TDT 4171 Artificial Intelligence Methods

Exercise 2

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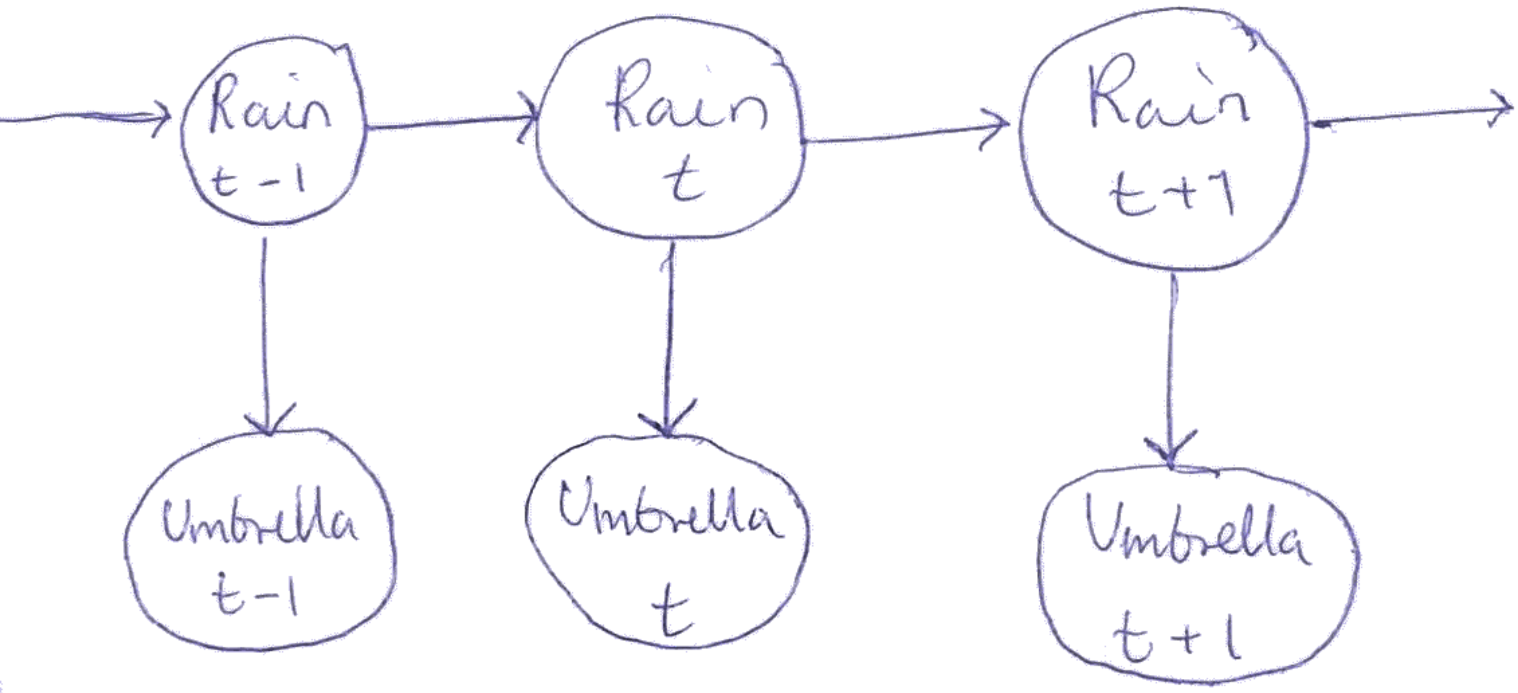
Part A

R_{t-1}	$P(R_t)$
t	0,7
f	0,3

R_t	$P(U_t)$
t	0,9
f	0,2

The "Umbrella domain"



The "Umbrella domain" as a Hidden Markov Model:

Unobserved variables: $X_t = R_t$ (Rain)

Observable variables: $E_t = U_t$ (Umbrella)

Dynamic model:

$$T = P(X_t \mid X_{t-1}) = P(R_t \mid R_{t-1}) = \begin{pmatrix} P(R_t = t \mid R_{t-1} = t) & P(R_t = f \mid R_{t-1} = t) \\ P(R_t = t \mid R_{t-1} = f) & P(R_t = f \mid R_{t-1} = f) \end{pmatrix}$$

$$\Rightarrow \quad T = \begin{pmatrix} 0,7 & 0,3 \\ 0,3 & 0,7 \end{pmatrix}$$

Observation model:

$$O = P(E_t \mid X_t) = P(U_t \mid R_t) = \begin{pmatrix} P(U_t = t \mid R_t = t) & P(U_t = f \mid R_t = t) \\ P(U_t = t \mid R_t = f) & P(U_t = f \mid R_t = f) \end{pmatrix}$$

$$\Rightarrow \quad O = \begin{pmatrix} 0,9 & 0,1 \\ 0,2 & 0,8 \end{pmatrix}$$

For mathematical convenience, we want O to be a square diagonal matrix. This can be achieved by constructing a matrix for both scenarios where E_t = true and E_t = false

E_t = true:

$$O_{true} = \begin{pmatrix} 0,9 & 0 \\ 0 & 0,2 \end{pmatrix}$$

E_t = false:

$$O_{false} = \begin{pmatrix} 0,1 & 0 \\ 0 & 0,8 \end{pmatrix}$$

- Assumptions encoded in this model

- Markov assumption: made about the dynamic model. This assumption states that the current state only depends on a finite fixed number of previous steps; in this case only the previous step. Making it a first-order Markov process, That is $P(X_t | X_{0:t-1}) = P(X_t | X_{t-1})$. the process is assumed to be stationary, which means the conditional probabilities are constant $\forall t$. The validity of this assumpten may depend on variables such as geographical location and current season.

- Sensor Markov assumption: states that the evidence variables E_t only depends on the current state variables X_t. That is

$$P(E_t | X_{0:t}, E_{0:t-1}) = P(E_t | X_t)$$

In reality the probability of bringing an umbrella would increase a long with an increasing set of coherent rainy days.

Part B

Results using forward-algorithm as described in Russel and Norvig:

$e_{1:2} = \{Umbrella_1 = true, Umbrella_2 = true\}$

- $P(X_2 | e_{1:2}) = \underline{0.883}$

$e_{1:5} = \{Umbrella_1 = true, Umbrella_2 = true, Umbrella_3 = false,$
$Umbrella_4 = true, Umbrella_5 = true\}$

	Rain	Not Rain
Day 1 $P(X_1 \| e_{1:5})$	0,8182	0,1818
Day 2 $P(X_2 \| e_{1:5})$	0,8834	0,1166
Day 3 $P(X_3 \| e_{1:5})$	0,1907	0,8093
Day 4 $P(X_4 \| e_{1:5})$	0,7308	0,2692
Day 5 $P(X_5 \| e_{1:5})$	0,8673	0,1327

Part ~~B~~C

Results using forward-backward-algorithm as described in Russel and Norvig:

$e_{1:2} = \{U_1 = true, U_2 = true\}$

- $P(X_1 | e_{1:2}) = \cancel{0,08}\ 0,883$

- $e_{1:5} = \{U_1 = true, U_2 = true, U_3 = false, U_4 = true, U_5 = true\}$

	Rain	Not Rain
Day 1 $P(X_1 \| e_{1:5})$	0,8673	0,1327
Day 2 $P(X_2 \| e_{1:5})$	0,8204	0,1796
Day 3 $P(X_3 \| e_{1:5})$	0,3075	0,6925
Day 4 $P(X_4 \| e_{1:5})$	0,8204	0,1796 ~~0,864~~
Day 5 $P(X_5 \| e_{1:5})$	0,8673	0,1327

Backward messages

	Rain	Not rain
Day 5 b_5	0,69	0,41
Day 4 b_4	0,4593	0,2437
Day 3 b_3	0,090639	0,150251
Day 2 b_2	0,06611763	0,04550767
Day 1 b_1	0,04438457	0,02422283