \gamma\, \hat{s}_{t-1} + \varphi_t. \tag{A.5}$$

A no-arbitrage argument: To induce finite demands on either type of asset (and thereby to close the model under the Taylor Rule on the nominal interest rate), which is necessary for the existence of an equilibrium, we posit that households demand equivalent *real* returns from both assets in log-deviations:

$$\hat{R}_t^n - \mathbb{E}_t \hat{\pi}_{t+1} = \hat{R}_t - (1-\theta)\mathbb{E}_t \hat{\pi}_{t+1}. \tag{A.6}$$

The left-hand side captures the real return on the nominal asset. The right-hand side captures the real return on the government-issued partially-indexed bond, where the inflation adjustment is weighted by $(1-\theta)$, the *non-indexed share* of that bond.

We can combine equations (A.2)+(A.3)+(A.6) to attain the following system of difference equations:

$$\mathbb{E}_t \hat{\pi}_{t+1} = \phi \hat{\pi}_t, \tag{A.7}$$

For the fiscal block, we remain with the same specification as in the main text:

$$\hat{s}_t = (1-\beta)\gamma \hat{s}_t + (1-\beta)\,\mathbb{E}_t \varphi_{t+1} + \mathbb{E}_t \beta \hat{s}_{t+1}. \tag{A.8}$$

Collecting, the system can be written as

$$\underbrace{\begin{pmatrix} 1 & 0 \\ 0 & \beta \end{pmatrix}}_{\equiv A_0} \mathbb{E}_t \begin{pmatrix} \hat{\pi}_{t+1} \\ \hat{s}_{t+1} \end{pmatrix} = \underbrace{\begin{pmatrix} \phi & 0 \\ 0 & 1-\gamma(1-\beta) \end{pmatrix}}_{\equiv A_1} \begin{pmatrix} \hat{\pi}_t \\ \hat{s}_t \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ -(1-\beta) \end{pmatrix}}_{\equiv C} \mathbb{E}_t \varphi_{t+1}. \tag{A.9}$$

The matrix relevant for the determinacy properties is

$$Z = A_0^{-1} A_1 = \begin{pmatrix} \phi & 0 \\ 0 & \dfrac{1-\gamma(1-\beta)}{\beta} \end{pmatrix}. \tag{A.10}$$

Its eigenvalues are ϕ and $\dfrac{1-\gamma(1-\beta)}{\beta}$. A *monetary-led* equilibrium requires $\phi > 1$ and $\gamma > 1$. A *fiscally-led* equilibrium requires $\phi < 1$ and $\gamma < 1$. In sum, inflation-indexed debt does *not* matter for the determinacy properties of this simple model when the Taylor Rule ignores the composition of government debt. This is a direct consequence of the no-arbitrage argument that forces the real return rate of the partially-indexed government bond to be equal to the return rate of the zero-supply outside asset.

B An argument ensuring exactly one root inside the unit circle in a 2-by-2 matrix

Proof. We here present a brief argument by which we can derive the conditions under which exactly one root of a 2×2 system lies inside the unit circle.

Recall that the characteristic equation of Z is given by

$$p(\lambda) = \lambda^2 - \mathrm{tr}(Z)\,\lambda + \det(Z).$$

Define the two following expressions for convenience:

$$p(1) \equiv 1 - \mathrm{tr}(Z) + \det(Z), \qquad p(-1) \equiv 1 + \mathrm{tr}(Z) + \det(Z).$$

Now, let the two eigenvalues, as defined by the characteristic equation, be given by λ_1 and λ_2. Then, $p(\cdot)$ evaluated at 1 and -1 yields:

$$p(1) = (1 - \lambda_1)(1 - \lambda_2), \qquad p(-1) = (1 + \lambda_1)(1 + \lambda_2),$$

while the determinant and trace can be expressed as $\det(Z) = \lambda_1\lambda_2$, $\mathrm{tr}(Z) = \lambda_1 + \lambda_2$.

Now, combining the insight of Woodford (2011) on the location of both eigenvalues outside the unit circle with results that allow us to express cases when both eigenvalues are inside the unit circle (e.g., Antsaklis and Michel (2006), Theorem 10.10), we can summarize the necessary and sufficient conditions for the existence of both eigenvalues either inside or outside the unit circle as follows:

- *Both inside* ($|\lambda_1|, |\lambda_2| < 1$) iff

$$\det(Z) < 1, \quad p(1) > 0, \quad p(-1) > 0.$$

- *Both outside* (*Case I*) iff

$$\det(Z) > 1, \quad p(1) > 0, \quad p(-1) > 0.$$

- *Both outside* (*Case II*) iff

$$p(1) < 0, \quad p(-1) < 0.$$

In addition, we now use a technical boundary to rule out that $p(-1) < 0$. That technical boundary is expressed as the condition under which $p(-1) > 0$, which is given by:

$$
\begin{aligned}
& p(-1) > 0 \\
\Leftrightarrow\ & 2\left(1+\frac{1}{\beta}\right)+\phi\sigma\left(1+\frac{1}{\beta}\right)-\frac{D^{SS}}{Y^{SS}}\frac{\kappa\sigma\theta}{\beta} > 0 \\
\Leftrightarrow\ & (2+\phi\sigma)\left(1+\frac{1}{\beta}\right) > \frac{D^{SS}}{Y^{SS}}\frac{\kappa\sigma\theta}{\beta} \\
\Leftrightarrow\ & \phi > -\frac{1}{\sigma}\left[2-\frac{\frac{D^{SS}}{Y^{SS}}\kappa\sigma\theta}{1+\beta}\right].
\end{aligned}
$$

Without indexed debt, this condition is always satisfied under the technical boundary ensuring a positive determinant, $\phi > -\frac{1}{\sigma}$. Here, for high levels of inflation-indexed debt (and a sufficiently high debt-to-GDP ratio or a sufficiently steep Phillips Curve), this might be an additional restriction to consider.

In sum total, we can express a total of eight cases in dependence of $\det(Z)$, $p(1)$, and $p(-1)$, for which six cases are covered so far as necessary and sufficient:

- $\det(Z) < 1$, $p(1) > 0$, $p(-1) > 0$: Both roots inside
- $\det(Z) < 1$, $p(1) > 0$, $p(-1) < 0$: Ruled out
- $\det(Z) < 1$, $p(1) < 0$, $p(-1) > 0$:
- $\det(Z) < 1$, $p(1) < 0$, $p(-1) < 0$: Both roots outside (case II)
- $\det(Z) > 1$, $p(1) > 0$, $p(-1) > 0$: Both roots outside (case I)
- $\det(Z) > 1$, $p(1) > 0$, $p(-1) < 0$: Ruled out
- $\det(Z) > 1$, $p(1) < 0$, $p(-1) > 0$:
- $\det(Z) > 1$, $p(1) < 0$, $p(-1) < 0$: Both roots outside (case II)

Then, all that must be proven is the restriction under which $p(1) = 1 - \mathrm{tr}(Z) + det(Z) = \left(\frac{1}{\beta}-1\right)\phi\sigma + \frac{D^{SS}}{Y^{SS}}\frac{\kappa\sigma\theta}{\beta}$ is smaller than zero. These are the remaining cases for which exactly one real root lies inside the unit circle. This restriction is given by:

$$\phi < -\frac{\frac{D^{SS}}{Y^{SS}}\kappa\theta}{1-\beta} \tag{B.1}$$

Therefore, the region ensuring a saddle-path stable equilibrium under a fiscally-led policy mix restricts the monetary policy parameter ϕ into the following space:

$$\max\left\{-\frac{1}{\sigma},\ -\frac{1}{\sigma}\left[2-\frac{\frac{D^{SS}}{Y^{SS}}\kappa\sigma\theta}{1+\beta}\right]\right\} \ < \ \phi \ < \ -\frac{\frac{D^{SS}}{Y^{SS}}\kappa\theta}{1-\beta}. \tag{B.2}$$

For $\theta = 0$, this reverts to the standard restriction for a fiscally-led policy mix in RANK models that $-\frac{1}{\sigma} < \phi < 0$.

■

C Criteria for monetary dominance in the RANK version of the model in section 3

Define again the matrix in equation (31) as A. Its characteristic polynomial is given by:

$$\begin{aligned}\det(A-\lambda I) &= \left[\frac{1}{\beta}(1-\tau_d)-\lambda\right]\left[\left(1+\phi\sigma-\frac{D^{SS}}{Y^{SS}}\frac{\kappa\sigma\theta}{\beta}-\lambda\right)\left(\frac{1}{\beta}-\lambda\right)+\frac{D^{SS}}{Y^{SS}}\frac{\kappa\theta}{\beta^2}\right] \\ &= \left[\frac{1}{\beta}(1-\tau_d)-\lambda\right]\left[\lambda^2-\lambda\left(\frac{1}{\beta}+1+\sigma\phi-\frac{D^{SS}}{Y^{SS}}\frac{\kappa\sigma\theta}{\beta}\right)+\frac{1}{\beta}(1+\sigma\phi)\right]\end{aligned} \tag{C.1}$$

In order to establish how indexed debt influences the system's stability properties, we consider the boundaries under which conventional monetary dominance is established.

Monetary dominance requires fiscal passivity, which here is ensured by $\tau_d > 1-\beta$, i.e., fiscal policy must be sufficiently reactive to deviations of the value of government debt from its steady-state. In that case, the root implied by the first bracket in equation (C.1) is inside the unit circle, so we require two further roots outside the unit circle.

Analysing those can be done equivalently by reducing our focus to the 2×2 sub-system on (y_t, π_t), characterized by the following matrix Z:

$$Z \equiv \begin{bmatrix} 1+\phi\sigma-\frac{D^{SS}}{Y^{SS}}\frac{\kappa\sigma\theta}{\beta} & \frac{D^{SS}}{Y^{SS}}\frac{\sigma\theta}{\beta} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta}\end{bmatrix}.$$

Following Woodford (2011) and Rachel and Ravn (2025), either of the following sets of conditions is sufficient to guarantee that two roots of Z lie outside the unit circle:

1. First case:
    - $\det(Z) > 1$
    - $\det(Z) - \mathrm{tr}(Z) > -1$
    - $\det(Z) + \mathrm{tr}(Z) > -1$
2. Second case:
    - $\det(Z) - \mathrm{tr}(Z) < -1$
    - $\det(Z) + \mathrm{tr}(Z) < -1$.

We focus on case 1, ruling out case 2 along the way. First, note that:

$$\det(Z) = \frac{1}{\beta}(1+\sigma\phi)$$
$$\text{tr}(Z) = (1+\sigma\phi) + \frac{1}{\beta}\left[1 - \frac{D^{SS}}{Y^{SS}}\kappa\sigma\theta\right].$$

Let active monetary policy be characterized by $\phi \geq 0$, as in the main body of the text. Then, $\det(Z) > 1$ is trivially true for any $\beta < 1$. Next, we consider the second condition of the first case:

$$\det(Z) - \text{tr}(Z) = \frac{1}{\beta}\left(\sigma\phi + \frac{D^{SS}}{Y^{SS}}\kappa\sigma\theta\right) - (1+\sigma\phi) > -1$$
$$\leftrightarrow \qquad \left(\frac{1}{\beta} - 1\right)\sigma\phi + \frac{D^{SS}}{Y^{SS}}\kappa\sigma\theta > 0,$$

which, again, holds trivially true for all values of θ, including $\theta = 0$ (which is the base case of Angeletos et al. (2024) and Rachel and Ravn (2025)). Therefore, the second set of conditions must not be considered, since the previous equation implies that $\det(Z) - \text{tr}(Z) < -1$ cannot hold true for $\phi > 0$.

Finally, consider the third condition of the first case. Here, we find that:

$$\det(Z) + \text{tr}(Z) = \left(\frac{1}{\beta} + 1\right)(1+\sigma\phi) + \frac{1}{\beta}\left(1 - \frac{D^{SS}}{Y^{SS}}\kappa\sigma\theta\right) > -1$$
$$\leftrightarrow \qquad (1+\beta)(1+\sigma\phi) + \left(1 - \frac{D^{SS}}{Y^{SS}}\kappa\sigma\theta\right) > -\beta$$
$$\leftrightarrow \qquad (1+\beta)(1+\sigma\phi) + > \frac{D^{SS}}{Y^{SS}}\kappa\sigma\theta - (1+\beta)$$
$$\leftrightarrow \qquad 1+\sigma\phi > \frac{D^{SS}}{Y^{SS}}\frac{\kappa\sigma\theta}{1+\beta} - 1$$
$$\leftrightarrow \qquad \phi > \frac{1}{\sigma}\left[\frac{D^{SS}}{Y^{SS}}\frac{\kappa\sigma\theta}{1+\beta} - 2\right],$$

i.e., the admissible bounds for ϕ that ensure monetary dominance *can* tighten when the share of indexed debt is sufficiently high. Tighter determinacy bounds under monetary dominance due to the presence of indexed debt are supported through steep Phillips Curves (high κ), high degrees of risk aversion (high σ), or high debt-to-GDP ratios (high $\frac{D^{SS}}{Y^{SS}}$). Note that without indexed debt ($\theta = 0$), this condition reduces to $\phi > -\frac{2}{\sigma}$, which is trivially true.

D Proof of the characterization of the parametric boundaries ensuring saddle path-stable equilibria

The trace is defined as:

$$\text{tr}(A) = 1+\phi \left(\sigma + \frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\right) - \frac{\kappa\theta}{\beta}\left(\sigma + \frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\right) - \frac{\tau_y}{\beta}(1-\beta\omega)(1-\tfrac{1}{\omega}) + \frac{2-\tau_d}{\beta}.$$

The determinant, in turn, is given by:

$$\begin{aligned}\det(A) = {}& a_{11}\left(\tfrac{1}{\beta}\right)\left(\tfrac{1-\tau_d}{\beta}\right) - \left(\tfrac{\theta}{\beta}\left(\sigma + \frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\right)\right)\left(-\tfrac{\kappa}{\beta}\right)\left(\tfrac{1-\tau_d}{\beta}\right) \\ &+ \left(\tfrac{(1-\beta\omega)(1-\frac{1}{\omega})}{\beta}(1-\tau_d)\right)\left(-\tfrac{\kappa}{\beta}\right)\left(\tfrac{\theta}{\beta}\tfrac{D^{SS}}{Y^{SS}}\right) - \left(\tfrac{(1-\beta\omega)(1-\frac{1}{\omega})}{\beta}(1-\tau_d)\right) a_{31}\left(\tfrac{1}{\beta}\right).\end{aligned}$$

Factoring out $\dfrac{1-\tau_d}{\beta^2}$, we obtain:

$$\det(A) = \frac{1-\tau_d}{\beta^2}\left[a_{11} + \frac{\theta\kappa}{\beta}\left(\sigma + \frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\right) - \frac{\theta\kappa}{\beta}\frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega}) - (1-\beta\omega)(1-\tfrac{1}{\omega})\, a_{31}\right].$$

Next, use $\left(\sigma + \frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\frac{1}{\omega})\right) - \dfrac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\frac{1}{\omega}) = \sigma$:

$$\det(A) = \frac{1-\tau_d}{\beta^2}\left[a_{11} + \frac{\theta\kappa}{\beta}\sigma - (1-\beta\omega)(1-\tfrac{1}{\omega})\, a_{31}\right].$$

Substitute a_{11} and a_{31}:

$$\begin{aligned}\det(A) = {}& \frac{1-\tau_d}{\beta^2}\Bigg[\underbrace{\left(1+\phi\left(\sigma + \frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\right) - \tfrac{\kappa\theta}{\beta}\left(\sigma + \frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\right) - \tfrac{\tau_y}{\beta}(1-\beta\omega)(1-\tfrac{1}{\omega})\right)}_{a_{11}} \\ &+ \frac{\theta\kappa}{\beta}\sigma - (1-\beta\omega)(1-\tfrac{1}{\omega})\underbrace{\left(\tfrac{D^{SS}}{Y^{SS}}\phi - \tfrac{\tau_y}{\beta} - \tfrac{\theta\kappa}{\beta}\tfrac{D^{SS}}{Y^{SS}}\right)}_{a_{31}}\Bigg] \\ = {}& \frac{1-\tau_d}{\beta^2}\Bigg[1+\phi\left(\sigma + \frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\right) - \frac{\kappa\theta}{\beta}\left(\sigma + \frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\right) \\ &- \frac{\tau_y}{\beta}(1-\beta\omega)(1-\tfrac{1}{\omega}) + \frac{\theta\kappa}{\beta}\sigma - (1-\beta\omega)(1-\tfrac{1}{\omega})\left(\tfrac{D^{SS}}{Y^{SS}}\phi\right) + (1-\beta\omega)(1-\tfrac{1}{\omega})\left(\tfrac{\tau_y}{\beta}\right) \\ &+ (1-\beta\omega)(1-\tfrac{1}{\omega})\left(\tfrac{\theta\kappa}{\beta}\tfrac{D^{SS}}{Y^{SS}}\right)\Bigg].\end{aligned}$$

Many elements cancel out:

$$-\frac{\tau_y}{\beta}(1-\beta\omega)(1-\tfrac{1}{\omega}) \;+\; (1-\beta\omega)(1-\tfrac{1}{\omega})\frac{\tau_y}{\beta} \;=\; 0,$$

$$\begin{aligned}
&-\frac{\kappa\theta}{\beta}\left(\sigma+\frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\right) \;+\; \frac{\theta\kappa}{\beta}\sigma+\frac{\theta\kappa}{\beta}\frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\\
=\;& -\frac{\kappa\theta}{\beta}(\sigma+\tfrac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega}))+\frac{\theta\kappa}{\beta}\sigma+\frac{\theta\kappa}{\beta}\frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})=0.
\end{aligned}$$

For the ϕ-terms,

$$\begin{aligned}
&\phi\left(\sigma+\frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\right)-(1-\beta\omega)(1-\tfrac{1}{\omega})\big(\tfrac{D^{SS}}{Y^{SS}}\phi\big)\\
&=\phi\Big(\left(\sigma+\frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\right)-\tfrac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})\Big)=\phi\,\sigma.
\end{aligned}$$

Hence,

$$\boxed{\det(A)\;=\;\frac{1-\tau_d}{\beta^2}\,(1+\sigma\,\phi).}$$

Let the characteristic polynomial be defined in the form:

$$\lambda^3+\Gamma_2\lambda^2+\Gamma_1\lambda+\Gamma_0=0.$$

Then, its elements are given by:

$$\Gamma_2=-\mathrm{tr}(A)=$$

$$-\left(\left(1+\phi\,\sigma+\frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})-\frac{\kappa\theta}{\beta}\,\sigma+\frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})-\frac{\tau_y}{\beta}\,(1-\beta\omega)(1-\tfrac{1}{\omega})\right)+\frac{2-\tau_d}{\beta}\right)$$

$$\begin{aligned}
\Gamma_1=\frac{2-\tau_d}{\beta}&\left[1+\phi\sigma+\frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})-\frac{\kappa\theta}{\beta}\sigma+\frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})-\frac{\tau_y}{\beta}(1-\beta\omega)(1-\tfrac{1}{\omega})\right]\\
&+\frac{1-\tau_d}{\beta^2}+\frac{\theta\kappa}{\beta^2}\sigma+\frac{D^{SS}}{Y^{SS}}(1-\beta\omega)(1-\tfrac{1}{\omega})-\frac{(1-\tau_d)(1-\beta\omega)(1-\frac{1}{\omega})}{\beta}\left[\frac{D^{SS}}{Y^{SS}}\phi-\frac{\tau_y}{\beta}-\frac{\theta\kappa}{\beta}\frac{D^{SS}}{Y^{SS}}\right].
\end{aligned}$$

$$\Gamma_0=-\det(A)=-\frac{1-\tau_d}{\beta^2}\,(1+\sigma\,\phi)$$

Then, in line with Rachel and Ravn (2025), the following sets of conditions are sufficient to claim that two roots of the characteristic polynomial lie outside the unit circle:

1. First set:
   - $1 + \Gamma_2 + \Gamma_1 + \Gamma_0 < 0$
   - $-1 + \Gamma_2 - \Gamma_1 + \Gamma_0 > 0$
2. Second set:
   - $1 + \Gamma_2 + \Gamma_1 + \Gamma_0 > 0$
   - $-1 + \Gamma_2 - \Gamma_1 + \Gamma_0 < 0$
   - $\Gamma_0(\Gamma_0 - \Gamma_2) + \Gamma_1 - 1 > 0$
3. Third set:
   - $1 + \Gamma_2 + \Gamma_1 + \Gamma_0 > 0$
   - $-1 + \Gamma_2 - \Gamma_1 + \Gamma_0 < 0$
   - $\Gamma_0(\Gamma_0 - \Gamma_2) + \Gamma_1 - 1 < 0$
   - $|\Gamma_2| > 3$.

We now start to check under which boundaries either of the sets of inequalities holds true. The first set of inequalities, in particular, must normally not be considered given our parametric boundaries on ϕ and τ_d, since the second inequality of the first set is always violated.

For simpler notation, let us define:

$$\mathcal{T} \equiv (1 - \beta\omega)\left(1 - \tfrac{1}{\omega}\right), \qquad \mathcal{S} \equiv \sigma + \frac{D^{SS}}{Y^{SS}}\mathcal{T}.$$

$\mathcal{T}$ is smaller then zero when there exists any mortality risk, and zero otherwise. The exact sign of $\mathcal{S}$ is unclear and depends on the relative strength of the risk aversion in relation to the debt-weighted wedge induced by the mortality risk. Usually, we can expect $\mathcal{S} > 0$ for common calibrations. We therefore impose that:

Assumption. The relative strength of the risk aversion channel is stronger than the effect of imperfect household insurance arising through mortality risk; formally,

$$\boxed{\mathcal{S} > 0}.$$

Recall that the entries from the matrix are given by:

$$\begin{aligned}
&a_{11} = 1 + \phi\,\mathcal{S} - \frac{\kappa\theta}{\beta}\mathcal{S} - \frac{\tau_y}{\beta}\mathcal{T}, \qquad a_{12} = \frac{\theta}{\beta}\mathcal{S}, \qquad a_{13} = \frac{\mathcal{T}}{\beta}(1 - \tau_d),\\
&a_{21} = -\frac{\kappa}{\beta}, \qquad a_{22} = \frac{1}{\beta}, \qquad a_{23} = 0,\\
&a_{31} = \frac{D^{SS}}{Y^{SS}}\phi - \frac{\tau_y}{\beta} - \frac{\theta\kappa}{\beta}\frac{D^{SS}}{Y^{SS}}, \qquad a_{32} = \frac{\theta}{\beta}\frac{D^{SS}}{Y^{SS}}, \qquad a_{33} = \frac{1 - \tau_d}{\beta}.
\end{aligned}$$

Proof.

Let
$$\mathcal{I} := (1-\beta\omega)\big(1-\tfrac{1}{\omega}\big) \le 0, \qquad \mathcal{S} := \sigma + \frac{D^{SS}}{Y^{SS}}\mathcal{I} > 0.$$

Additionally, define
$$C(\tau_d) := \sigma(\tau_d - (1-\beta)) + \beta\mathcal{I}\,\frac{D^{SS}}{Y^{SS}}, \qquad \tau_d^0 := (1-\beta) - \frac{\beta\mathcal{I}}{\sigma}\,\frac{D^{SS}}{Y^{SS}},$$
$$\phi^\star(\tau_d) := \frac{\tau_y\mathcal{I}}{C(\tau_d)} - \frac{\theta\kappa}{1-\beta},$$

and
$$\Delta(\phi,\tau_d) := 1 - S_2(A) + \mathrm{tr}(A)\det(A) - \det(A)^2.$$

Here, $C(\tau_d)$ can be interpreted as the "tax-finance wedge" of additional debt issuance, with τ_d^0 being its zero-bound and $\phi^*(\tau_d)$ being a related cut-off parameter of the monetary rule.

For
$$\tau_d^\sharp := 1 - \Big[\beta^2 + \frac{\beta^2}{\sigma}\frac{D^{SS}}{Y^{SS}}\mathcal{I}\Big],$$

the equation $\Delta(\phi,\tau_d) = 0$ has two parts $\phi_\pm(\tau_d;\theta)$ whenever $\tau_d > \tau_d^\sharp$, and $\Delta < 0$ if $\phi \in (\phi_-(\tau_d;\theta), \phi_+(\tau_d;\theta))$. The trace inside that policy space that is useful for our proof for ϕ is restricted by $|\mathrm{tr}(A)| \le 3$. Thus,

$$|\mathrm{tr}(A)| > 3 \iff \phi \notin [\phi_{\mathrm{tr}}^-, \phi_{\mathrm{tr}}^+], \qquad \phi_{\mathrm{tr}}^\pm(\tau_d;\theta) := \frac{\pm 3 - C_0(\tau_d;\theta)}{\mathcal{S}}, \quad C_0(\tau_d;\theta) := 1 + \frac{2-\tau_d}{\beta} - \frac{\kappa\theta}{\beta}\mathcal{S} - \frac{\tau_y}{\beta}\mathcal{I}.$$

Comparative statics in θ. The key boundaries move with θ as follows:
$$\frac{\partial\phi^\star}{\partial\theta} = -\frac{\kappa}{1-\beta} < 0$$
$$\frac{\partial\Delta}{\partial\theta} = \frac{\kappa(1-\tau_d)}{\beta^3}\big[\beta\sigma - \mathcal{S}(1+\sigma\phi)\big], \quad \Rightarrow \begin{cases} \text{for small } \phi:\ \Delta = 0 \text{ shifts left,} \\ \text{for large } \phi:\ \Delta = 0 \text{ shifts right,} \end{cases}$$
$$\frac{\partial\phi_{\mathrm{tr}}^\pm}{\partial\theta} = \frac{\kappa}{\beta} > 0 \quad \text{(the trace thresholds shift right one-for-one with } \theta\text{)}.$$

Some limits

- **As $\sigma \to 0^+$.** $\det(A) = \frac{1-\tau_d}{\beta^2}(1+\sigma\phi) \to \frac{1-\tau_d}{\beta^2}$ (virtually independent of ϕ); $C(\tau_d) \to \beta\mathcal{I}\,\frac{D^{SS}}{Y^{SS}}$, so

$$\phi^{\star}(\tau_d) \to \frac{\tau_y}{\beta \frac{D^{SS}}{Y^{SS}}} - \frac{\theta\kappa}{1-\beta}$$

and $\Delta(\phi, \tau_d)$ becomes nearly affine in ϕ.

- **As** $\kappa \to 0^+$**.** The inflation block tends to $\pi_{t+1} = (1/\beta)\pi_t$, yielding one unstable root $\lambda_\pi \to 1/\beta > 1$. Creating the second unstable root becomes difficult unless τ_d is small. Moreover, all θ-slopes are proportional to κ (e.g., $\partial_\theta \phi^\star = -\kappa/(1-\beta)$, $\partial_\theta \phi^{\pm}_{\text{tr}} = \kappa/\beta$), hence the determinacy boundaries become close to independent of θ as $\kappa \to 0^+$. On the flip side, $\kappa \uparrow$ thus seems to increase the relevance of θ.

We now provide the derivations related to the previous statement.

PART 1: Now, the inequality of interest defined on the elements of the characteristic polynomial is given by:

$$-1 + \Gamma_2 - \Gamma_1 + \Gamma_0 \;=\; -\Big(1 + \text{tr}(A) + S_2(A) + \det(A)\Big).$$

Thus, it is sufficient to show that:

$$1 + \text{tr}(A) + S_2(A) + \det(A) \;>\; 0,$$

where $S_2(A)$ is defined as the sum of all determinants of the (2×2) minors of the full model matrix:

$$S_2(A) \equiv \underbrace{(a_{11}a_{22} - a_{12}a_{21})}_{M_1} + \underbrace{(a_{11}a_{33} - a_{13}a_{31})}_{M_2} + \underbrace{(a_{22}a_{33} - a_{23}a_{32})}_{M_3}.$$

From the earlier derivations, we know that the determinant is given by:

$$\det(A) = \frac{1-\tau_d}{\beta^2}\,(1+\sigma\phi).$$

Therefore, under $\tau_d \in [0,1]$ and $\phi > -1/\sigma$, we have

$$\det(A) \;\geq 0, \qquad \text{so} \qquad -\det(A) \;=\; \Gamma_0 \;\leq 0.$$

Consider now the 2×2 minor defined on the output-inflation sub-system:

$$M_1 \;\equiv\; \det\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \;=\; a_{11}a_{22} - a_{12}a_{21}.$$

Using the entries above,

$$M_1 = \frac{1}{\beta}\Big(1 + \phi\mathcal{S} - \frac{\kappa\theta}{\beta}\mathcal{S} - \frac{\tau_y}{\beta}\mathcal{T}\Big) \;-\; \Big(\frac{\theta}{\beta}\mathcal{S}\Big)\Big(-\frac{\kappa}{\beta}\Big)$$
$$= \frac{1}{\beta}\Big(1 + \phi\mathcal{S} - \frac{\tau_y}{\beta}\mathcal{T}\Big),$$

that is, all terms related to θ cancel out.

Now impose that $\omega \in (0,1]$, which implies $\mathcal{T} = (1-\beta\omega)(1-1/\omega) \leq 0$. With $\tau_y \geq 0$, we obtain

$$-\frac{\tau_y}{\beta}\mathcal{T} \;\geq\; 0, \qquad \Rightarrow \qquad M_1 \;\geq\; \frac{1+\phi\mathcal{S}}{\beta}.$$

Moreover, since $\mathcal{S} = \sigma + \frac{D^{SS}}{Y^{SS}}\mathcal{T} \leq \sigma$ when $\mathcal{T} \leq 0$, we have

$$\phi > -1/\sigma \quad \Longrightarrow \quad 1 + \phi\,\mathcal{S} \;\geq\; 1 + \phi\,\sigma \;>\; 0.$$

Therefore

$$\boxed{M_1 > 0}.$$

Recall

$$S_2(A) = \underbrace{(a_{11}a_{22} - a_{12}a_{21})}_{M_1} + \underbrace{(a_{11}a_{33} - a_{13}a_{31})}_{M_2} + \underbrace{(a_{22}a_{33} - a_{23}a_{32})}_{M_3}.$$

We have $M_3 = a_{22}a_{33} \geq 0$ because $a_{22} = \frac{1}{\beta} > 0$ and $a_{33} = \frac{1-\tau_d}{\beta} \geq 0$. Hence,

$$S_2(A) \;=\; M_1 + M_2 + M_3 \;\geq\; M_1 + M_3 \;>\; 0.$$

We therefore know that:

$$\boxed{S_2(A) > 0}.$$

Finally, we combine the pieces:

$$1 + \mathrm{tr}(A) + S_2(A) + \det(A) \;\geq\; 1 \;+\; S_2(A) \;>\; 1,$$

where we used $S_2(A) > 0$ and $\det(A) \geq 0$.Therefore

$$-\left(1 + \mathrm{tr}(A) + S_2(A) + \det(A)\right) \;<\; -1 \;<\; 0.$$

And by this result we know that:

$$\boxed{-1 + \Gamma_2 - \Gamma_1 + \Gamma_0 \;<\; 0 \quad \text{for all} \quad \phi \in (-1/\sigma, \infty), \;\; \tau_d \in [0,1],}$$

Hence, the inequality

$$-1 + \Gamma_2 - \Gamma_1 + \Gamma_0 > 0$$

is never satisfied.

We can therefore focus on the second and third sets of inequalities, respectively.

—

PART 2: Now, we characterize when $1 + \Gamma_2 + \Gamma_1 + \Gamma_0 > 0$, which is a part of the second and third sets of conditions.

We work on the restricted parameter set $\phi \in (-1/\sigma, \infty)$, $\tau_d \in [0, 1]$, as defined before.

Recall that we defined the variables

$$\mathcal{T} \equiv (1 - \beta\omega)\left(1 - \tfrac{1}{\omega}\right), \qquad \mathcal{S} \equiv \sigma + \frac{D^{SS}}{Y^{SS}}\mathcal{T}.$$

Under $\omega \in (0, 1]$ we have $\mathcal{T} \leq 0$, and we additionally assumed that $\mathcal{S} > 0$.

Recall from before that we have

$$\boxed{\det(I\lambda - A) \; = \; \frac{1}{\beta^2}\Big(C(\tau_d)\,[\,\theta\kappa + (1-\beta)\phi\,] \; - \; \tau_y\,\mathcal{T}\,(1-\beta)\,\Big),}$$

where

$$\boxed{C(\tau_d) \; \equiv \; \sigma(\tau_d - (1-\beta)) + \beta\,\mathcal{T}\,\frac{D^{SS}}{Y^{SS}} = \sigma\Big(\tau_d - (1-\beta)\Big) + \beta\,\mathcal{T}\,\frac{D^{SS}}{Y^{SS}}.}$$

Therefore,

$$1 + \Gamma_2 + \Gamma_1 + \Gamma_0 > 0 \quad \iff \quad C(\tau_d)\,[\theta\kappa + (1-\beta)\phi] \; > \; \tau_y\,\mathcal{T}\,(1-\beta).$$

A direct solution for the parametric boundaries cannot be found, but in line with Rachel and Ravn (2025), we can restrict the severity of the conditions in the (τ_d, ϕ)-space.

Introduce the *vertical* boundary (zero-line of C):

$$\boxed{\tau_d^0 \; \equiv \; (1-\beta) - \frac{\beta\,\mathcal{T}}{\sigma}\,\frac{D^{SS}}{Y^{SS}} \; = \; 1 - \beta - \frac{\beta\,\mathcal{T}}{\sigma}\,\frac{D^{SS}}{Y^{SS}}.}$$

Since $\mathcal{T} \leq 0$ and $\frac{D^{SS}}{Y^{SS}} \geq 0$, we have $\tau_d^0 \geq (1-\beta)$. Moreover

$$C(1) = \sigma(1 - (1-\beta)) + \beta\mathcal{T}\frac{D^{SS}}{Y^{SS}} = \beta\Big(\sigma + \mathcal{T}\frac{D^{SS}}{Y^{SS}}\Big) = \beta\,\mathcal{S},$$

so if $\mathcal{S} > 0$, then $C(\tau_d)$ is *strictly increasing in* τ_d, negative for $\tau_d < \tau_d^0$ and positive for $\tau_d > \tau_d^0$, with a single crossing at $\tau_d^0 \in [0,1]$.

For $C(\tau_d) \neq 0$ we can solve for the zero-boundary for ϕ as a function of τ_d:

$$\boxed{\phi^\star(\tau_d) \;=\; \frac{\tau_y \mathcal{T}}{C(\tau_d)} \;-\; \frac{\theta\kappa}{(1-\beta)}.}$$

Note: $\dfrac{d\phi^\star}{d\tau_d} = -\tau_y \mathcal{T}\,\sigma / C(\tau_d)^2 \geq 0$ because $\tau_y \geq 0$, $\sigma > 0$, $\mathcal{T} \leq 0$, hence $\phi^\star(\tau_d)$ is *monotone increasing* in τ_d. It has a vertical asymptote at $\tau_d = \tau_d^0$ (since $C(\tau_d^0) = 0$); with $\mathcal{T} < 0$ we have

$$\lim_{\tau_d \nearrow \tau_d^0} \phi^\star(\tau_d) = +\infty, \qquad \lim_{\tau_d \searrow \tau_d^0} \phi^\star(\tau_d) = -\infty.$$

Inequality region: Because the characteristic polynomial is $\frac{1}{\beta^2}$ times an affine function of ϕ with τ_d-dependent slope $C(\tau_d)(1-\beta)$, the sign pattern is:

$$p(1) > 0 \iff \begin{cases} \phi \;>\; \phi^\star(\tau_d), & \text{if } C(\tau_d) > 0\,,\ (\tau_d > \tau_d^0), \\ \\ \phi \;<\; \phi^\star(\tau_d), & \text{if } C(\tau_d) < 0\,,\ (\tau_d < \tau_d^0). \end{cases}$$

On the set $C(\tau_d) = 0$ the characteristic polynomial is

$$-\frac{\tau_y \mathcal{T}\,(1-\beta)}{\beta^2},$$

which is > 0.

In sum, the set of parameters for which we have a determinate solution is defined through both an *increasing* curve $\phi^\star(\tau_d)$, and the vertical line $\tau_d = \tau_d^0$.

Summary: In sum, we can provide the following parametric boundaries implied by the first restriction on the second and third sets of conditions sufficient to postulate that the system is saddle-path stable:

$$\boxed{\begin{array}{l}
\text{Define} \quad \tau_d^0 = 1-\beta - \dfrac{\beta\mathcal{T}}{\sigma}\dfrac{D^{SS}}{Y^{SS}}, \qquad \phi^\star(\tau_d) = \dfrac{\tau_y \mathcal{T}}{\sigma(\tau_d - 1 + \beta) + \beta\mathcal{T}\,\frac{D^{SS}}{Y^{SS}}} - \dfrac{\theta\kappa}{1-\beta}. \\[2ex]
\text{On } (\phi, \tau_d) \in (-1/\sigma, \infty) \times [0,1]: \quad 1 + \Gamma_2 + \Gamma_1 + \Gamma_0 > 0 \iff \begin{cases} \phi > \phi^\star(\tau_d), & \tau_d > \tau_d^0, \\ \phi < \phi^\star(\tau_d), & \tau_d < \tau_d^0. \end{cases} \\[2ex]
\text{If } \mathcal{S} > 0 \text{ and } \mathcal{T} < 0 \text{ (typical), then } \phi^\star(\tau_d) \text{ is increasing with a vertical asymptote at } \tau_d^0.
\end{array}}$$

—

PART 3: In the last part, two distinctions must be made:

Case 1: $\Gamma_0(\Gamma_0 - \Gamma_2) + \Gamma_1 - 1 > 0$.

A convenient closed form for $S_2(A)$ is

$$S_2(A) = \frac{1}{\beta^2}\Big[\,\beta\,\frac{D^{SS}}{Y^{SS}}\mathcal{T}\,\phi - \mathcal{T}\,\tau_y - \beta\,\phi\sigma\,\tau_d + 2\beta\,\phi\sigma - \beta\,\tau_d + 2\beta + \kappa\sigma\theta(\tau_d - 1) - \tau_d + 1\Big].$$

We define now Δ as a function in ϕ and τ_d:

$$\Delta(\phi, \tau_d) \;\equiv\; 1 - S_2(A) + \mathrm{tr}(A)\ \det(A) - \det(A)^2.$$

Quadratic form in ϕ. For each $\tau_d \in [0, 1]$, Δ is a quadratic polynomial in ϕ:

$$\boxed{\Delta(\phi, \tau_d) = \frac{1}{\beta^4}\Big[\,\alpha(\tau_d)\,\phi^2 \;+\; b(\tau_d;\theta)\,\phi \;+\; c(\tau_d;\theta)\,\Big]\,,}$$

with the *leading coefficient* simplifying to

$$\boxed{\alpha(\tau_d) = \sigma(1 - \tau_d)\Big[\beta^2\,\frac{D^{SS}}{Y^{SS}}\,\mathcal{T} \;+\; \sigma(\beta^2 - (1 - \tau_d))\Big]\,.}$$

Two useful facts follow immediately:

- Since $1 - \tau_d \geq 0$ and $\mathcal{T} \leq 0$, there exists a mild threshold

$$\boxed{\tau_d > \tau_d^{\sharp} \;\equiv\; 1 - \left[\beta^2 + \frac{\beta^2\,\frac{D^{SS}}{Y^{SS}}\,\mathcal{T}}{\sigma}\right]}$$

  such that $\alpha(\tau_d) > 0$ for all $\tau_d > \tau_d^{\sharp}$ (note $\tau_d^{\sharp} \lesssim 1 - \beta^2$ and is typically small). When $\alpha(\tau_d) > 0$, $\Delta(\phi, \tau_d)$ is *convex* in ϕ.

- On that range, the boundary $\Delta = 0$ is given by:

$$\boxed{\phi_{\pm}(\tau_d;\theta) = \frac{-\,b(\tau_d;\theta) \;\pm\; \sqrt{b(\tau_d;\theta)^2 - 4\,\alpha(\tau_d)\,c(\tau_d;\theta)}}{2\,\alpha(\tau_d)}\,,}$$

  and the inequality $\Delta(\phi, \tau_d) < 0$ holds *between* the two curves:

$$\boxed{\alpha(\tau_d) > 0 \;\Rightarrow\; \Delta < 0 \;\Longleftrightarrow\; \phi_-(\tau_d;\theta) \;<\; \phi \;<\; \phi_+(\tau_d;\theta)\,.}$$

(If $\alpha(\tau_d) < 0$ on a small subset near $\tau_d = 0$, the inequality reverses.)

For compactness we separate the θ-dependence of the linear and constant coefficients:

$$b(\tau_d;\theta) = b_0(\tau_d) \ + \ \theta\, b_1(\tau_d), \qquad c(\tau_d;\theta) = c_0(\tau_d) \ + \ \theta\, c_1(\tau_d),$$

such that we can express the θ-related slopes as:

$$\boxed{b_1(\tau_d) = -\frac{\kappa\,\sigma}{\beta^3}\,(1-\tau_d)\,\mathcal{S}\,, \qquad c_1(\tau_d) = \frac{\kappa}{\beta^3}\,(1-\tau_d)\left[\beta\,\sigma - \mathcal{S}\right],}$$

while $b_0(\tau_d)$ and $c_0(\tau_d)$ collect the remaining θ-free terms (they are affine in τ_d and involve $\sigma, \beta, \mathcal{I}, \tau_y, \frac{D^{SS}}{Y^{SS}}$).

Monotone effect of θ on the region $\Delta < 0$. The previously defined function Δ co-moves with θ as follows:

$$\boxed{\frac{\partial \Delta}{\partial \theta} = \frac{\kappa\,(1-\tau_d)}{\beta^3}\left[\beta\,\sigma - \mathcal{S}\,(1+\sigma\phi)\right].}$$

Hence, for any fixed (ϕ, τ_d), we can find the following link between θ and $\phi_0^\star$:

$$\begin{cases} \dfrac{\partial \Delta}{\partial \theta} > 0 \quad \Longleftrightarrow \quad \phi \ < \ \phi_\theta^\star \ \equiv \ \frac{\beta}{\mathcal{S}} \ - \ \frac{1}{\sigma}, \\ \dfrac{\partial \Delta}{\partial \theta} < 0 \quad \Longleftrightarrow \quad \phi \ > \ \phi_\theta^\star. \end{cases}$$

(Here $\phi_\theta^\star$ is independent of τ_d; note $\mathcal{S} = \sigma + \frac{D^{SS}}{Y^{SS}}\mathcal{I} \le \sigma$ under $\mathcal{I} \le 0$.)

Therefore, *Increasing θ shifts the $\Delta = 0$ boundary* down *in ϕ for large ϕ, and* up *in ϕ for small ϕ.*

Equivalently, the values for which $\Delta < 0$ *expand* toward larger ϕ when $\phi < \phi_\theta^\star$, and *shrink* when $\phi > \phi_\theta^\star$.

Edges of the admissible parameter space. A special case can be quite illustrative:

Passive-fiscal edge $\tau_d = 1$. Here $\det(A) = 0$ and

$$\Delta(\phi, 1) = 1 - S_2(A) \ = \ -\frac{m + \mathcal{S}\,\phi}{\beta} + \frac{\mathcal{I}\,\tau_y}{\beta^2}.$$

Thus

$$\boxed{\Delta(\phi, 1) < 0 \iff \phi \ > \ -\frac{m}{\mathcal{S}} \ - \ \frac{\mathcal{I}\,\tau_y}{\beta\,\mathcal{S}}\,.}$$

Because $\mathcal{I} \le 0$ and $\tau_y \ge 0$, the second term is non-positive, making the condition even *less* restrictive than $\phi > -m/\mathcal{S}$ (a mild negative lower bound close to $-m/\sigma$).

Summary:

$$\boxed{\begin{array}{l}\text{The inequality } \Gamma_0(\Gamma_0-\Gamma_2)+\Gamma_1-1>0 \text{ holds exactly where } \Delta(\phi,\tau_d)<0.\\ \text{For most } \tau_d \text{ (specifically } \tau_d>\tau_d^{\#}\text{)}, \Delta(\cdot,\tau_d) \text{ is convex in } \phi,\\ \qquad \text{so the feasible region is the band } \phi_-(\tau_d;\theta)<\phi<\phi_+(\tau_d;\theta).\\ \text{The band shifts with } \theta \text{ according to } \partial_\theta\Delta=\dfrac{\kappa(1-\tau_d)}{\beta^3}[\beta\sigma-\mathcal{S}(1+\sigma\phi)]:\\ \qquad \text{increasing } \theta \text{ loosens (tightens) the inequality for } \phi<\phi_\theta^\star \ (\phi>\phi_\theta^\star).\end{array}}$$

—

Case 2: $\Gamma_0(\Gamma_0-\Gamma_2)+\Gamma_1-1<0$ AND $|\Gamma_2|>3$.

As for the first part, $\Gamma_0(\Gamma_0-\Gamma_2)+\Gamma_1-1<0$, the inverse of the previous statement holds true. As for the second part, the restriction that $|\operatorname{tr}(A)|>3$ is more restrictive. Recall that $\operatorname{tr}(A)=C_0(\tau_d;\theta)+\mathcal{S}\,\phi$, which is an affine function of ϕ. Thus, define

$$\phi_{\mathrm{tr}}^{+}(\tau_d;\theta) \equiv \frac{3-C_0(\tau_d;\theta)}{\mathcal{S}}, \qquad \phi_{\mathrm{tr}}^{-}(\tau_d;\theta) \equiv \frac{-3-C_0(\tau_d;\theta)}{\mathcal{S}}.$$

Since $\mathcal{S}>0$,

$$\boxed{|\operatorname{tr}(A)|>3 \iff \phi>\phi_{\mathrm{tr}}^{+}(\tau_d;\theta) \text{ or } \phi<\phi_{\mathrm{tr}}^{-}(\tau_d;\theta).}$$

Because $C_0(\tau_d;\theta)=C_0(\tau_d;0)-(\kappa\theta/\beta)\mathcal{S}$,

$$\frac{\partial\phi_{\mathrm{tr}}^{\pm}}{\partial\theta}=\frac{-(\partial C_0/\partial\theta)}{\mathcal{S}}=\frac{\kappa}{\beta} > 0,$$

so both thresholds shift to the right as θ increases.

Comparative statics in θ. Recall that:

$$\frac{\partial\Delta}{\partial\theta}=\frac{\kappa(1-\tau_d)}{\beta^3}\big[\beta\sigma-\mathcal{S}\,(1+\sigma\phi)\big].$$

Thus, for given τ_d,

$$\frac{\partial\Delta}{\partial\theta}\begin{cases}>0, & \phi<\phi_\theta^\star\equiv\dfrac{\beta}{\mathcal{S}}-\dfrac{1}{\sigma},\\ <0, & \phi>\phi_\theta^\star.\end{cases}$$

To keep $\Delta>0$ when θ rises, ϕ must increase; i.e. $\phi_+(\tau_d;\theta)$ shifts right with θ. Since also $\partial_\theta\phi_{\mathrm{tr}}^{+}=\kappa/\beta>0$, the boundary $\Phi^\star(\tau_d;\theta)$ moves *right* as θ increases.

Summary: Within $\phi\in(-1/\sigma,\infty)$, $\tau_d\in[0,1]$ and standard calibration ($\mathcal{S}>0$), the set where

$$\Gamma_0(\Gamma_0-\Gamma_2)+\Gamma_1-1<0 \quad\text{and}\quad |\Gamma_2|>3$$

is fulfilled is a wedge

$$\phi > \Phi^\star(\tau_d;\theta) = \max\{\phi_+(\tau_d;\theta),\ \phi_{\rm tr}^+(\tau_d;\theta),\ -1/\sigma\}, \quad \tau_d > \tau_d^\sharp,$$

and this wedge *shrinks* (shifts to larger ϕ) as θ increases. Simply put, more inflation-indexed debt (higher θ) places additional restrictions on the admissible (ϕ, τ_d) space *from below*.

■

E Additional empirical results

First, we replicate the results from section 5 without controlling for the central bank board index of Romelli (2024), which is closely related to the central bank independence index itself.

Dep. var.	Central Bank Independence Index (Romelli, 2024)							
Fiscal Rule Intensity	0.0456***	0.0511***	0.0129***	0.00170	0.0502***	0.1039***	0.0102**	-0.00030
	(0.0061)	(0.0066)	(0.0031)	(0.0029)	(0.0063)	(0.0069)	(0.0033)	(0.0029)
Indexed debt share	-0.1845*	-0.2348**	0.2445***	-0.0057	-0.3400***	0.9565***	0.2276**	-0.0638
	(0.0726)	(0.0734)	(0.0574)	(0.0449)	(0.0867)	(0.1269)	(0.0712)	(0.0513)
FisRules × IndexDebt			-0.1095***			-0.8851***	-0.1155***	
			(0.0260)			(0.0670)	(0.0277)	
Inflation					0.0014	0.0030	-0.0022***	-0.0022***
					(0.0020)	(0.0018)	(0.0006)	(0.0006)
IndexDebt × Inflation					0.0135*	-0.0176**	0.0025	0.0051**
					(0.0068)	(0.0065)	(0.0023)	(0.0020)
Constant	0.5720***	0.5677***	0.5991***	0.5554***	0.5642***	0.4938***	0.6127***	0.5777***
	(0.0124)	(0.0130)	(0.0056)	(0.0133)	(0.0152)	(0.0144)	(0.0074)	(0.0145)
Obs.	605	605	605	605	605	605	605	605
R^2	0.0938	0.1393	0.9355	0.9494	0.1134	0.3539	0.9373	0.9506
F	31.1719	35.0722	9.2438	6.1690	19.1892	56.5439	8.9450	6.3173
R^2_{adj}	0.0908	0.0943	0.9326	0.9445	0.1075	0.3166	0.9342	0.9457
RMSE	0.1856	0.1852	0.0505	0.0458	0.1839	0.1609	0.0499	0.0454
Year-FE	No	Yes	No	Yes	No	Yes	No	Yes
Country-FE	No	No	Yes	Yes	No	No	Yes	Yes

Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table E.1: Regressions on possible predictors of central bank independence (no CB board control), following Romelli (2024).

Dep. var.	AnyR	ER	BR	DR	AnyR	ER	BR	DR
CB independence	-0.9230	-0.3724	-1.5230	0.2394	-0.4363	6.8147**	-1.2566	-0.2508
	(0.9653)	(1.9799)	(0.9818)	(1.9364)	(1.2777)	(2.3326)	(1.2063)	(2.3983)
Indexed debt share	3.0495***	4.5955***	3.1201***	0.3687	6.2171***	12.0975***	6.1895***	1.9829
	(0.6649)	(0.9761)	(0.6850)	(0.8924)	(1.1265)	(1.5091)	(1.2104)	(1.3227)
Inflation	-0.0136	-0.0173	-0.0721*	-0.1561*	-0.0167	-0.1236***	-0.1319**	-0.2138*
	(0.0144)	(0.0181)	(0.0353)	(0.0651)	(0.0812)	(0.0255)	(0.0443)	(0.0975)
Constant	-3.0944***	-4.7794***	-2.6751***	-4.1030***	-1.2264	-8.0511***	-0.4638	-1.0064
	(0.5391)	(0.8917)	(0.5229)	(1.1886)	(0.9449)	(1.7184)	(0.8612)	(1.5822)
Obs.	652	652	652	652	285	261	285	95
ll	-104.2834	-53.1646	-91.7183	-42.2181	-59.2459	-19.6852	-56.3336	-24.7103
χ^2	24.9541	80.2726	24.3913	12.9324	72.2432	90.1966	71.7882	7.4952
p	0.0000	0.0000	0.0000	0.0048	0.0000	0.0000	0.0000	0.2775
R^2	0.0973	0.2555	0.1078	0.0217	0.3528	0.6571	0.3159	0.0998
Year-FE	No	No	No	No	Yes	Yes	Yes	Yes

Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table E.2: Rule Suspension Regression (no CB board control). The outcome variables are defined as: $AnyR$ = "Any Fiscal Rule Suspended", ER = "Expenditure Rule Suspended", BR = "Budget Balancing Rule Suspended", DR = "Deficit Rule Suspended".

In addition, we now provide results *including* all countries in the analysis, even those who have not issued any inflation-indexed debt at any point in time.

Dep. var.	Central Bank Independence Index (Romelli, 2024)							
CB board index	0.5595***	0.5172***	0.6292***	0.5358***	0.5654***	0.5114***	0.6235***	0.5255***
	(0.0085)	(0.0083)	(0.0075)	(0.0077)	(0.0086)	(0.0084)	(0.0076)	(0.0078)
Fiscal Rule Intensity	0.0164***	0.0096***	0.0198***	0.0095***	0.0139***	0.0091***	0.0202***	0.0092***
	(0.0015)	(0.0015)	(0.0012)	(0.0012)	(0.0015)	(0.0016)	(0.0012)	(0.0012)
Indexed debt share	-0.1972***	-0.2759***	0.4347***	0.1838***	-0.3216***	0.0779	0.4583***	0.1565***
	(0.0389)	(0.0373)	(0.0419)	(0.0292)	(0.0470)	(0.0745)	(0.0523)	(0.0336)
FisRules × IndexDebt			-0.0903***			-0.3081***	-0.0991***	
			(0.0235)			(0.0380)	(0.0256)	
Inflation					0.0000	0.0000	-0.0000	-0.0000
					(0.0000)	(0.0000)	(0.0000)	(0.0000)
IndexDebt × Inflation					0.0123***	0.0013	-0.0013	0.0017
					(0.0029)	(0.0031)	(0.0019)	(0.0016)
Constant	0.3144***	0.3415***	0.2725***	0.2826***	0.3184***	0.3505***	0.2798***	0.2879***
	(0.0044)	(0.0043)	(0.0037)	(0.0065)	(0.0044)	(0.0044)	(0.0038)	(0.0068)
Obs.	5568	5568	5568	5568	5282	5282	5282	5282
R^2	0.5092	0.5558	0.8560	0.8769	0.5166	0.5736	0.8549	0.8769
F	1924.2808	1554.2882	2458.8637	300.8337	1127.4948	768.2458	1580.1035	279.3330
R^2_{adj}	0.5089	0.5525	0.8518	0.8724	0.5161	0.5700	0.8505	0.8723
RMSE	0.1167	0.1114	0.0641	0.0595	0.1160	0.1094	0.0645	0.0596
Year-FE	No	Yes	No	Yes	No	Yes	No	Yes
Country-FE	No	No	Yes	Yes	No	No	Yes	Yes

Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table E.3: Regressions on possible predictors of central bank independence following Romelli (2024). Zero-imputation applied to missing values of IndexedDebtShare (and used where relevant).

Dep. var.	AnyR	ER	BR	DR	AnyR	ER	BR	DR
CB independence	0.7690	-1.7672	0.7520	5.9635**	0.3999	4.5296	0.1129	3.8548
	(1.2947)	(2.3993)	(1.3706)	(2.3014)	(1.4686)	(3.1602)	(1.4680)	(2.3100)
CB board index	0.6882	3.5521	0.4808	-2.0083*	-0.9293	-1.8822	-1.1405	-2.5309*
	(0.7764)	(1.9225)	(0.8092)	(1.0236)	(0.7915)	(1.8779)	(0.8058)	(1.0608)
Indexed debt share	4.2580***	5.8665***	4.0479***	1.3608	8.7290***	12.4760***	8.1679***	4.1236**
	(0.5268)	(0.6783)	(0.5412)	(0.7609)	(1.1160)	(1.3673)	(1.0264)	(1.2765)
Inflation	-0.0334**	-0.0264	-0.0516***	-0.0604***	-0.0789**	-0.1273***	-0.1500***	-0.1609***
	(0.0122)	(0.0169)	(0.0081)	(0.0086)	(0.0295)	(0.0243)	(0.0302)	(0.0351)
Constant	-5.4227***	-6.9312***	-5.3113***	-7.8902***	-1.3167	-5.3677***	-0.8528	-2.9907*
	(0.4746)	(0.4507)	(0.5022)	(1.0866)	(0.7109)	(1.3913)	(0.6873)	(1.1815)
Obs.	5329	5329	5329	5329	1771	1624	1771	590
ll	-282.8372	-99.4380	-259.3315	-163.5302	-154.6840	-50.1877	-147.8522	-92.4499
χ^2	105.5899	173.3730	95.7775	140.8960	169.8088	265.3820	203.2484	35.2012
p	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
R^2	0.1031	0.2754	0.0991	0.0650	0.3860	0.5526	0.3604	0.1794
Year-FE	No	No	No	No	Yes	Yes	Yes	Yes

Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table E.4: Rule Suspension Regression (all observations; zero-imputation applied where needed to include all rows). The outcome variables are defined as: $AnyR$ = "Any Fiscal Rule Suspended", ER = "Expenditure Rule Suspended", BR = "Budget Balancing Rule Suspended", DR = "Deficit Rule Suspended".

E.1 Rule suspension regressions against lagged levels of indexed debt

Dep. var.	AnyR	ER	BR	DR	AnyR	ER	BR	DR
CB independence	-0.9616	-0.2181	-1.5826	0.1545	-0.4310	7.7258***	-1.2557	-0.2000
	(0.9171)	(1.9145)	(0.9429)	(1.9678)	(1.2830)	(2.2862)	(1.2018)	(2.4362)
L. Indexed debt share	2.9801***	4.5868***	3.1124***	0.5533	6.4393***	12.5273***	6.0911***	2.0002
	(0.6474)	(0.9392)	(0.6645)	(0.8735)	(1.1373)	(1.6212)	(1.1368)	(1.1816)
L. Inflation	-0.0095	-0.0130	-0.0756*	-0.1814***	0.0346	-0.0858**	-0.1113**	-0.1331**
	(0.0198)	(0.0253)	(0.0356)	(0.0541)	(0.1189)	(0.0274)	(0.0409)	(0.0507)
Constant	-3.0050***	-4.8858***	-2.5595***	-3.9959***	-1.1878	-9.2778***	-0.6361	-1.6762
	(0.5224)	(0.8759)	(0.5040)	(1.1948)	(0.9295)	(1.7401)	(0.9002)	(1.5650)
Obs.	632	632	632	632	286	262	286	96
ll	-106.3518	-52.3364	-93.5731	-41.7471	-58.9689	-18.6168	-57.3783	-25.2193
χ^2	24.9385	81.4065	24.3770	24.1283	71.4016	75.5276	79.6409	11.7768
p	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0671
R^2	0.0963	0.2622	0.1109	0.0270	0.3717	0.6760	0.3234	0.0841
Year-FE	No	No	No	No	Yes	Yes	Yes	Yes

Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Table E.5: Rule Suspension Regression with lagged regressors. Specification includes *L*.Indexed debt share and *L*.Inflation. Outcomes: *AnyR* (Any Rule Suspended), *ER* (Expenditure Rule), *BR* (Budget-Balance Rule), *DR* (Deficit Rule). Columns 1–4 exclude year fixed effects; columns 5–8 include year fixed effects (coefficients not shown).

Dep. var.	AnyR	ER	BR	DR	AnyR	ER	BR	DR
CB independence	-0.0808	3.7843*	-0.5463	0.2099	-0.0274	8.6885**	-0.7570	0.0139
	(1.1800)	(1.8603)	(1.2741)	(2.1038)	(1.9263)	(2.7690)	(1.8040)	(3.3163)
L. Indexed debt share	1.7687*	3.1557***	1.8284*	0.6844	3.2555**	7.4810***	2.8992**	2.3885
	(0.7415)	(0.9130)	(0.8265)	(0.9016)	(1.1305)	(1.7710)	(1.1188)	(1.7064)
L. Inflation	-0.0045	-0.0055	-0.0376	-0.1959**	0.0430	0.0052	-0.0569*	-0.1041
	(0.0393)	(0.0605)	(0.0283)	(0.0723)	(0.0534)	(0.0298)	(0.0277)	(0.0548)
Lagged Rule	4.2898***	5.2069***	4.5895***	4.9449***	0.0000	0.0000	0.0000	0.0000
	(0.5175)	(0.8183)	(0.5701)	(0.8629)	(.)	(.)	(.)	(.)
Constant	-4.1893***	-8.3652***	-3.9760***	-4.7069***	-3.2621	-9.3682***	-2.5648	-1.8548
	(0.7992)	(1.2691)	(0.8350)	(1.3290)	(1.6873)	(2.0013)	(1.5199)	(1.9630)
Obs.	632	632	632	632	286	48	286	48
ll	-70.8248	-28.8303	-59.8161	-29.0105	-28.9850	-7.1589	-28.0778	-12.5956
χ^2	81.0217	66.7653	75.5655	47.9633	.	29.0637	.	5.1460
p	0.0000	0.0000	0.0000	0.0000	.	0.0000	.	0.2726
R^2	0.3982	0.5936	0.4317	0.3238	0.6912	0.4800	0.6689	0.0852
Year-FE	No	No	No	No	Yes	Yes	Yes	Yes

Robust standard errors in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. "LDV" denotes the lag of the column's dependent variable; dots (.) indicate omission.

Table E.6: Rule Suspension Regression with lagged dependent variable (LDV), lagged indexed debt share, and lagged inflation (no interaction terms). Outcomes: *AnyR* (Any Rule Suspended), *ER* (Expenditure Rule), *BR* (Budget-Balance Rule), *DR* (Deficit Rule). Columns 1–4 exclude year fixed effects; columns 5–8 include year fixed effects (coefficients not shown).