Comparing strategic voting incentives in plurality and instant-runoff elections

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4 Main Results

We now proceed to present and discuss our results. For each quantity of interest, we present the average *within* each CSES case (weighted by the respective survey weights) as a thin line across all iterations. We also compute a weighted average *across* all CSES cases for every iteration, which we plot with a thicker line.[1]

4.1 Convergence

We assume that a fixed proportion $\lambda = 0.05$ of all voters vote strategically in each iteration, and compute strategic incentives and ballot shares as described in Section 2.X.Y. The distribution of ballot shares quickly converges towards a fixed point under both Plurality and IRV in the vast majority of CSES cases. Figure 1 plots the Euclidean distance between the ballot shares from one iteration to the next for every case and iteration under both Plurality and IRV. The average Euclidean distance of ballot shares between the 59th to the 60th iteration is below 0.0014 for Plurality, and below 0.006 for IRV.[2] Put differently, we can obtain a voting equilibrium, where voters anticipate others' vote choices, and react accordingly, within about 60 iterations from the sincere voting profile.

In expectation, convergence towards the fixed point occurs faster under IRV than it does under Plurality. For more precise beliefs ($s \in 55, 85$), the shift away from the sincere ballot profile in the first few iterations is much larger under IRV than under Plurality; however,

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[1] We can interpret this as a 'worldwide' average, if you will...

[2] These averages are unweighted – need to recompile in the future.

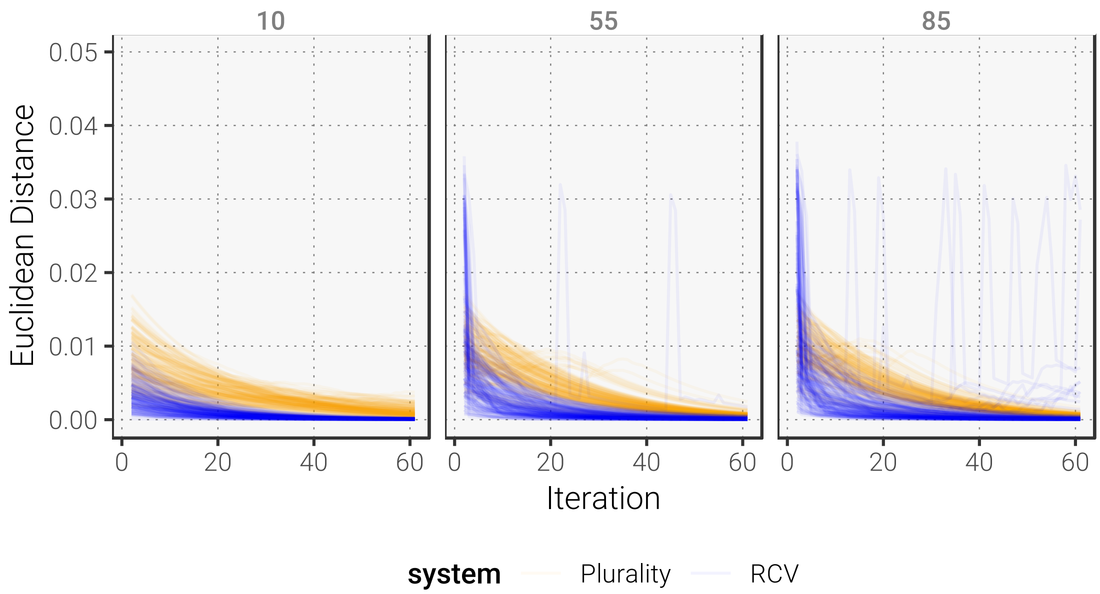

Figure 1: Euclidean distance between ballot share vectors from one iteration to another.

faster convergence does not necessarily mean that the fixed point is closer to the original ballot share vector.

Figure 2 plots the evolution of ballot shares with increasing iterations under both Plurality and IRV (with ballot shares under IRV aggregated to first preferences). Convergence in plurality is reached when all third-party voters have abandoned their first preference and are voting for the top-two candidates instead, resulting in a Duvergerian two-party equilibrium. There is a handful of cases where the idiosyncratic learning path is such that B is the candidate that ultimately gets abandoned in equilibrium. Under IRV, the fixed point ballot shares are closer to the original (sincere) voting profile and do not yield such extreme results as Plurality (e.g., the third party is not wiped out even if everyone is expected to vote strategically).

This result foreshadows a key insight that we discuss further below in Section 5.y: strategic incentives under Plurality are complements (the more I expect other likeminded voters to be strategic, the greater are my own incentives to do so) whereas they are substitutes (my own incentives to vote strategically decrease the more others do so). For now, the main takeaway from the result is that our approach converges on fixed points and we can analyse strategic voting incentives in a situation where the voter anticipates everyone to be sincere, or everyone to be strategic, or any point in between. We also show that, under full convergence and strategic voting, voting behaviour under Plurality converges to a two-party competition, whereas under IRV, first preferences are still cast for all three parties.

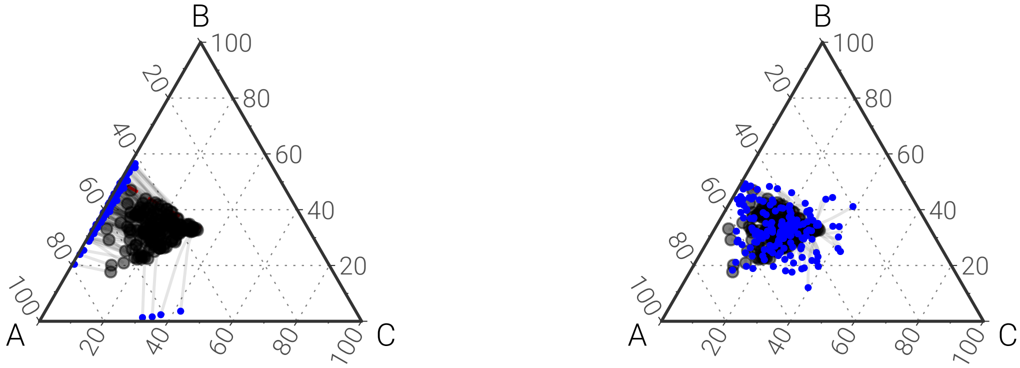

Figure 2: Evolution of ballot share vectors for all CSES cases over iterations, for both Plurality (left) and IRV (right), when $s = 85$. Grey dots indicate the initial ballot share vector before the first iteration; blue dots the ballot share vector after the 60th iteration.

4.2 Expected Benefit of Strategic Voting

Next, we report on the distribution of strategic voting incentives in the form of the expected benefit of doing so in each case.

Figure 3 shows the expected benefit of strategic voting (as defined in Section 2.x) for every case at every iteration.

We note the following characteristics in the results and will explain and discuss them below. First, the expected benefit is higher for Plurality than for IRV along the entire learning path, independent of the level of precision. This initial finding supports the 'folk' conjecture stated in the introduction that IRV offers fewer opportunities for strategic voting. Second, the expected benefit is (weakly) increasing in the number of iterations under Plurality and (weakly) decreasing under IRV. Furthermore, this behaviour becomes more pronounced in Plurality as beliefs decrease in precision. Lastly, the distribution of expected benefits across cases at any given iteration has much higher variance under Plurality than under IRV.

Strategic voting under Plurality is determined by three main pivotal events — which two candidates tie for first place. If the third-placed (in expectation) candidate C is performing poorly enough that she has a small chance of tying for first, her supporters will be better off voting for their second preference in case it comes to a tie between the top two candidates, A and B. There is little risk of this strategy backfiring: the more likely an AB tie is, the less

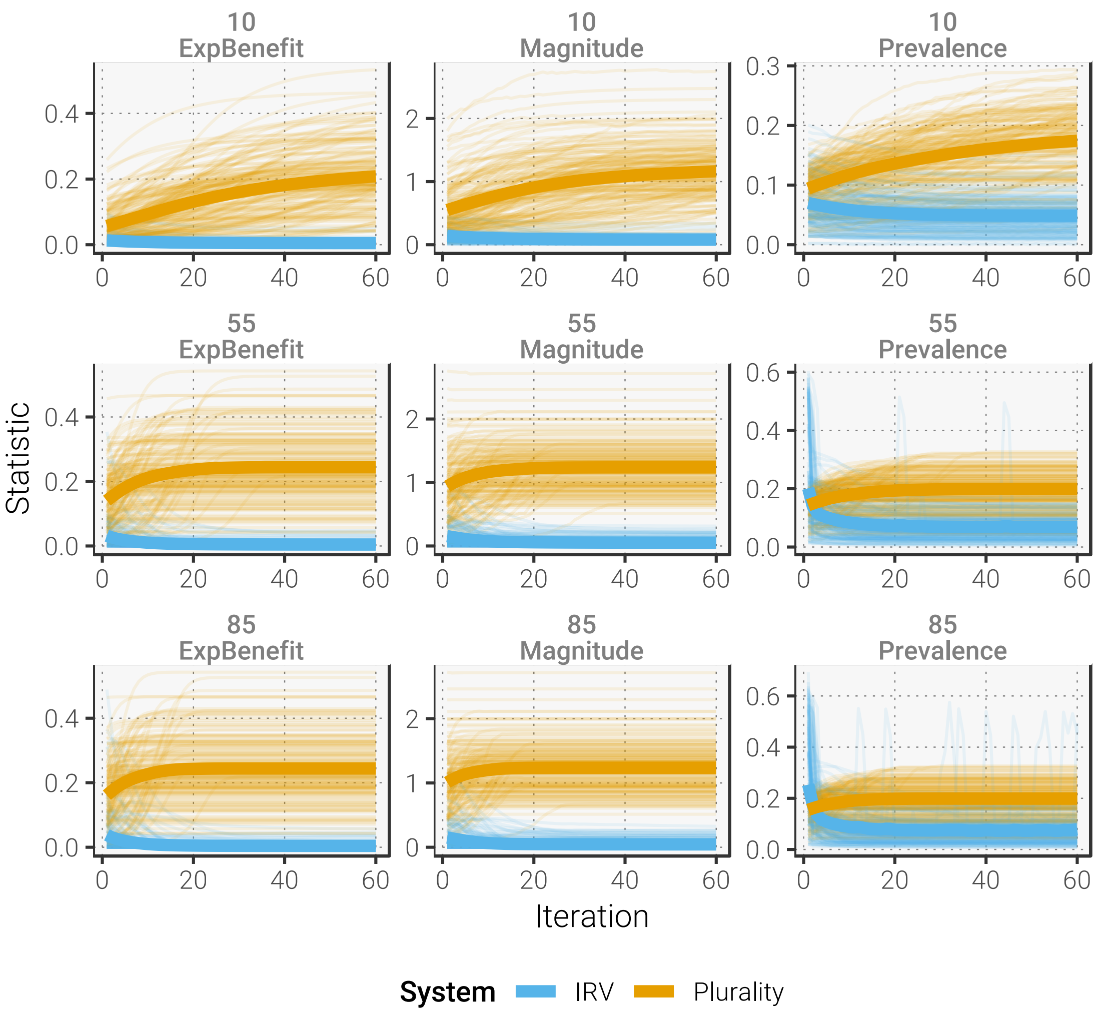

Figure 3: Main statistics

likely will a AC or BC tie be.[3] (In fact, we can assert that strategic voting in Plurality is predominantly driven by those who abandon their first preference that is expected to come third.)

Contrast this with IRV: because of the two-round nature of the competition, there are more pivotal events to consider, and also greater risks of a strategic vote 'backfiring'. A voter with *abc* preferences may want to vote strategically by casting a *bac* ballot; however, doing so carries the risk of backfiring if A and B are tied in either round, or if A and C are tied in the first round.[4] (It is much harder for us to characterise *who exactly* votes strategically in IRV.)

5 Further Results and Discussion

What explains the results reported above? We discuss two key differences in strategic voting between Plurality and IRV. Under Plurality, the predominant incentive comes from abandoning candidates thought to come third; this incentive bears few risks and is generally complementary: the more I expect fellow voters with similar preferences to do so, the larger is my incentive to follow suit. In IRV, incentives to cast a specific ballot bear a greater risk of helping elect a less preferred candidate; (for the most part) they are also strategic substitutes. The more I expect like-minded voters to vote strategically, the lower is my own incentive to do so. Below, we present supplementary evidence for these claims.

5.1 Decomposing Expected Benefit

We can decompose the Expected Benefit of strategic voting into the magnitude and prevalence of the incentive.[5] Figure 3 does exactly this.

The stochastic dominance of strategic voting incentives in Plurality over IRV is driven by both higher magnitude and (for the most part) prevalence. Similar to the expected benefit, we observe that in both quantities, the statistic (weakly) increases in Plurality, whereas the decrease in expected benefit under IRV seems to be driven by prevalence; the overall magnitude of strategic voting incentives under IRV is very small.

The fact that both elements of the decomposition share these characteristics can, once again, be explained by referring to the nature of strategic voting under either system. Strategic incentives under Plurality are more prevalent because (a) the pivotal events that make a

[3] Strictly speaking this is not true — if the initial vote share distribution is $(1, 0, 0)$, then all events become more likely as we move out of the vertex.

[4] In graphical terms, the pivotal events are much closer to one another within the ternary.

[5] Recall from Section 2.x that the expected benefit is the product of magnitude and prevalence.

strategic vote beneficial are more likely to occur under Plurality and (b) the pivotal events that would discourage a strategic vote are less likely to occur under Plurality. The magnitude is higher under plurality because there is little to gain from voting sincerely. The expected third is extremely unlikely to win the election, so voting for her is a wasted vote; the magnitude of the strategic incentive should increase with closeness between the top-two candidates.

Note that, when expecting everyone else to vote sincerely, prevalence of strategic incentives is actually higher under IRV given high enough precision of beliefs. As mentioned before, the drawback to strategic voting in IRV is the risk of backfiring; the more precise one's beliefs about the expected ballot shares are, the smaller is that risk (since it's less likely that we end up at a 'bad' pivotal event). However, even in these cases, the magnitude of these strategic voting incentives is very small: second-round pivotal events in IRV are conditional on the realisation of a particular top-two ranking. Consequently, these events can never be as likely as pivotal events in Plurality, and the magnitude of the benefit is lower.

5.2 The Risks of Strategic Voting

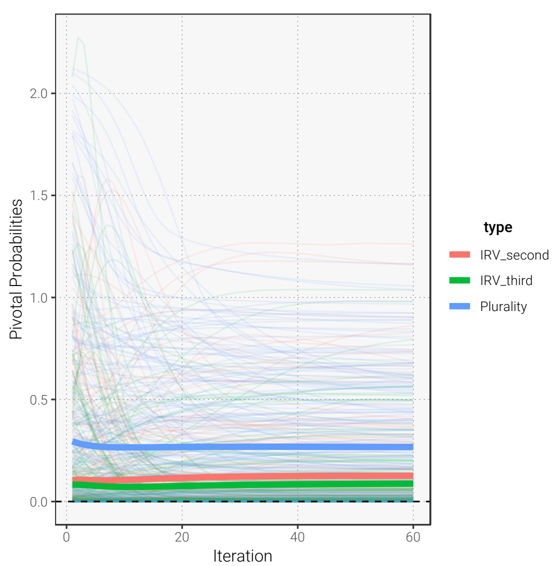

Figure 4: Pivotal probabilities relevant to each strategic vote

Really, this figure should show both probabilities relevant to each strategic vote AND the probabilities where these votes can backfire.

We now provide further evidence for the mechanisms that drive the above results. First, we show that the risk of one's strategic vote backfiring is indeed greater under IRV than it is under Plurality. This is in part because the pivotal events under Plurality are more likely, and in part because the benefit of voting strategically at these events is higher / the cost of voting strategically at 'discouraging' pivotal events is lower.

Figure 4 plots the average probability of all pivotal events (at $s = 85$) that render one's strategic vote beneficial. Across all iterations, pivotal events that benefit one's strategic vote under Plurality are more likely than those rendering a vote for one's second or third preference under IRV beneficial. Under plurality, the pivotal event is a tie between the one's second and third preferred candidate for first place. Under IRV, the pivotal events are much more complex and stipulate a second-round tie *conditional on* two particular candidates coming first and second in the first round.

Next, Figure 5 plots the correlation between average costs and benefits of strategic votes. For each type of strategic vote, we multiply the relevant pivotal probability with voters' utility conditional on the pivotal event and their ballot choice. We see that costs and benefits of voting for one's second preference in Plurality are negatively correlated: the higher my gross benefit from voting strategically is, the smaller, on average, will my gross cost be (the less will I have to worry about backfiring). For putting one's second preference first under IRV (IRV-second), the two are largely uncorrelated (but this tells us nothing about their respective magnitude...). Finally, under IRV-third, there is a positive correlation: on average, the higher the benefit, the higher the cost. This underlines the riskiness of putting one's third preference first: in the optimal scenario, it helps, but this also carries the danger of electing one's least favoured candidate if the wrong pivotal event occurs.

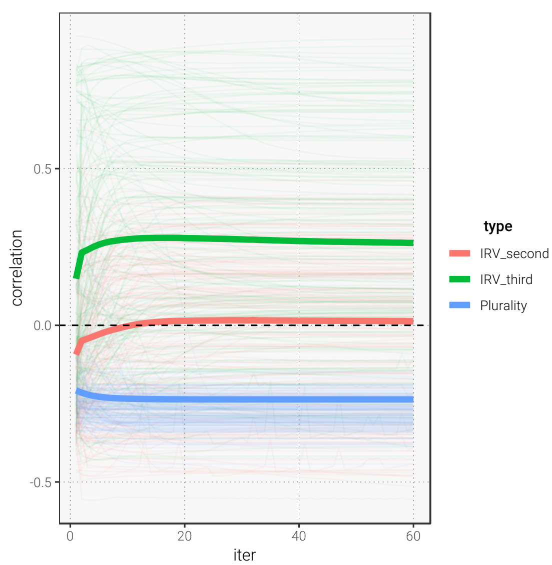

Figure 5: Pivotal probabilities relevant to each strategic vote

Together, these results highlight the different nature of strategic voting under Plurality and IRV. In the first, strategic incentives are strong if one expects one's first preference to have little chance of winning, and is better off deciding the tie between the top-two candidates instead. There is little risk in this strategy if one's first preference is truly uncompetitive in the race. In IRV, there is a manifold of pivotal events that need to be considered when voting strategically. Here, the chance of one's strategic vote 'paying off' is much smaller, and the risk of accidentally electing someone less preferred by voting strategically is much greater. This distinction contributes to the result that strategic voting has a higher expected benefit under Plurality than under IRV.

5.3 Substitutes and Complements in Strategic Voting

Finally, the distinct evolution of strategic incentives under Plurality and IRV, as well as their different convergence paths, can be explained by the fact that strategic incentives under Plurality are strategic complements, whereas they are strategic substitutes under IRV. As a consequence, when strategic incentives are complementary, the expected benefit rises if more people are doing so; the opposite is true if strategic incentives are substitutes.

Consider the following case under Plurality. I am a supporter of the expected third party C, my incentive to desert my preferred choice in favour of the top two increases the more other fellow voters do so, too, as the chance of my preferred C winning decreases even further. Consequently, the incentive to vote strategically increases the more I anticipate others doing so, too. Note that this is most prominent in the case with low precision ($s = 10$). Here, the initial prevalence and magnitude are both lower because with higher uncertainty, there is more of a risk of encountering a first-place tie between C and either A or B, in which case a non-sincere ballot for C-voters would backfire. As strategicness increases, however, the share of C voters decreases and so does the risk of backfiring.[6]

In contrast, in an IRV scenario, if my fellow like-minded voters already vote strategically, then my additional strategic vote may increase the risk of backfiring and accidentally electing the least-preferred option. This is especially dangerous in cases where the strategic incentive would suggest to put one's third preference first. As a result, the greater the share of anticipated strategic voters, the lower will my own incentives be.[7] To give an example, consider a voter with *abc* preferences who has an incentive to submit a *cab* ballot in order to keep B away from the second round. If the voter is, indeed, pivotal in a second-place first-round tie between B and C, then this strategic vote would be highly beneficial and

[6] Graphically, we are travelling from the centre of the vertex towards the A-B line, as C voters desert their most preferred candidate. It is easy to see how such a movement shifts the distribution away from the AC and BC pivotal lines.

[7] Since it is hard to judge the degree of strategicness *ex ante*, this poses a fascinating co-ordination problem in real life...

return A as the overall winner (if A beats C but not B in the second round). However, if everyone else with the same preference ordering *abc* does so, such behaviour risks abandoning A to the extent that they either lose to C in the second round or do not even have enough support to advance into the second round. Consequently, the more I expect other voters with the same preference to vote strategically, the smaller should my own incentive to do so be. This illustrates the necessity for co-ordination of strategic voting under IRV and its characterisation of strategic substitutability.

5.4 Summary of Results and Discussion

We calculated incentives to vote strategically in 160 different elections under both Plurality and IRV for a range of belief precisions and expectations about other voters' strategicness. Overall, we find the following. First, as the voters expect everyone else to vote more strategically, voting behaviour converges to a two-party equilibrium under Plurality, but moves less extremely under IRV. Second, the expected benefit of strategic voting is higher under Plurality compared to IRV. This holds for all levels of precision within our range of analysis and also along the entire 'learning path' about other voters' strategicness. Third, both magnitude and prevalence of strategic incentives are greater under Plurality. The only exception is the scenario where beliefs about the election outcome are very precise, and the expectation is that everyone else votes sincerely. In these circumstances, strategic incentives are more widespread under IRV, although they still have a smaller magnitude. Finally, we observe that the expected benefit (and magnitude) of strategic incentives increases in voters' strategicness under Plurality but decreases in voters' strategicness under IRV.

We discuss these findings and offer an explanation by characterising the different nature of strategic voting in the two electoral systems. Under Plurality, the pivotal events that incentivise strategic voting are more likely; furthermore, the risk of a strategic vote backfiring (causing an election outcome that is worse than if one had voted sincerely) is small, and strategic voting incentives are complementary. Under IRV, pivotal events that reward a strategic vote are comparatively less likely. The risk of 'backfiring' is also greater. These two characteristics reduce the prevalence, and magnitude of strategic voting, respectively. Morever, incentives under IRV can be strategic substitutes.

6 Conclusion