Introduction to general relativity: homogeneity and isotropy cont.

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Dynamics of homogeneous & isotropic universe

Idealise galaxies → "grains of dust"; assume galaxies move on worldlines of isotropic observers. Fluctuations of galaxy velocities is small → pressure is neglected. Thus stress-energy-tensor of galaxy content

$$T_{ab} = \rho u_a u_b$$

ρ is average density of matter.

To model background thermal radiation (@ 3K) we use perfect-fluid stress-energy tensor

$$T_{ab} = P_{rad}(g_{ab} + u_a u_b) + \rho_{rad} u_a u_b$$

where $P_{rad} = \rho_{rad}/3$. Thus most general stress-energy tensor consistent with homogeneity & isotropy is (ex.)

$$\boxed{T_{ab} = \rho u_a u_b + P(g_{ab} + u_a u_b)}$$

We aim to solve Einstein's equation

$$(*) \qquad \boxed{G_{ab} + \Lambda g_{ab} = 8\pi T_{ab}}$$

where we have added Λ, the cosmological constant, which is necessary to account for cosmic background radiation and dark energy surveys.

In principle we need to solve 10 unknowns & 10 equations

$$\begin{pmatrix} \cdot & \vdots & \vdots & \vdots \\ & & \cdot & \vdots \\ & & & \cdot \end{pmatrix}$$

Because of the symmetries of homogeneity & isotropy <u>only two equations are independent</u>: Firstly note, the $G^{ab} u_b$ & $T^{ab} u_b$ cannot have spatial components (isotropy). Similarly restricted to Σ_t, the same arguments we made for ${}^{(3)}R_{ab}{}^{cd}$ implies $G^{(3)}_{ab}$ is proportional to identity. Off-diagonal terms of (4) vanish. And diagonal space-space components are equal. Due to homogeneity implies space-time components vanish. Two independent components of (4) are

$$\boxed{\begin{aligned} G_{\tau\tau} + \Lambda g_{\tau\tau} &= 8\pi\rho \\ G_{**} + \Lambda g_{**} &= 8\pi P \end{aligned}}$$

Here $G_{\tau\tau} \equiv G_{ab} u^a u^b$ and $G_{**} = G_{ab} s^a s^b$ with $s^a u_a = 0$; $s^a s_a = 1$.

To carry out subsequent calculations we rewrite the FLRW metric as (ex)

$$ds^2 = -d\tau^2 + a^2(\tau)\left[\frac{dr^2}{1-kr^2} + r^2 (d\theta^2 + \sin^2\theta \, d\phi^2)\right]$$

where $k = +1 \equiv$ the sphere ; $k = 0$: flat space ; $k = -1 \equiv$ hyperboloid.