Comparing strategic voting incentives in plurality and instant-runoff elections

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5 Data and Methods

To assess strategic incentives under Plurality and IRV empirically, we rely on data from the Comparative Study of Electoral Systems (CSES) for a realistic set of preferences and beliefs. The dataset covers 160 surveys from xx different countries, administered shortly before or after an election.[1] We focus on the three largest parties (evaluated how?) and label them A, B, C in descending size, respectively. (Some more summary of the data set?)

We construct the utilities over candidates and beliefs over the electoral outcome directly from the CSES data. For (ordinal) utilities, we take CSES respondents' party like/dislike scores (on a scale from 0 to 10) for parties A, B, C. This also implies their preferences over the three parties and determines their voter type (e.g., *abc*). For beliefs about electoral outcomes, we proceed as if respondents were presented with full information about everyone else's ordinal utilities. A "Level-1" voter would then believe everyone else to vote sincerely, such that the expected electoral outcome is a vector of ballot shares, $\mathbf{v_j}$.[2] A "Level-2" voter would believe that everyone else in the sample is a "Level-1" strategic voter, i.e. votes strategically in expectation of a result with sincere voting. The respondent's belief about the probability distribution of electoral outcomes is then given by

$$f(\mathbf{v}, s) = \text{Dir}(s \times \mathbf{v}) \tag{1}$$

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[1] Two additional cases in the survey, Belarus (20xx) and Lithuania (20xx), are dropped because no respondent specified full preferences over more than two parties.

[2] I am not sure if we can think of this iterative process as just giving voters successive polls, or whether they need to have specific knowledge about others' utilities (probably not).

where s is a precision parameter (see above). Alternatively, we can think of a Level-2 voter as someone who has just been given a poll of everyone else voting ...(A better way to approach this is probably to start with sincere preferences and then work my way up...)

5.1 CSES summary statistics

Brief summary of the CSES dataset (countries, cases, three party dominance)

Also mentioned in earlier part of Andy's draft – decide where this belongs

5.2 Weights

Our objective is to characterise the general distribution of strategic incentives under Plurality and IRV under realistic distributions of voters. However, the sample of cases in the CSES dataset is not representative. Some countries have more elections surveyed than others, and there is large variance in countries' electorate. To account for these differences, we use two sets of weights:

- when calculating individual-level quantities and aggregating at the level of CSES cases, we use the CSES-provided survey weights for each individual observation.
- when calculating aggregate-level quantities and summarising across CSES cases, we use the following weight for each case:

$$w_j \equiv \tag{2}$$

5.3 Iterative Process

Having described our approach to constructing preferences and beliefs, next we apply the method provided in Section 2 to calculate strategic incentives and voter's optimal ballot if maximising expected utility under either electoral system. In the first instance, we assume that our voters are Level-1 strategic; that is, they expect everyone else to vote sincerely (or have been given a poll where everyone else declares their sincere vote). Let v_0 denote the vector of ballot shares if everyone voted sincerely, and $v_1(v_0)$ if everyone voted strategically while having a belief with an expected outcome at v_0.

Of course, when constructing respondents' utilities from like-dislike scores, we cannot know if they would report the same quantities had they been accustomed to a different electoral system (e.g., what if the UK had IRV, rather than plurality?). More generally, if I anticipate others voting strategically, too, I will update my expectations accordingly, and my optimal strategic vote may change as a result. This, in turn, will affect everyone else's optimal choice, and so forth. For that reason, we apply the above method iteratively, until the ballot shares converge onto a fixed-point equilibrium (i.e., where, after updating their expectation about others' strategic votes, no-one changes their vote anymore). [Interpreting the 'learning path'].

5.4 Quantities of Interest.

The iterative algorithm described above yields a large dataset of every respondent's strategic incentive and their optimal ballot for each iteration under either electoral system (all conditional on s).

For our analysis, we calculate and present the following quantities of interest in our results section. Each of these quantities is a weighted mean computed for each case within the CSES, conditional on iteration, system, and belief precision. (Express in formal language w/ conditionality).

Convergence of strategic voting. We are interested in what 'path' the iterative process described above takes. Specifically, we measure the distance between the iterated result (i.e., the vector of ballot shares) and the original ballot share vector (if everyone voted sincerely). Formally, for iteration k, the Euclidian distance is:

$$d \equiv \sqrt{(\mathbf{v}_k - \mathbf{v}_0)^\mathsf{T}(\mathbf{v}_k - \mathbf{v}_0)} \tag{3}$$

Prevalence of strategic incentives. We measure the proportion of voters who have a positive strategic incentive, that is, $\tau_i > 0$ in each setting. Formally, the measure is:

$$\text{Prevalence} \equiv \mathbb{P}(\tau > 0) = \frac{\sum_{i=1}^{N} I(\tau_i > 0)}{N} \tag{4}$$

Magnitude of strategic incentives. We measure the overall magnitude of these strategic incentives. This is the average of τ conditional on being greater than 0.

$$\text{Magnitude} \equiv \mathbb{E}[\tau|\tau > 0] = \frac{\sum_{i=1}^{N} I(\tau_i > 0) \times \tau_i}{\sum_{i=1}^{N} I(\tau_i > 0)} \tag{5}$$

Expected benefit of strategic incentives. We measure the expected benefit by asking: on average (across all respondents), what is the expected benefit of strategic voting?

$$\text{Expected Benefit} \equiv \text{Prevalence} \times \text{Magnitude} = ... \tag{6}$$

Likelihood of pivotal events. We measure the likelihood of pivotal events by taking the relevant pivotal probabilities from Section XX (see above). Note that these are conditional on the type of ballot cast and the type of voter – so for an *abc* voter, the probability of being pivotal with a *bac* ballot will be different than that of a *cba* voter being pivotal with a *bca* ballot.

(*Does this need formalisation?*)

Expected cost and benefit of specific ballots.

Paraphrase Andy's stuff here.

6 Results

We now proceed to present and discuss our results. For each quantity of interest, we present the average *within* each CSES case (weighted by the respective survey weights) as a thin line across all iterations. We also compute a weighted average *across* all CSES cases for every iteration, which we plot with a thicker line.[3]

6.1 Convergence

Main point: show that our cases converge on strategic voting equilibria; in the case of Plurality, two-party ones. Show that these fixed points are further away from original ballot shares under Plurality than under IRV.

When running our iterative procedure described in detail in Section 2.X.Y, the distribution of ballot shares quickly converges towards a fixed point in the vast majority of CSES cases under both Plurality and IRV. We assume that a fixed proportion $\lambda = 0.05$ of all voters vote strategically in each iteration. The average Euclidean distance going from the 59th to the 60th iteration is below 0.0014 for Plurality, and below 0.006 for IRV.[4] Put differently, we can obtain a voting equilibrium, where voters anticipate others' vote choices, and react accordingly, within about 60 iterations from the sincere voting profile.

Figure 1 plots the Euclidean distance between the ballot shares from one iteration to the next for every case and iteration under both Plurality and IRV. In expectation, convergence towards the fixed point occurs faster under IRV than it does under Plurality. As we discussed earlier, strategic incentives under Plurality are characterised by complementarity; this means that with every additional iteration, the incentive for supporters of the third party increases, until all of them have deserted the trailing candidate and the ballot shares are in a Duvergian (two-party) equilibrium.[5] In contrast, the substitutability of strategic voting incentives under

[3] We can interpret this as a 'worldwide' average, if you will...

[4] These averages are unweighted – need to recompile in the future.

[5] We could visualise this by plotting the share of third-party votes when $k = 60$.

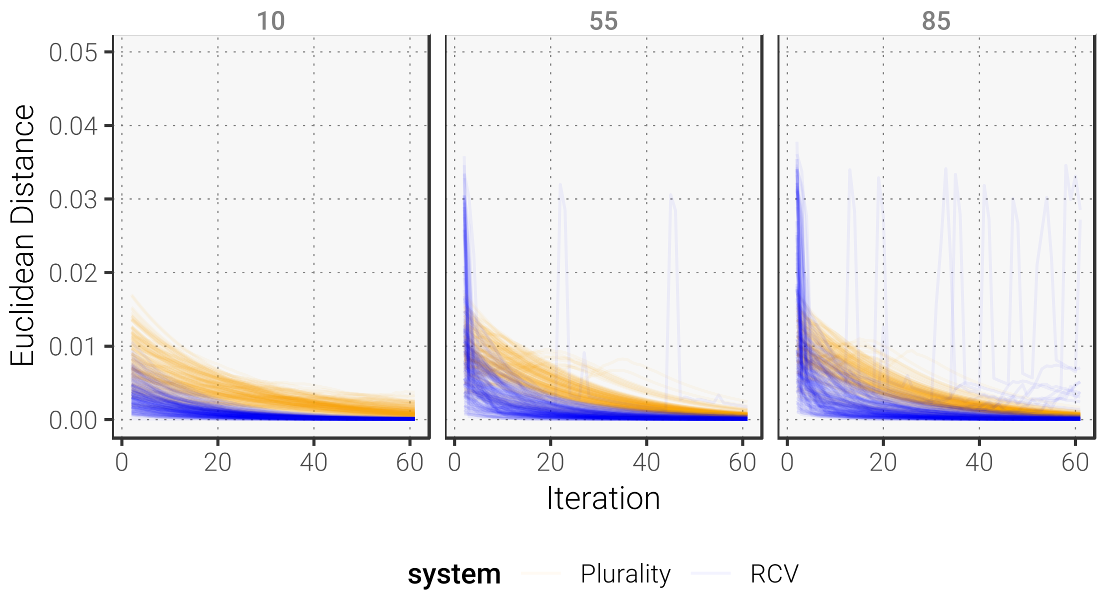


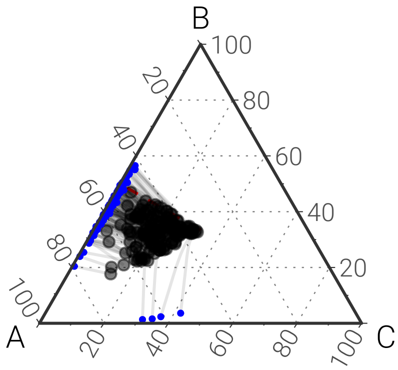
Figure 1: Euclidean distance between ballot share vectors from one iteration to another.

RCV allows them to reach a fixed point much sooner: as soon as some ABC voters switch to a strategic (*bac* or *cab*) ballot, the incentive for myself to do equally lessens. Note however, that, for more precise beliefs ($s \in 55, 85$), the shift away from the sincere ballot profile in the first few iterations is much bigger than under Plurality; quicker convergence does not necessarily mean that the fixed point is closer to the original ballot share vector. This is also related to the substitution / complementary difference: under Plurality, third-party supporters have an incentive to defect towards the top-two that only increases the more other third-party supporters do so. Consequently, the equilibria under Plurality will be such that all third-party supporters will have deserted their first choice and we end up further away from the original distribution of sincere ballots.[6]

In sum, when applying our iterative strategic voting procedure to all CSES cases, the ballot shares converge more quickly to a fixed point under IRV than under Plurality. Under IRV, these fixed points can occur anywhere in the ballot share space, whereas under Plurality, voters ultimately settle on a two-party Duvergian equilibrium. This is also illustrated by Figure 2, which maps the ballot share vectors before the first and the 60th iteration for $s = 85$.

(Figure about distance from sincere profile? – shows nicely that Plurality fixed points are further away from initial ballot shares.)

[6]This foreshadows a later result: with sufficiently high precision, the prevalence of strategic voting incentives under IRV will be higher in the first few incentives.



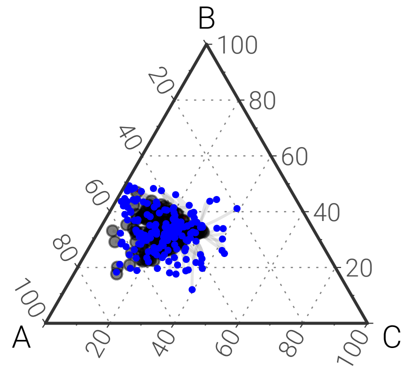


Figure 2: Evolution of ballot share vectors for all CSES cases over iterations, for both Plurality (left) and IRV (right), when $s = 85$. Grey dots indicate the initial ballot share vector before the first iteration; blue dots the ballot share vector after the 60th iteration.

6.2 Distribution of Strategic Incentives

two things to add: (a) Across all three quantities, plurality increases with higher precision; (b) discuss magnitude?

In this section, we characterise the distribution of strategic incentives along the iterative path. We focus on the prevalence, magnitude and expected benefit of strategic voting under either electoral system. Overall, we find that (a) strategic voting incentives are smaller under IRV than they are under Plurality; (b) as precision of beliefs increases, the prevalence of strategic voting incentives increases under IRV as long as voters believe that the vast majority are voting sincerely (lower number of iterations); it does not change much when voters anticipate widespread levels of strategic voting (higher number of iterations); (c)as the anticipation of other voters' strategicness increases, incentives decrease under IRV but increase under Plurality.

We present these results in the form of Figure 3, where we plot the weighted average of each quantity within every CSES case conditional on the level of precision (learning path) as a thin line across all iterations. The thicker lines represent the weighted averages aggregated across all CSES cases and are our main point of reference.

First, we note that the average expected benefit of strategic voting is unconditionally higher under Plurality than under IRV. In line with our previous discussion, strategic voting under Plurality is more straightforward and primarily affects those whose first prefer-

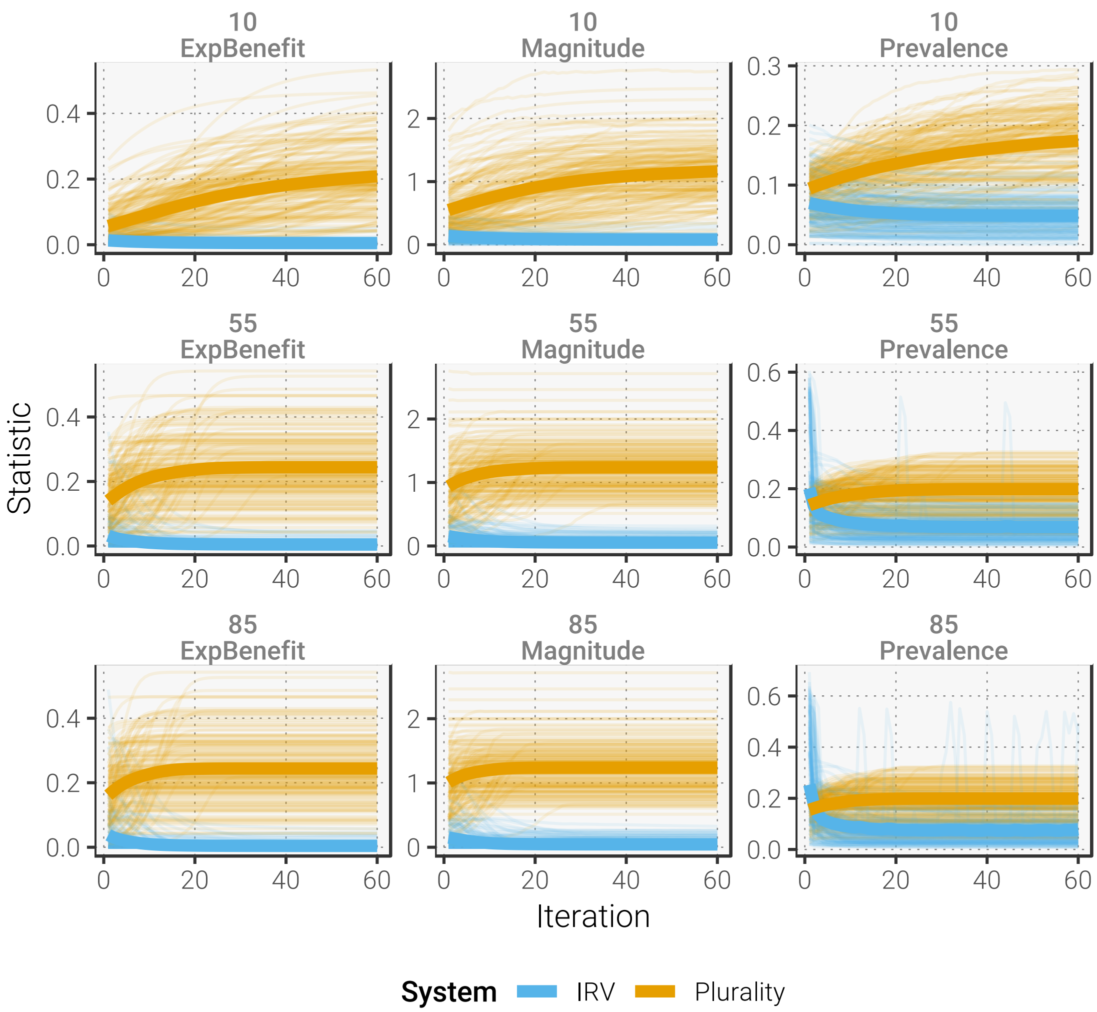


Figure 3: Main statistics

ence is the likely third-ranked candidate. These voters have a straightforward pivotal event (first/second tie) which, by construction, is much more likely than any other; thus, they face little drawback from voting strategically. Under IRV, on the other hand, voters need to be much more careful as the risk of ending up in a situation where the strategic vote backfires is much greater. This confirms the 'folk conjecture' that strategic voting incentives under IRV are less widely distributed.

Looking at the prevalence of strategic incentives underlines this intuition: For the most part, incentives are more common in Plurality. However, when the voter anticipates everyone else to vote sincerely (that is, we are at the first iteration) and precision of beliefs is high, the prevalence of strategic voting under IRV is actually quite high (a fifth of the voting population), and slightly higher than that of Plurality. These are the conditions under which a 'backfiring' of the strategic vote is least likely. Meanwhile, as we drop to low precision, that incentive becomes less common: with more uncertainty about where the result is going to end up, the risk of casting a 'backfiring' strategic ballot becomes greater. For Plurality, the prevalence of strategic voting does not change much across iterations under high precision: with precise beliefs, voters are more certain that their preferred candidate will come last from the onset;[7] thus already a large proportion of them will desert C from the onset and will continue doing so as more voters become strategic. With more uncertain beliefs, the initial prevalence of strategic incentives under Plurality is lower (as the probability of C tying with either competitor for first place is higher) and increases as strategicness increases and more voters desert C over the course of the iterations.

Finally, we note that as voters become more strategic (that is, we move further along the 'learning' path and look at a higher number of iterations), the expected benefit of strategic voting increases under Plurality, but decreases under IRV. This refers back to the difference in the nature of the strategic incentives. Under Plurality, strategic incentives are complements: if I am a supporter of the expected third party C, my incentive to desert my preferred choice in favour of the top two increases the more other fellow voters do so, too, as the chance of my preferred C winning decreases even further. Consequently, the incentive to vote strategically increases the more I anticipate others doing so, too. Note that this is most prominent in the case with low precision ($s = 10$). Here, the initial prevalence and magnitude are both lower because with higher uncertainty, there is more of a risk of encountering a first-place tie between C and either A or B, in which case a non-sincere ballot for C-voters would backfire. As strategicness increases, however, the share of C voters decreases and so does the risk of backfiring.[8]

[7] Unless the second and third are expected to finish very close to one another, of course.

[8] Graphically, we are travelling from the centre of the vertex towards the A-B line, as C voters desert

In contrast, under IRV, strategic incentives are mostly substitutes: if my fellow like-minded voters already vote strategically, then my additional strategic vote may increase the risk of backfiring and accidentally electing the least-preferred option. As a result, the greater the share of anticipated strategic voters, the lower will my own incentives be.[9]

In sum, the distribution of strategic incentives can be characterised as follows: the average expected benefit of strategic voting is higher in Plurality; with higher precision, the prevalence of strategic incentives improves under either electoral system but more strongly under IRV when voters anticipate everyone else to vote sincerely; finally, as voters' beliefs about others' strategicness increase, the expected benefit increases under Plurality but decreases under IRV.

We continue with the remaining quantities of interest to highlight the points made in this discussion further.

6.3 Probability of Pivotal Events

Figure 4 plots the pivotal probabilities for each ballot type as discussed in Subsection 5.4. Unsurprisingly, the probability is always positive. The average probability of being pivotal with one's strategic vote under Plurality is higher than that of an IRV ballot with one's second preference put first, or that of an IRV ballot with one's third preference put first.

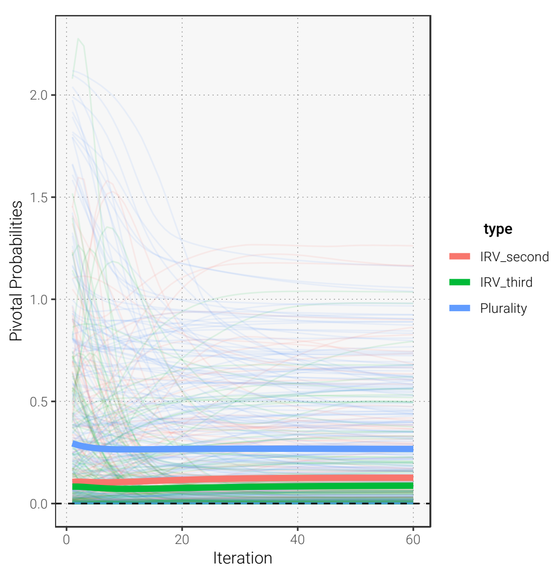


Figure 4: Pivotal probabilities relevant to each strategic vote

their most preferred candidate. It is easy to see how such a movement shifts the distribution away from the AC and BC pivotal lines.

[9]Since it is hard to judge the degree of strategicness *ex ante*, this poses a fascinating co-ordination problem in real life...

(*If we keep the conjectures in the text:*) This provides support for conjecture 1.
(*Otherwise:*) Explain / discuss meaning of this along the lines of conjecture 1.

6.4 Expected Costs and Benefits of Strategic Ballots

Finally, we report the average correlation between costs and benefits of strategic ballots in Figure 5.

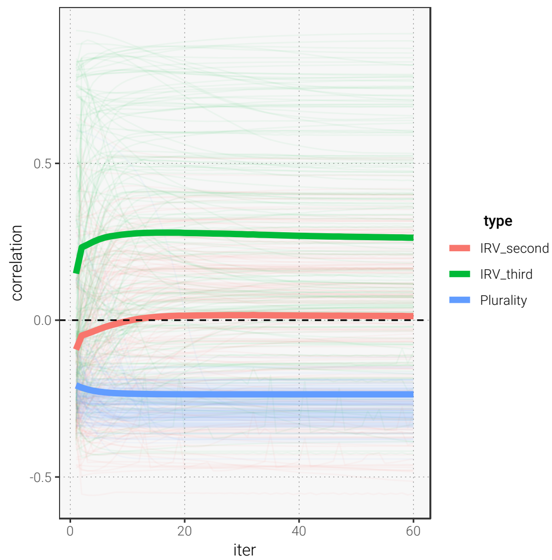


Figure 5: Correlation between costs and benefits of strategic ballots

On average, costs and benefits of a strategic ballot in Plurality (voting for one's second preference, rather than sincerely) are negatively correlated. The result supports conjecture 2. This is in line with our previous discussion: the more likely a tie between A and B is (which favours a strategic vote), the less likely will a tie between C and either of the other candidates be (these are the situations where a strategic ballot would be costly). The interpretation under IRV is somewhat less straightforward and depends on whether the strategic ballot puts one's second or third preference first. In the case of an "IRV-second" ballot, costs and benefits are, on average, uncorrelated at higher levels of strategicness. (Why?) Finally, in the case of an "IRV-third" ballot, costs and benefits are positively correlated. We can interpret this as an increased risk of "backfiring": the rewards of putting one's third preference in first position can be high if the right pivotal event occurs, but equally, so are the risks if one ends up with the worst candidate as the winner (and contributed to electing them). Clearly, backfiring carries a greater

7 Conclusion