NORMAL DISTRIBUTION

NORMAL DISTRIBUTION

MATHEMATICAL NOTATION

$$f(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Where:

- x is the random variable.
- μ is the mean of the distribution.
- σ is the standard deviation of the distribution.

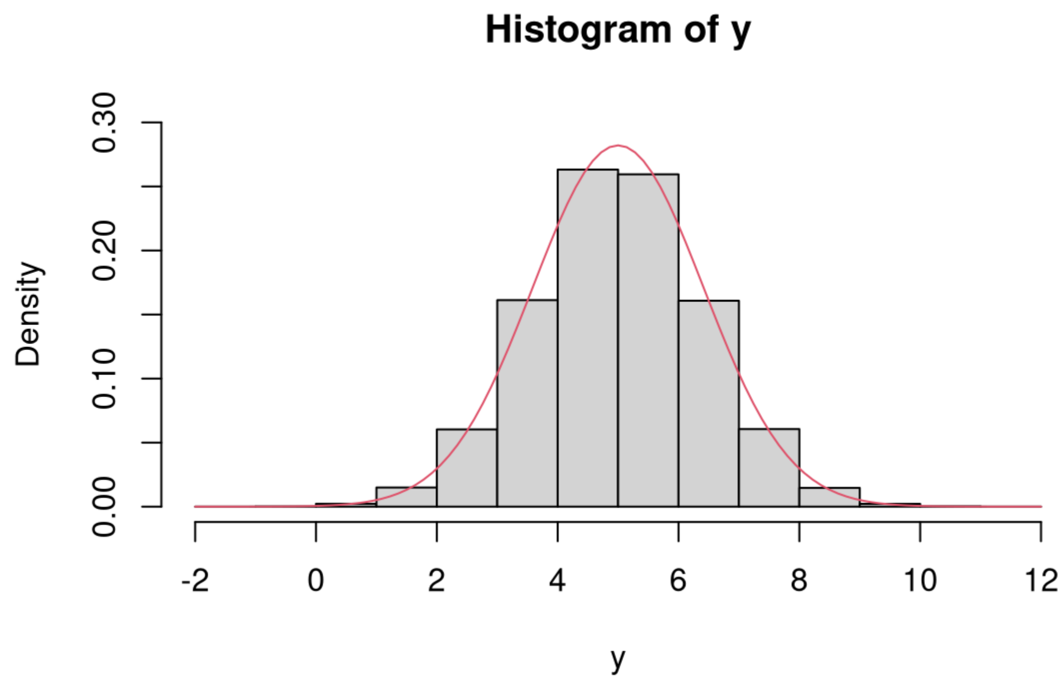
SAMPLE FROM NORMAL DISTRIBUTION

- generate a sample from a Gaussian distribution:

  `rnorm(n, mean = 0, sd = 1)`

Example:

```
y <- rnorm(n = 100000, mean = 5, sd = sqrt(2))
hist(y, freq = F, ylim = c(0, 0.3))
curve(dnorm(x, mean = 5, sd = sqrt(2)), col = 2, add = T)
```


Histogram of y
Density
0.00
0.10
0.20
0.30
-2
0
2
4
6
8
10
12
y

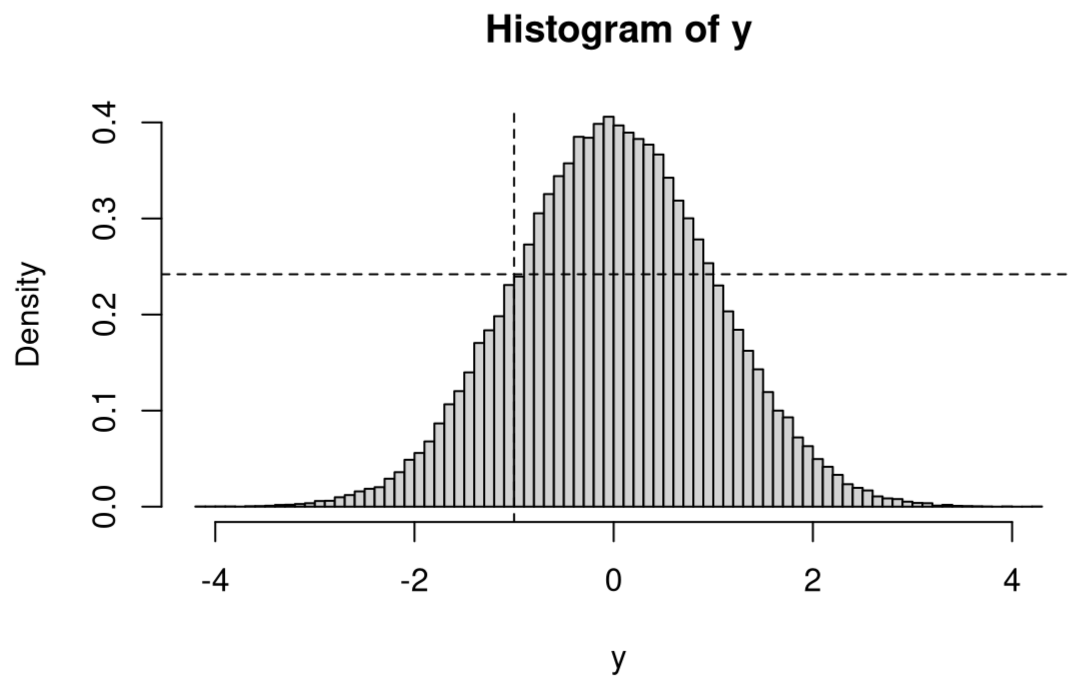
DENSITY FUNCTION (PDF) OF NORMAL DISTRIBUTION

Calculate pdf of Normal distribution:

- `dnorm(x, mean = 0, sd = 1)`

```r
y <- rnorm(n = 100000, mean = 0,sd = 1)
hist(y, freq=F, ylim = c(0, 0.4),breaks = 100)
dnorm(-1)
hist(y, freq = F, ylim = c(0, 0.4), breaks = 100)
abline(v = -1, lty = 2)
abline(h = dnorm(-1), lty=2)
```

```
[1] 0.2419707
```


Histogram of y
Density
0.0
0.1
0.2
0.3
0.4
-4
-2
0
2
4
y

CUMULLATIVE DENSITY FUNCTION (CDF) OF NORMAL DISTRIBUTION

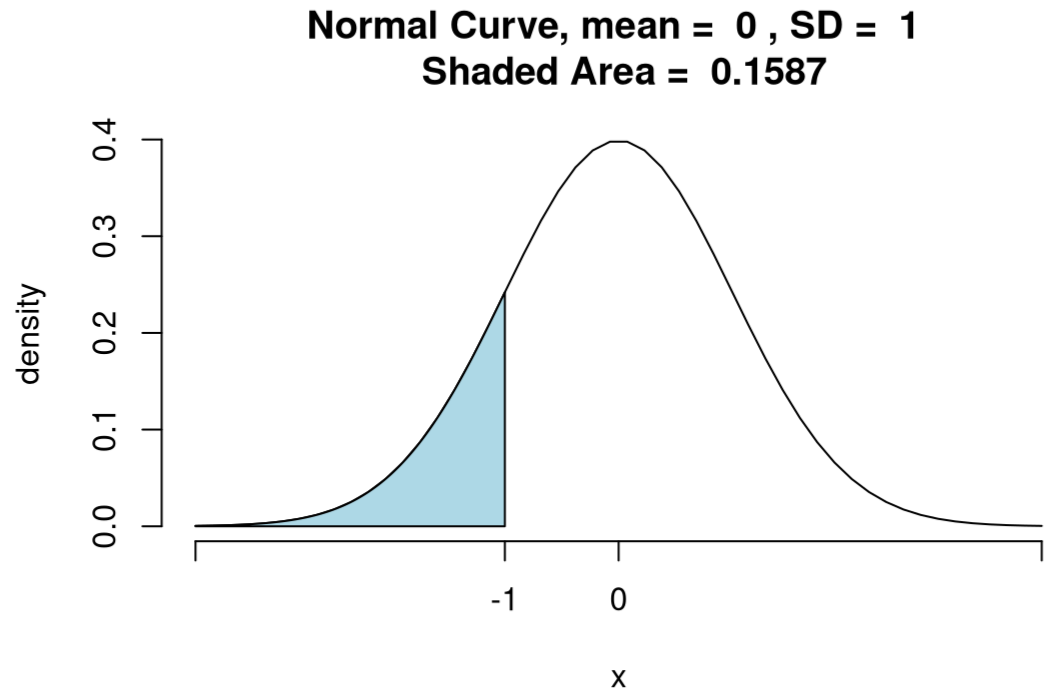
Calculate pdf of Normal distribution:

- `pnorm(q, mean = 0, sd = 1)`

```r
library(tigerstats)

pnorm(-1)
pnormGC(-1, region = "below", graph = T)
```

```
[1] 0.1586553
```

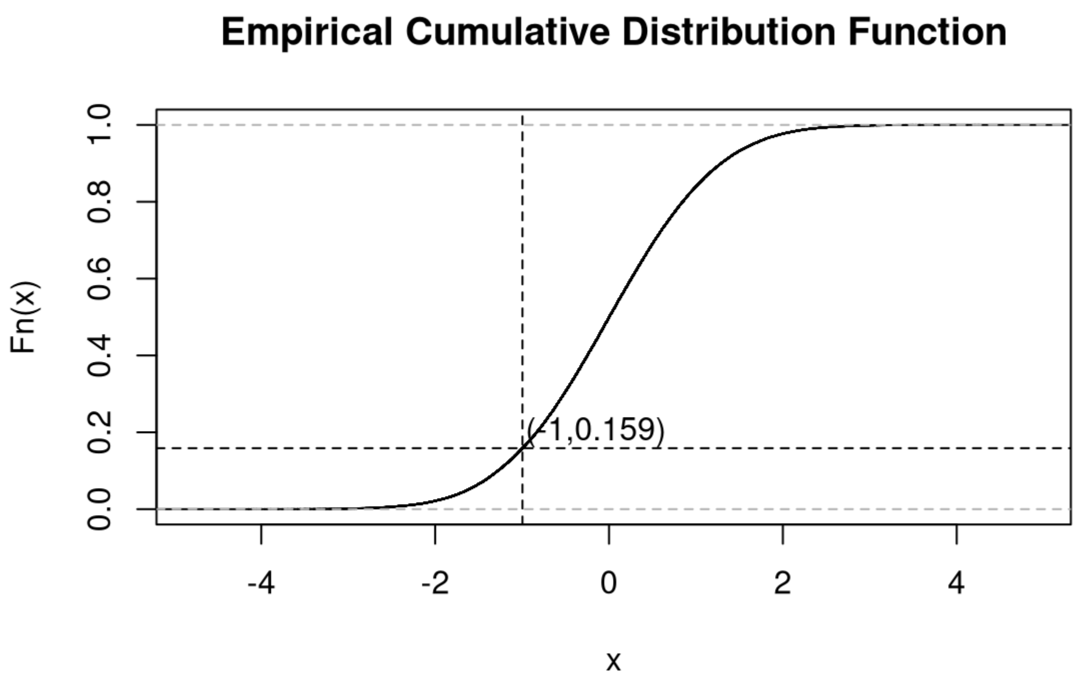

Normal Curve, mean = 0 , SD = 1
Shaded Area = 0.1587
density
0.0
0.1
0.2
0.3
0.4
-1
0
x

```
[1] 0.1586553
```

CUMULLATIVE DENSITY FUNCTION (CDF) OF NORMAL DISTRIBUTION

```
library(tigerstats)

plot(ecdf(y),main = "Empirical Cumulative Distribution Function")
abline(v= quantile(ecdf(y),0.158655254), lty = 2)
abline(h= pnorm(-1), lty = 2)
text(x = -0.15, y = 0.2,labels = "(-1,0.159)")
```



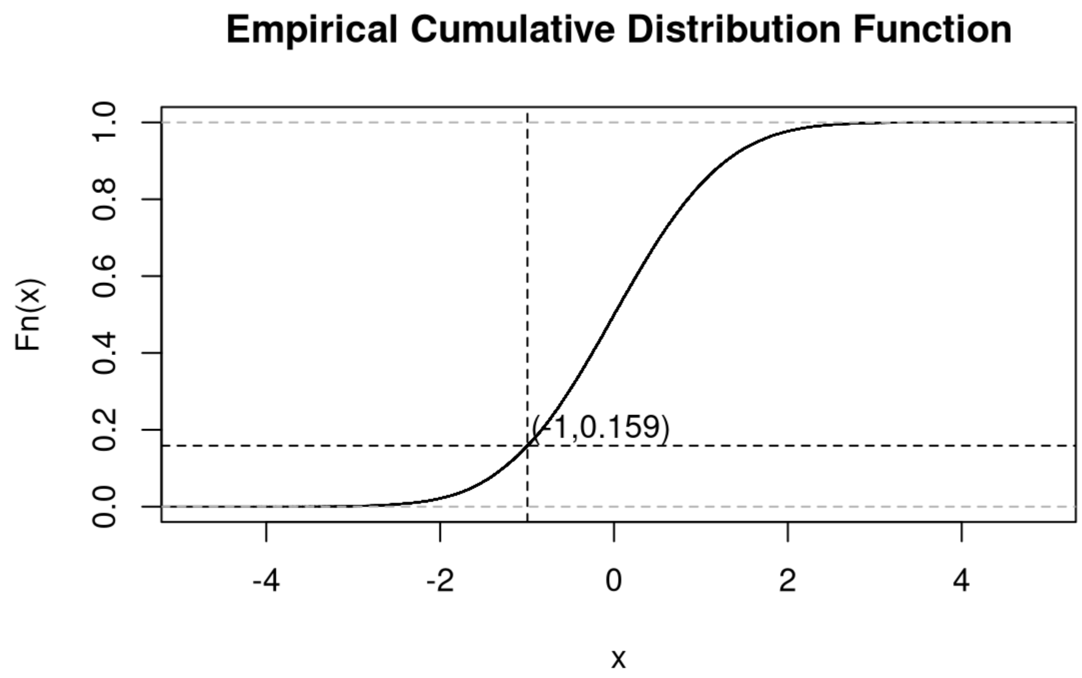
QUANTILES OF NORMAL DISTRIBUTION

- `qnorm(p, mean = 0, sd = 1)`

```
qnorm(0.158655254)
quantile(ecdf(y),0.158655254)
plot(ecdf(y), main = "Empirical Cumulative Distribution Function")
abline(v = quantile(ecdf(y), 0.158655254), lty = 2)
abline(h = pnorm(-1), lty = 2)
text(x = -0.15, y = 0.2,labels = "(-1,0.159)")
```

```
[1] -1
```

```
 15.86553%
-0.9955969
```



Empirical Cumulative Distribution Function
Fn(x)
0.0
0.2
0.4
0.6
0.8
1.0
(-1,0.159)
-4
-2
0
2
4
x

EXAMPLES

Consider $X \sim N(0, 1)$. It is very easy to compute the following probabilities with R:

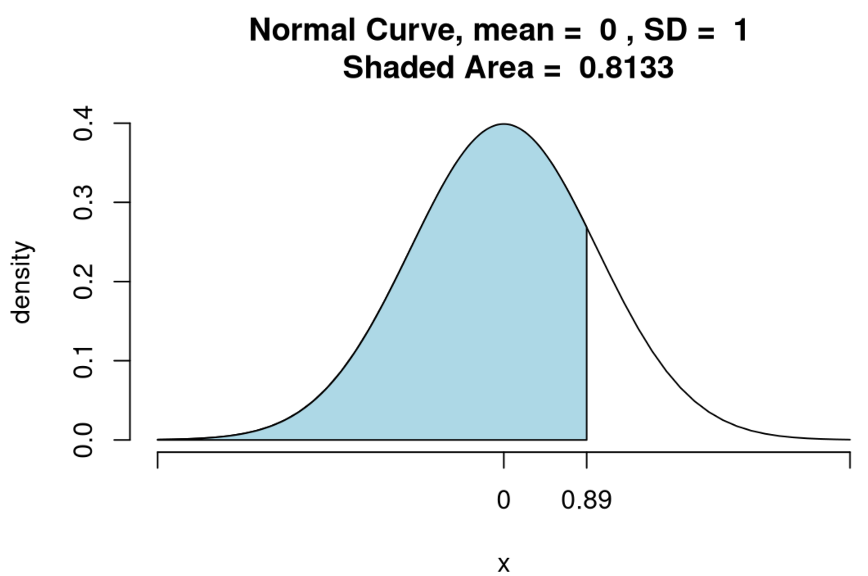
- $P(X \leq 0.89)$

```r
pnorm(0.89)
```

```
[1] 0.8132671
```

```r
pnormGC(0.89, region="below",graph=T)
```

```
[1] 0.8132671
```



- $P(X \geq 1.21)$

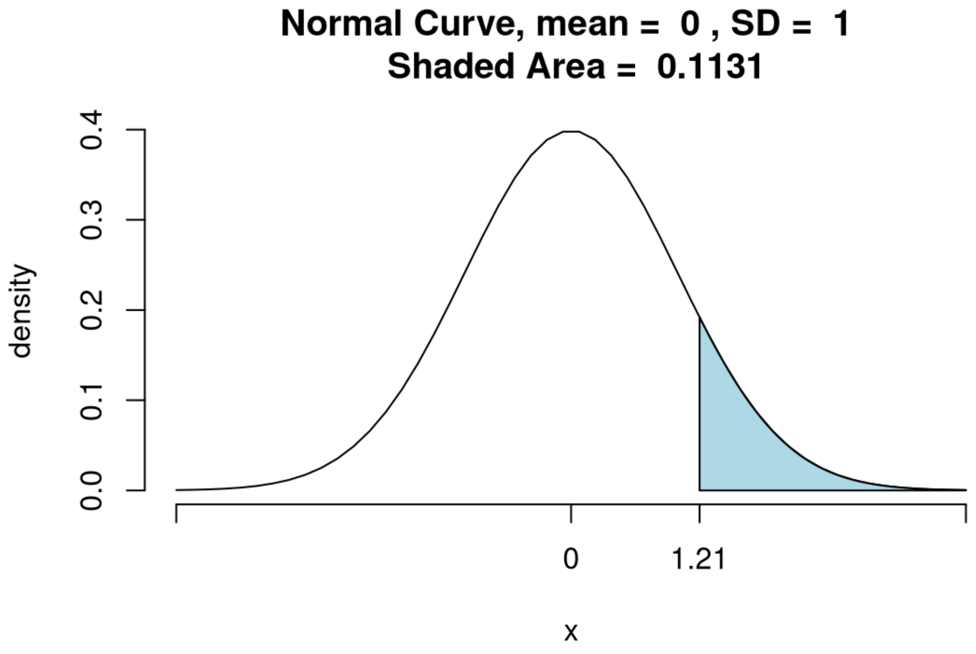
```r
#| fig-align: "center"
#| layout: [[100]]

1-pnorm(1.21)
```

```
[1] 0.1131394
```

```r
pnormGC(1.21, region="above",graph=T)
```

```
[1] 0.1131394
```



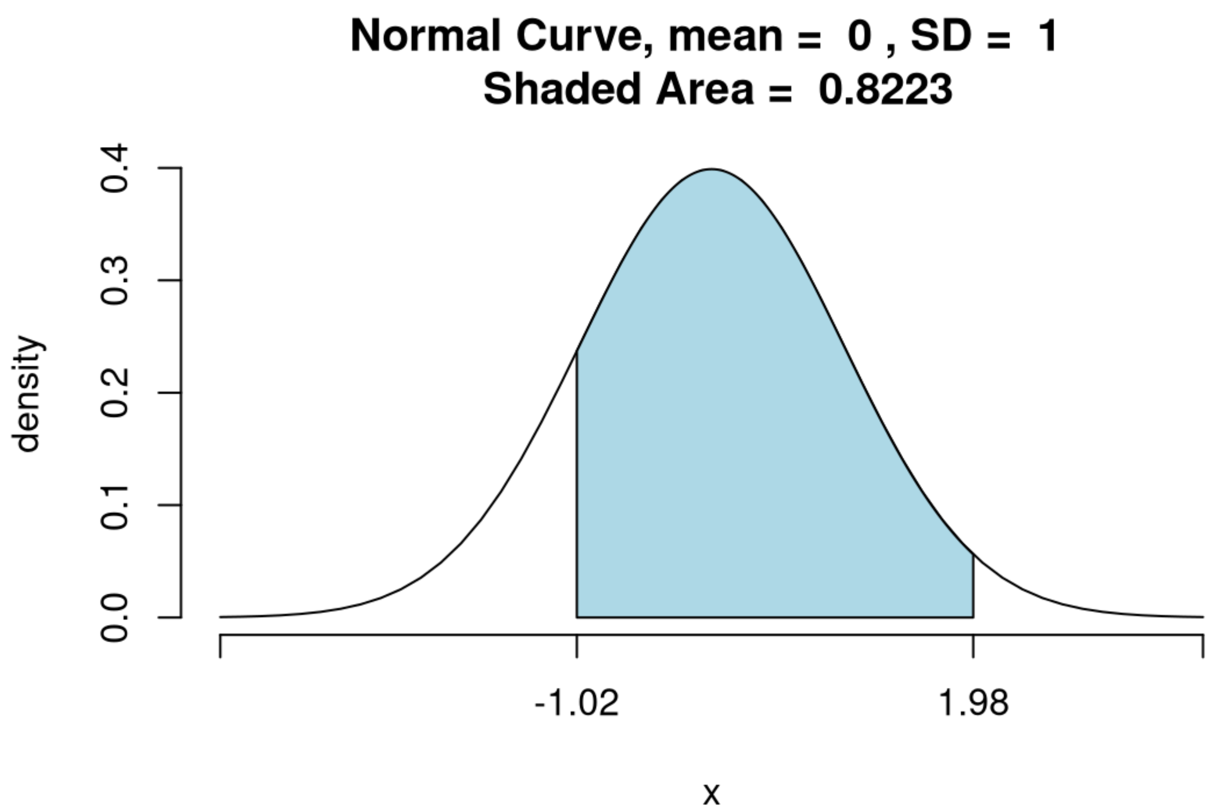
- $P(-1.02 \le X \le 1.98)$

```r
pnorm(1.98)-pnorm(-1.02)
```

```
[1] 0.822284
```

```r
pnormGC(c(-1.02,1.98), region="between",graph=T)
```

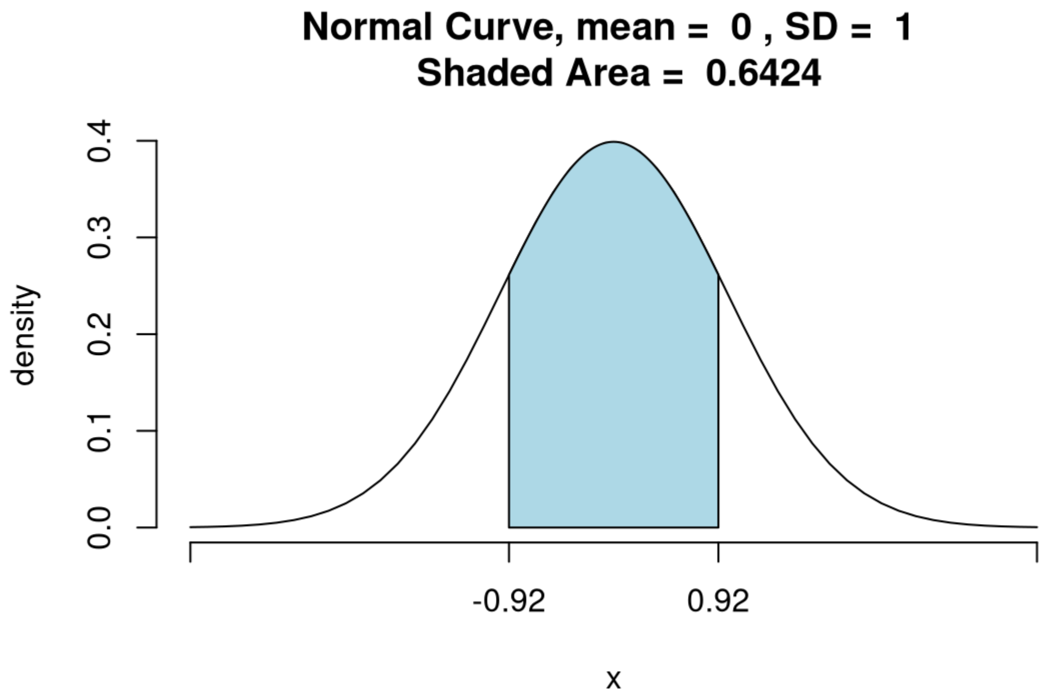
```
[1] 0.822284
```



- $P(|X| \leq 0.92)$

```
pnorm(0.92)-pnorm(-0.92)
pnormGC(c(-0.92,0.92), region="between",graph=T)
```

```
[1] 0.6424272
```




```
[1] 0.6424272
```

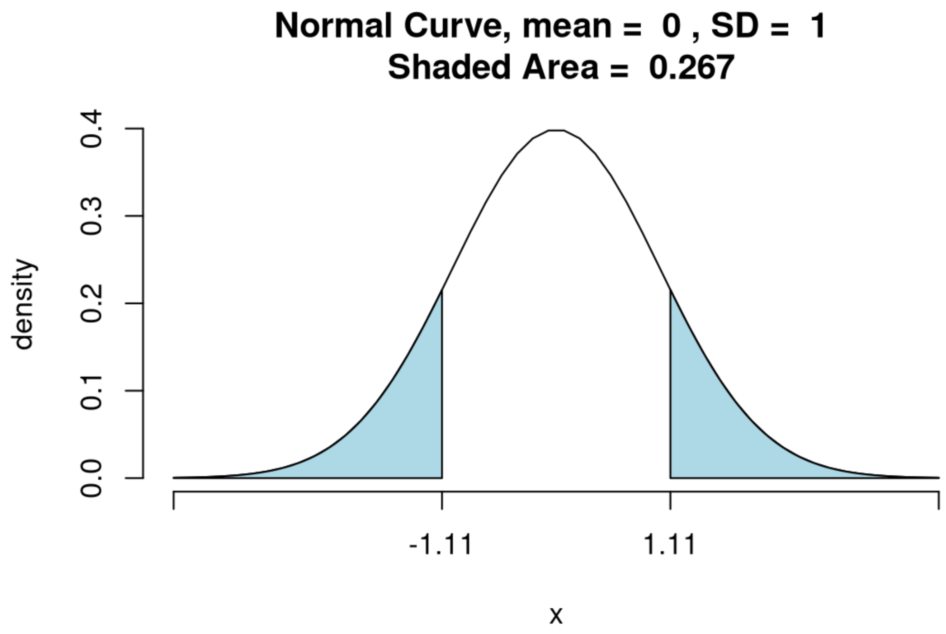
- $P(|X| \geq 1.11)$

```r
(1-pnorm(1.11))+pnorm(-1.11)
```

```
[1] 0.266999
```

```r
pnormGC(c(-1.11,1.11), region="outside",graph=T)
```

```
[1] 0.266999
```



OTHER DISTRIBUTIONS

We can use other distributions like the Normal distribution, for example
- binomial: rbinom, dbinom, pbinom, qbinom

- uniform: runif, dunif, punif, qunif
- Student's t: rt, dt, pt, qt
- …