Comparing strategic voting incentives in plurality and instant-runoff elections

Andrew C. Eggers* Tobias Nowacki†

30th May, 2019

5 Data and Methods

To assess strategic incentives under Plurality and IRV empirically, we rely on data from the Comparative Study of Electoral Systems (CSES) for a realistic set of preferences and beliefs. The dataset covers 160 surveys from xx different countries, administered shortly before or after an election.[1] We focus on the three largest parties (evaluated how?) and label them A, B, C in descending size, respectively. (Some more summary of the data set?)

We construct the utilities over candidates and beliefs over the electoral outcome directly from the CSES data. For (ordinal) utilities, we take CSES respondents' party like/dislike scores (on a scale from 0 to 10) for parties A, B, C. This also implies their preferences over the three parties and determines their voter type (e.g., *abc*). For beliefs about electoral outcomes, we proceed as if respondents were presented with full information about everyone else's ordinal utilities. A "Level-1" voter would then believe everyone else to vote sincerely, such that the expected electoral outcome is a vector of ballot shares, $\mathbf{v_j}$.[2] A "Level-2" voter would believe that everyone else in the sample is a "Level-1" strategic voter, i.e. votes strategically in expectation of a result with sincere voting. The respondent's belief about the probability distribution of electoral outcomes is then given by

$$f(\mathbf{v}, s) = \mathrm{Dir}(s \times \mathbf{v}) \tag{1}$$

*Nuffield College and Department of Politics and International Relations, University of Oxford, United Kingdom. aeggers@nuffield.ox.ac.uk

†Department of Political Science, Stanford University, CA, United States. tnowacki@stanford.edu

[1] Two additional cases in the survey, Belarus (20xx) and Lithuania (20xx), are dropped because no respondent specified full preferences over more than two parties.

[2] I am not sure if we can think of this iterative process as just giving voters successive polls, or whether they need to have specific knowledge about others' utilities (probably not).

where s is a precision parameter (see above). Alternatively, we can think of a Level-2 voter as someone who has just been given a poll of everyone else voting ...(A better way to approach this is probably to start with sincere preferences and then work my way up...)

5.1 CSES summary statistics

Brief summary of the CSES dataset (countries, cases, three party dominance)

5.2 Weights

Our objective is to characterise the general distribution of strategic incentives under Plurality and IRV under realistic distributions of voters. However, the sample of cases in the CSES dataset is not representative. Some countries have more elections surveyed than others, and there is large variance in countries' electorate. To account for these differences, we use two sets of weights:

- when calculating individual-level quantities and aggregating at the level of CSES cases, we use the CSES-provided survey weights for each individual observation.
- when calculating aggregate-level quantities and summarising across CSES cases, we use the following weight for each case:

$$w_j \equiv \tag{2}$$

5.3 Iterative Process

Having described our approach to constructing preferences and beliefs, next we apply the method provided in Section 2 to calculate strategic incentives and voter's optimal ballot if maximising expected utility under either electoral system. In the first instance, we assume that our voters are Level-1 strategic; that is, they expect everyone else to vote sincerely (or have been given a poll where everyone else declares their sincere vote). Let v_0 denote the vector of ballot shares if everyone voted sincerely, and $v_1(v_0)$ if everyone voted strategically while having a belief with an expected outcome at v_0.

Of course, when constructing respondents' utilities from like-dislike scores, we cannot know if they would report the same quantities had they been accustomed to a different electoral system (e.g., what if the UK had IRV, rather than plurality?). More generally, if I anticipate others voting strategically, too, I will update my expectations accordingly, and my optimal strategic vote may change as a result. This, in turn, will affect everyone else's optimal choice, and so forth. For that reason, we apply the above method iteratively, until the ballot shares converge onto a fixed-point equilibrium (i.e., where, after updating their expectation about others' strategic votes, no-one changes their vote anymore). [Interpreting the 'learning path'].

5.4 Quantities of Interest.

The iterative algorithm described above yields a large dataset of every respondent's strategic incentive and their optimal ballot for each iteration under either electoral system (all conditional on s).

For our analysis, we calculate and present the following quantities of interest in our results section. Each of these quantities is a weighted mean computed for each case within the CSES, conditional on iteration, system, and belief precision. (Express in formal language w/ conditionality).

Convergence of strategic voting. We are interested in what 'path' the iterative process described above takes. Specifically, we measure the distance between the iterated result (i.e., the vector of ballot shares) and the original ballot share vector (if everyone voted sincerely). Formally, for iteration k, the Euclidian distance is:

$$d \equiv \sqrt{(\mathbf{v}_k - \mathbf{v}_0)^\mathsf{T}(\mathbf{v}_k - \mathbf{v}_0)} \tag{3}$$

Prevalence of strategic incentives. We measure the proportion of voters who have a positive strategic incentive, that is, $\tau_i > 0$ in each setting. Formally, the measure is:

$$\text{Prevalence} \equiv \mathbb{P}(\tau > 0) = \frac{\sum_{i=1}^{N} I(\tau_i > 0)}{N} \tag{4}$$

Magnitude of strategic incentives. We measure the overall magnitude of these strategic incentives. This is the average of τ conditional on being greater than 0.

$$\text{Magnitude} \equiv \mathbb{E}[\tau|\tau > 0] = \frac{\sum_{i=1}^{N} I(\tau_i > 0) \times \tau_i}{\sum_{i=1}^{N} I(\tau_i > 0)} \tag{5}$$

Expected benefit of strategic incentives. We measure the expected benefit by asking: on average (across all respondents), what is the expected benefit of strategic voting?

$$\text{Expected Benefit} \equiv \text{Prevalence} \times \text{Magnitude} = ... \tag{6}$$

Likelihood of pivotal events. We measure the likelihood of pivotal events by taking the relevant pivotal probabilities from Section XX (see above). Note that these will be different for each voter type (e.g., an ABC ballot will be pivotal in different circumstances than a BAC one)

Expected cost and benefit of specific ballots.

Paraphrase Andy's stuff here.

- Convergence of strategic voting.
- Prevalence of strategic incentives.
- Magnitude of strategic incentives.
- Expected benefit of strategic incentives.
- Likelihood of pivotal events.
- Expected cost and benefit of specific ballots.

6 Results

We now proceed to present and discuss our results.

6.1 Convergence

Main point: show that our cases converge on strategic voting equilibria; in the case of Plurality, two-party ones. Show that these fixed points are further away from original ballot shares under Plurality than under IRV.

Under both IRV and Plurality, the distribution of ballot shares quickly converges towards a fixed point in the vast majority of CSES cases. The average Euclidean distance going from the 59th to the 60th iteration is below 0.0014 for Plurality, and below 0.006 for IRV.[3] Put differently, we can obtain a perfectly strategic voting equilibrium, where all voters anticipate others' vote choices, and react accordingly, within about 60 iterations from the sincere voting profile.

Figure 1 plots the Euclidean distance between the ballot shares for every case and iteration under both Plurality and IRV. In expectation, convergence towards the fixed point occurs faster under IRV than it does under Plurality. As we discussed earlier, strategic incentives under Plurality are characterised by complementarity; this means that with every additional iteration, the incentive for supporters of the third party increases, until all of them have deserted the trailing candidate and the ballot shares are in a Duvergian (two-party)

[3] These averages are unweighted – need to recompile in the future.

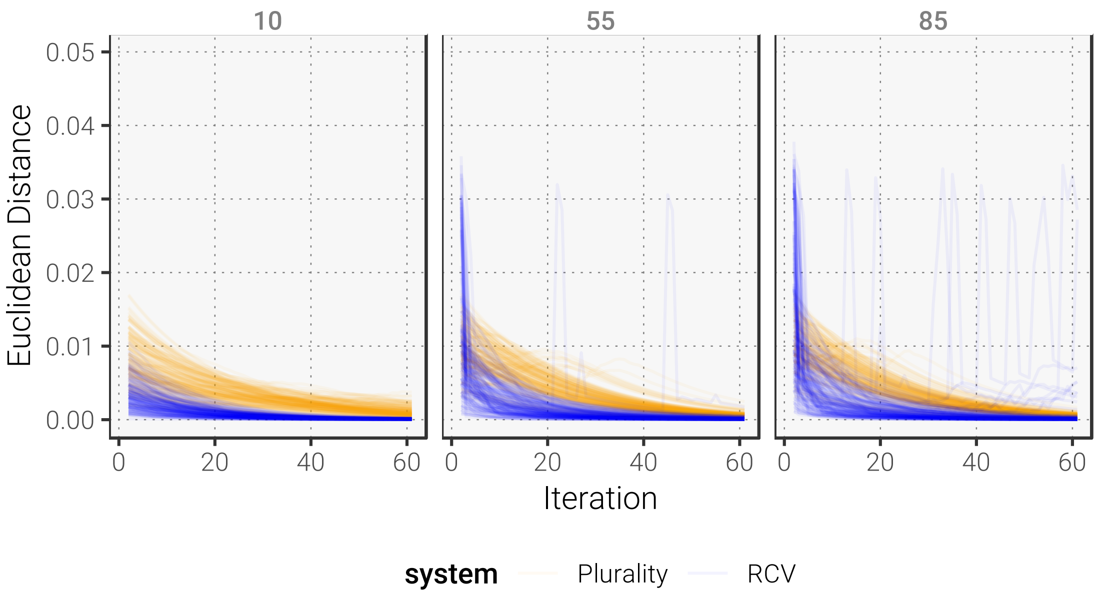

Figure 1: Euclidean distance between ballot share vectors from one iteration to another.

equilibrium.[4] In contrast, the substitutability of strategic voting incentives under RCV allows them to reach a fixed point much sooner. Note however, that, for more precise beliefs ($s \in 55, 85$), the shift away from the sincere ballot profile in the first few iterations is much bigger than under Plurality; quicker convergence does not necessarily mean that the fixed point is closer to the original ballot share vector.[5]

In sum, when applying our iterative strategic voting procedure to all CSES cases, the ballot shares converge more quickly to a fixed point under IRV than under Plurality. Under IRV, these fixed points can occur anywhere in the ballot share space, whereas under Plurality, voters ultimately settle on a two-party Duvergian equilibrium. This is also illustrated by Figure 2, which maps the ballot share vectors before the first and the 60th iteration for $s = 85$.

(Figure about distance from sincere profile? – shows nicely that Plurality fixed points are further away from initial ballot shares.)

6.2 Distribution of Strategic Incentives

Main takeaway: discuss expected benefit, magnitude, and prevalence. Show that strategic incentives are, generally speaking, higher under plurality than under IRV. With better infor-

[4] We could visualise this by plotting the share of third-party votes when $k = 60$.

[5] This foreshadows a later result: with sufficiently high precision, the prevalence of strategic voting incentives under IRV will be higher in the first few incentives.

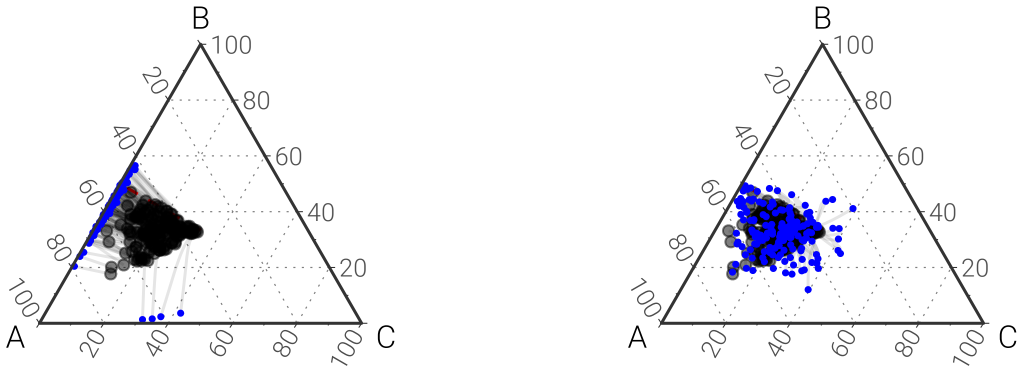

Figure 2: Evolution of ballot share vectors for all CSES cases over iterations, for both Plurality (left) and IRV (right), when $s = 85$. Grey dots indicate the initial ballot share vector before the first iteration; blue dots the ballot share vector after the 60th iteration.

mation, strategic incentives under IRV become more prevalent, but the benefit of acting on them does not really increase. –¿ complements and substitudes...

In this section, we present our main results. We focus on the prevalence, magnitude and expected benefit of strategic voting under either electoral system. Overall, strategic voting incentives are more prevalent, have a higher magnitude and higher expected benefit under Plurality than under IRV.

Figure 3 shows the quantities of interest for each case, as well as the weighted average.

6.3 Conjecture Tests

Continue here tomorrow and think more clearly about how to structure results section.

7 Conclusion

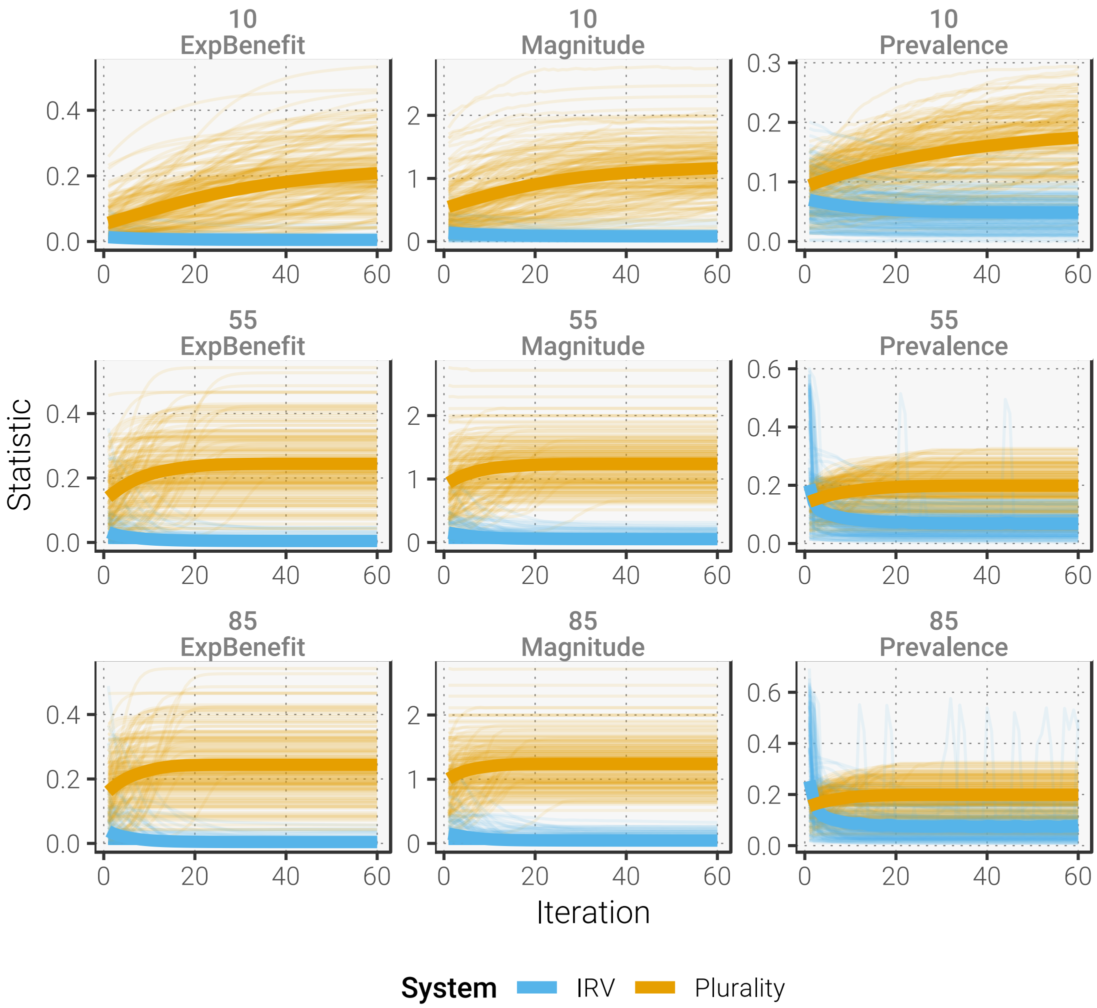

Figure 3: Main statistics