Problem 4: The Finite Free Stam Inequality

Critical Comparison of the Automated `af` Proof Attempt
with the Official Solution of Garza Vargas–Srivastava–Stier

First Proof Project — Adversarial Proof Framework Analysis

February 2026

Abstract

We present a critical comparison between the automated adversarial proof framework (`af`) attempt and the official correct solution for Problem 4 of the First Proof benchmark: the finite free Stam inequality, asserting that $1/\Phi_n(p \boxplus_n q) \geq 1/\Phi_n(p) + 1/\Phi_n(q)$ for monic real-rooted polynomials p, q of degree n. The `af` system explored three proof paths (subordination, de Bruijn/concavity, and entropy power inequality), proved the result for $n \leq 3$, and identified the $n \geq 4$ barrier as genuine. The official proof by Garza Vargas, Srivastava, and Stier succeeds via a fourth approach not attempted by `af`: exploiting the Jacobian of the convolution map, variance additivity, and the convexity of partial sums of eigenvalues of hyperbolic polynomials (Bauschke et al., 2001), combined with Blachman's classical argument. We analyze what the `af` system got right, where it went wrong, and what lessons this comparison yields for AI-assisted mathematical proof.

Contents

1 Problem Statement

For monic polynomials $p(x) = \prod_{i=1}^n (x-\alpha_i)$ and $q(x) = \prod_{i=1}^n (x-\beta_i)$ with all roots real, the *finite free additive convolution* $p \boxplus_n q$ is defined via the Marcus–Spielman–Srivastava (MSS) formula:

$$(p \boxplus_n q)(x) = \sum_{k=0}^{n} c_k \, x^{n-k}, \qquad c_k = \sum_{i+j=k} \frac{(n-i)!\,(n-j)!}{n!\,(n-k)!} \, a_i \, b_j,$$

where a_i, b_j are the coefficients of p, q. Equivalently, via the permanent-like formula:

$$(p \boxplus_n q)(x) = \sum_{\pi \in S_n} \prod_{i=1}^{n} (x - \alpha_i - \beta_{\pi(i)}).$$

Walsh (1922) showed that this preserves real-rootedness.

The *finite free score vector* is $\mathcal{J}_n(\alpha)_i = \sum_{j \neq i} \frac{1}{\alpha_i - \alpha_j}$, and the *finite free Fisher information* is $\Phi_n(p) = \|\mathcal{J}_n(\alpha)\|^2$. The conjecture, due to D. Shlyakhtenko, is:

Conjecture 1.1 (Finite free Stam inequality)**.** *For all monic real-rooted polynomials p, q of degree n,*

$$\frac{1}{\Phi_n(p \boxplus_n q)} \geq \frac{1}{\Phi_n(p)} + \frac{1}{\Phi_n(q)}. \tag{1}$$

This is the finite-n analogue of Voiculescu's free Stam inequality [5], proved in the continuum by Shlyakhtenko–Tao [6].

2 The Official Solution

The proof by Garza Vargas, Srivastava, and Stier [1] proceeds in three steps.

2.1 Step 1: Score vectors and the Jacobian

Let $\gamma = \Omega_{\boxplus_n}(\alpha, \beta)$ denote the ordered vector of roots of $p \boxplus_n q$, and let $J_{\boxplus_n}$ be the Jacobian of the map $\Omega_{\boxplus_n}$ at (α, β). The *heat flow* $p_t = \exp(-\frac{t}{2}\partial_x^2)p$ moves the roots via $\alpha_i'(0) = \mathcal{J}_n(\alpha)_i$ (Lemma 1.1 of [1]). Because finite free convolution commutes with differential operators, the identity $r_{(a+b)t} = p_{at} \boxplus_n q_{bt}$ holds, which upon differentiation at $t = 0$ yields

$$J_{\boxplus_n}(a\,\mathcal{J}_n(\alpha),\ b\,\mathcal{J}_n(\beta)) = (a+b)\,\mathcal{J}_n(\gamma). \tag{2}$$

This is the key structural identity relating the three score vectors through the Jacobian.

2.2 Step 2: The Jacobian is a contraction on mean-zero vectors

The central technical result is:

Proposition 2.1 (Proposition 2.1 of [1])**.** *If $u, v \in \mathbb{R}^n$ are both orthogonal to $\mathbf{1}_n$, then*

$$\|J_{\boxplus_n}(u, v)\|^2 \leq \|u\|^2 + \|v\|^2.$$

The proof of this proposition uses two ingredients:

Variance additivity and the Hessian identity. The subspace $\mathcal{V} = \{(u, v) : u^\top \mathbf{1} = v^\top \mathbf{1} = 0\}$ has the property that perturbations within it preserve the means of α and β. Differentiating the variance additivity relation $\mathrm{Var}(\gamma(t)) = \mathrm{Var}(\alpha(t)) + \mathrm{Var}(\beta(t))$ twice yields

$$w^\top J_{\boxplus_n} J_{\boxplus_n}^\top w = \|w\|^2 - \sum_{i=1}^{n} \gamma_i \, w^\top H_{\boxplus_n}^{(i)} w, \qquad w \in \mathcal{V}, \tag{3}$$

where $H_{\boxplus_n}^{(i)} = \mathrm{Hess}_{\Omega_{\boxplus_n, i}}$ is the Hessian of the i-th coordinate function.

Convexity from hyperbolic polynomials. The multivariate polynomial $f(x, a, b) = \sum_{\pi \in S_n} \prod_{i=1}^{n} (x - a_i - b_{\pi(i)})$ is hyperbolic in the direction $e_1 = (1, 0, \ldots, 0)$. A theorem of Bauschke, Güler, Lewis, and Sendov [2] asserts that for hyperbolic polynomials, the partial sums $\sigma_k(a) = \sum_{i=1}^{k} \lambda_i(a)$ of the ordered roots are convex functions of the parameters. Since the γ_i are ordered, any linear combination $\sum c_i \gamma_i$ with $c_1 \geq \cdots \geq c_n$ can be written as a positive combination of the σ_k, and is therefore convex. In particular, $\sum_i \gamma_i H^{(i)}_{\boxplus_n}$ is positive semidefinite (Corollary 2.1 of [1]). Combining with (3): $\|J_{\boxplus_n}(u, v)\|^2 = \|w\|^2 - \sum_i \gamma_i w^\top H^{(i)} w \leq \|w\|^2 = \|u\|^2 + \|v\|^2$.

2.3 Step 3: Blachman's argument

Since $\mathcal{J}_n(\alpha)$ is always orthogonal to $\mathbf{1}_n$ (each pair $(\alpha_i - \alpha_j)^{-1}$ and $(\alpha_j - \alpha_i)^{-1}$ cancels in the sum), Proposition 2.1 applies. Setting $a = 1/\Phi_n(p)$ and $b = 1/\Phi_n(q)$ in (2) and applying the contraction bound:

$$(a + b)^2 \Phi_n(r) \leq a^2 \Phi_n(p) + b^2 \Phi_n(q) = \frac{1}{\Phi_n(p)} + \frac{1}{\Phi_n(q)},$$

which rearranges to $1/\Phi_n(r) \geq 1/\Phi_n(p) + 1/\Phi_n(q)$. $\square$

3 The `af` Automated Attempt

The adversarial proof framework (`af`) system attacked this problem across two independent campaigns (designated "examples6" and "examples7") involving over 25 AI prover and verifier agents, 63 Python scripts totalling approximately 28,000 lines of verification code, and more than 2,000,000 random numerical trials. A third campaign ("examples8") synthesized the results into a 24-node hybrid proof tree with three independent proof paths.

3.1 What the `af` proved

(i) **Correct conjecture:** The system correctly identified the answer as TRUE, with zero violations in > 2M numerical trials.

(ii) **Base cases $n \leq 3$:** Two independent algebraic proofs for each of $n = 2$ (exact Pythagorean equality on root gaps) and $n = 3$ (via Cauchy–Schwarz on cumulants, and via subordination quadratic forms).

(iii) **$n = 4$ symmetric case ($e_3 = 0$):** Proved via strict concavity of a rational function and independently via a key lemma with vanishing third cumulant.

(iv) **Structural identities:** $\Phi_n(p) = 2 \sum_{i<j} (\lambda_i - \lambda_j)^{-2}$; S_2-additivity $S_2(p \boxplus_n q) = S_2(p) + S_2(q)$; the Gaussian splitting identity; the chain rule $H_r(\nu_k) = H_p(\lambda_{\sigma(k)}) - \alpha_k$ conditional on subordination.

3.2 Path A: Subordination + L^2 Contraction

This path adapted the Shlyakhtenko–Tao [6] continuum proof. The plan was:

1. Construct finite subordination functions ω_p, ω_q with $G_r(z) = G_p(\omega_p(z)) = G_q(\omega_q(z))$.

2. Use the chain rule to decompose $\Phi_n(p) = \Phi_n(r) + J_p$ where $J_p \geq 0$ is the Fisher decrease.

3. Prove the "Herglotz coupling lemma" $J_p \cdot J_q \geq \|\mathcal{J}_n(\gamma)\|^4$, from which Stam follows by AM–GM.

Where it stalled: The existence of finite subordination (Node 1.3.1) was verified numerically for $n = 2, 3, 4, 5$ but not proved. More critically, the continuum identity $\omega_p(z) + \omega_q(z) = z + 1/(nG_r(z))$ was shown to be *false* at finite n (verified at $n = 2$), blocking the coupling argument. The Herglotz coupling lemma (Node 1.3.4) remained open: expressing the Fisher decreases as quadratic forms in Herglotz residues was attempted but the coupling constraint between ω_p and ω_q could not be formulated without the continuum summation identity.

3.3 Path B: De Bruijn + $1/\Phi_n$ Concavity

This path followed the Costa–Villani entropy power template:

1. Establish the de Bruijn identity $dS/dt = \Phi_n(p_t)$ along the heat flow $p_t = p \boxplus_n G_t$.
2. Prove concavity of $1/\Phi_n(p_t)$ in t (equivalently $\Phi_n \cdot \Phi_n'' \geq 2(\Phi_n')^2$).
3. Derive Stam from concavity + Gaussian splitting.

Where it stalled: The concavity claim (Node 1.4.2) was numerically validated (0 violations in 590 trials) but the required inequality $\Phi_n \cdot \Phi_n'' \geq 2(\Phi_n')^2$, when expanded in terms of root dynamics, involved power sums $\sum_{i \neq j}(\lambda_i - \lambda_j)^{-k}$ of arbitrarily high order. A Cauchy–Schwarz argument was proposed but never completed for general n.

3.4 Path C: Finite Entropy Power Inequality

The most speculative path aimed to prove $N(p \boxplus_n q) \geq N(p) + N(q)$ where $N(p) = \exp(2S(p)/\binom{n}{2})$ is the entropy power, and derive Stam by differentiation. Approaches via the Harish-Chandra–Itzykson–Zuber formula and Brascamp–Lieb inequality were proposed but not developed. Numerical support was strong (0 violations in 13,770 trials).

3.5 Exhausted approaches

The `af` system systematically identified and refuted several false leads:

- $\langle \mathcal{J}_n(\gamma), \alpha \rangle \geq 0$: FALSE (counterexamples at $n \geq 3$).
- Partition of unity $\omega_p'(z) + \omega_q'(z) = 1$: FALSE (each derivative equals 1 independently at the roots, not summing to 1).
- Continuum summation $\omega_p + \omega_q = z + 1/(nG_r)$: FALSE at finite n.
- Shape factor monotonicity: FALSE (42.7% violation rate).
- Monotone gap along heat flow: FALSE (44% violation rate).
- Joint concavity of $-R_4$: FALSE (indefinite Hessian).
- SOS decomposition for $n \geq 4$: FAILS (mixed-sign cross terms).

4 Critical Comparison

4.1 Did any `af` path overlap with the successful approach?

The official proof is structurally closest to Path A (subordination/ L^2 contraction), but uses fundamentally different tools at the critical step. We can align the proofs as follows:

Official proof	**`af` Path A**
Score vectors as heat flow derivatives	Chain rule at roots (Node 1.3.2)
Jacobian relates score vectors via (2)	Subordination ω_p, ω_q relates score vectors (Node 1.3.2)
Jacobian contraction on $\mathcal{V}$ (Prop. 2.1)	Herglotz coupling lemma $J_p \cdot J_q \geq \|h\|^4$ (Node 1.3.4)
Blachman's choice of a, b to conclude	Harmonic mean from coupling (Node 1.3.5)

Both proofs express the Stam inequality through a contraction/monotonicity property of the map relating input and output score vectors. The `af` formulated this as a multiplicative coupling between Fisher decreases ($J_p \cdot J_q \geq \|h\|^4$), while the official proof formulates it as an L^2 contraction of the Jacobian. Both ultimately reduce to Blachman's classical trick of choosing parameters $a = 1/\Phi_n(p)$, $b = 1/\Phi_n(q)$.

However, the technical mechanism for establishing the contraction is entirely different:

- The `af` attempted to work through subordination functions and their Herglotz representations, trying to express everything in terms of the residues and poles of ω_p, ω_q. This is a complex-analytic approach.
- The official proof works entirely in real variables: the Jacobian $J_{\boxplus_n}$ is a real matrix, and its contraction property follows from variance additivity (a second-moment identity) and convexity of eigenvalue partial sums for hyperbolic polynomials (a result from convex algebraic geometry).

4.2 The key insight the `af` missed

The crucial insight of the official proof is the *Hessian identity* (3), which converts the contraction question into a positive-semidefiniteness question about weighted Hessians. This identity has two remarkable features:

1. **Variance additivity as the engine.** The fact that $\mathrm{Var}(\gamma) = \mathrm{Var}(\alpha) + \mathrm{Var}(\beta)$ under finite free convolution is a well-known, elementary property. The official proof differentiates this identity *twice* along perturbations in the mean-zero subspace to extract a Hessian identity. The `af` system *did know* this identity—it proved S_2-additivity (which is equivalent to variance additivity up to centering) as Node 1.1(ii). But it never considered differentiating it to obtain information about the Jacobian.
2. **Hyperbolic polynomial convexity.** The positive semidefiniteness of $\sum_i \gamma_i H^{(i)}_{\boxplus_n}$ follows from the theorem of Bauschke et al. [2] on convexity of partial sums of roots of hyperbolic polynomials. This is a result from convex algebraic geometry that is not standard in the free probability literature. The connection arises because the MSS convolution formula $\sum_{\pi \in S_n} \prod_i (x - a_i - b_{\pi(i)})$ is manifestly a hyperbolic polynomial in x (it has only real roots by Walsh's theorem). The `af` system never considered the convolution polynomial as a hyperbolic polynomial or invoked results from that theory.

In summary, the `af` missed the interaction between two ideas it partially possessed: (a) variance additivity (which it proved) and (b) the structure of the convolution as a sum over permutations (which it used for computations). The official proof's innovation is to connect these through the Jacobian and Hessian, then resolve the sign question via hyperbolic polynomial theory.

4.3 The `af`'s three paths in light of the solution

Path A (Subordination). This is the closest to the official approach at the strategic level (both use a contraction/decomposition of score vectors), but the `af` chose to route through subordination functions—a complex-analytic detour that introduces unnecessary difficulties. The official proof avoids subordination entirely. The existence of finite subordination functions remains unproved and appears to be a genuinely hard problem in its own right. By going through the Jacobian of the root map directly, the official proof sidesteps this entirely.

Path B (De Bruijn/Concavity). The concavity of $1/\Phi_n$ along the heat flow is a stronger statement than the Stam inequality itself. While the numerical evidence is compelling, no proof for general n was found. The official proof does not use heat flow concavity at all; it is a direct algebraic/geometric argument. Path B remains potentially viable as an alternative approach, but it requires establishing a strictly stronger result than necessary.

Path C (EPI). The finite entropy power inequality $N(p \boxplus_n q) \geq N(p) + N(q)$ is also a strictly stronger conjecture than the Stam inequality. The official proof does not address it. Whether the finite EPI is true remains open. If true, it would be a deeper result requiring different techniques.

5 Evaluation of Dead Ends

5.1 Genuine dead ends a mathematician would also encounter

Several of the `af`'s dead ends reflect traps that a human mathematician working from the free probability literature would also naturally fall into:

1. **Continuum identities failing at finite n.** The identities $\omega_p + \omega_q = z + 1/(nG_r)$ and $\omega_p'(z) + \omega_q'(z) = 1$ are *true* in the large-n limit (they are Voiculescu's subordination identities for free convolution). A mathematician familiar with free probability would naturally try to use these and would discover they fail at finite n. The `af`'s discovery that each $\omega_p'(\nu_k) = 1$ independently (rather than a partition of unity) is a genuine mathematical observation that clarifies the finite-n structure.

2. **SOS failure for $n \geq 4$.** Attempting a sum-of-squares proof is a standard approach for polynomial inequalities. The failure of SOS for $n \geq 4$ reflects the genuine algebraic complexity of the problem: the inequality is not a consequence of any simple polynomial identity.

3. **Direct polynomial manipulation at $n \geq 4$.** The combinatorial explosion of the MSS formula at $n = 4$ (24 terms in S_4) makes brute-force approaches genuinely intractable. A human mathematician would also find this route prohibitive.

4. **Subordination existence.** Constructing finite subordination functions is an open problem that several researchers have considered. The `af`'s difficulty here is shared with the human mathematical community.

5.2 Dead ends specific to the AI approach

1. **Excessive reliance on the continuum template.** The `af` organized all three proof paths as finite analogues of known continuum results (Shlyakhtenko–Tao, Costa–Villani, Shannon EPI). A human mathematician might also start here, but would more quickly recognize the need for genuinely finite-dimensional tools. The official proof, while inspired

by Blachman's argument, introduces fundamentally new ideas (the Hessian identity from variance additivity, hyperbolic polynomial convexity) that have no direct continuum analogue.

2. **Not exploiting the permutation formula.** The `af` used the MSS coefficient formula for computations but never exploited the permutation formula $\sum_{\pi \in S_n} \prod_i (x - \alpha_i - \beta_{\pi(i)})$ structurally. The official proof recognizes this as defining a hyperbolic polynomial in the (x, α, β) variables, which is the entry point for the Bauschke et al. convexity theorem.

3. **Searching for coupling constraints instead of convexity.** In Path A, the `af` tried to find algebraic constraints between ω_p and ω_q to force the coupling inequality. The official proof obtains the needed bound not from coupling but from *convexity*—specifically, convexity of eigenvalue sums, which is a property of the convolution map itself, not of any auxiliary objects.

6 What the `af` Got Right

Despite failing to find the complete proof, the `af` system demonstrated several strengths:

1. **Correct answer.** The conjecture is indeed true, and the system correctly identified this with overwhelming numerical evidence ($>$ 2M trials, zero violations).

2. **Valid base cases.** The proofs for $n = 2$ (Pythagorean identity) and $n = 3$ (two independent proofs) are correct and non-trivial. The symmetric $n = 4$ case was also proved.

3. **Structural identities.** The identities $\Phi_n = 2\sum_{i<j}(\lambda_i - \lambda_j)^{-2}$ and S_2-additivity are both used (implicitly) in the official proof. The official proof's variance additivity (Proposition 1.1(i)) is essentially the S_2-additivity that the `af` proved.

4. **Blachman's argument was anticipated.** The `af`'s Path A strategy of decomposing $\Phi_n(p) = \Phi_n(r) + J_p$ and then using $J_p \cdot J_q \geq \Phi_n(r)^2$ is algebraically equivalent to Blachman's argument. The system recognized this structure as the classical template.

5. **Identifying the true difficulty.** The system correctly identified that the hard step is establishing a contraction or coupling property for general $n \geq 4$. All three paths isolated a single hard lemma whose resolution would complete the proof.

6. **Efficient falsification.** Numerical testing quickly eliminated 6+ false conjectures (inner product non-negativity, shape factor monotonicity, partition of unity, etc.), preventing wasted effort. This is a genuine strength of the computational approach.

7. **Score vectors and heat flow.** The `af` identified the score vector $\mathcal{J}_n(\alpha)$ and its connection to heat flow derivatives. This is precisely Lemma 1.1 of the official proof. The `af`'s de Bruijn identity $dS/dt = \Phi_n(p_t)$ is the integral form of the same observation.

7 Lessons for AI Approaches to Inequality Proofs

7.1 The template trap

The `af` system organized its attack entirely around three templates from the continuum setting: the subordination proof (Shlyakhtenko–Tao), the concavity proof (Costa–Villani), and the entropy power proof (Shannon–Stam). Each template guided the discovery of useful partial results, but none yielded the final proof. The official proof uses a template (Blachman's argument) but supplies a genuinely new technical ingredient (hyperbolic polynomial convexity) that does not appear in any continuum analogue.

Lesson: Templates from analogous problems are valuable for structure, but solving genuinely new problems requires connecting to tools outside the template's domain. An AI system needs the ability to search across mathematical subfields for applicable results.

7.2 The abstraction barrier at $n \geq 4$

All three `af` paths proved $n \leq 3$ using direct computation and failed at $n \geq 4$ because the combinatorial complexity overwhelms algebraic manipulation. The official proof succeeds at general n because it works at the level of the Jacobian and Hessian—objects that exist for all n and whose properties can be established uniformly.

Lesson: For inequality proofs that must hold for all n, the key is often to find the right *abstraction* that makes the proof n-independent. Direct computation on specific-degree instances, while useful for building intuition, cannot substitute for finding the right structural viewpoint.

7.3 Known results as hidden tools

The `af` proved S_2-additivity and knew the permutation formula for $\boxplus_n$, but never combined them into the Hessian identity (3) or recognized the connection to hyperbolic polynomial theory. The Bauschke et al. theorem [2] is a published result from 2001, but it lives in the convex analysis literature rather than the free probability literature.

Lesson: The bottleneck in mathematical proof is often not proving new results but recognizing which *existing* results are relevant. AI systems would benefit from broader literature search capabilities and mechanisms for recognizing structural analogies across subfields.

7.4 The value of the Jacobian viewpoint

The `af` system worked with subordination functions ω_p, ω_q as the intermediary between input and output score vectors. The official proof works directly with the Jacobian of the root map $\Omega_{\boxplus_n}$. The Jacobian is a more elementary and more powerful object: it is a real matrix that can be analyzed with linear algebra, rather than a complex-analytic function requiring Herglotz representations.

Lesson: When a problem involves a smooth map between finite-dimensional spaces, the Jacobian is often the most natural analytical tool. The `af` introduced auxiliary complex-analytic objects (subordination functions) that, while natural from the free probability perspective, added unnecessary complexity.

7.5 Numerical evidence: strengths and limitations

The `af`'s numerical testing was a clear strength—it correctly predicted the truth of the conjecture and efficiently eliminated false leads. However, numerical evidence also gave confidence in subsidiary conjectures (concavity of $1/\Phi_n$, finite EPI) that turned out to be unnecessary for the proof.

Lesson: Numerical validation is excellent for falsification (disproving wrong conjectures) and for building confidence in the main claim, but it can lead to overinvestment in strong-but-unnecessary auxiliary conjectures. The official proof succeeds with a weaker intermediate statement (Jacobian contraction) than any of the three intermediate conjectures pursued by `af`.

7.6 Comparison with the ChatGPT Pro 5.2 attempt

The problem authors' commentary [1] notes that ChatGPT Pro 5.2 also attempted this problem and tried Blachman's approach via the random matrix model $r(x) = \mathbb{E} \det(xI - A - UBU^\top)$. That attempt failed because: (a) it only considered the score function of $r(x)$ without mentioning those of $p(x)$ and $q(x)$; (b) it did not exploit the preservation of real roots; (c) it asserted incorrect residue calculus facts.

The `af` system avoided these specific errors: it worked with all three score vectors simultaneously, it knew real-rootedness was essential, and its algebraic computations were verified. However, like ChatGPT Pro 5.2, the `af` was unable to identify the correct technical tool (hyperbolic polynomial convexity) for the key step. Both AI systems demonstrate that current LLMs can organize proof strategies and verify computations, but struggle to make the creative leap of connecting a problem to an unexpected area of mathematics.

8 Conclusion

The finite free Stam inequality admits a complete proof of approximately four pages, using:

- the connection between score vectors and heat flow derivatives,
- variance additivity for finite free convolution,
- convexity of eigenvalue partial sums for hyperbolic polynomials [2],
- Blachman's classical optimization trick.

The `af` system's investigation, while not reaching a complete proof, produced substantial partial results: correct base cases, important structural identities, efficient elimination of false leads, and a clear mapping of the proof landscape through three viable strategies. The system's principal failure was not recognizing that variance additivity—a fact it had already proved—could be differentiated to yield information about the Jacobian, and that the permutation formula for $\boxplus_n$ gives a hyperbolic polynomial whose eigenvalue convexity properties close the argument.

This case study illustrates both the current capabilities and limitations of AI-assisted mathematical reasoning: strong on computation, verification, and systematic exploration of known frameworks; weak on cross-domain insight and the creative recognition of how elementary known facts can be combined in unexpected ways.

References

[1] J. Garza Vargas, N. Srivastava, and Z. Stier, *The finite free Stam inequality*, in: First Proof Solutions and Comments, February 2026.

[2] H. H. Bauschke, O. Güler, A. S. Lewis, and H. S. Sendov, *Hyperbolic polynomials and convex analysis*, Canad. J. Math. **53** (2001), 470–488.

[3] N. M. Blachman, *The convolution inequality for entropy powers*, IEEE Trans. Inform. Theory **11** (1965), 267–271.

[4] A. W. Marcus, D. A. Spielman, and N. Srivastava, *Finite free convolutions of polynomials*, Probab. Theory Related Fields **182** (2022), 807–848.

[5] D. Voiculescu, *The analogues of entropy and of Fisher's information measure in free probability theory V*, Invent. Math. **132** (1998), 189–227.

[6] D. Shlyakhtenko and T. Tao, *A self-contained analytic proof of the free monotonicity of the free Fisher information*, arXiv:2009.01882, 2020.

[7] M. H. M. Costa, *A new entropy power inequality*, IEEE Trans. Inform. Theory **31** (1985), 751–760.

[8] J. L. Walsh, *On the location of the roots of certain types of polynomials*, Trans. Amer. Math. Soc. **24** (1922), 163–180.

[9] W. Nuij, *A note on hyperbolic polynomials*, Math. Scand. **23** (1968), 69–72.